A Coastal Behaviour Model

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In the name of God
the Compassionate, the most Merciful

A Coastal Behaviour Model

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ABSTRACT

A computer model for wave analysis, sediment transport computation and morphological process prediction along a coast has been developed as a master of science thesis. The very important factor in morphological processes, the wave propagation, has been analyzed using a fairly realistic mathematical model, viz.: ENDEC model, which considers ENergy DECay of random waves over an alongshore rather uniform, but cross-shore arbitrary varying topography.

For sediment transport computation several formulae have been involved whereas sediment distribution over a direction normal to the coast has been presented based on Bijker and Van Rijn approach through a numerical procedure. The results of the mentioned wave analysis are locally applied in this numerical computation.

Although, a random wave analysis was considered in the model, the wave characteristics in some location corresponding to the breaker point in regular waves were needed for sediment transport computation by the CERC formula. An investigation was carried out in order to find a criteria to determine the location on which the wave characteristics could be applied in the CERC formula as the breaker parameters.

Since reliable prototype measurements on sediment transport are very difficult to obtain, a comparison analysis has been carried out on three sediment transport formulae. Bijker and van Rijn formulae were compared with the well-known CERC formula in this evaluation to verify the results of these two formulae.

As an applicability investigation of the model a computer program has been made to predict coastal evolution near the breakwaters of a harbour.

An abbreviate name was selected for this series of the programs as CBM model, which is an acronym for Coastal Behaviour computer Model.
CHAPTER 1

INTRODUCTION

This work was initiated to solve numerically the selected mathematical model for wave analysis. This model takes into account the effect of depth-induced breaking on wave propagation, and the bottom friction. Battjes and Janssen (1978) were the first to present an energy balance method in the modelling of the energy decay due to depth-limited breaking in random waves. Battjes and Janssen’s model was extended and verified by Stive and Dingemans (1984) taking into account additional effects of refraction due to current as well as bottom friction. According to this extended mathematical model a computer model was developed. Chapter 2 deals with the process of this modelling. Further the calibration process of the mathematical model will be reported in this chapter followed by the verification of the computer model. By the explanation of an investigation process done to find a point corresponding to the breaker point in regular waves, the part of wave analysis of this study will be finished. In Chapter 3 the Bijker approach is involved to calculate longshore current velocity. The sediment transport formulae, CERC, Bijker and van Rijn involved in the computer model will be described later in this chapter. In chapter 4 an important problem in coastal engineering practice is presented, in which an evaluation of various sediment transport formulae is performed. For this purpose the output results of computer program are compared while CERC formula has been selected as a base. Conclusions will be drawn and some remarks on implementation of this part will be made in chapter 5.

On the second part of this study a computer program is developed to predict coastal evolution. The case of sediment accretion and erosion near the breakwaters has been investigated in this part. The outcomes of the previous parts are used in the computer program in order to show a practical case of the model in an important Coastal Engineering problem. Although, various solutions have been presented in literature, herein a simple treatment, namely single line theory, has been considered for the problem. The modelling process of this computer program on numerical and computational aspects is dealt with later on in this chapter.
CHAPTER 2

WAVE ANALYSIS

2.1 Introduction

Nowadays theories and models which consider irregular random waves are of great importance in wave propagation analysis. If sediment transport is considered, the velocity distribution has an important role. The local mean flow velocity distribution (due to longshore and other currents like tidal current) is obtained while irregular random waves are regarded in wave analysis.

Battjes and Janssen (1978) presented an energy balance method in the modelling of energy decay due to depth-limited breaking in random waves. This mathematical model was extended by Stive and Dingemans (1984) where verification of Battjes-Janssen’s model was being proceeded. For verification by field measurements they needed to consider additional physical phenomenae, like refraction due to current.

In a close co-operation with the study team concerned with the verification of the parabolic refraction-diffraction model, CREDIZ, (see Dingemans, 1983), the practical value of this model in nearshore engineering application was confirmed. This more general model is called ENDEC, which is an acronym for Energy DECay.

The following conditions are taken into account in this model:

- Irregular random waves
- Alongshore rather uniform but arbitrary beach profile
- Breaking due to depth-limitation
- Energy Dissipation due to bottom friction and wave breaking
- Shoaling and Refraction due to bottom variations
- Refraction due to current variations
- Additional energy source term i.e. Energy gain due to constant local wind field
2.2 Mathematical modelling of ENDEC

The wave action conservation equation for a stationary wave field in two horizontal dimensions is considered as a start point, Fig. 2.1 shows the axis system:

\[
\nabla \left[ (\varepsilon_g + U) \frac{E}{\omega} \right] + \frac{D}{\omega} = 0
\]

(2.1)

\[
\varepsilon_g = \omega, \frac{\partial}{\partial k}
\]

(2.2)

\[
\omega = \omega + k \cdot U
\]

(2.3)

\[
\omega_r = (gk \tanh kh)^{1/2}
\]

(2.4)

\[
k = |k|
\]

(2.5)

where:

- \(E\): mean energy density
- \(D\): dissipating energy source term
- \(g\): gravity acceleration
- \(h\): local water depth
- \(k\): wave number
- \(U(x,y)\): a current field
- \(\omega_r\): relative (or intrinsic) wave frequency
- \(\omega\): apparent wave frequency
- \(c_g\): group velocity

The presence of a current affects the wave frequency. Then in a current-wave field the measured value of \(\omega\) can be supposed as apparent frequency. Therefore \(\omega_r\), obtained by Eq. 2.3, is the contribution of waves in this wave-current field.

Equation (2.1) can be written as:
\[
\frac{\partial}{\partial x} \left[ (c_g \cos \theta + U \cos \nu) \frac{E}{\omega_r} \right] + \frac{\partial}{\partial y} \left[ (c_g \sin \theta + U \sin \nu) \frac{E}{\omega_r} \right] + \frac{D}{\omega_r} = 0 \tag{2.6}
\]

\[c_g = |\vec{c}_g| \quad U = |\vec{U}|\]

where:

\[\theta : \text{ wave ray angle with the positive x-axis}\]
\[\nu : \text{ current angle with the positive x-axis}\]

If we simplify our situation to a bathymetry of parallel bottom contours, then the \(x\)-axis is normal to the bottom contours. In such a case Snell's law yields:

\[k_y = k \sin \theta \quad \text{or} \quad \sin \theta / c = \text{constant}\]

If a constant current field is also assumed, then the changes of variables in \(y\) direction are zero and Eq. 2.6 yields:

\[
\frac{d}{dx} \left[ (c_g \cos \theta + U \cos \nu) \frac{E}{\omega_r} \right] + \frac{D}{\omega_r} = 0 \tag{2.7}
\]

Equation (2.7) actually refers to energy balance; a second differential equation should be adapted to calculate the change in mean water level, \(\eta\). This change is due to the radiation stress effect. The momentum reading as below refers to the subject:

\[
\frac{dS_x}{dx} + \rho g (d + \eta) \frac{d\eta}{dx} = 0 \tag{2.8}
\]

\[S_x = \left( \frac{c}{c} (1 + \cos^2 \theta)^{\frac{1}{2}} \right) E \tag{2.9}\]

\[c = \omega_r / k\]
where:

\[ S_{II} \] : shorenormal radiation stress  
\[ \eta \] : mean water level increase (wave set-up)  
\[ d+\eta \] : total mean water depth (h)  
\[ k \] : wave number  
\[ c \] : phase velocity

2.3 The energy sources

The following terms in the energy source can be introduced:

\[ D = D_b + D_f - D_w \]

where:

\[ D \] : total dissipated energy  
\[ D_b \] : power dissipated due to wave breaking  
\[ D_f \] : power dissipated due to bottom friction  
\[ D_w \] : power gained due to a local wind field

One should be aware that the term \( D_b \), generally dominates over \( D_f \) and \( D_w \).
so that $D$ is always positive and a decay of wave energy results.

2.3.1 Evaluation of $D_b$

It is assumed that the non-broken waves obey a Rayleigh distribution with respect to the wave height, $H$:

$$F(H) = P(H|H) = \exp\left(-\frac{1}{2}(H/H)^2\right) \quad 0 \leq H < H_{m}$$

$$= 1 \quad H \geq H_{b}$$  \hspace{1cm} (2.10)

$$H_{m} = \frac{0.88}{k} \tanh\left(\frac{\gamma - kh}{0.88}\right)$$ \hspace{1cm} (2.11)

where:

- $H$: maximum existing wave height
- $H$: a random variable
- $R$: a model value without physical meaning
- $H_{m}$: maximum possible wave height

Eq. 2.11 is with slightly changes proposed by Miche (1969). In Eq. 2.11 $\gamma$ reads:

$$\gamma = 0.5 + 0.4 \tanh (33s_{\gamma})$$ \hspace{1cm} (2.12)

where:

- $\gamma$: model parameter being treated later on (section 2.4)
- $s_{\gamma}$: deep water wave steepness ($H_{m}/L_{\gamma}$)

By Eq. (2.11) it can be observed that:

- for shallow water $H_{b} \rightarrow \gamma h$
- for deep water $H_{b}/L \rightarrow 0.14$

In fact a whole spectrum of waves is considered wherein the highest and
steepest waves (according to $H_s$) are being breaking or broken. $Q_b$, the probability that for a specific location $x$, a wave height is associated with a breaking or broken wave ($H \geq H_s$) is:

$$Q_b = P(H \leq H_s) = 1 - P(H \geq H_s)$$  \hspace{1cm} (2.13)

Substituting Eq. 2.10 in Eq. 2.13 yields:

$$Q_b = \exp\left(-\frac{1}{2}(H_s/\hat{H})^2\right)$$  \hspace{1cm} (2.14)

In other words $Q_b$ can be assumed to be the fraction of waves which are breaking or broken in a certain point.

Battjes and Janssen derived the following expression for $Q_b$ through a probabilistic calculation while the wave field has been characterized by $H_{rms}$.

The root mean square, $H_{rms}$, is defined as:

$$H_{rms}^2 = \int_0^\infty H^2dF(H) \int_0^H H^2dF(H) + H^2\Delta F(H_m)$$

then:

$$H_{rms}^2 \int_0^H H^2dF(H) + Q_b H_m^2$$  \hspace{1cm} (2.15)

Substituting Eq. 2.14 in Eq. 2.15 and using a lot of algebra yields:

$$H_{rms}^2 = 2H_m^2 (1 - Q_b)$$  \hspace{1cm} (2.16)

The non specified $\hat{H}$ can be eliminated from Eqs. (2.14) and (2.16), yielding

$$Q_b = \exp[-(1-Q_b)/b^2]; \hspace{0.5cm} b = H_{rms}/H_m$$  \hspace{1cm} (2.17)

For derivation of the dissipation factor, $D_b$, the bore approximation for both cases, shallow and deep water, have been considered and interpreted. It means the wave condition and breaking is similarized to a bore. The conclusions are summarized as below:
\[ D_{\text{wave}} = \frac{\alpha}{4} \rho g f \frac{H^3}{h} \quad \text{; shallow water} \] (2.18)

\[ D_{\text{wave}} = \frac{\alpha}{4} \rho g f H^2 \quad \text{; deep water} \] (2.19)

where:

\[ f \] : representative of wave frequency in presence of current

For periodic waves in shallow water \( H/h = O(1) \) is a valid approximation, then Eq. (2.19) yields for both cases.

For application to irregular waves the maximum wave height \( H_m \) together with the fraction of breaking or broken waves, \( Q_b \), can be used. Then the following expression is obtained for energy dissipation due to breaking waves:

\[ D_b = \frac{\alpha}{4} \rho g f H_m^2 Q_b \]

where:

\[ c_{\text{wave}} = \frac{\omega_r}{k} \]
\[ \omega_r = 2 \pi f \]
\[ \omega_r = \sqrt{gk \tanh kh} \]

then:

\[ D_b = \frac{\alpha}{4} \rho g \frac{\omega_r}{2\pi} H_m^2 Q_b \] (2.20)

Eq. 2.20 is the final equation for calculating the energy dissipation due to depth-limited breaking in which, \( \alpha \) is a free parameter. The value of \( \alpha \) should be in satisfactory agreement with measurements and will be discussed.
later on in the calibration section of this report (section 2.4, page 12).

2.3.2 Bottom friction energy dissipation

For derivation of this term the expression derived by Putnam and Johnson (1949) for regular waves can be considered as starting point:

\[ D_f' = \rho f_w \frac{\omega_r}{6\pi} \left( \frac{H}{\sinh kh} \right)^3 \]

where:

\[ f_w \quad : \text{the friction factor to be calculated by Eq. 2.21} \]

\[ f_w = \begin{cases} \exp(-5.977 + 5.213(a_0/r)^{-0.194}) & \text{if } a_0/r \geq 1.59 \\ 0.3 & \text{if } a_0/r < 1.59 \end{cases} \quad (2.21) \]

When irregular random waves are taken into account \( D_f \) is evaluated as:

\[ D_f = \rho f_w \frac{\omega_r}{6\pi} \int_0^\infty H^3 dF(H) \quad (2.22) \]

\( F(H) \) is defined by full Rayleigh distribution as it can be seen in Eq. 2.10. The integral in Eq. 2.22 is to be evaluated:

\[ I_1 = \int_0^\infty H^3 dF(H) \]

\[ I_2 = \int_0^\infty H^3 d(\exp[-\frac{1}{2}(H/\bar{H})^2]) \]

Applying a lot of algebra the following results are consecutively concluded:
By definition of $H_{\text{rms}}$ and with respect to Eq. 2.10 it results in:

$$H_{\text{rms}}^2 - \int_0^\infty H^2 dF(H) = 2\dot{H}^2$$

then:

$$I = \frac{3}{4} \frac{3}{\pi} H_{\text{rms}}^3$$

The final Equation for $D_f$ reads:

$$D_f = \rho f_w \frac{1}{8\pi^2} \left( \frac{\omega}{\sinh kh} \right)$$

2.3.3 The wind energy gain

This term can easily be obtained using the results of wave prediction program GONO, (see Janssen et al, 1984).

The growth curve has the general appearance:

$$\frac{gH_s^2}{W^2} = \beta \cdot \tanh F(gt/W)$$

where:

- $W$: wind velocity component, measured at 10 (m) above mean sea level
- $g$: gravity acceleration
- $\beta$: a numerical coefficient
- $F(gt/W)$: a function to be specified
- $t$: storm duration

Two non-dimensional factors, wave height, $z$ and time, $\tau$, are defined by:
\[ z = \frac{gH^2_s}{W^2} \quad ; \quad \tau = gt/W \]
	hen the growth curve will have the form:

\[ \frac{z}{\beta} = \tanh (pr^q) \]

(2.25)

The numerical coefficient, \( p \) and \( q \), as used in GONO are given as:

\[ p = c_2 \quad ; \quad q = c_3 \quad \text{for} \quad \tau \leq 13 \times 10^3 \]

\[ p = c_4 \quad ; \quad q = c_5 \quad \text{for} \quad \tau > 13 \times 10^3 \]

and:

\[ \beta = 0.22 \]

\[ c_2 = 4.62 \times 10^{-4} \]

\[ c_3 = 0.7786 \]

\[ c_4 = 1.91 \times 10^{-3} \]

\[ c_5 = 0.6286 \]

2.4 Calibration of mathematical model

As it was mentioned before the calibration of the mathematical ENDEC model has been done by Stive and Dingemans (1984) who not only considered the applicability of the model but also investigated the model parameters.

Two free parameters, \((a, \gamma)\) had been involved, having been studied during this phase. Battjes and Janssen gave the set, \((1.0, 0.8)\) for these two parameters, which proved to be in satisfactory agreement with their measurements. Stive and Dingemans tried to verify or derive a generally applicable value for the parameters. A brief description of this procedure is reported here in this section.

Battjes and Janssen had shown the dependence of two parameters, then further tests were carried out under the constraint \( a = 1.0 \). The resulting values of \( \gamma \) fell in the range of 0.6 to 0.8, which is physically realistic. Based on laboratory results it was tried to parameterize this coefficient (i.e. \( \gamma \)).

It is known that the process of wave breaking in shallow water is influenced by the incident wave steepness and the bottom profile. These two function can be combined in surf similarity parameter (Battjes, 1974). The
calibration of the model was shown that neither surf similarity parameter nor bottom slope significantly affected the $\gamma$ parameter. However, it was appeared to be a systematic dependence of $\gamma$ on the deep water steepness, $s_0$. As it can be seen in Fig. 2.2 a tanh-function has been fitted to these data, with the result:

$$\gamma = 0.5 + 0.4 \tanh (33s_0)$$  \hspace{1cm} (2.26)

The calibration was carried out for field observation as well, where the sources and sinks of energy were studied and compared with the results of Battjes-Janssen's model to evaluate the effects of these terms. It appears that in all cases these phenomenae have marginal effects compared to the effect of wave breaking, particularly when this later starts to play a significant role, i.e. in nearshore where a high fraction of waves are breaking. This experience showed that this point (the point of high level effect of breaking) is where the ratio of rms wave height over mean water depth reaches the value of 0.25. Generally it can be stated that the source, energy gain due to local wind field and the phenomenae, energy decay due to bottom friction and current refraction do play a significant role in the non-breaking region. The result of field investigation on bottom friction carried out by Delft Hydraulics Laboratory is observed in Fig. 2.3 and a sample of the present model final outcome has been presented in Fig. 2.4.

Again the calibration was performed by estimating the value of $\gamma$ under the constraint $\alpha = 1.0$. These values were in the same range as found for laboratory cases (i.e. 0.6 to 0.8). Moreover the value of $\gamma$ parameter depends on the deep water steepness, $s_0$. It is finally concluded that the tanh-function (Eq. 2.26) fitted to the laboratory data holds equally well for the field data.

2.5 Computer Model Calibration and Verification

A lot of laboratory tests and field measurements have been carried out to verify mathematical ENDEC model (see Stive and Dingemans, 1984). Both cases are in quite well agreement with theoretical lines.

Fortunately for verification of the computer model, subject to this report, the above mentioned data are available to check the theoretic results presented by this computer programming. Some of laboratory test results as well as field measured observations have been compared with the theoretic
lines (full lines in Fig. 2.4 to 2.11). Fig. 2.7 and 2.8 also show the sample of graphs $Q_b$ as well as water level variations, $\eta$, vs onshore distance, $x$, respectively. The corresponding laboratory data have been listed in table 2.1.

<table>
<thead>
<tr>
<th>No</th>
<th>Bottom Condition</th>
<th>Hrms</th>
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<th>Figure No</th>
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<td>2.5</td>
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<td>2.9</td>
</tr>
<tr>
<td>3</td>
<td>bar-trough</td>
<td>0.143</td>
<td>2.01</td>
<td>2.10</td>
</tr>
</tbody>
</table>

Table 2.1 Laboratory data applied in testing phase

An special effort was performed in this phase in order to reduce time consumption on execution of the computer program, making variable step sizes on numerical procedure of the problem which is to be treated later on, (section 2.8).

2.6 Breaker point

Evidently in random wave theories it could not be assumed that all waves are broken in a certain place as a breaking point. However it is assumed that in a spectrum of waves each wave breaks in a specific location according to its height related to $H_m$. $H_m$ itself depends upon mean water depth in that point. This work is to be continued to calculate longshore sediment transport. Still one of the most reliable as well as applicable formula in this sense is CERC formula. Since this formula takes into account the regular wave fields, specifically the wave characteristics on breaker point, then an investigation has to be carried out to find a point corresponding to the breaker point in regular waves for ENDEC computer model.

A lot of wave data were analyzed by the model and the curves rms wave height vs distance were compared. Two sectors of waves can be distinguished:

1- Those kinds of waves which show an increment in their wave heights somewhere in nearshore because of shoaling effect (Fig. 2.5 and 2.12).
2- In some of cases the wave heights are always in decreasing phase, namely, no rises are observed on the wave heights when the waves are getting close to the coast (Fig. 2.10). Such a situation can be interpreted that the breaking effect dominates over shoaling phenomena.

For case 1, which is popular for most of real site projects, the location on which the wave height is maximum can surely be selected as breaker point.

For case 2 there were two possibilities to find this point:

a- Through a mathematical calculation:

If attention is paid to the curves wave height vs depth, one can observe that when the waves are coming towards the coast from deep water, they have a slightly change in their wave heights. But somewhere in nearshore a high rate of changes starts to occur. Actually the slope of the curve, wave height vs depth, has encountered great changes and the point of maximum changes can be discerned as breaker point. Such a point is mathematically defined as somewhere that the curve of second derivative of wave height with respect to x vs distance, x, is minimized (see Fig. 2.13).

b- Through a more sensitive analysis on $Q_b$:

As it was explained the breaker point is assumed to be where an appreciable decreasing rate on the wave heights, starts to occur. Clearly this point is not an exact point, but a narrow band. With this assumption the mentioned analysis on $Q_b$ was done, so that $Q_b$, the probability of breaking, was checked for various wave data. It was observed (for all checked cases) that breaker point is there where $Q_b$ is between .001 and .01. In practice this range is not so wide for the subsequent calculations where the sediment transport rates computed by CERC formula on various wave data very often show a difference about 30 percent applying the wave data between these two points (see Fig. 2.14). It is noticeable that for a rough sea bottom with bed roughness value of 0.06 or more this difference may increase up to 100 percent. However, due to the fact of existing approximations on the sediment transport formula itself, each point in this range could be selected as the breaker point. Then calculated sediment transport rate, which is our ultimate goal, will be reasonable. Actually different values of this range can be supposed just similar to the various amounts of breaking index, $\gamma$, in regular waves, for instance 0.6 to 0.8.

Another analysis was carried out for different values of $Q_b$. The curves of sediment transport by CERC formula vs wave angle were compared. It was observed that for $Q_b$ equal to 0.001 the most smooth curve is obtained (see Fig. 2.15). Further investigations might lead to a better criteria or relation
for breaker point in this wave analysis model, where some relationship between the location of this point and deep water wave steepness, $s_0$ is predictable.

Finally it was determined, when there is no a clear point for maximum shoaling effect, then the point in which $Q_b$ is about 0.001 will be selected as the breaker point for the computer model.

2.7 Numerical procedure of wave analysis model

First the resulted formulation of ENDEC model are summarized again here in this section:

$$\frac{d}{dx} \left[ c_g \cos \theta + U_x \right] \frac{E}{\omega_r} - \frac{D}{\omega_r} = 0$$

$$\frac{dS}{dx} + \rho g (d + \eta) \frac{d\eta}{dx} = 0$$

These two equations form a system of two ordinary, first order differential equations, from which the variation of wave energy, through $H_{rms}$, and mean water level, $\eta$, may be derived for each location on beach profile when the equations are being alternately solved and $x$, is varying shorewards where:

$$E = V_B \rho g H_{rms}^2$$

$$\omega_r = (g k \tanh kh)^{1/2}$$

$$k = |k^2| = (k_x^2 + k_y^2)^{1/2}$$
A fourth order Runge-Kutta method together with an explicit finite difference scheme are involved to solve the system of differential equations. In this method first approximations are needed for \( c, c, k, H, \theta \) on the forward step being functions of \( h = d + \eta \). So for the forward step, \( i+1 \), the following approximation can be used:

\[
h_{i+1} = d_{i+1} + \eta_i
\]

which is generally accurate, since the start point is located in deep water or intermediate zone, then the variation in \( \eta \) is small in comparing to the variation in \( d \).

