Determination of the Hydrodynamics in a Bubble Column by means of LDA

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Graduation Thesis

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Abstract

The hydrodynamic behaviour of the liquid flow in a 3D 'air-water' bubble column 15 cm in diameter and a gas-fraction of 10 % was investigated. The study focussed on the low-frequency part (0-2 Hz) of liquid velocity time series measured close to the wall in order to get an indication as to how slow oscillations of the flow evolve in the column.

Laser Doppler Anemometry (LDA) was used to measure liquid velocities. When measuring with LDA in a two-phase flow, air bubbles were found to disturb the LDA measurements in several ways. Therefore, part of the investigation was focussed on validating the velocity time series and optimising the different parameters of the LDA-system. Multiple validation of Doppler signals was one of the main problems when measuring in a two phase flow. Multiple validation could be reduced by optimising the LDA parameters. Furthermore, an averaging procedure was used to compensate for the multiple validation.

Frequency spectra of the velocity time series were determined with the Lomb method, which can deal with unevenly sampled time series (measured with LDA). Joint time frequency spectra were calculated with the Short Time Fourier Transform.

Dominant frequencies ranging from 0.1 to 0.2 Hz occurred. The low frequencies were stronger in the bottom part of the bubble column and gradually became weaker when going up in the column. This means that vortices are stronger in the bottom part of the column and become weaker higher in the column. The joint time frequency analysis revealed that there were separate frequency peaks (0.1 to 0.2 Hz) spaced at irregular time intervals (30-50 s). The frequencies found can be linked to vortices moving through the column and slow oscillation waves. It was found that, in contrast to 2D bubble columns, the oscillatory motion in 3D bubble columns is irregular. An average frequency could be defined for the slow oscillations.
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$A$</td>
<td>amplitude of signal with Gaussian noise</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>design matrix</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>scale index frequency (Wavelet transform)</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>time shifting (Wavelet transform)</td>
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</tr>
<tr>
<td>$c$</td>
<td>constant</td>
<td>[-]</td>
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<tr>
<td>$c_n$</td>
<td>coefficients used for S-G filtering</td>
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<td>column diameter</td>
<td>[m]</td>
</tr>
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<td>$D_{beams}$</td>
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<td>[m]</td>
</tr>
<tr>
<td>$d_b$</td>
<td>bubble diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_{beam}$</td>
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<tr>
<td>$d_{window}$</td>
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<td>$[kgms^{-2}]$</td>
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<td>$F_s$</td>
<td>interaction force</td>
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<td>$f$</td>
<td>difference or differential Doppler frequency</td>
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<tr>
<td>$f_{d,b}$</td>
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<td>$\Delta f$</td>
<td>receiver pass band or filter range</td>
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</tr>
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<td>$g$</td>
<td>gravitational acceleration</td>
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<tr>
<td>$g_i$</td>
<td>linear combination of data points $h_i$</td>
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<td>$h_i$</td>
<td>value of single data point</td>
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<tr>
<td>$\bar{h}$</td>
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<tr>
<td>$I$</td>
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<td>$\frac{i^2}{2}$</td>
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</tr>
<tr>
<td>$\frac{i^2}{2}$</td>
<td>mean square noise current</td>
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<tr>
<td>$j$</td>
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<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
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<td>$N$</td>
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<td>$N(t)$</td>
<td>number of events in a given space of time $(0,t)$</td>
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<td>$N_{cycles}$</td>
<td>number of cycles per burst</td>
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<tr>
<td>$n$</td>
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<td>$n_{air-water}$</td>
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<td>$OO'$</td>
<td>aberration of flat window</td>
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<td>$P[.]$</td>
<td>probability</td>
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<td>$p$</td>
<td>pressure</td>
<td>$[kgm^{-1}s^{-2}]$</td>
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<tr>
<td>$R$</td>
<td>column radius</td>
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</tr>
<tr>
<td>$R_b$</td>
<td>bubble radius</td>
<td>$[m]$</td>
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<td>$R_c$</td>
<td>Reynolds stress tensor</td>
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<td>$r$</td>
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<tr>
<td>$S(\omega)$</td>
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<td>signal</td>
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<td>spacing between interference fringes</td>
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<td>$T$</td>
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<tr>
<td>$t$</td>
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<td>$\Delta t$</td>
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<td>$u$</td>
<td>bubble velocity</td>
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<td>$V$</td>
<td>bubble volume</td>
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<td>$V_D$</td>
<td>vortex descending velocity</td>
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<tr>
<td>$v$</td>
<td>velocity</td>
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<td>$v_{bot}$</td>
<td>velocity perpendicular to direction of observation</td>
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<td>$W$</td>
<td>wavelength of oscillation</td>
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<td>light power reaching the detector</td>
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<td>$X_s$</td>
<td>scattering cross-section of particle</td>
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<td>$\Delta x$</td>
<td>'operating window' for attaining bubble-caused velocity realization</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
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<td>--------</td>
<td>----------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>gas-fraction or hold up</td>
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<td>$\alpha$</td>
<td>angle between two laser beams</td>
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<tr>
<td>$\alpha$</td>
<td>constant related to size of window function</td>
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<td>$\bar{\alpha}$</td>
<td>cross-sectional averaged gas-fraction</td>
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<td>$\beta$</td>
<td>angle velocity makes with normal to bisector of laser beams</td>
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<td>window function</td>
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<td>$\Delta_\omega$</td>
<td>frequency bandwidth of signal</td>
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<td>$\delta_x$</td>
<td>width of scattering or measurement volume</td>
<td>[m]</td>
</tr>
<tr>
<td>$\delta_y$</td>
<td>effective beam diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$\delta_z$</td>
<td>length of scattering or measurement volume</td>
<td>[m]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency of detector</td>
<td>[-]</td>
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<tr>
<td>$\theta_1$</td>
<td>angle between first laser beam and seeding particle velocity</td>
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</tr>
<tr>
<td>$\theta_2$</td>
<td>angle between second laser beam and seeding particle velocity</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>angle between direction of observation and seeding particle velocity</td>
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</tr>
<tr>
<td>$\lambda$</td>
<td>mean rate of occurrence of an event</td>
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<tr>
<td>$\lambda$</td>
<td>wavelength of laser light</td>
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<td>$\mu$</td>
<td>refractive index</td>
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<td>$\nu$</td>
<td>optical frequency</td>
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<td>$\nu_m$</td>
<td>molecular kinematic viscosity</td>
<td>[m$^2$s$^{-1}$]</td>
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<td>$\nu_t$</td>
<td>turbulent kinematic viscosity</td>
<td>[m$^2$s$^{-1}$]</td>
</tr>
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<td>$\rho$</td>
<td>density</td>
<td>[kgm$^{-3}$]</td>
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<tr>
<td>$\bar{\rho}$</td>
<td>time averaged density</td>
<td>[kgm$^{-3}$]</td>
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<tr>
<td>$\sigma$</td>
<td>standard deviation</td>
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<td>$\sigma^2$</td>
<td>variance</td>
<td></td>
</tr>
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<td>$\sigma^2_N$</td>
<td>variance of noise</td>
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<tr>
<td>$\tau$</td>
<td>time used in integral</td>
<td>[kgm$^{-1}$s$^{-2}$]</td>
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<tr>
<td>$\tau_c$</td>
<td>shear stress tensor</td>
<td>[kgm$^{-1}$s$^{-2}$]</td>
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<tr>
<td>$\tau_M$</td>
<td>sum of the molecular and Reynolds stress tensor</td>
<td>[kgm$^{-1}$s$^{-2}$]</td>
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<td>$\tau_w$</td>
<td>shear stress at the wall</td>
<td>[kgm$^{-1}$s$^{-2}$]</td>
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<td>$\Phi(t)$</td>
<td>mother wavelet function</td>
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<tr>
<td>$\phi_1$</td>
<td>angle of incidence</td>
<td></td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>angle of refraction</td>
<td>[-]</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>solid angle aperture of the receiving optics</td>
<td>[-]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\delta\omega$</td>
<td>uncertainty in angular frequency $\omega$</td>
<td>[s$^{-1}$]</td>
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### Physical Constants and Dimensionless Numbers

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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$c$</td>
<td>speed of light</td>
<td>$[ms^{-1}]$</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant</td>
<td>$[kgm^2s^{-1}]$</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number</td>
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### Subscripts

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<thead>
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<tr>
<td>$ax$</td>
<td>axial component</td>
</tr>
<tr>
<td>$bottom$</td>
<td>bottom of the bubble column</td>
</tr>
<tr>
<td>$c$</td>
<td>continuous phase</td>
</tr>
<tr>
<td>$d$</td>
<td>dispersed phase</td>
</tr>
<tr>
<td>$dc$</td>
<td>drift (flux)</td>
</tr>
<tr>
<td>$g$</td>
<td>gas phase</td>
</tr>
<tr>
<td>$l$</td>
<td>liquid phase</td>
</tr>
<tr>
<td>$s$</td>
<td>slip (velocity)</td>
</tr>
<tr>
<td>$tan$</td>
<td>tangential component</td>
</tr>
<tr>
<td>$top$</td>
<td>top of the column (gassed height)</td>
</tr>
</tbody>
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Chapter 1

Introduction

1.1 Bubble Columns

A bubble column is a cylinder filled with water into which gas bubbles rise from the bottom to the top of the column. Bubble columns have been widely used as chemical and bioreactors. Examples of bioreactors are found in wastewater treatment and the production of antibiotics. Bubble column reactors are also used in the chemical process industry for carrying out gas-liquid reactions in large volumes. Their appealing features include low cost and simplicity of design. Scale up of bubble column reactors remains a problem, making CFD modelling an attractive approach for design. CFD modelling goals include the prediction of realistic qualitative and quantitative flows. Such predictions are non-trivial and require appropriate models describing the multi-phase physics of the gas-liquid interactions. Experimental data is required for comparison in order to validate the predictions made with CFD.

1.2 Flow inside a Bubble Column

In a bubble column one distinguishes often four different flow regimes: Homogeneous bubble flow, Turbulent bubble flow, Churn flow and Slug flow. Figure 1.1 shows an example of the four regimes. The flow regimes are determined empirical and are based on observations of the bubbles. For example, the flow is called homogeneous or turbulent bubbly flow when there is no coalescence, churn flow when coalescence appears and slug flow when slugs of air appear. The difference between homogeneous and turbulent bubbly flow is hard to make from bubble observations alone. One could argue whether the four regimes apply to the liquid flow. They are based on bubble observations and therefore tell not much about the liquid flow.

For many years a lot of research has been done at time averaged velocity and frequency or power spectra. Measurements of the radial gas-fraction profile and axial velocity profile were done by [Mulder(1996)] and [Oldeman(1995)] who...
Figure 1.1: Four different flow regimes.

gas fraction

low  

| Homogeneous bubble flow | Turbulent bubble flow | Churn flow | Slug or plug flow |

high

compared his results to [Hills(1974)]. Just by looking at the movement of the bubbles in the flow one can see that the liquid flow in bubble column is much more complicated than simple re-circulation model based on the time-average velocity profile, see figure 1.2. Movement of vortices cannot be described by the simple models based on time averaged behaviour of the flow, such as the models of [Ueyama and Miyauchi(1979)] or of [Geary and Rice(1992)]. The more complicated model of [Zehner(1986)] describes circulation cells, but also fails when the movement of the re-circulation vortices is considered. CFD simulations of two-phase flows considering bubble plumes, [Bauer(1999)] and [Lapin(1994)], clearly show that vortices are present and moving. Observations of the bubble column give an indication that vortices are moving through the column. Measurements and simulations done in 2D-bubble columns by the group of Fan, [Lin and Fan(1996)] show the movement of vortices in the columns. In order to 'measure' the vortices the time dependent behaviour of the liquid flow has to be known. In order to do this one needs a fast and accurate method.

1.3 Measuring the Liquid Flow

Laser Doppler anemometry (LDA) is a well known technique to measure velocity which is both fast and accurate. When measuring in fluids LDA has the advantage
that it is non-intrusive, which means that the fluid is not disturbed when measuring.

A few questions about the application of LDA in a bubble column remain: What are the limits of LDA when used in a bubble column and to which extend is it possible to measure the dynamic behaviour of the liquid flow in a bubble column.

A problem when measuring with LDA in a bubble column is: What is actually measured the seeding or the bubbles?

Bubbles are only measured under very specific circumstances, see [Groen(1999)].

The bubble related data rate is estimated to be:

\[ f_B = O(\alpha \frac{u \Delta x}{R^2}) \]  

(1.1)

where \( \alpha \) is the gas hold up, \( u \) bubble velocity, \( \Delta x \) 'operating window' for attaining a bubble-caused velocity realization and \( R \) is the bubble radius. Taking the bubble rise velocity to be 30 cm/s, Groen shows that the estimated bubble related data rate at a gas fraction of 10% is of the order of 1 Hz. Given that the 'normal' data rate is of the order of 150 Hz, it can be seen that bubbles are hardly measured. Only 0.7% of the data points are bubble related. Bubbles mainly cause the blocking of the signal (when a bubble crosses one of the laser beams, the signal is blocked, thereby preventing the LDA-measuring volume to be formed).

The full dynamic behaviour of the flow can only be determined close to the wall, due to the blocking of the bubbles. For frequency analysis, sufficient data points per time unit are necessary in order to satisfy the Nyquist criterium. For the time average behaviour the demands are less strict and can therefore be determined further into the column. (A sufficient number of points can be obtained by measuring
for a long time, since no time or frequency criteria have to be satisfied.) The LDA
technique is especially suitable for dynamic measurements close to the wall.
Signal processing of data obtained with LDA gives some difficulties, the data is not
evenly spaced in time and the signal is sometimes blocked by bubbles resulting in
gaps in the time series. A solution often used is re-sampling the unevenly spaced
data points to obtain evenly spaced ones. An alternative for the re-sampling method
was developed by astrophysicists. They too have unevenly spaced data-points with
large gaps. Some interesting solutions for these difficulties were developed over
time. One of particular interest was developed by [Lomb(1976)] and further elabor­
orated by [Scargle(1982)]. A method based on the classical Fourier periodogram
is investigated and used, the so-called Lomb periodogram.

