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Memorandum M-543

BRIEF NOTE ON THE ESTIMATION OF STRESS
CONCENTRATION FACTORS OF SHARP EDGE NOTCHES

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INTRODUCTION

In a recent paper Shin and Smith [1] have studied fatigue crack growth from sharp elliptical edge notches. Three configurations were used (Fig. 1), for which the stress concentration factors K_t were calculated by a finite-element method. The K_t -values obtained were 22.5, 17.0 and 12.4 respectively, if based on the nominal gross stress in the specimen. The K_t -values based on the nominal net stress are half the values quoted, which are still high K_t -values, thus illustrating the sharpness of the notches employed.

In the present report it will be shown that accurate K_t -values can be estimated for specimens with sharp elliptical edge notches by considering related geometries, for which K_t -solutions are available. Three different methods will be used.

K_t - ESTIMATES BASED ON NEUBER'S ANALYSIS

Values of K_t for double-edge notched flat bars in tension were calculated by Neuber [2] and reproduced in the book "Stress Concentration Factors" by Peterson [3]. The tip radius of the elliptical side notch is obtained from $\rho = b^2/a$. Values of K_{tn} (based on nominal net stress) can then be read from Fig. 18 of Peterson's book. The relevant data are compiled in Figure 2, where the Neuber K_{tg} -values (based on gross stress) are compared to the FEM-values obtained in [1]. The Neuber values are about 7 % lower. As pointed out by Peterson [3] the Neuber results are an ingenious interpolation between exact values for deep hyperbolic notches and shallow elliptical notches. However the values obtained are not exact, and according to Peterson, the results can be too low up to 12 % in the region $d/W = 0.5$ which applies here. The data in Figure 2 seem to confirm the deviation.

K_t - ESTIMATES BASED ON ELLIPTICAL HOLE DATA

Consider an infinite sheet with an infinite series of collinear shallow elliptical holes. Values of K_t were derived by Nisitani [4] (see also [3], Fig. 134). K_t can be written as:

$$K_t = \alpha \left(1 + 2 \frac{a}{b} \right) = \alpha \left(1 + 2 \sqrt{\frac{a}{\rho}} \right) \quad (1)$$

where $1 + 2\sqrt{a/\rho}$ is K_t for a single hole. It was shown by Nisitani that the correction factor α , for shallow ellipses, rapidly converges to the geometry correction factor of the stress intensity factor for an infinite row of collinear cracks (Westergaard solution). In other words:

$$\alpha = \sqrt{\frac{\operatorname{tg}(\pi a/W)}{\pi a/W}} \quad (2)$$

The specimen of Figure 1 is obtained from the infinite sheet in Figure 2 by making two cuts along AA' and BB'. These lines are carrying an equilibrium σ_x stress distribution. In view of $a/W = 0.5$ it is expected that these stresses will have a negligible effect on the stress distribution at the tip of the shallow elliptical hole. The expectation is supported by considering the same procedure for circular holes. In the entire range of $2a/W$ from 0 to 1 the K_t -values for an infinite row of circular holes and for strips with two semi-circular edge notches are very much similar, with differences of a few percent only. According to graphs in [3] the difference is only 1 % at $2a/W = 0.5$. Ignoring this effect K_t is then obtained as

$$K_t = \left(1 + \frac{2a}{b}\right) \sqrt{\frac{\operatorname{tg}(\pi a/W)}{\pi a/W}} \quad (3)$$

Because $a/W = 0.25$ for all three configurations the square root correction factor is constant ($\alpha = 1.128$). Calculated results for the three b-values are shown in Figure 3 in comparison to the FEM results of [1]. The agreement is excellent.

K_t - ESTIMATES BASED ON K OF A DOUBLE-EDGE CRACKED SPECIMEN

In Figure 4 a double-edge cracked specimen is shown, for which the K-solution in [5] is a modification of a formula by Benthem and Koiter [6]:

$$K = C S \sqrt{\pi a}$$

$$\text{with } C = \frac{1.122 - 1.122(a/W) - 0.82(a/W)^2 + 3.768(a/W)^3 - 3.04(a/W)^4}{\sqrt{1 - 2a/W}} \quad (4)$$

For $a/W = 0.25$ the result is $C = 1.184$. Assuming that the same correction applies to shallow elliptical edge notches, the K_t -value is:

$$K_t = C \left(1 + \frac{2a}{b}\right) \quad (5)$$

K_t -values thus calculated are given in Figure 4, which shows that the results are about 4 % higher than the FEM values. Equation (5) has a similar character as Eq. (3), but the correction factor is about 5 % lower for $a/W = 0.25$.

