Stellingen behorende bij het proefschrift:

CARRIER WAVE SIGNALS
INTERFERING WITH LORAN-C

door
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1. Het idee om verschillende radio-navigatie systemen te integreren, is een van de grootste bijdragen aan de wetenschappelijke vooruitgang op het gebied der plaatsbepaling.

2. Nauwkeurigheids-specificaties van radio navigatie systemen zijn meestal onnauwkeurig, betrouwbaarheids-gegevens zijn meestal onbetrouwbaar; slechts door gebruik te maken van fout-verdelings grafieken kan een goede indruk van de prestaties van een radio-navigatie systeem verkregen worden.

3. Loran-C ontvangers zijn nog teveel gericht op gebruikers met kennis van zaken; met de huidige stand der techniek is dit onnodig en het staat bovendien een meer algemeen gebruik van radio-navigatie m.b.v. Loran-C in de weg.

4. Het huidige streven naar efficiënte universitaire opleidingen is in directe tegenspraak met het belangrijkste element van deze opleidingen: het leren zelfstandig te denken, zonder een van boven opgelegd schema te volgen.


6. Er is dringend behoefte aan sterk verbeterde public relations naar het publiek toe op het gebied van de technische wetenschappen en de toepassing daarvan.
7. De voorstellen van de commissie-Dunning vormen een bedreiging voor de vooruitgang in de medische wetenschap.

8. Het ter beschikking stellen van dienst-auto's met chauffeur aan ministers en ambtenaren van de ministeries van Verkeer en Waterstaat en Justitie, is schadelijk voor een realistisch verkeersbeleid.

9. Er bestaat een opvallende overeenkomst tussen discussies over religies, computer operating systems en radio-navigatie systemen: argumenten zijn vaak gebaseerd op persoonlijke gevoelens en voorkeuren, en niet op technische argumenten.

10. Een ieder krijgt het operating system dat hij / zij verdient.
Carrier Wave Signals Interfering with Loran-C

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Carrier Wave Signals Interfering with Loran-C

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Summary

In this thesis, a thorough analysis of interference problems in the Loran-C radio navigation system (including possible solutions) is given. First, an explanation of the problems is offered, including a detailed description of the effects of interference on Loran-C. Part of this description is new; it fills a gap in the numerical analysis of the influence of interference on Loran-C. It shows that for further expansion of Loran-C especially in western Europe, solutions to the interference problems are needed.

Two types of interference to Loran-C can be distinguished: interference that comes from within the Loran-C system (Loran-C transmissions that are not used for positioning, but can be received), and interference that is generated by other activities, mostly legal transmissions in the frequency bands around the Loran-C spectrum. The effects of these interference types on positioning, will be described.

Once the effect of the different types of interference to Loran-C has been described, attention is focused solely on interference from non-Loran-C transmissions in the frequency bands around the Loran-C spectrum. This type of interference can be attacked at two levels: in the Loran-C system by properly choosing system parameters, and in Loran-C receivers by including all kinds of filtering techniques to remove the unwanted signals. For both levels, methods for interference reduction have been developed.

During the design of a Loran-C chain, many system parameters must be chosen. Most of these parameters have little or no use in the reduction of interference; only the so-called Group Repetition Interval (GRI) does have a noticeable influence on the amount of harmful interference to a Loran-C chain. Fortunately, this GRI can be chosen independently from other chain parameters, and can therefore easily be optimised for best interference rejection. An algorithm for selecting such an optimal GRI will be presented.

Even with a well-designed Loran-C chain, interference harmful to Loran-C operation must be expected to be present at a receiver antenna. Therefore, it is
very important to develop receivers that are more immune to interfering signals. It will be shown that the largest problem here is the proper detection of harmful interference signals, while rejection of signals (once they have been detected) is relatively easy with today's technology. This thesis presents a completely new concept for detecting all interference harmful to a Loran-C chain, based on Digital Signal Processing of antenna signals with very powerful processors.
Samenvatting

In dit proefschrift wordt een grondige analyse (inclusief mogelijke oplossingen) gepresenteerd van problemen in het Loran-C radio-navigatie systeem, veroorzaakt door stoorsignalen van diverse aard. Eerst zullen de effecten van stoorsignalen op Loran-C beschreven worden. Hieruit blijkt dat voor een succesvolle expansie van Loran-C vooral in West-Europa, oplossingen voor het stoorsignaal-probleem in Loran-C gezocht moeten worden.

Er kan een onderscheid gemaakt worden tussen twee soorten stoor-signalen: signalen die van andere Loran-C ketens afkomstig zijn, en signalen van andere radio-signalen (meestal legaal) vlak boven of onder de Loran-C frequentie band. Voor beide soorten interferentie zal beschreven worden wat hun effect is op het Loran-C systeem.

Nadat de effecten van beide soorten stoorsignalen, beschreven zijn, wordt de aandacht uitsluitend gericht op storingen veroorzaakt door signalen die niet van andere Loran-C ketens afkomstig zijn. Deze signalen kunnen bestreden worden op twee niveau’s: tijdens het ontwerp van een Loran-C keten (op systeem-niveau) en in Loran-C ontvangers. Voor beide niveaus zijn bestrijdingsmethodes ontwikkeld.

Tijdens het ontwerpen van een Loran-C keten moeten verschillende systeem-parameters gekozen worden. Van al deze parameters blijkt alleen de zg. Group Repetition Interval (GRI) een aanzienlijke invloed te hebben op de hoeveelheid schadelijke stoorsignalen. De keuze van een GRI voor een Loran-C keten is redelijk onafhankelijk van andere systeem-parameters, en het is derhalve mogelijk een GRI te kiezen met minimale hoeveelheden schadelijke stoorsignalen. Een algoritme voor een dergelijke GRI-keuze wordt beschreven.

Ook bij gebruik van een Loran-C keten met een optimaal gekozen GRI, moet rekening gehouden worden met het optreden van schadelijke stoorsignalen. Loran-C ontvangers zullen daarom "geimmuniseerd" moeten worden. Hierbij blijkt het grootste probleem de detectie van schadelijke stoorsignalen te zijn; het onderdrukken van stoorsignalen (na een succesvolle detectie) is met de huidige
stand van de techniek een veel geringer probleem. In dit proefschrift wordt een geheel nieuwe detectie-methode voor stoorsignalen gepresenteerd, gebaseerd op digitale verwerking van antenne-signalen met zeer krachtige processoren.
1 Introduction

1.1 Radio navigation systems - an overview

Currently, anyone considering the use of radio signals for positioning and navigation, has a wide variety of dedicated systems to choose from. Some well-known radio-navigation systems are listed in table 1.1, together with their most important properties.

<table>
<thead>
<tr>
<th>System</th>
<th>Range</th>
<th>Accuracy (95 %)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMEGA</td>
<td>World-wide</td>
<td>3 km</td>
<td>10.2 - 13.6 kHz</td>
</tr>
<tr>
<td>DECCA</td>
<td>200 km</td>
<td>300 m</td>
<td>70 - 130 kHz</td>
</tr>
<tr>
<td>Loran-C</td>
<td>1500 km</td>
<td>300 m</td>
<td>100 kHz</td>
</tr>
<tr>
<td>GPS / p-code with SA</td>
<td>World-wide</td>
<td>100 m</td>
<td>1.5 GHz</td>
</tr>
</tbody>
</table>

*Table 1.1: Comparison of some navigation systems.*

In 10 years from now, some of the systems in table 1.1 will be obsolete, while new ones will appear. For any research project on one of the systems in table 1.1 three questions should be answered first:

1) For what types of applications will the system be used in the future?

2) How large will be the user group (and therefore: will the system be obsolete in 10 years from now) ?

3) What will be the impact of future developments on receiver design?

This thesis deals with Loran-C, so an attempt will be made to answer the three questions posed above for Loran-C. First, however, a short description of Loran-C will be given.
1.2 The Loran-C system - a short description

The Loran-C radio-navigation system is currently in use in wide parts of the world. Loran-C has now been installed in the USA, Japan, People’s Republic of China, Saudi-Arabia, western and northern Europe and the USSR. It is one of the most likely candidates for the main civil radio navigation system in Europe, where plans for its expansion are developed right now.

The Loran-C system consists of many different so-called chains: groups of transmitters whose timing relations are tightly controlled. Each transmitter in a chain transmits a series of bursts with carrier frequency 100 kHz and a modulation as shown in fig. 1.1. One of the transmitters is designated the master (usually abbreviated to M): it is the first to transmit in the timing diagram of a chain, as illustrated in fig. 1.2. A master transmits 8 bursts, at intervals of 1 ms, and a ninth burst (useful for identification and status indication purposes) 2 ms later. After time interval TDX, which is tightly controlled, the first so-called secondary transmitter (secondary X) starts transmitting 8 pulses. These pulses are transmitted at intervals of 1 ms too. Then, again after a tightly controlled time interval, the second, third and further secondaries (called secondaries Y, Z, W etc.) will transmit their 8 pulses. A chain can contain 1 master and up to 6 secondaries. The repetition interval of the timing diagram is called the Group Repetition Interval (GRI) and is unique for each chain.

![Loran-C Burst Waveform](image)

*Fig. 1.1: Loran-C burst waveform.*
GRI’s can vary in 10 μs steps between 0.04 s and 0.09999 s. A commonly used notation method for GRI’s is to use multiples of 10 μs: a GRI of e.g. 0.0797 is written as 7970.

![Timing diagram of a Loran-C chain.](image)

In a Loran-C transmitter, the current in the antenna can be described mathematically as:

\[
S(t) = \begin{cases} 
0 & \text{for } t < 0 \\
\text{sign} \cdot A \cdot \sin(\omega t) \left( \frac{t}{t_p} \right)^2 e^{-\frac{2}{t_p} t} & \text{for } t \geq 0
\end{cases}
\]  

(1.1)

where:

- \( \text{sign} = -1 \) or \( +1 \)
- \( t \) = time
- \( t_p = 65 \mu s \)
- \( \omega = 2\pi \cdot 10^5 \text{ rad/s} \)

The electrical field at a large distance from the transmitter, can be approximated by shifting the carrier term in formula 1.1, by 90°. The definition in formula 1.1 and a detailed explanation of the transformation from antenna current to far electrical-field, can be found in [1.4].
The bursts of the master and secondaries in a chain are multiplied with 1 or -1; this is denoted in formula 1.1 by variable sign. The multiplication pattern is different for master and secondaries and repeats itself every 2 GRI [1.1]. The phase code repetition frequency $\frac{1}{2 \text{GRI}}$ is the lowest repetition frequency in a Loran-C transmission. This frequency plays a very important role in the remainder of this thesis; it will be called the Group Repetition Frequency (GRF) from now on:

$$\text{GRF} = \frac{1}{2 \text{GRI}} \quad (1.2)$$

The spectrum occupied by the Loran-C signal defined in formula 1.1, is located between 90 and 110 kHz: in this band 99% of the transmitted energy will be found.