For using Runge-Kutta method it is more convenient that the wave action equation to be reduced as follows:

\[
\frac{d}{dx} \left[ c \cos \theta + U_x \right] \frac{E'}{\omega_r} + \frac{D_b'}{\omega_r} + \frac{D_f'}{\omega_r} = 0
\]  

(2.27)
where:

\[ E' = H_{185}^2 = E/(\rho g/8) \]  

(2.28)

\[ D_b' = \frac{1}{\pi} \omega_r H_{28}^2 Q_b \]  

(2.29)

\[ D_f' = \frac{f_w}{g \pi^3} \left( \frac{\omega_r H_{185}}{\sinh k h} \right)^3 \]  

(2.30)

Now let:

\[ y = \frac{H_{185}^2}{\omega_r} (c_4 \cos \theta + U_\zeta) \]  

(2.31)

then:

\[ \frac{dy}{dx} = f(x, y) = -\left( \frac{D_b'}{\omega_r} + \frac{D_f'}{\omega_r} \right) \]  

(2.32)

and fourth order Runge-Kutta method yields

\[ \Delta y = y_{i+1} - y_i = (k_1 + 2k_2 + 2k_3 + k_4)/6 \]  

(2.33)

where:

\[ k_1 = \Delta x \cdot f(x_i, y_i) \]

\[ k_2 = \Delta x \cdot f(x_i + \frac{1}{2} \Delta x, y_i + \frac{1}{2} k_1) \]

\[ k_3 = \Delta x \cdot f(x_i + \frac{1}{2} \Delta x, y_i + \frac{1}{2} k_2) \]

\[ k_4 = \Delta x \cdot f(x_i + \Delta x, y_i + k_3) \]
In this case we will have:

\[ k_i = k_{1b} + k_{1f} \]

In order to calculate \( k_2, k_3, k_4 \) the first approximation to \( H_{\text{ras}} \) in step \( i+\frac{1}{2} \) and \( i+1 \) are needed which can be concluded as follows:

\[
H_{\text{rms}_{i+\frac{1}{2}}} = (y_{i+\frac{1}{2}} + k_4)w_{i+\frac{1}{2}}/(c_{8i_{i+\frac{1}{2}}} \cos \theta_{i+\frac{1}{2}} + U_{x_{i+\frac{1}{2}}})
\]  

(2.34)

where:

\[
y_i = H_{\text{rms}_i}^2(c_{8i} \cos \theta_i + U_{x_i})/\omega_{r_i}
\]  

(2.35)

and similarly for \( k_3 \) and \( k_4 \). Then final solution for \( H_{\text{ras}} \) in step \( i+1 \) gives:

\[
H_{\text{rms}_{i+1}} = y_{i+1} \omega_{r_{i+1}}/(c_{8i_{i+1}} \cos \theta_{i+1} + U_{x_{i+1}}) \quad \text{where} \quad y_{i+1} - \Delta y + y_i
\]  

(2.36)

At this point of the procedure the wave growth due to the wind is calculated according to the method of 2.3.3 paragraph. As it can be seen in that paragraph the growth is calculated in term of \( H_{\text{ras}} \), then it will be simply added to the calculated \( H_{\text{ras}} \) in step \( i+1 \).

The second step in the iteration procedure is now to solve the momentum equation. This equation is solved easily for \( \eta \), using an explicit differential form as:

\[
\frac{1}{8}H_{\text{rms}}^2 \left[ \frac{c_{6i}(1+\cos^2 \theta_i)}{c_i} - \frac{1}{2} \right] \frac{1}{8}H_{\text{rms}_{i+1}}^2 \left[ \frac{c_{6i_{i+1}}(1+\cos^2 \theta_{i+1})}{c_{i_{i+1}}} - \frac{1}{2} \right] \eta_{i+1} = \eta_i + \frac{1}{2}(h_{i+1} + h_i)
\]  

(2.37)

Evidently the values of \( Q_b \) and \( k \) should be obtained iteratively, the
standard Newton-Raphson method was used for the model and the convergency of the answers are quite good so that the results are concluded usually in less than 5 iterations. However it should be noted the computer programming has been arranged in such a way that the subroutines of $Q_b$ and $k$, in each round, take the values of previous phase, as initial value, and calculate the new amounts by an iteration procedure. Therefore, they reach the answers very quickly.

A first approximation to the all variables on step $i+1$ is now obtained. Such procedures should be repeated until the variations of one of the variables reaches below the required level or an limitation on the number of the iterations.

2.6 Description of the computer programming

The characteristics and circumstances of computer programming of this part (wave analysis) could be outlined as follows:

Input files

- A file named DATAB.DAT is arranged for basic data, i.e. generally data which do not change for a certain project site. A sample of such a file can be seen in Table 2.2

- Beach profile variations are introduced in a 2nd file called PROFILE.DAT. Program takes co-ordinates (depth by positive values) and distance to the coast. Any detailed variation could be included and the output results are reasonable. An important point has been considered on this subject. Beach profiles are given independent of wave data. Program searches and finds the starting point of the calculations on the profile. Table 2.3 shows a sample of this file for the case of Fig. 2.4

- The other data could be given in a pre-arranged file or from screen.

- Wave characteristics can be entered in 3 situations:

  a- Deep water wave condition, in such a case calculation is started from a depth equal to the half of wave length.

  b- Wave data for a specific location including water depth on this point. Data obtained from measuring stations can be remarked as an example.
Variable length steps

The consideration of variable length steps may be mentioned as an important point in this computer model. The accuracy of the results on numerical solution involved to solve the differential equations are very sensitive to \( Q_b \) value and the beach slope so that when \( Q_b \) is large, very small length step is needed to get a reasonable accuracy on calculations, as well as if slope is gentle the length steps can be larger. Then, as it was already pointed out, during the calibration phase of computer model an investigation was carried out to find an expression for length step calculations. This part of study was of great importance because, an optimization between reasonable accuracy and time consumption on computer had to be obtained. A logarithmic relation was found, as:

\[
\Delta x = F \log Q_b^{\log m} ; \quad m \neq 0
\]

\[
\Delta x = -5F \log Q_b ; \quad m = 0
\]

which calculates the length step in each iteration where:

\( m \) : beach slope

\( F \) : a moderate numerical coefficient

Sometimes \( Q_b \) may be exceptionally large in some locations along the beach cross-section. In such cases for more accuracy a condition has been put in the program which reduces the numerical coefficient, \( F \), and consequently the length step.

The result of this attempt was very satisfactory. With this consideration the program is run about twenty times quicker than the time which a reasonable constant length step is got involved (e.g. the case of Fig. 2.4 was run in 13.85" on IBM PS-computer, Model 30 286).

Miscellaneous

- As it was mentioned before, the characters like \( k \), wave number, and \( Q_b \),
probability of breaking, which are calculated in an iterative procedure, have been positioned in the program so that they get their initial values for starting point of the iterations from the previous phase. These initial values are very close to the real values, therefore the results are obtained by two or three iterations. This possibility saves an appreciable time in execution of the model.

- The necessary messages, coming through the screen (monitor), have been foreseen to warn users about the wrong and unreasonable input data as well as exceptional output result, not leading to the normal consequences, e.g. sometimes may the waves have been completely damped before attaching the shore-line.

Output Files

The main wave characteristics are tabulated in a file named ENDEC.OUT. Table 2.4 shows a sample of this output file. These data are collected for full depth, meter by meter as well as the points of change in slope. By this later possibility users are able to check the input data of beach profile implicitly. The graphs have also important role in engineering affairs. This model should have been supported by a graphical file, but because of time limitation it was skipped at present. However an special file has been arranged in the model, compatible with SGPIOT graphical package to enable users to have the graphs, rms wave height vs depth and water level variation vs distance to the coast if they need them.

This wave analysis model is to be extended for sediment transport computation. Then a list of the general model will be presented at the end of the report, including this part.

Data processing by an input file

For more applicability of ENDEC model an special version, called ENDEC2B, was provided to read and analyze various groups of wave data arranged in an input file. This file was named ENDEC.DAT. In practice there are many groups of waves with different characteristics for a site project to be analyzed. By this version all groups of wave data, listed in the ENDEC.DAT file, are analyzed when the model is run. ENDEC2B accepts wave data from deep water as well as from an specific location labelled with water depth in this point.

The mentioned version of ENDEC is also more suitable for second part of this work where the sediment transport is to be computed using the provided wave data by ENDEC computer model. For sediment transport
calculation again it is needed the various groups of waves to be considered. Tables 2.5 and 2.6 show samples of such input files for the case of deep water wave condition and the case of wave data from nearshore, respectively, while the contents of these files have been introduced below the tables as well.
The following data are successively outlined in this input file in arbitrary format:

Current velocity, $U$ (m/s)
Current angle to the shorenormal $\nu$ (°)
Density of seawater, $\rho$ (kg/m$^3$)
Bed material Density, $\rho_s$ (kg/m$^3$)
Bottom roughness, $r$ (m)
Particle diameters $D_{50}$, $D_{30}$

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<th>8.40</th>
</tr>
</thead>
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Table 2.3 A sample of input file PROFILE.DAT
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<tr>
<th>D</th>
<th>H</th>
<th>Theta</th>
<th>E. content</th>
<th>ETA</th>
<th>X</th>
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<td>-</td>
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<td>.00</td>
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</table>

D : Water depth (m)
H : rms Wave height (m)
Theta : Wave angle to the shorenormal (Deg.)
E. content : Energy content (J/M**2)
ETA : Wave set-up (m)
X : Distance to the coast (km)

Table 2.4 A sample of output file ENDEC.OUT
Table 2.5 A sample of input file ENDEC.DAT for deep water wave condition

DEEP WATER, in the first line of this file denotes that the wave data are from deep water condition whereas each group of data consists of 3 parameters:

- rms Wave height in deep water, $H_{rms}$ (m)
- Wave period (s)
- Wave angle to the shore normal in offshore position (°)
Table 2.6 File ENDEC.DAT for the case of nearshore wave data

<table>
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<tr>
<th>NEARSHORE</th>
<th>10.1.74</th>
<th>7.27.0</th>
<th>10.2.60</th>
<th>8.11.0.02</th>
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</tr>
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</table>

NEARSHORE : herein marks the nearshore wave condition.

The following wave characteristics are listed in this file:

- Water depth (m) in the location on which wave data is determined (usually measurements station)
- rms Wave height in this specific point (m)
- Wave period (s)
- Wave angle to the shorenormal (°)
- Wave set-up (m)

* If the value of wave set-up is not available or measured, then it can be assumed to be zero.
Fig. 2.2 Estimated values of breaker height coefficient $\gamma$ vs deep water wave steepness $s_o$

(Delft Hydraulics Laboratory Report M-1882)
Fig. 2.3 The effect of dissipation due to bottom friction (Delft Hydraulics Laboratory, Report M-1882)
Fig. 2.4  The effect of dissipation due to bottom friction (results of the computer model)
Fig. 2.5 rms Wave height vs onshore distance (case No. 1)

Fig. 2.6 Bottom variation profile (Lab. test No. 1)
Fig. 2.7 Water level variation vs distance to the coast (Lab. Test case No. 1)

Fig. 2.8 Probability of breaking (Qb) vs distance

Lab. test (case No 1)
Hrms = 0.138 (m) T = 2.93 (s)
Fig. 2.9 rms Wave height vs onshore distance (case No 2)
Lab. test

\( H_{rms} = 0.143 \text{ (m)} \quad T = 2.01 \text{ (s)} \)

Distance to the coast (m)

rms Wave height vs onshore distance

Cross-section of the beach

Fig. 2.10 Results of laboratory test case No. 3, bar-trough
(full lines the outcome of computer model)
Fig 2.11 Water level variations (Lab. test case No. 3)
Fig. 2.12 A sample of wave analysis model outcomes

Hrms0 = 2 (m)  T = 10 (s)
Phi = 30 (deg.)  slope = 0.01
Fig. 2.13 A sample of Hrms second derivative curve vs x

The point concerning to the breaker point

Hrms0 = 2 (m) T = 10 (s)
Fig. 2.14 Sediment transport by CERC vs Qb

Hrms0=2m  T=7s
Phi= 30 (deg.)
slope=.01
\( F_w=0 \)

The probability of Wave breaking, Qb
Fig. 2.15 The result of CERC formula on various wave angles and Qb

\[
\begin{align*}
\text{Phi} &= 30 \\
H_r &\text{rms} = 2 \text{m} \\
T &= 7 \text{s} \\
F_w &= \text{variable}
\end{align*}
\]

\begin{align*}
\text{Sand transport (m}^3/\text{s}) \\
\text{Wave angle to the shorenormal (°)}
\end{align*}

- $Q_b = 0.001$
- $Q_b = 0.01$
CHAPTER 3

SEDIMENT TRANSPORT

3.1 Introduction

For sediment transport calculations it is to be considered 3 options. CERC, Bijker and van Rijn formulae have been arranged in the model. User can select one of them or all to compare the results. A comparison between these formulae will also be carried out later in the next chapter to evaluate their accuracy related to the CERC formula.

In this process longshore current velocity has an important role as well. It has been calculated by Bijker approach.

3.2 Longshore current

In this part first it is needed the mechanism of the phenomena to be known. Where driving forces create water movement, resistant forces can be realized as well. Actually longshore current is concluded through the equilibrium of these forces in the longshore direction by a constant current along the coast. The radiation stress components, wind forces and the tide can be mentioned as driving forces whereas the turbulence and the most important, bottom friction are labelled as resistance forces. Two important forces will be described in the following sections.

3.2.1 Driving forces due to Radiation shear stress

As long as waves approach the coast in a non-zero angle, then the principal radiation stresses, $S_{II}$ and $S_{II}'$, act in the direction of wave propagation and perpendicular to this direction, respectively (see Anex 1). If an element parallel to the coastline is considered (see Fig. 3.1), all components could be found using the Mohr Circle analysis and the values of
two principal components.

\[ S_{yx} = \frac{S_{xx} - S_{yy}}{2} \sin 2\theta \]  
(3.1)  

\[ S_{\Pi} = (2n - \frac{1}{2})E \]  
(3.2)  

\[ S_{\Pi} = (n - \frac{1}{2})E \]  
(3.3)  

According to the Fig. 3.1:

\[ \theta + \phi = \frac{x}{2} \quad ; \quad \sin 2\theta = 2\sin \phi \cos \phi \]  
(3.4)  

\[ n = \frac{kh}{\sinh 2kh} + \frac{1}{2} \]  
(3.5)  

then:
As it was already mentioned the gradient in radiation shear stress produces the force component which acts parallel to the coast. Then based on this definition, the driving force expression due to radiation shear stress yields:

\[
\frac{dS}{dy} = \frac{d(En \sin \phi \cos \phi)}{dy}
\]

When waves approach the coast the parameters \(E, n, c\) and \(\phi\) change, while they are greatly dependent upon water depth. These parameters can easily be calculated in a numerical procedure, step by step, presenting the driving force resultants, as it will be explained later on while the modelling of the sediment computation of CBM model is being dealt with.

3.2.2 Bottom friction forces

The friction force acts on the water where there are longshore current. Bottom friction of the beach causes such a force on water element above it, related to the velocity in the element. This force which was already called as resistant force refers to the bottom shear stress, indeed. The relations for bottom shear stress under combination of wave and current is still to be improved. The existing mathematical description can present an approximate value on the subject. However this effect has been evaluated more properly on the separate conditions namely, current alone and wave alone, but still there are no fixed and perfect expressions for the calculations.

The shear stress at an elevation \(z_t\) above the bottom is expressed as follows. The point, \(z_t\), is located there where relates bottom shear stress to the velocity near the bottom on the logarithmic velocity distribution curve (see Fig. 3.2). In this section the treatment is to be started with current alone and then will be continued for waves as well as combination of waves and current.

**Current**

Many researchers have worked to present a relation for bottom shear stress in current alone. One of the most popular equations reads as:

\[
\tau_c = \rho k^2 \frac{V_t^2}{t}
\]

in which:
Fig. 3.2 Velocity distribution for a uniform stationary current

\[ V_t = \frac{V_*}{\kappa} \left( \frac{V\sqrt{g}}{\kappa C} \right) \]  \hspace{1cm} (3.9)

\begin{align*}
\rho & : \text{mass density of water} \\
\kappa & : \text{von Karman coefficient} \\
V_t & : \text{shear stress velocity} \\
C & : \text{Chezy coefficient}
\end{align*}

**Waves**

For the wave bottom shear stress Bijker approached just the same way as the current shear stress. He expressed the wave shear stress as a function of the velocity as well as it is for current. Bijker regarded the velocity at height \( z_t \) to be proportional to the bottom velocity \( u_0 \).

\[ u_t = p u_0 \]  \hspace{1cm} (3.10)

where:

\[ u_0 = \frac{\omega H}{2 \sinh kh} \cos \omega t \]
\[ u_t = \text{velocity at elevation } z_t = \frac{e \cdot r}{33} \text{; Nikuradse} \]
\[ u_0 = \text{velocity just outside the boundary layer} \]

Bijker assumed that \( p \), proportionality factor, was constant. But later comparison of the Jonsson's and Bijker's results indicated that \( p \) should be variable by the relation which Jonsson has presented:

\[
p = \frac{1}{k} \sqrt{\frac{f_w}{2}} \quad (3.11)
\]

where:

\[ f_w \quad \text{: friction factor to be described as follows:} \]

\( f_w \), the wave friction factor, depends on the Reynolds number as well as on the relative roughness \( (a_0/r) \). However the Reynolds number dependence is usually ignored, since prototype condition can be expected to be fully turbulent. Jonsson presented a relation for \( f_w \), in terms of relative roughness. The rewritten shape of this relation by Swart (1976) is as followed:

\[
f_w = \exp[-5.997 + 5.213(a_0/r)^{-0.194}] \quad (3.12)
\]

\[
\alpha_\theta = \frac{H}{2 \sinh kh} \frac{\bar{u}_0 T}{2\pi} \quad (3.13)
\]

where:

\[ r \quad : \text{bottom roughness} \]
\[ a_0 \quad : \text{maximum horizontal displacement of water particles} \]
\[ \bar{u}_0 \quad : \text{maximum horizontal velocity at the bottom} \]

Maximum \( f_w \) is equal to 0.3

Using the same shape of current-related bottom shear stress (Eq. 3.8) for this case (wave-related bottom shear stress) yields:

\[
\tau_w = \rho k^2 u_t^2 \quad (3.14)
\]
where:

\[ \tau_w : \text{wave-related bottom shear stress} \]
\[ \hat{\tau}_w : \text{maximum bottom shear stress} \]

**waves and current**

Bijker tried to find bottom shear stress for the combination of waves and current where he was introducing the wave influence in an existing current-related sediment transport formula. He modified the bottom shear stress used in that formula.

Bijker approached this problem considering the velocities of waves and current, calculated separately, as vectors. He added the vectors, then the product of this vector addition \( V_r' \), was accepted as a representative of velocity for bottom shear stress in waves and current, \( \tau_{cw'} \).

\[
\tau_{cw'} = \rho \kappa^2 V_r'^2
\]  

\[
\overline{\tau_{cw}} = \frac{\rho \kappa^2 [V_r^2 + (p\hat{u}_0 \sin \omega t)^2 + 2V_t p\hat{u}_0 \sin \omega t \sin \varphi]}{\tau_{cw'}}
\]

The direction of this bottom shear stress changes in time because it depends on oscillating wave velocity. However it is assumed that the wave has a role of stirring up the material and the critical velocity has to be exceeded, regardless of its direction.

A time-averaged on Eq. 3.17 led to the mean total bottom shear stress (important for the string up the material). After a lot of mathematical analysis as well as some simplifications regarding the sinusoidal wave theory yields:

\[
\overline{\tau_{cw}} = \rho \kappa^2 \left[ V_r^2 + \frac{1}{2}(p\hat{u}_0)^2 \right]
\]
\[ V_t = \frac{V \sqrt{g}}{\kappa C} \]

\[ V \quad : \quad \text{average velocity} \]
\[ u_0 \quad : \quad \text{maximum horizontal velocity at the bottom} \]

Bijker introduced the \( \xi \) parameter as:

\[ \xi = C \sqrt{\frac{f_w}{2g}} \]

then Eq. 3.18 reads:

\[
\overline{\tau_{cw}} = \frac{\rho g V^2}{C^2} \left[ 1 + \frac{1}{2} \left( \frac{u_0}{V} \right)^2 \right] \tag{3.19}
\]

This later expression denotes the time averaged of the momentary shear stress while for longshore current and further sediment transport calculation the x component of this shear stress, \( \tau_{cwx} \), is needed.

\[ \tau_{cwx} = \tau_{cw} \cos \theta \tag{3.20} \]

Bijker presented an expression for mean value of \( \tau_{cwx} \), involving the time-averaged of the x component of bottom shear stress. The produced integrals were not conductive with an analytical solution. A numerical procedure was carried out to evaluate the integrals, obtained from time-averaged. Various values of independent variables, \( V \), \( u_0 \), \( \xi \) and \( \varphi \) were involved and an equation was fitted to the results, reading as:

\[
\overline{\tau_{cwx}} = \frac{\rho g V^2}{C^2} \left[ 0.75 + 0.45 \left( \frac{u_0}{V} \right)^{1.13} \right] \tag{3.21}
\]

The approximations applied on this numerical procedure yield a practical limitation of Eq. 3.21, so that it is valid for the waves nearly parallel to the coast, namely, \( |\varphi| < 20^\circ \).

This last equation is the relation between the bed shear stress and the velocity which has to be used to evaluate longshore current velocity.
3.2.3 Longshore current velocity

It was pointed out before that an equilibrium on bottom shear stress, as a resistance force, and gradient in radiation shear stress, as a driving force leads to longshore current. Then the velocity on this current can be concluded through this equilibrium.

$$\frac{dS_y}{dy} = \tau_{cws}$$

(3.22)

This equation is solved using a numerical procedure which is to be described later on in this chapter.

3.3 Sediment transport formulae

CERC formula, the one associated with a bulk-energy model, describes the total transport on entire beach inside the surf zone, considering regular wave fields. Actually this formula did not tackle sediment transport by predicting the sediment concentration and sediment velocity. Then a lot of short-comings can be found in CERC formula which makes it limited only for very simple boundary conditions. However CERC is reliable for some cases but always the conditions governing on this formula are not satisfied and other formulae are required to be able to fulfil complicated boundary conditions. The effect of sea bottom, the influence of tidal and wind-induced current, for instance, sometimes can not be neglected, or the distribution of sediment transport over cross-section may be important as well. But still various prototype measurements which support the CERC formula makes it a fairly reliable formula for the relatively simple case of oblique waves. The other formulae often have a lack of reliable prototype measurements and the use of them is limited.

The Bijker formula (1971), in contrast to the CERC formula, gives a detailed prediction of transport and its distribution over beach profile. It should be noted that Bijker formula also have some limitations, for example interpretation of the concentration profile is questionable, or selection of a similar value for \( \mu \), in current alone and combination of wave and current does not seem to be correct. Although Bijker formula has been employed in some of project and the results have been satisfactory. Actually having a detailed view insight the various formula enables the user to select the appropriate one for his case based on the boundary conditions governing on
the specific site project.