1.4 Objectives

The objective of this investigation is to determine the dynamic behaviour of the
liquid flow. Since the flow in a bubble column is 'chaotic' this is not an easy task.
[Mudde and Groen(1997)] found low frequencies were present in the range of 0 to
2 Hz. Around 0.1 Hz. distinct frequencies were shown. This is an indication for the
presence of long time oscillations in the flow. Experiments in 2D-bubble columns
done by the group of Fan, [Mudde and Lee(1997)] also show frequencies in the
range of 0 to 2 Hz. Based on previous experiments in 2D-bubble columns the oscil­
lations seem to be vortices moving through the column, see [Lin and Fan(1996)].
The focus of this investigation is therefore the slow oscillations of the flow in the
range of 0 to 2 Hz.

The flow in a bubble column consists of coherent structures moving through the
column, see figure 1.3. One of the goals is to link the oscillations found to coherent
structures present in a bubble column. Understanding these coherent structures and
modelling them is useful for designing and scaling up bubble column reactors.
In order to look at time dependent phenomena occurring in the flow a method is
necessary which can identify the oscillations. The solution is found in the so-called
joint time frequency analysis (JTFA). With JTFA the frequency can be determined
as a function of time. The found frequencies represent oscillations occurring in the
flow. Oscillations close to the wall can be tracked in time with this method.
Figure 1.3: Classification of regions accounting for the macroscopic flow structures in a 3D-bubble column, Lin and Fan (1996)
Chapter 2

Theory: Flow Inside a Bubble Column

The flow in a bubble column is a dispersed two-phase flow. One of the two phases, the dispersed phase, is made up by segregated individual particles (gas bubbles) in the midst of a continuous (liquid) phase. The gaseous state in the form of air-bubbles causes the liquid state to move as a consequence of the action of gravity and density difference between the two states. The bubbles and their wakes are responsible for small scale liquid motion. Inhomogeneities in the gas-fraction distribution result in motion at a large scale. The two types of motion combined result in a long time circulation pattern. Generally, the circulation is responsible for an up-flow in the middle and a down flow at the wall. Understanding these large scale motions will lead to a better view of the flow field in a bubble column.

2.1 Modelling Two-Phase Flow

The relative proportions, in which the two phases are present, is denoted by the hold up or void fraction (volume fraction of the dispersed phase) and is indicated with \( \alpha \). In a bubble column the dispersed phase consists of gas bubbles and the continuous phase of liquid. The gas-fraction varies with the radial position. The time averaged density as a function of the radial position is defined as

\[
\bar{\rho} = (1 - \alpha(r))\rho_c + \alpha(r)\rho_d
\]  

(2.1)

where \( \rho_c \) is the density of the liquid and \( \rho_d \) of the gas.

The slip velocity is defined as

\[
\bar{v}_s = \bar{v}_d - \bar{v}_c
\]  

(2.2)

where \( \bar{v}_d \) is the velocity of the dispersed phase and \( \bar{v}_c \) of the continuous phase.

The superficial velocity for the continuous and dispersed phase are given by the following equations

\[
\bar{J}_c = (1 - \alpha)\bar{v}_c
\]  

(2.3)
and

\[ j_d^* = \alpha \bar{v}_d \quad (2.4) \]

### 2.1.1 Drift Flux Model

A well-known technique of modelling a two-phase flow is the so-called drift-flux modelling. The drift-flux model is 1-dimensional and it is supposed that both phases are distributed homogeneously over the apparatus. With equations 2.2, 2.3 and 2.4 the slip velocity becomes

\[ v_s = \frac{j_d}{\alpha} - \frac{j_c}{1 - \alpha} \quad (2.5) \]

A second relation between \( v_s \) and \( \alpha \) is given by

\[ \frac{v_s}{v_\infty} = f(\alpha) \quad (2.6) \]

Eliminating \( v_s \) leads to

\[ \frac{j_{dc}}{v_\infty} = \alpha(1 - \alpha)f(\alpha) \quad (2.7) \]

where

\[ j_{dc} = (1 - \alpha)j_d - \alpha j_c \quad (2.8) \]

is called the drift-flux.

In a bubble column the cross-sectional averaged superficial velocity of the continuous phase equals 0. Under this condition equations 2.7 and 2.8 lead to

\[ \frac{j_d}{v_\infty} = \alpha f(\alpha) \quad (2.9) \]

**Richardson and Zaki**

An empirical relation for function \( f(\alpha) \) is given by

\[ \frac{v_s}{v_\infty} = (1 - \alpha)^{n-1} = f(\alpha) \quad (2.10) \]

where \( n \) depends on the Reynolds number, \( Re \), based on the terminal velocity of a single bubble. This is shown in the table below.

<table>
<thead>
<tr>
<th>( Re )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Re &lt; 0.2 )</td>
<td>4.65</td>
</tr>
<tr>
<td>( 0.2 &lt; Re &lt; 1 )</td>
<td>( 4.35 Re^{-0.03} )</td>
</tr>
<tr>
<td>( 1 &lt; Re &lt; 500 )</td>
<td>( 4.45 Re^{-0.1} )</td>
</tr>
<tr>
<td>( Re &gt; 500 )</td>
<td>2.39</td>
</tr>
</tbody>
</table>

A bubble of a diameter of 3 mm results in a Reynolds number of about 700. It follows that \( n = 2.39 \).
2.1.2 Liquid Re-circulation

Ueyama and Miyauchi

[Ueyama and Miyauchi(1979)] model the bubble column by means of the equation of conservation of momentum. They consider a long, vertical bubble column. Both phases are supposed to be incompressible. The hold up, $\alpha$, is supposed to be a function of the radial position, $r$. The equation for the total momentum of the mixture reads as:

$$0 = -\nabla (\tau_c + R_c) - \nabla p + (1 - \alpha(r))\rho_c g + \alpha(r)\rho_d g$$  \hspace{1cm} (2.11)

with

$$\tau_c + R_c = \tau$$  \hspace{1cm} (2.12)

where $g$ is the gravitational acceleration, $p$ the pressure and $\tau$ the shear stress tensor with $\tau_c$ the deformation tensor and $R_c$ the Reynolds stress tensor.

In cylinder coordinates (1D) and with $\rho_d << \rho_c$ the momentum becomes:

$$\frac{1}{r} \frac{d}{dr} \left(r\tau\right) = \frac{dp}{dr} + (1 - \alpha(r))\rho_c g$$  \hspace{1cm} (2.13)

with

$$\tau = -\left(\nu_m + \nu_t\right)\rho_t \left(\frac{dv_c}{dr}\right)$$  \hspace{1cm} (2.14)

where $\nu_m$ is the molecular and $\nu_t$ the turbulent kinematic viscosity.

Integrating equation 2.13 over the cross-section of the column leads to the following pressure gradient:

$$-\frac{dp}{dz} = \frac{2}{R} \tau_w + (1 - \bar{\alpha})\rho_c g$$  \hspace{1cm} (2.15)

where $R$ is the radius of the column, $\tau_w$ is the shear stress at the wall and $\bar{\alpha}$ the cross-sectional averaged gas-fraction.

Combining equation 2.13, 2.14 and 2.15 gives the following differential equation for the velocity profile of the flow.

$$\frac{1}{r} \frac{d}{dr} (\nu_t r \frac{dv_c}{dr}) = \frac{2}{R \rho_c} \tau_w - (\bar{\alpha} - \alpha(r))g$$  \hspace{1cm} (2.16)

Ueyama and Miyauchi suppose that the turbulent viscosity is constant in the bubble column and is given by the following equation

$$\nu_t = 0.183 R^{1.77}$$  \hspace{1cm} (2.17)

Furthermore, Ueyama and Miyauchi suppose that the hold up is a known function of the radial position:

$$\alpha(r) = \bar{\alpha} \frac{m}{m} + \frac{2}{m} \left(1 - \left(\frac{r}{R}\right)^m\right)$$  \hspace{1cm} (2.18)
The following boundary conditions apply to the bubble column:
in the middle of the column: \( r = 0, \frac{\partial v_c}{\partial r} = 0 \)
and close to the wall there is a thin layer flow, which is calculated with the wall
stress: \( r = R, v_c = -11.63\sqrt{\frac{\tau_{wl}}{\rho_c}} \).
The problem is closed by stating that the net velocity of the flow is zero.
\[
\int_0^R 2\pi r (1 - \alpha) v_c dr = 0
\]
Solution of the liquid velocity (continue phase) is now fixed and after some basic
mathematics the following equation is found:
\[
v_c = v_{c,\text{wall}} + \left( \frac{\tau_w R}{2\nu_t \rho_c} - \frac{\alpha R^2}{4\nu_t} + \frac{(2 + n)R^2}{4n\nu_t} \right)(1 - \frac{r^2}{R^2}) + \frac{R^2}{n(2 + n)\nu_t} \left( 1^n - \frac{r^2 (\frac{r}{R})^n}{R^2} \right)
\]
with
\[
\tau_w = \frac{(-A \pm \sqrt{A^2 - 4BC})}{4B^2} \quad \tau_w > 0
\]
The constants A, B, and C are given by
\[
A = 11.63 \sqrt{\frac{1}{\rho_c} \left( 1 + 2\alpha \left( \frac{n + 2}{n} \right) \left( \frac{1}{2} - \frac{1}{2 + n} \right) \right)}
\]
\[
B = \frac{R}{8\nu_t \rho_c} + \frac{\alpha (n + 2)}{\nu_t \rho_c} \left( \frac{1}{4 - \frac{1}{2 + n} + \frac{1}{4 + n}} \right)
\]
\[
C = \frac{\alpha R^2}{\nu_t} + \frac{(2 + n)R^2}{8n\nu_t} + \frac{2R^2}{n(2 + n)\nu_t} \left( \frac{1}{4} - \frac{1}{2 + n} - \frac{1}{4 + n} \right) + \frac{2\alpha R^2}{n^2\nu_t} \left( \frac{1}{2} - \frac{1}{2 + n} - \frac{1}{4 + n} + \frac{1}{2(2 + n)} \right)
\]
This complex equation only describes the average velocity profile of the flow.
Other phenomena of the flow like vortices or oscillations cannot be modelled with
the Ueyama and Miyauchi model.
2.2 Equations of conservation of momentum

The models described above are appropriate for the average velocity. But they fail, when time dependent flow phenomena are considered. A mathematical or parameter model is hard if not impossible to make. Computational Fluid Dynamics or CFD is the key to 'model' or rather simulate the flow. In describing the hydrodynamic behaviour of many two-phase flow systems the equations of conservation of mass and momentum, the so-called continuity equations and momentum balances for the separate phases, are essential. They are the basis of many CFD-simulations. In order to validate the CFD-simulations, experimental data is needed. This is a goal of this investigation. A model often used, is the two fluid or Euler-Euler model. Both phases (gas, liquid) are supposed to be a continuous phase, scaled with the gas-fraction, $\alpha$ (gaseous state) or $1 - \alpha$ (liquid state).

The equation of conservation of mass for the gas phase is

$$\frac{\partial \alpha \rho_g}{\partial t} + \nabla \cdot (\alpha \rho_g \vec{v}_g) = 0 \tag{2.25}$$

The equation of conservation of mass for the liquid phase is

$$\frac{\partial (1 - \alpha) \rho_l}{\partial t} + \nabla \cdot (1 - \alpha) \rho_l \vec{v}_l = 0 \tag{2.26}$$

where $\rho$ is the density, $\vec{v}$ is the velocity vector and the subscripts l, g indicate the liquid and the gas phase respectively.

The equation of conservation of momentum for the gas phase is

$$\frac{\partial \alpha \rho_g \vec{v}_g}{\partial t} + \nabla \cdot (\alpha \rho_g \vec{v}_g \vec{v}_g) = -\alpha \nabla \tau^g_M - \alpha \nabla p + \alpha \rho_g \vec{g} - \vec{F}_s \tag{2.27}$$

where $\tau^g_M$ is the sum of the molecular and Reynolds stress tensor, $p$ is the pressure, $\vec{g}$ is the gravitational acceleration and $\vec{F}_s$ is the interaction force between the two fluids (gas, liquid).

The equation of conservation of momentum for the liquid phase is

$$\frac{\partial (1 - \alpha) \rho_l \vec{v}_l}{\partial t} + \nabla \cdot (1 - \alpha) \rho_l \vec{v}_l \vec{v}_l = -(1 - \alpha) \nabla \tau^l_M + (1 - \alpha) \nabla p + (1 - \alpha) \rho_l \vec{g} + \vec{F}_s \tag{2.28}$$

The interaction force $\vec{F}_s$ consists of the sum of the following forces

$$\vec{F}_s = F_{drag} + F_{am} + F_{lift} \tag{2.29}$$

where $F_{drag}$ is the drag force, $F_{am}$ is the added mass force and $F_{lift}$ is the lift or Magnus force.

These equations are basically the foundation for (Euler-Euler) CFD-work. They are able to describe time-dependent or transient flow phenomena. The focus of this experimental investigation was also to determine time dependent phenomena of the flow such as oscillations in order to support CFD-simulations.
2.3 Earlier Work on Bubble Columns

Investigations on the circulating liquid flow in a bubble column have been done by various researchers in the past decades. The experiments of [Hills(1974)] concerning time-averaged velocity-profiles are often used for model verification. Hills' measurements and other studies, [Devanathan and Dudukovic(1990)] have provided much insight into the overall averaged (steady-state and one-dimensional) flow field. This revealed the existence of the gross circulation flow field. Hills used a Pitot tube to measure the liquid velocity and gas-fraction profiles in a bubble column of 13.8 cm in diameter. He considered three different distributor-plates,

<table>
<thead>
<tr>
<th>Plate</th>
<th>Hole diameter</th>
<th>Number and arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.4 mm</td>
<td>32 (16 on each of 2 circles, 75 and 100 mm in diameter)</td>
</tr>
<tr>
<td>B</td>
<td>0.4 mm</td>
<td>61 (1 central, rest uniformly spaced on 3 circles)</td>
</tr>
<tr>
<td>C</td>
<td>1.6 mm</td>
<td>7 (1 central, 6 on circle, 115 mm in diameter)</td>
</tr>
</tbody>
</table>

An example of the liquid velocity profiles Hills found for the three different plates is illustrated in figure 2.1. A disadvantage of the method used by Hills is the fact that with his probes he is unavoidably disturbing the flow. Still his measurements form an important bases for one-dimensional, time invariant flow modelling.

