LECTURE NOTES ON
AIRPLANE STABILITY AND CONTROL I
PART I

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Delft - The Netherlands

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PREFACE

The present lecture notes on 'Airplane Stability and Control I' mainly deal with three subjects. In the first place, in Chapters 2, 3 and 7, some aerodynamic characteristics of the airplane are discussed. They relate in particular to the aerodynamic moments. These characteristics have a direct bearing on the stability and control characteristics of the airplane. To limit the scope of the discussions, varying the aerodynamic coefficients with airspeed, are not presented in any detail.

In the second place, in Chapters 3, 4, 5, 6 and 7 the conditions of equilibrium, static stability and the control characteristics in steady flight are discussed. Here, also, the presentation is restricted to those situations where airspeed has no influence on the aerodynamic coefficients. The third subject in chapter 8, is dynamic longitudinal and lateral stability. The discussion of control in non-steady flight, a logical continuation of this third subject, is given in the course 'Airplane Stability and Control II'.

Experience has shown that the comprehension of the meaning of the various notions, used to describe the stability and control characteristics of an airplane, often presents significant difficulties. In these notes an effort has been made to alleviate these difficulties, by making the text rather complete and self-supporting. In addition, emphasis is placed on simplified, linearized expressions representing the various aerodynamic characteristics of the airplane. These simplified expressions lead to relatively simple formulae for various stability and control characteristics. Using these expressions, the influence of the principal factors determining stability and control is studied.

The purpose of this procedure has been to provide a mainly qualitative insight in the principles of airplane stability and control. If however, in a given situation the purpose is to obtain more or less reliable quantitative data on stability and control characteristics of a particular aircraft by paper and pencil calculations, the linearized aerodynamic expressions may prove to be insufficiently accurate. In such situations experimentally obtained data have to be used.

The lecture notes on 'Airplane Stability and Control I' in their present form are the result of a close cooperation with ir. H. Binkhorst, ir. J.C. van der Vaart and mr. R.J. Kuil. I wish to express my sincere thanks for their
enduring assistance. Many thanks are due to Mr. Van Dam for his diligent and many faceted cooperation in bringing about this English version of the lecture notes. Besides I want to thank Mr. Hari Mohammad for his meticulous proof-reading and Miss Marian van der Vring for her punctual typing.

O.H. Gerlach
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References
0. Notations, definitions, reference frames and geometric parameters

0.1. Notations

\( \text{a} \) speed of sound
\( \text{a.c.} \) aerodynamic center
\( A = \frac{b^2}{S} \) wing aspect ratio
\( \text{b} \) wing span
\( b_f \) local fuselage width
\( B \) moment of momentum, angular momentum
\( \text{c} \) local wing chord
\( c_f \) chord wing flap
\( c_{a,e,r,t} \) chord of aileron, elevator, rudder or tab, behind hinge line
\( c_r \) chord length in the plane of symmetry
\( c_t \) chord length at wing tip
\( c_m \) mean wing chord (Eng. notation: \( \overline{c} \))
\( \overline{c} = \frac{S}{b} \) mean aerodynamic chord (Eng. notation: \( \overline{c} \))
\( c_{w,h,v} \) mean aerodynamic chord of wing, horizontal tailplane, or vertical tailplane
\( \overline{c}_{a,e,r,t} \) mean aerodynamic chord of control surfaces and tabs, behind hinge line
\( c_d = \frac{d}{\frac{1}{2} \rho V^2 c} \) drag coefficient, two-dimensional flow
\( C_D = \frac{D}{\frac{1}{2} \rho V^2 S} \) drag coefficient, three-dimensional flow
\( \text{c.g.} \) center of gravity
\( \overline{C_{h,e}} = \frac{H}{\frac{1}{2} \rho V^2 S \overline{c}} \) hinge moment coefficient of the elevator. The indices \( a, r \) or \( t \) are used to indicate the hinge moment coefficients of the aileron, the rudder or a tab respectively.
\[ C_{h\alpha} = \frac{\partial C_h}{\partial \alpha, h, v} \]
\[ C_{h\delta} = \frac{\partial C_h}{\partial \delta, a, e, r} \]
\[ C_{h\delta_\alpha} = \frac{\partial C_h}{\partial \delta_\alpha, a, e, r} \]
\[ C_{L} = \frac{L}{\frac{1}{2} \rho v^2 c} \]
\[ C_\lambda = \frac{L}{\frac{1}{2} \rho v^2 S_b} \]
\[ C_\mu = \frac{L}{\frac{1}{2} \rho v^2 S} \]

Coefficient of the additional lift distribution

Coefficient of the basic lift distribution

\[ c_\lambda = \frac{dc_\lambda}{d\alpha} \text{ derivative of the lift coefficient with respect to angle of attack, two-dimensional flow} \]

\[ C_{L\alpha} = \frac{dc_L}{d\alpha} \text{ derivative of the lift coefficient with respect to angle of attack, three-dimensional flow} \]

Coefficient of the rolling moment due to one inoperative engine

\[ C_{\mu} = \frac{\partial C_\mu}{\partial \frac{p_b}{2V}} \]
\[ C_{\lambda r} = \frac{\partial C_\lambda}{\partial \frac{r_b}{2V}} \]
\[ C_{\lambda p} = \frac{\partial C_\lambda}{\partial \beta} \]
\[ C_{L\delta_a} = \frac{\delta C_L}{\delta \delta_a} \]
\[ C_{L\delta_r} = \frac{\delta C_L}{\delta \delta_r} \]

\[ c_m = \frac{m}{\frac{1}{2} \rho V^2 c^2} \text{ coefficient of the aerodynamic moment about the Y-axis, two-dimensional flow} \]

\[ C_m = \frac{M}{\frac{1}{2} \rho V^2 Sc} \text{ coefficient of the aerodynamic moment about the Y-axis, three-dimensional flow} \]

\[ c_{m_{a.c.}} = \frac{c_m}{c_{m_0}} \text{ (}= \frac{c_m}{c_{m_0}} \text{ ) coefficient of the moment about the aerodynamic center, two-dimensional flow} \]

\[ C_{m_{a.c.}} \text{ coefficient of the moment about the aerodynamic center three-dimensional flow} \]

\[ C_{m_h} \text{ contribution of the horizontal tailplane to } C_m \]

\[ C_{m_q} = \frac{\delta C_m}{\delta \frac{ac}{V}} \]

\[ C_{m_u} = \frac{1}{\frac{1}{2} \rho V^2} \cdot \frac{\delta M}{\delta u} \]

\[ C_{m_w} \text{ contribution of the wing with fuselage and nacelles to } C_m \]

\[ C_{m_0} \text{ } C_m \text{ at } C_L = \delta_e = 0 \]

\[ C_{m_{\alpha_{\text{fix}}}} = \frac{\delta C_m}{\delta \alpha} \text{ at } \delta_e = \text{ constant} \]

\[ C_{m_{\alpha_{\text{free}}}} = \frac{\delta C_m}{\delta \alpha} \text{ at } F_e = 0 \]

\[ C_{m_{\alpha}} = \frac{\delta C_m}{\delta \frac{ac}{V}} \]
\( C_{m_{\beta}^2} = \frac{\partial C_m}{\partial \beta^2} \)

\( C_{m_\delta} = \frac{\partial C_m}{\partial \delta_e} \) elevator effectivity

\( c_n = \frac{n}{\frac{1}{4} \rho V^2 c} \) normal force coefficient, two-dimensional flow

\( C_n = \frac{N}{\frac{1}{4} \rho V^2 S_b} \) yawing moment coefficient

\( C_N = \frac{N}{\frac{1}{4} \rho V^2 S} \) normal force coefficient, three-dimensional flow

\( C_{n_e} \) yawing moment coefficient due to one inoperative engine

\( C_{n_p} = \frac{\partial C_n}{\partial \frac{p_b}{2V}} \)

\( C_{n_r} = \frac{\partial C_n}{\partial \frac{r_b}{2V}} \)

\( C_{n_\beta} = \frac{\partial C_n}{\partial \beta} \) static directional stability

\( C_{n_{\delta_a}} = \frac{\partial C_n}{\partial \delta_a} \)

\( C_{n_{\delta_r}} = \frac{\partial C_n}{\partial \delta_r} \)

\( C_{N_h} = \frac{N_h}{\frac{1}{4} \rho V^2 S_h} \) normal force coefficient of the horizontal tailplane

\( C_{N_{h_0}} = C_{N_h} \) at \( \alpha_h = \delta_e = \delta_r = 0 \)

\( C_{N_{h_{\alpha}}} = \frac{\partial C_{N_h}}{\partial \alpha_h} \)
\[ C_{N_{h\delta}} = \frac{\delta C_{N_{h\delta}}}{\delta e} \]

\[ C_{N_{h\delta t}} = \frac{\delta C_{N_{h\delta t}}}{\delta e} \]

\[ C_{N_P} = \frac{N_P}{\frac{1}{4} \rho V^2 S_p} \] coefficient of the normal force on the propeller

\[ C_{N_W} = \frac{N_w}{\frac{1}{4} \rho V^2 S} \] coefficient of the normal force on the wing with fuselage and nacelles

\[ c_r = \frac{r}{\frac{1}{4} \rho V^2_c} \] coefficient of the resultant aerodynamic force, two-dimensional flow

\[ C_R = \frac{R}{\frac{1}{4} \rho V^2 S} \] coefficient of the resultant aerodynamic force, three-dimensional flow

\[ c_t = \frac{t}{\frac{1}{4} \rho V^2_c} \] coefficient of the tangential force, two-dimensional flow

\[ C_T = \frac{T}{\frac{1}{4} \rho V^2 S} \] coefficient of the tangential force, three-dimensional flow

\[ C_{T_h} = \frac{T_h}{\frac{1}{4} \rho V^2_{h\delta t}} \] coefficient of the tangential force on the horizontal tailplane

\[ C_{T_w} = \frac{T_w}{\frac{1}{4} \rho V^2 S} \] coefficient of the tangential force on the wing with fuselage and nacelles

\[ C_X = \frac{X}{\frac{1}{4} \rho V^2 S} \]

\[ C_{X_q} = \frac{\delta C_X}{\delta \frac{q}{V}} \]

\[ C_{X_u} = \frac{1}{\frac{1}{4} \rho V S} \cdot \frac{\delta X}{\delta u} \]
\[ C_{X_0} = C_X \text{ in steady flight} \]

\[ C_{X_\alpha} = \frac{\partial C_X}{\partial \alpha} \]

\[ C_{X_\delta} = \frac{\partial C_X}{\partial \delta} \]

\[ C_Y = \frac{y}{\frac{1}{2} \rho v^2 S} \]

\[ C_{Y_p} = \frac{\partial C_Y}{\partial \frac{rb}{2V}} \]

\[ C_{Y_r} = \frac{\partial C_Y}{\partial \frac{rb}{2V}} \]

\[ C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta} \]

\[ C_{Y_\delta_a} = \frac{\partial C_Y}{\partial \delta_a} \]

\[ C_{Y_\delta_r} = \frac{\partial C_Y}{\partial \delta_r} \]

\[ C_Z = \frac{z}{\frac{1}{2} \rho v^2 S} \]

\[ C_{Z_d} = \frac{\partial C_Z}{\partial C} \]

\[ C_{Z_u} = \frac{1}{\frac{1}{2} \rho VS} \cdot \frac{\partial Z}{\partial u} \]

\[ C_{Z_0} = C_Z \text{ in steady flight} \]

\[ C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha} \]
\[ C_{Za} = \frac{\delta C_Z}{\delta V} \]

\[ C_{Z\delta_e} = \frac{\delta C_Z}{\delta \delta_e} \]

\[ C_{I} = \frac{T}{P} \]

d = \frac{1}{2} \rho V^2 c \text{ drag, two-dimensional flow}

d = \text{characteristic diameter of a jet engine}

D = \text{diameter of a propeller}

D = C_D \frac{1}{2} \rho V^2 S \text{ drag, three-dimensional flow}

\[ \frac{d}{dt} \text{ differential operator} \]

\[ \frac{d^2}{dt^2} \]

\[ D_b = \frac{d}{ds_b} = \frac{b}{V} \frac{d}{dt} \text{ dimensionless differential operator, asymmetric motions} \]

\[ D_c = \frac{d}{ds_c} = \frac{c}{V} \frac{d}{dt} \text{ dimensionless differential operator, symmetric motions} \]

e = \text{base of the natural logarithms}

\[ e = \text{distance of the center of pressure of a profile behind the leading edge} \]

F_{a,e,r} = \text{aileron, elevator or rudder control force exerted by the pilot}

h = \text{altitude}

\[ H_e = C_{he} \frac{1}{2} \rho V^2 S \text{ hinge moment about the elevator hinge line: subscripts a, e, r indicate the aileron, elevator or rudder respectively} \]

\[ H_{a,e,r} = \text{aerodynamic hinge moment} \]

\[ H_{a,e,r_f} = \text{hinge moment due to friction in the control mechanism} \]
\( H_{a,e,r_s} \) hinge moment due to a spring in the control mechanism

\( H_{a,e,r_w} \) hinge moment due to static unbalance of the control mechanism

\( \mathbf{i} \) unity vector along the X-axis

\( \mathbf{i}_{h,p} \) angle of incidence of the horizontal tail or the propeller axis, relative to the X-axis

\( I_x = \int (y^2 + z^2) \, dm \) moment of inertia about the X-axis

\( I_{x_0} \) principle moment of inertia

\( I_y = \int (z^2 + x^2) \, dm \) moment of inertia of the Y-axis

\( I_{y_0} \) principle moment of inertia

\( I_z = \int (x^2 + y^2) \, dm \) moment of inertia about the Z-axis

\( I_{z_0} \) principle moment of inertia

\( \mathbf{j} \) unity vector along the Y-axis

\( J_{xy} = \int xy \, dm \) product of inertia

\( J_{xz} = \int xz \, dm \) product of inertia

\( J_{yz} = \int yz \, dm \) product of inertia

\( k \) factor related to the variation of thrust with speed

\( \mathbf{k} \) unity vector along the Z-axis

\( k_x = \sqrt{I_x/m} \) radius of gyration about the X-axis

\( k_y = \sqrt{I_y/m} \) radius of gyration about the Y-axis

\( k_z = \sqrt{I_z/m} \) radius of gyration about the Z-axis

\( k_{xz} = \frac{J_{xz}}{m} \)
\[ K_X = \frac{k_X}{b} \text{ dimensionless radius of gyration about the X-axis} \]

\[ K_Y = \frac{k_Y}{c} \text{ dimensionless radius of gyration about the Y-axis} \]

\[ K_Z = \frac{k_Z}{b} \text{ dimensionless radius of gyration about the X-axis} \]

\[ K_{XZ} = \frac{k_{XZ}}{b^2} \text{ dimensionless product of inertia} \]

\[ l = c^2 \frac{1}{2} \rho V^2 c \text{ lift, two-dimensional flow} \]

\[ l_f \text{ fuselage length} \]

\[ l_h = x_h - x_w \text{ tail length, horizontal tail} \]

\[ l_v = x_v - x_w \text{ tail length, vertical tail} \]

\[ l_w \text{ length of the fuselage ahead of the wing} \]

\[ L = C_l \frac{1}{2} \rho V^2 S_b \text{ rolling moment about the X-axis} \]

\[ L = C_L \frac{1}{2} \rho V^2 S \text{ lift three-dimensional flow} \]

\[ m = c_m \frac{1}{2} \rho V^2 c^2 \text{ aerodynamic moment about the Y-axis, two-dimensional flow} \]

\[ m \text{ mass} \]

\[ m.a.c. \text{ mean aerodynamic chord, } \overline{c} \]

\[ m.p. \text{ manoeuvre point} \]

\[ M = C_m \frac{1}{2} \rho V^2 \overline{c} \text{ moment about the Y-axis, three-dimensional flow} \]

\[ M = \frac{a}{V} \text{ Mach number} \]

\[ M \text{ total moment} \]

\[ n = c_n \frac{1}{2} \rho V^2 c \text{ normal force, two-dimensional flow} \]

\[ n.p. \text{ neutral point} \]

\[ N = C_N \frac{1}{2} \rho V^2 S \text{ normal force, three-dimensional flow} \]

\[ N = C_n \frac{1}{2} \rho V^2 S_b \text{ yawing moment about the Z-axis} \]
\( N_h \) normal force on the horizontal tailplane
\( N_p \) normal force on the propeller
\( N_w \) normal force on the wing with fuselages and nacelles
\( O \) origin of the reference frame
\( p \) static pressure
\( p \) angular velocity about the \( X \)-axis
\( p_o \) static pressure in undisturbed flow
\( P \) period of an oscillation
\( q \) dynamic pressure
\( = \frac{1}{2} \rho V^2 \) dynamic pressure
\( q \) angular velocity about the \( Y \)-axis
\( r \) angular velocity about the \( Z \)-axis
\( r \) vector indicating a position
\( r \) resultant aerodynamic force, two-dimensional flow
\( R \) resultant aerodynamic force, three-dimensional flow
\( R \) Routh's discriminant
\( Re \) Reynolds number
\( s_{a,e,r} \) pilot's control deflection, aileron, elevator or rudder control respectively
\( s_b \) dimensionless parameter of time, asymmetric motions
\( s_c \) dimensionless parameter of time, symmetric motions
\( S \) wing area
\( S_a \) area of one aileron, behind hinge line
\( S_{e,r,t} \) area of elevator, rudder or tab respectively, behind hinge line
\( S_{h,v} \) area of horizontal or vertical tailplane
\( S_p \) area of the propeller disk
\( t \) time
\( t \) tangential force, two-dimensional flow
\( = \frac{1}{2} \rho V^2 c \) tangential force, two-dimensional flow
\[ T = C_T \frac{1}{4} \rho V^2 S \text{ tangential force, three-dimensional flow} \]

\[ T_c = \frac{T}{\rho V^2 d^2} \text{ thrust coefficient of a propeller} \]

\[ T_c' = \frac{T}{\frac{1}{4} \rho V^2 d^2} \text{ thrust coefficient of a jet engine} \]

\[ T_h \text{ tangential force on the horizontal tailplane} \]

\[ T_w \text{ tangential force on the wing with fuselage and nacelles} \]

\[ T_\frac{1}{2} \text{ time to damp to half amplitude} \]

\[ T_2 = -T_\frac{1}{2} \text{ time to double amplitude} \]

\[ \dot{u} = \frac{du}{V_o} \]

\[ u \text{ component of } V \text{ along the X-axis} \]

\[ u \text{ change in the component of } V \text{ along the X-axis} \]

\[ v \text{ component of } V \text{ along the Y-axis} \]

\[ V \text{ magnitude of the airspeed vector } V \]

\[ V \text{ velocity of the airplane's center of gravity relative to the undisturbed air or the earth} \]

\[ V_e \text{ equivalent airspeed} \]

\[ V_{m.c.} \text{ minimum control speed} \]

\[ V_{trim} \text{ trimmed airspeed, } V \text{ at which } F_e = 0 \]

\[ w \text{ component of } V \text{ along the Z-axis} \]

\[ W \text{ airplane weight} \]

\[ x \text{ x-coordinate, abscissa} \]

\[ x_{a.c.} \text{ abscissa of the a.c. of the wing} \]

\[ x_{c.g.} \text{ abscissa of the c.g.} \]

\[ x_d \text{ abscissa of the center of pressure} \]

\[ x_h \text{ abscissa of the a.c. of the horizontal tailplane} \]
$x^m_{\text{free}}$  
abscissa of the maneuver point, stick free

$x^m_{\text{fix}}$  
abscissa of the maneuver point, stick fixed

$x^n_{\text{free}}$  
abscissa of the neutral point, stick free

$x^n_{\text{fix}}$  
abscissa of the neutral point, stick fixed

$x_p$  
abscissa of the center of the propeller disk

$x_v$  
abscissa of the a.c. of the vertical tailplane

$x_w$  
abscissa of the a.c. of the wing with fuselage and nacelles

$x_o$  
abscissa of the leading edge of the m.a.c.

$X = C_X \frac{1}{2} \rho V^2 S$ component of the total aerodynamic force along the $X$-axis

$X$  
$X$-axis of the airplane body axes

$X_e$  
$X$-axis of the earth reference frame

$X_r$  
$X$-axis of the airplane reference frame

$X_s$  
$X$-axis of the stability reference frame

$y$  
y-coordinate, ordinate

$Y = C_Y \frac{1}{2} \rho V^2 S$ component of the total aerodynamic force along the $Y$-axis, lateral force

$Y$  
$Y$-axis of the airplane body axes

$Y_e$  
$Y$-axis of the earth reference frame

$Y_r$  
$Y$-axis of the airplane reference frame

$Y_s$  
$Y$-axis of the stability reference frame

$z$  
z-coordinate

$z_{\text{a.c.}}$  
z-coordinate of the a.c. of the wing

$z_{\text{c.g.}}$  
z-coordinate of the center of gravity

$z_h$  
z-coordinate of the a.c. of the horizontal tailplane

$z_p$  
z-coordinate of the center of the propeller disk

$z_v$  
z-coordinate of the a.c. of the vertical tailplane

$z_w$  
z-coordinate of the a.c. of the wing with fuselage and nacelles
$z_0$  
z-coordinate of the leading edge of the m.a.c.

$Z$  
$= C_z \frac{1}{2} \rho V^2 S$ component of the total aerodynamic force along the Z-axis

$Z_e$  
Z-axis of the airplane body axes

$Z_r$  
Z-axis of the earth reference frame

$Z_s$  
Z-axis of the airplane reference frame

$Z_{st}$  
Z-axis of the stability reference frame

$\alpha$  
angle of attack

$\alpha_0$  
angle of attack of the $X_a$-axis at $C_L = 0$

$\alpha_o$  
angle of attack in steady flight

$\alpha_{h,v,w}$  
angle of attack of the horizontal or vertical tailplane, or the wing

$\beta$  
angle of sideslip

$\gamma$  
flight path angle, angle between $V$, relative to the earth, and the horizontal plane

$\gamma_0$  
flight path angle in steady flight

$\Gamma$  
dihedral, angle between the $Y_r$-axis and the projection of the 1-chord line on the $Y_{r,oz}$-plane

$\Gamma_e$  
effective dihedral

$\delta$  
logarithmic decrement

$\delta_a = \delta_a - \delta_l$  
total aileron deflect-on

$\delta_{a_r}, \delta_{a_l}$  
deflection of the right or left aileron

$\delta_e$  
elevator deflection

$\delta_f$  
flap deflection

$\delta_r$  
rudder deflection

$\delta_s$  
spoiler deflection

$\delta_t$  
deflection of a trim tab

$\epsilon$  
wing twist or washout angle

$\epsilon$  
downwash angle, usually at the horizontal tailplane
\( \varepsilon \) angle between the \( X_r \)-axis and the principal inertial \( X \)-axis

\( \zeta \) damping ratio of an oscillation

\( \eta \) propeller efficiency

\( \eta \) imaginary part of a complex eigenvalue

\( \theta \) angle between the principal inertial \( X \)-axis and the \( X_s \)-axis of the stability reference frame

\( \theta_o \) angle of pitch, angle between the \( X_r \)-axis and the horizontal tail-plane

\( \lambda \) angle of pitch in steady flight

\[ \frac{c_t}{c_r} \] taper ratio

\( \lambda \) eigenvalue

\( \lambda_b \) dimensionless eigenvalue, asymmetric motions

\( \lambda_c \) dimensionless eigenvalue, symmetric motions

\( \Lambda \) angle of sweep, angle between the \( Y_r \)-axis and the projection of the 1/4-chord line on the \( X_r OY_r \)-plane

\( \mu_b \) relative density, asymmetric motions

\[ \frac{m}{\rho S_b} \]

\( \mu_c \) relative density, symmetric motions

\[ \frac{m}{\rho S_c} \]

\( \nu \) kinematic viscosity of the air

\( \xi \) real part of a complex eigenvalue

\( \rho \) air density

\( \sigma \) sideline angle, usually at the vertical tailplane

\( \tau \) time constant

\( \tau \) trailing edge angle of a profile

\( \varphi \) angle of roll, angle of the \( Y \)-axis and the intersection of the \( YOZ \)-plane and the horizontal plane

\( \phi \) angle of bank, angle between the \( Y \)-axis and the horizontal plane

\( x \) track angle
\( x_1 \) angle between the direction of \( \mathbf{C}_R \) and the Z-axis

\( x_2 \) angle between the neutral line and the Z-axis

\( \phi \) angle of yaw

\( \omega \) angular velocity

\( \omega \) circular frequency

\( \omega_0 \) undamped natural frequency

\( \Omega \) total angular velocity about the center of gravity
SUBSCRIPTS

a  aerodynamic
a  aileron
a.c. aerodynamic center
b  balance tab
c.g. center of gravity
e  earth reference frame
e  elevator
e  engine
f  flap
f  fuselage
f  friction
fix \( \delta^e = \text{constant} \)
free \( F^e = 0 \ (C^e_{n_e} = 0) \)
h  horizontal tailplane
i  inlet
i  initial
i  interference
n  nacelle
p  relating to the propulsive system
r  airplane reference frame
r  rudder
r  root of the wing
s  spring in the control mechanism
s  stability reference frame
t  tip
t  trim tab
u  ultimate, final
v  vertical tailplane
w  static unbalance
w  wing or wing with fuselage and nacelles
x  along the X-axis
y  along the Y-axis
z  along the Z-axis
o  initial, steady flight condition
0.2. Definitions of flight conditions and airplane configurations

Flight conditions:

Steady flight
An airplane is in steady flight, if the forces and moments acting on the airplane vary neither in direction nor in magnitude in time.

Straight flight
An airplane is in symmetric flight, if the velocity vector of any point of the airplane is parallel to the plane of symmetry.

Symmetric flight
An airplane is in symmetric flight, if the velocity vector of any point of the airplane is parallel to the plane of symmetry.

Slipping flight
An airplane is in slipping flight, if the velocity vector of the airplane's center of gravity is not parallel to the plane of symmetry of the airplane.

Airplane configurations
The airplane configuration is the ensemble of data describing the airplane's mass, the mass distribution, the external shape of the airplane and any parameter specifying the airflow around the airplane.

The elements of a full description of the airplane configuration comprise the airplane's weight or mass, center of gravity position and internal as well as external loading, undercarriage position, control surface deflections, flap angle, airbrake and spoiler deflections. A description of the engine operating condition, such as throttle position, engine speed etc. is also required.

In the Table 0.1 below some airplane configurations frequently used in American military requirements are briefly described (ref. 1.13).
<table>
<thead>
<tr>
<th>Aircraft configuration</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR (cruising flight)</td>
<td>Engine thrust or power for level flight at cruising speed, flaps in the position for cruising flight, undercarriage retracted.</td>
</tr>
<tr>
<td>L (landing)</td>
<td>Throttle closed, undercarriage down, flaps in the position for landing.</td>
</tr>
<tr>
<td>PA (powered approach)</td>
<td>Undercarriage down, flaps and airbrakes in the normal position for the powered approach, engine thrust or power for level flight at $1,15 V_{S, L}$ *) or the normal airspeed in the powered approach, if the latter is lower.</td>
</tr>
</tbody>
</table>

*) $V_{S, L}$ is the stalling speed in the airplane configuration for the landing.
### 0.3. Reference frames

<table>
<thead>
<tr>
<th>NAME</th>
<th>ORIGIN</th>
<th>X-AXIS</th>
<th>Y-AXIS</th>
<th>Z-AXIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Body axes:</td>
<td>a right handed system, OXYZ.</td>
<td>X-axis: in the plane of symmetry, in a direction fixed relative to the airplane, to be further defined on a case by case basis, positive X-axis points forward.</td>
<td>Y-axis: perpendicular to the plane of symmetry, positive Y-axis points to the right (starboard).</td>
<td>Z-axis: perpendicular to the XOY-plane, positive Z-axis points downward in normal flight.</td>
</tr>
<tr>
<td></td>
<td>airplane center of gravity.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Stability axes:</td>
<td>a particular system of body axes, OX Y Z.</td>
<td>X-axis: as in 1., but parallel to the velocity vector of the center of gravity in the steady flight preceding the disturbed motion; fixed to the airplane during the disturbed motion.</td>
<td>Y-axis: as in 1.</td>
<td>Z-axis: as in 1.</td>
</tr>
<tr>
<td></td>
<td>as in 1.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Earth axes:</td>
<td>a right handed system, OXYZ</td>
<td>X-axis: in the horizontal plane through the origin, in a fixed direction, e.g. pointing North.</td>
<td>Y-axis: in the horizontal plane through the origin, perpendicular to the X-axis, positive Y-axis is rotated 90 degrees to the right relative to the X-axis, if viewed in the direction of the positive Z-axis.</td>
<td>Z-axis: vertical, positive Z-axis points to the center of the earth.</td>
</tr>
<tr>
<td></td>
<td>fixed relative to earth, coinciding with the airplane's center of gravity at the beginning of the motion to be studied.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Airplane reference axes:</td>
<td>a left handed system, OXY T.</td>
<td>X-axis: parallel to the plane of symmetry, in a direction fixed and invariable relative to the airplane, positive X-axis points rearward.</td>
<td>Y-axis: perpendicular to the plane of symmetry positive Y-axis points to the left (port).</td>
<td>Z-axis: perpendicular to the XOY-plane positive Z-axis points upwards in normal flight.</td>
</tr>
<tr>
<td></td>
<td>in an arbitrary but fixed and invariable point of the airplane.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
0.4. Geometric parameters of the airplane

Fig. 0.1 illustrates the various parameters defining the geometry of the wing. The reference axes used in fig. 0.1 have been introduced in 0.3.

Wing area, $S$, is the area of the wing projection on the $X_r O Y_r$-plane. Often the wing is partially covered by the fuselage and the engine nacelles. The wing area is then calculated using straight line extrapolations of the wing leading and trailing edges through the fuselage and the nacelles.

The wing area can be expressed as:

$$ S = \int_{-b/2}^{+b/2} c \, dy $$

where $y$ is the coordinate in the $Y_r$-direction.

Wingspan, $b$, is the distance in $Y_r$-direction between the wing tips.

Mean aerodynamic chord, $\bar{c}$, (m.a.c.) is defined as:

$$ \bar{c} = \frac{1}{S} \int_{-b/2}^{+b/2} c^2 \, dy $$

Mean or geometric chord, $c_m$, sometimes used in the literature, is:

$$ c_m = \frac{S}{b} $$

Taper ratio, $\lambda$, is a measure of the variation in chord length along the span. It is expressed by:

$$ \lambda = \frac{c_T}{c_r} $$

Aspect ratio, $A$, is defined as:
\[ A = \frac{b^2}{S} \]

Fig. 0.1: Parameters defining the geometry of the wing.

Wing sweep, \( \Lambda \), is the angle between the \( Yu \)-axis and the projection of the 1/4-chord line on the \( X_{0r}Y_{r} \)-plane (fig. 0.1). In some cases such as delta wings, only the angle between the \( Yu \)-axis and the projection of wing leading edge on the \( X_{0r}Y_{r} \)-plane is given.

Dihedral, \( \Gamma \), of a wing is the angle between the \( Yu \)-axis and the projection of the 1/4-chord line on the \( Y_{o}Z_{r} \)-plane.

Washout or wing twist, \( \varepsilon \), expresses the variation in direction of the local wing chord relative to the direction of the chord line at the wing root.
Neither the gradient of the washout \( \frac{dz}{dy} \) nor the magnitude of wing sweep or dihedral need to be constant along the span. If necessary, these parameters are given as functions of the coordinate in span direction.

Wing profile is the shape of the cross section of the wing parallel to the plane of symmetry.

The above geometric parameters apply not only to wings, but to tailplanes as well. Definitions of the geometric parameters for the elevator, or control surfaces in general, are given in Chapter 3, see 3.2.

It proves to be difficult to define the geometry of vertical tailplanes in a way applicable to all aircraft. In particular the distinction between fuselage and vertical tailplane is often hard to make. In many instances the division between vertical tailplane and dorsal fin is also more or less arbitrary. Usually the surface of the dorsal fin is not considered to be part of the vertical tailplane.

Fig. 0.2 shows examples of definitions of vertical tailplanes, see ref. 1.1. Similar and other definitions are given in refs. 2.2 and 3.4.

Fig. 0.2: Examples of definitions of the vertical tailplane area.
(From NACA TN 775)
CHAPTER 1. Introduction to airplane flying qualities

1.1. Stability and control characteristics of an airplane

Flights mechanics can be divided into two general areas: performance and stability and control. The theory of airplane performance comprises the study of the motions of the airplane as influenced by the forces acting on it: the gravitational, the propulsive and other aerodynamic forces. It is essential that in these studies the airplane is considered point mass.

The results provide an insight in such characteristics as take-off and landing distances, rates of climb and descent, fuel consumption, airspeeds for economic cruise at a given flight altitude, maximum range and endurance, etc.

The study of stability and control characteristics, or flying qualities, or handling qualities, concerns itself with a problem less easily expressed in exact and intuitively comprehensible characteristics.

The main issue is the question whether the airplane can be brought and maintained safely and easily in desired flight conditions. To achieve this loosely defined goal, the airplane has to meet criteria, which can be expressed in a very general way as follows:

1. In any flight condition where equilibrium of the forces acting on the airplane is possible and desired, equilibrium of the moments should be possible as well. In steady flight both the forces and the moments acting on the airplane are in equilibrium.

2. The state of equilibrium of an airplane should be stable. This means, that after a small disturbance has been applied, the airplane should return to its original state of equilibrium on its own, i.e. without interference from the pilot.

3. Using his cockpit controls, the pilot should be able to obtain and to maintain any possible and desired flight condition without undue physical and mental effort. Also, he should be able to get out of undesired flight conditions sufficiently quickly and easily.

Note: Strictly speaking, stability is a characteristic of a state of equilibrium. It is customary, however, to speak of the stability or instability of the airplane, rather than of the state of equilibrium it may be in.

The ensemble of aerodynamic, structural and mass characteristics of the airplane, from which the behaviour of the airplane as regards the above criteria results, make up the airplane's flying qualities. To study these characteris-
tics, not only the translations of the center of gravity, due to the forces acting on the airplane are of importance, but the rotations about the c.g. due to the moments are of equal interest.

Apart from the forces on the airplane, the moments now have to be studied as well. No longer can the airplane be considered a point mass, it has become a real body having finite dimensions.

1.2. Static and dynamic stability

The study of the motions of an airplane after a disturbance has made it depart from an initial state of equilibrium, is a subject in dynamics. Since the airplane is free to move in three-dimensional space, it has six degrees of freedom if it is considered a rigid body: it has three translatory components of motion of the center of gravity, and three rotational components about the center of gravity. The resulting motions of the rigid airplane are described by six equations of motion derived in Chapter 8.

Suppose, the airplane has been disturbed from steady flight by a disturbance, such as an atmospheric gust or movements of passengers in the cabin. The result is a deviation from the equilibrium. The relation between the disturbance and the deviation is one of cause and effect. If the airplane returns to the original state of equilibrium after the disturbance has been taken away, the airplane — more precisely: its state of equilibrium — is said to be dynamically stable. The study of the disturbed motions of the airplane and of stability can be somewhat complicated, since several types of motions in the various degrees of freedom are possible. Fig. 1.1 shows some possible time responses of for instance the angle of attack, after a disturbance has been applied.

These complications have led already in the early days of aviation to the development of a much simpler approach to stability, based on the concept of static stability, which is defined as follows: an airplane is said to be statically stable in a given steady flight condition, if a change in attitude of the airplane relative to the airflow — i.e. a change in angle of attack or in angle of sideslip — causes a change of the aerodynamic moment about the center of gravity — i.e. a change in $C_m$ or a change in $C_n$ respectively — which tends to rotate the airplane back to its original attitude relative to the airflow.

It is shown in chapter 8 that static stability is a necessary, although not a sufficient, condition for dynamic stability. It does not tell very much about the dynamic motions of the airplane. But the study of static stability remains useful, because in many cases dynamic instability is due to static instability. In addition, some simple control
characteristics which are very important to the pilot, such as control displacements and control forces required to maintain the airplane in steady flight, are directly related to this concept of static stability. Ofter, however, a more detailed study of the non-steady flight behaviour and dynamic stability by means of the complete equations of motion will be necessary.

![Graphs](image)

**Fig.1.1: Characteristic time responses due to a temporary disturbance**

1.3. Longitudinal and lateral stability

In the following it is assumed that the airplane has a plane of symmetry. In symmetric flight, when the angles of sideslip and roll are zero, the resultant aerodynamic force and the airplane weight vector lie in the plane of symmetry. If in this situation the airplane is hit by a symmetric disturbance, such as a sudden change in angle of attack, the resultant aerodynamic force and the weight vector will remain in the plane of symmetry. As a consequence the
resulting motion of the airplane will also remain symmetric.

Conversely, as is shown in Chapter 8, an asymmetric disturbance, such as a sudden change in angle of roll or in angle of sideslip, as long as it remains small, induces only an asymmetric and no symmetric motion of the airplane to be studied quite separately from the asymmetric or lateral motions.

In real life the airplane is never perfectly symmetric. Asymmetry in the mass distribution may result from an uneven loading or fuel distribution. Aerodynamic asymmetry is caused mainly by the rotation in the slipstream of propeller driven airplanes. Usually, however, these effects are sufficiently small to be negligible. A further condition for the separation of longitudinal and lateral stability is the requirement that the symmetric and asymmetric disturbances and the resulting deviations from steady, symmetric flight remain small. Large disturbances might lead to non-negligible secondary effects, causing coupling of the longitudinal and lateral motions, as e.g. experiences with fast rolling airplanes have shown in the past. However, such situations remain the exception.

1.4. Airworthiness requirements

The requirements the airplane has to satisfy as regards handling qualities were formulated in very general terms in 1.1. These requirements are spelled out in much more detail in the airworthiness requirements drawn up by the civil and military airworthiness authorities in various countries. These airworthiness requirements refer not only to flying qualities but also to other fields such as performance and strength, stiffness and fatigue life of the structure. Airworthiness requirements provide only the minimal levels the airplane has to satisfy.

Usually, the requirements are made dependent on the purpose for which the airplane is to be used. As an example, distinction is made between the use for training, for carrying fare-paying passengers, or for experimental flights only. Refs. 1.12-1.18 give a number of sets of civil and military airworthiness requirements of several countries.

The most commonly used civil requirements are the Federal Air Regulations drawn up by the Federal Aviation Authority (FAA) of the United States of America and the Civil Air Regulations made by the Air Registration Board (ARB) of Great Britain. In the Netherlands it is the Rijksluchtvartdienst (RLD) who sets the requirements and ensures that they are observed.

The national requirements in the various countries show differences at
several points. This is the reason why the International Civil Aviation Organisation (ICAO) many years ago established international airworthiness requirements. Also in Europe, common requirements are gradually maturing. These are the Joint Airworthiness Requirements (JAR).

If the airplane satisfies the requirements with regard to the structure, the performance, the flying qualities etc., as imposed by the airworthiness requirements, it obtains a Certificate of Airworthiness (C. of A.). This C. of A. stipulates amongst other things the maximum allowable weights, the extreme permissible center of gravity positions as well as the types of flight for which the airplane may be used.

In the area of flying qualities, the requirements refer to such items as static stability, damping of oscillations, maximum and minimum allowable control forces in manoeuvres, control in take-off and landing, behaviour in stalls, control power at low airspeeds and the behaviour of the airplane with operating autopilot.