Conventional Loran-C receivers will try to find and follow a zero-crossing in the first part of the burst (less than 32.5 $\mu$s away from the start of the burst). This ensures a good separation of groundwave and skywave, because the skywave generally arrives more than 32.5 $\mu$s later at the receiver. The groundwave is used for range measurements; due to ionospheric propagation variations the skywave is not stable enough to be used for navigation. Note, too, that all Loran-C signal definitions are referenced to the amplitude of the modulation waveform of formula 1.1 at 25 $\mu$s, and not to the top of the burst as could be expected [1.7].

By tracking the zero-crossings of different signals, time-difference measurements can be made, which can be used to calculate the receiver position. Such a zero-crossing tracking process is split into two parts in Loran-C receivers:

1) First the proper cycle or cycles which are to be used for zero-crossing tracking, have to be identified. This so-called Cycle Identification process makes use of the modulation waveform of a Loran-C signal as defined in formula 1.1. This modulation waveform is commonly called the Loran-C envelope. The Cycle Identification process can be described as a locked loop, tracking amplitude information; this loop is appropriately called the envelope tracking loop, and will generally have a very small bandwidth: in the order of magnitude of 0.001 Hz.
2) After the proper cycle has been found, it is used to extract phase information from the Loran-C signal by following a zero-crossing within that cycle. Several techniques for phase tracking of Loran-C signals are in use today. One commonly used technique is based on linear processing of the incoming bursts (usually with an A/D converter and micro-processor); another option is to hard-limit the signal and track the transition of the signal from positive to negative or vice versa. The loop used to track the zero-crossing is commonly called the phase tracking loop; its bandwidth $f_b$ (the so-called tracking bandwidth) is usually less than 0.1 Hz.

1.3 Applications of Loran-C: now and in the future

In this section, an attempt will be made to describe the applications of Loran-C, now and in the future. The list of applications presented here, is certainly not complete: Loran-C is used in many ways and for many different purposes. Also, due to a lack of data about military applications, only civil use will be explained here. Three main civil application areas can be distinguished:

1) Marine use of Loran-C. This is a traditional application of Loran-C, as will become clear when looking at the coverage diagrams for most existing Loran-C chains. The largest marine user group of Loran-C is found in the U.S.A., probably due to the good coverage of the east- and west-coasts of the U.S. and the obligation to have a Loran-C receiver on board when entering U.S. coastal waters.

2) Loran-C used in civil aviation. Again, the largest aviation user group of Loran-C can be found in the U.S.A. There, three applications of Loran-C in aircraft can be distinguished:

- Using Loran-C stand-alone for en-route navigation. Until now, this has been the primary aviation application of Loran-C. Due to reliability requirements Loran-C as a stand-alone system is not certified for sole-means navigation; therefore, aircraft flying under Instrument Flight Rules using Loran-C, must also be equipped with a VOR/DME receiver.

- Using Loran-C for Non-Precision Approaches (NPA). In this application, Loran-C is be used to enable instrument approaches to airports not equipped with any other landing aid. While not comparable to systems
like ILS and MLS, Loran-C can increase the amount of time such airports can be used. In 1990, important test programs carried out in the U.S.A. of NPA-systems with Loran-C, have been concluded successfully [1.2]; this can be expected to lead to a wide-spread certification program of Loran-C Non-Precision Approaches.

- Using hybrid Loran-C and GPS as a sole-means navigation system to replace the current VOR/DME net. This option is currently under investigation in the U.S.A. for the U.S. national airspace system [1.5].

3) Land use of Loran-C. Because of the propagation properties of LF radio waves, Loran-C can be used quite well in vehicles. In one possible application, Loran-C can be used by trucking companies to get position information about their fleet of vehicles. This information can then be used to decrease the amount of driving without payloads. The advantage will be obvious: the trucking company can offer its services at a lower price.

1.4 User groups: size and growth

Currently, more than 1 million Loran-C receivers have been sold, mainly (again) in the U.S.A. This means that Loran-C is now by far the navigation system with the largest user community. Predicting how large this community will be in 10 years from now, is difficult. However, for the three main applications listed in section 1.3, an attempt will be made to do at least some general predictions:

1) Marine users. This is currently by far the largest user group (450000 users in 1990 [1.3]), consisting mainly of owners of pleasure crafts in the U.S.A. Growth of this group will probably be found outside the U.S., especially when Loran-C will replace DECCA as main civil navigation system in western and northern Europe.

2) Civil aviation. Currently 79500 Loran-C receivers are in use in (mainly small) aircraft in the U.S.A. [1.3]. When plans for Non-Precision Approaches with Loran-C and hybrid Loran-C/GPS have been implemented, this user group can grow considerably.

3) Land use of Loran-C. Development of this application of Loran-C is beginning only now: currently only 20000 land users of Loran-C are
counted [1.3]. Predictions on future growth are impossible to make, since no "track record" exists. However, there are several hundred million cars and trucks in the U.S.A. and western Europe. Even if only a very small percentage will be equipped with Loran-C, the total user group will be large compared to other systems or applications.

Loran-C, with its large and growing user community, will certainly not an obsolete system for years to come. This, together with the existence of unsolved problems, warrants the usefulness of research into Loran-C.

1.5 Requirements for Loran-C receivers in the future

The application areas listed in the previous sections, all have one important aspect in common: Loran-C is and will be used by people who are not aware of the existence of error sources in Loran-C or of methods used to combat errors. This has the following consequences for receiver designers:

1) All methods used to combat position errors must work completely independent of the user. Situations where a receiver must be re-aligned, because it is moved to a different area or because of component drift, are unacceptable.

2) If position errors are expected which lie outside the specified tolerances, a clear warning must be given to the user not to trust the data from the receiver. This requirement arises from the relative ignorance of the users about error sources: if errors occur too often without proper warning, Loran-C will be viewed as a "bad" system. Users will then turn to other systems, even if a perfectly valid explanation for the errors exists. A clear warning by Loran-C receivers of error conditions can help in avoiding such a situation.

3) With current low prices for consumer electronics, an uninformed user will have a hard time understanding why Loran-C receivers have to be much more expensive than today's consumer electronics equipment. Therefore, Loran-C receiver technology must be affordable to produce in large quantities.
As any navigation system, Loran-C has some good and some bad properties. Problems in Loran-C (including the receivers) are:

- Propagation anomalies. Loran-C waves propagate along the earth's surface. The propagation velocity will vary depending on the properties of the path between the transmitter and the receiver. For paths which pass over sea water, the propagation velocity is well known. For paths which include land masses, an accurate prediction of the average propagation velocity is difficult. Traditionally, users had to input propagation correction factors by hand, an error-prone process. With advancing technology in Read Only Memories, receivers will have more and more tables on board, containing position-dependent calibration data for propagation velocity prediction. This problem will not be dealt with in this thesis.

- Interference, here defined as all man-made signals which are received by Loran-C receivers and which can cause errors to position data. The different types of interference, as well as the different error types interference can cause, will be described in the next chapter. Traditionally, receivers have to be aligned carefully to solve this problem, while the alignment is dependent on the area the receiver is used in.

- Cycle Identification. This is a process in the receiver which determines the proper cycle of the Loran-C signal to be used for range measurements. If an error of one cycle is made in this process, a range error to a single station of $1 \lambda$ or 3 km at 100 kHz, is made. The resulting position error will be approximately 1.5 to 4 km, depending on the transmitter geometry (the positions of the transmitters relative to the receiver position) [1.6]. Traditional methods for cycle identification work well in receivers which are carefully aligned and checked regularly, and only in a specific area with known interference sources. Without proper alignment, the risk of cycle identification errors will increase considerably.

The remainder of this thesis will deal exclusively with interference: what types of interference can be found, which problems can be expected and how interference signals are combatted. Special attention will be given to those methods to deal with interference which require no knowledge of the problem by the user.
The decision to direct research only towards solving the interference problem, does not mean that the author views the other problems as unimportant. Other research projects are dealing with those problems; the interference problem happened to be a relatively neglected one, suitable for researching.

1.6 References


2 Signals interfering with Loran-C

2.1 Two types of interference signals

The Loran-C system suffers from two different types of interfering signals:

1) Signals generated by the Loran-C system itself: these signals originate from a different chain operating close to the chain used for positioning. This type of interference is called Cross Rate Interference (CRI).

2) All other man-made signals. The Loran-C spectrum runs from 90 to 110 kHz; other signals potentially harmful to Loran-C operation can be found in the surrounding spectrum from 50 to 150 kHz. These signals are called Carrier Wave Interference (CWI).

2.2 The Cross Rate Interference problem - an illustration

In order to give an indication of the potential severity of the CRI problem, fig. 2.1 is included. This figure shows one option in the Loran-C expansion plans currently under consideration in western Europe. This option includes a total of four chains, and one can expect to receive signals from all four chains simultaneously in large parts of Europe. At distances of more than 2000 km to the transmitter, the received signals will contain mostly skywaves and will therefore be useless for positioning, but interference with useful signals can certainly occur.

A fifth chain (not shown in figure 2.1) is a Tchaika chain in the western USSR, which cannot be used for navigation in western and northern Europe at all, but whose skywave signals are certainly present in this area. Also not shown in the Mediterranean Loran-C chain with a GRI of 7990, of which skywaves can also be received over large parts of the continent.

This means that signals of 6 different chains will be received in western Europe. If each chain would use a different carrier frequency, this would not have to be a
problem, since Loran-C uses time multiplexing: only one station of a chain is on the air at any given moment. However, as explained in chapter 1, Loran-C stations do all transmit at 100 kHz. Loran-C chains are usually not synchronised, and therefore a real risk exists that Loran-C pulses from stations in different chains are received at the same time. Without proper attention paid to the CRI problem, positioning errors might result.

2.3 Influence of Cross Rate Interference on positioning

As explained in the previous section, a fundamental property of Cross Rate Interference is that a receiver sees pulses from different Loran-C transmitters at its antenna at the same time. In such a case, the antenna signal $s_A(t)$ will be the sum of the signals $s_{L1}(t)$ and $s_{L2}(t)$ from the individual chains:

$$s_A(t) = s_{L1}(t) + s_{L2}(t)$$
$$= A_{L1} \sin(\omega_{L1} t + \varphi_{L1}) + A_{L2} \sin(\omega_{L2} t + \varphi_{L2})$$
$$= A_C \sin(\omega_C t + \varphi_C)$$

(2.1)

where:
\[ \omega_L = \text{Loran-C carrier frequency} \]
\[ A_{L1} = \text{Chain 1 signal amplitude} \]
\[ A_{L2} = \text{Chain 2 signal amplitude} \]
\[ \phi_{L1} = \text{Chain 1 signal phase} \]
\[ \phi_{L2} = \text{Chain 2 signal phase} \]
\[ A_C = \text{Composite signal amplitude} \]
\[ \phi_C = \text{Composite signal phase} \]

The composite amplitude \( A_C \) and the composite signal phase \( \phi_C \) can be written as:

\[
A_C = \sqrt{A_{L1}^2 + A_{L2}^2 + 2 A_{L1} A_{L2} \cos(\phi_{L1} - \phi_{L2})}
\]

\[
\phi_C = \tan^{-1}\left( \frac{A_{L1} \sin(\phi_{L1}) + A_{L2} \sin(\phi_{L2})}{A_{L1} \cos(\phi_{L1}) + A_{L2} \cos(\phi_{L2})} \right)
\]

(2.2)

and the tracking phase error \( \phi_e \) is then equal to \( \phi_C - \phi_{L1} \), assuming that chain 1 is the chain to be tracked:

\[
\phi_e = \tan^{-1}\left( \frac{A_{L1} \sin(\phi_{L1}) + A_{L2} \sin(\phi_{L2})}{A_{L1} \cos(\phi_{L1}) + A_{L2} \cos(\phi_{L2})} \right) - \phi_{L1}
\]

(2.3)

Looking at formula 2.3, it becomes clear that the tracking error due to CRI depends highly on the momentary amplitudes and phases of the Loran-C signals \( S_{L1}(t) \) and \( S_{L2}(t) \). These vary as functions of position and propagation, complicating the development of a generally valid description of the effects of Cross Rate Interference.

