SKYLLA: Wave motion in and on coastal structures

Implementation and verification of modified boundaries

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H.A.H. Petit, P. van den Bosch and M.R.A. van Gent

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<td>Wave motion on coastal structures can be simulated with the numerical program SKYLLA. The weakly reflecting boundaries, the incident waves and the free surface have been improved and verified. These improvements make the model more accurate and applicable on more types of coastal structures.</td>
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1 Introduction

1.1 Framework for the development of the model SKYLLA

The development of the numerical model SKYLLA started within the framework of the European research project "MAST-G6 Coastal Structures". The objective was to develop a physics-based numerical formulation for water motion on a smooth slope and also on-and-in permeable structures. This formulation led to the development of the numerical model SKYLLA\(^1\). Due to the inspiring results obtained with this European project, The Road and Hydraulic Engineering Division (Rijkswaterstaat) of the Dutch Ministry of Transport and Public Works were prepared to lead the continuation of the development (see contract DWW 743). This further development, outside the European MAST-project, started in 1993. The research is conducted in five phases:

1) Boundary conditions.
2) Rubble mound structures and overtopping.
3) Proposal for treatment of turbulence and air-entrapment.
4) Implementation of modifications of the treatment of roughness, turbulence and air-entrapment.
5) Optimization of efficiency for computations with overtopping.

These phases are described in detail in Klein Breteler and Petit (1993). This report describes the research for phase 1.

1.2 Considerations for the development of SKYLLA

Numerous coastal structures are studied using small-scale physical models. Physical modelling can be influenced by scale effects. Due to scale effects, various phenomena can be different under prototype conditions than under conditions present in small-scale physical models. In physical models variations in the lay-out of structures are often relatively laborious compared to numerical models. These problems, as well as the complexity of measurements in breaking waves, can be overcome by numerical modelling of the breaking waves on-and-in coastal structures. So, on the one hand the development of a numerical model as a research tool is very important. On the other hand, there are disadvantages such as the simplification and discretisation of the involved physical processes.

Existing one-dimensional models use simplified formulations of, for instance, the free surface. For many applications these simplifications are undesirable. The development of a three-dimensional model, able to simulate the complete breaking of a wave, within the near future seems unrealistic. Therefore, it was decided to develop a two-dimensional (vertical) numerical model that can simulate breaking waves on various types of coastal structures, first for wave motion on smooth impermeable slopes and at a later stage for wave motion on-and-in permeable structures. A proper representation of the wave impact will not yet be included.

\(^1\) The model SKYLLA is named after the sea monster from the Greek mythology, being a mistress of Poseidon and living on a rock devouring shipwrecked sailors.
1.3 Description of the numerical model SKYLLA

The studies performed within the European MAST-project resulted in the research tool SKYLLA, able to simulate breaking waves on smooth impermeable slopes. These studies are described in Broekens and Petit (1992) and Petit and Van den Bosch (1992) and are summarised by Van der Meer et al. (1992). A brief summary is given below.

The model uses Navier-Stokes equations in two dimensions with a constant turbulence viscosity. The technique for solving these Navier-Stokes equations in two dimensions, is based on the "Volume of Fluid method", see Nichols and Hirt (1980). The fluid is considered as incompressible. The model uses a staggered, non-equidistant grid where for each cell the fluid fraction can vary between zero (empty) and one (full) (Eulerian approach).

The model uses a complex description of the free surface based on an adapted flux-method known as "FLAIR", see Ashgriz and Poo (1991) and is capable of simulating free surfaces that can become multiple-connected whereas air-entrapment can be dealt with. Those two aspects are both essential for the simulation of plunging waves. The entrapped air is modelled as if it were vacuum.

The model includes the option to model the smooth impermeable slope with "no-slip" or with "free slip". The choice of the grid does not depend on the lay-out of the slope.

Before the present study was started, the incident waves were sinusoidal and generated at the left-hand side of the computational domain. This left-hand side boundary is weakly reflecting, enabling reflected waves to leave the computational domain with an acceptable small disturbance of the wave motion in the computational domain. The right-hand side boundary was closed.

1.4 Required program modifications

Broekens and Petit (1992) listed required program modifications for the applications which can be achieved with the model SKYLLA. In Klein Breteler and Petit (1993) the program modification for the near future has been scheduled. The first group of activities concerns modifications of boundaries. These activities will be described in this report.

For many practical applications, such as overtopped structures, submerged structures or sand bars, both the left-hand side boundary and the right-hand side boundary must be open and weakly reflecting as well. This was not the case for the right-hand side boundary. This has been implemented.

Incident waves were schematized as sinusoidal. For most applications this schematization is not satisfactory. Therefore, this schematization of the incident waves was improved. The incident wave signals, surface elevations and velocities as a function of time, are now generated based on the analytical solution of Rieneker and Fenton (1981).

The analytical solution of Rieneker and Fenton (1981) can also be used for verifying the surface elevations produced by SKYLLA. For this verification the surface elevations as a function of space for waves travelling along a horizontal (smooth) bottom are analyzed in
comparison with the SKYLLA results. The cause of occurring differences will be tracked down and this will be used to modify SKYLLA in such a way that the inaccuracies become acceptable.

1.5 Outline

The implementation of the weakly reflecting boundaries will be described in Chapter 2. The improvement of the incident wave by generating a more appropriate wave shape will be discussed in Chapter 3. The verification with the analytical solution of a wave travelling over a horizontal bottom and tracking down of error sources give rise to improvements of the model. As will be shown in a later stage, this appeared to become a relatively extensive effort resulting in considerable improvements of the free-surface description. This will be discussed in Chapter 4. To show that the model has become capable of dealing with new applications and of a more accurate modelling of breaking waves, a practical application of a wave breaking on a submerged structures will be performed and described in Chapter 5. The conclusions and recommendations in Chapter 6 will finish this study.
2 Weakly reflecting boundaries

2.1 Objective and approach

In the former state of the numerical model, only the left-hand side boundary was weakly reflecting enabling reflected waves to leave the computational domain. The right-hand side boundary was closed, see the upper sketch in Figure 1.

![Sketch of wave motion with open right boundary](image)

Figure 1 Extension with open right boundary

For many practical applications, such as overtopped structures, submerged structures or sand bars, both the left-hand side boundary and the right-hand side boundary must be open and weakly reflecting as well (see the second sketch in Figure 1). This was not the case for the right-hand side boundary. The new right-hand side boundary has been implemented. A preliminary verification of this implementation has been performed to study the accuracy of the new boundary in broad outlines. Whether the implementation of this weakly reflecting boundary causes acceptable small disturbance of the wave motion in the computational domain that is still a matter of verification.

Since the sinusoidal incident waves and inaccuracies in the other modules of the model contribute to the total deviations, the final verification of this implementation is performed after the improvement of the incident wave and the tracking of other deviation causing inaccuracies.
2.2 Implementation of the weakly reflecting boundary

At the right-hand side boundary the Navier-Stokes equations, the equation for continuity and the following equations need to be discretised:

\[
\frac{\partial \eta}{\partial t} + c_1 \frac{\partial \eta}{\partial x} = \frac{\partial \eta_{out}}{\partial x} + c_1 \frac{\partial \eta_{out}}{\partial x} \quad (1)
\]

\[
\frac{\partial u}{\partial t} + c_1 \frac{\partial u}{\partial x} = \frac{\partial u_{out}}{\partial x} + c_1 \frac{\partial u_{out}}{\partial x} \quad (2)
\]

\[
\frac{\partial v}{\partial t} + c_1 \frac{\partial v}{\partial x} = \frac{\partial v_{out}}{\partial x} + c_1 \frac{\partial v_{out}}{\partial x} \quad (3)
\]

Both the discretisation of this set of equations and the implementation of these equations near the boundary, are described in Appendix A. This discretisation leads to a first-order weakly reflecting boundary at the right boundary, allowing waves to propagate through this boundary.

Special attention has been paid to the problem of air-bubbles (vacuum) near the right boundary. These jeopardize the computation in case they travel through the boundary. Since the surface elevations are defined as the summation of fluid fractions in a certain column, partially filled cells in the bulk of the fluid near or at the boundary will cause difficulties. These partially filled cells may well be present in the fluid domain since air-bubbles (vacuum) are generated by the breaking process.

The option to prescribe an incident wave through the right-hand side boundary is made available. Even simultaneous incident waves from both the left-hand side and the right-hand side is made possible.

2.3 Preliminary test of the weakly reflecting boundary

A preliminary test of the implementation was performed to study the accuracy of the new boundary in broad outlines. Inaccuracies in the other modules of the model contribute to the total deviations. Sinusoidal incident waves cause also inaccuracies. Therefore, the final verification of this implementation will be performed after completion of the other activities of phase 1, the improvement of the incident wave and the tracking of other deviations causing inaccuracies.

The aim of the tests is to verify whether the weakly reflecting boundaries, especially at the right-hand side, allow fluid to leave the computational domain causing very limited reflection. If the reflections are not sufficiently small, this should be explicable. The final verification of the modified boundaries will be described later in this report.
Three cases are tested:

1. "Academic" test case in which the reflection caused by the boundary at the right hand side is determined.
2. A very similar case with partially filled cells to verify whether the boundary reacts correctly on the transport of partially filled cells through this boundary.
3. "Practical" test case with two open boundaries and a structure. It is shown that the boundary causes very limited reflection; from an animation it can be shown that the weakly reflecting boundary is assembled correctly.

The first two tests are described in Appendix B whereas the third test will be described in Chapter 5. This third one is a test after completion of the modifications of the other boundaries (phase 1).

Reflection can be determined by identifying the components from the surface elevation that travel to the left-hand side and to the right-hand side. An incoming wave at the left-hand side boundary travels to the right. If the boundary at the right-hand side causes reflection, a wave travelling to the left-hand side occurs. By identifying both components at the left boundary, the magnitude of the reflected wave can be determined \( \eta = \eta_i + \eta_r \). Incident waves \( \eta_i \) are described by a mutation of the free surface on the actual free surface. This input signal is known. If this input signal is equal to the actual movement of the free surface at the left-hand side boundary, no reflection occurs. If no dispersion takes place in the computational domain, the difference between \( \eta \) and \( \eta_i \) is a measure for the accuracy of the weakly reflecting boundary. The left-hand side boundary causes also reflections. The reflection of the reflected wave caused by the right-hand side boundary, can be neglected since this twice reflected signal is sufficiently small if the weakly reflecting boundary acts appropriately.

The first series of tests was performed with a wave height of 0.20 m, a wave period of 3.0 s and a water depth of 1.00 m. The computational domain was 0.30 m wide and 1.30 m high. This width is small compared to the wave length, see Figure 2. The grid was non-equipartition. The computations were done without a slope. Parameters such as the wave height, the grid size and the turbulence viscosity were varied. The incident wave has also once been generated at the right-hand side boundary.

This first series of tests showed reflections in the order of magnitude of a few percent. Although this was low, a weakly reflecting boundary should have given even lower reflections. These reflections are assumed to be partially caused by the type of single-sinusoidal incident waves, being single-sinusoidal. This is not a proper shape of a wave, since in reality waves tend to have also higher components. The single-sinusoidal wave in
the model will develop towards a wave with more components. This phenomenon causes fake-reflection coefficients; a part of the differences between generated and occurring signals is not caused by reflections but caused by this process.

Tests with varying wave heights show that the reflection (in percentage) is higher for higher wave heights. A weakly reflecting boundary will not show such an increase in reflection (in percentage) if the wave height is being increased. The development from a single-sinusoidal wave to a wave with more components, however, increases more than linear with an increasing wave height. This was the case in the tests and therefore the increased percentages are assumed to be due to this phenomenon. It can be concluded that these test cases do not indicate important inaccuracies in the treatment of the weakly reflecting boundaries although this must still be verified with a wave having a more realistic shape.

In the second series of tests, the fluid fractions in some cells in the bulk of the fluid were set at values lower than one to study whether or not air-bubbles (vacuum) at the boundary did not influence the functioning (1) of the weakly reflecting boundary.

The initial results with these tests did not give satisfactory results. The results became sufficient by redefining the free surface. Because the numerical model considers directly flow between cells, it does not primarily use a specified free surface. For the treatment near the weakly reflecting boundaries, a specified free surface is necessary. For this purpose, the free surface was defined as the summation of fluid fractions in a certain column, see Figure 1 in Appendix A. The new definition of the free surface at the boundary is the summation of the free-surface fractions in the upper cells whereas the other cells are not considered, see Figure 2 in Appendix A. This treatment provides a more realistic free surface which results in more accurate weakly reflecting boundaries for the case that air is transported through these boundaries. The results with this treatment were satisfactory, see Appendix B.