The experimental work of the group of Dudukovic involved the CARPT facility, see [Devanathan and Dudukovic(1990)]. CARPT stands for Computer Automated Radioactive Particle Tracking. The motion of a single neutrally buoyant radioactive particle is monitored by an array of scintillation detectors and analysed by an on-line computer to map the flow field. Some results obtained by the CARPT system are shown in figure 2.2.

Other work on bubble columns of the group of Dudukovic involves Computed Tomography (CT), [Sailesh and Dudukovic(1990)]. Sailesh used a CT-scanner to study the effects of various operating parameters (such as column diameter, superficial gas velocity and distributor type) on the gas holdup and its distribution in an air-water bubble column. The experimental investigation showed that the column dimensions had no significant effect on the void fraction, when the column diameter is greater than 0.15m. Differences in hold up distribution due to the kind of distributor used, are significant at low gas-flow rates. The advantages of both CARPT and CT is that is applicable to columns with large diameters, while PIV and LDA are limited to small bubble columns or measurements close to the wall. The limitations concerning the transparency of the system are absent with the CARPT and CT system. A disadvantage of CARPT and CT is that it involves radio-activity which makes it an expensive method.

Work on bubble columns concerning the instantaneous or transient behaviour of the flow is done by several groups. In next section the work of various research groups is discussed.
Figure 2.1: Experimental velocity profiles plotted against dimension-less radial position, Hills(1974).

Figure 2.2: i=(a) Radial variation of axial velocity at various z locations, and (b) Axial variation of radial velocity at various r locations, Devanathan and Dudukovic(1990).
2.3.1 Instantaneous or time dependent behaviour of the flow

An interesting study to dynamic behaviour of flows in 2D-bubble columns is done by the group of Fan, [Lin and Fan(1996)]. Lin used three 2-D columns made of transparent Plexiglass. A Particle Image Velocimetry or PIV-system was used to measure local flow structures in the bubble columns. Lin defined the wavelength as the distance between two crest points of the outer-most loop of the tracer trajectories, see figure 2.3. The vortex was identified by the outer-most closed loop of trajectories and was found to be approximately equal to twice the amplitude of the wave. An example of the found wavelengths and frequencies of the wave motions is illustrated in 2.4 a and b. It shows that the wavelength decreases with increasing gas velocity up to 3 cm/s beyond which it remains relatively constant. Furthermore, it is seen that the wavelength in a small column is less than that in a large column at the same gas velocity. The frequency increases with the gas velocity in columns of different width. Lin explains this by coalescence of the bubbles. Larger bubbles impart higher kinetic energy to the liquid phase and thus increase the frequency of the wave. From wave theory, the wave velocity, which defines the vortex descending velocity, is according to Lin equal to the generation frequency times the wavelength, given by:

\[ V_D = W \cdot f \]  

(2.30)

where \( V_D \), \( W \) and \( f \) are respectively the vortex descending velocity, wavelength and frequency. Lin illustrated that the vortex descending velocity is determined by the gas velocity rather than the width of the column. The trends found by Lin et al. may provide a qualitative link to frequencies found in 3-D bubble columns.

The group of Van den Akker is studying the dynamic behaviour of flows in 3-D bubble columns by means of LDA. This study implies also the applicability of LDA in bubble columns. Groen has studied the affects of bubble response, [Groen(1999)], by considering a bubble train and trying to measure bubbles with LDA. He found the number of bubble responses to be small compared to number of tracer particle responses. Measurements considering the transient behaviour of the flow were done in a 15.2, 23.4 and 38.4 cm column, [Mudde and Groen(1997)]. Mudde and Groen showed that the frequency spectra found, obeyed more or less the famous \(-\frac{5}{3}\) power law of Kolomogorov. They also found dominant frequencies in the range of 0.1 to 0.2 Hz. With joint time frequency analysis, (short time Fourier) the time/frequency spectrum was calculated. Figure 2.5 shows that contour lines are more or less parallel to the frequency axis. The investigation discussed in this report continues more or less the work started by Mudde and Groen and involves the measurements in the 15 cm column.
Figure 2.3: Definition of vortex size and wavelength, Lin(1996).
Figure 2.4: Variation of wavelength and frequency with gas velocity in columns of different width, Lin (1996)

Liquid velocity field in a bubble column

Figure 2.5: Short-time frequency contour plot of the axial velocity at $r/R=0.94$ and $U_{gs}=4.4$ cm/s, 23 cm column, Mudde and Groen (1997)
Chapter 3

Measuring Techniques

3.1 The Laser Doppler Technique

The so-called laser Doppler anemometry or LDA is based on the fact that monochromatic light reflected on a particle is shifted in frequency. Since the particle velocities commonly encountered are very small compared to the velocity of light, the corresponding Doppler shifts are very small. The small shifts are impossible to detect, because they are beyond the resolution of the highest resolution optical spectrometers. The only way to measure these small frequency shifts is by making use of a second or reference beam, while using the principle of optical beating of two frequencies in a device having a non-linear response. There are various ways to implement this, see [Drain(1980)].

When measuring in a fluid seeding particles have to be added to the flow. The seeding particles move along with the flow and should have the same speed as the flow. This is one reason why the seeding particles have to be very small. The speed of the flow is measured via the seeding particles, which pass the measurement volume. The measurement volume is formed by the crossing of the two laser beams. The passing of a particle will result in a so-called burst, which will be detected by the detector. From the burst the Doppler frequency is eventually obtained after a series of complex operations.

In a bubble column a difficulty arises, the bubbles. When the laser light encounters a bubble, the beam is blocked and prevents the measurement volume to be formed. As a result no signal is measured. When using LDA in a bubble column back scattering has to be used. This way the light encounters less bubbles. Backscattering is a differential Doppler technique. In this alternative Doppler technique a reference beam is not used: instead optical beating takes place between two beams scattered through different angles. The beat signal or burst has a frequency equal to the difference between the Doppler shifts for two angles of scattering. The scattering volume is illuminated simultaneously by two focussed beams of similar intensity and inclined at an angle, $\alpha$, as shown in figure 3.1. Doppler shifts on scattering to
the detector from the two beams are given by the following equations:

$$\Delta \nu = \frac{\nu v}{c} (\cos \theta_1 + \cos \theta_2)$$  \hspace{1cm} (3.1)

$$\Delta \nu' = \frac{\nu v}{c} (\cos \theta'_1 + \cos \theta'_2)$$  \hspace{1cm} (3.2)

where $\nu$ is the frequency and $c$ is the speed of light. The difference frequency, $f$, observed by the detector equals

$$f = \Delta \nu - \Delta \nu' = \frac{\nu v}{c} (\cos \theta_1 - \cos \theta'_1) = \frac{2\nu}{\lambda} \sin\left(\frac{\alpha}{2}\right) \cos \beta$$  \hspace{1cm} (3.3)

where $\alpha = (\theta'_1 - \theta_1)$, the angle between the two beams, $\beta = \frac{\pi}{2} (\theta_1 + \theta'_1 - \pi)$, the angle the velocity, $v$, makes with the normal to the bisector of the beams and $\lambda$ the wavelength of the laser light.

### 3.1.1 The Fringe Model

An easy way to understand the Laser Doppler technique is in terms of the fringe pattern. The fringe model assumes that two intersecting beams form a fringe pattern of high and low intensity, see figure 3.2. The spacing between the interference fringes, $s_{fringe}$, is given by equation 3.4

$$s_{fringe} = \frac{\lambda}{2 \sin\left(\frac{\alpha}{2}\right)}$$  \hspace{1cm} (3.4)

A particle passing the fringe pattern with velocity, $v$, at an angle $\beta$ normal to the fringe planes will thus experience a modulation of light intensity of frequency, $f$, which is in fact the differential Doppler frequency:

$$f = \frac{2v \cos(\beta) \sin\left(\frac{\alpha}{2}\right)}{\lambda}$$  \hspace{1cm} (3.5)
The velocity perpendicular to the direction of observation $v_\perp$ equals

$$v_\perp = v \cdot \cos(\beta)$$  \hspace{1cm} (3.6)

Thus, the scattered light fluctuates in intensity with a frequency equal to

$$f = \frac{v_\perp}{s_{\text{fringe}}}$$  \hspace{1cm} (3.7)

### 3.1.2 Measuring the Velocity

In order to determine the direction of the particle moving through the scattering volume (up or down) a pre-shift has to be added to one of the two beams. This way the beat signal equals the Doppler shifts plus the pre-shift. If a pre-shift is added, the measured difference frequency becomes $f_{++} = f + f_{\text{pre}}$. With equation 3.7 it follows that $f_{++}$ becomes:

$$f_{++} = \frac{v_\perp}{s_{\text{fringe}}} + f_{\text{pre}}$$  \hspace{1cm} (3.8)

It’s easy resolved that $v_\perp$ is:

$$v_\perp = s_{\text{fringe}} \cdot (f_{++} - f_{\text{pre}})$$  \hspace{1cm} (3.9)

This is the velocity to be measured. It can be seen that the sign of the velocity is now known thanks to the pre-shift frequency. One should notices that the pre-shift frequency has to be chosen in such a way that it captures the velocity range of the flow.
3.1.3 The Scattering or Measurement Volume

The region of fringes formed by the crossing of the beams from which signals can be obtained is often called the scattering or control volume. A particle moving in the control volume produces a signal proportional to the modulated component of the intensity distribution or fringe amplitude. For intersecting parallel Gaussian beams of equal width, the contours of equal fringe amplitude are ellipsoids centred on the central cross-over point as can be seen in figure 3.3. A convenient definition of the limit of the scattering volume is that contour corresponding to a fringe amplitude $\frac{1}{\sqrt{2}}$ of its maximum value.

A Gaussian beam focussed in water by a thin lens, propagating only through flat surfaces has the following effective diameter.

$$\delta_y = \sqrt{n_{air-water}} \frac{4\lambda F}{\pi d_{beam}}$$  \hspace{1cm} (3.10)

where $F$ is the focal length. It now follows that the length of the scattering volume is equal to

$$\delta_z = \frac{\delta_y}{\sin \frac{\alpha}{2}} = (1.3mm)$$  \hspace{1cm} (3.11)

and the width of the volume

$$\delta_x = \frac{\delta_y}{\cos \frac{\alpha}{2}} = (0.1mm)$$  \hspace{1cm} (3.12)
3.1.4 The Doppler Signal

A particle moving in the scattering volume produces a signal proportional to the modulated component of the intensity distribution or fringe amplitude. The Doppler signals are called bursts. An example of this is illustrated in figure 3.4. Each time a particle passes the scattering volume a bursts is detected. The resulting velocities and the time between them are stored. Together they form eventually a time series of the velocity from which all kinds of interesting properties can be derived.

3.1.5 Signal to Noise Ratios and Noise Sources

There are various noise sources present in the LDA system. Some noise sources are:

- electronic, thermal noise from electronic circuits.
- optical noise.
- light scattered from outside the measurement volume for example dirt, scratched windows, ambient light, etc.
- multiple particles in the measurement volume.
- unwanted reflections from windows, lenses, etc.

The way the various noise sources influence the signal can be described by the signal to noise ratio. It is obvious that one wants to maximise the signal to noise ratio. The signal to noise ratio for the differential LDA technique is given by the following equation:

\[
\frac{\sigma_s^2}{\sigma_N^2} = \frac{w\eta}{4hv\Delta f}
\]  

(3.13)
where \( \overline{r^2} \) is mean square signal current, \( \overline{r^2_N} \) the mean square noise current, \( h \) Planck's constant, \( \nu \) the optical frequency, \( \eta \) the efficiency of the detector and \( \Delta f \) the receiver pass band or filter range. We may substitute for \( w \), the light power reaching the detector from the particle, as follows:

\[
w = \frac{2W_0}{\pi r_0^2} X_s f_s(\theta) \Omega
\]  

(3.14)

where \( W_0 \) is the laser power, \( r_0 \) the radius of the focussed laser beam (to \( 1/e^2 \) intensity points), \( X_s \) is the scattering cross-section of the particle, \( f_s(\theta) \) is the angular scattering factor, and \( \Omega \) is the solid angle aperture of the receiving optics. In order to obtain good measurements it is necessary to select laser power, seeding, optical parameters etc. in such a way that the signal to noise ratio reaches a maximum. The following parameters can easily be influenced. The efficiency of the detector can be changed by the photo-multiplier current. A high photo-multiplier current leads to a higher efficiency of the detector and thus improves the signal to noise ratio. The same applies to the laser power (a high laser power improves the signal to noise ratio). It should be remarked that in practise very large values of photo-multiplier and laser power lead to higher noise levels. So an optimum has to be found experimentally.

The receiver band pass or filter range should be set as small as possible in order to get a good signal to noise ratio. But it cannot be set too small otherwise high velocities are filtered out. Thus, the receiver band pass should be set in combination with the pre-shift frequency in such a way that with a small receiver band pass the velocity range of the LDA-system lies just within the velocity range of the flow. This way the signal to noise ratio is optimised, see figure 3.5.

