Phase fluctuations in two-dimensional superconducting weakly coupled wire networks

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We have fabricated two-dimensional networks of superconducting weakly coupled wires in amorphous niobium-silicon films with high normal-state sheet resistance. Results of one representative network are presented in detail. Single wires have a width of 100 nm, and the lattice period is 500 nm. The magnetoresistance exhibits oscillations with a period of 1 flux quantum per unit cell. Near zero magnetic field a clear Kosterlitz-Thouless phase transition is present, in good agreement with theoretical predictions. Compared to Josephson-junction arrays, wire networks have a lower vortex mobility.

Two-dimensional (2D) superconducting weakly coupled networks provide an interesting experimental system to study phase fluctuations, vortex motion, and 2D phase transitions in a controlled way.¹ Arrays of superconducting islands, weakly connected by Josephson junctions, have been investigated intensively.^{1,2} A 2D wire network in a superconducting thin film with high normal-state resistivity can be viewed as a similar system. The nodes of the network form a regular lattice, which are weakly coupled by superconducting wires. In Josephson-junction arrays at zero field, a clear Kosterlitz-Thouless (KT) phase transition³ is found analogous to the transition in thin superconducting films with a high normal-state sheet resistance.⁴ The wire networks are expected to undergo a KT transition at lower temperatures than the continuous film of the same material. The sheet resistance is higher, as it is enhanced by the length/width ratio of the single wires.⁴ When a perpendicular magnetic field is applied, the discreteness of the networks becomes important. In junction arrays, the resistance and the critical current oscillate with a period of 1 flux quantum per unit cell of the array. For the temperatures of interest, the length of the single wires of superconducting weakly coupled wire networks is generally much larger than the Ginzburg-Landau coherence length. Consequently, the supercurrent depends linearly on the phase difference between adjacent nodes. The junction arrays, in contrast, have a periodic sinusoidal current-phase relation. Moreover, vortex flow in wire networks is different from that in junction arrays, as a vortex has to overcome the loss of condensation energy when crossing single wires of the network. These two points show that it is not *a priori* clear to what extent the properties of junction arrays and 2D weakly coupled wire networks are comparable.

Most experiments on wire networks so far have been performed on low resistance samples, where mean-field fluctuations near the BCS transition temperature are of

greatest importance.⁵ In the weakly coupled limit, dynamical ac measurements on wire networks in granular aluminum films have been published.⁶ In zero field, a KT transition has been observed. In a magnetic field oscillating behavior of measurable quantities has only been found in a narrow temperature range below the BCS transition temperature.

We have fabricated several superconducting weakly coupled wire networks. The networks are made in cosputtered niobium-silicon films, which are amorphous. This is a great advantage over granular films of high normal-state resistivity, which consist of superconducting islands coupled by irregular Josephson junctions. We found that the wire length is of great importance for the behavior in a field. In this paper, we present the dc measurements on one network in detail. The single wires have a width w of 100 nm, which is in the order of the Ginzburg-Landau (GL) coherence length $\xi(T)$. The lattice spacing s of the network is 500 nm. The resistive behavior of this wire network is qualitatively similar to that of Josephson-junction arrays, including a KT transition in zero field and oscillations as a function of applied field. At low temperatures the critical current of the wire network does not oscillate as a function of field.

In 2D weakly coupled superconducting systems, the resistance is due to vortex motion across the sample. For diffusive, uncorrelated vortex motion, the resistance is proportional to the product of the 2D density of free vortices $n_f(T)$ and their mobility $\mu(T)$:

$$R = c\Phi_0^2 n_f(T)\mu(T), \quad (1)$$

where c is a constant, and Φ_0 the flux quantum, $\Phi_0 = h/2e$. In zero field, free vortices may be present either by thermally activated dissociation of vortex-antivortex pairs (above the KT transition), by current-induced vortex pair unbinding (below the KT transition), or by thermal activation over the finite width of the sample.^{2,7} For

Josephson-junction arrays in zero field, the vortex mobility is determined by thermal activation over the energy barrier, to move from one cell to the next. In a square array expressed in units of the Josephson-coupling energy E_J , this energy barrier⁸ equals 0.2. The probability to cross is derived by using the result for phase slip in one single overdamped junction,⁹ yielding $\mu(T) = (SR_n/\Phi_0^2)I_0^{-2}(E_J/10k_B T)$, with R_n the normal-state resistance, S the area of a unit cell of the network, and I_0 a zero-order Bessel function.

To find the wire-network equivalent of the Josephson-coupling energy, we consider the energy U associated with a phase difference $\Delta\phi$ between nodes of a 2D network. For junction arrays in the case of small $\Delta\phi$, $U = \frac{1}{2}E_J(\Delta\phi)^2$. A similar expression can be found for wire networks. For wires with $s \gg \xi(T)$ and $w/\xi(T) < 1$ the supercurrent is given by $i_s = i_c(T)[3\sqrt{3}\xi(T)/2s]\Delta\phi$, where $i_c(T)$ is the Ginzburg-Landau pair-breaking current of the wire. Then, in a wire network the coupling energy U is equal to $\frac{1}{2}J(\Delta\phi)^2$, where

$$J = \frac{3\sqrt{3}\xi(T)}{2s} \frac{\hbar}{2e} i_c(T) = 3.69 \frac{k_B T_{c0}(T_{c0} - T)\hbar}{Tr_n e^2}. \quad (2)$$

Here, r_n is the normal-state resistance of one single wire.