A final validation will be discussed in Chapter 5.
3 Incident wave signal

3.1 Description of RF-WAVE

The program SKYLLA contained incident waves of a single sinusoidal wave shape. Since such waves are a rather rough approximation, more appropriate incident waves must be generated. For this purpose, the program RF-WAVE can be used.

The program RF-WAVE by G. Klopman can compute periodic waves on a horizontal bottom using the Fourier approximation method of Rienecker and Fenton (1981). A brief introduction is given here.

The flow field in periodic water waves on a horizontal bottom can be described by a stream function $\Psi(x,z,t)$ of the following form:

$$\Psi = (c+B_0)(z+h) + \sum_{j=1}^{N} (B_j \frac{\sinh jk(z+h)}{\cosh j(kh)} \cos jk(ct-x))$$

where

- $x$ : horizontal coordinate.
- $z$ : vertical coordinate, $z=0$ at the mean water surface.
- $t$ : time.
- $h$ : mean water depth.
- $k$ : wave number.
- $c$ : wave celerity.
- $N$ : number of Fourier components.
- $B_j$ : Fourier components of the stream function.

The surface elevation is described by:

$$\eta(x,t) = \sum_{j=1}^{N} A_j \cos jk(ct-x)$$

where

- $\eta$ : surface elevation.
- $A_j$ : Fourier components of the free surface.

The program computes the surface elevation and the stream function Fourier components for a periodic gravity wave propagating on a uniform current across a horizontal bottom, by applying the non-linear free surface boundary conditions in a number of free-surface points distributed uniformly over half a wavelength. This results in a system of non-linear equations, with the surface point elevations and the stream function Fourier components as unknowns. The resulting system of equations is solved by using Newton’s method. The program allows for the specification of an arbitrary mean velocity at any elevation, or of an arbitrary mass transport.
This solution seems useful for implementation in the model SKYLLA since for most applications the position of the left boundary will be such that non-disturbed waves (at least non-breaking) will occur at this boundary.

As the described approach does not account for reflected waves, which will occur in most applications of SKYLLA, the solution must be adapted. The adapted input signal is described in the next section.

### 3.2 Adapted RF-WAVE input signal

In using the weakly reflecting boundary conditions, problems can arise if the surface elevation of the incident wave differs significantly from the actual surface elevation at the boundary (sum of incident and reflected wave). This will in general be the case because most structures cause reflections.

The differential equations that have to be solved numerically, at for instance the left boundary, are:

\[
\frac{\partial (\eta - \eta_{in})}{\partial t} - c \frac{\partial (\eta - \eta_{in})}{\partial x} = 0
\]  \hspace{1cm} (6)

\[
\frac{\partial (u - u_{in})}{\partial t} - c \frac{\partial (u - u_{in})}{\partial x} = 0
\]  \hspace{1cm} (7)

\[
\frac{\partial (v - v_{in})}{\partial t} - c \frac{\partial (v - v_{in})}{\partial x} = 0
\]  \hspace{1cm} (8)

Using for instance a Rienecker and Fenton (1981) solution for the incident wave, the surface elevation, the velocity in the x-direction and the velocity in the y-direction, respectively \( \eta_{in} \), \( u_{in} \) and \( v_{in} \), are known. The values found for the velocity components are given only for the range \( 0 < y < \eta_{in} \). However, a reflected wave travelling through the boundary causes the actual location of the free surface \( \eta \) and the corresponding velocities to differ from \( \eta_{in} \), \( u_{in} \) and \( v_{in} \), see Figure 3. If the difference between \( \eta \) and \( \eta_{in} \) is small, the values found from the RF solution will not produce unrealistic values. If however, the difference is large, the velocities found from the RF solution cannot be used. The velocities show an exponential growth in the vertical

![Figure 3 Reflection causes deviations from the RF-WAVE velocity profile](image-url)
direction. Therefore, an extrapolation of the velocity profiles to the actual free surface can cause considerable deviations from reality.

In order to be able to give acceptable values for the velocities of the incident wave, a stretching method can be applied, see Appendix C. No extrapolation is performed but the profile prescribed by the RF-solution is stretched over the actual water depth instead. This gives the shape of the velocity profiles.

To determine the values of the velocities, another assumption must be made. On the new velocity profile it is possible to impose several conditions which express the conservation of certain quantities. Two conditions have been studied and both have been implemented (optional): conservation of mass flux and conservation of momentum flux, see Appendix C. In section 5.2 it is concluded that conservation of momentum yields slightly better results.

Equations 6 to 8 are used at the right-hand side boundary as well, except that the sign for the wave celerity \( c \) becomes positive. If the horizontal velocity at the boundary is in the direction of the computational domain, the expressions as shown in equation 6 are used to determine the surface elevations at the boundaries. If this horizontal velocity is in the opposite direction (outward), the kinematic boundary condition, as shown in equation 9, is used instead to determine \( \eta \):

\[
\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} - v = 0
\]  

(9)

This expression is theoretically advisable since no assumption concerning the wave celerity needs to be made. However, this expression cannot be applied in the case of inward horizontal velocities because of the discretisation of this expression which will lead to numerical instability.

The validation of the adapted input signal is described in Chapter 5.
4 Modified surface treatment

4.1 Objective and approach

Using two-dimensional Navier-Stokes equations is not a guarantee for a successful modelling of breaking waves: the surface treatment is essential as well. The applied method for solving the free surface determines whether complex breaking waves can be dealt with or not. This treatment, however, is rather complex and may easily cause inaccuracies.

As will be discussed in the next chapter, a first verification with an analytical solution was performed. The first results were not satisfactory. A major effort was made to improve this result. Various items were studied, see Appendix D. Most of these items led to small improvements but these will not be discussed in this report. The items that cause deviations are extremely difficult to trace since so many parameters and phenomena interact. Finally, it was concluded that it could not be avoided to modify the treatment of the surface.

Several treatments of the surface were studied. The main problem in the surface treatment is that for the discretisation of several equations, derivatives at the surface need to be determined. For these derivatives velocities at positions outside the fluid are needed. To determine these velocities additional information (equations) are necessary. Four methods are assessed:

1) Copying method
   Copying velocities from inside the fluid to these positions just outside the fluid domain.

2) Extrapolation method
   Extrapolation of velocities from the fluid domain to these positions.

3) Kinematic/dynamic method
   The use of a kinematic boundary equation and a dynamic boundary equation, see Petit (1993-a) and Petit (1993-b).

4) Irrotational method
   Using the assumption that the fluid (near the surface) is free of rotation, see Appendix E.

The first method was the original treatment. These four methods have been compared with the analytical solution. The fourth method gave the best results, see Appendix F.

The fourth method has the disadvantage that this assumption is not valid in areas where the wave is nearly or already breaking. Therefore, this method needs to be combined with another method. It is proposed to combine this method with the original method (copying method) since this method already has proved to give realistic results for breaking waves. Whether this is correct can only be proved after verification with a breaking wave in a physical model.
4.2 New free-surface conditions

As discussed in the previous section, it was decided to use a treatment of the free surface where two methods are combined. Near the in-flow boundary the irrotational method will be applied while in the area where the waves break the copying method will be applied. The irrotational method is discussed here.

In the case that the motion of the fluid is free of rotation, the following equations are valid:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}
\]

\[
\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \tag{11}
\]

As for all methods for the free surface, the discretisations depend on the actual situation at the free surface. This means the discretisation depends on the positions of empty cells with respect to the central cell. The discretisations are described in Appendix E. All discretisations used to determine the virtual velocities at the free surface (outside the fluid domain) are first-order accurate.

The validation and verification of this method and the combination of the two methods, will be discussed in the next chapter.
5 Tests and applications with modified boundaries

5.1 Objective and approach

This chapter describes a series of relevant tests performed after the implementation of the weakly reflecting boundary, the implementation of the adapted RF-WAVE signal as an incident wave and the implementation of the modified surface treatment.

The analytical solution of a wave over a horizontal bottom, as given by the program RF-WAVE is used as an incident wave (as a function of time). This analytical solution can also be applied as a description of waves propagating across a horizontal bottom (as a function of $x$). The waves computed with the model SKYLLA can therefore be compared with this analytical solution (as a function of $x$). The differences between both must be smaller than a few percent. This has been used as the criterion for accepting or rejecting certain treatments of several studied items.

This approach does not give any information of the cause of differences between the signals generated by SKYLLA and the signals from RF-WAVE. It should be noted that since this is the first quantitative verification of the model SKYLLA, errors can also be caused by other modules of the model than the mentioned extensions of the program. Therefore, all other inaccuracies must be diminished to a satisfactory degree before a fair verification of the two above mentioned extensions can be performed.

5.2 Verification with analytical solution

The implementation of the extensions, as discussed in Chapter 2 and Chapter 3, enables to verify the program SKYLLA with a theoretically derived wave shape. Certain aspects of the model have been verified separately before, but this verification is in fact the first verification of the complete program SKYLLA. Therefore, all existing inaccuracies will surface.

The first test presented in Appendix F shows a computation just after the implementation of the two above mentioned aspects. The comparison with the RF-WAVE signal is not satisfactory.

Various tests indicated that this bad performance is due to a combination of several inaccuracies. After checking the source-code, checking the existing modules of the model and checking the sensitivity to several parameters, it was concluded that the most likely sources of the inaccuracies were a combination of the modules "RF-WAVE input wave train", "Weakly reflecting boundaries" and the "Surface treatment"; see also Appendix D. Inaccuracies generated in one of these modules effect the other modules. Therefore, it is extremely difficult to trace these inaccuracies.
First the "RF-WAVE input wave train" is dealt with. The RF-WAVE program generates velocities from the bottom to the free-surface level which is calculated by RF-WAVE. However, this free-surface level from RF-WAVE may differ from the free surface in the SKYLLA model. A reflected wave causes a different free surface than the free surface corresponding to the incoming wave. This had to be considered.

The stretching method of the velocity profile as described in section 3.2 and Appendix C was implemented to be able to deal with the reflected waves. This stretching method stretches the RF-WAVE velocity profile to the actual free surface. However, this method does not allow both conservation of mass and conservation of momentum. Although the differences may be small, one of these had to be chosen. The differences are increasing in time. Based on this increase in differences (gradient in the differences), conservation of momentum was found to be slightly better. The tests presented here are those with the treatment of the free surface that was found to be the best in a later stage.

The description of the free surface has been discussed in Chapter 4. A comparison of the four methods of free surface treatment was performed, see test series 3 described in Appendix F. The fourth method, where the surface is assumed to be free of rotation, was chosen. This method appeared to be relatively sensitive to the discretisation at the weakly reflecting boundary. For this purpose, this discretisation was improved (higher order accuracy). The fourth method has the disadvantage that this assumption is not true in area where the wave is nearly or already breaking. Therefore, this method needs to be combined with another method.

The combination of two treatments was tested and described in test series 4 in Appendix F. The combination of the method where the velocities outside the fluid domain are derived by copying from the nearby velocities inside the fluid domain and the method where these velocities are derived with the assumption that the surface is free of rotation, gave satisfactory results.

The fourth treatment of the free surface (surface free of rotation) was studied by varying several parameters in similar situations. The results can be compared with the RF-WAVE signal and mutual, see test series 5 in Appendix F.

To exclude the effects of the adjustment time, a computation was done with an initial free surface and velocities derived from RF-WAVE, see test series 6 in Appendix F.

We concluded that the implementation of the RF-WAVE signal as an input wave-train and two weakly reflecting boundaries works satisfactorily in these principle tests (horizontal bottom).

Another test-case with a more practical situation (with a structure) is described in the next section.
5.3 Practical application

It was concluded that the implementation of the RF-WAVE signal as an input wave-train and two weakly reflecting boundaries works satisfactorily for the principle tests with a horizontal bottom.

Another test-case with a more practical situation was performed. The main difficulties for the previous tests occurred in combinations of two weakly reflecting boundaries (two permanently open boundaries). Therefore, a submerged structures was modelled with a front slope of 1:1 and a similar structure with a front slope 1:5. The second structure is shown in Figure 4.

