Application of Constrained Stochastic Simulation
to Determine the Extreme Response of Wind Turbines

Wim Bierbooms  
TU Delft  
Wind Energy Research Group  
P.O. Box 5058  
2600 GB Delft  
The Netherlands  
W.A.A.M.Bierbooms@tudelft.nl

Johan Peeringa  
ECN Wind Energy  
P.O. Box 1  
1755 ZG Petten  
The Netherlands  
peeringa@ecn.nl

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Summary

Up to now just deterministic gust shapes are specified in standards; e.g. the Extreme Operating Gust (IEC) is given by a Mexican hat like shape. In this paper gust shapes will be determined by application of so-called constrained stochastic simulation. This method specifies how to efficiently generate time series around some specific event (e.g. a local maximum) in a normal (Gaussian) process. In this way the generated gusts have the correct stochastic properties of turbulence (like generated wind fields for fatigue analysis).

In constrained stochastic simulation a random signal can be generated which satisfies some condition, e.g. a maximum value or jump at some time instant. Usually the condition is applied on the external condition, i.e. the wind or waves (for offshore wind turbines). In this paper the condition of a maximum value, at some time instant, will be applied to the response instead (the blade root flapping moment). Assuming a linearised model of the wind turbine, the accompanying wind input leading to the extreme response, can be derived. By performing load simulations the proposed method is validated.

In the future a probabilistic approach of extreme loading may replace the present deterministic procedure in standards.

1. Introduction

In this paper a linearised model (from hub wind speed to blade root flapping moment) of the reference turbine will be used in order to investigate ultimate loads. The reason to consider a linear model is that this allows validation of methods to arrive at ultimate loads. Namely, the 50-year response of a linear system can be arrived theoretically. This is treated in Section 2.

In Appendix F of the IEC standard a probabilistic method is given to determine the extreme response. The maximum values of 10-min. series have to be fitted to a distribution function and extrapolated to 50-year. In Section 3 the application of this method to the linearised turbine model will be discussed. An alternative approach based on so-called constrained simulation is given in Section 4.

In this paper just one mean wind speed (15.5 m/s) will be considered. So, the determined 50 year load is the load which would occur on average every 50 year in case the wind is always blowing with 15.5 m/s.

The response values, i.e. blade root flapping moment, are given with respect to the mean value at $V=15.5$ m/s and are normalized with the theoretical 50-year value.
2. Theoretical 50-year value of a linear model

The response of a linear system to a random input which is Gaussian is also Gaussian. This implies that the flapping moment can be considered to be Gaussian distributed since the (generated) wind input is also normal. For normal random variables a theoretical expression (after Rice, [3]) exists for the density of the local maxima (of level \( C \)) which depends on just one parameter (\( \varepsilon \)):

\[
f_{\eta}(\eta) = \eta \sqrt{1-\varepsilon^2} e^{-\frac{\eta^2}{2}} \Phi\left(\frac{\eta \sqrt{1-\varepsilon^2}}{\varepsilon} + \frac{1}{\sqrt{2\pi}} \varepsilon e^{-\frac{\eta^2}{2\varepsilon^2}}\right)
\]

(1)

with dimensionless level of the local maxima:

\[
\eta = \frac{C}{\sqrt{R_{rr}(0)}}
\]

(2)

and \( \Phi \) is the standard normal distribution function.

In the above \( R_{rr} \) represents the autocorrelation function of the response. The autocorrelation function can be determined via a Fourier Transform of the spectrum of the output. The latter one equals:

\[
S_{rr} = |H|^2 S_{uu}
\]

(4)

with \( H \) the transfer function from input to output which is obtained via system identification [4]. For this purpose, simulations of our reference turbine N6, see [1], are performed with the aid of Phatas (simulation package of ECN). The input \( S_{uu} \) represents the turbulence spectrum. Here the von Karman expression is used, but any other spectrum description can be taken as well. In Fig. 1 the distribution \( F_{n} \) belonging to the density \( f_{n} \) from Eq. (1) is shown, assuming a mean wind speed of 15.5 m/s and a turbulence intensity of 12%.