Formulas 2.1 to 2.3 were developed assuming that the Loran-C signals are pure carrier waves. Because a Loran-C station transmits bursts (and therefore amplitudes \( A_{Li} \) are time-dependent: \( A_{Li} = A_{Li}(t) \)), any analysis of Cross Rate Interference is more complicated than formula 2.3 indicates.

In the analysis of interference, the most important problem to be solved is: what happens inside a Loran-C receiver, and how are the tracking loops (which extract the information used for positioning from the Loran-C signal) influenced? In order to analyse the influence of CRI, fig. 2.2 shows a simplified picture of the antenna signal of a receiver suffering from CRI.

\[ \text{Signals interfering with Loran-C} \]
In fig. 2.2, transmissions of two Loran-C stations belonging to two different chains, are shown. During the first GRI, the pulses of the two stations will overlap. This will cause a phase error in the antenna signal, which can be calculated with formula 2.3.

During the next GRI, the time difference between the pulses from the two stations, has increased from 0 to GRI₂ - GRI₁. Depending on the GRI’s involved, the pulses can still overlap (with a minimum difference in GRI’s of 10 μs, the pulses will have shifted just one cycle of the Loran-C carrier, and a Loran-C groundwave burst is 250 μs long). After several GRI’s, however, overlapping of pulses will have ended and no phase errors will be found.

The sequence of overlapping / not overlapping will repeat itself with a repetition time \( t_{r,\text{GRI}} \) which is equal to the smallest common multiple of the two GRI’s. Within \( t_{r,\text{GRI}} \), however, several periods of overlapping pulses can occur, each with different phase errors. Such a situation is shown in fig. 2.3.

The signals drawn in figs. 2.2 and 2.3, show the antenna signal of a Loran-C receiver in the presence of CRI. This signal is fed into a phase tracking loop, which yields information about the relative phase of the wanted signal and therefore the relative distance to the transmitter. This phase tracking loop has a limited bandwidth and therefore a noticeable time constant [2.2], [2.10]. The actual value of the tracking bandwidth will depend on the application; some
tracking loops will even automatically adjust their bandwidth to noise levels, accelerations etc.

Changes in the phase of the incoming signal which are much faster than the main time constant of the tracking loop, will not lead to large phase errors at the output of the loop. With a tracking loop bandwidth of 0.1 Hz, the loop time constant will be approximately 1.5 seconds. Pulses from other Loran-C chains overlapping the wanted pulses for 1 or 2 GRI's only (0.2 s maximum), will not cause a large tracking error at the output of such a tracking loop. With overlapping periods of 1 to 10 s, however, tracking errors are bound to appear. An example, generated using computer simulation methods, is shown in fig. 2.4. In this simulation, the signal values of both the desired and the Cross Rate chains are calculated and fed into the tracking loop. On the computer screen the

![Fig. 2.4: An example of Cross Rate Interference.](image)

Signals interfering with Loran-C
zero crossing position estimated by the loop (called Sampling Point Position or SPP), is shown. In an ideal case (no interference), the zero crossing is located at 25 μs from the start of the burst. The deviation from the 25 μs line is a measure for the phase and range errors due to CRI.

Summarising:

1) Cross Rate Interference is characterised by a very complicated pattern of pulses from two Loran-C chain, which will overlap (and have phase errors) during some GRI's, and not overlap (and therefore not have phase errors) during most GRI's. Both the time interval between consecutive overlappings, and the pattern of the resultant phase errors, are highly dependent on the GRI's involved, and therefore highly dependent on the local Loran-C situation.

2) Cross Rate Interference on the antenna of a Loran-C receiver, will not necessarily lead to errors in the position indicated by the receiver. In case of short overlapping periods, virtually no errors will be experienced, due to the long time constants used in phase tracking loops of Loran-C receivers. In case of long overlapping periods, however, errors will result. The time constants of the loops used depend on the application of the receiver.

Since the effects of Cross Rate Interference are highly dependent on the actual situation (GRI's of the chains involved) and the receiver type (tracking loop bandwidth), developing general methods for CRI analysis is very difficult. In this thesis no new analysis methods will be described; the interested reader is referred to publications [2.9], [2.4] and [2.3] for descriptions of existing CRI analysis methods.

2.4 The CWI problem - why is it so difficult?

As an illustration of the CWI problem in Europe, figures 2.5 and 2.6 are included. Fig. 2.5 shows the power spectrum of a single Loran-C transmission without any interference between 50 and 150 kHz, measured with a spectrum analyser. This figure should be compared with fig. 2.6, which contains the result of a spectrum scan made at the Delft University of Technology, The Netherlands, of the spectrum from 50 to 150 kHz on a typical day. The frequency band from 90 to 110 kHz contains only Loran-C transmissions
Fig. 2.5: Spectrum of a pure Loran-C transmission.

Fig. 2.6: Received Loran-C and surrounding spectrum.

(in fig. 2.6 the Loran-C transmitter in Sylt will dominate the band between 90 and 110 kHz, since it is the Loran-C station closest to Delft), but the surrounding spectrum contains a large number of other signals. These signals are generated by a wide variety of sources: data-transmission stations, the DECCA navigation system, time references transmissions, etc.

From fig. 2.6 one might conclude that the Loran-C signal is always much weaker than the interference signals. This, however, is not generally true, since a Loran-C transmission is spread out over a wide bandwidth (99% of the energy is found between 90 and 110 kHz). In contrast, a CWI signal does usually occupy a small frequency band, typically much less than 2 kHz. This means that
the power density of a Loran-C signal is low compared to a CWI signal, and this can be seen clearly in fig. 2.6. However, the total Loran-C signal power usually is larger than the CWI signal power, especially if the Loran-C station is not far from the receiver in fig. 2.6.

One way to get rid of the interfering signals below 90 kHz and above 110 kHz, would be to use a very steep analog bandpass filter at the receiver input. An example of such a bandpass filter is shown in fig. 2.7: a seventh-order Chebyshev filter with a center frequency of 100 kHz and a bandwidth of 20 kHz.

![Bandpass filter useful to get rid of CWI.](image)

Fig. 2.7: Bandpass filter useful to get rid of CWI.

However, a bandpass filter also influences the Loran-C burst waveform: the carrier is delayed and the modulation waveform is distorted. Generally, steep bandpass filters yield more distortion of the modulation waveform than bandpass filters with gentle slopes [2.10]. This is illustrated in fig. 2.8, which shows a Loran-C groundwave burst filtered with the filter in fig. 2.7. The unfiltered Loran-C signal is shown as a dotted line in fig. 2.8; this gives an impression of the distortion to be expected with a steep bandpass filter.

One can see that the rising edge of the modulation waveform is much slower than the rising edge of an unfiltered Loran-C burst. The rising edge is used by receivers to distinguish the groundwave from the skywave, and for that reason should be rising steeply. A bandpass filter with steep slopes like fig. 2.7 will destroy this skywave rejection capability of a receiver. This can be proven easily
Fig. 2.8: Loran-C burst filtered with fig. 2.7.

by calculating the highest point in the filtered Loran-C burst, where the tracking error due to skywave is lower than a predefined minimum. This has been done in fig. 2.8 too for the worst-case skywave conditions as defined in [ 2.13 ]: skywave delay 32.5 μs and Skywave-to-Groundwave Ratio (SGR) 12 dB. Fig. 2.8 shows clearly that in the part of the burst where the tracking error is lower than 100 ns, there is almost no signal yet due to the slow burst rise-time.

Other filter types do exhibit much less distortion of the Loran-C envelope, as the example of fig. 2.9 shows. In this figure the unfiltered and the filtered Loran-C bursts are shown again. A third-order Butterworth filter has been selected as input bandpass filter, also with a center frequency of 100 kHz and a bandwidth of 20 kHz. Compared to the distortion caused by a high-order Chebyshev filter,
the Loran-C pulse is now much less changed. This is proven by the fact that
now an area in the burst can be found where the tracking error due to worst-case
skywave, is still less than 100 ns, and where the signal amplitude is already high
enough to be useful.

However, the amplitude transfer function of a third-order Butterworth filter does
have much less favorable CWI rejection capabilities, as fig. 2.10 shows.

![Image of Butterworth filter transfer function]

**Fig. 2.10: Amplitude transfer of 3rd-order Butterworth.**

The two filters presented in figures 2.7 and 2.10 do certainly represent extremes
in the choice of an input bandpass filter for a Loran-C receiver; such filters will
usually not be selected for implementation. However, they show that for any
input bandpass filter in a Loran-C receiver, the required CWI rejection
capabilities do conflict with the wanted skywave rejection properties:

- good CWI rejection requires steep filter slopes and with conventional filter
techniques this leads to large pulse distortion, while

- proper skywave rejection demands little pulse distortion, and with
  conventional filter techniques this leads to gentle slopes.

This conflict becomes especially difficult in areas with many CWI signals, like
western Europe. The problems of conventional Loran-C bandpass filters and
methods to optimize these filters, have been described in detail in [2.5] and
[2.10]. In this thesis, no further attention will be paid to this problem; in
chapter 7 a new receiver architecture will be described based on digital signal
processing. With this architecture, it is possible to reject a large part of the CWI spectrum, while leaving the Loran-C pulse intact.

One other conventional way to reject CWI signals, is to make use of notch filters. These take out a small part of the spectrum: preferably the part that contains CWI. It may be clear that with the number of interfering signals as shown in fig. 2.6, not all CWI signals can be filtered out with a notch filter. Any filter system using notches, will have to choose only those signals which lead to large position errors.

2.5 Influence of Carrier Wave Interference on positioning

Before the influence of CWI signals on positioning can be analysed, an explanation of the nature of these signals has to be given. The International Frequency List [2.12] contains all transmissions in the frequency band 50 - 150 kHz, as registered officially with the ITU. This list may not be complete; however, it can give a good indication of what types of interference signals can be expected.

