NUMERICAL STUDY ON CRACK DILATANCY. II: APPLICATIONS

By Peter H. Feenstra,¹ René de Borst,² and Jan G. Rots³

ABSTRACT: Discrete cracks have been modeled with interface elements in which the tractions are related to the relative displacements of the crack sides. Three stages in the cracking process can be distinguished: (1) The linear-elastic state; (2) the development state in which a tension-softening model is used; and (3) the open-crack state, for which five different crack-dilatancy models have been implemented. The predictions of these crack-dilatancy models have been compared mutually as well as with available experimental data. The discrete-crack elements have subsequently been used in an analysis of an experiment on crack dilatancy. Again, the results for the various crack-dilatancy models are compared. Finally, a non-linear analysis of a moderately deep shear-critical beam is presented including aggregate-interlock effects.

INTRODUCTION

In a companion paper (Feenstra et al. 1990), interface elements have been developed in which crack-dilatancy models have been implemented. Five different models have been considered, namely, those by Bažant and Gambarova (1980), Gambarova and Karakoç (1983), Li et al. (1989), Walraven (1980), and Walraven and Reinhardt (1981). While in the companion paper the emphasis was on the formulation of consistent tangent operators for crack-dilatancy models and the analysis of the stability of these models (which are governed by a nonsymmetrical tangential-stiffness relation), this contribution focuses on a comparison between the various crack models and on medium-scale analyses in which the discrete interface elements are combined with plasticity, for modeling crushing, and bond-slip elements.

The arrangement of this study is as follows. First, a comparison is made between the various crack-dilatancy models. This is done using simple assemblies of two plane-stress elements and one interface element representing the discrete crack. Homogeneous stress and strain states are assumed. Then, a typical crack-dilatancy experiment is discretized into a number of plane-stress and interface elements. Stresses and strains no longer have to be homogeneous along the discrete crack and the analysis effectively serves as a check whether the assumption of homogeneous stress and strain made in the translation of the results of such experiments into stress-strain laws is correct. Finally, a moderately deep, shear-critical beam is analyzed. This analysis serves the purpose of assessing whether the developed discrete-crack elements, including crack dilatancy, can be combined successfully with

¹Res. Engr., Delft Univ. of Tech., Dept. of Civ. Engrg., P.O. Box 5048, 2600 GA Delft, Netherlands.

²Prof. and Sr. Res. Fellow, Delft Univ. of Tech., Dept. of Civ. Engrg./TNO Building and Constr. Res., P.O. Box 5048, 2600 GA Delft, Netherlands.

³Res. Engr., Delft Univ. of Tech., Dept. of Civ. Engrg./TNO Building and Constr. Res., P.O. Box 5048, 2600 GA Delft, Netherlands.

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plasticity and bond-slip elements. The analysis also follows a first estimate of the importance of taking into account the effect of aggregate interlock on the failure behavior of shear-critical beams.

SINGLE ELEMENT TESTS

Tests with Constant Crack Width

These tests have been carried out to validate the implementation. A proper comparison of the numerical results with the analytical results is possible because the displacements have been fully prescribed. The computational model is shown in Fig. 1. It consists of two four-node plane-stress elements with a height of 190 mm, a width of 150 mm, and a thickness of 114 mm and one four-node interface element. Dependence relations have been added to ensure that the deformations of the interface element were uniform. The dimensions of the model have been chosen in agreement with the specimen used in the experiments of Paulay and Loeber (1974) to make a comparison with their experiments possible. These experiments have been performed for a range of maximum aggregate particles from 9.5 mm to 19.0 mm. To investigate the effect of the aggregate shape, crushed and rounded material has been used. The numerical tests have been performed with round gravel aggregate and material properties according to Table 1. The concrete tensile strength has been set equal to zero, because it is only required in the initial crack stage, which is no subject of interest with these tests.

