Acknowledgement

The financial support which made the publication of this translation possible was furnished by the National Science Foundation under Grant number GK - 607. This support is gratefully acknowledged.
HEAT TRANSFER FROM A HOT WALL TO DROPS IN A SPHEROIDAL STATE

by

L. H. J. Wachtors*

(Laboratorium voor Fysische Technologie, Technische Hogeschool, Delft, The Netherlands)

Translated by H. Houtsager

Chapter I. Statement of the Problem and History

Sect. 1.1. Introduction

It has been known for centuries that a liquid which is poured over a very hot solid surface forms drops which roll across it. This phenomenon, which was studied for the first time at the beginning of the 18th century by the German physician-physicist Leidenfrost, was later called "l'etat spheroidal" by the French pharmacist Boutigny [37]. Boutigny thought he had discovered a fourth state of matter. Since the end of the 19th century it has been known that this phenomenon is created by the vapor which forms on the bottom of the drop. The necessary heat must be transmitted through this vapor layer so that the heat transfer is poor; such a drop, therefore, evaporates very slowly.

The spheroidal state, also known as the phenomenon of Leidenfrost, has until recently been viewed mainly as a curiosity. Although as early as the beginning of the 19th century it was realized that the phenomenon was connected with film boiling, there was very little interest in it from a technical standpoint. For several years in England and Italy, however, a very intensive

* Present address: 3 M-Company, Central Research Laboratories, St. Paul, Minnesota, U. S. A.
research program has been underway in which the possibility of heat transfer in nuclear reactors by means of so-called spray-cooling is being investigated. This type of cooling occurs in a boiling water reactor if the steam quality if brought up to a very high value. In the second part of the cooling channels a mist of steam and very small water droplets if formed, while the walls are covered with a very thin evaporating liquid film, which constantly is being fed by drops which are deposited upon it. The thickness of the film is often not more than 10 microns, so that for the evaporation necessary the heat meets only a relatively low resistance. As a result, the heat transfer coefficient is relatively large. This method of cooling offers the advantages of a boiling water reactor whereas the chance for instability is rather slim.

However, if suddenly the power of the reactor increases, the liquid film on the pipe wall will completely evaporate and the wall temperature will increase rapidly. At a given moment the wall temperature will be so high that the wall can no longer be wetted by drops. Before they can touch the solid surface an insulating vapor layer is formed and the drops bounce like little rubber balls back into the mist stream. It is important to know under which conditions the spheroidal state exists, and this being the case, in what respect the bouncing drops still can aid in the heat transfer. This problem can be separated in two parts. The first part is how many drops per given unit of time and per given unit of surface impinge upon the wall, and in the second place one has to try to find out what happens if a particular liquid drop hits the hot wall. The research work described in this thesis will only try to answer the latter question.

We started with the most simple case - a motionless drop resting on its cushion of vapor on a horizontal hot plate. For the heat transfer to such
a drop theory was developed which was verified by measurements. The description of this is given in Chapter II. In the next chapter, the behavior of impinging drops is described. Their size was such that they could easily be photographed. In this way information was obtained about their hydrodynamic behavior, which was used to calculate how much heat is transmitted during the impingement of a drop. The results of these calculations were compared with experimental results. Finally, in Chapter IV the heat transfer to a mist of very small droplets is studied. By extrapolation of the results of Chapter III the hydrodynamic behavior of these small droplets could be found. The theoretically calculated heat transfer was again compared with experimental results.

Section 1.2 Some other Fields of Application

During the cooling of superheated steam by injection of water usually a large number of water drops hits the pipe wall. When the wall is not hot enough to form a complete vapor layer under the drops it will be exposed to very large temperature fluctuations. Practical experience has shown that the shock which is created in this way can severely damage the pipe. For this reason, immediately behind the place of injection a so-called shield is installed which easily can be replaced.

The same thing happens with the hot blades of a steam turbine. The velocity of these blades is usually very high so that except for the thermal shock also a great danger for erosion exists. Savic and Boul [68,69] have tried to calculate the heat transfer for this case. Their way of approach is far from complete and also not quite correct.
The first one who mentioned anything in a publication was Boerhaave (1). In his "Elementa Chemic" he expresses his amazement about the fact that a drop of alcohol can roll over a hot piece of iron without igniting. He does not get involved in explanation. Many, however, lay his incentive "nodus hic vestro dignus acumina" to their hearts. Leidenfrost (3) was the first one who thoroughly investigated the phenomenon. He studied the behavior of drops of water, alcohol, ammonia and mercury which he dropped on very hot surfaces of gold, silver, iron, copper, and lead, and discovered that the coarser the metal surface the faster the evaporation of the liquid.

During the first half of the 19th century many researchers have occupied themselves with the phenomenon of Leidenfrost. It was felt that there was a connection with the so often happening explosions of steam boilers. Boutigny (37) mentions that by these in the United States alone in the year 1840 about one thousand persons lost their lives. Tests made by Perkins (17) and Boutigny (37) showed that some connection did exist and that one had to do with a phenomenon which we now know as film boiling.

The fundamental cause of the phenomenon of Leidenfrost was not yet known during that time. In order to find out more about it very ingenious experiments were put together. Boutigny alone describes about 106 of them. The majority of these experiments can be summarized as follows.

1. One tried to obtain a spheroidal condition with various kinds of liquids, for instance water, alcohol, sulphuric acid, sulphur dioxide, trichloroethylene, turpentine, oil and mercury (see Faraday (18), Frankenheim (23) and Boutigny (37)).

2. The experiment was also tried with various kinds of hot surfaces, for instance gold, silver, platinum, iron, copper, and lead (see Klaproth (6),
Boutigny (37), Buff (41), Berger (45) and Moss (51). For some time a very hot discussion has been going on whether it was possible to do the experiment on glass or procelain (see Emsmann (30), Marchand (31), and Moritz (38)).

3. One succeeded also in forming drops of a given liquid on the surface of another hot liquid, for instance, ether on top of water, water on molten lead, mercury on molten iron (see Bellani (12), Thomlinson (27), Pelouze (28), Boutigny (37), Buff (41) and Van der Mensbrugghe (50)).

4. Only a few times the evaporation rate of the drops was measured. Usually the results could not be reproduced very well. It appeared that especially the nature of the surface was important (see Klaproth (6)).

5. Many have tried to measure the temperature of the drops (see Doebereiner (13), Baudrimont (25), Laurent (26), LeGrand, DeKramer, Belli (33), Peltier (34), Boutigny (37), Buff (41) and Berger (48)). The measured values varied considerably, both because of the rather poor measuring equipment and because of the role played by the evaporation at the top of the drops. (See section 2.7 of this thesis). In general one was very surprised about the low temperature of the drops. Really spectacular was Boutigny's experiment in which a little bit of water was frozen inside a drop of sulphur dioxide placed on a white glowing platinum plate.

6. Another point about which very heated discussions have been going was the temperature at which it was just about possible to deposit a drop on a surface without wetting it (see Boutigny (37), Buff (41), Budde (47), Berger (48), and Gossart (53)). Because the way of depositing was not always the same the measured values varied considerably. However, it became clear that a higher temperature was required when the surface was coarser and more corroded.
The following experiments proved that there is no direct contact between the liquid and the hot surface.

7. When a light source is placed behind a drop made opaque with a dye one can distinguish a beam of light between the drop and the plate (see Doebersiner (13), Person (32), Boutigny (37), Tyndall (145), Colley (146) and Hesehus (149)).

8. That no intermittent contact existed either was shown by the fact that no chemical reaction occurred between the liquid and the hot surface (see Boutigny (37), Church (149), and Berger (148)). The experiment, was done, for instance, with sulphuric acid on iron. However, confusion existed in some cases where the vapor of the liquid or a gas dissolved in the liquid would react with the metal surface, like e.g. drops of ammonia on copper (Church (145), Berger (149)).

9. A good solid proof was finally given by the fact that no electrical current could be established between the drop and the hot surface (see Poggendorff (22), Buff (41), and Hesehus (149)).

In the course of time various theories about the phenomenon of Leidenfrost have been developed. Perkins (17), e.g. thought that repulsive forces existed between all hot bodies. Muncke (19) and Frankenheim (23) stated that at high temperatures cohesion forces were much greater than adhesion forces. This hypothesis culminated in the theory of Boutigny (37) about "l'état spheroidal", which would be the fourth state of matter. Person (32) was the first one who developed the theory that the liquid and the hot surface can not touch each other because of a vapor layer which is formed between them. Around 1870 this theory was accepted almost everywhere.
It was felt that the slow evaporation rate of the drops was caused by reflection of the heat (see de Rumford (2)) or because of the thermal radiation transparency of the liquid (see Pouillet (15)). In 1888 Kristensen (22) stated that not heat radiation but heat conduction is the most important transport mechanism.

After 1900 the research does in fact not offer any new aspects. One concentrated mainly on the experiments called under the points 4 and 6. As far as necessary, we will go further into this in Chapter II.

**Chapter II. Sessile Drops**

Section 2.1 Introduction and Literature Survey

If one deposits very carefully a drop of liquid on a somewhat concave, sufficiently hot surface then immediately a continuous vapor layer is formed, which prevents direct contact between the liquid and the underlying surface. Usually the drop rolls up and down a little bit but eventually comes to rest at the lowest point of the surface. It obtains the shape of a somewhat sagged and on all sides rounded body such as shown in Figure 2.1. The evaporation rate of such a resting drop, especially when it is situated in an environment of saturated steam, is much lower than that of a liquid which is in direct contact with a hot surface.
In this chapter a theory is developed in order to describe the poor heat transfer (sections 2.2 to 2.4). Next a number of experiments are described in order to test this theory. The measuring apparatus and the measuring procedure are briefly described in section 2.5 after which in the sections 2.6 and 2.7 the measuring results and a discussion are given.

How many research workers have occupied themselves with the problem of a droplet on a hot plate is shown in Chapter I in which a summary of the literature until 1900 is presented. The major part of this research is, however, rather qualitative. After 1900 the information becomes much more quantitative. From the latter group of research workers the majority concentrated on the measurement of the evaporation rate of a sessile drop as a function of the temperature of the hot surface. Originally, one limited himself to measuring the time required for complete evaporation of a drop
of a given volume. Pleteneva and Retinder (61) found that for small drops the lifetime was proportional to the original diameter of the drops. Often various liquids were measured on various kinds of hot surfaces. Borishansky (62, 64) established, for instance, the lifetime of drops of water, alcohol, and tetra as a function of temperature of the hot surface. A typical picture is given in Figure 2.2 which shows some of his results.

Fig. 2.2 The lifetime of a water drop of 45,8 mm$^3$ on a brass plate as a function of the temperature of the surface $T_m$ (Borishansky (64)).

One sees that with decreasing surface temperature the lifetime of the drops first increases to a maximum and next drops very sharply to a minimum. The location of the maximum and minimum are characteristic for a given liquid and the nature of the hot surface. Between the maximum and the minimum nucleate boiling occurs, so that one cannot speak about a true
spheroidal state. At higher temperatures than those at which the maximum occurs there is a continuous layer of vapor underneath the droplet, at least when after deposition of the drop sufficient time has been allowed for the oscillatory motions of the drop to die out. For this part of the curve Borishansky developed some semi-empirical formulas for the lifetime for which he categorized the drops in three sizes: 1) Small and nearly round drops. 2) Large flat drops, and 3) Drops of such a size that vapor bubbles break through the liquid.

Gottfried and Bell [65] measured the lifetime of drops of water, alcohol, benzene, and tetra which were deposited on a stainless steel surface. They stated that the heat transfer only took place by means of radiation. Their calculations obtained a somewhat semi-empirical character by adjusting the geometric factor to the measuring results.

The method of finding the heat transfer to drops by measuring the total lifetime is rather questionable. Not only has the theoretically established evaporation rate to be integrated over all the sizes through which the evaporating drop has gone, but it also appears that the volume decrease of a drop as soon as it has become very small is no longer continuous. Thornton [73] discovered that with very small dimensions the evaporation rate may suddenly increase enormously. Sometimes the drop explodes with a soft crack. The condition of the surface determines the time at which this happens.

Other research workers used a steady state method to establish the evaporation rate. Gorton [63] for instance, used drops in which a quartz capillary was inserted from above. In this way he could
keep them on one location on a completely flat surface. Through the capillary a constant flow of liquid was fed to the drop. After some time, the size of the drop established itself at a certain value. In this steady state the amount of supplied liquid per unit of time is equal to the evaporation rate of the drop. Gorton calculated with the aid of the measured evaporation rate and the weight of the drop the width of the vapor layer and compared it with what he could measure from photos. For these calculations he assumed that the pressure built up underneath the drop was only caused by the acceleration of the vapor and that the viscous forces could be disregarded.

Kistemaker [67] also used a constant liquid flow which was introduced into the drop through a capillary. The number of his measurements is low so that only a rather faint picture can be obtained. Kistemaker also measured, with the use of thermocouples which were located in the metal surface, the amount of heat which was fed to the drop. The interpretation of the thus obtained experimental result is rather questionable. Finally, he determined the thickness of the vapor layer of drops of a dilute barium acetate solution with the use of x-rays. For the calculation of the flow within the vapor layer he disregarded the viscosity of the vapor and for the determination of the pressure at the edge of the vapor layer he neglected the influence of the surface tension. Therefore, his theory for the evaporation rate is not correct.

All above mentioned authors measured the evaporation rate of droplets under ambient conditions, i.e. in a non-saturated atmosphere. Only Gottfried and Bell, however, took into account the fact that under
these conditions evaporation also takes place on the top of the drops. The others only considered in their calculations the evaporation at the bottom. In section 2.7 it will appear that the thus introduced error may be quite serious.

Sect. 2.2 The shape of a sessile drop

A drop resting on a horizontal surface is radially symmetrical around a vertical axis. Hence, the question about the shape of the drop can be reduced to the question of the form of a meridian. For the present, we will limit ourselves to that part in Fig. 2.3 which is called OMN. The bottom is assumed to be completely flat which is actually not the case because the pressure in the vapor layer underneath the drop is not uniform (See section 2.3). It will appear, however, that this has no influence on the shape of OMN.

Fig. 2.3 Cross section of a drop in the spheroidal state resting on a hot surface.
Assume in Fig. 2.3 cylindrical coordinates with axes \( r \) and \( z \) meeting in the point \( O \) which lies in the top of the drop. The liquid surface \( z(r) \) has for every point the following pressure balance

\[
P_t + (\rho_l - \rho_g) g z = P_0 + \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right).
\]  

(2.1)

In this equation \( P_t \) is the pressure at the top of the drop, and \( P_0 \) the environmental pressure. \( \rho_l \) and \( \rho_g \) are the density of the liquid and the ambient gas, \( \sigma \) the surface tension, and \( R_1 \) and \( R_2 \) the main radii of curvature in each particular point. If the well-known values for \( R_1 \) and \( R_2 \) are filled in (see for instance Bouasse [1]), and if one considers that at the top of the drop \( R_1 \) and \( R_2 \) are the same, then one obtains

\[
2\sigma \frac{d^2z}{dr^2} \bigg|_{r=0} + (\rho_l - \rho_g) g z = \sigma \left[ \left[ 1 + \left( \frac{dz}{dr} \right)^2 \right]^{-\frac{3}{2}} \frac{d^2z}{dr^2} + \left[ 1 + \left( \frac{dz}{dr} \right)^2 \right]^{-\frac{1}{2}} \frac{1}{r} \frac{dz}{dr} \right].
\]  

(2.2)

This equation becomes dimensionless after the following substitutions

\[
z = \zeta L, \quad r = \rho L, \quad L = \left( \frac{2\sigma}{(\rho_l - \rho_g) g} \right)^{\frac{1}{3}}.
\]  

(2.3)
The equation becomes then

\[ \frac{d^2 \zeta}{d \rho^2} \bigg|_{\rho=0} + \zeta = \frac{1}{2} \left[ 1 + \left( \frac{d \zeta}{d \rho} \right)^2 \right]^{-1} \frac{d^2 \zeta}{d \rho^2} + \left[ 1 + \left( \frac{d \zeta}{d \rho} \right)^2 \right]^{-1} \frac{1}{\rho} \frac{d \zeta}{d \rho}. \] (2.4)

For \( \rho = 0 \) the following initial conditions apply

\[ \zeta(0) = 0, \quad \frac{d \zeta(0)}{d \rho} = 0. \] (2.5)

Assuming a certain value for the first term of (2.4), i.e., for the curvature at the top of the droplet, one can then find \( \zeta \) as a function of \( \rho \). The solution which can only be found numerically was calculated for 14 different values of \( \frac{d^2 \zeta}{d \rho^2} \bigg|_{\rho=0} \) with the use of a digital computer.\(^1\) Because in point \( M \) the first derivative of \( \zeta \) to \( f \) is infinite, we calculated in the area around \( M \) not \( \zeta \) a function of \( \rho \), but \( f \) as a function of \( \zeta \).

Also, the dimensionless volume \( \Phi = \frac{1}{2} \pi \rho^2 \zeta \) was calculated with the use of

\[ \Phi = \int_0^{\zeta_0} \pi \rho^2 d \zeta. \] (2.6)

Here it is assumed that the bottom of the drop is flat. The numerical calculations were checked by comparing the value of the dimensionless value found with (2.6) with that found from the pressure balance at the flat bottom:

---

\(^1\) The program for the ZEBRA was executed by A.A.M. Boeren and J. Emanuels. Both are employed by the Wiskundige Dienst of the Technische Hogeschool in Delft.
\[(\rho_1 - \rho_2)g V + \kappa R_0^2 \rho_0 = \kappa R_0^2 \{ \rho_1 + (\rho_1 - \rho_2)g h \} . \]  \hspace{1cm} (2.7)

If we fill in \( \rho_+ \) and we eliminate on the left and the right side of the 
equation \( R_0^2 \rho_0 \) and make it dimensionless with \( 2.3 \), we get

\[ \phi = \kappa \rho_0 \left\{ \frac{d^2 \zeta}{d \rho^2} \bigg|_{\rho = 0} + \zeta \right\} \]  \hspace{1cm} (2.8)

If we fill in the values of \( \phi \) and found from the integration of \( 2.4 \), we find a second value for \( \rho \). Besides the one given by \( 2.6 \). If with the 
numerical integration of \( 2.4 \) a suitable integration step was taken then 
the difference between the values of \( \rho \) found from \( 2.6 \) and \( 2.8 \) was not more 
than 2 units of the fourth decimal. In Table 2.1 the resulting main dimensions 
of the drops are listed. Because of the experiments that will be discussed 
later on, not the dimensionless values are given but the values associated 
with water drops at a temperature of \( 100^\circ C \). The value for \( L \) is in this case 
\( 3.5 \times 10^{-3} \) m.
Table 2.1

The dimensions of sessile water drops

<table>
<thead>
<tr>
<th>nr</th>
<th>$\frac{\text{d}^2 z}{\text{d}r^2}$ [m$^{-4}$]</th>
<th>$R_{\text{max}}$ [m]</th>
<th>$R_o$ [m]</th>
<th>$h$ [m]</th>
<th>$V$ [m$^3$]</th>
<th>$\pi R_o^2$ [m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0,1079,10$^3$</td>
<td>0,9261,10$^4$</td>
<td>0,2796,10$^5$</td>
<td>0,1849,10$^3$</td>
<td>0,3327,10$^4$</td>
<td>0,2456,10$^5$</td>
</tr>
<tr>
<td>2</td>
<td>0,5012,10$^4$</td>
<td>0,1994,10$^5$</td>
<td>0,1294,10$^4$</td>
<td>0,3954,10$^3$</td>
<td>0,3310,10$^4$</td>
<td>0,5239,10$^5$</td>
</tr>
<tr>
<td>3</td>
<td>0,2326,10$^5$</td>
<td>0,4280,10$^3$</td>
<td>0,5862,10$^4$</td>
<td>0,8301,10$^3$</td>
<td>0,3257,10$^4$</td>
<td>0,1080,10$^5$</td>
</tr>
<tr>
<td>4</td>
<td>0,1079,10$^6$</td>
<td>0,9066,10$^5$</td>
<td>0,2485,10$^3$</td>
<td>0,1640,10$^3$</td>
<td>0,2912,10$^4$</td>
<td>0,1940,10$^5$</td>
</tr>
<tr>
<td>5</td>
<td>0,5396,10$^7$</td>
<td>0,1719,10$^5$</td>
<td>0,7753,10$^3$</td>
<td>0,2688,10$^3$</td>
<td>0,1785,10$^4$</td>
<td>0,1888,10$^5$</td>
</tr>
<tr>
<td>6</td>
<td>0,3599,10$^8$</td>
<td>0,2410,10$^4$</td>
<td>0,1337,10$^2$</td>
<td>0,3326,10$^3$</td>
<td>0,4401,10$^3$</td>
<td>0,5617,10$^5$</td>
</tr>
<tr>
<td>7</td>
<td>0,1799,10$^9$</td>
<td>0,3926,10$^4$</td>
<td>0,2721,10$^3$</td>
<td>0,4223,10$^3$</td>
<td>0,1597,10$^4$</td>
<td>0,2326,10$^5$</td>
</tr>
<tr>
<td>8</td>
<td>0,1079,10$^{10}$</td>
<td>0,5225,10$^5$</td>
<td>0,3971,10$^3$</td>
<td>0,4669,10$^3$</td>
<td>0,2985,10$^4$</td>
<td>0,4956,10$^5$</td>
</tr>
<tr>
<td>9</td>
<td>0,2159,10$^{11}$</td>
<td>0,9618,10$^3$</td>
<td>0,8319,10$^3$</td>
<td>0,5222,10$^3$</td>
<td>0,1194,10$^4$</td>
<td>0,2170,10$^5$</td>
</tr>
<tr>
<td>10</td>
<td>0,1079,10$^{12}$</td>
<td>0,1152,10$^1$</td>
<td>0,1022,10$^4$</td>
<td>0,5268,10$^3$</td>
<td>0,4771,10$^3$</td>
<td>0,3294,10$^3$</td>
</tr>
<tr>
<td>11</td>
<td>0,5396,10$^{13}$</td>
<td>0,1341,10$^1$</td>
<td>0,1210,10$^4$</td>
<td>0,5275,10$^3$</td>
<td>0,2457,10$^3$</td>
<td>0,4599,10$^3$</td>
</tr>
<tr>
<td>12</td>
<td>0,2159,10$^{14}$</td>
<td>0,1588,10$^1$</td>
<td>0,1457,10$^4$</td>
<td>0,5260,10$^3$</td>
<td>0,3525,10$^3$</td>
<td>0,6668,10$^3$</td>
</tr>
<tr>
<td>13</td>
<td>0,1079,10$^{15}$</td>
<td>0,1775,10$^1$</td>
<td>0,1644,10$^4$</td>
<td>0,5243,10$^3$</td>
<td>0,4458,10$^3$</td>
<td>0,8487,10$^3$</td>
</tr>
<tr>
<td>14</td>
<td>0,2159</td>
<td>0,2210,10$^1$</td>
<td>0,2078,10$^1$</td>
<td>0,5200,10$^3$</td>
<td>0,7058,10$^3$</td>
<td>0,1357,10$^4$</td>
</tr>
</tbody>
</table>
In figure 2.4 to 2.6 the values for $R_0$, $h$, and $V$ are given versus $R_{\text{max}}$. At the opposing axes the values of the corresponding dimensionless parameters are given.

Fig. 2.4 The maximum radius of a sessile drop as a function of the radius of the bottom.
Fig. 2.5 The height of a sessile drop as a function of the maximum radius

Fig. 2.6 The volume of a sessile drop as a function of the maximum radius
Finally in Fig. 2.7 some of the calculated drop contours are given. It can be seen that small droplets are nearly spherical while large drops are rather flat. In the former the influence of the surface tension is dominant while for the latter gravity is important. For very small or very large drops one may ignore one of these two. For small droplets one may say that \( p_t - p_o > (\rho_f - \rho_g) gh \). If we ignore the last term in formula (2.7) and if we assume that the drop is a sphere with radius \( R_{\text{max}} \), so that \( V = \frac{4}{3}\pi R_{\text{max}}^3 \) and \( p_t - p_o = 2\sigma / R_{\text{max}} \), then we find

\[
R_o = \frac{2\sqrt{3} R_{\text{max}}^2}{3L}.
\] (2.9)

The height \( h \) of the droplet is of course equal to \( 2R_{\text{max}} \).