Due to the fast pace of developments in aeronautics, the airworthiness requirements have to be constantly revised and updated. This necessitates over the years a constant flow of research, in particular also with regard to the flying qualities. To an increasing extent for this purpose use is made of ground based and sometimes also of airborne flight simulators.

1.5. Airplane configurations and flight conditions, reference frames and dimensionless forces and moments

When studying stability and control characteristics, the topics of interest generally are the state of equilibrium or steady flight condition, the stability of that equilibrium and the control characteristics in a certain aircraft configuration. Definitions of some airplane configurations and flight conditions have been given in 0.2.

In the description of the aerodynamic forces and moments acting on the airplane, use will be made of the following three reference frames or coordinate systems:

1. The airplane body axes.
2. The stability system of axes.
3. The earth reference system of axes.
4. The airplane reference frame.
These four reference frames have been defined in 0.3, see page 20.

The aerodynamic forces and moments are commonly used in a dimensionless form, in order to make them primarily independent of the dynamic pressure \( q = \frac{1}{2} \rho v^2 \) and of the dimensions of the airplane. The dimensionless forces and moments in the system of body axes are:

\[
\begin{align*}
C_R &= \frac{R}{q.S} \\
C_X &= \frac{X}{q.S} \\
C_Y &= \frac{Y}{q.S} \\
C_Z &= \frac{Z}{q.S} \\
C_L &= \frac{L}{q.S.b} \\
C_m &= \frac{M}{q.S.c} \\
C_n &= \frac{N}{q.S.b}
\end{align*}
\]  \( (1-1) \)

where \( X, Y \) and \( Z \) are the components of the total aerodynamic force \( R \), \( L \) is the rolling moment (about the \( X \)-axis), \( M \) is the pitching moment (about the \( Y \)-axis) and \( N \) is the yawing moment (about the \( Z \)-axis). In addition, \( S \) is the wing area, \( b \) is the wing span and \( c \) the mean aerodynamic chord (m.a.c.). These geometry parameters have been introduced in 0.1 and 0.4.

In symmetric flight the angle of attack \( \alpha \) is defined as the angle between the negative \( X_e \) axis and the velocity vector of the center of gravity relative to undisturbed air. The more general definitions of angle of attack, \( \alpha \), and sideslip, \( \beta \), in asymmetric flight are given in Chapter 7.

When studying the equilibrium of the airplane in symmetric flight and the static stability, it is useful not to employ the components \( X \) and \( Z \) or \( R \), but rather the tangential force \( T \) and the normal force \( N \). These components \( T \) and \( N \) — in the body reference frame — have the same directions as \( X \) and \( Z \) respectively, but they are positive in the opposite sense.

\[
\begin{align*}
N &= -Z \\
T &= -X
\end{align*}
\]
In dimensionless form, see fig. 1.2:

\[
C_N = \frac{-N}{q \cdot S} = -C_Z
\]  \hspace{1cm} (1-2)

\[
C_T = \frac{T}{q \cdot S} = -C_X
\]

The aerodynamic forces acting on the airplane often become available as the results of theoretical calculations or of windtunnel measurements. Frequently they are then expressed as lift, \( L \), and drag, \( D \), the components of \( \mathbf{R} \) in the coordinate system of the wind axes. In dimensionless form they are:

\[
C_L = \frac{L}{q \cdot S}
\]

\[
C_D = \frac{D}{q \cdot S}
\]  \hspace{1cm} (1-3)

![Diagram of aerodynamic forces and moment on an airplane in symmetric flight.]

Fig. 1.2: The aerodynamic forces and the moment acting on the airplane in symmetric flight.
The relations between the components \( C_L \) and \( C_D \) and the components \( C_N \) and \( C_T \) are easily derived from fig. 1.2:

\[
C_R = \sqrt{C_L^2 + C_D^2} = \sqrt{C_N^2 + C_T^2}
\]

(1-4)

\[
C_L = C_N \cos \alpha - C_T \sin \alpha
\]

(1-5)

\[
C_D = C_N \cos \alpha + C_T \cos \alpha
\]

(1-6)

\[
C_N = C_L \cos \alpha + C_D \sin \alpha
\]

(1-7)

\[
C_T = -C_L \sin \alpha + C_D \cos \alpha
\]

(1-8)

The use of aerodynamic forces and moments in the form of dimensionless coefficients \( C_{R'} \), \( C_{N'} \), \( C_{T'} \), \( C_m \) etc. is so commonplace, that very often the normal force coefficient \( C_N \) is referred to simply as the 'normal force'. The 'pitching moment' signifies the pitching moment coefficient \( C_m \), etc.

1.6. The airplane in gliding flight at low airspeed

The dimensionless aerodynamic forces and moments of an airplane in a certain configuration depend in the first place on the attitude of the airplane relative to the undisturbed air flow. In straight and symmetric flight this attitude is characterized entirely by the angle of attack. There are, however, a number of physical phenomena that tend to make the dimensionless coefficients vary with airspeed as well.

1. Due to the **scale effect**, the coefficients may vary with Reynolds number \( (Re = \frac{V \cdot L}{\nu}) \). As far as the stability of an airplane in flight is concerned, this effect is negligible. If, however, the characteristics of an airplane - including those of stability and control - are compared with those of a scale model used for wind tunnel measurements, the scale effect may be a profound influence.

2. The **compressibility** of the air causes the coefficients to vary with Mach number \( (M = \frac{V}{a}) \). At low Mach numbers the effects of compressibility can usually be neglected. At high subsonic, transonic and supersonic speeds the coefficients may vary quite strongly with \( M \). As a consequence, in those
flight regimes the variation of the coefficients with airspeed cannot be neglected.

3. The elasticity of the airplane causes the external shape to vary with airspeed under the influence of the aerodynamic forces and moments acting on the airplane. This applies in particular to flight at high dynamic pressures \((pV^2)\) i.e. at low altitudes and high speeds. In those situations the aerodynamic coefficients will again vary with airspeed.

4. In the study of stability and control of an airplane the assumption is usually made, that during a disturbed motion the power setting of the engine(s), i.e. the setting of the cockpit controls of the engine(s) and - as the case may be - of the propeller(s) remain constant.

At a constant power setting, the effects of the slipstream and to a lesser degree the induced effects of the jet efflux, will vary with airspeed. As a consequence, the resultant dimensionless aerodynamic forces and moments acting on the entire airplane are also functions of airspeed.

In this course the aerodynamic coefficients will be assumed independent of \(Re\) and \(M\) and the airplane will be considered a rigid body. In addition, the slipstream and jet effects will not be considered. Under these simplifying assumptions the aerodynamic coefficients are independent of airspeed. This leads to a considerable simplification of the theory of airplane stability and control.

From the above discussion it might be concluded that the simplified theory to be set forth in the following, only strictly applies to rigid sailplanes. Experience shows, however, that the theory is reasonably valid also for powered airplanes in gliding flight in the entire subsonic flight regime. In addition, for many jet propelled airplanes the effects of the propulsive system on the flying qualities at subsonic speeds can largely be ignored. This is certainly not true for most propeller driven airplanes.

1.7. Outline of the course

As indicated in the previous paragraph, this course discusses the simple theory of stability and control of an airplane in gliding flight at subsonic airspeeds.

A number of general notions and definitions have been collected, together with the list of symbols and reference frames in this Chapter.
The first areas of interest are those of the *equilibrium of forces and moments*, as well as the *stability and control characteristics in steady symmetric flight*. They are discussed in the Chapter 2 to 6. Chapter 2 discusses the aerodynamic characteristics of the *wing with fuselage and nacelles*, with particular emphasis on the aerodynamic moments.

In the next Chapter, the airflow at the location of the horizontal tailplane and the characteristics of these tailplanes and of elevators are presented. In addition, the *equilibrium in steady, symmetric, straight flight* of the complete airplane is introduced. Chapter 4 discusses the *longitudinal static stability with fixed controls* and the associated *elevator trim curves*. Chapter 5 deals with *longitudinal static stability with free controls* and the closely related *elevator control force* as a function of airspeed, required in these steady flight conditions considered so far.

A different type of steady and quasi-steady flight conditions is introduced in Chapter 6. There, *longitudinal control in symmetric pull-up manoeuvres and in steady, level turning flight* is discussed. This ends the discussion of longitudinal static stability and control characteristics.

The second main subject is that of *lateral stability and control*. It is presented in Chapter 7, where by way of introduction the *aerodynamics* of the airplane in *asymmetric flight* are discussed. The Chapter continues with the *control characteristics in steady asymmetric flight conditions*.

The third principal subject is discussed in the final Chapter 8. It is *dynamic longitudinal and lateral stability*. The discussion is based on the *general equations of motion of the airplane* in non-steady flight. These equations are first derived in Chapter 8.
CHAPTER 2. The aerodynamic center and the moment about the a.c. of the wing with fuselage and nacelles

2.1. Introduction

When considering the stability and control characteristics of an airplane, the wing, the fuselage and the nacelles are usually taken as one unit. In this Chapter the characteristics of the wing alone are discussed first. Next, the influences of adding the fuselage and the nacelles are described. In this discussions the aerodynamic forces are touched upon only briefly since they are the subject of extensive discussions in other courses and textbooks. The aerodynamic wing moment, having a dominant influence on the equilibrium and the stability of the equilibrium, is discussed here in greater detail.

The aim of the following presentation is twofold:

1. to describe the aerodynamic forces and moments, supposed to be known, e.g. from wind tunnel measurements, see 2.2 to 4,
2. to calculate the aerodynamic forces and moments acting on a given wing with fuselage and nacelles, see 2.6.

Fig. 2.1: The aerodynamic forces and the moment on the wing in symmetric flight.
2.2. The aerodynamic forces and moments acting on the wing

a. The aerodynamic moment as a function of angle of attack

The forces and moment acting on a wing in symmetric flight are fully described by giving the components $C_L$ and $C_D$ or $C_N$ and $C_T$ of the total aerodynamic force $C_{R}$, together with the moment $C_m$ about a given, at this stage quite arbitrary, reference point, see fig. 2.1. The forces and the moment have been made dimensionless as indicated in 1.5, see also fig. 2.1:

$$C_N = C_L \cos \alpha + C_D \sin \alpha$$  \hspace{1cm} (1-7)

$$C_T = C_D \cos \alpha - C_L \sin \alpha$$  \hspace{1cm} (1-8)

Fig. 2.2 presents the familiar picture of $C_L$ and $C_D$ of a particular wing as functions of $\alpha$, as measured in the wind tunnel. In addition the $C_N-\alpha$ and $C_T-\alpha$ curves derived from the measured values have been given as well. From these figures it appears that the differences between $C_L$ and $C_N$ are negligible for the normal range of angles of attack. There are, however, considerable differences between $C_D$ and $C_T$. It is obvious that $C_N$ varies linearly with $\alpha$ over a large range of angles of attack, but the relation between $C_T$ and $\alpha$ is clearly nonlinear. Fig. 2.3 shows the relation between $C_N$ and $C_T$ of this wing. Due to the simplifying assumptions made in 1.8, $C_N$ and $C_T$ are functions of $\alpha$ only.

The magnitude and even the sign of $C_m$ vary strongly with the position of the reference point. The way in which $C_m$ changes with $\alpha$ for a given position of the reference point, will be shown to be of importance to the longitudinal stability. For this reason it is of interest to see how the wing moment varies with the position of the reference point. In order to compare the characteristics of different wings, it is necessary to calculate from the moments measured in the wind tunnel with respect to an arbitrary reference point - the suspension point of the model - the wing moment as it applies to a certain standard reference position. Very often this is the position of 25% of the wing m.a.c.

If for a given angle of attack the forces $C_N$ and $C_T$ as well as the moment $C_m$ about the point $(x_1z_1)$ are known, the moment about another point $(x_2z_2)$ in the plane of symmetry can be calculated, see fig. 2.4:

$$C_m(x_2z_2) = C_m(x_1z_1) + C_N \frac{x_2-x_1}{c} - C_T \frac{z_2-z_1}{c}$$  \hspace{1cm} (2-1)

where $x$ and $z$ are coordinates in the airplane reference frame.
Fig. 2.2a and b: $C_N$, $C_L$, $C_T$, $C_D$ as functions of $\alpha$ for the wing of the Fokker F-27.
(From ref. 2.21)
Fig. 2.3: $C_N$ as a function of $C_T$ for the wing of the Fokker F-27
(From ref. 2.21)

Fig. 2.4: The variation of the moment with changes in the reference point.
Fig. 2.5 shows the curves of \( C_m \) as a function of \( \alpha \) as derived from wind tunnel measurements, for various positions of the reference point relative to the m.a.c. The data apply to the same wing as used in the figs. 2.2 and 2.3. From this figure it appears that:

1. due to a shift of the reference point in the X-directions, the slope of the \( C_m-\alpha \) curve varies strongly;
2. for this wing \( C_m \) changes more or less linearly with \( \alpha \), up to the stall of the wing, in all cases where the reference point lies on the m.a.c.

Fig. 2.5 also shows the influence on the \( C_m-\alpha \) curve of a change in position of the reference point parallel to the \( Z_r \)-axis. The changes in the moment are much smaller in this case. This is easily explained by (2-1), where \( C_T \) is small relative to \( C_N \), see also fig. 2.3. It appears that when the reference point is situated far below or above the m.a.c., the moment no longer is a linear function of \( \alpha \). This is due the non-linear variation of \( C_m \) with \( \alpha \), see fig. 2.2.

The variations in the \( C_m-\alpha \) curve just discussed apply also to the corresponding curve of the entire airplane. The relevant reference point in that case is the airplane center of gravity. Often it is desirable to examine the influence on the \( C_m-\alpha \) curve of a shift in \( X \)- and \( Z \)-direction of the reference point, in order to study the stability characteristics at different center of gravity positions.

In the following a few notions are discussed, leading to a description of the aerodynamic moment as a function of angle of attack and the position of the reference point, more compact than could be achieved with a bundle of \( C_m-\alpha \) curves. The following items will be discussed:

1. the line of action of \( C_R \) and the center of pressure,
2. the aerodynamic center (a.c.) and the moment about the a.c., see the paragraphs 2.3 and 2.4.

*** b. The line of action of \( C_R \) and the center of pressure

A new way to describe the aerodynamic forces and the moment acting on the wing is obtained by giving the position and the direction of the line of action along which the resulting aerodynamic force vector \( C_R \) acts. The magnitude of the vector \( C_R \) is:

\[
C_R = \sqrt{C_L^2 + C_D^2} = \sqrt{C_N^2 + C_T^2}
\]  

(1-4)
Fig. 2.5: Moment curves for various positions of the reference point for the wing of the Fokker F-27. (From ref. 2.21)
The direction of $C_R$ is determined by the angle $\alpha_1$, see fig. 2.6, of the vector with the Z-axis:

$$\alpha_1 = \arctan \frac{C_T}{C_N}$$

The position of the line of action along which the resultant aerodynamic vector acts, is usually indicated by giving the point of intersection of the line of action of $C_R$ with the m.a.c. This point is called the center of pressure.

The position of the center of pressure $(x_1, z_1)$ relative to an arbitrary moment reference point $(x_o, z_o)$ can be derived from eq. (2-1), if $C_{m(x_1 z_1)}$ is again supposed to be known from measurements, and the coordinates $x_o, z_o$ indicate the leading edge of the m.a.c. Then:

$$C_{n(x_1 z_1)} = 0 = C_{m(x_1 z_1)} + C_N \frac{x_d - x_1}{c} - C_T \frac{z_o - z_1}{c}$$

(2-2)

If the arbitrary point $(x_1, z_1)$ coincides with $(x_o, z_o)$, it follows from (2-2), or directly from fig. 2.7:

$$0 = C_{m(x_o z_o)} + C_N \frac{x_d - x_o}{c}$$

from which the position of the center of pressure relative to the leading edge of the m.a.c. is obtained as:

$$\frac{x_d - x_o}{c} = - C_{m(x_o z_o)} C_N$$

(2-3)

If the reference point for $C_m$ lies on the m.a.c., measurements show, see e.g. fig. 2.5a, that the wing moment varies more or less linearly with angle of attack, and thus also with $C_N$. The relation between $C_{m(x_o z_o)}$, i.e. the wing moment about the leading edge of the m.a.c., and $C_N$ then is:
Fig. 2.6: Definition of the angle $X_1$

Fig. 2.7: Determination of the position of the center of pressure relative to the leading edge of the m.a.c.
\[ dC_m(x_o z_o) \]
\[ C_m(x_o z_o) = C_{m0} + \frac{dC_m(x_o z_o)}{dC_N} \cdot C_N \]  

(2-4)

where \( C_{m0} \) is the value of \( C_m \) at \( C_N = 0 \) and \( \frac{dC_m(x_o z_o)}{dC_N} \) is negative and very nearly constant. With good approximation, \( C_{m0} \) is independent of the reference point, see fig. 2.5a. Using (2-4), the position of the center of pressure is derived from (2-3) as:

\[ e = -\frac{C_{m0}}{C_N} - \frac{dC_m(x_o z_o)}{dC_N} \]  

(2-5)

Here again, \( C_{m0} \) is the moment at \( C_N = 0 \) which does not vary with the position of the reference point, see (2-4) and fig. 2.5. In the next paragraph this moment will be discussed in more detail. It will be shown that \( C_{m0} \) usually is negative, as it is in the present example of the Fokker F-27 wing. The change of \( C_m \) taken about leading edge of the m.a.c., with increasing \( C_N \) or \( \alpha \) is also negative. As a consequence, see (2-5), the center of pressure always lies behind the leading edge of the m.a.c. at positive values of \( C_N \).

Fig. 2.8 shows the coefficient of the total aerodynamic force vector for a number of angles of attack for the wind tunnel model of the Fokker F-27 wing. Fig. 2.9 gives the position of the center of pressure as a function for the same wing.

The two methods to describe the aerodynamic forces and moments on the wing introduced in this paragraph, i.e. the \( C_m - \alpha \)-curve, and the line of action of \( C_R \) and the center of pressure, both have their limitations. In the case of the \( C_m - \alpha \)-curve, both the components of the force, \( C_N \) and \( C_T \), and the moment \( C_m \) vary with angle of attack, while in the case of the line of action and the center of pressure, both \( C_R \) and the position of the center of pressure depend on \( \alpha \). As a consequence the picture of the aerodynamic forces and moments is not very clear and simple and it easily leads to mistakes. The \( C_m - \alpha \)-curve has the important advantage of remaining a correct and valid way to describe the aerodynamic moment under all circumstances.
Fig. 2.8: The magnitude of $C_R$ and the position of the line of action of $C_R$ as functions of the angle of attack of the wing of the Fokker F-27. [From ref. 2.21]

Fig. 2.9: The position of the center of pressure as a function of the angle of attack for the wing of the Fokker F-27. [From ref. 2.21]
In normal flight of 'more or less conventional' airplanes there is another way to describe the aerodynamic forces and moments, which is considerably more clear and simple. It is based on the concept of the aerodynamic center. In many situations of practical interest, the position of the aerodynamic center (a.c.) does not change with the angle of attack. In addition, the aerodynamic moment about the a.c. does not vary with angle of attack either. In the next paragraphs the aerodynamic center will be further discussed.

2.3. The a.c. and the $C_{m_{a.c.}}$: simplified discussion

The following method to describe $C_{m}$ as a function of angle of attack and the position of the reference point is based on the fact that wings having an aspect ratio which is 'not too low', over an appreciable range of angles of attack possess a very nearly invariant position of the moment reference point, about which $C_{m}$ is exactly constant. This position of the reference point is called the aerodynamic center (a.c.). The aerodynamic moment about the a.c. is indicated as $C_{m_{a.c.}}$. It is taken to be constant by definition. In this paragraph 2.3 the simplifying assumption is made that, in addition to the exactly constant $C_{m_{a.c.}}$, also the position of the a.c. does not vary with angle of attack.

*** a. The first metacenter, the neutral line and the neutral point

Fig. 2.10 shows the forces and moment about an arbitrary reference point, at two adjacent angles of attack, $\alpha$ and $\alpha + \Delta \alpha$. The position of the line of action of $\Delta C_R$ follows from the construction in fig. 2.11, where the resulting aerodynamic forces corresponding to $\alpha$ and $\alpha + \Delta \alpha$ have been placed along their respective lines of action.

For any point on the lines of action of $C_R$ and $C_R + \Delta C_R$ it is true that the moment about that point at the respective angle of attack, i.e. $\alpha$ and $\alpha + \Delta \alpha$, equals zero. Consequently, the moment about the point of intersection of the two lines, $M_1$, equals zero both at $\alpha$ and $\alpha + \Delta \alpha$. In the limiting case, where $\Delta \alpha \to 0$, this point of intersection of the two adjacent lines of action is called the first metacenter, $M_1$. In naval architecture the same name is used for the similar concept.
Fig. 2.10: The changes in the forces and the moments about an arbitrary point with a change in the angle of attack.

Fig. 2.11: The line of action of the change in the resultant aerodynamic force.
In the first metacenter the following two conditions hold:

\[
C_m = 0
\]

\[
\frac{dC_m}{d\alpha} = 0
\]  \hspace{1cm} (2-6)

For very small changes in the angle of attack, in any point of the line of action the change in aerodynamic force, \(dC_R\), the change of the moment equals zero. Consequently, the analytical expression for all points on the line of action of \(dC_R\) is:

\[
\frac{dC_m}{d\alpha} = 0
\]  \hspace{1cm} (2-7)

The line of action of \(dC_R\) is called the neutral line. The first metacenter turns out to be the point on the neutral line where \(C_m = 0\). This means that it is the point of intersection of the line of action of \(C_R\) and the neutral line.

If the aerodynamic forces and the moment about the leading edge of the m.a.c. \((x_o, z_o)\) are given, the expression for the neutral line can be extracted from (2-1) and (2-7):

\[
\frac{dC_m(x_o, z_o)}{d\alpha} + \frac{dC_N}{d\alpha} \cdot \frac{x-x_o}{c} - \frac{dC_T}{d\alpha} \cdot \frac{z-z_o}{c} = 0
\]  \hspace{1cm} (2-8)

The intersection of the neutral line with the m.a.c., where \(z = z_o\), is sometimes called the neutral point of the wing. The distance of this neutral point \((x_n, z_o)\) behind the leading edge of the m.a.c. follows from (2-8):

\[
\frac{x_n-x_o}{c} = - \frac{dC_m(x_o, z_o)}{dC_N}
\]  \hspace{1cm} (2-9)
whereas:

\[ z_n = z_0 \]

The direction of the neutral line follows from (see fig. 2.11):

\[ x_2 = \arctan \frac{dC_{m}}{dC_n} \]  \hspace{1cm} (2-10)

***

*** b. The aerodynamic center

Next, the change in the resulting aerodynamic force \( \Delta C_R \) is considered in two angles of attack, \( \alpha_1 = \alpha \) and \( \alpha_2 = \alpha + \Delta \alpha \). At both \( \alpha_1 \) and \( \alpha_2 \) there is a neutral line, as discussed earlier. The point where these two neutral lines intersect, as can be seen from fig. 2.12, follows from the conditions:

\[ \left( \frac{dC}{da} \right)_{\alpha_1} = 0 \]

\[ \left( \frac{dC}{da} \right)_{\alpha_2} = 0 \]

In the limiting case where \( \Delta \alpha \to 0 \), the point of intersection follows from:

\[ \frac{dC_m}{da} = 0 \]

\[ \frac{d^2C_m}{da^2} = 0 \]  \hspace{1cm} (2-11)
The point of intersection of two neutral lines belonging to two adjacent angles of attack is called the second metacenter, \( M_2 \), in naval architecture. In aeronautical engineering this point is referred to as the aerodynamic center or a.c.

In theoretical aerodynamics it is possible to prove that thin profiles in a potential flow have an a.c. which is exactly invariant with the angle of attack. According to this 'thin airfoil theory', the a.c. of all thin profiles is located at 25% of the profile chord.

In experimental aerodynamics, measurements on two-dimensional wing profiles show that in the range of angles of attack where no flow separation occurs, there is indeed an a.c. position which is very nearly independent of the angle of attack. Also in three-dimensional flow the a.c. of a complete wing turns out to have a constant position over a wide range of angles of attack in many cases - i.e. at aspect ratios not too small and sweep angles not too large.

***

The coordinates of the a.c. are \( x_{a.c.} \) and \( z_{a.c.} \). The position of the a.c. with respect to the leading edge of the m.a.c. follows from the conditions (2-11):

\[
\frac{dC_m}{d\alpha} = 0
\]
\[
\frac{dC_{m(x,z)}}{dx} + \frac{dC_N}{dx} \cdot \frac{x_{a.c.} - x_o}{c} + \frac{dC_T}{dx} \cdot \frac{z_{a.c.} - z_o}{c} = 0
\]

or:

\[
\frac{d^2C_{m(x,z)}}{dx^2} + \frac{d^2C_N}{dx^2} \cdot \frac{x_{a.c.} - x_o}{c} + \frac{d^2C_T}{dx^2} \cdot \frac{z_{a.c.} - z_o}{c} = 0
\]

(2-12)

\[
\frac{d^2C_{m(a,c)}}{dx^2} = 0
\]

Here \(x_{a.c.}\) and \(z_{a.c.}\) are assumed to be independent of \(x\). The two coordinates \(x_{a.c.}\) and \(z_{a.c.}\) follow from (2-12). The aerodynamic moment about the a.c., \(C_{m(a,c)}\), is calculated by substituting the coordinates \(x_{a.c.}\) and \(z_{a.c.}\) derived from (2-12) into (2-1). If the moment \(C_{m(x,z)}\) is again assumed to be given about the leading edge of the m.a.c., the resulting expression for \(C_{m(a,c)}\) is:

\[
C_{m(a,c)} = C_{m(x,z)} + C_{N} \cdot \frac{x_{a.c.} - x_o}{c} + C_{T} \cdot \frac{z_{a.c.} - z_o}{c}
\]

(2-13)

If, as a result of measurements, \(C_{N}\), \(C_{T}\) and \(C_{m(x,z)}\) are known as functions of the angle of attack, it is possible in principle to obtain the coordinates \(x_{a.c.}\) and \(z_{a.c.}\) using the equations (2-12). To this end the experimental data of \(C_{N}\), \(C_{T}\) and \(C_{m}\) for a given angle of attack have to be differentiated twice. It is common knowledge that it is rather difficult, if not quite impossible, to obtain sufficiently reliable second derivatives from experimental data.

According to the foregoing, however, the position of the a.c. can, as a first approximation, be considered to be independent of the angle of attack.

Therefore, the determination of the position of the a.c. is usually performed by using the expressions for the neutral line (2-7), i.e. the first of the equations (2-12), for two adjacent angles of attack. The coordinates \(x_{a.c.}\) and \(z_{a.c.}\) follow from the point of intersection of these two neutral lines.

The model of the aerodynamic forces and moments acting on a wing can be
described in a very simple way by using the invariable a.c. As the neutral line, i.e. the line of action of the change in aerodynamic force, passes through the a.c. at any angle of attack, there is no change in the aerodynamic moment about the a.c. With $\alpha$. By definition the moment $C^m_{a.c.}$ is constant, independent of angle of attack. The model of the aerodynamic forces and moments acting on the wing thus consists of the constant moment $C^m_{a.c.}$ about the invariable a.c. of the wing, and the total aerodynamic force $C_R$ which passes through the a.c. at all angles of attack (see fig. 2.13).

![Diagram](image)

Fig. 2.13: The aerodynamic forces and the moment, using the a.c. as the reference point.

c. Replacing the a.c. by the neutral point

We have seen that the aerodynamic center is the moment reference point about which $C^m$ is exactly constant. It turns out that usually a range of angles of attack exists where the position of the a.c. is more or less constant. Furthermore, it appears that this constant position is usually located very close to the wing m.a.c. In studies of stability and control characteristics it is quite common to assume that the a.c. is located exactly on the m.a.c. This implies that the a.c. then coincides with the neutral point discussed in 2.3a. The following shows that this simplification is usually quite acceptable.
The fact that \( C_{m \cdot a \cdot c.} \) is constant by definition means:

\[
\frac{dC_{m \cdot a \cdot c.}}{d\alpha} = 0
\]

if \( x_o \cdot z_o \) is again the position of the leading edge of the m.a.c. and assuming that, from measurements, \( C_m^{(x_o \cdot z_o)} \) is known as a function of \( \alpha \), (2-1) implies that:

\[
\frac{dC_{m \cdot a \cdot c.}}{d\alpha} = \frac{dC_m^{(x_o \cdot z_o)}}{d\alpha} + \frac{dC_N}{d\alpha} \cdot \frac{x_{a \cdot c.} - x_o}{c} - \frac{dC_T}{d\alpha} \cdot \frac{z_{a \cdot c.} - z_o}{c} = 0 \quad (2-14)
\]

In this expression \( \frac{dC_T}{d\alpha} \) is relatively small compared to \( \frac{dC_N}{d\alpha} \).

In addition, the distance from the a.c. to the m.a.c. is small (a few percents of \( c \) at most). Then it can safely be put that:

\[
z_{a \cdot c.} = z_o
\]

But then the \( x \)-coordinate of the a.c., \( \frac{x_{a \cdot c.}}{c} \), follows immediately from (2-14):

\[
\frac{x_{a \cdot c.} - x_o}{c} = -\frac{dC_m^{(x_o \cdot z_o)}}{dC_N} \quad (2-15)
\]

This equation (2-15) agrees entirely with the expression for the position of the neutral point (2-9), meaning that if the contribution of the tangential force is neglected in (2-14), the a.c. will coincide with the neutral point.

The magnitude of \( C_{m \cdot a \cdot c.} \) in this situation, where \( z_{a \cdot c.} = z_o \), follows, with (2-1), from:

\[
C_{m \cdot a \cdot c.} = C_m^{(x_o \cdot z_o)} + C_N \cdot \frac{x_{a \cdot c.} - x_o}{c} \quad (2-16)
\]

Apparently:

\[
C_{m \cdot a \cdot c.} = C_m^{(x_o \cdot z_o)} = C_m^{(z_o \cdot z_o)} \quad C_N = 0
\]
As has been shown, the constant position of the a.c. and the magnitude of the moment about the leading edge of the m.a.c. can be obtained simply from experimental data by first calculating the moment from the slope of the \( C_m - C_N \) curve. According to (2-17), \( C_{m_{a.c.}} \) is equal to the moment coefficient \( C_m \) at \( C_N = 0 \). In many cases in literature this simplified an approximated position of a.c. is used, see for instance 2.6.

Finally, if \( \frac{x_{a.c.}}{c} \) and \( C_{m_{a.c.}} \) are known, the moment coefficient about an arbitrary reference point near the m.a.c. is given by:

\[
C_m(x) = C_{m_{a.c.}} + C_N \cdot \frac{x - x_{a.c.}}{c} \frac{dx}{c} \quad (2-15)
\]

where:

\[
\frac{x_{a.c.} - x_o}{c} = \frac{dC_m(x_o z_o)}{dC_N} \]

and:

\[
C_{m_{a.c.}} = C_m(x_o z_o) \quad C_N = 0 \quad (2-17)
\]

Note: The definition of the a.c. and the \( C_{m_{a.c.}} \) if both vary with angle of attack.

The description of \( C_m \) using the a.c. and the \( C_{m_{a.c.}} \) derives its practical value from the fact \( C_{m_{a.c.}} \) is exactly constant and the position of the a.c. is, to a sufficient approximation, independent of the angle of attack over the range of angles of attack occurring in normal flight. According to various theoretical calculation methods, both the position of the a.c. and the magnitude of the \( C_{m_{a.c.}} \) are exactly constant. However, for wings with a low aspect ratio and/or a large sweep angle the variations in the position of the a.c. and possibly the change in \( C_{m_{a.c.}} \) with angle of attack may not always be negligible, not even at those angles of attack occurring in normal flight.

In order to enlarge the applicability of the a.c. and the \( C_{m_{a.c.}} \), a more
general definition, more complete than that given in 2.3, is needed. In this case the position of the a.c. does not have to be constant anymore.

***

*** a. The complete definition of the a.c. and the $C_{m_{a.c.}}$

Distinction has to be made between the situation at $\alpha_{C_L=0}$ and at any other angle of attack. At $\alpha_{C_L=0}$ the a.c. and the $C_{m_{a.c.}}$ are defined as in 2.3 using (2-12) and (2-13). This means that at $\alpha_{C_L=0}$ the a.c. is the second metacenter. It should be remembered, that with good approximation:

$$\alpha_{C_L=0} = \alpha_{C_N=0}$$

At all other angles of attack, $C_{m_{a.c.}}$ is defined to be constant and equal to the value at $\alpha_{C_L=0}$. The position of the a.c. at an arbitrary angle of attack is derived from the two conditions:

$$C_m = C_{m_{a.c.}}$$

$$\frac{dC_m}{d\alpha} = 0$$

The a.c. at angles of attack different from $\alpha_{C_L=0}$ is thus defined as a 'modified first metacenter'.

The calculation of the position of the a.c. and of $C_{m_{a.c.}}$ from given forces and moments about the foremost point of the m.a.c. ($x_z$) proceeds as follows:

a. Determination of $x_{a.c.}$ and $z_{a.c.}$ at $\alpha_{C_L=0}$ from eqs. (2-12); from the results $C_{m_{a.c.}}$ is obtained with (2-13) as discussed in 2.3.

b. Determination of $x_{a.c.}$ and $z_{a.c.}$ for all other angles of attack from the conditions:

$$C_m = C_{m_{a.c.}} = C_m(x_o z_o) + C_N \frac{x_{a.c.} - x_o}{c} - C_T \frac{z_{a.c.} - z_o}{c}$$  \hspace{1cm} (2-18)
\[
\frac{dC_m}{d\alpha} = \frac{dC_{m,a.c.}}{d\alpha} = 0 = \frac{dC_m(x_o z_o)}{d\alpha} + \frac{dC_N}{d\alpha} \cdot \frac{x_{a.c.} - x_o}{c} - \frac{dC_T}{d\alpha} \cdot \frac{z_{a.c.} - z_o}{c} + d \frac{x_{a.c.}}{c} - C_N \frac{x_{a.c.}}{c} - C_T \frac{z_{a.c.}}{c}
\]

(2-19)

In (2-19) the last two terms are neglected relative to the other terms. It will be shown later on that the a.c. loses its practical value if the variation of the position of the a.c. with \( \alpha \) is so large that this approximation is no longer permissible.

The NASA, and before that its predecessor, the NACA uses a slightly different definition of the a.c. and the \( C_{m,a.c.} \). At \( \alpha_{CL}=0 \) both the a.c. and the \( C_{m,a.c.} \) are defined exactly as described in this paragraph. At other angles of attack, however, the position of the a.c. is kept constant by definition. The constant coordinates are obtained from (2-18) and (2-19) at \( \alpha_{CL}=0^\circ \).

In principle \( C_{m,a.c.} \) can now very well with \( \alpha \). From (2-18) \( C_{m,a.c.} \) can be calculated for any angle of attack.

The advantage of the definitions of \( C_{m,a.c.} \) and the a.c. as given in this paragraph as compared to the one used by the NASA, lies in the fact that now the a.c. can be considered to be the point through which act both the resultant aerodynamic force - with the inclusion of the constant moment \( C_{m,a.c.} \) - and the change of the aerodynamic force with \( \alpha \), as can be seen from the eqs. (2-18) and (2-19).

***

b. The approximate position of the a.c., using the complete definition

As in 2.3c, the position of the a.c. and the magnitude of \( C_{m,a.c.} \) at \( \alpha_{CL}=0 \) may be approximated by assuming the a.c. to lie on the m.a.c., so \( z_{a.c.} = z_o \). As a consequence only \( \frac{x_{a.c.}}{c} \) and \( C_{m,a.c.} \) need to be determined. From eq. (2-15) follows \( \alpha_{CL}=0 = \alpha_{CN}=0^\circ \).
\[
\frac{x_{a.c.} - x_o}{c} = - \left( \frac{dC_m(x_o z_o)}{dC_N} \right)_{\alpha_C=0}
\]

\[
\frac{z_{a.c.} - z_o}{c} = 0
\]

and, see (2-17):

\[
C_{m_{a.c.}} = (C_{m(x_o z_o)})_{C_N=0} = C_{m_o}
\]

where \(C_{m(x_o z_o)}\) again is the aerodynamic moment relative to the foremost point of the m.a.c.

If \(C_{m_{a.c.}}\) is kept constant by definition when going to other angles of attack, the approximated position of the a.c. on the m.a.c. can be obtained from (2-18) in either of two ways.

a) From (2-18) follows for the \(C_m\) about the \(C_{m_{a.c.}}\):

\[
C_m = C_{m_{a.c.}} + C_{m(x_o z_o)} \frac{x_{a.c.} - x_o}{c}
\]

(2-20)

where the constant \(C_{m_{a.c.}}\) has already been obtained at \(\alpha = 0^\circ\).

From (2-20) follows:

\[
\frac{x_{a.c.} - x_o}{c} = - \frac{1}{C_{N_{m(x_o z_o) - m_{a.c.}}}} \cdot (C_{m(x_o z_o)} - C_{m_{a.c.}})
\]

(2-21)

Equation (2-21) can be compared to eq. (2-5) - derived in 2.2 - for the position of the center of pressure. The position of the a.c. thus approximated may be
considered to be the point on the m.a.c. through which the force $C_N$ acts, if the measured moment $C_{m(a,c.)}^{(x_0\,z_0)}$ is reduced by $C_{m(a,c.)}^{m(a,c.)}$.

b) Considering $x_{a,c.}$ again as variable, differentiating (2-20) with respect to $C_N$ gives:

$$\frac{x_{a,c.} - x_0}{C} = -\frac{dC_{m(x_0\,z_0)}}{dC_N} - C_N \frac{d\frac{x_{a,c.}}{C}}{dC_N}$$

If the variation of the a.c. position with $C_N$ is not too large, the last term in the above equation is again neglected:

$$\frac{x_{a,c.} - x_0}{C} = -\frac{dC_{m(x_0\,z_0)}}{dC_N} \quad (2-22)$$

This appears to be the expression for the position of the neutral point, see (2-9).

If large shifts in the position of the a.c. occur with changes in angle of attack, the term containing $\frac{d\frac{x_{a,c.}}{C}}{dC_N}$ in (2-22) cannot be neglected. In such a situation the position of the a.c. approximated by (2-21) still gives the point on the m.a.c. through which the normal $C_N$ acts — under the addition of the constant moment $C_{m(a,c.)}^{m(a,c.)}$ — but this point is no longer also the neutral point, i.e. the point through which acts the change of the normal force $dC_N$.

The foregoing is depicted in fig. 2.14. It can be seen that this simplified use of the complete definition - assuming the a.c. to lie on the m.a.c. — apparently does not give a unique position of the a.c. In general it can be stated that the a.c. can only be approximated by the neutral point, if both $C_m$ and $C_N$ vary approximately linearly with $\alpha$.

The above simplified use of the complete definition of a variable position of the a.c. and a constant $C_{m(a,c.)}^{m(a,c.)}$ is not suitable to determine with great accuracy small shifts in the a.c. at small angles of attack, as the approximation $z_{a,c.} = z_0$ is too coarse for such an application.
\[\tan \psi_1 = \frac{x_{a.c} \cdot x_0}{c} = \frac{1}{C_N} (C_m(x_0z_0) - C_{m_{a.c.}}) \quad (2-21)\]

\[\tan \psi_2 = \frac{x_{a.c} \cdot x_0}{c} = \frac{dC_m(x_0z_0)}{dC_N} \quad \quad (2-22)\]

Fig. 2.14: The approximated position of the a.c. according to the complete definition if the slope of the moment curve is not constant.