The experimental results in Figs. 2(a-c) are presented as the shear traction-tangential crack displacement relations expressed by the mean experimental data. These are defined as the average of the readings at the top and

FIG. 1. Computational Model for Tests with Constant Crack Width

TABLE 1. Test on Specimen with Constant Crack Width (Paulay and Loeber 1974)

Test (1)	Δu_n (mm) (2)	f_{cc} (N/mm ²) (3)	D_{max} (mm) (4)
1	0.13	35.6	19
2	0.25	36.6	19
3	0.51	36.7	19

FIG. 2(a). Comparison between Experimental and Computational Results for Tests with Constant Crack Width $\Delta u_n = 0.13$ mm

the bottom of the specimen. The normal traction-tangential crack displacement relations have not been given in the experimental data, so a comparison can only be made between the analytical results. The relation between the shear and the normal traction is given by Paulay and Loeber as the envelope of 75% of all test results. Paulay and Loeber have observed that the shear stress-tangential crack displacement relation essentially depends on the crack width. They concluded that shape and size of the aggregate have no noticeable influence.

FIG. 2(b). Comparison between Experimental and Computational Results for Tests with Constant Crack Width $\Delta u_n = 0.25$ mm

FIG. 2(c). Comparison between Experimental and Computational Results for Tests with Constant Crack Width $\Delta u_n = 0.51$ mm

The numerical results have been obtained by first giving the left plane-stress element an initial normal displacement of the required crack width and then increasing the tangential crack displacement monotonically at a constant crack width until the maximum tangential crack displacement of 0.8 mm has been reached. The comparison with the experimental results shows that the numerical results are in reasonable agreement, but that there is a rather large deviation between the different models. A remarkable observation is the fact that this deviation becomes less when the crack width is larger.

Tests with Constant Restraint Stiffness

The experiments described in the previous section do not provide a proper reflection of the real structural behavior. Cracks generally do not open to their final width and shear thereafter, but open and shear simultaneously. In order to test this, structural behavior tests have been performed by Walraven et al. (1979).

The experimental setup is very appropriate to further validate the implemented models and derived expressions for the tangential stiffness matrix. An iterative procedure is necessary to obtain equilibrium because the normal crack displacement is now a degree of freedom. Here and in the calculations that will be presented in the sequel to this paper, a full Newton-Raphson procedure has been used, unless indicated otherwise. The stability of this iterative procedure will give an impression of the behavior of these models governed by a nonsymmetrical stiffness matrix.

The computational model is shown in Fig. 3. The model consists of two four-node plane-stress elements with height of 300 mm, width of 200 mm and thickness of 120 mm and one four-node interface element. The displacements have been assumed to be uniform. This has been ensured by adding dependence relations. The restraint bars are modeled with two truss elements between the outer nodes of the plane-stress elements with a stiff-

FIG. 3. Model with External Restraint Bars

ness necessary to obtain a normal stress of 7.8 N/mm^2 , respectively, 1.6 N/mm^2 at a crack width of 0.6 mm .

The left plane-stress element has been given an initial displacement equal to the required crack width. Then the tangential crack displacement is increased monotonically until the maximum tangential crack displacement of 2.0 mm has been reached. To make possible a comparison with the experimental results, the material properties of Table 2 have been used. The values between braces are the actual values of the experiments. The concrete tensile strength is set to zero with the numerical tests, as has been explained for the tests with constant crack width. The initial tangential crack displacement $\Delta u_{t0} = 0.04 \text{ mm}$ in test 1/.2/1.6 has not been modeled in the numerical test because the influence of this initial crack sliding is very small.

All the numerical results represent converged solutions, and it can be concluded that the iterative procedure is stable. The comparison with the experimental results shows that the models are capable of simulating the experiments qualitatively, but the exact path is not simulated by any of the models, see Figs. 4(a) and 4(b). The restraint stiffness is taken constant in the calculations. This is in agreement with the initial design of the experimental setup. However, the restraint stiffness was lower than expected during the experiments; this was blamed on the bad fitting of the restraint plates on the concrete surface. The restraint stiffness is only indicated with the average restraining stress normal to the crack plane for an arbitrary crack width of 0.6 mm . This might be the cause of the differences between experiment and calculations in the shear traction-tangential crack displacement diagrams.

