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DOI

[10.1016/j.actaastro.2018.08.008](https://doi.org/10.1016/j.actaastro.2018.08.008)

Publication date

2018

Document Version

Accepted author manuscript

Published in

Acta Astronautica

Citation (APA)

Vergaaij, M., & Heiligers, J. (2018). Time-optimal solar sail heteroclinic-like connections for an Earth-Mars cycler. *Acta Astronautica*, 152, 474-485. <https://doi.org/10.1016/j.actaastro.2018.08.008>

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IAC-17-C1,6,6,x37912

TIME-OPTIMAL SOLAR SAIL HETEROCLINIC-LIKE CONNECTIONS FOR AN EARTH-MARS CYCLER

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This paper investigates solar sail Earth-Mars cyclers, in particular cyclers between libration point orbits at the Earth-Moon L_2 point and the Sun-Mars L_1 point. In order to facilitate cyclers in as few Earth-Mars synodic periods as possible, the overall objective is to minimize the time of flight. These time-optimal cyclers are obtained by using a direct pseudospectral method and exploiting techniques from dynamical systems theory to obtain an initial guess. In particular, heteroclinic connections between the unstable and stable manifolds of the target libration point orbits at the Earth-Moon L_2 point and the Sun-Mars L_1 point are sought for. While such connections do not exist in the ballistic case, they can be achieved by complementing the dynamics with a solar sail and assuming a constant attitude of the sail with respect to the direction of sunlight. These trajectories are sub-optimal due to the assumed constant sail attitude as well as minor discontinuities in position and velocity at the linkage of the manifolds, which are overcome by transferring the initial guess to the direct pseudospectral optimal control solver. For near- to mid-term sails, results show time-optimal round-trip trajectories that span three synodic Earth-Mars periods, with a few months to one year stay times at the libration point orbits, depending on the time of departure within a five-month window. Through the propellant-less nature of solar sailing, these Earth-Mars cyclers can, in theory, be maintained indefinitely.

I. INTRODUCTION

The notion of solar sailing has been around for over a hundred years, with initial writings on the subject by Tsiolkovsky and his co-worker, Tsander, dating back to the 1920s¹. However, it was not until the 1960s that solar sailing gained momentum with the first design effort in 1976, when the Jet Propulsion Laboratory proposed a mission to rendezvous with comet Halley. This mission was shortly thereafter canceled due to the high risks involved with the deployment of the proposed 800 x 800 m² sail¹. It was only recently that the first successful missions were deployed, namely JAXA's IKAROS mission (2010)², NASA's NanoSail-D2 mission (2010)³, and The Planetary Society's LightSail-1 mission (2015)⁴. Contrary to conventional methods of propulsion, solar sailing does not rely on the expulsion of mass: solar sails generate continuous thrust by reflecting solar photons off a large, highly reflective membrane. As a propellant-less form of propulsion, they have in principle unlimited ΔV at their disposal, which enables high- ΔV and long-duration missions¹. Promising examples of such missions include highly non-Keplerian orbits⁵, multiple near-Earth asteroid rendezvous missions⁶, displaced geostationary orbits⁷, missions for the high-latitude observation of the Earth and Moon⁸⁻¹⁰ as well as other planets^{11,12}, and an interstellar he-

liopause probe mission¹³.

This paper aims at extending this list of high-potential solar sail missions with a solar sail Earth-Mars cycler. A cycler is a type of orbit that encounters two bodies and can be repeated after an integer number of synodic periods. The most well-known cycler is the Aldrin cycler: a ballistic trajectory that visits the Earth and Mars once every synodic Earth-Mars period (approximately 2.14 years)¹⁴. Though enabling fast round trips, the fly-by velocities at the planets are high for the ballistic Aldrin cycler, which can be reduced to some extent by using low-thrust propulsion¹⁵⁻¹⁸, including solar sail propulsion¹⁸. However, the work presented in this paper will be the first investigation into solar sail Earth-Mars cyclers that reduce the relative planet-spacecraft velocity to such extent that rendezvous with libration point orbits become feasible, allowing the spacecraft to dwell for extended periods of time in close proximity of Earth and Mars. In particular, cyclers between libration point orbits at the Earth-Moon L_2 point (EM- L_2) and the Sun-Mars L_1 point (SM- L_1) will be discussed. The former is chosen as a departure and arrival location in Earth's vicinity, because a space station at the EM- L_2 point has been proposed as departure location to Mars as well as to other interplanetary destinations¹⁹. A solar sail Earth-Mars

cycler, starting from EM-L₂, will therefore fit into ongoing research and will be of great benefit to the scientific exploration - and potential habitation - of Mars. For example, the cycler may provide a continuous (near)propellant-less cargo transport link between Earth and Mars.

The work in Ref. 20 has shown that solar sail heteroclinic connections between libration point orbits in the Sun-Earth and Sun-Mars systems are feasible for reasonable times of flight, but the extension to the Earth-Moon system and the return trajectory have not yet been investigated. Furthermore, in order to facilitate as many cycler round trips as possible within a given time span, the overall objective of this paper is to minimize the time of flight, which requires the solution to an optimal control problem. Techniques from dynamical systems theory similar to the ones in Ref. 20 are exploited to obtain an initial guess, which is subsequently optimized using a direct pseudospectral method to obtain the solution to the optimal control problem.

The structure of this paper is as follows. First, the elements of the dynamical model used in this work are discussed, which includes the circular restricted three-body problem, the solar sail model, and the modeling of fourth-body perturbations. Next, the problem at hand is defined, which comprises the target libration point orbits, the phasing of the problem and the associated optimal control problem. Subsequently, the method to obtain first guess solutions for the time-optimal cycler is introduced. This is followed by a presentation of the time-optimal results and a set of sensitivity analyses. The paper ends with the conclusions.

II. DYNAMICAL MODEL

The dynamical framework employed in this paper is that of the circular restricted three-body problem (CR3BP). Then, following the concept of the patched restricted three-body problem approximation^{21,22}, both the outbound and the inbound legs of the cycler are modeled as three patched phases. For the outbound transfer, these are:

1. Departure from a halo orbit at the EM-L₂ point in the Earth-Moon CR3BP (EM-CR3BP), up to the sphere of influence of the Earth.
2. Propagation of the state in the Sun-Earth CR3BP (SE-CR3BP).
3. Arrival at a halo orbit at the SM-L₁ point in the Sun-Mars CR3BP (SM-CR3BP).

The inbound transfer is the reverse of this sequence, denoted by phase 4, 5, and 6.