There are some negative effects on the signal to noise ratio when measuring in a bubble column. Especially the noise caused by laser light scattered from outside the measurement volume increases. There is a lot of extra unwanted scattering light scattered by the bubbles. In particular the bubbles directly behind the measurement volume are causing trouble. Their scattering light is almost in focus and is scattered directly in the lens of the measuring probe. The unwanted reflections lead to an increase in quantum noise in the photo-multipliers. One can imagine that if the laser power is increased, there are stronger reflections of the bubbles and therefore the quantum noise in the photo-multipliers increases. So the laser power should be set with care.

### 3.2 LDA in bubble columns

LDA in a bubble column differs somewhat from LDA in a 'normal' one-phase flow. LDA is used to measure the liquid-phase in the bubble column. The gas-phase in the form of air bubbles disturbs the LDA measurements in several ways.

- bubbles sometimes block the laser beams, preventing the measurement volume to be formed.
bubbles are sometimes measured by the LDA system, as if they were a seeding particle.

- bubbles and their wakes disturb the seeding particles, the particles are therefore no longer distributed homogeneously over the liquid flow.

- reflections of the bubbles cause extra (disturbing) scattering light.

The blocking of the bubbles mainly causes the data-rate to decrease. Bubble gaps in the velocity time series affect the time interval distribution (discussed in next chapter).

Groen showed, [Groen(1999)] that generally the velocity realizations of the bubbles are negligible with respect to the high data-rate of velocity realizations from the liquid-phase.

The fact that seeding particles are not distributed homogeneously over the fluid has consequences for the time interval distribution (discussed in the next chapter). The extra scattering light mainly causes an increase in noise in the detector. Extra noise leads to an increase in so-called multiple validation (discussed in the next chapter).

Especially, when applying LDA in a bubble column or multi-phase system in general, one should always keep the following question in mind: is the flow correctly measured? Therefore, validating the measurements is of great importance, when using LDA in a bubble column.
Chapter 4

Signal Analysis

4.1 Distribution Function of Time between Data

The time between data is defined as the time between two successive Doppler signals. Due to the fact that the passing of seeding particles through the measurement volume is a random process, the time between data varies for each two Doppler signals. The data is unevenly sampled, see figure 4.1.

One could calculate the distribution function of the time between data. This distribution function of the time intervals is an important indication for the quality of the measured time series.

In a single phase flow it is assumed that the seeding particles are distributed homogeneously across the fluid. If this is the case, the process of measuring the velocity of the passing seeding particles is a Poisson process. Theoretically the time between data should be Poisson distributed as is described in the next section. In a bubble column (two-phase flow) seeding particles are not distributed homogeneously over the fluid. So the time between data distribution will not be an exact Poisson distribution.

![Figure 4.1: Difference between evenly and unevenly sampled data.](image-url)
4.1.1 Poisson Distribution

Poisson processes describe the pattern of occurrences of random events. Some examples of Poisson processes are:

• the emission of α-particles from a radio-active substance
• the arrival of cars at a particular road junction
• the data collected by means of LDA.

A process is called a Poisson process, when it has an exact Poisson distribution. Let $N(t)$ be the number of events in a given space of time $(0, t)$ then the probability of finding $i$ events is

$$P[N(t) = i] = e^{-\lambda t} \frac{\lambda^i}{i!}, i = 0, 1, ... \quad (4.1)$$

where $\lambda$ is the mean rate of occurrence of an event. As mentioned before the process of collecting data by means of LDA is a Poisson process. However, when the data is collected in a bubble column the LDA-signal is now and then blocked by bubbles. Instead of one random process there are now two ‘random’ processes: the ‘random’ occurrence of a burst and the blocking of the LDA-signal by the bubbles. Thus, the burst received by the detector will therefore probably not have an exact Poisson distribution, but will most likely have a distribution which will be a combination of on the one hand a Poisson distribution caused by the random occurrence of bursts and on the other hand a disturbing distribution caused by the blocking bubbles.

In fact one can consider two time scales the ‘burst-burst’ (small) time scale, which indicates the time between two consecutive bursts and the ‘bubble-block’ (large) time scale, which indicates the blocking time of the bubbles.

In a one-phase flow the ‘burst-burst’ time scale is directly related to the average data rate (reciprocal of data-rate). In a two-phase flow the burst-burst time scale is also related to the average data-rate, but the data-rate has to be compensated for the blocking of the bubbles. The blocking of the bubbles causes the average data-rate to be smaller than it would be in a single-phase flow. [Ohba and Ogasawara(1976)] have shown that the probability for a laser beam to penetrate into a bubbly flow decreases exponentially with gas-fraction and path length. The ratio of the received (time-averaged) light intensity with bubbles $I$ to that without bubbles $I_0$ is given by

$$\frac{I}{I_0} = e^{\frac{-3 l \alpha}{2 d_b}} \quad (4.2)$$

where $l$ is the path length through the bubbly flow, $d_b$ the bubble diameter and $\alpha$ the gas-fraction along the path. [Mudde and Groen(1997)] showed that the average data-rate follows a similar relation:

$$\frac{\bar{f}_{d,b}}{\bar{f}_{d,0}} = e^{\frac{-l \alpha}{d_b}} \quad (4.3)$$
where \( f_{d,b} \) is the data-rate measured in a bubble column, \( f_{d,0} \) the data-rate if the flow where without bubbles and \( c \) a constant of order 2. An example of the exponential drop in data rate is illustrated in figure 4.2.

The 'bubble-block' time scale is more difficult. It could be estimated by the passing of a bubble through the laser-beam. Given a bubble of diameter \( d_b \) travelling at a velocity \( v \) perpendicular to a laser beam, it can be estimated that the passing of the bubble takes about \( \frac{d_b}{v} \) seconds. This is only valid for low gas-fractions or close to the wall. At higher gas-fractions or further into the column more than one bubble could block the laser beams. This results in larger gaps in the time series. Since the data-rate drops, when the number of bubbles, which are blocking increases, the number of bubbles blocking the laser beams could very well follow a similar relation as the data-rate. When measuring further into the column or at higher gas-fractions the 'bubble-block' time will increase exponentially.

Another way to describe the time interval distribution is by looking at the rate of occurrence of a certain time interval instead of by looking at the rate of occurrence of bursts in a time interval. In that case the time interval distribution becomes exponential.

\[
P[\Delta t] = \frac{1}{t_0} \exp \left( -\frac{\Delta t}{t_0} \right)
\]

in which: \( \Delta t \) is the time interval between two successive Doppler signals or time between data and \( t_0 \) is the average time interval between two successive Doppler signals. In the ideal case (one phase flow), \( t_0 \) is equal to the reciprocal value of the average data-rate of the time series. The advantage of this representation of the
time interval distribution is the fact that if plotted on a log-scale the time interval distribution function becomes a straight line, see figure 4.3. At least this is the case, when the time between data is Poisson distributed. In a bubble column the distribution will most likely not be an exact Poisson distribution as is explained earlier on.

Deviations from the ideal time interval distribution function are caused by various effects and can often tell what is wrong with the time series or measurements. A lot of these effects are described extensively by [Maanen(1999)]. The main effect looked upon in this investigation is the effect of multiple validation.

**Multiple Validation**

What is multiple validation? Basically it is the following: One Doppler signal results in multiple measuring points. The multiple measuring points result in compact clusters of points. Since the time between the multiple measuring points is very small, the small time intervals in the time interval distribution will increase. This is illustrated in figure 4.4.

To compensate for multiple validation the clusters of points are averaged. This way the small time intervals are filtered out. A disadvantage is that high frequencies are filtered out as a result of the averaging. In practice this means frequency above 1000 Hz are lost. Since in this investigation only the low-frequency domain (0-2 Hz.) is considered, the averaging doesn’t seem to be a problem. Another option is removing the clusters of points showing multiple validation from the data set. Removing the entire cluster means a great loss of data, since a lot of points show multiple validation. So, this option isn’t used.

Another way of dealing with the clusters is keeping the first point and discarding
the other points of a cluster. However, since there is no reason why the first point would be the correct point or better than the other points, the average of the cluster of points is taken.

Remains the question what is causing the multiple validation. In order to find the cause of multiple validation, test measurements were done in a controlled situation. A small stirred tank was used for this purpose. These test measurements indicated that multiple validation was most likely caused by a bad signal to noise ratio. A detailed overview of test measurements can be found in the next chapter, 'Test Measurements'.

Figure 4.4: example of multiple validation, Maanen(1999)
4.2 Spectral Analysis

As mentioned before the data obtained by means of LDA are unevenly sampled. FFT techniques are based on evenly sampled data. There are some obvious ways to get from unevenly spaced to evenly spaced time intervals. An easy way to achieve this by laying a grid of evenly spaced times on the data and interpolating the values onto that grid. It's then possible to use FFT methods. There are a few disadvantages of re-sampling the data. [Adrian and Yao(1987)] showed that re-sampling at the mean data rate with sample and hold is essentially low-pass filtering the data which affects the high frequency contents of the signal. In the case of bubbly flow the re-sampling also has to close bubble gaps. [Mudde and Groen(1997)] compared sample and hold (solid line between two data points) with linear interpolation at the averaged data rate. No difference between the two auto-power spectral density was found. This is not surprising since for larger gaps linear interpolation and sample and hold is almost the same. Both linear interpolation and sample and hold add velocities to the time series while filling the bubble gaps. The real question is: is one not adding extra information to the time series by filling the bubble gaps with velocities and as a result cause extra frequencies in the power spectra? The comparison made by Mudde and Groen does not answer this question. Since no clear answer is found another method for calculating the frequency spectrum was investigated.

A completely different method of spectral analysis for unevenly sampled data was developed by [Lomb(1976)]. The Lomb method evaluates data only at times \( t_i \), that are actually measured. Given that there are \( N \) data points \( h_i = h(t_i), i = 1, \ldots, N \), with mean and variance:

\[
\bar{h} = \frac{1}{N} \sum_{i=1}^{N} h_i, \quad \sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (h_i - \bar{h})^2
\]  

The Lomb normalised periodogram, which is the spectral power as a function of angular frequency \( \omega \) is defined by

\[
P_N(\omega) = \frac{1}{2\sigma^2} \left( \frac{\sum_j (h_j - \bar{h}) \cos \omega (t_j - \tau))^2}{\sum_j \cos^2 \omega (t_j - \tau)} + \frac{\sum_j (h_j - \bar{h}) \sin \omega (t_j - \tau))^2}{\sum_j \sin^2 \omega (t_j - \tau)} \right)
\]

\( \tau \) is defined by the relation

\[
\tan(2\omega \tau) = \frac{\sum_i \sin 2\omega t_i}{\sum_i \cos 2\omega t_i}
\]  

J.D. Scargle gives a detailed description of the Lomb method and it's properties in comparison with other statistical methods such as the Fourier method, see [Scargle(1982)]. The Lomb periodogram is based on the classical Fourier periodogram. It has a simple statistical behaviour, and is equivalent to the reduction of
the sum of squares in least squares fitting of sine waves to the data. It reduces to
Fourier if the spacing is even, and has time-translation invariance. The computation
of the Lomb periodogram is not substantially more difficult than that of the Fourier
periodogram. So that even though the numerical differences are small, the Lomb
periodogram is theoretically preferable. Especially, in a bubble column Lomb has
the advantage that the bubble gaps are not filled with non-existing velocities as a
result of re-sampling. Therefore, when using Lomb no extra information is added
to the time series.

4.2.1 Calculating Lomb periodogram

In order to obtain an averaged spectrum, the time series were divided into 100
different blocks. The blocks had an overlap of 75%. From each block the Lomb
spectrogram was calculated and all spectra found, were summed and averaged.
This way the average spectrum of the 100 blocks was obtained. A time series
consisted of about 50-100k points for the axial and 15-25k points for the tangential
component. As a result the block size was 2-4k points for the axial and 0.6-1k
points for the tangential component.

Only low frequencies up to 2 Hz were considered for the calculation of the power
spectra. The resolution of the frequency, \( f_{\text{res}} \), based on the number of data-points
of the signal, \( N \), and the Nyquist frequency, \( f_{\text{Ny}} \), is equal to

\[
 f_{\text{res}} = \frac{2f_{\text{Ny}}}{N} \tag{4.8}
\]

The uncertainty in the determination of a single frequency, \( \delta \omega \), is given by

\[
 \delta \omega = \frac{3\pi \sigma N}{2N_0^{\frac{3}{2}} T A} \tag{4.9}
\]

where \( A \) is the amplitude of a single signal with Gaussian noise, \( \sigma^2 \), the variance
of the noise after the signal has been subtracted and \( T \) the total length of the data
set. The derivation of 4.9 assumes a single signal (sinus) with Gaussian noise and
even data spacing. [Horne and Baliunas(1986)] showed that uneven spacing does
not degrade the uncertainty to any noticeable degree. Multiple signals, on the other
hand, can cause shifts in detected frequencies if they are closely spaced. Since
the signal detected in the bubble column consist of a bandwidth of multiple fre­
quencies, no clear uncertainty of the different frequency peaks can be derived with
equation 4.9. Therefore, the average spectrum of multiple blocks was calculated.
The uncertainty of an average spectrum is related to the reciprocal value of the root
of the number of spectra used. For example 100 spectra lead to an uncertainty in
the average spectrum of about 10%.
4.3 Joint Time Frequency Analysis

The Fourier transform is a well known tool to study a signal's frequency properties. A disadvantage of the classical Fourier transform is the fact that it cannot distinguish frequency changes over time. It is impossible to tell how the frequencies evolve in time. Since one of the goals in this investigation is to measure oscillations which evolve in time, another method for the calculation of the power spectrum is required. With joint time frequency analysis it is possible to calculate the frequency spectrum as a function of the time. In this section two methods are described: the Short Time Fourier Transform and Wavelet Transform. First some basic properties of the classical Fourier Transform are explained in the next section.

