Optimal leg compliance for rejecting disturbances in bipedal running

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Optimal leg compliance for rejecting disturbances in bipedal running

Master of Science Thesis

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by

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Chapter 1

Optimal leg compliance for rejecting disturbances in bipedal running
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Abstract—22nd June 2012— ...

I. INTRODUCTION

In the construction of bipedal running robots, it is important to implement leg compliance because leg compliance helps decrease the cost of transport [1] and reduces damaging impact forces. However, too much leg compliance might negatively affect the ability of a robot not to fall under the influence of external disturbances. To preserve the functionality of a bipedal running robot, the influence of leg compliance on this ‘disturbance rejection behavior’ must be considered.

Most bipedal running robots are equipped with point feet presumably because point feet introduce less inertia than flat feet. The influence of leg compliance on the disturbance rejection behavior of bipedal running robots with point feet was considered by Seyfarth et al. (2002) [2], Rummel et al. [3] and Owaki et al. [4]. Seyfarth et al. (2002) consider the SLIP model, which is an accepted model for running with linear leg stiffness [5]. This study shows that disturbance rejection behavior improves w.r.t. linear leg stiffness. Rummel et al. and Owaki et al. consider a model with non-linear leg stiffness that is parameterized with a set of parameters, of which the values are fixed. With this ‘fixed’ non-linear leg stiffness, these studies indicate that non-linear leg stiffness can cause a further improvement in the disturbance rejection behavior of a bipedal running robot w.r.t. linear leg stiffness.

Although previous studies consider leg compliance, they do not consider the optimal leg compliance w.r.t. disturbance rejection behavior. A recent study by Karssen et al. [6] consider this optimal leg compliance. Karssen et al. consider a model with a point foot and non-linear leg stiffness that is parameterized with a set of parameters of which the values are left free. This ‘free’ non-linear leg stiffness is optimized and the resulting disturbance rejection behavior is a factor 7 better than the disturbance rejection behavior of the SLIP model with optimal linear leg stiffness w.r.t. disturbance rejection behavior.

To decrease the chance of slipping [7], to increase the resistance against yaw and to make a robot more human-like, flat feet can be implemented instead of point feet. The whole leg compliance of a bipedal running robot with flat feet consists of leg compliance and ankle compliance and depends on the ankle angle and the leg length. This ‘whole-leg compliance’ is considered by Hosoda et al. [8], Cham [9], Seyfarth et al. (2006) [10] and Iida et al. [11]. Hosoda et al. and Cham consider fixed leg stiffness and fixed ankle stiffness that are not interdependent, of which Hosoda et al. consider non-linear stiffness and Cham considers linear stiffness. Seyfarth et al. and Iida et al. consider fixed non-linear leg stiffness and fixed non-linear ankle stiffness that are interdependent. Although these four studies consider whole-leg compliance, they do not show the influence of whole-leg compliance on the disturbance rejection behavior of bipedal running robots with flat feet. In addition, all these studies only investigated a single type of whole-leg compliance and did not look at all possible options for whole-leg compliance.

In this paper, we present the results of an optimization study in which whole-leg compliance is optimized to maximize disturbance rejection behavior. The goal of this study is to find the optimal whole-leg compliance, so that we know by how much whole-leg compliance can maximally improve the disturbance rejection. To find the optimal whole-leg compliance w.r.t. disturbance rejection behavior, we consider free non-linear leg stiffness and free non-linear ankle stiffness that are interdependent. We use free stiffness in models because it is not possible to implement free stiffness in bipeds. To find a model with a flat foot and free leg compliance, we extend a model with a point foot and free leg compliance. We continue by optimizing both the whole-leg compliance of the ‘FALC’ model (Flat foot Arbitrary Leg Compliance) and the leg compliance of the ‘PALC’ model (Point foot Arbitrary Leg Compliance) w.r.t. disturbance rejection behavior. Finally, we map the optimal disturbance rejection behavior by comparing both models with optimal leg compliance to the SLIP model as is used by Seyfarth et al. (2002).

The remainder of this paper is organized as follows: Section II presents the models that are used in this study and Section III discusses the approach that we followed to find the optimal leg compliance. The paper continues by presenting results in Section IV, Section V and Section VI. These sections are followed by a discussion in Section VII and a conclusion in Section VIII.
II. MODELS WITH ARBITRARY LEG COMPLIANCE

The introduction showed an example of a biped with arbitrary leg compliance, which has a point foot. The introduction also showed an example of a biped with a flat foot, which has specific leg compliance. In this study, we consider a biped that has both arbitrary leg compliance and a flat foot. This section shows that this model is an extension from a model with a point foot and arbitrary leg compliance.

A. Models

Two models used in this study are similar except for the foot type and the leg compliance. Both models consist of a point mass $m$ attached to a massless leg of maximum length $L_0$ and are in a gravitational field $g$. When the models are off the ground, their legs make a fixed angle w.r.t. the ground $\alpha_0$, which is named ‘angle of attack’. The leg stiffness is modelled as the second order derivative of the leg potential $P$ w.r.t. position. For the point foot model, this leg potential is implemented as a field that is rotational symmetric in the point foot. This field is parameterized as a cubic Catmull-Rom spline that is shaped by a grid of 7 control points $b_1 \ldots b_7$ that are equally spaced between $-0.5$ m and 1.5 m. The potential of all control points is free except the potential of control point $c_4$, which is a reference potential of zero. For the flat foot model, which has a foot that extends by length $d$ to the front and to the back of the ankle, the potential is implemented as an asymmetric field. To ensure that the flat foot model is energy conservative, we make the crude assumption that the flat foot model is in a flight phase and follows a ballistic trajectory until reaching an apex point. After touchdown, the leg compresses and exerts a force that pushes against the point mass and the ground $F_{leg}$. This ‘leg

\[ z = 1 - \frac{1}{L_0^2} \left( x_{ma}^2 + y_{ma}^2 \right)^{\frac{3}{2}} \]  

In which $(x_{ma}, y_{ma})$ is the position w.r.t. the ankle. The bicubic spline is shaped by a grid of 7 x 7 control points $b_1 \ldots b_{49}$ that are equally spaced within a 3 m wide rectangle that has its center at the ankle. The potential of these control points is free.

For both models, Table I gives the parameters. We chose $g$, $L_0$, $\alpha_0$ and $m$ the same as the SLIP model [5] that was used in Seyfahrt et al. [2] and which is also the model to which we compare our results.

B. Motion

The running motion consists of sequential steps. Figure 1 shows an example of one step in the running motion. The motion has two distinct phases: a flight phase and a stance phase. A step begins in the apex point of the flight phase, at which the state of the model can be fully described by the height of the point mass $y_{a1}$ and the total energy in the system $E$. During the flight phase, the model follows a ballistic trajectory described by the following equations of motion (EoMs),

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \end{bmatrix} \]  

Of the initial conditions that accompany these EoMs, the initial horizontal position $x_{a1}$ does not affect the running motion and can therefore be chosen free. The initial horizontal velocity $\dot{x}_{a1}$ depends on the initial conditions for a step,

\[ \dot{x}_{a1} = \sqrt{\frac{2}{m} \left( E - mgy_{a1} \right)} \]  

The model follows the ballistic trajectory until the ankle or point foot touches the ground. Assuming that the angle or point foot is in the origin at touchdown, the point mass enters the working range of the leg in $(x_{td}, y_{td})$,

\[ x_{td} = -L_0 \cos(\alpha_0) = -0.400 \]  

\[ y_{td} = L_0 \sin(\alpha_0) = 0.917 \]

After touchdown, the leg compresses and exerts a force that pushes against the point mass and the ground $F_{leg}$. This 'leg

<table>
<thead>
<tr>
<th>Model</th>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Gravitational acceleration</td>
<td>$g$</td>
<td>9.81</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td></td>
<td>Maximum leg length</td>
<td>$L_0$</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>Angle of attack</td>
<td>$\alpha_0$</td>
<td>66.44</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Point mass</td>
<td>$m$</td>
<td>80</td>
<td>kg</td>
</tr>
<tr>
<td>Point foot</td>
<td>Control points</td>
<td>$c_1, c_2, c_3, c_5$</td>
<td>free</td>
<td>J</td>
</tr>
<tr>
<td></td>
<td>Control point</td>
<td>$c_4$</td>
<td>0</td>
<td>J</td>
</tr>
<tr>
<td>Flat foot</td>
<td>Control points</td>
<td>$b_1 \ldots b_9$</td>
<td>free</td>
<td>J</td>
</tr>
<tr>
<td></td>
<td>Foot length</td>
<td>$d$</td>
<td>0 \ldots 1</td>
<td>m</td>
</tr>
</tbody>
</table>

Table I

MODEL PARAMETERS

![Figure 1](image-url)

