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The behaviour of bar and steel-fibre-reinforced concrete beams in static testing

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THE BEHAVIOUR OF BAR-REINFORCED STEEL FIBRE
CONCRETE BEAMS IN STATIC TESTING

by

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## Contents

1. Introduction .......................... 1
2. Analysis for the ultimate stage ........ 1
3. Analysis for service stage ............. 5
   3.1. Before cracking .................. 5
   3.2. After cracking .................. 8
4. Experimental tests ..................... 10
5. Comparison of calculated and test data . 12
   5.1. The ultimate stage .............. 12
   5.2. The service stage .............. 13
6. Summary and conclusions ............... 15
7. Acknowledgements ..................... 16
8. References ........................... 17
9. Notations ................................ 18
10. Appendix ............................. 20

The tables are given in the appendix.
b) The tensile contribution of the fibres is represented by a rectangular stress block.

c) The effective tensile stress in the fibrous concrete ($\sigma_t$) is given according to R.N. Swamy [1] and C.H. Henager [3] by equation:

$$\sigma_t = 0.00772 \frac{l}{d_f} p F_{be}$$  \hspace{1cm} (1)

in which:

- $l$ - fibre length,
- $d_f$ - fibre diameter,
- $p$ - percentage by volume of fibres,
- $F_{be}$ - bond efficiency factor.

d) The tension bar reinforcement is reaching the yield strength of steel ($f_a$).

![Diagram](image)

**Fig. 1.** Assumed stress distribution (1) and simplified (2) representation.

The equation for the equilibrium of compressive force ($C$), tensile force of fibrous concrete ($T_{fc}$) and tensile force of bar reinforcement ($T_a$) from fig. 1 is:

$$C = T_{fc} + T_a$$  \hspace{1cm} (2)
or:

\[ 0.85 \beta \frac{f'_b}{f'_b} = b(h - x)\sigma_t + A_a f'_a \]  

(2a)

With the notations:

\[ \omega = \frac{A_a}{bh}, \xi = \frac{x}{h} \]  

(3)

the position of neutral axis is given by the equation:

\[ \xi = \frac{\omega f'_a + \sigma_t}{0.85 \beta f'_b + \sigma_t}. \]  

(4)

Equating moments about C we obtain the ultimate moment strength:

\[ M = A_a f'_a (h_o - \frac{x}{2}) + b(h - x)\sigma_t (\frac{h}{2} - \frac{x}{2}) = \]

\[ = \left[ \omega f'_a (\delta - 0.5 \xi) + 0.5(1 - \xi)\sigma_t \right] bh^2 \]  

(5)

where:

\[ \delta = \frac{h_o}{h}. \]  

(6)

In the equation (5), the second term represents the tensile contribution of the fibres to increasing of moment strength. For B 37,5 concrete and FeB 400 steel [5] this contribution is shown in figure 2. It's easy to see that the tensile contribution of the fibres is greater when the effective tensile stress in the fibrous concrete (\( \sigma_t \)) is greater and when reinforcement percentage ratio (\( \omega_o = 100 \omega \)) is smaller.
Fig. 2. The tensile contribution of the fibres: a - absolute values; b - relative values (\(\Delta M\) - the moment capacity increasing due to steel fibres).
3. **Analysis for service stage**

3.1. **Before cracking**

The following assumptions can be made for the analysis method:

a) The strains in the concrete and reinforcing steel are directly proportional to the distance from the neutral axis.

b) The stress distribution in both compressive and tensile zone is triangular (fig. 3).

c) The modulus of deformation of concrete in tension is the same as in compression.

![Assumed stress distribution and strain diagram](image)

**Fig. 3. The assumptions for the analysis before cracking.**

The equation for the equilibrium of forces shown in fig. 3 is:

\[
\frac{1}{2} b x \sigma_b' = \frac{1}{2} b (h - x) \sigma_b + A_a \sigma_a.
\]  

(7)
By similar triangles in strain diagram we find:

\[ \epsilon'_b = \epsilon_a \frac{x}{h_o - x} \]  
\[ \epsilon_b = \epsilon_a \frac{h - x}{h_o - x} \]

and further we obtain the values of stresses:

\[ \sigma'_b = \epsilon_a \frac{x}{h_o - x} E \]  
\[ \sigma_b = \epsilon_a \frac{h - x}{h_o - x} E. \]

Above, \( E \) represents the modulus of deformation according to \( \sigma-\epsilon \) diagram of fibre reinforced concrete. Such a diagram is obtained by S.P. Shah, P. Stroeven, D. Dalhuisen and P. van Stekelenburg \(^4\).