![Submerged structure](image)

Figure 4 Submerged structure

It appeared that for the computation with the steep 1:1 (submerged) slope, the results were worse than for the 1:5 slope, see Appendix F, test-cases 7.1 and 7.2. The computation with the 1:1 slope shows that the surface elevations are not periodical; differences of a few centimetres per wave period occur. This somewhat disappointing result was probably caused by the influence of relatively large reflections caused by the steep slope (1:1). These relatively large reflections cause relatively large differences at the boundaries. These differences at the boundaries eventually generate an unwanted constant flow in the computational domain. The computation with the more gentle sloping section 1:5 showed a relatively large improvement of the results; after four wave periods the signals were not periodical but the differences were much smaller. For most practical purposes the differences were diminished to an acceptable degree. However, it should be verified whether the differences are acceptable for the intended application.

The wave height at the right hand side boundary (after breaking) is 0.61 times the local depth. This is not an unrealistic wave height since for many practical applications a first estimation of the wave height after breaking is in fact 0.6 times the local water depth.

In Figure 5 a few snapshots of the computation with the submerged slope 1:5 are shown. The scale in Figure 5 is in the vertical direction about 40% smaller than in the horizontal direction.
Figure 5 Snapshots of breaking waves on a submerged structure
6 Conclusions and recommendations

It can be concluded that a correct incident wave can be imposed on the boundaries instead of the single sinusoidal signal that was generated before. The treatment of the free surface has been improved while various small inaccuracies in the model were solved. A second weakly reflecting boundary has been implemented. This second weakly reflecting boundary can be used in combination with the first weakly reflecting boundary. This enables the computation of breaking waves on submerged structures although the modelling of the structure must end at the position where the downward slope starts (downward slopes are not possible yet). The verification with the analytical solution for smooth horizontal bottoms gave satisfactory results.

The combination of the two weakly reflecting boundaries (two permanently open boundaries) in the case of a breaking wave on a submerged structure, however, showed inaccuracies. It seems as if these differences depend on the amount of reflection caused by the submerged structure. For steep submerged structures the inaccuracies may be too large for some applications (very steep slopes). Therefore, it should be verified whether the differences are acceptable for the intended application with submerged structures. For submerged structures with more gentle sloping sections, as well as for structures that are not permanently submerged, no problems are expected.

The model gave satisfactory results for non-breaking waves over a smooth horizontal bottom. This is not the intended application for the model so another verification has to be performed with breaking waves. No analytical solutions exist for breaking waves and, therefore, a verification with a physical model is recommended. A breaking wave on a submerged structure or sand bank causing minor reflections can be used for a first verification. The model is technically able to compute breaking waves on this kind of structures while the results indicate that the process is simulated rather realistically (qualitative verification). The verification with the physical model can lead to a quantitative appreciation. A verification of breaking waves on a submerged impermeable structure with a gentle slope is already planned.
References


Petit, H.A.H. and P. van den Bosch (1992), SKYLLA: Wave Motion in and on Coastal Structures; Numerical analysis of program modifications, MAST-G65 report, DELFT HYDRAULICS.


Petit, H.A.H. (1993-b), SKYLLA: Note 2 on free surface treatment in SKYLLA, Note (1-6-93), see Documentation SKYLLA '93.

Appendix A

Implementation of the weakly reflecting boundary
Aim:

- First-order weakly reflecting boundary at the outflow boundary at the right-hand side of the computational domain, allowing fluid to move through this boundary with little reflection back to the computational domain.
- Option to prescribe incoming fluid (extra) through this boundary still open.
- Solve difficulties that are to be expected with partially filled cells in the bulk of the fluid.

The Navier-Stokes equations are:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial x} - \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0
\]

(1)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial y} - \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g = 0
\]

(2)

The equation for Continuity is:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(3)

Other equations to be dealt with at the outflow-boundary:

\[
\frac{\partial \eta}{\partial t} + c_1 \frac{\partial \eta}{\partial x} = \frac{\partial \eta_{out}}{\partial x} + c_1 \frac{\partial \eta_{out}}{\partial x}
\]

(4)

\[
\frac{\partial u}{\partial t} + c_1 \frac{\partial u}{\partial x} = \frac{\partial u_{out}}{\partial x} + c_1 \frac{\partial u_{out}}{\partial x}
\]

(5)

\[
\frac{\partial v}{\partial t} + c_1 \frac{\partial v}{\partial x} = \frac{\partial v_{out}}{\partial x} + c_1 \frac{\partial v_{out}}{\partial x}
\]

(6)
For discretizing the equations the following additional notation has been used:

- Computational domain contains cells from \( i = 1 \) to \( i = \text{imax} \) in the \( x \)-direction and from \( j = 1 \) to \( j = \text{jmax} \) in the \( y \)-direction. The first \((i = 1 \text{ and } j = 1)\) and last \((i = \text{imax} \text{ and } j = \text{jmax})\) rows/columns are virtual.
- Velocities \( u \) at the right border of a cell have the same number as the cell. Velocities \( v \) at the top of a cell have the same number as the cell.
- \( \hat{u} \) denotes a virtual velocity.
Using the discretisation of the Navier-Stokes equation, the explicit expression for \( u_{(\text{imax-2,j})}^{k+1} \) for time level \( k+1 \) becomes (for positive velocities):

\[
\begin{align*}
&u_{\text{imax-2,j}}^{k+1} = u_{\text{imax-2,j}}^k + \Delta t \left( \frac{1}{\Delta x_{\text{imax-2}}} \left( \frac{\Delta x_{\text{imax-2}}}{\Delta x_{\text{imax-1}}} \left( u_{\text{imax-1,j}}^k - u_{\text{imax-2,j}}^k \right) + \Delta x_{\text{imax-1}} \left( u_{\text{imax-2,j}}^k - u_{\text{imax-3,j}}^k \right) \right) \right) \\
&- \Delta t \left( 1 - \beta \right) \left( \frac{u_{\text{imax-2,j}}^k - u_{\text{imir-3,j}}^k}{\Delta x_{\text{imax-2}}} \right) \\
&- \Delta t \left( \frac{1}{2} \right) \left( \frac{\Delta x_{\text{imax-2}}}{\Delta x_{\text{imax-1}}} \left( u_{\text{imax-2,j}}^k - u_{\text{imir-2,j}}^k \right) + \Delta x_{\text{imax-1}} \left( u_{\text{imir-2,j}}^k - u_{\text{imir-3,j}}^k \right) \right) \\
&+ \left( 1 - \beta \right) \left( \frac{\Delta y_j + \Delta y_{j+1}}{0.5(\Delta y_j + \Delta y_{j+1})} \right) \left( \frac{u_{\text{imir-2,j}}^k - u_{\text{imir-3,j}}^k}{0.5(\Delta y_j + \Delta y_{j+1})} \right) \left( \frac{u_{\text{imir-2,j}}^k - u_{\text{imir-3,j}}^k}{0.5(\Delta y_j + \Delta y_{j+1})} \right) + \beta \left( \frac{u_{\text{imir-3,j}}^k - u_{\text{imir-3,j}}^k}{0.5(\Delta y_j + \Delta y_{j+1})} \right) \\
&- \Delta x_{\text{imax-2}} \left( \frac{P_{\text{imax-1,j}}^{k+1} - P_{\text{imir-2,j}}^{k+1}}{\Delta x_{\text{imax-1}}} \right) \\
&+ \frac{\Delta t}{\Delta x_{\text{imax-2}} + \Delta x_{\text{imax-1}}} \left( \frac{\Delta x_{\text{imax-2}}}{\Delta x_{\text{imax-1}}} \left( u_{\text{imir-2,j}}^k - u_{\text{imir-3,j}}^k \right) \right) \\
&+ \frac{\Delta t}{\Delta y_j} \left( \frac{1}{2} \left( \Delta y_{j-1} + \Delta y_j \right) \right) \\
&- \frac{1}{2} \left( \Delta y_{j+1} + \Delta y_j \right) \\
&\left( \frac{1}{2} \left( \Delta y_{j+1} + \Delta y_j \right) \right)
\end{align*}
\] (7)

The Navier-Stokes equation can be rewritten using the sign \( - \) for the velocity where the discretisation of all terms is included except for the pressure gradient. For \( u_{\text{imax-2,j}} \) this yields:

\[
\begin{align*}
&u_{\text{imax-2,j}}^{k+1} = u_{\text{imax-2,j}}^k - \frac{2 \Delta t}{\Delta x_{\text{imax-2}} + \Delta x_{\text{imax-1}}} \left( P_{\text{imax-1,j}}^{k+1} - P_{\text{imir-2,j}}^{k+1} \right)
\end{align*}
\] (8)

The explicit expression for \( v_{(\text{imax-1,j})}^{k+1} \) for \( k+1 \) can be derived in similar way as for \( u_{(\text{imax-2,j})} \). For \( v_{\text{imax-1,j}} \) the expression becomes:

\[
\begin{align*}
v_{\text{imax-1,j}}^{k+1} = v_{\text{imax-1,j}}^k - \frac{2 \Delta t}{\Delta y_j + \Delta y_{j+1}} \left( P_{\text{imax-1,j+1}}^{k+1} - P_{\text{imir-1,j}}^{k+1} \right)
\end{align*}
\] (9)

The explicit expression for \( v_{(\text{imax-1,j-1})}^{k+1} \) for \( k+1 \) can be derived in similar way as for \( u_{(\text{imax-2,j})} \). For \( v_{\text{imax-1,j-1}} \) the expression becomes:

\[
\begin{align*}
v_{\text{imax-1,j-1}}^{k+1} = v_{\text{imax-1,j-1}}^k - \frac{2 \Delta t}{\Delta y_j + \Delta y_{j+1}} \left( P_{\text{imax-1,j}}^{k+1} - P_{\text{imir-1,j-1}}^{k+1} \right)
\end{align*}
\] (10)
Using a first-order accurate up-wind discretisation of equation 5 at the outflow boundary, the equation becomes:

\[
\frac{u_{imax-1,j}^{k+1} - u_{imax-1,j}^k}{\Delta t} + c_1 \frac{u_{imax-1,j}^k - u_{imax-2,j}^k}{\Delta x_{imax-1}} = S_j^k
\]

where

\[
S_j^k = \frac{u_{out}(x_{imax-1}, y_j, -\frac{1}{2} \Delta y_{j+1}) - u_{out}(x_{imax-1}, y_j, -\frac{1}{2} \Delta y_{j+1})}{\Delta t} + c_1 \frac{u_{out}(x_{imax-1}, y_j, -\frac{1}{2} \Delta y_{j+1}) - u_{out}(x_{imax-2}, y_j, -\frac{1}{2} \Delta y_{j+1})}{\Delta x_{imax-1}}
\]

wherein \(u_{out}\) is a known function of \((x, y, t)\) that describes the horizontal velocity component of an outgoing wave through the right-hand side of the computational domain. This velocity is zero if no wave is being generated at this boundary.

For equation 6 an implicit discretisation using time-levels \(k\) and \(k+1\) will be used:

\[
\frac{v_{imax-1,j}^k - v_{imax-1,j}^{k-1}}{\Delta t} + c_1 \frac{v_{imax,j}^k - v_{imax-1,j}^k}{\frac{1}{2} (\Delta x_{imax-1} + \Delta x_{imax})} = R_j^k
\]

and

\[
\frac{v_{imax-1,j}^{k-1} - v_{imax-1,j-1}^{k-1}}{\Delta t} + c_1 \frac{v_{imax,j-1}^{k-1} - v_{imax-1,j-1}^{k-1}}{\frac{1}{2} (\Delta x_{imax-1} + \Delta x_{imax})} = R_{j-1}^k
\]

where

\[
R_j^k = \frac{v_{out}(x_{imax-1}, y_j, -\frac{1}{2} \Delta x_{imax-1}, y_{j+1}) - v_{out}(x_{imax-1}, y_j, -\frac{1}{2} \Delta x_{imax-1}, y_{j+1})}{\Delta t} + c_1 \frac{v_{out}(x_{imax-1}, y_j, -\frac{1}{2} \Delta x_{imax-1}, y_{j+1}) - v_{out}(x_{imax-1}, y_j, -\frac{1}{2} \Delta x_{imax-1}, y_{j+1})}{\frac{1}{2} (\Delta x_{imax-1} + \Delta x_{imax})}
\]

wherein \(v_{out}\) is a known function of \((x, y, t)\) that describes the vertical velocity component of an outgoing wave through the right-hand side of the computational domain.