TABLE 2. Test on Specimen with Constant Restraint Stiffness (Walraven et al. 1979)

Test (1)	Δu_{n0} (mm) (2)	Δu_{t0} (mm) (3)	f_{cc} (N/mm ²) (4)	D_{max} (mm) (5)
1/0.0/7.8	0.01	0.00	38.5	16
1/0.2/1.6	0.18	0.00 (0.04)	38.5	16

FIG. 4(a). Comparison between Experimental and Computational Results for Test 1/.0/7.8 with Constant Restraint Stiffness

The normal traction-normal crack displacement diagram is in agreement with the constant stiffness perpendicular to the crack plane. The normal tractions should make equilibrium with the tensile forces in the external bars with constant normal stiffness. For all models the numerical result is a straight line, which is agreement with the constant restraint stiffness.

FIG. 4(b). Comparison between Experimental and Computational Results for Test 1/.2/1.6 with Constant Restraint Stiffness

EXAMPLES FOR ELEMENT ASSEMBLIES

The previous section dealt with the evaluation and stability of crack-dilatancy models in a single interface element. For practical applications it is necessary to validate the models for assemblies of interface elements and plane-stress elements. Here, the stabilizing influence of the boundary conditions may differ significantly from the situation for single element tests and this may result in an instability of the solution procedure. The comparison with the experimental results has been performed in order to simulate the experiments in a qualitative sense, since this study is focused on the aspects of stability and not on an exact replication of the experiments.

Experiment 1/.0/7.8 (Walraven et al. 1979)

The experiments with external restraint bars are very convenient for testing the crack-dilatancy models in a finite element mesh because the only interface problem is aggregate interlock. Softening, dowel action, or bond slip of embedded reinforcement have no influence on these experiments. Test 1/.0/7.8 has been simulated because the restraint stiffness obtained in the experiment is approximately constant, which is convenient for the numerical modeling. It is also an experiment that has some agreement with rough cracks in concrete structures. The initial crack width is 0.01 mm.

The computational setup is given in Fig. 5 and consists of eight-node plane-stress elements and six-node interface elements. The plane-stress elements are integrated with a nine-point Gauss quadrature, while the interface elements are integrated with a three-point Gauss integration rule. Dependence relations have been added to ensure that the long sides of the specimen be

FIG. 5. Computational Setup for Test with External Restraint Bars

TABLE 3. Test 1/0/7.8: Specimen with Constant Restraint Stiffness (Walraven et al. 1979)

f_{ec} (N/mm ²) (1)	c (N/mm ²) (2)	ϕ (3)	f_{cr} (N/mm ²) (4)	D_{max} (mm) (5)
38.5	11.1	30	0.0 (2.8)	16

kept straight. Stiff plates have been fixed to these sides, connected by restraint bars. The specimen is supported on roller bearings. At the top, the load has been supplied by giving the fixed nodes a vertical displacement until an average shear displacement has been reached of 2.0 mm. Before shearing the specimen a discrete crack has been formed by applying loads at the ends of the truss elements until an average crack width of 0.01 mm, the initial crack width in the experiment.

During the experiments small regions of compressive spalling have been observed. In order to investigate the influence of this spalling on the traction distribution along the interface, the spalling has been modeled with a Mohr-Coulomb plasticity model with a cohesion c and a friction angle ϕ . The material properties are given in Table 3.

The major goal of this numerical experiment is to investigate the behavior of the interface elements with crack-dilatancy models in a finite element mesh in a qualitative sense, rather than to obtain an exact simulation of the experiments. Therefore, attention has been focused on (1) The stability of the numerical solution; and (2) the stress and the displacement distributions along the discrete crack. The numerical solution is considered to be stable because a fully converged solution has been obtained at all stages of the loading process. This holds true for all five crack-dilatancy models that have been used in the analysis.

The stress distribution and the distribution of the relative displacements are shown in Figs. 6(a) and 6(b) respectively, for the first 10 loading steps. These distributions have been obtained with the (linear) aggregate-interlock

FIG. 6(a). Stress Distribution along Interface Elements

FIG. 6(b). Distribution of Relative Displacements along Interface Elements

relation. The six interface elements are separated by the lines, with the integration points marked by a circle. It appears that the distribution of tractions and relative displacements across the crack are almost uniform. This observation lends support to the correctness of these types of experiments. If the tractions and the relative displacements are averaged with a weighted sum, where the Gauss factors are the weight factors, the traction-shear displacement should be equal to the input constitutive relations on element level, see Fig. 7.