Large changes in the slope of the \(C_m - \alpha\) and \(C_N - \alpha\) curves occur at the stall of the wing. Swept wings may experience important changes in the slope of the \(C_m - \alpha\) curves already at relatively small angles of attack. This phenomenon, called 'pitch-up' is discussed further in 2.6. The \(C_m - \alpha\) curves of delta wings may be non-linear over the entire range of angles of attack.

Although the complete definitions of the a.c. and the \(C_{m_{a.c.}}\) remain valid in such situations, the a.c. as a reference point for the aerodynamic moment largely loses its practical value to describe experimentally obtained moments. The \(x\) and \(z\)-coordinates of the a.c. become strongly dependent on angle of attack. An accurate determination of \(x_{a.c.}\) and \(z_{a.c.}\) from measurements having a limited accuracy, may turn out to be unfeasible in such situations. For design
purposes — to the extent that a prediction of such characteristics can be made at that stage — one often reverts to the results of measurements on similar wings. Sometimes such results are presented in the form of the position of the center of pressure and the lift coefficient as functions of \( \alpha \). The center of pressure may be used in such situations, because it is easier to derive the center of pressure with a reasonable accuracy from measured data, rather than the aerodynamic center, see 2.2.

In the assessment of the characteristics of a wing at large angles of attack the curves of \( C_m \) versus \( \alpha \) are commonly used, rather than the a.c. and \( C_{\text{m.a.c.}} \). When studying the (static) stability, see also 2.7, the change in the slope of the \( C_m - \alpha \) curve is important. This change characterizes the variations in the position of the neutral point with \( \alpha \). As previously discussed, the neutral point is the position on the m.a.c. through which acts the change in total aerodynamic force due a change in \( \alpha \). In the presentation of test-results, often the position of the neutral point is given as a function of \( \alpha \). Unfortunately, however, in literature, this neutral point is often called the aerodynamic center. As was shown previously, the neutral point only provides an indication of the position of the true aerodynamic center at those angles of attack where \( C_m \) and \( C_N \) vary approximately linearly with \( \alpha \).

At all other angles of attack the position of the neutral point is merely a measure of the slope of the \( C_m \) curve for moment reference points lying on or near the m.a.c.

Finally it should be noted that confusion is still further increased by the fact that sometimes in literature the neutral point is called the center of pressure.

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Table 2.1 on page 100 summarizes the principal notions discussed in the previous paragraphs.

2.5. The importance of the a.c. and the \( C_{\text{m.a.c.}} \) to static stability

If the \( C_{\text{m.a.c.}} \) and the position of the a.c. \( (x_{\text{a.c.}}, z_{\text{a.c.}}) \) of a wing are known, the moment about an arbitrary point \( (x, z) \) is given by, see fig. 2.15:

\[
C_m(x, z) = C_{\text{m.a.c.}} + C_N \frac{x - x_{\text{a.c.}}}{c} - C_T \frac{z - z_{\text{a.c.}}}{c} \tag{2-23}
\]
For qualitative discussions it usually is permissible to simplify (2-23), as was already indicated in 2.3c, by neglecting the contribution of the tangential force:

\[ C_m = C_{m_{a.c.}} + C_N \frac{x - x_{a.c.}}{c} \]  

(2-24)

This simplified expression \( C_m \) will be used many times in the following discussions.

Fig. 2.15: The moment about an arbitrary point \((x,z)\) if the position of the a.c. and the \( C_{m_{a.c.}} \) are known.

Suppose a model of a wing has been mounted in an airstream, so that the model is free to rotate about an axis through the moment reference point and perpendicular to the plane of symmetry. Furthermore, it is supposed that the center of gravity of the model coincides with this axis of rotation. In such a situation the attitude of the model relative to the airflow is determined only by the aerodynamic moment about the axis of rotation. Equilibrium exists if \( C_m = 0 \).

It is now assumed that due to a disturbance, a deviation in model attitude or angle of attack from the equilibrium situation occurs. If such a deviation causes an aerodynamic moment acting against the change in angle of attack, the original equilibrium situation was a stable one. This means that an increase in angle of attack must give rise to negative, nose-down, change in aerodynamic
moment. The condition for stability then is:

\[ \frac{dC_m}{d\alpha} < 0 \text{ at } C_m = 0 \]  \hspace{1cm} (2-25)

It is easy to see that if \( \frac{dC_m}{d\alpha} > 0 \) at \( C_m = 0 \), the equilibrium is unstable. If a change in angle of attack causes no change in the aerodynamic moment, the equilibrium is said to be indifferent or neutrally stable. In that case \( C_m = 0 \) at all angles of attack and there will be equilibrium at any value of \( \alpha \).

The importance of the position of the moment reference point, relative to the a.c. of the wing, is illustrated by considering the equilibrium of the moment and the stability of the equilibrium of the wing for different positions of the moment reference point. From (2-24) follows for the equilibrium (\( C_m = 0 \)):

\[ C_m = C_m^{a.c.} + C_N \frac{x - x_{a.c.}}{c} = 0 \]  \hspace{1cm} (2-26)

and for the change of the moment with angle of attack:

\[ \frac{dC_m}{d\alpha} = \frac{dC_N}{d\alpha} \cdot \frac{x - x_{a.c.}}{c} \]  \hspace{1cm} (2-27)

where \( \frac{dC_N}{d\alpha} \) is in the range of linear variation of \( C_N \) with \( \alpha \) is always positive.

Three cases are considered:

a. Moment reference point ahead of the a.c. \( \frac{x - x_{a.c.}}{c} < 0 \)

According to (2-26), equilibrium is possible for either positive or negative values of \( C_N \) depending on the magnitude and the sign of \( C_m^{a.c.} \), see figs. 2.16 and 2.17. From (2-27) follows that \( \frac{dC_m}{d\alpha} < 0 \). This means that the equilibrium is stable, as can be seen from the figs. 2.16 and 2.17.
b. Moment reference point in the a.c. \[ \frac{x - x_{a.c.}}{c} = 0 \]

In this situation, equilibrium is possible only if \( C_{m_{a.c.}} = 0 \), see (2-26). If this is the case, there is equilibrium at any value of \( C_N \), or \( a \) (fig. 2.18). The equilibrium is indifferent \( \left( \frac{dC_m}{da} = 0 \right) \). If \( C_{m_{a.c.}} \neq 0 \), no equilibrium situation is possible and as a consequence there can be no discussion of stability either (fig. 2.19).

c. Moment reference behind the a.c. \[ \frac{x - x_{a.c.}}{c} > 0 \]

If \( C_{m_{a.c.}} < 0 \), equilibrium will exist at a certain positive \( C_N \) and for \( C_{m_{a.c.}} > 0 \) at a negative value of \( C_N \) (fig. 2.20 and 2.21). In both cases the equilibrium is unstable, \( \frac{dC_m}{da} > 0 \).

In summary it can be said that in the general case, i.e. where \( C_{m_{a.c.}} \neq 0 \), a condition of equilibrium \( (C_m = 0) \) is possible for any position of the moment reference point ahead or behind the a.c. The value of \( a \) or \( C_N \) for which \( C_m = 0 \), depends also on \( C_{m_{a.c.}} \), see (2-26), but \( C_m \) itself has no direct influence on the stability of the equilibrium. At a given \( C_{N_a} \), the stability is determined only by the position of the moment reference point relative to the a.c., see (2-27).

If the moment reference point is situated ahead of the a.c. the model is always stable, \( \frac{dC_m}{da} < 0 \). If the moment reference point moves back, the model becomes less stable, \( \frac{dC_m}{da} \) becomes less negative.

If the moment reference point coincides with the a.c., the slope of the moment curve is zero. In that case equilibrium is possible only if \( C_{m_{a.c.}} = 0 \). This equilibrium is said to be indifferent or neutrally stable.

The equilibrium is always unstable if the moment reference point lies behind the a.c., \( \frac{dC_m}{da} > 0 \).
Fig. 2.16: Equilibrium and stability of a wing, reference point ahead of the a.c., \( C_{m_{a.c.}} \) negative.
Fig. 2.17: Equilibrium and stability of a wing; reference point ahead of the a.c., $C_{m_{a.c.}}$ positive.
Fig. 2.18: Equilibrium and stability of a wing; reference point in the a.c., $C_{m_{a,c}} = 0$. 

- Equilibrium at any $\alpha$.
- $C_{m_{a,c}} = 0, \frac{dC_{m}}{dC_N} = 0$.
- Neutrally stable equilibrium.
Fig. 2.19: Equilibrium and stability of a wing: reference point in the a.c., $C_{m_a.c.}$ negative.
Fig 2.20: Equilibrium and stability of a wing: reference point behind the a.c., $C_{m_{a.c.}}$ negative.

$C_{m_{a.c.}} < 0; \frac{dC_m}{dC_N} > 0$

unstable equilibrium

$\Delta C_m > 0$
Fig. 2.21: Equilibrium and stability of a wing: reference point behind a.c., \( C_{m,a,c.} \) positive.
In the foregoing the influence of a change in the x-coordinate of the moment reference point (the rotation axis) was considered. In quite a similar way, the influence of a change in the position of the a.c. at a constant position of the moment reference point can be studied. Such a shift in a.c.-position may be caused for instance by the influence of the fuselage on \( C_m \) as will be discussed in 2.8. From (2-27) it follows, that a forward shift of the a.c. \( x_{a.c.} \) decreases always has a destabilizing influence and a rearward shift of the a.c. a stabilizing effect.

In chapter 4 it is shown, see 4.2, that the discussion given here for a wing model, in a more extended form applies to the complete airplane as well. For an airplane in flight, all rotations are considered about an axis passing through the center of gravity. The center of gravity of an airplane has in this respect the same role as the axis of rotation of the previous wing model. The slope of the moment curve \( (C_m-a) \), where the center of gravity is the moment reference point, indicates the (static) stability of the airplane.

2.6. The calculation of the position of the a.c. and the \( C_{m a.c.} \) of a wing

In the foregoing paragraphs -- in 2.2 - 2.4 -- it was shown, how the aerodynamic moment acting on a wing can be described. Paragraph 2.5 discussed the importance of the concepts thus introduced, to the study of airplane stability. As mentioned already in 2.1, this paragraph describes methods to calculate the aerodynamic moment for a wing of given shape and dimensions.

The calculation methods to be discussed are closely related to the way in which the lift distribution over the wing is determined. Before going into these matters, a brief description is given of the model of aerodynamic forces and moments to be used in the following.

The two-dimensional lift coefficient \( c_L = \frac{L}{\frac{1}{2} \rho V^2 c} \) is always assumed to be equal to \( c_n \), where \( c_n \) is the two-dimensional normal force coefficient. It is also assumed that the contribution of the tangential force coefficient \( c_t \) to the moment coefficient \( c_m \) is negligible.

In this way a model of the forces and moments results, where the local force acting at a certain spanwise location passes through the local a.c., while the local \( c_{m a.c.} \) acts about this same a.c. With this formulation, the following discussion uses precisely the simplified model of the aerodynamic forces and moments developed in 2.3c for the complete wing.
Fig. 2.22: The basic, the additional and the total lift distribution of a wing having negative twist.
It is assumed to be known, that the lift distribution across the span of a wing can be divided into two parts: a basic lift distribution \( c_{L}^{b}(y) \), which is the lift distribution at \( C_{L}=0 \) and depends on the shape of the wing but does not vary with \( \alpha \), and an additional lift distribution \( c_{L}^{a}(y) \), depending on the wing shape as well, but varying with the angle of attack, see fig. 2.22.

The a.c. of the wing has been defined in the foregoing, see 2.3c, as the point through which, with varying angle of attack, passes the change in the aerodynamic force. As a consequence, the a.c. of the wing is the point through which acts the resultant force of the additional lift distribution.

The \( C_{m}^{\text{a.c.}} \) is the total moment at \( C_{L}=0 \). This implies that \( C_{m}^{\text{a.c.}} \) caused by the torque resulting from the basic lift distribution together with the \( c_{m}^{\text{a.c.}} \)'s of the wing profiles.

If the characteristics of the wing profiles as well as the shape of the wing are known, it is possible to calculate the basic and the additional lift distribution, using methods available from literature. The way to derive from these data the position of the a.c. and the magnitude of \( C_{m}^{\text{a.c.}} \) is discussed in the following.

The lift acting on a strip of the wing having a width \( dy \) is:

\[
dL = dL_{a} + dL_{b} = (c_{x}^{a} + c_{x}^{b}) \cdot \frac{1}{2} \rho V^{2} \cdot c \cdot dy
\]

This force acts on the local a.c., if the constant - i.e. independent of \( \alpha \)-moment \( C_{m}^{\text{a.c.}} \) is applied about this point, see fig. 2.23.

Assuming, as in fig. 2.23, that the foremost point of the m.a.c. has the abscissa \( x_{0} \), the moment due to the additional lift distribution about this leading edge is:

\[
M_{a} = -2 \int_{0}^{b/2} (c_{x}^{a} \cdot \frac{1}{2} \rho V^{2}c) \cdot (x - x_{0}) \cos \alpha dy
\]

As previously mentioned in this discussion the a.c. of the wing is the point on the m.a.c. through which passes the resultant force of the additional lift distribution. For this reason, the moment about the leading edge of the m.a.c. due to the additional lift distribution is:
Fig. 2.23: The calculation of the position of the a.c. and the magnitude of $C_{m\text{a.c.}}$ of a wing of arbitrary plan form.

\[ M_a = -L_a \cdot (x_{a,c} - x_0) \]

Combining the above two expressions leads to:

\[ L_a (x_{a,c} - x_0) = 2 \int_0^{b/2} (c_{\chi_a} \cdot \frac{1}{2} \rho \upsilon^2 c) \cdot (x - x_0) \cos \alpha \, dy \]

Using:

\[ L = L_a = C_L \cdot \frac{1}{2} \rho \upsilon^2 S \]
and assuming \( \cos \alpha = 1 \), results in the required position of the a.c. of the wing:

\[
\frac{x_{a.c} - x_0}{c} = \frac{1}{c} \frac{b/2}{Sc} \int_{0}^{c} \left[ \frac{c}{\alpha} \cdot c \cdot (x - x_0) \right] dy \quad (2-28)
\]

The contribution of a wingstrip of width \( dy \) to the total moment about the leading edge of the m.a.c. follows from Fig. 2.23:

\[
dM = \left[ c_{m_{a.c.}} \cdot c - c_{\alpha} \cdot (x - x_0) \cdot \cos \alpha \right] \frac{1}{2} \rho V^2 \cdot c \cdot dy
\]

The total dimensionless moment of the entire wing about the \( Y_m \)-axis, if again \( \cos \alpha = 1 \), is obtained by integration along the wing span and division by \( \frac{1}{2} \rho V^2 Sc \):

\[
C_m = \frac{2}{Sc} \left[ \int_{0}^{b/2} \left\{ \int_{0}^{c} c_{m_{a.c.}} \cdot c^2 \cdot dy - \int_{0}^{c} c_{\alpha} \cdot c \cdot (x - x_0) \cdot dy \right\} \right] \quad (2-29)
\]

or since:

\[
c_{\alpha} = c_{\alpha_a} + c_{\alpha_b}
\]

\[
C_m = \frac{2}{Sc} \left[ \int_{0}^{b/2} \left\{ \int_{0}^{c} c_{m_{a.c.}} \cdot c^2 \cdot dy - \int_{0}^{c} c_{\alpha_b} \cdot c \cdot (x - x_0) \cdot dy - \int_{0}^{c} c_{\alpha_a} \cdot c \cdot (x - x_0) \cdot dy \right\} \right]
\]

\[
(2-30)
\]

According to (2-24) of 2.5, the \( C_m \) about the leading edge of the m.a.c. \((x=x_0)\) can be written, if \( C_N = C_L \), as:

\[
C_m = C_{m_{a.c.}} + C_L \cdot \frac{x_0 - x_{a.c.}}{c}
\]

\[
(2-31)
\]

Substituting (2-28) in (2-31) and equating (2-30) to (2-31) results in the
expression of $C_{m\text{ a.c.}}$:

$$C_{m\text{ a.c.}} = \frac{2}{\text{Sc}} \left[ \int_{0}^{b/2} c_{m\text{ a.c.}} \cdot c^2 \, dy - \int_{0}^{b/2} c_{b} \cdot c \cdot (x - x_0) \cdot dy \right] \quad (2-32)$$

In this way the position of the a.c. of the wing has been expressed in (2-28) and the value of $C_{m\text{ a.c.}}$ in (2-32). These results are subject to the same simplifying assumptions as were made in 2.3c:

a) the contribution of the tangential force to the moment is neglected and as a consequence, the a.c. is placed on the m.a.c.

b) the position of the a.c. is assumed to be independent of $\alpha$.

Ref. 2.5 provides for a large number of wing shapes the values of $x_{a.c.}$ according to (2-28), in addition to the magnitudes of the first and second integral in (2-32).

Data on the position of the a.c. and the magnitude of $C_{m\text{ a.c.}}$ can also be found in refs. 2.9, 2.10 and 2.11.

2.7. The influence of the wing shape on the moment curve, the position of the a.c. and the magnitude of $C_{m\text{ a.c.}}$.

It is quite difficult to describe the influence on the moment curve, due to the various parameters describing the wing shape, such as aspect ratio and wing sweep. The reason is, that the influence of one parameter strongly depends on the value of the other parameters. An additional complication is that few systematic series of measurements are available. Those that have been published often apply to wing-fuselage combinations, so that variations of the influence of the fuselage with wing sweep, wing taper ratio etc. are included in the experimental results. For these reasons it is not feasible to aim for completeness and a description of only a few of the more general effects of wing shape on the aerodynamic moment has to suffice.

In the foregoing it has been assumed - albeit tacitly - that when the wing aspect ratio is not 'too small' and the swing sweep angle is not 'too large', there is a range of angles of attack where both $C_m$ and $C_N$ vary more or less linearly with $\alpha$. In such situations the moment characteristics of the wing can be very well expressed by the $C_{m\text{ a.c.}}$ and the approximated position of the a.c., on the wing m.a.c. Relative to an arbitrary reference point on the m.a.c. $(x_1, z_0)$ the moment then is, see (2-23):
Fig. 2.24: The influence of camber on the moment curves. (From ref. 2.22 and 2.23)
(a) The moment due to the basic lift distribution.

(b) \( \Delta C_{m_{a,c}} \) as a function of \( A \) and \( \lambda \)

Fig. 2.25a and b: The variation of \( C_{m_{a,c}} \), with wing sweep and twist.
(From ref. lit. 2.6)
\[
C_{m}(x_{1},x_{0}) = C_{m}^{a.c.} + C_{N}^{x_{1} - x_{a.c.}}
\]

Using the expressions (2-28) and (2-32) for the calculation of \(x_{a.c.}\) and \(C_{m}^{a.c.}\) from theory, the following general remarks on the influences of wing shape on the a.c.-position and the \(C_{m}^{a.c.}\) can be made.

The \(C_{m}^{a.c.}\) of a complete wing has been derived as:

\[
C_{m}^{a.c.} = \frac{b/2}{Sc} \left[ \int_{0}^{c} c^{2} \, dy - \int_{c_{b}}^{c} c \cdot x \, dy \right] \tag{2-32}
\]

From this expression it is seen that the \(C_{m}^{a.c.}\) is partly determined by the \(c_{m}^{a.c.}\) in the first integral, that means by the camber of the wing profiles.

From the second integral in (2-32) it appears that \(C_{m}^{a.c.}\) depends also on the wing twist if the wing is swept, since the combination of these two factors influences the contribution made by the basic lift distribution.

Straight wings having no sweepback or sweepforward, have a constant factor in the second integral in (2-32). This reduces the integral to zero and \(C_{m}^{a.c.}\) is then determined only by the camber of the wing profiles. Increasing the camber leads to a more negative \(C_{m}^{a.c.}\), see fig. 2.24. If a straight wing has a constant \(C_{m}\) across the wing span, the result is \(C_{m}^{a.c.} = C_{m}^{a.c.}\), due to the definition of the a.c.c.

For sweepback wings having a negative wing twist, the positive \(c_{b}\) at the wing root and the negative \(c_{b}\) at the tip result in a positive, tail-heavy, contribution to \(C_{m}^{a.c.}\), see fig. 2.25a. This contribution increases with wing sweep and aspect ratio, see fig. 2.25b. Decreasing the taper ratio (\(\lambda = c_{t}/c_{r}\)) decreases this effect.

The deflection of landing flaps causes a considerable increase of the camber of the wing cross-section over part of the wing span. For straight wings the result is a more negative \(C_{m}^{a.c.}\) of the entire wing.
Fig. 2.26: The positions of the local a.c.'s of swept wings and delta wings.

Fig. 2.27: The influence of wing sweep on the moment curves of wings of aspect ratio $A = 4$  
(From ref. 2.24)
Flap deflection also causes a change in the basic lift distribution as would be caused by a more negative twist of the wing tips. If the wing has sweep back, this change in the basic lift distribution itself causes a positive change in $C_{m\text{a.c.}}$. As a consequence, the total change in $C_{m\text{a.c.}}$ due to flap deflection may even be positive. Calculations of this effect have been made e.g. in refs. 2.12 and 2.13.

The position of the a.c. has been derived as:

$$\frac{x_{\text{a.c.}} - x_o}{c} = \frac{2}{b/2} \int_{c_o}^{c} \frac{c}{c_{\text{a}}} \cdot c \cdot (x - x_o) \cdot dy$$

(2-28)

Straight wings having an aspect ratio which is not too small, show positions of the local a.c.'s corresponding well with the positions of the a.c.'s of the wing profiles in two-dimensional flow. As a consequence, the a.c. of such wings will lie around the 25% position on the m.a.c. Straight wings having a low aspect ratio ($A < 3$) show an appreciable forward shift with further decreasing aspect ratio. Flap deflection causes no shift of the a.c. of any importance.

The lift distribution in chordwise direction of swept wings and delta wings is strongly influenced by the wing sweep. The result is, that the local a.c.'s of the wing no longer agree with the a.c.'s of the local wing profiles in two-dimensional flow, see fig. 2.26. The a.c. position now has to be calculated using lifting surface theory.

*** The local a.c.'s at the tips of swept back wings lie ahead of the a.c.'s of the corresponding wing profiles. Since for swept back wings the additional lift distribution is concentrated more near the wing tips, the a.c. of the complete wing will be situated ahead of the 25% m.a.c. position. This forward shift increases with increasing sweepback angle. In 2.5 it has been argued that such a forward shift of the a.c. has an destabilizing influence.

The combination of a swept back wing with a fuselage, however, often shows a rearward shift of the a.c. with increasing sweepback angle, see fig. 2.27. This is due to the influence of wing sweepback on the wing-fuselage interference. In 2.8 this effect will be further discussed.

If the taper ratio ($\lambda = c_L/c_r$) decreases, the contribution of the wing tips to the total aerodynamic force decreases as well. This leads to an a.c. position of a swept back wing less far ahead of the 25% m.a.c. with decreasing taper ratio. Decreasing the $\lambda$ of swept back wings thus has a stabilizing influence, see fig. 2.28.
Wing aspect ratio has only a small influence on the shift of the a.c., expressed as a percentage of m.a.c. Fig. 2.29 shows for 45° swept back wings the position of the a.c. as a function of aspect ratio. Fig. 2.30 presents some measured moment curves.

Delta wings have local a.c.'s situated behind the quarter-chord position over a large part of the wing span, see fig. 2.26. In addition, the lift distribution of delta wings shows a concentration more towards the center parts of the wing. This causes the wing a.c. to be situated behind the 25% m.a.c. position.

Wing aspect ratio and sweep back of delta wings cannot be varied independently. Increasing sweepback decreases aspect ratio, leading to a rearward shift of the a.c., see fig. 2.29. The change of the fuselage influence with decreasing wing aspect ratio - and thus with increasing sweepback - has a stabilizing influence also for delta wings. This causes a larger rearward shift of the a.c. with decreasing aspect ratio.

Fig. 2.28: The influence of taper ratio on the moment curves of swept wing.
(From ref 2.26)
Fig. 2.29: The position of the a.c. of delta wings and concept wings as a function of aspect ratio, ($\Lambda = 45^\circ$).
(From ref. 2.25)
Fig. 2.30: The influence of aspect ratio on the moment curve of swept wings.
(From ref. 2.27)
Fig. 2.31: The moment curve of a straight wing and a swept wing. (DUT measurements)

At sufficiently large angles of attack various causes - but mostly flow separation - lead to non-linear moment curves for straight wings as well as for swept back and delta wings.

It has been explained in 2.4 that the characteristics of the wing in such situations can best be judged from the moment curves, rather than from an a.c. position and a $C_{m_{a.c.}}$. The slope of the moment curve is of special interest for stability, as previously discussed.
Fig. 2.32: Boundaries for the combinations of aspect ratio and sweep for which destabilizing changes in the moment curves may be expected. 
(From ref. 2.28 and 2.29)
Straight wings usually exhibit large negative changes in \( \frac{dC_{L}}{dC_{m}} \) at lift coefficients near \( C_{L_{\text{max}}} \), due to flow separation. Figs. 2.31 and 2.24 give examples of the phenomenon. Since the changes in \( C_{m} \) in the negative, nose-down, sense promote a decrease in angle of attack, this aspect of the moment curve points to a favourable characteristic of straight wings.

Swept wings with not too small an aspect ratio in many cases possess a moment curve having a considerable positive increase in slope with increasing angle of attack. The change in slope of the moment curve may start already at relatively small values of \( \alpha \). Figs. 2.27, 2.28, 2.30 and 2.31 give examples.

These positive changes in \( C_{m} \) of swept wings are caused by premature flow separation at the outer wings, predominantly caused by the high tip loading and the cross flow towards the tips. The result is an increasing thickness of the boundary layer and flow separation. This effect occurs at the wing tip at lower values of \( C_{L} \) and the effect on \( C_{m} \) is stronger, at larger sweep back angles and larger aspect ratios. Fig. 2.32 gives an indication of the combinations of aspect ratio and wing sweep for which destabilizing changes in the slope of the moment curves of the wing alone may be expected.

If a sudden change of the slope of the moment curve with increasing \( \alpha \) occurs for the complete airplane, the phenomenon is called 'pitch up'. When the angle of attack is increased beyond the value for which this 'pitch up' effect starts, a tail-heavy moment will promote a further increase in \( \alpha \). Evidently this is highly undesirable and even potentially dangerous.

The contribution of the horizontal tailplane to the aerodynamic moment \( C_{m} \) has an important effect on the 'pitch-up' characteristics of an airplane. But a positive change of the wing moment with increasing angle of attack certainly gives an undesirable contribution to the 'pitch-up'.

To avoid the lift concentration near the wing tips, negative wing twist is often used. Camber and other shape parameters of the wing profiles may be further adjusted to obtain desired values of local \( c_{l_{\text{max}}} \) without undue drag increases in cruising flight. To prevent cross-flow towards the wing tips, several means may be applied, such as layer fences or a 'saw tooth' leading edge. Ref. 2.14 provides an extensive compilation of the various methods and tools employed for this purpose.

2.8. The characteristics of a wing with fuselage and nacelles

The model of the aerodynamic moment of the combination of a wing with
fuselage and nacelles on the wing is in principle the same as for the wing alone. At small angles of attack, $C_m$ varies linearly with $C_N$ as for the wing alone. As a consequence, changes in the longitudinal moment caused by the fuselage and the nacelles are commonly expressed as a shift in the position of the a.c. and a change in $C_{m_{a.c.}}$. Neglecting again the contribution of the tangential force, the moment coefficient of the wing can be written as, see (2-24):

$$C_{m_w} = C_{m_{a.c.}} + C_N \frac{x - x_{a.c.}}{c}$$

(2-24)

In the same way, the moment of the combination of the wing with fuselage and nacelles can be expressed as:

$$C_{m_{w+f+n}} = C_{m_{a.c.}} + C_N \frac{x - x_{a.c.}}{c}$$

At constant $C_N$, the change in the moment results as:

$$\Delta C_m = C_{m_{w+f+n}} - C_{m_{w+a.c.}} = \Delta C_{m_{a.c.}} - C_N \frac{\Delta x_{a.c.}}{c}$$

(2-33)

Like $C_{m_{a.c.}}$, also $C_{m_{w+f+n}}$ is defined as constant, independent of angle of attack. This means that $\Delta C_{m_{a.c.}}$ also is independent of $\alpha$. It is the change of $C_m$ at $C_N = 0$ due to the addition of the fuselage and the nacelles.

The shift in the a.c. position then follows from (2-33):

$$\frac{\Delta x_{a.c.}}{c} = -\frac{d(C_m)}{dC_N} = -\frac{1}{C_{N_0} \frac{d(C_m)}{d\alpha}}$$

(2-34)

and the change in $C_{m_{a.c.}}$:

$$\Delta C_{m_{a.c.}} = (\Delta C_m)_{C_N=0}$$

(2-35)
The change in the longitudinal moment due to the presence of the fuselage and the wing nacelles can be thought of as consisting of the contributions of the fuselage and the nacelles each separately, the contribution of the wing-fuselage interference and the contribution of the wing-nacelle interference. Nacelles mounted at the rear fuselage are commonly regarded as part of the fuselage. Wing-mounted nacelles exert an influence on the longitudinal moment, equal in principle to that of the fuselage. For these reasons, only the effect of adding a fuselage to the wing is discussed in the following.

The change in the moment due to the presence of the fuselage, \( \Delta C_m \), can be written in the following expression:

\[
C_m^{w+f} = C_m^w + \Delta C_m
\]

where:

\[
\Delta C_m = C_m^f + C_m^i
\]

Here \( C_m^w \) is the longitudinal moment of the free fuselage in undisturbed flow, while \( C_m^f \) expresses the wing-fuselage interference.

From windtunnel tests the total \( \Delta C_m \) can be simply obtained by comparing the moment curves of the wing alone and the wing-fuselage combination at equal values of \( C_N \). If, in addition, measurements have been made on the fuselage alone, to determine \( C_m^f, C_m^i \), can be found as well.

For theoretical studies it appears to be useful to divide the total interference effect \( C_m^i \) into two parts: \( \Delta C_m^{f,i.} \), the change in the fuselage moment caused by situating the fuselage in the field of flow around the wing, and \( \Delta C_m^{w,i.} \), the change in the wing moment caused by placing the wing in the field of flow around the fuselage. So:

\[
\Delta C_m = C_m^f + \Delta C_m^{f,i.} + \Delta C_m^{w,i.}
\]

Usually, the total moment \( C_m^{f,i.} \) contributed by the fuselage in the field of flow around the wing, is calculated as one effect:
\[ C_{m_{f.i.}} = C_{m_{f}} + \Delta C_{m_{f.i.}} \]

This results in:

\[ \Delta C_{m} = C_{m_{f.i.}} + \Delta C_{m_{w.i.}} \]

In the following, the latter two contributions to the change in \( C_m \) and the subsequent changes in the position of the a.c. and the \( C_{m_{a.c.}} \) are discussed.

a. The moment on the fuselage placed in the field of flow around the wing, the magnitude of \( C_{m_{f.i.}} \).

Suppose a fuselage is placed under a non-zero angle of attack in an inviscid and incompressible flow. Considering the impulse of the airflow, it is possible to prove that the resultant aerodynamic force acting on the fuselage is zero, but there will be a resultant moment, a torque. The origin of the torque, i.e. a moment without a resultant force, is illustrated in fig. 2.33. The magnitude of this moment acting on a slender fuselage of circular cross-section in an incompressible, inviscid flow has been derived by Munk (ref. 2.15):

\[ M_f = \rho V^2 \cdot a_f \quad \text{(Volume of the fuselage)} \]

Fig. 2.33: Pressure distribution over a fuselage in inviscid flow.
From this expression follows quite simply:

\[
C_m = \frac{m \alpha_f}{2Sc} \int_0^{\ell_f} b^2_f(x) \, dx
\]  

(2-36)

Where:
- \(b_f(x)\) - local fuselage width
- \(\ell_f\) - fuselage length
- \(\alpha_f\) - fuselage angle of attack, in radians
- \(S,c\) - characteristic parameters of the wing to which \(C_m\) is referred.

According to (2-36) the moment is positive (tail-heavy) at positive angles of attack. It increases with increasing angles of attack. The implication is, that the idealized fuselage alone is statically unstable at \(\alpha_f = 0^\circ\) and cannot be in equilibrium at \(\alpha_f \neq 0\).

If the wing and the fuselage are combined, the fuselage is situated in the airflow influenced by the wing. This causes the angle of attack along the fuselage axis to be variable rather than constant, as for the fuselage alone. As a consequence, the pressure distribution over the fuselage experiences important changes as compared to the fuselage alone situation. The variation of the local angle of attack and the normal force along the fuselage axis is shown schematically in fig. 2.34. Ahead of the wing, at positive values of \(C_{L'}\), exists an upflow, causing an increase in \(\alpha_f(x)\) until very close to the wing leading edge. Across the wing chord the airflow is fully guided parallel to the wing chord, making \(\alpha_f(x)\) with good approximation equal to zero. Behind the wing, the downwash determines the local angle of attack along the fuselage axis.

Mainly due to the upwash in front of the wing, the increase in tail-down moment with increasing angle of attack becomes larger. The extra change in the moment caused by the presence of the wing is thus seen to be destabilizing as well. The part of the fuselage behind the wing contributes only little to the destabilizing moment with changes of the angle of attack. This leads to the conclusion that increasing the fuselage length forward of the wing may have a pronounced destabilizing effect. Many modern transport airplanes have relatively long forward fuselages. It will be clear that they have an unfavourable influence on static longitudinal stability.

In analogy with (2-36), see also ref. 2.16, the moment acting on a slender fuselage in an inviscid, incompressible flow having a variable angle of attack along the fuselage axis - now \(\alpha_f(x)\) - can be written as:
Fig. 2.34. The variation of the angle of attack and the normal force along the fuselage axis in the flow field disturbed by the presence of the wing.
\[
C_{m_{f.i.}} = \frac{\pi}{2Sc} \int_{0}^{l_{f}} b_{f}^2(x) \cdot \alpha_{f}(x) \cdot dx
\]  
(2-37)

The relation between \(\alpha_{f}(x)\) in (2-37) and the wing angle of attack is given by:

\[
\alpha_{f}(x) = \alpha_{f_{o}} + \frac{d\alpha_{f}(x)}{da} \cdot (\alpha - \alpha_{o})
\]  
(2-38)

where \(\alpha_{o}\) is an abbreviated notation for \(\alpha_{C_{N}=0}\) and \(\alpha_{f_{o}}\) is the fuselage angle of attack at \(\alpha = \alpha_{o}\).

With (2-38) follows for \(C_{m_{f.i.}}\):

\[
C_{m_{f.i.}} = \frac{\pi \alpha_{f_{o}}}{2Sc} \int_{0}^{l_{f}} b_{f}^2(x) \cdot dx + \frac{\pi(\alpha-\alpha_{o})}{2Sc} \int_{0}^{l_{f}} b_{f}^2(x) \cdot \frac{d\alpha_{f}(x)}{da} \cdot dx
\]  
(2-39)

The first term on the right hand side in (2-39) is the fuselage moment at \(C_{N} = 0\). An approximation of \(\Delta C_{m_{a.c.}}\) — and usually not a very accurate one — evidently is:

\[
\Delta C_{m_{a.c.}} = \left(\Delta C_{m}\right)_{C_{N}=0} = \frac{\pi \alpha_{f_{o}}}{2Sc} \int_{0}^{l_{f}} b_{f}^2(x) \cdot dx
\]  
(2-40)

The shift in a.c. position due to the moment of the fuselage placed in the airflow disturbed by the wing, is also obtained from (2-39), using (2-34):

\[
\frac{\Delta x_{a.c.}}{C_{f.i.}} = -\frac{1}{C_{N}} \frac{dC_{m_{f.i.}}}{da} = -\frac{1}{C_{N}} \cdot \frac{\pi}{2Sc} \int_{0}^{l_{f}} b_{f}^2(x) \cdot \frac{d\alpha_{f}(x)}{da} \cdot dx
\]  
(2-41)

In ref. 2.17 formulae have been derived, expressing the influence of the finite fuselage length and the friction on the contribution of the fuselage on the moment. They apply to the fuselage in undisturbed flow as well as to the situation where the flow around the fuselage is disturbed by the presence of the wing. Applying this theory to (2-40) and (2-41) results in:
Fig. 2.35: The correction factor $k$ for three-axial ellipsoids (From ref. 2.17)
\[\Delta C_{m_{a.c.}} = \frac{\pi}{2Sc} k \left\{ \int b_f^2(x) \, dx - b_f^2 \cdot (\ell_f - x) \right\} \quad (2-42)\]

\[\frac{\Delta x_{a.c.}}{c_{f.i.}} = - \frac{1}{C_n} \frac{\ell_f}{2Sc} \left\{ \int b_f^2(x) \cdot \frac{d}{da} \frac{d_x f(x)}{dx} \, dx - \right\}
+ b_f^2 \cdot \frac{d}{da} \frac{d_x f(x)}{dx} \cdot (\ell_f - x) + b_f^2 \int \frac{d}{da} \frac{d_x f(x)}{dx} \cdot \ell_w \} \quad (2-43)\]

Where:
- \(k\) is the correction factor given in Fig. 2.35 for the finite fuselage length.
- \(b_f\) is the active width at the fuselage rear end given in Fig. 2.36.
- \(b_f^{**}\) is the replacement width at the fuselage nose, see Fig. 2.37.
- \(\ell_f - x\) is the distance from the fuselage rear end to the chosen moment reference point.
- \(\ell_w\) is the length of the fuselage ahead of the wing.
- \(\frac{d}{da} \frac{d_x f(x)}{dx}\) is the value of \(\frac{d_x f(x)}{dx}\) at the fuselage rear end.
- \(\frac{d}{da} \frac{d_x f(x)}{dx}\) is the value of \(\frac{d_x f(x)}{dx}\) at the fuselage nose.

According to the foregoing, the calculation of the component \(C_{m_{f.i.}}\) of the fuselage moment can be performed quantitatively, if the variation of \(\frac{d}{da} \frac{d_x f(x)}{dx}\) along the fuselage axis is known. For slender straight wings the calculation can be made using Ref. 2.16. But this method cannot be used in more accurate calculations. For the latter, the upflow in front of the wing can be calculated employing the lifting line theory of Prandtl. For swept and delta wings the upflow in front of the wing and the downwash behind it are also commonly obtained with the aid of lifting line theories, see for instance Refs. 2.7 and
Fig. 2.36: The active fuselage width at the fuselage rear end.

<table>
<thead>
<tr>
<th>shape of the fuselage rear end</th>
<th>active fuselage width</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangular</td>
<td>1</td>
</tr>
<tr>
<td>parabolic</td>
<td>0.6</td>
</tr>
<tr>
<td>elliptic</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Fig. 2.37: The replacement width $b_f^{**}$ at the fuselage nose as a function of $\frac{b_{f_{\text{max}}}}{l_f}$. (From ref. 2.17)
Fig. 2.38: The change in active wing planform of swept wings and delta wings in combination with a fuselage.

Fig. 2.39: The influence of aspect ratio on the variation of \( \frac{d\alpha_1}{d\alpha} \) along the fuselage axis \( \Lambda = 0 \).