Rather than all local maxima, one may be interested in the 10-min. maxima (i.e. the maxima of each 10-min. time series). The distribution \( F_{10} \) of these 10-min. maxima follows from the distribution \( F_{n} \) of all maxima:

\[
F_{10}(\eta) = F_{n}^{N}(\eta)
\]

(5)

with \( N \) the number of local maxima in 10-min. Eq. (5) is valid only in case the local maxima are independent. This is probably not the case for small values but since we are primarily interested in extremes we can use this expression.

The mean frequency \( \nu \) of all local maxima equals (also after Rice, [3]):

\[
\nu = \frac{1}{2\pi} \sqrt{\frac{-R_{rr}(0)}{R_{rr}(0)}}
\]

(6)

thus \( N \) can be easily obtained via \( N=600 \nu \).

The distribution of the 10-min. maxima is given in the graph by the red line. In the bottom graph the same distributions are shown on a logarithmic scale in order to focus on the upper tail.

The 50 year event can be determined on basis of the graphs. By definition, the 50-year event is on average exceeded once in 50 years. So, on average the 50 year response happens once every \( T_{50} \) 10-min. time series; with \( T_{50}=50\cdot365\cdot24\cdot6 \) the number of 10-min. series in 50 year. In terms of probability of non-exceedence this can be expressed as:

\[
F(r_{50}) = 1 - \frac{1}{T_{50}} = 0.99999962
\]

(7)
This quantile is indicated by a red dashed line.
Note: the quantile of the 50-year event for all maxima corresponds to $T_{50}=50*365*24*3600*\nu$; see blue dashed line.

![Distribution of the response](image1.png)

**Figure 1**: Theoretical distribution of local maxima as well as 10-min. maxima.

From the graph the theoretical 50-year value is obtained. This value is used throughout the paper to normalize the loads; so $r_{50}=1$.

3. Unconstrained simulations

In Section 4 a special kind of simulation, so-called constrained stochastic simulation, will be treated. In this Section normal, unconstrained stochastic simulations of 10-min. length are discussed. An example of such a simulation is given in Fig. 2.

**Distribution of the response**

In total 10000 10-min. simulations (so in total almost 70 days) are performed from which the (10-min.) maxima in the response are taken. Since we are dealing with a linear model of limited order, this can be done within several minutes computational time. From the results an empirical distribution can be obtained by ordering the values to magnitude. The corresponding distribution value of the i-th ranked value equals: $F_i=i/(N+1)$, with $N$ the total number of simulations. So, the largest value in all 10000 simulations is connected to a distribution value of $10000/10001$; on the bottom graph this corresponds to a value of $\log(10001)\approx 9.2$. One could opt to extrapolate from this point onwards. Here it is chosen to extrapolate above the 90th percentile since the scatter (from one data set to another) of the 90th percentile is far less. The extrapolation is done by fitting the results to a Generalised Extreme Value (GEV) distribution. The GEV comprises all three possible limiting Extreme Value (EV) distributions with left endpoint, right endpoint or no endpoint at all. A standard Matlab routine (gevfit.m) is used to fit the data. This routine also provides upper and lower bounds of the estimate; in the figure indicated by dashed lines. A confidence level of 0.68 is taken and as a result the confidence interval (range between lower and upper limit) equals about two times the standard deviation (assuming that the estimated values are normal distributed).
In Fig. 3 the result is compared to the theoretical distribution from Section 2. Note: Eq. (5) only holds for independent events. Small maxima are not independent so for small values of the response (say less than 0.6) the theoretical curve is not correct.

The 50-year estimate based on normal, unconstrained simulations can be read from the graph (the 0.99999962 quantile): 0.96±0.02, which is smaller than the theoretical value of 1. This is due to the
fact that here the fitted GEV happens to have an endpoint and the theoretical one not. So, the estimate could probably be improved by considering some other distribution function, e.g. the 3-parameter Weibull. Note: in general (non-linear systems) it is NOT known if the to be fitted distribution has a right endpoint or not.

