A Three-Dimensional Iterative Scheme for an Electromagnetic Inductive Applicator

Maciej J. Sowiński and Peter M. van den Berg

Abstract—An efficient iterative method for solving quasi-static electromagnetic field problems is presented. The electromagnetic field is generated by an inductive applicator and is represented as a superposition of two constituents, viz. a primary field in absence of the tissue configuration and a secondary field generated by the presence of the tissue. Then, for the secondary field a quasi-static approximation is employed. In the quasi-static field equations a relaxation function is introduced, such that the resulting equations can be solved iteratively. For a realistic three-dimensional model of a human hand numerical results are presented.

I. INTRODUCTION

THERE exists a variety of research papers dealing with the inductive applicators. Some number of these applicators have been analyzed with an idealized configuration, e.g., semi-infinite slabs of homogeneous tissue [1] or rotationally symmetric media [2]. Exact analytical solutions for these latter cases are obtained. Unfortunately, these tissue models are far from the realistic ones and this also applies to the resulting electromagnetic heat power distributions [3], [4]. On the other hand, it is possible nowadays to solve numerically the full set of Maxwell equations for more realistic tissue configurations [5]. However, the methods based on a full solution of Maxwell equations [6] are still too expensive for the designer use in view of computing facilities requirements.

The approach presented in this paper deals with the quasi-static approximation of Maxwell equations but the model of the tissue configurations is taken as realistic as possible. Our quasi-static approximation is derived directly from Maxwell's equations and differs slightly from the impedance method [7]. The quasi-static equations are solved numerically by an iterative method which has been proposed for problems with capacitive applicators [3]. The method is similar to the one of solving a Laplace equation from a parabolic (diffusion-type) partial differential equation by using an explicit stable scheme. When a stationary state of this parabolic equation is reached the solution of

our quasi-static problem is achieved. In the present case of inductive excitation, the method proofs also here to be efficient.

A multiturn "pancake" coil with circular turns is considered. Each turn of the coil can have a different diameter and a different position in this pancake model. These types of coils are of interest in the design of new applicators for physiotherapy treatment. The objective of the latter treatment is to induce heat in the human tissue in such a way that the local temperature will increase a few degrees. It is expected that this has a positive effect to the blood transfer and the movability of bone joints. As a numerical example, the electromagnetic (EM) power distribution in a human-hand model has been computed.

II. FORMULATION OF THE QUASI-STATIC PROBLEM

The configuration to be investigated is shown in Fig. 1. A pancake coil applicator acts as a source of the electromagnetic field irradiating the body. Each circular turn of the applicator carries the electric current $I = I_0 \exp(j\omega t)$, where $\omega = 2\pi f$ (f = frequency of operation).

In order to model the electromagnetic field in the tissue we start with Maxwell's equations:

$$\nabla \times \mathbf{H} - (\sigma + j\omega\epsilon)\mathbf{E} = \mathbf{J}^{\text{source}} \quad (1a)$$

$$\nabla \times \mathbf{E} + j\omega\mu_0\mathbf{H} = 0 \quad (1b)$$

with $\epsilon = \epsilon_r\epsilon_0$, where ϵ_r is the relative permittivity and σ is the electrical conductivity of the medium, ϵ_0 and μ_0 are the permittivity and permeability of free space. $\mathbf{J}^{\text{source}}$ is the volume density of the electric current in the sources which generate the electromagnetic field. We have assumed that the permeability of human tissue is equal to the permeability of free space. The electromagnetic field is represented as a superposition of two constituents, viz.

$$\mathbf{E} = \mathbf{E}^p + \mathbf{E}^s, \mathbf{H} = \mathbf{H}^p + \mathbf{H}^s. \quad (2)$$

The primary field denoted by a superscript p satisfies Maxwell's equations (1) in free space

$$\nabla \times \mathbf{H}^p - j\omega\epsilon_0\mathbf{E}^p = \mathbf{J}^{\text{source}} \quad (3a)$$

$$\nabla \times \mathbf{E}^p + j\omega\mu_0\mathbf{H}^p = 0. \quad (3b)$$

The secondary field denoted in (2) by a superscript s is the field which is generated by the primary field due to the presence of the body. From (1)–(3) we can derive the

Manuscript received July 6, 1990; revised February 10, 1992. This work was supported in part by Enraf-Nonius, Delft, The Netherlands.

M. J. Sowiński is with the Laboratory for Electromagnetic Research, Department of Electrical Engineering, 2600 GA Delft, The Netherlands, on leave of absence from the Department of Electromagnetic Wave Theory, Institute of Fundamental Technological Research, Warsaw, Poland.

P. M. van den Berg is with the Laboratory for Electromagnetic Research, Department of Electrical Engineering, Delft University of Technology, 2600 GA Delft, The Netherlands.

IEEE Log Number 9204048.

set of equations which are satisfied by the secondary fields $\{E^s, H^s\}$:

$$\nabla \times H^s - (\sigma + j\omega\epsilon)E^s = J^p \quad (4a)$$

$$\nabla \times E^s + j\omega\mu_0 H^s = 0 \quad (4b)$$

$$J^p = [\sigma + j\omega(\epsilon - \epsilon_0)]E^p \quad (4c)$$

where J^p is the volume density of the primary contrast-source current. In the range of frequencies ($f \leq 30$ MHz), in most cases the magnitude of the second term of (4b) satisfies the relation

$$|j\omega\mu_0 H^s| \ll |\nabla \times E^s|. \quad (5)$$

Assuming that the quasi-static condition (5) is satisfied we can neglect the term $j\omega\mu_0 H^s$ in (4b) and we obtain the quasi-static approximation $\nabla \times E^s = 0$, hence we may write