Put \( \sigma'_b \) from (10), \( \sigma_b \) from (11) and \( \sigma_a = \epsilon_a E_a \) (\( E_a \) - elastic modulus of steel) in equation (7), use the notations (3) and (6) and solve for \( \xi \):

\[ \xi = \frac{1 + 2\omega \frac{E_a}{E}}{2(1 + \omega \frac{E_a}{E})}. \]

The sum of moments in respect to the axis of the compressive force of concrete is:

\[ M = A_a \sigma_a (h_o - \frac{x}{3}) + \frac{1}{2} b(h - x)\sigma_b \left[ \frac{2}{3} (h - x) + \frac{2}{3} x \right]. \]

With (3), (6) and (11), the equation (13) becomes:

\[ M = \left[ \omega_o (\delta - \frac{\xi}{3}) + \frac{1}{3} \frac{(1 - \xi)^2 E_a}{E} \right] bh^2 \sigma_a. \]
The curvature of a deformed element (see fig. 4) can be calculated, as it is known, using the equation:

\[
\frac{1}{R} = \frac{d\theta}{dx} = \frac{\varepsilon'_{b}}{x} = \frac{\varepsilon_{a}}{(h_{o} - x)} = \frac{\varepsilon_{b}}{(h - x)}.
\]  \hspace{1cm} (15)

With \(\sigma_{a}\) from (14), the equation becomes:

\[
\frac{1}{R} = \frac{\varepsilon_{a}}{h_{o} - x} = \frac{\sigma_{a}}{E_{a}(h_{o} - x)} = \frac{M}{K}
\]  \hspace{1cm} (16)

where the modulus of rigidity (\(K\)) is given by the equation:

\[
K = \left[\omega(\delta - \frac{\xi}{3}) + \frac{1}{3} \left(1 - \frac{\xi}{\delta - \xi}\right)\frac{E}{E_{a}}\right](\delta - \xi)bh^{3}E_{a}.
\]  \hspace{1cm} (17)

The value of deflection (\(f\)) can be obtained with the relation:

\[
f = S \frac{M_{I}^{2}}{K}
\]  \hspace{1cm} (18)
in which:

- $S$ - coefficient depending on the type of load and on the kind of supporting [6],
- $l$ - span of the element.

3.2. After cracking

The analysis method, after cracking, is based on the next assumptions (see fig. 5):

a) In the compressive zone the stress distribution is triangular.

b) The bottom of cross section is reaching the tensile strength ($f_a$). Thus the fibre concrete has an elastic-plastic behaviour and the stress distribution in the tensile zone is a second degree parabola.

c) After cracking, only the compressive strain of the concrete and the strain of reinforcing steel are directly proportional to the distance from the neutral axis.

d) The deformation modulus ($E$) in the compressive zone depends on the shape of the $\sigma$-$\varepsilon$ diagram of fibre concrete [1].

Fig. 5. The assumptions for the analysis after cracking.
From the equilibrium of compressive and tensile forces shown in fig. 5 we get:

\[
\frac{1}{2} bx \sigma_b' = \frac{2}{3} b(h - x) f_b + \sigma_a A_a. \tag{19}
\]

With the equations (8) and (10) resulted from similar triangles in strain diagram and with the notations (3) and (6) we obtain the position of the neutral axis:

\[
\left( \frac{E}{E_a} - \frac{h}{3} \frac{f_b}{\sigma_a} \right) \xi^2 + \left[ \frac{h}{3} \frac{f_b}{\sigma_a} (1 + \delta) + 2\omega_o \right] \xi - \left( \frac{h}{3} \frac{f_b}{\sigma_a} \delta + 2\omega_o \delta \right) = 0 . \tag{20}
\]