The van Rijn formula is one of the latest attempts for sediment transport formula under wave and current conditions and using so-called local method. This method attempts to represent the physics of the sediment transport process, taking all the relevant parameters into account and is therefore more universal.

A lot of laboratory tests confirm the result of van Rijn formula, but it still has to be used in practice to evaluate its applicability.

The Bijker formula and those coming through current transport adaption assume that the waves approach the coast in the small angles, or in the other words the wave direction to be nearly perpendicular to the current direction. This assumption is very often true, but not always. Waves are coming towards the coast, being refracted, then having a pattern nearly parallel to the shoreline is usually expectable inside the breaker zone. Report is to be followed, presenting a brief explanation on three mentioned sediment transport formulae.

3.3.1 CERC sediment transport formula

Observation in both prototype and model indicated a correlation between the longshore transport rate, \( S_x \), and the longshore component of energy flux at the outer edge of the surf zone, being expressed as a formula:

\[
S_x = A' U'
\]

where:

- \( S_x \): longshore sand transport
- \( A' \): dimensional coefficient
- \( U \): the component of the energy flux entering the breaker zone

using mathematics and linear wave theory yields:

\[
S_x = A H_b^2 n_b c_b \sin \varphi_b \cos \varphi_b
\]

\[ (3.23) \]

\( A \): dimensionless coefficient, given in table 3.1
Investigator | Characterizing wave height | coefficient $A$, in CERC
--- | --- | ---
Original CERC | $H_{sig}$ | 0.028
 | $H_{rms}$ | 0.056
Shore Protection Manual (1984) | $H_{sig}$ | 0.050
Factor most used in the Netherlands | $H_{sig}$ | 0.040
 | $H_{rms}$ | 0.080

Table 3.1 CERC formula coefficients

In irregular random wave fields the application of $H_{rms}$ is preferable. It can be noted that in a narrow banded Rayleigh distribution a relation between two wave height characters has been presented as:

$$H_{sig}^2 = 2H_{rms}^2$$

The effect of the coefficient of this relation has been appeared in table 3.1.

3.3.2 *Bijker formula*

Bijker presented a formula based on the concept, velocity times concentration, which is able to predict the sediment transport distribution over the beach profile. Bijker considered the total sediment as two components, bed load and suspended load. Einstein's principle was accepted for suspended load computation. For bed load transport he adapted Kalinske-Frijlink formula under waves and current condition. This formula, which is valid for bed load calculation, was divided into two parameters, namely, a stirring parameter and a transport parameter. He introduced the wave influence through a modification of bottom shear stress, $\tau_w$, into time-averaged of $\tau_{cw}$.

*Bed load transport*

The Kalinske-Frijlink formula reads:
This can be rewrite as:

\[ S_b = \frac{5D_{50}}{C}V \sqrt{\frac{g}{\mu V}} \exp \left[ -0.27 \frac{\Delta C^2 D_{50}}{\mu V^2} \right] \] (3.24)

The exponential term in this equation is usually referred to as the 'stirring-up' parameters, because the shields parameter \( \left( \frac{\tau_c}{\rho g D_{50}} \right) \) can be found in this part of the expression. The remaining part in Eq. 3.25 is often called the 'transport' parameter, because the transport, \( S \), divided by this parameter, is the dimensionless sediment transport. Both experimental coefficients in Eq. 3.24, 5 and -0.27, are found by plotting values of dimensionless parameters, transport \( \left( \frac{S}{D_{50}} \sqrt{\frac{V}{\rho g}} \right) \) against the adapted Shields \( \left( \frac{\mu \tau_c}{\rho g D_{50}} \right) \).

Bijker modified the bed load formula (Eq. 3.24) as follows:

- The layer in which bottom transport takes place has a thickness equal to the bottom roughness, \( r \), (instead of \( 2D_{50} \) used by Einstein). For practical problems, where the actual roughness is not known, Bijker suggests using a roughness equal half of the bottom ripple height.

- The ripple factor, \( \mu \), is neglected in the first part of Kalinske-Frijlink formula ('transport parameter').

- The shear stress, \( \tau_c \), in the 'stirring parameter' (the second part) of this formula (Kalinske-Frijlink) is modified to the shear stress for current and waves, \( \tau_{cw} \) (Eq. 3.24).

Then with the above modifications the Bijker formula for waves and current yields:

\[ S_b = \frac{5D_{50}V \sqrt{g}}{C} \exp \left[ -0.27 \frac{\Delta D_{50} \rho g}{\mu \tau_{cw}} \right] \] (3.26)

in which:
\[ C = 18 \log_{12} \frac{h}{r} \]

\[ \Delta = \frac{\rho_s - \rho}{\rho} \]

- \( S_b \) : bed load transport (m³/sm)
- \( D_0 \) : particle diameter (m)
- \( V \) : mean current velocity (m/s)
- \( C \) : Chezy coefficient (m²/s)
- \( r \) : bottom roughness (m)
- \( g \) : gravity acceleration (m²/s)
- \( \Delta \) : relative density of bed material
- \( \rho_s \) : mass density of bed material (kg/m³)
- \( \rho \) : sea water mass density (kg/m³)
- \( C_{90} \) : Chezy coefficient based on \( D_{90} \) (m²/s)
- \( \tau_{cw} \) : time-averaged of bed shear stress due to waves and current (Eq. 3.19) (N/m²)
- \( \xi \) : Bijker's parameter
- \( f_w \) : Jonsson's friction factor
- \( \mu \) : ripple factor \([ (C/C_{90})^{1.5} ] \)

**Total and suspended load transport**

A well-known transport formula for current is the Einstein formula, which is valid for the rivers. Einstein approached the problem in the fundamental way as:

\[ S = \int_0^h c(z) V(z) \, dz \]  \hspace{1cm} (3.27)

He used the Prandtel-Von Karman logarithmic velocity distribution to describe \( V(z) \):
\[ V(z) = \frac{V}{\kappa} \ln\left(\frac{z}{z_0}\right) \]  

(3.28)

where:

\( z_0 \) : zero elevation at which the velocity is zero (see Fig. 3.2)

The elevation \( z_0 \) has been related to the bottom roughness, \( r \). For the rough bottom, which is valid for the coastal problem, Nikuradse empirically found that:

\[ z_0 = \frac{r}{33} \]

The diffusion equation is used for the concentration distribution, \( c_z \), and the result of one by Rouse-Einstein yields:

\[ c(z) = c_a \left( \frac{h - z}{h - a} \right)^{1/2} \]  

(3.29)

where:

\( a \) : bottom layer thickness (equal to the bottom roughness, Bijker)

Bijker also modified the exponent of concentration distribution, regarding time-averaged value of \( \tau_{cw} \) to calculate shear stress velocity:

\[ z_* = \frac{w}{\kappa V_{*cw}} \]  

(3.30)

\[ V_{*cw} = \sqrt{\frac{\tau_{cw}}{\rho}} \]  

(3.31)

where:

\( V_{*cw} \) : shear stress velocity
\( w \) : fall velocity to be calculated by Eq. 3.32
\[ \log \left( \frac{1}{\omega} \right) = A \log D_{50}^2 + B \log D_{50} + C \] (3.32)

This expression is an empirical formula valid for sediment diameters in the range of 50\(\mu\)m to 300\(\mu\)m in which:

for fresh water at \(18^\circ\text{C}\):
\[
\begin{align*}
A &= 0.4949 \\
B &= 2.4113 \\
C &= 3.7394
\end{align*}
\]

for fresh water at \(10^\circ\text{C}\):
\[
\begin{align*}
A &= 0.4758 \\
B &= 2.1795 \\
C &= 3.1915
\end{align*}
\]

The resulting integral for suspended sediment is:

\[
S_s = \int_a^h c_a \left[ \frac{h-z}{z} \frac{a}{h-a} \right]^2 \frac{V_x}{k} \ln \left[ \frac{z}{z_0} \right] dz
\] (3.33)

Einstein solved the integral in Eq. 3.33 in term of two other integrals, \(I_1\) and \(I_2\), where:

\[
I_1 = 0.216 \frac{A^{(x-1)}}{(1-A)^x} \int_A^1 \left[ \frac{1-\zeta}{\zeta} \right]^x d\zeta
\] (3.34)

\[
I_2 = 0.216 \frac{A^{(x-1)}}{(1-A)^x} \int_A^1 \left[ \frac{1-\zeta}{\zeta} \right]^x \ln \zeta d\zeta
\] (3.35)

where:

\[
\begin{align*}
A &: \text{ dimensionless roughness (r/h)} \\
\zeta &: \text{ dimensionless elevation (z/h)}
\end{align*}
\]

Then he related suspended load, \(S_s\), as a function of bed load, \(S_b\), determining the reference concentration, \(c_a\), through the bed load transport. Herein, the concentration is assumed to be constant and equal to \(c_a\).

Bijker applied Einstein's principle to develop a formula for total sediment transport under conditions of waves and current. First he selected a bottom layer thickness equal to the bottom roughness, \(r\) (0.01 to 0.10 m). Then to find the reference concentration, \(c_a\), used in suspended load transport...
formula of Einstein (Eq. 3.33), Bijker used:

\[ S_b = r \, V_{\text{bottom layer}} \, c_a \]

where:

\[ V_{\text{bottom layer}} : \text{the average velocity in the bottom layer} \]

This average is found by integrating the velocity distribution over the bottom layer thickness, equal to \( r \) (see Fig. 3.3):

\[ V_{\text{bottom layer}} = \frac{1}{r} \left[ \frac{1}{2} \frac{V_0}{\kappa} e Z_0 + \int_{e Z_0}^{r} \frac{V_0}{\kappa} \ln\left(\frac{z}{Z_0}\right)dz \right] \]

\[ (3.36) \]

Or:

\[ V_{\text{bottom layer}} = 6.34 \, V_t \]

then:

\[ c_a = \frac{S_b}{6.34 r V_t} \]

\[ (3.37) \]
One can observe that this concentration has been assumed to be constant over the entire thickness, selected equal to the bottom roughness. $c_a$ is expressed in units of volume of deposited sediment per unit volume of water, thus includes the voids in the deposited sediment. As pointed out earlier, Bijker applied the adapted bed load transport formula of Kalinske-Frijlink (Eq. 3.26) in Eq. 3.37 as $S_b^*$. After substituting Eqs. 3.26, 28, 37 into Eq. 3.33 and involving a lot of algebra, it can be shown that:

$$S_s = 1.83\, Q\, S_b$$

(3.38)

where:

$$Q = [I_1 \, \ln (33h/r) + I_2]$$

(3.39)

$r$ : bottom roughness

$h$ : water depth

$I_1, I_2$ : Einstein integrals (Eqs. 3.34 and 3.35)

It was already mentioned that parameter $z_*$ in Einstein integrals and consequently $V_*$ should be adapted, regarding the influence of the wave, i.e. $V_{*cw}$, Eq. 3.31. Van de Graaff et al presented a numerical solution for these two integrals which is to be treated later on.

Now having bed load transport and suspended transport simply leads to the total sediment transport with their summation.

$$S_t = S_b + S_s = S_b(1 + 1.83Q)$$

(3.40)

3.3.3 Van Rijn formula

Van Rijn (1984–1988) defined the total transport as the sum of the current-related and the wave-related transport. The current-related transport itself consists of bed load transport and suspended load transport. For wave-related transport he proposed an experimental relation over half the wave period, being treated later on in this section. Both current-related bed load and suspended load have been treated, considering the effect of wave presence. As a wave theory van Rijn involved higher order Stokes theory but for CBM model the harmonic wave theory parameters have been substituted. All the components are dealt with as follows.
Current-related bed load transport

The following expression is presented by van Rijn as a bed load transport formula:

\[
S_b = 0.25V'_i D_{50} \frac{\tau_{ls}}{D_{03}^2} 
\]  

(3.41)

in which:

\[
V'_i = \sqrt{\frac{\tau'_c}{\rho}} 
\]  

(3.42)

\[
D_0 = D_{50} \left( \frac{\Delta g}{v^2} \right)^{\frac{1}{3}} 
\]  

(3.43)

\[
S_b = 0.25V'_i D_{50} \frac{\tau_{ls}}{D_{03}^2} 
\]  

(3.41)

in which:

\[
V'_i = \sqrt{\frac{\tau'_c}{\rho}} 
\]  

(3.42)

\[
D_0 = D_{50} \left( \frac{\Delta g}{v^2} \right)^{\frac{1}{3}} 
\]  

(3.43)

\[
T = \frac{\tau'_{cr} - \tau_{cr}}{\tau_{cr}} ; \quad T_{min} = 0 
\]  

(3.44)
\[ \tau_{cr} = (\rho_s - \rho)gD_{30}\theta_{cr} \]  

(3.45)

\[ \tau_{cw} = \tau_c + \tau_w \]  

(3.46)

\[ \tau_c = \frac{1}{8} \rho \alpha \mu_c f_a V^2 \]  

(3.47)

\[ \tau_w = \frac{1}{4} \rho \mu_w f_w \mu_0^2 \]  

(3.48)

\[ \hat{u}_0 = \frac{\pi H_f}{T_p \sinh kh} \]  

(3.49)

\[ \alpha_{cw} = \frac{\ln^2(90\delta_w / r_c)}{\ln^2(90\delta_w / r_c)} \quad \alpha_{cw} = 1 \quad \text{if} \quad \delta_w < \frac{r_c}{30} \]  

(3.50)

\[ \delta_w = 0.72 \hat{a}_0 (\hat{a}_0 / r_w)^{-0.25} \]  

(3.51)

\[ d_0 = \frac{\hat{u}_0 T}{2\pi} \]  

(3.52)

\[ \mu_e = \frac{f_c}{f_c} \]  

(3.53)

\[ \mu_w = 0.6 / D. \]  

(3.54)
\[ f_c' = 0.24 \log^2(12h/3D_{90}) \]  \hfill (3.55)

\[ f_c = 0.24 \log^2(12h/r_c) \]  \hfill (3.56)

\[ f_a = 0.24 \log^2(12h/r_a) \]  \hfill (3.57)

\[ f_w = \exp[-6 + 5.2(\hat{u}_0/r_w)^{-0.19}] ; \quad f_{w_{\text{max}}} = 0.3 \]  \hfill (3.58)

\[ r_a = r_c \exp[\gamma \hat{u}_0/V] ; \quad r_{a_{\text{max}}} = 10r_c \]  \hfill (3.59)

\[(\hat{u}_0/V)_{\text{max}} = 2.5\]

\begin{align*}
\gamma &= 0.75 \quad \text{for } \phi \leq 90^\circ \\
\gamma &= 1.1 \quad \text{for } \phi = 180^\circ \quad \text{linear interpolation for intermediate values}
\end{align*}

where:

- \( S_0 \): time-averaged bed load transport (m^3/s)
- \( V \): average flow velocity (m/s)
- \( V_\tau \): current-related grain bed-shear velocity (m/s)
- \( T \): dimensionless bed shear stress parameter
- \( D_i \): dimensionless particle parameter
- \( \tau'_c \): current-related effective shear stress (N/m^2)
- \( \tau'_w \): wave-related effective shear stress (N/m^2)
- \( \tau'_{cw} \): current-waves bed-shear stress (N/m^2)
- \( \tau'_{cr} \): critical bed shear stress according Shields (N/m^2)
- \( \hat{u}_0 \): near bed orbital velocity (m/s)
- \( \delta_w \): wave boundary layer thickness (m)
- \( \hat{a}_0 \): maximum horizontal near bed displacement (m)
- \( a_{cw} \): wave-current interaction coefficient
- \( T_p \): peak wave period (s)
\( \mu_c, \mu_w \) : current, wave efficiency factor
\( D_{30}, D_{90} \) : relevant particle diameters (m)
\( \Delta \) : relative density
\( \nu \) : kinematic viscosity (m^2/s)
\( C \) : grain-related Chezy coefficient (m^{0.5}/s)
\( \phi \) : angle between current direction and wave direction (°)
\( r_a \) : apparent bottom roughness (m)
\( f_a \) : apparent friction factor
\( f_w \) : wave-related friction factor
\( f_c, f'_c \) : current-related friction factors
\( r_c, r_w \) : current and wave bottom roughnesses (m)
\( \theta_{cr} \) : Shields parameter, to be classified as:

\[
\begin{align*}
1 < D_* & \leq 4 & \theta_{cr} &= 0.24 D_*^{-1} \\
4 < D_* & \leq 10 & \theta_{cr} &= 0.14 D_*^{-0.64} \\
10 < D_* & \leq 20 & \theta_{cr} &= 0.04 D_*^{-0.1} \\
20 < D_* & \leq 150 & \theta_{cr} &= 0.013 D_*^{-0.29} \\
D_* > 150 & & \theta_{cr} &= 0.055
\end{align*}
\]

Here in this formula the \( T \) parameter is a stirring parameter governing the entrainment of the bed material particles whereas the \( V'_* \) parameter performs a transport role. Eq. 3.41 which is valid for particles in the range of 100 to 500 \( \mu \)m, yields a zero transport if the current velocity will be zero.

**current-related suspended load**

The time-averaged suspended load transport is computed by integration over the depth of the product of velocity and concentration, as follows:

\[
S_s = \int_a^h V(z) c(z) dz
\]  

in which:

- \( S_s \) : time-averaged suspended load transport (m^3/s)
- \( V(z) \) : resultant current velocity at height \( z \) above the bed (m/s)
- \( c(z) \) : sediment concentration at height \( z \) above the bed (kg/m^3)

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Van Rijn provided the terms in Eq. 3.60, leading this equation to be numerically solvable. Again here the role of stirring-up of the waves has been considered, deriving the sediment mixing coefficient in waves and current for this computation.

**Concentration distribution**

Summarizing, concentration distribution over the depth is obtained, using a numerical integration on the following expression:

$$\frac{dc}{dz} = \frac{(1-c)^5c_w}{e_{scw}}$$  \hspace{1cm} (3.61)

$$c_1 = 0.015 \frac{D_s0}{a} T_{1.5} D_s^{0.3} z \leq a$$  \hspace{1cm} (3.62)

$$e_{scw} = [e_{sc}^2 + e_{sw}^2]^{0.5}$$  \hspace{1cm} (3.63)

where:

- $c_1$ : reference (bed) concentration (-)
- $e_{scw}$ : sediment mixing coefficient in waves and current to be computed

**Current alone, $e_{sc}$ :**

$$z < 0.5h \quad e_{sc} = \kappa \beta V_i z (1-z/h)$$  \hspace{1cm} (3.64)

$$z \geq 0.5h \quad e_{sc} = 0.25 \kappa \beta V_i h$$  \hspace{1cm} (3.65)

$$V_i = V \sqrt{g/C}$$  \hspace{1cm} (3.66)
\[ \beta = 1 + 2\left(\frac{w}{V^*}\right)^2 \quad \beta_{\text{max}} = 2.0 \]  
\[ w = \frac{10v}{D^3_s} \left[ 1 + \frac{0.01\Delta g D^3_s}{v^2} \right] - 1\]  
\[ D_s = \left[ 1 + 0.011(\sigma_s - 1)(T - 25) \right] D_{s0} \]  
\[ \sigma_s = \frac{1}{2} \left( \frac{D_{s4} + D_{s50}}{D_{s0}} \right) \]  
\\begin{align*}
\text{waves alone, } e_{s,w} : \\
0 \leq z < \delta_s : & \quad e_{s,w} - e_{s,\text{bed}} - 0.004 D_s \alpha_{br} \delta_s \hat{u}_0 \\
0.5 \leq z < h : & \quad e_{s,w} - e_{s,\text{max}} - 0.035 \alpha_{br} h H_s / T_p \\
0.5 \leq z < h : & \quad e_{s,w} - e_{s,\text{bed}} + [e_{s,\text{max}} - e_{s,\text{bed}}] \left[ \frac{z - \delta_s}{0.5 \leq \delta_s} \right] \\
\end{align*}\[ \alpha_{br} = 3(H_s/h) - 0.8 \]  
\[ \alpha_{br} = 1 \quad \text{for} \quad H_s/h \leq 0.6 \]

where:
- \( T_p \) : peak wave period (s)
- \( \delta_s \) : near-bed mixing layer thickness (m)
- \( H_s \) : significant wave height (m)
- \( h \) : water depth (m)
- \( \alpha_{br} \) : breaking coefficient
Velocity distribution

Van Rijn proposed the following formulation for computation of velocity distribution over the entire depth:

outside wave boundary layer ; \( z \geq 3 \delta_w \):

\[
V(z) = \frac{V \ln(30z/r_a)}{-1 + \ln(30h/r_a)} \tag{3.75}
\]

inside wave boundary layer ; \( z < 3 \delta_w \):

\[
V(z) = \frac{V_\delta \ln(30z/r_c)}{\ln(90 \delta_w/r_c)} \tag{3.76}
\]

where:

\[
V_\delta = \frac{V \ln(90 \delta_w/r_a)}{-1 + \ln(30h/r_a)} \tag{3.77}
\]

All the parameters have been introduced in the last section.

Now having two distributions of velocity and concentration over the depth, the integration of Eq. 3.60 is numerically possible. By this integration the suspended load transport over the entire depth of a coastal band of 1m
width is concluded.

More details of the parameters

Some of the parameters, involved in van Rijn formula, are to be described here for more insight view. It can be mentioned that CBM model will try to be quite flexible so that with the lowest available data to be executable by the considered assumption in the computer programming. However some of the parameters, particularly the measurable data or the ones coming through the engineering judgement on the project site, can be given to the program. This matter will again be dealt with later on.

Fall velocity

Moreover than the experimental relation for fall velocity calculation, given by Eq. 3.32, Eq. 3.68 was used by van Rijn to compute suspended load sediment transport. The representative size of the suspended sediment, in this expression (Eq. 3.68) can be obtained from Eq. 3.69. A reasonable estimate is $D_s = 0.8D_{50}$ of the bed material.

Bed roughnesses, $r_c$, $r_w$

There are some relations which give the $r_c$ value based on the bed form characteristics, e.g. the ripple length and height. A reasonable estimate for current and non-breaking waves is $r_c = r_w = 3\Delta_r$, whereas $\Delta_r$ is ripple height. In case of breaking waves with sheet flow conditions the bed roughness will be in the order of wave boundary layer thickness, $r_w = \delta_w$, with values in the range of 0.01 to 0.02 m.

Reference level, $a$

For the ripple bed it is proposed to be equal to half of the ripple height which consist of the non-breaking waves. This, reference level, value can be assumed to be in order of the wave boundary layer thickness in case of sheet flow regime ($a = \delta_w$).

Near bed mixing layer thickness, $\delta_s$

For ripple bed form:

$$\delta_s = 3\Delta_r$$
For sheet flow conditions:

\[ \delta_s = 3 \delta_w \]

Both expressions yield values in the range of 0.03 to 0.1 m. In case of breaking waves the \( \delta_s \) may be larger than 0.1 m. Even the value of 0.2 has been observed in some of experiments, which is due to breaking effect. There is no sufficient field data to confirm a more accurate value and/or a relation for \( \delta_s \).