$$E^s = -\nabla V. \quad (6)$$

The scalar continuous function V can be considered as a secondary electric potential generated by the presence of the tissue in the primary electromagnetic field. Subsequently, applying the divergence operator to both sides of the (4a) we arrive at

$$-\nabla \cdot J^s = \nabla \cdot J^p \quad (7)$$

where the secondary current J^s is defined as

$$J^s = (\sigma + j\omega\epsilon)E^s. \quad (8)$$

Let us introduce the total generalized current density J for the secondary field problem formulated above. According to (4a) we define this current as

$$J = J^s + J^p. \quad (9)$$

Finally, we can formulate our quasi-static problem as follows:

$$\nabla \cdot J = 0, \quad (10a)$$

$$J = -\kappa(x, y, z)\nabla V + J^p, \quad (10b)$$

$$J^p = [\kappa(x, y, z) - j\omega\epsilon_0]E^p, \quad (10c)$$

$$\kappa(x, y, z) = \sigma(x, y, z) + j\omega\epsilon(x, y, z). \quad (10d)$$

Equations (10) have to be supplemented with the boundary conditions at the interfaces where the secondary potential V and the component of the generalized current J normal to the interface are continuous.

To solve this problem numerically we have to bound our domain of investigation. We assume some external boundary surface Σ where we prescribe an impedance boundary condition of the type

$$V = \beta J^s \cdot n_\Sigma \quad (11)$$

where n_Σ is an outward normal vector to Σ and $\beta = \beta(x, y, z)$ is a spatially dependent specific impedance at the point (x, y, z) of the surface Σ .

Fig. 1. The applicator configuration.

As soon as the generalized current J is obtained, the total electric field E can be evaluated via

$$E = \frac{J + j\omega\epsilon_0 E^p}{\kappa} \quad (12)$$

and subsequently the volume density distribution of dissipated EM power can be obtained from the relation

$$\dot{w}(x, y, z) = \frac{1}{2} \sigma(x, y, z) [|E_x|^2 + |E_y|^2 + |E_z|^2] \quad (13)$$

where E_x , E_y , E_z are the components of the total electric field E in the Cartesian coordinate system shown in Fig. 1.

To complete the present formulation we will specify the primary electric field E^p of the pancake coil applicator as

$$E^p = \sum_{n=1}^{N_T} E_n^p \quad (14)$$

where N_T is the number of turns in the coil and E_n^p is rotationally symmetric primary electric field of each current loop n . These primary electric fields have been evaluated in the local, circular cylinder coordinate system (r_n, φ_n, z_n) of each particular turn.

$$E_n^p = -j\omega A_n = -j\omega i_{\varphi n} A_{\varphi n} \quad (15a)$$

$$A_{\varphi n} = \frac{\mu_0 I_0 a_n}{2\pi} \int_0^\pi \frac{\exp(-jk_0 R(\phi))}{R(\phi)} \cos \phi d\phi \quad (15b)$$

$$k_0 = \omega[\mu_0 \epsilon_0]^{1/2} \quad (15c)$$

$$R(\phi) = [z_n^2 + r_n^2 + a_n^2 - 2r_n a_n \cos \phi]^{1/2}. \quad (15d)$$

In (15), A_n is the magnetic vector potential and $A_{\varphi n}$ is the only nonzero component of this vector in the local coordinate system of the n th turn. Further, k_0 is the wave number of free space, I_0 is the amplitude of the electric current in the pancake coil, a_n radius of the n th turn, and $i_{\varphi n}$ is the unit vector in the circular cylinder coordinate system of a turn.

III. IMPLEMENTATION OF THE ALF ITERATIVE METHOD

To work with strongly inhomogeneous media and to handle the large number of unknowns in a three-dimensional problem we will employ the averaging-leap-frog iterative method (ALF) which has been developed and successfully tested for the quasi-static problems with ca-

capacitive applicators [3]. The general approach in this method is an extension of the present three-dimensional problem to the four-dimensional problem:

$$\frac{\partial V}{\partial \tau} = -F(x, y, z) \nabla \cdot \mathbf{J}(x, y, z; \tau) \quad (16a)$$

$$\mathbf{J}(x, y, z; \tau) = -\kappa(x, y, z) \nabla V(x, y, z; \tau) + \mathbf{J}^p(x, y, z) \quad (16b)$$

where τ is the iteration coordinate and $\mathbf{J}^p$ is our primary current density defined by (10c). Equations (16) form the initial boundary value problem with boundary condition at the external surface Σ :

$$V(x, y, z; \tau) = \beta(x, y, z) \mathbf{J}^s(x, y, z; \tau) \cdot \mathbf{n}_\Sigma \quad (17)$$

and the initial value condition

$$V(x, y, z; 0) = V_0(x, y, z), \quad (18a)$$

$$\mathbf{J}(x, y, z; 0) = \mathbf{J}_0^s(x, y, z) + \mathbf{J}^p(x, y, z). \quad (18b)$$

Further, the relaxation function F is a suitably chosen complex function such that

$$F(x, y, z) \neq 0 \text{ for all } (x, y, z). \quad (19)$$

The four-dimensional initial boundary value problem formulated above usually has a solution which possesses the property of reaching some equilibrium ($\partial V / \partial \tau = 0$, $\tau \rightarrow \infty$). This equilibrium solution is the desired solution of the three-dimensional problem defined by (10) and (11).