The energy barrier for a vortex in a wire network to move from one cell to the next is different from the one in junction arrays. In junction arrays, there is only one contribution to the energy barrier, which is connected to the extra kinetic energy associated with phase differences between lattice nodes.⁸ In wire networks, there is a similar contribution to the energy barrier, which will be approximately equal to $0.2J$. However, there is a second, additive contribution to the barrier. When a vortex crosses the single wires of the network the amplitude of the order parameter vanishes locally, leading to a loss of condensation energy. For wires with $d, w < \xi(T)$, this additional barrier to vortex crossing can be estimated from the barrier against phase slip in a 1D wire¹⁰

$$U_{1D} = \frac{4\sqrt{2}}{3} \mu_0 H_c^2 w d \xi(T) = 0.94 \frac{s}{\xi(T)} J, \quad (3)$$

where H_c is the thermodynamical critical field. The total-energy barrier U_{bar} for a vortex to cross from one cell to the next is the sum of the two contributions:

$$U_{\text{bar}} = \left(0.94 \frac{s}{\xi(T)} + 0.2 \right) J. \quad (4)$$

In our networks with $s \gg \xi(T)$, the second term can be neglected. Although the width of our single wires is of the order of $\xi(T)$, we think that Eq. (4) is still a reasonable approximation of the energy barrier.

Two different types of networks have been fabricated with lattice parameters $s = 2$ and $0.5 \mu\text{m}$. Well below T_{c0} , the $2 \mu\text{m}$ networks do not show oscillating behavior as a function of applied perpendicular magnetic field because U_{bar} is so large that vortices are effectively pinned. In the $0.5\text{-}\mu\text{m}$ networks, on the other hand, the barrier is small enough that resistance oscillations are observed. These

oscillations become more and more pronounced at lower temperatures. In the following, we will discuss the results of one $0.5\text{-}\mu\text{m}$ network in detail.

The Nb-Si films are made with a cosputter technique. The niobium is dc magnetron sputtered, and the silicon is rf magnetron sputtered in argon gas of 1 Pa. The background pressure before deposition is typically 3.10^{-5} Pa. The substrate, oxidized silicon, rotates under the Nb and Si targets at 23.4 cycles per minute. The films reported on in this paper contain 70% niobium and 30% silicon. The thickness d is 10 nm (90-s sputtering time). The normal-state sheet resistance is 240Ω , yielding a normal-state resistivity $\rho = 2.4 \mu\Omega\text{m}$. At 4.2 K, the same value for ρ is found. The Nb-Si film fabrication method is found to be highly reproducible. In x-ray diffraction, there is no indication for the formation of multilayers. Moreover, high-resolution TEM photographs of the films do not show long-range order, while some small-range order on the scale of 1 nm seems to be present. From this, we conclude that our films are amorphous. Similar Nb-Si films have been fabricated by others, who also find them to be amorphous.¹¹

In the films, networks are fabricated using a two-layer resist system, consisting of 100 nm of a high-resolution silicon-based negative resist (SNR) on top of 125-nm hard-baked AZ 1470 photoresist. Patterns are made by direct writing in the SNR resist with an electron-beam pattern generator and subsequent development. The AZ resist and the Nb-Si films are etched by reactive ion etching in O_2 and SF_6 plasmas, respectively. A completed network is shown in Fig. 1. From that photograph, the width of the single wires is about 100 nm. The lattice parameter of the network $s = 500$ nm. The array has a length L of 200 unit cells and a width W of 99.

The normal-state resistance of the array and the BCS transition temperature are obtained from a 2D Aslamazov-Larkin fit,¹² giving $R_n = 2450 \Omega$ and $T_{c0} = 2.135$ K. With $R_n = 2450 \Omega$, the resistance per single wire r_n equals 1225Ω . From the sheet resistance and the estimated length-width ratio of one wire, one calculates a value of 1190Ω . If we assume a ρl product of $10^{-15} \Omega\text{m}^2$, the mean-free path is about 0.4 nm. We estimate the BCS coherence length to be 800 nm. The zero-

FIG. 1. A SEM photograph of a completed array. The white bar at the bottom indicates $1 \mu\text{m}$. The lattice spacing of the network is 500 nm, the width of the single wires is about 100 nm.

FIG. 2. The resistance as a function of applied magnetic field, indexed with the frustration parameter f for seven different temperatures: curve a , $T=2.3$ K; curve b , $T=2.19$ K; curve c , $T=2.16$ K; curve d , $T=2.12$ K; curve e , $T=2.09$ K; curve f , $T=2.06$ K; and curve g , $T=2.03$ K.

temperature GL coherence length is 15 nm. In the temperature range of interest, $\xi(T)$ varies between 60 and 200 nm. The measured array critical current follows the temperature dependence of the depairing current $i_c(T)$ above 1.75 K. The measured absolute value is 30% below the value calculated with the previous estimates. We consider this a very satisfactory agreement.