Figure 1. One step in the running motion. The step begins at the apex point in the flight phase. After apex the model follows a ballistic trajectory until touchdown, at which the point mass enters the working range of the leg. During the stance phase the point mass moves through the working range of the leg while the foot stays fixed on the ground. The point mass leaves the working range of the leg when the foot lifts off the ground. After liftoff the model is in a flight phase and follows a ballistic trajectory until reaching an apex point, from which a new step begins.
force’ is the negative gradient of the leg potential $P$ w.r.t. position for both models,

$$
\mathbf{F}_{\text{leg}} = - \left[ \frac{\partial P}{\partial \phi} \right]
$$

(6)

The leg force combined with the gravitational force is the total force that acts on the point mass $\mathbf{F}$,

$$
\mathbf{F} = \mathbf{F}_{\text{leg}} + \left[ \begin{array}{c} 0 \\ -mg \end{array} \right]
$$

(7)

From this total force the EoMs for the stance phase follow,

$$
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \frac{1}{m} \mathbf{F}
$$

(8)

Numerically integrating these EoMs results an in integration error that causes the system energy to vary in time. For an example of this time-varying system energy, see the appendix. To remove this integration error, the number of state variables is reduced from four to three by using the angle of the velocity vector w.r.t. the horizontal $\phi$ instead of the horizontal and vertical speed. This reduction results in a state $\mathbf{q}$ that writes,

$$
\mathbf{q} = \begin{bmatrix}
\phi \\
x \\
y
\end{bmatrix}
$$

(9)

The time derivative of this state is,

$$
\begin{bmatrix}
\dot{\phi} \\
\dot{x} \\
\dot{y}
\end{bmatrix} = \mathbf{V}_{\text{abs}} \begin{bmatrix}
-F_x \\
-mV_y \cos(\phi) \\
-mV_y \sin(\phi)
\end{bmatrix}
$$

(10)

in which $F_x$ is the magnitude of the force perpendicular to the direction of motion and $V$ is the magnitude of the velocity,

$$
F_\perp = - \left[ \sin(\phi) \cos(\phi) \right] \mathbf{F}
$$

(11)

$$
V = \sqrt{\frac{2}{m} (E - P(x, y) - mgy)}
$$

(12)

During the stance phase, the point mass follows the trajectory described by (10) until the foot lifts off the ground, which is given by,

$$
\sqrt{x^2 + y^2} = L_0
$$

(13)

After liftoff, the model is in the flight phase and follows a ballistic trajectory as described by (6). The model follows this trajectory until reaching the apex point.

### III. Approach to Find Optimal Leg Compliance

After having introduced the PALC model and the FALC model, we now optimize leg compliance w.r.t. disturbance rejection behavior. To optimize w.r.t. disturbance rejection behavior, we first quantify disturbance rejection behavior with a cost function and continue by discussing the constraints. Finally, we introduce the optimization method.

#### A. Cost function

**Step disturbance:** The step disturbance [15, 16] is the most common disturbance on running bipeds [17]. The step disturbance changes both the apex height, which is measured relative to the ground, and the system energy. First, we optimize the models response to changes in apex height and next, we optimize the response both to changes in the apex height and to changes in the system energy.

**Apex return map:** The response to changes in apex height shows from the apex return map $S$, which is the mapping from the apex height at which a step begins into the apex height at which the step ends,

$$
y_{\text{ref}} = S(y_{\text{ref}})
$$

(14)

Figure 3 shows an example of the apex return map of two models. At the intersection point of an apex return map with the diagonal, the model is in a limit cycle, at which it has an apex height that is fixed over successive steps and which we refer to as ‘limit cycle apex height’. If the slope of the apex return map is between $-1$ and $1$, the limit cycle is stable, and the apex height will converge to this ‘stable’ limit cycle apex height after a small change in apex height. When this slope is smaller than $-1$ or larger than $1$, the limit cycle is unstable, and the apex height will diverge from this ‘unstable’ limit cycle apex height after a small change in apex height. An energy-neutral step down disturbance corresponds to a shift to the right and an energy-neutral step up disturbance (not shown in the Figure 3) corresponds to a shift to the left. By successively mapping one apex height into the next, the propagation of apex height over multiple steps is obtained.

**Reference apex state:** To simplify the implementation of a step disturbance, we only consider step disturbances w.r.t. a fixed reference state at apex, which consists of a reference system energy of $E_{\text{ref}}$ and a reference limit cycle apex height of $y'_{\text{ref}}$, which is a desired apex height for the model. Because of the conservative nature of the models, the changed system energy w.r.t. $E_{\text{ref}}$ after a step disturbance is fixed, and the apex height propagates on the apex return map that accompanies
Figure 3. An example of two apex return maps indicated with solid lines. The apex return maps, which are defined for apex heights larger than $y_{td}$ intersect the diagonal at $y'_{stab}$. At this intersection point, the apex return maps have a slope that is between $-1$ and $1$, which indicates a stable limit cycle. Additionally to this limit cycle, apex return map II indicates another limit cycle, which is at $y'_{stab}$ and which is unstable because the slope of the return map in this point is not between $-1$ and $1$. The dashed arrows show how an energy-neutral step down disturbance of $\Delta y_{a1}$ form $y'_{stab}$ propagates over multiple steps. The range of apex heights $y_{a1}$ in which the running motion is stable, is indicated by $\Delta y_{stab}$ for apex return map II. Although the ‘stable range’ for apex return map I does not follow from this figure, this range is larger than the stable range for apex return map II.

B. Constraints

Constraints are added to the optimization to ensure that the running motion is physically correct. In both models, the vertical component of the leg force $F_{legy}$ must be positive to ensure that the foot stays on the ground during stance,

$$F_{legy} > 0$$

(16)

To ensure that the foot of the FALC model stays flat during stance, the line of action of the leg force must intersect the foot,

$$-d \leq x - y\frac{F_{legx}}{F_{legy}} \leq d$$

(17)

in which $F_{legx}$ is the horizontal component of the leg force. Constraints (16) and (17) are checked 50 times during each step, equally divided over the stance phase.

Two more constraints were implemented in both models: firstly, to ensure that the running motion is directed forwards, the angle of the velocity vector at liftoff $\phi_{lo}$ should be between $-\frac{1}{2}\pi$ and zero,

$$-\frac{1}{2}\pi \leq \phi_{lo} \leq 0$$

(18)

Secondly, to ensure that the foot of the models are clear from the ground at the apex point, the velocity in vertical direction must be large enough,

$$\frac{1}{2}y_{lo}^2 \geq g(y_{td} - y_{lo})$$

(19)

in which $y_{lo}$ and $\dot{y}_{lo}$ are the vertical position and speed at liftoff.

C. Optimization approach

Optimization problem: To solve the optimization problem of finding optimal compliance w.r.t. disturbance rejection behavior, we use fmincon, which is a local solver in MATLAB-R2010b to minimize the cost (15) by varying the potential at the control points that have free potential. To consider the majority of the step disturbances implemented in current bipedal running robots, we consider apex return maps only on the domain between $y_{td}$ and 1.3 m. To allow for easy comparison, we choose the reference apex state equal to the state at apex that was used by Seyfarth et al. (2002), which results in $y_{ref} = 1$ m and $E_{ref} = 1785$ J. To increase the chance of finding the global minimum w.r.t. exclusively using a local solver, we use the globalsearch function, which is a global solver that uses the scatter search method described by Ugray et al. [18]. We use this function because Ugray et al. show that this global solver converges faster to the global optimum than current other global solvers.

The optimization problem is solved both for energy-neutral step disturbances and step disturbances. For energy-neutral step disturbances, we only consider the PALC model. To make the cost function (15), in which $m = 1$ for this optimization problem smooth w.r.t. the optimization parameters, we used 20 grid points. The resulting model with optimal leg compliance will be referred to as ‘PALC$^{opt\text{fix}}$ model’.

\[ f = \left\| v - v_{ref} \right\|_2 \]  

(15)

in which $v$ and $v_{ref}$ are in $\mathbb{R}^{n \times m \times 1}$, in which $n \times m$ is the number of sample points, of which $n$ sample points are equally spaced along the direction of $y_{lo}$, and $m$ sample points are equally spaced along the direction of $E$. 

This changed system energy. Although the changed system energy remains fixed over successive steps, the apex height might converge back to the reference limit cycle apex height. We consider a disturbance rejected when a stable limit cycle apex height is reached. We consider a disturbance optimally rejected if this disturbance is rejected in one step.

