RELIABILITY-BASED ANALYSIS OF RIVER DIKES DURING FLOOD WAVES

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Abstract: Reliability and risk analyses are basic approaches, nowadays, for flood risk assessments in European countries such as the Netherlands, Germany, United Kingdom, etc. The methods also receive a lot of interest in developing countries. By considering uncertainties for both load and strength variables, the probability of failure for each element of the flood defence system can be formulated. This paper focuses on the reliability analysis of the Red river dike during a typical flood wave of 1/500 year frequency in which the soil properties (unit weight, grain size, its thickness under the dike, etc) are modelled probabilistically. Harmonic functions are fitted for the predicted flood wave and the unsteady groundwater flow under the river dike is considered with delay and decay detection from piezometer observation data. Uncertainties are analysed and failure probabilities of some geotechnical phenomena under the river dike such as piping and instability have been calculated with different water levels.

Keywords: Reliability analysis, flood defence, river dike, probabilistic design.

1. INTRODUCTION

Flood defences play an importance role in our lives through centuries especially in delta areas with the influence of sea level rise. Probabilistic theory is applicable in this field, by considering into uncertainties for both load and strength, the probability of failure for each element could be formulated. However, with a given probability of failure, the risk could be evaluated due to the certain consequences. In Red river delta area, flooding used to be a daily problem in the past 50 years because Vietnam has the monsoon weather with very high river discharge in rainy season, in August and September every year, and the dike system were not strong enough. Nowadays, flood risk becomes smaller due to some efforts fighting against the nature such as strengthened dike, constructed reservoirs at upstream of the river... but the researchers also figured out that the dike should be considered with an extreme hydraulic boundary condition such as flood wave with frequency of 1/500 year and long duration. Dike becomes less safe in this situation, so that in this paper, the authors try to determine the reliability analysis of river dike with the uncertainties comes from geotechnical engineering aspects during a typical flood wave. Some expert opinion and assumptions will be made to give us an overview of the dike safety, however, the piezometer measurement data during flood wave would be a strong evidence for analysis work. In the first part, a general framework will be demonstrated before going further to the application.

2. RELIABILITY ANALYSIS OF RIVER DIKE DURING FLOOD WAVE

2.1. General framework

In civil engineering, generally speaking, we approach with deterministic methods which calculate the structures, materials or loads with certain values following current codes and standards. In this approach, loads and strengths, mostly, are assumed to be determined and the structures is safe when
the margin between the design value of the load and the characteristic value of the strength is large enough for all limit states of all elements. Therefore, the safety level of a structured system is not explicitly known, see [1].

Theoretically, the limit state function is defined as:

\[ Z = R - S \]  

where: \( Z \) is the limit state function; \( R \): strength; \( S \): load. Both \( R \) and \( S \) combine many uncertainties, for instance, the inherent and epistemic uncertainty, so that probability of failure \( P_f \{ Z < 0 \} \) is defined as the probabilistic failure if \( Z \leq 0 \) and \( Z = 0 \) is the boundary between safe and unsafe area, see figure 1. The probability of failure is calculated with the following integral, see [2]:

\[ P_f = P_{Z < 0} = \int_{-\infty}^{+\infty} F_R(x)f_S(x) \, dx \]  

in which \( F_R(x) \) is the cumulative distribute function of the strength \( R \); \( f_S(x) \) is the probability density function of the load \( S \), \( x \) is the random variable.

Mathematically, in \( R - S \) space, joint probability density function of \( R \) and \( S \) is \( f_{RS}(R, S) \), therefore, formula (2) can be written by:

\[ P_f = P_{Z < 0} = \iint_{Z < 0} f_{RS}(R < S) \, dRdS \]  

Limit state function is formulated for each element of structures and analyzing system with number of elements will be considered, in each case, stochastic variables are considered.

There are 3 levels of reliability calculation: The following four levels of approach were distinguished in determination of the safety of a structure, [2]:

- **Level 0**: Deterministic approach, the design is based on an average situations and an appropriated safety factor is introduced for obtaining a safe structure;

- **Level I**: Semi-probabilistic approach, a characteristic value is used in the design, like the load which is not exceeded in 95% of the cases, or the strength which is available for 95% of the construction material;

- **Level II**: Probabilistic approach with statistical distributions of all variables are taken into account. Level II comprises a number of approximate methods in which the distribution functions are transformed into standard normal or standard Gaussian distributions. In order to approximate the probability of failure, mathematical formulation of the problem has to be linearized;
• Level III: a highest level probabilistic approach and the probability distribution functions of the stochastic variables are fully taken into account. In this calculation level, the problem is solved for both linear and nonlinear functions, for the dependent or independent variables . . . etc.