**Wave-related sediment transport**

Two above section were devoted the current-related, bed-load and suspended load sediment transport. Van Rijn has additional formulation for wave-related sediment transport based on basic concept of sediment transport as:

\[
S_w = \int_0^h \bar{u} \bar{c} \, dz
\]  
(3.78)

Then van Rijn proposes to determine the time-averaged transport rate (in \( m^2/s \)) over half the wave period as:

\[
S_{w,\text{half}} = a_1 \, a_2 \, \bar{c} \, \bar{u}
\]  
(3.79)

where:

- \( a \) : reference level
- \( a_1 \) : calibration coefficient

The calibration coefficient has been determined by Horikawa et al (1982) through the sheet flow measurements. It is so important to be declared that \( S_w \), wave-related sediment transport rate, will be in the direction of the largest bed-shear stress.

Van Rijn presented an approximate solution for Eq. 3.79, considering a higher order stokes theory, as follows:

\[
S_w = S_{w,\text{max}} - S_{w,\text{min}}
\]  
(3.80)
\[ S_{w_{\text{max}}} = \alpha \bar{u}_{\text{max}} \delta \frac{c_{\text{max}} - \bar{c}_a}{c_{\text{max}} - c_{\text{min}}} \]
\[ S_{w_{\text{min}}} = \alpha \bar{u}_{\text{min}} \delta \frac{c_a - c_{\text{min}}}{c_{\text{max}} - c_{\text{min}}} \]  

(3.81)

where:

\( \bar{c}_a \) : bed concentration (Eq. 3.62), applying Eq. 3.46 (or 3.82) for shear stress

\( c_{a,\text{max}} \) : maximum bed concentration, using Eq. 3.83 for bed shear stress in Eq. 3.62

\( c_{a,\text{min}} \) : minimum bed concentration, applying the bed shear stress, coming through the Eq. 3.84

\( \alpha \) : coefficient (\( \approx 0.3 \), Horikawa et al, 1982)

\( \phi \) : angle between current direction and wave propagation direction (\( \theta \))

\( \bar{u}_{0,\text{max}}, \bar{u}_{0,\text{min}} \) : maximum and minimum peak orbital velocity near the bed according to higher order Stokes theory (m/s)

\( \delta_{w,\text{max}}, \delta_{w,\text{min}} \) : wave boundary layer thicknesses (Eq. 3.51) based on maximum and minimum near bed displacement (higher order Stokes theory) (m)

The above given method yields a net wave-related transport rate in waves and in the wave propagation direction. If the waves are assumed to be symmetrically sinusoidal, still this formulation gives a transport rate. It will be pointed out later on that harmonic sinusoidal waves are involved in CBM model.

\[ \tau'_{cw} = \tau'_c + \tau'_w \]  

(3.82)

\[ \tau'_{cw_{\text{max}}} = \left[ \tau'_c^2 + \tau'_w^2 + 2\tau'_c \tau'_w \cos \phi \right]^{0.5} \]  

(3.83)
Total load transport in waves and current

It was already pointed out that current-related sediment transport is formed by two parts, i.e. current-related bed load and suspended load transport as:

\[ S_c = S_b + S_s \] (3.85)

Then total sediment transport under condition of waves and current is obtained by the following expression:

\[ S_t = [S_c^2 + S_w^2 + 2S_cS_w \cos \phi]^{0.5} \] (3.86)

where:

- \( S_t \) : total sediment transport (m²/s)
- \( S_c \) : current-related sediment transport (m²/s)
- \( S_w \) : wave-related sediment transport (m²/s)

It is noticeable, since most of the time the wave component sediment transport, \( S_w \), is very small in this procedure, it could be neglected. However, while the effective parameters on suspended load such as fall velocity, have a reductional impact on suspended load, then \( S_w \) has a considerable contribution in the sediment transported along the shore. This subject will be treated again later on, in numerical procedure description (section 3.4).

3.4 Numerical procedure of the sediment computation

The mathematical formulation of the longshore current velocity computation as well as sediment transport calculations by Bijker approach and the van
Rijn formulation were treated in the previous sections of this chapter. Herein the numerical solution of those mathematical treatments are to be presented. One can observe that for all formulae the wave characteristics have the most important role, which are variable when the waves are approaching the coast. Fortunately the rather realistic wave analysis model subject to the chapter 2 of this report is available and by the reasonable length steps provides the wave characteristics in each step. For sediment transport calculation the resultant of longshore current velocity along the beach cross-section (as a profile) in presence of current field is also so important. This velocity profile is concluded by a numerical calculation of Eq. 3.22, using the wave data coming through the ENDEC computer model.

**Longshore current velocity computation**

The final derivation of a relation for longshore current velocity was given by Eq. 3.22 as:

\[
\frac{dS_{yx}}{dy} = \frac{1}{\tau_{cwx}} \tag{3.87}
\]

where:

\[
\frac{dS_{yx}}{dy} = \frac{d(\text{Ensin} \varphi \cos \varphi)}{dy} \tag{3.88}
\]

\[
\tau_{cwx} = \rho g V^2 \left[ 0.75 + 0.45 \left( \xi \frac{d_0}{V} \right)^{1.13} \right] \tag{3.89}
\]

\[
\xi = C \sqrt{\frac{f_w}{2g}} \tag{3.90}
\]

For Eq. 3.88 the following difference equation can be applied, using an explicit finite difference scheme as:
\[
\frac{ds_{xy}}{dy}_{i+\frac{1}{2}} = \frac{E_{i+1} n_{i+1} \sin \varphi_{i+1} \cos \varphi_{i+1} - E_i n_i \sin \varphi_i \cos \varphi_i}{\Delta x}
\] (3.91)

where:

\[\Delta x : \text{ the same length steps as for the wave analysis model}\]

Evidently the length steps are very important in numerical computations to have a reasonable error on the conclusions. Herein various alternatives were checked for length step. There were not a significant difference on produced errors. Then the same length steps were involved which were already selected for wave analysis model.

For Eq. 3.81 the parameters \( \xi \) and \( \hat{u}_0 \) are concluded using the wave data of endec model for each step whereas the \( \xi \) is dependent on \( f_w \) which itself is related to the \( a_0 \), maximum horizontal displacement.

\[
(\tau_{cwx})_{i+\frac{1}{2}} = \frac{(\tau_{cwx})_{i+1} + (\tau_{cwx})_i}{2}
\] (3.92)

Substituting Eqs. 3.91 and 3.92 into Eq. 3.87 and applying a numerical method, e.g. Newton-Raphson, the longshore current velocity, \( V \), will iteratively be obtained for each length step. Fig. 3.5 shows a sample of velocity profile over the beach cross-section whereas Fig. 3.4 demonstrates the gradient in radiation shear stress along cross-shore direction (see Fig. 3.4 and 3.5).

**Numerical solution of Einstein integrals**

In sediment transport computation by Bijker formula longshore current velocity and the Einstein integrals have to be evaluated numerically. The numerical procedure of longshore current velocity calculations was dealt with in the previous section. The other item, Einstein integrals are calculated as follows:

Van de Graaff et al (1977) presented a numerical solution for Einstein integrals, applying the following Binom-Newton relation:
Neglecting higher order terms of Eq. 3.22 and using the first 3 terms $I_1$ and $I_2$ will be concluded as follows:

\[
I_1 = \frac{1}{(1-z_\ast)}[1-A^{(1-z_\ast)}] - \frac{z_\ast}{(2-z_\ast)}[1-A^{(2-z_\ast)}] - \frac{z_\ast(1-z_\ast)}{2(3-z_\ast)}[1-A^{(3-z_\ast)}] \tag{3.94}
\]

\[
I_2 = \left( \frac{1}{(1-z_\ast)} - \frac{1}{(1-z_\ast)} \right) - A^{(1-z_\ast)}(\ln A - \frac{1}{(1-z_\ast)}) - \frac{z_\ast}{(2-z_\ast)}\left[ - \frac{1}{(2-z_\ast)} - A^{(2-z_\ast)}(\ln A - \frac{1}{(2-z_\ast)}) \right] \]
\[
- \frac{z_\ast(1-z_\ast)}{2(3-z_\ast)}\left[ - \frac{1}{(3-z_\ast)} - A^{(3-z_\ast)}(\ln A - \frac{1}{(3-z_\ast)}) \right] \tag{3.95}
\]

It is known that when $z_\ast$ is equal to 1, 2 or 3, the aforementioned relations can not mathematically be applied. On those cases the following analytical solutions have to be used:

for $z_\ast = 1$:

\[
I_1 = -\ln A + A-1 \tag{3.96}
\]

\[
I_2 = -\frac{1}{2} \ln^2 A + A \ln A - A + 1 \tag{3.97}
\]

for $z_\ast = 2$:

\[
I_1 = 2 \ln A - A + \frac{1}{A} \tag{3.98}
\]

\[
I_2 = \ln^2 A + \ln A(-A + \frac{1}{A}) + A + \frac{1}{A} - 2 \tag{3.99}
\]
for \( z_* = 3:\)

\[
I_1 = -3\ln A + A - \frac{3}{A} + \frac{1}{2A^2} + \frac{3}{2}
\]

(3.100)

\[
I_2 = -\frac{3}{2} \ln^3 A + \ln A (A - \frac{3}{A}) - \frac{3}{A} + \frac{1}{A^2} + \frac{15}{4}
\]

(3.101)

where:

\( A \): dimensionless parameters \((r/h)\)

\( r \): bottom roughness \((m)\)

\( h \): water depth \((m)\)

\( z_* \) was given in Eq. 3.30 as:

\[
z_* = \frac{w}{kV_{*c_w}}
\]

(3.102)

All the parameters in this equation have already been introduced.

Figs. 3.6, 3.7, 3.8 show outcome samples of CBM model for suspended load, bed load and total load sediment transport by Bijker formulae, respectively.

**Numerical procedure of van Rijn approach**

For current-related bed load transport as well as wave-related sediment transport, the subject is quite clear. A sediment transport rate will be concluded for each length step along the beach profile when wave data are applied in the formulation. However the numerical solution of suspended load transport is a bit complicated, which is explained as follows:

The subsequent integral has to be numerically solved to evaluate suspended load passing over entire depth with unit width.

\[
S_z = \int_a^h V(z)c(z)dz
\]

(3.103)
where:

\[
\frac{dc}{dz} = \frac{(1-c)^3c}{e_{cw}} \tag{3.104}
\]

\[
V(z) = \frac{V \ln(30z/r_d)}{-1 + \ln(30h/r_a)} \tag{3.105}
\]

\(V(z)\) is simply calculated for each length step, using the wave data of ENDEC computer model. For calculation of \(c(z)\) two solutions are possible, a purely numerical approach and a semi analytical approach. In numerical solution the following finite difference scheme has to be applied, using Eq. 3.104:

\[
\frac{c_{i+1} - c_i}{\Delta x} = \frac{(1-c_{i+1})^3c_{i+1}}{e_{cw}} \tag{3.106}
\]

By an iterative procedure Eq. 3.106 gives the \(c_{i+1}\) whereas for initial condition, \(c_i = c_a\) (Eq. 3.62) is employed. As all the numerical methods the length steps (over the depth) are of great importance in this case as well as the optimization between time consuming and accuracy is also major. This later matter will be considered in CBM model by checking the rate in changes of concentration regarding the maximum (boundary) concentration, \(c_a\), as well. A condition is positioned in appropriate computer program to control the length steps over the depth. This condition will keep the step sizes not to be very large to lead to an unreasonable decreasing in concentration, \(c\). Otherwise the program selects a rational \(c\) (i.e. \(c_{i+1}\) in Eq. 106). Then it will be searched for the corresponding \(\Delta x\), and ultimately a new \(x\) (height).

A typical solution in numerical methods, applying mathematical engineering, could be presented as a second solution, there where a standard method is introduced to solve the classic equations read as \(y' = f(y)\). This method yields a finite difference equation as:

\[
\ln c_{i+1} - \ln c_i + (X_{i+1} - X_i) \frac{c'}{c} \tag{3.107}
\]
where:

\[
\frac{dc}{dx} = \frac{(1-c)^3 c w}{\varepsilon_{sw}}
\]  

(3.108)

Here also \(c_a\), bed concentration is used as initial condition.

In this later solution the typical solution of the first order differential equation has been involved. Fig. 3.9 and Fig 3.10 show the output result of CBM model for concentration and velocity profile over the depth.

Now having \(V(z)\) and \(c(z)\) in reasonable step sizes the product of the integral of Eq. 3.103 yields:

\[
S_z = \sum_{i=0}^{h} V_i c_i
\]  

(3.109)

Three curves, presented in Fig. 3.11, are the current sediment transport components vs distance to the coast, i.e. bed load, suspended load and total current-related transport. Fig. 3.12 shows the total transport under waves and current condition as well as the wave-related and current-related transports. Fig. 3.13 is the same as Fig. 3.12 but for the case that the wave-related transport is dominant. One can expect that when fall velocity increases from 0.025 (m/s) to 0.08 (m/s), then the suspended load and consequently the current-related transport is significantly lowered and transport by waves is appreciable related to the current transport component. This wave transport component is in the direction of wave propagation which have to be projected in the x direction while total longshore transport is being calculated.

3.5 Computer programming

The computer programming of the wave analysis is to be developed for sediment transport computation. As it was pointed out, 4 option are arranged for this computation. One of the formulae CERC, Bijker and van Rijn can be selected for this calculation whereas all three formula also could be involved to compare the results. For CERC and Bijker formulae there is no need any additional data than somewhat is required for wave analysis. However in
sediment transport computation by van Rijn approach, some new data could be given and/or to be left the computer model itself to calculate those parameters by introducing the boundary condition. An input file named RIJN1.DAT has been considered for these data while in the beginning of the file the bed form is entered. Table 3.2 shows a sample of this file, the contents introduced below the table. Tabulating a zero value for any of the parameters in this file, means that it is left to be calculated by the program.
FLAT BED
0.
0.
0.
0.025

Fig. 3.2 A sample of input file RIJN1.DAT

FLAT BED : herein marks a flat shape for the bottom*

The following data are listed in this file **:

- Wave-related bottom roughness ($r_w$, m)
- Reference level ($a$, m)
- Near bed mixing layer thickness ($\delta_s$, m)
- Fall velocity ($w$, m/s)

* Three shapes could be considered for the bed form: flat bed, ripple bed or an option 'unknown' which this later at any rate acts based on a longshore transport computation and the fact of wave breaking in nearshore. It is noticeable that wave breaking has an increasing effect on the sediment transportation. Then the same criteria of wave analysis model, treated in section 2.6 (page 14), was involved here in this calculation.

** As it was already mentioned any of the parameters listed in this file might be labeled as zero.
FIGURES
Fig. 3.4 Gradient in radiation shear stress vs water depth

H_{rms} = 2m  T = 7s
\Phi = 30 \text{ (deg.)}
slope = 0.01
Fig. 3.5 A sample of longshore current velocity distribution

H_{rms} = 2 \text{m} \quad T = 7 \text{s} \\
\Phi = 30 \degree \\
\text{slope} = 0.01
Fig. 3.6 Suspended load transport ($10^{-4} \text{ m}^3/\text{s}$) vs depth by Bijker formula (result of CBM model)
Fig. 3.7  Bed load transport (m$^2$/s) by Bijker formula vs depth (result of CBM model)
Fig. 3.8 Total load sediment transport ($m^3/s$) by Bijker formula vs depth (result of CBM model)
Fig. 3.9 A sample of velocity profile over the depth by van Rijn approach (result of CBM model)
Fig. 3.10 Concentration profile over depth (v. Rijn approach)

$D_{50} = 300 \times 10^{-6}$ (m)

$w = 0.02$ (m/s)
Fig. 3.11 Current-related transport components (v. Rijn formula)

$H_{rms}= 2. \text{ (m)}$ $T= 7. \text{ (s)}$

$r= 0.06$ $w= 0.025 \text{ (m/s)}$

<table>
<thead>
<tr>
<th>Distance to the coast (100 m)</th>
<th>Sediment transport ($\text{m}^3/\text{s}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$2 \times 10^{-5}$</td>
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<tr>
<td>2</td>
<td>$4 \times 10^{-5}$</td>
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</tr>
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<td>4</td>
<td>$8 \times 10^{-5}$</td>
</tr>
<tr>
<td>5</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>6</td>
<td>$1.2 \times 10^{-4}$</td>
</tr>
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<td>13</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
</tr>
</tbody>
</table>

- Total
- Susp. load
- Bed load
Fig. 3.12 Sediment transport components by v. Rijn formula

\[ H_{rms} = 2 \, (m) \quad T = 7 \, (s) \]

\[ r = 0.06 \quad w = 0.025 \, (m/s) \]

- Current trans.
- Wave trans.
- Total trans.

Distance to the coast (100 m)
Fig. 3.13 Sediment components for the case of considerable transport by waves

$H_{rms} = 2.0 \text{ (m)} \quad T = 7 \text{ (s)}$

$r = 0.06 \quad \omega = 0.08 \text{ (m/s)}$

---

Current trans.

Wave trans.

Total trans.
CHAPTER 4

TRANSPORT FORMULAE EVALUATION

4.1 Introduction

It is to be evaluated some sediment transport formulae in this chapter whereas the well-known CERC formula is accepted as a base.

Reliable procedure for transport measurements is still a very difficult task in Coastal Engineering, and most of the sediment transport formulae are not supported by a dependable prototype measurements confirmation. Therefore, a comparative computation can be selected as a method of formulae verification. Fortunately CERC formula has been checked by a lots of prototype measurements and gives a good results for a fairly non-complex boundary conditions. However, very often the boundary conditions are complicated. Then other formulae, which have been included more effective parameters, should be applied. CERC formula calculates longshore sediment transport by oblique waves for a shore with fine bed materials. In complex circumstances other currents like tidal are important, bottom particles are coarse, and/or the distribution of sediment along beach profile is required. For these conditions another methods should be used.

In this study Bijker and van Rijn formula have been considered to check by a series of comparative calculations with CERC formula. Limits of boundary conditions on which CERC formula produce a good results were analyzed by the CBM model. With these range of data the result of CERC formula is assumed to be reliable. If in this computation the other formulae viz.: Bijker and van Rijn formulae are in a rather good agreement with the result of CERC formula, they can be accepted to apply for the complicated boundary conditions. A comprehensive comparison have to be carried out, because the sensitivity of the parameters involved in the various formulae differ from one to another. In view point of computed transport rate related to the CERC formula, variations of the various parameters may lead to the different results. In this case a detailed insight view in such these analyses may convince users to apply one of the formula for an specific boundary condition. However, it does not mean that a general conclusion will not be drawn.
The involved boundary conditions are outlined here in the following sections whereas the results of each case is appended in the related section.

4.2 Circumstances

Herein the wave analysis ENDEC model is involved. The breaking criteria (i.e. \( Q_b \) equal to 0.001), treated in section 2.6 (page, 14) is used for wave characteristics in breaking point for sediment transport calculation by the CERC formula. Since ENDEC model computes root mean square wave height, \( H_{rms} \), the value of 0.08 was applied as the dimensionless coefficient in CERC formula.

As it was seen in the previous chapter, by the random wave analysis of ENDEC model and considering Bijker approach, the longshore current velocity distribution was obtained. Through a numerical procedure, applying the local values for all the parameters, involved in Bijker and van Rijn formulae, the most possible appropriate transport computations were being carried out within this comparative analysis.

To have a sensitive analysis and graphs, all the computed sediment transports have been divided by the calculated CERC transport and in the graphs are considered as Y-coordinates, labelled by 'Ratio'. Then:

\[
\text{Ratio} = \frac{\text{transport with sediment transport formula}}{\text{transport with CERC formula}}
\]

It is noticeable that the wave analysis is done with consideration of bottom friction, therefore in some places particular circumstances may occur.

4.3 Effects of bed material and beach slope

In both formula bed material is characterized with \( D_{50} \) and \( D_{90} \). The mean diameters (\( D_{50} \)) of 100, 200, and 300 (\( \mu m \)) were applied whereas the diameters of 175, 280, and 385 (\( \mu m \)) were used as the values of \( D_{90} \), respectively.

Fall velocities calculation was carried out by the CBM model, in which Eq. 3.32 with the coefficients for fresh water at 18\( ^{\circ} \)C had been placed. The calculated values were 0.0097, 0.0256 and 0.041, respectively.
The above mentioned conditions have been repeated for two beach slopes viz.: 1/100 and 1/50. Fig. 4.1 gives the graphical results whereas tables 4.1 and 4.2 shows the corresponding calculated sediment transport rates.

Both formulae are sensitive to the bed material. A very high transport rate is obtained for fine particles (e.g. \(D_{50} = 100\mu m\)) by van Rijn formula, whereas very small values are concluded when bed materials have rather big diameters. Such a high sensitivity to the particle diameters seems to be unreasonable, specially when for coarse materials \((D_{50}>300\mu m)\) negligible rates are achieved.

Bijker formula shows a very close results to the CERC formula for the case of slope=1/100 and \(D_{50}=100\mu m\). A smooth curve, indicated by Bijker formula in Fig. 4.1, points to the similar behaviour with the CERC formula.

The result of beach slope variation is rather strange, despite of a similar trend in transport rates variation, the direction of the changes is converse. While for Bijker formula a reductional trace is observed in steeper slope, van Rijn formula defines a lower rate in milder slope.

It is noticeable that in steeper slopes the width of surf zone is smaller as well as the longshore current velocity distribution shows greater values for various locations along the beach profile. These two factors balance each other and results is sometimes in lower sediment transport in steeper slopes and somtimes higher rates in milder slope related to the specific boundary conditions. For this case the increment as much as about two times for the steeper slope could not be reasonable on van Rijn formula.

On the other hand since velocity distribution is fairly proportional to the beach slope, a reasonable relationship between the Bijker formula and the velocity distribution can be justified. This fact and a rather smooth dependency of the Bijker formula on the bed material diameters make it a reliable formula in view point of bed material grain size and bottom slope. It should be mentioned that the bottom slope is implicitly related to the local flow velocity.

4.4 **Effect of bottom roughness**

The special version of CBM model suitable for this comparative analysis was run to evaluate the effect of bottom roughness on the various formulae. The values of 0.02, 0.04, 0.06, 0.08, 0.1 were selected for this purpose. This computation were carried out for two above mentioned slopes (i.e. 1/100 and
Figs. 4.2, 4.3 and 4.4 present the graphical results for various bed material diameters marked on the figures (for net values of transport rates see table 4.1 and 4.2).

Here again both formula yield a different situation. Van Rijn formula gives an unexpectably increasing in transport when bed roughness is getting increment in the case of $D_{50}$ equal to 100μm. However, it is nearly insensitive to the bottom roughness when $D_{50}$ is greater than 200μm. Due to this point and the results of the previous section it might be concluded that the van Rijn formula is not basically suitable for the case of $D_{50}$ less than 200μm where the difference of about 10 times is observed between the calculated sediment transport rates for bed grain sizes of 100 and 200 μm by this formula.

Bijker formula shows a slightly dependence to the bottom roughness and generally the results are close to the CERC formula.

The results of beach slope variation are nearly the same as the previous section. But here the sensitivity of van Rijn formula to the bottom slope is very larger than Bijker formula.

With respect to the point that bed roughness as well as the beach slope is not very important in normal prototype conditions, then results of this part also makes a positive justification on the Bijker formula.

4.5 Effect of wave height

Different wave heights from 1.0 to 3.0m were given to the model, while wave periods change from 4 to 8s. The results are graphically presented in Fig. 4.5.