DISCUSSION

The K_t -estimates in comparison to the FEM results of [1] are shown in the table below for the three estimation methods.

b (mm):	3.75	5	7	3.75	5	7
Estimation method	K_t -values			Differences with FEM results (%)		
Neuber analysis	21.9	16.0	11.5	-6.7	-5.9	-7.3
Elliptical hole estimate	22.2	16.9	12.4	1.3	0.6	0
K-solution estimate	23.3	17.8	13.0	3.6	4.7	4.8
FEM - result	22.5	17.0	12.4	-	-	-

If the K_t -estimates are considered to be engineering approximations all results are satisfactory. However, the Neuber analysis gives an under-estimation (6 to 7 %) in all three cases. According to Peterson [3] this should be expected in view of the Neuber K_t -values being too low.

The estimates based on the elliptical hole analysis agree almost exactly with the FEM results, while the results based on a geometry correction factor for edge cracks give an overestimation of 4 to 5 %. Before any conclusion is drawn from these trends, it should be pointed out that there are some differences between the finite-element model and the geometries assumed in Figures 3 and 4. For instance, the finite length of the finite-element model and the boundary conditions at the ends of the model can be responsible for small differences. In Figures 3 and 4 the length was assumed to be infinite. Further the finite-element model by its nature can underestimate the peak stress at the root of the notch. Aspects like these can easily explain a few percent differences. It then appears that both estimates, based on elliptical holes and on a K-solution respectively, have given quantitatively

good indications of the K_t -values. It would hardly be justified to express a general preference for one of the two methods if applied to other geometries. However, it is encouraging to see that with some engineering judgement, good estimates are possible. Such estimates can also be helpful as a check on the order of magnitude of K_t if FEM calculations have to be made for geometries, for which K_t -values are not yet available.

Another illustration of obtaining a K_t -value with simple means has recently been given in [7] for the case of a small hole in the inhomogeneous stress field of a larger notch. A satisfactory agreement with a FEM calculation was obtained afterwards.

CONCLUSIONS

1. Simple K_t -estimates for sharply notched specimens were in quantitatively good agreement with results of finite-element calculations.
2. Three estimation methods were employed. The method based on Neuber K_t -data gave values 6 to 7 % too low, which is supposed to be due to incorrect Neuber values. More reliable K_t -estimates were obtained by employing K_t -geometry correction factors of relevant elliptical hole cases, or by adopting K-geometry correction factors of relevant cracked specimen cases.

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3 Values of b

b (mm)	K_t (FEM) [1]
3.75	22.5
5	17.0
7	12.4

K_t based on gross stress

Figure 1: Specimen with sharp elliptical edge notch used in [1]

(mm)			ratio's		K_t		
a	b	ρ ($=b^2/a$)	d/W	ρ/W	Neuber K_{tn}	K_{tg}	FEM[1] K_t
35	3.75	0.402	0.5	0.00286	10.5	21.0	22.5
	5	0.714		0.0051	8.0	16.0	17.0
	7	1.400		0.0100	5.75	11.5	12.4

Figure 2 K_t -values derived from Neuber's equations [2],[3]

$a = 35$ mm
 $W = 140$ mm

$$K_t = \left(1 + 2 \frac{a}{b}\right) \sqrt{\frac{t_g (\pi a/w)}{\pi a/w}}$$

b	K_t	
	$E_q(3)$	FEM[1]
3.75	22.2	22.5
5	16.9	17.0
7	12.4	12.4

Figure 3: Estimated K_t -values for an infinite row of elliptical holes (Nisitani [4])

Edge cracked specimen

$$K = C S \sqrt{\pi a} \text{ with}$$

$$C = 1.184 \text{ for } 2a/W = 0.5$$

Edge notched specimen

$$K_t = C \left(1 + 2 \frac{a}{b} \right)$$

$$\text{with } C = 1.184$$

$$a = 35 \text{ mm}$$

$$W = 140 \text{ mm}$$

b	K_t	
	$C \left(1 + 2 \frac{a}{b} \right)$	FEM[1]
3.75	23.3	22.5
5	17.8	17.0
7	13.0	12.4

Figure 4 : Estimated K_t - values based on an edge crack correction factor.

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