The equation for continuity (equation 3) can be discretised for cell \((imax-1,j)\):

\[
\Delta y_j (u_{imax-1,j}^{k+1} - u_{imax-2,j}^k) + \Delta x_{imax-1} (v_{imax-1,j}^{k+1} - v_{imax-1,j-1}^k) = 0
\]
Equations 8, 9 and 10 show explicit expressions for \( u_{(i\text{max}-2,j)} \), \( v_{(i\text{max}-1,j)} \) and \( v_{(i\text{max}-1,j-1)} \). These can be substituted in equation 16. This yields:

\[
\Delta y_j = \frac{1}{2} \left( \frac{\Delta x_{\text{max}-2} + \Delta x_{\text{max}-1}}{\Delta y_{j-1} + \Delta y_j} \right) \left( p_{\text{max}-1,j}^{k+1} - p_{\text{max}-2,j}^{k+1} \right) \\
+ \frac{1}{2} \left( \frac{\Delta x_{\text{max}-1}}{\Delta y_j + \Delta y_{j+1}} \right) \left( p_{\text{max}-1,j+1}^{k+1} - p_{\text{max}-2,j}^{k+1} \right) \\
- \frac{1}{2} \left( \frac{\Delta x_{\text{max}-1}}{\Delta y_j + \Delta y_{j-1}} \right) \left( p_{\text{max}-1,j-1}^{k+1} - p_{\text{max}-2,j}^{k+1} \right) \\
- \frac{1}{2} \left( \frac{\Delta x_{\text{max}-1}}{\Delta y_{j-1} + \Delta y_j} \right) \left( p_{\text{max}-1,j}^{k+1} - p_{\text{max}-2,j-1}^{k+1} \right) = \frac{\Delta y_j}{\Delta t} (\Delta x_{\text{max}-1,j}^{k+1} - \Delta x_{\text{max}-1,j}^{k}) + \frac{\Delta x_{\text{max}-1}}{\Delta t} (\Delta v_{\text{max}-1,j}^{k+1} - \Delta v_{\text{max}-1,j-1}^{k})
\] (17)

The virtual velocities indicated with \(^\wedge\), can be calculated using equations 11, 13 and 14.

For cell \((i\text{max}-1,j)\), the F-value is assumed to be 1 (except for the most upper cell in this column). This implies that no "air bubbles" are supposed to be present at the boundary.

In the most upper cell containing fluid \((0 < F < 1)\), the surface is assumed to be approximately horizontal. To determine the Dirichlet boundary condition for the pressure in this cell, the F-values of the 8 surrounding cells are needed. Three of those surrounding cells are virtual. The F-values in those three cells can be calculated with equation 4. The discretisation is as follows:

\[
\frac{\eta_{\text{max}-1}^{k+1} - \eta_{\text{max}-1}^k}{\Delta t} + c_1 \frac{1}{2} \left( \frac{\Delta x_{\text{max}-1}}{\Delta y_{\text{max}}} \right) \Delta x_{\text{max}} - \eta_{\text{max}-1}^k = Q^k
\] (18)

where

\[
Q^k = \frac{\eta_{\text{out}}(x_{\text{max}} - \frac{1}{2} \Delta x_{\text{max}}, y_k) - \eta_{\text{out}}(x_{\text{max}} - \frac{1}{2} \Delta x_{\text{max}}, y_{k-1})}{\Delta t} \\
+ c_1 \frac{\eta_{\text{out}}(x_{\text{max}} - \frac{1}{2} \Delta x_{\text{max}}, y_k) - \eta_{\text{out}}(x_{\text{max}} - \frac{1}{2} \Delta x_{\text{max}}, y_{k-1})}{\frac{1}{2} \left( \Delta x_{\text{max}} + \Delta x_{\text{max}} \right)}
\] (19)

Equation 18 gives the new virtual surface \( \tilde{\eta}_{\text{max}-1}^{k+1} \). In this equation the surface in \( i=1 \) is defined as:

\[
\eta_{\text{max}-1}^k = \sum_{j=2}^{j_{\text{max}-1}} F_{\text{max}-1,j} \Delta y_j
\] (20)
The F-values in the virtual column at the outflow boundary can then be found using the relation between the virtual surface $\hat{\eta}^{k+1}_{i_{\text{max}}}$ and the virtual $F^{k+1}_{i_{\text{max}}}$-values:

$$\hat{\eta}^{k+1}_{i_{\text{max}}} = \left( \sum_{j=2}^{j_{i-1}} \Delta y_j \right) + \Delta y_{j_0} F_{i00}$$

(21)

where:

For $j$ from 2 to $j(i)-1$ the $F_{ij}$ is 1.
For $j=j(i)-1$: $0 \leq F_{ij} < 1$.
For $j > j(i)-1$: $F_{ij} = 0$.

**Alternative for definition of last, not-virtual, surface elevation (equation 20)**

In Figure 1, the situation with a "partially filled area" reaching the right hand side boundary (velocity towards the boundary) has been sketched (1A). The definition of the free surface as described with equation 20 has been sketched as well (1B). It can been seen that a discontinuity occurs in case a "partially filled area" reaches the boundary. For some practical cases this may cause problems. Therefore an alternative definition of the free surface in this last column has been developed. This definition will only be used in the treatment of the boundary, not for fluxing.

![Figure 1](image)

**Figure 1**  
A) Actual situation  
B) Definition of the $\hat{\eta}^{k}_{i_{\text{max}}-1}$ (for eq.23).

The free surface in the last, not virtual column, $\eta^{k}_{i_{\text{max}}-1}$ is needed to determine the free surface in the virtual column for the new time step ($\hat{\eta}^{k+1}_{i_{\text{max}}}$).

For the free surface in the last, not virtual column, an additional condition can be defined with respect to the maximum slope of the free surface at the boundary. This gives a limited y-range in which the free surface in column $i_{\text{max}}-1$ can appear. The most upper cell in this y-range is called $j_{\text{up}}$, the lowest cell in this y-range is called $j_{\text{down}}$. The maximum angle of the free surface at the boundary is called $\varphi$. 
\( j_{\text{up}} \) and \( j_{\text{down}} \) are the (largest) numbers of the cells where the following expressions are valid:

\[
\sum_{j=2}^{j_{\text{up}}-1} \Delta y_j \leq (\eta_{j0} + \tan \varphi \Delta x_{\text{imax}-1})
\] (22)

\[
\sum_{j=2}^{j_{\text{down}}-1} \Delta y_j \leq (\eta_{j0} - \tan \varphi \Delta x_{\text{imax}-1})
\] (23)

where \( j(i) \) is the number of the cell where the free surface occurs in the virtual column (see equation 21).

\[
\eta_{\text{imax}-1}^k = \sum_{j=2}^{j_{\text{down}}-1} \Delta y_j + \sum_{j_{\text{up}}}^{j_{\text{imax}}} (\Delta y_j F_{\text{imax}-1,j})
\] (24)

Equation 24 is the alternative definition, replacing equation 20. The procedure of determining the free surface for the last, not virtual column, is illustrated in Figure 2. Figure 2A shows the actual situation and the determination of \( j_{\text{down}} \) and \( j_{\text{up}} \) for a certain angle \( \varphi \). Figure 2B shows the surface elevation \( \eta_{\text{imax}-1}^k \) that is used in equation 18.

---

**Figure 2**

A) Actual situation

B) Alternative definition of \( \eta_{\text{imax}-1}^k \) (eq.24)
Appendix B

Preliminary validation of the weakly reflecting boundary
Aim:

- Testing whether the weakly reflecting boundaries, especially at the right hand side, allows fluid to leave the computational domain causing very limited reflection.

Three cases have to be tested:

1. "Academical" test case wherein the reflection caused by the boundary at the right hand side is determined.
2. A very similar case with partially filled cells to verify whether the boundary reacts correctly on the transport of partially filled cells through this boundary.
3. "Practical" test cases wherein it is shown that the boundary causes very limited reflection; From an animation it must be clear that the weakly reflecting boundary is assembled correctly.

Test case 3 will be done in a later stage, after the improvements and extensions of other boundaries.

Reflection can be determined by distinguishing the components from the surface elevation that travel to the left-hand side and to the right-hand side. An incoming wave at the left-hand side boundary travels to the right. In case the boundary at the right-hand side causes reflection, a wave travelling to the left-hand side occurs. By distinguishing both components, the magnitude of the reflected wave can be determined \( \eta = \eta_1 + \eta_r \). Incident waves (\( \eta_1 \)) are described by a mutation of the free surface on the actual free surface. This input signal is known. In case this input signal is equal to the actual movement of the free surface at the left hand side boundary, no reflection occurs. In case no dispersion takes place in the computational domain, the difference between \( \eta \) and \( \eta_1 \) is a measure for the accuracy of the weakly reflecting boundary. The left-hand side boundary causes reflection of the reflected signal caused by the right hand side boundary. This contribution can be neglected since this twice reflected signal is sufficiently small in case the weakly reflecting boundary acts appropriate.
Test case 1

The incoming signals are determined by:

\[ \eta_1(x,t) = a \sin(\omega t - kx) \]

\[ u(x,y,t) = \omega a \frac{\cosh(ky)}{\sinh(kh)} \sin(\omega t - kx) \]

\[ v(x,y,t) = \omega a \frac{\sinh(ky)}{\sinh(kh)} \cos(\omega t - kx) \]

Wave height \( H = 0.2 \) m : \( a = 0.1 \) m
Wave period \( T = 3.0 \) s : \( \omega = 2\pi/T = 2.094 \) s\(^{-1}\)
Wave length \( L = 8.72 \) m : \( k = 2\pi/L = 0.72 \) m\(^{-1}\)
Water depth \( h = 1.0 \) m : 0.30 m
Length of the flume : 1.30 m
Height of the flume : 9 (including 2 virtual cells)
Number of cells in x-direction : \( \Delta x = 0.30/9 = 0.0333 \)
Number of cells in y-direction : 40 (including 2 virtual cells)
Number of cells in free surface y-range : 20 (12-20-8)
Computation period \( t_c \) : 3.5 s
Upwind fraction \( \beta \) : 0.30
Viscosity \( \nu \) : 0.025 m\(^2\)/s

The computation was done without slope.

TEST CASE 1a : As described.
TEST CASE 1b : As test case 1a except: \( \Delta x = 0.30/18 = 0.0167 \).
TEST CASE 1c : As test case 1a except: \( H = 0.40 \) (a = 0.20).
TEST CASE 1d : As test case 1a except: incoming wave from the right hand side boundary.
TEST CASE 1e : As test case 1a except: \( \nu = 0.0125 \); Upwind fraction \( \beta = 0.038 \).
Results test case 1

SKYLLA - WEAKLY REFLECTING BOUNDARY - TEST 1a

SKYLLA - WEAKLY REFLECTING BOUNDARY - TEST 1b

12-03-93
In Table 1 the reflection coefficients, calculated by $|\eta_{\text{max}} - \eta_{\text{min}}| \text{ difference} / |\eta_{\text{max}} - \eta_{\text{min}}| \text{ analytical}$, have been printed.

<table>
<thead>
<tr>
<th>TEST CASE</th>
<th>REFLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>2.39%</td>
</tr>
<tr>
<td>1b ($\Delta x/2$)</td>
<td>2.15%</td>
</tr>
<tr>
<td>1c ($H^2$)</td>
<td>4.90%</td>
</tr>
<tr>
<td>1d (incoming right)</td>
<td>2.44%</td>
</tr>
<tr>
<td>1e ($\omega=0.0125; \beta=0.038$)</td>
<td>2.40%</td>
</tr>
</tbody>
</table>

Table 1 Reflection coefficients
Discussion test case 1

As can be seen in the figures for test cases 1a-1e, the difference have a frequency twice as high as the input signals. The average is not zero.

The computations started with a wave crest. After a certain adjustment period, the average water level becomes constant. However, this new average water level is higher than the original one. This has been observed in previous tests as well. This causes the deviation from the zero elevation for the "difference"-signal.

The incoming wave was sinusoidal. In reality a free surface wave has higher order components as well (Stokes-wave). In the numerical model, the surface elevation reacts as in reality. Therefore in surface elevation becomes, a "Stokes-wave" rather than staying a single-sinusoidal wave; a higher component is generated. This causes a difference between the incoming wave (analytical) and the actual wave (SKYLLA). As can be seen in the figures, just after starting the computation relatively large deviations occur. In the period till roughly 0.2 s, the computed signal is not effected by a possibly reflected wave since its "travelling time" to the right hand side border and back, is longer. In case of a "Stokes" wave, the surface elevation is lower than for a single-sinusoidal wave. The SKYLLA-signal and the analytical-signal show respectively the same trend.