II.I Circular restricted three-body problem

The CR3BP describes the motion of an infinitesimally small mass, m , under the influence of two much larger masses, m_1 and m_2 , where m_1 is the larger mass of the two primaries. The gravitational influence of the small mass on the primaries is neglected and the two primaries are assumed to move in circular orbits around the barycenter of the system²³.

The reference frame employed, $A(x, y, z)$, is a synodic frame, rotating about the barycenter of the two primaries, see Fig. 1. The x -axis connects the two primaries and points from the origin to m_2 , the z -axis is in the direction of the angular momentum vector of the primaries, and the y -axis completes the right-handed reference frame. The frame rotates around the z -axis at a constant angular velocity, $\omega = \omega \hat{z}$. The position vectors from the first and second primaries to m are denoted by $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively.

Fig. 1: Schematic of the CR3BP.

The dynamics of the CR3BP are made dimensionless using the sum of the two primary masses as the unit of mass, the distance between the primaries, λ , as the unit of length, and $1/\omega$ as the unit of time, τ . The latter implies that one dimensionless orbital period of the primaries around the barycenter equals 2π . Using the mass ratio $\mu = m_2/(m_1 + m_2)$, the dimensionless masses of the primaries become $m_1 = 1 - \mu$ and $m_2 = \mu$, and their locations along the x -axis become $-\mu$ and $1 - \mu$, respectively. Values for μ , λ , and τ can be found in Tab. 1 for the Sun-Earth, Sun-Mars, and Earth-Moon CR3BPs.

Table 1: Circular restricted three-body problem parameters.

	SE-CR3BP	SM-CR3BP	EM-CR3BP
μ [-]	$3.003460 \cdot 10^{-6}$	$3.226835 \cdot 10^{-7}$	0.01215
λ [km]	$1.4960 \cdot 10^8$	$2.2794 \cdot 10^8$	384400
τ [s]	$5.0163 \cdot 10^6$	$9.4461 \cdot 10^{-6}$	$3.7570 \cdot 10^5$

The motion of a body in the CR3BP is prescribed

by the following system of equations of motion²⁴:

$$\ddot{\mathbf{r}} + 2\boldsymbol{\omega} \times \dot{\mathbf{r}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \nabla V = \mathbf{a}, \quad [1]$$

where the left-hand side represents the ballistic CR3BP, for which $\mathbf{r}$ is shown in Fig. 1 and V is the gravitational potential:

$$V = \frac{1 - \mu}{r_1} - \frac{\mu}{r_2}. \quad [2]$$

The right-hand side of Eq. [1] contains the perturbing acceleration, $\mathbf{a}$, which in this work includes the solar sail induced acceleration, $\mathbf{a}_s$, as well as fourth-body gravitational perturbations, $\mathbf{a}_4$, see Sections II.II and II.III, respectively, such that $\mathbf{a} = \mathbf{a}_s + \mathbf{a}_4$.

II.II Solar sail model

This paper adopts a perfectly reflecting (= ideal) sail, which assumes pure specular reflection of the incident radiation. The resulting solar sail induced acceleration vector then acts normal to the sail surface, in the direction of $\hat{\mathbf{n}}$ defined in Fig. 2, where $\hat{\mathbf{r}}_S$ is the unit vector from the Sun to the sail. The attitude of the sail is expressed through the cone angle, α , and the clock angle, δ , which fix the attitude of the sail *with respect to the Sun-sail line*:

$$\hat{\mathbf{n}}|_{\hat{\mathbf{r}}_S} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \sin(\delta) \\ \sin(\alpha) \cos(\delta) \end{bmatrix}, \quad [3]$$

which can be transformed to the CR3BP frame through:

$$\hat{\mathbf{n}} = \mathbb{R} \hat{\mathbf{n}}|_{\hat{\mathbf{r}}_S}, \quad [4a]$$

$$\mathbb{R} = \begin{bmatrix} \hat{\mathbf{r}}_s & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \end{bmatrix}, \quad [4b]$$

where definitions for $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\phi}}$ can be found in Fig. 2. The solar sail induced acceleration can then be defined as:

$$\mathbf{a}_s = a_s \hat{\mathbf{n}}, \quad [5]$$

where a_s is the magnitude of the solar sail acceleration. Both a_s and $\mathbf{r}_s$ are defined differently in, on the one hand, the Sun-Earth and Sun-Mars CR3BPs (hereafter in general referred to as a Sun-planet CR3BP) and on the other hand the EM-CR3BP. This is due to the difference in their relative size and because the inclusion of a solar sail acceleration makes the EM-CR3BP non-autonomous, while a Sun-planet CR3BP is time-independent. Therefore, the governing equations will be discussed separately for both systems.

Fig. 2: Attitude of the solar sail.

Sun-planet system

In a Sun-planet system, the Sun-sail line is given by $\mathbf{r}_1$, therefore $\hat{\mathbf{r}}_S = \hat{\mathbf{r}}_1$. This results in the following solar sail acceleration magnitude:

$$a_s = \beta \frac{1 - \mu}{r_1^2} (\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}})^2, \quad [6]$$

where it can be seen that the acceleration is proportional to the solar gravitational acceleration, $\frac{1 - \mu}{r_1^2}$, and is scaled by β , the lightness number. The lightness number is then defined as the ratio of solar radiation pressure and solar gravitational acceleration¹. Finally, the term $(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}})^2$ accounts for a reduction in the solar sail acceleration magnitude when the sail is not oriented perpendicular to the Sun-sail line.

Earth-Moon system

As the distance from the Sun to the Earth is much larger than the distance from the Earth to the Moon, the instantaneous direction of the Sun-sail line in the EM-CR3BP is assumed uniform throughout the Earth-Moon system, i.e., not dependent on the position within the EM-CR3BP. However, over time, the Sun moves around the EM-CR3BP once per synodic lunar period. When defining $\Omega_s = 0.9252$ as the ratio of the synodic lunar period and the sidereal lunar period, the direction of the Sun-sail line in the EM-CR3BP is defined as^{25,26}:

$$\hat{\mathbf{S}}(t) = \begin{bmatrix} \cos(\Omega_s t + \Omega_0) & -\sin(\Omega_s t + \Omega_0) & 0 \end{bmatrix}, \quad [7]$$

where Ω_0 is the angle between the x -axis and the Sun-sail line at $t = 0$, see Fig. 3. Note that Eq. [7] ignores the small inclination difference between the Sun-Earth and Earth-Moon orbital planes. The rotation matrix $\mathbb{R}$ in Eq. 4b to determine the attitude of the sail in the Earth-Moon system is then computed using $\hat{\mathbf{r}}_S = \hat{\mathbf{S}}$. In addition to a uniform direction of the Sun-sail line throughout the EM-CR3BP, the magnitude of the solar radiation pressure is also assumed uniform and equal to the solar radiation pressure at a solar distance equal to that of the Earth (1

astronomical unit, AU), resulting in:

$$a_s(t) = a_0 \left(\hat{\mathbf{S}}(t) \cdot \hat{\mathbf{n}} \right)^2, \quad [8]$$

with¹:

$$a_0 = \frac{\beta \frac{G m_{Sun}}{(1 \text{ AU})^2}}{\frac{\lambda}{\tau^2}}, \quad [9]$$

where G is the gravitational constant and m_{Sun} the mass of the Sun.