Cost function: Considering the apex height after a step $y_{a2}$ a surface w.r.t. the apex height before a step $y_{a1}$ and the system energy $E$, the disturbance rejection behavior is optimal when this ‘apex height surface’ is a horizontal plane of which the height is the reference limit cycle apex height $y_{ref}$. Disturbance rejection behavior is quantified by the distance between the apex height surface and this ‘reference plane’. A measure for this distance is cost $f$, which is the euclidean distance between vector $v$, which consists of a grid of sample points on the apex height surface, and vector $v_{ref}$, which consists of this grid but with sample points on the reference plane.
For step disturbances, we consider both models, in which we determine the cost function (15) smooth w.r.t. the optimization parameters, we use a grid of 20 points along the direction of \( y_{a1} \) and 10 points along the direction of \( E \). Firstly, we solve the optimization problem for the PALC model and next, we solve the optimization problem 20 times for the FALC model with different foot lengths. These foot lengths are equally spaced between \( d = 0.05 \) m and \( d = 0.35 \) m. Finally, we optimize the FALC model with infinite foot length. To reduce the dimension of the parameter space, only the potential at the 24 control points on the rectangular surface \( \{(x, y) \mid -1 \leq x \leq 1.5 \land 0 \leq y \leq 1.5\} \) is varied. The resulting models with optimal leg compliance will be referred to as ‘\( \text{PALC}^{\text{opt}} \)’ and ‘\( \text{FALC}^{\text{opt}} \)’.

Starting points: To reduce the computation time and to increase the chance of finding a feasible solution w.r.t. starting from an arbitrary leg potential, we selected specific leg potentials as starting points. The starting point for the optimization of the PALC model w.r.t. energy-neutral step disturbances was the leg potential of a leg with constant stiffness of \( 20000 \) N/m, which is the stiffness that was used in Seyfarth et al. As a starting point for the optimization of the PALC model w.r.t. step disturbances and the FALC model with a foot length of 0.05 m, we used the leg potential of the \( \text{PALC}^{\text{fix}} \) model. To find the start potential at the control points \( b_{-1,1.5}, b_{-0.5,1.5}, b_{0,0.5}, b_0, b_{0.5}, b_{1,1.5}, b_{1.5}, b_{1.5,1.5}, b_1, b_{1.5,1} \) and \( b_{1.5,0} \), we linearly extrapolated from the leg potential of the \( \text{PALC}^{\text{opt}} \) model.

IV. RESULTS: POINT FOOT AND A FIXED SYSTEM ENERGY

After having optimized the leg compliance in the PALC model w.r.t. rejecting energy-neutral step disturbances, we map the disturbance rejection behavior of the resulting model, which we refer to as the \( \text{PALC}^{\text{fix}} \) model.

Apex return map: Figure 4 shows the apex return map of the \( \text{PALC}^{\text{fix}} \) model, which has a cost (15) of 0.16 m and the apex return map of the SLIP model, which has a cost of 4.39 m. The apex return map of the \( \text{PALC}^{\text{fix}} \) model is flat and is close to the reference limit cycle apex height w.r.t. the apex return map of the SLIP model. The apex return map of the \( \text{PALC}^{\text{fix}} \) model intersects the diagonal, at \((0.99, 0.99)\) with a slope of 0.05, indicating that the model has a stable limit cycle apex height of 0.99 m. The apex return map of the SLIP model intersects the diagonal twice, once at \((1.00, 1.00)\) with a slope of 0.78, indicating that this model has a stable limit cycle apex height of 1.00 m, and once at 1.11 m with a slope of 1.19, indicating that this model has an unstable limit cycle apex height of 1.11 m. Where the stable range of the SLIP model is 0.19 m, the stable range of the \( \text{PALC}^{\text{fix}} \) model is at least 0.38 m.

Disturbance rejection behavior: The disturbance rejection behavior of the \( \text{PALC}^{\text{fix}} \) model is significantly improved w.r.t. the disturbance rejection behavior of the SLIP model. The significance of the improvement follows from the ratio between the cost of the SLIP model and the cost of the \( \text{PALC}^{\text{fix}} \) model, which is 27. That the disturbance rejection behavior is improved is confirmed from the largest energy-neutral step disturbance that the models can reject. Where the \( \text{PALC}^{\text{fix}} \) model can reject energy-neutral step down disturbances of at least 0.3 m, the SLIP model can reject energy-neutral step down disturbances of ‘only’ up to 0.1 m.

Although the apex return map of the \( \text{PALC}^{\text{fix}} \) model is flat, it is not completely flat. We expect that with a finer grid of control points, the cost can be minimized to zero because the apex height after a step depends on two variables of which one can be chosen free. We expect that the apex height after a step \( y_{a2} \), which depends on the apex height before a step \( y_{a1} \) and the leg potential at maximum leg compression \( \Delta P_{L_{\text{max}}} \), can be made any desired value by choosing the leg potential at maximum leg compression w.r.t. \( y_{a1} \). However, this leg potential can only be chosen free w.r.t. \( y_{a1} \) if there is a distinct maximum leg compression for all \( y_{a1} \). Although we cannot check for this distinct maximum leg compression, at least we can show a distinct maximum leg compression for the four trajectories in Figure 5.

Point mass trajectories: Figure 5 shows four point mass trajectories of the \( \text{PALC}^{\text{opt}} \) model and the leg potential, which is symmetric in the origin. The point mass trajectories enter the working range of the leg in the touchdown point. The horizontal distance from this point to the point at which
the trajectories begin increases w.r.t. the height at which a trajectory begins, which is a consequence of the fixed system energy. The maximum leg compression, which is 0.28 m for the trajectory that begins at 1.30 m, is distinct w.r.t. the height at which the trajectories begin. Although there are no trajectories that share the same liftoff point, the height at which the trajectories end is the about the same, which is in agreement with the flatness of the apex return map.

**Leg stiffness profile:** Karssen et al. [6] show an optimal leg stiffness profile which is similar to the leg stiffness profile that Figure 6 shows. Similar to the figure, Karssen et al. show a distinct peak in the optimal leg force w.r.t. leg compression. The trend is similar but not the same because Karssen et al. show a peak of 29038 N and the leg stiffness decreases to 6200 N/m. As the leg is compressed, the leg potential increases to 630 J and the leg stiffness decreases to −6200 N/m. The leg force shows a peak of 2910 N at a leg compression of 0.22 m.

**V. RESULTS: POINT FOOT AND A SYSTEM ENERGY RANGE**

We showed the disturbance rejection behavior of the PALC model with optimal leg compliance w.r.t. rejecting energy-neutral step disturbances. In this section we show the disturbance rejection behavior of the PALC model with optimal leg compliance w.r.t. step disturbances.

**Apex return maps:** Figure 7 shows apex return maps of the PALC model and apex return maps of the SLIP model. The apex return maps of the PALC model, which has a cost (15) of 3.67 m, are more flat than the apex return maps of the SLIP model, which has a cost of 14.44 m. All apex return maps of the PALC model intersect the diagonal between (0.97, 0.97) and (1.04, 1.04) with a slope between −0.22 and 0.37, which indicates that this model has stable limit cycle apex heights between 0.97 m and 1.04 m. The intersection point of an apex return map with the diagonal shifts in the direction of the origin w.r.t. system energy, which indicates that the stable limit cycle apex height decreases w.r.t. system energy. Only the apex return maps of the SLIP model that accompany system energies between 1768 J and 1935 J intersect the diagonal. These apex return maps intersect the diagonal twice. The first intersection is between (0.96, 0.96) and (1.05, 1.05) with a slope between 0.20 and 1, which indicates that the SLIP model has stable limit cycle apex heights between 0.96 m and 1.05 m. The second intersection is at a point further on the diagonal than (1.05, 1.05), with a slope that is larger than 1, which indicates that the model has unstable limit cycle apex heights that are higher than 1.05 m. Notice that the apex return maps of both models spread out w.r.t. $y_{\alpha 1}$. 

---

*Figure 5. Example of point mass trajectories of the PALC model that begin with heights of 0.92 m, 1.04 m, 1.17 m and 1.30 m. The boundary of the working range of the leg is indicated by a thick gray semicircle and the relevant leg potential is indicated by semicircular contour lines. The trajectories enter the working range of the leg at the touchdown point, which is $y = -0.92$ and each trajectory exits the working range of the leg at a liftoff point. All trajectories end with about the same height.*

*Figure 6. Leg potential, leg force and leg stiffness of the PALC model w.r.t. leg compression $\Delta L$. When the leg is not compressed, the leg potential is at the reference value of zero, the leg has a pretension of 400 N and the leg stiffness is 2260 N/m. As the leg is compressed, the leg potential increases to 630 J and the leg stiffness decreases to −6200 N/m. The leg force shows a peak of 2910 N at a leg compression of 0.22 m.*
**Velocity angle:** Figure 8, which shows the angle of the velocity at touchdown of the PALC\opt model w.r.t. \(y_{a2}\), which shows a similar trend w.r.t. Figure 7. Similar to the apex return maps, the velocity angle at touchdown spreads out w.r.t. \(y_{a1}\). Based on this similarity, we expect that the spread-out of the velocity angle at touchdown is a main reason why the apex return maps spread out.

