NUMERICAL MODELING OF LONG-TERM OSCILLATING FLOWS IN MATERIAL PROCESSING OPERATIONS

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Abstract. Long-term oscillating behaviour of turbulent recirculating flow fields are observed in many material processing operations even with steady-state boundary conditions. Such unsteady large scaled flow behaviour is of particular importance for process control and process performance. Hence, it is important to understand the underlying mechanisms in order to improve the process performance when appropriate.

Therefore, models which are based on the unsteady Reynolds averaged flow equations (URANS approach) are employed in order to investigate long-term oscillations of flows in different material processing operations (electromagnetic stirring and continuous casting) in the present paper. The flows are also investigated in corresponding model experiments. The results of the simulations agree well with observations from model experiments when the numerical parameters are properly chosen. Long-term transient phenomena like vortex shedding are resolved in the Reynolds averaged flow field.

1 INTRODUCTION

Long-term oscillating behaviour of turbulent recirculating flow fields are observed in many material processing operations, e.g. in stirred vessel flows [1, 2] or in the continuous casting process of steel [3] even with steady-state boundary conditions. Frequently these oscillations are of particular importance for process control and process performance. Therefore, detailed knowledge of the underlying mechanisms is important in order to improve the process performance and/or to avoid oscillations of the flow when appropriate.

As an example, figure 1 shows the free surface of a continuous casting mold flow. The flow is studied in a water model experiment. In the flow, a large recirculating eddy is present which exhibits an oscillating behaviour. This oscillation generates periodic waves at the mold surface which can lead to slag entrainment into the melt.

Unfortunately, the possibilities of experimental research at material processing operations are often limited due to the physical and chemical properties of the liquids. For
example, an opaque melt at a temperature of some hundred Kelvin is only poorly controlled in experiments and the flow field cannot be analysed by conventional measuring techniques.

As an alternative numerical analysis - in combination with model experiments - is very helpful to investigate the flows which tend to long-term oscillations. Here it is necessary that an appropriate numerical model is chosen which describes the turbulence characteristics of the flow, resolves the oscillations and reflects the long-term behaviour of the flow correctly. Suitable candidates for high Reynolds number flows should be the large eddy simulation and the unsteady Reynolds averaged (URANS) approach of the fundamental equations. URANS models seem to be better suited for the investigation of industrial-scale flows from the engineers point of view, because they can give a cost-effective and appropriate description of the unsteady flow.

2 FUNDAMENTALS OF THE MODEL

2.1 URANS equations

The unsteady Reynolds averaged equation of continuity and Navier-Stokes equations are

\[
\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial}{\partial x_j} (u_j u_i) = -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \langle u_i \rangle}{\partial x_j} \right) + \langle f_i \rangle
\]

Here, \( u \) is the velocity and \( p \) is the pressure. The material parameters are density \( \rho \) and kinematic viscosity \( \nu \). The force density \( f \) is imposed by external sources, e. g. by gravity or electromagnetic fields. The brackets \( \langle \ldots \rangle \) represent a long-term time filter, which has been applied to the flow equations. With the turbulent stress tensor

\[
\tau_{ij} = (\langle u_j u_i \rangle - \langle u_j \rangle \langle u_i \rangle)
\]

2
the unknown time-filtered velocity products \( \langle u_j u_i \rangle \) in equation (2) are replaced by the product of the resolved time-filtered velocities \( \langle u_j \rangle \langle u_i \rangle \):

\[
\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial}{\partial x_j} \left( \langle u_j \rangle \langle u_i \rangle \right) = -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \langle u_i \rangle}{\partial x_j} + \tau_{ij} \right) + \langle f_i \rangle \tag{4}
\]

Equations (1) and (4) give the resolved flow field, where the velocity \( \langle u_i \rangle \) and the pressure \( \langle p \rangle \) are assumed to be implicitly time-filtered. The turbulent stress tensor \( \tau_{ij} \) in equation (4) gives the contributions of the unresolved turbulent fluctuations on the resolved flow. The individual stresses must be provided by a suitable turbulence model.

### 2.2 Turbulence model

Here, the Reynolds stress model of Launder, Reece and Rodi [4] is employed because the flows under investigation show high streamline curvatures. Eddy viscosity models, like the \( k-\varepsilon \) model of Launder and Spalding [5], have been also tested in the simulations, but they fail to describe this type of flow correctly. The flow quantities close to rigid walls are determined according to the non-equilibrium wall function approach [6].