Since 10000 10-min. maxima are available it is also possible to do 10 times a fit to 1000 10-min. maxima. The result is given below. The spread in the 10 results (black lines) is an excellent indication of the uncertainty of a fit to 1000 values. In the figure also the confidence level as determined by the Matlab routine of just one of the 10 fits is shown (black dashed lines): $0.95 \pm 0.03$. Again a confidence level of 0.68 is taken so one should expect that about 6 to 7 out of 10 fits will be inside the confidence interval. Indeed, from Fig. 4a it can be seen that there are 6 inside this range. Please note that the uncertainty is inherent to the stochastic process so cannot be reduced. Or to put it in other words: if one performs 1000 simulations and estimates the 50-year value based on the obtained results, one could have ended up with any of these 10 different fits and thus 10 different values of the 50-year estimate. The only way to decrease the uncertainty is to increase the number of simulations.

**Figure 4:** The distribution of the response based on 10000 or 1000 unconstrained simulations.

**Figure 4a:** A close up of Fig. 4.
4. Constrained simulations

By means of system identification, a linear model (of order 8) has been fitted to time domain simulations, [4]. On basis of the linear model it is possible to generate wind gusts which will lead to a local maxima in the response (at $t=0$). This way of wind simulation is called constrained stochastic simulation. It is based on a conditional distribution. For a full treatment of the method see [2].

An example of a $-5\sigma$ gust is given in the figure below (a turbulence intensity of 12% is taken). Due to the negative correlation between input and response the constrained wind is a dip rather than a peak!

![Constrained wind and response](image)

Figure 5: Example of the flapping response to constrained turbulent wind.

Distribution of the response for given gust amplitude

In total 1000 simulations (with different random seeds) are performed for the gust amplitude of $-5\sigma$ (since we are dealing with a linear system this is done in seconds on an ordinary PC). The results, the response maxima at $t=0$ s, can be fitted to a distribution. Below the $90^{th}$ percentile the empirical distribution is used ($F_{\text{e}}=i/(N+1)$ with $i=1$ to $N$ and $N$ is the number of simulations). Above the $90^{th}$ percentile a GEV fit is applied (which may have a right endpoint). The uncertainty range of the fit is given by dashed lines, see Fig. 6.

In [2] an analytical expression can be found for this conditional distribution.
The distribution of the flapping response to constrained turbulent gusts with an amplitude of \(-5\sigma\).  

**Distribution of the response**

Above the distribution of the flapping response is treated for constrained gusts with amplitude of \(-5\sigma\). Similar, the distribution of the response on different gust amplitudes can be determined. Finally, the overall distribution of the response is obtained through averaging. This will be outlined below.

The procedure can be formalized as follows. Say, the gust amplitude of the stochastic wind input of the wind turbine is denoted as random variable \(x\) and the local maxima of the response as random variable \(y\). The marginal densities are \(f_x(x)\) and \(f_y(y)\); the joint density is \(f(x, y)\) and \(f_y(y | x)\) is the conditional density of \(y\) upon observing \(x=x\). The following well-known relations exist:

\[
f_y(y) = \int_{-\infty}^{\infty} f(x, y) \, dx \tag{8}
\]

\[
F_y(y) = \int_{-\infty}^{y} f_y(\beta) \, d\beta \tag{9}
\]

\[
f_c(y) = \frac{f(x, y)}{f_x(x)} \tag{10}
\]

\[
F_c(y) = \int_{-\infty}^{y} f_c(\beta) \, d\beta \tag{11}
\]

Combination leads to:

\[
F_y(y) = \int_{-\infty}^{y} \int_{-\infty}^{\infty} f(\alpha, \beta) \, d\alpha \, d\beta = \int_{-\infty}^{\infty} \int_{-\infty}^{y} f_c(\beta) f_x(\alpha) \, d\alpha \, d\beta = \int_{-\infty}^{\infty} F_c(y) f_x(\alpha) \, d\alpha = \sum F_c(y) n_x \tag{12}
\]

with \(n_x\) the probability (‘fraction of time’) that a gust amplitude is within the discretized amplitude intervals of random input \(x\). An analytical expression of \(n_x\) can be found in [2].

