Dispersive multi-modal mud-roll elimination using feedback-loop approach

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Summary

In a shallow water environment, mud-rolls are often dominant and appear as a prevailing coherent noise in OBC seismic data. Their complex properties make the noise elimination notably challenging in seismic processing. To address these challenges, we propose a dispersive multi-modal mud-roll elimination method using a feedback-loop approach with a sparse inversion of focal/Radon transformation. In this paper, we illustrate the proposed method, and show some examples on synthetic seismic data to demonstrate its virtues.

Introduction

In a shallow water environment like the Gulf region in the Middle East, mud-rolls are often dominant and appear as a prevailing coherent noise in OBC seismic data. Compared to the primary signal, they are present with higher amplitude, lower frequency and lower apparent velocity. They are dispersive, i.e., different frequencies propagate with different phase velocities, and multi-modal, i.e., each frequency propagates with several phase velocities simultaneously, or different modes exist all together. In addition, their characteristics are spatially variable, i.e., the dispersion properties change from one shot to another across the whole seismic survey area. Furthermore, they are usually aliased due to a large spatial sampling interval. These complex properties make the noise elimination for obtaining high data quality with respect to S/N, resolution and prestack amplitude fidelity a challenging step in seismic processing.

Conventionally, seismic processing is implemented as an open-loop process, in which information about the inconsistency between output and input is not taken into account. Recently, the so-called feedback-loop process has been introduced, see, e.g., Kutscha and Verschuur (2012) and Verschuur et al. (2012) for data reconstruction, Lopez and Verschuur (2012) for multiple elimination and primary estimation, Soni et al. (2012) and Davydenko et al. (2012) for full wavefield migration, and Berkhou (2012) for joint migration inversion. The feedback-loop is described by an inversion problem of its own model parameters in each processing. It contains a feedback-path which connects output with input via a forward modeling module and closes the loop itself, making it possible to evaluate the residual between the forward modeled data and the input measurements, to update the model parameters, and to obtain an optimal solution after several iterations of the procedure (Figure 1).

To address the challenges of mud-rolls and fully adopt the recent advances in seismic processing, we propose a

dispersive multi-modal mud-roll elimination method using the feedback-loop approach with a sparse inversion of focal/Radon transformation.

Figure 1: Feedback-loop diagram.

Theory and Method

The proposed method is described by the feedback-loop in Figure 1, in which mud-roll parameters are found such that the forward modeled mud-rolls should match the input ones. It consists of two stages in fact: first several iterations of the feedback-loop for estimating the essential mud-roll parameters and establishing the mud-roll model; and later iterations for extracting the mud-roll elements from the input measurements and reconstructing these elements, using an operator based on the mud-roll model.

Mud-roll parameterization

First several iterations work for estimating the essential mud-roll parameters and establishing the mud-roll model. Mud-rolls are well parameterized by summing each mode in each frequency band, and the space-frequency dependent mud-roll model N can be formulated by:

$$N(\vec{r} - \vec{r}_s, \omega) = \sum_m \sum_n A_{mn}(\vec{r} - \vec{r}_s, \omega; \alpha_{mn}) N_{mn}(\vec{r} - \vec{r}_s, \omega; V_{mn}) S_{mn}(\omega), \quad (1)$$

$$A_{mn} = \exp(-\alpha_{mn}(\omega) \|\vec{r} - \vec{r}_s\|) \frac{1}{\sqrt{\|\vec{r} - \vec{r}_s\|}}, \quad (2)$$

$$N_{mn} = \exp(-j\varphi_{mn}^N(\|\vec{r} - \vec{r}_s\|, \omega)) = \exp(-j\omega \frac{\|\vec{r} - \vec{r}_s\|}{V_{mn}(\omega)}), \quad (3)$$

$$S_{mn} = \alpha_{mn}^S(\omega) \exp(-j\varphi_{mn}^S(\|\vec{r}_s\|, \omega)), \quad (4)$$

where m is the number of modes, n is the number of frequency bands, and $\vec{r}_s$ is the source location. A_{mn} is the amplitude attenuation term, α_{mn} is the attenuation constant, N_{mn} is the phase term, V_{mn} is the velocity, and S_{mn} is the source signature term including the amplitude and phase. A measured or synthesized source signature S_{mn} is usually