According to [2.12], two main types of signals can be found in the frequency band 50 to 150 kHz:

1) Signals which are amplitude modulated in a very slow rhythm (modulation frequency < 1 Hz). This includes time reference signals, which contain a "marker" for every second, and signals from the DECCA navigation system, which are switched on/off in a sequence of (in total) 10 seconds.

2) Signals which are used for data transmission: binary data (e.g. telex connections) and fax pictures are transmitted with this type of signal. Frequency Shift Keying modulation is usually employed by these stations.

Common to all signals listed above, is that their transmitted spectrum will consist of a set of discrete spectral lines. This means that they can be modelled as a finite set of pure sine waves with different frequencies and amplitudes. The CWI analysis, detection and rejection methods introduced in the remainder of this thesis, can be described as linear processes (with some exceptions, which will be dealt with separately). In that case, understanding the influence of
modulated interference on receiver operation can be reached by analysing the general case of one sine-wave.

In order to understand the properties of CW interference signals, first an explanation of the spectrum transmitted by a Loran-C station is necessary. As explained in section 1.2, a Loran-C station transmits sequences of 8 pulses, with a repetition time equal to the GRI. Every individual pulse in a sequence is multiplied with either +1 or -1. This multiplication is called phase coding; the pattern of +1's and -1's is different for master and secondary stations and repeats itself every 2 GRI's. The first sequence of 8 pulses is called the A GRI, the second sequence the B GRI. Fig. 2.11 shows the phase coding functions for both master and secondaries.

![Diagram](image)

**Fig. 2.11:** Phase coding functions of Loran-C.

In the time domain, the Loran-C signal generation function $s_{\text{Loran-C}}(t)$ can be described as a double convolution, according to (2.4):

1) First three signals are generated separately:

- a Loran-C burst $s_{\text{burst}}(t)$:

\[
s_{\text{burst}}(t) = \begin{cases} 0 & \text{for } t < 0 \\ A \cdot \sin(\omega t) \cdot \left( \frac{t}{t_p} \right)^2 \cdot e^{-2 \left( \frac{t}{t_p} \right)} & \text{for } t \geq 0 \end{cases}
\]

(2.4)

- a phase coding sequence $s_{\text{phase code}}(t)$:

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\[ s_{phasecode}(t) = \sum_{i=0}^{7} \left( pc(i) \cdot \delta(t-i \cdot T_p) \right) + \sum_{i=8}^{15} \left( pc(i) \cdot \delta(t-GRI-(i-8) \cdot T_p) \right) \] (2.5)

with \( pc(i) \) (\( i = 0..15 \), \( pc(i) = -1 \) or +1) describing the phase code of the station, and \( T_p \) the distance between pulses in one GRI of 1 ms.

- an infinite impulse series \( s_{impulse}(t) \) with repetition interval \( GRF = \frac{1}{2 \text{ GRI}} \):

\[
s_{impulse}(t) = \begin{cases} 
  s_{impulse}(t) = 0 \text{ for } t \neq N \cdot 2 \text{ GRI} \\
  s_{impulse}(t) = 1 \text{ for } t = N \cdot 2 \text{ GRI}
\end{cases} \] (2.6)

2) The Loran-C burst function \( s_{burst}(t) \) is convolved with the phase coding function \( s_{phasecode}(t) \), thus generating a sequence of 16 bursts with the proper phase coding.

3) This sequence is convolved with the impulse series \( s_{impulse}(t) \), in order to generate an infinite signal \( s_{Loran-C}(t) \) as transmitted by a Loran-C station:

\[
s_{Loran-C}(t) = s_{burst}(t) * s_{phasecode}(t) * s_{impulse}(t) \] (2.7)

Fig. 2.12 contains a suitable illustration of this Loran-C signal generation process.

**Fig. 2.12: Loran-C signal generation in time domain.**
In the frequency domain, a convolution of two functions becomes a multiplication. So, in order to find an expression for the transmitted Loran-C spectrum $S_{\text{Loran-C}}(\omega)$, first the Fourier transforms $S_{\text{burst}}(\omega)$, $S_{\text{phasecode}}(\omega)$ and $S_{\text{impulse}}(\omega)$ of the time domain functions $s_{\text{burst}}(t)$, $s_{\text{phasecode}}(t)$ and $s_{\text{impulse}}(t)$ must be calculated. Then these three frequency domain functions are multiplied to get the transmitted Loran-C spectrum.

This mathematical description of the generation of a Loran-C signal, has been explained in detail in [2.4]. Fig. 2.13 provides an illustration of Loran-C signal generation in the frequency domain.

![Diagram showing Loran-C signal generation in frequency domain](image)

**Fig. 2.13: Loran-C signal generation in freq. domain.**

Since the Fourier transform of an infinite impulse series is a line spectrum, the transmitted Loran-C spectrum will also consist of many lines, spaced at a distance of GRF (= $\frac{1}{2\text{GRF}}$). An example of this spectrum has already been shown in fig. 2.5.

In a receiver, the incoming antenna signal is sampled. It is assumed here that only one sample is taken per burst. The sample is multiplied with +1 or -1, depending on the station status (master or secondary) and the burst number (0..15). The sample is then used in a phase-locked loop (usually implemented in software) to find and track the proper zero-crossing of the Loran-C signal.

Mathematically, this process can be described with two functions:

1) the phase code function $s_{\text{phasecode}}(t)$, already described in formula 2.5;
2) the infinite impulse series \( s_{\text{impulse}}(t) \) with repetition interval 2 GRI, as described in formula 2.6.

The two functions \( S_{\text{phasecode}}(t) \) and \( s_{\text{impulse}}(t) \) are again convolved to yield the proper sampling function \( s_{\text{sampling}}(t) \). This process is illustrated in fig. 2.14.

\[ s_{\text{sampling}}(t) = s_{\text{impulse}}(t) \ast S_{\text{phasecode}}(t) \]

\[ \ast = \text{convolution} \]

**Fig. 2.14: Receiver sampling process in time domain.**

Again, in the frequency domain the receiver sampling function \( S_{\text{sampling}}(\omega) \) is obtained by multiplying the Fourier transforms \( S_{\text{phasecode}}(\omega) \) and \( S_{\text{impulse}}(\omega) \). Since \( S_{\text{impulse}}(\omega) \) is a line spectrum (with lines at intervals GRI) and \( S_{\text{phasecode}}(\omega) \) a continuous function, the result is a sampling function which can be described as:

\[ S_{\text{sampling}}(\omega) = S_{\text{phasecode}}(N \cdot \text{GRF}), \quad N = 1, 2, 3 \ldots \]  

(2.8)

and by actually writing out the Fourier transform of \( S_{\text{phasecode}}(t) \), formula 2.8 can be written as:

\[ S_{\text{sampling}}(\omega) = \sum_{i=0}^{7} \left[ pc(i) e^{-j\omega iT_p} \right] + e^{-j\omega \text{GRI}} \cdot \sum_{i=8}^{15} \left[ pc(i) e^{-j\omega (i-8)T_p} \right] \]  

(2.9)

with \( pc(i) \) \((i = 0..15, \ pc(i) = -1 \ or \ +1)\) describing the phase code of the station, \( T_p \) the distance between pulses in one GRI of 1 ms, and \( \omega = N \cdot \text{GRF} \). This receiver sampling process has been described in [2.4] too.
In a receiver, the incoming signal is multiplied in the time domain with the sampling function $S_{\text{sampling}}(t)$. In the frequency domain, the incoming spectrum is convolved with the sampling function $S_{\text{sampling}}(\omega)$. In fig. 2.15 an attempt is made to illustrate what happens when a spectrum containing a single CWI signal at $f_{\text{CWI}}$, is convolved with an impulse function like $S_{\text{sampling}}(\omega)$.

Fig. 2.15 illustrates that the result of the receiver sampling pattern, is that the whole spectrum between $-\infty$ and $\infty$ is folded back onto a spectrum between $-\frac{1}{2}$ GRF and $\frac{1}{2}$ GRF. The Loran-C carrier of 100 kHz is folded back to 0, and the tracking loop will adjust sampling timing such that the DC value at $f = 0$, is 0 too (sampling at a zero-crossing of the Loran-C carrier). In reality, of course, the spectrum that is folded back, is limited by the input bandpass filter of the receiver.

![Diagram showing the process of spectrum convolution.](Image)

*Fig. 2.15: Result of spectrum convolution.*

Note that the amplitude of the receiver sampling function $|S_{\text{sampling}}(\omega)|$ as defined in formula 2.9, is not 1, but varies between -10 and -18 dB. This effect can be illustrated by representing the CWI signal as a vector, and calculating this vector for all 16 bursts in one period of 2 GRI (see fig. 2.16, where vectors A belong to the A-GRI and vectors B to the B-GRI). Then these 16 vectors are averaged, and the resultant vector turns out to be smaller than the original CWI vector. This means that in linear Loran-C receivers, the phase coding pattern
Fig. 2.16: CWI rejection through phase coding.

built into Loran-C does already provide a partial CWI rejection capability of at least 10 dB. In hard-limiting receivers, calculating the CWI rejection due to the phase coding pattern is much more difficult, because of the non-linear signal averaging process [2.8].

After a receiver has taken samples of the incoming signal, these samples are used in two loops:

- An envelope tracking loop which determines the zero-crossing of the Loran-C signal to be tracked: the so-called Cycle Identification process. Chapter 3 contains more information about the operation of the Cycle Identification process in Loran-C receivers.

- A phase tracking loop, which is used for actual propagation time measurements. Many different loop types have been proposed or implemented. [2.10] contains descriptions of well-established phase-tracking algorithms for hard-limiting receivers; in contrast [2.1] describes a completely different approach to phase tracking, based on a linear receiver architecture.

Envelope and phase tracking loops in fact constitute very narrow lowpass filters, with bandwidths ranging from 0.001 Hz for envelope tracking, to 0.1 Hz for phase-tracking loops. Only those interference frequencies in fig. 2.15 that fall
inside the filter passband, will be seen at the output of the tracking loops. This leads to a separation of CW interference signal types:

1) Synchronous interference signals are those signals which fold back to \( f = 0 \). These signals can be found on frequencies that are exact multiples of GRF:

\[
f_{\text{CWI}} = N \cdot \text{GRF} = N \cdot \frac{1}{2 \ \text{GRI}}, \quad N = 1, 2, 3, \ldots \tag{2.10}
\]

2) Near-synchronous interference signals are those signals which fold back to frequencies smaller than the tracking bandwidths \( f_b \) of the tracking loops (with \( f_b \) much smaller for envelope tracking than for phase tracking). These signals can be found on frequencies near to exact multiples of GRF:

\[
f_{\text{CWI}} = N \cdot \text{GRF} + \Delta f = N \cdot \frac{1}{2 \ \text{GRI}} + \Delta f, \quad N = 1, 2, 3, \ldots, \quad |\Delta f| < f_b \tag{2.11}
\]

3) A-synchronous interference signals are those signals which fold back to frequencies larger than the tracking bandwidth \( f_b \):

\[
f_{\text{CWI}} = N \cdot \text{GRF} + \Delta f = N \cdot \frac{1}{2 \ \text{GRI}} + \Delta f, \quad N = 1, 2, 3, \ldots, \quad |\Delta f| \geq f_b \tag{2.12}
\]

For hard-limiting receivers, a fourth class of signals has been defined in [2.8]: sub-synchronous CWI. These signals can be found on frequencies:

\[
f_{\text{CWI}} = (N + \frac{1}{n}) \cdot \text{GRF} = (N + \frac{1}{n}) \cdot \frac{1}{2 \ \text{GRI}}, \quad N = 1, 2, 3, \ldots, \quad n = 2, 3, 4, \ldots \tag{2.13}
\]

According to [2.8], sub-synchronous CWI can be safely neglected for linear receivers in general and for hard-limiting receivers under noisy conditions. Since the remainder of this thesis will deal only with linear receiver architectures, sub-synchronous CWI will not be discussed in detail here.