### 2.3 Numerical methods, boundary conditions

Equations (1) and (4) are solved by the finite-volume method. The commercial code FLUENT is employed for this purpose. The discretization procedures within the model are second order implicit time-stepping, QUICK [7] for interpolation and the central differencing scheme for the discretization of derivatives. Pressure interpolation on the non-staggered grid is performed due to the method by Rhie and Chow [8]. The SIMPLE algorithm [9] is used for the iterative solution of all equations.

The steady boundary conditions of the flows are as follows: (i) At inlets, fixed values for velocity and turbulence intensity are given. The velocity values correspond to a constant flow rate in the tundish or the mold. Model specific values are given in tables 3 and 5. (ii) At outlets, zero gradients normal to the outlet for all flow variables are assumed. (iii) At rigid walls, no-slip conditions are applied.

Simulations with all flow variables in steady-state (RANS approach) generate the initial values for the URANS simulations.

### 3 ELECTROMAGNETIC STIRRING

#### 3.1 Induction furnace configuration

The geometry of an induction furnace is sketched in figure 2. The physical parameters of the molten metal (iron) are summarised in table 1. A crucible contains the melt pool, which possesses a free surface. The liquid metal is heated and stirred by an oscillating magnetic field \( B \). Due to the oscillation \( B \) is condensed into a skin just beneath the outer surfaces of the melt pool, which is denoted on the right side of figure 2. \( B \) induces a current density \( j \) in the melt. The product of \( j \) and \( B \) yields the Lorentz force density
$f_L$ which is the driving force of the melt flow. The Reynolds number of the melt flow is $Re = [u]_m d/\nu \sim 5000$ where $[u]_m$ is the mean melt velocity. The mean flow is dominated by a counter-rotating toroidal vortex pair, which is indicated in figure 2, too.

<table>
<thead>
<tr>
<th>magnetic permeability</th>
<th>$\mu$</th>
<th>$1.257 \times 10^{-6}$ H/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>electrical permittivity</td>
<td>$\epsilon$</td>
<td>$8.854 \times 10^{-12}$ F/m</td>
</tr>
<tr>
<td>electrical conductivity</td>
<td>$\sigma$</td>
<td>$7 \times 10^5$ S/m</td>
</tr>
<tr>
<td>density</td>
<td>$\rho$</td>
<td>$7000$ kg/m$^3$</td>
</tr>
<tr>
<td>viscosity</td>
<td>$\eta$</td>
<td>$6 \times 10^{-3}$ kg/(m·s)</td>
</tr>
</tbody>
</table>

Table 1: Physical parameters of the molten pure iron

The URANS model is employed to resolve the melt flow in a laboratory-scale induction furnace. The parameters of the furnace are given in table 2. It has been used in order to study the mass transfer at the free surface of the melt bath [10]. A more detailed description of the numerical model is given in [2]. Figure 3 pictures the numerical grid for the calculation of the flow field.

Figure 3: Grid of the melt bath in the induction furnace crucible.
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| electric current in the induction coil | $I$ | 1100 A |
| frequency of the electromagnetic field | $f$ | 2500 Hz |
| number of coil windings | $n$ | 8 |

Table 2: Process parameters of induction heating and stirring

3.2 Equations of the magnetic field

Due to the parameters of the problem the flow does not affect the electromagnetic field. This approach is called the diffusion limit. Here the induction equations of the magnetic field can be written as

$$\frac{\partial A_i}{\partial t} - \frac{1}{\mu \sigma} \frac{\partial^2 A_i}{\partial x_j^2} = \frac{1}{\sigma} j_{S,i}$$

(5)

where $A$ and $j_S$ are the magnetic vector potential and the coil current, respectively. The material parameters are given in table 1. The Lorentz force density $f_L$ can be deduced from the solution of equation (5) by means of Ohm’s law and Maxwell’s equations [11]. Then, $f_L$ is added as a force density into eq. (4).

Figure 4: Streamlines in the RANS (left) and the URANS simulation (right) in the vertical cross section of the flow field in the induction furnace crucible. Streamlines are coloured due to velocity. The velocity scale is from 0 m/s (blue) to 0.3 m/s (red).

3.3 Results

Results of the transient URANS simulation are compared with the initial RANS results in figure 4. The figures show the streamlines in the vertical cross section of the flow field. The overall flow structure is similar, but also some marked differences are obvious. The RANS simulation shows a symmetrical flow field. The toroidal vortex pair can be clearly identified which corresponds to reported findings of other simulation studies, e. g.
No connection between the upper and the lower vortex by streamlines can be seen.