**Disturbance rejection behavior:** The disturbance rejection behavior of the PALC\opt model is significantly improved w.r.t. the disturbance rejection behavior of the SLIP model. How significant this improvement is, follows from the ratio between the cost of the SLIP model and the cost of the PALC\opt model, which is 3.9. That the disturbance rejection behavior is improved is confirmed by the stable range and the system energy range in which the models run stable. The PALC\opt model can reject larger step disturbances because its stable range and the system energy range in which this model is stable, is larger w.r.t. the SLIP model.

Although the disturbance rejection behavior is improved, it is suboptimal. We expect that there is one main reason for this suboptimal disturbance rejection behavior: The maximum leg compression in a step is not distinct w.r.t. the apex height at which a step begins \(y_{a1}\) and the system energy \(E\). If this maximum leg compression is not distinct w.r.t. \(y_{a1}\) and \(E\), the potential at maximum leg compression \(P_{\Delta L_{\text{max}}}\) cannot be chosen free for every \((y_{a2}, E)\) such that the apex height after a step, which depends on \((y_{a2}, E, P_{\Delta L_{\text{max}}})\), is any desired value.
height difference indicates that the apex return maps spread out w.r.t. $y_{a1}$.

Concluding from this section, we state that the disturbance rejection behavior of a bipedal running robot can improve with at least a factor 3.93 by implementing optimal leg compliance w.r.t. disturbance rejection behavior instead of linear leg stiffness of 20000 N/m. However, this optimal leg compliance does not make the disturbance rejection behavior optimal.

VI. RESULTS: FLAT FOOT AND A SYSTEM ENERGY RANGE

We showed the disturbance rejection behavior of the PALC model with optimal leg compliance w.r.t. disturbance rejection behavior. In this section we show the disturbance rejection behavior of the FALC model with optimal whole-leg compliance w.r.t. disturbance rejection behavior.

Apex return maps: Figure 10 shows apex return maps of the SLIP model and apex return maps of the FALC$^{\text{opt}}_{0.13}$ model, which is the FALC model with leg compliance optimized w.r.t. the distance between the surface $y_{a2}(y_{a1}, E)$ and the plane $y_{a2} = 1$ m expressed in (15). The dashed horizontal lines and the dashed vertical lines indicate the touchdown height while the dotted horizontal line indicates the reference limit cycle apex height. The apex return maps of the FALC$^{\text{opt}}_{0.13}$ model are more flat relative to the apex return maps of the SLIP model.

Leg force: Figure 11 shows the leg force at three point mass trajectories of the FALC$^{\text{opt}}_{0.13}$ model with a system energy of 1935 J and apex heights of 0.92 m, 1.11 m and 1.30 m. The point mass trajectories, indicated with solid lines are in the area enclosed by thick black dots, which is the area in which the leg potential is optimized. The leg force at the point mass trajectories is indicated with thick arrows, of which the tails are extended with thin dotted lines. The extended tails intersect the horizontal axis between $-0.13$ m and 0.13 m. Arrows on the boundary of the working range of the leg have an extended tail that intersect the origin.

Foot length: Figure 12 shows the cost of the FALC$^{\text{opt}}_{0.13}$ model as a function of the system energy and apex height. The cost of the FALC$^{\text{opt}}_{0.13}$ model is significantly lower than for the SLIP model.

Disturbance rejection behavior: The disturbance rejection behavior of the FALC$^{\text{opt}}_{0.13}$ model is significantly improved w.r.t. the disturbance rejection behavior of the SLIP model. How significant this improvement is, follows from the ratio between the cost of the SLIP model and the cost of the FALC model, which is 6.0. That the disturbance rejection behavior is improved is confirmed by the stable range and the system energy range in which the models are stable. The stable range and this system energy range are larger for the FALC model than for the SLIP model.

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model w.r.t. foot length and the cost of the PALC opt model. As foot length increases, the cost of the FALC opt model decreases up to a maximum decrease of 2.46 m at infinite foot length w.r.t. the PALC opt model. This decrease is larger at small foot lengths than at large foot lengths. At a foot length of 0.13 m, the cost has ‘already’ decreased 50.6% of the maximum decrease.

**Disturbance rejection behavior:** Irrespective of the foot length, the disturbance rejection behavior of the FALC opt model improves relative to the disturbance rejection behavior of the PALC model, in which the significance of this improvement follows from the ratio between the cost of the models. We expect that the reason of this relative improvement is that the leg potential of the FALC opt model depends on two variables instead of one variable, as is the case in the PALC model. We expect that because the leg potential of the FALC model depends on two variables, there is a larger range over which the leg potential can vary freely before the foot constraint (17) is violated w.r.t. the range in which the leg potential of the PALC model can vary.

The disturbance rejection behavior of the FALC opt model improves w.r.t. foot length. We expect that by increasing the foot length, increases the range over which the leg potential can vary. This increased range allows for more freedom to choose the leg potential to reduce the effect of the apex height before the step and the system energy on the apex height after the step.

**Concluding** from this section, we state that the disturbance rejection behavior of a bipedal running improves up to a factor 3.0 by implementing a flat foot and optimal whole-leg compliance w.r.t. disturbance rejection behavior instead of a point foot and optimal leg compliance w.r.t. disturbance rejection behavior. With flat feet that have a foot length up to 35% of the leg-length, the disturbance rejection behavior improves up to a factor 2.3.

**VII. DISCUSSION**

**Measure for disturbance rejection behavior:** In this study, we used the flatness of apex return maps as a measure for disturbance rejection behavior. We found that in all three models, a decrease in our measure of disturbance rejection behavior is accompanied by an increase in stable range. Additional to this increase, in the PALC opt model and the FALC opt model, we found a significant increase in the range in system energies in which the models runs stable w.r.t. the SLIP model, from which we expect that the cost is indeed a measure for disturbance rejection behavior.

**Similar leg potential:** The leg potential of the PALC opt model is similar to the leg potential of the PALC ran model, which follows from comparing Figure 5 to Figure 9. Because the leg potential is similar, the apex return maps should be similar. Figure 13 shows the apex return maps of the PALC opt model in the same range of system energies that is considered in the PALC ran model. The apex return maps of Figure 13 are similar to the apex return maps of the PALC ran model, which are shown in Figure 7. The main difference between the apex return maps is that the apex return map of the PALC opt model that accompanies a system energy of 1935 J is partly below the touchdown height, which indicates that constraint 19 is not sufficed.
Application of results: The results of this study do not apply to step disturbances only. Because a state in the flight phase maps into a magnitude and an angle of the velocity at touchdown of $V_{td}$ and $\phi_{td}$, a step disturbance affects the angle and the magnitude of the touchdown velocity. The results are applicable to any disturbance during the flight that changes the angle and magnitude of the velocity at touchdown within the domain that this study considers, which is \{$(\phi_{td}, V_{td}) | 0 \leq \phi_{td} \leq 35.0 \land 4.78 \leq V_{td} \leq 5.12$\} with $\phi_{td}$ in Deg and $V_{td}$ in m/s.

VIII. CONCLUSION

In this paper we presented a simulation study on the influence of leg compliance on the disturbance rejection behavior of bipedal running robots. Based on the results of this study, we conclude that:

- The disturbance rejection behavior of a bipedal running robot improves by a factor 27 after implementing optimal leg compliance w.r.t. disturbance rejection behavior instead of point feet and linear leg stiffness of 20000 N/m, but only w.r.t. rejecting energy-neutral step disturbances.
- The disturbance rejection behavior of a bipedal running robot with point feet improves by a factor 3.8 after implementing optimal leg compliance w.r.t. disturbance rejection behavior instead of linear leg stiffness of 20000 N/m.
- The disturbance rejection behavior of a bipedal running robot improves up to a factor 3.0 after implementing flat feet and optimal whole-leg compliance w.r.t. disturbance rejection behavior instead of point feet and optimal leg compliance w.r.t. disturbance rejection behavior.
- The disturbance rejection behavior of a bipedal running robot improves up to a factor 2.3 after implementing flat feet with a foot length of up to 35% of the leg length and optimal whole-leg compliance w.r.t. disturbance rejection behavior instead of point feet and optimal leg compliance w.r.t. disturbance rejection behavior.