Beside the reliability analysis for each element (as mentioned before), in reality, we dealt with system analysis which the components are connected serially or parallel. In this cases, we could determine the probability of failure for system following formulas:

\[ P_{\text{serial}} = P(Z_1 < 0 \text{ or } Z_2 < 0 \text{ or } \ldots \text{ or } Z_n < 0) \] (4)

in which \( Z_i < 0 \) denotes at least one of \( n \) failure mechanisms occurs, the system will collapse. Boundary of probability of failure can be estimated by:

\[ \max P_f(i) \leq P_f \leq \sum_{i=1}^{n} P_f(i) \] (5)

or narrower boundary given by Ditlevsen in [3] as following:

\[ P(R_1 - S_1) + \sum_{i=2}^{n} \max \{P(R_i - S_i) - \sum_{j=1}^{n-1} P((R_i - S_i) \cap (R_j - S_j))\} \leq P_f \] (6)

\[ P_f \leq \sum_{i=1}^{n} P(R_i - S_j) \max_{j<i} P((R_i - S_i) \cap (R_j - S_j)) \] (7)

In term of parallel system, if all elements are not functioned, the whole system will be failed, so probability of system, in this case, can be formulated as following:

\[ P_{\text{parallel}} = P(Z_1 < 0 \text{ and } Z_2 < 0 \text{ and } \ldots \text{ and } Z_n < 0) \] (8)

and the failure boundary can be calculated by:

\[ \prod_{i=1}^{n} P_f(i) \leq P_{\text{parallel}} \leq \min(P_f(i)) \] (9)

The reliability analysis has been developed for flood defense since 1950s in United State and European countries (such as the Netherlands, Germany . . . etc). By evaluating the reliability of existing levees system of Mississippi river, in [4], Wolff created a general framework of risk-based analysis for U.S flood defense system. In the Netherlands, Delta commission was found in 1953, after a flooding disaster, to prevent similar situation happening again. Group 10 Probabilistic method of the Technical Advisory Committee on Waters Defenses (TAW) has been assigned the task of making the results of this development applicable to flood defense structures, see [5]. Recently, many researchers also developed more detail for the application of reliability analysis not only in coastal flood defense but also through river dike.

In the light of reliability analysis, dike ring is considered as series system with number of dike reaches, which can be evaluate as a mono-system with many failure mechanism, see figure 2. Basically, for each dike section, failure mechanism can be named as: slope instability, piping, over topping/over flow or other anomalies, following analysis for each failure mechanism base on some research results from [6].

2.2. Hydraulic boundary condition

Red river (known as Song Hong) flows from mountain areas of southern China to the Gulf of Tonkin with the total length over 1150km (the length in Vietnam area is about 510km). At the border of Vietnam and China, Red river enters Laocai province (northern of Vietnam) and runs through the mountain areas to Viettri where Red river is contributed by two other tributaries named as Da river and Lo river respectively. In the past, [7] Red river Delta area used to be flooded in rainy season
with a lot of human and property damage. From the 11th century, people started paying attention to flood protection works such as: built up the dike along the river, declare the policy to deal with flooding situation … etc. The more development of the economy the higher of flood risk and the more damage people there would had. The fight again the flood has dramatical progress in the 1970s, after the Vietnam war, people strengthened the river dike system, constructed reservoirs in the upstream area…etc, so that, the safety level increased so far.

Following current maintenance guide and some researches results, the design water level is 13.4m in Hanoi (at Hanoi hydrology station), see figure 3. Following [8], this water level could be kept for hundreds hours if it happened the flood with frequency of 1/500 year in Hanoi area. By analyzing the flood statical data from 1960 to 2002, over 120 flood wave with different shape (water level and duration), we chose the flood wave of year 1996 as the typical flood, see figure 4 for more detail.