(From ref. 2.1)
Fig. 2.40: The influence of sweep on the variation of $\frac{d\alpha_f}{d\alpha}$ along the fuselage axis ($A = \infty$)
(From ref. 2.1)

Fig. 2.41: The influence of the presence of the fuselage on the variation of the angle of attack along the wing span.
2.8. In the calculation of $\frac{d\alpha_f}{d\alpha}$ along the fuselage axis for such wings, the shape of the wing across the fuselage is usually taken as shown in Fig. 2.38.

According to (2-39) the influence of the wing shape $C_{mf,i}$ is determined largely by the values of $\frac{d\alpha_f}{d\alpha}$ along the fuselage axis. Figs. 2.39 and 2.40 show the influence of the wing aspect ratio and wing sweep respectively on $\frac{d\alpha_f}{d\alpha}$. It is apparent that both a decrease of aspect ratio and an increase of sweepback decrease $\frac{d\alpha_f}{d\alpha}$ in front of the wing. As a consequence, the contribution of the fuselage nose to $\frac{dC_{mf,i}}{d\alpha}$ becomes less stabilizing, see (2-41). The downwash angle behind the wing increases with decreasing wing aspect ratio and increasing sweepback angle, $\frac{de}{d\alpha}$ increases, and $\frac{d\alpha_f}{d\alpha}$ comes closer to zero. In this way the destabilizing contribution to $C_{mf,i}$ made by the part of the fuselage behind the wing decreases as well. In summary, it turns out that both a decrease in wing aspect ratio and an increase in sweepback angle lead to a less destabilizing contribution of the fuselage to $C_m$.

b. The influence of the fuselage on the wing moment, the magnitude of $\Delta C_{mw,i}$.

The influence of the fuselage on the characteristics of the wing itself consists of two parts. The first part arises from the extra upflow induced by the presence of the fuselage over the wing root, next to the fuselage, see Fig. 2.41. The second part is caused by the change in lift distribution over the part of the wing covered by the fuselage, see Fig. 2.42.

A further look at the induced upflow over the wing shows that the influence on the wing moment is relatively small. It can be neglected, also for swept and delta wings, if compared to other effects, see Ref. 2.1. Only for wing-mounted nacelles, whose diameter may vary appreciably over the wing chord, the induced upflow may have a non-negligible influence on the wing moment, see Ref. 2.16.

Fig. 2.42 shows the results of measurements on the lift distribution across the span of a wing with and without fuselage. It appears that the loss in lift due to the fuselage occurs only in the immediate vicinity of the fuselage. For straight wings it can be assumed, that this loss in lift $\Delta C_{lw}$ causes no change in the moment about the a.c. of the wing.

For swept back wings and also for delta wings, the a.c. of the entire wing lies behind the local a.c. of the wing center part, see Fig. 2.43. The loss in lift $\Delta C_{lw}$, acting through the latter a.c., will induce in this situation a nose-
Fig. 2.42: The $c_{L,c}$-distribution along the span of a wing with and without a fuselage. (From ref. 2.30)

Fig. 2.43: The change in wing moment due to the loss in lift over the wing center part.
down change in moment. This means that $\Delta C_{m_{w,i}}$ is negative. As $\Delta C_{L_w}$ is more or less proportional with $C_L$, the change in the wing moment acts in the stabilizing sense, it will shift the a.c. of the wing to a more rearward position. For swept forward wings, however, the $\Delta C_{L_w}$ causes a tail-down change in the wing moment, acting in a destabilizing sense. It will shift the a.c. of the wing to a more forward position.

On the basis of the preceding discussion, ref. 2.20 presents an approximative method to determine $\Delta C_{m_{w,i}}$ as caused by the presence of the fuselage. The influence of the angle of sweep on the position of the a.c., calculated from:

$$\frac{\Delta x_{a.c.}}{c} = - \frac{1}{C_{N\alpha}} \frac{d\Delta C_{m_{w,i}}}{d\alpha}$$

is shown in fig. 2.44 for a certain series of idealized airplane configurations. The figure also gives a comparison with the contribution to $\Delta x_{a.c.}$ caused by $C_{m_{f,i}}$ as derived from (2-39) and with the total shift in the a.c. position caused by the fuselage, both calculated and measured.

![Diagram showing the variation in a.c. position due to the fuselage, as a function of wing sweep. (From ref. 2.20)]
From the figure appears once again the destabilizing influence of the fuselage and the stabilizing influence of wing sweepback. As has already been noted in 2.7, this stabilizing influence of wing sweepback may compensate the destabilizing influence of the fuselage. The resultant change in a.c. position may well be in the stabilizing sense. Wind tunnel measurements of swept wings are usually made on models including the fuselage.
**TABLE 2.1. Definitions of the center of pressure, the neutral line, the neutral point and the aerodynamic center.**

<table>
<thead>
<tr>
<th>CONCEPT</th>
<th>DESCRIPTION</th>
<th>CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of pressure</td>
<td>The point of intersection of the line of action of the total aerodynamic force $C_R$ and the m.a.c.</td>
<td>$\frac{dc_R(x_i, z_j)}{dx_i} = 0$ if $z_j = 0$</td>
</tr>
<tr>
<td>1st Metacenter</td>
<td>The point of intersection of two neighbouring lines of action of $C_R^1$ for angles of attack $\alpha$ and $\alpha + \Delta \alpha$, in the limiting case where $X \alpha = 0$.</td>
<td>$C_R^1 = 0$ relative to 1st metacenter</td>
</tr>
<tr>
<td>Neutral line</td>
<td>The line of action of $C_R^1$ at a certain angle of attack $\alpha$.</td>
<td>$\frac{dc_R^1}{d\alpha} = 0$ relative to all points on the neutral line</td>
</tr>
<tr>
<td>Neutral point</td>
<td>The point of intersection of the neutral line with the m.a.c.</td>
<td>$\frac{dc_R^2(x_i, z_j)}{dx_i} = 0$ if $z_j = 0$</td>
</tr>
<tr>
<td>2nd Metacenter</td>
<td>The point of intersection of two neighbouring lines of action of $C_R^2$ (neutral lines) for angles of attack $\alpha$ and $\alpha + \Delta \alpha$in the limiting case where $\Delta \alpha = 0$.</td>
<td>$\frac{dc_R^2}{d\alpha} = 0$ relative to 2nd metacenter</td>
</tr>
<tr>
<td>Aerodynamic center</td>
<td>A. Exact</td>
<td>From these expressions follow x.a.c. and z.a.c., $C_a = 0$ is found using $x_a$ and $z_a$.</td>
</tr>
<tr>
<td>I. a.c. position</td>
<td>independent of $\alpha$ (possible only in theory)</td>
<td>See (2-12)</td>
</tr>
<tr>
<td>II. a.c. position</td>
<td>varying with $\alpha$</td>
<td>See (2-13)</td>
</tr>
<tr>
<td>B. Approximated</td>
<td>the a.c. is the invariant point on the m.a.c. about which the $C_R^1$ does not vary with angle of attack ($C_R^1 = constant$). According to this definition, the a.c. is identical to the 2nd metacenter.</td>
<td>From these expressions follow x.a.c. and z.a.c.</td>
</tr>
<tr>
<td></td>
<td>A. Exact</td>
<td>See (2-12)</td>
</tr>
<tr>
<td></td>
<td>1. At $\alpha = 0^\circ$: the a.c. is identical to the 2nd metacenter.</td>
<td>See (2-13)</td>
</tr>
<tr>
<td></td>
<td>2. At $\alpha = 0^\circ$: the point on the line of action of $C_R^1$ about which $C_R^1$ equals the value of $C_R^1$ determined at $\alpha = 0$.</td>
<td>See (2-18) and (2-19)</td>
</tr>
<tr>
<td>B. Approximated</td>
<td>1. As for I.A.</td>
<td>See (2-15) and (2-17)</td>
</tr>
<tr>
<td></td>
<td>2. $C_{a,c} = 0$</td>
<td>See (2-15) and (2-17)</td>
</tr>
<tr>
<td></td>
<td>a. the a.c. is the point on the m.a.c. about which $C_R^1$ equals to $C_R^1$ found at $\alpha = 0^\circ$.</td>
<td>See (2-21)</td>
</tr>
<tr>
<td></td>
<td>b. the a.c. coincides with the neutral point.</td>
<td>See (2-22)</td>
</tr>
</tbody>
</table>

**Remark:**
If the definitions given in II B.2 for the a.c. at $\alpha = 0^\circ$ lead to appreciable differences in a.c. positions and $C_{a,c}$, this approximated definition can no longer be used.
CHAPTER 3: The equilibrium in steady, symmetric straight flight

3.1. Introduction

Stability is a characteristic of a state of equilibrium of the airplane. The states of equilibrium to which studies of the stability of airplanes refer, are conditions of steady symmetric, straight flight. In the study of flying qualities, the equilibrium of the longitudinal moment is of utmost importance, although the aerodynamic forces enter into the discussions as well.

Ideally it should be possible to obtain equilibrium of the aerodynamic moment at any angle of attack and center of gravity position at which equilibrium of the forces is possible, see 1.1. To achieve this aim, use is commonly made of a horizontal tailplane with an elevator. The horizontal tailplane serves the additional purpose of contributing to changes in the aerodynamic moment in the stable sense, if the airplane experiences a deviation from the equilibrium due to a disturbance.

This chapter discusses the equilibrium of the forces and moments in steady, symmetric, straight flight. The contributions of the horizontal tailplane and the elevator to the longitudinal equilibrium of moments is also considered. The stability of the equilibrium is the subject of the next chapters.

3.2. The conditions of equilibrium

Since steady flight only is considered here, forces and moments due to inertia of the airplane can be omitted. The airplane weight and the aerodynamic forces and moments are in equilibrium:

\[ W + R = 0 \]
\[ M = 0 \]

The forces and the moment are resolved in components along the airplane body axes. In this way the conditions for equilibrium are written as:

\[ W_x + X = 0 \]
\[ W_y + Y = 0 \]
\[ W_z + Z = 0 \]
\[ L = 0 \]
\[ M = 0 \]
\[ N = 0 \]
In the following, it is always assumed that the airplane is symmetric and that in the flight conditions to be considered, the flow around the airplane has a plane of symmetry, coinciding with that of the airplane. In these symmetric flight conditions, the resultant aerodynamic force $\mathbf{R}$ and the weight vector $\mathbf{W}$ lie in the plane of symmetry of the airplane, which means:

\[
\begin{align*}
W_y &= Y = 0 \\
L &= N = 0
\end{align*}
\]

The three symmetric conditions of equilibrium are then, see fig. 3.1:

\[
\begin{align*}
-W \sin \theta + X &= 0 \\
W \cos \theta + Z &= 0 \\
M &= 0
\end{align*}
\]

where $\theta$ is the angle of pitch of the airplane.

Fig. 3.1: The equilibrium in steady, symmetric flight.
When considering the longitudinal equilibrium in steady, symmetric flight, the components $X$ and $Z$ are commonly replaced by the components $N$ and $T$ which are equal in magnitude but have the opposite sign, see 1.5. In addition, to simplify the discussion, the airplane body axes are chosen parallel to the airplane reference axes. The origin of the body axes remains in the airplane center of gravity. In this situation the symmetric conditions of equilibrium read as:

\[
T = -W \sin \theta \quad (3-1)
\]
\[
N = W \cos \theta \quad (3-2)
\]
\[
M = 0 \quad (3-3)
\]

In the following these conditions are further analyzed.

The resultant aerodynamic force $R$ is built up from the contributions of the different parts of the airplane. These parts also provide the elements of the resultant aerodynamic moment $M$. Three parts of the airplane will be distinguished.

1. **The wing with fuselage and nacelles**
   The contributions of these parts of the airplane, usually taken as one entity, are commonly indicated only with the index $w$. This means, that from now on in this text the index $w$ refers to the combination of the wing with fuselage and nacelles. The model of the forces acting on this part of the airplane is given by the components $N_w$ and $T_w$ of the total aerodynamic force $R_w$, acting in the a.c. of the wing with fuselage and nacelles (a.c. $w$), and the moment $M_{a.c. w}$ acting about the a.c. $w$.

2. **The horizontal tailplane**
   As for the wing with fuselage and nacelles, the model of the forces acting on the horizontal tailplane is given by the components $N_h$ and $T_h$, acting in the a.c. of the tailplane (a.c. $h$), and the moment $M_{a.c. h}$.

3. **The propulsion unit**
   The contribution of the propeller, or the jet engine, consists of the thrust $T_p$ along the propeller or engine-axis and the normal force $N_p$ in the plane of the propeller or in the plane of the engine inlet. The normal force originates when the propeller, or the engine inlet as the case may be, has a local angle of attack different from zero.
Fig. 3.2: The forces and moments in steady, straight, symmetric flight.
Fig. 3.2 depicts the contributions of the various parts of the airplane. The airplane reference axes are used to indicate the distances from the center of gravity to the action points of the various forces.

The symmetric conditions of equilibrium (3-1), (3-2) and (3-3) can now be written as follows:

Forces along the X-axis:

\[ T_x = T_w + T_h - T_p \cos \theta_p + N_p \sin \theta_p = -W \sin \theta \]  \hspace{1cm} (3-4)

Forces along the Z-axis:

\[ N = N_w + N_h + N_p \cos \theta_p + T_p \sin \theta_p = +W \cos \theta \]  \hspace{1cm} (3-5)

Moments about the Y-axis:

\[
M = M_{a.c.} + N_w (x_{c.g.} - x_w) - T_w (z_{c.g.} - z_w) + \\
+ N_{a.c.} + N_h (x_{c.g.} - x_h) - T_h (z_{c.g.} - z_h) + \\
+ (N_p \cos \theta_p + T_p \sin \theta_p) (x_{c.g.} - x_p') + \\
+ (T_p \cos \theta_p - N_p \sin \theta_p) (z_{c.g.} - z_p') = 0
\]  \hspace{1cm} (3-6)

Before bringing the expressions (3-4), (3-5) and (3-6) in a dimensionless form, various new dimensionless coefficients have to be defined.

The normal force and the tangential force acting on the horizontal tailplane and the moment about the a.c. of the tailplane can be written as:

\[ N_h = C_{N_h} \frac{1}{2} \rho v^2 S_h \]  \hspace{1cm} (3-7)

\[ T_h = C_{T_h} \frac{1}{2} \rho v^2 S_h \]  \hspace{1cm} (3-7)

\[ M_{a.c.} = C_{m_{a.c.}} \frac{1}{2} \rho v^2 S_h \]  \hspace{1cm} (3-7)
Fig. 3.3a: Geometry parameters of the horizontal tailplane, the elevator, and the tab

Fig. 3.3b: The positive senses $\alpha_h$, $\delta_e$, and $\delta_{te}$. 
The dimensionless coefficients $C_{N_h}$, $C_{\alpha}$, and $C_m$ are thus referred to the surface area $S_h$ and the m.a.c. $c_h$ of the horizontal tailplane, see fig. 3.3a, and the average local dynamic pressure $\frac{1}{2} \rho v^2_h$ at the horizontal tailplane. In general, this dynamic pressure is not exactly equal to that of the undisturbed flow. This subject is further discussed in 3.5.

The thrust of the propeller and the normal force in the plane of the propeller are written as:

$$ T_p = T_c \cdot \rho v^2 \cdot D^2 $$  \hspace{1cm} (3-8)

$$ N_p = C_{N_p} \cdot \frac{1}{2} \rho v^2 \cdot S_p $$  \hspace{1cm} (3-9)

where $D$ is the propeller diameter and $S_p = \frac{\pi}{4} D^2$ is the area of the propeller disc. The equations of equilibrium are made dimensionless in the usual way by dividing (3-4) and (3-5) by $\frac{1}{2} \rho v^2 S$ and (3-6) by $\frac{1}{2} \rho v^2 S_\alpha$. Using the approximation:

$$ - \frac{T_p \cos \alpha}{p} + \frac{N_p \sin \alpha}{p} = - T_p $$

the result is:

$$ C_T = C_{T_w} + C_{T_h} \left( \frac{V_h}{V} \right)^2 \frac{S_h}{S} - T_c \frac{2D^2}{S} = - \frac{W}{\frac{1}{2} \rho v^2 S} \sin \theta $$  \hspace{1cm} (3-10)

$$ C_N = C_{N_w} + C_{N_h} \left( \frac{V_h}{V} \right)^2 \frac{S_h}{S} + T_c \frac{2D^2}{S} \sin \alpha + C_{N_p} \frac{S_p}{S} = \frac{W}{\frac{1}{2} \rho v^2 S} \cos \theta $$  \hspace{1cm} (3-11)

$$ C_m = C_{m.a.c.} + C_{N_w} \frac{x_{c.g.} - x_w}{c} - C_{T_w} \frac{z_{c.g.} - z_w}{c} + $$

$$ + C_{m.a.c.} \left( \frac{V_h}{V} \right)^2 \frac{S_h}{S} \frac{c_h}{c} + C_{N_h} \left( \frac{V_h}{V} \right)^2 \frac{S_h}{S} \frac{x_{c.g.} - x_h}{c} + $$

$$ + C_{T_h} \left( \frac{V_h}{V} \right)^2 \frac{S_h}{S} \frac{c_{g.} - z_h}{c} + \left( T_c \frac{2D^2}{S} \sin \alpha + C_{N_p} \frac{S_p}{S} \frac{x_{c.g.} - x_p}{c} + $$

$$ + T_c \frac{2D^2}{S} \frac{z_{c.g.} - z_p}{c} = 0 $$  \hspace{1cm} (3-12)
These equations of equilibrium may be used in various ways. If the aim is to find the equilibrium situation of an airplane under certain given conditions by calculation as accurately as possible, these equations may indeed be used in the form as just given. To obtain a satisfactory accuracy, it will often be necessary to have reliable results of measurements on a wind tunnel model of the airplane.

In the following, however, the purpose is slightly different. The aim is to provide an insight into the various concepts and phenomena related to flying qualities. To this end it is permissible and perhaps even desirable to simplify the equations of equilibrium in such a way that only the most important contributions remain. This is why the following simplifications are made.

1. Gliding flight is assumed. The terms proportional to $T_c$ and $C_{p}$ are thus neglected.

2. The contribution of $C_{h}$ to the force in $X$-direction in (3-10) and to the moment in (3-12) is neglected. This simplification is nearly always permissible.

3. The contribution of $C_{w}$ to the moment in (3-12) is omitted. At large angles of attack, or in cases where the center of gravity is situated far above or below the a.c.w, this simplification may not always be permissible.

4. If the horizontal tailplane has a symmetric profile, $C_{m_{a.c.\_h}} = 0$, if the elevator is in the neutral position. Even if this restriction on the shape of the horizontal tailplane is not met, the contribution of the moment about the a.c. of the tailplane will remain small, relative to the other contributions to the total moment about the center of gravity. As a consequence, the coefficient $C_{m_{a.c.\_w}}$ in (3-12) is always neglected. The index $w$ of $C_{m_{a.c.\_w}}$ will therefore be omitted as well.

As a result of the assumptions the equations of equilibrium now read as:

$$C_T = C_{T_w} = -\frac{W}{\frac{1}{2} \rho \gamma^2 S} \sin \theta \quad (3-13)$$

$$C_N = C_{N_w} + C_{N_h} \left(\frac{V_h}{V}\right) \frac{S_h}{S} = \frac{W}{\frac{1}{2} \rho \gamma^2 S} \cos \theta \quad (3-14)$$
\[ C_m = C_{m_{a.c.}} + C_{N_w} \frac{x_{c.g.} - x_w}{c} + C_{N_h} \left( \frac{V_h}{V} \right) \frac{V^2}{h} \frac{x_{c.g.} - x_h}{c} = 0 \]  
(3-15)

The assumptions made in the model of the equilibrium lead to a very simple picture, see fig. 3.4.

The distance between the a.c. of the horizontal tailplane and the a.c. of the wing with fuselage and nacelles \((x_h - x_w)\) is called the tail length \(l_h\). The distance between the center of gravity and the a.c. of the tailplane is approximately equal to this tail length,

\[ l_h = x_h - x_w = x_h - x_{c.g.} \]  
(3-16)

where \(x_h\) is often taken as the distance to the \(x\)-coordinate of the position at 25% of \(c_h\). Using (3-16) the equation for the equilibrium of the moments (3-15) becomes:

\[ C_m = C_{m_{a.c.}} + C_{N_w} \frac{x_{c.g.} - x_w}{c} + C_{N_h} \left( \frac{V_h}{V} \right) \frac{V^2}{h} \frac{l_h}{S.c} = 0 \]  
(3-17)

or:

\[ C_m = C_{m_w} + C_{m_h} = 0 \]

The combination \( \frac{S_h \cdot l_h}{S \cdot c} \) is called the 'tailplane volume'.

The contribution \(C_{m_w}\) of the wing with fuselage and nacelles to the moment in (3-17) is usually not equal to zero. Equilibrium of the total moment about the lateral axis at a given angle of attack and center of gravity position is commonly obtained by means of an appropriate deflection of the elevator or, for some airplanes, by suitably adjusting the angle of incidence of the entire horizontal tailplane.

In this way the contribution \(C_{m_h}\) from the horizontal tailplane to the moment, generated by \(C_{N_h}\) is made equal but opposite in sign to the contribution of the wing with fuselage and nacelles.

In the following paragraphs the characteristics of the horizontal tailplane and the effects of the elevator will be studied in more detail. Also,
expressions will be derived for the elevator angle and the pilot's control force, necessary to obtain equilibrium under given conditions.

The equations of equilibrium (3-13), (3-14) and (3-15) have been derived for the most common airplane configuration, where the horizontal tailplane is placed behind the wing. In the given form the expressions are valid also for airplanes where the horizontal 'tailplane' is situated ahead of the wing. In that case $l_h$ in (3-17) has a negative sign, see (3-16).

![Diagram showing forces and moments](#)

**Fig. 3.4**: Simplified version of the equilibrium of forces and moments.

For tailless airplanes the contribution of the horizontal tailplane obviously does not exist. The expression for the equilibrium of the moment then is:

$$
C_m = C_{m_{a.c.}} + C_N \frac{x_{c.g.} - x_w}{c} = 0
$$

(3-18)
See also fig. 2.17.

The elevator of the tailless airplane is situated at the trailing edge of the wing. Deflection of the elevator of such an airplane may be thought of as to cause in the first place a change in the $C_{m_{a.c.}}$ of the wing with fuselage and nacelles. The elevator deflection also has an influence on $C_N$ which is non-negligible in many cases. For tailless airplanes, equilibrium of the moment at a given value of $C_N$ is obtained by making $C_{m_{a.c.}}$ in (3-18) equal but opposite in sign to the moment about the center of gravity due to $C_N$.

3.3. The normal force on the horizontal tailplane

The horizontal tailplane usually consists of a stabilizer and the control surface. In many cases a small auxiliary surface has been added to the control surface, the tab. This may be a trim tab, a balance tab or a servotab. The dimensionless pressure distribution over the horizontal tailplane is completely determined by three angles:

1. the angle of attack of the horizontal tailplane, $\alpha_h$,
2. the control deflection angle, $\delta_e$ (index $e$ = elevator),
3. the tab angle, $\delta_{te}$ (index $t$ = tab).

Fig. 3.3b shows the positive sense of the three angles $\alpha_h$, $\delta_e$ and $\delta_{te}$.

Fig. 3.5 shows schematically the changes in the chordwise pressure distribution caused by changes in the angle of attack, the control surface deflection and the tab angle.

The normal force on the horizontal tailplane $N_h$, is obtained by integrating the pressure distribution both in the chordwise and the spanwise directions of the tailplane.

The dimensionless normal force $C_{N_h}$ has already been defined in (3-7). $C_{N_h}$ is determined by the three angles just discussed:

$$C_{N_h} = C_{N_h}(\alpha_h, \delta_e, \delta_{te})$$
Fig. 3.5: Pressure distributions over the tailplane chord, due to $\alpha_h$, $\delta_e$ and $\delta_t e$. 
In fig. 3.6 results are given of measured $\frac{C_{N_h}}{\delta_t}$ as a function of $\alpha_h$, $\delta_e$ and $\delta_t$. It is shown that for small values of the three angles, $\frac{C_{N_h}}{\delta_t}$ varies approximately linearly with $\alpha_h$, $\delta_e$ and $\alpha_t$. In the range of those small angles a reasonable approximation of $\frac{C_{N_h}}{\delta_t}$ is obtained by:

$$C_{N_h} = C_{N_h_0} + \frac{\partial C_{N_h}}{\partial \alpha_h} \cdot \alpha_h + \frac{\partial C_{N_h}}{\partial \delta_e} \cdot \delta_e + \frac{\partial C_{N_h}}{\partial \delta_t} \cdot \delta_t$$  \hspace{1cm} (3-19)

Various shorthand notations exist for the aerodynamic partial derivatives introduced in (3-19), see table 3.1.

Table 3.1: Abreviated notation for the normal force derivatives on a tailplane

<table>
<thead>
<tr>
<th>Derivative</th>
<th>$\frac{\partial C_{N_h}}{\partial \alpha_h}$</th>
<th>$\frac{\partial C_{N_h}}{\partial \delta_e}$</th>
<th>$\frac{\partial C_{N_h}}{\partial \delta_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dutch and American notation</td>
<td>$C_{N_h\alpha}$</td>
<td>$C_{N_h\delta}$</td>
<td>$C_{N_h\delta_t}$</td>
</tr>
<tr>
<td>British notation</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
</tr>
</tbody>
</table>

1) based on the norm V 995
2) see ref. 3.2.
Fig. 3.6: The normal force coefficient $C_{N_h}$ as a function of $\delta_e$, $\delta_{e_0}$ and $\delta_{e_0}$ for the tailplane of the Fokker F-27. (Wind tunnel measurements from ref. 3.7.)
The expression (3-19) can be somewhat simplified. In the first place the influence of $\delta_e$ on $C_{N_h}$ is usually small, see for example fig. 3.6. The contribution of $\delta_e$ to $C_{N_h}$ is therefore commonly neglected. Furthermore, tailplanes often, although not always, possess a symmetric profile. Then $C_{N_h}$ is zero.

Using the American notation and the given simplifications, the normal force coefficient of the horizontal tailplane is expressed as:

$$C_{N_h} = C_{N_h \alpha} \cdot \alpha_h + C_{N_h \delta} \cdot \delta_e$$  \hspace{1cm} (3-20)

Calculations intended to have some accuracy, require the availability of the experimentally derived function $C_{N_h}$ ($\alpha_h$, $\delta_e$, $\delta_e$). To obtain this function, wind tunnel measurements are usually required, employing a model of the tailplane made at as large a scale as possible.

Both a positive change in the angle of attack $\alpha_h$, and in the control deflection $\delta_e$, cause positive changes in $C_{N_h}$, which means:

$$C_{N_h \alpha} > 0 \quad \text{and} \quad C_{N_h \delta} > 0$$

A calculation of $C_{N_h \alpha}$ and $C_{N_h \delta}$ for a given tailplane and control surface is possible, using refs. 3.1 to 3.5. The calculation methods given in these publication are based almost entirely on empirical data, see also ref. 3.6. The normal force gradient $C_{N_h \alpha}$ can be obtained in principle using one of the available wing theories. The presence of the fuselage and the existence of an open gap between the stabilizer and the control surface are to be taken into account. Both will reduce the value of $C_{N_h \alpha}$.

From measurements on two- and three-dimensional models it has been shown that the derivative $C_{N_h \delta}$ largely depends on $C_{N_h \alpha}$. In addition, it is determined by the ratio of the control surface chord to the tailplane chord, $\frac{c_\delta}{c_h}$. Fig. 3.7
Fig. 3.7: $\frac{C_{Nh_\delta}}{C_{Nh_\alpha}}$ as a function of $\frac{\bar{e}_e}{\bar{e}_h}$ for various trailing edge angles $\tau$.

(From ref. 3.6)
presents a summary of the measured relation between \( \frac{C_{N_{h}\delta}}{C_{N_{h}\alpha}} \) and \( \frac{C_{e}}{C_{h}} \) for a large number of three-dimensional tailplane configurations. From this figure it can be seen that the trailing edge angle \( \tau \) of the profile also has an important influence on \( \frac{C_{N_{h}\delta}}{C_{N_{h}\alpha}} \). This is true for aerodynamically unbalanced control surfaces extending over the entire span of the horizontal tailplane, having a closed gap between the stabilizer and the control surface. In addition, \( C_{N_{h}\delta} \) is influenced by several other factors, such as cut-outs in the control surface at the fuselage and the vertical tailplane, the gap between the stabilizer and the tailplane and, if the control surface is aerodynamically balanced, by the size and the shape of the nose balance, see refs. 3.4 and 3.5.

3.4. The hinge moment of the elevator

The aerodynamic hinge moment about the elevator hinge line, \( H_{e} \), is determined by the pressure distribution over the elevator. This hinge moment is expressed by a dimensionless coefficient, \( C_{h_{e}} \); the index \( a \), to be mentioned later in this chapter, is omitted.

\[
C_{h_{e}} = \frac{H_{e}}{\frac{1}{2} \rho v^2 S \frac{c_{e}}{h_{e}}}
\]  
(3-21)

The hinge moment coefficient is thus referred to the average local dynamic pressure at the horizontal tailplane and to the surface \( S_{e} \) and the m.a.c. \( \bar{c}_{e} \) of the part of the control surface behind the hinge line, see fig. 3.3a. The hinge moment is taken as positive if it acts in the sense of increasing a positive control surface deflection.

Like \( C_{N_{h}} \), \( C_{h_{e}} \) also depends on the three variable angles, \( \alpha_{h} \), \( \delta_{e} \), and \( \delta_{t_{e}} \), see the pressure distributions of fig. 3.5, this means:

\[
C_{h_{e}} = C_{h_{e}}(\alpha_{h}, \delta_{e}, \delta_{t_{e}})
\]
Measurements of \( C_{h_e} \) as a function of \( \alpha_h, \delta_e \) and \( \delta_t \) are depicted in fig. 3.8. From this figure it is clear — as could be expected — that \( \delta_t \), has an important influence on \( C_{h_e} \).

Contrasting with the negligible effect on \( C_{N_h} \) at small angles of attack, control deflections and tab angles, \( C_{h_e} \) varies more or less linearly with \( \alpha_h, \delta_e \) and \( \delta_t \). In the range of small angles, an expression can be given for \( C_{h_e} \) in analogy to that for \( C_{N_h} \):

\[
C_{h_e} = C_{h_o} e^{\alpha_h \cdot \delta_e + \delta_t \cdot \delta_t} + \frac{\partial C_{h_e}}{\partial \alpha_h} \cdot \alpha_h + \frac{\partial C_{h_e}}{\partial \delta_e} \cdot \delta_e + \frac{\partial C_{h_e}}{\partial \delta_t} \cdot \delta_t
\]  

(3-22)

If a symmetric profile is used for the tailplane, \( C_{h_o} = 0 \). Using in addition the shorthand notation, see tabel 3.2, \( C_{h_e} \) can be written as:

\[
C_{h_e} = C_{h_\alpha} \cdot \alpha_h + C_{h_\delta} \cdot \delta_e + C_{h_\delta_t} \cdot \delta_t \]  

(3-23)

In (3-23), \( C_{h_\delta_t} \) is the derivative of the moment about the elevator hinge axis with respect to the tab deflection. Table 3.2 also mentions the derivatives \( C_{h_\alpha}, C_{h_\delta_t} \) and \( C_{h_\delta_t} \). They refer to the hinge moment about the hinge axis of the tab. These derivatives are made dimensionless, using \( S_t \) and \( c_t \) of the tab.
Fig. 3.8: The hinge moment coefficient $C_{he}$ as a function of $\alpha_h$, $\delta_e$, and $\delta_{le}$ for a tailplane of the Fokker F-27.

(Wind tunnel measurements from ref. 3.7)
Table 3.2: Abbreviated notation for the hinge moment derivatives

<table>
<thead>
<tr>
<th>Derivative</th>
<th>$\frac{\partial C_{he}}{\partial \alpha_h}$</th>
<th>$\frac{\partial C_{he}}{\partial \delta_e}$</th>
<th>$\frac{\partial C_{he}}{\partial \delta_t}$</th>
<th>$\frac{\partial C_{he}}{\partial \alpha_h}$</th>
<th>$\frac{\partial C_{he}}{\partial \delta_e}$</th>
<th>$\frac{\partial C_{he}}{\partial \delta_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dutch and American notation 1)</td>
<td>$C_{h\alpha}$</td>
<td>$C_{h\delta}$</td>
<td>$C_{h\delta_t}$</td>
<td>$C_{ht\alpha}$</td>
<td>$C_{ht\delta}$</td>
<td>$C_{ht\delta_t}$</td>
</tr>
<tr>
<td>British notation 2)</td>
<td>$b_1$</td>
<td>$b_2$</td>
<td>$b_3$</td>
<td>$c_1$</td>
<td>$c_2$</td>
<td>$c_3$</td>
</tr>
</tbody>
</table>

1) based on the norm V 995
2) see ref. 3.2

For more accurate calculations, $C_{he}$ has to be obtained like $C_{Nh}$ from wind tunnel measurements, using a model of the tailplane at as large a scale as possible.

If the hinge line of the control surface is situated right at the leading edge of the surface, the change in hinge moment, due to both an increase in angle of attack and an increase in control deflection, will be negative. This can be seen from the pressure distributions given in fig. 3.5. In such a situation both derivatives are negative:

$$C_{h\alpha} < 0 \quad \text{and} \quad C_{h\delta} < 0$$

It follows from (3-2) that the hinge moment of (hypothetical) airplanes having identical shape but various sizes, varies with the square of the airspeed and the cube of the dimensions of the airplanes. Fast and large airplanes would soon have variations in the hinge moment with varying flight conditions which could become quite large. The pilot, who has to exert through the (manual) control system a hinge moment, equal but opposite in sign to $H_e$ in steady
flight, would be required to produce rather large control forces. In order to keep the required control forces and their variations with the flight condition sufficiently small, the designer has to aim at values of \( C_h \) and \( C_{h \delta} \) sufficiently small in the absolute sense. This can be done by aerodynamically balancing the control surfaces.

One of the means to change \( C_h \) and \( C_{h \delta} \) in the positive sense, is to employ a rearward shift of the hinge axis, or a forward extension of the nose of the control surface in front of the hinge line. As long as both \( C_{h \alpha} \) and \( C_{h \delta} \) are negative, the control surface is called aerodynamically underbalanced. Increasing the rearward shift of the hinge-line or forward extension of the balance nose will make the control surface overbalanced with respect to the angle of attack, or the control deflection, or both. \( C_h \) and \( C_{h \delta} \) or both will then be positive see fig. 3.9.

A quantitative calculation of the hinge moment derivatives of a control surface of given shape and dimensions is not always possible with sufficient accuracy. This is mainly due to the relatively large influence of viscous effects, which are difficult to account for quantitatively. Estimates of the two derivatives \( C_{h \alpha} \) and \( C_{h \delta} \) of an unbalanced control surface are determined mainly by the ratio of the control surface chord to the tailplane chord. Fig. 3.10 gives an example for a two-dimensional profile. The shape of the tailplane, the profile and the detailed form of the gap between stabilizer and control surface also influences \( C_h \) and \( C_{h \delta} \).

3.5. The direction of the airflow and the average dynamic pressure at the horizontal tailplane

If the horizontal tailplane is located as usual behind the wing, it experiences the field of flow disturbed by the wing, the fuselage, the nacelles, the propellers etc. Mainly due to the downwash behind the wing, the average angle of attack \( \alpha_h \) of the horizontal tailplane is decreased. If the horizontal tailplane lies in the wake produced by the wing or in the flow retarded by the fuselage, also the local average dynamic pressure at the horizontal tailplane is lower than that of the undisturbed flow. This is expressed in a value of \( \frac{\rho_h}{\rho} < 1 \) in (3-17).
Fig. 3.9: The occurrence of overbalance with respect of angle of attack and elevator angle due to elongation of the elevator nose.
Fig. 3.10: \( c_{h\alpha} \) and \( c_{h\delta} \) as functions of \( \frac{c_e}{c_h} \).

\( \alpha \) and \( \delta \) in rad.

Fig. 3.11: The angle of attack \( \alpha_h \) of the horizontal tailplane.

\[ \alpha_h = \alpha - \varepsilon + \iota_h \]
Both effects cause a change in the contribution of the horizontal tailplane to the moment about the airplane lateral axis. The slipstream of the propeller-driven airplanes and the exhaust stream of jet-propelled airplanes may also cause changes in $\alpha_h$ and $\left(\frac{V}{V_h}\right)^2$.

If the average downwash angle at the location of the horizontal tailplane is designated $\varepsilon$, and the tailplane incidence setting relative to the $X_m$-axis is $i_h$, the average angle of attack of the horizontal tailplane follows from, see fig. 3.11:

$$\alpha_h = \alpha - \varepsilon + i_h$$  \hspace{1cm} (3-24)

The downwash angle is approximately proportional to $C_L$, in the range where $C_L$ varies linearly with $\alpha$:

$$\varepsilon = (\alpha - \alpha_o) \frac{dc}{d\alpha}$$

Where $\alpha_o = C_{L_o} = 0$ and $\frac{dc}{d\alpha}$ is constant to a first approximation. Thus $\alpha_h$ can be written as:

$$\alpha_h = (\alpha - \alpha_o) \left(1 - \frac{dc}{d\alpha}\right) + (\alpha_o + i_h)$$  \hspace{1cm} (3-25)

From (3-25) the variation of $\alpha_h$ with $\alpha$, which is of interest for the static stability, can easily be obtained:

$$\frac{d\alpha_h}{d\alpha} = \left(1 - \frac{dc}{d\alpha}\right)$$  \hspace{1cm} (3-26)

The downwash angle $\varepsilon$ and the factor $\left(\frac{V}{V_h}\right)^2$ depend in the first place on the shape of the wing and on the location of the horizontal tailplane relative to the wing. It is assumed to be well known, that the downwash behind the wing is induced by vortices, both bound and free, originating at the trailing edge of the wing. The lifting vortex creates an upwash in front of the wing and a downwash behind the wing.

The free vortices reinforce the downwash, and reduce the upwash, see fig. 3.12. Under the influence of the vertical velocities induced by the vortices, the vortex plane itself is displaced downward. It assumes the form depicted in fig. 3.13. The vortex plane leaves the trailing edge of the wing tangential to the plane of the wing. Further downstream the induced downwash decreases, the vortex plane is bent in the direction of the undisturbed flow. The upwash, the
Fig. 3.12: The contributions of the lifting and the free vortices to the induced vertical velocities in front of and behind the wing.

Fig. 3.13: The position of the vortex plane behind the wing.
downwash and the depression of the vortex plane are approximately proportional to $C_L$ and inversely proportional to the wing aspect ratio.

The position of the wing wake generally coincides with the position of the vortex plane behind the wing. The reduction of the dynamic pressure and the height of the region of reduced flow speed both depend on the profile drag and the distance behind the wing.

The depression of the wing wake relative to the undisturbed flow, increases with angle of attack. But it is smaller than the vertical displacement of the horizontal tailplane itself, see fig. 3.14. As a consequence, a horizontal tailplane mounted high up at the vertical tail will cross the wing wake at relatively large angles of attack. This leads to the conclusion that the horizontal tailplane should preferably be mounted so high that it remains above the wing wake, or so low as to remain below the wake at all angles of attack occurring in flight.