The effects of the three main types of CWI signals on loop operation, are quite different:
1) Synchronous signals fold back to $f = 0$, with a signal phase that is a random variable. For a signal phase of approximately 90° relative to the Loran-C carrier signal, a maximum DC component at $f = 0$ will be introduced. The phase tracking loop will adjust its output phase to follow the phase of the Loran-C signal plus the interference, and therefore will make a time-invariant error in the propagation time measurement. This leads to a time-invariant error in the range measurement and consequently to a stationary position error. The amplitude of the error will depend on the filtered Signal-to-Interference Ratio (SIR), according to the following formula:

$$E_{\text{track}} = \frac{T_L}{2\pi} \sin^{-1} \left( \frac{1}{\text{SIR}} \right)$$  \hspace{1cm} (2.14)

with $E_{\text{track}}$ the time measurement error and $T_L$ the Loran-C cycle time of 10 μs. The method used to derive formula 2.14 can be found in [2.8], which contains an in-depth analysis of the effects of all types of CWI interference.

2) The DC-component introduced by synchronous signals also severely affects the Cycle Identification capabilities of a Loran-C receiver. This is due to two different effects:

- As described above, synchronous CWI changes the sampling moment used for zero-crossing tracking. In conventional Loran-C receivers, the sampling clock used for Cycle Identification is derived from the zero-crossing sampling clock. Therefore, any error in the zero-crossing tracking mechanism will show up in the Cycle identification too.

- Cycle Identification depends on accurate amplitude information of the folded-back Loran-C signal. Any synchronous CWI adds a DC level to this folded-back Loran-C signal, and therefore changes the amplitude information.

Up until now no analysis method has been presented in literature, dealing with this Cycle Identification problem. Chapter 3 describes a first step to deal with this omission.

3) Near-synchronous signals fold back to $f \leq f_b$. The phase tracking loop will see a signal with low frequency at its input, and will follow the phase of the
sum of the Loran-C signals and the CWI signal. This leads to an oscillating phase error and therefore also an oscillating position error. Again, the amplitude of the error will depend on the filtered SIR, and can also be calculated with formula 2.14. Envelope tracking loops can also be affected by near-synchronous CWI; the analysis method described in chapter 3 for synchronous signals, can be used to describe the effect of near-synchronous CWI on Cycle Identification as well. Note, though, that envelope tracking loops are much less likely to experience near-synchronous CWI than phase tracking loops, since their bandwidth $f_b$ is much lower.

4) A-synchronous signals fold back to $f \geq f_b$. Such signals will be attenuated by the low-pass nature of the tracking loops. However, in practice tracking of the Loran-C zero-crossing will become less quiet; therefore the effect of a-synchronous signals can be described as an increase in the noise level seen by the loop [2.8]. Similarly, Cycle Identification will also experience larger signal variations; this leads to an increased risk of improper operation.

Generally speaking, synchronous and near-synchronous interference signals are the most dangerous signal types, since these lead directly to position errors. Fortunately, the risk that a single CWI signal becomes (near-)synchronous, is quite low due to the small tracking bandwidths $f_b$ employed. The frequency band from $-\frac{1}{2}$ GRF to $\frac{1}{2}$ GRF is minimally 5 Hz wide. With a tracking bandwidth of 0.01 Hz and a randomly distributed CWI frequency, the risk that the CWI signal becomes synchronous, is $\frac{2 \cdot 0.01 \text{Hz}}{5 \text{Hz}} = 0.4\%$ worst-case.

In an area with little CWI, such a (relatively low) risk may be acceptable. However, in western Europe actual measurements have produced 68 CWI signals [2.6], of which 43 probably are not controlled by atomic standards; this means that their exact frequency can be considered random due to oscillator drift. In this case, the chance that none of these signals is (near-)synchronous, becomes $(1 - 0.004)^{43} = 84\%$. This limits the reliability of Loran-C in western Europe to 84\%, if nothing is done about (near-)synchronous signals! Clearly this is unacceptable. Note too, that other sources have reported many more CWI signals [2.7], indicating that the reliability problem of Loran-C due to (near-)synchronous CWI might even be worse. This clearly shows that counter-measures against (near-)synchronous CWI are needed.
2.6 Strategies for solving interference problems

In the previous paragraphs, the nature and effects of the interference problem have been shown. As explained, two types of interference can be distinguished: Cross Rate Interference (from other Loran-C chains) and Carrier Wave Interference (from all other transmissions in and around the Loran-C band). These two types have different properties and different effects on position accuracy, and will require separate strategies to get rid of them. Due to limitations in time and space, the remainder of this thesis will deal solely with the reduction of CWI interference. Solutions to Cross Rate Interference problems will not be described.

CWI Interference can best be combatted with a combination of different methods, because of both the size of the problem (in Europe, there are many CWI signals) and the potential dangers. Generally speaking, two levels can be found where steps can be taken to reduce or remove interference: at the system level (when designing Loran-C chains) and at the receiver level. Both levels, however, have a common problem: for any CWI signal an expression has to be found for the expected risk to proper Loran-C operation, preferably in the form of a single number. As explained in the previous paragraphs, the risk posed by a CWI signal depends on two signal parameters: the interference signal strength and the interference frequency. A method for generating a single number describing the expected harm done by a CWI signal, will be described in chapter 4.

Interference reduction at system level is done by properly choosing the parameters of Loran-C chains. During the design of a new Loran-C chain, values for the following parameters have to be chosen:

1) Transmitter positions (and therefore chain coverage). These parameters are determined by coverage requirements, and cannot be changed much in order to improve interference rejection.

2) Chain timing: the transmitting order in a chain and the selection of Emission Delays (ED) for the transmitters. These parameters are closely linked to:

3) Group Repetition Interval. The GRI of a Loran-C chain can be chosen freely between two values:

Signals interfering with Loran-C
- a minimum GRI determined by the transmitter positions and chain timing requirements defined by the United Stated Coast Guard (USCG) [2.11];

- and a maximum GRI of 0.09999 seconds (which is also defined by the USCG in [2.11]).

In chapter 5 a description will be given of selection procedures for chain timing and GRI which yield minimum interference susceptibility.

Interference reduction in receivers can (and should) be done in two steps:

1) Detection of interference. Methods for CWI detection in receivers will be explained in chapters 6 and 7.

2) Rejection of interference. CWI is usually filtered out with notch filters. Conventional notch filters work, but can be improved; methods for improving their overall performance will be dealt with in chapter 6.

2.7 References


3 Cycle Identification and CWI

3.1 Introduction

Though much effort has been put into understanding the influence of CWI on phase tracking, little attention has been paid to the distortion caused to the Loran-C envelope by CWI. The envelope is at least as important for proper Loran-C receiver operation as proper phase tracking, since the envelope is used by the Loran-C receiver to determine the proper carrier cycle to be tracked. Errors in this Cycle Identification (CI) process immediately yield errors of multiples of 3 km in range measurements. A CWI signal does always distort the envelope of a Loran-C burst, as is illustrated in the example of fig. 3.1. In this figure, the signal drawn with solid lines represents a Loran-C signal contaminated with a CWI signal at 85 kHz, 3 dB stronger than the Loran-C signal according to the MPS definition of Signal-to-Interference Ratio [ 3.8 ]. The pulse drawn with dotted lines, is a pure Loran-C signal.

Fig. 3.1: Envelope distortion caused by CWI.

In the presence of a-synchronous signals, the envelope distortion will be different for every pulse, with an average distortion of zero. With properly designed Cycle Identification systems, these signals are removed due to the long
integration times that are used for Cycle Identification anyway. However, the relative phase $\varphi_I$ between a synchronous CWI signal and the Loran-C pulse is equal for all pulses spaced 2 GRI apart, as demonstrated in fig. 3.2.

![Diagram showing phase relation of synchronous CWI and Loran.]

**Fig. 3.2: Phase relation of synchronous CWI and Loran.**

This means that after 2 GRI seconds, the receiver will see exactly the same antenna signal (consisting of a Loran-C pulse and a synchronous CWI signal) again. Any distortion in the Loran-C envelope due to the CWI signal, can therefore not be removed by integrating samples of the antenna signal over the intervals commonly used for Cycle Identification, which are much longer than 2 GRI. The next section will show that the Cycle Identification process depends heavily on correct envelope information; since synchronous CWI changes the Loran-C signal envelope, it poses a heavy threat to the reliability of Cycle Identification.

Until now no generally valid analysis method has been developed, describing the deterioration of Cycle Identification due to synchronous CWI signals. Existing literature ([3.7], [3.5]) has always described only cases of particular CWI signals: one CWI frequency $f_{CWI}$ has been selected, with a pre-defined Signal-to-Interference Ratio and relative phase $\varphi_I$. It is then relatively easy to generate a plot of the combined signal, and see the distortion of the Loran-C signal due to the synchronous CWI. Computer simulations of Cycle Identification mechanisms can be carried out to get more information about the influence of the chosen set of CWI conditions on Cycle Identification. However, this approach does not guarantee at all that the obtained results describe a worst-case situation, or that results obtained for one set of $f_{CWI}$, $\varphi_I$ and SIR are even approximately related to the results obtained for a different set of CWI conditions.
In this chapter, a new method will be introduced for analysing the influence of CWI on Cycle Identification. This method calculates the deterioration of the reliability of the most commonly used Cycle Identification methods in the presence of one synchronous CWI signal, with pre-determined $f_{CWI}$ and SIR. The CWI phase $\phi_l$ will be unpredictable under real-world circumstances; therefore, the proposed method finds the value of $\phi_l$ for which Cycle Identification reliability is most affected. Since the proposed method is not based on (computer) simulations but rather on solving formulas, it can be used easily to calculate the deterioration of Cycle Identification reliability for a large number of different CWI frequencies and Signal-to-Interference Ratios, and thus to generate plots of Cycle Identification reliability versus $f_{CWI}$ or SIR. Such plots will greatly enhance the understanding of the influence of synchronous CWI on Cycle Identification.