As can be seen from the figure 4, the predicted flood wave is flatter than the actual one in year 1996 because the redistributed effect of the reservoirs in upstream area. As the result, the water level from alarm level 3, equal to 14.4m at study location, will be remained longer than the flood in 1996 by over 6 times. So it is a challenge for the dike safety assessment in that situation because the longer the water level remains the more dangerous it would has. In figure 4(b), the flood wave is fitted as a half harmonic function in this form:

$$h(x, t) = h_o + \Delta h \cos(\omega t + \phi)$$  \hspace{1cm} (10)

where, $h(x, t)$ is the harmonic function of water level depending on distance $x$ and time $t$; $h_o$ is the initial water level (here is alarm level 3); $\Delta h$ is the amplitude and $\omega, \phi, t$ are the parameters of harmonic function.

It would be easier if we use the hydraulic boundary condition as the harmonic function so that the delay and decay of the groundwater flow could be figured out. The data from the fitted harmonic function will be used as the input parameters for reliability analysis of some failure mechanisms in some next parts as follows:

$$h(x, t) = 14.4 + 1.33 \cos(0.00595t - 1.4989)$$  \hspace{1cm} (11)

2.3. Groundwater flow

Groundwater flow has strongly influence to the failure mechanism of the river dike during flood wave [9], especially, if the probability of overtopping is small, geotechnical failure mechanisms such as
piping or instability will dominate the failure mechanism of the dike section. Following approach was described in [10], [11] and a part refer from [12], there is a delay and decay of the groundwater flow in sand layer and the impermeable layer under the dike which could be calculated. If the boundary condition imposed to the aquifer with the harmonic function:

$$P_1 = P_0 \cos(\omega t)$$  \hspace{1cm} (12)

The corresponding of groundwater flow in the aquifer becomes:

$$P = P_0 \exp\left(-\frac{x}{\lambda_\omega}\right) \cos(\omega t - \frac{ax}{\lambda_\omega})$$  \hspace{1cm} (13)
and the pore water pressure responded in the aquitard will becomes:

\[ p = P_o \exp\left(-\frac{x}{\lambda \omega} - \frac{z \delta}{d}\right) \cos(\omega t - \frac{ax}{\lambda \omega} - \frac{z \delta}{d}) \]  

(14)

in which, \( P_1, P, p \) are water pressure at the boundary condition, aquifer and aquitard respectively; \( P_o \) is the amplitude and \( \omega, \phi, t \) are the parameters of harmonic function; \( \lambda \omega \) is the cyclic leakage factor; \( a, d, z, \delta \) are parameters follow [10].

Equation (13) and (14) show that the cyclic boundary pressure is conveyed into the aquifer system with an amplitude decay \( \exp(-x/\lambda \omega) \) and a retardation (delay) \( ax/(\lambda \omega) \) for the aquifer pressure and an additional amplitude decay \( \exp(-z \delta/d) \) and additional retardation time (delay) \(-z \delta/(d\omega)\) for the aquitard pressure. The value of decay and delay also could be calculated by the observation data from piezometer during flood wave with ellipse shape demonstration. From the actual observation data at the study location in flood season in year 1996, we could calculate the parameters for cyclic response as follows:

\[ \lambda \omega = 100(m); a = 0.3; x = 95(m) \]

(15)

and the decay and delay could be figured out as: Delay = 47.9(\text{hours}) and Decay = 0.39(\text{m}), see figure 5 for more detail.

2.4. Reliability analysis of river dike

As presented before, in this paper, the failure analysis is focused on some geotechnical failure mechanisms only, for instance instability, piping, uplift . . . and some suggested judgement from expert opinion. Hereafter, we will investigate more detail to each mechanism.

2.4.1. Piping

Basically, following [13], [14] and [15] piping under the dike can be divided, draftly, in to 4 phases as follow:

1. Phase 1 - Initial and seepage: Water level in river increasing lead to the expansion of pore water pressure under the dike, this period last due to the speed of water level.
2. Phase 2 - Consolidation and backward erosion: Pore water pressure responses continuously as the result of the phenomenon. If there is no cover layer, backward erosion could be started with the sand transported as sand boil. In the location with the existing of impermeable layer, due to the crack or inhomogenous in its structures, sand boil could takes place with a delay depending on the difference water head and properties of cover layer. If there is no damage in this layer, it needed a very high water pressure to break the impervious layer, [16].