To solve this initial boundary value problem we need to define a proper discretization in the x , y , z , and τ variable. Let us introduce in the spatial x , y , z domain a rectangular grid with mesh sizes h_x , h_y , h_z and nodal points $P = P_{k,l,m}$ located in the baricenters of each elementary block of the grid, see Fig. 2. To simplify further calculations we will assume that each elementary block of our grid contains only one kind of the tissue. Using an appropriate finite difference discretization of (16b) in space, based on its integral form [3], the mean value of the normal component $\mathbf{J} \cdot \mathbf{n}$ on the surface S_x^+ (see Fig. 2) is obtained as

$$(\mathbf{J} \cdot \mathbf{n})|_{S_x^+} = \frac{\kappa_{k+(1/2),l,m}^{(x)}}{h_x} (V_{k,l,m} - V_{k+1,l,m}) + (J_x^p)_{k+(1/2),l,m} \quad (20)$$

where the effective admittance between the nodal points (k, l, m) and $(k+1, l, m)$ is given by

$$\kappa_{k+(1/2),l,m}^{(x)} = \frac{2\kappa_{k+1,l,m}\kappa_{k,l,m}}{\kappa_{k+1,l,m} + \kappa_{k,l,m}} \quad (21)$$

and the mean value of x -component of the primary current density at S_x^+ is defined as

$$(J_x^p)_{k+(1/2),l,m} = \frac{\kappa_{k+(1/2),l,m}^{(x)}}{2} \left[\frac{J_x^p(k+1/4, l, m)}{\kappa_{k,l,m}} + \frac{J_x^p(k+3/4, l, m)}{\kappa_{k+1,l,m}} \right]. \quad (22)$$

Fig. 2. Elementary block of the rectangular grid in the spatial domain.

Here $V_{k,l,m}$ and $\kappa_{k,l,m}$ denote the value of κ and V at the nodal point $P_{k,l,m}$, respectively, and $(k+1/4, l, m)$ denotes the point located $(1/4)h_x$ above the nodal point (k, l, m) while $(k+3/4, l, m)$ denotes the point located $(1/4)h_x$ below the nodal point $(k+1, l, m)$. Similar equations can be obtained for all six rectangular sides of the surface $S_{k,l,m}$ of our elementary block. Subsequently, these results are used in the discretized version of the integral form of (16a), viz.

$$\frac{\partial V_{k,l,m}}{\partial \tau} = -\frac{F_{k,l,m}}{h_x h_y h_z} \iint_{S_{k,l,m}} \mathbf{J} \cdot \mathbf{n} ds \quad (23)$$

where $F_{k,l,m}$ is the value of the relaxation function F at the nodal point (k, l, m) .

In the case when an elementary block is adjacent to the external boundary surface Σ , the density of secondary current $\mathbf{J}^s \cdot \mathbf{n}$ flowing through Σ is found from (17) and (20) as

$$\mathbf{J}^s \cdot \mathbf{n}_\Sigma = \frac{\kappa_B^R}{h} V_B, \text{ with } \kappa_B^R = \frac{2\kappa_B h}{2\beta\kappa_B + h}. \quad (24)$$

In (24), V_B is the value of the potential V in the nodal point of the boundary block, h is the mesh size in the direction normal to Σ , κ_B is the effective admittance between nodal point of the boundary block, and a point outside Σ and β is the specific impedance which occur in the boundary condition (17).

In order to obtain the computational iterative scheme we apply some finite-difference formula for the τ -derivative in the left-hand side of (23). For this purpose we introduce the discretization of the τ -coordinate as follows:

$$\tau^{(0)}, \tau^{(1)}, \dots, \tau^{(n)}, \tau^{(n+1)}, \dots, \quad (25)$$

where

$$\tau^{(0)} < \tau^{(1)} < \dots < \tau^{(n)} < \tau^{(n+1)} < \dots$$

Further, we use the notation $V_{k,l,m}^{(n)}$, $\mathbf{J}_{k,l,m}^{(n)}$ for the discretized values of the electric potential V and the current $\mathbf{J}$ in a nodal point $P = P_{k,l,m}$ evaluated at $\tau = \tau^{(n)}$. Replacing in (23) the derivative with respect to the τ -coordinate by a finite difference approximation [3] and applying an averaging procedure:

$$V_{k,l,m}^{(n)} = \theta V_{k,l,m}^{(n+1)} + (1 - \theta) V_{k,l,m}^{(n-1)} \quad (26)$$

where the weight factor θ is the real number satisfying $1/2 \leq \theta \leq 1$, we obtain a numerically stable iterative scheme:

$$V_P^{(n+1)} = \frac{\delta^{(n)}(\Delta V^{(n)})_P}{1 + \theta \delta^{(n)}\Lambda_P} + \frac{1 - (1 - \theta)\delta^{(n)}\Lambda_P}{1 + \theta \delta^{(n)}\Lambda_P} V_P^{(n-1)} - \frac{\delta^{(n)}C_P}{1 + \theta \delta^{(n)}\Lambda_P} \quad (27)$$

where $\delta^{(n)} = \tau^{(n+1)} - \tau^{(n-1)}$ is the "leap-frog" step along the τ -coordinate, $P = P_{k,l,m}$ is the nodal point of the mesh, Δ is the spatial finite-difference operator of the form

$$\begin{aligned} (\Delta V)_{P_{k,l,m}} = & \frac{F_{k,l,m}}{h_x^2} (\kappa_{k-(1/2),l,m}^{(x)} V_{k-1,l,m} \\ & + \kappa_{k+(1/2),l,m}^{(x)} V_{k+1,l,m}) \\ & + \frac{F_{k,l,m}}{h_y^2} (\kappa_{k,l-(1/2),m}^{(y)} V_{k,l-1,m} \\ & + \kappa_{k,l+(1/2),m}^{(y)} V_{k,l+1,m}) \\ & + \frac{F_{k,l,m}}{h_z^2} (\kappa_{k,l,m-(1/2)}^{(z)} V_{k,l,m-1} \\ & + \kappa_{k,l,m+(1/2)}^{(z)} V_{k,l,m+1}) \end{aligned} \quad (28)$$