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available, or even can be extracted from the input measurements, and thereby it is a set of known parameters. The attenuation constant α_{mn} and the velocity V_{mn} are unknown parameters, however, an estimate can be obtained from the input measurements by picking the mud-rolls along offset for each mode in each frequency band. Furthermore, the velocity V_{mn} can be updated by the so-called differential time-shift analysis (Berkhout, 1997) in the focal domain (Berkhout and Verschuur, 2006).

Mud-roll velocity estimation

The focal transformation is based on a decomposition of seismic data in certain basis functions, e.g., hyperbolic events, representing the basis events in seismic data. The focal transformation can be regarded as a multi-shift correlation. Forward focal transformation removes spatial phase from the basis event, i.e., it maps the basis event to the time zero and offset zero in the focal space, and inverse focal transformation restores the removed spatial phase, i.e., it maps the basis event back to the original position in the original data space. These transformations are described as a matrix multiplication in the x - f domain with the focal operator representing the basis function. In our case, making use of the mud-roll model $\mathbf{N}$ in the matrix form as the focal operator, the focal transformation can be formulated for each monochromatic component by:

$$(\tilde{\mathbf{P}}^S + \tilde{\mathbf{P}}^N) = \mathbf{N}^{-1}(\mathbf{P}^S + \mathbf{P}^N), \quad (5)$$

$$(\mathbf{P}^S + \mathbf{P}^N) = \mathbf{N}(\tilde{\mathbf{P}}^S + \tilde{\mathbf{P}}^N), \quad (6)$$

$$\mathbf{N}^{-1} = \mathbf{N}^H \mathbf{B}, \quad (7)$$

where $(\mathbf{P}^S + \mathbf{P}^N)$ is the recorded seismic data; $\mathbf{P}^S$ is the primaries; and $\mathbf{P}^N$ is the mud-rolls. $\mathbf{B}$ is an amplitude scaling operator for equalizing the amplitude spectra after the roundtrip of forward (equation 5) and inverse (equation 6) focal transformations. The superscript H is used for denoting conjugate transposition; and the tilde symbol $\sim$ is used for indicating the focal domain here. In this case, the mud-rolls are focused around the focal point, localized in terms of the energy level and areal position, and well distinguished from other events. To sharply focus and effectively enhance the mud-rolls, it is mandatory to properly establish an optimal focal operator, thus an optimum mud-roll model.

Following the differential time-shift concept, with a correct focal operator representing mud-rolls with their correct velocities, the time-reversed focal operator and its related mud-rolls have equal traveltimes for all offsets, and thereby the focal point occurs exactly at the time zero and offset zero in the focal space. Otherwise, with an incorrect focal operator, the focal point is positioned away from the origin. Therefore, it is possible to evaluate the velocity errors based on the time-shift between the origin and the position of the focal point or the energy maximum in the focal

space; to estimate correct velocities from the errors; and to update the velocities and the resulting focal operator. Updating the velocity for each mode in each frequency band, the final velocities fully address the dispersive multi-modal properties of the mud-rolls.

In Figure 1, the input to the feedback-loop is the recorded seismic data containing both primaries and mud-rolls, $(\mathbf{P}^S + \mathbf{P}^N)$. Forward focal transformation is used for the parameter estimation module; the differential time-shift analysis is for the parameter selection module; and inverse focal transformation is for the forward modeling module. An initial focal operator representing the mud-rolls with their initial velocities is roughly estimated from the recorded seismic data, and it is updated as stated above. This procedure is iterated with the updated operator until the differential time-shift has been minimized. After several iterations, the optimally updated velocities V_{mn} and the resulting mud-roll model $\mathbf{N}$ are obtained, which should explain the input mud-rolls in the recorded seismic data.