The new features of the URANS simulation are that the upper and the lower vortex interact. Streamlines connect both vortices. Therefore exchange of species and heat between the upper and the lower vortex are not only due to turbulent diffusion. It is also driven by mean convection in the flow. This is of fundamental importance for the mixing characteristics especially in larger induction furnace crucibles. This influence has to be studied in future simulations.

4 CONTINUOUS CASTING OPERATIONS

4.1 Tundish flow

The geometry and the basic structure of the flow in a one-strand continuous casting tundish without flow modifying devices is pictured in figure 5. The inlet stream generates an impinging jet, which drives a large horse shoe vortex. The structure of the flow is well-known and has been resolved in many experimental and numerical investigations.

![Figure 5: Geometry and flow features of a one-strand tundish, lengths are given in mm.](image)

The periodic generation of funnel-shaped vortex tubes in the wake of the shroud has been observed recently in water model experiments [14]. Here, this phenomenon is investigated by a URANS simulation of the flow. The parameters of the flow are given in table 3, they correspond to a water model experiment at the authors institute.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>flow rate</td>
<td>$V$</td>
</tr>
<tr>
<td>inlet turbulence intensity</td>
<td>$t_i$</td>
</tr>
<tr>
<td>density</td>
<td>$\rho$</td>
</tr>
<tr>
<td>viscosity</td>
<td>$\eta$</td>
</tr>
</tbody>
</table>

Table 3: Boundary conditions and material parameters for tundish flow.

The periodic generation of funnel-shaped vortex tubes in the wake of the shroud has been observed recently in water model experiments [14]. Here, this phenomenon is investigated by a URANS simulation of the flow. The parameters of the flow are given in table 3, they correspond to a water model experiment at the authors institute.
Results of the simulations are analysed in a horizontal surface $S$ just beneath the top surface of the tundish in order to resolve eddy structures in the flow. Additionally, time-series of the lateral velocity $u_y$ are recorded in a point, which is located 75 mm above the midpoint of the tundish. Corresponding time series are also measured in the water model experiment.

Figure 6 shows streamlines in surface $S$ at two different times. Due to the action of the horse shoe vortex, the upwelling flow at the walls is directed towards the centerline of the tundish. The wake region is on the right side of the the submerged entry nozzle (SEN). Here large vortex structures exists, which oscillate around the tundish centerline. These vortex structures generate funnel-shaped vortex tubes, when the vorticity exceeds a certain limit.

The time series of the fluctuating flow quantities are analyzed by Fast Fourier transforms. Distinct peaks are evident in the the spectra of the water model experiment and the simulations for the flow rate $\dot{V}_2 = 1.6 \text{l/s}$, figure 7. The simulations have been performed with three different time step widths, $\Delta t_1 = 0.2 \text{s}$ (denoted as SIM1), $\Delta t_2 = 0.1 \text{s}$ (denoted as SIM2) and $\Delta t_3 = 0.05 \text{s}$ (denoted as SIM3). Obviously, the numerically obtained peak frequencies converge toward a value of $f_{\text{sim}} = 0.16 \text{Hz}$, which is near to the experimental result of $f_{\text{exp}} = 0.15 \text{Hz}$.

A similar spectrum is obtained for the flow rate $\dot{V}_2 = 2.2 \text{l/s}$ in the tundish. The peak frequencies for both flow rates are given in table 4. Good agreement between the experimental observation and the numerical prediction is evident.

In the simulations, the number of iterations per time step $n_x$ must be larger than the maximum local Courant number in the flow in order to resolve the oscillating flow.
Figure 7: Fast Fourier spectra in experiment and simulation, $\dot{V} = 1.6 \, l/s$.

<table>
<thead>
<tr>
<th>$V$ [l/s]</th>
<th>$f_{exp}$ [Hz]</th>
<th>$f_{sim}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>2.2</td>
<td>0.18</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 4: Measured and predicted oscillation frequencies in the tundish flow.

4.2 Mold flow

Next the flow in a continuous casting mold is examined. Figure 8 pictures basic features of the mold flow. The inlet stream which comes out of the SEN builds two jets downstream of the SEN ports. The jets drive two upper and two lower recirculating eddies. The whole flow field tends to long-term large scale oscillations.