The bending moment is given by the equation:

\[
M = A_a \sigma_a (h_o - \frac{x}{3}) + \frac{2}{3} b(h - x) f_b \left[ \frac{5}{6} (h - x) + \frac{2}{3} x \right] = \left[ \omega_o (\delta - \frac{\xi}{3}) + \frac{2}{3} (1 - \xi) \left( \frac{5}{6} + \frac{1}{24} \xi \right) \frac{f_b}{\sigma_a} \right] bh^2 \sigma_a . \tag{21}
\]

The curvature and the deflection can be obtained by the equations (16) and (18) respectively, taking for the modulus of rigidity the following value:

\[
K = \left[ \omega_o (\delta - \frac{\xi}{3}) + \frac{2}{3} (1 - \xi) \left( \frac{5}{6} + \frac{1}{24} \xi \right) \frac{f_b}{\sigma_a} \right] (\delta - \xi) bh^3 E_a . \tag{22}
\]

In the equations (20), (21) and (22) the \( \frac{f_b}{\sigma_a} \) ratio is known only if the \( \sigma_a \) value is known, for example by experimental measuring of the steel strain. Otherwise an iteration procedure is necessary, taking primarily an approximate value for the \( \frac{f_b}{\sigma_a} \) ratio according to the 3.1. a assumption (see fig. 3):

\[
\frac{f_b}{\sigma_a} = \frac{1 - \xi}{\delta - \xi} \frac{E}{E_a} . \tag{23}
\]
Example

For the beam number 8 in table 5 we have the following data: BK fibres, $E_t = 20800 \text{ N/mm}^2$, $\omega_o = 2.09\%$, $E_a = 21000 \text{ N/mm}^2$, $M_{\text{test}} = 7.51 \times 10^6 \text{ N.mm}$ and $f_{\text{test}} = 6.113 \text{ mm}$.

Put the equation (23) in (20):

$$
\left( \frac{E_t}{E_a} - 1.33 \frac{E_t}{E_a} \right) \xi^2 + 2 \left( 1.33 \frac{E_t}{E_a} + \omega_o \right) \xi - \left( 1.33 \frac{E_t}{E_a} + 2 \omega_o \delta \right) = 0
$$

and with above data solve for $\xi$ obtain $\xi = 0.59$.

From the equation (21) we obtain $\sigma_a = 190 \text{ N/mm}^2$.

A new value for $\xi$ we obtain from the equation (20): $\xi = 0.474$.

Using the equation (22), we find $K = 6.145 \times 10^{11} \text{ N.mm}^2$ and further using the equation (18) with $S = \frac{23}{216}$ we obtain $\sigma_{\text{calc}} = 5.205 \text{ mm}^2$.

The ratio $\frac{f_{\text{calc}}}{f_{\text{test}}} = 0.85$ and the value of this ratio obtained with known value of $\sigma_a$ (by measuring) are very close to each other (see table 5).

4. Experimental tests

The validity of the analytical method was verified by a testing program carried out in the Stevin-Laboratory, Delft University of Technology [8].

Four sets of beams were tested in flexure. First a conventional set was made without fibres. The types of fibres used in random distribution for other three sets were a straight one (AR), a hooked one (BK) and a straight one with paddles on both ends (TH), like is shown in figure 6.
The percentage by volume of fibres (p) was 0.89% for AR; 1.27% for BK and 1.53% for TH fibres.

Each set of beams included one beam without bar reinforcement and three beams reinforced with 2 \( \phi 4 \), 4 \( \phi 6 \), and 4 \( \phi 10 \) respectively. The steel quality was a FeB 500 Hi bond.

The concrete mix consisted of 400 kg/m\(^3\) type A portland cement, round sand, gravel aggregate with a maximum size of 16 mm and 0.48 water-cement ratio. Data of concrete are given in table 1 (see appendix).

The beams were 100 x 150 mm\(^2\) in cross-section and 2200 mm long. The free span during testing was 2000 mm with a pure bending zone of 800 mm long.

The top and bottom concrete strain was measured by electrical extensometers and the deflection by dial gauges.