4.3.1 The Fourier Transform

The Fourier transform is a very well known and commonly used technique to analyse a signal's frequency behaviour. The frequency representation of a signal \( s(t) \) is given by

\[
s(t) = \frac{1}{2\pi} \int S(\omega) \exp(\imath \omega t) d\omega \tag{4.10}
\]

where

\[
S(\omega) = \int s(t) \exp(-\imath \omega t) dt \tag{4.11}
\]

is named the continuous-time Fourier transform.

Since a signal is often only defined at a finite time and the Fourier transform goes from \(-\infty\) to \(\infty\), there is an uncertainty in the frequency resolution. This is described by the following.

Uncertainty Principle

If

\[
\sqrt{\|s\|} \to 0 \tag{4.12}
\]

for \(\|t\| \to \infty\), then

\[
\Delta_t \Delta_\omega \geq \frac{1}{2} \tag{4.13}
\]

where \(\Delta_t\) is the time duration of the signal and \(\Delta_\omega\) the frequency bandwidth. 

Note: The equality only holds when \(s(t)\) is a Gaussian function, i.e.,

\[
s(t) = A \exp(-\alpha t^2) \tag{4.14}
\]
4.3.2 Short Time Fourier Transform

The Short Time Fourier Transform (STFT) or windowed Fourier transform is defined as

$$\text{STFT}(t, \omega) = \int s(\tau)\gamma_{t,\omega}(\tau) d\tau = \int s(\tau)\gamma(\tau - t)e^{-j\omega t} d\tau \quad (4.15)$$

The function $\gamma(t)$ usually has a short time duration and is named the window function. Figure 4.5 explains the procedure of computing the STFT: first multiply the function $\gamma(t)$ with the signal $s(t)$ and compute the Fourier transform of the product $s(\tau) \cdot \gamma(\tau - t)$. Because the window function $\gamma(t)$ has a short time duration, the Fourier transform of $s(\tau) \cdot \gamma(\tau - t)$ reflects the signal's local frequency properties.

By moving the window function along the signal and repeating the same process, the evolving of the signal's frequencies in time is obtained.

Suppose the function $\gamma(t)$ is centred at $t = 0$ and its Fourier transform is centred at $\omega = 0$. If the time and frequency duration of the the window, $\gamma(t)$ are $\Delta_{t}$ and $\Delta_{\omega}$, then the STFT$(t, \omega)$ indicates the signal's behaviour in the vicinity of $[t - \Delta_{t}; t + \Delta_{t}] \times [\omega - \Delta_{\omega}; \omega + \Delta_{\omega}]$.

In order to have a good time and frequency resolution $\Delta_{t}$ and $\Delta_{\omega}$ should be as narrow as possible. Unfortunately, $\Delta_{t}$ and $\Delta_{\omega}$ are not independent. They are related via the Fourier transform and linked via the uncertainty principle.

$$\Delta_{t}\Delta_{\omega} \geq \frac{1}{2} \quad (4.16)$$

So, there is a trade-off between the selection of the time and frequency resolution. A small time resolution leads to larger frequency resolution and vice versa. The
equality only holds, when the window function is a Gaussian function. The square of STFT is named STFT spectrogram and is the most simple and most used time-dependent spectrum, which roughly depicts a signal's energy distribution in the joint time-frequency domain.

The window function

For the short time Fourier transform a Gaussian window was used. The Gaussian window is described by the following equation:

\[
\gamma(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{2}} \exp\left(-\frac{\alpha}{2} t^2\right)
\]

the size of the window is determined by \( \alpha \). The Gaussian window is zero in \(-\infty\) and \(\infty\) and still is a local function. Since the Fourier transform has a domain from \([-\infty, \infty]\), the Gaussian window is suitable function for the short time Fourier transform. In other words the window runs over the entire time series. The effective size of the window, \( t_{\text{win}} \), is defined as 0.1% of the top value of the window.

\[
t_{\text{win}} = \sqrt{\frac{2}{\alpha} \ln\left(\frac{1}{100}\right)}
\]

For example the value \( \alpha = 0.01 s^{-2} \) leads to an effective size of \( t_{\text{win}} = 37 s \).

Savitzky-Golay Filters

Since only low frequencies are considered in the frequency analysis a low pass (moving average) filter is applied to the data. This way the spectra found, become more smooth. The Savitzky-Golay filter is used for this purpose. It is a low pass filter with good properties in the Fourier domain, but still the filtering takes place in the time domain. Rather than having its properties defined in the Fourier domain, and then translated to the time domain, Savitzky-Golay filters derive directly from a particular formulation of the data smoothing problem in the time domain. The difference between Savitzky-Golay (S-G) and a normal moving average filter lies in the fact that S-G uses coefficients to give each data point a certain weight.

If the digital filter is applied to a series of equally spaced data values, \( h_i = h(t_i) \), where \( t_i = t_0 + i \Delta \). (for constant sample spacing \( \Delta \) and with \( i = \ldots -2, -1, 0, 1, 2, \ldots \) ) The simplest type of digital filter replaces each data value \( h_i \) by a linear combination \( g_i \) of itself and some number of nearby neighbours,

\[
g_i = \sum_{n=-n_L}^{n_R} c_n h_{i+n}
\]

where \( n_L \) is the number of points used "to the left" of a data point \( i \), i.e. earlier than it, while \( n_R \) is the number used to the right, i.e. later than it. For some fixed
\( n_L = n_R \), the coefficients, \( c_n \), are given by the following equation:

\[
c_n = \{ (A^T \cdot A)^{-1} \cdot (A^T \cdot e_n) \}_0 = \sum_{m=0}^{M} \{ (A^T \cdot A)^{-1} \}_{0m} n^m
\]  

(4.20)

where \( e_n \) is the unit vector and \( A \) the design matrix:

\[
A_{ij} = \delta^j_i \quad i = -n_L, ..., n_R \quad j = 0, ..., M
\]  

(4.21)

For a more detailed description see [Press(1992)].

4.3.3 The Wavelet Transform

Traditionally time signals are compared with a harmonically related complex sinusoidal functions, \( \exp(j2\pi nt/T) \). Each harmonic function, \( \exp(j2\pi nt/T) \), corresponds to a particular frequency \( 2\pi n/T \), the Fourier transform. By changing \( n \) all different frequency tick marks are obtained. It's also possible to build frequency tick marks by scaling the time index \( t \) of a given elementary function \( \Phi(t) \). Scaling a signal in the time domain results in inverse scaling in the frequency domain. When scaled and time-shifted elementary functions \( a^{-1}(t - b) \) are employed to analyse the given signal, the resulting presentation is known as wavelets. The elementary function \( \Phi(t) \) is known as the mother Wavelet.

The continuous Wavelet transform is described as

\[
CWT(a, b) = \frac{1}{\sqrt{|a|}} \int s(t) \Phi \left( \frac{t - b}{a} \right) dt
\]  

(4.22)

where \( \Phi(t) \) denotes the mother Wavelet, the parameter \( a \neq 0 \) represents the scale index that is the reciprocal of the frequency and the parameter \( b \) indicates the time shifting. If the signal only needs to be analysed and doesn’t have to be reconstructed from the transform, then the mother Wavelet \( \Phi(t) \) could be any function. Since different mother wavelets lead to different frequency tick marks or Wavelet numbers, selecting an appropriate mother Wavelet is of great importance. An example of a mother Wavelet are the Daubechies wavelets. The Daubechies 4 Wavelet is illustrated in figure 4.6. A more detailed description of the way wavelets work can be found in [Qian(1996)] and [Press(1992)].

Just like the Gaussian window at the STFT, the Wavelet function is shifted over the time series. Instead of frequencies the Wavelet Transform gives Wavelet numbers. The Wavelet numbers have to be translated to time/frequency information. In practice this proves to be rather difficult.
Figure 4.6: Wavelet function: Daubechies 4 (x-axis represents the time [s])
Chapter 5

Experimental Setup

5.1 The Bubble Column

The apparatus consist of a column with diameter $D_{column} = 15.2$ cm filled with tap water and aerated via a sintered polyethylene porous plate as can be seen in figure 5.1. The pores are 40 $\mu m$ in diameter and the porosity is 40%. The gas (air) enters the column via the porous plate and the gas flow is measured via gas flow meters mounted next to the column. This way bubbles were formed with a diameter of approximately 3 mm. The laser beams enter the column through a flat faced window mounted into the column wall. This way the beams are not distorted by the curved column wall. The bubbles are responsible for the motion of the liquid (water), which is measured by the LDA system. Therefor seeding has to be added to the fluid. The seeding consisted of aluminium coated particles with a diameter of approximately 5 $\mu m$.

5.1.1 Bubble Size

As mentioned the bubbles had a diameter of about 3 mm. The variation in bubble size is discussed in this section. The ideal gas equation shows that the bubble size is directly related to the pressure:

$$P_{top}V_{top} = P_{bottom}V_{bottom}$$  \hspace{1cm} (5.1)

where $V$ is the bubble volume and $p$ is the pressure. If a bubble column is filled with water and the un-gassed height is 1.35 m, it follows that the pressure difference between the top and bottom is approximately 0.135 atm. As a consequence the bubble volumes vary only a factor 1.135 from bottom to top ($V_{top} = 1.135V_{bottom}$). The radius varies at maximum a factor 1.04. It follows that the variation in bubble size is of the order of 0.1 mm. So, the bubble size is assumed to be constant in the entire column.
Figure 5.1: Overview of the experimental setup.
5.2 The LDA System

The LDA equipment consists of a 4W Spectra-Physics \( \text{Ar}^+ \) laser and a TSI 9201 colorburst multicolour beam separator. Figure 5.2 shows a schematic overview of the LDA system. The beam pairs are focussed with a 350mm lens. The beam spacing, \( D_{\text{beams}} \), is 50mm. The beam diameter, \( d_{\text{beam}} \), is 2.10mm. The green laser beam had a wavelength, \( \lambda=514.5 \) nm and the blue beam pair of \( \lambda=488.0 \) nm. The detected light is sent to the TSI 9230 colorlink. The Doppler signals are processed with processor type IFA 750, controlled by a 486/66MHz PC. The Doppler signals were monitored by an oscilloscope.

5.2.1 Propagation of the Laser Beams through the System

The propagation of light is determined by the Maxwell equations. However, in practise the exact solutions of Maxwell's equations are seldom necessary in optical problems. Approximations valid for distances of propagation large compared with the wavelength will be used. Elementary geometrical optics are all that is needed to determine the propagation of light beams. In this approximation the laws are as follows:

- In a uniform medium light travels in straight lines.
• When rays are reflected at a smooth surface, the angle of incidence is equal to the angle of reflection.

• When passing from one medium to another, light is refracted, the angles of incidence and refraction $\phi_1$ and $\phi_2$ (see figure 5.3) being related by Snellius' law:

$$\frac{\sin \phi_1}{\sin \phi_2} = \frac{\mu_2}{\mu_1} = n_{\text{air-water}}$$  \hspace{1cm} (5.2)

where $\mu_1$, $\mu_2$ are the refractive indices of the media for the particular wavelength used and $n_{\text{air-water}} = 1.33$ is the refractive index of air to water for wavelength $\lambda$ of approx. 500 nm at water temperature of $20^\circ$ C.

The angle of incidence, $\phi_1$ is easily obtained from the distance between the two laser beams and the focal length of the lens. The angle of refraction, $\phi_2$ can be calculated with Snellius' law. With equation (3.4) the fringe spacing can be calculated. The fringe spaces for the 350mm lens are equal to 3.629$\mu$m (green beam pair) and 3.425$\mu$m (blue beam pair). The 250mm lens has a fringe spaces of 2.608$\mu$m (green beam pair) and 2.452$\mu$m (blue beam pair).

A lens is used for focussing the laser beams. A lens may give rise to the problem of spherical aberration. Although the use of well corrected lenses will minimise the beam crossing errors, it is necessary to project the beam through a perspex window. Apart from perturbations introduced by insufficiently flat surfaces, there can be a focussing error similar to that produced by the spherical aberration of a lens. The aberration of a flat window is equal to

$$OO' = \frac{d_{\text{window}} \phi_2}{2\mu} \left(1 - \frac{1}{\mu^2}\right)$$  \hspace{1cm} (5.3)
where \( d_{\text{window}} \) is the diameter, \( \mu \) the refractive index of the window and \( \phi_2 \) the angle of refraction. The refractive index of the perspex window to air is 1.49, the diameter is 17.1 mm and the angle of refraction was \( \phi_2 \).

The medium behind the window is water, so instead of the refractive index to air the refractive index from perspex to water is needed, \( \mu = 1.12 \). Thus, with equation 5.3, it follows that the aberration of the window, \( OO' \) is of the order of 0.01 mm. This is small compared to the beam diameter (2.10 mm).

### 5.3 Measurement Positions

Figure 5.4 shows an overview of the different measurement positions. For the main measurements the gas-fraction was fixed at 10 %. Both the axial and tangential component were measured at 9 different heights (44, 54, ..., 124 cm) and one radial position (6.6 cm). Measurements of only the axial component were taken at different gas-fractions (0.1 to 10 %), 3 different heights (49, 64 and 84 cm) and different radial positions (2.0 to 7.5 cm). Further measurements without seeding were conducted at a gas-fraction of 7.7 % and different radial positions (5.0 to 7.5 cm).
5.4 Data processing

The measured time series consisted of about 100k-200k data-points and the time of the series ranged from 800 to 900 s. The time series used for the average radial velocity profile were smaller (about 420 s). The data-rate ranged from 150 to 250 Hz for the axial component and 20 to 80 Hz for the tangential component. The LDA-system had a time-out of 900 s. The time series were stored in binary format on the hard disk of the 486 PC. After copying the data, the data was converted to ASCII. An example of the ASCII data-set is shown in the table below.