Fig. 2.7 The calculated contours of sessile drops of different sizes. For each contour the number associated with Table 2.1 has been given.
Very large drops are very flat at the upper side, so that \( p_t - p_o \).

From equation (2.7) one obtains using \( V = R_{\text{max}}^2 h \) the trivial result

\[
R_o = R_{\text{max}}^2 .
\] (2.10)

The height of the drops can be found with the method indicated by Bouasse (74). If we say that in equation (2.1) \( p_t - p_o = 0 \) and the radius of curvature in the plane perpendicular to a meridian plane is everywhere infinitely large, then this relation becomes

\[
(\rho_t - \rho_o)gz = \sigma \left[ 1 + \left( \frac{dz}{dr} \right)^2 \right] - \frac{d^2 z}{dr^2} .
\] (2.11)

Using the substitution \( \frac{dz}{dr} = tg \), we get

\[
(\rho_t - \rho_o)gz = \sigma \sin \varphi \frac{d\varphi}{dz} .
\] (2.12)

Integration from \( z = 0 \) to \( z = h \) finally gives

\[
h = L \sqrt{2} .
\] (2.13)

These approximations for big and small drops are given as dotted lines in Figs. 2.4 to 2.6. One sees that the approximation for small drops is fairly good for \( R_{\text{max}} = 1 \) mm. The approximation for large drops is much coarser. This approximation is only true for drops which are much larger than the calculated ones.

In the foregoing it was assumed that the drop bottom was flat. Based on this the volume \( V \) was calculated. In reality this is not the case.
with a drop in the spheroidal state. The vapor production underneath such a drop implies a radial pressure gradient. The bottom surface, therefore, will obtain a more or less curved shape. For the rest of the drop surface this does not matter though. There the shape is determined by the pressure balance (2.1) and this is in no respect depending upon the shape of the bottom of the drop. It thus follows that the calculated value \( V \) is not the true volume. However, it will appear later on that \( V \) as a number is very useful. Otherwise \( V \) is the volume of a drop resting on a dry surface, for instance a drop of mercury on glass or a drop of water on a sooty surface.

The above treated problem is not new. In the past the shape of a sessile drop is often used to determine the surface tension of a liquid especially that of liquid metals, (See for instance Heydweiler [75] Siedentopf [76]). Usually one limited himself to the upper half which can be calculated with the use of an approximation method. Bashforth and Adams [77] calculated the entire shape for a number of drops. Their results which already were published in 1883 but had fallen into oblivion for a while, are quite close to the ones which were obtained in the above described way. The later ones are, however, more accurate and cover a much larger field of drop sizes.

Sect 2.3 The evaporation Rate of Sessile Drops in a Saturated Atmosphere

The calculation of the evaporation rate of a drop floating on its vapor cushion can be separated into two parts:

1. The evaporation at the bottom, i.e. the area at which \( r = R_o \) (See Fig. 2.3). The distance between the drop and the hot surface is determined by the amount of vapor that is produced at this area.

2. Evaporation at the sides and the top of the drop (Section OMN in Fig. 2.3). Although this part of the drop is further away from the hot
surface it will appear later on that the evaporation there cannot be ignored, especially in the case of small drops where the bottom surface area \((rR_0)\) is relatively small.

In order not to complicate the calculations too much it is wise to make a number of simplifying assumptions. A discussion whether these are correct or not will be given in section 2.4. We assume now:

a. The thickness of the vapor layer underneath the droplet is so small with respect to the radius \(R_0\) that the vertical, velocity of the vapor can be disregarded with respect to the horizontal velocity.

b. The flow of the vapor in the layer is laminar and is only determined by viscous forces. Inertial forces may be disregarded.

c. The viscosity \(\eta_g\), the heat conductivity \(\lambda_g\) and the density of the vapor are constant and equal to those associated with the mean temperature between the drop and hot surface.

d. The influence of the vapor stream which comes off the liquid surface on the heat transport is unimportant.

e. The influence of the radiation on the heat transport may be ignored.

f. The influence of the pressure gradient underneath the droplet on the boiling temperature of the liquid at the bottom surface of the drop may be ignored, so that we may assume that all heat which is transferred from the hot surface to the drop is used for local production.

1. The evaporation at the bottom side of the drop.

It is assumed that the thickness of the vapor layer under the drop is known at every given spot. Then it is possible to calculate the amount of heat which per unit of time is transported from the hot surface
to the drop. If all the heat is locally used for evaporation then it is possible to calculate the vapor velocity and the pressure gradient caused by the vapor flow for every location in the vapor layer. In the steady state the pressure underneath the droplet has to be in balance with the sum of the pressure in the drop and the pressure \( p \) which is caused by the surface tension in the liquid surface. The formation of an equation for this pressure balance will enable us to calculate the distance between the drop and the hot surface at every location. After this it will be easy to calculate the total amount of vapor which is produced at the bottom. Then also the distance between the drop and the surface for \( r = R_0 \) is fixed, so that also the heat transfer to the sides and the top can be found.

In the above considerations it was assumed that the pressure gradient only exists in the region where \( r < R_0 \). In \( r = R_0 \) the pressure is assumed to be equal to the ambient pressure. Because the drop surface in \( r = R_0 \) curves rather fast away from the hot surface, and because the vapor velocity under the drop will appear to be relatively low so that the dynamic pressure difference is not large, this assumption is acceptable.

Assume in Fig. 2,8 a cylindrical coordinate system with axes \( r \) and \( z \) and with origin \( O \) at the hot surface. Assume that the vertical velocity with which the vapor leaves the bottom of the drop is \( v_0 \) and assume that this is a function of \( r \). Then for the horizontal radially directed vapor velocity \( u \) the following mass balance can be formed, in which \( d(r) \) represents the thickness of the vapor layer at \( r \).
\[ \int_0^{d_0} 2\pi r' \rho_g v_{u'} \, dr' = \int_0^{\infty} 2\pi r \rho_g u \, dr. \tag{2.14} \]

For the velocity \( u \) also the following momentum balance may be written:

\[ \frac{dp}{dr} = \eta \left( \frac{\dot{r}^2 u}{r^2} + \frac{1}{r} \frac{\dot{r}}{r} \frac{u}{r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial r^2} \right). \tag{2.15} \]

If we assume that the liquid is not dragged along by the vapor flow, then the following boundary conditions hold true:

\[ u(r, 0) = 0 \quad u(r, d) = 0 \quad p(R_o) = p_o. \tag{2.16} \]
Since the dimensions of the vapor layer in direction \( z \) are much smaller than those in direction \( r \) (see assumption a. on page 23) the last three terms on the right hand side of (2.15) may be disregarded. For (2.15) the solution is then

\[
\frac{dp}{dr} = \frac{dp}{dr} \frac{z(d-z)}{2\eta_v},
\]

(2.17)

in which \( \frac{dp}{dr} \) is still a function of \( r \).

If we fill the mass balance (2.14) in (2.17) we obtain

\[
\frac{dp}{dr} = -\frac{12\eta_v}{rd^3} \int_0^r r^2 r'^2 dr'.
\]

(2.18)
so that

\[ u = \frac{6z(d-z)}{r^3} \int_0^r r' v_s dr'. \]  \hspace{2cm} (2.19)

Integration of (2.18) gives the pressure distribution underneath the drop,

\[ p - p_0 = 12\eta \int r \int_0^r \frac{1}{r'^3} \int_0^{r'} r'' v_s dr' dr''. \]  \hspace{2cm} (2.20)

The velocity \( v_o \) of the vapor directed from the drop bottom is given by the amount of heat that arrives there per unit of time and per unit of area. If the heat transfer through the vapor layer is only by heat conduction and if all transferred heat is used locally for evaporation and finally, if the bottom side of the droplet is not too strongly curved, then one may write

\[ v_o = \frac{\dot{\lambda}_g(T_m - T_o)}{\rho \Delta H d}. \]  \hspace{2cm} (2.21)

Here \( \dot{\lambda}_g \) is the heat conductivity of the vapor, \( \Delta H \) the heat of evaporation of the liquid, \( T_m \) the temperature of the hot surface, and
$T_s$ the temperature of the drop. When we use (2.21) in (2.20) we find for the pressure distribution under the drop

$$p - p_o = \frac{12\eta_o \xi (T_m - T_s)}{\rho_a \Delta H} \int_0^r \frac{1}{r'd^3} \int_0^r \frac{r''}{d} \, dr' \, dr''. \tag{2.22}$$

In this expression $\eta_g$, $\lambda_g$, and $\rho_g$ are chosen at the mean temperature in the vapor layer, i.e. $\frac{1}{2}(T_m + T_s)$. In order to know the complete dependence of $p - p_o$ on $T_m$ and $T_s$ one has still to introduce temperature dependence of $\eta_g$, $\lambda_g$, and $\rho_g$ into formula (22.2). If $\eta_s$, $\lambda_s$, and $\rho_s$ are the values of $\eta_g$, $\lambda_g$, and $\rho_g$ at the boiling point $T_s$ of the liquid, then one may say that approximately

$$\eta_s = \eta_0 \sqrt{\frac{T_m + T_s}{2T_s}},$$

$$\lambda_s = \lambda_0 \sqrt{\frac{T_m + T_s}{2T_s}},$$

$$\rho_s = \rho_0 \left( \frac{2T_s}{T_m + T_s} \right). \tag{2.23}$$

where $T_m$ and $T_s$ are supposed to be expressed in degrees Kelvin. The pressure distribution underneath the drop is then finally
\[ p - p_0 = \frac{3\eta \lambda (T_1 - T_0)(T_1 + T_0)^2}{\rho_0 \mu T_0^2} \int_0^{R_0} \frac{1}{r} \int_0^r \frac{r^2}{d} dr' dr''. \]  

(2.24)

This is the excess pressure which is created underneath the drop by the viscous vapor flow to the outside edge of the slit.

At any point at the bottom one has inside the drop a pressure

\[ p_i + \sigma (\rho_i - \rho_v)(h - d + d_o). \]

In this expression \( p_i \) is the pressure in the top of the drop. The second term is the hydrostatic pressure difference between the top and the above mentioned point at the bottom. The difference in pressure between the inside and the outside of the drop is determined by the surface tension forces. Hence, the pressure balance becomes

\[ p_i + (\rho_i - \rho_v)gh - (\rho_i - \rho_v)h(d - d_o) = p + \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right). \]  

(2.25)

According to (2.7) the first two terms in the lift part of the equation may be replaced by
\[ \frac{(\rho_1 - \rho_2) dV}{\pi R^2} + p_r. \]  \hspace{1cm} (2.25a)

The second term in the right hand part is for small values of \( d \) \( dr \) equal to

\[ \sigma \left( \frac{d^2 d}{dr^2} + \frac{1}{r} \frac{dd}{dr} \right). \]  \hspace{1cm} (2.25b)

Finally \( p \) is given by (2.24). If we insert all these expressions in (2.25) we obtain

\[
\frac{\rho_1 \sigma V}{\pi R^2} - \rho_2 (d - d_0) = \frac{3\eta \beta_1 (T_m - T_i)(T_m + T_i)^2}{\rho_1 A H T_i^2} \left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{dd}{dr} \right] + \sigma \left( \frac{d^2 d}{dr^2} + \frac{1}{r} \frac{dd}{dr} \right).
\]  \hspace{1cm} (2.26)

The boundary conditions are here

\[
\frac{dd(0)}{dr} = 0, \quad \frac{dd(R_o)}{dr} = 0.
\]  \hspace{1cm} (2.27)

Since for the solution of the equation, no analytic method is available, this has to be done numerically. Therefore, assume in a first approximation a solution of the form

\[ d(r) = a_0 + a_1 r^2 + a_2 r^4. \]  \hspace{1cm} (2.28)

For \( r = R_o \) one has \( d \left( R_o \right) = d_o \). When we use this together with the boundary condition for \( r = R_o \) given by (2.27) we find for (2.26)

\[ \frac{d^2 d(R_o)}{dr^2} = \frac{\rho_1 \sigma V}{\pi R^2}. \]
From all these expressions follows for (2.28)

\[ d(r) = d_o + \frac{\rho_g V}{8\kappa} \left( 1 - \frac{r^2}{r^2_o} \right)^2. \]  

(2.29)

In this expression \( d_o \) is still unknown. If we, however, could find a suitable estimate for \( d_o \) then it would be possible, after substitution of (2.29) in the integral of (2.26), to find a first approximation of \( d \) as a function of \( r \) by integration of (2.26) with the initial conditions \( d(R_o) = d_o \) and \( \frac{d(R_o)}{dr} = 0 \). The same can be done with another value of \( d_o \) that differs only a little from the first estimate. It is probable that in neither of the two cases the boundary condition (2.27) in \( r = 0 \) is satisfied. If for \( d_o \) a too large value is chosen then the pressure underneath the drop becomes too low. Then the surface tension forces have to compensate the pressure in the drop, so that \( \frac{d^2}{dr^2} \) on the path from \( r = R_o \) to \( r = 0 \) will remain positive too long. Then \( \frac{d}{dr} \) in \( r = 0 \) will be negative. If, however, \( d_o \) is chosen too small then \( \frac{d}{dr} \) becomes positive. The two values of \( \frac{d}{dr} \) that are found from the two estimated values of \( d_o \) may, however, be used for a linear inter-or extrapolation between the contours \( d(r) \) that in the two cases were found. This gives a new and better approximation of \( d(r) \). If this approximation is used for calculation of the integral in equation (2.26), then the integration of this equation gives again a better approximation. Also this time \( \frac{d}{dr} \) will most likely not be equal to zero. Linear inter-or extrapolation between the contour that was used to calculate the integral and the contour that is found from the integration of (2.26) with the aid of the corresponding values of \( \frac{d}{dr} \) gives, however, again a better approximation of \( d(r) \).
If this iterative process is convergent then after not too many iterations a solution can be found which is very close to the exact solution.

The above mentioned procedure was programmed for calculation on a digital computer*. In this way the shape of the bottom of four water drops with sizes of 2, 9, 12, 17, 85, 141, 01 and 150, 7 mm³ was calculated for T_s = 100° C and 15 different values of T_m between 120° and 100° C. For the first two drop sizes we could use as first approximations for d_o respectively d_p and \( \frac{3}{2} d_p \), where d_p is the (constant) thickness of the vapor layer underneath the drop which can be calculated with equation (2.3b). This equation is derived with the assumption that the bottom of the drop for \( r \leq R_o \) is absolutely flat. From the calculated contours given in Fig. 2.10 it appears that for very small droplets d(R_o) is indeed about equal to d_p.

For large drops a more accurate approximation was required. However, it turned out that d(R_o) increased so slowly with the increase of the drop volume that for the first approximation of d(R_o) the value could be used that was found for the larger one of the already calculated small drops. This was only done for T_m = 120° C. For the other values of T_m for the first approximation always the value of d(R_o) corresponding to the preceding (lower) value of T_m was taken. For the second approximation for d(R_o) in both cases a value was chosen which was a factor of 1.2 larger than the first approximation.

In Figure 2.9 the shape of the bottom of a \( h \cdot 01 \text{ mm}^3 \) drop is given at two different temperatures. The dotted lines correspond to the layer

* The programming for the TR h was carried out by P. J. Van der Hoff.
thickness $d_p$ that one finds with $(2.34)$ when the drop bottom is supposed to be completely flat. It appears that the true shape of the bottom side is quite different from a flat one. In order to compare the curvature of the bottom side at various drop sizes in Figure 2.10 the ratio $d(r)/d_p$ at $T_m = 200^\circ\text{C}$ is plotted as a function of $r/R_o$. From this it is seen that large drops depart much more from a flat bottom than small droplets.

Finally, in Table 2.2 all calculated values of $d(0)$ and $d(R_o)$ are given. It is seen that while with constant $T_m$ $d(R_o)$ only slowly increases with the drop size, $d(0)$ is nearly linear with $R_o$. The values of $d(R_o)$ given in this table will be used in the second half of this section for the calculation of the evaporation at the sides and the top of the drops.

<table>
<thead>
<tr>
<th>$10^4 V [\text{m}^3]$</th>
<th>2.942</th>
<th>17.85</th>
<th>44.01</th>
<th>150.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^5 R_{\text{max}} [\text{m}]$</td>
<td>0.007</td>
<td>0.72</td>
<td>2.41</td>
<td>3.93</td>
</tr>
<tr>
<td>$10^4 R_o [\text{m}]$</td>
<td>0.245</td>
<td>0.7753</td>
<td>1.337</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T_m [^\circ\text{C}]$</th>
<th>$d(0)$</th>
<th>$d(R_o)$</th>
<th>$d_p$</th>
<th>$d(0)$</th>
<th>$d(R_o)$</th>
<th>$d_p$</th>
<th>$d(0)$</th>
<th>$d(R_o)$</th>
<th>$d_p$</th>
<th>$d(0)$</th>
<th>$d(R_o)$</th>
<th>$d_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>14.7</td>
<td>5.71</td>
<td>7.51</td>
<td>43.0</td>
<td>9.29</td>
<td>15.0</td>
<td>74.2</td>
<td>11.3</td>
<td>20.5</td>
<td>161</td>
<td>13.9</td>
<td>30.7</td>
</tr>
<tr>
<td>140</td>
<td>16.7</td>
<td>7.07</td>
<td>9.08</td>
<td>48.4</td>
<td>11.6</td>
<td>18.0</td>
<td>83.2</td>
<td>14.2</td>
<td>24.7</td>
<td>179</td>
<td>17.5</td>
<td>37.0</td>
</tr>
<tr>
<td>160</td>
<td>18.1</td>
<td>8.05</td>
<td>10.2</td>
<td>52.1</td>
<td>13.3</td>
<td>20.2</td>
<td>89.3</td>
<td>16.3</td>
<td>27.8</td>
<td>202</td>
<td>20.2</td>
<td>41.5</td>
</tr>
<tr>
<td>180</td>
<td>19.2</td>
<td>8.85</td>
<td>11.0</td>
<td>55.0</td>
<td>14.7</td>
<td>22.0</td>
<td>94.1</td>
<td>18.0</td>
<td>30.2</td>
<td>202</td>
<td>22.4</td>
<td>45.1</td>
</tr>
<tr>
<td>200</td>
<td>20.1</td>
<td>9.56</td>
<td>11.8</td>
<td>57.4</td>
<td>15.9</td>
<td>23.5</td>
<td>98.2</td>
<td>19.6</td>
<td>32.3</td>
<td>210</td>
<td>24.3</td>
<td>48.3</td>
</tr>
<tr>
<td>220</td>
<td>21.0</td>
<td>10.2</td>
<td>12.5</td>
<td>59.6</td>
<td>17.1</td>
<td>24.8</td>
<td>102</td>
<td>21.0</td>
<td>34.2</td>
<td>217</td>
<td>26.1</td>
<td>51.2</td>
</tr>
<tr>
<td>240</td>
<td>21.7</td>
<td>10.8</td>
<td>13.1</td>
<td>61.6</td>
<td>18.1</td>
<td>26.2</td>
<td>105</td>
<td>22.3</td>
<td>35.9</td>
<td>224</td>
<td>27.8</td>
<td>53.8</td>
</tr>
<tr>
<td>260</td>
<td>22.5</td>
<td>11.3</td>
<td>13.7</td>
<td>63.4</td>
<td>19.1</td>
<td>27.3</td>
<td>108</td>
<td>23.5</td>
<td>37.6</td>
<td>230</td>
<td>29.3</td>
<td>56.2</td>
</tr>
<tr>
<td>280</td>
<td>23.1</td>
<td>11.9</td>
<td>14.3</td>
<td>65.1</td>
<td>20.0</td>
<td>28.4</td>
<td>111</td>
<td>24.7</td>
<td>39.1</td>
<td>236</td>
<td>30.8</td>
<td>58.5</td>
</tr>
<tr>
<td>300</td>
<td>23.8</td>
<td>12.4</td>
<td>14.8</td>
<td>66.7</td>
<td>20.9</td>
<td>29.5</td>
<td>114</td>
<td>25.8</td>
<td>40.6</td>
<td>241</td>
<td>32.3</td>
<td>60.7</td>
</tr>
<tr>
<td>320</td>
<td>24.4</td>
<td>12.9</td>
<td>15.4</td>
<td>68.3</td>
<td>21.7</td>
<td>30.5</td>
<td>116</td>
<td>26.9</td>
<td>42.1</td>
<td>246</td>
<td>33.7</td>
<td>62.9</td>
</tr>
<tr>
<td>340</td>
<td>24.9</td>
<td>13.3</td>
<td>15.9</td>
<td>69.7</td>
<td>22.6</td>
<td>31.5</td>
<td>118</td>
<td>27.9</td>
<td>43.4</td>
<td>251</td>
<td>35.0</td>
<td>64.9</td>
</tr>
<tr>
<td>360</td>
<td>25.5</td>
<td>13.8</td>
<td>16.3</td>
<td>71.1</td>
<td>23.4</td>
<td>32.5</td>
<td>212</td>
<td>28.9</td>
<td>44.7</td>
<td>255</td>
<td>36.3</td>
<td>66.9</td>
</tr>
<tr>
<td>380</td>
<td>26.0</td>
<td>14.2</td>
<td>16.8</td>
<td>72.5</td>
<td>24.2</td>
<td>33.4</td>
<td>123</td>
<td>29.9</td>
<td>46.0</td>
<td>260</td>
<td>37.6</td>
<td>68.8</td>
</tr>
<tr>
<td>400</td>
<td>26.6</td>
<td>14.6</td>
<td>17.2</td>
<td>73.8</td>
<td>24.9</td>
<td>34.3</td>
<td>125</td>
<td>30.9</td>
<td>47.3</td>
<td>264</td>
<td>38.9</td>
<td>70.7</td>
</tr>
</tbody>
</table>

Table 2.2 Calculated vapor slit thickness under non-circulating, sessile water drops
Fig. 2.9 The calculated shape of the bottom of a non-circulating sessile water drop \((V = 1.01 \times 10^{-9} \text{ m}^3)\)

Fig. 2.10 The calculated shape of the bottom of a non-circulating water drop \((T_m = 200^\circ C)\)
Table 2.3 Calculated evaporation rate at the bottom of non-circulating sessile water drops

With the function $d(r)$ found above one can now calculate the amount of liquid which per unit of time evaporates at the bottom side of the drop. This is

$$
\Phi_v = \int_0^{r_v} \frac{8\pi r^4 \rho_v}{\rho_l} dr \quad (2.30)
$$

If we fill in the expression (2.21) for $v_0$ and we recall that the temperature dependence of $\lambda_g$ is given by (2.23), one obtains
\[ \psi_{ev} = \frac{\sqrt{2} \pi \lambda_s (T_m - T_b)(T_m + T_b)^{\frac{1}{2}}}{\rho_l \Delta H T_s} \cdot \int_0^{R_o} \frac{r \, dr}{d} . \quad (2.31) \]

The values calculated with (2.31) for the evaporation at the bottom side are given in Table 2.3. They correspond to the four above mentioned drop sizes and are for the same temperatures \( T_m \) between 120 and 400°C.

Above we mentioned a couple of times the results which one would obtain for a completely flat bottom. This is not completely fictitious. Because especially large drops are almost always oscillating a little bit, it is very well possible that a flat bottom side gives a much better picture of reality than the above calculated curved shape. Assume now that the drop at the bottom side, i.e. for \( r \leq R_o \), is absolutely flat. Then one can calculate the now constant value of the vapor layer thickness \( d_p \) from the force balance in the steady state has to exist between the weight of the drop and the integrated pressure in the vapor layer:

\[ \rho_o g V = \int_0^{R_o} 2 \pi r (p - p_o) dr . \quad (2.32) \]

The pressure distribution underneath the drop here again can be found with (2.24), but \( d \) is now a constant and equal to \( d_p \).

\[ p - p_o = \frac{3 \eta_s \lambda_s (T_m - T_b)(T_m + T_b)^3}{4 \rho_s \Delta H T_s d_p^2} \cdot (R_o^2 - r^2) . \quad (2.33) \]
Substitution in (2.32) finally gives for the vapor layer thickness

\[ d_r = \left\{ \frac{3n_H \lambda_s (T_m - T) (T_m + T)^2 R^2_c}{8 \rho \mu g \delta \bar{H} T_r^2 V} \right\}^{1/4} \]  

(2.34)

With (2.21) and (2.30) this gives the evaporation rate at the flat bottom

\[ \phi_{vop} = \left\{ \frac{2\pi^3 \rho \lambda_s^3 g (T_m - T)^3 V^2}{3 \rho \mu \bar{H}^2 \bar{H}^3} \right\}^{1/4} \]

(2.35)

From these expressions \( d_p \) and \( \phi_{vop} \) were calculated for the above mentioned drop sizes and temperatures. The results are given in the Tables 2.2 and 2.3. From these follows that \( \phi_{vop} \) for very small droplets is nearly equal to \( \phi_{vo} \). It is striking that the difference between \( \phi_{vop} \) and \( \phi_{vo} \), which is small for little drops, increases very strongly for larger drop volumes. This is evidently related to the fact that for larger drops the quotient \( d(r)/d_p \) as a function of \( r \) shows a larger variation (see Figure 2.10).