Fig. 3: Schematic to support the definition of the solar sail dynamics in the Earth-Moon CR3BP.

II.III Fourth-body perturbation

As stated at the beginning of this section, the dynamics of the problem at hand are defined in three different solar-sail perturbed CR3BPs. To ensure that the dynamics are more consistent across the different phases, fourth-body perturbations are included in Eq. 1. The perturbing acceleration from a fourth body is calculated as²⁷:

$$\mathbf{a}_4 = \frac{\partial \Omega_4}{\partial \mathbf{r}_4} \quad [10]$$

where

$$\Omega_4 = \mu_4 \left(\frac{1}{|\mathbf{r}_{s,4}|} - \frac{\mathbf{r} \cdot \mathbf{r}_4}{|\mathbf{r}_4|^3} \right) \quad [11]$$

where $\mathbf{r}_4$ is the position vector from the barycenter of the CR3BP to the fourth body and $\mathbf{r}_{s,4}$ the position vector from the sail to the fourth body. Subsequently, $\mathbf{r}_{s,4} = \mathbf{r}_4 - \mathbf{r}$. Finally, μ_4 is the dimensionless gravitational parameter of the fourth body, with values in Tab. 2.

The equations above can be applied to both the Sun-planet CR3BPs and the EM-CR3BP. However, not all bodies are included in each system: Mars is included as fourth body in the SE-CR3BP, the Earth in the SM-CR3BP, and the Sun in the EM-CR3BP.

To calculate the location of the fourth body in the CR3BP, $\mathbf{r}_4$, a different method is used for the Sun in the EM-CR3BP and Mars and the Earth in the SE- and SM-CR3BP, respectively. For the case of the Sun in the EM-CR3BP, the direction of the Sun at an instantaneous time t is given by Eq. 7, and the distance is assumed constant and equal to 1 AU (389.1779 in non-dimensional units). The location of Mars and the Earth in the SE- and SM-CR3BP is defined in Section II.IV.

Table 2: Fourth-body perturbation parameter.

	SE-CR3BP	SM-CR3BP	EM-CR3BP
Perturbing body	Mars	Earth	Sun
μ_4 [-]	$3.226827 \cdot 10^{-7}$	$3.003468 \cdot 10^{-6}$	328903

II.IV Ephemerides

In order to compute the fourth-body perturbation, the relative position of the planets has to be known. In this paper it is assumed that the bodies move in circular orbits about the barycenter of their respective CR3BPs and in the ecliptic plane. This allows for simplification of the ephemerides, by expressing the position of the planets in the heliocentric inertial frame.

To calculate the state vector of Mars and the Earth as fourth bodies in the CR3BP, $\mathbf{r}_4$, first, the state vector of the fourth body is given in its own CR3BP by:

$$\mathbf{x} = [\mathbf{r} \quad \dot{\mathbf{r}}]^T = [(1 - \mu) \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T, \quad [12]$$

which is transformed to the heliocentric inertial frame and subsequently to the target CR3BP, using the transformations given in Section II.V.

II.V Reference frame transformations

To ensure linkage between the three trajectory phases mentioned in the introduction of Section II (i.e., linkage between the three CR3BPs systems in their accompanying reference frames), transformations between these reference frames have to be carried out. First, to link phases 2 and 3 in the outbound leg of the cycler, a transformation from a Sun-planet CR3BP to a heliocentric inertial frame is required. To link phases 1 and 2, a transformation from the EM-CR3BP to the SE-CR3BP is required. Furthermore, transformations between the heliocentric inertial and CR3BP frames need to be defined to include the ephemerides in the dynamics. All required transformations will be discussed below.

Sun-planet CR3BP to heliocentric inertial frame

This section describes the transformation from the Sun-planet synodic frame, A (with $\mathbf{x} = [\mathbf{r} \ \dot{\mathbf{r}}]^T$), to a heliocentric inertial frame, $H(\tilde{X}, \tilde{Y}, \tilde{Z})$ (with $\tilde{\mathbf{X}} = [\tilde{\mathbf{R}} \ \dot{\tilde{\mathbf{R}}}]^T$), see Fig. 4. In the heliocentric reference frame, the $\tilde{X}$ -axis points towards the vernal equinox, the $\tilde{Z}$ -axis is directed perpendicular to the ecliptic plane, and the $\tilde{Y}$ -axis completes the right-handed reference frame. Note that the notation $(\dot{\cdot})$ denotes a parameter in dimensional units, including t , whose unit is in seconds after 1-1-2000 12:00 (noon).

The first step is to translate the barycenter of the Sun-planet CR3BP to the Sun:

$$\mathbf{r}' = \mathbf{r} + [\mu \ 0 \ 0]^T, \quad [13]$$

after which the state and time are dimensionalized using the variables from Tab. 1. Then,

$$\dot{\tilde{\mathbf{R}}} = \mathbb{T}_z(\phi) \left(\dot{\mathbf{r}} + \boldsymbol{\omega} \times \tilde{\mathbf{r}}' \right), \quad [14a]$$

$$\tilde{\mathbf{R}} = \mathbb{T}_z(\phi) \tilde{\mathbf{r}}' \quad [14b]$$

where $\mathbb{T}_z$ is the rotation matrix around the z -axis and ϕ the angle between the Sun-planet line and the vernal equinox:

$$\phi = \phi_0 + t, \quad [15]$$

where t is the non-dimensional time discussed in Section II.I and ϕ_0 the initial angle between the Sun-planet line and the vernal equinox at $t = 0$, corresponding to 1-1-2000 12:00 (noon), see Tab. 3.

Fig. 4: Heliocentric inertial frame $H(\tilde{X}, \tilde{Y}, \tilde{Z})$ and Sun-planet synodic reference frame $A(x, y, z)$.

Table 3: Initial angle between Sun-planet line and vernal equinox. Values for ϕ_0 are based on the NASA/JPL Horizons online ephemeris system²⁸.