and Λ_P is the complex number given by

$$\begin{aligned} \Lambda_{P_{k,l,m}} = & F_{k,l,m} \left(\frac{\kappa_{k-(1/2),l,m}^{(x)} + \kappa_{k+(1/2),l,m}^{(x)}}{h_x^2} \right. \\ & + \frac{\kappa_{k,l-(1/2),m}^{(y)} + \kappa_{k,l+(1/2),m}^{(y)}}{h_y^2} \\ & \left. + \frac{\kappa_{k,l,m-(1/2)}^{(z)} + \kappa_{k,l,m+(1/2)}^{(z)}}{h_z^2} \right). \end{aligned} \quad (29)$$

$$\text{ERR}_P^{(n)} = \frac{\left| \sum_{k,l,m} \iint_{S_{k,l,m}} \nabla V^{(n)} \cdot \mathbf{J}^{(n)*} dv - \iint_{\Sigma} V^{(n)} \mathbf{J}^{(n)*} \cdot \mathbf{n} ds \right|}{\frac{1}{2} \iiint_{\mathcal{V}_T} \sigma \mathbf{E}^{(n)} \cdot \mathbf{E}^{(n)*} dv} \quad (33)$$

The source terms C_P are evaluated in each nodal point as

$$\begin{aligned} C_{P_{k,l,m}} = & \frac{F_{k,l,m}}{h_x} [(J_x^p)_{k+(1/2),l,m} - (J_x^p)_{k-(1/2),l,m}] \\ & + \frac{F_{k,l,m}}{h_y} [(J_y^p)_{k,l+(1/2),m} - (J_y^p)_{k,l-(1/2),m}] \\ & + \frac{F_{k,l,m}}{h_z} [(J_z^p)_{k,l,m+(1/2)} - (J_z^p)_{k,l,m-(1/2)}] \end{aligned} \quad (30)$$

where (J_x^p) , (J_y^p) , and (J_z^p) are the mean values of the primary contrast-source current density in the x , y , and z -

direction computed via (22). Equations (28)–(30) are valid also in the boundary nodal points provided the boundary admittance κ_B^R defined by (24) will be used instead of the corresponding effective admittance defined by (21), and provided the value of the potential V in a point outside the grid will be set equal to zero.

IV. NUMERICAL IMPLEMENTATION

To start the averaging-leap-frog iterative scheme (ALF) described in Section III we need to know the initial values of the electric potentials $V^{(0)}$ and $V^{(1)}$ in all mesh points and the size $\delta\tau^{(1)} = \tau^{(1)} - \tau^{(0)}$ of the first τ -step. One of the simplest possibilities is to start the ALF iterative scheme with

$$\begin{cases} V^{(0)} = V^{(1)} = 0 & \text{in all grid points} \\ \delta\tau^{(1)} = 0. \end{cases} \quad (31)$$

The potential $V^{(2)}$, obtained from (27) with initialization of (31) and with some $\delta\tau^{(2)} = \tau^{(2)} - \tau^{(1)} > 0$, will have nonzero values in the points where the $C_{k,l,m}$ terms are different from zero. When the scheme continues the electric potential $V^{(n)}$ will penetrate to the other points of the grid ("diffusion") until the equilibrium situation is arrived at.

As a measure of the convergence, the normalized integrated square error with respect to the satisfaction of the original three-dimensional problem is introduced as

$$\text{ERR}_V^{(n)} = \left(\frac{\sum_{k,l,m} \left| \iint_{S_{k,l,m}} \mathbf{J}^{(n)} \cdot \mathbf{n} ds \right|^2}{\sum_{k,l,m} \left| \iint_{S_{k,l,m}} \mathbf{J}^{(0)} \cdot \mathbf{n} ds \right|^2} \right)^{1/2} \quad (32)$$

where the surface integrals are evaluated using expressions of the type of (20). In the case when the initialization of (31) is used, the denominator in (32) is equal to the value of the volume integral of the divergence of the primary contrast-source current density $\mathbf{J}^p$. As a second measure, the normalized power balance error is introduced as

The quantity in the denominator is the power dissipated in the tissue domain $\mathcal{V}_T$ by the total electric field $\mathbf{E}^{(n)}$ found from (12) after n iterations. This global error measure is based on the quasi-static version of the power balance identity, $\nabla \cdot (V\mathbf{J}^*) = \nabla V \cdot \mathbf{J}^*$, provided that V and $\mathbf{J}$ are the solution of our quasi-static problem formulated in Section II. In the case when the initialization of (31) is used, the power balance error does not start from the unit value like the previous, local error measure but start with a zero value. Then, after the first few iterations, it reaches its maximum and when the convergence process continues $\text{ERR}_P^{(n)}$ decreases.

In the numerical program, first we have to compute the primary electric field E^p which provide the sources for the problem. This field is computed as a scalar quantity using local coordinate systems introduced in (15) for each turn of the pancake coil. To evaluate this field in some point of our three-dimensional grid the coordinates of a point are first transformed to the local coordinate system. Next, the bilinear approximation formula is used to evaluate the value of E^p and finally, this field is transformed back to the grid coordinate system. After the primary electric field has been calculated the source terms $C_{k,l,m}$ are computed via (30) with the mean values of the primary current density $(J_{x,y,z}^p)$ found from (22). Finally, after some number of iterations we need to evaluate the total electric field defined by (12). This field is necessary when we want to compute the volume density of dissipated EM power from (13), and the power balance error $ERR_P^{(n)}$ from (33). It is important to notice that in Section III we have assumed that the generalized current J is constant on the line connecting two adjacent nodal points and equal to the value of the normal component of the current on the block surface perpendicular to this line between the two nodal points. This means that the generalized current following between the points $P_{k-1,l,m}$ and $P_{k,l,m}$ is equal to the value of $(J \cdot n)|_{S_x^-}$ while the current flowing between the points $P_{k,l,m}$ and $P_{k+1,l,m}$ is equal to the $(J \cdot n)|_{S_x^+}$, see Fig. 2. This network-like grid model causes different values of the total electric field components found from (12) in different parts of this same elementary block. Using (12), (20), and (22) we find these electric fields as