- Sub-phase 2a: Pore pressure increase in impervious layer due to high water in river with a certain decay and delay. However, there are number of disturbance in impervious layer such as: man-made hole, drought crack, animal buries... so water will be leakage out of this aquitard. At the same time, water pressure in sand layer causes the re-arrangement of sand particals.
- Sub-phase 2b: During pore pressure exceed in both sand and clay layer, as result, backward erosion increases step by step. This is a complicated process with partial moving separately or mass erosion. This phenomena was observed in the laboratory test clearly. If the water head is still increasing, the seepage length will be reduced time by time so that pipe will be formed through the dike foundation at the end of this sub-phase.

3. Phase 3 - Enlargement channel: Channel will be eroded and widened quickly during backward effect especially, after pipes are fully formed. This phenomenon will be recognised easily with high gradient of water flows out of dike foundation. Consequently, the subsidence of dike crest can be seen clearly, duration of this phase is around 1-6 hours or longer.

4. Phase 4 - Dike failure: Dike will be collapsed immediately in minutes by erosion effect of over flow. As a result, a huge lake will be formed in the land side at the dike failure position.

It is very difficult to describe the piping process happening during flood wave, until now, there is no model to deal with this situation. In [17], Sellmeijer - a Dutch man, described detail the piping mechanism mathematically, in [18], Foekje Buijs tried to formulate the length reduction of the pipe under the dike for time-dependent reliability analysis for the dike ring in UK. In this paper, the authors follow that approach with the assumptions of the length decreasing follow this formula:

$$L_t = L_{t-1} - \Delta L_t$$

with $\Delta L_t$ could be determine as follows:

$$\Delta L_t = \frac{\gamma'}{\gamma_w} D_{70}.K.C.\frac{\Delta H}{L_{t-1}}$$

in which: $\gamma' / \gamma_w$ is submerge density of soil/unit weight of water, $D_{70}$ is diameter which 70% passing, K is permeability of sand layer, $\Delta H$ is the different water level between river and land side, C is the correlated coefficient, t is the duration and $L_{t-1}$ is the seepage length at the time step t-1.

As can be seen from the equation (17), the recommended of Sellmeijer (1988), Fokej Buijs (2005) have already included. In [16], B.X.Truong proved that, the failure mechanism dominating in Thai Binh river dike area is the failure of cover layer with its cracks and inhomogeneous. Because of its importance, from the statistical data, the thickness of clay layer ($d$) is analyzed and fitted with probabilistic distribution function. Basically, we found $d$ being normal distributed with mean value $\mu = 2.5m$ and standard deviation $\sigma = 0.57m$. By back calculating from the field tests on the duration of sand boil and piping phenomena in this dike system, we figure out the correlated coefficient C is 74.96. Other input parameters for calculation is used in table 1.

The calculation result for piping failure mechanism is presented in figure 6(a). As we can see from the figure, reliability index of piping phenomenon decreases dramatically again the increasing of water level. The highest point of water level has a delay around 2 days with the lowest reliability index, for detail result, see figure 8.
Table 1: Input parameters for piping calculation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter passing 70%, D&lt;sub&gt;70&lt;/sub&gt;</td>
<td>mm</td>
<td>0.62</td>
</tr>
<tr>
<td>Submerged density of sand, γ'</td>
<td>KN/m&lt;sup&gt;3&lt;/sup&gt;</td>
<td>16.5</td>
</tr>
<tr>
<td>Permeability coefficient, K</td>
<td>m/s</td>
<td>0.001</td>
</tr>
<tr>
<td>Initial seepage length, L</td>
<td>m</td>
<td>95</td>
</tr>
<tr>
<td>Thickness of sand layer, D</td>
<td>m</td>
<td>20</td>
</tr>
<tr>
<td>Water level in land side</td>
<td>m</td>
<td>10</td>
</tr>
</tbody>
</table>

2.4.2. Instability and uplift

Stability and uplift are analyzed together due to their combination effect to the failure of the dike, in [19], M.Van performed a framework for uplift calculation. In [9], W.ter Horst also mentioned to the increasing of phreatic line in saturated and unsaturated zone in the dike embankment. In this case, we ignored the influenced of possible precipitation to the dike surface in flood season and only the macro-instability is considered.

For geotechnical condition, in [20] simply, there are three main soil layers such as: dike embankment (layer 1): greyish brown, yellow, stiff to very stiff, sandy clay; cover layer (layer 2): reddish yellow, yellowish brown, firm to stiff, clay; under cover layer (layer3): greyish yellow clayey sand and permeable layer (layer 4): brownish yellow, grayish brown, medium sand or coarse sandy gravel.