<table>
<thead>
<tr>
<th>Processor</th>
<th>Number of Cycles</th>
<th>Burst time</th>
<th>Time between Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>proc</td>
<td>( N_{cycles} )</td>
<td>( t_{burst} ) [ns]</td>
<td>( tbd ) [( \mu s )]</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>.309280e+5</td>
<td>8979</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>.307040e+5</td>
<td>926</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>.314880e+5</td>
<td>1577</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>.301120e+5</td>
<td>451</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>.313920e+5</td>
<td>1372</td>
</tr>
</tbody>
</table>

The Doppler frequency plus frequency pre-shift, \( f_{++} \) is calculated from the data-set with the following equation.

\[
f_{++} = \frac{N_{cycles}}{t_{burst}} \tag{5.4}
\]

The velocity, \( v_\perp \) is obtained with equation 3.9. This results in the following data-set:

<table>
<thead>
<tr>
<th>Time between Data</th>
<th>Processor</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( tbd ) [( \mu s )]</td>
<td>proc</td>
<td>( v_\perp ) [m/s]</td>
</tr>
<tr>
<td>8979</td>
<td>1</td>
<td>0.18023</td>
</tr>
<tr>
<td>926</td>
<td>1</td>
<td>0.19478</td>
</tr>
<tr>
<td>1577</td>
<td>1</td>
<td>0.14475</td>
</tr>
<tr>
<td>451</td>
<td>2</td>
<td>-0.00637</td>
</tr>
<tr>
<td>1372</td>
<td>1</td>
<td>0.15075</td>
</tr>
</tbody>
</table>

The axial (1) and tangential (2) components were separated and the time was derived from the time between data, resulting in two data-set containing the velocity at a given time.

<table>
<thead>
<tr>
<th>Axial Component</th>
<th>Tangential Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Velocity</td>
</tr>
<tr>
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<td>( v_{ax} ) [m/s]</td>
</tr>
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<td>0.18023</td>
</tr>
<tr>
<td>0.009905</td>
<td>0.19478</td>
</tr>
<tr>
<td>0.011482</td>
<td>0.14475</td>
</tr>
<tr>
<td>0.013305</td>
<td>0.15075</td>
</tr>
</tbody>
</table>
The axial component contained more data due to the higher data-rate. After this elementary processing of the data the time series have to be validated. When looking at the time series one notices that some points seem to have an unreal velocity, see figure 5.4. They are probably caused by light scattered of bubbles or other unwanted scattering light. To filter these points out, all points outside the band $\bar{u}_L \pm 3\sigma_{u_L}$ are discarded, see figure 5.4. It must be remarked that only a few velocities are filtered out this way. The Lomb periodogram was calculated of the filtered data to obtain a frequency diagram. The data still showed spurious data, due to noise. A Savitzky-Golay filter, which is a moving average filter was used, see figure 5.4. After filtering the data the Short Time Fourier Transform was taken resulting in a joint time frequency diagram of the time series. The joint time frequency diagram was compared to the time series to link the frequency peaks to velocity changes, which might indicate the presence of a vortex.

5.4.1 Validating the Time Series

The average velocity and standard deviation of the time series were calculated in order to calculate the average velocity profile. An important measure for the quality of a data-set obtained by LDA is the distribution of the time between data. Deviations from the ideal distribution (equation 4.4) might indicate poor quality of the time series and should be explained in order to validate the time series. In practise the ideal distribution is not found, when measuring in a bubble column, due to the fact that the bubbles are disturbing the LDA-measurements.
Figure 5.6: Savitzky-Golay filtering
Chapter 6

Test Measurements

6.1 Test Measurements in small stirred tank

The data-sets obtained with the LDA-system showed clustered data. In a small time-interval (order of 0.1 ms) multiple bursts are measured with a spread in velocity of 0.05 m/s. Questions are: 'What is the effect of the clusters on the reliability of the data?' and 'What causes the clustered data?' A cluster is in fact multiple validation of a Doppler signal. To determine whether the phenomena of clustered data is typical for a bubble column or is caused by the LDA-system, measurements are taken in a stirred tank and compared to data measured in a bubble column. The time interval distribution of the time between data is investigated and compared to the theoretical value.

6.1.1 Experimental Setup

The measurements were conducted in a stirred tank and the bubble column. The stirred tank consisted of a small glass cup with a magnetic stirrer. All other settings were the same as the ones of the experiments with the LDA-system in the bubble column. An overview of the experimental setup is illustrated in figure 6.1.

The following parameters were checked: threshold-level, seeding-level, single burst on or off, cycles per burst, filter range and preshift-level.

6.1.2 Results

The time interval distribution of the axial component measured in the bubble column shows abnormal behaviour at small time intervals. Figure 6.2(a) shows the same peak for small time intervals as figure 4.4, which shows an example of multiple validation of the Doppler signal. Thus, multiple validation seems to be the cause of the abnormal time interval distribution measured in the bubble column. The measurements in the bubble column were compared to the same sort of measurements in the small stirred tank. The results in figure 6.2(b) show that the measurements in the stirred tank have a normal exponential pattern for the time interval.
Experimental setup.

Figure 6.1: Setup of LDA measurements in small stirred tank

Figure 6.2: Time interval distribution of bubble column compared to stirred tank
Figure 6.3: Time interval distribution of the blue beam pair, i.e. tangential velocity distribution. It matches the model. The spread in the bins at larger time intervals can be explained by the fact that there are less points per time-interval for the calculation of the probability, as a result the uncertainty will be larger.

As could be seen in figure 6.2(b) the axial component (green beam) had a linear course. In other words the data seems to be reliable. The tangential component (blue beam) showed a peak at small time intervals just like the data of the bubble column, see figure 6.3. This raises the following question: why is blue beam pair performing less? The answer to the question can be found, when the cause of the multiple validation is known. In order to find the cause various parameters were checked.

**Different Threshold levels**

The influence of different threshold levels on the time-interval distribution was examined in the stirred tank. The axial component performs good. At higher threshold levels the slope decreases, see figure 6.4. Which is logical, because the average data-rate reduces at higher threshold levels.

The tangential component performs badly at all measured threshold levels, see figure 6.5. The average data-rate decreases not as much as one would expect. This could indicate that more noise was measured. But multiple validation seems not to increase or decrease. It seems that the threshold level has no measurable effect on multiple validation. It should be noted that on the oscilloscope the axial component had a low noise band (300 mV) with distinct bursts much larger in amplitude than the noise band. While the tangential component showed a large noise band (900 mV) and relative weak bursts. This could be an indication that the data is affected by multiple validation, due to a bad signal to noise ratio.
Figure 6.4: Time interval distribution: different threshold levels (axial)
Figure 6.5: Time interval distribution: different threshold levels (tangential)
The measurements at different seeding levels show the same pattern as the measurements at different threshold levels. Due to a lower data-rate the slope decreases. The axial component performs good at all seeding levels, see figure 6.6. The tangential component performs bad at all seeding levels, see figure 6.7. It can be concluded that at least for the measured seeding concentrations the amount of seeding does not affect the time interval distribution in a negative way.

**Different Seeding Levels**

The measurements at different seeding levels show the same pattern as the measurements at different threshold levels. Due to a lower data-rate the slope decreases. The axial component performs good at all seeding levels, see figure 6.6. The tangential component performs bad at all seeding levels, see figure 6.7. It can be concluded that at least for the measured seeding concentrations the amount of seeding does not affect the time interval distribution in a negative way.

**Single Bursts on/off**

With single burst per measurement set to off, more than one validation per burst is possible. In this case there should be multiple validation of the data. That this is the case is shown in figure 6.8. The increase in small time intervals is clearly
Figure 6.7: Time interval distribution: different seeding levels (tangential)
Axial component: single burst on (stirred tank)

Figure 6.8: Time interval distribution: single burst on/off (axial and tangential)

present. The time interval distribution with single bursts off, looks a lot like the other abnormal distributions (for example: measurements in the bubble column and with the blue beam pair). The measurements of the tangential component show that the time interval distribution shows a larger peak at small time intervals, when single bursts is set to off. It seems plausible that multiple validation is the cause of the abnormal time interval distribution.

Effects of Signal to Noise Ratio

Remains the question what causes the multiple validation. When looking at the axial and tangential component the following is noted. The axial component (green beam pair) doesn’t show multiple validation, while the tangential (blue beam pair) does. The tangential component had a smaller signal to noise ratio than the axial component. So noise seems to be the cause. In order to test this hypothesis some
other measurements are conducted. The axial component showed a normal time interval distribution in the previous test measurements. To check, whether noise could be the cause of the multiple validation, the settings of the axial component were deliberately set in such a way that the signal to noise ratio was small (The noise-band on the scope was 900mV in stead of the 300mV in the earlier experiments). Figure 6.9 shows that multiple validation now takes place in the axial component (green beam pair) too. It seems that an increase in noise leads to an increase of multiple validation. Equation 3.13 shows, that an easy way to influence the signal to noise ratio is by changing the filter range. Measurements were conducted at different filter ranges. A larger filter range should lead to a smaller signal to noise ratio and thus to an increase of multiple validation. Figure 6.10 shows that this is the case. For small filter ranges there is almost no multiple validation, while at larger filter ranges the multiple validation increases as was expected.

Photomultiplier of blue beam

The test measurements showed, that the blue beam pair was performing significantly less than the green beam pair. Suspected was, that the photomultiplier components of the blue beam pair caused the small or bad signal to noise ratio. The photomultiplier was changed and new test measurements of the blue beam pair (tangential) were conducted. Different photomultiplier voltages were tested. Figure 6.11 shows, that no multiple validation occurs. The switching of the photomultiplier has resulted in a far better or larger signal to noise ratio. The noise-band of the blue beams pair was varied from 300 to 1700mV at photomultiplier voltages of 800 V to 1000 V. In all three cases there was no multiple validation. In previous measurements (with old components) the noise band was already 1100mV at 800 V and beyond 2000mV at 1000 V. So noise was reduced significantly. The time
Figure 6.10: Time interval distribution: different filter ranges (axial)
Figure 6.11: Time interval distribution: different photomultiplier voltages (tangential)

interval distribution was also determined at different filter ranges. At a range of 100-1000kHz no multiple validation took place, but starting from 300-3000kHz the multiple validation arose again. Although the blue beams are performing better with the new components, a large filter range may still give rise to multiple validation.

At first the data-rate of the blue beam pair was almost zero in the bubble column, as a result only one component (axial) could be measured. After the changing of the photomultiplier it was now possible to measure the tangential and axial component simultaneously.
6.1.3 Compensating for Multiple Validation of the Doppler Signal

It seemed that the multiple validation problem was solved now. Unfortunately, in the bubble column multiple validation still occurred. The bubbles are responsible for a dramatic increase in noise, due to extra scattering light of the bubbles. Two questions remain: is data showing multiple validation still reliable and is it possible to compensate for multiple validation? One possible way is removing the clusters of points showing multiple validation from the data set. But removing means a great loss of data, since a lot of points show multiple validation. So, this isn’t an option. (There would not be enough data left for good frequency analysis.) Another way to compensate for multiple validation might be averaging clusters of points. In this case the small time intervals are filtered out. Looking at the figures measured in the bubble column showing multiple validation, 6.13 (a) and (c), it seems that time intervals smaller than 1 ms are causing the trouble.

The solution could be averaging all points spacing less than 1 ms from each other. In order to do this every points in the data set is checked, whether it is spacing less than 1 ms from the previous point. If this is the case the point is added to the cluster, else a new cluster begins with the point and the previous cluster is averaged. This raises the following question: how does averaging affect the time interval distribution of the data? In order to answer this question several cases showing multiple validation are compensated. The time interval distribution is calculated and compared with the distribution based upon the average data-rate. First the measurements with 'single burst set to off' are corrected for the effect of multiple validation. The results of the correction are shown in figure 6.12. Both the axial and tangential component show that the theoretical values of the time interval distribution based upon the new data-rate (without time intervals below 1 ms) match the corrected time interval distribution of the data. Though it should be noted that the spread in the bins is quite large. This is caused by the fact that the small test-sets (2.5-7.5k) become even smaller (0.5k), so that the calculated time interval distribution has a larger uncertainty.

The same correction was applied to data measured in the bubble column. Figure 6.13 shows that the corrected data do not lie on a straight line. The theoretical line based upon the average data-rate does not match the points of the corrected data. This is expected, because in reality the measured average data-rate is not a good measure for the time interval distribution, because of the blocking of the bubbles. The fact that bubbles block the LDA-signal now and then, decreases the average data-rate of a time series. The average data-rate turns out to be lower than the rate one would expect based on the 'small' time interval distribution. The blocking of the bubbles affects the 'large' time interval distribution (> 0.1s) while leaving 'small' time interval distribution (< 50ms) more or less in tact. In other words the 'small' time interval distribution is less affected by bubble gaps, while the average data-rate is.
Figure 6.12: Time interval distribution: compensation of multiple validation
Figure 6.13: Time interval distribution: compensation of multiple validation
6.1.4 Conclusions

Multiple validation occurs, when the signal to noise ratio is small. Improving the signal to noise ratio reduces multiple validation. Selecting a small filter range increases the signal to noise ratio and thus will reduce multiple validation. Multiple validation couldn't be eliminated from the data-sets measured in the bubble column. Light scattered of the bubbles increases the noise in the system dramatically. An optimum for the signal to noise ratio has to be found by selecting the different LDA-settings in such way that the signal is maximised and the noise is minimised. The only way to remove the multiple validation was by averaging clusters of points. This correction seems to be possible, but leads to loss of information of the flow (high frequencies above 1000Hz are lost). In general testing the measurement settings with small test-sets can be beneficial. Optimising the different parameters of the LDA-system should be done before any real measurements can be conducted. This way multiple validation can in most cases be avoided. In the case of the bubble column it was impossible to reduce multiple validation to zero.
6.2 Test Sets

The algorithms used for the spectral analysis of the data are tested with some test-sets. The test-sets consisted of white noise (sampled from LDA-system noise band) and two known frequencies ($\frac{2}{2\pi}$ and $\frac{1}{2\pi}$ Hz). The amplitude ratio was 1:1. The test sets were made in such a way that they represent the same frequency range (0 to 2 Hz) as could be found in the bubble column, [Mudde and Groen(1997)]. Like the real data sets the test sets were unevenly sampled (noise from the LDA system was used). This way the test sets were very comparable to real data-sets.