	SE-CR3BP	SM-CR3BP	EM-CR3BP
ϕ_0 [°]	100.307	-0.986	-142.710

Heliocentric inertial frame to Sun-planet CR3BP

The reverse of the transformation in Section II.V.1 is defined as:

$$\tilde{\mathbf{r}}' = \mathbb{T}_z(-\phi) \tilde{\mathbf{R}}, \quad [16a]$$

$$\dot{\tilde{\mathbf{r}}}' = \mathbb{T}_z(-\phi) \left(\dot{\tilde{\mathbf{R}}} - \boldsymbol{\omega} \times \tilde{\mathbf{R}} \right), \quad [16b]$$

after which the state and time are non-dimensionalized using the units of the target CR3BP from Tab. 1, and

$$\mathbf{r} = \mathbf{r}' - [\mu \ 0 \ 0]^T. \quad [17]$$

Earth-Moon CR3BP to the Sun-Earth CR3BP

The transformation from the EM-CR3BP to the SE-CR3BP is achieved using an intermediate, dimensional, geocentric inertial reference frame, $I(\tilde{X}_I, \tilde{Y}_I, \tilde{Z}_I)$, with $\tilde{\mathbf{X}}_I = [\tilde{\mathbf{R}}_I \ \dot{\tilde{\mathbf{R}}}_I]^T$. Frame I is defined as follows: the $\tilde{X}_I$ -axis is in the direction of the vernal equinox, the $\tilde{Z}_I$ -axis is directed perpendicular to the ecliptic, and the $\tilde{Y}_I$ -axis completes the right-handed reference frame. Figure 5 supports the required transformations:

- First, from the EM-CR3BP (frame A_{EM} , with $\mathbf{x}_{EM} = [\mathbf{r}_{EM} \ \dot{\mathbf{r}}_{EM}]^T$) to the dimensional frame I , according to the transformation sequence as described in Eq. [13] to [14], using $\phi = \phi_{EM}$, defined in Fig. 5.
- Second, from the dimensional frame I to the SE-CR3BP (frame A_{SE} , with $\mathbf{x}_{SE} = [\mathbf{r}_{SE} \ \dot{\mathbf{r}}_{SE}]^T$), starting with transforming the state and time from step 1 to non-dimensional SE units, resulting in $\mathbf{X}_I = [\mathbf{R}_I \ \dot{\mathbf{R}}_I]^T$. Followed by:

$$\dot{\mathbf{r}}_{SE} = \mathbb{T}_z(-\phi_{SE}) \left(\dot{\mathbf{R}}_I - \boldsymbol{\omega} \times \mathbf{R}_I \right), \quad [18a]$$

$$\mathbf{r}' = \mathbb{T}_z(-\phi_{SE}) \mathbf{R}_I, \quad [18b]$$

$$\mathbf{r}_{SE} = \mathbf{r}' + [1 - \mu_{SE} \ 0 \ 0]^T. \quad [18c]$$

Fig. 5: Schematic to support the transformation from EM-CR3BP to SE-CR3BP, centered at the Earth.

III. PROBLEM DEFINITION

The ballistic CR3BP as discussed in Section II.I exhibits five libration points, which can be found by setting the time derivatives of the position vector to zero: $\ddot{\mathbf{r}} = \dot{\mathbf{r}} = 0$. Two collinear libration points are used in this work: EM- L_2 and SM- L_1 . A range of periodic orbits can be found in the vicinity of these libration points, but this paper focuses on northern halo orbits around these collinear libration points. The halo orbits used in this paper are found through a differential corrector method²⁹ using Ref. 30 for the initial guess of the initial conditions. The halo orbits are created assuming an out-of-plane amplitude of 20,000 km in the Earth-Moon system and 350,000 km in the Sun-Mars system.

The aim of this paper is to find a cycler trajectory that abides by the following:

- Outbound transfer from the EM- L_2 halo orbit to the SM- L_1 halo orbit.
- Remain in SM- L_1 halo orbit for at least one month.
- Inbound transfer back from SM- L_1 to EM- L_2 .
- Remain in EM- L_2 halo orbit for at least one month.
- Depart from EM- L_2 halo orbit exactly n_p synodic Earth-Mars periods after the initial departure date at the Moon and repeat.

If such a trajectory can be found with a solar sail, it is in theory possible to maintain a cycler between the EM- L_2 and SM- L_1 points indefinitely. The cycler would only be limited by the lifetime of the solar sail in the space environment. Note that the position of the Moon in the Sun-Earth system is likely not exactly the same after n_p synodic Earth-Mars periods. However, the relative size of the Earth-Moon system with respect to the Sun-Earth and Sun-Mars systems suggests that only small changes in the control profile will be able to accommodate this change in the initial conditions of the next outbound transfer.

Both the inbound and outbound transfers will first be optimized separately, after which they will be optimized simultaneously for an overall objective function, see Section III.II.

During all phases, the state of the spacecraft is expressed in frame $A(x, y, z)$ as:

$$\mathbf{x} = [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^T \quad [19]$$

and the controls as:

$$\mathbf{u} = [\alpha \ \delta]^T \quad [20]$$

where $0^\circ \leq \alpha \leq 90^\circ$ and $-180^\circ \leq \delta \leq 180^\circ$.

III.I Constraints

In order to ensure a continuous trajectory, smooth linkage of the phases is required for the states, controls, and time. Linkage for the states takes place in the heliocentric inertial frame, using the transformation described in Section II.V. Linkage for the controls can be done without a transformation, as the cone and clock angle are defined with respect to the Sun-sail line and thereby consistent if the states are correctly linked. Time linkage is done in dimensional time, as described in Section II.V. This results in the following set of linkage constraints:

$$\tilde{\mathbf{x}}_{f_{p_n}} = \tilde{\mathbf{x}}_{0_{p_{n+1}}} \quad [21a]$$

$$\mathbf{u}_{f_{p_n}} = \mathbf{u}_{0_{p_{n+1}}} \quad [21b]$$

$$\tilde{\mathbf{t}}_{f_{p_n}} = \tilde{\mathbf{t}}_{0_{p_{n+1}}} \quad [21c]$$

where p_n denotes phase n , subscript $(\cdot)_0$ the initial state of the phase, and subscript $(\cdot)_f$ the final state of the phase. During optimization of the separate outbound trajectory, $n = 1, 2$, and for the separate inbound trajectory, $n = 4, 5$. During optimization of the complete cycler, $n = 1, 2, 4, 5$.