2. The evaporation on the rest of the drop surface.

The heat transfer to the rest of the drop surface can in principle only be determined if one knows the vapor flow pattern around the drop. However, if we assume that the stream lines of the vapor flow coincide with the isotherms which one would have if the vapor was at rest, then the determination of the heat transfer is reduced to a pure conduction problem. This assumption is especially acceptable in the relatively important part of the surface for which \( r \sim R_0 \). For radial symmetry around the z-axis the heat balance in the vapor is now
\[
\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} + \frac{d^2 T}{dz^2} = 0,
\]

(2.36)

for which the boundary conditions say that the temperature \( T \) at the hot surface is equal to \( T_h \) and at the drop surface equal to \( T_0 \). From this equation one would like to calculate the temperature gradient at the drop surface in order to determine the total integrated heat flow to the drop. This is however completely impossible because of the complexity of the analytical expression for the shape of the drop. Therefore, we used an analog model, namely an electrolytic bath, in which for the electrical potential an equation is true that is completely analogued to the heat balance (2.36). An electric current between two equipotential surfaces in this bath is analogued to the heat flow between two isothermal surfaces in a similarly shaped configuration in the heat transfer case.

Electrical analog models were made for the drops numbered 4, 5, 6, and 7 in Table 2.7. These are the drops for which the Tables 2.2 and 2.3 are true. A rather strong enlargement (140 to 560 times) was necessary to obtain a reasonable accuracy in the measurements. This is clear if one realizes that the distance between the drop bottom and the hot surface is often not more than a few microns. And it is just as a function of this distance that one wants to determine the contribution of the sides and the top of the drop to the total evaporation rate.
Fig. 2.11 Sector model of the drop

Because of the large dimensions which the drop models would get and the difficulties of fabricating these we did not use models which covered the entire drops but only sectors of $20^\circ$ (See Figure 2.11). Besides, these sectors only showed the lower half of the drops. Comparision of measurements on a model of a completely spherical drop with values known from potential theory (see for instance Buchholz [78]) showed that the elimination of the top half had hardly any influence and was consequently permitted.

The sectors were fabricated in a rather simple way from two plexiglass plates which were cut in the correct shape and then glued together at a $20^\circ$ angle. They were covered at the circumference with copper sheet.

The model of the hot surface was made up of a sector-shaped flat plate of copper with a radius about twice as large as the maximum radius of the largest drop model.
Fig. 2.12 Electrolytic bath

In a container which was tilted at an angle of 20° and filled with a solution of copper sulphate the drop sectors could be located in such a way opposite the sector shaped copper plate that their two "vertical" surfaces coincided with the bottom of the tank and the liquid surface. (See Figure 2.12). Their distance to the sector-shaped plate, which corresponded to the vapor layer thickness of the sessile drop, could with the aid of a screw spindle be varied very accurately.

The side of the models
which corresponded to the bottom side of the drops were kept flat, because of the fact that our problem only dealt with the heat transfer to the sides and the top of the drops. The effect of the heat transfer to the flat bottom side alone was determined by measurements on models corresponding to only this part of the drop surfaces. For these measurements we used flat sector-shaped copper plates with radii equal to those of the flat bottom sides of the corresponding complete drop models. This also gave us the possibility to relate the electrical resistance measured in the electrolytic bath to the heat resistance in case of the real drops.

The results of the measurements are shown in Figure 2.13. Here the ratio of the evaporation at the sides and the top of the drop, \( \frac{\phi_{vz}}{\phi_{vcp}} \), and the evaporation at the supposedly flat bottom side, \( \phi_{vcp} \), which ratio according to point f. on page 21 is equal to the ratio of the corresponding heat transports, is plotted against the vapor slit thickness in \( r = R_0 \). For the drop volumes for which these graphs are true, one can now easily calculate the evaporation rate at the sides and the top of a flat bottomed drop. Suppose that \( T_m \) is equal to one of the temperatures mentioned in the Tables 2.2 and 2.3. Then, for any of the drop volumes mentioned in these tables \( \frac{\phi_{vz}}{\phi_{vcp}} \) and \( d_p \) can be found. With the latter one the ratio \( \frac{\phi_{vz}}{\phi_{vcp}} \) can be determined from one of the graphs in Fig. 2.13. Multiplication by the value found above for finally gives the requested evaporation rate at the sides and the top,

For a drop with a curved bottom side the calculation becomes somewhat more difficult. It can be done as follows: Look first in Table 2.2 for the value of \( d \left( R_0 \right) \) which belongs to the considered drop size and temperature \( T_m \). Next look for the corresponding value of \( \frac{\phi_{vz}}{\phi_{vcp}} \) in one of the graphs in Figure 2.13. Now \( \phi_{vz} \) has still to be found. This is the evaporation rate at the flat bottom side if this side would be at a distance \( d \left( R_0 \right) \) from
Fig. 2.13 The ratio between the evaporation at the sides and at the flat bottom for water drops of different sizes.
from the hot surface. The easiest way to find this evaporation rate is to take the value of \( \Phi_{\nu_C} \) at the corresponding drop size and temperature \( T_m \) from Table 2.3 and to multiply this by the quotient \( d_p \frac{d(R_o)}{d} \). Multiplication by the above found value of \( \Phi_{\nu_2}/\Phi_{\nu_C} \) finally gives the requested \( \Phi_{\nu_2} \).

For the fifteen temperatures \( T_m \) between 120 and 180°C the value of corresponding to the above mentioned drop volumes was calculated. This was done for both curved and flat bottom sides. The results are given in Table 2.4.

<table>
<thead>
<tr>
<th>( 10^3 \cdot V \text{ [m}^3\text{]} )</th>
<th>2.942</th>
<th>17.85</th>
<th>44.01</th>
<th>150.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_m \text{ [°C]} )</td>
<td>( \Phi_{\nu_2} )</td>
<td>( \Phi_{\nu_C} )</td>
<td>( \Phi_{\nu_2} )</td>
<td>( \Phi_{\nu_C} )</td>
</tr>
<tr>
<td>120</td>
<td>0.00644</td>
<td>0.00625</td>
<td>0.0261</td>
<td>0.0274</td>
</tr>
<tr>
<td>140</td>
<td>0.0129</td>
<td>0.0124</td>
<td>0.0647</td>
<td>0.0490</td>
</tr>
<tr>
<td>160</td>
<td>0.0196</td>
<td>0.0185</td>
<td>0.0899</td>
<td>0.0603</td>
</tr>
<tr>
<td>180</td>
<td>0.0255</td>
<td>0.0247</td>
<td>0.114</td>
<td>0.0910</td>
</tr>
<tr>
<td>200</td>
<td>0.0316</td>
<td>0.0306</td>
<td>0.137</td>
<td>0.111</td>
</tr>
<tr>
<td>220</td>
<td>0.0384</td>
<td>0.0366</td>
<td>0.158</td>
<td>0.130</td>
</tr>
<tr>
<td>240</td>
<td>0.0445</td>
<td>0.0427</td>
<td>0.181</td>
<td>0.149</td>
</tr>
<tr>
<td>260</td>
<td>0.0511</td>
<td>0.0488</td>
<td>0.200</td>
<td>0.168</td>
</tr>
<tr>
<td>280</td>
<td>0.0577</td>
<td>0.0522</td>
<td>0.225</td>
<td>0.187</td>
</tr>
<tr>
<td>300</td>
<td>0.0632</td>
<td>0.0615</td>
<td>0.248</td>
<td>0.207</td>
</tr>
<tr>
<td>320</td>
<td>0.0706</td>
<td>0.0680</td>
<td>0.271</td>
<td>0.227</td>
</tr>
<tr>
<td>340</td>
<td>0.0774</td>
<td>0.0743</td>
<td>0.290</td>
<td>0.246</td>
</tr>
<tr>
<td>360</td>
<td>0.0840</td>
<td>0.0804</td>
<td>0.316</td>
<td>0.267</td>
</tr>
<tr>
<td>380</td>
<td>0.0905</td>
<td>0.0871</td>
<td>0.331</td>
<td>0.285</td>
</tr>
<tr>
<td>400</td>
<td>0.0969</td>
<td>0.0929</td>
<td>0.356</td>
<td>0.305</td>
</tr>
</tbody>
</table>

Table 2.4 Calculated evaporation rate at the sides of non-circulating, sessile water drops
It is remarkable that for constant temperature and increasing drop size $d_n$ first increases and then decreases. At first sight this would not be expected. The decrease is partly due to the increase of $d_p$ and $d(R_o)$. For the rest it can only be caused by the increase of the curvature of the meridian of the drop surface in $r = R_o$. A larger curvature in $r = R_o$ results in a more rapid bending away of the drop surface from the hot plate.

<table>
<thead>
<tr>
<th>$10^6 V [\text{m}^3]$</th>
<th>2.942</th>
<th>17.85</th>
<th>44.01</th>
<th>150.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_m [\text{C}]$</td>
<td>$\psi_{ct}$</td>
<td>$\psi_{v/p}$</td>
<td>$\psi_{ct}$</td>
<td>$\psi_{v/p}$</td>
</tr>
<tr>
<td>120</td>
<td>0.0117</td>
<td>0.0122</td>
<td>0.0592</td>
<td>0.0565</td>
</tr>
<tr>
<td>140</td>
<td>0.0219</td>
<td>0.0224</td>
<td>0.104</td>
<td>0.0980</td>
</tr>
<tr>
<td>160</td>
<td>0.0319</td>
<td>0.0322</td>
<td>0.144</td>
<td>0.136</td>
</tr>
<tr>
<td>180</td>
<td>0.0409</td>
<td>0.0416</td>
<td>0.182</td>
<td>0.174</td>
</tr>
<tr>
<td>200</td>
<td>0.0499</td>
<td>0.0505</td>
<td>0.218</td>
<td>0.209</td>
</tr>
<tr>
<td>220</td>
<td>0.0594</td>
<td>0.0595</td>
<td>0.252</td>
<td>0.242</td>
</tr>
<tr>
<td>240</td>
<td>0.0682</td>
<td>0.0684</td>
<td>0.287</td>
<td>0.275</td>
</tr>
<tr>
<td>260</td>
<td>0.0774</td>
<td>0.0772</td>
<td>0.318</td>
<td>0.307</td>
</tr>
<tr>
<td>280</td>
<td>0.0864</td>
<td>0.0862</td>
<td>0.351</td>
<td>0.339</td>
</tr>
<tr>
<td>300</td>
<td>0.0944</td>
<td>0.0951</td>
<td>0.388</td>
<td>0.371</td>
</tr>
<tr>
<td>320</td>
<td>0.104</td>
<td>0.104</td>
<td>0.422</td>
<td>0.403</td>
</tr>
<tr>
<td>340</td>
<td>0.113</td>
<td>0.113</td>
<td>0.452</td>
<td>0.434</td>
</tr>
<tr>
<td>360</td>
<td>0.122</td>
<td>0.121</td>
<td>0.488</td>
<td>0.468</td>
</tr>
<tr>
<td>380</td>
<td>0.131</td>
<td>0.130</td>
<td>0.513</td>
<td>0.496</td>
</tr>
<tr>
<td>400</td>
<td>0.140</td>
<td>0.138</td>
<td>0.549</td>
<td>0.526</td>
</tr>
</tbody>
</table>

Table 2.5 Calculated total evaporation rate of non-circulating sessile water drops

Finally, the total evaporation rate of drops with both curved and flat bottom sides can be calculated by addition of the corresponding results in the Tables 2.3 and 2.4. The total evaporation rate thus calculated is given in Table 2.5. It is striking that for small droplets the end results for curved and flat bottom sides differ only a little bit. What evaporates more at the bottom side with a "flat" droplet is compensated for with a "curved" droplet.
by the larger evaporation rate at the rest of the surface. This is the result of the fact that for any \( T_m \) \( d(R_0) < d_p \).

The results of Table 2.5 will be compared with experimental results in section 2.6.

In the above considerations we assumed in the boundary conditions (2.16) for the impulse balance (2.15) that the liquid in the drop is absolutely motionless, there being no drag at the drop surface by the vapor flow. However, if its viscosity is not too high, the liquid will always be dragged along by the vapor flow. It is difficult to calculate how large this drag will be. One can however easily find the evaporation of a drop for complete drag, i.e., the drag which one would obtain if the viscosity of the liquid would be zero. Then for the impulse balance (2.15) the boundary condition at the liquid surface is

\[
\frac{\partial u(r, d)}{\partial z} = 0.
\]

(2.37)

so that the velocity profile in the vapor slit becomes half parabolic.

For the pressure distribution underneath the drop one finds in the same way as on page 28

\[
p - p_0 = \frac{3mL(T_m - T_0)(T_m + T_0)^2}{4\rho AVT_0^2} \left[ r \int_0^r \frac{1}{r} \int_0^r \frac{r''}{d} dr' dr'' \right].
\]

(2.38)

For the rest the terms in the equation for the pressure balance (2.26) remains the same. In exactly the same way as was done above one can now find the variation of \( d \) as a function of \( r \) and the evaporation at the bottom side of the droplet \( \Phi_{\nu_2} \). The evaporation at the sides and the top, \( \Phi_{\nu_2} \), can again be found with the aid of Figure 2.13. The summation
of \( \Phi_{v0} \) and \( \Phi_{v2} \) finally gives the total evaporation \( \Phi_{v2} \).

Here also an analytic expression analog to (2.34) can be developed for the slit thickness under a flat bottomed drop,

\[
d_p = \left( \frac{3\pi \eta \lambda (T_m - T_0)(T_m + T_0) R_s^2}{32 \rho \rho_d AH T_s V} \right)^{\frac{1}{3}}.
\]

(2.39)

For this follows the evaporation at the bottom side of such a drop,

\[
\Phi_{v2} = \left( \frac{8 R^3 \rho \lambda^3 \rho (T_m - T_0) R_s^4 V}{3 \rho_d \eta AH^3} \right)^{\frac{1}{4}}.
\]

(2.40)

It is seen that \( \Phi_{v2} \) is now a factor of \( \sqrt{\frac{T}{T_m}} \) larger than for a completely motionless liquid surface. As \( d_p \) has become a factor of \( \sqrt{\frac{T}{T_m}} \) smaller also \( \Phi_{v2} \) increases somewhat.

<table>
<thead>
<tr>
<th>10^6 V [m^3]</th>
<th>2.942</th>
<th>17.85</th>
<th>44.01</th>
<th>150.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_m [\degree C] )</td>
<td>( \Phi_{v2} )</td>
<td>( \Phi_{v2p} )</td>
<td>( \Phi_{v2} )</td>
<td>( \Phi_{v2p} )</td>
</tr>
<tr>
<td>120</td>
<td>0.0179</td>
<td>0.0119</td>
<td>0.0739</td>
<td>0.0744</td>
</tr>
<tr>
<td>140</td>
<td>0.0238</td>
<td>0.0272</td>
<td>0.137</td>
<td>0.130</td>
</tr>
<tr>
<td>160</td>
<td>0.0372</td>
<td>0.0388</td>
<td>0.192</td>
<td>0.181</td>
</tr>
<tr>
<td>180</td>
<td>0.0482</td>
<td>0.0502</td>
<td>0.243</td>
<td>0.227</td>
</tr>
<tr>
<td>200</td>
<td>0.0589</td>
<td>0.0607</td>
<td>0.289</td>
<td>0.270</td>
</tr>
<tr>
<td>220</td>
<td>0.0698</td>
<td>0.0719</td>
<td>0.335</td>
<td>0.313</td>
</tr>
<tr>
<td>240</td>
<td>0.0799</td>
<td>0.0820</td>
<td>0.379</td>
<td>0.358</td>
</tr>
<tr>
<td>260</td>
<td>0.0908</td>
<td>0.0926</td>
<td>0.421</td>
<td>0.398</td>
</tr>
<tr>
<td>280</td>
<td>0.101</td>
<td>0.104</td>
<td>0.464</td>
<td>0.441</td>
</tr>
<tr>
<td>300</td>
<td>0.112</td>
<td>0.114</td>
<td>0.509</td>
<td>0.480</td>
</tr>
<tr>
<td>320</td>
<td>0.122</td>
<td>0.124</td>
<td>0.550</td>
<td>0.518</td>
</tr>
<tr>
<td>340</td>
<td>0.133</td>
<td>0.136</td>
<td>0.587</td>
<td>0.559</td>
</tr>
<tr>
<td>360</td>
<td>0.143</td>
<td>0.146</td>
<td>0.630</td>
<td>0.599</td>
</tr>
<tr>
<td>380</td>
<td>0.153</td>
<td>0.155</td>
<td>0.669</td>
<td>0.635</td>
</tr>
<tr>
<td>400</td>
<td>0.164</td>
<td>0.165</td>
<td>0.706</td>
<td>0.671</td>
</tr>
</tbody>
</table>

Table 2.6 The calculation total evaporation rate of sessile water drops with circulating liquid
Summation of $\phi_{vop}$ and $\phi_{vzp}$ gives finally the total evaporation, $\phi_{vtp}$.

For the same temperatures as above the calculated values for the total evaporation rate of both curved and flat bottomed drops are given in Table 2.6. It is seen that they are always a factor of 1.2 to 1.4 larger than the total evaporation of drops of which the bottom surface is not dragged by the vapor flow.

Since the viscosity of water is neither zero nor infinite one may expect a partial drag of the liquid in the experiment. It depends upon the flow pattern in the drop where the real total evaporation lies between the two calculated extremes. Refinement of the theory by calculation of this flow pattern is not only very difficult but it has also very little use. In section 2.6 we will see that for not too large a viscosity of the liquid the drop always vibrates a little bit so that there is always a deviation from the above given theory.

Section 2.4 Discussion of the assumptions made in the preceding section

The effect of the simplifying assumptions a to f in the preceding section is small. The magnitude of each of them was calculated for a flat-bottomed drop with non-circulating liquid. One may assume that the deviations are also small for a curved-bottomed drop and circulating liquid.

Under point a it was assumed that the thickness of the vapor slit was much smaller than the radius of the bottom side of the drop. From Table 2.2 it is seen that for the investigated drop sizes this is true at not too high temperatures of the hot surface.
Under point b it was assumed that the vapor flow in the slit was laminar, and that the acceleration of the vapor in its way to the outer edge of the slit has no influence on the pressure distribution.

The maximum value of the Reynolds number that characterizes the flow underneath the drop is reached in $r = R_0$:

$$Re_{\text{max}} = \frac{\rho \phi_{\text{vap}}}{2 \eta \nu \rho \nu}.$$  \hspace{1cm} (2.41)

For the drops of Table 2.3 the highest $Re_{\text{max}}$ is found for the largest drop size being about equal to 5. This is low enough to guarantee a laminar flow.

Another point concerns the influence of the acceleration of the vapor during its way underneath the droplet to the outside. One can calculate this with the aid of the method used by Jackson (79) for the determination of the force which has to be applied to two flat round plates between which a liquid or gas is squeezed. For this calculation also the inertia term $\rho g u \frac{\partial u}{\partial r}$ is used in the impulse balance (2.15). This can now no longer be solved in a simple way. One may, however, use an iteration method which for $Re \rightarrow 1$ (where $Re = \frac{\rho g \nu}{\eta} \frac{d}{\rho \nu}$

$\lambda_{\eta} \left( \frac{T_m - T_s}{\eta \Delta H} \right)$ converges rather rapidly. The effect of the acceleration of the vapor is only a function of $Re$. For $T_m - T_s < 300^\circ C$ and for water drops the deviation in $\frac{d}{\rho \nu}$ and $\frac{\phi_{\text{vap}}}{\rho}$ is only about 0.5 per cent. This shows that the influence of the acceleration of the vapor is rather small. This is not only the case for water but for all other "regular" liquids. Hence, the calculation method used by Gorton (63) and by Kistemaker (67), who assumed that the acceleration forces dominate
the friction forces, is not correct.

Under point c it was assumed that the effect of the temperature dependence of the viscosity, the heat conductivity, and the density of the vapor may be ignored. For the calculations in the preceding paragraph \( \eta_j \), \( \lambda_j \) and \( \rho_j \) were chosen at the average temperature in the slit \( 1/2 (T_m + T_s) \). The error made in this way can be exactly calculated if one assumes that the vapor behaves as an ideal gas, i.e. if \( \lambda_j \) and \( \eta_j \) are proportional to the square room of the absolute temperature and that is inversely proportional to this. One finds that even for \( T_m = 100^\circ \text{C} \) for water drops the deviation in \( d_p \) and \( \phi_{vop} \) is not larger than about 0.1 per cent. Hence the influence of the temperature gradient in the slit underneath the drop may certainly be ignored. One may expect that this is also the case with the influence upon the heat transport to the sides and the top of the drop.

Under point d in section 2.3 it was assumed that the vapor flow perpendicular to the bottom surface has no influence on the heat transfer. We mean with this the reduction in heat transfer to the drop as a result of the fact that the heat which comes from the hot surface has to row upstream against the vapor flow coming from the liquid surface.

If in the slit underneath the drop a linearly varying vertical vapor speed \( v = v_0 z/d \), is assumed then the influence of this can rather easily be calculated. The deviation in \( d_p \) and \( \phi_{vop} \) can be shown to be a function of \( c_g (T_m - T_s) / \Delta H \). In the next table this deviation is given for water drops at 1 atm.
<table>
<thead>
<tr>
<th>$T_m - T_s$ [°C]</th>
<th>$c_p(T_m - T_s)/\Delta H$</th>
<th>$-\Delta d_p/d_p$ [%]</th>
<th>$-\Delta \Phi_{vap}/\Phi_{vap}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0187</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>100</td>
<td>0.0926</td>
<td>0.7</td>
<td>2.0</td>
</tr>
<tr>
<td>200</td>
<td>0.185</td>
<td>1.4</td>
<td>4.5</td>
</tr>
<tr>
<td>300</td>
<td>0.277</td>
<td>2.2</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Table 2.7 The influence of the vertical vapor velocity upon the heat conduction to the bottom of a sessile water drop

If $d_p$ would not change, then the relative deviation of the evaporation at the sides and the top, $\Phi_{vap}$, would be equal to that of $\Phi_{vap}$. But because of the fact that the vapor split thickness $d_p$ decreases a little bit, the true deviation is somewhat smaller. It depends upon the drop size how much this is. Therefore, also the deviation in the total evaporation rate depends upon the drop size. For $T_m - T_s = 300^\circ$ C this deviation can be shown to be 6 to 7 per cent. From this follows that the deviation may not completely be ignored. It will however be seen that in the experiments a much larger deviation arises from the fact that a sessile drop is never completely motionless. In this connection ignoring of the deviation is permitted.

Under point e in section 2.3 it was assumed that for the calculations of the heat transfer the influence of the heat radiation may be ignored. The contribution of the heat radiation to the heat transport to the bottom side of a water drop (absorption coefficient 0.95 - see e.g. McAdams (80)-) on a smooth polished gold surface (absorption coefficient 0.03) turns out to be about 0.3 per cent. If the hot surface would be a black radiator then
the deviation in $v_{op}$ would be equal to 10 per cent. Obviously, also in this case the influence of the radiation is still relatively small.

The contribution of the heat radiation to the heat transport to the rest of the drop surface can be estimated by assuming the drop to be a sphere situated in the middle of a circular flat surface. If one takes the radius of the surface equal to 0.025 meter and $T_0 = 673^\circ K$, then it can be shown that for a drop volume between 2.9 and 150 mm$^3$ the contribution of the heat radiation to the total heat transport is only 0.51 to 0.75 per cent. If the surface would be a black radiator, then these values would be about 30 times as large. The influence of the radiation may then no longer be ignored. Also then however the heat conduction is still much more important than the heat radiation. From this follows that the calculation method of Gottfried and Bell (65), who assumed that the radiation is the only important heat transfer mechanism is not correct.