Mud-roll elements extraction/reconstruction

Even after obtaining the optimal mud-roll model, some difference between the mud-roll model and the input measurements is still expected to be present because the model is never perfect. Therefore, following iterations have the objective of extracting the mud-roll elements from the recorded seismic data and reconstructing these elements, using the optimal mud-roll model as an operator. This is in such a way that the forward modeled mud-rolls should match the input ones. For this purpose, data reconstruction concept can be used, and the feedback-loop can be re-described for a better perspective in Figure 2.

Many domains for data reconstruction exist with their transformations based on a decomposition of seismic data in certain basis functions, such as linear events with Fourier and linear Radon transformations, curvelets with curvelet transformation, and certain seismic responses with focal transformation (as stated above). A least-squares inversion method is commonly used since these transformations are usually ill-posed. In addition, certain constraints like sparsity are commonly used in the transformed domain for

Figure 2: Feedback-loop diagram at the mud-roll elements extraction/reconstruction stage. α is the iterative scaler here.

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obtaining a unique solution. The essence of these transformations is to find optimal model elements in the transformed domain under certain constraints such that the reconstructed data match the input measurements in the original data domain by minimizing the objective function. In our case, a suitable transformation brings mud-rolls from the original data domain to another domain where the mud-rolls and other events are well distinguished in terms of their energy level and areal separation. Sparsity constraint makes the distinguishability more robust. Additional constraints like amplitude thresholding and areal muting in the transformed domain for enhancing mud-rolls make it possible that only the mud-rolls are reconstructed. Following this concept and making use of the mud-roll model $\mathbf{N}$ for the operator, the transformations can be formulated by:

$$(\widetilde{\mathbf{P}}^S + \widetilde{\mathbf{P}}^N) = \mathbf{F}(\mathbf{P}^S + \mathbf{P}^N), \quad (8)$$

$$\widehat{\mathbf{P}}^N = \mathbf{G}\widehat{\mathbf{P}}^N, \quad (9)$$

$$\mathbf{F} = \mathbf{G}^H \mathbf{B}, \quad (10)$$

with

$$\mathbf{G} = \mathbf{N}, \quad \text{for focal transformation,}$$

$$\mathbf{G} = \mathbf{L}^H, \quad \text{for Radon transformation,}$$

$$\mathbf{G} = \mathbf{L}^H \mathbf{N}, \quad \text{for cascaded focal/Radon transformation,}$$

where $\mathbf{F}$ is the forward transformation operator; $\mathbf{G}$ is the inverse transformation operator; and $\mathbf{B}$ is an amplitude scaling operator for equalizing the amplitude spectra after the roundtrip of forward (equation 8) and inverse (equation 9) transformations. $\mathbf{L}$ is the linear Radon operator; and $\mathbf{L}^H$ is the adjoint operator of $\mathbf{L}$. The tilde symbol $\sim$ is used for indicating the transformed domain; and the hat symbol $\wedge$ is used for indicating “estimated” under certain constraints in the transformed domain. Based on equation (9), an optimal solution is found by minimizing the objective function:

$$J = \|\mathbf{P}^S + \Delta \mathbf{P}^N\|^2 = \|(\mathbf{P}^S + \mathbf{P}^N) - \widehat{\mathbf{G}}\widehat{\mathbf{P}}^N\|^2 = \min. \quad (11)$$

With proper constraints in the transformed domain, only the mud-rolls are reconstructed, and thereby the residual between the reconstructed mud-rolls and input ones is expected to contain only the resulting primaries.

In Figure 2, forward transformation is used for the parameter estimation module; amplitude thresholding and areal muting in the transformed domain are for the parameter selection module; and inverse transformation is for the forward modeling module; followed by the adaptive subtraction module. To solve the inversion problem, a greedy sparse algorithm (Wang et al., 2010) is used, in which a conventional conjugate gradient scheme is employed, but sparsity is promoted in the transformed

domain based on prioritization of the energy level. This procedure is iterated with the updated mud-rolls until a stopping criterion or a maximum number of iterations has been reached. The output is the final mud-rolls in the transformed domain, which should explain the input ones in the recorded seismic data. The output can be transformed to the original data domain by equation 9, and adaptively subtracted from the input to obtain the final result: the mud-roll-eliminated data.