In order to obtain a supplementary confirmation, the author's method was applied on the experimental results of C.H. Henager [3].
5. Comparison of calculated and test data

5.1. The ultimate stage

The moment strength of beams calculated by the proposed method ($M_{\text{calc}}$) are given in table 2 and compared with test values ($M_{\text{test}}$).

Note that, for non-reinforced beams we used the assumption shown in figure 7 and the following equation:

$$M = (1 - \xi) \frac{3 + \xi}{6} bh^2 f_b$$

(24)

where:

$$\xi = \frac{X}{h} = 0.5,$$

$$f_b = (1 + \frac{1}{20} f'_{ck}) \times 0.87, \, \text{N/mm}^2, \, \text{according to \cite{5}.}$$

Fig. 7. The assumptions for the calculation of ultimate bending moment of non-reinforced beams.
Therewith, for non-fibrous beams the $\sigma_t$ value in the equations (4) and (5) is set equal to zero.

For the bond efficiency factor ($F_{be}$) in the equation (1) we used the following values: 1.1 for BK fibres, 1.0 for AR fibres and 1.2 for TH fibres.

The value of $\delta = \frac{h_0}{h}$ for reinforced beams was equal to 0.93.

The results, given synthetically in table 2 by $\frac{M_{calc}}{M_{test}}$ ratio, show good agreement of the calculated moments with test moments. The average value of this ratio was 1.04 for the fibrous beams and 1.09 for all the beams. The main difference between the calculated and test values presented the non-fibrous beams without reinforcement (number 1 in table 2). Note that such beams don't represent actual cases in practice.

In table 3 are presented the calculated values of ultimate moment strength by the author's method comparatively with the experimental values obtained by C.H. Henager [3]. Calculated strength agree well with experimental values, the average $\frac{M_{calc}}{M_{test}}$ ratio being 1.02.

So, we can conclude that the method presented in paragraph 2 predicts the ultimate moment strength of fibre reinforced concrete beams with a good accuracy.

5.2. The service stage

In table 4 are given the values of the top and bottom concrete strain measured at a medium level of loading. The values of compressive and tensile concrete stress and tensile steel stress, obtained from the measured strains multiplied by the moduli of deformation, are given also in this table.

The values of bending moment obtained according to paragraph 3 ($M_{calc}$) are presented in the table 4 comparatively with test values ($M_{test}$).
Fig. 8. The assumptions for the analysis of non-reinforced elements under service loads.

For the non-reinforced beams we used the assumptions shown in fig. 8 and the following equation:

\[ M = \frac{1}{3} (1 - \xi)bh^2\sigma_b \]  \hspace{1cm} (25)

in which:

\[ \xi = 0.5 \text{ - when } \sigma_b < 0.5 f_b \]

\[ \xi^2 \left( \frac{E}{E_t} - 1 \right) + 2 \xi - 1 = 0 \text{ - when } \sigma_b > 0.5 f_b. \]

Calculated moment values agree well with experimental values. The average \( \frac{M_{\text{calc}}}{M_{\text{test}}} \) ratio is 1.02.

In table 5 a comparison of calculated and test deflections under service loads (about half of ultimate load) is done. For the non-reinforced beams, according to assumptions from fig. 8, the stiffness was calculated with the equation:

\[ K = \frac{1}{3} (1 - \xi)^2 bh^3E_t. \]  \hspace{1cm} (26)
The agreement between calculated deflections and experimental values is also good. The $\frac{f_{\text{calc}}}{f_{\text{test}}}$ ratio has an average value equal to 1.01.

This means that according to the method of analysis, presented in paragraph 3 we can predict accurately the value of bending moment or the value of deflection under service loads.

6. **Summary and conclusions**

The method of analysis described here is suitable for predicting the behaviour of reinforced steel fibre concrete beams, regarding:

- the ultimate moment,
- the moment value in service stage when the values of stresses are known (e.g. by measuring),
- the magnitude of deflection caused by a given bending moment.

There is a good agreement between predicted values and experimental values in ultimate stage as well in service stage.

It can be seen that fibre reinforcement (in random distribution) increases the ultimate moment strength of bending elements. The influence is the more distinct the smaller the steel bar reinforcement ratio and the higher the effective tensile stress in the fibrous concrete.