The reliability of the Cycle Identification mechanism is of course directly linked to the deviation of the antenna signal envelope from the ideal Loran-C envelope: a higher deviation yields less reliable Cycle Identification. The proposed method therefore concentrates on calculating this envelope deviation. Performance standards have been defined for the maximum amount of envelope distortion which a Loran-C receiver must be able to tolerate [3.8]. The proposed method generates results which can be compared directly to these standards, showing immediately whether a Loran-C receiver can be expected to perform reliable Cycle Identification under the chosen synchronous CWI conditions. In section 3.3 this issue will be dealt with in more detail. First, however, the two most widely used Cycle Identification methods will be explained briefly and it will be shown, that both methods work on the same principle.

3.2 Cycle Identification - the two most common methods

In the previous section, it has been shown that CWI signals influence the Cycle Identification abilities of a Loran-C receiver, and that a general model describing the deterioration, is needed. In order to be able to develop such a model, first attention has to be focused on Cycle Identification mechanisms.

Receivers use phase tracking loops in order to get Time-of-Arrival data from the received Loran-C signal. Conventionally, these loops find and track a zero crossing of the incoming signals, i.e. the moment in time when the incoming signal changes from positive to negative or vice versa. Within each Loran-C
cycle of 10 μs there is one positive and one negative zero crossing (see fig. 3.3). It is assumed here that receivers use either the positive or the negative crossing. Therefore, one Loran-C cycle has one zero crossing useful to the receiver.

![Zero crossing](image)

Fig. 3.3: Loran-C pulse with tracked zero crossing.

Cycle Identification will be defined here as the mechanism by which the receiver finds the cycle of the Loran-C carrier it wants to use for Time-Of-Arrival (TOA) or Time-Difference (TD) measurements. Note that this definition does not fix the cycle to be used: this could be any cycle within the Loran-C burst.

Traditionally, Loran-C receivers have found the cycle to be used for tracking by internally generating two signals derived from the incoming antenna signal $s_1$ (see fig. 3.4): a signal $s_2$ attenuated with factor $A$, and a signal $s_3$ delayed by 5 μs.

![Circuit diagram](image)

Fig. 3.4: Traditional Cycle Identification circuit.
These two signals are summed; this summation will result in a phase reversal somewhere in the combined signal, as shown in the example of fig. 3.5. The position of this phase reversal in the combined signal depends on the attenuation factor $A$ and the envelope of the Loran-C signal, which is dependent on the bandpass filter system used in the front-end of the receiver. The phase reversal moment can be detected easily, since it is the only time interval in the burst where two samples taken 2.5 $\mu$s before and 2.5 $\mu$s after the zero crossing (i.e. on the signal peaks) will have the same sign. The attenuation factor $A$ is adjusted until the phase reversal occurs at the position of the zero-crossing that has been selected for proper phase tracking. This so-called delay-and-add method works well for linear and hard-limiting receivers. It has been described in more detail in [3.6].

A second method useful for cycle identification in linear receivers is shown in fig. 3.6. Here, two samples are taken: one 2.5 $\mu$s before and another 2.5 $\mu$s after a zero crossing of the incoming signal. The amplitudes of the two samples are then used to form an amplitude ratio $R_{amp}$:

---

**Fig. 3.5:** Combined signal in a traditional Cl circuit.

**Fig. 3.6:** Cycle Identification in linear receivers.
\[ R_{\text{amp}} = \frac{A_2}{A_1} \]  

(3.1)

A table can be calculated, giving the amplitude ratio \( R_{\text{amp}} \) for every zero crossing of a loran-C burst.

A Loran-C receiver in its initialization phase will track a zero crossing found at random. It then uses the following mechanism for its Cycle Identification:

1) determine \( R_{\text{amp}} \) for the zero crossing being tracked at the moment;

2) look up the position of the zero-crossing in the table;

3) calculate the time difference between the zero crossing that is tracked and the zero crossing that should be tracked;

4) and finally jump to the correct zero crossing.

Table 3.1 shows an example of a ratio table, calculated with a 5th order Butterworth input bandpass filter.

| Filter | Butterworth 5th order |
| Center Frequency | 100 kHz |
| Bandwidth | 18 kHz |
| Zero number | Position of zero | Ratio | Zero number | Position of Zero | Ratio |
| 1. | 7.6942 \( \mu \)s | 25.47 | 6. | 30.8915 \( \mu \)s | 2.44 |
| 2. | 12.1665 \( \mu \)s | 9.12 | 7. | 35.7264 \( \mu \)s | 2.12 |
| 3. | 16.7180 \( \mu \)s | 5.27 | 8. | 40.5939 \( \mu \)s | 1.90 |
| 4. | 21.3693 \( \mu \)s | 3.72 | 9. | 45.4860 \( \mu \)s | 1.74 |
| 5. | 26.1006 \( \mu \)s | 2.92 | 10. | 50.3968 \( \mu \)s | 1.62 |

*Table 3.1: Example of a ratio table.*
Basically, the delay-and-add method makes use of amplitude ratios as well:

- In a traditional CI circuit as shown in fig. 3.4, the output signal represents the difference between the attenuated signal \( s_2 \) and the delayed signal \( s_3 \). This is due to the delay time of 5 \( \mu s \), which inverts the carrier and therefore converts the addition of the signals into a subtraction of the envelopes.

- The phase inversion shown in fig. 3.5 takes place at the moment that \( s_2 \) and \( s_3 \) have equal amplitudes (difference zero). This phase inversion is used to mark the zero crossing to be tracked.

- Since signal \( s_2 \) in fig. 3.4 is attenuated with a factor \( A \), the phase inversion will occur at the moment where signals \( s_1 \) and \( s_3 \) have an amplitude ratio \( \sqrt{A} \). Because \( s_1 \) and \( s_3 \) represent the same signal with a time difference of 5 \( \mu s \), the phase inversion in fact marks a zero crossing with a pre-defined amplitude ratio \( R_{amp} = \sqrt{A} \).

Because the Cycle Identification methods shown in figs. 3.4 and 3.6 are both based on finding the position on the Loran-C pulse with a fixed ratio \( R_{amp} \), it is interesting to analyze the change of a ratio \( R_{amp} \) due to synchronous CWI. This will be done in the next section.

### 3.3 Calculating amplitude ratios in the presence of CWI

We are interested in the worst-case ratio error, i.e. the worst-case difference between the ratio belonging to a zero-crossing with synchronous CWI interference and the ratio belonging to the same zero-crossing, without synchronous CWI interference present. For the calculation of this ratio difference, the following assumptions were made:

- A decision has been made on whether input bandpass filter systems are to be taken into account, and if so, what filter specifications are used (filter type, center frequency, bandwidth, etc.). This defines the pure Loran-C waveform used in the analysis.

- Within the pure Loran-C burst, one negative-going zero crossing is currently being (phase-)tracked. Around this zero-crossing, the Loran-C signal is
composed as shown in fig. 3.7. It consists of two half sine waves around a negative going zero crossing, with two different amplitudes $A_1$ and $A_2$. This is in fact an approximation of a Loran-C burst, since the amplitude of a Loran-C burst is a continuous function of time $t$, while the amplitude of the signal in fig. 3.7 is constant during each half sine wave cycle. The zero crossing of the pure signal in fig. 3.7 provides the time reference $t = 0$ for the calculations.

![Diagram of Loran-C signal](image)

**Fig. 3.7: Definition of Loran-C signal for analysis.**

This Loran-C signal can be expressed mathematically as:

$$ S_{Loran} = \begin{cases} 
-A_1 \cdot \sin(\omega_L \cdot t) & t \leq 0 \\
-A_2 \cdot \sin(\omega_L \cdot t) & t \geq 0 
\end{cases} \quad (3.2) $$

with $\omega_L$ the Loran-C carrier frequency of $2\pi \cdot 100$ kHz. Note that a change in the bandpass filter system definition, or the selection of a different zero-crossing for phase-tracking, implies that different values for $A_1$ and $A_2$ have to be used.

- The CWI signal is shown in fig. 3.8. It is a pure sine wave with a negative going zero crossing with a phase difference $\phi_I$ between the Loran-C signal and the CWI signal:

$$ S_I = A_I \cdot \sin(\omega_I \cdot t + \phi_I) \quad (3.3) $$
Fig. 3.8: Definition of CWI signal for analysis.

The phase difference $\varphi_i$ is assumed to be random. This corresponds with real-world conditions: due to propagation effects and position shifts the phase of a received CWI signal is impossible to predict. The ratio found with a CWI signal present, can be smaller or larger than the ratio of the pure signal depending on phase shift $\varphi_i$. Therefore, both the maximum and minimum ratios as function of $\varphi_i$ should be found.

In order to be able to calculate the maximum and minimum ratios as function of $\varphi_i$, the following steps have to be made:

1) First the phase tracking error due to the CWI signal has to be calculated. This is necessary since a Loran-C receiver determines ratios by taking samples 2.5 $\mu$s before and 2.5 $\mu$s after the zero crossing it is tracking. This implies that if a tracking error is made, the sampling moments used for ratio determination will be shifted in time too; this alone will already cause an error in the measured ratio.

2) Then the measured ratio $R_{\text{measured}}$ belonging to the tracked zero crossing has to be calculated. This ratio is a function of the Loran-C parameters as defined in equation 3.2 and of the CWI signal parameters as defined in equation 3.3, including $\varphi_i$.

3) Next the first derivative $\frac{dR_{\text{measured}}}{d\varphi_i}$ has to be found and set equal to zero. This will yield all values of $\varphi_i$ where $R_{\text{measured}}(\varphi_i)$ has a local maximum or minimum.
4) For all values of $\phi_t$ where $R_{measured}$ has a local maximum or minimum, that maximum or minimum has to be calculated and the overall highest maximum and lowest minimum have to be found. This yields the maximum and minimum ratios and, by subtracting the constant ratio of the pure signal, also the maximum and minimum ratio errors.

For ease of interpretation it was decided to include a possibility to recalculate a ratio found with CWI present, into an apparent zero-crossing. This is done by converting the ratio table calculated for the pure Loran-C signal (which will be similar to the example in table 3.1) into a continuous function, using interpolation techniques:

$$R_{\text{amp}} = f_{\text{ratio}}(t).$$

and $f_{\text{ratio}}(t)$ is of course dependent on the analog input bandpass filter chosen for the analysis, too. With the inverse of function 3.4, a time $t_{\text{zero}}$ can be assigned to a ratio $R_{\text{amp}}$:

$$t_{\text{zero}} = f_{\text{ratio}}^{-1}(R_{\text{amp}}).$$

This time $t_{\text{zero}}$ can be interpreted as being the position at which the receiver thinks the zero crossing currently used for phase tracking, is located, after it has measured ratio $R_{\text{amp}}$. With a pure Loran-C signal, the measured value for $R_{\text{amp}}$ must correspond to the zero-crossing currently being tracked. If, however, the Loran-C burst envelope is distorted (due to ECD or synchronous CWI), a value for $R_{\text{amp}}$ will be found which is different from the value found for a pure signal, and no zero crossing at $t_{\text{zero}}$ will exist.