The vortex plane is not stable. It changes its shape and downstream it is transformed into two tip vortices, see fig. 3.15. This process depends on the lift distribution over the wing. It is more intense at large lift coefficients and small aspect ratios, see refs. 3.8 and 3.9. For wings having a large aspect ratio and a small sweep angle, the curling up of the vortex plane is of little importance in the range of practical angles of attack. For these wings the calculation of the downwash at the horizontal tailplane can be made with good approximation on the basis of an undeformed, curled vortex plane. On wings having a small aspect ratio or a large sweep angle or both, flow separation often occurs in one form or another at small angles of attack.

According to ref. 3.10, the influence of the deformation and the curling up of the vortex plane on the downwash close to the wing plane of symmetry may as well be neglected at low values of $C_L$. In general the consequences of flow separation are more important form the downwash than the curling up of the vortex plane, as will be discussed later.

Fig. 3.16 gives the theoretical value $\frac{dc}{da}$ in the core of the vortex plane, as a function of aspect ratio and distance behind the wing, for wings having an elliptical lift distribution and unseparated flow. The dominant influence of the wing aspect ratio is apparent.
\[
\Delta z_h \int \Delta \alpha \, dx = \Delta \alpha (x_h - x_w)
\]

\[
\Delta z_{\text{wake}} \int \Delta \varepsilon(x) \, dx
\]

\[
\Delta z_{\text{wake}} = \Delta z_h \text{ as } \Delta \varepsilon(x) \text{ is smaller then } \Delta \alpha
\]

With increasing angle of attack the stabilizer moves from top to bottom through the wake.

Fig. 3.14: The vertical displacement of the wake and the tailplane relative to the flow field, due to an increase in angle of attack \( \Delta \alpha \).

A rough estimate of \( \frac{d\varepsilon}{d\alpha} \) at sufficiently large distances behind the wing is possible for straight wings of not too small an aspect ratio.

The formula to be used is:

\[
\frac{d\varepsilon}{d\alpha} = 2 \frac{C_L \alpha}{\pi A} = \frac{2}{\pi A} \cdot 2\pi \frac{A}{A+2}
\]

\[
= \frac{4}{A+2}
\]
Fig. 3.15: The deformation and curling up of the vortex plane behind the wing.
Fig. 3.16: Calculated values of \( \frac{d\psi}{d\alpha} \) behind a wing of elliptical lift distribution, as a function of aspect ratio and distance behind the wing.
Fig. 3.17: The influence of wing taper ratio on the calculated downwash distribution in span wise direction ($C_L = 1.0; A = 6.0; \Lambda = 0^\circ; \frac{x}{b/2} = 1.0$). Calculated with lifting line theory, see ref. 2.1.
Fig. 3.18: The influence of wing sweep on the calculated downwash distribution in spanwise direction 
\( (C_L = 1.0; \alpha = 6.0; \lambda = 1.0; \frac{x}{b/2} = 1.0) \). Calculated with extended lifting line theory; see ref. 21.
Fig. 3.19: Downwash angles in the plane of symmetry behind the wing (From ref. 3.12).
This expression results in values of $\frac{dc}{da}$ for $A > 5$ which are in very good agreement with the more exact values given in fig. 3.16 for the elliptical wing.

Fig. 3.17 shows the variation in spanwise direction of the downwash behind wings of different taper ratios. For wings having small values of $\lambda (= c_t/c_r)$ the average downwash angle near the plane of symmetry is larger than for wings having larger values of $\lambda$ at equal values of $C_L$. This is caused by the higher lift loading near the wing center of former wings. As a consequence $\frac{dc}{da}$ at the horizontal tailplane increases with decreasing $\lambda$. Sweepback of the wing causes a lower loading near the wing center. As a consequence, $\frac{dc}{da}$ decreases with increasing sweep angle $\Lambda$, see fig. 3.18.

The downwash angle decreases with increasing distances above and below the vortex plane. Fig. 3.19 shows the results of measurements of the downwash angle in the plane of symmetry behind the tapered wing. From this figure it is clear that the downwash in a certain point above the wake is larger than in the point at an equal distance below the wake. This is due to the fact that as a result of the depression of the wake and of the deformation of the vortex plane the free vortices and the lifting vortex contribute more to the downwash above the wake than below the wake. In addition, a certain inflow occurs into the wake. Due to this inflow, the downwash increases above the wake and decreases below the wake.

The field of flow behind wings having separated flow is considerably different from the field of flow behind wings showing attached flow. Flow separation may cause appreciable changes in $\frac{dc}{da}$ with increasing angle of attack. A detailed discussion of these changes related to flow separation can be found in ref. 3.10. For straight wings having little taper, i.e. having a large $\lambda$, flow separation starts near the wing root. The reduction in lift over the wing center results in a decrease of $\frac{dc}{da}$. In addition a broad and deep wake is formed, causing a strong inflow into the wake. Flow separation near the wing tips, occurring with highly tapered and swept back wings, causes an increase in downwash at the tailplane, due to the decreased effective wing aspect ratio caused by the flow separation.

Swept wings and delta wings having thin profiles often exhibit flow separation near the wing leading edge, already at small angles of attack. This leads to characteristic conical vortices above the wing. Such separation vortices may considerably increase the downwash.

Variations in the camber and nose shape of the profiles, wing twist, boundary layer fences, nose flaps and slats etc. all aim to influence the separation pattern. It will be clear that in this way also a considerable influence is exerted on the downwash at the horizontal tailplane.
The fuselage causes a change in the flow behind the wing. This is due to the flow around the fuselage itself and due to the change in the lift distribution caused by the fuselage. The resultant influence on the average downwash at the horizontal tailplane is relatively small for slender wings. For wings of small aspect ratio the influence of the fuselage can be considerable and is strongly dependent on the position of the horizontal tailplane relative to the fuselage. If the tailplane is mounted on the fuselage, a decrease in the average value of $V \frac{h^2}{V}$ occurs, as part of the tailplane is in the boundary layer of the fuselage. In such cases a reduction in $V \frac{h^2}{V}$ of some .05 to .10 is often taken into account.

Flap deflection causes a concentration of the lift in the center part of the wing. This produces an increase in downwash. As a consequence, but certainly also due to the decrease in $\alpha_0$, the angle of attack of the horizontal tailplane decreases.

A calculation of the downwash behind wing-fuselage combinations with deflected landing flaps is generally difficult, due to the important influence of the wing-fuselage interference.

In take-off and landing the field of flow around and in particular behind the wing is strongly influenced by the proximity of the ground. As is well-known, the ground effect can be modelled by the use of additional lifting vortices and trailing vortices, employing the ground as a mirror plane. Then the influence of the mirror vortices on the various parts of the airplane is calculated, see fig. 3.20.

The mirror vortices cause a decrease in the induced angles of attack and a corresponding decrease in the effective angles of attack, proportional to the strength of the mirror vortices and thus with $C_L$.

The result of the ground effect is an increase in $C_{N_w\alpha}$ and $C_{N_h\alpha}$. As $C_{N_h\alpha}$ is proportional to $C_{N_h\delta}^2$, $C_{N_h\delta}$ increases as well.

The downwash angles behind the wing decrease under the influence of the mirror vortices. This effect can also be understood directly, as the presence of the ground inhibits the downward flow behind the wing. Fig. 3.21a gives an example of the change in $\varepsilon$ at the horizontal tailplane caused by the ground influence. This figure also demonstrates that the reduction in $\frac{d\varepsilon}{d\alpha}$ due to the ground effect can be quite important.
An additional consequence of the reduced downwash angles behind the wing is a smaller downward displacement of the wake, as compared with the situation without ground effect, see fig. 3.21b. When choosing the vertical position of the horizontal tailplane in the course of the design process, this effect may have to be seriously considered.

A reasonably accurate theoretical determination of the average downwash angle and the average dynamic pressure at the horizontal tailplane is in general only possible for configurations having a slender wing and in conditions of attached flow. In addition, the influence of the fuselage has to be small.

For slender, straight wings without and with deflected flaps refs. 3.2 and 3.11 provide calculation methods, see also ref. 3.13. The downwash behind swept wings can be determined with refs. 3.14-3.18. An extensive discussion of the ground effect is given in ref. 3.19. The effect of the ground on the downwash and the wake can be calculated with ref. 3.20.

For wings having a small aspect ratio or a large sweep angle or both, where the fuselage has a considerable influence on the field of flow at the horizontal tailplane and where in addition flow separation occurs already at low angles of attack, the flow field at the horizontal tailplane can be derived only from measurements in the wind tunnel. An extensive summary and analysis of experimental data is given in ref. 3.10, where also a method is described to estimate the characteristics of the horizontal tailplane.
Fig. 3.21: The ground effect on the downwash angle $\varepsilon$ and the location of the wake relative to the horizontal tailplane of a Siebel 204 D-1 airplane.
3.6. The tail load as a function of airspeed and c.g. position

As discussed in 3.2, a normal force acts on the horizontal tailplane to produce the equilibrium of the longitudinal moment in an equilibrium situation. This normal force is called the tail load. The magnitude of the tailload \( N_h \) follows from the expression with \( \frac{1}{2} \rho V^2 S \)

\[
M = 0 = C_{m_{a.c.}} \frac{1}{2} \rho V^2 Sc + N_w \cdot (x_{c.g.} - x_w) - N_h \cdot \lambda_h \quad (3-27)
\]

or:

\[
N_h = \frac{1}{\lambda_h} \left[ C_{m_{a.c.}} \frac{1}{2} \rho V^2 Sc + N_w (x_{c.g.} - x_w) \right] \quad (3-28)
\]

At small values of the angle of pitch \( \theta \), if in addition \( N_h \ll N_w \), it follows with (3-14):

\[
N_w \approx W
\]

The result then is:

\[
N_h = \frac{1}{\lambda_h} \left[ C_{m_{a.c.}} \frac{1}{2} \rho V^2 Sc + W (x_{c.g.} - x_w) \right] \quad (3-29)
\]

This also follows directly from fig. 3.22, where the model of the equilibrium of the forces and the moment has been given in a strongly simplified form.

It can be seen from (3-29) that the tail load \( N_h \) is needed:

1. to compensate the moment caused by the coefficient \( C_{m_{a.c.}} \); this moment depends on the airspeed, but not on the center of gravity position; in addition, it varies with flap deflection \( \delta_f \);

2. to balance the moment of \( N_w (\approx W) \) about the center of gravity; this moment is independent of the airspeed, but varies with the center of gravity position.

From (3-29) it can be seen, that \( N_h \) varies quadratically with \( V \) and increases in the upward sense with a rearward shift of the center of gravity. Fig. 3.23 gives a sketch of the variations of the tail load with airspeed for various c.g. positions and signs of \( C_{m_{a.c.}} \).
Fig. 3.22: Simplified picture of the equilibrium of the moments.

Fig. 3.23: The variation of the tail load with airspeed at different c.g. positions and values of $C_{m_{a,c}}$. 
Fig. 3.24: The tail load as a function of airspeed at two c.g. positions for the De Havilland "Mosquito II F."
(From ref. 3.21)
The point of intersection of the parabola with the line $V = 0$ - in actual flight the airspeed can not of course drop below $V_{\text{min}}$ - follows from (3-29):

$$N_h = \frac{W \cdot \frac{X_{c-g} - X_w}{l_h}}{c_h}$$

Fig. 3.23 illustrates that due to the usually negative sign of $C_{n_{a,c}}$ of the wing fuselage and nacelles, the tail load reaches its largest positive value at the rearmost c.g. position and at minimum airspeed. The largest negative, downward, tail load occurs for a given negative $C_{n_{a,c}}$ at the most forward c.g. position and at a maximum airspeed.

Fig. 3.24 presents tail loads measured in flight on a De Havilland 'Mosquito'. The influence on $N_h$ of airspeed as well as of c.g. position agrees in principle with (3-29). From the figure can be derived for the range of airspeeds $350 < V_e < 500 \text{ km/h}$: $x_w = 0.08 \bar{c}$ and $C_{n_{a,c}} = -0.021$.

3.7. The elevator deflection required for equilibrium of the moment

The way in which the elevator is used to produce the equilibrium of the longitudinal moment, follows from the expression (3-17) for that moment:

$$C_m = C_{n_{a,c}} + C_{N_w} \frac{X_{c-g} - X_w}{c} - C_{N_h} \left( \frac{V_h}{v} \right)^2 \frac{S_h \cdot l_h}{S \cdot c} = 0$$

(3-17)

$C_{N_w}$ can be written in the linear range as:

$$C_{N_w} = C_{N_w \alpha} \cdot (\alpha - \alpha_0)$$

(3-30)

In 3.3 $C_{N_h}$ has been expressed by:

$$C_{N_h} = C_{N_h \alpha} \cdot \alpha + C_{N_h \delta} \cdot \delta_e$$

(3-20)
while in 3.5 the expression for the angle of attack of the horizontal tailplane $\alpha_h$ was found to be:

$$\alpha_h = (\alpha - \alpha_o) \left(1 - \frac{d\alpha}{da}\right) + \left(\alpha_o + \alpha_h\right)$$

(3-25)

Substituting (3-30), (3-20) and (3-25) in (3-17) results in the condition for equilibrium of the moment:

$$c_m = c_{m_{a.c.}} + c_{N_{w_a}} \cdot \frac{x_{c.g. - x_w}}{c} \cdot \left[\left(\alpha - \alpha_o\right)\left(1 - \frac{d\epsilon}{da}\right) + \left(\alpha_o + \alpha_h\right)\right] + c_{N_{h_o}} \cdot \delta_e \right] \cdot \frac{V_h^2 \cdot S_{h \cdot \dot{x}_h}}{S \cdot c} = 0$$

(3-31)

or, in an ordered form:

$$c_m = c_{m_o} + c_{m_a} \cdot \left(\alpha - \alpha_o\right) + c_{m_o} \cdot \delta_e = 0$$

(3-32)

where:

a. $c_{m_o}$ is a constant:

$$c_{m_o} = c_{m_{a.c.}} - c_{N_{h_a}} \left(\alpha_o + \alpha_h\right) \frac{V_h^2 \cdot S_{h \cdot \dot{x}_h}}{S \cdot c}$$

(3-33)

b. $c_{ma}$ is the static longitudinal stability, stick fixed, see 4.2:

$$c_{ma} = c_{N_{w_a}} \frac{x_{c.g. - x_w}}{c} - c_{N_{h_a}} \left(1 - \frac{d\epsilon}{da}\right) \frac{V_h^2 \cdot S_{h \cdot \dot{x}_h}}{S \cdot c}$$

(3-34)
c. $C_{m_{\delta}}$ is the elevator effectivity:

$$\frac{C_{m_{\delta}}}{C_{N_{h_{\delta}}}} = - \frac{V}{V} \left( \frac{S_{h_{\delta}}}{S_{c}} \right) \frac{S_{h_{\delta}}}{S_{c}}$$

(3-35)

For a given airplane the pilot can obtain equilibrium of the moment at a certain angle of attack, or $\alpha - \alpha_0$, and c.g. position, or $C_{m_{\alpha}}$, see (3-34), by deflecting the elevator, i.e. by choosing $\delta_{e}$, such that the resultant $C_{m}$ is equal to zero.

If the total moment $C_{m}$ is written as the sum of the moment at $\delta_{e} = 0$ and the moment due to the elevator deflection:

$$C_{m} = (C_{m_{\delta}})_{\delta_{e}=0} + C_{m_{\delta}} \cdot \delta_{e}$$

(3-36)

the condition of equilibrium of the total moment, $C_{m} = 0$, results in the required elevator deflection:

$$\delta_{e} = - \frac{1}{C_{m_{\delta}}} (C_{m})_{\delta_{e}=0}$$

(3-37)

Since, according to (3-32):

$$C_{m} = C_{m_{\delta}} + C_{m_{\alpha}} (\alpha - \alpha_0)$$

(3-38)

$\delta_{e}$ is:

$$\delta_{e} = - \frac{1}{C_{m_{\delta}}} \left[ C_{m_{\delta}} + C_{m_{\alpha}} (\alpha - \alpha_0) \right]$$

(3-39)

The expressions derived above are based on the strongly simplified equation for the moment as derived in 3.2. In addition, use has been made of several linearizations. As a consequence, the above expressions are valid, strictly speaking, only for those angles of attack and elevator angles where $C_{N_{w}}$ and $C_{N_{h}}$
are proportional to $\alpha$, or $\alpha_h$ and $\delta_e$ respectively. For accurate calculations it will nearly always be necessary to fall back on the results of wind tunnel measurements made on models at a sufficiently large scale, where $C_m$ has been measured as a function of $\alpha$, $i_h$ and $\delta_e$.

The expressions for the moment and the elevator deflection required for equilibrium apply in the given form only to an airplane having a fixed stabilizer and a movable elevator. In many cases, however, the available power $|C_{m_0,\delta_e}|$ is insufficient to obtain equilibrium of the moment at all desired c.g. positions over the entire range or airspeeds.

Often an adjustable stabilizer, having a variable $i_h$, is employed. For such airplanes, $C_m$ in (3-33) no longer is a constant for all airplane configurations and flight conditions. Sometimes use is also made of a movable horizontal tailplane having no separate elevator to obtain equilibrium, the 'flying-tail'. In a way analogous to the above, expressions can be derived for such cases, resulting, in the value of $i_h$ required for equilibrium.

With regard to the variables occurring in (3-31) and (3-35) the following remarks can be made:

$C_m$, $C_{N_{\alpha/\alpha}}$ and $\alpha$ are fixed by the aerodynamic design of the wing, the fuselage and the nacelles.

The required tailvolume $\frac{S_h i_h}{S c}$ is determined primarily by the requirement that the airplane must be statically stable at the rearmost permissible c.g. position. The derivative $C_{N_{\alpha/\alpha}}$ is fixed in substance after the choice of the geometry of the tailplane, see 3.3.

If the angle of incidence of the horizontal tailplane, $i_h$, is not adjustable, $i_h$ is chosen preferably such, that in cruising flight at an average c.g. position the elevator angle is zero, to minimize the drag of the tailplane.

Both $\frac{dc}{d\alpha}$ and $\frac{V^2}{V_h}$ are determined by the characteristics of the wing and the location of the horizontal tailplane relative to the wing and the fuselage, see 3.5.

From the variables just discussed the value of $C_{m_{\delta_e=0}}$ follows for every angle of attack and c.g. position. This moment has to be compensated by the moment generated by the elevator $C_{m_\delta \delta_e}$, see (3-36).

For most airplanes the maximum, positive value of $C_{m_\delta \delta_e}$ (elevator up)
required in any steady flight condition, is larger than the minimum, negative (elevator down) value.

The largest positive values of \( C_{m e} \delta \) will be needed to compensate the largest value of \( C_{m e} \delta = 0 \). The latter occurs in principle at the most forward c.g. position and the maximum angle of attack.

Ground effect usually produces a further negative contribution to \( C_{m e} \delta = 0 \). As a consequence, for many airplanes the required size of the elevator is determined by the requirement that the aircraft can take-off and land at the most forward c.g. position. This implies also that the size of the elevator is dictated mainly by an aircraft configuration and a flight condition in which the airflow at several places on the airplane is separated, such as for instance on the elevator itself.

As a consequence the linearized expressions for the aerodynamic coefficients are merely a rough approximation of reality in those conditions. This increases the difficulty of estimating the required size of the elevator, if no measurements on wind tunnel models are available. Similar difficulties arise in principle in the determination of the required sizes of the ailerons and the rudder.

3.8. The longitudinal control force required for equilibrium about the lateral axis

To obtain equilibrium of the moment about the lateral axis, the pilot has to apply and maintain a certain elevator deflection. To this end he has to exert a certain force on the control stick or wheel. In many modern aircraft the connection between the pilot's control manipulator and the elevator is such, that this effort provides only part of the total required hinge moment. Sometimes there is even no direct mechanical connection at all between the manipulator and the control surface. Such more complex situations will not be considered here.

In designing the control system, the aim is always to arrive at control forces varying in the same way with airspeed, c.g. position etc., as is the case in the elementary situation to be further discussed in these notes. In fig. 3.25 a normal manual control system is depicted. The positive directions of the control deflections \( s \), control forces \( F \), surface angles \( \delta \) and hinge moments \( H \) have been indicated. In the following the longitudinal control force required for equilibrium about the lateral axis, \( F_{e} \), will be considered in more detail.
Fig. 3.25: The positive direction of control deflections, control forces, control surface deflections and hinge moments.

\[ F_e \Delta s_e + H_e \Delta \delta_e = 0 \]

Fig. 3.26: The relation between the control force and the hinge moment.
Similar discussions can be given in principle for the control about the lateral axis, \( F_a \), and for the control force on the rudder pedals, \( F_r \).

If the longitudinal control manipulator, for short: the control stick, is displaced a distances \( ds_e \), the control force \( F_e \) delivers an amount of work \( F_e ds_e \). At the same time the elevator deflects through the angle \( d\delta_e \), and the hinge moment \( H_e \) produces an amount of work \( H_e d\delta_e \). If the internal friction in the control system as well as stretch in the cables is neglected, the control system absorbs no work internally and the total amount of external work has to be zero, if the stick and the elevator are again at rest after the displacement.

This implies:

\[
F_e ds_e + H_e d\delta_e = 0 \tag{3-40}
\]

or, see also fig. 3.26:

\[
F_e = -\frac{d\delta_e}{ds_e} H_e \tag{3-41}
\]

where the gear ratio \( \frac{d\delta_e}{ds_e} \) is positive in any normal control system, see fig. 3.25.

The total hinge moment to be balanced by the control force, see fig. 3.27, consists of several components: the aerodynamic hinge moment \( H_{ea} \), a hinge moment caused by friction in the control system \( H_{ef} \) and, if the control surface is not statically balanced, a hinge moment \( H_{ew} \) caused by the static unbalance. In many airplanes a spring is applied in the control system, giving rise to yet another component of the total hinge moment, \( H_{es} \).

![Diagram of hinge moments](image-url)
The total hinge moment then is:

\[ H_e = H_{e_a} + H_{e_f} + H_{e_w} + H_{e_s} \]

In order to simplify the discussions, the hinge moments due to friction, static unbalance and the spring will be omitted in the following. This results in:

\[ H_e = H_{e_a} \]

Substituting (3-21) and (3-41) the longitudinal control force can then be written as:

\[ F_e = - \frac{d\delta_e}{ds_e} \cdot \frac{1}{2} p V^2 S \bar{c} e \cdot C_{h_e} \]

and, using the linearized expression (3-23) for \( C_{h_e} \):

\[ F_e = - \frac{d\delta_e}{ds_e} \cdot \frac{1}{2} p V^2 S \bar{c} e \left( C_{h_{\alpha}} \alpha + C_{h_{\delta}} \delta_e + C_{h_{\delta_t}} \delta_t e \right) \]

Suppose now, that for a given c.g. position and angle of attack, or airspeed, the elevator angle, \( \delta_e \), required for equilibrium of the moment about the lateral axis has been determined according to (3-39). The longitudinal control force required to maintain this elevator angle, for a given tab angle \( \delta_t e \), follows from (3-42).

Chapter 5 deals in more detail with this control force as a function of airspeed, c.g. position and tab angle.

A few remarks can be made on the required sign and the magnitudes of \( C_{h_{\delta_t}} \), \( C_{h_{\delta}} \) and \( C_{h_{\alpha}} \) based on the requirement to obtain certain control characteristics.

The magnitude of \( C_{h_{\delta_t}} \) is determined by the requirement that in steady flight conditions which may have to be maintained for some time, it has to be possible to trim the control forces down to zero, by suitably adjusting the trim tab angle.

A more detailed definition of the aircraft configurations and flight conditions for which this requirement holds, can be found in the airworthiness require-
ments, see refs. 1.12-1.18. The required magnitude of \( C_{h_{\delta_t}} \) also depends on the permissible tab deflecting angles, \( \delta_t \), and on the values of \( C_{h_{\alpha}} \) and \( C_{h_{\delta}} \).

As has been mentioned before, it is generally desirable that both \( C_{h_{\alpha}} \) and \( C_{h_{\delta}} \) are small in the absolute sense, in order to keep the required control forces small when changing the aircraft configuration, the steady flight condition or when manoeuvring the aircraft.

It is shown in 5.2 to be necessary that \( C_{h_{\delta}} \) is negative. Usually \( C_{h_{\alpha}} \) is negative as well, but small positive values of \( C_{h_{\alpha}} \) are permissible and are indeed used in practice. When studying the control forces in more detail in 5.6, this will be further elaborated.
4. STATIC LONGITUDINAL STABILITY, STICK FIXED

4.1. Introduction

In the preceding chapter the equilibrium of the forces and moment acting on the airplane was discussed. The function of the horizontal tailplane and elevator in maintaining the equilibrium of the moment was also described. The expressions for the equilibrium may be repeated here, see (3-13), (3-14) and (3-15):

\[ C_T = \frac{C_{T_w}}{\frac{W}{\frac{1}{2} \rho V^2 S}} \sin \theta \quad (3-13) \]

\[ C_N = \frac{C_{N_w}}{\frac{W}{\frac{1}{2} \rho V^2 S}} + \frac{C_{N_h}}{\frac{V_h}{V}} \left( \frac{S_h}{S} \right) \cos \theta \quad (3-14) \]

\[ C_m = C_{m_{a.c.}} + \frac{C_{N_w}}{\frac{V_h}{V}} \frac{x_{c.g.} - x_w}{c} + \frac{C_{N_h}}{\frac{V_h}{V}} \frac{S_h}{S} \frac{x_{c.g.} - x_h}{c} = 0 \quad (3-15) \]

Also used for the equilibrium of the moments was (3-17):

\[ C_m = C_{m_{a.c.}} + \frac{C_{N_w}}{\frac{V_h}{V}} \frac{x_{c.g.} - x_w}{c} - \frac{C_{N_h}}{\frac{V_h}{V}} \frac{S_h}{S} \frac{x_{c.g.} - x_h}{c} \quad (3-17) \]

It has been noted in 1.1 that the equilibrium of an airplane in flight has to be a stable one. This means that after the occurrence of a small disturbance the airplane should return to the original equilibrium situation. If this is the case, the equilibrium is called dynamically stable. Although, as remarked in 1.1, stability is strictly a characteristic of an equilibrium, it is common practice to call an airplane dynamically stable if its equilibrium is stable.

The motions of an airplane after a disturbance, and with it the dynamic stability, are determined by the equations of motion of the airplane. A discussion of the airplane motions, using these equations of motion, is postponed at this stage, see however Chapter 8.

There is, as in 1.2, a simpler way to study stability, based on the concept of static stability. Experience has shown, that the most critical requirements
to be satisfied for dynamic stability can be expressed for most airplanes by the condition that the airplane must be statically stable.

The discussion of static stability is based on the equations for the equilibrium of the airplane (3-13), (3-14) and (3-17). These equations are in fact special cases of the more general equations of motion.

Static longitudinal stability is considered using only the expression for the longitudinal moment (3-17) or (3-15). The two expressions for the equilibrium of the forces do not have to be taken into consideration. This means that the change in moment is studied, caused by a change in angle of attack, i.e. caused by a change in attitude of the airplane relative to the airstream.

In a way analogous to the study of a wing in 2.5, in this chapter the static stability of the entire airplane is studied. The static stability on both the stick fixed and the stick free situation is considered.

In the present chapter it will be shown that a close relationship exists between the way in which the stick position varies with airspeed or angle of attack in steady, straight flight and the static stability, stick fixed.

It makes sense to study also the stability of the equilibrium in a steady flight condition, in the situation where pilot does not hold the control stick. In actual flight this so called 'stick free' situation is quite common. It is desirable that the equilibrium is stable, also in the stick free case. It will be shown in Chapter 5 that the static stability, stick free, is closely related to the way in which the stick force varies with airspeed in steady, straight flight.

In order to simplify the discussion, only gliding flight will be considered. The aerodynamic coefficients are assumed to be independent of airspeed.

4.2. Static longitudinal stability, stick fixed, in gliding flight

In paragraph 2.5 the concept of static stability was presented for a wing alone. This discussion is now extended to a complete airplane. Suppose the airplane is in steady flight, which means that the moment about the center of gravity is zero: \( C_m = 0 \). If in this situation an external disturbance causes an increase in angle of attack, the result of this increase in \( \alpha \) should be a negative, nose-down change in \( C_m \).

The condition for static longitudinal stability then is:
\[
\frac{dC_m}{d\alpha} = C_{m_\alpha} < 0, \text{ at } C_m = 0 \tag{4-1}
\]

If \(C_m = 0\) the airplane possesses static neutral stability. If \(C_{m_\alpha} > 0\) the airplane is statically unstable.

To study the various contributions to the derivative \(C_m\) of the complete airplane, the expression for the moment coefficient is written once more as (3-31):

\[
C_m = C_{m_{a.c.}} + C_{N_{w_\alpha}} \cdot (t - \alpha_0) \cdot \frac{X_{c.g} - X_{w}}{c} - \left[ C_{N_{h_{\alpha}}} \cdot \{(\alpha - \alpha_0) \left(1 - \frac{dc}{d\alpha}\right) + (\alpha_0 + t_{h_1})\} + \right.
\]

\[
+ C_{N_{h_\delta}} \cdot \delta_e \right] \cdot \frac{V_h}{V} \cdot \frac{S_h \cdot \delta_h}{S \cdot c} \tag{4-2}
\]

Eq. (4-2) can be rearranged to combine the contributions of the wing with fuselage and nacelles (indicated by the index \(w\)) and the contribution of the horizontal tailplane (indicated by index \(h\)):

\[
C_m = C_{m_w} + C_{m_h} \tag{4-3}
\]

where:

\[
C_{m_w} = C_{m_{a.c.}} + C_{N_{w_\alpha}} \cdot (\alpha - \alpha_0) \cdot \frac{X_{c.g} - X_{w}}{c} \tag{4-4}
\]

\[
C_{m_h} = - \left[ C_{N_{h_{\alpha}}} \cdot \{(\alpha - \alpha_0) \left(1 - \frac{dc}{d\alpha}\right)(\alpha_0 + t_{h_1})\} + C_{N_{h_\delta}} \cdot \delta_e \right] \cdot \frac{V_h}{V} \cdot \frac{S_h \cdot \delta_h}{S \cdot c} \tag{4-5}
\]
Fig. 4.1: The contributions of the various parts of the airplane to the moment curve $C_m - \alpha$.

The figs. 4.1a, b and c schematically show the $C_m - \alpha$-curves of the contributions according to (4-4) and (4-5) and of the complete airplane according to (4-2). Fig. 4.2 presents the contributions of the various parts of the airplane to the $C_m - \alpha$-curve as measured in the wind tunnel on a model of the Fokker F-27.

The slopes of the moment curves follow from (4-4) and (4-5) by differentiating with respect to $\alpha$. Because the 'stick fixed' situation is considered, the elevator angle $\delta_e$ is left constant.

$$C_{m_a} = C_{N_w \alpha} \cdot \frac{x_c - x_W}{c} \quad (4-6)$$
Fig. 4.2: Measured contributions of the various airplane parts to the moment curve of the Fokker F-27. Reference point at 0.346\%.
(From ref. 4.1).

\[ C_{m_{\alpha}} = -C_{N_{\alpha}} \cdot (1 - \frac{dC}{d\alpha}) \cdot \frac{V^2}{\rho} \cdot \frac{S_{h} \cdot L_{h}}{S \cdot c} \quad (4-7) \]

Because the c.g. usually lies behind the a.c. of the combination of wing with fuselage and nacelles, the derivative \( C_{m_{\alpha}} \) is positive, see (4-6).

This means, that if the airplane without the horizontal tailplane could be at equilibrium at all (\( C_{m_{w}} = 0 \)), the equilibrium would be unstable, see also 2.5.

The contribution of the tailplane has a stabilizing effect, according to (4-7): \( C_{m_{\alpha}} = 0 \). This contribution must be large enough, so that \( C_{m_{\alpha}} \) is, see
\[ C_m = C_{Nw} \cdot \frac{x_{cg} - x_w}{c} - C_{Nh} \cdot (1 - \frac{de}{da}) \left( \frac{v_h}{V} \right)^2 \cdot \frac{S_{h} \cdot z_h}{S \cdot c} \] (3-34)

is negative:

\[ C_{m\alpha} < 0 \]

The above shows the important function of the tailplane in obtaining static stability. This is additional to the function discussed in 3.2, in assuming equilibrium of the longitudinal moment.

Figs. 4.3 and 4.4 show another few measured moment curves. \( C_m \) varies linearly with \( \alpha \) over a large range of angles of attack. But at large values of \( \alpha \), \( C_{m\alpha} \) is no longer constant. This is due to the contribution of \( C_T \) to the longitudinal moment and to the non-linearity caused by flow separation.

---

Fig. 4.3: The influence of the tailplane incidence on the moment curve of the Fokker F-27. Reference point at 0.346\( \overline{c} \). (From ref. 4.1)
Strictly speaking, static stability is the slope \( C_m \) of the moment curve, only at \( C_m = 0 \). By varying \( i_h \) and \( \delta_e \) the moment curves can be shifted up and down. If the angle of attack is not too large, the shifted moment curves are all parallel, see fig. 4.3. A change in \( i_h \) influences \( C_m \) in the expression:

\[
C_m = C_m^{\alpha} + C_m^{\alpha_0} (\alpha - \alpha_0) + C_m^{\delta_e} \cdot \delta_e = 0
\]

(3-32)

only via the term \( C_m^{\alpha_0} \), which is independent of \( \alpha \):

\[
C_m^{\alpha_0} = C_m^{\alpha_0} \cdot \frac{N_h^{\alpha} (\alpha_0 + i_h) \left( \frac{V}{h} \right)^2}{S \cdot c} \cdot \frac{S_h \cdot h}{S \cdot c}
\]

(3-33)

A change in \( \delta_e \) influences \( C_m \) via the term \( C_m^{\delta_e} \), again independent of in principle:

\[
C_m^{\delta_e} \cdot \delta_e = -C_m^{N_h^{\delta_e}} \left( \frac{V}{h} \right)^2 \frac{S_h \cdot h}{S \cdot c} \cdot \delta_e
\]

(3-35)

If \( C_m \) can be made zero at any angle of attack by a suitable choice of \( i_h \) or \( \delta_e \) or both without influencing \( C_m^{\alpha_0} \), the slope of the moment curve at an angle of attack where \( C_m \neq 0 \) may be considered the static stability at that particular \( \alpha \).

4.3. The neutral point, stick fixed

It follows from (3-34) and fig. 4.4 that the c.g.-position has a strong influence on the static stability. This influence is studied in more detail here. In order to arrive at slightly more elegant expressions, the starting point is the formula for \( C_m^{\alpha_0} \), obtained by differentiating (3-15) with respect to \( \alpha \):

\[
C_m^{\alpha_0} = C_N w_{\alpha} \cdot \frac{x_{c.g.} - x_h}{c} + C_{N_h^{\alpha}} \cdot \left( 1 - \frac{dx}{d\alpha} \right) \left( \frac{V}{h} \right)^2 \frac{S_h}{S} \cdot \frac{x_{c.g.} - x_h}{c}
\]

(4-8)
The only difference between (4-8) and the expression (3-34) following from (3-17) is, that \( l_h \) has been replaced by \( x_h - x_{c.g.} \). If the shifts in c.g. position are small relative to the tail length - they are typically about 20% of \( c \) and \( l_h \) is 3 to 4 - the contribution of the tailplane to \( C_{m\alpha} \) may be considered as constant to a first approximation.

The principle effect of a change in c.g. position will be a change in the contribution of the wing-fuselage and nacelles to \( C_{m\alpha} \). Shifting the c.g. rearward (\( x_{c.g.} \) increases) will change the contribution \( C_{m\alpha} \), usually already positive, to grow further in the positive sense. The result is, that \( C_{m\alpha} \) of the complete airplane becomes less negative. A rearward shift of the center of gravity thus has a destabilizing effect, see also fig. 4.4.

At a c.g. position sufficiently far aft, the positive contribution of the wing-fuselage combination just compensates the negative contribution of the horizontal tailplane. Then:

\[
C_{m\alpha} = 0
\]

This center of gravity position, at which the equilibrium of the moment is neutrally stable, stick fixed, is called the neutral point, stick fixed (n.p. fix). This is the first interpretation of the neutral point, stick fixed. The abscissa of this point is \( x_{n_{fix}} \). If the center of gravity coincides with this neutral point, it follows from (4-8) that:

\[
C_{m\alpha} = 0 = C_{N_{\alpha}} \cdot \frac{x_{n_{fix}} - x_w}{c} + C_{N_{\alpha}} \cdot (1 - \frac{d\alpha}{d\alpha}) \frac{V}{V} \frac{S}{S} \cdot \frac{h}{h}
\]

\[
C_{m\alpha} = 0 = C_{N_{\alpha}} \cdot \frac{x_{n_{fix}} - x_w}{c} + C_{N_{\alpha}} \cdot (1 - \frac{d\alpha}{d\alpha}) \frac{2V}{V} \frac{s}{S} \frac{h}{h}
\]

(4-9)

From (4-9) the position of the neutral point is derived. To this end \( C_{N_{\alpha}} \) is eliminated from (4-9) by multiplying the expression for the normal force gradient:

\[
C_{N_{\alpha}} = C_{N_{\alpha}} + C_{N_{\alpha}} \cdot (1 - \frac{d\alpha}{d\alpha}) \frac{V}{V} \frac{2S}{S} \frac{h}{h}
\]

(4-10)
Fig. 4.4: The influence of the position of the reference point (center of gravity) on the moment curves of the Fokker F-27. (From ref. 4.1)
\[ \frac{x_{n_{\text{fix}}} - x_w}{c} \]

The result is:

\[ \frac{x_{n_{\text{fix}}} - x_w}{c} = \frac{C_{N_{\alpha}}}{C_{N_{\alpha}}} (1 - \frac{d\epsilon}{d\alpha}) \left( \frac{V}{W} \right)^2 \left( \frac{S_{n_f} \cdot h}{S \cdot c} \right) \]  \hspace{1cm} (4-11)

where, as in (3-16):

\[ l_h = x_h - x_w \]

Next, (4-9) is subtracted from (4-8). Using (4-10), the resulting expression for \( C_m \) is:

\[ C_{m_{\alpha}} = C_{N_{\alpha}} \cdot \frac{x_{c.g.} - x_{n_{\text{fix}}}}{c} \]  \hspace{1cm} (4-12)

Consider now by means of (4-12) a change in the moment \( dC_m \) due to a change in angle of attack \( d\alpha \), such that \( dC_m = C_{m_{\alpha}} \cdot d\alpha \). Since the accompanying change in the normal force is \( dC_N = C_{N_{\alpha}} \cdot d\alpha \), (4-12) may be written as:

\[ dC_m = dC_N \cdot \frac{x_{c.g.} - x_{n_{\text{fix}}}}{c} \]  \hspace{1cm} (4-13)

According to (4-13), the change in the moment \( dC_m \) may be interpreted as being caused by a change in the normal force \( dC_N \), acting a distance \( \frac{x_{c.g.} - x_{n_{\text{fix}}}}{c} \) from the center of gravity, see fig. 4.5. From this follows the second interpretation of the neutral point, stick fixed. It is the point on the m.a.c. through which
acts the change in normal force, caused by a change in angle of attack at constant elevator angle.

In concurrence with this second interpretation of the neutral point, the dimensionless distance \( \frac{x_{\text{nfix}} - x_{\text{c.g.}}}{c} \) is often called the 'stability margin', stick fixed.

In the present situation, where the contributions of the tangential forces to the longitudinal moment have been neglected, the second interpretation of the neutral point, stick fixed, agrees entirely with that of the neutral point as introduced in 2.3a.

There the neutral point was defined as the point of intersection of the mean aerodynamic chord and the line of action of the change in the resultant aerodynamic force \( dC_R \).