The fibre reinforcement increases also the post-cracking stiffness of the reinforced fibrous beams. However, an increase of deflections is caused by the higher moment values.

The crack width and the crack spacing of fibre reinforced concrete, analysed in the report [6], were less than in non-fibrous concrete. The crack load in fibrous concrete is greater than in conventional concrete. Thus, the crack width of fibre reinforced concrete will be probably not a condition for the serviceability limit state.
7. Acknowledgements

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The tests used in order to establish the analysis method were carried out in the Stevin-Laboratory, Concrete Structure. The author wish to thank Dr. Ing. H.W. Reinhardt for his generous help in this investigation. Thanks are also extended to Ir. H.A. Körmeling for useful discussions pertaining to experimental results.
8. References

1. X Fiber Reinforced Concrete, symposium held at Ottawa, Canada. ACI Publication SP-44, 1974.


8. Reinhardt, H.W., Körmeling, H.A. Static and Dynamic Testing of Concrete Beams Reinforced with Steel-Fibres and Continuous Bars. FIP 8th Congress 1978 - written contribution to the discussion.
9. Notations

\[ A_a = \text{area of tension bar reinforcement, mm}^2 \]
\[ b = \text{width of cross section, mm} \]
\[ C = \text{compressive force in concrete, N} \]
\[ d_f = \text{fibre diameter, mm} \]
\[ E = \text{modulus of deformation of concrete, N/mm}^2 \]
\[ E_o = \text{modulus of elasticity of concrete, N/mm}^2 \]
\[ E_t = \text{modulus of elasticity-plasticity of concrete, N/mm}^2 \]
\[ E_s = \text{modulus of elasticity of tensile steel, N/mm}^2 \]
\[ f = \text{deflection of beam, mm} \]
\[ f_a = \text{yield strength of steel (characteristic value), N/mm}^2 \]
\[ f_b = \text{tensile strength of concrete, N/mm}^2 \]
\[ f'_b = \text{cylindrical compressive strength of concrete, N/mm}^2 \]
\[ f_{bk} = \text{characteristic cylindrical compressive strength of concrete, N/mm}^2 \]
\[ F_{be} = \text{bond efficiency factor} \]
\[ h = \text{depth of cross section, mm} \]
\[ h_o = \text{effective depth, mm} \]
\[ K = \text{modulus of rigidity, Nmm}^2 \]
\[ l = \text{span of beam as well as fibre length, mm} \]
\[ M = \text{applied moment as well as internal moment, Nmm} \]
\[ M_u = \text{ultimate moment, Nmm} \]
\[ p = \text{percentage by volume of fibres, \%} \]
\[ R = \text{radius of curvature, mm} \]
\[ \frac{1}{R} = \text{curvature of beam, mm}^{-1} \]
\[ S = \text{coefficient for calculus of deflection} \]
\[ T_a = \text{tensile force of bar reinforcement, N} \]
\[ T_{fc} = \text{tensile force of fibrous concrete, N} \]
\( x \) = distance from extreme compression fibre to neutral axis, mm
\( \delta \) = ratio \( \frac{h_0}{h} \)
\( \varepsilon_a \) = steel strain, mm/m
\( \varepsilon_b \) = tensile strain of concrete, mm/m
\( \varepsilon'_b \) = compressive strain of concrete, mm/m
\( \omega_o \) = steel bar reinforcement ratio
\( \omega_o \) = steel bar reinforcement percentage ratio, %
\( \sigma_a \) = tensile stress of bar reinforcement, N/mm^2
\( \sigma_b \) = tensile stress of concrete, N/mm^2
\( \sigma'_b \) = compressive stress of concrete, N/mm^2
\( \sigma_t \) = effective tensile stress of fibrous concrete, N/mm^2
\( \zeta \) = ratio \( \frac{x}{h} \).
10. Appendix