Finally, it was assumed under point $f$ in section 2.3 that the temperature of the liquid surface at the bottom of the drop was equal to the boiling temperature. In reality, the pressure underneath the droplet is higher than the ambient pressure, so that also the temperature at the bottom surface is higher. The average excess pressure is equal to $\frac{V p_x q}{\eta R_o^2}$. If the drop bottom is flat, then the pressure distribution is parabolic with a maximum excess pressure $\Delta p_{\text{max}}$ which is two times as large as the average value. For the first seven drop sizes of Table 2.1 the maximum excess pressure and the corresponding boiling temperature increase $\Delta T_b$ are given in Table 2.8. Obviously, the temperature in the middle of the drop bottom is only slightly higher than at the rest of
Table 2.8 Elevation of the boiling point in the middle of the drop bottom

the surface. Therefore, one may assume with good approximation that all heat which is transported through the vapor slit to the drop surface is locally used for the evaporation of the liquid.

Sect. 2.5 Experimental Method and Apparatus

As was mentioned in section 2.1 the literature about the phenomenon of Leidenfrost gives two methods for determining the evaporation rate of a spheroidal drop. The first one is the measurement of the total lifetime of the drop, i.e. the time which elapses between the moment when a drop is deposited on the hot surface till the time when all the liquid is evaporated. The second one is a steady state method. Here the drop after deposition on the hot surface is continuously fed by a fresh stream of liquid which is supplied through a capillary. The drop obtains such a size that the flow rate of the supplied liquid is equal to the evaporation rate.

In our research first the second method was tried. This method however showed rather large disadvantages especially for drops in a saturated environment. When the liquid fed through the capillary was too cold, then the vapor from the environment would condense on the outside of the capillary,
so that a very serious error was introduced in the determination of the feeding stream. When, however, the liquid in the capillary was at the boiling temperature, then it could easily become so hot that at the end of the capillary vapor bubbles were formed which disturbed the drop very seriously. In addition to this, also the capillary itself could influence the measurement. If it was too thick, then the surface tension of the liquid would cause an important upward force on the drop. Was it too thin then the entering liquid stream would seriously influence the streaming pattern within the droplet. Therefore, we finally changed over to the following non-stationary method. With the aid of a pipet a certain amount of a pure liquid was deposited on a somewhat concave hot surface. Next sufficient time was allowed for the oscillations of the drop to die out. This could be speeded up by inserting a very thin metal wire into the drop. Now at regular intervals (five to twenty seconds, depending upon the evaporation rate) the drop was photographed from aside, till it had evaporated completely. From the photos, the height $h$ and the diameter $2R_{\text{max}}$ of the drop were measured for each particular moment. In principle, the volume $V$ could now be determined with the aid of Figure 2.1 to 2.6. Very often, however, the drop oscillated still a little bit. This oscillation was usually the first mode of the drop (see for instance Lamb (81), or Rayleigh (82)). Therefore, the determination of $h$ or $R_{\text{max}}$ alone would not give sufficient information. However, the repeated determination of the volume of the drop by measuring the drop contour from each successive picture followed by graphic integration would be rather time consuming. Fortunately, after measuring the contours of around 25 drops we found that
The factor $h R_{\text{max}}^2$ was a good measure for the volume $V$ of the drop, even when this was still oscillating. The volume which was determined in this way differed in the average only about 1 per cent and maximally only $2\frac{1}{2}$ per cent from the true volume. It was therefore sufficient to determine $h R_{\text{max}}^2$. The corresponding value of $V$ could be found from Figure 2.14.

![Graph](image)

Figure 2.14 The relationship between the function $h R_{\text{max}}^2$ and the volume $V$ of a sessile water drop.
Fig. 2.15  Example of a measured curve of the volume of a sessile water drop as a function of time

When the volume thus calculated was plotted versus the time then a curve like the one in Figure 2.15 was obtained. From this $\Phi_{vt} = -dV/dt$ could easily be found by differentiation.
The first experimental results were obtained with the setup which is sketched in Fig. 2.16. An asbestos box containing an electric element is covered with a thick copper plate, in the middle of which a solid copper cylinder of 5 cm diameter is mounted on top of which a 2 mm thick 24 karat gold plate is soldered. The gold plate is somewhat concave so that when the whole setup is perfectly leveled a drop stays on it.

In a little hole in the gold surface a chromel-alumel thermocouple is soldered for temperature measurements. The voltage of this is measured with a potentiometer. In order to prevent false readings of the temperature caused by heat conduction through the thermocouple wires, these wires are made very thin and where possible run along the copper cylinder.
In order to be able to work in a saturated environment the asbestos box with everything which is mounted on top of it, is placed in a metal box with one glass wall, in which saturated steam can be introduced. In order to prevent too much condensation the walls of the box are electrically heated. The glass window, through which the drops have to be photographed from the outside, is heated by a stream of hot air which comes from a hot plate which is located underneath it.

In order to prevent the vapor around the drop from overheating an annular container filled with water is located around the copper cylinder. It turned out that this also very strongly reduced the oscillations of the drop, probably because the free convection of vapor around the droplet is decreased. Water is run through the annular container at such a rate that it does not boil so that the drop cannot be disturbed by spray droplets.

The water for the drop was deposited on the golden surface with the aid of glass pipet. This was then turned away from the surface in order to prevent disturbance of the measurements by falling drops of condensed water.

When working with the above described apparatus it turned out that the gold surface had to have a minimum temperature in order to get the liquid into the spheroidal state. When the temperature was lower than around 170°C, the liquid usually started to boil very violently. Then a quiet spheroidal drop could not be obtained. This phenomenon can also be found in the literature. It has even been a hot point of discussion in the literature at which temperature of the surface a drop still could be
deposited. Drew and Macler (28) give an extensive survey of the results obtained with different liquids on various kinds of surfaces. They connect these with the temperature at which a heated wire immersed in a liquid film boiling occurs. On account of the values of the vapor slit thickness calculated in section 2.3, we suspected, however, that the above mentioned temperature is not really the limit for the Leidenfrost phenomenon but that we have to do with disturbances created during the deposition of the drop which initiate local contact between the liquid and the solid surface.

For the measurement of the evaporation rate of the drop at low temperatures we therefore changed to the apparatus of Fig. 2.17. Here one can deposit a drop on the hot surface at a temperature above the above mentioned minimum temperature and after that perform the measurements at a temperature which is considerably lower. The most important part of the apparatus is a vertical copper pipe with an outside diameter of 25 mm on which at the top a somewhat concave gold plate of 1 mm is soldered. The pipe is heated with an electrical element which is wound around its lower half. Halfway the pipe, a stream of cold air can tangentially be blown into it, which escapes through a small pipe at the bottom. After a drop has been deposited on the gold surface, at a temperature of about 170° C, air is blown into the pipe so that this, because of its relatively small heat capacity, rapidly cools down to the desired measuring temperature.

The rest of the apparatus shows a great similarity to the previous apparatus. The only difference actually is that the metal box is replaced by two wide concentric glass pipes which rest in an annular container.
around the gold surface. At the bottom of each of these pipes a gate-shaped aperture is provided through which the droplets can be photographed. The loss of steam through these openings is not serious.

The temperature of the gold surface is measured with a thermocouple which is soldered onto the middle of the bottom. The temperature gradient in the gold plate which is caused by the heat transport to the drop may be ignored. In order to prevent a false indication because of the heat transfer through the thermocouple wires the upper part of the pipe is closed off with an asbestos plate. The voltage of the thermocouple is determined by a Brown recorder. The cold junction of the thermocouple is located in a thermostat with adjustable temperature, so that a large temperature range can be measured. The temperature of the surface can be compared with the boiling temperature $T_b$ which is measured in the flask in which steam is produced for the saturation of the environment of the drop.

It appeared now that it is really possible to obtain the Leidenfrost phenomenon also below $170^\circ C$ and with special care even down to a few tenths of a degree above the boiling point. The way of measuring was the same as with the first apparatus, on the understanding that after the deposition of the liquid with the surface at a temperature above $170^\circ C$, the pipe and the gold surface were cooled down to the measuring temperature by means of the pressurized air stream. After that the temperature was easily kept constant with the aid of the pressurized air stream.
Fig. 2.17 Experimental set-up with easily variable plate temperature

Sect. 2.6 Measuring Results with Droplets in a Saturated Environment*  

The measurements were done with drops of distilled water with an initial volume of about 500 mm$^3$. Usually one had to wait for some time till the liquid had come to a rest so that with the first photo the volume was usually not more than 200 mm$^3$. If after development the photos showed that the drop has oscillated too strongly, then they were discarded. The shape of the drops corresponded quite closely with the results of the calculations in section 2.2. This already turned out to be true for the values

* The measurements were done by H. J. van Nohuis and H. Boume
of the main dimensions given by Borishansky, at least for the relationship between the volume and the diameter. The height which he measured is generally about 0.5 mm too large. This can probably be explained by the vibrations of the drops.

Because of the severe vibration of large drops the evaporation rate was only determined for drops smaller than 200 mm$^3$. The results are given in Figs. 2.18 to 2.29. The measurements at surface temperatures above 180°C were carried out with the apparatus described first in the previous section. With the second apparatus it became clear that the Leidenfrost phenomenon could be maintained at surface temperatures down to very close to the boiling point of the liquid. The drops were however not very stable at these low temperatures. The slightest vibration could initiate wetting of the surface followed by the collapse of the drop. Though, even at $T_m = 120°C$ an evaporation curve could still be obtained.

The accuracy of the determination of the drop volume was about 2 per cent. By drawing a smooth curve through a large number of measuring points the accuracy could however be improved appreciably, so that the final accuracy of $\dot{V}_{tot}$ was only determined by the possible error introduced with differentiation of the curve. This error was not more than 3 per cent.

One would expect that the results would lie between the values for $\dot{V}_{tot}$ which were calculated for respectively circulating and non-circulating liquid. In reality they are situated between the two extreme values which were calculated for $\dot{V}_{top}$, i.e. for a flat-bottomed drop. This most likely is because of the fact that the drops during their evaporation were constantly a little bit in motion. This was even the case for drops which at the first glance seemed to be quiet. With the aid of the
photos it was found that, although the shape of the drops was always a body of revolution, the height and width would vary periodically. The drops obviously vibrated in their first mode. For small deviations from the spherical shape Rayleigh (82) determined the following formula for this frequency,

\[ f = \frac{1}{\pi} \sqrt{\frac{2\sigma}{\rho_1 R^3}} \]  

(2.42)

in which \( R \) is the radius of the drop when spherical.

From a dynamic measurement of the capacitance of the system formed by the sessile drop and the gold surface\(^*\), it turned out that the oscillation frequency of the bottom of the drop completely agrees with the frequency calculated with (2.42) if one takes for \( R \) the radius of a sphere with the same volume as that of the sessile drop (see Fig. 2.30). The results of Stark (54) already indicated this. He connected the hot surface and the sessile water drop in series with a battery and a headphone. In the latter a sound was heard of which the frequency became higher when the drop size decreased. It is seen that the frequency \( f \) for regular drop dimensions is indeed in the audible range.

From the fact that the frequency of the vibrations at the drop bottom corresponds with the first mode of the drop one may conclude that the bottom side is alternately concave and convex. Because in addition the relationship between the vapor production and the distance between the

\* The measurements were executed by E. Ruentol.
liquid and the metal surface is non-linear, it is conceivable that the flat bottom theory suits better than the exact theory.

Fig. 2.18 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 120^\circ C$
Fig. 2.19 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 140^\circ C$

Fig. 2.20 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 160^\circ C$. 
Fig. 2.21 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 180^\circ$ C

Fig. 2.22 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 197^\circ$ C
Fig. 2.23 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 224^\circ C$

Fig. 2.24 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 254^\circ C$
Fig. 2.25 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 285^\circ C$.

Fig. 2.26 Measured and calculated evaporation rate of a sessile water drop in a saturated atmosphere at $T_m = 317^\circ C$. 

[Graph showing evaporation rate vs. time for different drop shapes and conditions]
Fig. 2.27 Measured and calculated evaporation rate of a sessile water drop of $17.85 \times 10^{-9} \text{ m}^3$ in a saturated atmosphere.

Fig. 2.28 Measured and calculated evaporation rate of a sessile water drop of $44.0 \times 10^{-9} \text{ m}^3$ in a saturated atmosphere.
Fig. 2.29 Measured and calculated evaporation rate of a sessile water drop of $150.7 \times 10^{-9} \text{ m}^3$ in a saturated atmosphere.

In Fig. 2.21 the influence of the vibrations can readily be observed. Here the drop was disturbed when it had a volume of around $100 \text{ mm}^3$. At a volume of $30 \text{ mm}^3$ it recovered. However, after that it again started to vibrate. The vibration was probably caused by a small piece of dirt on the gold surface.

It is hard to determine from the results whether they are influenced by the drag on the liquid by the vapor flow underneath the drop. The possible effect of this disappears completely in the effect of the vibrations of the drop.
Taking into account what was found about the influence of the vertical vapor speed on the heat transmission in section 2.4, one would expect that at higher temperatures the heat transfer would be noticeably less than the one calculated with the simplified theory. The deviation which one in this way obtains in the evaporation rate disappears however also within the effect of the drop vibrations.

Fig. 2.30 The frequency of a vibrating sessile water drop at 100° C

In order to find out if there is a minimum temperature for the existence of the Leidenfrost phenomena, we did the following experiment with the second apparatus of section 2.5. With the temperature of the gold surface above 170° C we deposited around 500 mm$^3$ of water. After this had come to nearly complete rest, the temperature of the surface was reduced at a constant rate. When at a certain surface temperature
the liquid made contact with the surface and the drop collapsed a sharp bend in the cooling curve of the surface was observed. The wetting temperature found in this way depended upon the roughness of the metal surface and on the vibration intensity of the drop. On a very smooth surface a vibrationless drop could be maintained down to the boiling point of the liquid. Was this reached, then it collapsed all of a sudden. This agrees completely with the theory of section 2.3. From Table 2.2 it appears that \( d_p \) for \( T_m = 120^\circ C \) for a droplet of volume of 2.9 mm\(^3\) is still 7.51 micron. Therefore, \( d_p \), being about proportional to \( (T_m - T_s)^{\frac{1}{3}} \), for \( T_m - T_s = 0.01^\circ C \) is still larger than 1 micron. For small values of \( T_m - T_s \) \( d_p (R_o) \) is nearly equal to \( d_p \), so that its behavior may be expected to be similar. The spheroidal state can therefore be maintained down to very low values of \( T_m - T_s \).

If there was a small piece of dirt on the surface or did this have a little scratch then the wetting of the surface would start at a temperature which was appreciably higher than the boiling point. Even the little residue of the dirt left on the surface from a drop which was evaporated completely before was here sufficient. If after the surface was wetted its temperature was still high enough then nucleate boiling was observed.

The cooling rate of the gold surface turned out only to be important for vibrating drops. Was the cooling very rapid, then the wetting would already take place above the boiling point. Was the cooling very slow, then the drop would become more and more quiet as the temperature was reduced. Apparently the vibration was damped by the diminishing of the vapor slit underneath the droplet.
From the above it follows that a wetting temperature or a "minimum temperature" for the Leidenfrost phenomenon is hard to define. Could we obtain a completely quiet drop, for instance with a very viscous liquid, then there had to be such a temperature. This temperature would then be dependent upon the roughness of the surface. For less viscous liquids the drop is always more or less in vibration. Because the amplitude is not always the same a wetting temperature cannot be determined in an unambiguous way. Hence, the wetting temperatures given in the literature are arbitrary, the more so as because they usually were affected by the phenomena that occur with the deposition of the liquid upon the surface.

Sect. 2.7 Sessile droplets in a non-saturated environment*

Also the evaporation rate of drops in atmospheric air was measured. Because at 20°C the saturated vapor pressure for water is only 0.02 atmosphere, one may assume that the environment is completely dry. The results of the measurements are given in Fig. 2.31. It is not possible to compare the results with those in the literature. Measurements of the total life time of the drop have little value, particularly if they are done on a coarse surface. The other known measurements, i.e., those

* The experiments were carried out by H. J. van Nouhuis and H. Bonne.
using the stationary method (see section 2.1) are done with much larger droplets than those used for this research.

It turns out that now the evaporation rate is much larger than in an environment of saturated vapor with otherwise the same conditions. From this one may conclude that the distance between the drop and the hot surface is smaller. In section 2.3 we have seen that this distance is determined by the volume of the drop and the amount of vapor which is produced at the bottom. If we keep the first one constant, then the second has to be smaller in this case. From this follows that the evaporation rate at the top and on the sides is extra large. One can understand this if one considers that the drop at the top and the sides is completely surrounded by dry air.

![Graph showing evaporation rate](image)

**Fig. 2.31** Measured evaporation rate of a sessile water drop in dry air
The average temperature of the liquid will establish itself at such a value below the boiling point that the heat of evaporation corresponding with the amount of vapor which diffuses into the ambient air is equal to the heat which is transferred through the drop. This heat comes from the bottom and from the sides. The heat transport through the drop is mainly done by a circulation flow. Because at the top the drop is colder than at the bottom there exists a surface tension gradient in the drop surface, so that the area with high surface tension at the top pulls the liquid surface at the bottom constantly upward. This can quite easily be visualized with some soot (see for instance the experiments of Stark (54)).

It is interesting to find out what the ratio is of the evaporation rates at the bottom and at the rest of the surface. This can be done with the aid of the measured total evaporation and the results for drops in a saturated environment which are given in the preceding sections. Take for instance a drop with volume $V$, resting in a non-saturated environment on a hot surface with temperature $T_m$. The evaporation rate of this can be separated in two parts, the one for the bottom $\dot{E}_{vob}$, and the one for the sides and the top $\dot{E}_{vb}$ The heat which is required has to be transferred from the hot surface by conduction. However, only a small portion of this heat is directly used for evaporation at the place of arrival. The larger part is absorbed by the circulation stream in order to be used for evaporation somewhere else on the surface.
Compare now the above described drop with a drop of the same volume but resting in an environment of saturated vapor on a hot surface with temperature $T_m'$. Choose now the temperature $T_m'$ in such a way that the thickness of the vapor slit underneath the drop is equal to that of the first drop. If in both cases the temperature of the drop is equal to the boiling temperature $T_S$, then the heat flow that goes from the hot surface to the second drop will be equal to the heat flow to the first drop multiplied by $(T_m' - T_S) / (T_m - T_S)$. The amount of vapor which is produced underneath the second drop is however equal to the amount which is formed underneath the first drop, because the distance to the hot surface is determined by the vapor production at the bottom. It was assumed that this distance would be the same in both cases.

**Fig. 2.32** The construction of the evaporation rate at the bottom of a sessile drop in dry air with the aid of the calculated evaporation rate of a sessile drop with circulating liquid in a saturated atmosphere.
Considering the above, one can quite easily determine the required temperature $T_m'$ in a graphic way. Assume that in Fig. 2.32 the original drop in the dry environment is indicated by point A. Draw in the same figure also the curve for the evaporation in a saturated environment. Because of the results from the preceding sections it seems recommendable to take for this the evaporation rate calculated for drops with a flat bottom, i.e. $F_{vtp}$. Then there are still two possibilities. First, the evaporation rate of a rigid drop and second the evaporation rate of a drop with circulating liquid. Of these the last one seems to be preferable because of the circulation stream within the drop due to the surface tension gradient. Draw also the corresponding curve for $F_{vop}$. Draw next the line OA. If one ignores the temperature dependence of $\eta$, $\lambda$ and $\rho$, this line indicates the locus of all drops of the considered size which are located at the same distance from the hot surface. This is true because on OA the number $F_{v} / (T_m - T_s)$ is constant, and this is for constant $\eta$, $\lambda$, and $\rho$ only a function of the distance between the drop and the hot surface. Suppose that the line OA meets the curve for $F_{vtp}$ in point B. The corresponding temperature is the requested temperature $T_m'$. Draw now also the line BD, which meets the curve for $F_{vop}$ in point C. Because the drops which correspond to the points A and B are located at the same distance from the hot surface, they will both show an evaporation at the bottom which is equal to that of DC. Thus the evaporation rate at the bottom of the drop in a dry environment $F_{vod}$ has been found.

Finally we can draw the lines OCF, AG, and CE in Fig. 2.32. The
line OGF divides AG in two pieces. One of these, FG, is equal to the evaporation which results from the heat supplied through the bottom of the drop. Because CD=FG, EF is the heat which is not directly used for evaporation, at the bottom of the drop, but which is transferred to the rest of the surface.

The above described procedure was applied for two drop volumes, \( V = 17.85 \, \text{mm}^3 \) and \( V = 44.01 \, \text{mm}^3 \). The results are plotted in Fig. 2.33 and 2.34. Obviously, the effect of the fact whether the environment is saturated or non-saturated is very important. It is especially striking that the evaporation at the sides and the top of the drop is now much larger than the evaporation at the bottom. A small uncertainty in the above has still to be mentioned. For the graphic determination it was implicitly assumed that the shape of the bottom of the drop would not change. With this we have in fact assumed that the part of the heat which is not right on the spot used for evaporation is equal for every point. Probably this is not the case in reality. Hence, the drop surface might obtain a different shape. However, because of the fact that due to the vibrations at the bottom we actually have to do with an average slit thickness, the approximation with a flat bottom seems to be acceptable.

Besides, in section 2.3 we have already found that for drops with a volume smaller than 50 mm\(^3\) the fact whether the bottom is flat or curved has little influence on the total evaporation rate. Hence, the location of point B in Fig. 2.32 would change very little. Only the location of C and F could be different.
From the above it is clear that the evaporation in a dry atmosphere differs very strongly from that in a saturated environment. Hence, theories to explain measuring results in a dry atmosphere while assuming a saturated atmosphere are definitely not correct. Although in the first instance the difference in the total evaporation is not large, there is always an essential difference in the evaporation at the bottom, and thus also in the thickness of the vapor slit.

\[ V = 17.85 \times 10^{-9} \text{ m}^3 \]

- \( \Phi \) measure of total evaporation rate (non-saturated)
- \( \Phi \) constructed evaporation rate at the bottom (non-saturated)
- \( \Phi \) measured total evaporation rate (saturated)

**Fig. 2.33** The total evaporation rate and evaporation at the bottom for a water drop of 17.85x10^{-9} m\(^3\) in saturated and non-saturated atmospheres.
Fig. 2.3h: The total evaporation rate and evaporation at the bottom for a water drop of 14.01 x 10^{-9} m^3 in saturated and non-saturated atmospheres.

In addition to the strongly increased evaporation at the top of the drop there is an extra complication for the lower values of $T_m$ which changes the total physical picture of the phenomenon. For in a non-saturated atmosphere resting drop one has to realize that in fact on all sides of the drop, and hence also at the bottom side, there is a one way diffusion of vapor into the surrounding gas. Also here the drop is pushed upwards from the "hot" surface. Note that the one way diffusion introduces a drift velocity which creates the pressure build up underneath the droplet.
The result is that as long as there is a difference between the temperatures of the drop and the "hot" surface — and this will be the case as long as the temperature of the surface is higher than the "wet bulb temperature" of the environment — the drop in principle will show the phenomenon of Leidenfrost. Of course in practice a limit is set by the coarseness of the surface and the possible vibrations of the drop.

![Diagram of one-way diffusion under a sessile drop in a non-saturated atmosphere.](image)

**Fig. 2.35** One-way diffusion under a sessile drop in a non-saturated atmosphere.

The same phenomenon was already noticed by Derjagin and Prokhorov (83) with the coalescence of drops in a dry atmosphere. This turned out to be much slower than the coalescence in pure vapor. For the calculation of the pressure build up under a sessile drop a method can be used which is analogue to that of Derjagin and Prokhorov. Assume that the drop has the shape of the dotted line in Fig. 2.35. In Fig. 2.9 one can see that this shape is acceptable. Assume next that in the narrow slit underneath
the edge no vapor is produced. The molar flow through the slit to the outside is then

\[ \Phi_{\text{mol}} = 2\pi r_d C \left( \ln \frac{r_2}{r_1} \right)^{-1} \ln \left( \frac{C - C_2}{C - C_1} \right). \]  
(2.43)

In this equation \( C_1 \) and \( C_2 \) are the concentrations of the vapor at \( r = r_1 \) and \( r = r_2 \), and \( C \) is the sum of the concentrations of the vapor and the ambient gas. If, except in the slit, the concentration gradient is everywhere zero, then \( C_1 \) and \( C_2 \) are the concentrations of the vapor respectively in the area underneath the drop and in the environment.