By using the GeoSlope packages the stability of each time step with a certain water level from the river will be figured out with the integrated between SEEP and SLOPE. Input data for calculation will be performed in table 2 and the detail calculation could be seen in the figure 6(b).

It can be seen from the figure 6(b), the reliability index of stability analysis drop slightly compare to the raise of flood wave, from \( \beta = 3.68 \) to \( \beta = 3.33 \) in 13 days, for detail result, see figure 8. The typical cross section analyzed and critical circle failure will be demonstrated in figure 7.

Table 2: Input parameters for instability calculation

<table>
<thead>
<tr>
<th>Layer</th>
<th>Distribution</th>
<th>Unit weight ( (\gamma_w, kN/m^3) )</th>
<th>Cohesion ( (C, kN/m^2) )</th>
<th>Internal friction ( (\phi, Degree) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal, ( \mu )</td>
<td>19.40</td>
<td>14.40</td>
<td>10.36</td>
</tr>
<tr>
<td></td>
<td>( \sigma )</td>
<td>0.61</td>
<td>1.62</td>
<td>1.86</td>
</tr>
<tr>
<td>2</td>
<td>Normal, ( \mu )</td>
<td>18.89</td>
<td>15.20</td>
<td>10.80</td>
</tr>
<tr>
<td></td>
<td>( \sigma )</td>
<td>0.73</td>
<td>2.67</td>
<td>1.38</td>
</tr>
<tr>
<td>3</td>
<td>Normal, ( \mu )</td>
<td>18.98</td>
<td>10.00</td>
<td>12.07</td>
</tr>
<tr>
<td></td>
<td>( \sigma )</td>
<td>2.50</td>
<td>2.20</td>
<td>2.15</td>
</tr>
<tr>
<td>4</td>
<td>Normal, ( \mu )</td>
<td>18.51</td>
<td>28.63</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>( \sigma )</td>
<td>0.53</td>
<td>2.17</td>
<td>2.17</td>
</tr>
</tbody>
</table>

2.4.3. Judgement of other failure mechanisms

Expert opinion plays an important role in risk assessment, in case, there is no data available, [21]. In this situation, besides all mentioned failure mechanisms, there are numbers of uncertainties that could result in the failure of river dike, for instance: animal burry, anomaly in dike embankment, poor maintenance, human error etc. Unfortunately, these influences can not be accounted qualitatively, so...
it should be judged by expert opinions combining with field inspection. It is assumed the reliability index and probability of failure for all components are given in figure 8.

2.4.4. Reliability analysis of the dike section

Basically, dike section failure due to the failure from each related mechanism as the serial system. In this paper, there are three failure mechanisms will be analyzed, after that the failure probability of the dike section could be determined following approach in [3]. The final result for the probability of failure of the dike section is presented in figure 8.

3. DISCUSSION

Generally, the reliability analysis for the dike during flood wave is a difficult issue because their complicated physical behaviour. Our understanding of the nature is limited so by analytical and numerical model, we have a part-view about the process with an actual case study. By the modelled flood wave frequency of 1/500 year, the water level in Red river around Hanoi area does not increase over 13.4m because of the redistributed discharge from the reservoirs in upstream area. Of course, in this case, we ignore the system analysis for the flood defence system included reservoirs, dam, dike, maintenances work . . . and the human error as well.
As we can see from the calculation result, the piping phenomenon dominates the failure of the dike section with the failure probability change from 0 to 0.4767. The lowest reliability index is 2 days later than the maximum of water level. In contrast, the fluctuation of water level does not influence so much on the instability analysis, the failure probability changes from $1.2 \times 10^{-4}$ to $4.3 \times 10^{-4}$. Expert opinion is needed for risk assessment of flood defense structure because all uncertainty cannot be taken into account. In this case, by collecting experiences data, a judged probability during flood wave has been given. The failure probability of the dike section could be seen in figure 8 with high probability of failure for dike section during flood wave predicted with frequency 1/500 year by the highest value of 53.6%. It means that, the dike could fail during the flood with frequency of 1/500 year with probability of over 50% but the delay could be predicted around 2 days later than the peak. Nevertheless, beside some assumptions in calculation due to lack of data but the research result give us an inside understanding about the failure mechanism of the river dike during flood wave. Work should be done not only in laboratory but also in numerical modelling to figure out the physical process and their behavior in the future.

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