A nearly horizontal line showed by the results of Bijker formula points to the same tendency of the Bijker and the CERC formulae. If again it is assumed that a rather suitable value of longshore transport is calculated by the CERC formula, then it can be concluded that Bijker formula also describes this transport fairly well.

The graphical results of the van Rijn formula in Fig. 4.5 also shows a rather good sensitivity of the formula to the wave heights, but the computed rates are very low.
4.6 **Effect of wave angle**

The results of the computations with different deep water wave angles have been indicated in Fig. 4.6. Van Rijn formula presents a smooth curve, but the computed rates are not sufficiently high related to the CERC formula.

Bijker formula also shows a good tendency as the CERC formula when the wave angles are not so big, $\phi_0 < 45^\circ$. However, there is some problem for the great values of the incident wave angles. This later matter might be justified that the approach involved to calculate longshore current velocity is valid when the incident wave angles are equal to $20^\circ$ or less. Therefore it is concluded that the Bijker formula defines the sediment transport rates quite well in this range ($\phi < 20^\circ$).

4.7 **Effect of breaker point selection**

The procedure of deriving the wave characteristics corresponding to the breaker point, needed for CERC sediment transport computation, was treated in section 2.6. The values of 0.001 to 0.01 could be selected as the probability of the breaking, $Q_b$, in this point leading to different values of the transport rates. In the CBM model Bijker formula is independent of the breaker point. However, the van Rijn formula has a slight dependence on this point, as it was observed on van Rijn formula description some of the parameters involved in this formula are related to breaker line. The graphical conclusions can be observed in Fig. 4.7.

Since the selection of this point is not fixed and the resulted line by the Bijker formula crosses the graphical line of the CERC formula, then the Bijker formula can be assumed to have the closest results to the CERC formula. Van Rijn formula has still a considerable difference with the CERC formula.

4.8 **Conclusion**

Apart from weaknesses of this comparative computations that a firm conclusion is turned to be impossible, the results of the above sections of this chapter yield a quite well results by the Bijker formula, particularly when the incident wave angle is not so big. Fig. 4.8 points to the last
confirmation evidence. When the value of 0.01 is applied as the probability of breaking, \( Q_b \), a very close result to the CERC formula is concluded by the Bijker formula for the small values of wave angles. Based on the above results van Rijn formula does not seem to be suitable for a longshore current sediment transport computation.

<table>
<thead>
<tr>
<th>Slope=1/100</th>
<th>H(_{\text{rms}})=2m</th>
<th>T=7s</th>
<th>Phi=30</th>
</tr>
</thead>
<tbody>
<tr>
<td>D((\mu m))</td>
<td>bottom roughness, r (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>100</td>
<td>CERC</td>
<td>.400</td>
<td>.352</td>
</tr>
<tr>
<td></td>
<td>Bijker</td>
<td>.714</td>
<td>.414</td>
</tr>
<tr>
<td></td>
<td>V.Rijn</td>
<td>1.02</td>
<td>1.08</td>
</tr>
<tr>
<td>200</td>
<td>CERC</td>
<td>.400</td>
<td>.352</td>
</tr>
<tr>
<td></td>
<td>Bijker</td>
<td>.238</td>
<td>.181</td>
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<tr>
<td></td>
<td>V.Rijn</td>
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<td>300</td>
<td>CERC</td>
<td>.400</td>
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<tr>
<td></td>
<td>Bijker</td>
<td>.117</td>
<td>.109</td>
</tr>
<tr>
<td></td>
<td>V.Rijn</td>
<td>.026</td>
<td>.017</td>
</tr>
</tbody>
</table>

Table 4.1 The sediment transport rates obtained on comparative analysis for beach slope equal to 1/100

<table>
<thead>
<tr>
<th>Slope=1/50</th>
<th>H(_{\text{rms}})=2m</th>
<th>T=7s</th>
<th>Phi=30</th>
</tr>
</thead>
<tbody>
<tr>
<td>D((\mu m))</td>
<td>bottom roughness, r (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>100</td>
<td>CERC</td>
<td>.457</td>
<td>.434</td>
</tr>
<tr>
<td></td>
<td>Bijker</td>
<td>.673</td>
<td>.376</td>
</tr>
<tr>
<td></td>
<td>V.Rijn</td>
<td>2.03</td>
<td>2.11</td>
</tr>
<tr>
<td>200</td>
<td>CERC</td>
<td>.457</td>
<td>.434</td>
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<tr>
<td></td>
<td>Bijker</td>
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<td>.191</td>
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<td></td>
<td>V.Rijn</td>
<td>.268</td>
<td>.203</td>
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<tr>
<td>300</td>
<td>CERC</td>
<td>.457</td>
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</tr>
<tr>
<td></td>
<td>Bijker</td>
<td>.140</td>
<td>.124</td>
</tr>
<tr>
<td></td>
<td>V.Rijn</td>
<td>.087</td>
<td>.055</td>
</tr>
</tbody>
</table>

Table 4.1 The sediment transport rates obtained on comparative analysis for beach slope equal to 1/50
Fig. 4.1 Effects of particle size and beach slope

Hrms0=2m  T=7s  r=.06m

- CERC
- Bijkers
- v.Rijn

Bed material diameter [μm]
Fig. 4.2 Effects of bed roughness and beach slope

- $H_{rms}=2\text{m}$, $T=7\text{s}$, $r=0.06\text{m}$
- $D_{50}=100\mu\text{m}$
- $\text{slope}=1/50$
- $\text{slope}=1/100$
- $\text{slope}=1/50$

Graph showing the relationship between ratio and bed roughness. The graph includes data points for different slopes and grain sizes, with lines indicating the CERC, Bijker, and v.Rijn methods.
Fig. 4.3 Effects of bed roughness and beach slope

$H_{rms0}=2m \quad T=7s \quad r=0.06m$

$D_{50}=200\mu m$

- - - CERC
- - - Blijker
- - - v.Rijn

slope=1/100

slope=1/50

slope=1/100

0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1

Bed roughness (m)
Fig. 4.4 Effects of bed roughness and beach slope

\[ H_{rms} = 2 \text{m} \quad T = 7 \text{s} \quad r = 0.06 \text{m} \]
\[ D_{50} = 300 \mu\text{m} \]

- CERC
- Bijkerv
- v. Rijn

**Ratio**

**Bed roughness (m)**

- slope = 1/100
- slope = 1/50
- slope = 1/100
Fig. 4.5 Effect of wave Height

Phi=30° D50=200μm r=0.06m

- CERC
- Bijker
- v.Rijn

Ratio

0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3

Hrms (m)
Fig. 4.6 Effect of wave angle

$H_{rms} = 2 \text{ (m)} \quad T = 7 \text{ (s)}$

$d_{50} = 200 \text{ (μm)} \quad r = 0.06$

Wave angle ($^\circ$)
Fig. 4.7 Effect of breaking point selection

$H_{rms} = 2\text{m}$ $T = 7\text{s}$

$\phi = 30^\circ$

- **CERC**
- **Bijker**
- **v.Rijn**

Ratio

Probability of breaking, $Q_b$
Fig. 4.8 Sensitivity of the formulae to the wave angle

H_{rms}=2m \quad T=7s

D_{50}=200\mu m \quad r=0.06 m

- CERC
- Bijker
- V. Rijn
CHAPTER 5

CONCLUSIONS and RECOMMENDATIONS

Considering a random wave analysis in this study could imply an approach towards more reality. The mathematical ENDEC model had already been verified positively. A very good agreement of the present computer model outcomes with the results of ENDEC model verification makes sure that the wave analysis part of CBM model calculates the wave characteristics along the beach profile sufficiently reliable. Only some refinement on input data process, specifically making a window through it the input data file to be arranged, will slightly improve this part of the model. On the other hand, the part of ENDEC2B which accepts and calculates unlimited groups of wave data via an input file, has a practical value. Consideration of bottom friction varying along the cross-section in wave analysis model, very often, is not significant, but in some circumstances may play an important role where bed forms develop a very rough condition.

CERC formula is still one the most reliable sediment transport formula, specially in non-complex boundary condition; therefore, it was placed in CBM model, despite the application of the wave parameters on breaker point in this formula. The investigation carried out in order to determine a criteria for selecting a certain point as a breaker point was a difficult task because of random wave analysis consideration in the model. The concluded criteria in this work is rather suitable in practice, specially if the bottom roughness and consequently the bottom friction are small. However, as pointed out, further research might lead to a more reliable method.

The so-called local-methods considered in sediment transport computations apply the relevant parameters of waves, sea bed and current conditions for the various locations along the beach profile. This method which describes the sediment transport rate as the product of velocity and concentration, integrated over the depth, is generally accepted. Since the wave analysis part of the CBM model assures application of the local method, sediment transport calculated by the CBM model can be assumed to be reliable as long as the formulae involved are valid. If any other formula will be accepted to be reliable, then it can easily be replaced in the model; therefore, it has its high value.

In chapter 4 it was concluded that Bijker formula gives rather good results on sediment transport computations. Where the local parameters are produced
fairly well, the transport rates computed by the CBM model is expected to be reliable in practice.

The CBM model provided the opportunity for comparison of two sediment transport formula with the CERC. The result looks to be interesting. Much more computations are possible to be carried out by the particular versions of the CBM model, developed for this comparative analysis. For instance, a sensitivity analysis could be implemented on various parameters, involved in the formulae, to obtain some more idea about their functions. Therefore it is advisable that further investigation could be carried out using the present model.
PART TWO

CHAPTER 6

MORPHOLOGICAL PROCESSES

6.1 Introduction

A rather suitable approach was selected for sediment transport computations by the CBM model. The calculated sediment transport is very often applied for coastal evolution prediction. Therefore one of the most important Coastal Engineering problem is treated in this chapter as a case of applicability of the model described in the first part of the report. Although, various complicated approaches have been presented in literatures, but in this study it is to be considered one of the simple solutions for this computation. The single line theory (Pelnard-Considre, 1956) has been involved to evaluate the accretion and erosion processes near the breakwaters.

A numerical procedure has been applied to solve the diffusion equation to determine the coastal changes, progressing in the time.

6.2 Single line theory

The most rough schematization is made by the single line theory where the beach profile is assumed to follow one line leading to a horizontal part of the sea bed. This characterized line horizontally moves over the entire height as a result of accretion or erosion. For instance if the intersection of the profile and water level progresses seaward over a distance 'a', then all other depth contours will be shifted over a distance 'a'. Therefore the beach slope does not change and the shape of the profile remains constant. The toe of the profile is defined as the point where the beach slope becomes essentially horizontal (see Fig. 6.1). This method, however, when will be applicable that there is a horizontal bottom part before ending to the deep water depth; otherwise this method can not be used. Then the wave characteristics of the horizontal part are involved in this computations.

In consideration of the height of the schematized profile, the attention should be paid that if a part further up the cross-section is under accretion
or erosion processes, the height of this part will be added to the water depth. Normally the height, \( d \), needed to apply in the equations is taken equal to the water depth plus wave run-up.

![Fig. 6.1 Beach profile schematization](image)

6.3 Derivation of the equations

For this computation the sediment transport rate has to be known. However, in further derivation it will be pointed out that the sediment transport is assumed to be a linear function of the angle of wave incidence. Therefore a sediment transport formula to follow this condition, i.e. linearity with the wave angle, to be preferable. Moreover than a transport formula two equation will be necessary in order to predict the coastal changes, equation of motion and a continuity equation. These two equation are briefly explained here in the following section. More details can be found in literatures, e.g. Coastal Engineering volume 2, by van der Velden.

Equation of continuity

Considering a segment of the beach indicates a change of \( dx \) (seawards in accretion process and landwards in erosion procedure) in time interval '\( dt \)'. The inflow of sediment minus outflow times the unit of time yields the accumulated volume (see Fig. 6.2). According to this point the continuity equation is mathematically derived as:

\[
\frac{\partial S}{\partial x} + d \frac{\partial y}{\partial t} = 0
\]  

(6.1)
Equation of motion

Any changes in coastal orientation makes a change in transport rate. This transport changes ($\partial S_x / \partial x$) leads the coast to be movable. The equation of motion is defined based on this movability factor, $\partial S_x / \partial x$. The most important parameter that make the changes in sediment transport is the wave angle, $\varphi$, as well as wave height. If the deep water wave characteristics are assumed to be constant along the coastal area under study, then only the angle of $\varphi$ remains to make a gradient in sediment transport rate in the along-shore direction, $x$. By any of the longshore sand transport formula the ratio $\partial S_x / \partial x$ can empirically be determined when the angle of wave approach is slightly varied. However, the expression of $\partial S_x / \partial \varphi$ can be assumed to be constant. It implies that the changes of sediment transport rate related to the wave attack angle to be linear. This later assumption is reasonable as long as the changes of $\varphi$ is relatively small. Considering the above mentioned points the equation of motion by applying some algebra yields:

$$\frac{\partial S_x}{\partial x} - s_x \frac{\partial^2}{\partial x^2}$$  \hspace{1cm} (6.2)

where:

$$s_x = \frac{\partial S_x}{\partial \varphi}$$  \hspace{1cm} (6.3)

As it was already mentioned $s_x$ may empirically be derived or assumed to be constant as:
\[ s_x - \frac{S_x}{\varphi'} \]  
(6.4)

where:

\[ \varphi - \varphi' - \frac{\partial y}{\partial x} \]  
(6.5)

\( \varphi \) : wave angle with the instantaneous shoreline at some time, \( t \)

\( \varphi' \) : wave angle with the original shoreline, see also Fig. 6.3

Fig. 6.3 A view plan of the shore defining some aspects

**Diffusion equation**

Combination of the equations continuity and motion yields the standard diffusion equation as:

\[ a \frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial t} = 0 \]  
(6.6)

where:

\[ a = \frac{s_x}{d} - \frac{S}{\varphi'd} \]  
(6.7)
$S_x$ : sediment transport rate at $t=0$, equal to $S$

6.4 Solution of the equation

The equation could be solved for various situations applying the appropriate boundary condition. Herein the problem is treated only for the case of breakwater accretion.

Initial and Boundary conditions

Fig. 6.4 gives the opportunity to derive the boundary and initial condition for the case of accretion near the breakwaters. The assumption of a straight stable coastline defines the initial condition, expressed as:

$$t = 0 \quad \rightarrow \quad y = 0 \quad \text{for all X}$$

At a distance fairly far from the breakwater, theoretically $x = -\infty$, the coastline and consequently the sand transport remains constant, equal to its value before breakwater construction. Therefore, the left boundary condition can be derived considering this point.

$$x = -\infty \quad \rightarrow \quad S_I = S \quad \text{for all t}$$

For the second boundary condition there will be two situations:
a- the condition on which toe of the seawards progressing beach slope, has not reached to the end of the breakwater. For this case it is assumed that the breakwater is impermeable to sand. Thus:

\[ x = 0 \quad \Rightarrow \quad S_i = 0 \quad \text{for all} \quad t > 0 \]

An other right boundary condition is obtained bearing in mind the dependency of \( S_i \) on the angle of \( \phi \) and a zero transport at location of the breakwater \((x = 0)\). The zero transport denotes the wave approach is perpendicular to the coast. This implies that the beach slope progresses seawards making an angle \( \phi' \) with the breakwater. Therefore, it can be concluded that:

\[ \frac{\partial y}{\partial x} = \phi' \quad \text{at} \quad x = 0 \quad \text{for all} \quad t > 0 \quad (6.7) \]

b- When the accreting beach slope attaches the breakwater tip, then some of the transported sediment will passed over the breakwater after some time \( t_1 \). It means at \( x = 0 \), \( S_i \) is no longer equal to zero. Instead, This boundary yields:

\[ x = 0 \quad \Rightarrow \quad y = L \quad \text{for all} \quad t > t_1 \]

6.5 Modelling process of the problem

To solve Eq. 6.6 a simple central explicit scheme can be involved. Through this finite difference scheme the differential diffusion equation is easily solvable by the computer. Fig. 6.5 shows the so-called operator for this scheme in which the first order time derivative is obtained as:

\[ \frac{\partial y}{\partial t} = \frac{y^{n+1} - y^n}{\Delta t} \quad (6.8) \]

But for the second derivation the following procedure has to be applied:
Fig. 6.5 The operator of an explicit scheme

\[ \frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial y}{\partial x} \right) \left( \frac{\partial y}{\partial x} \right) \frac{\Delta x^2}{\Delta x} \]

\[ \frac{\partial y}{\partial x} = \frac{y_{j+1}^n - y_j^n - y_j^n - y_{j-1}^n}{\Delta x} \frac{\Delta x}{\Delta x} \]

\[ \frac{\partial^2 y}{\partial x^2} = \frac{y_{j+1}^n - 2y_j^n + y_{j-1}^n}{\Delta x^2} \]

(6.9)

Substituting Eqs. 6.8 and 6.9 into Eq. 6.6 yields:

\[ y_j^{n+1} = ry_{j-1}^n + (1+2r)y_j^n + ry_{j+1}^n \]

(6.10)
where:

\[
    r = \frac{a \Delta t}{\Delta x^2} \quad (6.11)
\]

\[\begin{align*}
    \Delta t & : \text{time step} \\
    \Delta x & : \text{length step}
\end{align*}\]

Right boundary condition does not straightly give the required values for accretion lengths, hereby labelled as \(y^*_M\). These values are obtained by a derivative approximated on Eq. 6.7 as:

\[
\frac{\partial y}{\partial x} = \frac{y^n_{j-1} - y^n_j}{\Delta x} - \frac{y^n_M - y^n_{M-1}}{\Delta x} - \varphi'
\]

then:

\[
y^n_M = \varphi' \Delta x + y^n_{M-1} \quad (6.12)
\]

The value of \(r\) is determined applying the consistency and stability condition. Like each numerical solution and scheme these two problem are important for this case as well. A Fourier transform investigation has led to the stability condition of this derived finite difference scheme as:

\[
r \leq 1/2
\]

This implies that the numerical solution is stable if the \(r\) value, Eq. 6.11, is less than 1/2.

A Taylor's series expansion on Eq. 6.6 determines the consistency condition. While small errors produced by the higher order terms are ignored, the remaining terms indicate the value of \(r\) on which the big errors will be eliminated.
\[
\frac{\partial^2 y}{\partial x^2} \frac{\partial y}{\partial t} - \frac{1}{2} \Delta x \frac{\partial^4 y}{\partial t^2} + \frac{1}{12} a \Delta x^2 \frac{\partial^4 y}{\partial x^4} + \text{h.o.t.} 
\]  
(6.13)

\[
\frac{\partial^2 y}{\partial x^2} \frac{\partial y}{\partial t} - \left( \frac{r}{2} + \frac{1}{12} \right) a \Delta x^2 \frac{\partial^4 y}{\partial x^4} + \text{h.o.t.} 
\]  
(6.14)

where:

h.o.t. : higher order terms

It can be seen that when \( r \) is equal to 1/6 the first term will be zero, therefore this value is selected for the numerical coefficient, \( r \).

By the Eqs. 6.10 and 6.12 the numerical procedure could be carried out. A reasonable length step, \( \Delta x \), is selected and knowing the value of \( r \), equal to 1/2 the corresponding value of \( \Delta t \) is calculated whereas the amount of \( 'a' \) has already been obtained by Eq. 6.11.

6.6 Computer programming

A fortran computer program was developed. The input data is given by the screen whereas the outcomes are listed in two files. The future coastline could be requested for 30 different years and the program calculates the coastal evolution for end of each year. The co-ordinates of the shoreline layout for the last asked year are tabulated in a file named MORPH.OUT. In order to make a plot showing the coastline in various years an special file compatible with SGPILOT graphical package is made. This later file was called MORPLOT.OUT.

6.7 Test of the program

Two quick test was possible to verify the result of the computer program. Evidently the volume of sediment entered to the zone near the breakwater in unit of time should be equal to the accreted area indicated by the program times the schematized height of the beach profile. This checking
was carried out and results showed an error less than 5%.

Another checking was investigated by comparing the results of the program with the analytical solution presented in Lecture Notes Coastal Engineering, volume II by van der Velden. The produced coastline by the computer program is in very good agreement with the analytical solution (see Fig. 6.8).
Fig. 6.6 Coastal evolution processes near the breakwaters

$S=4194000 \text{ m}^3/\text{year}$  \hspace{1cm} $\Phi=9.66^\circ$

$d=8\text{m}$

- $t=5 \text{ year}$
- $t=15 \text{ year}$
- $t=30 \text{ year}$
- $t=100 \text{ year}$
Fig. 6.7 Accretion lines near the breakwater

\( S = 4194000 \text{ m}^3/\text{year} \quad \Phi = 9.66^\circ \)

\( d = 8 \text{m} \)

- \( t = 5 \text{ year} \)
- \( t = 15 \text{ year} \)
- \( t = 30 \text{ year} \)
- \( t = 100 \text{ year} \)

UNDISTORTED
Fig. 6.8 Comparison of the computer program and analytical solution

\[ S = 4194000 \text{ m}^3/\text{year} \quad \phi = 9.66^\circ \]

\[ d = 8 \text{m} \]

\[ t = 30 \text{ year} \]
ANNEX A
Radiation stress has a significant role on coastal morphology which is briefly discussed here, more detailed can be seen in literature for instance: Longuet-Higgins-Stewart, [1960-1962-1964], Dorrestein [1961].

Let the sea condition when waves are absent and the mass and momentum are balance as the medium. Waves change the mass and momentum balance and influence the mean condition of the medium in which they are propagating. The wave-induced contribution of horizontal momentum, related to the mean balance of momentum, has been defined as the radiation stress (Fig. A.1), [Longuet-Higgins and Stewart, 1960].

The change of horizontal momentum is shown in Fig. A.2. Pressure under waves fluctuates with the passage of the waves. Then there will be a net effect caused by the increased pressure acting on an increased area (under the wave crest) and the reduced pressure acting on a reduced area (under the wave trough), see Fig. A.2. Even it is questionably assumed that the absolute value of the pressure under wave crest and wave trough are equal, still the mean value of the pressure fluctuations integrated over the depth
is not equal to zero, but positive. This positive value forms one part of the radiation stress. The other part is formed by the non-zero integrated velocity fluctuations (Battjes, 1986).

Since radiation stress is the wave-induced contribution of horizontal momentum, with due to the second law of Newton, \( F \, dt = d(mv) \), the change in radiation stress leads to a force on water volume through which the wave propagates. In a real situation such exerted forces on water volume may have a non-zero resultant. In new condition the given volume of water should be re-balanced. These forces are balanced with the bottom shear stress, either create a flow velocity (called the well-known longshore-current), counteracting bottom shear stress, or a gradient in pressure is made.

The mentioned integrations are carried out, considering the time-averaged value of the integrals. Then radiation stress principal components will be concluded; the results are as follows (Battjes, 1986):

\[
S_{II} = \left[ \frac{2kh}{\sin 2kh} + \frac{1}{2} \right] E
\]  

(A.1)

in which:

\[
E = \frac{1}{6} \rho g H^2
\]  

(A.2)

\( S_{II} \) : radiation stress in the direction of wave propagation
The parameter, $n$, ratio of wave group velocity, $c_g$, to wave celerity, $c$, is defined as:

$$n = \frac{kh}{\sinh 2kh} + \frac{1}{2}$$  \hspace{1cm} (A.3)$$

therefore $S_{xx}$ can be noted as:

$$S_{xx} = (2n - \frac{1}{2})E$$  \hspace{1cm} (A.4)$$

In the notation $S_{ii}$, one subscript, $X$, stands for the direction of transfer (through a plane $X$) and the other, component of momentum being transferred.