These oscillations are investigated in a URANS simulation. The parameters of the flow are given in table 5. A more detailed description of the numerical model is given in [15]. Corresponding water model experiments have been conducted by Yuan et al. [16].

Features of the flow in the RANS and the URANS simulation are displayed in figure 9. The results of the RANS simulation are nearly symmetric and fit very good to the mean flow data of the water model experiments in [16].

The structures of time-filtered velocity fields in the transient flows of the URANS simulation differ markedly from the steady state. Basic components, i. e. the jet streams,

<table>
<thead>
<tr>
<th>flow rate</th>
<th>$\dot{V}$</th>
<th>0.71 l/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>inlet turbulence intensity</td>
<td>$t_i$</td>
<td>0.1</td>
</tr>
<tr>
<td>density</td>
<td>$\rho$</td>
<td>1000 kg/m$^3$</td>
</tr>
<tr>
<td>viscosity</td>
<td>$\eta$</td>
<td>$1 \times 10^{-3}$ kg/(m·s)</td>
</tr>
</tbody>
</table>

Table 5: Boundary conditions and material parameters for mold flow.
Figure 8: Geometry and flow features of a mold, lengths are given in mm.

Figure 9: Streamlines in the vertical center plane of the mold flow field. Results are from the RANS (left) and two different times of the URANS simulation (middle, right). Streamlines are coloured due to velocity. The velocity scale is from 0 m/s (blue) to 0.2 m/s (red).
the upper and the lower rotating eddies, are still present. But they are not symmetrical. At time $t_1$, the upper right eddy is elongated, whereas the upper left eddy is shrinked with respect to the steady solution, figure 9 (middle). Later at time $t_2$, the alternations of the eddies are nearly inverted, figure 9 (right).

It is found, that the complete mold flow takes part in the oscillatory motion with a constant frequency. This result is in good agreement with the qualitative observations of Gupta et al. [17]. They also report, that the complete mold flow pattern is mostly asymmetrical and oscillating.

An important process parameter of the mold flow is the mean velocity at the free mold surface. When the velocity is too high, problems can arise due to slag entrainment. The URANS approach offers the possibility to study the influence of the mold flow oscillations on the surface velocity.

Figure 10 shows the mean horizontal velocity $\bar{u}(x)$ in the right half of the horizontal centerline just beneath the free surface. Experimental data from the measurements of Yuan et al. [16] are compared to numerical data. The experimental data (denoted as EXP1 and EXP2) are derived from PIV measurements at two different flow rates. The numerical data corresponds to the mean of the transient URANS simulation and from a steady-state simulation (RANS).

The results of the URANS simulation fit very good to the experimental findings. Only for $x < 0.15 \, m$ the vertical velocity is slightly overpredicted. It can be assumed, that geometrical simplifications are responsible for this error: In the simulations, the free surface is flat. In the experiment, the surface is vertically elongated due to the surface waves, figure 1. This elongation is not resolved in the simulation, hence the numerically predicted acceleration of the fluid towards the SEN will be higher.
Contrary the steady-state solution overpredicts $\overline{u}(x)$ markedly. The numerical values are about 25% higher than the experimental data. The maximum amplitude in the RANS simulation is $\overline{u}_{\text{max}}^{\text{RANS}} = -0.20 \, \text{m/s}$, whereas it is only $\overline{u}_{\text{max}}^{\text{EXP}} = -0.15 \, \text{m/s}$ in the measurements.

The results for tundish and mold flows show, that URANS simulations can correctly resolve transient behaviour of the flows even at steady-state casting conditions. Measures for reducing or avoiding flow oscillations can be economically tested in URANS simulations, too.

5 CONCLUSIONS

A numerical model for oscillating flows in material processing operations is presented. It is based on the unsteady Reynolds averaged equations (URANS) in combination with a Reynolds stress turbulence model. Non-equilibrium wall functions are employed in order to describe the near-wall region of the turbulent flows.

The model is applied to the flows in a vacuum induction furnace, a continuous casting tundish and a continuous casting mold. All flow exhibit oscillating behaviour in the simulations. Numerical results are in good agreement with experimental findings.

The numerical model can be used in order to examine the effects of the long-term flow oscillations on the performance of the different processes in more detail. This would permit e. g. the investigation of surface wave generation in tundish mold flows, which are frequently suspected to promote unwanted slag entrainment into the melt. Measures for reducing or avoiding flow oscillations can be economically tested in URANS simulations, too.

REFERENCES