A reflecting boundary would not show an increase in reflection (in %) if the wave height is increased. With the above explained phenomenon, it may be the case.

Rather small differences occur comparing case 1a and 1d; R = 2.39\% and R = 2.44\%. The conclusion is that allowable small differences occur between a wave generated from the right hand side boundary and the same wave generated from the left hand side boundary.

The effect of the varied viscosity is very small; R = 2.39\% and 2.40\%. Therefore viscosities in this order of magnitude are supposed not to effect the described phenomenon.

The conclusion is that the observed differences are not caused by reflection. The reflection coefficients in Table 1 are in fact "apparent"-reflections rather than real reflections.
Test case 2

The same input-data as for test case 1 can be used. Now the fluid fraction in some cells have to be adapted. In the centre of the fluid the fluid fraction of some cells must be initiated not equal to 1. The sinusoidal wave has to start with a crest which causes that the partially filled cells leave the computational domain within a half wave-period. Computational length \( t_e = 3.5 \) s. The computation can be done without slope. For the wave height 0.40 m has been taken.

| TEST CASE 2A | Cells (i=6, j=5 to 20) F=0.99. |
| TEST CASE 2B | Cells (i=6, j=5 to 20) F=0.90. |
| TEST CASE 2C | Cells (i=6, j=5 to 20) F=0.50. |
| TEST CASE 2D | Cells (i=4 to 6, j=5 to 20) F=0.50. |
| TEST CASE 2E | Cells (i=4 to 6, j=5 to 20) F=0.00. |
| TEST CASE 2F | Cells (i=4 to 6, j=5 to 20) F=0.00 with the alternative definition of the free surface in the last, not virtual, column for the boundary treatment. |
Results test case 2

SKYLLA - WEAKLY REFLECTING BOUNDARY - TEST 2a

SKYLLA - WEAKLY REFLECTING BOUNDARY - TEST 2b

12-03-93
Discussion test case 2

In test case 2, some cells in the middle of the computational domain have been initialized with fluid fractions smaller than 1, representing air-bubbles. Those "air-bubbles" must be able to leave the computational domain and must cause allowable disturbance of the working of the weakly reflecting boundary.

Comparison of test cases 1c and 2a shows minor differences. The position of the cells with fluid fractions of \( F = 0.99 \) moves to the right hand boundary in case the velocities are in that direction. Near the right hand side boundary, the cells with fluid fraction \( F = 0.99 \) become totally filled, \( F = 1 \).

Test case 2b shows that the boundary is not insensitive to not-completely filled cells, travelling through the boundary. The free surface is determined by enumerating the fluid-fractions (multiplied by \( \Delta y \)). The virtual cells from the boundary are supposed to be completely filled (\( F = 1 \)). Due to the definition of the free surface, a discontinuity occurs in the "defined" free surface at the boundary if some cells are not completely filled. This is illustrated in Figure 1 (velocity is to the right hand side).

![Figure 1](image)

Figure 1 A) Actual situation B) Definition of the free surface at the boundary

This discontinuity causes that the signals deviate from the analytical signals. The analytical signals are determined as if all cells are completely filled. A part of the deviations is caused by the phenomena as described in the discussion of test case 1 ("Stokes-wave" and "increase of the average surface elevation").

Test case 2b shows that near \( t = \pm 1.2 \) s, the influence of the "partially filled area" (\( F = 0.9 \)) reaches the (left) boundary. After this "partially filled area" is disappeared, the computation proceeds similar as for in test case 1c. The average surface elevation is decreased due to the "partially filled area". The new average surface elevation will be slightly lower than the average surface elevation in test case 2c and slightly higher than the original still water level.
Except for the slightly lower average surface elevation, the difference with test case 1c will disappear after sufficient long continuation of the computation. Test case 2c (F=0.5) shows a more distinct case of what happened in test case 2b (F=0.9).

In test case 2d, more rows with fluid fractions smaller than one (F=0.5) have been prescribed. This causes a clear discontinuity at the point where the influence of the "partially filled area" reaches the (left) boundary (t = ±1.2 s). After t = ±2 s, the computation seems to proceed as if there were no partially filled cells, except for the mutation of the average surface elevation.

In test case 2e (F=0), the mutation of the average surface elevation is considerably. This effect is so large that the remaining water depth causes "breaking" of the wave. However, this breaking does not happen since the computational domain is insufficient large. After ± 2 s, it seems as if the computation is not really disturbed by the "partially filled area" except for the large mutation in water depth. From test 2e, it can be concluded that the boundary handles empty cells without "exploding" or other radical numerical aspects. Test 2e does not give enough information about the disturbance in the reflection due to empty cells. The mutation of the water depth is unallowable large for this case.
Test case 2f (Alternative definition of the free surface in last, not-virtual, column)

For the problems that occurred in test 2e with respect to the decreasing average surface elevation, an alternative treatment is regarded.

Due to the definition of the free surface in the last (not-virtual) column (see Figure 1), the average surface elevation decreases depending on the amount of "partially filled area" travelling through the boundary. This may lead to problems in case a relatively large amount of "air bubbles" are generated near the boundary with respect to the occurring water depth.

The original definition of the free surface in the last, not-virtual, column has been described in Appendix A (Equation 20). An alternative definition is also described in this Appendix A (Equation 24).

In test case 2f the average water level still decreased due to passing of the "partially filled area" through the boundary. Comparing this result with those from the original definition, test case 2e, shows that the average surface elevation now decreased ± 0.1 m while in test case 2e this was ± 0.5 m. Test case 2f shows that near t = 3.0 s the differences with the original signal decrease again (the analytical solution is without air-bubbles). This is caused by a further development of the wave towards a "Stokes wave". The crest becomes higher; this diminishes the differences under the crest while the differences during a the wave trough rise.
Appendix C

Description of adapted RF-WAVE input signal
Aim:

- Adapting the RF-WAVE input signal so that it can be used for the SKYLLA model.

In using the weakly reflecting boundary conditions problems can arise when the surface elevation of the incoming wave differs significantly from the actual surface elevation at the boundary.

The differential equations that have to be solved numerically at the left boundary are:

\[
\frac{\partial \eta - \eta_{in}}{\partial t} - c \frac{\partial \eta - \eta_{in}}{\partial x} = 0
\]  
\[\text{(1)}\]

\[
\frac{\partial u - u_{in}}{\partial t} - c \frac{\partial u - u_{in}}{\partial x} = 0
\]  
\[\text{(2)}\]

\[
\frac{\partial v - v_{in}}{\partial t} - c \frac{\partial v - v_{in}}{\partial x} = 0
\]  
\[\text{(3)}\]

When we use for example a Rieneker and Fenton (RF) solution for the incoming wave, we know the surface elevation the velocity in x direction and the velocity in y direction \(\eta_{in}, u_{in}\) and \(v_{in}\) respectively. The values found for the velocity components are given only for the range \(0 \leq y \leq \eta_{in}\). When there is a reflected wave going out at the boundary however, the actual location of the free surface \(\eta\) may differ from \(\eta_{in}\) and velocities for the incoming wave have to be prescribed in Eq. (2) and Eq. (3) for the range \(0 \leq y \leq \eta\). When the difference between \(\eta\) and \(\eta_{in}\) is small the values found from the RF solution will not produce unrealistic values. If however, the difference is large, for instance when a standing wave is to be simulated and the wave height becomes twice that of the incoming wave, the velocities found from the RF solution cannot be used. The reason for this is that the Fourier approximation used in the RF solution shows an exponential growth in the vertical direction. Velocities found outside the fluid domain of the incoming wave can for that reason easily become very large.

In order to still be able to give acceptable values for the velocities of the incoming wave we stretch the velocity profile of the incoming wave.

On the new velocity profile we can impose several condition which express the conservation of quantities of the RF solution. In SKYLLA we built in two options, conservation of mass flux and conservation of momentum flux.

**Mass flux conservation**

For the velocity in x direction we require:

\[
\int_0^{\eta_{in}(x,t)} u_{in}(x,y,t) \, dy = \alpha(x,t) \int_0^{\eta(x,t)} u_{in}(x, \frac{\eta_{in}(x,t)}{\eta(x,t)} y) \, dy
\]

from which we find that

\[
\alpha(x,t) = \frac{\eta_{in}(x,t)}{\eta(x,t)}.
\]
We therefore replace the RF solution at and near the boundary \( u_{in}(x,y,t) \) which is valid on \( 0 \leq y \leq \eta_{in}(x,t) \) by:

\[
u_{in}^*(x,y,t) = \frac{\eta_{in}(x,t)}{\eta(x,t)} \frac{\eta_{in}(x,t)}{\eta_{in}(x,t)} u_{in}(x, \frac{\eta_{in}(x,t)}{\eta(x,t)} y, t)\]

which is defined in the range \( 0 \leq y \leq \eta(x,t) \).

As there is no mass flux in vertical direction over the (vertical) weakly reflecting boundary we can choose another condition to be satisfied by the vertical velocities at the boundary. We chose conservation of vertical momentum flux over the boundary:

\[
\int_0^{\eta(x,t)} u_{in}(x,y,t) v_{in}(x,y,t) \, dy = \alpha \beta \int_0^{\eta(x,t)} u_{in}(x, \frac{\eta_{in}(x,t)}{\eta(x,t)} y, t) v_{in}(x, \frac{\eta_{in}(x,t)}{\eta(x,t)} y, t) \, dy
\]

from which we find \( \beta = 1 \). We therefore replace \( v_{in}(x,y,t) \) by \( v_{in}(x, \frac{\eta_{in}(x,t)}{\eta} y, t) \).

**Momentum flux conservation**

Here the equations for the momentum flux become:

\[
\int_0^{\eta(x,t)} u_{in}^2(x,y,t) \, dy = \alpha^2 \int_0^{\eta(x,t)} \frac{\eta_{in}(x,t)}{\eta(x,t)} y, t) \, dy
\]

\[
\int_0^{\eta_{in}(x,t)} u_{in}(x,y,t) v_{in}(x,y,t) \, dy = \alpha u_{in}(x, \frac{\eta_{in}(x,t)}{\eta(x,t)} y, t) \beta v_{in}(x, \frac{\eta_{in}(x,t)}{\eta(x,t)} y, t) \, dy
\]

from which we find \( \alpha = \beta = \frac{\eta_{in}(x,t)}{\eta(x,t)} \).

We can therefore use as incoming velocity signals in SKYLLA:

\[
u_{in}^*(x,y,t) = \alpha u_{in}(x, \frac{\eta_{in}(x,t)}{\eta} y, t)
\]

\[
v_{in}^*(x,y,t) = \beta v_{in}(x, \frac{\eta_{in}(x,t)}{\eta} y, t)
\]
Appendix D

List of studied error causes
Aim:

- Find the errors that contribute to the deviations between the analytical signals and the signals derived with the numerical model after implementation of the weakly reflecting boundaries and the RF-WAVE signal as an input signal.

The errors and weak modules were traced by studying the following items:

- Grid dependence,
- Initial process and adjustment period,
- Viscosity effects,
- Choice for mass transport in RF solution,
- Stretching of the incoming wave (mass flux or momentum flux as in RF),
- Choice of small \( v \) at the free surface (in RF solution \( v \) is zero),
- Influence of upwind fraction in discretisation of the momentum equations,
- Setting velocities just outside the fluid domain:
  - Copying from within the fluid domain,
  - Linear extrapolation from within the fluid domain,
  - Combination of dynamic and kinematic boundary condition,
  - Combination of zero curl and incompressibility,
- Replacing the simple wave equation for \( \eta \) at the boundaries by the kinematic boundary condition in the case of outflow,
- Influence of the chosen velocity in the simple wave equations used at the boundary,
  - Velocity based on linear dispersion relation,
  - Velocity as in RF,
  - Velocity calculated from quantities inside the fluid domain (Orlanski).
Appendix E

Description of adapted surface treatment
Aim:

- Improvement of the surface treatment.

The treatment of the surface can be improved by prescribing new surface boundary conditions.

The conditions we want to impose at the free surface are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

and

\[
\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad (2)
\]

All discretisations used to determine the virtual velocities at the free surface are first order accurate.

The notation for the staggered velocity components in SKYLLA is shown in Fig. 0 for the case of cell (i,j).