The molar flow creates in the slit the following average radial drift velocity

\[ \langle u \rangle = \frac{\Phi_{\text{mol}}}{2\pi r C d}. \]  
(2.44)

Hence, the pressure increase in the area underneath the drop is in case that the liquid in the drop does not circulate

\[ \Delta p = \frac{12\eta_0 r \langle u \rangle}{d^2} \ln \frac{r_2}{r_1} = \frac{12\eta_0 D}{d^2} \ln \left( \frac{p_0 - p_2}{p_0 - p_1} \right). \]  
(2.45)

In this equation \( p_1 \) and \( p_2 \) are the vapor pressures underneath the drop and in the environment. If \( r_2 - r_1 \ll R_0 \), then the following
force balance is true

\[ V_{p, \beta} = \pi R_e^2 \Delta \rho , \] \hspace{1cm} (2.46)

so that the slit thickness finally becomes

\[ d = \left( \frac{12 \eta \Delta R_e^2}{V_{p, \beta} \ln \frac{p_o - p_2}{p_o - p_1}} \right)^{\frac{1}{4}}. \] \hspace{1cm} (2.47)

If the gold surface was smoothly polished then it was possible to keep a droplet with a cross section of 1 mm in the spheroidal state down to \( T_m = 76^\circ C \). This demonstrates that, as was expected, the Leidenfrost phenomenon in a dry environment can also be maintained below the boiling point of the liquid.

For the calculation of \( d \) for the above mentioned droplet one would need the temperature at the bottom of the droplet. If we assume that this was \( 50^\circ C \) and that the relative humidity of the ambient air at \( 20^\circ C \) was 60 per cent, then we find with (2.47) a split thickness \( d \) of 1 micron. This turned out to be the order of magnitude of the surface inequalities of the gold surface.
Chapter III

IMPINGING DROPLETS

When a drop approaches a very hot surface with a certain velocity a vapor layer is formed as soon as the distance between them is small enough. This vapor layer prevents direct contact between the drop and the hot surface. The drop is forced against the surface but is always separated from it by the vapor layer. If the velocity with which the drop impinges upon the hot wall is not too high then the drop behaves as an elastic ball. It rebounds from the surface after a certain residence time. If the impact velocity is high then the surface tension cannot keep the liquid together so that the drop breaks up in a larger number of smaller droplets. These too leave the surface after a short time.

The analytical description of what takes place in the drop during the impact turns out to be very difficult. Although it is rather easy to find the equations for the velocity and the pressure in the liquid, until now a solution of these has not yet been found. Therefore, we have tried to obtain some insight into the impact process by photographing drops during their stay at the surface. We will go deeper into this in Section 1 of this chapter.

Just like with a sessile spheroidal drop the heat to an impinging drop has to be transferred through a thin vapor layer. The thickness of this layer is, however, not constant. Both the size of the "contact"
surface area between the drop and the hot wall, and the force with which the liquid presses itself against the surface vary during the impact continually. The calculation of the heat transfer is therefore much more complicated than for sessile drops. The least complicated is the calculation for drops of which the liquid before the impact is at its boiling point and where the environment contains saturated vapor. In that case, with a number of simplifying assumptions and the information obtained from photos of the impact process an estimate could be made of the amount of heat which during the impact is transferred from the hot surface to the drop. This estimate was compared with experimental results obtained with the aid of a heat flow meter. This will be described in the Sections 3.2 and 3.3.

If the droplet before the impact is not at the boiling temperature then during its stay on the hot surface heat is needed both for the formation of vapor and for the heating of the liquid. Because we do not have a mathematical description of the liquid flow in the drop - and, due to the surface tension gradient this flow is even more complicated than for droplets of homogeneous temperature - it is impossible for us to give an analytical description of the heat transfer through the droplet. It is therefore also impossible to calculate the thickness of the vapor layer and the amount of the vapor vapor. The same problem was met before for sessile drops in an non-saturated environment.

Some information about the heat transport could however be obtained by measurement of the weight loss and the temperature increase that the droplet undergoes during the impact. From this followed that the behavior of the drop as a function of temperature of the surface is more complicated than one would expect at first glance. In Section 3.4 we will go deeper into this problem.
Sect. 3.1 The hydrodynamic behavior of a drop during the impact.

Assume that a drop impinges perpendicularly upon a completely flat surface that is not wetted by the liquid. We will leave aside whether the lack of wetting is a result of the formation of an insulating vapor layer or is caused by relatively large cohesive forces within the liquid as for instance is the case with mercury drops on a glass plate. If the density of the surrounding gas is not too high, then in the impact process the following factors are of importance: the radius $R$ and the impact velocity of the droplet, $v_a$, and also the viscosity $\eta$, the density $\rho$, and the surface tension $\sigma$ of the liquid. A dimension analysis shows us that the impact process is characterized by the Reynolds number,

$$Re_a = \frac{2\rho v_a R}{\eta} \quad (3.1)$$

and the Weber number,

$$We_a = \frac{2\rho v_a^2 R}{\sigma} \quad (3.2)$$

Later on it will appear that $Re_a$ in the cases considered by us is always so large that the friction forces can be ignored. Therefore, one
may assume that $\eta_\psi = 0$. One may now introduce a velocity potential in order to describe the flow within the droplet.

![Diagram of a drop with coordinates r, $\theta$, and $\psi$](image)

Fig. 3.1 Cross section of a drop during impact

If one chooses in Fig. 3.1 a polar coordinate system with coordinates $r$, $\theta$, and $\psi$ and if one writes $v_r = -\partial \psi / \partial r$ and $v_\psi = -1/r \partial \psi / \partial \psi$ and one considers the fact that there is a circular symmetry around the vertical axis, then the mass balance becomes

$$\sin \phi \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{\partial}{\partial \phi} \left( \sin \phi \frac{\partial \psi}{\partial \phi} \right) = 0. \quad (3.3)$$

The velocity potential in this equation is a function of $r$, $\psi$, and time $t$.

Because of the circular symmetry the velocity perpendicular to the vertical axis is equal to zero, so that

$$\left. \frac{\partial \psi}{\partial \phi} \right|_{\phi = 0} = 0. \quad (3.4)$$
If one assumes that the bottom of the droplet is a flat surface which goes through the origin 0, then one may write

$$\frac{\partial \psi}{\partial \varphi}_{r=0} = 0. \quad (3.5)$$

Also the velocity in the origin 0 is equal to zero, so that

$$\frac{\partial \psi}{\partial r}_{r=0} = 0, \quad \frac{\partial \psi}{\partial \varphi}_{r=0} = 0. \quad (3.6)$$

The motion of the liquid surface $s(\psi, t)$ is related to the local speed according to

$$\frac{\partial s}{\partial t} = \frac{1}{s^2} \frac{\partial s}{\partial \varphi} \frac{\partial \psi}{\partial r} - \frac{\partial \psi}{\partial t}. \quad (3.7)$$

Finally, for the liquid surface the following pressure balance is true

$$\rho_l \frac{\partial \psi}{\partial t} + \frac{1}{2} \rho_l \left( \frac{\partial \psi}{\partial r} \right)^2 + 1 \left( \frac{\partial \psi}{\partial \varphi} \right)^2 \left|_{r_1}^{r_2} \right. = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right). \quad (3.8)$$

in which $R_1$ and $R_2$ are the main radii of curvature of the liquid surface in a point $(s, \psi)$. For a cylindrical coordinate system these are given in Section 2.2. Gravity forces are ignored in equation (3.8).

When the drop arrives on the solid surface it has the shape of a sphere with radius $R$, so that
\[ s(\varphi, 0) = 2R \cos \varphi. \quad (3.9) \]

The velocity potential is then

\[ \psi(r, \varphi, 0) = r_r \cos \varphi. \quad (3.10) \]

Until now it was not possible to find a solution of the above given system of equations. The reason for this is that the time dependence of \( \Upsilon \) and \( s \) is determined by the boundary conditions. One can however predict what will happen if the surface tension is zero or negligibly small - in other words for larger values of \( \text{We}_g \). If in equation (3.7) the initial conditions (3.9) and (3.10) are filled in then it turns out that for \( \psi \rightarrow \frac{3}{2} \pi \) the expansion velocity of the liquid surface becomes infinite. However, in the origin \( 0 \) the velocity of the liquid is equal to zero. If the surface tension is ignorable then the motion of the lowest liquid layer in the drop will not be impeded by higher located layers. As a result, immediately after the first contact of the drop with the solid surface a very thin liquid layer will shoot over the surface. In the following we will see that this really happens with high impact velocities. If, however, the viscosity and the surface tension are not ignorable then the radial velocity of the lowest liquid layer will communicate its velocity to the higher located layers, so that the motion in the liquid will be less local.
Savic and Boult (68) dropped liquid droplets with a diameter of 2.9 mm from a rather large height (about 1.50 m) upon a hot silver surface using an environmental pressure of 0.5 cm Hg, and filmed the impact with an ultra fast camera. They calculated in a very complicated way the shape of the droplet during the first part of the impact. For that purpose they assumed that before the impact the drop was at rest and that the solid surface impinged against the drop with a constant velocity. They also assumed that the top half of the drop retained a spherical shape. Only the radius of the sphere would change. With a simple geometric analysis one can however show that the top half of the droplet never can retain its spherical shape if one, like Savic and Boult did, supposes that in the drop a toroidal flow pattern exists and that the drop surface is always an equipotential surface. Most likely they did not notice this because they only calculated the radius of curvature in the top of the droplet and next assumed that the radius of the spherical taken drop surface was equal to this.

In order to find out a little bit yet about the hydrodynamics of the collision process we took pictures of the droplet in the various stages of its impact. Because of the fact that the behavior of a impinging drop is very reproducible we could in this way obtain remarkably useful results.

The first information was obtained with a few series of pictures made with stroboscopic illumination. Droplets with a diameter of 1.7 mm were formed at a thin stainless steel capillary. Under the influence of gravity they fell upon a hot polished metal surface which made a small
angle with the horizontal. The temperature of the metal surface was about 400°C. In this case the collisions were almost noiseless so that we might expect that a fully developed spheroidal state was obtained. In an absolutely dark environment a photo camera was located with its optical axis in the plane of the hot surface. Just before the droplet fell the shutter of the camera was opened and the stroboscopic light source was turned on.

Figure 3.2 shows a representative result for this experiment. By making a series of similar pictures we could follow the behavior of the drop during its impact. It turned out that the drop during the first part of its collision collapses to a flat disk. Next it straightens again and finally departs from the surface with vigorous vibrations. The period during which the droplet stays on the surface turned out to be equal to the period of these vibrations. For our drop size this period was equal to 7.1 x 10^-3 sec. It is remarkable that the equation that Rayleigh (82) found for the first made of a vibrating droplet,

\[ \tau = \pi \sqrt{\frac{p R^3}{2\sigma}} \]  

(3.11)

predicts the same value, although it was derived for very small deviations from a sphere with radius R.
Fig. 3.2 Stroboscopic picture of an impinging drop (drop volume $2.7 \times 10^{-9} \text{m}^3$, frequency of the stroboscopic lighting 100s$^{-1}$)

It is seen in Fig. 3.2, that the droplet during its successive impacts upon the surface gradually loses its kinetic energy. At each impact a part of this energy is directly dissipated through viscous forces in the liquid. Another part is still available after the impact in the form of surface energy. This energy is partially dissipated during the vibrations which the drop performs during its trajectory through the air. The rest of the energy gives the droplet its translation energy from the surface. By measuring the parabolic trajectories which the drop performed between its impacts we found the relationship between this remaining kinetic energy and the original energy. It turned out that the component which is parallel to the hot surface does not change during the collision. The perpendicular component, however, decreases appreciably at higher velocities (see Fig. 3.3).
A more refined insight into the collision process could be obtained with the aid of the apparatus shown in Fig. 3.1. With this it was possible to take a picture of an impinging droplet at a previously chosen moment during the impact process. It works as follows. Very thoroughly cleaned and by prolonged boiling desorated water is forced by the pressure of a nitrogen cylinder B from the container K into a thin stainless steel capillary. The pressure is kept very constant in order to prevent the diameter of the drops and the frequency (about 0.5 sec\(^{-1}\)) with which they fall from the end of the capillary from varying. For the same reason a heat exchanger, controlled by a thermostat, was mounted around the capillary.

Fig. 3.3 Relation between velocities of arrival and departure of an impinging drop
When the drop catcher (see Fig. 3.4) is pulled aside for a moment a drop will fall on a very smooth polished gold plate. This is soldered upon a thick brass plate which is heated electrically from the bottom side. The temperature of the gold plate is measured by a thermo-couple which is soldered into a small hole in the surface. The task of the drop catcher is to prevent too many drops from falling on the soft gold surface so that this would be eroded.

During its fall the drop passes through a very thin light beam which is aimed at a phototransistor. The transistor then sends a small negative pulse to the following apparatus. The task of this apparatus is to provide an adjustable time delay for the triggering of a light flash given by a high pressure mercury lamp, so that at a certain moment a picture is obtained with an open camera. The camera is situated in such a way that its axis falls nearly in the plane of the hot surface.

The time delay is obtained as follows. After amplification, signal a. (see also Fig. 3.5.) is fed via a Schmitt-trigger and a bistable multivibrator into an univibrator. This changes signal d into a long pulse the length of which can be varied by means of a 10-turn potentiometer. Differentiation of this signal gives a positive and a negative pulse. After elimination of the first pulse by rectification of the signal the second pulse is fed to the trigger electrode of the mercury lamp. A large capacitor which is charged to about 2000 volts is then discharged over the lamp. The duration of the light flash is about $10^{-5}$ sec.
Fig. 3.4 The apparatus for the photographing of impinging drops.

The time which elapses between the moment at which the drop passes through the light beam and the moment at which the mercury lamp flashes is equal to the duration of the pulse produced by the univibrator. By also feeding this pulse to an electronic gate which controls the passing of a 100 kc/s-signal to an electronic counter one can measure the time delay with an accuracy of $10^{-5}$ sec.

When the drop rebounces from the surface it passes again through the light beam. The bistable multivibrator prevents now that signal c again go through, so that no second light flash can follow. A new signal coming from the photo transistor can only pass through after a reset switch is turned.
A large number of photos of impinging water drops were taken with this apparatus. The diameter of these drops was always about $2.3 \times 10^{-3}$ m. Figures 3.6 to 3.8 give a picture of the impact process for drops in the pure spheroidal state. The photographs were taken with a horizontal hot surface of which the temperature was about 1000°C. With an inclined surface the development of the collision was similar. Then the behavior was completely determined by the perpendicular component of the impact velocity $v_{an}$.

One notices right away from these photos that the collision process can be reproduced fairly well. Every picture which was taken at a later time during the impact does usually show a drop which is more advanced in the impact process.

It is seen in Fig. 3.8 that for high velocities the drop shoots sideways a thin liquid film over the hot surface. This is in agreement with what was predicted in the beginning of this section. In a later stage
of the collision the thin layer breaks up into small droplets. Further experiments with liquids with various viscosities (water, methyl-alcohol, acetylationcohol, n-butylalcohol, isobutylalcohol, and tetra) showed that this desintegration always takes place if the Weber number corresponding to the velocity component perpendicular to the hot surface is larger than 80*. Whether the environment was saturated or not was not important. Apparently, only the Weber number determines the hydrodynamic behavior of the drop, at least at high velocities. That this is probably also the case for low velocities follows from the fact that the dimensionless group \( \frac{\eta V_{an}}{\sigma} \), which determines the ratio of the viscous forces and the surface tension forces in the droplet, decreases with decreasing \( v_{an} \). If, for instance, \( v = 0.1 \text{ m/s} \), then this dimensionless group is equal to \( 5 \times 10^{-4} \), while the Weber number, which gives the ratio between the inertial and the surface tension forces, for droplets with a diameter of \( 2 \times 10^{-3} \text{ m} \) is about 0.3. In the rest of this chapter we will therefore, when possible, use the Weber number for the characterization of the hydrodynamic behavior of the drops.

For \( 30 < We_{an} < 80 \) the drop gives off one or more small drops by constrictions in its upper part during the later part of the impact. These little drops are shot upwards with a high velocity (see Fig. 3.7). The phenomenon is a result of the instability of the thin liquid stem

* These experiments were done by G.F. M. Seeke and J. F. Henning.
(see Rayleigh (82)) which is formed when the drop rises from the surface. The diameter of the small drops depends upon the dimensions of this liquid stem.

In the above we found the residence time of the drop on the hot plate is about equal to the vibration period of the first mode of the drop. In figure 3.9 the ratio between the measured residence time \( t_{3:m} \) and the vibration period \( T \), which was calculated at 100°C from (3.11) \( (= 11.1 \times 10^{-3} \text{ sec.}) \), is plotted against \( W_{\text{an}} \). It turns out that for small \( W_{\text{an}} \) a smaller value is found. For the latter it could be expected that the Rayleigh equation derived for small deviations from the spherical shape would no longer be true.
Fig. 3.6 The impact of a water drop on a polished hot surface (drop diameter $2.3 \times 10^{-3}$ m, $W_{an} = 15$). Time has been measured from the first "contact."
Fig. 3.7 The impact of a water drop on a polished hot surface.
(drop diameter \(2.3 \times 10^{-3}\) m, \(We_{an} = 7h\)). Time has been measured from the first "contact."
Fig. 3.8 The impact of a water drop on a polished hot surface.
(drop diameter $2.3 \times 10^{-3} \text{m}$, $We_{an} = 184$.) Time has been measured from the first "contact."
The point in Fig. 3.9 that corresponds to $W_{an} = 10^4$ is somewhat out of proportion because for high values of $W_{an}$ in the second part of the impact the drop is dispersed into a number of small droplets. Since these small droplets have a smaller vibration period they leave the surface faster.

![Graph](image)

Fig. 3.9 Residence time of the drop on the hot surface

It appears from the photos that the drop during the impact maintains rotational symmetry. Therefore, it is possible to determine the force with which the drop during the impact presses itself against the surface. Using also the radius of the drop bottom that is measured from the photos one can now calculate the evaporation rate at the bottom of the drop in the same way as was done in Chapter II. In the next section we will go deeper into this.
Sect. 3.2. The Calculation of the Evaporation of Impinging Drops.

In Chapter II we derived for non-circulating sessile drops with a flat bottom, the expressions (2.3h) and (2.35) respectively for the split thickness $d_p$ and the evaporation rate $\Phi_{evp}$. In these expressions a group $\rho g \sqrt[4]{\mu}$ appeared which represented the force with which a sessile drop presses on the hot surface. However, when we drop a droplet on a horizontal surface then this force is equal to

$$\rho_i V \left( g - \frac{dx}{dt} \right).$$ \hspace{1cm} (3.12)

In this equation $v_z$ is the velocity which the center of gravity of the drop has in the direction of the surface. If we in (2.3h) and (2.35) replace the group $\rho g \sqrt[4]{\mu}$ by (3.12), then we obtain for the split thickness and the evaporation rate at the bottom of a falling drop.

$$d_p = \left\{ \frac{3\pi \eta T(T_m - T_i)(T_m + T_i)^2}{8 \rho \rho_i \Delta H T_i^2} \right\}^{-1} \cdot R_0 V^{-1} \left( g - \frac{dx}{dt} \right)^{-1}. \hspace{1cm} (3.13)$$

and

$$\Phi_{evp} = \left\{ \frac{2\pi^3 \rho_i \lambda_i^2(T_m - T_i)^2}{3 \rho_i \eta_i \Delta H^2} \right\}^{-1} \cdot R_0 V \left( g - \frac{dx}{dt} \right)^{-1}. \hspace{1cm} (3.14)$$
These expressions are, however, only true if the heat transport through the vapor slit is established fast enough. It will appear later on that the calculated vapor slit thickness is in the order of $5 \times 10^{-5} \text{m}$. Thus the time which is necessary for a complete heat penetration, $t = \frac{d^2}{r^2/\dot{a}_g}$ is about $10^{-1} \text{sec}$. This is short compared with the total time that the drop is on the surface, which is about $10^{-2} \text{sec}$, so that it is permissible to ignore the thermal inertia of the vapor.

During the derivation of the above given expressions it was assumed that the bottom of the drop is flat. In reality this will most likely not be the case because of the pressure distribution underneath the impinging drop. We will go deeper into this at the end of this section.

From the above it follows that the total evaporation of the drop during the collision can be calculated by integration of (3.14) if the velocity of the center of gravity as a function of time can be found. This turns out to be possible with the aid of the photos which were obtained as described in Sect. 3.1 Since from these photos the distance of the center of gravity to the hot surface, $h_z$, can be determined if one assumes that the drop during its collision maintains its rotational symmetry. Especially in the beginning of the collision this assumption holds fairly well. Also the radius of the bottom, $R_0$, can be measured from the photos.
Fig. 3.10 The measurement of the photographed drops.

The calculations were carried out as follows. The photos were enlarged a hundred times, projected and copied. After that for n values of the vertical coordinate $z$, which all differed a constant value $\Delta z = 5 \times 10^{-3} \text{m}$ (thus in fact $5 \times 10^{-5} \text{m}$) from each other, the width $2R_1$ was measured (see Fig. 3.10). The number $n$ was chosen in such a way that $z_t - n\Delta z < \Delta z$, where $z_t$ is the total height of the drop.
Fig. 3.11 The height of the center of gravity above the hot surface as a function of time (drop diameter $2.3 \times 10^{-3} m$).

Fig. 3.12 The radius of the drop bottom as a function of time (drop diameter $2.3 \times 10^{-3} m$).
The height of the center of gravity, $h_z$, was calculated with

$$h_z = \frac{R_z \left( z^2 - x^2 (dz)^2 \right) + (dz)^2 \sum_{i=1}^{n} (R_{i-1}^2 + R_{i-1}R_i + R_i^2)(2i-1)}{2R_x \left( z_i - z_{i-1} \right) + 2dz \sum_{i=1}^{n} (R_{i-1}^2 + R_{i-1}R_i + R_i^2)}$$

(3.15)

Because we knew the time of each photo in respect to the entire impact process, we could now plot $h_z$ as a function of time $t$. Double differentiation of the obtained relationship between $h_z$ and $t$ gave finally the acceleration of the center of gravity $dv_z/dt$ as a function of $t$.

The results for three different values of the impact velocity $v_{an}$ are given in Figs. 3.11 to 3.13. In figures 3.11 and 3.12 one sees that in the beginning of the impact the points obtained for respectively $h_z$ and $R_o$ are located on a smooth curve. At the end they show a fairly large spread. In Fig. 3.11 the curves are drawn in such a way that they correspond with Fig. 3.3.

Fig. 3.13 shows in addition to the with (3.13) calculated vapor slit thickness of an impinging drop also the slit thickness of a sessile drop. Both were calculated for a hot surface temperature of 300°C. It is seen that $d_p$ at the initial "contact" of the drop with the surface increases very fast. This is mainly a result of the fast increase of $R_o$. Despite the fast increase of $d_p$ there is yet at the beginning of the collision a large chance of direct contact between the liquid and the hot surface. Also just before the drop rebounds from the surface this will be the case. We will recur to this in the following two sections.
Fig. 3.13 Vapor slit thickness $d_p$ at $T_u = 300^\circ C$ as a function of time (drop dimension $2.3 \times 10^{-3} m$).

Finally, in Fig. 3.14 the quotient of the relative volume loss of the drop during the impact due to evaporation at the bottom, $\Delta V_{op}/V_0$, and $(T_u - T_S)^{3/4}$ is plotted as a function of the Weber number $We_{an}$ that corresponds to the perpendicular component of the impact velocity. In the extreme case where $We_{an} = 0$ the drop undergoes only the acceleration of gravity. In fact, one has then to do with a sessile drop which stays just as long on the surface as a drop which impinges upon the surface with a finite speed. In this case the evaporation at the bottom is equal to $\int \phi_{vop}$, in which $\phi_{vop}$ is determined by (2.35).
Fig. 3.1h  The calculated evaporation at the bottom of an impinging water drop with a diameter of $2.3 \times 10^{-3} \text{m}$

With the calculation of the evaporation it turned out that for not very low impact velocities the acceleration of gravity did not play any important part. If it had been equal to zero then the calculations would have given almost the same results. Thus, the points in Fig. 3.1h are also true for $g = 0$. However, if both $W_{an}$ and $g$ are zero the radius of the bottom, $R_0$, is equal to zero, so that also $\rho \nabla p = 0$. In this case the curve goes through the origin.