Boundary constraints have to be defined such that the transfers depart from and arrive at the designated halo orbits. The exact departure/arrival position along the halo orbit is not determined a priori and is optimized using static optimization parameters, κ_{EM-L_2} and κ_{SM-L_1} , one for each halo orbit:

$$0 \leq \kappa_{EM-L_2} < P_{EM-L_2} \quad [22a]$$

$$0 \leq \kappa_{SM-L_1} < P_{SM-L_1}, \quad [22b]$$

where P is the period of the halo orbit. These parameters are used both during the optimization of the separate transfers and the optimization of the full cycler, but in a different way. During optimization of the separate transfers, for example, the boundary conditions for the outbound trajectory are defined as:

$$\mathbf{x}_{0_{p_1}} = \mathbf{x}_{EM-L_2}(\kappa_{EM-L_2}) \quad [23a]$$

$$\mathbf{x}_{f_{p_3}} = \mathbf{x}_{SM-L_1}(\kappa_{SM-L_1}), \quad [23b]$$

where, for example, $\mathbf{x}_{EM-L_2}(t)$ is the state along the halo orbit at time t (with $0 \leq t < P_{EM-L_2}$). A similar set of boundary conditions can be defined for the inbound trajectory, resulting in two *different* sets of optimized static parameters.

During the optimization of the full cycler, only one set of static parameters is used, which can be explained as follows. A value for κ determines the arrival position along the halo orbit, after which the dwell time at that halo orbit, t_d , in combination with the period of that halo orbit, determines the departure position along that same halo orbit. This results

in the following boundary conditions for the full cyc-
cler:

$$\mathbf{x}_{0_{p_1}}^O = \mathbf{x}_{EM-L_2}(\kappa_{EM-L_2} + t_{d_{Moon}} \mod P_{EM-L_2}) \quad [24a]$$

$$\mathbf{x}_{f_{p_3}}^O = \mathbf{x}_{SM-L_1}(\kappa_{SM-L_1}) \quad [24b]$$

$$\mathbf{x}_{0_{p_4}}^I = \mathbf{x}_{SM-L_1}(\kappa_{SM-L_1} + t_{d_{Mars}} \mod P_{SM-L_1}) \quad [24c]$$

$$\mathbf{x}_{f_{p_6}}^I = \mathbf{x}_{EM-L_2}(\kappa_{EM-L_2}), \quad [24d]$$

with

$$t_{d_{Mars}} = t_{0_{p_4}}^I - t_{f_{p_3}}^O, \quad [25a]$$

$$t_{d_{Moon}} = t_{0_{p_1}}^O + n_p \cdot P_{syn} - t_{f_{p_6}}^I, \quad [25b]$$

where n_p is the number of synodic Earth-Mars periods, given by P_{syn} , $(\cdot)^O$ denotes the outbound trajectory, and $(\cdot)^I$ the inbound trajectory. Note that t and P_{syn} have to be given in the same unit. Equation [25a] thus computes the dwell time at the SM-L₁ halo orbit as the difference between the arrival time of the outbound trajectory and the departure time of the inbound trajectory.

To guarantee that the resulting trajectory is continuous in time, extra constraints are enforced to ensure that the dwell time at the halo orbits is positive, such that arrival at the halo orbit occurs before departure.

III.II Objective function

As mentioned at the beginning of this section, the outbound- and inbound transfers will first be optimized separately, after which the resulting time-optimal transfers will be optimized together in one large optimal control problem spanning the full cyc-
cler. This means that different objective functions are required. For the separate transfers, the transfer time, t_t , is minimized:

$$J^O = t_t^O = t_{f_{p_3}} - t_{0_{p_1}}, \quad [26a]$$

$$J^I = t_t^I = t_{f_{p_6}} - t_{0_{p_4}}, \quad [26b]$$

where

$$t_t = t_{f_{p_3}} - t_{0_{p_1}}, \quad [27]$$

and for the full cyc-
cler the sum of the dwell times at the halo orbits is maximized:

$$J^{O+I} = -(t_{d_{Mars}} + t_{d_{Moon}}). \quad [28]$$

Note that when using Eq. [28], the total cyc-
cler time, i.e., the number of synodic periods, has to be fixed. In that case, maximizing the time spent at the halo orbits automatically minimizes the transfer times.

III.III Optimal control solver

The optimal control problem defined in the previ-
ous subsections is solved using a direct pseudospec-
tral method, implemented in the software package PSOPT³¹. The infinite dimensional optimal control problem is discretized into a finite number of col-
location points and Legendre polynomials are used to approximate and interpolate the time dependent variables at and between the collocation points. The resulting finite dimensional non-linear programming (NLP) problem will be solved using IPOPT, an open source interior point optimizer for NLP problems.

IV. FIRST GUESS TRANSFERS

As with all direct optimization methods, PSOPT requires a sufficiently accurate initial guess. This section will elaborate on the method used to find this guess. Note that the resulting first guesses are sub-optimal due to the following: a constant sail attitude is assumed, fourth body perturbations are neglected, and minor discontinuities at the linkage between the consecutive phases are allowed. These suboptimality will be overcome by PSOPT in Section V.

IV.I Solar sail assisted manifolds

As an initial guess, connections between the un-
stable and stable invariant manifolds of the target libration point orbits at the EM-L₂ point and SM-L₁ point are sought for. An invariant manifold can be seen as a set of trajectories that form a surface, where the trajectories either depart from or arrive asymptotically to n spaced locations along the periodic orbit. The invariant manifolds are generated by slightly perturbing the state along the periodic orbits and propagating the perturbed state over time. This perturbation has to be applied in the direction of the stable/unstable eigenvectors along the halo orbit, $\mathbf{v}$, which are determined from the monodromy matrix M . The monodromy matrix is produced by propagating the state transition matrix for one orbital period from $t = t_0$ to $t = t_0 + P$, thereby containing information about the stability of the entire halo orbit. Using the state transition matrix, the stable and unstable eigenvectors at time $t = t_i$, where $i = 1, 2, \dots, n$, can be computed from³²:

$$\mathbf{v}_i^S = \Phi(t_i, t_0) \mathbf{v}^S, \quad [29a]$$

$$\mathbf{v}_i^U = \Phi(t_i, t_0) \mathbf{v}^U, \quad [29b]$$

where $\mathbf{v}^S$ denotes the stable eigenvector, $\mathbf{v}^U$ the un-
stable eigenvector, and $\Phi(t_i, t_0)$ the state transition matrix from $t = t_0$ to $t = t_i$. The unstable eigenvector corresponds to the largest absolute real eigenvalue and the stable eigenvector to the smallest absolute real eigenvalue³².