A major concern is the numerical stability of the models. The models showed considerable differences in the convergence characteristics, but all solutions are truly converged. For the Rough Crack model it was not possible to use a tangential stiffness matrix until a displacement at the top of the specimen had been reached of approximately 0.14 mm. Although the step size was very small, approximately 0.01% of the ultimate tangential displacement, an iteration method with a linear-elastic stiffness matrix was necessary to obtain a converged solution. A full Newton-Raphson method did not result in converged solutions. The other models showed better convergence characteristics with a Newton-Raphson method, although for the two-phase model small steps were necessary when the crack width was still small. The contact density model showed somewhat better behavior and the aggregate-interlock relation showed the best behavior, since loading steps of 5% of the ultimate tangential displacement were possible, even for small crack widths.

Calculation Beam A2

The moderately deep beam A2 has been tested experimentally at Delft University of Technology (Walraven 1978) in order to investigate the ultimate shear resistance of lightweight concrete beams without shear reinforcement. Within this research program, experiments have also been performed on normal aggregate concrete beams, in which aggregate interlock is an important component of the shear transfer. During the experiment, it was observed that vertical bending cracks developed. However, failure ultimately

FIG. 7. Comparison Experimental and Computational Results Constant Restraint Stiffness

FIG. 8. Beam A2 Stevin Laboratory: Position of Discrete Cracks

occurred due to diagonal shear cracks.

This beam has been analyzed before using a smeared-crack approach (de Borst and Nauta 1985) and a predictor-corrector method (Wang 1989). Following the latter procedure, proposed by Rots (1988), a smeared-crack analysis is followed by a discrete-crack corrector analysis. The smeared-crack analysis predicts the position and direction of the dominant cracks. With the discrete-crack corrector, it is possible to investigate the ultimate load of the structure more accurately. The position of the discrete cracks is given in Fig. 8. The position has been chosen by assessing the smeared-crack patterns

TABLE 4. Material Properties Beam A2

Property (1)	Value (2)
(a) Concrete	
E_c (N/mm ²)	28,000
ν	0.2
f_{cc} (N/mm ²)	34.2
f_{ct} (N/mm ²)	2.49
G_f (N/mm ²)	0.06
D_{max} (mm)	16.0
(b) Reinforcement	
E_s (N/mm ²)	210,000
σ_y (N/mm ²)	440.0
A_s (mm ²)	150.8

and the experimentally observed crack pattern at impending failure. The discrete-crack analysis has been carried out with eight-node plane-stress elements with four-point Gauss quadrature. The discrete cracks have been modeled with six-node interface elements with three-point Gaussian integration. The reinforcement has been modeled with three-node truss elements. Six-node line-interface elements have been used for modeling the bond slip between concrete and reinforcement. The material properties are given in Table 4. The discrete cracks have been modeled with a linear tension-softening model, followed by a crack-dilatancy model; in this analysis the aggregate-interlock relation has been used. The initial stiffnesses of the interface elements k_n and k_t have been chosen low, namely 10^{+3} , to avoid oscillation in the linear-elastic traction profile, see also Feenstra et al. (1990) or Schellekens (1990). The slip between the reinforcement and the concrete has been modeled using the relation of Dörr (1980).

The structure has been analyzed using a full Newton-Raphson method with indirect displacement control (de Borst 1986). The constraint equation for this control method is the crack opening displacement (CMOD) at the mouth of all discrete cracks. It was possible to obtain a converged solution, with a variation of 10^{-4} on the reference energy norm, until the maximum load had almost been obtained. At this stage, the inclined shear crack opened and the constraint equation was applied to the crack sliding displacement (CMSD) of the mouth of the inclined discrete crack. The load-displacement curve of Fig. 9(a) shows that the opening of the inclined crack is attended by a snap-back in the load-displacement curve. Also beyond the maximum load, converged solutions were obtained, although the load had to be decremented very carefully and with small steps.