---

**Fig. 4.5:** The change in the moment \( dC_m \) due to a change in the angle of attack \( d\alpha \).
4.4. The elevator trim curve and the elevator control position stability

a. The elevator trim curve

In 3.7 the elevator angle required for equilibrium - i.e. for \( C_m = 0 \) - was expressed in (3-39):

\[
\delta_e = -\frac{1}{C_{m_0}^m} \left[ C_{m_0} + C_{m_0}^a \cdot (\alpha - \alpha_0) \right]
\] (3-39)

where \( C_{m_0}^m \) is the static stability according to (3-34) or (4-8). The constant \( C_{m_0} \) and the elevator effectivity \( C_{m_0}^a \) were already expressed in (3-33) and (3-35) respectively, see 4.2.

If \( C_{m_0}^m \) and \( C_{m_0}^a \) are constant, the elevator angle required for equilibrium varies in a linear way with \( \alpha \), according to (3-39). The graphic representation of the elevator angle required for equilibrium as a function of \( \alpha \) or airspeed \( V \) is known as the elevator trim curve.

The relation between the angle of attack \( \alpha \) and the airspeed is derived from the equilibrium in Z-direction, or in a slightly modified form:

\[
C_N = C_{N_0}^m \cdot (\alpha - \alpha_0) = \frac{W}{\frac{1}{2} \rho V^2 S}
\] (4-14)

From (3-39) follows with (4-14) the expression for the elevator trim curve \( \delta_e \) as a function of \( V \):

\[
\delta_e = -\frac{1}{C_{m_0}^m} \left[ C_{m_0} + C_{m_0}^a \cdot \frac{W}{\frac{1}{2} \rho V^2 S} \right]
\] (4-15)

Fig. 4.6 presents schematically some elevator trim curves and the corresponding moment curves for a statically stable \( C_{m_0}^m < 0 \), a neutrally stable \( C_{m_0}^m = 0 \) and a statically unstable \( C_{m_0}^m > 0 \) airplane. In the graphic representation it is common practice to plot an 'elevator-up' deflection in the upward direction. This means that the negative \( \delta_e \)-axis points upwards.
From these figures and also from the expressions (3-39) and (4-15) it can be seen that a close relation exists between the moment curve and the elevator trim curve.

Fig. 4.6: The moment curves and the corresponding trim curves.
In particular the slope of the elevator trim curve is directly related to the slope of the moment curve, i.e. the static longitudinal stability, if $C_{m\delta}$ is exactly constant.

b. The elevator control position stability

The following discussion shows how the slope of the elevator trim curve is important for the pilot's opinion of the handling qualities of the airplane.

The slope of the elevator trim curve $\delta_e - \alpha$ follows from (3-39) by differentiating with respect to $\alpha$:

$$\frac{d\delta_e}{d\alpha} = -\frac{C_m}{C_{m\delta}}$$  \hspace{1cm} (4-16)

Considering that $C_{m\delta}$ is always negative, it follows that for a statically stable airplane ($C_m < 0$) the slope of the trim curve $\delta_e - \alpha$ is always negative:

$$\frac{d\delta_e}{d\alpha} < 0$$  \hspace{1cm} (4-17)

The slope $\frac{d\delta_e}{dV}$ of the elevator trim curve $\delta_e - V$ follows from (4-15) by differentiating with respect to $V$:

$$\frac{d\delta_e}{dV} = \frac{4W}{\rho y^2 S} \cdot \frac{1}{C_{m\delta}} \cdot \frac{C_m}{C_N \alpha}$$  \hspace{1cm} (4-18)

For a statically stable airplane ($C_m < 0$) it follows from (4-18):

$$\frac{d\delta_e}{dV} > 0$$  \hspace{1cm} (4-19)

If the relation between the elevator angle and the airspeed in steady, straight flight is such that (4-19) is satisfied, the airplane is said to possess elevator control position stability, see also ref. 4.2.

This concept of the elevator control position stability can be interpreted in two different ways.
According to the first interpretation, elevator control stability — the slope \( \frac{d\delta_e}{dV} \) of the elevator trim curve \( \delta_e - V \) — provides an indication of the static longitudinal stability of the airplane.

Elevator control position stability is a convenient characteristic to obtain the static stability both qualitatively and quantitatively from measurements in actual flight. In 4.7 this subject is discussed in more detail.

The second interpretation of elevator position stability lies in the fact that an airplane possessing such stability is pleasant for the pilot and safe to fly. This will be further illustrated in the following.

Suppose the airplane is in a situation of steady flight. Now the pilot wants to change to another steady flight condition, say at a slightly lower airspeed and a corresponding larger angle of attack. To this end he must initiate a nose-up rotation of the airplane over the small angle \( \Delta \theta \), to obtain the required new angle of attack. This initial rotation is obtained through an initial moment \( \Delta C_{m_i} \) about the lateral axis, generated by an initial elevator deflection \( \Delta \delta_{e_i} \), see fig. 4.7. If \( \alpha \) and \( \theta \) are intended to increase and as a consequence \( V \) is intended to decrease, the initial elevator movement must always be an upward deflection. The initial movement of the cockpit control — stick or wheel — is directed aft. For any airplane at any c.g. position the following is true:

\[
\frac{\Delta \delta_{e_i}}{\Delta V} > 0 , \quad \frac{\Delta s_{e_i}}{\Delta V} > 0
\]

Now it is known that it is pleasant for the pilot, because it eases the flying of the airplane, if the initial control displacement \( \Delta s_{e_i} \) and the ultimate control displacement \( \Delta s_{e_i} \) are in the same direction:

\[
\frac{\Delta s_{e_i}}{\Delta s_{e_i}} > 0 , \quad \frac{\Delta \delta_{e_i}}{\Delta \delta_{e_i}} > 0
\]

(4-20)

There is no need for \( \Delta \delta_{e_i} \) and \( \Delta s_{e_i} \) to be of equal magnitude as well. In order to speed up the response of the airplane, the pilot will generate an initial control deflection larger than the ultimate control displacement.
**Fig. 4.7:** The initial control surface deflection.

**Fig. 4.8:** The initial and the ultimate control displacement and control surface deflection for the transition to a lower airspeed.
This has been indicated by the dashed line in fig. 4.8a.

Evidently:

\[
\frac{\Delta \delta_{e_u}}{\Delta \delta_{e_l}} = \frac{\Delta \delta_{e_u}}{\Delta \delta_{e_l}} = \frac{\Delta \delta_{e_l}}{\Delta \delta_{e_l}}
\]

Trim curve of an airplane possessing control position stability

trajectory of \(\delta_e\) during the non-steady transition from \(\text{1 to 2}\)

\(\Delta \delta_{e_{u_l}}\)

\(\Delta \delta_{e_{u_i}}\)

\(V_2\)

\(V_1\)

\(\Delta \delta_{e_{i_l}}\)

\(\Delta \delta_{e_{i_i}}\)

trajectory of \(\delta_e\) during the non-steady transition from \(\text{1 to 2}\)

trim curve of an airplane possessing control position instability

---

Fig. 4.9: Airplane response to a control deflection.

\[
\Delta \delta
\]

It was argued, that always \(\frac{\Delta \delta_{e_l}}{\Delta \delta_{e_u}} > 0\). As a consequence (4-20) corresponds to the requirement \(\frac{\Delta \delta_{e_u}}{\Delta \delta_{e_l}} > 0\) or, for infinitesimally small changes: \(\frac{d \delta_{e_u}}{d \delta_{e_l}} > 0\).
Fig. 4.10: Time histories of elevator angle and airspeed during the transition to a lower airspeed. Lockheed 1079 C-"Super Constellation."
Fig. 4.11: Airplane response to a pulse-shaped control deflection. Loheed 1049 C-"Super Constellation".
It is thus seen, that (4-20) leads to the requirement (4-19) for elevator control position stability.

Fig. 4.9 may serve as a further illustration of the importance of this desired control characteristic. The figure shows in a highly schematic way how the pilot handles the elevator control, if he wants to change the airplane from one steady flight condition into another steady condition. The case of a trim curve showing control position stability is considered first.

Suppose again the pilot wants to reduce the airspeed, starting from a steady flight condition (indicated 1 in the figure). He applies an initial control deflection, directed aft, causing \( \alpha \) and \( \theta \) to increase, \( V \) starts decreasing. To the new control position \( \delta_{e2} \) belongs, for an airplane possessing elevator control position stability, a new steady flight condition at a lower airspeed \( V_2 \) (see the flight condition indicated as 2 in fig. 4.9). The response of the airplane to the initial control displacement is in the direction of the desired new equilibrium situation.

Fig. 4.9 also shows the case of an airplane possessing elevator control position instability. It can be seen, that the response of the airplane to the initial control displacement is in the direction away from the desired new equilibrium situation. As a consequence a much more complicated time-history of the control displacement is needed to arrive at the steady flight condition, 2. It requires a great deal more of the pilot's attention than if the airplane were stable.

Fig. 4.10 shows recordings of the time-histories of the elevator angle and airspeed as they occur when the pilot wants to reduce the airspeed. The recorded airplane motions were calculated on an analog computer. The motions relate to an airplane possessing elevator control position stability (fig. 4.10a), neutral stability (fig. 4.10b) and instability (fig. 4.10c). It can be seen that for the airplane possessing elevator control position stability, the initial and the ultimate control deflection are in the same direction. In the case of control position instability, the two control deflections are in opposite directions. The required control deflection is much more difficult to apply.

The figs. 4.11a, b and c show the responses of the airplane to a pulse-shaped disturbance at a fixed elevator angle, corresponding to the three situations studied in fig. 4.10. If the airplane possesses elevator control position instability, it is indeed dynamically unstable.
Fig. 4.12: Influence of the c.g. position on the trim curve.
4.5. The influence of various parameters on the elevator trim curve and the elevator control position stability

a. The influence of the center of gravity position

Using (4-12) the expression (4-15) for the elevator trim curve can be written as:

$$\delta_e = -\frac{1}{C_m \delta} \left( C_{m_\alpha} + \frac{W}{\frac{1}{2} \rho V^2 S} \cdot \frac{x_{c.g.} - x_{n\text{fix}}}{c} \right)$$  (4-21)

The slope of the trim curve, the control position stability, follows from (4-18) and (4-12):

$$\frac{d\delta}{dV} = \frac{4W}{\rho V^3 S} \cdot \frac{1}{C_m \delta} \cdot \frac{x_{c.g.} - x_{n\text{fix}}}{c}$$  (4-22)

Eqs. (4-21) and (4-22) bring to light the direct influence of the position of the center of gravity relative to the neutral point, stick-fixed, on the elevator angle required for equilibrium and on the elevator control position stability.

A rearward shift of the c.g. increases the elevator angle required for equilibrium, the elevator trim curve, $\delta_e - V$, in fig. 4.12 shifts downward. In addition, when the c.g. shifts aft, the stability margin decreases. The elevator control position stability, i.e. slope of the trim curve, decreases, see also fig. 4.12.

b. The influence of the stabilizer angle of incidence

In the expression for the trim curve the part of the elevator angle independent of airspeed is obtained, by letting in (4-21) $V = \infty$ (and by consequence $\alpha = \alpha_0$). Using (3-33), the resulting $\delta_{e_{V=\infty}}$ is:

$$\delta_{e_{V=\infty}} = -\frac{C_{m_\alpha}}{C_m \delta} \cdot \frac{1}{C_{m_\delta}} \left( C_{m_{a,c}} - C_{N_{h_2}} \left( \alpha + \alpha_0 \right) \left( \frac{V}{V_h} \right)^2 \frac{S_h \cdot 2h_1}{s \cdot c} \right)$$  (4-23)
Fig 4.13: Influence of the tailplane angle of incidence on the trim curve (airplane has control position stability).
In many airplanes the stabilizer angle of the incidence $i_h$ can be varied in flight. From (4-23) it follows that such a change has an influence on $\delta_{e_{V\infty}}$. The stabilizer setting is adjusted in flight to reduce the elevator control force to zero in a given steady flight condition, see Chapter 5.

In such an airplane the stabilizer thus has a function comparable to that of a trim tab. It follows from (4-23) that an increase of $i_h$ (the stabilizer leading edge moves up) causes an increase in the negative sense of the part of $\delta_e$ independent of airspeed. The trailing edge of the elevator moves up as well. The simple explanation is, that at constant airspeed and c.g. position the value of $C_{N_h}$ required for equilibrium of the moment remains constant. This has been indicated schematically in fig. 4.13.

c. The influence of the trim tab angle

Many airplanes, especially the smaller and slower ones, are provided with a trim tab at the trailing edge of the elevator. If the tab angle $\delta_{e_{t}}$ is changed, the trim curve shifts parallel to itself. This shift is entirely comparable, in the qualitative sense, to the shift caused by a change in the stabilizer angle of incidence $i_h$. Usually, the influence of a change in $\delta_{e_{t}}$ on the elevator trim curve is neglected, see also page 115. But fig. 5,5a shows a few trim curves measured at constant c.g. positions and different trim tab angles.

d. The shape of the elevator trim curve

The shape of the trim curve depends not only on the variables discussed so far: the c.g. position, the stabilizer angle of incidence and the trim tab angle.

The influence of the engine power setting can be appreciable, in particular for propeller-driven airplanes. Usually, an increase in engine power decreases the elevator control position stability. The compressibility of the air and the aeroelastic deformation can also have an important influence on the elevator trim curve. These influences are due, in principle, to the fact that they render the aerodynamic coefficient functions of airspeed. In 1.6 it was already noted that these phenomena will not be considered here.

4.6 The static longitudinal stability of tailless airplanes

The stability of a tailless airplane has already been discussed in 2.5. It was shown, see fig. 2.17, that equilibrium - i.e. - at positive values of
$C_N$ is possible only if $C_{m_{a.c.}} > 0$.

The equilibrium is stable -- i.e. $C_m < 0$ -- if the center of gravity lies ahead of the a.c.:

$$x_{c.g.} < x_{a.c.} = x_w$$

By choosing a sufficiently forward c.g. position this condition can be satisfied.

The requirement for a positive $C_{m_{a.c.}}$ needs a further discussion. In 2.6 the following expression was derived for the $C_{m_{a.c.}}$ of a wing:

$$C_{m_{a.c.}} = \frac{2}{S_c} \left[ \int c^2 \cdot dy - \int_0^{b/2} c \cdot x \cdot dy \right]$$

(2-32)

It is possible to obtain a positive $C_{m_{a.c.}}$ by choosing wing profiles having a positive $c_{m_{a.c.}}$. Profiles showing a S-shaped camber line possess at $c_x = 0$ a positive value of the moment coefficient. For wings having such profiles the first integral in (2-32) is positive.

A second possibility to obtain a positive $C_{m_{a.c.}}$ is offered by the second integral in (2-32). As was discussed in 2.6, this integral represents the moment due to the basic lift distribution. Fig. 4.14 may show, that this moment is positive at $C_L = 0$ for a wing having a sweep back and negative wing twist or washout.

Although it is thus shown to be quite feasible to obtain static longitudinal stability without resorting to a horizontal tailplane, most airplanes nevertheless have a horizontal tailplane.

The elevator of a tailless airplane is placed at the trailing edge of the wing. The two parts of the elevator are commonly used as ailerons ('elevons') as well. Due to the relatively small distance to the airplane center of gravity, elevons are less effective for pitch control than an elevator mounted on a tailplane. When the airplane is in a trimmed condition, in general $\delta_e \neq 0$. The increased drag due to the elevator deflection -- called 'trimdrag' -- of the tailless airplane may be one of the factors influencing the choice in favour of a design with a tailplane.
Fig. 4.14: A positive $C_{\text{m}_{a.c.}}$ is obtained by combining sweepback and negative wing twist.

The horizontal tailplane also contributes to a considerable degree in the damping of the so-called short-period oscillation, occurring after a symmetric disturbance of the equilibrium, see 8.4.1. This is an additional factor to the advantage of the horizontal tailplane.

Summarizing, it can be said very schematically, that the horizontal tailplane contributes to:

1. the equilibrium of the moment about the airplane c.g.
2. the static longitudinal stability
3. the aerodynamic damping about the lateral axis.

The price to be paid is however:

1. an increased aerodynamic drag
2. an increased airplane weight.

4.7. The determination of several characteristic variables from measurements in flight

In 3.7. the condition for equilibrium of the moment was written as:
\[ C_m = (C_m)_{\delta e = 0} + C_{m_{\delta}} \cdot \delta e = 0 \] (3-36)

Suppose \( C_{m_{\delta}} \) were known. From a trim curve, \( \delta e - V \), measured in flight the moment coefficient \( (C_m)_{\delta e = 0} \) can be obtained as a function of \( V \) or \( \alpha \). In many cases \( C_m \) varies markedly with the flight condition, for instance under the influence of the wing wake or the slipstream. It follows that it is necessary to know \( C_{m_{\delta}} \) in the flight conditions of interest. A measured trim curve is not a straight reflection of the moment curve \( (C_m)_{\delta e = 0} \).

Methods to determine the elevator effectiveness \( C_m \) from measurements in flight are all based on the principle of applying a known moment about the lateral axis in the flight conditions to be investigated. The extra elevator deflection needed to compensate for this moment is a measure of \( C_{m_{\delta}} \) in that flight condition. The simplest manner to generate a known moment employs a shift of the center of gravity in the X-direction, for instance by shifting a known amount of ballast over a known distance.

In fig. 4.15 c.g.-positions (c.g.\(_1\) and c.g.\(_2\)) have been indicated as well as the equilibrium at c.g.\(_1\). At constant elevator angle \( \delta e = \delta e_1 \), no equilibrium exists about c.g.\(_2\). At constant \( \alpha \) and \( \delta e \), \( C_N \) does not change and it still passes through c.g.\(_1\). Equilibrium of the moment c.g.\(_2\) at constant \( \alpha \) is obtained by changing the elevator angle.

The change of the moment about c.g.\(_2\) caused by \( \delta e_2 - \delta e_1 \) is:

\[
\Delta C_m = C_{m_{\delta}} \cdot (\delta e_2 - \delta e_1) \\
= C_{m_{\delta}} \cdot \Delta \delta e
\]

This moment is balanced by the moment of \( C_N \) about c.g.\(_2\). This means:

\[
\frac{x_{c\cdot g\cdot 2}}{c} - \frac{x_{c\cdot g\cdot 1}}{c} + C_{m_{\delta}} \cdot (\delta e_2 - \delta e_1) = 0
\] (4-24)
The influence of the elevator deflection $\Delta \delta_e$ on $C_N$ is neglected here, or:

$$\Delta C_N = C_{N\delta_0} \cdot \Delta \delta_e \cdot \left( \frac{V_h}{V} \right)^2 \cdot \frac{S_h}{S} \ll C_N$$

**Fig. 4.15**: The determination of elevator effectiveness from flight tests.

From (4-24) follows:

$$C_{m\delta} = -\frac{1}{\Delta \delta_e} \cdot C_N \cdot \frac{\Delta x_{c.g.}}{c} \quad \text{(4-25)}$$

The above means, that if the trim curves have been measured at two c.g. positions differing in their $x$-coordinates, $C_{m\delta}$ can be obtained with (4-25). In the foregoing the influence of the c.g. position on $C_{m\delta}$ has been neglected.

Because the tailvolume $\frac{S_{h \cdot h}}{S \cdot c}$ is obtained directly from the dimensions of the airplane, use of (3-35) gives:
\[
C_{N_{h_0}} \cdot \left( \frac{V}{V} \right)^2 = -C_{m_0} \cdot \frac{S \cdot c}{S_h \cdot \delta_h} \tag{4-26}
\]

The normal force gradient \( C_{N_{h_0}} \) proper can only be obtained from these flight tests, if in addition \( \left( \frac{V}{V} \right)^2 \) has been measured along the span of the horizontal tailplane.

Once the elevator effectiveness \( C_{m_0} \) has been measured as just described, the stability margin, stick fixed can be derived from the slope of the trim curve, using (4-22).

If the airplane is equipped with an adjustable stabilizer, the derivative \( C_{m_{i_h}} \) can be obtained as well, once \( C_{m_0} \) has been measured.

To this end, the elevator angle required for equilibrium is measured in two flight conditions, differing only in the choice of the stabilizer angle of incidence but identical in all other respects.

In 3.5 the angle of attack \( \alpha_h \) of the horizontal tailplane was derived as:

\[
\alpha_h = \alpha - \varepsilon + i_h \tag{3-24}
\]

From this expression follows that if the flight condition does not change (both \( \alpha \) and \( \varepsilon \) remain constant) a change in \( i_h \) influences only \( \alpha_h \), as expressed by:

\[
\Delta \alpha_h = \Delta i_h
\]

At constant \( \alpha \) and \( \varepsilon \), \( C_{N_{h_0}} \) has to remain constant as well to maintain the same steady flight conditions, where \( C_m = 0 \):

\[
C_{N_{h_0}} = C_{N_{h_0}} \cdot \alpha_h + C_{N_{h_0}} \cdot \delta_e = \text{constant}
\]

or:

\[
C_{N_{h_0}} \cdot \Delta \alpha_h + C_{N_{h_0}} \cdot \Delta \delta_e = 0
\]
The result is:

\[ C_{N_{h\alpha}} = - \frac{\Delta \delta}{\delta i_h} \cdot C_{N_{h\delta}} \]

As mentioned before, in general \( C_{N_{h\delta}} \cdot \left( \frac{V_h}{V} \right)^2 \) will be determined, rather than \( C_{N_{h\delta}} \). If \( \left( \frac{V_h}{V} \right)^2 \) is not known, the procedure just described can give only \( C_{N_{h\alpha}} \cdot \left( \frac{V}{V_h} \right)^2 \):

\[ C_{N_{h\alpha}} \cdot \left( \frac{V}{V_h} \right)^2 = - \frac{\Delta \delta}{\delta i_h} \cdot C_{N_{h\delta}} \cdot \left( \frac{V}{V_h} \right)^2 \]

By way of illustration, the following gives an example of the determination of stability characteristics from measured elevator trim curves. The airplane in the example is the Fokker F-27, the measured trim curves were taken from ref. 4.3. Fig. 4.16 shows two elevator trim curves \( \delta_e - V_e \). They were measured at an engine power of 420 hp (approximately gliding flight) and a trim tab angle \( \delta_{e_{trim}} = +4.5^\circ \). Apart from the c.g. positions also the airplane weights were different. For this reason, the trim curve for \( x_{c.g.} = 0.227 \) c, pertaining to an airplane weight of 14500 kg, was corrected to a weight of 13000 kg.

At a constant flight condition, i.e. at constant \( C_N \) and \( \delta_e \) is:

\[ V_{e_2} = V_{e_1} \cdot \sqrt{\frac{W_2}{W_1}} \]

The trim curve thus corrected is also shown in fig. 4.16. In the first place \( C_{m_\delta} \) is derived from the two trim curves belonging to the same airplane weight of 13000 kg.

At \( V_e = 240 \text{ km/hr.} \), fig. 4.16 shows that \( \Delta \delta_e = 1.4^\circ \). The shift in c.g. position is \( \frac{\Delta x_{c.g.}}{c} = 0.058 \).
Fig. 4.16: Trim curves for two c.g. positions of the Fokker F-27.  
(From ref. 4.3)

At an airplane weight of 13000 kg the airspeed of $V_e = 240$ km/hr. corresponds to a normal force coefficient of $C_N = 0.681$.  From (4-25) then follows:

$$C_{m_\delta} = \frac{1}{1.4} \cdot 0.681 \cdot 0.058 = -0.0282 \ (\delta_e \ \text{in degrees})$$

The tailplane volume of the present airplane is $\frac{S_{h, h_e}}{S \cdot c}$. Using this value, at $V_e = 240$ km/h follows with (4-26):

$$C_{N_{h, \delta}} \cdot \left(\frac{V_h}{V}\right)^2 = +0.0303 \ (\delta_e \ \text{in degrees})$$

$$= +1.735 \ (\delta_e \ \text{in radians})$$
Fig. 4.17: Some stability characteristics derived from the measured trim curves shown in fig. 4.16 (Fokker F-27).
Next, the position of the neutral point, stick fixed is derived by means of (4-22).

Again at \( V_e = 240 \text{ km/h} \) is \( \frac{4M}{\rho_0 V^3 e} = 0.0204 \) and at \( x_{c.g.} = 0.277 \overline{c} \) the slope of the elevator trim curve is: \( \frac{d\delta_e}{dV_e} = +0.052^*/\text{km/h} \) or \(+0.187^*/\text{m/sec}.\)

This results in:

\[
\frac{x_{c.g.} - x_{n\text{fix}}}{\overline{c}} = -0.187 \times 0.0282 \times 48.9 = -0.258
\]

With \( x_{c.g.} = 0.277 \overline{c} \) follows \( x_{n\text{fix}} = 0.485 \overline{c}. \)

If the same calculation is repeated for the trim curve measured at \( x_{c.g.} = 0.285 \), the result is: \( x_{n\text{fix}} = 0.490 \overline{c}. \)

Using the data obtained thus far the moment curves \( C_m - V \) and \( C_m - C_N \) can be derived. If in addition the relation \( C_m - \alpha \) of the airplane is known, the moment curves \( C_m - \alpha \) can be calculated as well.

Fig. 4.17a shows the calculated values of \( C_{mN} \times \left(\frac{V_h}{V}\right)^2 \) as a function of \( \alpha \). Fig. 4.17b gives the \( C_m - \alpha \)-curves and fig. 4.17c shows the position of the neutral point, stick fixed, as a function of \( \alpha \) and \( C_{mN} \).
5.1. Introduction. The behaviour of the free elevator

It was already noted in the previous chapter that it makes sense to study the static stability in the situation where the pilot does not hold the elevator control in a fixed position. Two reasons can be given to study the static stability in the 'control free' condition. The first is the requirement for the airplane to be stable, also in the stick-free situation. The second reason lies in the close relationship between the static stability, stick free, and a certain characteristic of the way in which the elevator force required to maintain the elevator in the position for steady flight, varies with airspeed. This characteristic will be called the elevator control force stability.

As in Chapter 4, the study of the static stability is based entirely on the variation of the pitching moment with angle of attack. Leaving the elevator free only changes the contribution of the horizontal tailplane - and in particular the elevator - to the pitching moment.

\[ C_{m_{e_{free}}} = C_{h_{\alpha}} \alpha + C_{h_{\delta}} \delta_{e_{free}} + C_{h_{\delta_t}} \delta_t = 0 \]

Fig. 5.1: The equilibrium of the free elevator.
The first subject to be studied is the behaviour of the elevator in the control free situation. If the control stick or wheel is free, the control force \(F_e\) is zero. Neglecting the friction in the control mechanism this means that in the stick free situation the hinge moment is zero: \(H_e = 0\).

Or:

\[
C_{h_{e_{\text{free}}}} = 0
\]

In this situation the elevator angle assumes a certain value, depending on the way the elevator is aerodynamically balanced and on the trim tab angle. This elevator angle, stick free, indicated as \(\delta_{e_{\text{free}}}\), see fig. 5.1, varies with angle of attack. If the profile of the tailplane is symmetric, see (3-23), \(C_{h_0} = 0\) and then:

\[
C_{h_{e_{\text{free}}}} = C_{h_{\alpha}} \cdot \alpha_h + C_{h_{\delta}} \cdot \delta_{e_{\text{free}}} + C_{h_{\delta_t}} \cdot \delta_t = 0
\]

This means:

\[
\delta_{e_{\text{free}}} = -\frac{C_{h_{\alpha}}}{C_{h_{\delta}}} \cdot \alpha_h - \frac{C_{h_{\delta_t}}}{C_{h_{\delta}}} \cdot \delta_t
\]

(5-1)

Differentiation with respect to \(\alpha_h\) results in the variation of \(\delta_{e_{\text{free}}}\), with \(\alpha_h\) at constant trim tab angle:

\[
\frac{d\delta_{e_{\text{free}}}}{d\alpha_h} = -\frac{C_{h_{\alpha}}}{C_{h_{\delta}}}
\]

and the variation with \(\alpha\):

\[
\frac{d\delta_{e_{\text{free}}}}{d\alpha} = -\frac{C_{h_{\alpha}}}{C_{h_{\delta}}} \cdot \frac{d\alpha_h}{d\alpha}
\]

(5-2)

Using (3-26):

\[
\frac{d\alpha_h}{d\alpha} = 1 - \frac{d\alpha}{d\alpha}
\]

(3-26)
results in:

\[
\begin{align*}
\left( \frac{d\delta_e}{da} \right)_{\text{free}} &= - \frac{C_h}{C_{h}\delta} \left( 1 - \frac{d\xi}{da} \right) \\
\end{align*}
\]

(5-3)

5.2. The static longitudinal stability, stick free, in gliding flight

The contribution of the horizontal tailplane to the pitching moment is, see (3-20) and (4-5):

\[
C_{m_h} = -(C_{N_{h\alpha}} \cdot \alpha_h + C_{N_{h\delta}} \cdot \delta_e) \left( \frac{V}{V} \right)^2 \frac{S_h \cdot \lambda_h}{S_c} \\
\]

(5-4)

When studying the static longitudinal stability in Chapter 4, the elevator control was assumed to be held fixed, the elevator angle \( \delta_e \) was constant and did not change when \( \alpha \) was slightly varied. If however in (5-4) the constant \( \delta_e \) is replaced by the variable \( \delta_{e_{\text{free}}} \), differentiation with respect to \( \alpha \) results in the contribution of the horizontal tailplane with free elevator to the static longitudinal stability, stick free, \( C_{m \alpha_{\text{free}}} \):

\[
\begin{align*}
\frac{dC_{m_h}}{da}_{\text{free}} &= C_{m \alpha_{\text{free}}} \cdot \left[ \frac{d\alpha_h}{da}_{\text{free}} + \frac{d\delta_e}{da}_{\text{free}} \right] \left( \frac{V}{V} \right)^2 \frac{S_h \cdot \lambda_h}{S_c} \\
\end{align*}
\]

(5-5)

Substituting (3-26):

\[
\frac{d\alpha_h}{da} = (1 - \frac{d\xi}{da}) \\
\]

(3-26)

and (5-3):

\[
\left( \frac{d\delta_e}{da} \right)_{\text{free}} = - \frac{C_h}{C_{h}\delta} \cdot (1 - \frac{d\xi}{da}) \\
\]
results in:

$$C_m = -\left( C_{N_h} - C_{N_h}^\alpha \right) \cdot \frac{C_h^\alpha}{C_h^\delta} \cdot \left( 1 - \frac{d\alpha}{da} \right) \frac{V_h^2}{V} \cdot \frac{S_h \cdot \beta_h}{S \cdot c} \quad (5-6)$$

If (5-6) is compared with the corresponding expression (4-7) for the contribution of the horizontal tailplane in the stick fixed situation, when $\beta_e$ is constant:

$$C_m = -C_{N_h} \left( 1 - \frac{d\beta}{da} \right) \frac{V_h^2}{V} \cdot \frac{S_h \cdot \beta_h}{S \cdot c} \quad (4-7)$$

the concept of the normal force gradient of the tailplane in the stick free situation arises:

$$C_{N_h}^\alpha = C_{N_h} - C_{N_h} \cdot \frac{C_h^\alpha}{C_h^\delta} \quad (5-7)$$

Using this gradient in (5-6) results in close analogy with (4-7)

$$C_m = -C_{N_h} \cdot \left( 1 - \frac{d\beta}{da} \right) \frac{V_h^2}{V} \cdot \frac{S_h \cdot \beta_h}{S \cdot c} \quad (5-8)$$

The static stability, stick free, $C_{m_{\alpha_{free}}}$, follows by adding to (5-8) the contribution of the wing fuselage and nacelles:

$$C_m = C_{N_w} \cdot \frac{X_{\alpha_{free}} - \frac{X_w}{c}}{c} \cdot \left( 1 - \frac{d\alpha}{da} \right) \frac{V_h^2}{V} \cdot \frac{S_h \cdot \beta_h}{S \cdot c} \quad (5-9)$$

Previously, the static stability, stick fixed, was derived as:

$$C_m = C_{N_w} \cdot \frac{X_{\alpha_{free}} - \frac{X_w}{c}}{c} - C_{N_h} \left( 1 - \frac{d\alpha}{da} \right) \frac{V_h^2}{V} \cdot \frac{S_h \cdot \beta_h}{S \cdot c} \quad (3-34)$$
The expressions (3-34) and (5-9) and the expression (5-7) for $C_{N_h\alpha}^{free}$ indicate that the difference between the static stability stick fixed and stick free is entirely due to the term $C_{h\alpha}^{N_h\alpha}$. In this term $C_{h\alpha}$ is positive. The hinge moment derivatives $C_{h\alpha}^{h\alpha}$ and $C_{h\delta}^{h\delta}$ were already discussed in Chapter 3. They depend on the way the elevator is aerodynamically balanced. To keep the control forces at reasonably low levels, it is essential that both $C_{h\alpha}$ and $C_{h\delta}$ are made small in the absolute sense.

The required sign of $C_{h\delta}$ follows from the behaviour of the free elevator in steady, trimmed flight, i.e. the situation where $F_e = 0$. In that situation is $C_{h\delta} = 0$. Suppose the elevator now obtains a small deviation $d\delta_e$ from the equilibrium position, due to some disturbance. The immediate effect is a hinge moment:

$$dC_{h_e} = C_{h\delta}^{h\delta} \cdot d\delta_e$$

If $C_{h\delta} > 0$, the hinge moment will be positive if $d\delta_e$ is positive. Due to the hinge moment the change in elevator angle will further increase. The general conclusion is, that an equilibrium position of a control surface in the control free condition will be unstable if $C_{h\delta}$ is positive: the control surface does not return to the equilibrium position after a disturbance has occurred. For this reason it is strictly necessary that $C_{h\delta}$ be negative.

The above argument needs a slight extension. Suppose a control surface has a $C_{h\delta}$ nearly equal to zero. In such a case actually two hinge moments are present, of nearly equal magnitude but of opposite sign, see the pressure distribution in fig. 3.9. The resultant value of $C_{h\delta}$ is due to the difference of these two hinge moments. A small variation in one of these moments, as may be caused by a small change in the shape of the control surface, due for instance to the tolerance in the manufacturing process, will have a large effect on $C_{h\delta}$ if compared to the intended value. For this reason $C_{h\delta}$ has to be not only negative, but should not be allowed to approach zero to closely. Common values are: $-0.2$ to $-0.3 < C_h < -0.1$ ($\delta_e$ in rad).
The sign of \( C_{h_\alpha} \) will be discussed in detail in 5.4 in relation to the elevator control force stability. Here it can be said that \( C_{h_\alpha} \) may be either negative or positive. Too large positive values of \( C_{h_\alpha} \), e.g. \( C_{h_\alpha} \approx 0.1 \) (\( \alpha_h \) in rad), can not be used because they may lead to dynamic instability, stick free. In the decision on the required values of \( C_{h_\alpha} \) and \( C_{h_\delta} \), the influence which ice accretion on the stabilizer and elevator has on the aerodynamic balance of the control surface, has to be taken into account as well. In the final choice of the elevator balance, the non-linear relation between the hinge moment and \( \alpha_h \) and \( \delta_e \) at large angles is an additional important consideration.

From (5-7) and (5-9) it can be seen, that depending on the sign of \( C_{h_\alpha} \), the airplane with free elevator control will be less statically stable (\( C_{h_\alpha} < 0 \)), equally stable (\( C_{h_\alpha} = 0 \)) or more stable (\( C_{h_\alpha} > 0 \)) than it is in the control fixed situation.

5.3. The neutral point, stick free

As for the airplane with fixed elevator control a certain center of gravity position exists also in the control free situation where \( C_{m_\alpha} \), here \( C_{m_\alpha} \), is equal to zero. This center of gravity position is called, the neutral point, stick free, indicated as \( n.p._{\text{free}} \). The abscissa of this point is \( x_{n_{\text{free}}.h} \). As in (4-11) the position of the neutral point, stick free, is obtained by letting \( C_{m_\alpha} = 0 \), see (5-9):

\[
\frac{x_{n_{\text{free}.h}}}{c} = \frac{C_{N_{h_\alpha}}}{C_{N_\alpha}} \left( 1 - \frac{d \alpha}{da} \right) \frac{V_h^2}{S. c} \cdot \frac{S_{h.h}}{S. c}
\]

(5-10)

Just like the neutral point, stick fixed, the neutral point, stick free, can be interpreted in a second way, in addition to the one just given. It is the point of action (on the m.a.c.) of the total change in normal force coefficient \( dC_N \), due to a change in angle of attack \( d \alpha \) if the elevator control is left free.
Fig. 5.2: The change in the moment $dC_{m\text{free}}$ due to a change in angle of attack $d\alpha$.

In analogy with (4-12) it follows:

$$C_{m\text{free}} = C_{N\text{free}} \cdot \frac{x_{c.g.} - x_{n\text{free}}}{c}$$

where:

$$C_{N\text{free}} = C_{N\text{fix}}$$

The latter expression is obtained by neglecting in (4-10) the difference in normal force gradient $C_{N\alpha}$ due to the difference between $C_{N\text{free}}$ and $C_{N\text{fix}}$, as the contribution of the tailplane to $C_{N\alpha}$ is relatively small if compared to the contribution of the wing.

The change in pitching moment $dC_{m\text{free}}$ due to a change in angle of attack $d\alpha$ with free elevator control then is:
\[ \frac{dC_{m_{\text{free}}}}{dC_{N_{\text{free}}}} = \frac{x_{c.g.} - x_{n_{\text{free}}}}{c} \]  

(5-11)

Fig. 5.2 gives an illustration of this second interpretation of the neutral point, stick free.

The relative position of the two neutral points, stick fixed and stick free, follows from (4-11) and (5-10):

\[ \frac{x_{n_{\text{free}}} - x_{n_{\text{fix}}}}{c} = \frac{C_{N_{h_{a_{\text{free}}}}} - C_{N_{h_{a_{\text{fix}}}}}}{C_{N_{a}}} \left(1 - \frac{dc}{da}\right) \frac{V_{h}}{\nu} \frac{V_{h}}{S_{h} \cdot h} \frac{S_{h} \cdot h}{S \cdot c} \]  

(5-12)

Using (5-7):

\[ C_{N_{h_{a_{\text{free}}}}} - C_{N_{h_{\delta}}} = C_{h_{\alpha}} \frac{C_{h_{\delta}}}{C_{h_{\delta}}} \]  

(5-7)

the result is:

\[ \frac{x_{n_{\text{free}}} - x_{n_{\text{fix}}}}{c} = \frac{C_{N_{h_{\delta}}}}{C_{N_{a}}} \cdot \frac{C_{h_{\alpha}}}{C_{h_{\delta}}} \left(1 - \frac{dc}{da}\right) \frac{V_{h}}{\nu} \frac{V_{h}}{S_{h} \cdot h} \frac{S_{h} \cdot h}{S \cdot c} \frac{C_{m_{\delta}}}{C_{N_{a}}} \frac{C_{h_{\delta}}}{C_{h_{\delta}}} \]  

(5-13)

According to this latter expression, the relative position of the two neutral points is determined primarily by the sign of \( C_{h_{\alpha}} \).

A simple explanation of the influence of the sign of \( C_{h_{\alpha}} \) on the static stability, stick free, is possible by looking at the behaviour of the free elevator, see fig. 5.3.