The initial conditions for the stable and unstable manifolds at time $t = t_i$, $\mathbf{x}_i^S$ and $\mathbf{x}_i^U$ respectively, are then found by applying a small perturbation, ϵ , to the state of the orbit at that time:

$$\mathbf{x}_i^S = \mathbf{x}_i \pm \epsilon \frac{\mathbf{v}_i^S}{|\mathbf{v}_i^S|}, \quad [30a]$$

$$\mathbf{x}_i^U = \mathbf{x}_i \pm \epsilon \frac{\mathbf{v}_i^U}{|\mathbf{v}_i^U|}, \quad [30b]$$

The sign of the perturbation makes a distinction between the interior and exterior manifold³².

The unstable manifolds are generated by forward propagation of the perturbed states of the departure orbits. The stable manifolds are generated using backward propagation of the perturbed states along the arrival halo orbit. Connections in configuration space between the unstable and stable manifolds do not exist in the ballistic case, but can be achieved by complementing the dynamics with a solar sail, resulting in *solar sail dedicated sets*²².

The solar sail dedicated sets are composed of two segments, a ballistic segment and a solar-sail perturbed segment. This is required to ensure that the separate trajectories of the solar sail dedicated set still form a topological tube. If the trajectory would be perturbed by the sail at the halo orbit, the trajectories would fan out in different directions, resulting in a relatively chaotic set of trajectories. Therefore, the trajectory is built up as follows:

1. A ballistic segment up to a hyperplane defined by $\Sigma : x = x_p$, which is a plane perpendicular to the (x, y) -plane at a distance of x_p along the x -axis. The dynamics used are those in Eq. [1] with $\mathbf{a} = 0$. Note that for the EM-L₂ halo orbit $x_p = 1.2$ and for the SM-L₁ halo orbit $x_p = 0.9925$.
2. A segment where the attitude of the sail is controlled by a control law $R(\alpha, \delta, t)$, from surface $\Sigma : x = x_p$ up to $t = t_{end}$. This segment is created by propagating the final state from segment 1 using the dynamics in Eq. [1] with $\mathbf{a} = \mathbf{a}_s$ where $\mathbf{a}_s$ is given through Eq. [5] in combination with Eq. [6] (Sun-planet) or Eq. [8] (Earth-Moon).

For example, Fig. 6 shows the first segment and part of the second segment for an unstable solar sail dedicated set departing from SM-L₁ using $\beta = 0.05$, and a constant attitude control law with $\alpha = 50^\circ$ and $\delta = -90^\circ$.

IV.II Design technique

The technique used to find a feasible heteroclinic connection using the solar sail dedicated sets explained in Section IV.I consists of five steps as given

Fig. 6: Example of an unstable solar sail dedicated set departing from SM-L₁.

below for the outbound trajectory. A similar procedure applies to the inbound trajectory.

1. Create the departure unstable solar sail dedicated set from the EM-L₂ halo orbit, starting from $t = t_{dep}$ up to $t = t_{link}$. At $t = t_{link}$, linkage will occur between the Sun-Earth phase and the Sun-Mars phase, as explained in Section III. Note that integration from $t = t_{dep}$ up to $t = t_{link}$ includes a switch in the dynamical system from EM-CR3BP to the SE-CR3BP at the sphere of influence of the Earth.
2. Create the arrival stable solar sail dedicated set from the SM-L₁ halo orbit, starting from $t = t_{arr}$ up to $t = t_{link}$. Note that this propagation is done backwards in time.
3. For all n trajectories of both sets, transform the state at $t = t_{link}$ to the heliocentric inertial frame, as explained in Section II.V.
4. Calculate the error in position, Δr , and the error in velocity, Δv , at the linkage for all $n \times n$ combinations of the departure and arrival sets.
5. For the first guess, all trajectories satisfying $\Delta r \leq 10^{-3}$ and $\Delta v \leq 10^{-2}$ in Sun-Earth non-dimensional units (i.e., $1.5 \cdot 10^5$ km and 0.30 km/s) are considered feasible enough for further optimization.

For the first guess, the control law $R(\alpha, \delta, t)$ considers a constant attitude of the sail with respect to the Sun-sail line for both the departure and the arrival set, with $\delta = 90^\circ$ for the outbound transfer and $\delta = -90^\circ$ for the inbound transfer.

The above method assumes a given t_{dep} , t_{link} , t_{arr} , and α . This means that there is an infinite number of combinations of these parameters and not all combinations will satisfy the constraints on Δr and Δv at

linkage of the unstable and stable solar sail dedicated sets. How to find combinations of these parameters that will satisfy the constraints will be discussed in Section IV.III.

IV.III Systematic search

This section will deal with finding combinations of t_{dep} , t_{link} , t_{arr} , and α that produce first guesses that are feasible enough for further optimization, both for the outbound and the inbound transfer.

First, a suitable α will have to be found. For this, we remove the Earth-Moon phase to make the problem autonomous and, while varying α , search for a heteroclinic connection between a Northern *Sun-Earth* L_2 halo orbit with an out-of-plane amplitude of $650 \cdot 10^3$ km (SE- L_2) and the previously defined SM- L_1 halo orbit. Again, note that this problem is autonomous (i.e., we can assume any configuration of Earth and Mars), because any relative orientation will occur once every synodic period. Therefore, t_{dep} and t_{arr} do not have to be considered, reducing the search to two parameters, α and t_{link} .

Fig. 7 shows the minimum Δv among 200 trajectories along both manifolds for varying α with travel times in the order of 4-6 years. It should be noted that all transfers in Fig. 7 have $\Delta r < 10^{-5}$. In Fig. 7 it can be seen that the minimum Δv occurs for $\alpha = 52^\circ$ and is therefore the optimal constant sail attitude to transfer from SE- L_2 to SM- L_1 . Because of the relative size of the Earth-Moon system compared to the Sun-Earth/Sun-Mars systems, it is assumed that this constant sail attitude is also optimal to create an initial guess transfer from the EM- L_2 halo orbit to the SM- L_1 halo orbit.

Fig. 7: Δv at linkage and travel time for a transfer from SE- L_2 to SM- L_1 , for varying α .

Now that α is fixed, we once again include the Earth-Moon phase and the parameters that are left are: t_{dep} , t_{link} , and t_{arr} , where $t_{dep} < t_{link} < t_{arr}$. A systematic search has been performed to find suitable values for these parameters. One important thing to note is that not all departure times and arrival times in the Earth-Moon system produce a solar sail dedicated set that can be used as described in Section IV.II. This is because during intervals of the synodic Sun-Earth-Moon period, the direction of the solar sail

acceleration is unfavorable and the sail takes too long to reach the sphere of influence of the Earth, resulting in a chaotic set of trajectories rather than a topological tube. This is therefore taken into account during the systematic search.