The distribution of response \(y\) can thus be obtained through a weighted summation (convolution) of the conditional distributions.
Here, five gust amplitudes, from -3σ to -7σ, have been considered. The results are presented in Fig. 7. The overall distribution (for U=15.5 m/s) is determined by the weighted average, Eq. (12). In the figure this is indicated by 'convol' (short for convolution). The uncertainty range is again indicated by dashed lines. The theoretical distribution is the one given by Eq. (5), Section 2.

Assuming that the mean wind speed of 15.5 m/s happens all the time the 50-year response is obtained by the 1-T50 percentile; with T50=50*365*24*6 the number of 10-min. periods in 50 year (log T50 is about 15). The resulting 50-year value is r_{50}=1.02±0.01 which is close to the theoretical one: r_{50t}=1.

Figure 7: The distribution of the response based on constrained simulations; gust amplitudes from -3σ to -7σ.

Figure 8: Top: The value of the conditional distribution (exceedance probability) for r=1.02 for gust amplitudes of -3σ to -7σ.
Middle: The fraction of time for each gust amplitude.
**Bottom:** Contribution of each gust amplitude to the tail estimation of the response (i.e. the normalized product of the values of the top and middle graph).

The contribution of each gust amplitude to the 50-year value, based on Eq. (12), is shown in Fig. 8. On basis of this graph the procedure is repeated for gust amplitudes in the range of -4σ to -7σ (with a step of 0.3σ) are considered, Fig. 9 and 10. This leads to an improved 50 estimate of 0.99±0.01, just about 1% deviation from the theoretical value, which validates the method.

![Figure 8: Distribution of the response based on Eq. (12).](image)

**Figure 9:** The distribution of the response based on constrained simulations; gust amplitudes from -4σ to -7σ.

![Figure 10: Top: The value of the conditional distribution (exceedance probability) for r=0.99 for gust amplitudes of -4σ to -7σ.](image)

**Figure 10:** Top: The value of the conditional distribution (exceedance probability) for r=0.99 for gust amplitudes of -4σ to -7σ. Middle: The fraction of time for each gust amplitude. Bottom: Contribution of each gust amplitude to the tail estimation of the response (i.e. the normalized product of the values of the top and middle graph).
5. Conclusions

Comparison constrained en unconstrained simulations

In total 16 gust amplitudes have been considered in Section 4; 5 for the determination of Fig. 7 (and Fig. 8) and 11 for Fig. 9 (and Fig. 10). For each amplitude 1000 simulations are done of a length of 200 s. So, in total this corresponds to 5300 10-min. simulations. In fact, this could be reduced since the constrained simulation can be much shorter than 200 s. since just care should be taken of transient response.

The results of Section 3 concerning unconstrained simulations are based on 10000 10-min. simulations.

The estimated 50-year response is $0.96 \pm 0.02$ according to the analysis with constrained simulations and $0.99 \pm 0.01$ based on unconstrained simulations. The theoretical value equals 1. So, constrained stochastic simulation outperforms normal, unconstrained simulation: the 50-year estimate is closer to the theoretical one, the uncertainty margin is less and the required computational effort is just half.

Final remarks

The constrained simulations of Section 4 should be performed in regular simulation tools (Bladed, Phatas, Flex) in order to determine the 50-year response of a non-linear wind turbine model. The procedure should be done for different mean wind speeds and combined on basis of the probability of these mean wind speed bins (i.e. Weibull distributed).

Section 4 considered simulations of a linear model and just done for demonstrating the probabilistic method. A side effect is that a theoretical expression of the distribution function is available so this may serve as a reference case of fit procedures to an extreme value distribution.

6. References