The required information for the algorithm is domain, thresholding parameter, areal muting parameter, and stopping criterion or maximum number of iterations. The domain can be decided based on the mud-roll shape to be transformed in a shot gather which is affected by the perpendicular distance between the shot and the receiver line. Focal transformation is suitable for hyperbolic events, Radon transformation is superior for linear events, and cascaded focal/Radon transformation is reasonable for both events. The areal muting parameter can be determined straightforward since the mud-rolls are mapped around a particular position while other events are distributed all over the transformed space. The number of iterations can be estimated to be larger without degrading results but with a trade-off for the computation time. Besides, the thresholding parameterization is not trivial due to a trade-off between the sparsity and the residual. Higher threshold provides higher-resolution solution, however, results in more residual, and vice-versa. Therefore, some parameter testing should determine it.

Examples

Figures 3 and 4 show two examples of the proposed method on synthetic seismic data. Figure 4 includes the inputs to this method: synthetic shot gathers in the x - t and k - f domains. They simply consist of three events: a mode of mud-roll with 900 m/s velocity in the 0-5 Hz frequency band; a related mud-roll but with 750 m/s velocity in the 5-20 Hz frequency band; and a primary from a depth of about 3000 m in the 0-50 Hz frequency band. They are generated with zero phase wavelets and geometrical spreading effects. The perpendicular distance between the shot and the receiver line is 1000 m, and the receiver point interval (Δx_d) is 12.5 m and 50 m, respectively for two examples. The mud-rolls appear as hyperbolic events due to the perpendicular distance. Figure 3 shows that the proposed method satisfactorily updates the mud-roll velocities from

Figure 3: Mud-roll initial (green) and updated (blue) velocities for each 5 Hz frequency band from the example with Δx_d of 12.5 m.

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initial velocities (randomly assigned here) to the correct ones. Figure 4 shows the results of the proposed method with cascaded focal/Radon transformation: the input; the output; and the final result in the original x - t and k - f domains. The results from Δx_d of 12.5 m demonstrate that this method satisfactorily separates the mud-rolls and the primary, and thereby successfully produces the mud-roll-eliminated data. The results from Δx_d of 50 m reveal that a larger spatial sampling interval deteriorates the resulting data quality in general. However, still in this case, this method faithfully reconstructs even spatially aliased mud-rolls, satisfactorily separates them from the primary, and thereby successfully produces the mud-roll-eliminated data beyond aliasing.

Conclusions and Remarks

We proposed a dispersive multi-modal mud-roll elimination method. The essence of this method is:

- This method fully addresses the dispersive multi-modal properties of mud-rolls.
- This method is fully data-driven and data-adaptive. This makes the method practicable for any receiver component type of seismic data, thus, not only for hydrophone and vertical geophone data but also for horizontal geophone data.
- This method is implemented in common shot gathers, i.e., shot by shot. This makes the method practicable for variable perpendicular distance between the shot and the receiver line by choosing a suitable transformation. Furthermore, the method automatically takes into account the spatial variability of mud-roll characteristics by the optimal mud-roll model established in each common shot gather.
- This method is implemented for both regularly and irregularly undersampled seismic data. This offers a possibility in relaxing the spatial sampling interval without degrading the resulting data quality in terms of mud-rolls.

The proposed method is targeting the mud-rolls which are often dominant in OBC seismic data in a shallow water environment. It should be noted that recent advances in OBC seismic acquisition, like single point receivers and a large amount of equipment, make this method more effective because of the proper sampling of mud-roll wavefields without negative array effects.

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Figure 4: The results from the examples with Δx_d of 12.5 m (a) and 50 m (b) in the x - t and k - f domains.

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EDITED REFERENCES

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