The first test-set just contained the two frequencies over the whole range. The second test-set consisted of the same two frequencies, but alternating in time. To construct the second test-set the data-set containing the white noise was divided into 20 blocks. To block 1 a frequency of $\frac{2}{2\pi}$ Hz was added, to block 2 one of $\frac{1}{2\pi}$ Hz, to block 3 one of $\frac{2}{2\pi}$ Hz, to block 4 one of $\frac{1}{2\pi}$ Hz, etc.

Figure 6.14 shows the Lomb spectrogram of the test-sets. The two pictures (a and b) are roughly the same. They show clearly that the two frequencies are present, but no time-information can be obtained from the Lomb spectrogram. (This the reason why the pictures look the same.)

The Short Time Fourier Transform or STFT of the two test-sets was taken for 3 different window sizes: $\alpha=0.1 \ s^{-2}$ (≈12 s), $\alpha=0.05 \ s^{-2}$ (≈17 s) and $\alpha=0.01 \ s^{-2}$ (≈37 s), see equation 4.18. The results are shown in the figures 6.15, 6.16 and 6.17.

On notices that a small window size leads to good time resolution, see figure 6.15 (b) and a poor frequency resolution, see figure 6.15 (a). The opposite applies for a large window size, see figure 6.17 (a),(b). A compromise has to be found between frequency and time resolution, thus a window size of $\alpha=0.05$ was taken. It has a reasonable time and frequency resolution, see figure 6.16.

Another approach is the Wavelet Transform (WT), instead of frequencies Wavelet
Figure 6.15: STFT of constructed test-sets, $\alpha = 0.1 \text{ s}^{-2}$

Figure 6.16: STFT of constructed test-sets, $\alpha = 0.05 \text{ s}^{-2}$
coefficients are calculated. Generally a good time resolution can be obtained with a Wavelet function smaller in time than a comparable STFT-window would be, see figure 6.18(b). The problem is the interpretation of the diagrams. Especially the frequency information is difficult to extract from the Wavelet diagram. At this stage no information of the flow can be obtained with the Wavelet transform. Since it is possible to choose any Wavelet one likes, a suitable Wavelet has to be designed and understood in order to translate the Wavelet coefficients to reliable time-frequency information.
Figure 6.18: WT of constructed test-sets, Wavelet function size = 5 s.
6.3 Test Measurements Bubble Column

The superficial velocity of the dispersed phase was calculated from the gas-flow at different hold-ups. The result is compared with the empirical model of Richardson & Zaki. The terminal velocity for a bubble in tap water of 3 mm is 23 cm/s. The corresponding Reynolds-number was about 690. So \( n \) in the equation of R&Z is 2.39. The error in the superficial velocity, \( j_d \), is obtained by reproducing the measurements of the flow several times. The error in the hold up increases with the gas-fraction. Since at higher gas-fractions the surface of the liquid is harder to define, the determination of the gassed height becomes less accurate. The error was estimated to be about 4\% of the measured value of the hold up. Figure 6.19 shows the measured superficial velocity in comparison with the model for different gas-fractions. For higher gas-fractions the superficial gas velocity found is slightly higher than the superficial gas velocity calculated with the model. However, it can be concluded, that at least for gas-fractions smaller than 15\%, the measured velocities are within the margin of error compared to Richardson and Zaki.
Table 6.1: Average data-rate at different radial positions

<table>
<thead>
<tr>
<th>radial position in cm</th>
<th>average data-rate in Hz</th>
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<tbody>
<tr>
<td>7.2</td>
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</tr>
<tr>
<td>6.8</td>
<td>4.3</td>
</tr>
<tr>
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<td>5.7</td>
</tr>
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<td>2.9</td>
</tr>
<tr>
<td>4.8</td>
<td>0.3</td>
</tr>
</tbody>
</table>

6.3.1 Measurements without Seeding

For different radial positions the data-rate was measured 7 times. The average data-rate and the corresponding radial position are given in table 6.1. It shows that the data-rate is very low (about 50 times lower) compared to 'normal' measurements. The found burst can be caused by bubbles or by dust-particles. However, the data-rate is so low that even if all the bursts are caused by bubbles, they will hardly affect the measurements.
Chapter 7

Results

7.1 Measurements directly through the column wall

In order to make the measurements at different heights more easy, it was tried to measure directly through the column wall. This way measurements could be taken from different directions without replacing the flat faced window. The profiles didn't make any sense. Therefore, the measurements were discarded. LDA in bubble column directly through a curved wall proves to be rather hard. The main problem is the fact that the light may get distorted by the curved wall, which might lead to wrong velocities. Another problem is the fact that the axial and tangential component cannot be measured simultaneously.

7.2 Average radial velocity profiles

Some measurements of only the axial component were done. Measuring the tangential component was impossible at that stage, due to bad photomultiplier components. The measurements were conducted at an un-gassed height of 120 cm and at three different heights \(h=49.0, 64.0 \text{ and } 84.0 \text{ cm}\). The hold up was kept constant at four values \((a=1.64, 2.44, 5.51 \text{ or } 9.09 \%\) ), while the radial position was varied \((r=2.0-7.5 \text{ cm}\). The frequency pre-shift was set to 1MHz (which is in fact too high but at this stage of the investigation multiple validation was not known). So the signal to noise ratio was not optimal. Therefore, no frequency analysis of these data-sets was done. All other variables were set as described in the experimental setup.

At all three heights the velocity profiles at different gas-fractions show the same pattern, see figures 7.2, 7.3 and 7.4. This gives confidence in the measured profiles. It can be seen that at low gas-fractions \((\alpha = 1.64 \text{ and } 2.44\%)\) the flow is most likely asymmetric. According to Ueyama and Miyauchi the average velocity should be zero at 0.7 times the radius, which in this case is at \(r = 5.3 \text{ cm}\). It can be seen that at higher gas-fractions \((\alpha = 9.09\%)\) the velocity profile more or less approaches this value. The measurements close to the wall for determining time dependent
phenomena of the flow were therefore done at a gas-fraction of 10%. These measurements were conducted at nine different heights \((h=44,54,64,\ldots 124\text{ cm})\). Both the axial and tangential component were measured, see figures 7.5 and 7.6. The fact that the velocity profiles do not look as 'nice' as the ones in the figures 7.2, 7.3 and 7.4, is caused by the fact that less data points (10k instead of 100K) were used to determine the average velocities, so the spread in the velocities is greater. The reason for this was simple: to save time, since the goal of this investigation was not the determination of the velocity profile, but the time dependent phenomena of the flow close to the wall. Therefore, the main effort was put in getting good and longer time series (200k) close to the wall.

When comparing the measurements of the average axial velocity profiles to other data like the ones of [Hills(1974)] (see figure 2.1), it seems that the average velocities are too low. However, it should be noted that Hills used a different kind of distributor plate. Hills used plates with a number of holes in it, while the distributor plate used in this investigation is a porous plate. Furthermore, [Sailesh and Dudukovic(1990)] showed that at low gas flow rates the type of distributor plate can have significant influence on the hold up distribution. One can imagine that with different hold up distributions the average flow pattern could just as well vary. Since at different heights and at different measurements the same profiles were found, it would be very unlikely that the profiles are wrong or that the multiple validation affected the time series in such a way that it had a significant effect on the average velocity.

The averaged tangential velocity is negative at \(h=84\) and 125 cm, which might indicate that there is an average spiral motion. However, this averaged negative velocity can also be the results of imperfect alignment of the laser beams. In figure 7.1 it is shown that a small rotation around the optical axis of the beams causes the axial velocity to 'leak' into the tangential velocity. The same applies to an axial rotation in that case the radial component 'leaks' into the tangential component. (There is almost no flow in the radial direction, so this effect is negligible.) It remains the question whether the negative averaged tangential velocity is indeed a spiral motion or just caused by imperfect alignment of the laser beams. It should be noted that the time series for the tangential component consisted of only 2-5k data points (which is low compared to the 100k points used for the other profiles). Another interesting feature is the fact that the standard deviation increases closer to the wall, see figures 7.2, 7.3 and 7.4. At first one might guess that the flow is more turbulent close to the wall. A better explanation for the increase in standard deviation is the fact that vortices cause the flow to oscillate. [Lin and Fan(1996)] showed that such oscillations occurred in a 2D bubble column, due to vortices moving downwards. It's likely that such flow structures are also present in a 3D bubble column and cause oscillations in the flow, which in their turn lead to an increase in standard deviation. To relate the standard deviation to flow properties is therefore tricky and can probably only be done correctly, when one understands the oscillations of the flow.
Figure 7.1: Effect of radial rotation on tangential velocity.

Figure 7.2: Velocity profiles at h=49 cm and $\alpha = 1.6\% \ +$, 2.4\% \ - , 5.5\% \ *, 9.1\% \ □$
Figure 7.3: Velocity profiles at h=64cm and $\alpha = 1.6\% '+'$, 2.4\% 'x', 5.5\% '+' $\times$, 9.1\% '$\square$'

Figure 7.4: Velocity profiles at h=84cm and $\alpha = 1.6\% '+'$, 2.4\% 'x', 5.5\% '+' $\times$, 9.1\% '$\square$'
Figure 7.5: Axial Velocity profile at h=44, 84 and 125 cm, $\alpha = 10.0\%$
Figure 7.6: Axial velocity profile at h=44, 84 and 125 cm, $\alpha = 10.0\%$
The average velocity as a function of the gas-fraction measured at a fixed position close to the wall is measured at three heights (49, 64 and 84 cm), see figures 7.7, 7.8 and 7.9. The hold up was varied ($\alpha=1-15\%$), while the radial position was fixed at $r=6.6$ cm. The plots show an almost linear decrease in speed towards zero except for the measurements at a height of 84 cm. At a gas fraction of 12 % the velocity drops suddenly to much higher values. The corresponding standard deviation increases with the gas-fraction at a constant slope. When the drop in velocity occurs (at $h=84$ cm), it increases more rapidly. When comparing the results to earlier measurements done in the bubble column, it can be seen that [Mudde(1999)] also found a drop in velocity, but at a different gas fraction (8-10 %), see figure 7.10. It seems that the drop in velocity does not occur at the same gas-fraction each time. No explanation is found in literature for this sudden drop in velocity. It should be noted that the measurements of [Mudde(1999)] were done in the same bubble column, as a result this behaviour might occur only in this specific bubble column. It could be that this behaviour might have something to do with the air distributor. It is possible that from a certain superficial gas velocity the distributor starts leaking from the side. The leaking causes bigger bubbles to escape from the side. Bigger bubbles cause a stronger liquid re-circulation (because bigger bubbles have more 'inertia'), which could be an explanation for the sudden drop in velocity. Since the drop in velocity is quite sudden and occurs around gas-fractions of 10 %, very low and high velocities can be measured at this range. If the drop in velocity has not yet occurred, velocities in the radial profile tend to be lower, then if the drop in velocity has occurred. In fact lower velocities should be trusted more, since no possible leaking has yet occurred.
Figure 7.8: Velocity as a function of gas-fraction at h=64cm and r = 6.6cm (1) '†', (2) '×'

Figure 7.9: Velocity as a function of gas-fraction at h=84cm and r = 6.6cm (1) '†', (2) '×'
Figure 7.10: Mean axial liquid velocity and st.dev. versus gas fraction, Mudde(1999)
7.4 Measurements at different heights at a fixed position close to the wall.

Measurements were taken of both the axial and tangential velocity at a gas-fraction of 10% and radial position of $r = 6.54$ cm. The pre-shift and filter range were set in such a way that multiple validation is minimal, $f_{pre} = 0.5$ MHz at a filter range of 100-1000 kHz. This way more reliable time series were obtained and a joint time frequency analysis could be performed of the series. The tangential component and axial component were measured simultaneously and all positions were measured two times.

7.4.1 Distribution functions

An example of the probability density functions (pdf) of the measured velocities is illustrated in the figures 7.11. The probability density function of the axial velocity shows a net down flow as is expected for this radial position. From the asymmetric shape of the distribution one can see that the velocity fluctuates towards positive velocities at this position. The probability density function of the tangential velocity is symmetric and situated around zero. Still the standard deviation is quite large, so it seems the tangential velocity oscillates in both positive and negative direction. When comparing the pdf's found to literature, [Mudde and Groen(1997)], the distribution functions seem to be correct. Furthermore, it can be seen that the oscillating behaviour of the flow also shows in the pdf's of the velocity.

The time between data distributions of both the axial and tangential component do not correspond to the theoretical line based on the Poisson distribution, see figure 7.12. This is expected, because the passing of the seeding particles is only a Poisson process under specific circumstances as is explained before, see section 4.1.1.

7.4.2 (Joint Time) Frequency Diagrams

Figure 7.13 shows the Lomb periodogram of the axial and tangential component at a height of 44 cm. Figure 7.14 shows the same for the duplicated measurement. The axial component shows a distinct peak around 0.1 Hz with a spectral power of 180. The tangential component also shows a peak around 0.1 Hz with a power of 30. The tangential spectrum is 6 times weaker than axial spectrum, which seems logical, since the time series of the tangential component also consisted of 6 times less points. Low frequencies have also been reported in 2D bubble columns, [Lin and Fan(1996)]. Lin found frequencies of 0.1-0.3 Hz in the 15 cm 2D column. The frequencies were based on vortices descending along the column wall. In order to link the frequencies found in the 3D bubble column to possible flow phenomena such as vortices, a joint time frequency analysis of the time series was performed.
Figure 7.11: Power density functions of the axial and tangential velocity at $h=44\text{cm}$, where $N$ is number of velocities in a bin.