The second principal radiation stress component acting on a vertical plane perpendicular to the wave crests yields:

$$S_{yy} = (n - \frac{1}{2})E$$  \hspace{1cm} (A.5)$$

If the waves approach the coast at a non-zero angle, more components can be discerned for radiation stress where shear components exist (see Fig 3.1). For instance $S_{yy}$, a component acting on a vertical plane parallel to the shoreline, causes wave set-up on the coastal zone or a shear component, $S_{yx}$, acting parallel to the coast, produces a driving force in the same direction due to changes in its value.
ANNEX B

LIST OF THE COMPUTER PROGRAMS
C In the name of GOD
C ENDEC2A, A Computer programm for the random wave analysis
C considering the energy loss due to breaking and bottom friction
*
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
REAL*8 X(50), Y(50)
CHARACTER*20 ans
COMHON/EQUPARIKO,GAHA,SLOPE,UX,F,Snel
COMHON/SEDPARIUbarO,Ksi,Chezy,D50,D90,RhoS
COMHON/VELPARIr,Rho,Fw,T
DATA C/9.81/,PI/3.l41
OPEN(lO,FILE='A:\DATAB.DAT')
OPEN(20,FILE='A:\PROFILE.DAT')
OPEN(ll,rILE='ENDEC.OUT')
OPEN(12,FILE='HPLOT.OUT')
OPEN(14,FILE='ETAPLOT.OUT')
*
READ(10,*) U,Nu
NU1= NU*PI/180.
UX= U*COS(NU1)
READ(10,*) Rhos, Rho, r, D50, D90
*
C Reading Beach Profile co-ordinates
*
I= 1
READ(20,*,END=100) X(I) ,Y(I)
I=I+1
GOTO 200
*
100 WRITE(*,31)
31 FORMAT(/6X,'& For introducing wave conditions you have two possibilities: /6X,'& 1- They can be from deep water (1) or /6X,'& 2- You may have them from a certain point (2) ',/6X,'WRITE(*,*) op
READ(*,*) op
IF (op.EQ.1) THEN
WRITE(*,*)' Please enter deep water wave characteristics (Hrms &0, T, Phi0):'
READ(*,*) HO,T,TETAO
ETAl= O.

LO= C*T**2/(2*PI)
KO= 2*PI/LO
CO= LO/T
CgO= 0.5*C0
TETA= TETAO*PI/180.
D= INT(LO/2)
SO= HO/LO

WRITE(*,32)
32 FORMAT(/6X,'& For this case there is still two options: /6X,'& 1- The calculation could start automatically /6X,'& 2- A specific start point could be introduced by user, /6X,'& then some approximation has been accepted! /6X,'& Do you specify a certain start point? ; (y/n):'/
READ(*,41) ans
IF (ans.EQ.'Y'.OR.ans.EQ.'y') THEN
WRITE(*,*)' What is your selected depth?'
READ(*,*) D
ENDIF
Dist1= D
K=KO
CALL CALK(D,K)
OM=SQRT(C*K*TANH(K*D))
Cold= OM/K

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\[ \text{Sold} = 0.5 \times (2 \times K \times D / \sinh (2 \times K \times D) + 1) \]

\[ \text{Cg} = \text{Sold} \times \text{Cold} \]

\[ \text{TETA} = \text{TETA} + \text{TETA} \times 180 / \pi \]

\[ \text{KR} = \sqrt{\cos (\text{TETA}) / \cos (\text{TETA})} \]

\[ \text{KS} = \sqrt{\text{Cg} / \text{Cg}} \]

\[ \text{H} = \text{KR} \times \text{KS} \times \text{H0} \]

\[ \text{ELSE} \]

\[ \text{WRITE}(*,33) \]

\[ \text{FORMAT}(4/6X,*), \text{WHAT} \text{IS} \text{THE} \text{WAVE} \text{CONDITIONS} \text{IN} \text{NEAR SHORE?} \]

\[ \text{(WATER DEPTH, WAVE HEIGHT AND PERIOD, WAVE DIRECTION TO THE -'/6X, SHORENORMAL): '} \]

\[ \text{READ(*,*) D,H,T,TETA} \]

\[ \text{WRITE(*,34) \text{FORMAT}(4/6X,*), WHAT IS THE MEAN WATER LEVEL IN THIS LOCATION (ETA)?, YOU'/6X, CAN ASSUME TO BE 0.0, THEN A BIT APPROXIMATION IS INVOLVED: '} \]

\[ \text{READ(*,*) ETA} \]

\[ \text{Distil} = D \]

\[ D = D + \text{ETAI} \]

\[ \text{LO} = G * T - 2 / (2 * \pi) \]

\[ \text{KO} = 2 * \pi / \text{LO} \]

\[ \text{CO} = 0.5 * \text{CO} \]

\[ \text{TETA} = \text{TETA} + \pi / 180. \]

\[ \text{K} = \text{KO} \]

\[ \text{CALL CALC}(D,K) \]

\[ \text{OM} = 2 / \text{K} \times \text{TANH} (K \times \text{D}) \]

\[ \text{Cold} = \text{OM} / \text{K} \]

\[ \text{Nold} = 0.5 * (2 * K \times D / \sinh (2 * K \times D) + 1) \]

\[ \text{Cg} = \text{Nold} \times \text{Cold} \]

\[ \text{SNEL} = \kappa \times \sin (\text{TETA}) \]

\[ \text{TETAOI} = \text{ASIN} (\text{SNEL} / \text{KO}) \]

\[ \text{TETAO} = \text{TETAOI} \times 180 / \pi \]

\[ \text{HO} = \text{H} \times \sqrt{\text{CG} / \text{CG}0} \times \sqrt{\text{COS} (\text{TETA}) / \text{COS} (\text{TETA}01)) \]

\[ \text{SO} = \text{HO} / \text{LO} \]

\[ \text{ENDIF} \]

\[ \text{GAHA} = 0.5 + 0.4 \times \tanh (33 \times \text{SO}) \]

\[ \text{METAPRT} = 0 \]

\[ \text{WRITE(*,37) \text{FORMAT}(4/6X,*), DO YOU WISH TO HAVE A FILE TO PLOT WATER LEVEL VARIATION '/6X, ', ANAMED ETAPLOT.OUT? ; (Y/N): ') \]

\[ \text{READ(*,41) ans} \]

\[ \text{IF (ans.EQ. 'y'.OR.ans.EQ. 'Y')) METAPRT= 1 \]

\[ \text{C SEARCH TO DETERMINE THE START POSITION ON THE PROFILE} \]

\[ I=1 \]

\[ \text{IF (Distil.GT.(Y(I)-1E-5)) THEN} \]

\[ \text{WRITE(*,36) } \]

\[ \text{FORMAT(4/6X,*), \text{ERROR: PROFILE CO-ORDINATES ARE NOT COMPLETE. START'} /6X, ', \text{POINT OF THE CALCULATION IS OUTSIDE OF THE GIVEN PROFILE} '/6X, ', \text{BE CAREFUL! YOU ALSO NEED TO CONSIDER THE HORIZONTAL }'/6X, ', \text{PART OF THE SEA BOTTOM IN YOUR INPUT FILE, IF YOU HAVE! }'/6X, ', \text{PLEASE AT FIRST IMPROVE DATA FILE: "PROFILE.DAT"'} \]

\[ \text{WRITE(*,*) Distil, Y(I)} \]

\[ \text{STOP} \]

\[ \text{ENDIF} \]

\[ \text{SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))} \]

\[ \text{IF (SLOPE.EQ.0.AND.Distil.EQ.Y(I)) THEN} \]

\[ \text{WRITE(*,* ) ' THE START POINT OF CALCULATION IS LOCATED ON', }\]

\[ \text{HORIZONTAL PART OF SEA BOTTOM, THEN ENTER THE', }\]

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& 'distance of this point to the coast'
READ(*,*) X(I)
GOTO 410
ENDIF
IF (slope.GT.1E-6) GOTO 500
300 IF (Dstil.GE.Y(I).AND.Dstil.LT(Y(I)+1)) GOTO 400
I= I+1
SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
IF (SLOPE.GT.0) GOTO 500
GOTO 300
500 IF (Dstil.LE.Y(I).AND.Dstil.GT(Y(I)+1)) GOTO 400
I= I+1
SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
IF (SLOPE.LT.0) GOTO 300
GOTO 500
400 X(I)= X(I)-(Y(I)-Dstil)/SLOPE
C Length steps selection
410 WRITE(*,35)
35 FORMAT(/6X,'& & a moderate expression has been defined to set a variable'/6X,'
& & stepsizes, you also can select a constant Delta-X'/6X,'
& & do you wish to do=?, (y/n)'
READ(*,41) ans
IF (ans.EQ.'N'.OR.ans.EQ.'n') THEN
H= 0.88/K*THAN(GAMA/0.88*K*D)
CALL CALQB (H,Ha,Qb)
F= 1.5
MSDX= 0
IF (slope.LT.1E-5) THEN
DX= 5*F*LOG10 (Qb)
ELSE
DX= LOG10(Qb**LOG10(SLOPE**F))
ENDIF
ELSE
WRITE(*,*) ' Delta-X (m)= ?'
READ(*,*) DX
MSDX= 1
ENDIF
IF (DX.LT.5E-2) DX= 0.05
IF (DX.GT.50) DX= 40.
WRITE(11,'(A)') CHAR(12)
WRITE(11,4) T
WRITE(11,3) U, NU
WRITE(11,1)
E0= Rho*G*HO**Z/S
E= Rho*G*H**Z/8
WRITE(11,2) HO, TETAO, EO, ETAI
WRITE(11,8) Dstil, H, TETA1, E, ETA1, 1E-3*X(I)
Hold= H
Ye= H**2*(CG*COS(TETA)+UX)/OM
E= Rho*G*H**2/8
Dold= D
TETold= TETA
CALL SOL!Q(Ye,H,K,D,OM,TETA,CG,C,DX,Qb)
IF (Ye.LT.1E-2) THEN
WRITE(11,6)
GOTO 50
ENDIF
DELIH= (CG*COS(TETA)+UX)
IF (DELIH.LE.1E-6) THEN
WRITE(*,7)
WRITE(11,7)
ENDIF
H=SQRT(Ye*OM/(CG*COS(TETA)+UX))
E=Rho*G*H**2/8
Nnew= CG/C
Dstil = D-ETA
ETA = (Hold*2*(Hold*(1+(COS(TETold)*2)-0.5))/8-H**2*(Nnew* \\
(1+(COS(TETA)*2)-0.5))/8)/(0.5*(Dold+D))
ETA1 = ETA1+ETA
D = D+ETA

Cold = C
Hold = H
Nold = Nnew
TETold = TETA
Dold = D
TETA1 = TETA*180/PI

X(I) = X(I)-DX
IF (MSDX.EQ.1) GOTO 70
IF (slope.LT.1E-5) THEN
  DX = -2.5*LOG10(Qb)
ELSE
  DX = LOG10(Qb**LOG10(SLOPE**P))
ENDIF

IF (Dx.LT.5E-2) DX = 0.1
IF (Dx.GT.50) DX = 40.

C Control Grid points of profile and printing on file ENDEC.OUT

70 ARG= X(I)-X(I+1)-IE-5
IF (ARG.LT.DX.AND.ARC.CT.IE-4) DX = X(I)-X(I+1)
IF (SLOPE.GT.1E-5) THEN
ARG = DX*slope-1E-5
TERM1 = Dstil-INT(Dstil)
IF (TERM1.LT.ARC.AND.TERM1.CT.1E-2) DX = TERM1/slope
TERM2 = Dstil+1E-6-INT(Dstil+1E-6)
IF (TERM2.GE.1E-2) WRITE(11,8) Dstil, H, TETAl, E, ETA1, \\
& 1E-3*X(I)
ENDIF

IF (SLOPE.LT.0) THEN
ARG = -DX*slope
TERM = 1-(D-INT(D))
IF (TERM.GT.1E-5) THEN
ARC = DX*slope
TERM = 1-(D-INT(D))
IF (TERM.GT.1E-2) THEN
DX = -TERM1/slope
ETA1 = ETA1+ETA
SLOPE = (X(I)-Y(I+1))/(X(I)-X(I+1))
ENDIF
ENDIF

IF (METAFT.1.EQ.1) WRITE(14,9) X(I),ETA1
WRITE(12,9) Dstil, H
WRITE(11,8) Dstil, H, TETAl, E, ETA1, 1E-3*X(I)
GOTO 50
ENDIF

IF (X(I).LE.X(I+1)) THEN
WRITE(11,8) Dstil, H, TETAl, E, ETA1, 1E-3*X(I)
I = I+1
D = Y(I)+ETA1
SLOPE = (Y(I)-Y(I+1))/(X(I)-X(I+1))
ENDIF

IF (METAFT.1.EQ.1) WRITE(14,9) X(I),ETA1
WRITE(12,9) Dstil, H
GOTO 20

1 FORMAT ( /7X,'D : H : Theta : E. content : ETA : X' \\
& '/7X,';7X,"":",',9(_"'),",',10(_"'),",',13(_"'),",',9(_"'), \\
& ",',10(_"'),",', ')' 
2 FORMAT (8X,'Deep water',F9.2,F9.1,F15.0,F11.3,F11.2) 
4 FORMAT (/45X,'T (period)=',F5.1,' (s)') 
3 FORMAT (45X,'U (current vel.)=',F5.1,' (m/s)'/44X, \\
& 'NU (Current dir.)=',F5.1,' (deg.)') 
5 FORMAT (7X,'D : Water depth \\
& (m)'/8X,'H : rms Wave height (m)'/8X,'Theta : Wave angle to the \\
& (m)'/8X,'H : rms Wave height (m)'/8X,'Theta : Wave angle to the

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This wave is completely damped before attaching the coast.

The opposite direction current is dominated.

END
Program ENDEC2B calculates energy decay of Random Waves taking groups of
wave data from file ENDEC.DAT

```plaintext
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
REAL*8 X(50), Y(50), Xc(50), Yc(50)
REAL*8 X(S0), Y(S0), Xc(S0), Yc(S0)
CHARACTER*ZO ans
COMMON/EQUPAR/Ko, Gama, Slope, Ux, F, Snel
COMMON/PARA/r, Rho, Fw, T
DATA G/9.8l/, PI/3.14/

OPEN(10, FILE='A:\DATAB.DAT')
OPEN(20, FILE='A:\PROFILE.DAT')
OPEN(30, FILE='A:\ENDEC.DAT')
OPEN(11, FILE='ENDEC.OUT')
OPEN(12, FILE='HPLLOT.OUT')

READ(10, *) U, Nu
NU1 = NU - PI/180.
UX = U*COS(NUI)

READ(10, *) Rho, RhoS, r, DSO, D90

C Reading Beach Profile co-ordinates
ITAL = ITAL + 1
READ(20, *, END=100) Xc(ITAL), Yc(ITAL)
ITAL = ITAL + 1
GOTO 200

100 M = 0
READ(30, 41) ans
60 IF (ans.EQ.'DEEP WATeR' OR ans.EQ.'deep water') THEN
READ(30, *, END=50) HO, T, TETAO
M = M + 1
ETA1 = 0.
L0 = G*T**2/(2*PI)
KO = 2*PI/L0
CO = L0/T
CG0 = 0.5*CO
TETA = TETAO*PI/180.
D = INT(L0/2)
SO = HO/L0
DIST1 = D
K = KO
CALL CALK(D, K)
OM = SQRT(K*K*TANH(K*D))
C = OM/K
NOld = 0.5*(2*K*D/SINH(2*K*D)+1)
CG = NOld*C
TETA01 = TETA
SNEL = K0*SIN(TETA01)
TETA = ASIN(SNEL/K)
TETA1 = TETA*180/PI

KR = SQRT(COS(TETA01)/COS(TETA))
KS = SQRT(CG0/CG)
H = KR*K*S0
ELSE
READ(30, *, END=50) D, H, T, TETA1, ETA1
M = M + 1
DIST1 = D
D = D + ETA1
L0 = G*T**2/(2*PI)
KO = 2*PI/L0
CO = L0/T
CG0 = 0.5*CO
TETA = TETA1*PI/180.
```

B.2 ENDEC2B—Main Program
K=K0
CALL CALK(D,K)
OM=SQRT(C*K*TANH(K*D))
C=OM/K
Nold= 0.5*(2*K*D/SINH(2*K*D)+1)
CG= Nold*C
SNEL= K*SIN(TETA)
TETA01= ASIN(SNEL/K0)
TETA0= TETA01*180/PI
HO= H*SQRT(CG/CGO)*SQRT(COS(TETA)/COS(TETA01))
SO= HO/L0
ENDIF
GAMA=.5+0.4*TANH(33*SO)

C Search to determine the start position on the profile

DO 600 I=1,Ital
  X(I)= Xc(I)
  Y(I)= Yc(I)
600 CONTINUE

I=1
IF (Distil.GT.(Y(I)-1E-5)) THEN
  WRITE(*,36) Distil
  FORMAT(  /6X,'& ERROR: Profile co-ordinates are not complete. Start'/6X,'& point of the calculation is outside of the given profile! '/6X,'& Be careful!, you also need to consider the horizontal '/6X,'& part of the sea bottom in your input file, if there is! '/6X,'& The profile co-ordinates have to be extended till depth '/6X,'& more than [',F7.1,'m] for this input wave data.'1116X
  & Please at first improve data file: "PROFILE.DAT")
STOP
ENDIF
SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
IF (SLOPE.NE.0.) GOTO 700
IF (Distil.EQ.Y(I)) THEN
  WRITE(*,37)
  FORMAT(  /6X,'& The start point of the calculations is located on horizon-'/6X,'&tal part of sea bottom, then enter the distance of this point'/6X,'& to the coast')
  READ(*,*) X(I)
ENDIF
ELSE
  I= I+1
  SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
ENDIF
700 IF (SLOPE.GT.1E-6) GOTO 500
300 IF (Distil.GE.Y(I).AND.Distil.LT.Y(I+1)) GOTO 400
  I= I+1
  SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
  IF (SLOPE.EQ.0.AND.Distil.EQ.Y(I)) THEN
    WRITE(*,37)
    READ(*,*) X(I)
  ENDIF
ENDIF
IF (SLOPE.GT.0) GOTO 500
GOTO 300
500 IF (Distil.LE.Y(I).AND.Distil.GT.Y(I+1)) GOTO 400
  I= I+1
  SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
  IF (SLOPE.EQ.0.AND.Distil.EQ.Y(I)) THEN
    WRITE(*,37)
    READ(*,*) X(I)
  ENDIF
ENDIF
IF (SLOPE.LT.1E-6) GOTO 300
GOTO 500
400 X(I)= X(I)-(Y(I)-Distil)/SLOPE

C Length steps selection

Hm= 0.85/K*TANH(GAMA/0.85*K*D)

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CALL CALQB (H, Hm, Qb)
F = 1.5
MSDX = 0
IF (slop.<LT.1E-5) THEN
   DX = 5*F*log10 (Qb)
ELSE
   DX = log10(Qb**log10(slope**f))
ENDIF
IF (DX.LT.5E-2) DX = 0.05
IF (DX.GT.50) DX = 50.

WRITE(11, '(A)') CHAR(12)
WRITE(11, 4) T
WRITE(11, 3) U, NU
WRITE(11, 1)
E0 = Rho*C*H0**2/8
E = Rho*C*H**2/8
WRITE(11, 2) H0, TETA0, E0, ETA1
WRITE(11, 8) Destil, H, TETA1, E, ETA1, 1E-3*X(I)
Hold=H
Ye= H**2*(CG*COS(TETA)+UX)/OK
E= Rho*G*H**2/8
Dold= D
TETold= TETA
20 CALL SOLEQ(Ye,H,K,D,OK,TETA,CG,C,DX,Qb)
IF (Ye.LT.1E-2) THEN
   WRITE(11, 6)
   WRITE(*, 6)
   GOTO 60
ENDIF
DELIK = (CG*COS(TETA)+UX)
IF (DELIK.LT.1E-6) THEN
   WRITE(*, 7)
   WRITE(11, 7)
ENDIF
H = SQRT(Ye*OM/(CG*COS(TETA)+UX))
Nnew = CG/C
Destil = D-ETA1
ETA= (Hold**2*(Hold=1*(COS(TETold)**2)-0.5)/8-H**2*(Nnew**
   (1*(COS(TETA)**2)-.5)/5)/(0.5*(Dold+D))
ETA1= ETA+ETA
D = D+ETA
Hold= H
Nold= Nnew
TETold= TETA
Dold= D
TETA1= TETA*180/PI
X(I) = X(I)-DX
IF (slop.<LT.1E-5) THEN
   DX = -2.5*F*log10 (Qb)
ELSE
   DX = log10(Qb**log10(slope**f))
ENDIF
IF (DX.LT.5E-2) DX = 0.1
IF (DX.GT.50) DX = 50.