$$E_{x;k,l,m}^- = \frac{1}{\kappa_{k,l,m}} \left[\kappa_{k-1/2,l,m}^{(x)} \cdot \frac{V_{k-1,l,m} - V_{k,l,m}}{h_x} + (J_x^p)_{k-1/2,l,m} \right] + \frac{j\omega\epsilon_0}{\kappa_{k,l,m}} E_x^p \left(k - \frac{1}{4}, l, m \right), \quad (34a)$$

$$E_{x;k,l,m}^+ = \frac{1}{\kappa_{k,l,m}} \left[\kappa_{k+1/2,l,m}^{(x)} \cdot \frac{V_{k,l,m} - V_{k+1,l,m}}{h_x} + (J_x^p)_{k+1/2,l,m} \right] + \frac{j\omega\epsilon_0}{\kappa_{k,l,m}} E_x^p \left(k + \frac{1}{4}, l, m \right), \quad (34b)$$

where the superscripts denote the values of the x -component of electric field E at the left-hand “-” and the right-hand side “+” of the block. In a similar way we can find the y - and z -components of the total electric field E and complete all other calculations.

To have some particular choice of β in (24) we have established a physical meaning of this parameter as follows: let $d_{\Sigma,P}$ be the distance measured in the direction normal to the external surface Σ from a point P located outside the grid. Let us assume that the potential V and

the secondary electric field E^s decay outside our grid exponentially with factor $\exp(-d_{\Sigma,P}/d)$. With these assumptions the rate between the value of potential V and its normal derivative at a point of the surface Σ is equal to d . Using (6) and (11), we have found that to enforce the assumed decay of V and E^s we should work with the specific impedance β satisfying

$$\beta = \frac{d}{\kappa_B} \quad (35)$$

where d (measured in meters) is the distance from Σ in which the potential V and the secondary electric field E^s decay with a factor of $\exp(-1)$.

We finally consider the spatially dependent relaxation function F . Very large differences in the admittance κ in the different tissues and the fact that all these values are complex numbers reduce the convergence rate of our scheme dramatically. To accelerate the convergence, extensive numerical experiments have shown, that as in [3], a suitable choice for this function appears to be

$$F_{k,l,m} = (\kappa_{k,l,m})^{-1/2} \frac{\min(h_x, h_y, h_z)}{\max(|\kappa|)} \quad (36)$$

where $\max(|\kappa|)$ denotes the maximum value of $|\kappa_{k,l,m}|$ in the configuration. The second factor in (36) is added for normalization purposes, while the first, spatially dependent factor, reduces large differences of the values of $(\Delta V)_{P_{k,l,m}}$ and $\Lambda_{P_{k,l,m}}$ in different points of the grid. This choice of relaxation function F overcomes the decrease of convergence and facilitates an iterative solution for problems where the quantity $\max(|\kappa|)/\min(|\kappa|)$ exceeds the value of 1000 and where the complex value of κ in one of the media of our model, namely air, is almost imaginary.

V. IMPEDANCE OF THE INDUCTIVE APPLICATOR

The impedance of the applicator is the only one parameter which is “seen” by the transmitter connected to the terminals of the coil. For this reason it is important to know how much this impedance changes when the human tissue is placed in its primary field. Let U_0 be the electric potential difference between the terminals of the transmitter and I_0 the electric current driven through the coil of the applicator; the impedance of the applicator is then given by

$$Z = R + j\omega L = \frac{U_0}{I_0} \quad (37)$$

where R is the resistance and L the self-inductance of the applicator. Note that the input power delivered by the transmitter is given by $1/2Z|I_0|^2$. This means that power balance considerations lead to the desired expression for the impedance Z .

When the tissue is not present, the applicator generates the primary field only and the impedance is then denoted as the primary impedance

$$Z^p = R^p + j\omega L^p \quad (38)$$

Fig. 3. Electric circuit analog for the applicator configuration.

where

$$R^p = \frac{1}{|I_0|^2} \operatorname{Re} \iint_{\Sigma} (\mathbf{E}^p \times \mathbf{H}^{p*}) \cdot \mathbf{n}_{\Sigma} ds \quad (39a)$$

$$L^p = \frac{1}{|I_0|^2} \iiint_{\mathcal{V}} (\mu_0 \mathbf{H}^p \cdot \mathbf{H}^{p*} - \epsilon_0 \mathbf{E}^p \cdot \mathbf{E}^{p*}) dv + \frac{1}{\omega |I_0|^2} \operatorname{Im} \iint_{\Sigma} (\mathbf{E}^p \times \mathbf{H}^{p*}) \cdot \mathbf{n}_{\Sigma} ds \quad (39b)$$

where $\mathcal{V}$ is the domain enclosed by the external surface Σ .

When the tissue is placed in the primary field, the impedance is changed as

$$\Delta Z = \Delta R + j\omega \Delta L = Z - Z^p \quad (40)$$

where

$$\Delta R = \frac{1}{|I_0|^2} \iiint_{\mathcal{V}_T} \sigma \mathbf{E} \cdot \mathbf{E}^* dv \quad (41a)$$

$$\Delta L = -\frac{1}{|I_0|^2} \iiint_{\mathcal{V}_T} (\epsilon \mathbf{E} \cdot \mathbf{E}^* - \epsilon_0 \mathbf{E}^p \cdot \mathbf{E}^{p*}) dv \quad (41b)$$

where $\mathcal{V}_T$ is the domain occupied by the tissue. In (41) we have neglected the contributions from the external surface Σ . Further, we have assumed that the presence of the tissue does not change the electric current I_0 . It means that the transmitter maintains the current amplitude I_0 changing only the potential difference U_0 as a reaction to the presence of the tissue.