REFERENCES

Chapter 2

The influence of feet on the disturbance rejection behavior of bipedal-running robots
The influence of feet on the disturbance rejection behavior of bipedal-running robots

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Abstract—18th January 2012- Bipedal running robots generally have point feet whereas bipedal walking robots generally have arc-shaped feet or flat feet with compliant ankle joints. The purpose of the feet of bipedal walking robots is to improve the disturbance rejection behavior of these robots. How feet affect the disturbance rejection behavior in running robots was not clear. In this study, the influence of arc radius and ankle stiffness on the disturbance rejection behavior of running robots was investigated. First a literature study was performed to get an overview of what is known about the effects of feet on walking and running robots. Next, a simulation study was performed to fill the knowledge gaps found in the literature study. In the literature study we found maximum factors of improvement reduce to $\sqrt{2}$ and $\sqrt{7}$ for walking with arc-shaped feet and for walking with compliant ankles respectively. When compensating for the increase in walking speed that accompanies both arc-shaped feet and flat feet with compliant ankle joints, the maximum factors of improvement reduce to 2 and 1 for walking with arc-shaped feet and for walking with flat feet and compliant ankle joints respectively. Both for increasing arc radius and for increasing ankle stiffness the disturbance rejection behavior of bipedal running robots degrades.

I. INTRODUCTION

Nature shows that bipedal gaits can be fast [1], efficient [2], and robust to perturbations [3]. These characteristics make bipedal gaits a topic of research of which the results are potentially beneficial for the use in vehicles and other moving machines, prostheses and rehabilitation techniques. One way of validating results found by research on bipedal gaits is by implementing these gaits in robots.

The successful implementation of the bipedal walking gait was shown in several bipedal-walking robots [2, 4–8]. A disadvantage of these robots is that their speed is confined to $\sqrt{2}$ [9]. To reach higher speeds, a robot requires distinct flight phases in its gait. The most common example of such a gait is running.

In the implementation of the running gait in a bipedal robot, the most important performance requirement should be considered. This requirement is the ability of the robot not to fall under external disturbances and is called the ‘disturbance rejection behavior’ of the robot [10]. If its disturbance rejection behavior is too poor, a robot fails because it falls.

The disturbance rejection behavior of bipedal-walking robots is thought to improve by adding feet to these robots.

What the effect is of feet on the disturbance rejection behavior of bipedal-running robots was not clear. This effect was investigated in this study.

Feet in bipedal robots can be either passive or active. In this paper, we focus on passive feet because we believe they are more practically applicable than active feet. There are three reasons for this conviction. Firstly, passive feet have in general less mass than active feet, which results in lower inertia of the leg and therefore lower torques in the hip necessary to accelerate the leg. Secondly, no extension of the control laws implemented in a robot is necessary when using passive feet as opposed to active feet. Finally, passive feet do not need an external energy supply as opposed to active feet.

In this study, the influence of passive feet on the disturbance rejection behavior of bipedal-running robots was investigated. For this study, first literature was reviewed and then a simulation was made. From the reviewed literature it became clear that the amount of research on the effect of feet on the disturbance rejection behavior of bipedal-running robots is limited. To escape this limitation, we extended the scope to incorporate bipedal-walking robots. We chose bipedal-walking robots as a subject for the scope extension for two reasons. The first reason is that the walking gait and the walker show similarities to the running gait and the runner. The main similarity between the gaits is that they are both characterized by a single stance period and the main similarities between the robots is that they are both bipedal and they can both be equipped with the same type of passive feet. The second reason for considering bipedal-walking robots is that they have been researched thoroughly.

In general, two types of passive feet are used in bipedal robots. These feet are either arc-shaped feet that are rigidly connected to the leg or flat feet that are connected to the leg by a compliant ankle joint. The two most important design parameters accompanying these feet are arc radius and ankle stiffness. From these notions, the following main question arose: To what extend does arc radius and ankle stiffness affect the disturbance rejection behavior of bipedal-running robots? The answer to this main question is presented in this paper.

The remainder of this paper is organized as follows: Section II discusses disturbance rejection measures, Section III
discusses the literature study, Section IV discusses the simulation study, Section V discusses the results, and Section VI concludes upon these results.

II. MEASURES FOR DISTURBANCE REJECTION BEHAVIOR

To answer the main question in-depth, disturbance rejection behavior must be quantified. This section first introduces three measures that capture the results presented in this paper and then discusses their effects on determinants for the disturbance rejection behavior of bipedal robots.

The disturbance rejection behavior of a bipedal robot depends on the Convergence rate, which is the speed by which the biped recovers from a disturbance and the maximum deviation, which is the maximum deviation from the limit cycle before the gait of a robot becomes unstable [11]. If on the one hand the convergence rate of a biped is too small, successive small disturbances will cause it to fall. If on the other hand, the maximum deviation of a biped is too small, a single large disturbance will cause it to fail.

Three measures that quantify the convergence rate and the maximum deviation are: (1) the distance from the Floquet multipliers [12–14] - which are the eigenvalues of the Jacobian of the linearized step-to-step map - to the unit circle: \(1 - |\lambda_i|\), (2) the maximum allowable disturbance \(\max |e|\) [15–17] - which is the smallest disturbance \(e\) for which a biped falls within a number of steps \(n\) - and (3) the inverse of the Gait sensitivity norm \(\frac{1}{\|\partial g/\partial e\|_2}\) [10, 18] - which is the inverse of the Euclidean distance between the value of a gait indicator after the biped encounters a disturbance and the value of that gait indicator for the biped walking in the limit cycle over a sequence of steps [10].

The three measures that quantify disturbance rejection behavior have each their own characteristics. The first measure quantifies convergence rate, but only for small disturbances and only for the mode that accompanies the Floquet multiplier. These characteristics make this measure correlate poor to disturbance rejection behavior but well to stability [19]. A value of 1 indicates maximally stability, a value of 0 indicates marginal stability and a negative values indicates instability. The second measure quantifies maximum deviation, but only in the direction in state space of the states that are disturbed. This measure correlates well to disturbance rejection behavior but poor to stability [20]. The third measure weighs the relevance of the eigenmodes to the actual fail modes of a biped [21]. If the contribution of an eigenmode to the fail modes is low, its influence on the gait sensitivity norm is also low and vice versa. This measure correlates well to the disturbance rejection behavior of bipedal walking robots [18].

III. LITERATURE REVIEW

This section considers the literature that we found and from which the role of arc radius and ankle stiffness on the disturbance rejection behavior of bipedal robots follows. The section first discusses walking and then discusses running.

A. Role of passive feet on the disturbance rejection behavior of bipedal-walking robots

There are 8 studies that consider the role of passive feet on the disturbance rejection behavior of bipedal-walking robots. These studies were performed by Aoi et al. [22], Hobbelen et al.(2005) [23], Hobbelen et al.(2007) [10], Hobbelen et al.(2008) [24], McGeer(1990a) [25], Wang et al. [26], Wisse et al.(2003) [27] and Wisse et al.(2006) [28].

1) Arc-shaped feet: Hobbelen et al.(2007), Wisse et al.(2006), Wisse et al.(2003), Wang et al., Aoi et al. and McGeer(1990a) considered the role of arc-shaped feet in walking. They considered 2-dimensional walking with no knees. Wisse et al.(2003) used an actual prototype whereas the remainder of the studies considered models for simulation. The model by Aoi et al. is active, which means that this model is actuated, and walks on the flat. The remainder of the models and the prototype are passive, which means that these models are powered by gravity, and walk down slopes. The models by Hobbelen et al.(2007), Wang et al., McGeer(1990a) and Aoi et al. have in common that the mass in their hips is significant. The models by Wang et al., McGeer(1990a), Aoi et al. and Wisse et al.(2006) and the prototype by Wisse et al.(2003) have in common that the mass in their legs is significant.

The efficiency of a walker depends on the radius of its arc-shaped feet. A walker with feet that have a larger arc radius walks more efficiently than the same walker but with feet that have a smaller arc radius [29]. This dependency causes passive walkers with larger arc radius to walk faster down a slope. The increase in speed can be compensated for by decreasing the slope angle. The walking speed of the model by Aoi et al. does not increase because an actuator makes the angle between the legs track a sinusoid that is fixed in time.

Figures 1 till 4 show results for the walkers discussed previously. The results are expressed w.r.t. relative arc radius \(\frac{R}{\ell}\) which is defined on the domain \(\{0 \leq \frac{R}{\ell} \leq 1\}\) to be the arc radius of the foot of a walker normalized to the length of the leg of this walker. The figures 1 and 3 express the results in the measure that correlates well to stability and the figures 2 and 4 express the results in the measures that correlate well to disturbance rejection behavior. The figures 1 and 2 consider passive walkers that walk down slopes with constant slope angles and the figures 3 and 4 consider models of which the walking speed is constant w.r.t. relative arc radius.

In figure 1, Hobbelen et al.(2007), Wisse et al.(2003) and Wang et al., show that for increasing relative arc radius the stability first improves and then degrades. The relative arc radii for which stability is maximal depends on the model for walking.