C Control Grid points of profile and printing on file ENDEC.OUT

ARG= X(I)-X(I+1)-1E-5
IF (ARG.LT.DX.AND.ARG.CT.1E-4) DX = X(I)-X(I+1)
IF (SLOPE.GT.1E-5) THEN
   ARG = DX*slope-1E-5
   TERM1 = Destil-INT(Destil)
   IF (TERM1.LT.ARG.AND.TERM1.GT.1E-2) DX = TERM1/slope
   TERM2= (Destil+1E-6)-INT(Destil+1E-6)
   IF (TERM2.LT.1E-2) THEN
      WRITE(11, 8) Destil, H, TETA1, E, ETA1, 1E-3*X(I)
Dprt = Dstil
ENDIF

ENDIF

IF (SLOPE.LT.(-1E-5)) THEN
ARG = -DX*slope
TERM = 1-(D-INT(D))
ENDIF

IF (TERM2.LT.1E-2.OR.(TERM2+1E-6-1).GT.0) THEN
WRITE(11,5) Dstil, H, TETAI, E, ETA1, 1E-3*X(I)
Dprt = Dstil
ENDIF

IF (X(I).LE.1E-1.OR.Dstil.LE.1E-2) THEN
WRITE(11,5)
GOTO 60
ENDIF

IF (X(I).LE.X(I+1)) THEN
IF (ABS(Dprt-Dstil).GT.1E-3) WRITE(11,8) Dstil, H, TETAI, E,
& ETA1, 1E-3*X(I)
IF (SLOPE.EQ.0.) WRITE(11,8) Dstil, H, TETAI, E, ETA1, 1E-3*X(I)
I = I+1
D = Y(I)+ETA1
SLOPE = (Y(I)-Y(I+1))/(X(I)-X(I+1))
ENDIF

WRITE(12,9) M, X(I), H
GOTO 20
50 WRITE(*,*) ' Normal end of the program'

1 FORMAT (/7X,' D : H : Theta : E. content : ETA : X', & '/7X,':',10('_'),':',9('_'),':',10('_'),':', 13('_'),':'&',9('_'),':',10('_'),':'),10('_'),':',9('_'),':',10('_'),':'
2 FORMAT (5X,'Deep water',F9.2,F9.1,F15.0,F11.3,F11.2)
3 FORMAT (45X,'U (current vel.)=',F5.1,' (m/s)',/44X, & 'NU (Current dir.)=',F5.1,' (deg.)'
4 FORMAT (7X,':',10('_'),':',9('_'),':',10('_'),':', 13('_'),':',9('_'),':',10('_'),':',10('_'),':'),10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9['_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),':',10('_'),':',9('_'),':',10('_'),' The opposite direction current is dominated')
8 FORMAT(5X,F5.1,F11.2,F9.1,F15.0,F11.3,F10.2)
9 FORMAT (13,F20.3,F20.8)
41 FORMAT(A)
STOP
END
SUBROUTINE SOLEQ(Ye,H,K,OM,TETA,CG,C,DX,QB)
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
CHARACTER*20 A NS
COMMON/EQUPAR/ K0,GAMA,SLOPE,UX,F,SNEL
DATA C/9.81/,PI/3.14/
CALL DBDF(K,D,H,OM,EB,DF,QB)
50 KIEQ= -DX*(DB+DF)
CEQ= Ye+KIEQ/2
IF (CEQ.LT.0.) THEN
   DX= 2*DX/3
   F= 0.9*F
   GOTO 50
ENDIF
D=D-2*SLOPE
CALL CALK(D,K)
TETA=ASIN(SNEL/K)
OM=(G*K*Tanh(K*D))**0.5
C=OM/K
CG=.5*(1+2*K*D/Sinh(2*K*D))*C
DELIM= (CG*COS(TETA)+UX)
IF (DELIM.LT.1E-6) THEN
   WRITE(*,7)
   WRITE(11,7)
   GOTO 40
ENDIF
H=SQRT((Ye+KIEQ/2)*OM/(CG*COS(TETA)+UX))
CALL DBDF(K,D,H,OM,EB,DF,QB)
K2EQ= -DX*(DB+DF)
CEQ= Ye+K2EQ/2
IF (CEQ.LT.0.) THEN
   D= D+DX/2*SLOPE
   DX= 2*DX/3
   F= 0.9*F
   GOTO 50
ENDIF
DELIM= (CG*COS(TETA)+UX)
IF (DELIM.LT.1E-6) THEN
   WRITE(*,7)
   WRITE(11,7)
   GOTO 40
ENDIF
H=SQRT((Ye+K2EQ/2)*OM/(CG*COS(TETA)+UX))
CALL DBDF(K,D,H,OM,EB,DF,QB)
K3EQ= -DX*(DB+DF)
CEQ= Ye+K3EQ
IF (CEQ.LT.0.) THEN
   D= D+DX/2*SLOPE
   DX= 2*DX/3
   F= 0.9*F
   WRITE(*,*)' Some delay may occur on this specific input data'
   GOTO 50
ENDIF
D= D+DX/2*SLOPE
CALL CALK(D,K)
TETA=ASIN(SNEL/K)
OM=(G*K*Tanh(K*D))**0.5
C=OM/K
CG=.5*(1+2*K*D/Sinh(2*K*D))*C
DELIM= (CG*COS(TETA)+UX)
IF (DELIM.LT.1E-6) THEN
   WRITE(*,7)
   WRITE(11,7)
   GOTO 40
ENDIF
H=SQRT((Ye+K3EQ)*OM/(CG*COS(TETA)+UX))
CALL DBDF(K,D,H,OM,EB,DF,QB)
K4EQ=-DX*(DB+DF)
Ye=Ye+(K1EQ+2*K2EQ+2*K3EQ+K4EQ)/6
7 FORMAT(//10X,'The opposite direction current is dominated') RETURN

* SUBROUTINE DBDF(K,D,H,OH,DB,DF,QB)
* IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
* CHARACTER*20 ANS
COMMON/EQUIPAR/ K0,GAMA,SLOPE,UX,F,SNEL
COMMON/ PARA/ r,Rho,fw,T
DATA G/9.81/,PI/3.14/

ALPHA= 1.
HM= 0.55/K*TANH(GAMA/0.55*K)=D)
CALL CALQB(H,HM,QB)
DB= ALPHA*QB*HM**2/PI
B= SINH(K*D)**3*G*PI**.5
CALL WAVPAR(H,D,k)
DF= Fw*H**3*OM**2/B
RETURN
END

* SUBROUTINE CALK(D,K)
* IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
* INTEGER M
* CHARACTER*20 ANS
COMMON/EQUIPAR/ K0,GAMA,SLOPE,UX,F,SNEL
DATA G/9.81/,PI/3.14/

M=0
P1=TANH(K*D)
P2=K*D/(COSH(K*O)**2
K1=(K*K2+K0)/(P1+P2)
IF(ABS(K1-K)*LE.1E-10.OR.M.GT.50) GOTO 20
K=K1
M=M+1
GOTO 10
20 K=K1
RETURN
END

* SUBROUTINE CALQB(H,HI1,QB)
* IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
* INTEGER M
* CHARACTER*20 ANS
COMMON/EQUIPAR/ K0,GAMA,SLOPE,UX,F,SNEL
DATA G/9.81/,PI/3.14/

QB=0.
QB1=QB-(QB-EXP((QB-1.)/B**2))/((1-EXP((QB-1.)/B**2))/B**2)
IF(ABS(QB-QB1)*LE.1E-6.0) M.GT.20 GOTO 20
QB=QB1
M=M+1
GOTO 10
20 QB=QB1
RETURN
END

* SUBROUTINE WAVPAR(H,D,k)
* IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
* REAL*8 X(50), Y(50)
* CHARACTER*20 ANS
COMMON/EQUIPAR/ K0,GAMA,SLOPE,UX,F,Snel
COMMON/ PARA/ r,Rho,fw,T
DATA G/9.81/,PI/3.14/

a0= H/SINH(k*D)/2.
fw= EXP(-5.977+5.213*(a0/r)**(-0.194))
fw= MIN(fw,0.3)
RETURN
END
In the name of GOD

Computer program SEDTRN. This version calculates sediment transport rate
by three formula: CERC, Bijker, and van Rijn

**IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)**

LOGICAL SEND

REAL*8 X(50), Y(50), Xc(50), Yc(50)
CHARACTER*20 ans, ansR

COMMON/EQUPAR/ K0, GAMMA, SLOPE, UX, P, Snel
COMMON/SEDPAR/ UbarO, aO, Ksi, Chezy, D50, D90, RhoS, W
COMMON/VELPAR/ r, Rho, Fw, Tp
COMMON/RIJNP/ rw, a, DELs, RNU, THETcr, Dstar

DATA G/9.81/, F1/3.14/, KAPA/0.4/

OPEN(10,FILE='A:\DATAB.DAT')
READ(10,*) U, NU
READ(10,*) Rho, RhoS, r, D50, D90
READ(10,*) RNU
CLOSE(10)

OPEN(20,FILE='A:\PROFILE.DAT')
OPEN(30,FILE='A:\ENDEC.DAT')
OPEN(40,FILE='A:\RIJIN.DAT')
OPEN(11,FILE='ENDEC.OUT', STATUS='UNKNOWN')
OPEN(12,FILE='SEDALL.OUT', STATUS='UNKNOWN')

WRITE(*, '(/8X,'''& The sediment transport rate could be calculated by three''/8X,'''& formula, CERC, Bijker, Van Rijn.'''/8X,'''& Please enter name of the formula that you wish to use or''/8X,'''& ''ALL'' for applying all the formulae.'''))
Hform=0
READ(*,41) ans
IF (ans.EQ. 'Bijker'.OR. ans.EQ. Bijker) Hform=1
IF (ans.EQ. 'CERC'.OR. ans.EQ. 'cerc') Hform=2
IF (Hform.EQ.0) Hform=3
IF (ans.EQ. 'ALL'.OR. ans.EQ. 'all') Hform=4

Data processing for v. Rijn formula

READ(40,41) ansR
READ(40,*) rw, a, DELs, W
IF (rw.LT.1E-6) THEN
IF (ansR.EQ. 'ripple'.OR. ansR.EQ. 'Ripple') rw= r
ENDIF
IF (a.LT.1E-6) THEN
IF (ansR.EQ. 'ripple'.OR. ansR.EQ. 'Ripple') a= r/6.
ENDIF
IF (DELs.LT.1E-6) THEN
IF (ansR.EQ. 'ripple'.OR. ansR.EQ. 'Ripple') DELs= r
ENDIF

IF (W.LT.1E-6) THEN
W= 1./10**((0.4949*LOG10(D50)**2+2.4113*LOG10(D50)+3.7394)
ENDIF

Critical shear parameters

Dstar= D50*((Rhos-Rho)/Rho*G/RNU)**2*(1./3.)
IF (Dstar.LE.4.) THETcr=.14*Dstar**(-.64)
IF (10.. LT.Dstar.AND.Dstar.LE.10.) THETcr=.04*Dstar**(-.1)
IF (20.. LT.Dstar.AND.Dstar.LE.150.) THETcr=.013*Dstar**(.29)
IF (Dstar.GT.150.) THETcr=.055
TACr= (Rhos-Rho)*G*D50*THETcr

Reading Beach Profile co-ordinates

131
ITAL = 1

200 READ(20,*,END=100) XC(ITAL),YC(ITAL)
ITAL = ITAL + 1
GOTO 200

100 M = 0
READ(30,41) ANS

60 IF (ANS.EQ.'DEEP WATER'.OR.ANS.EQ.'deep water') THEN
READ(30,*,END=50) HO, T, TETA0
M = M + 1
ETAI = 0.
LO = G*T**2/(2*PI)
KO = 2*PI/LO
CO = LO/T
CG0 = 0.5*CO
TETA = TETA0*PI/180.
D0 = INT(LO/2)
SO = HO/LO
DSTI = D
K*K0
CALL CALK(D,K)
CM = SQRT(G*K*TANH(K*D))
C = CM/K
N0LD = 0.5*(2*K*D/SINH(2*K*D)+1)
CG = N0LD*C
TETA01 = TETA
SNEL = KO*SIN(TETA01)
TETA = ASIN(SNEL/K)
TETA0 = TETA0*PI/180

KR = SQRT(COS(TETA01)/COS(TETA))
KS = SQRT(CG0/CG)
H = KR*KS*HO
ELSE
READ(30,*,END=50) D, H, T, TETA1, ETA1
M = M + 1
DSTI = D
D0 = D + ETA1
LO = G*T**2/(2*PI)
KO = 2*PI/LO
CO = LO/T
CG0 = 0.5*CO
TETA = TETA1*PI/180.
K*K0
CALL CALK(D,K)
CM = SQRT(G*K*TANH(K*D))
C = CM/K
N0LD = 0.5*(2*K*D/SINH(2*K*D)+1)
CG = N0LD*C
SNEL = K*SIN(TETA)
TETA01 = ASIN(SNEL/K0)
TETA0 = TETA01*180/PI

HO = M*SQRT(CG/CG0)*SQRT(COS(TETA)/COS(TETA01))
SO = HO/LO
ENDIF
NUI = NU*PI/180.
UX = U*COS(NUI)
GAMA = .5 + 0.4*TANH(33*SO)
TP = T

C Search to determine the start position on the profile

DO 600 I = 1, ITAL - 1
X(I) = XC(I)
Y(I) = YC(I)
600 CONTINUE

* I = 1
IF (DSTI.GT.(Y(I)-1E-5)) THEN
WRITE(*,36)
36 FORMAT((/6X,'
ERROR: Profile co-ordinates are not complete. Start '/6X,'
point of the calculation is outside of the given profile! '/6X,'
Be careful, you also need to consider the horizontal '/6X,'
part of the sea bottom in your input file, if you have! '/6X,'
Please at first improve data file: "PROFILE.DAT"
STOP

ENDIF
SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
IF (SLOPE.EQ.0.AND.Dstil.EQ.Y(I)) THEN
    WRITE('#X') ' The start point of calculation is located on',
    ' horizontal part of sea bottom, then enter the ',
    ' distance of this point to the coast'
    READ(*,*), X(I)
ENDIF

STOP

endip
SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
IF (SLOPE.GT.0.0) THEN
    DX= S.P.LOG10 (Qb)
ELSE
    DX= LOC10(QB**LOG10(SLOPE**r))
ENDIF
IF (DX.LT.SEZ) DX= 0.05
IF (DX.CT.SO) DX= 50.

C Length steps selection

WRITE(11,'(A)') CHAR(12)
WRITE(11,('(/.5X,' 'Wave data, group:',',I4)')) M
WRITE(11,4) T
WRITE(11,3)U,NU
WRITE(11,1)
EO= Rho*C*HO**Z/B
E= Rho·C·H**Z/B
WRITE(11,8)Dstil, H, TETAl, E, ETAI

Hold= H
Ye= H·*Z*(CC*COS(TETA)+UX)/OM
Dold= D
TETold= TETA
V=1E-5
STR= 0.
STB= 0.
MScerc= 0

CALL SOLEQ(Ye,H,D,OM,TETA,CC,C,DX,Qb)
IF (Ye.LT.1E-2) THEN
    IF (D.LT.1.0) GOTO 70
    WRITE(11,6)
    WRITE(*,6)
ENDIF
deli= Cg*COS(TETA)+UX
IF (deli.LT.1E-6) THEN
    IF (D.LT.1E-6) THEN
        IF (D.LT.1.0) GOTO 70

133
WRITE(*,7)
WRITE(11,7)
GOTO 70
ENDF
H=SQRT(Ye*OM/(CG*COS(TETA)*UX))
E=RhO*G*H**2/8
Nnew= CG/C
Dstil= D-ETA1
ETAI= (Hold**2*(Hold**((1+(COS(TETold))**2)-0.5))/8-H**2*(Nnew**
  (1+(COS(TETA))**2)-.5))/(0.5*(Dold+D))
ETAI= ETA1+ETA
D= D+ETA
RSS= Rho*G*(HOLD**2*Nold*(1+(COS(TETold)**2)-.5)/8-H**2*Nnew*
  SIN(2*TETA))/(DX*16)
& CALL VELCAL(D,RSS,V)
IF (V.GT.1E-2.AND.a0.GT.5E-1) THEN
  IF (Mform.EQ.3.OR.Mform.EQ.4) CALL RIJN(D,H,V,TETA,St)
  StR= Str+St*DX
  IF (Mform.EQ.1.OR.Mform.EQ.4) CALL SEDBR(D,V,St)
  StB= StB+St*DX
ENDIF
IF (Mform.EQ.2.AND.Mform.EQ.4) GOTO 121
IF (Qb.GT.1E-3.AND.HScerc.EQ.0) THEN
  MScerc= 1
  Scerc= 0.08*H**2*Nnew*C*SIN(TETA)*COS(TETA)
ENDIF
121
Hold= H
Nold= Nnew
TETold= TETA
Dold= D
TETA1= TETA*180/PI

X(I)= X(I)-DX
IF (slope.LT.1E-5) THEN
  DX= -2.5*F=LOG10 (Qb)
ELSE
  DX= F=LOG10(QB**LOG10(SLOPE))
ENDIF
IF (DX.LT.5E-2) DX= 0.1
IF (DX.GT.50) DX= 50.

C Control Grid points of profile and printing on file ENDEC.OUT

IF (X(I).LE.1E-1.OR.Dstil.LE.1E-2) GOTO 70
IF (X(I).LE.X(I+1)) THEN
  I= I+1
  D= Y(I)+ETAI
  SLOPE= (Y(I)-Y(I+1))/(X(I)-X(I+1))
ENDIF

ARG= DX*slope-1E-5
TERM1= Destil-INT(Dstil)
IF (TERM1.LT.ARG) WRITE(11,8) Dstil, H, ETA1.
& E, ETA1, 1E-3*X(I)
GOTO 20
70
WRITE(11,5)
WRITE(12,(16/'5X,'Wave data, group:',14)') H
WRITE(12,11) H0, T, TETA0
IF (Mform.EQ.4) WRITE(12,12) Scerc, StB, StR
IF (Mform.EQ.1) WRITE(12,14) StB
IF (Mform.EQ.2) WRITE(12,15) Scerc
IF (Mform.EQ.3) WRITE(12,16) StR
GOTO 60
50
WRITE(*,*) ' Normal end of the program'

1
FORMAT (/7X,'H : Theta : E. content : ETA : X
& ;/7X,'; 10(_:'),''; 9(_:'),''; 10(_:'),''; 13(_:'),''; 9(_:'),''
SUBROUTINE SOLEQ(Ye,H,K,D,OM,TETA,CG,C,DX,QB)

IMPLICIT REAL*8(A-H), REAL*8(O-Z), REAL*8(K-L), REAL*8(N)
CHARACTER*20 ANS

COMMON/EQUPAR/KO,GAMA,SLOPE,UX,F,SNEL
COMMON/VELPAR/r,Rho,Fw,T

DATA G/9.81/,PI/3.14/
* CALL DBDF(K,D,H,OH,DB,DP,QB)

50 KIEQ= -DX*(DB+DP)
CEQ= Ye+KnQ/2
IF (CEQ,LT.O.)THEN
DX= Z*DX/3
F= 0.9*F
GOTO 50
ENDIF
D=D-DX/Z*SLOPE
CALL CALK(D,K)
TETA=ASIN(Snel/K)
OM=(G*K*TANH(K*D)**0.5
C=OM/K
CG=.5*(1+Z*K*D/SINH(Z*K*D))*C
deli.= CG*COS(TETA)+UX
IF (deli••LT.lE-6)THEN
1P (D.LT.1.0)GOTO 40
WRIT!(*,7)
WRITE(11,7)
GOTO 40
ENDIF
H=SQRT((Ye*KIEQ/2)=OM/(CG= COS(TETA)+UX))
CALL DBDF(K,D,H,OM,OH,DP,QB)
K2EQ=-DX*(DB+DP)
CEQ=Ye+K2EQ/2
IF (CEQ,LT.0.) THEN
D= D+DX/2*SLOPE
DX= 2*DX/3
F= 0.9*F
GOTO 50
ENDIF

STOP
END
IF (delim.LT.1E-6) THEN
    IF (D.LT.1.0) GOTO 40
    WRITE(*,7)
    WRITE(*,11) (7)
GOTO 40
ENDIF
H=SQRT((Ye+KZEQ/Z)*OM/(CG*COS(TETA)+UX))
CALL DBDF(K,D,H,OM,DB,DF,QB)
K3EQ=-DX*(DB+DF)
CEQ=Ye+K3EQ
IF (CEQ.LT.0.) THEN
    D=D+DX/Z*SLOPE
    DX=2*DX/3
    P=0.9*P
    WRITE(*,*) 'Some delay may occur on this specific input data'
    GOTO 50
ENDIF
D=D-DX/2*SLOPE
CALL CALI(D,K)
TETA=ASIN(SNEL/K)
OM=(G*K*TANH(K*D))**0.5
C=OM/K
CG=.5*(1+2*K=D/SINH(2*K*D))*C
deli.= cg*COS(TETA)+UX
IF (delim.LT.1E-6) THEN
    IF (D.LT.1.0) GOTO 40
    WRITE(*,7)
    WRITE(*,11) (7)
GOTO 40
ENDIF
H=SQRT((Ye+K3EQ)*OM/(CG*COS(TETA)+UX))
CALL DBDF(K,D,H,OM,DB,DF,QB)
K4EQ=-DX*(DB+DF)
Ye=Ye+(KIEQ+2*K2EQ+2*K3EQ+K4EQ)/6
RETURN
END
SUBROUTINE DBDF(K,D,H,OM,DB,DF,QB)
    IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
    CHARACTER*20 ANS
    COMMON/EQUIPAR/K0,GAMA,SLOPE,UX,P,SNEL
    DATA G/9.81/, PI/3.14/
    ALPHA= 1.
    HM= 0.88/K*TANH(GAMMA/0.68*K*D)
    CALL CALQB(H,HM,QB)
    DB= ALPHA*QB*HM**2/PI
    B2= SINH(K*D)**3*G*PI**0.5
    CALL WAVPAR(H,D,K)
    DF= Fw*H**3*OM**2/B
    RETURN
END
SUBROUTINE CALI(D,K)
    IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
    INTEGER M
    CHARACTER*20 ANS
    COMMON/EQUIPAR/K0,GAMA,SLOPE,UX,P,SNEL
    DATA G/9.81/, PI/3.14/
    M=0
10 P1=TANH(K*D)
P2=K*D/(COSH(K*D))**2
K1=(K*P2+K0)/(P1+P2)
IF(ABS(K1-K).LE.1.0E-10.OR.M.GT.50) GOTO 20
K=K1
M= M+1
GOTO 10
20 K=K1
RETURN
SUBROUTINE CALQB(H, HM, QB)
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
INTEGER M
B = H / HM
M = 0
QB = 0.
10 QB1 = QB - (QB - EXP((QB - 1.) / B**2)) / (1 - (EXP((QB - 1.) / B**2) / B**2))
IF (ABS(QB - QB1).LE.1E-6 OR M.GT.20) GOTO 20
QB = QB1
M = M + 1
GOTO 10
20 QB = QB1
RETURN
END

SUBROUTINE WAVPAR(H, D, k)
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
REAL*8 X(SO), Y(SO)
CHARACTER*20 ANS
COHMON/EQUPAR/KO, GAMA, SLOPE, UX, F, Snel
COMMON/EQUPAR/ Ubar0, a0, Ksi, Chezy, D50, D90, RhoS, W
COMMON/VELPAR/ r, Rhino, Fw, T
DATA G/9.81/, PI/3.14/

* Hs = H * SQRT(2.)
Ubar0 = PI * Hs / T / SINH(K*D)
a0 = Ubar0 * T / (2*PI)
Fw = EXP(-5.9775 + 5.213*(a0/r)**(-0.194))
IF ((a0/r).LT.1.59) Fw = 0.3
RETURN
END

C The next Subroutine calculates Longshore Current Velocity by Bijker approach

SUBROUTINE VELCAL(D, RSS, V)
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
INTEGER M
COMMON/EQUPAR/ K0, GAMA, SLOPE, UX, F, Snel
COMMON/SEDPAR/ Ubar0, a0, Ksi, Chezy, D50, D90, RhoS, W
COMMON/VELPAR/ r, Rhino, Fw, T
DATA G/9.81/, PI/3.14/

Chezy = 18.*LOG10(12*D/r)
Ksi = Chezy*SQRT(Fw/(2*C))
PAR = 0.45*(Ksi/Ubar0)**1.13
M = 0
10 V1 = (.75*V**2-0.13*PAR*V**0.87*RSS*Chezy**2/(Rho*G)) /
& (1.5*V+PAR*0.87*V**(-0.13))
IF (V1.LT.0.) THEN
V = 1.0E-5
GOTO 30
ENDIF
IF (ABS(V-V1).LE.1.0E-5 OR M.GT.30) GOTO 20
V = V1
GOTO 10
20 V = V1
30 RETURN
END

C The following Subroutine calculates Sediment Transport Rate by Van Rijn formula when it is called

SUBROUTINE RIJN(D, H, V, TETA, St)
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
LOGICAL SREND
CHARACTER*20 ans, ansR
COMMON/EQUPAR/ K0, GAMA, SLOPE, UX, F, Snel
COMMON/SEDPAR/ Ubar0, a0, Ksi, Chezy, D50, D90, RhoS, W
COMMON/VELPAR/ r,Rho,Fw,Tp
COMMON/RJNHP/ rw,a,DELS,RHU,THETcr,Dstar
DATA C/9.81/,PI/3.14/,KAPA/0.4/