It should be observed that the change of the applicator resistance ΔR is always a positive quantity while the change of applicator self-inductance ΔL seems to be always negative due to the high values of the tissue permittivity $\epsilon \gg \epsilon_0$. It means that when the body is placed in the primary field the resistance of the applicator will increase and the self-inductance will decrease. The change of the applicator impedance can be expressed in the form of the "load" impedance Z^l which simulates the body presence for the transmitter. This impedance is connected parallel to the primary impedance of the applicator. Assuming the electric circuit analog as illustrated in Fig. 3 we arrive at the formulas for the "load" impedance:

$$R^l = \operatorname{Re} Z^l + \frac{(\operatorname{Im} Z^l)^2}{\operatorname{Re} Z^l} \quad (42a)$$

$$L^l = \frac{1}{\omega} \left[\operatorname{Im} Z^l + \frac{(\operatorname{Re} Z^l)^2}{\operatorname{Im} Z^l} \right] \quad (42b)$$

where

$$Z^l = -\frac{(Z^p + \Delta Z)Z^p}{\Delta Z} \quad (43)$$

VI. NUMERICAL RESULTS

For a one-turn current loop above a lossy dielectric half-space, we have tested our model and compared the numerical results with analytical ones. Although they are not presented here, good agreement has been observed.

As an example of the tissue configuration we have developed the model of a human hand. The three-dimensional rectangular blocks model have been built with the help of the Cross-Sectional Anatomy Atlas [9] and the pictures found in [10].

The left human hand was modeled in a flat position like resting on the horizontal flat surface with its palm touched to this surface. The model starts from the lower part of the forearm and continues to the ends of the fingers. The middle finger was aligned with the longitudinal z -axis (see Fig. 1). First, the $20 \times 33 \times 35 = 23100$ blocks model was developed with fat, bone, and skin tissue filling the elementary blocks. After the shape of the hand model and its skeleton has been designed as close as possible to the realistic ones, the ligament, muscle, and bone marrow tissues were added to the model with the help of pictures found in [9] and [10]. Finally some free space was added to the each side of the hand model except the wrist to lower forearm side where the model continues up to the external surface Σ . Then, the whole hand model configuration consist of $30 \times 43 \times 40 = 51600$ elementary blocks with mesh sizes $h_x = 2.5$ mm, $h_y = 4$ mm, and $h_z = 6.65$ mm. The electromagnetic parameters used in the computations are taken from Johnson and Guy [11] and Smith and Foster [12], and are listed in Table I.

It is difficult to say which value of the specific impedance β defined at the surface Σ by the boundary condition (11) will ensure the best approximation of the results to the real ones. During numerical experiments we have found that the value of β provides a kind of damping to the "oscillations" which can occur in the iterative process. For the presented examples the specific impedance β was calculated from (35) with the distance $d = 0.2$ m in the longitudinal z -direction and $d = 0.15$ m in x - and y -direction. With this choice, in all numerical experiments, the value of the surface integral over Σ in the power balance error definition of (33) was found to be a few hundred times smaller than the dissipated power defined by the denominator of (33). This means that the outflow of energy through the surface Σ is negligibly small and the impedance calculated from the approximate versions of (41) can be assumed to be accurate enough.

We performed some experiments with the choice of the τ -step $\delta\tau_n = \tau^{(n+1)} - \tau^{(n)}$ and the weight factor θ examining the speed of convergence of the method. A strong dependence on the $\delta\tau^{(n)}$ has been found. It was found that the appropriate growing of $\delta\tau^{(n)}$ at the beginning of the iterations has a big influence on the whole iterative pro-

TABLE I
PARAMETERS USED IN THE COMPUTER PROGRAM FOR THE MODEL OF A
HUMAN HAND ($f = 27.12$ MHz)

Medium	Relative Permittivity ϵ	Conductivity σ [S/m]
air	1.0	0.0000001
marrow	30.0	0.46
bone	7.3	0.028
fat	20.0	0.047
muscle	113.0	0.61
skin	102.0	0.5
ligament*	17.0	0.15

*data estimated from the water content

cess. With the choice of the initialization of (31) the best results have been obtained with the smooth growth of $\delta\tau^{(n)}$ from the value almost equal to zero. In practice, the most typical choice of $\delta\tau^{(n)}$ within the first few iterations was as follows: $\delta\tau^{(2)} = 0.02$, $\delta\tau^{(3)} = 0.06$, $\delta\tau^{(4)} = 0.15$, $\delta\tau^{(5)} = 0.3$, $\dots$, $\delta\tau^{(10)} = 1.5$. Then $\delta\tau^{(n)}$ was sinusoidally grown to $\delta\tau^{(50)} = 4$ and next to $\delta\tau^{(350)} = 22$. After this growth, $\delta\tau^{(n)}$ was varying sinusoidally between the value of 16 and 22. The weight factor θ was parabolically decreased within first 40 iterations from $\theta = 0.54$ to $\theta = 0.5$ and next, only slightly varied within the range $\theta \in (0.5, 0.51)$. The computer program was written in Fortran. The CPU computation time of one iteration on a VAX-8250 computer amounts to 32 s.

Four different pancake coils applicators have been tested during numerical experiments. Parameters of these coils and some numerical results obtained for these applicators are listed in Table II. All these results have been computed with the coil positioned horizontally, 10 mm above the center of the hand model. The best convergence has been achieved for the small flat coil with six turns designated in Table II as Fl6 and the worst for the big coil with three turns designated as B3. The resulting errors during the convergence process for these two coil applicators are presented in Table III.