In the above approximation it is assumed that the motion of the liquid has no influence on the resistance which the vapor finds on its way towards the outside. It appears from the photos that the velocity in the liquid surface is in the order of about $1 \text{ m/s}$, whereas the
average vapor velocity at the outermost edge of the slit is in the order of 10 m/s. From this follows that the influence of the motion of the liquid surface is so small that it may be ignored.

We may probably not ignore the evaporation at the sides of the drop. We saw in Chapter II that for sessile drops this could be relatively important. But we have to consider here that the heat penetration depth, $\sqrt{\alpha t}$, for water vapor of 1 atm and for $t = 10^{-2}$ sec. is only about 0.5 mm. However, since the evaporation at the sides mainly takes place at the lower part of the drop, the heat penetration into the vapor is probably fast enough to allow us to ignore the thermal inertia. Nevertheless, the exact determination of the evaporation at the side remains a difficult problem. In principle, it would of course be possible to find this by measurement on electric models of the drop in an electrolytic bath, just like was done in Chapter II for sessile drops. The models which would have to represent all the different stages in the impact process could be made with the aid of the photos. This would require, however, a tremendous amount of work. Therefore, we will here only estimate the evaporation at the sides by comparing the shapes which the drops get during the impact process with those of sessile drops in Chapter II. During this time the ratio $\Phi_{\text{vz.p}} / \Phi_{\text{vop}}$ is very small. Only during the very beginning and just before the departure of the drop from the surface we may expect a large value of $\Phi_{\text{vz.p}} / \Phi_{\text{vop}}$. Therefore, from Fig. 2.13 we may say that for the three
impact velocities that were considered the evaporation at the sides will certainly not be larger than half of that at the bottom. Only for very small impact velocities we will find a larger contribution. The extreme case is met when both $W_{an}$ and $g$ are zero. Then the quotient $\frac{\phi_{wp}}{\phi_{wp}}$ is infinitely large.

In the derivation of 2.34 and 2.35 a number of assumptions were made which were investigated later in Sect. 2.4. It turns out that also for impinging drops the influence is not very large. The largest deviation is caused by the heat radiation. This one is however for a polished surface smaller than 5 per cent. This can be ignored in comparison to the uncertainties which otherwise are inferred into the analysis.

One of these uncertainties is the influence of the shape of the bottom of the drop. As was mentioned before, the bottom is probably not completely flat. Note that during the first part of the impact when the originally spherical drop is transformed into a flat disk the pressure underneath the still spherical part in the center is much larger than under the flat rim that surrounds it. The spherical part of the drop contains liquid of which the vertical velocity has not yet been reduced very much. The stagnation pressure under this part will therefore be large. Thus the slit thickness under the middle of the drop may be thinner than at the circumference. From Chapter II where just the opposite was the case we know that the evaporation rate may therefore be larger than for a completely flat surface.
Whether there are any disturbances at the bottom surface of the drop is very difficult to find out. From a simple analysis follows that a Taylor- or Helmholtz-instability (see E. G. Lamb(61)) will not take place. It is of course possible that the liquid in the beginning of the impact process at a few spots will stick to the solid surface. This could disturb the drop bottom quite severely. The contact in the beginning of the impact is made easier by the fact that the hot surface is never completely smooth. If direct contact really takes place then the evaporation rate will be much higher than was estimated above.

Finally, we assumed above that the temperature of the hot surface during the collision is constant. In reality the wall temperature will drop because heat is withdrawn from it by the droplet. We can easily calculate the order of magnitude of this temperature drop with the information of Fig. 3.13. It follows that the average slit thickness is usually larger than 2×10⁻⁵m. Assume now that during the impact the heat flux is constant corresponds with a slit thickness of 2×10⁻⁵m. In that case the heat flux \( \dot{Q} \) is equal to 2.5W/m². The total temperature drop of the surface is then

\[
\Delta T_m = 2\phi'' \sqrt{\frac{\tau}{\alpha_{m} p_{m} c_{m}}}
\]  

(3.16)
in which \( \Lambda_m \), \( \rho_m \), and \( c_m \) are respectively the heat conductivity, the specific gravity and the specific heat of the solid wall. Table 3.1 below gives this maximum temperature drop for the collision of a drop of 2.3 mm on some materials which we will meet later on.

Table 3.1 The maximum temperature decrease of the metal surface.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sqrt[3]{k_m/2c_m} ) [J/m² °C s²/³]</th>
<th>( \Delta T_m ) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>steel</td>
<td>2.7 ( \times ) 10¹</td>
<td>1.1</td>
</tr>
<tr>
<td>copper</td>
<td>3.8 ( \times ) 10¹</td>
<td>0.7</td>
</tr>
<tr>
<td>platinum - 10 % Ir</td>
<td>0.64 ( \times ) 10¹</td>
<td>3.0</td>
</tr>
<tr>
<td>roestvrij staal</td>
<td>1.3 ( \times ) 10¹</td>
<td>2.2</td>
</tr>
</tbody>
</table>

We see that the assumption of a constant surface temperature is fully permissible.

Sect. 3.3 The measurement of the evaporation of impinging drops in saturated vapor*

In order to find out to what extent the results of the preceding section correspond with reality, an apparatus was built with which the heat transport during the collision of a drop could be measured. A picture of this is given in Fig. 3.15. A thick-walled aluminum box which inclines at an angle of 30° is heated by a surrounding electric heating element and insulated by asbestos sheet from the outside. Inside this box a heat flow meter is mounted between a small aluminum block which is screwed

* The measurements were done by N. A. J. Westerling
to the bottom and a little thin-walled silver container*. A gold cylinder with a diameter of 15 mm and height of 5 mm. is placed in the silver container. By pulling aside a metal plate we can admit a droplet into the aluminum box through a little hole in the cover. This droplet falls upon the polished top of the gold cylinder. If the temperature of the box is high enough then the drop rebounds from the gold surface and finally arrives at the bottom of the box. This is covered with an asbestos plate in order to prevent splashing of the drop.

Fig. 3.15 Apparatus for the measurement of the evaporation of an impinging drop.

* This meter was developed by the Technisch Physische Dienst T. H./T.N.O. It was made out of nickel iron wire with a diameter of 0.1 mm., which was partly covered with a thin layer of gold. Fortafix was used as a refractory cement (see also De Jong and Marquenie (85)).
During the impact the top of the gold cylinder is cooled a little bit due to the heat transfer to the drop. The height of the cylinder is chosen such that at the end of the impact (i.e., after about $10^{-2}$ sec.) its bottom has not yet noticed anything of the heat transfer. It thus behaves towards the drop as a half-infinite medium. Only after the drop has left the gold surface the cold will penetrate all the way down to the bottom. At that moment a temperature difference develops across the heat flow meter, so that this one starts to produce a voltage.

Next to the heat flow meter which is mounted underneath the hole in the cover of the box, another heat flow meter is placed which is connected in compensation with the first one. In this way one compensates for the small heat flows which also without a drop landing on the gold surface always will exist, for instance, as a result of a small temperature difference between the bottom and top of the aluminum box. The differential voltage from the two heat flow meters is amplified 1000 times by a linear amplifier. Because of the fact that the two heat flow meters, e.g., because of the small differences in construction, never completely compensate each other, the amplifier output is fed into an extra compensation circuit. The differential voltage of this circuit can be read on voltmeter $V_1$. Before every measurement the compensator is adjusted in such a way that the reading on this voltmeter is zero.

The reading on voltmeter $V_1$ is thus a measure of the amount of heat which per unit of time from the bottom of the aluminum box through the heat
flow meter is fed to the gold cylinder. Because this is in fact the only way that heat is fed to the cylinder, we can find the total amount of heat which is withdrawn by the drop from the cylinder by integration of the voltage $V_1$. Therefore the output of the compensation circuit is fed into an electronic integrator. The integrated signal which is proportional to the total amount of heat transferred is finally read on a recorder and a voltmeter ($V_2$).

The feeding water for the drops, which by prolonged boiling is released of dissolved air, is forced from a glass container to a thin stainless steel capillary with the aid of pressurized nitrogen. A very constant pressure is maintained so that the drops are deposited at regular intervals from the capillary. In this way a fairly uniform drop size is obtained.

In order to keep the droplets at the boiling point during their fall around the capillary an electrically heated jacket is mounted. Steam is fed into the upper side of the jacket. The lower side of the jacket goes down to the asbestos cover on the top of the aluminum box. In the lower part two holes are drilled so one can see from the outside when a drop is falling from the capillary.

In order to prevent that the feeding water due to the presence of the surrounding steam start boiling, which would disturb a regular production of drops, a heat exchanger is installed around the supply line of the water, which is connected to a thermostat with a temperature of about $99^\circ$ C. The condensed water which is formed at the outside of the heat
exchanger is collected at the bottom in a little container, which is continually drained through a little tube.

The steam atmosphere in the aluminum box is obtained by dropping a few water drops through a second hole in the cover a few hours before every measurement. Because of the fact that the temperature in the box is above the boiling point the steam in the box is superheated. One may expect that this does not give any serious error.

Before, with the above described apparatus, the heat transfer during the impact of a drop could be determined, the whole setup had to be calibrated. To that purpose a small brass box of the same dimensions as the gold cylinder was placed in the little silver container on top of the first heat flow meter. This little box contained an electric heating element, of which the electrical resistance was accurately determined. The calibration was done as follows. After both voltmeters $V_1$ and $V_2$ were adjusted to zero, respectively through adjustment of the compensation circuit and through short circuiting of the integrating capacitance in the integrator, a calibrated capacitor which had been charged to an accurately determined voltage was discharged over the heating element and a resistance bank connected in series. In order to come as close as possible to the conditions of the real experiment for the calibration the time constant of the circuit was chosen in the order of the residence time of an impinging drop on the hot surface. A representative picture of the voltages $V_1$ and $V_2$ is given in Fig. 3.16. After voltmeter $V_1$ had returned to its zero position, and thus the balance in the aluminum box was restored the voltage $V_{2\infty}$ was read.
The calibration was carried out in an energy range between 0.01 and 0.2 J for various temperatures of the aluminum box. Because of the large heat capacity of this box we had to wait a day every time we had adjusted a new value of the current in the heating element around the box in order to make sure that the temperature had established itself at a constant value. It turned out that for every temperature a fixed ratio existed between the finally measured voltage \( V_2 \) and the amount of heat developed in the little heating element. For 200°C this ratio was equal to 23.8 V/J. With increasing temperature it dropped linearly with 0.109 V/°C.

![Diagram](image)

Fig. 3.16 The voltages \( V_1 \) and \( V_2 \) as a function of time.

The measurement of the heat transfer to the impinging drop was originally done in the same way as the calibration. It appeared however that the time constant of the system consisting of the gold cylinder and the heat transfer meter was so large that the drift in the electrical circuit sometimes created an error which could not be ignored. Therefore,
we determined this time constant \( t_c \) from a number of measurements in which there was a relatively large heat transfer. With the following measurements we did not wait any more with reading \( V_2 \) till \( V_1 \) was returned to its zero position, but \( V_1 \) and \( V_2 \) were read some time before. This time was chosen such that the temperature of the gold cylinder was sufficiently homogenized and thus \( V_1 \) decreased according to an exponential function. If the voltages that were read were \( V_1' \) and \( V_2' \), then we could calculate with

\[
V_{20} = V_2' + \int_{0}^{\infty} V_1' e^{-t/t_c} dt = V_2' + t_c V_1'.
\]

(3.17)

With the results of the calibration we could now find the amount of heat which the drop withdrew from the gold cylinder.

The mass of the droplets was determined by collecting a number of them in a little container that was filled with a non-volatile oil and weighing before and after. The mass appeared to be about \( 5.4 \times 10^{-6} \text{ kg} \).

<table>
<thead>
<tr>
<th>( 10^3 V/V )</th>
<th>( 15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10 )</td>
<td></td>
</tr>
<tr>
<td>( 5 )</td>
<td></td>
</tr>
<tr>
<td>( 0 )</td>
<td></td>
</tr>
</tbody>
</table>

- Drop volume \( V = 5.6 \times 10^{-9} \text{ m}^3 \)
- Impact velocity \( V_a = 1.08 \text{ m/s} \)
- Inclination angle = 30°

---

- Calculated evaporation at the drop bottom
- Maximally possible total evaporation (\( V_{xp} / V_{op} = 0.5 \))

Fig. 3.17 Relative volume decrease of an impinging water drop (\( \text{We}_{an} = 33 \)).
drop volume \( V = 5.6 \times 10^{-9} \text{ m}^3 \),

impact velocity \( v_a = 1.25 \text{ m/s} \),

inclination angle \( \theta = 30^\circ \).

--- calculated evaporation at the drop bottom.

--- maximally possible total evaporation (\( \Delta V_{zp}/\Delta V_{cp} = 0.5 \)).

--- maximal mogelijke totale verdamping (\( \Delta V_{zp}/\Delta V_{cp} = 0.5 \)).

--- berekende verdamping aan de onderzijde.

Fig. 3.18 Relative volume decrease of an impinging water drop (\( W_{an} = 144 \)).

--- Relative volume decrease of an impinging water drop (\( W_{an} = 61 \)).
Figures 3.17 to 3.19 show the results of the measurements of the heat transport to impinging droplets. Because of the fact that the temperature of the drops when they arrived upon the surface was equal to the boiling point of the liquid all the heat withdrawn from the gold cylinder was used for evaporation. Therefore, in the figures the relative volume decrease could be plotted. Also shown are the with Fig. 3.14 calculated curves for the evaporation at the bottom and the maximally possible total evaporation (for $\Delta V \eta / \Delta V c_p \approx 0.5$).

It is seen that for low temperatures the heat transport is rather large and decreases with increasing $T_m$. Apparently, a completely spheroidal state does not yet exist. The drops break up in this region because not all vapor that is produced at the bottom can find a way out to the sides. The fluctuation in the measuring results is rather large. The heat transfer coefficient which is calculated from the time $\tau$ that the drop stays on the surface and the average "contact" surface area, is for a relative evaporation of about 1 per cent equal to $2 \times 10^4 \text{W/m}^2\text{OC}$.

Also when for higher temperatures the drop no longer breaks apart the heat transport appears to decrease with increasing $T_m$. According to the theory it had to increase however. Apparently, also for higher temperatures there is still contact between the liquid and the hot surface. We have mentioned in the preceding paragraph that because of the very thin thickness of the vapor slit and the finite coarseness of the metal surface this would indeed be very probable in the beginning of the impact process. Would however the drop bottom always be completely flat then direct contact could never exist very long. The created vapor underneath the drop would
build up such a pressure that it would lift the drop from the hot surface. If however the drop bottom is not absolutely rigid then there is a great chance that if once in the beginning of the impact the liquid has wetted the metal on a few spots, the liquid will stick there to the surface for some time. How long this lasts depends upon the evaporation rate of the sticking liquid and on the pressure differences underneath the drop which try to eliminate the obstacles for a free vapor escape. Therefore, with increasing surface temperature the chance for the creation of spots of direct contact and also their duration will decrease.

Comparison of the Figures 3.17 to 3.19 shows that the influence of the velocity of the drop for high $T_m$ is small. This corresponds with what was predicted by the theory. For lower temperatures, however, the influence of the velocity of the droplet on the heat transfer is rather large. Apparently, the drop velocity mainly determines how strongly the metal is wetted by the liquid.

It was not possible to use the heat flow meter for still higher temperatures. Thus the region where a fully-developed spheroidal state exists could in fact not be reached. From Figures 3.17 to 3.19 it appears that this probably will only be the case as $T_m$ is larger than 400°C.

Par. 3.4 Measurements on cold impinging drops in air*

* The experiments were done by W. H. van Houwelingen and J. P. van de Water.
During the research work described in this chapter we originally tried to measure the evaporation of an impinging drop by determining its weight loss. Therefore we built the apparatus that is shown in Fig. 3.20. The drops here again are formed with a very thin stainless steel capillary. The feeding water, which was freed of air by boiling, is forced to the capillary from a glass container in which during the measurements it was continually kept at 100°C. A pressure reducing valve which is kept at a constant temperature by a thermostat maintains a constant pressure on the liquid. The pressure is delivered by a nitrogen cylinder. Around the capillary a small heat exchanger is built which is connected to a thermostat. Moreover, a little container filled with water is installed around the capillary. This guarantees us that the capillary is always surrounded with saturated vapor which is in balance with temperature of the drop. In this way during the measurements the average drop volume could be kept constant within 0.1 per cent.

The drops fall with a frequency of around 0.5 s⁻¹ from the capillary on a hot metal plate which is installed at a 32° angle with the horizontal and which is heated from below by an electric heating element. The plate temperature is measured by a thermocouple that is welded on the surface. Its voltage is measured by a potentiometer.

A little glass container partly filled with paraffine oil is weighed in advance and located in such a way that all droplets which rebound from the hot surface are deposited in it. By collecting the drops in this way one prevents that they evaporate in the unsaturated environment. By weighing the container after collecting a known number of drops the average
drop weight can easily be determined. By dropping a known number of
drops into another container without first having them impinge upon the
hot metal surface one determines the average drop weight before impact.
In order to make the counting of the drops easier we installed directly
underneath the capella a counting system based on the interruption of a
light ray. As to the number of droplets that had to be collected we had
to find a compromise between on the one hand the fluctuation of the drop
size and the accuracy of the weighing procedure, and on the other hand the
small drift in the drop size that occurred despite the measures described
above. A measuring accuracy of about 0.1 per cent of the drop weight could
be obtained by depositing in turn in the containers a and b two times fifty
drops.

Fig. 3.20 The apparatus for the measurement of the relative volume decrease,
the temperature increase and the velocity of departure of an
impinging drop.
Most measurements were done on drops with a temperature of 20°C. Originally, we chose such a low value because we were afraid that the evaporation which the drops in their trajectory through the air would be larger than the evaporation on the hot surface which we were looking for. It appeared however that the temperature increase of the liquid during the collision was quite substantial, so that the evaporation of the drops in their trajectory from the hot surface to the collecting container was of the same order of magnitude as that during the collision. Thus, the above described measuring method did not in fact answer the set purpose. However, during the impact of the cold drops some other phenomena appeared to occur which could also result in a weight reduction. In this way we could still obtain some very interesting information about the collision process. We will discuss this in this section together with the results from the measurement of respectively the temperature increase of the liquid during impact and the velocity with which the drop rebounds from the surface.

The temperature increase was measured with the apparatus called c in Fig. 3.20. A heating wire is wound around a brass rod. The current within the wire can be controlled by a variable resistance. A thermocouple and its cold junctions are connected in series with the compensation circuit d. The differential signal is amplified and then displayed on an oscilloscope. If the temperature of a drop that falls on the thermocouple is not equal to that of the brass rod, then we see on the oscilloscope a pulse-like signal. Since in a set condition of this apparatus the drops
that rebounce from the hot surface all have about the same temperature, the current in the heating element around the brass rod can be adjusted such that on the oscilloscope no longer a pulse is observed. Then the temperature of the rod is equal to that of the drop falling upon it. This temperature is finally measured with potentiometer e. In order to prevent the liquid from sticking to the end of the rod a little tube was mounted next to it to withdraw the liquid by capillary action. A detailed sketch of the apparatus is given in Fig. 3.21.

Fig. 3.21. The instrument for the measurement of the temperature of a rebounding drop.

The velocity with which the drops leave the surface was determined by measuring the distance x (See Fig. 3.22) that they jump away after their impact on the surface. Since we may ignore the resistance that the drops endure during their flight through the air, we may write
\[ x = 2H \sin \theta \cos \left( \sin^2 \theta - \frac{v_{en}}{v_{an}} \cos^2 \theta \right) \left( 1 + \frac{v_{en}}{v_{an}} \right) x \times \left\{ \sqrt{1 + \frac{y}{H} \left( \sin^2 \theta - \frac{v_{en}}{v_{an}} \cos^2 \theta \right)^2} - y \right\}. \] (3.18)

In this equation \( H \) is the height of the capillary above the hot surface, \( y \) the height from which the drop falls after the impact, and \( \theta \) the inclination angle of the hot surface. \( v_{an} \) and \( v_{en} \) are the components perpendicular to the surface of respectively the arrival and departure velocities of the drop. For the derivation of (3.18) it is assumed that, in correspondence to what was found in section 3.1, the velocity component parallel to the surface remains constant during the collision.

Fig. 3.22 The determination of the ratio between \( v_{en} \) and \( v_{an} \)

As during the investigations with the above described apparatus we had to look for a material of which the surface would not age during the collision of ten thousands of drops, measurements were carried out on
various kinds of metals. It will appear later on that this has given us some nice additional information about the impact process.

A representative picture is given by the results which are shown in Fig. 3.23. These were obtained with a 1 mm thick gold plate which was soldered on a thick brass plate. As well the relative volume decrease, as the temperature increase of the drop, and the quotient of the perpendicular velocity components were determined as a function of the surface temperature. Besides, for the same conditions, a number of photos were made by means of the method described in Section 3.1. Some of them are shown in Fig. 3.24.

One can divide the results into the four regions of the surface temperature:

\[ T_m < 210^\circ C \]. In this area we can distinguish a definite wetting. The liquid sticks at many places to the surface so that it is difficult for the vapor which is formed at the bottom to find a way out. Therefore, some of it breaks through the liquid, which separates the drop and creates a hissing sound. With the measurement of the relative evaporation only the largest pieces of the drop were collected. Thus, the value of \( \Delta V/V \) indicated in Fig. 3.23 represents not only the evaporation during the impact and thereafter, but also the volume of all small pieces of the drop which were not collected. The temperature of the large pieces is in general rather high. Often the temperature increase \( \Delta T \) is more than 40^\circ C.

Because the liquid sticks on many places to the surface, a large part of the original kinetic energy is dissipated, so that \( v_{en}/v_{an} \) is low. That there is really a direct contact between the liquid and the surface can also
be seen in the photos. When the largest part of the dispersed drop has left the surface, we can see that at some places there is still some liquid sticking to the metal. It is striking however that the time between the first contact of the drop with the surface and the departure of the largest pieces, just as in the completely spheroidal state, is about equal to the vibration period of the first mode of the drop (ca. $10^{-2}$ sec.).

If the temperature of the surface is lower than $170^\circ$C then all liquid will stick to it. Large vapor bubbles are formed within the liquid which break out after some time.

$210^\circ$C $< T_m < 250^\circ$C. With increasing surface temperature the average thickness of the vapor layer underneath the drop gradually increases, so that now all vapor formed underneath the drop can find a way out. The drop therefore stays together and the relative volume decrease obtains a very low value. As was mentioned in the beginning of this section, a part of the measured volume decrease has to be blamed on the evaporation of the drop during its trajectory from the hot surface to the collecting container. Therefore, the evaporation at the surface will at any rate be less than the measured volume decrease. The latter one is for $T_m = 240^\circ$C between 0.1 and 0.2 per cent. This is less than was found for droplets at the boiling temperature (see Fig. 3.17 to 3.19). The reason for this is that a large part of the heat which arrives at the drop bottom is transported into the drop for heating of the liquid. A similar situation was already met in Section 2.7, where the evaporation at the bottom of the sessile drops in an unsaturated environment turned out to be smaller than that in a saturated environment.
Fig. 3.23 Relative volume decrease, ratio between the normal velocity components and temperature increase of a water drop impinging upon polished gold.
Because of the fact that the liquid is no longer sticking to the surface the velocity with which the drop leaves the surface is now much higher than the one found at lower temperatures. During the collision one hardly hears any sound, either. From the photos it follows that the shape of the drop during the collision is rather regular and that its behavior is similar to that shown in Figs. 3.6 to 3.8 for much higher values of $T_m$.