Many combinations of t_{dep} , t_{link} and t_{arr} have to be explored, which is done in a grid search. To limit computation time, one parameter is varied in relatively large steps, while using a finer grid for the remaining two parameters. This results in a set of images like Fig. 8 (outbound transfer) and Fig. 9 (inbound transfer), one for each value of the parameter for which large steps are taken, with n_{link} steps on the x-axis and n_{time} steps on the y-axis. Each grid point shows the Δr at linkage, Δv at linkage and travel time corresponding to the two trajectories that show the closest match in position at linkage. For all combinations of the n_{time} and n_{link} steps, the design technique from Section IV.II results in the most feasible transfer. Subsequently, the discontinuities at the linkage between the Sun-Earth and Sun-Mars phase and the travel time can be used to trade-off the resulting transfers.

Finally, the systematic search is conducted multiple times to iteratively find more suitable values for t_{dep} and t_{arr} of both transfers:

1. The first grid search is performed to find a range of departure dates in the synodic Earth-Mars period for which the relative orientation of Earth and Mars allows for a transfer. This search is done by using a range of t_{dep}^O or t_{arr}^I one week apart over one synodic Earth-Mars period (2021-2023 for the outbound transfer and 2026-2028 for the inbound transfer). This grid search can be very coarse, using $n = 20$, $n_{time} = 10$, and $n_{link} = 10$.
2. The second grid search is performed in the resulting time frame of the first grid search, to find the most suitable t_{dep} within one synodic Earth-Moon period. Over the course of two months, two day offsets are used for t_{dep} . This grid search has a finer mesh than step 1, using $n = 200$, $n_{time} = 200$, and $n_{link} = 200$.
3. The last grid search once again opens up the search space to the one found in step 1 and evaluates t_{dep} values that are one or multiple synodic Earth-Moon periods apart starting from the t_{dep} found in step 2. The mesh for this grid search is the same as for step 2, i.e., $n = 200$, $n_{time} = 200$, and $n_{link} = 200$.

These steps have been carried out for both the outbound and the inbound transfer, using $\beta = 0.05$, $\alpha = 52^\circ$, and $\delta = 90^\circ$ (outbound) or $\delta = -90^\circ$ (inbound). The results can be found in Section IV.IV

Fig. 8: Example of intermediate result of grid search for the outbound transfer for departure at the EM-L₂ halo orbit at October 10th, 2021. The twenty smallest values for Δr_{min} are marked by the bold red crosses.

Fig. 9: Example of intermediate result of grid search for the inbound transfer for arrival at the EM-L₂ halo orbit at February 21st, 2028. The twenty smallest values for Δr_{min} are marked by the bold red crosses.

and IV.V, for the outbound and inbound transfer, respectively.

IV.IV Outbound trajectory

Step 1 of the grid search for the outbound transfer results in possible departure dates between July and December 2021. Subsequently, step 2 is performed in the middle of this window, in the months of September and October. The results of step 2 can be found in Fig. 10a. The shaded areas indicate that for half a synodic Earth-Moon period the direction of the solar sail acceleration is unfavorable and the resulting solar sail dedicated set is not usable. The point in time within the synodic Earth-Moon period where Δv is the smallest is marked with a bold red cross. Step 3 is carried out using this knowledge, resulting in Fig. 10b.

Fig. 10: Grid search for the outbound trajectory. Result of step is denoted by a bold red cross.

From Fig. 10b it can be seen that the outbound transfer departing on October 10, 2021 has the smallest Δv and is therefore the most feasible transfer using the previously assumed constant attitude control law. The first guesses for the three phases are shown in orange in Fig. 14a through 14c. Note that the ballistic portion of the initial guess is illustrated using $\alpha = 90^\circ$, meaning that the sail is aligned with the incoming solar radiation pressure. The complete outbound trajectory is visualized in the heliocentric inertial frame in Fig. 11. Finally, the characteristics

of the first guess for the outbound transfer can be found in the second column of Tab. 4.

Table 4: First guess transfers characteristics.

	Outbound	Inbound
t_{dep}	Oct 10, 2021	Feb 11, 2024
t_{arr}	Oct 10, 2025	Feb 21, 2028
t_t [days]	1461	1471
Δr [km]	57464	110308
Δr [-, SE]	$3.8 \cdot 10^{-4}$	$7.4 \cdot 10^{-4}$
Δv [km/s]	0.21725	0.18066
Δv [-, SE]	$7.3 \cdot 10^{-3}$	$6.0 \cdot 10^{-3}$

Fig. 11: First guess for outbound transfer in the heliocentric inertial frame. Arrows indicate the induced solar sail acceleration vector.

IV.V Inbound trajectory

Conducting the same process for the inbound transfer results in a possible time window from December 2027 to June 2028. Step 2 is performed in February and March, and provides the results as shown in Fig. 12a. Using the transfer marked by a bold red cross as a starting point in step 3 provides the final results of the grid search in Fig. 12b.

From Fig. 12b it can be seen that the inbound transfer arriving on February 10, 2021 has the smallest Δv and is therefore the most feasible transfer. The first guesses for the three phases is shown in orange in Fig. 14d through 14f. The inbound trajectory

Fig. 12: Grid search for the inbound trajectory. Result of step is denoted by a bold red cross.

is visualized in the heliocentric inertial frame in Fig. 13. The characteristics of the first guess for the inbound transfer can be found in the third column of Tab. 4. Note that departure at Mars occurs *before* arrival of the first guess for the outbound transfer. However, this issue will be solved during the optimization.

V. RESULTS

Results are given for the separate inbound and outbound trajectories in Section V.I and for the full cycler in Section V.II. A sensitivity analysis is given in Section V.III.

V.I Separate transfers

With the first guesses for both transfers known, the problem defined in Section III can be solved using PSOPT. As mentioned in Section III, the two transfers will first be optimized separately, resulting in two time-optimal transfers with non-constant $R(\alpha, \delta, t)$, of which the numerical results can be found in Tab. 5 for $\beta = 0.05$. Note that the departure and arrival at the halo orbits are not yet according to Eq. [24]. These resulting transfers are subsequently used as initial guess for one larger optimal control problem with six phases, where also the relative timing of the two transfers can be optimized and Eq. [24] will be satis-

Fig. 13: First guess for inbound transfer in the heliocentric inertial frame. Arrows indicate the induced solar sail acceleration.

fied.

Table 5: Numerical results of optimizing the outbound and inbound trajectories separately.