From the results shown in Table III we can observe that the local error measure ERR_L defined by (32) usually stops its decay after 200–300 iterations while the global error measure ERR_P defined by (33) continues its decay. Up to the authors knowledge this is caused by the small oscillations imposed on the iterative solution. These oscillations have the spatial period equal to the double mesh size of the grid and they are excited by the non-optimum growth of $\delta\tau^{(n)}$ at the beginning of the iterative process. Fortunately, the amplitude of these ripples is so small that they do not have any influence on the results presented.

The EM power distribution patterns, presented in Figs. 4–6, have been normalized to the average dissipated power in the tissue domain of the hand model which is enclosed between two transversal planes. These transversal planes are positioned in this way that the coil applicator is fit between them as close as possible. The results obtained for the hand model are presented in the three

TABLE II
PARAMETERS AND RESULTING CHARACTERISTICS OF THE PANCAKE COIL
USED IN NUMERICAL EXAMPLES

Coil	B3	Fl6	CV6	Cx6
Number of Turns	3	6	6	6
Diameters of Turns [mm]	109 96 84	56 48 40 32	56 48 40 32	16 24 32 40
Height [mm]	0	0	15	15
L^p [μ H]	1.3	1.1	1.0	1.0
R^l [k Ω]	191	280	310	571
L^l [μ H]	3.0	4.8	5.2	9.6
$U(1\text{ W})$ [Volts]	437	528	554	755

TABLE III
CONVERGENCE OF ALF ITERATIVE SCHEME FOR TWO DIFFERENT COIL
APPLICATORS

Number of Iterations	ERR _L from (32)		ERR _P from (33)	
	B3	Fl6	B3	Fl6
50	3.16%	2.40%	5.66%	2.19%
100	1.55%	1.08%	4.10%	1.67%
150	0.68%	0.54%	2.12%	1.06%
200	0.30%	0.15%	1.05%	0.45%
300	0.22%	0.06%	0.21%	0.06%
400	0.21%	0.05%	0.04%	0.01%
500	0.20%	0.05%	0.01%	0.003%

cross-sections, each of different type. A horizontal cross-section (called ZY) shown in Fig. 4(b) is parallel to the hand surface and it is crossing the skin layer on the top of the hand model. The thumb is placed below this layer and is not visible in this horizontal cross-section. A longitudinal cross-section (called ZX) is shown in Fig. 4(a) and Fig. 6. A transversal cross-section (called XY) is shown in Fig. 5. The strange protrusion near the end of the middle finger is due to our discretization procedure. The positions of the applicator are visible in the longitudinal and transversal cross-sections. On the top of each figure we present the tissue configuration in the current cross-section with other cross-sections marked by the solid lines. All results shown in Figs. 4–6 were obtained after 500 iterations and they have been compared with the results obtained after 300 iterations. No visible difference was found when examining the normalized distribution patterns.

A big difference can be observed between the EM power distributions shown in the Fig. 4(a) and (b). On both figures the middle pattern is obtained from the knowledge of the primary electric field of the applicator acting in free space. Assuming that the electric field is not changed when the tissue is present we have evaluated this EM power distribution pattern via $1/2 \sigma \mathbf{E}^p \cdot \mathbf{E}^{p*}$. The patterns obtained in this way are called the patterns obtained

Fig. 4. (a) A longitudinal cross-section of the hand model and EM power distributions obtained from the primary field prediction ($1/2 \sigma E^p \cdot E^p$, middle pattern) and computed by the ALF method ($1/2 \sigma E \cdot E$, bottom pattern). Applicator type B3, 27.12 MHz. (b) A horizontal cross-section of the hand model and EM power distributions obtained from the primary field prediction ($1/2 \sigma E^p \cdot E^p$, middle pattern) and computed by the ALF method ($1/2 \sigma E \cdot E$, bottom pattern). Applicator type B3, 27.12 MHz.

from the primary field prediction. The patterns placed at the bottom in the Fig. 4(a) and (b) are obtained via $1/2 \sigma E \cdot E^*$. These patterns have been computed by the ALF method presented here. From the results shown in these figures we can observe that the EM power deposition penetrate much deeper in the computed patterns and cause some heating also in the bone tissue, while the bone marrow is not heated, see Fig. 4(a). There are also hot spots in the skin layer which are not predicted from the knowl-

edge of the primary field, while the "primary-field" heating of the skin on the fingers does not occur in the computed total EM power distribution pattern, see Fig. 4(b).

The difference between the EM power distribution patterns obtained with different types of the pancake coil applicator are presented in Fig. 5. The only significant difference occurs between the B3 coil pattern and the patterns obtained for the other coil applicators which have the external diameter twice as small than B3. The pattern for

Fig. 5. A transversal cross-section of the hand model and EM power distribution obtained with different types of coil applicator. Patterns computed by the ALF method; frequency of operation 27.12 MHz.

Fig. 6. Longitudinal cross-section of the hand model and EM power distribution obtained for different locations of coil applicator. Patterns computed by the ALF method; Applicator type F16, 27.12 MHz.

the convex coil designated in Table II as Cx6 is not presented because it looks almost the same as the pattern for the concave Cv6 applicator. The only difference between these two coil applicators is that Cx6 is much less efficient than Cv6. This fact can be seen when examining the input voltage $U(1\text{ W})$ necessary to dissipate 1 W power in the whole volume of the model, see Table II.

Finally, the most efficient flat coil F16 have been tested in three different positions to the hand model, see Fig. 6. Once again we can observe quite significant EM power deposition inside the bones of the fingers and a smaller deposition inside the metacarpal bones. The carpal bones, which form one almost solid bone in our hand model, are not heated in any of the applicator positions presented in Fig. 6. It should be also noticed that the bone marrow tissue gets always much less of the EM power deposition than the surrounding bone tissue.