In figure 2 Hobbelen et al.(2007), Wang et al., Wisse et al.(2006) and Wisse et al.(2003) show that for increasing relative arc radius the disturbance rejection behavior improves, Hobbelen et al.(2007) and Wisse et al.(2006) additionally
show that for larger relative arc radii the disturbance rejection behavior degrades. They also show that the relative arc radii for which this behavior is maximal is model-dependent. This model-dependency is of a lesser degree than the model-dependency of the relative arc radii for which the stability is maximal. Wang et al. and Wisse et al. (2003) do not show a degradation in the disturbance rejection behavior, due to the limited domains that they consider.

Notice that, for the three smallest relative arc radii of the prototype, the disturbance rejection behavior of this prototype strokes with the disturbance rejection behavior of the simulation models. For a relative arc radius of 1 the disturbance rejection behavior of the prototype and the models do not stroke. For this arc radius, a value for the maximum allowable disturbance is found for the prototype but no value is found for the models because their gaits are unstable. We hypothesize that this finding could have two reasons. The first reason is that the number of steps that the prototype took after it was disturbed might have been too little for the state to diverge enough from the limit cycle to make the prototype fall. The second reason is that the prototype might have hit the edge of its feet at the beginning and the end of its steps.

Figures 1 and 3 show three main differences between the stability of models for passive walking of which the walking speed increases w.r.t. relative arc radius and the stability of models for passive walking of which the walking speed is constant w.r.t. relative arc radius. The first two differences show from the model by Hobbelen et al. (2007). Firstly, this model walks more stable when the walking speed increases than when the walking speed is constant. Secondly, the relative arc radius for which the stability is maximal is larger when the speed increases than when the speed is constant. The third difference shows from all passive models. Models for which the walking speed increases become unstable as their relative arc radii approach 1 whereas models for which the walking speed is constant become marginally stable as their relative arc radii approach 1.

Figure 3 shows that the stability of the active model by Aoi et al. is poor w.r.t. the stability of the passive models by Hobbelen et al. (2007) and McGeer (1990a). We suspect that this difference is caused by the means of actuation. This actuation does not permit swing leg retraction whereas actuation by gravity for the passive models does permit swing leg retraction. Since swing leg retraction is a large determinant for the stability of a biped [21] we assume that the absence of
this characteristic in the active walker causes the poor stability.

![Graph](image)

Figure 4. Maximum allowable disturbance for a model for passive walking of which the walking speed is constant. The maximum allowable disturbance is an order of magnitude smaller than that of the model in which walking speed increases w.r.t. arc radius (cf. figure 2), which indicates that increased walking speed is the main cause by which arc radius improves disturbance rejection behavior.

Figure 4 shows the disturbance rejection behavior of the walker of which the walking speed is constant w.r.t. relative arc radius by Hobbelen et al.(2007). Comparing this figure to 2 shows that the optimal maximum allowable disturbance of the walker by Hobbelen et al.(2007) degrades by an order of magnitude when the walking speed of this model is kept constant w.r.t. relative arc radius. In addition, this optimum occurs for a smaller relative arc radius when the model is walking at constant speed than when the model is walking at increased speed.

The main finding in this section is that the disturbance rejection behavior of the robots improves by adding arc-shaped feet. This improvement goes up to a factor of 52. The majority of this improvement is due to the increase in walking speed for increasing relative arc radius. Compensating for this increase in walking speed results in an improvement factor of 'only' 2.

2) Compliant ankle joints: Wisse et al.(2006), Hobbelen et al.(2005) and Hobbelen et al.(2008), considered walking on flat feet. These feet are investigated for the following three reasons: flat feet provide more resistance against yaw, they make the walker less sensitive to irregularities in the walking surface and they are more human-like [28].

The studies consider different models. Hobbelen et al.(2008) consider a ‘complex’ seven-link model that is similar to the model which Hobbelen et al.(2005) consider but for a different parameter set. Additional to this model, Hobbelen et al.(2008) consider two more modes; a ‘simple’ model, which is the extension of the simplest walking model by flat feet, and an actual prototype which resembles their complex model. Wisse et al.(2006) consider a model that is similar to their model, which was discussed in the previous section, but for flat feet and compliant ankle joints. This is the only model that is passive and together with the simple model by Hobbelen et al.(2008) this is the only model in which foot scuffing was ignored. All models are two-dimensional.

The walking speed of a model or prototype increases for increasing ankle stiffness unless compensated for. Hobbelen et al.(2005) and Wisse et al.(2006) did not compensate for this increased speed whereas Hobbelen et al.(2008) did compensate for this increased speed. Hobbelen et al.(2008) performed this compensation by adjusting the actuation pattern of the models and the prototype.

Figures 5 till 7, capture results for the models and the prototype discussed above. These results are expressed w.r.t. relative ankle stiffness $k_{hr}$, which is the torsional stiffness in the ankle, normalized to the values that are shown in the legends. These values are approximates of the smallest stiffness for which the heel of the stance leg lifts off the ground before the toes of the leading leg contact the ground after heel strike. This event is called premature heel rise. Since normalizing factors did not follow for any of the models except for the simple model by Hobbelen et al.(2008), we estimated them by using (1).

$$k_{hr} = \frac{mgL_f}{\phi}$$

In this equation, $m$ is the total mass of the walker, $g$ is the gravitational acceleration, $L_f$ is the distance between the ankle and the toe of the foot and $\phi$ is the angle that the trailing leg makes w.r.t. the vertical during the double support phase.

Figure 5 shows that for increasing relative ankle stiffness the stability of the model by Wisse et al.(2006) first improves and then degrades. Comparing figure 5 to figure 1 shows that for the optimal relative ankle stiffness and for the optimal relative arc radius the walker with flat feet is equally stable to the walker with arc-shaped feet respectively.

Figure 6 shows that for increasing relative ankle stiffness the disturbance rejection behavior of the walker by Wisse et al.(2006) first improves and then degrades. Comparing figures 6 and 2 shows that this course is similar to the course which was found for the same walker but with arc shaped feet except for one main difference: The disturbance rejection behavior of the walker with flat feet is poorer than the disturbance rejection behavior of the walker with arc-shaped feet. The figure also shows that the disturbance rejection behavior of the model by Hobbelen et al.(2005) improves for increasing relative ankle stiffness. This course is shown on a relatively small domain and compared to the course that Wisse et al.(2006) show, the one by Hobbelen...
Figure 5. The distance of the largest Floquet multiplier to the unit circle for a model for passive walking that has flat feet and of which the walking speed increases w.r.t. relative ankle stiffness which is the ankle stiffness, normalized to the smallest ankle stiffness for which premature heel rise occurs. For increasing relative ankle stiffness stability first improves and then degrades.

Wisse et al. (2006). $k_{hr} = 5.6 \text{ Nm/Rad}$

Figure 6. Maximum allowable disturbance and inverse of gait sensitivity norm for two models for walking of which walking speed increases w.r.t. relative ankle stiffness, which is the ankle stiffness, normalized to the smallest ankle stiffness for which premature heel rise occurs. For increasing relative ankle stiffness, disturbance rejection behavior first improves and then degrades.

et al. (2005) is more rough. This roughness originates from discrete changes in walking behavior. An example of a discrete change in walking behavior is tripping over the tips of the feet.

Figure 7 shows that the disturbance rejection behavior of the 'complex' walker and the prototype in the models by Hobbelen et al. (2008), does not significantly depend on relative spring stiffness. This trend is also shown for the simple walker for relative ankle stiffness below one. For values higher than one, the disturbance rejection behavior first degrades. Hobbelen et al. (2008) hypothesize that this trend is an effect of the simplicity of the model. For relative ankle stiffnesses above 1.46, Hobbelen et al. (2008) reported that the walking behavior is not realistic.

The main finding of this section is that ankle stiffness improves the disturbance rejection behavior of walking bipedal robots because it increases the walking speed of these robots. The improvement can at least run up to a factor of 7.2. Compensating for the increase in walking speed showed that ankle stiffness does not directly affect the disturbance rejection behavior.

B. Role of passive feet on the disturbance rejection behavior of bipedal-running robots

We did not find any studies that considered the effect of passive feet on other measures for disturbance rejection behavior than the measure that quantifies stability. The studies that we did find and which are discussed in this section are by McGeer (1990b) [30], Yun jun et al. [31], Ringrose [32] and Cham [33].

1) Arc-shaped feet: McGeer (1990b), Ringrose and Yun jun et al. considered running on arc shaped feet. For this consideration they used 4 simulation models which are constrained to move in a single plane. McGeer (1990b) considered a passive bipedal model, Ringrose considered a ‘simple’ and a ‘complex’ model of a monopedal hopper and Yun jun et al. used an extension of the most basic model for running.