In the four steps described above, a ratio has been calculated for a selected zero-crossing in the presence of CWI. This ratio is called $R_{\text{amp, CWI}}$; it can be converted into a time $t_{\text{zero, apparent}}$ with formula 3.5:

$$t_{\text{zero, apparent}} = f_{\text{ratio}}^{-1}(R_{\text{amp, CWI}}).$$

By subtracting the zero crossing of the pure signal used for tracking (defined as $t = 0$ in fig. 3.7) from $t_{\text{zero, apparent}}$, it is possible to calculate the apparent shift of the envelope due to CWI (see the example of fig. 3.9). This shift is called the
apparent ECD due to CWI. Note that in the example of fig. 3.9, the shape of function $f_{\text{ratio}}(t)$ is completely fictitious.

![Graph showing calculation of apparent ECD](image)

**Fig. 3.9: Calculation of the apparent ECD.**

For the calculation of the tracking error due to CWI (step 1 on page 43), the phasor diagram method described in [3.4] has been used. Fig. 3.10 shows the phase relations between a Loran-C signal (with zero crossing at $t = 0$), a CWI signal and the combined signal.

![Phasor diagram](image)

**Fig. 3.10: Phasor diagram used in analysis.**

The phase difference $\phi_I$ and therefore the angle $\alpha$, as well as the Loran-C and CWI amplitudes $L$ and $I$ are known. First the amplitude of the combined signal is calculated:

$$S = \sqrt{I^2 + L^2 - 2 \cdot I \cdot L \cdot \cos(\alpha)}$$  \hspace{1cm} (3.7)

*Cycle Identification and CWI*
The angle \( \alpha \) is written as: \( \alpha = \pi - \phi_0 \), and so equation 3.7 becomes:

\[
S = \sqrt{I^2 + L^2 - 2 \cdot I \cdot L \cdot \cos(\pi - \phi_0)}
\]
\[
= \sqrt{I^2 + L^2 + 2 \cdot I \cdot L \cdot \cos(\phi_0)}
\]

(3.8)

Angle \( \phi_{err} \) (the phase tracking error angle) can then be calculated with:

\[
\frac{I}{\sin(\phi_{err})} = \frac{S}{\sin(\alpha)} \rightarrow \sin(\phi_{err}) = \frac{I}{S} \cdot \sin(\alpha) \rightarrow
\]

\[
\phi_{err} = \sin^{-1}\left(\frac{I}{S} \cdot \sin(\alpha)\right)
\]
\[
= \sin^{-1}\left(\frac{I}{S} \cdot \sin(\pi - \phi_0)\right)
\]
\[
= \sin^{-1}\left(\frac{I}{S} \cdot \sin(\phi_i)\right)
\]

(3.9)

This tracking error angle can be converted into a time shift error:

\[
t_{err} = \frac{T_L}{2 \cdot \pi} \cdot \phi_{err}
\]
\[
= \frac{T_L}{2 \cdot \pi} \cdot \sin^{-1}\left(\frac{I}{S} \cdot \sin(\phi_i)\right)
\]

(3.10)

and \( S \) in equation 3.10 can be calculated with equation 3.7. For simplicity's sake we will write equation 3.10 often as:

\[
t_{err} = t_{err}(\phi_i)
\]

(3.11)

Samples for the measurement of the ratio are now taken at \( t_1(\phi_i) = -2.5 \mu s - t_{err}(\phi_i) \) and \( t_2(\phi_i) = +2.5 \mu s - t_{err}(\phi_i) \). The corresponding amplitude ratio is:
\[ R_{\text{amp}}(\varphi_l) = \frac{A_2 \cdot \sin(\omega_L \cdot t_2) + A_I \cdot \sin(\omega_I \cdot t_2 + \varphi_l)}{A_1 \cdot \sin(\omega_L \cdot t_1) + A_I \cdot \sin(\omega_I \cdot t_1 + \varphi_l)} \]  

(3.12)

and of course in equation 3.12 \( t_1 \) and \( t_2 \) are functions of \( \varphi_l \) too.

As \( \varphi_l \) is randomly and evenly distributed between \(-\pi\) and \(\pi\), the first derivative \( \frac{dR_{\text{amp}}}{d\varphi_l} \) has to be calculated and set to zero to find all local minima and maxima:

\[ \frac{dR_{\text{amp}}}{d\varphi_l} = 0 \]  

(3.13)

Getting an expression for \( \frac{dR_{\text{amp}}}{d\varphi_l} \) is complex but standard mathematics, which will not be shown here. The result of this mathematical exercise is that the following equation has to be solved:

\[
\left( A_2 \cdot \frac{dt_2}{d\varphi_l} \cdot \cos(\omega_L \cdot t_2) + A_I \cdot \left( 1 + \frac{dt_2}{d\varphi_l} \cdot \cos(\omega_I \cdot t_2 + \varphi_l) \right) \right) \cdot \\
\left( A_1 \cdot \sin(\omega_L \cdot t_1) + A_I \cdot \sin(\omega_I \cdot t_1 + \varphi_l) \right) - \\
\left( A_1 \cdot \frac{dt_1}{d\varphi_l} \cdot \cos(\omega_L \cdot t_1) + A_I \cdot \left( 1 + \frac{dt_1}{d\varphi_l} \cdot \cos(\omega_I \cdot t_1 + \varphi_l) \right) \right) \cdot \\
\left( A_2 \cdot \sin(\omega_L \cdot t_2) + A_I \cdot \sin(\omega_I \cdot t_2 + \varphi_l) \right)
\]

= 0  

(3.14)

with \( t_1 \) and \( t_2 \) functions of \( \varphi_l \). Equation 3.12 has a local maximum or minimum at all zeros of equation 3.14. These zeros can be found with well-known numerical methods.

From the set of local maxima and minima the global maximum and minimum ratios can be selected. The maximum and minimum ratios can be converted into maximum (positive) and minimum (negative) apparent ECD shifts with the method shown in fig. 3.9 and equation 3.10.
In theory, an analytical solution to equation 3.14 could be developed. However, it should be realised that this equation is more complicated than it seems at first sight:

- $t_1$ and $t_2$ are non-linear functions of $\varphi_i$;

- $A_1$ and $A_2$ depend on the filter type and the zero crossing chosen for the analysis, and can be calculated only with elaborate mathematical expressions (see appendix A).

A simpler approach to solving equation 3.14, is to use a numerical method, implemented in a computer program. Such a program should also contain bandpass filter algorithms, for the calculation of $A_1$ and $A_2$. This approach has been chosen in the work described in this chapter; the next section will describe a suitable program.

### 3.4 The Delft CWI analysis software

As stated in the previous section, equation 3.14 is too complex to be solved analytically. Therefore, it was decided to use the Bisect method [3.2], which numerically calculates the roots of an equation with a single variable, to solve equation 3.14. For that purpose, the following Loran-C and CWI signal parameters have to be known:

- $A_1$ and $A_2$: the amplitudes of the Loran-C half cycles just before and after the zero crossing of the pure signal which is tracked;

- $A_i$: the interference signal amplitude;

- $\omega_i$: the interference signal angular frequency.

The amplitudes of the Loran-C and CWI signals can be found easily when it is assumed that these signals are not filtered. This, however, is rather unrealistic, since every Loran-C receiver uses a bandpass filter system in its front-end. This filter system influences the Loran-C burst envelope (and thereby the amplitudes $A_1$ and $A_2$) even though it usually has no attenuation in the Loran-C band. It also changes the CWI signal amplitude $A_i$. 
For the calculation of $A_1$ and $A_2$, algorithms for the simulation of bandpass filtering of Loran-C pulses have to be available; such algorithms are described in [3.1] and appendix A. In their original form, these algorithms only calculate the filtered Loran-C pulse $s_{\text{Loran-C, filtered}}(t)$. With numerical methods the zero-crossings of $s_{\text{Loran-C, filtered}}(t)$ can be found, and by calculating the signal values at 2.5 $\mu$s before and 2.5 $\mu$s after the zero crossing, the amplitudes $A_1$ and $A_2$ are found.

Once filter algorithms are available, calculating the filtered CWI amplitude $A_I$ is easy:

$$ A_I = \left| H_{BPF}(\omega_I) \right| \cdot 10^{\frac{\text{SIR}}{20}} $$

(3.15)

where

- SIR is the Signal-to-Interference Ratio of the CWI signal as defined in the Minimum Performance Specifications [3.8];

- $\omega_I$ is the angular frequency of the CWI signal;

- $\left| H_{BPF}(\omega_I) \right|$ is the amplitude transfer of the filter system at frequency $\omega_I$.

It was decided to include equations 3.12 and 3.14 and the Bisect method used to solve these equations, into the existing receiver simulation program LOSP. This program is described in detail in appendix A and includes all necessary filter calculation algorithms as well as facilities easing the implementation of equations 3.12 and 3.14:

- Calculation of zero-crossings, amplitudes and ratios of filtered Loran-C bursts, for a wide selection of different bandpass filter systems.

- Calculation of amplitudes of filtered CWI signals.

- Easy conversion of ratios into zero-crossing positions, as defined in equation 3.5 and fig. 3.9.
- A good user interface and a program structure which enables easy and fast adaptation.

The output of the calculations in equations 3.12 and 3.14 is given in graphics representation. Three functions are defined:

1) Calculation of apparent ECD as function of variable SIR, with the CWI frequency and the zero crossing to be tracked, having fixed values.

2) Calculation of envelope shift as function of the zero-crossing, with the CWI frequency and the SIR having fixed values.

3) Calculation of envelope shift as function of the CWI frequency, with the SIR and the zero crossing having fixed values.

Fig. 3.11 gives an example of a typical screen output of the calculations in equations 3.12 and 3.14 in LOSP. It shows that apparent ECD shift is represented in LOSP as a filled area. This area contains all possible ECD shifts that can be found as a function of the CWI phase $\phi$, between the maximum positive and the minimum negative shift found with equations 3.12 and 3.14. The upper window has the apparent ECD phase calculated as function of the Signal-to-Interference Ratio, with a pre-defined value for the interference frequency and one zero-crossing of the pure Loran-C burst selected. In the lower

Fig. 3.11: Typical LOSP screen output of CI analysis.
window the apparent ECD is calculated for different zero-crossings of the pure burst, with the interference frequency and the SIR having pre-defined values. A third option, which displays the apparent ECD as function of the interference frequency, with a pre-defined value for the SIR and one zero-crossing of the pure Loran-C signal selected, is not shown in fig. 3.11.

The next paragraphs will discuss examples and limitations of the analysis capabilities now provided with equations 3.12 and 3.14 and their implementation in LOSP.

3.5 An example

An interesting question to be answered is whether synchronous interference causes more harm to Cycle Identification or to phase tracking in a Loran-C receiver. An analysis was carried out in order to find an answer to this question under the following conditions:

- a GRI of 8940, which belongs to the French Loran-C chain;

- a synchronous interference signal at 85000 Hz, which comes from one of the UK DECCA chains;

- a SEIKO bandpass filter with a bandwidth of 20 kHz with an amplitude transfer function as shown in fig. 3.12, and a filtered Loran-C burst as shown in fig. 3.13.