![Figure 0](image-url)
Empty cells at top:

![Diagram of empty cells at top]

Figure 1

We discretise Eq.(1) at the centre of cell (i,j).

\[ \Delta x_i (v_{ij} - v_{ij-1}) + \Delta y_j (u_{ij} - u_{i-1,j}) = 0 \]  

(3)

From this equation \( v_{ij} \) can directly be determined.

At \((x_i, y_j)\) we discretise Eq.(2):

\[ \frac{u_{ij+1} - u_{ij}}{\frac{1}{2} \Delta y_{j+1} + \Delta y_j} - \frac{(\Delta y_j + \Delta y_{j-1})(v_{i+1,j-1} - v_{ij-1}) - \Delta y_j (v_{i+1,j-2} - v_{ij-2})}{\frac{1}{2} \Delta y_{j-1} (\Delta x_{i+1} + \Delta x_i)} = 0 \]  

(4)

From this equation we can solve \( u_{ij+1} \).
Empty cell at top:

Here \( v_{ij} \) can now directly be determined from Eq. (3)

Empty cell at top and left:

Eq. (2) is now discretised in \( (x_{ij}, y_{j-1} + \frac{1}{2} \Delta y_j) \).

\[
\frac{\partial u}{\partial y} = u_y = \frac{(h_1 + h_2)^2 (u_{ij} - u_{ij-1}) - h_1^2 (u_{ij} - u_{ij-2})}{h_1 h_2 (h_1 + h_2)}
\]
where we have defined:

\[ h_1 = \frac{1}{2}(\Delta y_j + \Delta y_{j-1}) \quad \text{and} \quad h_2 = \frac{1}{2}(\Delta y_{j-1} + \Delta y_{j-2}). \]

\[ \frac{v_{i+1,j} - v_{i,j} + v_{i,j+1} - v_{i,j-1}}{\Delta x_{i+1} - \Delta x_i} - u_y = 0 \]

From this equation \( v_{ij} \) can now be determined. 
\( u_{i-1,j} \) is found from Eq. (3).

**Two empty cells at top and one left:**

![Figure 4](image)

we define:

\[ m_1 = \frac{1}{2}(\Delta x_i + \Delta x_{i+1}) \]

\[ m_2 = \frac{1}{2}(\Delta x_{i-1} + \Delta x_i) \]

\[ \frac{\partial v}{\partial x} = v_x = \frac{(m_1 + m_2)^2(v_{i+1,j-1} - v_{j-1}) - m_1^2(v_{i,j-1} - v_{j-1})}{m_1 m_2(m_1 + m_2)} \]

The discretisation of Eq.(2) now becomes:

\[ \frac{u_{i-1,j} - u_{i,j-1} + u_{i+1,j} - u_{i,j-1}}{\Delta y_j + \Delta y_{j-1}} - v_x = 0 \]

from which \( u_{i-1,j} \) can be determined.
$v_{i-1j}$ can now be determined from Eq. (3).

We redefine $v_x$.

$$v_x = \frac{(v_{i+1j-1} - v_{ij-1})(\Delta y_j + \Delta y_{j-1}) - (v_{i+1j-2} - v_{ij-2}) \Delta y_j}{m_1 \Delta y_{j-1}}$$

$u_{ij+1}$ is now determined from a discretisation of Eq.(2):

$$\frac{u_{ij+1} - u_{ij}}{1/2 (\Delta y_{j+1} + \Delta y_j)} - v_x = 0$$

Three empty cells top-right corner:

![Diagram](image)

Figure 5

We define:

$$n_1 = \frac{1}{2} (\Delta x_i + \Delta x_{i-1})$$

$$n_2 = \frac{1}{2} (\Delta x_{i-1} + \Delta x_{i-2})$$

$$\frac{\partial v}{\partial x} = v_x \left( \frac{(n_1 + n_2)(v_{ij-1} - v_{i-1j-1}) - n_1^2 (v_{ij-1} - v_{i-2j-1})}{n_1 n_2 (n_1 + n_2)} \right)$$
The discretisation of Eq.(2) becomes:

\[
\frac{u_{ij} - u_{i-1,j} + u_{i+1,j} - u_{i,j-1}}{\Delta y_j + \Delta y_{j-1}} - v_x = 0
\]

From which we find \( u_{ij} \).

\( v_{ij} \) can be found from Eq. (3).

**Two empty cells top-right corner:**

![Diagram of two empty cells top-right corner](image)

Figure 6

We define:

\[
n_1 = \frac{1}{2} (\Delta x_i + \Delta x_{i-1})
\]

\[
m_1 = \frac{1}{2} (\Delta y_j + \Delta y_{j-1})
\]

\[
m_2 = \frac{1}{2} (\Delta y_{j-1} + \Delta y_{j-2})
\]

\[
\frac{\partial u}{\partial y} = u_y = \frac{((m_1 + m_2)^2 - m_1^2)u_{i-1,j} - (m_1 + m_2)^2u_{i-1,j-1} + m_1^2u_{i-1,j-2}}{m_1m_2(m_1 + m_2)}
\]

The discretisation of Eq.(2) now becomes:

\[
\frac{v_{ij} - v_{i-1,j} + v_{ij-1} - v_{i-1,j-1}}{2n_1} - u_y = 0
\]
From this equation we can solve $v_{ij}$.

$u_{ij}$ can be found from Eq. (3).

**Empty cell at the right:**

\[
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\end{array}
\]

Figure 7

We define:

\[
\begin{align*}
n_0 &= \frac{1}{2} (\Delta x_{i,1} + \Delta x_i) \\
n_1 &= \frac{1}{2} (\Delta x_i + \Delta x_{i,-1}) \\
n_2 &= \frac{1}{2} (\Delta x_{i,-1} + \Delta x_{i,-2}) \\
m_1 &= \frac{1}{2} (\Delta y_{j-1} + \Delta y_{j-1}) \\
m_2 &= \frac{1}{2} (\Delta y_{j-1} - \Delta y_{j-2})
\end{align*}
\]

$u_{ij}$ can directly be found from Eq.(3).

\[
\frac{\partial v}{\partial x} = v_x = \frac{1}{2} \left[ \frac{n_1^2 v_{i+1j} - (n_1^2 - n_0^2) v_{ij} - n_0^2 v_{i-1j}}{n_0 n_1 (n_0 + n_1)} + \frac{((n_1 + n_2)^2 - n_1^2) v_{ij-1} - (n_1 + n_2)^2 v_{i-1j-1} + n_1^2 v_{i-2j-1}}{n_1 n_2 (n_1 + n_2)} \right]
\]
which contains the unknown $v_{i+1j}$.

\[
\frac{\partial u}{\partial y} = u_y = \left( \frac{1}{2} \frac{\Delta x_i}{\Delta x_{i-1}} + 1 \right) \frac{((m_1 + m_2)^2 - m_1^2)u_{i-1j} - (m_1 + m_2)^2u_{i-1j-1} + m_1^2u_{i-1j-2}}{m_1m_2(m_1 + m_2)}
\]

\[
\frac{1}{2} \frac{\Delta x_i}{\Delta x_{i-1}} \frac{((m_1 + m_2)^2 - m_1^2)u_{i-1j} - (m_1 + m_2)^2u_{i-2j-1} + m_1^2u_{i-2j-2}}{m_1m_2(m_1 + m_2)}
\]

From

\[v_x - u_y = 0\]

we can now solve $v_{i+1j}$.

At the boundary we can redefine $u_y$ and $v_x$:

\[
v_x := \frac{v_{i+1j} - v_{ij}}{n_0}
\]

\[
u_y := \frac{u_{i-1j} - u_{i-1j-1}}{m_1}
\]

Empty cell at the left:

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</tbody>
</table>
```

Figure 8
We define:

\[ n_0 := \frac{1}{2} (\Delta x_{i-1} + \Delta x_i) \]

\[ n_1 := \frac{1}{2} (\Delta x_i + \Delta x_{i+1}) \]

\[ n_2 := \frac{1}{2} (\Delta x_{i+1} + \Delta x_{i+2}) \]

\[ m_1 := \frac{1}{2} (\Delta y_i + \Delta y_{i-1}) \]

\[ m_2 := \frac{1}{2} (\Delta y_{i-1} + \Delta y_{i-2}) \]

\( u_{i-1j} \) can be solved from Eq.(3).

\[
\frac{\partial v}{\partial x} = v_x := \frac{1}{2} \left[ \frac{n_0^2 v_{1+i,j} - (n_0^2 - n_1^2) v_{i,j} - n_1^2 v_{i-1,j} +}{n_0 n_1 (n_0 + n_1)} \right]
\]

\[
+ \frac{(n_1 + n_2)^2 v_{1+i,j-1} - n_1^2 v_{1+i,j-1} + (n_1^2 - (n_1 + n_2)^2) v_{i,j-1}}{n_1 n_2 (n_1 + n_2)} \]

\[
\frac{\partial u}{\partial y} = u_y := \left( \frac{1}{2} \frac{\Delta x_i + 1}{\Delta x_{i+1}} \right) \frac{((m_1 + m_2)^2 - m_1^2) u_{i,j} - (m_1 + m_2)^2 u_{i-1,j} + m_1^2 u_{i-1,j-2}}{m_1 m_2 (m_1 + m_2)}
\]

\[
- \frac{1}{2} \frac{\Delta x_i}{\Delta x_{i+1}} \frac{((m_1 + m_2)^2 - m_1^2) u_{i+1,j} - (m_1 + m_2)^2 u_{i+1,j-1} + m_1^2 u_{i+1,j-2}}{m_1 m_2 (m_1 + m_2)}
\]

From

\[ v_x - u_y = 0 \]

we can find \( v_{i-1j} \).
Appendix F

Test report "modified boundaries"
Test documentation

Aim:

- Testing whether the model SKYLLA gives satisfactory results after the implementation of the weakly reflecting boundaries and the implementation of input signals derived from the program RF-WAVE and an adapted surface treatment.

This appendix describes a series of relevant tests starting from the implementation of the weakly reflecting boundary and the implementation of input signals derived from the program RF-WAVE.

State of SKYLLA before the implementation of these extensions:

- The model contained a weakly reflecting boundary at one side (left) of the computational domain (one open boundary). For many purposes, the model has to deal with a second open boundary at the right hand side as well. This boundary has to be weakly reflecting as well.
- The model contained an input signal with a simple single-sinusoidal shape. This is not realistic since in reality waves do not have such a simple shape; they can be seen as a summation of sinusoidal signals with different phases and different amplitudes. The input (regular) wave train that is being used in SKYLLA must be more realistic. The program RF-WAVE generates such a realistic wave. This is implemented.

The implementation of these two extensions at the boundaries can be compared with the theoretically correct signal from the program RF-WAVE. However, differences between the signals generated by SKYLLA and the signals from RF-WAVE can be caused by other inaccuracies than the above mentioned extensions of the program. Therefore, all other inaccuracies must be diminished to a satisfactory degree before a fair verification of the two above mentioned extensions can be made.

The implementation of these two extensions enables to verify the program SKYLLA with a theoretically derived wave shape. Certain aspects of the model have been verified before but this verification is in fact the first verification of the complete program SKYLLA. Therefore, all existing inaccuracies will surface.

The implementation of the extensions have been described in Appendix A, B, C and E.

TEST-CASE 1 shows a computation with the implementation of the two above mentioned aspects. The comparison with the RF-WAVE signal is not satisfactory.

Various tests indicated that this not-functioning of the model is a combination of several inaccuracies. After checking of the source-code and checking of the existing modules of the model it was concluded that the most likely sources of the inaccuracies were a combination of the modules "RF-WAVE input wave train", "Weakly reflecting boundaries" and the "Surface treatment". Inaccuracies generated in one of these modules effect the other modules. Therefore, it is extremely difficult to trace these inaccuracies.
First the "RF-WAVE input wave train" is dealt with. The RF-WAVE program generates velocities from the bottom to the free surface which is the free surface calculated by RF-WAVE. However, this RF-WAVE free surface may differ from the free surface in the SKYLLA model. A reflected wave causes a different free surface than the free surface corresponding to the incoming wave. This had to be dealt with. A stretching method of the velocity profile has been proposed. This stretching method stretches the RF-WAVE velocity profile to the actual free surface. However, this method does not allow both conservation of mass and conservation of momentum. Although the differences may be small, one of these had to be chosen. This choice is based on TEST-CASES 2.1-2.2. Conservation of momentum has been chosen. The test-cases 2, presented here, have been done with a treatment of the surface that was found to be the best in a later stage. The choice of this treatment is described now.

The "Surface treatment" is discussed now. Several treatments of the surface have been studied. The main problem in the surface treatment is that for the discretisation of several equations, derivatives at the surface need to be determined. For these derivatives velocities at positions outside the fluid are needed. To determine these velocities additional information (equations) is necessary. This can be done in several ways:

1) Copying velocities from inside the fluid to these positions just outside the fluid domain.
2) Extrapolation of velocities from the fluid domain to these positions.
3) The use of a kinematic boundary equation and a dynamic boundary equation, see Note Petit (1993).
4) Using the assumption that the fluid (near the surface) is free of rotation, see Appendix E.

The first method was the original treatment. These four methods have been compared in TEST CASE 3.1-3.4. The fourth method has been chosen.

The fourth method for the surface treatment has the disadvantage that this assumption is not true in area where the wave is nearly or already breaking. Therefore, this method need to be combined with another method. It is proposed to use the first method (copying velocities) in this area because previous computations of breaking waves provide a realistic process. Whether this is correct can only be verified after verification of a breaking wave using a physical model. TEST-CASE 4 shows a combination of these two methods where the left hand side is treated with the fourth method and the right hand side with the first method.

The fourth treatment of the surface treatment has been studied by varying several parameters in similar situations. The results can be compared with the RF-WAVE signal and mutual, see TEST-CASE 5.1-5.9.

To exclude the effects of the adjustment time, a computation has been done with the initial free surface and velocities derived from RF-WAVE, see TEST-CASE 6.

It is concluded that the implementation of the RF-WAVE signal as an input wave-train and two weakly reflecting boundaries works satisfactorily for these principle tests.
Another test-case with a more practical situation has been performed. See TEST-CASE 7. A submerged structure is modelled with a front slope of 1:1 and a similar structure with a front slope 1:5. Figure 7.1 C shows that after nine wave periods the mass flux through the boundaries do not become periodical. Figure 7.1 D shows that the surface elevations are not periodical either; differences of a few centimetres per wave period occur. This somewhat disappointing result is probably caused by the influence of relatively large reflections caused by the steep slope (1:1). A more gentle sloping section is implemented for a new computation: 1:5. Figure 7.2 C shows a relatively large improvement of the results; after four wave periods the signals are not periodical but the differences are much smaller. For most practical purposes the differences are diminished to an acceptable degree. However, it should be verified whether the differences are acceptable for the intended application.

It can be concluded that a more correct incident wave is generated, that the treatment of the free surface is improved and that a second weakly reflecting boundary is implemented. The combination of the two weakly reflecting boundaries (two permanently open boundaries) is however, causing differences that might be too large for some applications.
TEST CASE 1

Description of the test: "Test with 'original' surface treatment"

Input data:

- Surface treatment: 1) Copying
- Input wave train: RF-WAVE with cons. of momentum
- Grid: Equidistant
- Spacestepl Δx: 0.06 m
- Timestep Δt: 0.001 s
- Viscosity ν: 0.025 m²/s
- Wave height H = 0.2 m
- Wave period T = 3.0 s
- Wave length L = 8.835 m
- Water depth h = 1.0 m
- Length of the flume: 0.30 m
- Height of the flume: 1.60 m
- Number of cells in x-direction: 7 (including 2 virtual cells)
  Δx = 0.30/(7-2) = 0.06
- Number of cells in y-direction: 40 (including 2 virtual cells)
  Δy = 0.02 at SWL
- Number of cells in free surface y-range: 10
- Computation period t_e: 30.0 s
- Upwind fraction β: 0.30

The computation has been done without slope.

Results test case 1

Figure 1.A shows the difference between the free surface from the RF-WAVE signal that is imposed as an input wave train in the model and the free surface from SKYLLA as a function of time.

Figure 1.B shows the free surface elevation at the left and the right boundary (calculated by SKYLLA) and the difference between the RF-WAVE signal and the SKYLLA signal at the left boundary as a function of time.
SURFACE ELEVATION AT LEFT AND
RIGHT BOUNDARY AND DIFFERENCE
BETWEEN RF WAVE AND SKYLLA
dt = .001
TEST CASE 1
DELFIT HYDRAULICS
CP_1A FIG 1B
TEST CASES 2.1-2.2

Description of the test: "Comparison of stretching methods"

Input data:

- Surface treatment: 4) Free of rotation
- Input wave train: RF-WAVE
- Grid: Equidistant
- Spacstep $\Delta x$: 0.06 m
- Timestep $\Delta t$: 0.001 s
- Viscosity $\nu$: 0.025 m$^2$/s
- Wave height $H=0.2$ m
- Wave period $T=3.0$ s
- Wave length $L=8.835$ m
- Water depth $h=1.0$ m
- Length of the flume: 0.30 m
- Height of the flume: 1.60 m
- Number of cells in x-direction: 7 (including 2 virtual cells)
  $\Delta x = 0.30/(7-2) = 0.06$
- Number of cells in y-direction: 40 (including 2 virtual cells)
  $\Delta y = 0.02$ at SWL
- Number of cells in free surface y-range: 10
- Computation period $t_c$: 30.0 s
- Upwind fraction $\beta$: 0.30

The computation has been done without slope.

Results test cases 2.1-2.2

TEST 2.1 is done with conservation of momentum as the stretching method.
TEST 2.2 is done with conservation of mass as the stretching method.

Figures 2.1A and 2.2A show the difference between the free surface from the RF-WAVE signal that is imposed as an input wave train in the model and the free surface from SKYLLA as a function of time.
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DELFIT HYDRAULICS

dt = .005
TEST CASE 2.1

RT_1 FIG 2.1A
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DELFT HYDRAULICS

\( dt = 0.005 \)

TEST CASE 2.2

RT.2 FIG 2.2A
TEST CASES 3.1-3.4

Description of the test: "Comparison of methods of surface treatment"

Input data:

- Surface treatment : Varied
- Input wave train : RF-WAVE with cons. of momentum
- Grid : Equidistant
- Space step $\Delta x$ : 0.06 m
- Time step $\Delta t$ : 0.001 s
- Viscosity $\nu$ : 0.025 $m^2/s$

- Wave height $H = 0.2$ m : $a = 0.1$ m
- Wave period $T = 3.0$ s : $\omega = 2\pi/T = 2.094$ s$^{-1}$
- Wave length $L = 8.835$ m : $k = 2\pi/L = 0.71$ m$^{-1}$
- Water depth $h = 1.0$ m
- Length of the flume : 0.30 m
- Height of the flume : 1.60 m
- Number of cells in x-direction : 7 (including 2 virtual cells)
  $\Delta x = 0.30/(7-2) = 0.06$
- Number of cells in y-direction : 40 (including 2 virtual cells)
  $\Delta y = 0.02$ at SWL
- Number of cells in free surface y-range : 10
- Computation period $t_e$ : 30.0 s
- Upwind fraction $\beta$ : 0.30

The computation has been done without slope.

Results test cases 3.1-3.4

TEST 3.1 is done with 1) copying as the method for the surface treatment.
TEST 3.2 is done with 2) extrapolation as the method for the surface treatment.
TEST 3.3 is done with 3) kin+dyn as the method for the surface treatment.
TEST 3.4 is done with 4) free of rotation as the method for the surface treatment.

Figures 3.A show the difference between the free surface from the RF-WAVE signal that is imposed as an input wave train in the model and the free surface from SKYLLA as a function of time. Note that the vertical scale is adjusted.
Figures 3.B show the free surface elevation at the left and the right boundary (calculated by SKYLLA) and the difference between the RF-WAVE signal and the SKYLLA signal at the left boundary as a function of time.