$250^\circ \text{C} < T_m < 380^\circ \text{C}$. As soon as the temperature of the surface is higher than $250^\circ \text{C}$ a striking phenomenon appears. Some drops will produce during their collision a high ticking sound. At the same time around them at the surface a very fine mist is observed. The photos give a picture of what actually takes place. It turns out that immediately after the first contact with the hot surface in the ticking drops small vapor bubbles (diameter less than 100 microns) are formed. A circulation flow along the liquid surface which most likely is caused by the surface tension gradient, moves the small bubbles to the top of the drop. There, a part of them break through the drop surface while thereby long thin liquid jets are formed. Because of the fact that these jets are not stable (see Rayleigh (82)) they break apart and thus form the observed mist of very small drops (diameter about 50 microns).*

* The same phenomenon can be observed at the liquid surface of a carbonated drink. The loose breaking of the CO$_2$ bubbles from the liquid surface forms liquid drops that jump often up to 10 centimeters high.
This also accounts for the increase of $\Delta V/V$ shown in Fig. 3.23a. Only at about 300°C all drops show the phenomenon. Thus Fig. 3.23a gives in fact the region between 250°C and 300°C only the average of the volume loss of respectively ticking and non-ticking drops.
Fig. 3.2a The impact of a water drop at 20°C upon a hot polished gold surface (drop diameter $2.17 \times 10^{-3}$ m, $v_0 = 1.25$ m/s, $\epsilon = 30^\circ$, so that $We_{en} \approx 1$). In all four cases the pictures have been respectively taken 0.63, 1.88, 4.84 and 9.27 milliseconds after the first "contact" of the drop with the hot surface.
The temperature of ticking drops is after the impact higher than that of non-ticking drops. Apparently, the bubbles increase the heat transfer in the drop. This may be caused by their stirring action, but also by a process of evaporation and condensation (see E. G. Bankov (86)).

The velocity with which ticking drops leave the surface after the impact turns out to be smaller than that of non-ticking drops. Apparently, the presence of bubbles in the liquid results in the dissipation of a larger part of the original kinetic energy than normally is the case. Fig. 3.23b shows that drops of both kinds can exist together over a large temperature region. In that region the percentage of ticking drops increases with increasing temperature.

The cause of the ticking sound could be the result of a large number of pressure pulses which are produced during the very fast growth of the bubbles. It is however also possible that it is caused by the separation of the bubbles from the liquid surface.

The actual cause of the creation of bubbles will be discussed later on in this section after also the results obtained with other surfaces have been presented.

$T_m > 380^\circ C$. Here finally a somewhat more complete spheroidal condition appears. In the beginning there are still once in a while some bubbles in the liquid. Their number decreases however with the increase of the surface temperature. This goes together with the decrease of heating of the drops. The quotient of the normal components of the arrival and departure velocities approaches a value which corresponds with Fig. 3.3.
Figs. 3.25 to 3.28 show in principle the same behavior as the one described above. By comparing these figures one can however come to a better understanding about the observed phenomena.

Fig. 3.25 and Fig. 3.26 show the results which were obtained with a very smooth polished copper plate which was covered electrolytically with a thin layer of gold. After the first measurements, of which the results are given in Fig. 3.25, it turned out that the surface was fouled by oxidation of the copper that had diffused through the gold layer. It was next cleaned with a polishing compound which made it coarser than it originally was. Fig. 3.26 shows the results of the measurements which were done after that. By the increase of the coarseness of the surface the maximum in the curve at which the ticking drops appear has become substantially higher. With the second series of measurements it turned out that the surface was again fouled by the oxidation at high temperature. Now an even higher maximum was found.

Fig. 3.27 gives the results obtained with a 1 mm thick polished plate of platinum, alloyed with 10 per cent iridium and soldered upon a thick stainless steel plate. This material appeared to be hard enough to withstand the collision of ten thousands of drops so that now reproducible results could be obtained. One sees that the curves in Fig. 3.27 compared to those of Fig. 3.23 are moved over about 50°C to higher temperatures. They also have obtained a somewhat smoother shape. This has resulted in an overlapping of the first and third of the above described temperature regions for the two higher speeds. The ticking drops often broke up into a number of smaller drops.
copper with 10 μ gold (very smooth),
drop volume $V = 4.7 \times 10^{-9} \text{m}^3$,
impact velocity $V_i = 1.25 \text{ m/s}$,
drop temperature $T_{da} = 200^\circ\text{C}$,
inclination angle $\theta = 32^\circ$.

Fig. 3.25 Relative volume decrease with impact upon very smooth gold-plated copper.

copper with 10 μ gold
drop volume $V = 5.4 \times 10^{-9} \text{m}^3$,
impact velocity $V_i = 1.25 \text{ m/s}$,
drop temperature $T_{da} = 200^\circ\text{C}$,
inclination angle $\theta = 32^\circ$.

○ polished surface
○ surface covered with copper oxide

Fig. 3.26 Relative volume decrease with impact upon polished and corroded gold-plated copper.
The minimum that occurs in the heating curve in Fig. 3.27c is fairly strange.

Finally, in Fig. 3.28 the results are given of measurements for various values for the drop temperature $T_{\text{in}}$ before the impact. It is seen that for the highest values of $T_{\text{in}}$ the maximum in the curve for the relative volume decrease has disappeared. Most likely the maximum is completely lost in the rather large volume decrease which the drop experiences on its trajectory from the hot surface to the collecting container. Experiments during which the collecting container was located at various heights indicated that this volume decrease for $T_{\text{in}} = 100^\circ C$ was in the order of 1 per cent. The Figs. 3.17 to 3.19 show that probably also for higher values of $T_{\text{in}}$ a maximum appears. For a $T_{\text{m}}$ of about 290°C we do distinguish a diffuse maximum. It is however possible that this, due to the small number of bubbles that appears or due to the absence of a surface tension gradient, is not accompanied by an expulsion of little drops.

Also, Fig. 3.28b shows that for high $T_{\text{in}}$ there is still some bubble formation within the drop. The combination of a maximum and a minimum in the curve is only absent for drops at the boiling point. Also from the curves for the drop temperature increase in Fig. 2.28c it is seen that something special takes place for temperatures over 250°C.

From the above experimental results one does not get much quantitative information about the evaporation of cold drops during their impact upon a hot surface. The only conclusion that can be made with some certainty is that the comparison of Figs. 3.17 to 3.19 with Fig. 3.23 shows that for $T_{\text{m}} \leq 240^\circ C$ the evaporation of cold drops is smaller than that of drops at
the boiling point. This is after all in accordance with what we might expect.

If the hot surface is wetted by the liquid or if bubbles are formed within the drop then the evaporation will most likely be larger. Note that this already followed from the results in Section 3.3. How much this increase in evaporation is for cold drops, cannot be determined from the above given measuring results.

If the amount of heat which is used for the evaporation is compared with that which is used for the heating of the drop, then it turns out that the latter one is many times larger. From Fig. 3.23 for instance it follows that the amount of heat which is needed for the evaporation is in the order of 0.05 J, whereas the heat used for the heating of the drop lies between 0.5 and 1 J. The heat transfer coefficient which for this heating is needed at the bottom of the drop can with a certain approximation be calculated with the following heat balance,

\[ \rho_c c V \frac{d\langle T \rangle}{dt} = \sigma \frac{R^2}{\delta} (T_b - \langle T \rangle). \]  

(3.19)

Here \( \langle T \rangle \) is the average temperature within the drop, \( T_b \) the temperature of the bottom, and \( \overline{\delta R^2} \) the average surface area of the bottom during the collision. From Section 3.2 one finds that this surface area for the conditions of Fig. 3.23 is about 7x10^{-6} m^2. If one assumes that the temperature of the bottom of the drop is equal to 100°C then it appears from
(3.19) that $\alpha$ lies between 1 and $2.5 \times 10^5$ W/m$^2$°C. If, however, the temperature at the bottom would be equal to that of the hot surface, which during the largest part of the collision is definitely not the case, then we find for $\alpha$ values between 0.2 and $0.9 \times 10^5$ W/m$^2$°C. According to the theory of heat penetration in a solid medium the heat transfer coefficient could not be larger than $0.2 \times 10^5$ W/m$^2$°C. Apparently, one way or the other the heat transfer into the drop is substantially increased.

This large heat transfer could in the first place be caused by flow of liquid in the drop. One can distinguish two kinds of flow. On the one hand one has the flow which is inherent with the collision, and on the other hand the flow which is caused by the surface tension gradient due to the temperature differences in the drop. From the pictures one can however see that in both cases velocities of at most 1 m/s occur, while moreover these velocities do not have the same size and direction during the entire impact process. The explanation of for instance a five times larger heat transfer than is predicted by the penetration theory requires during the collision a twenty five fold liquid renewal, which corresponds with a constant circulation speed of 2.5 m/s.

Therefore, the most important cause for the large heat transfer is probably the appearance of boiling within the drop. We saw that in the first temperature region of Fig. 3.23 some kind of boiling did take place. Probably the vapor bubbles which are formed are for a larger part condensed after a short time. In this way the liquid can be heated up appreciably.
platinum - 10% iridium (smooth)
drop volume \( V = 5.1 \times 10^{-9} \text{m}^3 \)
drop temperature \( T_{\text{IA}} = 20^\circ \text{C} \).
inclination angle \( \theta = 32^\circ \).

- \( u = 0.99 \text{ m/s} \)
- \( u = 1.25 \text{ m/s} \)
- \( u = 1.47 \text{ m/s} \)

**Fig. 3.27** Relative volume decrease, ratio between the normal velocity components and temperature increase with impact upon polished platinum-iridium.
With increasing $T_m$ one notices first a strong decrease in the temperature drop. This is accompanied by the fact that the drop no longer is dispersed and with a sporadic appearance of bubbles in the drop during its departure from the surface. The value of $\alpha$ calculated with (3.19) is now for $T_{lc} = 100^\circ C$ equal to $10^5 W/m^2^0C$. If the liquid would touch the hot surface, so that $T_{lc} = T_m = 245^\circ C$, it would be about $0.3\times10^5 W/m^2^0C$. From this follows that probably also in the second temperature region vapor bubbles are formed within the drop. In the pictures we see however nothing of this. Yet in the beginning of the impact the drop is not very transparent because of disturbances on the surface. Therefore it is very well possible that during that time a small number of vapor bubbles develop which before the drop is clear again are already condensed.

We see in Fig. 3.23c that above $250^\circ C$ the temperature increase of the drop quite strongly increases. This goes together with the already mentioned explosion-like formation of vapor bubbles. A remarkable fact is here that this takes place for surface temperatures for which based on the results of the second temperature region we can expect that there is no good contact between the liquid and the hot metal.

It is however known that common nucleate boiling of a liquid at a hot wall occurs because of the presence of little cavities and scratches in the surface. These cavities and scratches contain gas enclosures which serve as a nucleus for the formation of vapor. Also spots where the liquid only poorly wets the hot surface can serve for this (See for instance Griffith and Wallis
(87)). If, however, these nuclei on the surface are absent, then it is possible to heat a thoroughly purified liquid up to temperatures far above the boiling point. Kendrick et al. (88) found that at atmospheric pressure water can be heated up to 280°C before it all of a sudden explodes. Something similar is not for cavitation of a liquid. Briggs (90) found that in the absence of external nuclei cavitation in water only would take place at a "pressure" of -281 atm. In both cases one has to do with a nucleus diameter of about 100 Å, which is about the size of dust particles which are probably always present in the liquid. Probably one has to do with a similar phenomenon during the sudden bubble formation at the beginning of the third temperature region. In order to verify this we will now calculate the real surface temperature during the collision. Note that on the average this will be somewhat lower than that before the collision, because during the stay of the drop on the surface a certain amount of heat will be taken away from it.
platinum - 10% iridium (smooth),
drop volume $V = 6.4 \times 10^{-9} m^3$,
inclination angle $\theta = 32^\circ$,
impact velocity $v_\text{impact} = 0.99 $ m/s,
drop temperature $T_\text{initial} [^\circ C]$.

- $\circ - 20$  
- $\bullet - 50$  
- $\ast - 90$  
- $\triangle - 100$

--- $T_\text{m} [^\circ C]$ ---

$\frac{\Delta V}{V}$  

$\circ - 20$  
$\bullet - 50$  
$\ast - 90$  
$\triangle - 100$

--- $T_\text{m} [^\circ C]$ ---

$\Delta T_\text{i}$  
$\circ - 20$  
$\bullet - 50$  
$\ast - 90$  
$\triangle - 100$

--- $T_\text{m} [^\circ C]$ ---

Fig. 3.28 Relative volume decrease, ratio between the normal velocity components and temperature increase with impact upon polished platinum-iridium.
Assume now that the effective surface area along which the metal transfers heat to the drop is equal to $\pi R_o^{\text{max}}^2$, in which $R_o^{\text{max}}$ represents the maximum value of $R_o$ during the impact, which is found from Fig. 3.12. Assume also that the heat flux through the surface is constant during the collision. If $Q$ is the total amount of heat which is transferred to the drop during the collision, then the average temperature decrease of the metal surface can be calculated from

$$T_m - T_e = \frac{4Q}{3\pi R_o^{\text{max}}^3 \sqrt{\frac{\pi}{100} c_m^3}} \quad (3.20)$$

From Fig. 3.23 it is seen that the drop temperature increase $\Delta T_e$ in the third temperature region for $v_o = 1.15$ m/s is equal to $47^\circ\text{C}$. The above given equation gives in this case an average surface temperature decrease of $20^\circ\text{C}$.

The transition to the region of ticking drops lies according to Fig. 3.23 at about $260^\circ\text{C}$. From this follows that here the average temperature of the surface is $240^\circ\text{C}$. The same can be done with the other measuring results. Table 3.2 gives a survey of this.

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_o$ [°C]</th>
<th>$v_o$ [m/s]</th>
<th>$\Delta T_e$ [°C]</th>
<th>$T_m$ [°C]</th>
<th>$T_e$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au</td>
<td>20</td>
<td>1.15</td>
<td>47</td>
<td>260</td>
<td>240</td>
</tr>
<tr>
<td>Au</td>
<td>20</td>
<td>1.31</td>
<td>45*</td>
<td>260</td>
<td>244</td>
</tr>
<tr>
<td>Cu + 10% Au</td>
<td>20</td>
<td>1.25</td>
<td>45*</td>
<td>260</td>
<td>238</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>20</td>
<td>0.99</td>
<td>63</td>
<td>320</td>
<td>228</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>20</td>
<td>1.25</td>
<td>61</td>
<td>305</td>
<td>241</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>20</td>
<td>1.47</td>
<td>58</td>
<td>295</td>
<td>243</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>20</td>
<td>0.99</td>
<td>60</td>
<td>335</td>
<td>247</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>53</td>
<td>0.99</td>
<td>44</td>
<td>291</td>
<td>245</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>60</td>
<td>0.99</td>
<td>22</td>
<td>280</td>
<td>247</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>80</td>
<td>0.99</td>
<td>9</td>
<td>269</td>
<td>256</td>
</tr>
<tr>
<td>Pt - 10% Ir</td>
<td>98</td>
<td>0.99</td>
<td>0.5</td>
<td>265</td>
<td>257</td>
</tr>
</tbody>
</table>

Table 3.2 Average temperature of the metal surface during the impact.
In the above given table always that temperature was chosen for $T_m$ where about 50 per cent of the drops showed bubble formation. For a number of cases this could easily be done with the aid of the curves for $\nu_{en}/\nu_{an}$. In the other cases an estimate had to be made with the aid of the curves for the relative volume decrease $\Delta V/V$.

It turns out that $T_m$ is always about 240°C. Apparently, there is a characteristic temperature for the explosive bubble formation. It would appear that we have to do with a kind of spontaneous nucleation. Probably the following takes place. As long as the slit thickness under the drop is not yet thick enough, roughness peaks of the metal surface pierce into the liquid during the impact. The metal surface will despite how well it is polished always show the characteristics of a mountainous terrain. On the metal peaks the liquid will be heated fast. Because there are no ordinary nuclei present as we have in regular boiling, the liquid has to be heated up to far above the boiling point in order to form vapor bubbles. From the above it follows that the minimum temperature for this is in the neighborhood of 240°C. It is now also clear that the coarseness of the surface plays such an important role as observed in Figs. 3.24 and 3.25. With a coarse surface more metal peaks will stick during a longer time into the liquid so that more bubbles will be formed. Because the contact between a given metal peak and a given packet of liquid is always short, the diameter of the bubbles is restricted. Especially in the beginning of the collision the majority will condense immediately. A relatively small number does not condense and is
transferred along the surface to the top by the circulation flow.

When the slit thickness under the drop increases the contact between the liquid and the metal surface becomes shorter. The bubble formation decreases therefore which results in a reduction of the heating of the drop. Probably, also for very high temperatures there is still some contact between the liquid and the hot surface. An indication for this is the very weak noise which even then can be heard during a collision.

One can in the same way as in the above also try to determine a criterion for the wetting of the surface by the liquid. With the aid of the results given in Fig. 3.28a one finds then that the requested "wetting temperature", which is defined as that temperature below which the volume decrease is larger than a certain value, increases with the bubble temperature. This is exactly the opposite of what one would expect at first glance. One has to consider however that the vapor bubbles which are formed in the liquid will grow faster as this is warmer. A drop with a higher temperature will therefore faster be dispersed. From this follows that the measurement of the relative volume decrease is actually not suitable for the search into a wetting criterion. It is after all most likely that the decrease of the wetting with increasing temperature is only gradual. Therefore, one can hardly speak of a real transition.
Chapter IV

HEAT TRANSFER TO A MIST OF VERY SMALL WATER DROPS

Finally, we will investigate how large the heat transfer is if very small drops impinge upon a hot surface. We will consider drops with a diameter of 20 to 200 microns, like one finds in a mist.

It will be clear that the experimental determination of the evaporation of these cannot be done for individual drops. Therefore, a mist of water droplets in steam was made with the aid of a nozzle. A thin metal plate was heated to a high temperature and then suddenly exposed to the mist flow. By measuring the cooling rate of the plate the percentage of liquid that evaporated during the collision on it was obtained. The advantage of this non-stationary method is that in a simple and fast way a large amount of information about the heat transport can be obtained.

Sect. 4.1 Calculation of the heat transport to impinging mist drops of 100°C in a saturated environment.

In Section 3.2 equation (3.14) was derived for the evaporation rate at the bottom of an impinging drop. If the gravitational acceleration is supposed to be small in comparison with the acceleration of the drop because of its impact on the hot surface, then integration of the equation
over the residence time $\tau$ gives for the relative weight loss by evaporation,

$$\Delta W / W = A \left( \frac{2 \pi \rho \lambda^3}{5 \rho_0 H} \right)^{4/3} V^{2/3} (T_u - T_v)^2 \int_0^r R_r \left( \frac{\partial r}{\partial t} \right)^2 dr.$$ (4.1)

The factor $A$ in this equation pertains to the evaporation at the side of the drop.

In the experiments which will be discussed in the Sections 4.2 and 4.3 the diameter of the drops was about $6 \times 10^{-5}$ m, while their velocity was about $5$ m/s. The order of magnitude of the inertia forces in the droplets ($\approx \rho f v_{an}^2 R^2$) was therefore $2 \times 10^{-5}$ N, while the friction forces ($\approx \eta f v_{an} R$) and the surface tension forces ($\approx \sigma R$) were respectively $4 \times 10^{-8}$ N and $2 \times 10^{-6}$ N. Apparently, also in this case the friction forces may be ignored so that the Weber number characterizes the behavior of the drop and the results of Sect. 3.2 may be used.

We know that for equal values of $\text{We}_{an}$ the behavior of the drops on the hot surface is geometrically similar. From this follows that for constant $\text{We}_{an}$ the radius of the bottom, $R_0$, is proportional to the radius $R$ of the drop. If besides $\text{We}_{an}$ also $\sigma$ and $\rho_f$ are kept constant, then $v_{an}$ is proportional to $R^{-1/2}$. Note that this is true for all velocities in the drop during the collision. Because of the fact that now all dimensions of the drop are proportional to the radius $R$ and all velocities to $R^{-1/2}$, the time measure during the collision is proportional to $R^{3/2}$. This can also be found from the expression for the vibration
period of the drops,

\[ \tau = \frac{\pi}{4} \frac{p_i R^3}{2 \sigma} \]  \hspace{1cm} (4.2)

to which the residence time of a drop on the surface is about equal (see Sect. 3.1).

The above argument can be applied to the equation for the relative volume loss, (4.1). Factor \( A \) does not change because of the geometric similarity of the collision process for equal \( \text{We}_\text{an} \). For equal \( \text{We}_\text{an} \) is thus

\[ \frac{\Delta V}{V} \sim R^{-1}. \] \hspace{1cm} (4.3)

Apparently, the relative volume loss changes only slightly with a variation in the radius \( R \).

In the preceding chapter drops were investigated which had a diameter of about \( 2.3 \times 10^{-3} \) m. In the mist that was investigated the average drop diameter was however \( 6 \times 10^{-5} \) m. The ratio of the diameters is thus 1:38 so that according to (4.3) the relative volume loss of small drops is about 2.5 times as large as that of drops of \( 2.3 \times 10^{-3} \) m with the same Weber number. The theoretically calculated evaporation at the bottom of these larger drops was given in Fig. 3.14.
Multiplication of this with the factor 2.5 gives the evaporation at the bottom of the small drops. In Figs. 4.3 and 4.4 the thus calculated evaporation at the bottom and the \( \frac{\Delta V_{tp}}{\Delta V_{op}} = 1.5 \) estimated total volume decreases are compared with the measured evaporation.

In the preceding chapter equation (3.13) was derived for the slit thickness. If here also the gravitational acceleration is supposed to be relatively small, then one obtains

\[
d_p = \left\{ \frac{3\pi \eta \Delta v (T_m - T_s) (T_m + T_s)^2}{8 \rho \rho_1 \Delta H \Delta t} \right\} \frac{1}{2} R_p V^{-\frac{1}{2}} \left( -\frac{dr}{dt} \right)^{-\frac{1}{2}}.
\]

In the same way as for (4.1) one finds now that for constant \( W_e \),

\[
d_p \sim R^2.
\]

In Chapter III for drops with a diameter of \( 2.3 \times 10^{-3} \) m a slit thickness in the order of \( 10^{-5} \) m was found. Thus, for drops of \( 6 \times 10^{-5} \) m the slit thickness would be about \( 10^{-6} \) m. The roughness of polished metal surfaces lies between \( 10^{-6} \) and \( 10^{-7} \) m. This is thus of the same order of magnitude as the calculated slit thickness. The chance is therefore great that during the collision there is contact between the liquid and the hot surface. In Sect. 4.3 we will discuss this more extensively.

In the above it was assumed that the temperature of the surface during the collision is constant. In Sect. 3.2 we derived for the maximum
temperature decrease of the surface during the collision the following equation.

\[ \Delta T_m = 20 \tau \left( \frac{\tau}{\alpha \rho_0 c_m} \right)^{3/2} \left( \frac{\tau}{\alpha \rho_0 c_m} \right)^{-3/4} \]  \hspace{1cm} (4.6)

According to (4.2) and (4.5) \( \tau \) and \( \Phi_w \) are here proportional to respectively \( R^{3/2} \) and \( R^{-3/4} \). If we fill this in in (4.6), then it appears that \( \Delta T_m \) does not depend on the size of the drop. From this follows that for drops of different dimensions but with the same Weber number the calculated values of Table 3.1 are true, provided that the drops show the fully developed spheroidal state.

In the same way as in Sect. 2.4 one can prove that the assumptions of Sect. 2.3 are also valid for very small impinging droplets. The heat radiation is now also for non-polished surface completely ignorable because of the very short time that the drops spend on the surface.

Sect. 4.2 Experimental apparatus and way of measuring

The experimental setup is shown in Fig. 4.1. A circular metal test plate with a diameter of 20 mm and a thickness of 1 mm is mounted on the point of a thin needle in the middle of a wide glass pipe. The length of the pipe is about 1.5 m and the diameter 80 mm. The

* The measurements were done by J. R. Vermeulen and R. Smulders.
temperature of the thin plate is measured by a thermocouple which is soldered at its back side. The voltage of this thermocouple is registered by a recorder. With the aid of a nozzle of the type used by Alexander and Coldren [91] a fine mist of water and steam is blown into the pipe. In order to prevent any scale formation on the test plate distilled water was used in the experiments.

At the other end a long metal tube is inserted into the pipe onto which an electric heating element is mounted. The tube can be inserted so far into the glass pipe that the heating element makes contact with the back side of the test plate.