VII. CONCLUSIONS

We have adopted an efficient ALF iterative scheme to solve realistic three-dimensional quasi-static electromagnetic field problems with inductive excitation and strongly inhomogeneous media. The number of iterations amounts to about 300 for our present problem with 50 000 unknowns. Double precision arithmetic is not needed in the case of a required accuracy of about 1%. Together with our model for capacitive applicator [3], we are now able to determine the EM power distribution in realistic three-dimensional models, provided that the frequency of operation is such that the quasi-static approximation may be applied.

The analyzed pancake coil applicators are useful for physiotherapy treatment where the high EM power deposition in the skin layer guarantees that any damage to

the inner tissue can be prevented if this skin heating activates the patient to react to the physiotherapist. The most significant differences between the patterns obtained for different types of the applicators occur in the case of different diameters.

REFERENCES

- [1] A. W. Guy, J. F. Lehmann, and J. B. Stonebridge, "Therapeutic applications of electromagnetic power," *Proc. IEEE*, vol. 62, pp. 55-75, 1974.
- [2] N. Morita and J. B. Andersen, "Near-field absorption in a circular cylinder from electric and magnetic line sources," *Bioelectromagnet.*, vol. 3, pp. 253-274, 1982.
- [3] M. J. Sowiński and P. M. van den Berg, "A three-dimensional iterative scheme for an electromagnetic capacitive applicator," *IEEE Trans. Biomed. Eng.*, vol. BME-37, pp. 975-986, 1990.
- [4] J. R. Oleson, "A review of magnetic induction methods for hyperthermia treatment of cancer," *IEEE Trans. Biomed. Eng.*, vol. BME-31, pp. 91-97, 1984.
- [5] R. J. Spiegel, M. B. Fatmi, S. S. Stuchly, and M. A. Stuchly, "Comparison of finite-difference time-domain SAR calculations with measurements in a heterogeneous model of man," *IEEE Trans. Biomed. Eng.*, vol. BME-36, pp. 849-855, 1989.
- [6] P. M. van den Berg, "Computer modelling of electromagnetic fields in human bodies," in *Proc. 1989 URSI Symp. Electromagnetic Theory*, Stockholm, Sweden, August 14-17, 1989, pp. 135-137.
- [7] N. Orcutt and O. P. Gandhi, "A 3-D impedance method to calculate power deposition in biological bodies subjected to time varying magnetic fields," *IEEE Trans. Biomed. Eng.*, vol. BME-35, pp. 577-583, 1988.
- [8] N. N. Yanenko, *The Method of Fractional Steps*. Berlin: Springer, 1971, pp. 10-11.
- [9] R. S. Ledley, H. K. Huang and J. C. Mazziotta, *Cross-Sectional Anatomy—An Atlas for Computerized Tomography*. Baltimore: Waverly 1977.
- [10] H. O. Kendall, F. P. Kendall, and G. E. Wadsworth, *Muscles, Testing and Function*. Baltimore: Williams and Wilkins 1971.
- [11] C. C. Johnson and K. R. Guy, "Nonionizing electromagnetic wave effects in biological materials and systems," *Proc. IEEE*, vol. 60, pp. 692-718, 1972.
- [12] S. R. Smith and K. R. Foster, "Dielectric properties of low-water-content tissues," *Phys. Med. Biol.*, vol. 30, pp. 965-973, 1985.

Maciej J. Sowiński was born in Warsaw, Poland, on July 24, 1947. He received the M.S. degree with first class honors in mathematics from Warsaw University of Technology in 1971 and the Ph.D. degree in 1980 from the Institute of Fundamental Technological Research, Warsaw, Poland.

In 1972 he joined the Department of Electromagnetic Wave Theory, Institute of Fundamental Technological Research, Warsaw, Poland, where he was appointed as an Assistant Professor. He has been engaged in research on waveguides and microwave devices, dielectric optical waveguides, and applications of null-field method and eigenfunction expansion method to field problems.

From 1987 to 1991 he was with the Laboratory of Electromagnetic Research, Delft University of Technology, Delft, The Netherlands first as a Research Fellow and then as an Assistant Professor. During this time he was working with computational methods based on fast Fourier transform algorithm, finite difference representation and iterative methods. All these methods were focused on applications in hyperthermia. From 1991 he is employed as a Research Engineer by the company Tecnomatix Medical n.v., Antwerpen, Belgium where he is engaged in the development of new applicators for intracavitary microwave hyperthermia and numerical programs estimating the temperature distribution during the hyperthermia treatment.

Peter M. van den Berg was born in Rotterdam, The Netherlands, on November 11, 1943. He received the degree in electrical engineering from the Polytechnical School of Rotterdam in 1964, the B.Sc. and M.Sc. degrees in electrical engineering, and the Ph.D. degree in technical sciences, all from the Delft University of Technology, in 1966, 1968, and 1971, respectively.

From 1967 to 1968 he was employed as a Research Engineer by the Dutch Patent Office. Since 1968 he has been a member of the Scientific Staff of the Electromagnetic Research Group of the Delft University of Technology. During these years he carried out research and taught classes in the area of wave propagation and scattering problems. During the academic year 1973-1974 he was Visiting Lecturer in the Department of Mathematics, University of Dundee, Scotland. During a three-month's period in 1980-1981, he was a Visiting Scientist at the Institute of Theoretical Physics, Goteborg, Sweden. He was appointed as Professor at the Delft University of Technology in 1981. At present, his main research interest is the efficient computation of field problems using iterative techniques based on error minimization, the computation of fields in strongly inhomogeneous media, and the use of the wave phenomena in seismic data processing.