In Fig. 2, we show the resistance versus applied field. The frustration parameter f is defined as the applied flux per unit cell of the array divided by Φ_0 . In Fig. 2, temperature decreases from a to g . At $T \approx T_{c0}$ small oscillations in the resistance become visible. With decreasing temperature, the oscillations become more pronounced and are found up to higher values of f , superimposed on a parabolic background. For the lowest temperatures, the oscillations are periodic up to the maximum applied field, which is about $f=16$ (0.13 T). In contrast, oscillations in the critical current are only observed in a narrow temperature range of 90 mK below T_{c0} . Below 2.04 K, only the parabolic dependence of the critical current on the field remains.

The magnetoresistance at a low temperature shows values of R at $f=0$ and 1 that are almost equal. The maximum resistance is about 1 order of magnitude larger than at $f=0$. A clear minimum in the resistance is present at $f=\frac{1}{2}$. Small dips also occur at other fractional values of f . This behavior is qualitatively very similar to that observed in Josephson-junction arrays,² though the dips for the wire networks are less pronounced.

In Fig. 3, we give the resistive transition of the network for $f=0$ as a function of the normalized temperature τ , which is defined as $\tau=k_B T/J$. The $f=0$ transition should be a Kosterlitz-Thouless transition at a temperature $\tau_{KT}=\pi/(2\epsilon_c)$, with ϵ_c a nonuniversal constant slightly larger than 1. Just above τ_{KT} , the density of free vortices follows a square-root cusp dependence^{8,9} $n_f(\tau)=(c_1/S) \times \exp[-b/(\tau-\tau_{KT})^{1/2}]$, where c_1 and b are constants. For wire networks there is no reason that the probability for a vortex to cross the single wires is determined by a Bessel function dependence. Instead, it seems reasonable

to use a Boltzmann expression

$$\begin{aligned} \mu(\tau) &= (SR_n/\Phi_0^2) \exp(-U_{\text{bar}}/k_B T) \\ &= (SR_n/\Phi_0^2) \exp(-u_{\text{bar}}/\tau), \end{aligned}$$

where $u_{\text{bar}}=0.94s/\xi(T)$. With Eq. (1), we now obtain

$$\frac{R}{R_n} = c_2 \exp[-b/(\tau-\tau_{KT})^{1/2}] \exp(-u_{\text{bar}}/\tau). \quad (5)$$

In Fig. 3, the dashed line shows the fit to Eq. (5). The parameters are $c_2=1.5$, $b=4.3$, and $\tau_{KT}=1.05$. These values are all in reasonable agreement with the theoretical predictions. However, in practice the fitting procedure requires three parameters, which leaves a certain degree of freedom. As a result, it is difficult to obtain clear indications for the presence of a KT transition from the resistance versus temperature curves.²

A more severe test for the KT transition is the nonlinear resistance. In this measurement, the voltage is measured as a function of transport current for various temperatures near the transition. When plotted as $\log V$ vs $\log I$, below the transition temperature a power-law dependence $V \sim I^{a(\tau)}$ should be present, above the transition the resistance should be linear with $a(\tau)=1$. At the transition the nonlinear resistance exponent $a(\tau)$ jumps from 3, which is a universal value, to 1. We have measured the nonlinear resistance and derived $a(\tau)$ from the $\log V$ - $\log I$ plots at a voltage level of 50 nV. In the inset of Fig. 3 the result is given. Clearly, a jump in the exponent is present, in good agreement with the universal jump prediction. The transition takes place at $\tau_{KT}=1.05$, $T_{KT}=1.985$ K. With $\tau_{KT}=1.05$, $\epsilon_c=1.49$, which is a reasonable value.

In conclusion, we have made superconducting weakly coupled wire networks in amorphous niobium-silicon films. We showed that the $s/\xi(T)$ ratio is of crucial importance for the behavior in a field. The energy barrier U_{bar} for a single vortex to move from one unit cell of the network to the next is approximately equal to $s/\xi(T)$,

FIG. 3. The resistive transition for $f=0$, plotted as a function of normalized temperature. The dashed line is a fit to Eq. (5). In the inset, the nonlinear resistance exponent is given vs normalized temperature, showing the universal jump from 1 to about 3 at the transition.

when expressed in units of the vortex coupling energy J [Eq. (4)]. At low temperatures, oscillations in the magnetoresistance can be observed if the single vortex energy near $f=0$ ($15\text{--}18J$ for arrays of 100 unit cells wide²) is larger than U_{bar} , i.e., when the $s/\xi(T)$ ratio is not too large. Then, the resistive behavior is qualitatively very similar to that observed in Josephson-junction arrays, including a Kosterlitz-Thouless phase transition near $f=0$.

We found that at low temperature the critical current does not oscillate as a function of applied magnetic field.

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FIG. 1. A SEM photograph of a completed array. The white bar at the bottom indicates $1\ \mu\text{m}$. The lattice spacing of the network is $500\ \text{nm}$, the width of the single wires is about $100\ \text{nm}$.