A more detailed inspection of Fig. 9(a) allows us to draw a parallel with the tension-pull specimen analyzed by Rots (1988). The load-elongation curve of this specimen shows local maxima corresponding with the development of successive cracks. This serrated type of curve is quite similar to experimental results for a long-embedment tension-pull specimen (Rots 1988). Indeed, the development of the discrete cracks in the beam should show some resemblance to the previously mentioned load-elongation curve. It appears

FIG. 9(a). Load versus Deflection at Midspan Beam A2

that the tiny peaks in the load-deflection curve [Fig. 9(a)] correspond to the opening of the discrete cracks. If this part of the load-deflection curve is plotted on a larger scale [Fig. 9(b)] it becomes apparent that although there is no local snapback behavior as in the tension-pull specimen of Rots (1988), the local load-deflection curve has a descending branch whenever a discrete crack opens. During this state, i.e., when a discrete crack develops, the load steps have to be taken small, approximately 0.1 kN, in order to obtain a converged solution.

After the development of the four bending cracks, the load-deflection curve is almost linear until the peak load of 58.0 kN has been reached. This peak load is 18% lower than the ultimate load measured in the experiment. While this ultimate load is not predicted very accurately, the numerically computed collapse mode is in better agreement with the experimentally observed collapse mode. The experiment has been performed with load steps of 2.5–

FIG. 9(b). Load versus Deflection at Midspan Beam A2: Development of Four Discrete Bending Cracks

FIG. 10. Beam A2 Stevin Laboratory. Total Displacements before Peak Load: $F = 56.32$ kN

FIG. 11(a). Beam A2 Stevin Laboratory. Total Displacements after Peak Load: $F = 48.61$ kN

FIG. 11(b). Beam A2 Stevin Laboratory. Incremental Displacements after Peak Load: $F = 48.61$ kN

10.0 kN and failure occurred as a sudden collapse of the beam because of development of a diagonal crack. The numerically computed global snap-back behavior, which accompanies structural failure, is in full agreement with the experimentally observed explosive collapse of the beam.

The prepeak displacements in Fig. 10 show that the first four cracks are open and that the diagonal crack is still closed. In this stage, just prior to

FIG. 12(a). Beam A2 Stevin Laboratory. Total Displacements after Peak Load: $F = 48.14$ kN

FIG. 12(b). Beam A2 Stevin Laboratory. Incremental Displacements after Peak Load: $F = 48.14$ kN

the peak load, the stress transmitted in the four bending cracks has almost been reduced to zero. The postpeak part of the load-deflection curve is the result of the development of the diagonal crack, as indicated in Figs. 11(a) and 11(b). It is clear from the incremental displacement pattern, Fig. 11(b), that the first four cracks are closing and the diagonal crack is opening at this stage.

Figs. 12(a) and 12(b) show the results at the stage where the calculations have been terminated. The load-deflection curve is again ascending and from the total displacements, Fig. 12(a), it is clear that the diagonal crack has opened further. The incremental displacement pattern, Fig. 12(b), shows that the first four cracks are opening again but that failure is localized at the diagonal crack. The principal stresses at this stage show a further stress relief at the bending cracks and at the diagonal crack. Only at the first, vertical part of the diagonal crack shear tractions are transmitted due to aggregate interlock. Stress transfer due to aggregate interlock at the inclined part of the diagonal crack appeared to be negligible.

CONCLUSIONS

From the computations on the simple tests, in which homogeneous stress and strain states are assumed a priori, it appears that the crack-dilatancy

models that have been tested show a reasonable agreement with the experimental data. Nevertheless, a large scatter is mutually observed, especially for smaller crack widths.

All computations on homogeneous stress configurations with a full Newton-Raphson procedure resulted in a good convergence behavior, in spite of the fact that the tangential stiffness matrix of the interface element is non-symmetric and becomes indefinite at some stage of the loading process. For the medium-scale calculations, convergence was sometimes slow. For some models, load steps as small as 0.01% of the ultimate load had to be taken in order to obtain a converged solution. Even then, iterating with a tangential stiffness matrix was not always possible.

From the numerical simulation of the aggregate-interlock experiment it appeared that the assumptions that are usually made in translating such experiments into stress-strain laws are justified, since the stress concentrations at the notches have a negligible influence on the average behavior of the specimen.

The analysis of the moderately deep beam revealed that the failure mechanism, namely a diagonal shear crack, of such beams is highly explosive. Simulations under quasi-static loading conditions then show a severe snap-back behavior. Surprisingly, the aggregate interlock had only a minor effect on the stress distribution and load-carrying capacity of this shear-critical beam.

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