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

Delft Hydraulics

dt = .001
Test Case 3.1
CP_1A
FIG 3.1A
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DELFT HYDRAULICS

dt = .001
TEST CASE 3.2
EX_1A FIG 3.2A
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLLA

dt = .001

TEST CASE 3.2

DELFIT HYDRAULICS

EX_1A FIG 3.2B
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKULLA AS
A FUNCTION OF TIME

DELFT HYDRAULICS

\[ dt = 0.001 \]

TEST CASE 3.3

KIN_1A FIG 3.3A
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLLA

DELFT HYDRAULICS
DIFFERENCE BETWEEN SURFACE ELEVATION FROM RF WAVE AND SKYLLA AS A FUNCTION OF TIME

DELFT HYDRAULICS

dt = .001
TEST CASE 3.4
RT.1A FIG 3.4A
TEST CASE 4

Description of the test: "Combination of two methods for the surface treatment"

**Inputdata:**

<table>
<thead>
<tr>
<th>Surface treatment</th>
<th>1) Copying (right) and 4) Free of rotation (left)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input wave train</td>
<td>RF-WAVE with cons. of momentum</td>
</tr>
<tr>
<td>Grid</td>
<td>Equidistant</td>
</tr>
<tr>
<td>Spacstep $\Delta x$</td>
<td>0.06 m</td>
</tr>
<tr>
<td>Timestep $\Delta t$</td>
<td>0.001 s</td>
</tr>
<tr>
<td>Viscosity $\nu$</td>
<td>0.025 $m^2/s$</td>
</tr>
<tr>
<td>Wave height $H=0.2$ m</td>
<td>$a=0.1$ m</td>
</tr>
<tr>
<td>Wave period $T=3.0$ s</td>
<td>$\omega=2\pi/T=2.094 , s^{-1}$</td>
</tr>
<tr>
<td>Wave length $L=8.835$ m</td>
<td>$k=2\pi/L=0.71 , m^{-1}$</td>
</tr>
<tr>
<td>Water depth $h=1.0$ m</td>
<td></td>
</tr>
<tr>
<td>Length of the flume</td>
<td>0.60 m</td>
</tr>
<tr>
<td>Height of the flume</td>
<td>1.60 m</td>
</tr>
<tr>
<td>Number of cells in x-direction</td>
<td>14 (including 2 virtual cells) $\Delta x=0.60/(14-2)=0.05$</td>
</tr>
<tr>
<td>Number of cells in y-direction</td>
<td>40 (including 2 virtual cells) $\Delta y=0.02$ at swl</td>
</tr>
<tr>
<td>Number of cells in free surface y-range</td>
<td>10</td>
</tr>
<tr>
<td>Computation period $t_c$</td>
<td>30.0 s</td>
</tr>
<tr>
<td>Upwind fraction $\beta$</td>
<td>0.30</td>
</tr>
</tbody>
</table>

The computation has been done without slope.

**Results test case 4**

TEST-CASE 4.1 can be compared with TEST-CASE 3.4.

Figures 4.A show the difference between the free surface from the RF-WAVE signal that is imposed as an input wave train in the model and the free surface from SKYLLA as a function of time.

Figures 4.B show the free surface elevation at the left and the right boundary (calculated by SKYLLA) and the difference between the RF-WAVE signal and the SKYLLA signal at the left boundary as a function of time.
DIFFERENCE BETWEEN SURFACE ELEVATION FROM RF WAVE AND SKYLLA AS A FUNCTION OF TIME

dt = .001
TEST CASE 4.1

DELFt HYDRAULICS

RTCP_1A FIG 4.1A
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLLA

dt = .001

TEST CASE 4.1

RTCP_1A FIG 4.1B
DIFFERENCE BETWEEN SURFACE ELEVATION FROM RF WAVE AND SKYLLA AS A FUNCTION OF TIME

DELFT HYDRAULICS

dt = .001

TEST CASE 4.2

RTCP_L1A  FIG 4.2A
SURFACE ELEVATION AT LEFT AND
RIGHT BOUNDARY AND DIFFERENCE
BETWEEN RF WAVE AND SKYLLA

DELFT HYDRAULICS

$dt = 0.001$

TEST CASE 4.2

RTCP_L1A FIG 4.2B
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DELFT HYDRAULICS

dt = .001
TEST CASE 4.3
RTCP_L2A FIG 4.3A
TEST CASES 5.1-5.9

Description of the test: "Sensitivity of results on several parameters".

**Inputdata:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface treatment</td>
<td>4) Free of rotation</td>
</tr>
<tr>
<td>Input wave train</td>
<td>RF-WAVE with cons. of momentum</td>
</tr>
<tr>
<td>Grid</td>
<td>Varied</td>
</tr>
<tr>
<td>Spacestep Δx</td>
<td>Varied</td>
</tr>
<tr>
<td>Timestep Δt</td>
<td>Varied</td>
</tr>
<tr>
<td>Viscosity ν</td>
<td>Varied.</td>
</tr>
<tr>
<td>Wave height H=0.2 m</td>
<td>a=0.1 m</td>
</tr>
<tr>
<td>Wave period T=3.0 s</td>
<td>ω=2π/T=2.094 s⁻¹</td>
</tr>
<tr>
<td>Wave length L=8.835 m</td>
<td>k=2π/L=0.71 m⁻¹</td>
</tr>
<tr>
<td>Water depth h=1.0 m</td>
<td></td>
</tr>
<tr>
<td>Length of the flume</td>
<td>0.30 m (except 5.8-5.9)</td>
</tr>
<tr>
<td>Height of the flume</td>
<td>1.60 m</td>
</tr>
<tr>
<td>Number of cells in x-direction</td>
<td>7 (including 2 virtual cells) (5.1-5.7)</td>
</tr>
<tr>
<td></td>
<td>Δx=0.30/(7-2)=0.06</td>
</tr>
<tr>
<td>Number of cells in y-direction</td>
<td>40 (including 2 virtual cells)</td>
</tr>
<tr>
<td></td>
<td>Δy=0.02 at swl</td>
</tr>
<tr>
<td>Number of cells in free surface y-range</td>
<td>10</td>
</tr>
<tr>
<td>Computation period t_c</td>
<td>30.0 s</td>
</tr>
<tr>
<td>Upwind fraction β</td>
<td>0.30</td>
</tr>
</tbody>
</table>

The computations have been done without slope.
<table>
<thead>
<tr>
<th>Test 5.1</th>
<th>Test 5.2</th>
<th>Test 5.3</th>
<th>Test 5.4</th>
<th>Test 5.5</th>
<th>Test 5.6</th>
<th>Test 5.7</th>
<th>Test 5.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>done with</td>
<td>done with</td>
<td>done with</td>
<td>done with</td>
<td>done with</td>
<td>done with</td>
<td>done with</td>
<td>done with</td>
</tr>
<tr>
<td>&quot;BASIC TEST&quot;</td>
<td>&quot;LARGER Δt&quot;</td>
<td>&quot;SMALLER Δt&quot;</td>
<td>&quot;SMALLER Δx&quot;</td>
<td>&quot;NOT EQUIDISTANT-SYMMETRIC&quot;</td>
<td>&quot;NOT EQUIDISTANT-NOT SYMMETRIC&quot;</td>
<td>&quot;SMALLER VISCOSITY&quot;</td>
<td>&quot;LARGER FLUME:L=λ&quot;</td>
</tr>
<tr>
<td>Grid</td>
<td>Grid</td>
<td>Grid</td>
<td>Grid</td>
<td>Grid</td>
<td>Grid</td>
<td>Grid</td>
<td>Grid</td>
</tr>
<tr>
<td>Spacestep Δx</td>
<td>Spacestep Δx</td>
<td>Spacestep Δx</td>
<td>Spacestep Δx</td>
<td>Spacestep Δx</td>
<td>Spacestep Δx</td>
<td>Spacestep Δx</td>
<td>Spacestep Δx</td>
</tr>
<tr>
<td>0.06 m.</td>
<td>0.06 m.</td>
<td>0.06 m.</td>
<td>0.025 m.</td>
<td>0.03 m at sides, 0.10 in the middle.</td>
<td>0.025 m left, 0.080 m right, 0.085 middle.</td>
<td>0.06 m.</td>
<td>0.06 m.</td>
</tr>
<tr>
<td>Timestep Δt</td>
<td>Timestep Δt</td>
<td>Timestep Δt</td>
<td>Timestep Δt</td>
<td>Timestep Δt</td>
<td>Timestep Δt</td>
<td>Timestep Δt</td>
<td>Timestep Δt</td>
</tr>
<tr>
<td>0.001 s.</td>
<td>0.001 s.</td>
<td>0.001 s.</td>
<td>0.001 s.</td>
<td>0.001 s.</td>
<td>0.001 s.</td>
<td>0.001 s.</td>
<td>0.001 s.</td>
</tr>
<tr>
<td>Viscosity μ</td>
<td>Viscosity μ</td>
<td>Viscosity μ</td>
<td>Viscosity μ</td>
<td>Viscosity μ</td>
<td>Viscosity μ</td>
<td>Viscosity μ</td>
<td>Viscosity μ</td>
</tr>
<tr>
<td>0.025 m²/s.</td>
<td>0.025 m²/s.</td>
<td>0.025 m²/s.</td>
<td>0.025 m²/s.</td>
<td>0.025 m²/s.</td>
<td>0.025 m²/s.</td>
<td>0.005 m²/s.</td>
<td>0.025 m²/s.</td>
</tr>
</tbody>
</table>
TEST 5.9 is done with "LARGER FLUME: L = 2λ"

Grid : Equidistant.
Spacestep Δx : 0.06 m.
Timestep Δt : 0.001 s.
Viscosity ν : 0.025 m²/s.

Results test cases 5.1-5.9

Figures 5.A show the difference between the free surface from the RF-WAVE signal that is imposed as an input wave train in the model and the free surface from SKYLLA as a function of time.

Figures 5.B show the free surface elevation at the left and the right boundary (calculated by SKYLLA) and the difference between the RF-WAVE signal and the SKYLLA signal at the left boundary as a function of time.

Figures 5.C show the mass flux through the left boundary and through the right boundary as a function of time.
DIFFERENCE BETWEEN SURFACE ELEVATION FROM RF WAVE AND SKYLLA AS A FUNCTION OF TIME

dt = .001

TEST CASE 5.1

DELFIT HYDRAULICS

RT_1A FIG 5.1A
MASS FLUX THROUGH THE LEFT AND RIGHT BOUNDARY

DT = .001
TEST CASE 5.1
DELFIT HYDRAULICS
FL_RT_1A FIG 5.1C
DIFFERENCE BETWEEN SURFACE ELEVATION FROM RF WAVE AND SKYLLA AS A FUNCTION OF TIME

Dt = .005
TEST CASE 5.2

DELFt HYDRAULICS
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLLA

dt = .005
TEST CASE 5.2

DELFIT HYDRAULICS

RT_1  FIG 5.2B
MASS FLUX THROUGH THE
LEFT AND RIGHT BOUNDARY

dt = .005

TEST CASE 5.2

DELFIT HYDRAULICS

FL_RT_1  FIG 5.2C
SURFACE ELEVATIONS AT THE
LEFT AND RIGHT BOUNDARY
dt = 0.005
TEST CASE 5.2
DELFIT HYDRAULICS
RT_1 FIG 5.2D
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DELFT HYDRAULICS

dt = .002
TEST CASE 5.3
RT_1B FIG 5.3A
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLIA

dt = .002

TEST CASE 5.3

DELFt HYDRAULICS

RT.1b  FIG 5.3B
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DELFT HYDRAULICS

dt = .001
TEST CASE 5.4
RT.3 FIG 5.4A
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

Delft Hydraulics
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY, AND DIFFERENCE BETWEEN RF, WAVE, AND SKYLLA.

\[ \Delta t = 0.001 \]

TEST CASE 6.5

FIG 5.38
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DELFT HYDRAULICS
SURFACE ELEVATION AT LEFT AND
RIGHT BOUNDARY AND DIFFERENCE
BETWEEN RF WAVE AND SKYLLA

DELF'T HYDRAULICS

dt = .001
TEST CASE 5.6
RT_4A FIG 5.6B
DIFFERENCE BETWEEN SURFACE ELEVATION FROM RF WAVE AND SKYLLA AS A FUNCTION OF TIME

dt = .001

TEST CASE 5.7

DELFt HYDRAULICS

RT.1C FIG 5.7A
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLLA

dt = .001
TEST CASE 5.7

DELFIT HYDRAULICS

RT.1C   FIG 5.7B
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

dt = .001

TEST CASE 5.8

DELFt HYDRAULICS

RT_L1A  FIG 5.8A
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLLA

DELFT HYDRAULICS
DIFFERENCE BETWEEN SURFACE ELEVATION
FROM RF WAVE AND SKYLLA AS
A FUNCTION OF TIME

DT = .005
TEST CASE 5.9

DELFIT HYDRAULICS
RT_L2  FIG 5.9A
SURFACE ELEVATION AT LEFT AND RIGHT BOUNDARY AND DIFFERENCE BETWEEN RF WAVE AND SKYLLA

DELFT HYDRAULICS

dt = .005
TEST CASE 5.9
RT.L2 FIG 5.9B
TEST CASE 6

Description of the test: "Initial situation from RF-WAVE"

Inputdata:

Surface treatment : 4) Free of rotation
Input wave train : RF-WAVE with cons. of momentum
Grid : Equidistant
Space step $\Delta x$ : 0.06 m
Timestep $\Delta t$ : 0.001 s
Viscosity $\nu$ : 0.025 m$^2$/s

Wave height $H=0.2$ m : $a=0.1$ m
Wave period $T=3.0$ s : $\omega=2\pi/T=2.094$ s$^{-1}$
Wave length $L=8.835$ m : $k=2\pi/L=0.71$ m$^{-1}$
Water depth $h=1.0$ m
Length of the flume : 0.30 m
Height of the flume : 1.60 m
Number of cells in x-direction : 7 (including 2 virtual cells)
$\Delta x=0.30/(7-2)=0.06$
Number of cells in y-direction : 40 (including 2 virtual cells)
$\Delta y=0.02$ at SWL
Number of cells in free surface y-range : 10
Computation period $t_c$ : 30.0 s
Upwind fraction $\beta$ : 0.30

The computation has been done without slope.

Results test case 6

This test can be compared with TEST-CASE 5.1.

Figure 6.A shows the difference between the free surface from the RF-WAVE signal that is imposed as an input wave train in the model and the free surface from SKYLLA as a function of time.
DIFFERENCE BETWEEN SURFACE ELEVATION FROM RF WAVE AND SKYLLA AS A FUNCTION OF TIME

DELFT HYDRAULICS

dt = .001
TEST CASE 6
BCROT18  FIG 6A
TEST CASE 7

Description of the test: "Demonstration test"

Input data:

- **Surface treatment**: Combination 1) Copying and 4) Free of rotation
- **Input wave train**: RF-WAVE with cons. of momentum
- **Grid**: Equidistant
- **Spacetime Δx**: left 0.02 m; middle 0.15 m; right 0.02 m
- **Timestep Δt**: 0.0035 s
- **Viscosity ν**: 0.025 m²/s

- **Wave height H=0.4 m**: a = 0.2 m
- **Wave period T=3.0 s**: \( \omega = \frac{2\pi}{T} = 2.094 \text{ s}^{-1} \)
- **Wave length L=9.22 m**: \( k = \frac{2\pi}{L} = 0.68 \text{ m}^{-1} \)
- **Water depth h=1.0 m**
- **Length of the flume**: 17.0 m
- **Height of the flume**: 1.50 m
- **Number of cells in x-direction**: 250
- **Number of cells in y-direction**: 50
- **Number of cells in free surface y-range**: 10
- **Computation period tₑ**: 30.0 s
- **Upwind fraction β**: 0.30

Test-case 7.1 has been done with a slope 1:1; test-case 7.2 with a slope 1:5.

![Sketch of the structure in test-case 7.2](image)

Results test case 7

Figures 7.C show the mass flux through the left boundary and through the right boundary as a function of time.

Figures 7.D show the surface elevations at the left and the right boundaries as a function of time.
Figures 7.E show the volume of water inside the computational domain as a function of time.

A few "snapshots" of TEST-CASE 7.2, with the slope of 1:5, are given. The scale in the vertical direction is about 40% smaller than in the horizontal direction.
MASS FLUX THROUGH THE LEFT AND RIGHT BOUNDARY

dt = .0035

TEST CASE 7.1

DELFt HYDRAULICS
SURFACE ELEVATIONS AT THE
LEFT AND RIGHT BOUNDARY

DELFT HYDRAULICS
MASS FLUX THROUGH THE
LEFT AND RIGHT BOUNDARY

DELFT HYDRAULICS

dt = .0035
TEST CASE 7.2
FLUX_TSC  FIG 7.2C
SURFACE ELEVATIONS AT THE
LEFT AND RIGHT BOUNDARY

DELFT HYDRAULICS

dt = .0035
TEST CASE 7.2
TST_TSC FIG 7.2D
VOLUME OF WATER IN THE
COMPUTATIONAL DOMAIN

DELFT HYDRAULICS

d = .0035
TEST CASE 7.2
FLUX_TSC FIG 7.2E
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