The measurements are carried out as follows. First the test plate is heated up to a temperature of about 500°C with the aid of the heating element. Next the steam and water supply are opened so that a mist is formed in the pipe. At the outset the mist production is still somewhat irregular, but after a short period it becomes stationary. Next the heating element is taken away. Now the test plate cools down because heat is transferred to the mist flow. The cooling curve is recorded on the recorder. It is obvious that because of the heat transfer the temperatures of the front and the back side of the plate are not the same. The temperature difference is however small since the plate is rather thin. One can easily prove (see for instance Carslaw and Jaeger [92]) that for the highest measured heat flux \( 3.8 \times 10^4 \text{ W/m}^2 \) and the poorest
conducting metal (heat conductivity of platinum -10 per cent iridium $\approx 30 \text{ W/m}^0\text{C}$). The temperature difference is not larger than $0.6^0\text{C}$.

Fig. 4.1 The apparatus for the measurement of the heat transfer to a mist flow.

Because of the fact that both the temperature and the time derivative of the temperature can be determined from the cooling curve, the heat flow from the test plate can be found as a function of the test plate temperature with the heat balance,

$$\rho_m c_m \dot{V} \frac{dT_m}{dt} = \Phi_w \quad (4.7)$$

This heat flow can be separated in two pieces, one part used for heating
the steam that flows along the plate and one part for the partial evaporation of the drops that impinge upon the plate. It will appear that in our experiments both the liquid concentration in the mist flow and the coverage of the surface with drops were very small. The two heat flow components may therefore be assumed as independent of each other. If one further assumes that the flow pattern of the steam is the same with and without the mist drops then one can calculate the heat transport from the plate to the colliding drops with

\[ \phi_{\text{w}} = \rho_{\text{w}} c_{\text{w}} V_{\text{m}} \left( \frac{dT_{\text{m}}}{dt} - \frac{dT_{\text{w}}}{dt} \right) \]  

(4.8)

Herein \( dT_{\text{m}} / dt \) and \( dT_{\text{m}} / dt \) are the slopes of cooling curves obtained with flows of saturated steam respectively with and without drops.

In order to be able to determine the relative evaporation of the impinging drops we still have to know the amount of liquid which per given unit of time is deposited on the plate. Therefore, instead of the plate a thin-walled nozzle was installed in the glass tube with the same diameter as the plate. The mist caught by the nozzle was condensed and the liquid was collected in a measuring glass. The water collected in this way thus originated partly from the mist drops and partly from condensed steam. By adding to the feeding water of the nozzle a little bit of an organic salt (in this case we used nickel sulphate because this could also be used for the determination of the drop size distribution) and by titrating both the
feeding water and the collected water the part in the latter that originated from mist droplets could be determined.

In the above described way we determined in fact the liquid flow $\Phi_v$ which would be deposited on the plate if this would not influence the flow pattern in the tube. In reality especially very small drops are bent away and dragged along by the vapor flow without touching the surface. Geist, York and Brown [93] calculated for different drop diameters and velocities how much liquid an oblong 2.5 cm wide plate would collect. We have used their results in the determination of the amount of liquid that impinged upon our test plate. The error made in this way is at the most 5 per cent.*

If the collecting efficiency of the plate is called $E$ then the relative volume loss of the impinging droplets by evaporation is given by

$$\frac{AV}{V} = \frac{\rho_v c_v V_{ei}}{\rho_i \Delta H L \Phi_v} \left\{ \frac{dT_m}{dt} - \left( \frac{dT_m}{dt} \right)_l \right\}.$$

One may expect that every drop which has once impinged upon the plate is dragged along by the vapor flow so that it does not collide a second time with the surface. The value of the relative evaporation given by (4.9) is thus equal to the relative evaporation per impact.

In the above it is assumed that no drops impinge upon the back side of the plate. One could imagine that by the turbulence behind the plate some drops might eventually end up on the back side. Calculation of

* In a later stage of the investigations it turned out that the estimate made in this way corresponded very closely with the results of measurements of a downward mist stream in a vertical pipe. In this experiment the collecting nozzle was respectively filled with liquid or empty so that the steam was respectively bent or not bent away by the nozzle.
the forces that may work on a single drop shows however that the chance that this will happen with drops of about 60 microns in 1 atm steam is fairly slim.

The following method was applied for the determination of the drop size distribution in the mist. A microscope slide was covered with cellophane and then dipped into a solution of dimethylglyoxime in ethylalcohol. After the alcohol had evaporated the surface was wiped with tissue in order to remove possible crystals left on the tape. Next the plate was rapidly moved through the mist flow at the location of the test plate. Because a few weight percents of nickel sulphate were added to the feeding water small red dots appeared on the tape. The diameter of these dots was determined under a microscope.

The relationship between the diameter of the dots and the real diameter of the drops was found in the following way*. A very fine water jet was squirted from an about 10 cm long and very thin glass capillary of which the end was bent at a right angle. A piece of iron was attached to the capillary so that this could be brought into vibration electromagnetically. If the capillary vibrated in one of its characteristic modes then a number of very fine jets were formed, each consisting of very small drops (30 to 200 microns) of a very constant size. After collection of the drops of one of these jets in an oil film on a microscope slide their diameter could be determined under a microscope.

* These measurements were carried out by H. C. Kleiweg.
The diameter of the spots formed by the same drops on the cellophane tape was compared with these. The latter one always turned out to be exactly twice as large as the real diameter of the drops.

The drop size distribution found in the above described way turned out to be dependent upon the water flow rate so that in this way with the heat transfer measurements the average drop size could somewhat be varied. For not too large water flow rates the standard deviation of the drop diameter was about 20 per cent.

The drop velocity was determined by taking a picture of the mist flow using an open camera and two separate spark discharges which followed each other very closely. In this way each droplet in the mist was represented twice on one photographic plate. Because the short time between the light flashes was known, the measurement of the distance between the two representations gave us the velocity of a specific drop. The velocity distribution determined in this way turned out to have a rather large divergence. (Standard deviation about 50 per cent) Just like the average diameter the average velocity showed a small increase with increasing water flow rate.

Sect. 4.3 Measuring results and discussion

For all measurements the steam mass flow rate was $4.8 \times 10^{-3}$ kg/s.

If no condensation would take place on the glass wall of the tube then
the average steam velocity in the pipe would be about 1.8 m/s. The water flow rate was varied between 0.4 and $2.0 \times 10^{-6}$ m$^3$/s. Hence, the amount of liquid which per unit of time would impinge upon the plate, $\phi_{vl}$, turned out to vary between 1.3 and $8.8 \times 10^{-8}$ m$^3$/s. At the same time the average drop diameter varied between 50 and 75 microns, and the average drop velocity between 4.5 and 6 m/s. Hence the collecting efficiency $E$ varied between 0.80 and 0.89.

Because of the fact that according to (4.2) a droplet of 60 microns stays only for $5.10^{-5}$ sec. on the surface the percentage of the surface of the plate that is covered with drops was for the largest liquid flow rate still only 0.03 per cent. From this follows that the chance of interference between the droplets on the surface was small.

Measurements were done on test plates of gold, platinum, stainless steel and copper. Two cooling curves are shown as an example in Fig. 4.2. It is seen that from $T_m = 200^\circ$C the curve relating to the mist flow starts to diverge from the one for dry steam. One has to consider however that the difference between the curves is cumulative.
Fig. 4.2 Some examples of cooling curves

The final results are given in Figs. 4.3 and 4.4. Here the relative evaporation of the liquid which impinges upon the plate is plotted against the temperature \( T_m \). The measurements which were done for various water flow rates turned out to give always about the same curves for \( \Delta V/V \). The differences between the curves were never larger than the possible error (about 20 per cent).

In the figures also the theoretically calculated evaporation at the flat assumed drop bottom and the estimated total evaporation (with \( \Delta V_{zp}/\Delta V_{op} = 0.5 \)) are given for a fully developed spheroidal state.
Fig. 4.3 Relative volume decrease of droplets impinging upon polished gold ($W_{\text{an}} = 25$).

Fig. 4.4 The influence of roughness and corrosion ($W_{\text{an}} = 25$).
One sees in Fig. 4.3 that probably the spheroidal state, just like for large drops, (see Fig. 3.17) only takes place above $400^\circ$C. At first glance this appears to be strange. One would expect that because of the much thinner vapor slit underneath the little drops there would be more contact during the impact between the liquid and the metal surface so that for $400^\circ$C still a large difference between the measured and the theoretically calculated evaporation would exist. One has to consider however that small drops are much stiffer. That is to say that they show a much larger resistance against a change in their shape than larger drops. If the size of the ripples at the bottom of the drop would only depend upon the Weber number associated with the perpendicular component of the impact velocity then for a constant Weber number the size of the ripples would be proportional to the radius $R$ of the drop.

In Par. 4.1, however, we have seen that the slit thickness $d$ was $^{3/4}$ proportional to $R^{3/4}$. Thus, with increasing radius $R$ the height of the ripples on the bottom of the drop would diminish faster than the slit thickness. Therefore, the chance of direct contact between the liquid and the hot surface and hence also the ratio between the measured and theoretically predicted evaporation is smaller for little drops. This is however only true for a completely smooth hot surface. In reality even after very careful polishing some small inequalities will be left. Therefore, it is understandable that the Figs. 3.17 and 4.4 show about
the same ratio between experimental and theoretical results.

If the surface is not very smoothly polished then the difference between the measured and theoretically calculated evaporation is appreciably larger as is seen quite clearly from Fig. 4.4. One also sees that for stainless steel and platinum the transition from perfect wetting of the surface to the fully-developed spheroidal state is more gradual than for gold. This is caused by the fact that the product $\lambda \rho c_m$ is lower so that the metal surface undergoes about a three times larger temperature decrease during the impact.

The curve found for copper in Fig. 4.4 lies much higher than the other ones. This is caused by the oxide layer which was formed upon the plate. Due to the very low values of $\lambda \rho c_m$ the surface locally cools down appreciably during the impact, so that the chance of contact between liquid and hot wall increases during the stay of the drop on the surface. Therefore, the entire curve, which without the oxide skin would lie at the same level as the curve for gold, has moved up to higher temperatures.

Completely analogous results were obtained by Dougherty [94] with transition boiling of methyl alcohol on a horizontal, steam-heated copper pipe (see Fig. 4.5). Also in this case one sees a movement of the curve up to higher temperatures as the surface becomes rougher or dirtier. The only difference between both cases is that with impinging
Fig. 4.5 The influence of the nature of the surface upon transition boiling of methanol

drops in the beginning of each impact the liquid may touch the hot surface even at very high values of $T_m$, whereas with boiling of a liquid at a hot wall direct contact between the liquid and the wall already ceases at comparatively low temperatures. From there on the heat transfer again increases with increasing temperature.

Par. 4.4 Conclusion

We saw that mist drops of about 60 microns impinging upon a not very smooth hot surface with a velocity of about 5 m/s
lose about one percent of their volume. This means that during a
time interval of \(5 \times 10^{-5}\) s about \(2.5 \times 10^{-6}\) J are withdrawn from the
metal. If the temperature of the surface is \(300^\circ C\) then it follows
that the heat transfer coefficient during the stay of the drop on the
surface is about \(10^5\) W/m\(^2\) °C. In spite of this high heat transfer
coefficient is the cooling of the hot wall by the drops rather small.
The reason is that their residence time on the surface is so extremely
short. Hence, the cause of the poor heat transfer from a hot wall to
a flow of mist drops (see e.g. Parker and Grosh [95]) is in fact not
the vapor layer which forms between the liquid and the wall but the
elasticity of the drops.

If with constant \(W\), the diameter of the drops increases
then both the residence time and the "contact" surface area increase.
From equation (4.3) follows that this results in an increase of the
amount of heat which is withdrawn from the hot wall by each drop,
but in a decrease in the amount of heat which is withdrawn from the hot
wall to per unit of volume of impinging liquid. At the same time the
vapor slit thickness increases so that at least for a not completely
smooth surface the chance of direct contact between the liquid and the
wall and hence also the chance of an increase in the heat transfer by
this contact decreases. Therefore, if per unit of time a given amount
of liquid impinges upon a hot wall, then for equal \(W\) the cooling
with small drops is better than that with large drops. How the situation
is with equal $v_{an}$ can be found with the aid of Fig. 3.14 and equation (4.3).

What takes place at a higher pressure and thus at a higher value of $T_s$ can easily be found from (4.1). It is easily seen that if the pressure is increased from 1 to 100 atmospheres and the diameter and the velocity of the water drops is kept constant the factor in (4.1) containing the material constants becomes about three times as large. Also the Weber number increases with a factor of 3, so that $R_o$ somewhat increases. At the same time $\tau$ increases with a factor of 1.75. It follows then that when $(T_m - T_s)$ is kept constant the relative volume loss increases with a factor of 5. The slit thickness stays about the same. It is seen that for high pressures the heat transfer does increase but is still not very large. Summarizing the results one may say that in general

1. the relative evaporation of a water drop that impinges upon a hot wall is for the fully-developed spheroidal state not larger than a few tenths of a percent;

2. there is a transition region between complete wetting of the surface and the fully-developed spheroidal state; in this transition region the relative evaporation decreases when the wall temperature increases;

3. the limit for the appearance of the fully-developed spheroidal state can be moved up to higher wall temperatures by
a. increase of the drop velocity, b. increase of the roughness of the wall, and c. the existence of a poorly conducting layer on the wall.

In Chapter I we saw that when in a spray-cooled nuclear reactor the continuous liquid film on the wall of the cooling channels was destroyed, at a sufficiently high heat flux the temperature of the wall could rise to such a level that the drops impinging upon the wall would take on the spheroidal state. It turned out that the question concerning the cooling that could be obtained by drops under this condition had to be separated in two parts, namely the question of how many drops per given unit of time and per given unit of surface area are deposited upon the hot wall, and next the question concerning the amount of liquid which evaporates per drop per collision. The research work described in this thesis only answers the latter question.

However, based on results in the literature (see e.g. Parker and Grosh [95]) from which it appears that the transport coefficient for drops from a mist flow to the wall of a round pipe for $Re \approx 10^5$ is in the order of 0.1 m/s, an estimate can be made of the cooling which is obtained by the impacts of the mist drops upon the wall. It then turns out that for values of $(T_m - T_s)$ larger than 100°C the cooling by the drops is negligible with respect to the convective heat transfer by the vapor.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>thermal diffusivity</td>
<td>m²/s</td>
</tr>
<tr>
<td>A</td>
<td>constant associated with the evaporation at the sides of the drop</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>specific heat</td>
<td>J/kg °C</td>
</tr>
<tr>
<td>C</td>
<td>vapour concentration</td>
<td>kmol/m³</td>
</tr>
<tr>
<td>d</td>
<td>thickness of the vapour cushion under the drop</td>
<td>m</td>
</tr>
<tr>
<td>D</td>
<td>molecular diffusivity</td>
<td>m²/s</td>
</tr>
<tr>
<td>E</td>
<td>efficiency for capture of droplets</td>
<td>-</td>
</tr>
<tr>
<td>f</td>
<td>frequency of first order vibration of a drop</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>h</td>
<td>height of the drop</td>
<td>m</td>
</tr>
<tr>
<td>h₂₀</td>
<td>position of the centre of gravity of the drop</td>
<td>m</td>
</tr>
<tr>
<td>H</td>
<td>see fig. 3.22</td>
<td>m</td>
</tr>
<tr>
<td>AH</td>
<td>heat of evaporation</td>
<td>J/kg</td>
</tr>
<tr>
<td>l</td>
<td>lifetime of a sessile drop</td>
<td>s</td>
</tr>
<tr>
<td>L</td>
<td>characteristic length</td>
<td>m</td>
</tr>
<tr>
<td>L</td>
<td>pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>p</td>
<td>amount of heat</td>
<td>J</td>
</tr>
<tr>
<td>r</td>
<td>radial coordinate</td>
<td>m</td>
</tr>
<tr>
<td>R</td>
<td>radius of a spherical drop</td>
<td>m</td>
</tr>
<tr>
<td>Rₘₐₓ</td>
<td>maximum radius of a sessile drop</td>
<td>m</td>
</tr>
<tr>
<td>R₀</td>
<td>radius of the bottom of the drop</td>
<td>m</td>
</tr>
<tr>
<td>R₀ₘₐₓ</td>
<td>maximum value of R₀ during impact</td>
<td>m</td>
</tr>
<tr>
<td>s</td>
<td>radial distance to the drop surface</td>
<td>m</td>
</tr>
</tbody>
</table>
t  time  s

t_c  time constant  s

T  temperature  °C or °K

T_l  temperature of the drop  °C

T_m  temperature of the hot surface  °C or °K

T_s  saturation temperature  °C or °K

ΔT_l  temperature increase of the drop during impact  °C

ΔT_m  temperature decrease of the hot surface during impact  °C

u  radial vapour velocity  m/s

v  velocity  m/s

v_o  vertical vapour velocity at the bottom of the drop  m/s

V  volume of the drop  m^3

ΔV  volume decrease of the drop during impact  m^3

V_m  volume of a small metal disc  m^3

x  see fig. 3.22  m

y  see fig. 3.22  m

z  vertical coordinate  m

α  heat-transfer coefficient  W/m^2 °C

ε  angle of inclination of the hot surface  degrees

η  dynamic viscosity  Ns/m^2

ζ = z/L  --

ζ = h/L  --

ψ  angle  radians

λ  thermal conductivity  W/m °C

ε  density  kg/m^3
\( c = r/L \)
\( \bar{c} = R_0/L \)
\( c_{\text{max}} = R_{\text{max}}/L \)

- \( \sigma \) surface tension \( \text{N/m} \)
- \( \tau \) period of the first order vibration of a drop \( \text{s} \)
- \( \tau_{\text{gem}} \) measured residence time of a drop on the hot surface \( \text{s} \)
- \( \Phi \) angle \( \text{radians} \)
- \( \Phi = \frac{v}{L}^3 \)
- \( \Phi_{\text{mol}} \) molar flow rate \( \text{kmol/s} \)
- \( \Phi_v \) volumetric liquid flow rate \( \text{m}^3/\text{s} \)
- \( \Phi_{vl} \) volumetric flow of liquid which would impinge upon the metal disc were the droplets not deflected \( \text{m}^3/\text{s} \)

- \( \Phi_w \) heat flow rate \( \text{W} \)
- \( \Phi_w'' \) heat flux \( \text{W/m}^2 \)
- \( \Phi_{wl} \) heat flow only for the evaporation of droplets \( \text{W} \)
- \( \Psi \) velocity potential \( \text{m}^2/\text{s} \)
- \( \text{Re} \) Reynolds number
- \( \text{We} \) Weber number

**SUBSCRIPTS**
- \( a \) at the start of the impact
- \( b \) at the top of the drop
- \( d \) in dry atmosphere
- \( e \) at the completion of the impact
- \( g \) of vapour or gas
- \( l \) of the liquid
- \( m \) of the hot surface
- \( n \) normal to the hot surface
at the bottom of the drop, or at \( r = R_0 \)

for a flat-bottomed drop

in radial direction

at the boiling point of the liquid

total (unless indicated otherwise)

at the sides of the drop, or of the centre of gravity of the drop

in tangential direction

OTHER SYMBOLS

\( \bar{\cdot} \) time average

\( \langle \cdot \rangle \) cross sectional or volumetric average
1. H. Boerhaave  

2. J. Th. Ellison  
   Mem. Acad. Berlin (1766) 43.

3. J. G. Leidenfrost  
   De aquae communis nonnullis qualitatis us tractatus, Duisburgi ad Rhenum (1756) Pars II, 30-63.

4. Ziegler  
   De digestorii Poplini (1769).

5. J. H. Lambert  
   Pyrometrie (1779) 130.

6. Kloprath  

7. Lichtenberg  

8. T. Meyer  

9. B. de Rumford  
   Philos. Trans. (1804) 23.

10. H. C. Oersted  
    Gehrten N. Journ. d. Phys. 3 (1807) 323.

11. Belli  

12. Bellani  
    Giornale di fisica di Pavia (1816) 255.

13. J. W. Döbereiner  

14. Hazard  
    Sillim. Journ. 13 (1827) 58.

15. Pouillet  

16. M. Dumas  
    Traité de chimie appliqué aux arts, t. I, Paris (1828) 32.

17. Perkins  

18. M. Faraday  

19. G. W. Mencke  

20. N. W. Fischer  

21. Lechevalier  

22. R. Johnson  

23. M. L. Frankenheim  
    Die Lehre von der Cohäsion, Breslau (1835) 123-134.

24. R. Böttger  

25. A. Baudrimont  

26. A. Laurent  

27. C. Thomson  
    The student's manual of natural philosophy (1838) 553.
32. Person Compt. rend. 15 (1842) 492.
33. Le Grand, de Kramer, Belli Giornale dell’ Instituto Lombardo (1844) 192.
34. Peltier Archives supplément à la Bibliothèque Universelle de Genève (1844).
35. A. Boucharlat Physique élémentaire avec principales applications, Paris (1845).
41. H. Buff Arch. Ph. Nat. 31, 75.
44. Church Phil. Mag. 10 (1855) 350-353.
45. J. Tyndall Phil. Mag. 10 (1855) 353-354.
50. G. van der Mensbrugge Phil. Mag. 4 (1876) 40.
57. W. Świętałowski, S. Bąkowiński, C. Karpinski
   Przemysł Chem. 14 (1930) 518-529.

58. T. B. Drew, A. C. Mueller
   Trans. AIChE 33 (1937) 649.

59. S. Th. Holst Weber

60. N. Frechling
   Götlands Bölt. Geophys. 52 (1938) 170.

61. N. A. Platensiew, P. A. Rehinder

62. V. M. Borishansky, S. S. Kutateladze
   Zhurnal Tekhnicheskoy fiziki 17 (1947) 891-903.

63. C. W. Gorton
   Heat transfer to drops of liquid in the spheroidal state, Ph. D.
   Thesis, Purdue Univ. (1953).

64. V. M. Borishansky
   Heat transfer to a liquid freely flowing over a surface heated
to a temperature above the boiling point. U.S. AEC transl. ser.,
   AEC-tr-3405, (1953).

65. B. S. Gottfried, K. J. Bell
   The evolution of small drops on a flat plate in the film
   boiling regime, paper pres. at 5th ann. meeting, N.Y. AIEE,
   dec. 1961, prep. nr. 52.

66. B. S. Gottfried
   The evolution of small drops on a flat plate in the film
   boiling regime, Ph. D. Thesis, Case Inst. Techn., Cleveland
   (June 1962).

67. J. Kistemaker
   Physica 29 (1963) 96-104.

68. P. Savic, G. J. Boult
   The fluid flow associated with the impact of liquid drops with

69. P. Savic
   The cooling of a hot surface by drops boiling in contact with it,

70. J. Meurer

71. W. T. Lyn
   Study of burning rate and nature of combustion in Diesel
   engines, Paper pres. at 9th Internat. Symp. on Combustion,

72. J. K. Adavadjov, O. A. Uyehara, P. S. Myers
   Droplet vaporization under pressure on a hot surface, SAE
   rep. nr. 701 B (June 1963).

73. J. D. Thornton

74. H. Bousac

75. A. Heydweiler

76. H. Stoeveloog
   Ann. Phys. Chem. 61 (1897) 255-266.

77. F. Bashforth, H. Adams
   An attempt to test capillary action by forms of drops of fluid,
   Cambridge (1883).

78. H. Buchholz

79. J. D. Jackson

80. W. H. McAdams

81. H. Lamb
   Hydrodynamics, 6th ed. (1932) 473.

82. Lord Rayleigh

83. K. V. Derjagin, P. S. Prokhorov
   New studies in the field of aerosols; Acad. o. Sci., Inst. Phys.
   Chem., USSR (1949) 84-101, U.S. AEC transl., rep. nr. UCRL-
   trans - 601.

84. H. Lamb
   Hydrodynamics, 6th ed. (1932) 461.

85. J. de Jong, L. Marquenna

86. S. G. Bankov

87. P. Griffith, J. D. Wellis

118
   K. L. Winsor

89. J. W. Westwater

Boiling of liquids, Advances in Chem. Eng., ed. T.B. Drew,

90. L. J. Briggs

Science 112 (1950) 427.
Science 113 (1951) 483, (zie ook 89).

91. L. G. Alexander,
   C. L. Coldren

Ind. Eng. Chem. 43 (1951) 325.

92. H. S. Carslaw, J. C. Jaeger


93. J. M. Geist, J. L. York,
   G. G. Brown

Ind. Eng. Chem. 43 (1951) 1372.

94. E. L. Dougherty


95. J. D. Parker, R. J. Grosh

Heat transfer to a mistflow U.S. AEC rep. nr. ANL-6291
(1951).