	Outbound	Inbound
t_{dep}	Nov 7, 2021	June 29, 2025
t_{arr}	June 2, 2024	Jan 15, 2028
t_t [days]	937	930

V.II Full cycler

Using the objective function in Eq. [28], the results of Fig. 14 are obtained for the full cycler. The figure shows both the initial guess obtained in Section IV and the time-optimal trajectory. Numerical data can be found in Tab. 6, where the departure of the next cycle (Apr 6, 2028) is exactly 3 synodic Earth-Mars periods after the previous departure of the outbound transfer (Nov 16, 2021). The next cycle has been calculated by off-setting the original initial guess by three synodic periods, changing the bounds for time in the optimal control solver, and running the optimal control solver. These results show that the transfer and dwell times of the next cycle are within one day of the cycle in Tab. 6. By comparing the transfer times in Tab. 5 and 6, it becomes clear that the optimal transfer is significantly faster than the first guess as it

achieves a 36% reduction in both the outbound and inbound transfers. However, note that part of this decrease is due to the fact that the first guess assumes a ballistic (and therefore slow) departure from/arrival at the halo orbits. The optimal trajectory is also visualized in the heliocentric inertial frame in Fig. 15.

Table 6: Numerical data for time-optimal Earth-Mars cyler.

	Total time [days]
Outbound transfer	936 (Nov 16, 2021 - June 9, 2024)
Dwell time at Mars	377 (June 9, 2024 - June 22, 2025)
Inbound transfer	937 (June 22, 2025 - Jan 15, 2028)
Dwell time at Moon	83 (Jan 15, 2028 - Apr 6, 2028)

V.III Sensitivity analysis

One of the main advantages of low-thrust propulsion is that one is not constrained to one single departure time, but that in many cases the departure time is flexible. In order to see if this is the case for the cyler, a sensitivity analysis has been performed, where t_{dep} is forced to differ from the baseline in Tab. 6. Fig. 16 shows the results of this sensitivity analysis. In general, it can be seen that, when t_{dep} differs from the baseline, the relative orientation of Earth and Mars is sub-optimal, resulting in longer transfer times and therefore a shorter dwell time at Mars. Interesting to note is that when the departure time at the EM-L₂ halo orbit is forced to change, the arrival time at that halo orbit only changes slightly, leading to longer dwell times at the Moon. An explanation for this is that when there is a shift in time in the Earth-Moon system, the state at the sphere of influence is changed significantly, since the orientation of the Moon can vary over 360° within one synodic Earth-Moon period. With these different initial (or final) conditions, the boundary and linkage constraints are not met, causing the optimizer to shy away from a transfer that leaves (or enters) the Earth-Moon phase at a different time. Only when forced by a boundary constraint on the departure date, the optimizer will find such a transfer, but in general not by itself. Concluding, Fig. 16 shows that it is indeed possible to depart earlier or later than the baseline, with a penalty on the dwell time at Mars.

So far, all transfers have been calculated using $\beta = 0.05$, consistent with near-term solar sail technology³³. However, mid-term to far-term solar sails are expected to achieve β values up to 0.1³³. It is therefore interesting to see how the cyler behaves for larger values for β . In addition, the minimum lightness number for which a cyler is still possible will also be of interest. Fig. 17 shows the results of

this sensitivity analysis, which are obtained through a continuation on the lightness number, where the result for a slightly smaller value for β is used as an initial guess for a slightly larger value for β and vice versa. The figure shows how an increase or decrease of the lightness number results in faster/slower transfers. It can be seen that using $\beta = 0.09$ or larger, it is possible to achieve a cyler in only two synodic Earth-Mars periods. On the other hand, when β is smaller than 0.04, four synodic periods are required, increasing to even more synodic periods for $\beta < 0.035$. The analysis is therefore truncated at $\beta = 0.035$ as the cyler would take too long. The same behavior for the departure and arrival times at the Moon as for the sensitivity analysis on the departure date at EM-L₂ is observed here: only marginal changes, see the dash-dot line in Fig. 17. The same explanation holds: the optimizer shies away from leaving/entering the Earth-Moon system at times different from the initial guess when it is not forced to do so.

VI. CONCLUSIONS

In this paper, solar sail optimal trajectories for an Earth-Mars cyler have been investigated. The cyler consists of solar sail heteroclinic-like connections between halo orbits at the Earth-Moon L₂-point and the Sun-Mars L₁-point with dwell times at both halo orbits. The objective of the optimization has been to maximize the dwell times at the halo orbits, while minimizing the number of synodic Earth-Mars periods required for the cyler. To accomplish this, an optimal control problem has been formulated and solved using a particular implementation of a direct pseudospectral optimization method, PSOPT.

PSOPT requires a first guess for the optimization, which has been generated using the patched circular restricted three-body problems approximation. For both the outbound and the inbound transfer, three separate phases are used, e.g., for the outbound transfer: departure in the Earth-Moon system, propagation in the Sun-Earth system, and arrival in the Sun-Mars system. The first guess is sub-optimal in the sense that a constant sail attitude is used, fourth-body perturbations are neglected, and small discontinuities at the linkage between the phases are allowed. During the optimization the sail attitude is optimized and linkage constraints are enforced, resulting in a continuous trajectory. For near-term sails ($\beta = 0.05$), this results in a time-optimal cyler that spans three synodic Earth-Mars periods, with a dwell time at Mars of 400 days and at the Moon of 80 days. A sensitivity analysis has shown that it is possible to depart a few months earlier or later than the optimal departure day, with a penalty on the dwell time at Mars or the Moon. Finally, based on the obtained

Fig. 14: Trajectories and control profiles for first guess and time-optimal cyclers. Trajectories are given in the respective CR3BPs, where arrows indicate the solar sail normal vector. The controls are provided through the cone angle, α , and the clock angle, δ . The left column shows the outbound transfer; the right column shows the inbound transfer.

(a) Outbound trajectory.

(b) Inbound trajectory.

Fig. 15: Optimal trajectory of the full cycle in the heliocentric frame. Arrows indicate the induced solar sail acceleration vector.

Fig. 16: Transfer and dwell times resulting from a different departure date in the EM-system, t_{dep} .

Fig. 17: Transfer and dwell times resulting from changing the solar sail lightness number, β .

results, it is expected that using mid-term to far-term sails ($\beta \geq 0.09$) a cyler that spans only two synodic periods can, be maintained indefinitely.

VII. ACKNOWLEDGEMENTS

Jeannette Heiligers acknowledges support from the Marie Skłodowska-Curie Individual Fellowship 658645 - S4ILS: Solar Sailing for Space Situational Awareness in the Lunar System.

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