If \( C_{h_{\alpha}} = 0 \), the elevator angle \( \delta_{e_{\text{free}}} \) does not vary with angle of attack, according to (5-3). It is as if the control were fixed, see fig. 5.3b. There is no difference between the static stabilities stick fixed and stick free, the two neutral points coincide.
If $C_{h_{\alpha}} < 0$, the elevator turns with the direction of the flow as the angle of attack varies, see fig. 5.3a. With an increase in angle of attack the trailing edge of the elevator moves up. The effect of freeing the elevator control is an extra downward force on the tailplane. This extra force promotes a further increase in angle of attack. As a consequence the static stability stick free is less than stick fixed and $x_{n_{\text{free}}}^{n_{\alpha}}$ is smaller than $x_{n_{\text{fix}}}^{n_{\alpha}}$, if $C_{h_{\alpha}} < 0$.

The influence of a positive $C_{h_{\alpha}}$ can be understood in the same way, see fig. 5.3c.
5.4. The elevator control force curve and the elevator control force stability

a. The control force curve

The elevator control force needed for equilibrium was derived in Chapter 3 as:

\[
F_e = -\frac{d\delta_e}{ds_e} \cdot \frac{1}{2} \rho V^2 \frac{S}{e} \frac{e}{c} \cdot C_{h_e}
\]

The following discussion is intended primarily for qualitative purposes. It is, therefore, permissible to express \( C_{h_e} \) here as a linear function of \( \alpha_h, \delta_e \) and \( \delta_t \), see (3-23). This leads to the following expression for \( F_e \), see also (3-42):

\[
F_e = -\frac{d\delta_e}{ds_e} \cdot \frac{1}{2} \rho V^2 \frac{S}{e} \frac{e}{c} (C_{h_a} \alpha_h + C_{h_b} \delta_e + C_{h_d} \delta_t) \quad (3-42)
\]

For a more accurate calculation of the control force in a given flight condition, \( C_{h_e} \) has to be obtained from wind tunnel measurements for the correct values of \( \alpha_h, \delta_e \) and \( \delta_t \).

In Chapter 3, \( \alpha_h \) was derived as:

\[
\alpha_h = (\alpha - \alpha_o) \left(1 - \frac{d\alpha}{d\alpha_o}\right) + (\alpha_o + i_h) \quad (3-23)
\]

The elevator angle \( \delta_e \) required for equilibrium about the lateral axis is, see (2-39):

\[
\delta_e = -\frac{1}{C_{m_{\delta}^{\mu}}} \left[C_{m_{\mu}} + C_{m_{\mu}^{\alpha}} \right] (\alpha - \alpha_o) \quad (3-39)
\]

Substitution of (3-25) and (3-39) in (3-42) results after some elaboration in:

\[
F_e = -\frac{d\delta_e}{ds_e} \frac{1}{2} \rho V^2 \frac{S}{e} \frac{e}{c} \left[C_{h_a} \alpha + C_{h_b} (\alpha - \alpha_o)\right] \quad (5-14)
\]
where:

\[
C'_{h_o} = - \frac{C_{h_o \delta}}{C_{m_o \delta}} \cdot C_{m_o \alpha \cdot c} - \frac{C_{h_o \delta}}{C_{N_{h_o \delta \alpha \text{free}}}} \cdot (a_o + i_h) + C'_{h_o \delta t} (5-15)
\]

\[
C'_{h_a} = C_{h_a} (1 - \frac{d \delta}{d a}) - \frac{C_{h_o \delta}}{C_{m_o \delta}} \cdot C_{m\alpha \text{fix}} = - \frac{C_{h_o \delta}}{C_{m_o \delta}} \cdot C_{m\alpha \text{free}}
\]

\[
= - \frac{C_{h_o \delta}}{C_{m_o \delta}} \cdot C_{N_{h_a}} \frac{x_{c.g.} - x_{n_{\text{free}}}}{c} (5-16)
\]

Furthermore:

\[
(a - a_o) \approx \frac{W}{\frac{1}{2} \rho V^2 S} \cdot \frac{1}{C_{L_a}} \approx \frac{W}{\frac{1}{2} \rho V^2 S} \cdot \frac{1}{C_{N_a}} (5-17)
\]

Substituting (5-17) in (5-14) results in:

\[
P_e = - \frac{d \delta}{d s} e_c \frac{V^2}{e_c} \left[ C'_{h_o} \cdot \frac{1}{2} \rho V^2 + C'_{h_a} \cdot \frac{W}{S} C_{N_a} \right] (5-18)
\]

According to (5-18) the elevator control force required to maintain steady flight consists of two parts. One is proportional to the dynamic pressure and thus varies with airspeed. The other part is independent of airspeed. The part depending on airspeed is considered first. It can be seen from (5-18) and (5-15) that this part varied by changing the trim tab angle or the stabilizer angle of incidence or both. Suppose for the moment that \( i_h \) is constant. At a certain value of \( \delta_{t_e} \), \( C'_{h_o} \) will be equal to zero.

This particular trim tab angle is called \( \delta_{t_e} \). It is derived from (5-15) by letting:

\[
C'_{h_o} = 0
\]

The result is:
\[
\delta_{te} = \frac{C_{h\delta} + C_{N_h\delta} \cdot (\alpha_0 + 1_h)}{C_{m\delta} C_{m,c.} + C_{N_h\delta} \cdot \alpha_{free}}
\]

\[5-19\]

\[\frac{F_e}{V=0} \quad 1 \frac{1}{2} p V^2_{\min} \quad \frac{1}{2} p V^2 \quad F_e \quad \delta_t = \delta_{te_0} \quad \delta_t > \delta_{te_0} \quad \delta_t < \delta_{te_0} \quad \delta_t > \delta_{te_0} \quad \delta_t = \delta_{te_0} \quad \delta_t < \delta_{te_0} \]

(a) \( F_e \) as a function of dynamic pressure \( \frac{1}{2} p V^2 \)
(b) \( F_e \) as a function of equivalent airspeed \( V_e \)

Fig. 5.4: The schematic form of the elevator control force curve.
Using (5-15) and (5-19), \( C_{h_o} \) can now be written as:

\[
C'_{h_o} = C_{h_o t} \cdot (\delta_{e} - \delta_{e_o})
\]  \( \text{(5-20)} \)

After some elaboration, the final expression for the elevator control force can now be obtained from (5-18), using (5-20) and (5-16). The result is:

\[
F_e = \frac{d\delta_{e}}{ds_{e}} \cdot S_e \cdot c_e \cdot \frac{V^2}{\mathcal{V}} \cdot \left[ \frac{C_{h_o}}{S} \cdot \frac{C_{x_{c.g.}} - C_{x_{n_{free}}}}{\mathcal{C}} - \frac{1}{2} \rho V^2 \cdot C_{h_o t} \cdot (\delta_{e} - \delta_{e_o}) \right]
\]  \( \text{(5-21)} \)

The variation of the elevator control force with dynamic pressure or airspeed as expressed by (5-21) is shown schematically in fig. 5.4. Negative elevator control forces, i.e. pull forces exerted by the pilot, are plotted upward in the figure, just like negative elevator angles. If the various aerodynamic characteristics in (5-21) may indeed be considered as constants, then \( F_e \) varies linearly with \( \frac{1}{2} \rho V^2 \), see fig.. 5.4a, or quadratically with \( V \), see fig. 5.4b. The measured trim curves and elevator control force curves shown in fig. 5.5 show that in reality the various simplifying assumptions are not always satisfied.

The second part of the elevator control force was independent of airspeed. It is indicated as \( F_{e_{V=0}} \). According to (5-21) this part is determined by the position of the c.g. relative to the n.p. free, or by the sign of the static stability, stick free. If the airplane is statically stable, stick free \( (x_{c.g.} < x_{n_{free}}) \), \( F_{e_{V=0}} \) is negative.

The variation of the part of \( F_e \) depending on airspeed is determined by \( \delta_{e} \). If \( \delta_{e} = \delta_{e_o} \), \( F_e \) does not vary with airspeed. If \( \delta_{e} > \delta_{e_o} \), the control force increases in the positive sense with increasing airspeed, see fig. 5.4, independent of the c.g. position. In this discussion the assumption was made that \( C_{h_o} \), \( C_{m_o} \), and \( C_{h_{o_t}} \) have the normal, negative sign.

The airspeed at which the control force is zero, is called the trim speed, indicated as \( V_{tr} \). So:

\[
V_{tr} = V_{F_e=0}
\]
If the elevator control is let free at this airspeed, the control position does not change.

b. The elevator control force stability

If the elevator control force curve at the trim speed satisfies the condition:

\[
\frac{dF}{dV} \bigg|_{F_e=0} > 0 \quad (5-22)
\]

the airplane is said to possess elevator control force stability in that particular flight condition. Elevator control force stability is thus seen to depend only on the slope of the control force curve in the trimmed, i.e. \( F_e=0 \), condition.

The expression will now be further analysed. Differentiating (5-21) with respect to \( V \) gives the slope of the control force curve as:

\[
\frac{dF}{dV} = -\frac{d\delta_e}{ds_e} \cdot S_e \frac{c}{c_e} \cdot \left( \frac{V}{V} \right)^2 \cdot \rho V \cdot C_{h_{\delta_t}} (\delta_{t_e} - \delta_{t_o}) \quad (5-23)
\]

This expression holds for both trimmed and untrimmed conditions. The elevator control force stability follows by substituting in (5-23) the values of \( \delta_{t_e} \) pertaining to \( V_{tr} \). The trim speed \( V_{tr} \) is obtained from (5-21) by letting \( F_e=0 \):

\[
\frac{1}{2} \rho V^2 \ C_{h_{\delta_t}} (\delta_{t_e} - \delta_{t_o}) = \frac{W}{S C_{h_{\delta}}} \frac{c_g \cdot x - x_{free}}{c} \quad (5-24)
\]

Combining (5-23) and (5-24) results in the elevator control force stability:

\[
\frac{dF}{dV} \bigg|_{F_e=0} = -2 \frac{d\delta_e}{ds_e} \cdot S_e \frac{c}{c_e} \left( \frac{V}{V} \right)^2 \frac{W}{S C_{h_{\delta}}} \frac{c_g \cdot x - x_{free}}{c} \cdot \frac{1}{V_{tr}} \quad (5-25)
\]

The direct relation between the control force stability as defined in (5-22) and the position of c.g. relative to the n.p. \( x_{free} \) can be seen from (5-25). If the
Fig. 5.5: Measured trim curves and elevator control force curves of the De Havilland D.H.98 "Mosquito" MIII E in gliding flight. (From ref. 5.2)
airplane is statically stable, stick free, it possesses control force stability according to (5-25) and vice versa.

Just like the control position stability discussed in Chapter 4, control force stability is important in two respects.

In the first place, the control force stability provides the possibility to ascertain from measurements in flight if the airplane is statically stable, stick free, in a certain airplane configuration and flight condition, see (5-25).

In the second place — and this is perhaps more important — an airplane possessing control force stability is more pleasant and safer to fly by the pilot. The latter subject will now be discussed.

As a general rule it can be stated that it is highly desirable, if not imperative that control displacements and the control forces required to generate and maintain these displacements both in manoeuvres and in steady flight conditions, have equal directions. This requirement can be expressed simply as:

\[ \frac{dF_e}{ds_e} > 0 \]  \hfill (5-26)

---

Fig. 5.6: Schematic representation of the concept of "positive feel".

\[ \frac{dF_e}{ds_e} > 0 \]
The control manipulator - be it a stick or a wheel - then behaves as if it were pulled back to the neutral position by springs, see fig. 5.6. The effect to the pilot is, that he feels through the force he has to exert on the control manipulator, in which direction and to what extent he has deflected the control. Generally it can be said, that the pilot is far more sensitive to changes in the control forces he has to exert, than to changes in the control displacement.

Due to the adapted sign convention $\frac{d\delta}{ds_e} > 0$, the requirement (5-26) can then be written as:

$$\frac{dF_e}{d\delta_e} > 0$$

It was argued in 4.4b that the initial and the ultimate control displacement should be in the same direction. From the above it follows that this applies equally to the accompanying changes in the exerted control force. On the basis again of the chosen sign convention for control displacements and control forces and on the requirement for elevator control position stability, see (4-19):

$$\frac{d\delta_e}{dv} > 0$$  \hspace{1cm} (4-19)

the corresponding requirement for elevator control force stability is written as in (5-22):

$$\frac{dF_e}{dv} \bigg|_{F_e=0} > 0$$  \hspace{1cm} (5-22)

It is inevitable that the control mechanism of the airplane has a certain friction, which has been neglected so far. The magnitude of this friction, which is primarily static friction, has been expressed in 3.8 in an equivalent hinge moment $h_e$ . A certain control force $F_e$ is required to overcome this friction. If the airspeed in steady flight differs only slightly from $V_{tr}$, the required control may be small as well. If $|F_e| < |F_{eF}|$, no control force is required from the pilot. He can leave the control stick or wheel free, the required control position is maintained due to the friction in the control mechanism, see fig. 5.7.
Fig. 5.7: The lack of definition of trim speed due to friction in the control mechanism.

The result is, that in actual flight not one single trim speed exists. Any airspeed within the range where $F_e > F_{f}$ can act as a trim speed.

In order to keep this lack of definition of the trim speed within acceptable limits, the elevator control force stability must have a certain minimum positive value and, in addition, $F_{f}$ has to be sufficiently small.

The U.S. military regulations, see ref. 1.13, require:

$$\frac{dF_e}{dV} \bigg|_{F_e=0} > 0.5 \text{ lbs/3kts}$$

corresponding with:
\[
\frac{dF_e}{dv} \bigg|_{v=0} > 0.041 \text{ kg/km/h}
\]

According to the same regulations, the control force required to overcome the friction is limited to:

**light airplanes:**

- control stick \( F_e < 1.3 \text{ kg} \)
- control wheel \( F_e < 1.8 \text{ kg} \)

**heavy airplanes:**

- control stick \( F_e < 2.3 \text{ kg} \)
- control wheel \( F_e < 3.2 \text{ kg} \)

This control force is to be measured on the ground, the engines not running. In flight the control force needed to overcome the friction may be considerably less, due to the vibrations of the airplane. The exact definitions of the terms 'light' and 'heavy' airplanes will be found in ref. 1.13.

5.5. The influence of various parameters on the elevator control force curve and the elevator control force stability

a. The influence of the center of gravity position

In the foregoing it was shown that the center of gravity has an influence only on the part \( F_{e, v=0} \) of the control force, which is independent of airspeed, see (5-21). If the c.g. is shifted in the X-direction, this constant term in (5-21) varies and - at a constant trim tab angle - the entire control force curve will move up or down, parallel to itself.

If the c.g. is moved forward, the control force curve will move up (a larger pull force will be required) at constant \( \delta_e \) and \( i_e \). This agrees with the measured control force curves shown in figs. 5.5 and 5.8.
Fig. 5.8: Measured elevator control force curves of the North American "Harvard" II B in gliding flight. (From ref. 5.1)
The slope of the control force does not change at constant $\delta_{te}$, but with the new c.g. position corresponds another, higher trim speed. If the original trim speed is re-established by an additional downward deflection of the trim tab, it turns out from e.g. fig. 5.8d that the elevator control force stability at constant trim speed has increased. Strictly speaking, the elevator control force stability cannot be considered independent of the trim speed.

b. The trim tab angle and the stabilizer angle of incidence

From (5-21) it can be concluded, that for an airplane that is statically stable, stick free, any trim tab angle $\delta_{te} > \delta_{te_0}$ results in a certainairspeed $V_{tr}$, where $F_e = 0$. The trim tab angle corresponding to a given $V_{tr}$ is obtained from (5-24):

$$\delta_{te} = \delta_{te_0} + \frac{W}{C_{h\delta}} \cdot \frac{1}{\frac{1}{4} \rho V_{tr}^2 S} \cdot \frac{C_{h\delta}}{C_{m\delta}} \cdot \frac{x_{c.g.} - x_{n.free}}{c}$$

(5-27)

It follows from (5-27), that for a given c.g.-position the pilot can choose a certain trim speed, where he can fly 'hands-off', by selecting the appropriate $\delta_{te}$. If the stability margins, stick free, is positive - c.g. in front of the tail-no free - the trim speed is reduced by increasing $\delta_{te}$, i.e. with increasing downward tab deflection. The trim wheel in the cockpit is turned backward to this end. Once a certain trim speed has been chosen, the variation of the control force with airspeed in steady flight is fixed, according to (5-25), and so is the elevator control force stability.

It can be clearly seen in fig. 5.8 from the measured elevator control forces, that the slope of the control force curve increases at a given airspeed by a downward deflection of the trim tab.

From the foregoing it becomes clear that for a given c.g. position a fixed relation exists between the trim tab angle and the correspondig trim speed. The airplane possesses elevator control force stability, if according to (5-27):

$$\frac{d\delta_{te}}{dV_{tr}} < 0$$
Fig. 5.9: Trim curves, hinge moment coefficients and required trim tab angles as functions of $C_L$ for the Siebel 204-D-1. (From ref. 5.3)
Fig. 5.10: The influence of a bobweight in the control mechanism.

Fig. 5.11: The influence of a spring in the control mechanism.
The trim tab angle curve may be used to investigate the elevator control force stability. An example of a measured trim tab angle curve is shown in fig. 5.9.

As said already in 3.7, in some airplanes the elevator control forces are reduced to zero by adjusting the stabilizer angle of incidence. In those cases the elevator does not need to have a trim tab. It follows from (5-15) and (5-18) that both the angle of incidence \( i_h \) and the trim tab angle \( \delta_{te} \) figure in \( C^t_{h_0} \) and \( F_{e_{v=0}} \).

It also follows from (5-15) and (5-18) that it makes no difference whether the stabilizer or the trim tab is used to trim the control force to zero. If the stabilizer is used, \( C^t_{h_0} \) can be written in analogy with (5-20) as:

\[
C^t_{h_0} = C_{h_0} (i_h - i_{h_0}) 
\]

(5-28)

where \( i_{h_0} \) is the value of \( i_h \) for which \( C^t_{h_0} = 0 \). The influence of \( i_h \) on the elevator control force stability is entirely comparable to that of \( \delta_{te} \).

---

**Fig. 5.12**: The influence of a spring or bobweight on the elevator control force curve.
c. The influence of a spring or an unbalanced mass in the control mechanism

Sometimes it is not possible to obtain satisfactory control forces and elevator control force stability in all required airplane configurations and flight conditions, using aerodynamic balancing of the elevator as the only tool.

If at a given c.g. position and trim speed the elevator control force stability should become too low, an unbalanced mass — a 'bobweight' —, see fig. 5.10, or a spring, see fig. 5.11, may be used in the control mechanism, see also fig. 3.27. These devices increase \( \frac{F_{e}}{e_{V=0}} \) by a constant force \( \Delta F_{e_{V=0}} \), see fig. 5.12. A spring to be used for this purpose is chosen in principle such, that the applied force remains approximately constant over the entire range of control deflections.

The entire control force curve will then be shifted upward, parallel to itself. A more detailed analysis shows, that now \( C_{m_{\text{free}}} \) has become a function of airspeed. It also turns out, that the n.p. \( C_{\text{free}} \) has moved backwards and consequently the stability margin, stick free, has been increased. The use of a spring or an unbalanced spring in the control mechanism is thus seen to have the same effect — as far as the control force curve is concerned — as a forward shift of the center of gravity.

5.6. The influence of the design variables on the control forces

Another look at (5-21):

\[
F_{e} = \frac{d\delta_{e}}{ds_{e}} e_{c_{e}} (v_{e})^{2} \left( \frac{h_{c}}{S} \cdot \frac{c_{\text{\delta}}}{c_{m_{\delta}}} \cdot \frac{X_{c_{\text{g}}} - X_{\text{free}}}{c} \cdot \frac{1}{2} \rho V^{2} \frac{C_{\text{\delta}_{e}}}{\delta_{e}} (\delta_{t} - \delta_{t_{e}}) \right) \tag{5-21}
\]

shows that the variables of direct interest to the pilot: \( 1/2 \rho V^{2}, X_{c_{\text{g}}}, \delta_{t} \) and \( \delta_{t_{e}} \) have already been discussed in the previous paragraphs. Other variables occurring in (5-21) are of interest in the design of the airplane. These variables are:

\[
\frac{d\delta_{e}}{ds_{e}} e_{c_{e}}, \text{ the tail volume} \frac{X_{h}}{S c_{e}} \text{ and } C_{m_{\text{\delta}}}, \text{ (or } C_{m_{\delta}}), C_{\text{\delta}_{e}}, C_{\text{\delta}_{t}}, \frac{X_{\text{free}}}{c}.
\]

Concerning these variables, the following can be said. The control forces increase with the wing loading \( \frac{h_{c}}{S} \), the control forces are in addition proportional to \( s_{e} \) and \( c_{e} \). For airplanes of identical shape the control forces vary
with the third power of the dimensions.

The control gearing \( \frac{d\delta_e}{d\alpha_e} \) is determined by the fact that the extreme deflection of the control surface must correspond with the extreme displacement of the cockpit control manipulator. The range of elevator deflections \( \delta_{e_{max}} \) is restricted aerodynamically. It amounts to about 50° or 60° (25° to 30° to either side). The permissible range of control displacements \( \delta_{e_{max}} \) is restricted by the dimensions of the human pilot and sometimes by the dimensions of the cockpit. The maximum value is about 40 cm. The resulting value of \( \frac{d\delta_e}{d\alpha_e} \) turns out to be approximately 1.25 to 1.5°/cm (2.2 to 2.6 rad/m). Some airplanes have a variable gear ratio, depending on the airplane configuration.

The required value of \( C_{n_0} \) is determined by the maximum required elevator power. Usually either the take-off or the landing at the most forward c.g. position is the most critical condition; see 3.7. The derivative \( C_{\delta} \) determines the required size of the trim tab, taking into account the (limited) range of the trim tab angles. For a trim tab to be effective at any occurring value of \( \alpha \) and \( \delta \), the deflection should not be larger than \( \pm 15° \). The required size of the trim tab then follows from the requirement that, according to the regulations, it must be possible to reduce the control force to zero in certain airplane configurations—including the c.g. position—and over certain ranges of airspeed, see 3.8.

The only remaining variables which the designer can use to influence the control forces, are the stability margin, stick free and \( C_{h_0} \). Earlier was found:

\[
\frac{X_{n_{free}} - X_{n_{fix}}}{C} = -\frac{C_{N_{h_0}}}{C_{N_{\alpha}}} \cdot \frac{C_{h_0}}{C_{h_0}} \cdot (1 - \frac{d\delta}{d\alpha}) \left( \frac{V}{h} \right)^2 \frac{S_{h_0}}{S_c} \tag{5-13}
\]

This means that at a given position of the neutral point, stick fixed, \( C_{h_0} \) and \( C_{h_0} \) are the only remaining variables with which the designer can influence the control forces.

From the foregoing it will have become abundantly clear, that the general requirement is for the center of gravity to be positioned in front of the neutral points, both stick fixed and stick free. The airplane then possesses both elevator control position and force stability. If \( C_{h_0} \) of the elevator as well as \( C_{h_0} \) are negative, the n.p.\( _{\text{free}} \) lies in front of the n.p.\( _{\text{fix}} \). The range
of permissible c.g. positions is limited to the rear by the \( n_p^{free} \). It is possible to shift this \( n_p \) aft by reducing \( |C_h| \). The \( n_p^{free} \) will even lie behind the \( n_p^{fix} \) if \( C_h > 0 \). This is one reason why a slight overbalance with respect to angle of attack is sometimes aimed for when balancing the elevator.

For many modern airplanes it turns out to be no longer possible to obtain acceptable control forces in all required aircraft configurations and flight conditions, merely by choosing \( C_{h\alpha} \) and \( C_{h\delta} \). This is in particular true for airplanes having:

a) a high wing loading \( \frac{W}{S} \)

b) a high maximum airspeed \( V_{max} \)

c) a large ratio of maximum to minimum airspeed \( \frac{V_{max}}{V_{min}} \) (V/STOL-airplanes).

If purely aerodynamic means do not suffice, additional use can be made in the first place of a spring or an unbalanced mass or both in the control mechanism. These devices also influence the dynamic stability, stick free, and sometimes in the unfavourable sense.

In more advanced cases, such as transonic, supersonic and some V/STOL-airplanes, springs and masses cannot offer a satisfactory solution. In those cases the aerodynamic hinge moments are balanced in total or in part by - usually hydraulic - control force amplifiers: control boosters, or by servo controls.

When hydraulic servo controls are used, it would be sufficient for the pilot to apply only the very small forces needed to operate the servo valve. Experience has shown, however, that in the interest of the safety of flight the pilot must have to exert forces in his control stick or wheel having the usual levels of magnitude and varying in the normal way with airspeed, control deflection and trim deflection. This requirement is met by the use of a separate installation in the flight control system, a so called 'artificial feel unit'. This mechanism applies a force in the control manipulator which has to be balanced by the pilot's control force. These artificially generated control forces are made to vary with \( \frac{1}{4} \rho V^2 \), \( \delta_e \) and \( \delta_e \) in such a way, that the pilot's control force shows indeed the variations found in an airplane where the control manipulator has a direct mechanical linkage with the control surface.

5.7. The control forces a pilot can exert

The discussions in the previous paragraphs centered around the control forces required to fly the airplane. The following deals with forces a pilot can
exert on his controls. Evidently the designer must ensure that the required control forces remain within the capabilities of even the least strongest amongst the pilot population.

Appropriate regulations aim to safeguard this requirement. The maximum control force a pilot can apply depends primarily on:

a. the individual person; strength and endurance differ appreciably from one person to another;

b. the type of control manipulator (control wheel, center stick, side stick, rudder pedals);

c. the time during which the force must be exerted;

d. the location of the control manipulator relative to the pilot.

a. and b. The variations between individual pilots and the type of control manipulator

Fig. 5.13 clearly shows the large differences in the maximum possible control forces, due to individual differences in strength and endurance of the test subjects.

It appears that the largest force a pilot can exert is the rudder pedal force (curve a), the side force for aileron control is the smallest (curves d and e). It is also seen that the forces on a control wheel can be larger than those on a control stick.

In the measurements reported in fig. 5.13 the control manipulators were all handled by the right hand. This is in agreement with the situation in most single seat cockpits, where the pilot uses his left hand to operate the engine throttle.

c. The time during which the force must be exerted

Fig. 5.13 gives an indication of the maximum possible control forces as a function of the uninterrupted time period over which these forces have to be applied. The maximum possible control force decreases approximately logarithmically with increasing time period.

The location of the control manipulator is not only important because it influences the maximum possible control forces. It also influences the pilot's comfort. Reference is made to refs. 5.4 to 5.6, 5.17 and especially 5.22. The latter publications give a large amount of information on the preferable locations of controls in the cockpit.

Occasionally the airplane flight condition can experience a rather sudden change. In view of such occurrences it is necessary to know how fast a pilot can change the control deflections under various levels of control force gradients.
Information on this subject can be found in refs. 5.8 to 5.10. It is evident that the required control forces may not surpass the possible control forces. The various Airworthiness Regulations contain requirements concerning the maximum permissible control forces under various circumstances. Table 5.1 on p. gives the maximum forces as stipulated in the U.S. and the British Airworthiness Regulations, refs. 1.14 and 1.18 respectively.

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**Fig. 5.13: Maximum control forces as functions of the duration.**

(From ref. 5.4)
5.8. Regulations on the static longitudinal stability and the control characteristics in steady, straight and symmetric flight

The Airworthiness Regulations, see refs. 1.12 to 1.18, require that the airplane is statically stable in the most important airplane configurations and flight conditions. This means that the c.g. must be in front of the neutral point in those conditions. Sometimes the requirement is expressed in terms of an equivalent requirement regarding the control characteristics.

In view of the fact that the pilot is in general sensitive to control forces and changes in control forces, rather than to control positions and control displacements, the requirement usually stipulates that the airplane possesses elevator control force stability in the relevant airplane configurations and flight conditions at all permissible center of gravity positions. As discussed in 5.3, this requirement can easily be reformulated into the condition that the center of gravity must be positioned in front of the neutral point stick free, in the described circumstances.

For the designer the emphasis may be slightly different. In 5.5 it was seen that several possibilities exist to influence the control force through non-aerodynamic means. The arsenal of available tools is certainly not exhausted with the springs and unbalanced masses discussed in 5.5. It is of great practical interest that several of these means may still be applied relatively easily in a late stage of the development of the airplane, if the need arises, without great expense or drastic changes in the design. To change the elevator control position stability or the static stability, stick fixed, usually turns out to be much more difficult since it requires considerably more drastic modifications in the design, see 4.3.

This argument leads to an important design rule. Already in an early stage of the design of an airplane it is essential to ensure that - in addition to other requirements to be met - the neutral point, stick fixed, in the prescribed airplane configurations and flight conditions will lie behind the envisaged rearmost center of gravity position.

Experience has shown, that during the development of an airplane the center of gravity often shows the tendency to move a bit due to the successive design modifications. For this reason it may be wise to provide an additional margin when choosing the position of the neutral point, stick fixed, in the various aircraft configurations and flight conditions.

In the next Chapter 6 some control characteristics will be discussed concerning turning flight. Requirements to be met by the control characteristics in such flight conditions will prove to influence also the forward and rear
limits of the permissible center of gravity positions.

Table 5.1.: Maximum permissible values of the required control forces, according to the U.S. and British Civil Airworthiness Regulations

<table>
<thead>
<tr>
<th>Requirements according to</th>
<th>U.S. civil (ref. 1.18)</th>
<th>British civil (ref. 1.14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rudder pedals</td>
<td>82 kg (short intervals)</td>
<td>82 kg (short intervals)</td>
</tr>
<tr>
<td></td>
<td>9 kg (long intervals)</td>
<td>22.5 kg (long intervals)</td>
</tr>
<tr>
<td>elevator control</td>
<td>35 kg (short intervals)</td>
<td>22.5 kg (short intervals)</td>
</tr>
<tr>
<td></td>
<td>4.5 kg (long intervals)</td>
<td>26 kg (long intervals)</td>
</tr>
<tr>
<td>aileron control</td>
<td>27 kg (short intervals)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.3 (long intervals)</td>
<td></td>
</tr>
</tbody>
</table>

1) a few seconds
2) a few minutes
CHAPTER 6. LONGITUDINAL CONTROL IN TURNING FLIGHT

6.1. Introduction

The previous two chapters dealt with the longitudinal control characteristics in steady straight flight. Control in turning and non steady flight, i.e. in manoeuvres, was not considered so far. It is by no means certain that an airplane showing good control characteristics in steady, straight flight, also has similarly good control characteristics in manoeuvres. It is thus necessary to study these latter control characteristics and to this end express them quantitatively in appropriate terms. This way also allows to formulate quantitative requirements for desired control characteristics.

Of the many possible turning manoeuvres only two of the greatest general interest are considered: the pull-up from a dive and the steady, coordinated - i.e. slip-free - horizontal turn.

During these two manoeuvres the airplane has an angular, pitching, velocity about the lateral axis. In turns the pitching velocity \( q \) about the lateral axis is always accompanied by a yawing velocity \( r \) about the top axis, but because of the symmetry of the airplane, the influence of the asymmetric component \( r \) on the symmetric forces and moments can be neglected, see Chapter 8.

An important purpose of the following discussion is to make longitudinal control in these manoeuvres accessible to quantitative study. To this end the control characteristics have to be expressed in characteristic variables, sufficiently simple to be manageable for the designer. To arrive at such characteristic variables some simplifying assumptions have to be made.

Suppose the pilot of a normal stable airplane wants to pull his plane up from a steady, straight, symmetric dive. In this context any descending flight will be called a dive. To achieve his purpose, he moves the elevator control back to a new position, which is assumed to be constant to simplify the discussion.

The effect of this schematic control movement can be described as follows. Assuming a stable airplane, first the pitching velocity \( q \) and the angle of attack \( \alpha \) quickly increase to new approximately constant values depending on the ultimate control displacement. This motion occurs usually within a few seconds, often via a more or less well damped oscillation during which the airspeed remains very nearly constant.

The airplane motion, usually called the 'short period oscillation' will be discussed in more detail in 8.4.
Fig. 6.1: Response curves corresponding to an elevator step deflection. Lockheed 1049 C - "Super Constellation".
Fig. 6.2: Response curves corresponding to an elevator step deflection, Auster J-5B "Autocar". (From ref. 6.1).
Fig. 6.1 gives an example of such a symmetric motion caused by a step elevator deflection. The motion was calculated using an analog computer.

Fig. 6.2 presents the various measured components, the elevator deflection and the control force for a pull-up manoeuvre of an Auster J-5B 'Autocar':

Due to the increase in angle of attack the total lift on the airplane becomes larger than the airplane weight. This causes the trajectory of the c.g. to be curved, the airplane is pulled up from the dive.

If the pilot keeps the elevator angle constant, a second oscillation will occur. In contrast with the first oscillation, the second one is relatively slow and usually has a very low damping. After the second oscillation has also died away, the pitching velocity $q$ is zero and the airspeed as well as the angle of attack have assumed the new constant values corresponding to the new elevator angle in steady, straight flight. In actual flight, when pulling up from a dive the pilot will suppress the second, long-period, oscillation by means of suitable, small corrective elevator movements. This second oscillation, usually called 'the slow oscillation', or 'phugoid' will also be discussed in more detail in 8.4.

In the following, only the first and relatively fast part of the motion is studied. The assumption is made that the airspeed remains constant at the level of the original steady flight condition.

In addition to the assumption of a constant airspeed another simplification can be made. The normal load factor, $n$, is equal to 1 in the original straight flight.

Due to the control displacement initiating the pull-up from the dive, the value of $n$ increases. If the short-period oscillation is sufficiently damped, the normal load factor remains very nearly constant during a brief time interval, see figs. 6.1 and 6.2. This fact allows the normal load factor to be considered constant in the following, just like the airspeed.

Due to these two assumptions, the pull-up trajectory is idealized to a steady flight condition. For steady turns, the two simplifying and approximating assumptions concerning the airplane motion need not to be made. The airplane has by definition in a steady turn not only a constant airspeed, but also a constant normal load factor.

In the following two notions will be derived characterizing longitudinal control in the 'steady' pull-up and in the steady, horizontal, coordinated turns.
6.2. Characteristic notions for longitudinal control in turning flight

A notion characteristic for longitudinal control in turning flight should relate two aspects of the manoeuvre: on the one side the pilot's action, the control displacement and the control force, and on the other hand a characteristic element of the resultant airplane motion. For the latter the normal load factor \( n \), mentioned before, is used:

\[
\frac{N}{W} = n \quad (6-1)
\]

In the initial steady flight condition is \( N = W \), or \( n = 1 \). Using this load factor, the 'stick displacement per g' and the 'stick force per g' are obtained, where \( g \) is the acceleration due to gravity.

Just as with the elevator control position stability, the elevator angle is used rather than the stick displacement itself.

\[
\frac{d\delta}{dn}^e \quad \text{stick displacement per g}
\]

\[
\frac{dF}{dn}^e \quad \text{stick force per g}
\]

The load factor in these two expressions is not only characteristic for the change in the trajectory of the airplane, it is also closely related to the loads imposed on the airplane. The two characteristics \( \frac{d\delta}{dn}^e \) and \( \frac{dF}{dn}^e \) express the extent to which the control position and control force have changed after the airplane has changed at constant airspeed from a condition of steady, straight flight to another condition of steady, but turning flight or to steady turn at a load factor \( n \). For the case of the pull-up manoeuvre it can be argued that \( \frac{d\delta}{dn}^e \) and \( \frac{dF}{dn}^e \) should have the negative sign. This argument is as follows.

In order to achieve a pull-up from a dive at a certain airspeed, an increase in angle of attack is needed. This requires an increase in the angle of pitch of the airplane. This is the initial airplane motion, the transition from the steady dive to the 'steady' pull-up manoeuvre. For this initial motion a tail-heavy moment (\( \Delta C_m > 0 \)) is needed and because \( C_m < 0 \), an initial backward elevator control movement and an elevator deflection 'trailing edge up'
(\Delta \delta^e < 0) are required.

It is argued in 4.4 that a very general desirable control characteristic requires the ultimate control displacement to have the same sign as the initial displacement. During the 'steady' pull-up manoeuvre at a constant, positive \Delta \alpha this general characteristic requires that also the ultimate \Delta \delta^e is negative. This implies that the required sign is negative:

\[
\frac{d\delta^e}{dn} < 0
\]

In Chapter 6 it is argued that the change in control force required for a control displacement should have the same direction as the control displacement itself. As a consequence, since \( \frac{d\delta^e}{ds} > 0 \):

\[
\frac{dF^e}{dn} < 0
\]

It is of course desirable, that also in a turn both the change in control position and control force are negative. In the pull up manoeuvre as well as in the turn, a backward control movement and an extra pull force should be needed.

The stick force per g is generally assigned both a maximum and a minimum permissible value. If the stick force per g is too large in the absolute sense, the pilot will become tired too soon if he has to perform many manoeuvres. If, on the other hand, the stick force per g is too low, the airplane is too sensitive to small unintentional variations in the control force. There is the additional risk that the airplane may become overstressed due to a sudden control force.

The dynamic characteristics of the airplane are also important here. They have a direct bearing on the desired values of \( \frac{d\delta^e}{dn} \) and \( \frac{dF^e}{dn} \), see ref. 6.10.

The following Table 6.1. gives the maximum and minimum permissible values of the stick force per g, derived from the U.S. military requirements, see ref. 1.13. It will be seen that these values are related to the maximum permissible load factor \( n_L \) and thus to the strength of the airplane.
Fig. 6.3: Description of the character of the control feel for various ratios of the control force per g to the control displacement per g. (From ref. 6.11).

Fig. 6.4: The influence of the stick displacement per g and the stick force per g on the pilot's opinion of the control characteristics at constant airspeed. (From ref. 6.10).
Table 6.1: Permissable values of the stick force per g, in kg

<table>
<thead>
<tr>
<th></th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trainer and fighter aircraft</td>
<td>$-\frac{25.4}{n_L-1}$</td>
<td>$-\frac{9.5}{n_L-1}$</td>
</tr>
<tr>
<td>Transport aircraft and bombers</td>
<td>$-\frac{54.4}{n_L-1}$</td>
<td>$-\frac{20.4}{n_L-1}$</td>
</tr>
</tbody>
</table>

A general rule is, that the absolute value of the stick force per g should not be lower than 1.4 kg.

If the requirements of Table 6.1 are applied to fighter airplanes having $n_L = 6$, the result is that the stick force per g should be between $-5.1$ kg and $-1.9$ kg.

For transport airplanes, usually having $n_L$ of 2.5, the boundaries are $-32.2$ kg and $-13.6$ kg.

It is not sufficient to give a maximum and a minimum permissable value of the stick force per g. The relation between the stick displacement per g and the stick force per g is also important to achieve likeable control characteristics. Fig. 6.3 shows how various combinations of the stick displacement per g and the stick force per g were judged. Fig. 6.4 is another example of the influences of the two criteria on the pilot's opinion on the control characteristics of a fighter airplane at constant airspeed. Quantitative requirements for the stick displacement per g do not exist.

In the following the stick displacement per g and the stick force per g are expressed as functions of the aerodynamic coefficients and the mass of the airplane for the two cases:

a. The idealized pull-up manoeuvre previously discussed, where $n$ and $V$ are assumed to be constant.