Figure 7.12: Time between Data distributions (axial) at $h=44\text{ cm}$, where N is normalised number of time intervals in a bin.
Figure 7.13: Frequency spectrum of the axial and tangential component at h=44 cm, (1)

Figure 7.14: Frequency spectrum of the axial and tangential component at h=44 cm, (2)
Figure 7.15 shows the joint time frequency plot of the axial and tangential component at a height of 44 cm (indicated as 1). The results are duplicated, see figure 7.16 (indicated as 2). For the window size $\alpha=0.05 \text{ s}^{-2}$ was used (17 s). The tangential component of figure 7.15 shows two very dominant peaks which causes the rest of the spectrum more or less to disappear. When comparing the axial to the tangential plot (in figure 7.15), one notes that the two frequency peaks more or less match the 'axial' peaks. However, in the duplicated series the tangential spectrum does not show any real peaks, but more a sort of noise bands. The existence of the bands is explainable, since a spectrum has not always the same power density (depends on signal to noise ratio and number of points the spectrum is based on). It is not unlikely that the signal to noise ratio varies over the time series (e.g. bubble gaps could have an influence on it, there are less points in that case for the determination of the spectrum). One can also see that the highest peaks of the axial component are approximately 30, while for tangential component it is only about 1. This in fact means that the spectrum of the tangential component is worthless and probably is only showing noise. No conclusion can be drawn from the tangential spectrum. Therefore, only the axial component is considered for explaining possible flow phenomena.

Some very distinct frequency peaks appear around 0.1 Hz at the axial component, see figures 7.15 a and 7.16 a. The time between the peaks is about 30-50 s. How can this be linked to vortices or other flow phenomena?

If one compares the results to the findings of [Lin and Fan(1996)] in the 2D bubble column, one could state that the time between the frequency peaks could be the time between two passing vortices. and frequency of 0.1 Hz could be a measure for the oscillation or rotation of the vortex. Lin calculated the frequency of the wave motion of the flow by counting the vortices passing in a certain time. He found frequencies around 0.3 Hz. When doing the same in the 3D column (under the assumption that the frequency peaks are vortices) one comes to the following. There are about 5 'vortices' in 150 s. This results in a frequency of 0.033 Hz, which is ten times lower than the frequency found by Lin. It is possible that the vortices move slower in the 3D bubble column. Since the average velocity profiles show also relative low velocities compared to those of Lin, this is not unlikely. Furthermore, it should be noted that the frequency peaks occur at irregular spaced times in the 3D bubble column, so the wave motion as described by [Lin and Fan(1996)] is no longer valid for the 3D bubble column. One can only speak of an 'average' wave motion. The frequency of 0.033 Hz is therefore the averaged frequency of the 'average' wave motion.

Measurements at different gas-fractions are necessary to check whether a similar relation between frequency and superficial gas velocity occurs in the 3D bubble column as Lin found, see figure 2.4.

Since there are no models which can describe the 3D bubble column accurate enough or other results of 3D bubble columns concerning vortices in literature, at this point no hard conclusion can be drawn, whether the frequency peaks are indeed vortices. But there is also no evidence that the peaks are not vortices. Since
Figure 7.15: Joint time frequency plot of the axial and tangential component at h=44 cm. (1)

descending vortices are found in 2D bubble columns, for now it is assumed that the peaks are vortices moving downwards.

Measurements at different heights

The (joint time) frequency plots of the axial component and the Lomb periodograms of both the axial and tangential velocity of the other heights can be found in appendix A. When looking at the different Lomb spectrograms one notices the following. The power spectra of the axial velocity show strong peaks around 0.1 Hz at h=44 cm. The tangential velocity shows also peak at 0.1 Hz at this height, but the peak is six times weaker. When going up from h=44-74 cm the spectrum and frequency peaks become weaker. But it still shows frequency peaks of 0.1-0.2 Hz for both the axial and tangential spectrograms. In the range from h=84-125 cm the tangential spectrum shows no frequency peaks anymore. However, the axial component still shows a relative weak frequency peak at 0.1-0.2 Hz. It seems that the oscillations are stronger in the low part of the column. This might mean that when vortices are moving downwards, they gain in strength. Or that in contrast to the 2D column the vortices are formed in the entrance region. The joint time frequency diagrams are only shown for those which had a significant power.
Figure 7.16: Joint time frequency plot of the axial and tangential component at h=44 cm, (2)
Since the tangential component just was showing noise (the maximum power of the frequency peaks was less than 1), only the time/frequency diagrams of the axial components, for which the power of the frequency peaks was at least 5, are shown. Only at the heights of 44 cm and 54 cm the time/frequency had a sufficient power (of at least 5). The spectra of the measurements higher on the column proved to be too weak for a joint time frequency analysis. The found spectra at those other heights were only showing noise. The (joint time) frequency peaks at \( h=54 \) cm are also very weak (3-6 in power). With some imagination some peaks can be noticed, but one should keep in mind that the spectrum is quite weak. Thus, the value of these spectra seems therefore to be at least questionable. Only the joint time frequency spectra measured at \( h=44 \) cm seem to be valid and trustful.

### 7.5 Reliability of the data

One of the questions when measuring with LDA (especially in a two-phase flow) was: Is the data reliable? The time between data distributions indicated that there was some multiple validation. However, the multiple validation doesn’t seem to have a great impact on the average velocity profiles if the data sets are large enough (+100k points). The found profiles look reliable and are reproduced for the different heights.

The effect of multiple validation was reduced significantly by optimising the LDA parameters (especially filter range and pre-shift frequency). The measurements close to the wall showed therefore far less multiple validation. But still they showed some. To minimise the influence of multiple validation on the power spectra, points spacing less than 1 ms (time intervals in which multiple validation occurred) were averaged. The frequency spectra obtained the with Lomb method were reproduced and had a logical relation in respect to the different heights. This gives confidence in the way multiple validation was dealt with. The fact that the joint time frequency diagrams were only reliable for the axial component at a height of 44 cm, has to do with the strength of the oscillation signals compared to the noise. Since the Lomb spectra are weaker for the tangential component and at higher heights, due to weaker oscillations in the flow, it is logical that the same is the case for the joint time frequency spectra. Further, the joint time frequency spectra are about 6-10 times weaker in comparison to the Lomb spectra, due to the fact that less points are used for the determination of the frequency spectrum. The fact that only reliable joint time frequency spectra were found at \( h=44 \) cm is explainable and not surprising.
Chapter 8

Conclusions and Recommendations

The objective of this investigation was to determine the dynamic behaviour of the liquid flow in a bubble column. The focus was on the low-frequency part of the spectrum of time series measured with LDA. The frequencies found were linked to flow phenomena such as vortices found by other researchers in 2D bubble columns, [Lin and Fan(1996)].

Laser Doppler Anemometry

During the investigation, some problems and difficulties involving measuring with the LDA system were encountered. It was found that measuring with LDA is not as straightforward as one at first might think. Clusters of points showed up in the data sets. The clusters had a large spread in velocity. With the help of the work of [Maanen(1999)] the clusters were linked to multiple validation. Part of the investigation was focussed on the influence of the different LDA settings on the time between data distribution.

It was found that multiple validation corrupts a data set by causing clusters of points with a large spread in velocity. However, since reasonable radial profiles of the average velocity are measured with data sets infected by multiple validation (during this investigation and in the past), it seems that the average velocity of such a data set is still reliable if the set is sufficiently large (100k+ points). This does not mean that multiple validation can be ignored.

Conclusion: it is reasonable to assume that multiple validation has an effect on time dependent features and on the standard deviation of a time series. (It has for sure a negative impact on the signal to noise ratio.) If one is interested in more than the average velocity, multiple validation should definitely be avoided.

Multiple validation could not totally be excluded from the measurements in the bubble column, because of the noise caused by extra scattering light reflected of the bubbles. It was reduced significantly by optimising LDA settings. To restrict the effect of multiple validation on the time series, points spacing smaller than 1
ms from each other were averaged. The fact that results were reproducible gave confidence in the way multiple validation was dealt with.

Time/Frequency Analysis

The frequency spectrum was calculated with the Lomb method rather than with the Fourier Transform. An advantage of this method is that the bubble gaps have a smaller effect on the frequency spectra. For the Fourier Transform the bubble gaps have to be filled with points, which lead to 'unreal' frequencies in the spectrum. The Lomb method (using unevenly sampled data) doesn't suffer from this problem. A disadvantage of the Lomb method is the long computation time compared to Fourier. Only recently computer power is strong enough to use the Lomb method for large time series. Another problem of Lomb is that filtering the (unevenly spaced) data is more difficult. The difficulties are solvable and computation time is also no longer a problem. 

Conclusion: the Lomb method is favourable over Fourier, when the data have large gaps and are unevenly spaced.

The joint time frequency plots were calculated with the so-called Short Time Fourier Transform. It gives an indication as to how frequencies evolve over time. Since Fourier is used, the bubble gaps could have some influence on the frequency spectra. Short time Lomb would be worth considering. Although the filter problem becomes substantially more difficult, the 'bubble gap' problem would be dealt with in a better way.

Conclusion: the Short Time Fourier Transform can only distinguish between different frequency peaks, when oscillations are strongly present (in bottom part of the column). If the spectrum becomes too weak frequency peaks can no longer be distinguished from the noise.

Flow Phenomena

Some insight in time dependent flow phenomena occurring in a 3D bubble column was gained. The wave motion found in a 2D bubble column by [Lin and Fan(1996)] is not equally well defined in the 3D bubble column. The wave motion is more irregular and vortices move more slowly through the column at comparable gas flow rates. Based on the average time spacing between the frequency peaks, it was found that the average frequency of the wave motion is of the order of 0.03 Hz. This is ten times smaller in the 2D bubble column.

The frequencies around 0.1 Hz found in the 3D bubble column are not related to the wave motion, but could be caused by the revolving motion of the vortices. Higher into the column the low frequencies (0.1 Hz) are weaker. If the situation is compared to the 2D column, it seems that vortices gain in strength when they move downwards. Another explanation is that the vortices are formed in the entrance region and move in an irregular way through the column. One problem remains the fact that so little about vortices and other secondary flow phenomena is known.
for 3D bubble columns. There are no satisfactory models or other results available to compare the results with.

Conclusion: Frequencies were found at \( f \approx 0.1 \) Hz. The frequencies are stronger in the lower part of the column and could be linked to vortices moving through the column with a wave motion in the order of 30-50 s.

8.1 Recommendations

Laser Doppler Anemometry

As stated before, measuring with LDA in a two-phase flow is far from straightforward. One should carefully choose the different parameters of the system. Further, test measurements give great insight in the way the system performs and therefore help optimising the different settings of the LDA system. Especially the pre-shift frequency and filter range should be set in such a way that all velocities occurring in the flow can be measured at small filter ranges. A high laser power in general leads to a stronger signal. However, in two-phase flow, a high laser power also leads to an increase in scattering noise of the bubbles. The laser power has to be chosen with care and an optimum between laser power and scattering light of the bubbles has to be found. Dedicated experiments concerning the light intensity of a laser and bubble reflections are necessary in order to get a better insight as to the way the scattering light of the bubbles affects the noise of the LDA system.

Air distributor

It should be noted that various researchers use different kinds of air distributors. Especially at low gas flow rates this leads to different results. An investigation to the effect of different air distributor types on the liquid flow can give more insight. Furthermore, measurements of the radial profile at a wide range of gas fractions range may certainly help in understanding the effects of different air distributor types on the liquid flow.

General Remarks

One of the problems is that there are no models available which describe the dynamic behaviour of the flow satisfactorily. One reason for this is that the flow in a bubble column is behaving chaotically even at low gas flow rates. Simple steady-state models are therefore no longer sufficient to describe the flow in a bubble column. Tracking the vortices in the bubble column with for example PIV or LIF could give insight in the way vortices move through the column and help understand the frequency spectra on the different heights. Further, CFD-simulations can play an important role in understanding the dynamic behaviour of the flow in bubble column, especially when compared with experimental data.
Bibliography


Appendix A

Time-Frequency diagrams of the Different Heights

In this appendix in overview is given of all frequency spectra (calculated with Lomb) at the different heights. The joint time frequency spectra the axial component of the heights of 44 and 54 cm. are plotted. The spectra of the tangential components and spectra of the other heights were too weak to draw any conclusions of them. Measurements are duplicated, indicated with (1) and (2).
Figure A.1: Frequency spectrum of the axial and tangential component at h=44 cm, (1)

Figure A.2: Frequency spectrum of the axial and tangential component at h=44 cm, (2)
Figure A.3: Frequency spectrum of the axial and tangential component at h=54 cm, (1)

Figure A.4: Frequency spectrum of the axial and tangential component at h=54 cm, (2)
Figure A.5: Frequency spectrum of the axial and tangential component at h=64 cm, (1)

Figure A.6: Frequency spectrum of the axial and tangential component at h=64 cm, (2)
Figure A.7: Frequency spectrum of the axial and tangential component at h=74 cm, (1)

Figure A.8: Frequency spectrum of the axial and tangential component at h=74 cm, (2)
Figure A.9: Frequency spectrum of the axial and tangential component at \( h=84 \) cm, (1)

Figure A.10: Frequency spectrum of the axial and tangential component at \( h=84 \) cm, (2)
Figure A.11: Frequency spectrum of the axial and tangential component at h=94 cm, (1)

Figure A.12: Frequency spectrum of the axial and tangential component at h=94 cm, (2)
Figure A.13: Frequency spectrum of the axial and tangential component at h=104 cm, (2)

Figure A.14: Frequency spectrum of the axial and tangential component at h=114 cm, (2)
Figure A.15: Frequency spectrum of the axial and tangential component at h=125 cm, (1)

Figure A.16: Frequency spectrum of the axial and tangential component at h=125 cm, (2)
Figure A.17: Joint time frequency plot of the axial at $h=44$ cm, (1)

Figure A.18: Joint time frequency plot of the axial at $h=44$ cm, (2)
Figure A.19: Joint time frequency plot of the axial at $h=54$ cm, (1)

Figure A.20: Joint time frequency plot of the axial at $h=54$ cm, (2)