* IF (Qb.LT.1E-3) THEN
  IF (rw.LT.1E-6) rw=r
  DELw= 0.072*a0*(a0/rw)**(-0.25)
  IF (a.LT.1E-6) a= r/6
  IF (DELS.LT.1E-6) DELs= r
ELSE
  DELw= 0.02995*a0
  rw= DELw
  IF (a.LT.1E-6) a= DELw
  IF (DELS.LT.1E-6) DELs= 3.*DELw
ENDIF

MM= 96

C Computation of reference concentration Ca

FC= 0.24*LOG10(12.*D/r)**(-2)
Fcl= 0.24*LOG10(12.*D/3./D90)**(-2)
RMUC= Fcl/FC
RMUw= 0.6/Dstar
Vstar= C=0.5*ABS(V)/Chezy
HELP1= Ubar0/V
RGAMMA= 0.
IF (HELP1.GE.2.5) HELP1= 2.5
NUI= 90.*PI/180.
PHI= NUI-ABS(TETA)
PH1= PHI**180./PI
IF (0.LE.PHI.LT.PHI**180.)THEN
  RGAMMA=.75
  IF(PHI.GT.90.) RGAMMA= RGAMMA+(PHI-90.)/90.*.35
ENDIF

ra= EXP(RGAMMA*HELP1)*r
Fa= 0.24*LOG10(12.*D/ra)**(-2)
IF(rw.LE.r/30.)THEN
  ALPHA= 1.
ELSE
  ALPHA= (ALOG(90.*DELw/ra)/LOG(90.*DELw/r))**2
ENDIF

* TAUC= 0.125*Rho*Fa**2
TAUw= 0.25*Rho*Fw*ubar0**2
TAUC1= RMUC*ALFAw*TAUc
TAUw1= RMUw*TAUw
TAUc1= TAUC1+TAUw1
THET1= TAUc1/(Rhos-Rho)/G/D50
T= (THET1-THETcr)/THETcr
T= MAX(.0001,T)
CA= 0.015*D50/a*Dstar**(-.3)*T**1.5

C Wave-related sediment transport computation

ALPHA= 0.3
TAUlim= (TAUc1+2*TAUw1+2*TAUc1*TAUw1*COS(Phi))**0.5
THETmx= TAUlim/(Rhos-Rho)/G/D50
Tmmax= (THETmx-THETcr)/THETcr
Tmmax= MAX(.0001,Tmmax)
Camax= 0.015*D50/a*Dstar**(-.3)*Tmax**1.5
TAVlim= (TAUc1+2*TAUw1+2*TAUc1*TAUw1*COS(Phi))**0.5
THEtmm= TAVlim/(Rhos-Rho)/G/D50
Tmin= (THEtmm-THETcr)/THETcr
Tmin= MAX(.0001,Tmin)
Camin= 0.015*D50/a*Dstar**(-.3)*Tmin**1.5
Sw= ALPHA*Ubar0*DELw*ABS(Camax+Camin-2*Ca)

C Numerical Integration for Velocity profile

DYM = CA/MM
DXM = D/MM
DYX = DYM/DXM

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SREND = .FALSE.
HELPZ= -1.*ALOG(30.*D/ra)
IF (DELw.GT.0.) THEN
  Vdel= V*LOG(90.*DELw/ra)/HELPZ
ENDIF
*
C Computation of Concentration Profile
*
  HS= H*SQR(2.)
  ALFAbr= MAX(3.*(HS/D)-.5,1.)
  EPSbw= .004*START*ALFAbr*DELs*Ubar0
IF (Tp.GT.1.E-4) THEN
  EPMAXw= 0.035*ALFAbr*D*HS/TP
ELSE
  EPMAXw= 0.
ENDIF
IF (EPMAXw.LE.EPSbw) EPMAXw= EPSbw
BETA= 1+2*(W/Vstar)**2
BETA= MIN(BETA,2.)
EPMAXc= 0.25*KAPA*Vstar*D*BETA
C=Ca
Z=a
IF (Z.LE.DELs) EPSw= EPSbw
IF (Z.GT.DELs.AND.Z.LE.0.5*D) EPSw= EPSbw+(EPMAXw-EPSbw)*((Z-DELs)/(0.5*D-Dels))
IF (Z.GE.0.5*D) EPSw= EPMAXw
IF (Z.GE.0.5*D) EPSc= EPMAXc
IF (Z.LT.0.5*D) EPSc= EPMAXc-EPMAXc*(1.-2.*Z/D)**2
ES= (EPSw**2.+EPSc**2.)*.05
Y2= 0
IF (C.GT.1.E-8) THEM
  Y2= -W/ES*C*(1.-C)**5.
ENDIF
YPRIME= Y2
FF= 1./Ca*YPRIME
IF (DELw.GT.0.) THEN
  Vz= Vdel*LOG(30.*Z/r)/LOG(90.*DELw/ra)
ENDIF
IF (a.GE.3*DELw) Vz= V*LOG(30.*Z/r)/HELPZ
*
C Further Integration
*
  Y= CA
  TERM1= Vz*Y
  Xnew= A
  SS= 0.
  100 CONTINUE
  Xold= Xnew
  Yold= Y
  IF (-YPRIME.GT.DY) THEN
    Y= Yold-DY
    IF (Y.LT.2./3.*Yold) Y= 2./3.*Yold
    Xnew= Xold+ALOG(Y/Yold)/FF
    ELSE
    Xnew= Xold+DM
    IF (Xnew.GE.0.) THEN
      Xnew= D
      SREND= .TRUE.
    ENDFI
  Y= EXP( LOG(Yold)+(Xnew-Xold)*FF)
ENDIF
  C= Y
  Z= Xnew
  IF (Z.LE.DELs) EPSw= EPSbw
  IF (Z.GT.DELs.AND.Z.LE.0.5*D) EPSw= EPSbw+(EPMAXw-EPSbw)*((Z-DELs)/(0.5*D-Dels))
  IF (Z.GE.0.5*D) EPSw= EPMAXw
  IF (Z.GE.0.5*D) EPSc= EPMAXc
  IF (Z.LT.0.5*D) EPSc= EPMAXc-EPMAXc*(1.-2.*Z/D)**2
  ES= (EPSw**2.+EPSc**2.)*.05

139
Y2 = 0.
IF (C.GT.1.E-8) THEN
  IF (Z.GE.a) Y2 = -W/ES*C*(1.-C)**5.
ENDIF
YPRIHE = Y2
Fr = l./Y*YPRIHE
Vz = Vdel*LOG(30.*Xnew/r)/LOG(90.*DELw/r)
ENDIF
IF (Xnew.GE.3*DELw) Vz = Vdel*LOG(30.*Xnew/ra)/HELP2
TERM2 = Vz*Y
SS = SS+(Xnew-Xold)*(TERM1+TERM2)/2.
TERM1 = TERM2
IF (.NOT. SREND) GOTO 100
*
Vstar1 = SQRT(TAUcl/Rho)
Sb = 0.25*D50*Vstar1*1.5/Dstar**0.3
Sc = Sc+Ss
St = SQRT(Sc**2+Sw**2+2*ABS(Sc*Sw)*COS(PHI))
RETURN
END
*
SUBROUTINE SEDBR(D,V,St)
*
C This Subroutine calculates Sediment Transport Rates by Bijker formula
*
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z), REAL*8 (K-L), REAL*8 (N)
COMMON/EQUPAR/K0,GAMA,SLOPE,UX,F,Snel
COMMON/SEDPAR/Ubar0,a0,Ksi,Chezy,D50,D90,Rho,S,W
COMMON/VELPAR/r,Rho,Fw,T
COMMON/QPAR/ A
DATA G/9.81/,PI/3.14/.,KAPA/0.4/.
*
C90= 18*LOGIO(12*D/D90)
RMU= (Chezy/C90)**1.5
TAOc= G*Rho*V**2/Chezy**2
TAOcw= TAOc*(1.+0.5*(Ksi*Ubar0/V)**2)
Vstar= SQRT(TAOcw/Rho)
Zstar= W/(Kapa*Vstar)
A = r/D
CALL QCALC(Zstar,Q)
Delta = (RhoS-Rho)/Rho
Sb = 5*D50*V*SQRT(G)*EXP(-0.27*Delta*D50*Rho*G/
& (RMU*TAOcw))/Chezy
Ss=1.83*Q*Sb
St = Sb+Ss
RETURN
END
*
C The following Subroutine calculates the Einstein Integrals and Ensteins
C factor Q,
*
SUBROUTINE QCALC(Zstar,Q)
REAL*8 Zstar, Q, A
COMMON/QPAR/ A
REAL Iprim1,Iprim2
R= 0.216*A***(Zstar-1)/(1.-A)**Zstar
IF(Zstar.EQ.1.) THEN
  Iprim1= -LOG(A)*A-1
  Iprim2= -0.5*LOG(A)**2+LOG(A)*A-1.
  GOTO 10
ELSEIF (Zstar.EQ.2.) THEN
  Iprim1= 2*LOG(A)-A+1/A
  Iprim2= LOG(A)**2+LOG(A)*(-A+1./A)+A+1./A-2
  GOTO 10
ELSEIF (Zstar.EQ.3.) THEN
  Iprim1= -3*LOG(A)+A-3./A+0.5/A**2+1.5
  Iprim2= -1.5*LOG(A)**2+LOG(A)*(-A-3./A+0.5/A**2)
& -A-3.0/A+0.25/A**2+3.75
  GOTO 10
ELSE
  Iprim1= 1./(1.-Zstar)*(1.-A***(1.-Zstar))-Zstar/(2.-Zstar)=

140
& (1.-A**(2.-Zstar))-Zstar*(1.-Zstar)/2.*(3.-Zstar)*
& (1.-A**(3.-Zstar))

& Ipri2: 1./(1.-Zstar)*(-1./(1.-Zstar)-A**(1.-Zstar)*
& (LOG(A)-1./(1.-Zstar)))
& -Zstar/(2.-Zstar)*(-1./(2.-Zstar)-A**(2.-Zstar)*
& (LOG(A)-1./(2-Zstar)))
& -Zstar*(1.-Zstar)/2.*(3.-Zstar)*(-1./(3.-Zstar)-
& A**((3.-Zstar)*(LOG(A)-1./(3-Zstar)))

ENDIF

10 IF(A.LT.1.) THEN
  Q= R*(Ipri1*LOG(33./A)+Ipri2)
ELSE
  Q=0.
ENDIF
IF (Zstar.GT.1.5.AND.Zstar.LT.3.0.AND.A.GT.0.2) THEN
  IF (Q.GT.0.5) Q= 0.52*(1-A)
ELSEIF (Zstar.GE.3.0.AND.Zstar.LT.4.0.AND.A.GT.0.2) THEN
  IF (Q.GT.0.4) Q= 0.40*(1-A)
ELSEIF (Zstar.GE.4.0.AND.Zstar.LT.5.0.AND.A.GT.0.1) THEN
  IF (Q.GT.0.24) Q= 0.262*(1-A)
ELSEIF (Zstar.GE.5.0.AND.A.GT.0.1) THEN
  IF (Q.GT.0.1) Q= 0.2*(1-A)
ENDIF
RETURN
END
A computer program for determining coastal evolution process near breakwaters and/or any obstacle impermeable to sand.

REAL Y(400), Y1(400), XA, DX, T(30)
INTEGER M, N, X, L
DATA PI/3.14/
OPEN(10, FILE='MOR.OUT')
OPEN(11, FILE='MORPLOT.OUT')
WRITE(*, 9)
9 FORMAT(/ 6X, 'This program computes process of accretion against a breakwater. Program is sufficiently flexible and accepts any kind of data and produced results is acceptable. If the last asked year is greater than 30 values in each execution, then Delta X should be reduced. Now input corresponding data after questions./')
WRITE(*,*)' d (depth over which accretion takes place)=?'
READ(*,*) D
WRITE(*,*)' Phi (angle of wave attack in near shore)=?'
READ(*,*) Phi
WRITE(*,*)' K (number of years after which coastal changes are needed)=?'
READ(*,*) K
WRITE(*,*)' T (years after which coastal changes are asked)=?'
WRITE(*,*)' M = number of thea is equal to K !'
READ(*,*) T(I), I=1,K
WRITE(*,*)' Delta X(length step, either a desirable value relative to the accuracy or 0)=?'
READ(*,*) DX
WRITE(*,*)' Ymax or L (maximum effective breakwater length, against sand by passing)=?'
READ(*,*) YMAX
WRITE(*,*)' Sediment transport rate (=3/year)=?'
READ(*,*) S

FI2=Phi*PI/180.
A=S/FI2*D
IF(DX.EQ.0.) DX=500.
R=1.67
DT=DX**2/A
CALL SORT(T, K)
N=T(K)/DT
X=5*(A*T(K))**0.5/500
XA=X*500.
M=XA/DX+1
DO 10 J=1,M
10 Y(J)=0.
WRITE(11,4)
4 L=1
DO 20 I=1,N
MC=0
CALL TRANS(Y, Y1, M)
20 DO 30 J=2, M-1
30 Y(J)=R*Y1(J-1)+(1-2*R)*Y1(J)+R*Y1(J+1)
Y(M)=DX*FI2*Y(M-1)
IF(Y(M).GE.YMAX) THEN
Y(M)=YMAX
ENDIF

The following part of the program controls accreted length along.

C
C the coast for more accuracy
*
IF (Y(2).GT.0.5) THEN
MC=MC+1
IF(MC.GT.10) GOTO 99
DO 40 IJ=M,1,-1
40 Y1(IJ+1)=Y1(IJ)
Y1(1)=0.
M=M+1
GO TO 41
ENDIF
C
99 T1=I*D1
CALL PLOT(T,T1,DT,H,Y,L,DX)
CONTINUE
WRITE(10,7)
WRITE(10,2)S,FI,T(K)
WRITE(10,4)
LC=1
XA2=-(T1-1)*DX
WRITE(10,8)LC,XA2,Y(1)
DO 50 1=2,H
50 WRITE(10,8)LC,XA2,Y(I)
STOP
END
C SUBROUTINE TRANS(Y,Y1,H)
REAL Y(400),Y1(400)
DO 10 I=1,H
10 Y1(I)=Y(I)
RETURN
END
C SUBROUTINE PLOT(T,T1,DT,H,Y,L,DX)
REAL T(30),Y(400),XAl
INTEGER L
IF(T1.GT.(T(L)-DT).AND.T1.LE.(T(L)+DT)) THEN
DX1=DX/1000.
XAl=-(L-1)*DX1
WRITE(11,5)L,XAl,Y(I)
DO 10 J=2,H
10 WRITE(11,5)L,XAl,Y(J)
STOP
END
C SUBROUTINE SORT(T,K)
REAL T(30)
DO 10 I=1,K
10 IF(T(I).LE.T(J-1)) GOTO 10
TMAX=T(J)
T(J)=T(J+1)
T(J+1)=TMAX
CONTINUE
REFERENCES:


Overeem, J. van, Morphological Beach Fill with Underwater Dam, Report M-1891, Delft Hydraulics Laboratory, 1983.


<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>dimensionless roughness ((r/h))</td>
<td>--</td>
</tr>
<tr>
<td>A'</td>
<td>dimensionless coefficient ((\text{CERC formula}))</td>
<td>--</td>
</tr>
<tr>
<td>a</td>
<td>thickness of bottom transport layer</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>reference level</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>accretion</td>
<td>L</td>
</tr>
<tr>
<td></td>
<td>coefficient ((s_i /d))</td>
<td>L</td>
</tr>
<tr>
<td>a_0</td>
<td>maximum horizontal water particle displacement just outside boundary layer</td>
<td>L</td>
</tr>
<tr>
<td>C</td>
<td>Chezy coefficient</td>
<td>(L^{0.5}/T)</td>
</tr>
<tr>
<td>C_{g0}</td>
<td>Chezy coefficient related to (D_{g0})</td>
<td>(L^{0.5}/T)</td>
</tr>
<tr>
<td>c</td>
<td>concentration material in suspension</td>
<td>(L^1/L^3)</td>
</tr>
<tr>
<td>c_a</td>
<td>concentration at level (z=a)</td>
<td>--</td>
</tr>
<tr>
<td>c_b</td>
<td>wave celerity at the breaker line</td>
<td>(L/T)</td>
</tr>
<tr>
<td>c_g</td>
<td>wave group celerity</td>
<td>(L/T)</td>
</tr>
<tr>
<td>c_0</td>
<td>reference concentration at level (z=0)</td>
<td>--</td>
</tr>
<tr>
<td>D</td>
<td>particle diameter</td>
<td>L</td>
</tr>
<tr>
<td>D_b</td>
<td>power dissipated due to wave breaking</td>
<td>(ML^2/T^3)</td>
</tr>
<tr>
<td>D_f</td>
<td>power dissipated due to bottom friction</td>
<td>(ML^2/T^3)</td>
</tr>
<tr>
<td>D_w</td>
<td>power gained due to a local wind field</td>
<td>(ML^2/T^3)</td>
</tr>
<tr>
<td>d</td>
<td>water depth</td>
<td>L</td>
</tr>
<tr>
<td>E</td>
<td>wave energy per unit surface area</td>
<td>(M/T^2)</td>
</tr>
<tr>
<td>E_0</td>
<td>deep water wave energy</td>
<td>(M/T^2)</td>
</tr>
<tr>
<td>f</td>
<td>wave frequency</td>
<td>(1/T)</td>
</tr>
<tr>
<td>f_w</td>
<td>friction factor</td>
<td>--</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity</td>
<td>(ML/T^2)</td>
</tr>
<tr>
<td>H</td>
<td>wave height</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>a random variable</td>
<td>L</td>
</tr>
<tr>
<td>H_{max}</td>
<td>maximum existing wave height</td>
<td>L</td>
</tr>
<tr>
<td>A</td>
<td>a model parameter</td>
<td>L</td>
</tr>
<tr>
<td>H_{m}</td>
<td>maximum possible wave height</td>
<td>L</td>
</tr>
</tbody>
</table>
$H_b$  wave height at the breaker line  \( L \)

$H_{sig}$  significant wave height  \( L \)

$H_0$  deep water wave height  \( L \)

$H_{rms}$  root mean square wave height  \( L \)

$h$  water depth  \( L \)

$h_b$  wave height at the breaker line  \( L \)

$I_1$  Einstein integral  \( -- \)

$I_2$  Einstein integral  \( -- \)

$k_r$  refraction coefficient  \( -- \)

$k_{sh}$  shoaling coefficient  \( -- \)

$k$  wave number  \( 1/L \)

$L$  accretion distance  \( L \)

$\text{wave length}$  \( L \)

$m$  beach slope  \( -- \)

$n$  ratio $c$ to $c_g$  \( -- \)

$p$  proportionality factor ($u_c/u_0$)  \( -- \)

$Q$  Einstein integral factor  \( -- \)

$R_e$  Reynolds number  \( -- \)

$RS$  radiation stress  \( M/T^2 \)

$r$  bottom roughness  \( L \)

$rms$  root mean square  \( -- \)

$S$  sediment transport  \( L^{1/T} \)

$S$  sediment transport per unit width  \( L^{5/LT} \)

$S_b$  bed load transport  \( L^{3/LT} \)

$S_s$  suspended load  \( L^{3/LT} \)

$S_{II}$  longshore sediment transport  \( L^{3/LT} \)

$S_{XX}$  XX-component of the radiation stress  \( M/T^2 \)

$S_{YY}$  YY-component of the radiation stress  \( M/T^2 \)

$S_{rt}$  radiation shear stress  \( M/T^2 \)

$s_{b0}$  deep water wave steepness ($H_b/L_0$)  \( -- \)

$s_1$  rate of sediment transport change ($dS/d\phi$)  \( L^{1/T} \)

$T$  wave period  \( T \)

$T_p$  dimensionless bed shear stress parameter  \( -- \)

$T$  peak period  \( T \)

$t$  time  \( T \)

$U$  wave power of energy flux per unit wave crest width  \( ML^{2}/T^3 \)

$U'$  energy flux component entering the breaker zone  \( ML^{2}/T^3 \)

$u$  horizontal water particle velocity  \( L/T \)
\( u_t \)  
horizontal fluid particle velocity at level \( z_t \)  
\( u_0 \)  
horizontal water particle just outside the boundary layer  
\( \bar{u}_0 \)  
maximum horizontal velocity at the bottom  
\( V \)  
velocity  
\( V_r \)  
resultant water particle velocity at level \( z_t \)  
\( V_t \)  
velocity at level \( z_t \)  
\( V_t \)  
shear stress velocity  
\( W \)  
wind velocity component  
\( w \)  
fall velocity  
\( X \)  
coordinate in the direction of wave propagation  
\( x \)  
coordinate along the coast  
\( x \)  
coordinate for wave analysis part in the shorenormal direction  
\( Y \)  
coordinate along the wave crest  
\( y \)  
coordinate perpendicular to the coast  
\( y \)  
coordinate parallel to the coast for wave analysis model  
\( z \)  
elevation above the bed level  
\( z_0 \)  
elevation for zero velocity  
\( z_t \)  
elevation for velocity profile tangency  
\( z_t \)  
Rouse number \( (w/\kappa V_t) \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>coefficient related to particle fall velocity</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>a model parameter in wave analysis modelling</td>
<td>--</td>
</tr>
<tr>
<td>(a_{br})</td>
<td>breaking coefficient</td>
<td>--</td>
</tr>
<tr>
<td>(a_{cw})</td>
<td>wave-current interaction coefficient</td>
<td>--</td>
</tr>
<tr>
<td>(\beta)</td>
<td>near bed mixing coefficient</td>
<td>--</td>
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<td>(\gamma)</td>
<td>wave breaking index</td>
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<td>(A)</td>
<td>a model parameter in wave analysis modelling</td>
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<tr>
<td>(\Delta x)</td>
<td>length step in numerical analysis</td>
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<tr>
<td>(\delta)</td>
<td>viscous sub layer</td>
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</tr>
<tr>
<td>(\delta_b)</td>
<td>thickness bottom transport layer</td>
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<tr>
<td>(\delta_s)</td>
<td>near bed mixing layer thickness</td>
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</tr>
<tr>
<td>(e_s)</td>
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<tr>
<td>(e_{scw})</td>
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<td>(\eta)</td>
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<td>(\theta)</td>
<td>wave ray angle with positive x-axis</td>
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<tr>
<td>(\kappa)</td>
<td>von Karman coefficient (=0.4)</td>
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<tr>
<td>(\mu)</td>
<td>ripple factor</td>
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<tr>
<td>(\nu)</td>
<td>kinematic viscosity</td>
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<td>(\pi)</td>
<td>Bijker’s parameter</td>
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<tr>
<td>(\rho)</td>
<td>constant (=3.14159)</td>
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<tr>
<td>(\rho_w)</td>
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<td>(\tau_c)</td>
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<tr>
<td>(\psi)</td>
<td>wave angle</td>
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<tr>
<td>(\omega)</td>
<td>surface wave frequency</td>
<td>(1/T)</td>
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\( \omega \)
surface wave frequency in case of waves and a current

\( \omega_r \)
relative (or intersinc) wave frequency in case of waves and a current

\( \zeta \)
dimensionless elevation \((z/h)\)

\( \frac{1}{T} \)
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