The passive model by McGeer (1990b) consists of a hip joint of significant mass, that connects two legs by a pin joint and a torsional spring. The legs have significant mass and are connected to massless feet by linear springs. Running exists by the combination of a bouncing motion on the stance leg and a scissor action of the legs. The angle of attack, which
is the angle between the leg and the ground at touchdown, is kept constant. This model is conservative and therefore it can run for indefinite time on flat grounds.

Both models for hopping by Ringrose have a hip with significant mass and inertia. This hip is attached to the ‘leg’, which is a damper that is attached parallel to a spring and linear actuator in series. To this leg, a foot is attached which, for the simple model, does not have mass but for the complex model does have mass. The models do not actually move forward significantly; they 'run in place'.

In contrast to the models for hopping by Ringrose, the model by Jun et al. has significant forward speed. This model is based on the 'Spring Loaded Inverted Pendulum' (SLIP) model, which is the simplest model for running that still captures elementary characteristics of bipedal running such as bouncing frequency, vertical displacement and vertical ground reaction force [34, 35]. Jun et al. extended this SLIP model by a massless arc-shaped foot and called it the ‘SLIP-R’ model.

There are four differences between the stability of the simple model for hopping and the stability of the complex model for hopping. The first difference is that the relative arc radii for which the simple model is stable is smaller than those for which the complex model is stable. The second difference is that the domain in relative arc radius for which the simple model is stable, is smaller than this domain for the complex model. The third difference is that the optimal stability is better for the complex model than for the simple model and the final difference is that the stability of the complex model for hopping is much more irregular w.r.t. the stability of the simple model for hopping. The exact mechanisms that work behind the four main differences do not become clear from Ringrose. He mentions that the complex model has an error between the actual leg length and the desired leg length during flight and that the simple model does not have an error between these leg lengths. He states that this effect changes the touchdown time of the complex model w.r.t. that of the simple model. According to Ringrose, this change is a cause of the differences between the stability of both models. How this difference is exactly affected, together with finding the other causes, is left for future research.

In contrast to the arc radii of the feet of the models for hopping, the arc radii of the feet of the bipedal model are always smaller than the distance of the COM of the model to the foot. McGeer(1990b) showed that increasing this radius causes the running speed to decrease and that this decrease is accompanied by a degradation in stability. Although he does not explain why for a relative arc radius of 0.5 stability is better than for one of 0.4, he does mention that for relative arc radii of over 0.5 the stability of the model degrades.

Figure 8 does not show any results of Yun jun et al. because their paper only provides qualitative results on stability. From these results follows that there are sets of parameters that make the running motion unstable for small relative arc radii, but stable for larger radii. This difference in stability suggests that this property improves for models that incorporate these sets of parameters.

All studies considered, it can be concluded that arc radius has a definite effect on the stability of the models that this section considers. For the hoppers, this property improved significantly but for the bipedal model, it degraded and for the extended SLIP model, an effect was seen but not quantified.

In previous studies, the effect of arc radius on the disturbance rejection behavior of bipedal-running robots has not yet been investigated. This investigation is presented in this paper in Section IV.

2) Compliant ankle joints: This section considers bipedal running on flat feet. Although we did not find any studies on this exact subject, we found a study of which the subject shows enough similarities to be discussed here. This is the study by Cham.
Cham considered hexapedal running which is not exactly the same as bipedal running, but both types of gait show enough similarities to be modeled by the same template; the SLIP model \cite{36-38}. Cham considered a specific type of hexapedal running robot, namely that which has a sprawled posture and of which the legs are inclined w.r.t. the vertical \cite{39}. To capture the running behavior of this type of robot, cham used an adaptation of the SLIP model that he called the 'Sprawled-Type SLIP' model.

Figure 9 shows the 'Sprawl-Type SLIP' model. This model consists of a point mass in the 'hip', which is attached to a massless 'leg' that consists of a spring, a damper and a force-producing element in parallel. On the other side of the leg, a point foot is attached which, when in contact with the ground, shows a rotational spring damper characteristic w.r.t. the difference between the leg angle and the angle of attack. This results in an 'ankle' torque between the ground and the leg.

To capture the sprawled posture of the hexapods that Cham considers, the Sprawled-Type SLIP model has an angle of attack $\alpha_0$ that is larger than 90 Deg. A consequence of this large angle is that the ankle torque plays a significant role in decelerating the vertical speed of the CoM from the speed, which it has at touchdown. For this deceleration, the ankle torque itself must also be significant and since the necessity for large ankle torques in bipedal robots is accompanied by large feet in these robots, we doubt that the Sprawl-Type SLIP model contains damping elements which dissipate energy during gait. This dissipated energy is injected back into the model in the form of work, created by the force-producing element. This force-producing element exerts a constant force during thrust, which initiates at a fixed time after touchdown and endures until the moment of lift-off. Because thrust initiation is clock-based, the dynamics of the model are time-dependent. This time-dependency extends the state of the model w.r.t. the state of the SLIP model by one state variable.

For the Sprawled-Type SLIP model, Cham considered stability w.r.t. the parameters ankle stiffness, normalized to the inertia of the leg around the foot at moment of touchdown, angle of attack and thrust initiation time span. For these three parameters, he first finds a limit cycle by varying the first two initial state variables and then determines the norms of the Floquet multipliers in this fixed point. Using this approach, he considers angles of attack between 95 Deg and 130 Deg and normalized ankle stiffnesses between 15 Rad/s and 35 Rad/s.

The results by Cham show that the Sprawl-Type SLIP model has one unstable mode for small angles of attack and large stiffnesses if thrust initiates just after maximum leg compression. He explains that thrust initiation time span has a large contribution to this mode and shows that it can be stabilized by decreasing the ankle stiffness and increasing the angle of attack. If thrust however is initiated before or well after maximum leg compression, both ankle stiffness and angle of attack do not affect the stability of the model significantly. It follows that stability in this model is highly related to the means by which it is actuated and since this type of actuation is specific to the Sprawl-Type model, we doubt that this model is a template for bipedal running robots that are actuated otherwise.

The study by Cham considered, leads us to two main conclusions. Our first conclusion is that smaller ankle stiffness positively influences the stability of the Sprawl-type SLIP model where thrust is initiated just after maximum leg compression. If thrust is activated before or well after maximum leg compression stability is not influenced by the ankle stiffness. Our second conclusion is that the Sprawl-type SLIP model is not representable for any bipedal-running robot that we know of. Therefore, we cannot generalize the results by Cham.

Since no study has researched the influence of ankle stiffness on the disturbance rejection behavior in bipedal running, we performed this research. The results of this research are presented in section IV.

C. Discussion and conclusion on the literature review

The main question 'To what extent does arc radius and ankle stiffness affect the disturbance rejection behavior of bipedal-running robots?' was not answered in this literature review.

From the study by Cham \cite{33}, which is the only study available that considered bipedal running with flat feet and
compliant ankles, the influence of ankle stiffness on the disturbance rejection behavior does not become clear because of two reasons. The first reason is that the quality of the model as a template for bipedal running is poor and the second reason is that Cham only considered stability and did not consider disturbance rejection behavior.

From the studies by Ringrose [32], McGeer(1990b) [30] and Jun et al. [31], which consider arc shaped feet in running, it does not become clear how arc radius affects the disturbance rejection behaviour of bipedal-running robots because these studies only considered stability.

From the studies discussed in Section III-A-1 and Section III-A-2, which considered bipedal walking, the answer to the main question does not follow because it is not known how well the results for walking can be generalized to running. We suspect that a generalization is not justified because of two reasons. The first reason is that Section III-A-1 and Section III-B-1 showed that the influence of arc radius on the stability of walkers is not in agreement with the influence of arc radius on the stability of runners. The second reason is that walkers and runners differ significantly in structure and therefore differ significantly in functioning. A structural difference is that walkers have rigid legs whereas runners have spring-like legs. This difference causes walking to be necessarily dissipative whereas running can be conservative. Due to this difference,walkers dissipate any additional energy that is introduced by a disturbance whereas runners do not necessarily dissipate this energy.

Because the available literature did not answer the main question we performed a simulation study. This simulation study is presented in the following section.

IV. SIMULATION STUDY

This section presents a simulation study in which the influence of arc radius and ankle stiffness on the disturbance rejection behavior of bipedal robots was investigated.