The trajectory of the airplane then is an arc of a circle in the vertical plane. The extent to which this idealized pull-up manoeuvre agrees with the actual motion of the airplane may be judged from figs. 6.1 and 6.2. It will be seen that $n$ and $V$ indeed remain very nearly constant over a brief time interval.

b. The steady, horizontal, coordinated turn.
6.3. The stick displacement per g

The calculation is based on the forces and moments acting on the airplane both before and after the transition from the steady, straight flight to the idealized pull-up manoeuvre or the steady, horizontal turn. During both manoeuvres the airspeed is assumed to be constant. This implies, that the forces in the X-direction are assumed to be in equilibrium and need not further be considered.

In the steady initial condition is:

\[ N_0 = W \]

and:

\[ M_0 = 0 \]

or in dimensionless form:

\[ C_{N_0} = \frac{W}{\frac{1}{2} \rho V^2 S} \]

and:

\[ C_{m_0} = 0 \]

The index  \( o \) here indicates the steady initial condition.

After the transition is, using (6-1):

\[ \Delta C_N = \frac{W}{\frac{1}{2} \rho V^2 S} \Delta \alpha \]  \hspace{1cm} (6-2)

and:

\[ \Delta C_m = 0 \]  \hspace{1cm} (6-3)

After the transition the angle of attack has increased an amount \( \Delta \alpha \) and the airplane has obtained an angular velocity \( q \) about the lateral axis. The elevator angle has changed by \( \Delta \delta_e \).

The variables describing the airplane motion now are:
$\alpha$, $q$ or $\frac{qc}{V}$ and $\delta_e$ \hspace{1cm} (V = constant)

whereas in straight flight, considered so far, the variables are:

$\alpha$, $V$ and $\delta_e$ \hspace{1cm} ($\frac{qc}{V} = 0$)

In Chapter I the assumption was made that $V$ would have no effect on the aerodynamic coefficients.

Due to the change in angle of attack the airplane experiences an extra normal force $C_N \cdot \Delta \alpha$ and an extra moment $C_m \cdot \Delta \alpha$. The moment due to the elevator deflection is $C_m \cdot \Delta \delta_e$.

The pitching velocity $q$ causes an aerodynamic force along the $Z$-axis. In 8.1a a dimensionless 'stability derivative' $C_{Zq}$ is introduced:

$$C_{Zq} = \frac{\Delta C_N}{\Delta q} - \frac{\Delta C_m}{\Delta q}$$

Using this stability derivative, the extra force along the $Z$-axis can be expressed as $C_{Zq} \cdot \frac{qc}{V}$. Here $\frac{qc}{V}$ is the dimensionless pitching velocity, see also 8.1.

A positive force $C_{Zq} \cdot \frac{qc}{V}$ is directed along the positive $Z$-axis (downward).

Usually $C_{Zq}$ is negative.

The pitching velocity $q$ gives also rise to an aerodynamic pitching moment about the lateral axis. This moment can be expressed using another 'stability derivative', $C_{mq}$, also to be discussed in 8.1:

$$C_{mq} = \frac{\Delta C_m}{\Delta q}$$

The pitching moment due to $q$ is $C_{mq} \cdot \frac{qc}{V}$. Usually $C_{mq}$ is negative.

From the foregoing follows for (6-2) and (6-3):

$$\Delta C_N = C_N \cdot \Delta \alpha - C_{Zq} \cdot \frac{qc}{V} = \frac{W}{\frac{1}{2} \rho V^2 S} \cdot \Delta \alpha$$

(6-4)
\[ \Delta C_m = C_{m_a} \Delta \alpha + C_{m_q} \frac{\Delta q}{V} + C_{m_b} \Delta \theta = 0 \]  

(6-5)

(6-5) gives:

\[ \Delta \delta_e = -\frac{1}{C_{m_b}} \left( C_{m_a} \Delta \alpha + C_{m_q} \frac{\Delta q}{V} \right) \]

The stick displacement per g results from differentiating this latter expression with respect to \( n \):

\[ \frac{d\delta_e}{dn} = -\frac{1}{C_{m_b}} \left( C_{m_a} \frac{d\alpha}{dn} + C_{m_q} \frac{d\frac{q}{V}}{dn} \right) \]

\[ \frac{d\alpha}{dn} \] follows from (6-4):

\[ \frac{d\alpha}{dn} = \frac{1}{C_{N\alpha}} \cdot \frac{W}{\frac{1}{2} \rho V^2 S} + C_{N_q} \frac{d\frac{q}{V}}{dn} \]  

(6-6)

This results in:

\[ \frac{d\delta_e}{dn} = -\frac{1}{C_{m_b}} \left( C_{N\alpha} \frac{C_{m\alpha}}{C_{N\alpha}} \frac{W}{\frac{1}{2} \rho V^2 S} + \frac{C_{m\alpha} C_{N_q}}{C_{N\alpha} C_{N\alpha}} + C_{m_q} \right) \cdot \frac{d\frac{q}{V}}{dn} \]  

(6-7)

The relation between the dimensionless pitching velocity \( \frac{q}{V} \) and the normal load factor \( n \) is yet to be determined. A distinction has to be made here between the idealized pull-up manoeuvre and the steady, horizontal turn.

a. Derivation of \( \frac{d\frac{q}{V}}{dn} \) for the pull-up manoeuvre

In the pull-up manoeuvre the bottom of the trajectory is considered, see fig. 6.5. In this lowest point of the trajectory, the \( Z \)-axis is very nearly vertical and the total force along the \( Z \)-axis is \( N - W \). The centripetal acceleration along the \( Z \)-axis is \( V \cdot q \). As a consequence:
\[ N - W = m \cdot V \cdot q \]

or, using (6-1):

\[
\frac{\bar{q}c}{V} = \frac{\bar{g}c}{V^2} (n-1)
\]

This results in:

\[
\frac{d \, \frac{\bar{q}c}{V}}{dn} = \frac{\bar{g}c}{V}
\]  (6-8)

In 6.1 a dimensionless measure of the airplane's mass, \( \mu_c \), is introduced. This so-called 'relative density', is defined as:

\[
\mu_c = \frac{m}{p \bar{S}c} = \frac{W}{g \rho \bar{S}c}
\]

Using \( \mu_c \), (6-8) can be written as:

\[
\frac{d \, \frac{\bar{q}c}{V}}{dn} = \frac{1}{2 \mu_c} \cdot \frac{W}{\frac{1}{2} \rho V^2 g}
\]  (6-9)

b. Derivation of \( \frac{d \, \frac{\bar{q}c}{V}}{dn} \) for the steady, horizontal turn

In this steady flight condition the resultant force along the Z-axis, see fig. 6.6, is \( N - W \cdot \cos \varphi \). The centripetal acceleration along the Z-axis is again \( V \cdot q \), so:

\[ N - W \cos \varphi = m \cdot V \cdot q \]
Fig. 6.5: The forces and angular velocity $q$ in the idealized pull-up manoeuvre.

Fig. 6.6: The forces and angular velocity in a steady horizontal turn.
From fig. 6.6 follows also:

\[ N \cos \varphi = W \]

This relation and (6-1) result in:

\[ \frac{\bar{q} \bar{c}}{v} = \frac{8 \bar{c}}{v^2} \left( n - \frac{1}{n} \right) \]

and, after differentiation:

\[ \frac{d \bar{q} \bar{c}}{dn} = \frac{1}{2 \mu_c} \cdot \frac{W}{\frac{1}{2} \rho v^2 s} \cdot \left( 1 + \frac{1}{n^2} \right) \]  \hspace{1cm} (6-10)

It follows from (6-10) that in steady, horizontal turns \( \frac{\bar{q} \bar{c}}{v} \) is a non-linear function of \( n \).

If (6-9) is substituted in (6-7), the resulting expression for the stick displacement per g in the pull-up manoeuvre can be written as:

\[ \frac{d \delta e}{dn} = -\frac{1}{C_{m_0}} \cdot \frac{W}{\frac{1}{2} \rho v^2 s} \left( \frac{C_m}{C_N} \right) \left( 1 + \frac{C_Z}{2 \mu_c} \right) \]  \hspace{1cm} (6-11)

Using (6-10) and (6-7) the resulting expression for the steady turns reads:

\[ \frac{d \delta e}{dn} = -\frac{1}{C_{m_0}} \cdot \frac{W}{\frac{1}{2} \rho v^2 s} \left\{ \frac{C_m}{C_N} \left( 1 - \frac{C_Z}{2 \mu_c} \right) \left( 1 + \frac{1}{n^2} \right) \right\} \]  \hspace{1cm} (6-12)

A few simplifications can now be made. In (6-11) \( 2 \mu_c \) is relatively large if compared with \( |C_{Z,q}| \). Also, in (6-12) \( C_m \) is large in the absolute sense, if compared to \( \frac{C_m}{C_N} \cdot C_{Z,q} \). This permits both (6-11) and (6-12) to be simplified to:
pull-up manoeuvres:

\[
\frac{d\delta_e}{dn} = -\frac{1}{c_{m_0}} \frac{W}{\frac{1}{2} \rho V^2 S} \left( \frac{C_m}{C_{N_{\alpha}}} + \frac{C_m}{2\mu_c} \right) \quad (6-13)
\]

steady turns:

\[
\frac{d\delta_e}{dn} = -\frac{1}{c_{m_0}} \frac{W}{\frac{1}{2} \rho V^2 S} \left( \frac{C_m}{C_{N_{\alpha}}} + \frac{C_m}{2\mu_c} \right) \left( 1 + \frac{1}{n^2} \right) \quad (6-14)
\]

It follows from (6-13) that in the pull-up manoeuvres the control deflection and the elevator angle are linear functions of \( n \). In steady, horizontal turns this is, according to (6-14), not the case.

Then \( \frac{d\delta_e}{dn} \) varies with \( n \). The difference between \( \frac{d\delta_e}{dn} \) pull-up manoeuvres and \( \frac{d\delta_e}{dn} \) turns decreases with increasing \( n \).

From (6-13) and (6-14) it follows also, that both for the pull-up manoeuvres and the turns \( \frac{d\delta_e}{dn} \) is proportional to the wingloading \( \frac{W}{S} \) and to \( \frac{1}{\frac{1}{2} \rho V^2 S} \).

Figs. 6.7 and 6.8 show measurements of the stick displacement per \( g \) in pull-up manoeuvres and steady turns of the Auster J-5B 'Autocar'.

For both types of manoeuvres \( \frac{d\delta_e}{dn} \) appears to have the desired negative sign. For this airplane the relation between \( \delta_e \) and \( n \) in the pull-up manoeuvres is non-linear. From the measurements in figs. 6.7 and 6.8 it is clear, that the required control displacement for a steady turn is larger than for the pull-up manoeuvre at the same load factor \( n \).

6.4. The manoeuvre point, stick fixed

In (6-13) and (6-14) the first term between the parantheses is \( \frac{c_{m_0}}{C_{N_{\alpha}}} \). From the discussion of the static longitudinal stability and the neutral point, stick fixed, in 4.3 follows:

\[
\frac{c_{m_0}}{C_{N_{\alpha}}} = \frac{x_{c^{*}S^*} - x_{n_{f_{x}}}}{c} \quad (6-15)
\]
Fig. 6.7: The incremental elevator deflection $\Delta \delta_e$ as a function of the incremental load factor $\Delta n$ in pull-up manoeuvres. Auster J-5B "Autocar".
(From ref. 6.1).

Fig. 6.8: The incremental elevator angle $\Delta \delta_e$ as a function of the incremental load factor in turns. Auster J-5B "Autocar".
(From ref. 6.1).
In (6-15) \( x_{n_{\text{fix}}} \) is the \( x \)-coordinate of the neutral point, stick fixed, in gliding flight. The first contribution to \( \frac{d\delta}{dn} \) then appears to be proportional to the distance of the center of gravity to the neutral point.

As a consequence, the stick displacement per \( g \) will decrease in the absolute sense if the c.g. is shifted backward. This can clearly be seen in figs. 6.7 and 6.8. It follows that there is a certain c.g. position where the stick displacement per \( g \) is zero. This is the case for the pull-up manoeuvres, if, see (6-13),

\[
\frac{C_{m_d}}{C_{N\alpha}} + \frac{C_{m_q}}{2\mu_c} = \frac{x_{\text{c.g.}} - x_{n_{\text{fix}}}}{c} + \frac{C_{m_q}}{2\mu_c} = 0
\]  

(6-16)

The c.g. position at which the stick displacement per \( g \) is zero, is called the manoeuvre point, stick fixed, abbreviated as m.p.\( _{\text{fix}} \). The \( x \)-coordinate of this point is \( x_{m_{\text{fix}}} \).

From (6-16) follows, if \( x_{\text{c.g.}} = x_{n_{\text{fix}}} \):

\[
\frac{x_{m_{\text{fix}}} - x_{n_{\text{fix}}}}{c} = -\frac{C_{m_q}}{2\mu_c}
\]  

(6-17)

Here \( C_{m_q} \) has been assumed independent of the c.g. position, see Chapter 8. Combining (6-15) and (6-17) results in:

\[
\frac{x_{\text{c.g.}} - x_{m_{\text{fix}}}}{c} = \frac{C_{m_d}}{C_{N\alpha}} + \frac{C_{m_q}}{2\mu_c}
\]  

(6-18)

Next, (6-18) is substituted in (6-13). This results in an expression for the stick displacement per \( g \) as a function of the c.g. position relative to the m.p.\( _{\text{fix}} \):

\[
\frac{d\delta}{dn} = -\frac{1}{C_{m_{\delta}}} \cdot \frac{W}{\frac{1}{2} \rho V^2 S} \cdot \frac{x_{\text{c.g.}} - x_{m_{\text{fix}}}}{c}
\]  

(6-19)

But the manoeuvre point, stick fixed, is not only the c.g. position at which the stick displacement per \( g \) vanishes. The manoeuvre point can be interpreted in yet another way, as is explained in the following.

The normal force \( N \) can generally be written as, see (6-1):
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\[ N = n \cdot W \]  

(6-1)

Then:

\[ \frac{dC_N}{dn} = \frac{W}{\frac{1}{2} \rho V^2 S} \]  

(6-20)

Substitution of (6-20) in (6-19) results in:

\[ \frac{d\delta_e}{dn} = - \frac{1}{C_{m_{\delta}} \cdot C_{N_{\delta}}} \cdot \frac{dC_N}{dn} \cdot \frac{x_{c.g.} - x_{\text{fix}}}{c} \]

or:

\[ C_{m_{\delta}} \cdot d\delta_e = -dC_N \cdot \frac{x_{c.g.} - x_{\text{fix}}}{c} \]  

(6-21)

Here \( C_{m_{\delta}} \cdot d\delta_e \) is the change in the pitching moment due to the elevator deflection \( d\delta_e \), and \( dC_N \) is the change in the normal force coefficient. According to (6-21), \( C_{m_{\delta}} \cdot d\delta_e \) must be balanced by \( dC_N \cdot \frac{x_{c.g.} - x_{\text{fix}}}{c} \). This means that \( \frac{x_{c.g.} - x_{\text{fix}}}{c} \) is the arm of the force \( dC_N \). Evidently, the second interpretation of the manoeuvre point, stick fixed, is the point where the resultant change in normal force acts, after the transition from a condition of steady, straight flight to a 'steady' pull-up manoeuvre, if the elevator angle were kept constant in this transition.

Since \( C_m \) is negative, it follows from (6-17) that the m.p.\( F_{\text{fix}} \) always lies behind the n.p.\( F_{\text{fix}} \). With increasing \( \mu_c \), i.e. with increasing flight altitude, the manoeuvre point moves forward. This can also be seen in fig. 6.9, where the calculated position of the m.p.\( F_{\text{fix}} \) of the Fokker F-27 'Friendship' is shown. Fig. 6.9a clearly shows the forward shift of the m.p.\( F_{\text{fix}} \) and also the limiting position, the n.p.\( F_{\text{fix}} \). The latter position is attained when the term \( \frac{C_m}{2 \mu_c} \) becomes vanishingly small.
Fig. 6.9: Calculated positions of manoeuvre point, stick fixed, and the stick displacement per g of the Fokker F-27 "Friendship".
Thus far the manœuvre point was discussed only in relation to the pull-up manoeuvres. Both interpretations of the manœuvre point hold equally for the steady turns.

The difference is, however, that then the position of the manœuvre point is a function also of the load factor \( n \). This can be seen as follows.

From (6-14) and (6-15) follows for the steady turn:

\[
\frac{x_{m_{\text{fix}}} - x_{n_{\text{fix}}}}{c} = -\frac{C_{m}}{2\mu_{C}} \cdot \left( 1 + \frac{1}{n^2} \right)
\]

(6-22)

and by consequence:

\[
\frac{x_{c.g} - x_{m_{\text{fix}}}}{c} = \frac{C_{m}}{C_{N_{g}}} + \frac{C_{m} q}{2\mu_{C}} \cdot \left( 1 + \frac{1}{n^2} \right)
\]

(6-23)

The expression (6-19) is valid also for the stick displacement per \( g \) in steady turns, although the actual position of the manœuvre point will be different from the position in the pull-up manoeuvre. From (6-22) it can be seen that \( x_{m_{\text{fix}}} \) now depends not only on the flight altitude, but also on the load factor.

If \( n = 1 \):

\[
\frac{x_{m_{\text{fix}} \text{ turns}} - x_{n_{\text{fix}}}}{c} = 2 \cdot \frac{x_{m_{\text{fix}} \text{ pull-up}} - x_{n_{\text{fix}}}}{c}
\]

(6-24)

With increasing \( n \) the manœuvre point in turns shifts forward. At high values of \( n \) the manœuvre points in the pull-up and in the turn very nearly coincide. This can be seen also in fig. 6.9b. It should be remembered, that \( n = 2 \) in a steady, horizontal turn corresponds to an angle of roll \( \varphi = 60^\circ \), \( n = \frac{1}{\cos \varphi} \).

Apart from the stick displacement per \( g \), the stick force per \( g \) is of particular interest. It is discussed in the following paragraph.
6.5. The stick force per g

The elevator control force was written in 3.8 as:

\[ F_e = - \frac{d\delta}{ds_e} \cdot \frac{1}{2} \rho V_h^2 S_e c_e \left( C_{h_\alpha} \alpha_h + C_{h_\delta} \delta_e + C_{h_{\delta_t}} \delta_{te} \right) \]  \hspace{1cm} (3-42)

The stick force per g is the derivative of the elevator control force with respect to the load factor in a pull-up manoeuvre or a steady turn. It is a measure of the change in elevator control force after the transition from a condition of steady, straight flight to a 'steady' pull-up manoeuvre or a steady turn. In (3-42) both \( \alpha_h \) and \( \delta_e \) are functions of the load factor \( n \). It is assumed that the trim tab angle does not vary in the pull-up manoeuvre or the turn: \( \delta_{te} \) is constant. The airspeed \( V \) and thus also \( V_h \) in (3-42) are assumed to be constant.

The stick force per g is then obtained from (3-42):

\[ \frac{dF_e}{dn} = - \frac{d\delta}{ds_e} \cdot \frac{1}{2} \rho V_h^2 S_e c_e \left( \frac{d\alpha}{dn} + \frac{d\delta}{dn} \right) \]  \hspace{1cm} (6-25)

The stick displacement per g — the factor \( \frac{d\delta}{dn} \) on the right hand side of (6-25) — has already been determined in 6.3. In order to find \( \frac{dF_e}{dn} \), the derivative \( \frac{d\alpha_h}{dn} \) needs to be determined.

The derivation of \( \frac{d\alpha_h}{dn} \)

The angle of attack \( \alpha_h \) of the horizontal tailplane varies with \( n \) in the steady manoeuvres, because both \( \alpha \) and \( q \) or \( \frac{q}{V} \) vary with \( n \) and:

\[ \alpha_h = f (\alpha, \frac{q}{V}) \]

The partial variation of \( \alpha_h \) with \( \alpha \) alone was already discussed in 3.5. For steady, straight flight it was found that:

\[ \alpha_h = (\alpha - \alpha_o) \left( 1 - \frac{d\alpha}{d\alpha} \right) + (\alpha_o + \delta_h) \]  \hspace{1cm} (3-25)
The partial variation of $\alpha_h$ with $q$ alone will now be derived. In the idealized pull-up manoeuvre and in the steady turn the symmetric motion of the airplane relative to the air can be considered as a pure rotation - with angular velocity $q$ - about a center of rotation situated on the $z$-axis above the center of gravity, see fig. 6.10.

This position of the center of rotation is such that the local angle of attack at the center of gravity does not change due to the rotation.

When discussing in 8.2 the stability derivatives with respect to pitching velocity, it will be shown that the principal effect of such a 'q-motion' is a variation of the local, geometric angle of attack, proportional to the angular velocity $q$ and the distance in $x$-direction to the center of gravity, see fig. 6.10:

$$\Delta \alpha = \frac{x - x_{c.g.}}{R} = \frac{x - x_{c.g.} - q \bar{c}}{c} \cdot \frac{q \bar{c}}{V}$$  \hspace{1cm} (6-26)

Fig. 6.10: The variation of the angle of attack in an arbitrary point of the airplane caused by a pure q-motion.
At the horizontal tailplane the change in geometric angle of attack due to a \( q \)-motion is:

\[
\Delta \alpha_h = \frac{x_h}{c} - \frac{x_c q_c}{c} \cdot \frac{q_c}{V} = \frac{l_h}{c} \cdot \frac{q_c}{V} \tag{6-27}
\]

In steady, turning flight the total \( \alpha_h \) is found by adding (3-25) and (6-27):

\[
\alpha_h = (\alpha - \alpha_a) \left( 1 - \frac{d\epsilon}{d\alpha} \right) + (\alpha_a + l_h) + \frac{l_h}{c} \cdot \frac{q_c}{V} \tag{6-28}
\]

From (6-28) follows the derivative with respect to \( n \):

\[
\frac{d\alpha_h}{dn} = \left( 1 - \frac{d\epsilon}{d\alpha} \right) \cdot \frac{d\alpha}{dn} + \frac{l_h}{c} \cdot \frac{d\frac{q_c}{V}}{dn} \tag{6-29}
\]

The derivatives \( \frac{d\alpha}{dn} \) and \( \frac{d\frac{q_c}{V}}{dn} \) were already obtained previously, see (6-6), (6-9) and (6-10).

All variables needed to determine \( \frac{d\alpha_h}{dn} \) are now known. Substituting (6-6), (6-9) and (6-10) results in:

**Pull-up manoeuvres:**

\[
\frac{d\alpha_h}{dn} = \frac{W}{\frac{1}{2} \rho V^2 S} \left\{ \left[ 1 - \frac{d\epsilon}{d\alpha} \right] \frac{1}{C_{N\alpha}} + \frac{l_h}{c} \cdot \frac{1}{2\mu_c} \right\} \tag{6-31}
\]

**Steady turns:**

\[
\frac{d\alpha_h}{dn} = \frac{W}{\frac{1}{2} \rho V^2 S} \left\{ \left[ 1 - \frac{d\epsilon}{d\alpha} \right] \frac{1}{C_{N\alpha}} + \frac{l_h}{c} \cdot \frac{1}{2\mu_c} \cdot \left( 1 + \frac{1}{n^2} \right) \right\} \tag{6-32}
\]

All factors in (6-25) are now known. Substituting and using the expressions
already derived in 3.7 and 5.2:

\[ C_{m_{\delta}} = - C_{N_{h_{\delta}}} \left( \frac{V}{V} \right)^2 \frac{S_h \cdot \lambda_h}{S \cdot c} \]  

(3-35)

and:

\[ C_{m_{\alpha}} = C_{N_{\alpha}} \cdot \frac{x_{c \cdot s_{\alpha}} - x_{\alpha}}{c} = C_{N_{h_{\delta}}} \left( 1 - \frac{dc}{d\alpha} \right) \left( \frac{V}{V} \right)^2 \frac{S_h \cdot \lambda_h}{S \cdot c} \]  

(5-9)

and:

\[ C_{N_{h_{\delta}}} = C_{N_{h_{\alpha}}} - C_{N_{h_{\delta}}} \cdot \frac{C_{h_{\alpha}}}{C_{h_{\delta}}} \]  

(5-7)

the stick force per g is obtained after some reformulation:

**pull-up manoeuvres:**

\[ \frac{dF}{dn} = \frac{d\delta}{ds} \frac{V}{S} \left( \frac{V}{V} \right)^2 \frac{S_c}{C_{m_{\delta}}} \left( C_{m_{\alpha}} \frac{C_{m_{\delta}}}{C_{m_{\alpha}}} + \frac{1}{2} \frac{C_{m_{\delta}}}{C_{m_{\alpha}}} \left( C_{m_{\alpha}} - C_{m_{\delta}} \cdot \frac{C_{h_{\alpha}}}{C_{h_{\delta}} \cdot \frac{\lambda_h}{c}} \right) \right) \]  

(6-33)

**steady-turns:**

\[ \frac{dF}{dn} = \frac{d\delta}{ds} \frac{V}{S} \left( \frac{V}{V} \right)^2 \frac{S_c}{C_{m_{\delta}}} \left( C_{m_{\alpha}} \frac{C_{m_{\delta}}}{C_{m_{\alpha}}} + \frac{1}{2} \frac{C_{m_{\delta}}}{C_{m_{\alpha}}} \left( C_{m_{\alpha}} - C_{m_{\delta}} \cdot \frac{C_{h_{\alpha}}}{C_{h_{\delta}} \cdot \frac{\lambda_h}{c}} \right) \right) \]  

(6-34)

The first term between the brackets of the expressions (6-33) and (6-34) is due to the static stability, stick free. The second term between the brackets in (6-33) can be simplified considerably. From (6-27) it can be seen that \( \frac{\lambda_h}{c} \) is a measure of the change in angle of attack of the horizontal tailplane caused by the \( q \)-motion:

\[ \frac{d\alpha_h}{d_q} = \frac{\lambda_h}{c} \]  

(6-35)
The variation of the elevator angle with the angle of attack of the horizontal tailplane in the stick free situation was expressed in (5-1) of 5.1:

\[
\left( \frac{d\delta}{d\alpha_h} \right)_{\text{free}} = -\frac{C_{h\alpha}}{C_{h\delta}}
\]

Combining (6-35) and (5-1) results in:

\[
\left( \frac{d\delta}{d\alpha_h} \right)_{\text{free}} = \left( \frac{d\delta}{d\alpha_h} \right) \cdot \frac{d\alpha_h}{d\frac{ac}{V}} = -\frac{C_{h\alpha}}{C_{h\delta}} \cdot \frac{\alpha}{c}
\]

Evidently, the term \(-\frac{C_{h\alpha}}{C_{h\delta}} \cdot \frac{\alpha}{c}\) in (6-33) and (6-34) represents the moment due to the \(q\)-movement, caused by freeing the elevator for:

\[
\Delta \left( \frac{d\delta}{d\frac{ac}{V}} \right)_{\text{free}} = \Delta C_m q = C_{m\delta} \cdot \left( \frac{d\delta}{d\frac{ae}{V}} \right)_{\text{free}} = -C_{m\alpha} \cdot \frac{C_{h\alpha}}{C_{h\delta}} \cdot \frac{\alpha}{c}
\]

If a new derivative, \(C_m q_{\text{fix}}\), is introduced it can be written as:

\[
C_m q_{\text{free}} = C_m q_{\text{fix}} + \Delta C_m q
\]

or, with (6-36):

\[
C_m q_{\text{free}} = C_m q_{\text{fix}} - C_{m\delta} \cdot \frac{C_{h\alpha}}{C_{h\delta}} \cdot \frac{\alpha}{c}
\]

With (6-37), (6-33) and (6-34) this can be written as:
pull-up manoeuvres:

\[
\frac{dF_e}{dn} = \frac{d\delta}{dn} \frac{w}{S} \left( \frac{V}{V} \right)^2 \frac{C_h}{C_m \alpha} \left[ \frac{C_m^{\alpha \text{free}}}{C_m \alpha} + \frac{C_m^{\delta \text{free}}}{2\mu_c} \right]
\]  (6-38)

steady-turns:

\[
\frac{dF_e}{dn} = \frac{d\delta}{dn} \frac{w}{S} \left( \frac{V}{V} \right)^2 \frac{C_h}{C_m \alpha} \left[ \frac{C_m^{\alpha \text{free}}}{C_m \alpha} + \frac{C_m^{\delta \text{free}}}{2\mu_c} \left( 1 + \frac{1}{n^2} \right) \right]
\]  (6-39)

From (6-38) and (6-39) the influence of the c.g. position on the stick force per g can be understood. If the c.g. moves aft, \( C_m^{\alpha \text{free}} \) decreases in the absolute sense and as a consequence the stick force per g decreases in the absolute sense.

From the above expressions it will be seen, that contrary to the stick displacement per g, the stick force per g is independent of airspeed, if the aerodynamic derivatives in (6-38) and (6-39) are invariant with airspeed.

The variation of the stick force per g with the c.g. position is also evident from the measurements shown in figs. 6.11 and 6.12. The stick force per g in steady turns is indeed - in the absolute sense - larger than in pull-up manoeuvres at the same load factor n. The relatively large scatter in the data points, visible mainly in fig. 6.11, is thought to be caused by the friction in the control mechanism.

6.6. The manoeuvre point, stick free

In 6.4, it was shown that at a certain c.g. position corresponding with the manoeuvre point, stick fixed, the stick displacement per g becomes zero. \( \frac{dF_e}{dn} \) is zero. This point is called the manoeuvre point, stick free. It is also indicated as \( m.P^{\text{free}} \); the abscissa of this point is \( n^{\text{free}} \). From the discussion in 5.3, follows:

\[
\frac{C_m^{\alpha \text{free}}}{C_m \alpha} = \frac{x_{\text{c.g.}} - n^{\text{free}}}{c}
\]  (6-40)
Fig 5.11: The incremental control force $\Delta F_e$ as a function of the incremental load factor $\Delta n$ in pull-up manoeuvres. Auster J-5B "Autocar". (From ref 6.1).

Fig 6.12: The incremental control force $\Delta F_e$ as a function of the incremental load factor $\Delta n$ in turns. Auster J-5B "Autocar". (From ref 6.1).
If \( x_{\text{c.g.}} = x_{m_{\text{free}}} \), is \( \frac{dF}{dn} = 0 \), then:

\[
\frac{C_{m_{\text{a,free}}}}{C_{N_\alpha}} + \frac{C_{m_{q_{\text{free}}}}}{2\mu_c} = 0
\]

For the pull-up manoeuvre, the above can be written, see (6-38) and (6-40), as:

\[
\frac{x_{m_{\text{free}}}}{c} - \frac{x_{n_{\text{free}}}}{c} = \frac{C_{m_{q_{\text{free}}}}}{2\mu_c}
\]

(6-4)

Since \( C_{m_{q_{\text{free}}}} \) is again negative, the manoeuvre point, stick free, lies behind the neutral point, stick free in gliding flight.

According to (6-40) and (6-41) is for the pull-up manoeuvres:

\[
\frac{x_{c.g.} - x_{m_{\text{free}}}}{c} = \frac{C_{m_{a_{\text{free}}}}}{C_{N_\alpha}} + \frac{C_{m_{d_{\text{free}}}}}{2\mu_c}
\]

(6-42)

For the stick force per g follows then with (6-38) and (6-42):

\[
\frac{dF}{dn} = \frac{d\delta}{dn} \frac{W}{S} \left( \frac{h}{V} \right)^2 \frac{C_{n_\delta}}{S} \frac{C_{m_{c.g.}} - x_{m_{\text{free}}}}{c}
\]

(6-43)

For the steady turns it can be derived in the same way:

\[
\frac{x_{n_{\text{free}}}}{c} - \frac{x_{n_{\text{free}}}}{c} = \frac{C_{m_{q_{\text{free}}}}}{2\mu_c} \left( 1 + \frac{1}{n^2} \right)
\]

(6-44)

and consequently:

\[
\frac{x_{c.g.} - x_{m_{\text{free}}}}{c} = \frac{C_{m_{a_{\text{free}}}}}{C_{N_\alpha}} + \frac{C_{m_{d_{\text{free}}}}}{2\mu_c} \left( 1 + \frac{1}{n^2} \right)
\]

(6-45)
Fig. 6.13: Calculated positions of the manoeuvre point, stick free, and the stick force per g of the Fokker F-27 "Friedship".
This means, that (6-43) is true also for the stick force per $g$ in steady turns, although the position of the manoeuvre point, stick free, differs for the two types of manoeuvres, according to (6-41) and (6-44).

Just as in the case of the manoeuvre point, stick fixed, a second interpretation can be given of the manoeuvre point; stick free. To this end, the transition from a condition of steady, straight flight to a manoeuvre is separated in two phases. These will be considered separately one after another.

In the first phase, the airplane is supposed to change with free elevator, from steady, straight flight to a turning manoeuvre. As far as the forces along the $z$-axis are concerned this is a steady manoeuvre: equilibrium exists along the $z$-axis. The change in normal force $dC_N$ due to this transition acts in a certain point, the abscissa of which is provisionally indicated as $x_{dC_N}^{\text{free}}$.

The accompanying change in the pitching moment due to this transition then is:

$$dC_{m_{\text{free}}} = dC_N \frac{x_{c.g.} - x_{dC_N}^{\text{free}}}{c}$$

The second and subsequent phase takes place at constant $\alpha$, $\gamma$ and $V$. In this phase equilibrium of the pitching moment is obtained. The moment $dC_{m_{\text{free}}}$ is balanced by applying an appropriate elevator deflection $d\delta_e$ and the control force $dF_e$. The required elevator angle $d\delta_e$ is:

$$d\delta_e = -\frac{dC_{m_{\text{free}}}}{c_{m_{\delta}}} = -\frac{1}{c_{m_{\delta}}} \cdot dC_N \frac{x_{c.g.} - x_{dC_N}^{\text{free}}}{c}$$

(6-46)

The required elevator control force $dF_e$ follows from the change in the hinge moment occurring in the second phase. Note that in the first phase the elevator was free: $F_e = 0$. Thus:

$$dF_e = \frac{d\delta_e}{ds_e} \frac{1}{2} \rho V^2 S \frac{c_{m_{\delta}}}{c_{m_{\delta}}} \cdot dC_{\delta_e}$$

(6-47)
Fig. 5.14: Influences of attitude and magnitude and sign of $C_{h\alpha}$ on the position of the manoeuvre point, stick free, and the stick force per g.
Since $a_h$ is constant in the second phase ($\alpha$ and $q$ remain constant), \( dC_{h_e} \) is:

\[
dC_{h_e} = C_{h_{\delta}} \cdot d\delta_e
\]  \hspace{1cm} (6-48)

Substituting \( d\delta_e \) according to (6-46) in (6-48) and subsequently in (6-47) results in:

\[
dF_e = \frac{d\delta_e}{ds_e} \cdot \frac{1}{\rho V_h^2} \cdot \frac{S_e}{c_e} \cdot \frac{C_{h_{\delta}}}{C_{m_{\delta}}} \cdot \frac{dC_N}{N_{\text{free}}} \cdot \frac{x_{c.g.} - x_{dC_N_{\text{free}}}}{c}
\]  \hspace{1cm} (6-49)

Next, the change in elevator control force \( dF_e \) is considered, following from the expression (6-43) for the stick force per g. With (6-20), where \( dC_N = dC_{\text{N\_free}} \), from (6-43) follows for both types of manoeuvres:

\[
dF_e = \frac{d\delta_e}{ds_e} \cdot \frac{1}{\rho V_h^2} \cdot \frac{S_e}{c_e} \cdot \frac{C_{h_{\delta}}}{C_{m_{\delta}}} \cdot \frac{dC_N}{N_{\text{free}}} \cdot \frac{x_{c.g.} - x_{m_{\text{free}}}}{c}
\]  \hspace{1cm} (6-50)

If the expressions (6-49) and (6-50) are now compared, it will be clear that the manoeuvre point, stick free, in (6-50) corresponds with the point of action of the change in normal force \( dC_{\text{N\_free}} \) in (6-49), caused by the transition with free elevator from steady, straight flight to a manoeuvre. This explains the second interpretation of the manoeuvre point, stick free.

Using (6-41) and (6-44) the position of the manoeuvre point was calculated for the Fokker F-27. The results are shown in fig. 6.13. The \( C_{h_{\alpha}} \) of the elevator of this airplane is approximately zero. As a result, there is no difference between the situations: control free and control fixed. It can be seen from the figure that the manoeuvre point moves forward with increasing flight altitude, approaching in the limit the neutral point. With increasing load factor \( n \), the manoeuvre point in steady turns approaches the manoeuvre point in pull-up manoeuvres.

Fig. 6.14 indicates schematically how the position of the manoeuvre point, \( dF_e \) stick free, and \( \frac{dF_e}{dn} \) change with flight altitude and with \( C_{h_{\alpha}} \). Fig. 6.15 shows the influence of \( C_{h_{\delta}} \) on the range of permissible center of gravity positions.
The discussion in 6.2 referred to the fact that the stick force per g has an upper and a lower limit of permissible values. Other requirements remain also in force, such as those relating to the elevator control position and force stability. As a consequence it is sometimes not an easy matter to satisfy all requirements on the stability and control characteristics in all airplane configurations and flight conditions stipulated in the regulations.

6.7. Non-aerodynamic means to influence the stick force per g

Sometimes springs or unbalanced masses are installed in the control mechanism, to influence the elevator control force stability or the stick force per g or both.

In 5.5 the influence was discussed of a spring or an unbalanced mass on the elevator control force stability and the position of the neutral point, stick free. The spring was assumed to exert a constant hinge moment.

Such a spring has no influence on the stick force per g, but an unbalanced mass has.

Suppose, a mass has been installed in the control mechanism, see fig. 6.16, such that in straight and level flight a static hinge moment $H_{eW}$ is generated.
\[
\Delta H \frac{d\delta_e}{ds_e} \Delta F_e \text{ due to bobweight.}
\]

\[\Delta H_e = (n-1) \mathcal{H}_e^W\]

Fig. 6.16: The incremental hinge moment due to a bobweight in the control mechanism, in a pull-up manoeuvre at a load factor \(n\).

In a manoeuvre at a load factor \(n\) an extra hinge moment \((n-1) \mathcal{H}_e^W\) occurs. This hinge moment must be balanced by an extra control force \(\Delta F_e\):

\[
\Delta F_e = - \frac{d\delta_e}{ds_e} \Delta n \cdot \mathcal{H}_e^W
\]

The influence of the mass - sometimes called a 'bobweight' - on the stick force per g is:

\[
\frac{dF}{dn} = - \frac{d\delta_e}{ds_e} \mathcal{H}_e^W \quad (6-51)
\]

In this way it is seen that a modification of the stick force per g can be obtained which is independent of e.g. position and flight altitude.

\[
\frac{dF}{dn}
\]

In some cases this may bring \(\frac{dF}{dn}\) within the required limits.

By choosing the right combination of springs and bobweights, it is possible in principle to vary both the elevator control force stability and the stick force per g independent of one another, using non-aerodynamic means. If however, the hinge moment generated by springs or bobweights or both becomes too large, the dynamic stability, stick free, may be influenced unfavourably. In addition, the possibility of the occurrence of flutter has to be investigated.

It was shown in this Chapter, that the stick displacement per g and the
stick force per g must have the negative sign, whereas the magnitude of the stick force per g is additionally limited to lie within a certain range. These requirements may restrict both the forward and the rear limits of permissible c.g. positions. This remark is supplementary to what was noted in 5.8.

There the rear limit of c.g. positions was said to be determined by the requirement that the airplane must possess elevator control position and - force stability. The forward limit of permissible c.g. positions was there shown to be dictated by the available elevator power in take-off or landing.
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<td>2.9.</td>
<td>V. Holmboe</td>
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