Table 1. Data of concrete

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<th>type of concrete</th>
<th>$f'_{bk}$ N/mm$^2$</th>
<th>$E_o$ N/mm$^2$</th>
<th>$E_t$ N/mm$^2$</th>
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<td>31.15</td>
<td>34 800</td>
<td>22 800</td>
</tr>
<tr>
<td>with BK fibres</td>
<td>35.88</td>
<td>36 100</td>
<td>20 800</td>
</tr>
<tr>
<td>with AR fibres</td>
<td>35.17</td>
<td>34 200</td>
<td>19 000</td>
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<td>with TH fibres</td>
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<td>37 100</td>
<td>15 800</td>
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Table 2. Comparison of calculated and test values of the ultimate bending moment

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<th>Nr.</th>
<th>$\omega$ %</th>
<th>fibres</th>
<th>$p$ %</th>
<th>$\sigma_t$ N/mm²</th>
<th>$f_a$ N/mm²</th>
<th>$\xi$</th>
<th>$M_{calc} \times 10^{-6}$ Nmm</th>
<th>$M_{test} \times 10^{-6}$ Nmm</th>
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Table 3. Comparison of calculated moments and C.H. Henager's test values

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<th>( d_f ) (mm)</th>
<th>( l ) (mm)</th>
<th>( p ) (%)</th>
<th>( F_{be} ) N/mm²</th>
<th>( \sigma_t ) N/mm²</th>
<th>( f_a' ) N/mm²</th>
<th>( \varepsilon )</th>
<th>( M_{calc} ) N.m</th>
<th>( M_{test} ) N.m</th>
<th>( \frac{M_{calc}}{M_{test}} )</th>
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Table 4. Comparison of calculated and test moments under service loads

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<th>$\sigma'_b$ N/mm²</th>
<th>$\varepsilon_b$ mm/m</th>
<th>$\sigma_b$ (N/mm²)</th>
<th>$\xi$</th>
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Table 5. Comparison of calculated and test deflections under service loads

| Nr. | fibres | $\omega_0$ % | $f_{\text{c}}$ $f_{\text{t}}$ M $\sigma_a$ $M_u$ $\xi$ $K$ $f_{\text{calc}}$ $f_{\text{test}}$ | $f_{\text{calc}} / f_{\text{test}}$ |
|-----|--------|-------------|-----------------|-----------------|-----------------|------------------|-----------------|-----------------|------------------|
| 1   | -      | 0.64 0.43   | (1.14) 0.53 0.45 | 7.759 0.290 0.189 | 1.53 1.65 |
| 2   | -      | 0.17 0.46   | 26.54 1.12 0.51  | 6.543 0.729 0.517 | 1.41 |
| 3   | -      | 0.75 0.45   | 247.00 7.63 0.43  | 5.826 5.578 5.669 | 0.98 |
| 4   | BK     | 0.97 0.81   | (2.43) 1.11 0.43  | 7.603 0.621 3.035 | 0.204 |
| 5   | BK     | 0.17 0.45   | 33.87 1.34 0.41  | 7.371 0.774 0.819 | 0.94 |
| 6   | BK     | 0.75 0.48   | 216.99 3.93 0.42  | 3.615 4.630 4.524 | 1.02 |
| 7   | BK     | 2.09 0.43   | 245.97 7.51 0.47  | 5.966 5.362 6.113 | 0.87 |
| 8   | AR     | 0.75 0.85   | (2.40) 1.12 0.43  | 6.945 0.686 0.663 | 1.03 |
| 9   | AR     | 0.17 0.53   | 40.91 1.64 0.43  | 7.088 0.985 1.054 | 0.93 |
| 10  | AR     | 0.75 0.57   | 361.20 5.41 0.34  | 3.324 6.932 7.660 | 0.90 |
| 11  | AR     | 2.09 0.52   | 267.20 8.96 0.48  | 5.741 6.647 6.993 | 0.95 |
| 12  | TH     | 0.93 0.90   | (2.42) 1.12 0.39  | 6.614 0.721 1.082 | 0.66 |
| 13  | TH     | 0.17 0.52   | 69.03 2.01 0.33  | 4.780 1.791 1.524 | 1.17 |
| 14  | TH     | 0.75 0.58   | 315.36 5.41 0.39  | 3.058 7.535 6.670 | 1.13 |
| 15  | TH     | 2.09 0.57   | 301.48 10.50 0.51 | 5.180 8.634 7.465 | 1.15 |