A. Method

Two models for conservative running are considered in this study. These models are the 'SLIP-R' model, which has an arc-shaped foot, as was first used by Jun et al. and a model with flat feet and ankle compliance, which we call the 'SLIP-C' model. Figure 10 shows the SLIP-R and the SLIP-C models. Both models have a massless 'leg' that is a spring with stiffness $k$ and natural length $C_0$. On one side of this leg, a point mass $m$ is attached on which a gravitational force $F_g = mg$ acts. On the other side of this leg, a massless 'foot' is attached, which is arc-shaped with radius $R$ for the SLIP-R model and flat with ankle compliance $C$ for the SLIP-C model. The motion of the models during stance is described by (2) till (5) respectively, in which $q = [\zeta \ \psi]^T$ holds with $\zeta$ and $\psi$ the length of the leg and the angle of the leg w.r.t. the vertical. Adapted from [31]

$$0 = M(q)\ddot{q} + N(q, \dot{q}) + G(q)$$ (2)

The running motion of the SLIP-R and the SLIP-C models consists of a succession of steps. These steps begin and end in the flight phase with 'apex' which occurs if the speed of the CoM in vertical direction is zero: $\dot{z} = 0$. At this moment, the leg is instantaneously set to a predefined angle of attack. After apex, the model continues in a parabolic trajectory until the moment of touchdown causes the transition from flight phase to stance phase. During the stance phase, the motion of the CoM of the SLIP-R model is described by (2) till (5). In (2) till (5), $q = [\zeta \ \psi]^T$ is a vector of polar coordinates in which $\zeta$ and $\psi$ represent the instantaneous length of the leg and the instantaneous angle of the leg w.r.t. the vertical (cf. figure 10). The motion of the CoM of the SLIP-C model during the stance phase is described by (6) in which $p = [x \ y]^T$ is a vector of Cartesian coordinates in which $x$ is the horizontal position of the CoM and $y$ is the vertical position of the CoM. In (6), $x_{rel} = x - x_{foot}$ is the horizontal position of the CoM w.r.t. the horizontal position of the foot and $L = \sqrt{x_{rel}^2 + y^2}$ is the length of the leg.

$$0 = M(q)\ddot{q} + N(q, \dot{q}) + G(q)$$ (2)

### Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SLIP-R</th>
<th>SLIP-C</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>80</td>
<td>80</td>
<td>kg</td>
</tr>
<tr>
<td>$g$</td>
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<td>m/s^2</td>
</tr>
<tr>
<td>$k$</td>
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<td>20e3</td>
<td>N/m</td>
</tr>
<tr>
<td>$R$</td>
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<td>0.45</td>
<td>m</td>
</tr>
<tr>
<td>$C_0$</td>
<td>1 - $R$</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>$C$</td>
<td>-</td>
<td>-1200…1200</td>
<td>Nm/Rad</td>
</tr>
</tbody>
</table>

The direction of travel is depicted in Figure 10. On one side of the leg, a point mass $m$ is attached on which a gravitational force $F_g = mg$ acts. On the other side of this leg, a massless 'foot' is attached, which is arc-shaped with radius $R$ for the SLIP-R model and flat with ankle compliance $C$ for the SLIP-C model. The motion of the models during stance is described by (2) till (5) respectively, in which $q = [\zeta \ \psi]^T$ holds with $\zeta$ and $\psi$ the length of the leg and the angle of the leg w.r.t. the vertical. Adapted from [31]

Figure 10. Left: SLIP-R model. Right: SLIP-C model. On one side of a massless 'leg' that is a spring with stiffness $k$ and natural length $C_0$, a point mass $m$ is attached on which a gravitational force $F_g = mg$ acts. On the other side of this leg, a massless 'foot' is attached, which is arc-shaped with radius $R$ for the SLIP-R model and flat with ankle compliance $C$ for the SLIP-C model. The motion of the models during stance is described by (2) till (5) and (6) respectively, in which $q = [\zeta \ \psi]^T$ holds with $\zeta$ and $\psi$ the length of the leg and the angle of the leg w.r.t. the vertical. Adapted from [31]
For every stable limit cycle, the maximum allowable disturbance, as defined in Section II, was determined with a precision of $10^{-4}$ m. For this determination, the disturbance $e$ was chosen to be an increase in apex height and an accompanying decrease in apex speed, such that total energy was not disturbed and the number of steps was chosen to be $n = 50$.

### B. Results and discussion

Figure 11 shows stability and disturbance rejection behavior of the SLIP-R and the SLIP-C model. The figures 11(a) and 11(b) show the distance of the smallest Floquet multiplier to the unit circle for the SLIP-R and the SLIP-C model respectively. The figures 11(c) and 11(d) show the maximal allowable disturbance for these models respectively. In the figures 11(a) and 11(d) stability and disturbance rejection behavior is expressed w.r.t. relative arc radius $R/L$ and in the figures 11(b) and 11(d) these measures are expressed w.r.t. absolute ankle stiffness $C$.

Figure 11 shows that for increasing relative arc radius and for increasing ankle stiffness stability and disturbance rejection behavior degrade similarly. For the SLIP-C model the degradation in stability and disturbance rejection behavior is approximately linear.

From this simulation study, it can be concluded that the disturbance rejection behavior of bipedal-running robots degrades both by implementing arc-shaped feet and by implementing flat feet and compliant ankles. It can also be concluded that...
stability is a good measure to quantify disturbance rejection behavior in bipedal-running robots.

V. DISCUSSION

From the literature study, we suspect that results for walking cannot be generalized to running (cf. Section III-C). The simulation study confirms this suspicion because the results from this study, which concern running, do not stroke with the results from the literature study that concern walking. Differences between these results show from comparing figures 1 till 7 to figures 11(a) till 11(d).

Except for the results on the stability of the model for walking with arc shaped feet by Aoi et al. [22], the results from the literature study on walking with arc shaped feet do not stroke with the results from the simulation study on running with arc shaped feet. Figures 1 till 4 show that both the stability of the walking gaits and the disturbance rejection behavior of the models by Hobbelen et al. [10], Wang et al. [26] and Wisse et al. [28] can improve by implementing arc-shaped feet. Figure 2 additionally shows that the disturbance rejection behavior of the prototype by Wisse et al. [2003] can improve by implementing arc-shaped feet and figure 3 additionally shows that the stability of the walking gait of the model by McGeer [1990a] can improve by implementing arc shaped feet. In contrast to what figures 1 to 4 show, figure 11(a) and figure 11(b) show that both the stability of the running gait and the disturbance rejection behavior of the SLIP-R model degrade by implementing arc shaped feet. Figure 3 shows that the stability of the walking gait of the model by Aoi et al. degrades by implementing arc shaped feet. Comparing the stability of the walking gait of the model by Aoi et al. to the stability of the running gait of the SLIP-C model as shown in figure 11(a) shows one difference. Whereas the walking gait is marginally stable for a relative arc radius of 1, the running gait is unstable for this arc radius.

The results from the literature study on walking with flat feet and compliant ankle joints do not stroke with the results from the simulation study on running with flat feet and compliant ankle joints. Figures 5 and 6 show that both the stability of the walking gait and the disturbance rejection behavior of the model by Wisse et al. [2006] can improve by implementing flat feet and compliant ankle joints. Figure 6 additionally shows that the disturbance rejection behavior of the model for walking by Hobbelen et al. [2005] can improve by implementing arc shaped feet. Figure 7 shows that the disturbance rejection behavior of the models for walking and the prototype by Hobbelen et al. [2008] does not improve nor degrade by implementing flat feet and compliant ankle joints. In contrast to what figures 5 to 7 show, figures 11(b) and 11(d) show that both the stability of the running gait and the disturbance rejection behavior of the SLIP-C model degrade by implementing arc shaped feet.

In future work we will investigate negative ankle stiffness, because simulations on the SLIP-C model showed that this stiffness positively affects the disturbance rejection behavior. We consider this subject only briefly because negative ankle stiffness might not be practically applicable in bipedal-running robots. We will also investigate the influence of biarticular structures on the disturbance rejection behavior of bipedal-running robots. These structures pose more freedom to design the forces on the CoM of the runner during stance, which could be beneficial for the disturbance rejection behavior. Finally, if pilot tests show that the negative ankle stiffness and biarticular
structures do not improve the disturbance rejection behavior we will look into ankle actuation. With ankle actuation, it might be possible to design the forces on the CoM during stance such that the disturbance rejection behavior of a runner improves.

VI. CONCLUSION

In this study, the effect of arc radius and ankle compliance on the disturbance rejection behavior of bipedal-running robots was investigated. For this investigation, also bipedal walking was considered. From this study the following conclusions can be drawn:

- Both for increasing relative arc radius and for increasing ankle stiffness the disturbance rejection behavior of bipedal-walking robots first improves and then degrades.
- The improvement of the disturbance rejection behavior of bipedal-walking robots is mainly due to the increase in walking speed that accompanies feet.
- For increasing the arc radius of the arc-shaped feet of bipedal-walking robots the disturbance rejection behavior degrades until the gait becomes unstable.
- For increasing the ankle stiffness of bipedal-running robots with flat feet and compliant ankles the disturbance rejection behavior degrades approximately linearly.

REFERENCES