![Amplitude transfer function of SEIKO filter.](image)

Fig. 3.12: Amplitude transfer function of SEIKO filter.
Fig. 3.13: Loran-C burst filtered with SEIKO filter.

- a zero-crossing as far up in the pulse without having a phase tracking error of more than 100 ns, with a worst-case skywave as defined in the MPS [3.8]: skywave delay 32.5 \( \mu \)s, Skywave-to-Groundwave Ratio 12 dB.

First the appropriate zero-crossing has to be found. LOSP can calculate the zero-crossings of signals with and without skywaves. By comparing the generated lists, the last zero-crossing with an error less than 100 ns can be found easily. Two lists, one calculated with and the other without skywaves, are shown in table 3.2.

<table>
<thead>
<tr>
<th>Bandpass filter</th>
<th>SEIKO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>20 kHz</td>
</tr>
<tr>
<td>Center frequency</td>
<td>100 kHz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Zero of pure signal</th>
<th>Zero with skywave</th>
<th>Zero of pure signal</th>
<th>Zero with skywave</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.4242 ( \mu )s</td>
<td>40.4242 ( \mu )s</td>
<td>65.1269 ( \mu )s</td>
<td>65.0614 ( \mu )s</td>
</tr>
<tr>
<td>45.3419 ( \mu )s</td>
<td>45.3410 ( \mu )s</td>
<td>70.0914 ( \mu )s</td>
<td>69.9760 ( \mu )s</td>
</tr>
<tr>
<td>50.2739 ( \mu )s</td>
<td>50.2709 ( \mu )s</td>
<td>75.0607 ( \mu )s</td>
<td>74.8787 ( \mu )s</td>
</tr>
<tr>
<td>55.2168 ( \mu )s</td>
<td>55.2047 ( \mu )s</td>
<td>80.0344 ( \mu )s</td>
<td>79.7690 ( \mu )s</td>
</tr>
<tr>
<td>60.1683 ( \mu )s</td>
<td>60.1365 ( \mu )s</td>
<td>85.0119 ( \mu )s</td>
<td>84.6474 ( \mu )s</td>
</tr>
</tbody>
</table>

Table 3.2: Zero's of pure and contaminated signals.
From table 3.2, it can be seen that the last zero crossing without skywave contamination is found at 65 μs. This zero crossing was used in the rest of the analysis.

The next step is to generate a picture containing the maximum and minimum envelope shift due to CWI under the chosen conditions ($f_{CW1} = 85$ kHz, zero-crossing at 65 μs), as a function of the unfiltered SIR. This was done again with LOSP and the results are shown in fig. 3.14.

![Diagram](image)

**Fig. 3.14: Envelope shift as function of SIR.**

We can define a synchronous CWI signal to be harmless to phase tracking if the tracking error it causes, is smaller than 100 ns. With equation 2.14, the tracking error as function of Loran-C and CWI amplitudes can be calculated directly:

$$t_{err} = \frac{T_L}{2\pi} \cdot \sin^{-1}\left(\frac{A_I}{A_1}\right)$$

(3.16)

where:

- $T_L$ is the Loran-C carrier cycle time of 10 μs;

- $A_1$ is the Loran-C signal amplitude before the zero crossing (this amounts to worst-case conditions);

- $A_I$ is the filtered CWI signal amplitude.

Equation 3.16 can be converted to calculate the filtered CWI signal amplitude $A_I$ as function of the tracking error $t_{err}$:
\[ A_t = A_1 \cdot \sin(2 \cdot \pi \cdot \frac{t_{err}}{T_L}) \]  

(3.17)

With equation 3.17 and the maximum allowable \( t_{err} \), we can then calculate the corresponding amplitude \( A_t \):

\[ A_t = A_1 \cdot \sin(2 \cdot \pi \cdot \frac{0.1 \mu s}{10 \mu s}) \approx 0.064 \cdot A_1 \]  

(3.18)

LOSP gives an amplitude \( A_1 \) (relative to the Loran-C pulse peak) for a SEIKO filter at 65 \( \mu \)s of 0.25, so the maximum permissible \( A_t \) is 0.016 (also relative to the Loran-C pulse peak). With the amplitude transfer of the SEIKO filter at 85 kHz, this amplitude can be recalculated into a Signal-to-Interference Ratio as defined in the MPS [ 3.8 ]. This simple calculation yields a SIR of 7 dB; this SIR should not get lower if the tracking error caused by the CWI signal on 85 kHz is to remain below 100 ns.

In fig. 3.14, we can see that a CWI signal at 85 kHz with a SIR of 7 dB, will cause an apparent ECD shift between 4 \( \mu \)s maximum and -4 \( \mu \)s minimum. This is already much more than specified in the MPS [ 3.8 ]; the MPS require a receiver to lock on the proper zero crossing with a maximum ECD of \( \pm 2.4 \mu \)s. In principle Cycle Identification is possible with ECD up to \( \pm 5 \mu \)s, but with the presence of noise and "real" ECD (due to propagation effects), the chance of detecting the proper cycle with an apparent ECD shift of 4 \( \mu \)s due to synchronous CWI, are quite slim. This illustrates that synchronous CWI signals are a higher risk to Cycle Identification than to phase tracking.

### 3.6 Conclusions

A model has been presented describing the effects of synchronous CWI interference on Loran-C Cycle Identification, for the two most common Cycle Identification mechanisms described in section 3.2. An example has been given of the usefulness of the implemented model in analyzing the problems synchronous CWI can cause.

The model presented here, is certainly not perfect. Some possible improvements are:
1) Due to the Loran-C transmission sequence with its irregular phase coding pattern, the relative phase $\phi_t$ of a synchronous CWI signal has a different value for each of the 16 Loran-C pulses within a cycle of 2 GRI. After 16 pulses (2 GRI seconds), $\phi_t$ has the same value again. For linear receivers, this effect can probably be modeled as a reduction in the interference amplitude $A_t$.

2) Equations 3.12 and 3.13 are valid if one CWI signal is present. As fig. 2.6 shows, this is not the case in western Europe, even if only synchronous signals are selected from all signals present. Therefore, an expansion of equations 3.12 and 3.13 is necessary to include the effects of more than one synchronous CWI signal.

3) Equations 3.12 and 3.13 are developed for pure Loran-C bursts without ECD. Future versions of the model can include "real" ECD (due to propagation effects) and skywaves, to see the total envelope shift under real-world conditions.

Another future development is the inclusion of the model presented here, into the coverage prediction software described in [ 3.3 ]. This should lead to a further improvement in real-world coverage prediction, especially under European conditions.

3.7 References


4 CWI signal weighting

4.1 Introduction

In chapter 2 an overview of the Carrier Wave Interference signal problem has been given, including the different CWI signals and the effects these signals have on positioning accuracy and reliability. In this (short) chapter, a method will be introduced, which can be used to determine the relative danger a CWI signal presents to Loran-C positioning. This method is useful both in interference reduction methods during chain design, and in interference detection and rejection schemes in receivers.

4.2 Which signals are most dangerous - weighting factors

In chapter 2, three different CWI signal types were described:

1) Synchronous interference signals are located on an exact multiple of the lowest Loran-C repetition frequency GRF:

\[ f_{CWI} = N \cdot \text{GRF} = \frac{N}{2 \text{GRI}}, \quad N = 1, 2, 3, \ldots \]  \hspace{1cm} (4.1)

2) Near-synchronous interference signals are located in a band around an exact multiple of the lowest Loran-C repetition frequency GRF, with the frequency difference \( |\Delta f| \) between the multiple of GRF and the interference frequency smaller than the tracking bandwidth \( f_b \) of the receiver:

\[ f_{CWI} = N \cdot \text{GRF} + \Delta f = \frac{N}{2 \text{GRI}} + \Delta f, \quad N = 1, 2, 3, \ldots , \quad |\Delta f| \leq f_b \]  \hspace{1cm} (4.2)

3) A-synchronous interference signals are all other signals:

\[ f_{CWI} = N \cdot \text{GRF} + \Delta f = \frac{N}{2 \text{GRI}} + \Delta f, \quad N = 1, 2, 3, \ldots , \quad |\Delta f| > f_b \]  \hspace{1cm} (4.3)
Of these three CWI signal types, synchronous and near-synchronous interference is usually considered most harmful, since the resulting errors cannot be distinguished in any way from changes in the receiver position.

In the next chapters, methods will be described for automatic selection of the most harmful CWI signals. These methods are used for very different purposes (Loran-C chain design resp. automatic filtering in Loran-C receivers). However, they have one common problem: a selection criterion in the form of a single number has to be generated for each CWI signal, describing the expected harm done by the signal to proper Loran-C operation.

At first sight the amplitude of an interfering signal seems to be the most important selection criterion of whether a signal is harmful or not. This certainly holds within one class of interference signals: a strong synchronous interference signal is more harmful than a weaker synchronous one. However, comparing amplitudes of different CWI signal types can lead to large errors: a weak synchronous signal, causing a tracking error of e.g. 0.5 µs, is more harmful than a stronger a-synchronous signal, which might cause a momentary phase error of e.g. 1 µs in the antenna signal, but which will not generate any noticeable error in the output of a receiver phase tracking or envelope loop.

This problem can be solved by multiplying the amplitude $A_{CWI}$ of each CWI signal found, with a weighting factor $W$. This weighting factor depends on the CWI signal frequency: synchronous and near-synchronous signals are multiplied with a high weighting factor; a-synchronous signals are multiplied with a low weighting factor. The result of this multiplication can be called the modified CWI amplitude $A_{CWI, \text{mod}}$:

$$A_{CWI, \text{mod}} = W(f_{CWI}) \cdot A_{CWI} \tag{4.4}$$

The modified CWI amplitude is then used as a criterion for selecting the most harmful CWI signals. This focuses filtering efforts mainly on synchronous and near-synchronous signals, but still keeps very large a-synchronous signals from blocking receiver tracking loops. Theoretically speaking, such a weighting function should represent the suppression of a-synchronous CWI signals by the tracking loop; this suppression is caused by the inability of the loop to follow the fast phase changes in the composite Loran-C / CWI signal due to the interference.
Fig. 4.1 shows a graphical representation of a weighting function, which gives the weighting factors for all types of interference signals. Note: only a small part of the Loran-C spectrum and its adjacent parts is shown, but the function will repeat itself around every spectral line of Loran-C, spaced at GRF Hz.

![Graphical representation of a weighting function](image)

*Fig. 4.1: Example of a CWI weighting function.*

4.3 Selection of weighting functions: how error-sensitive are they?

The principle of using a weighting function to determine which CWI signals are harmful and which are not, has been chosen for one reason: a Loran-C receiver employs a tracking loop which has a small tracking bandwidth. This loop reduces the tracking error due to a-synchronous signals, but not the error due to (near-)synchronous signals. The weighting function is used to do something similar with information about the CWI spectrum: reduce the importance of a-synchronous signals compared to (near-)synchronous signals.

Ideally, the weighting function should be equal to the amplitude transfer function of the tracking loop between \( f = -\frac{1}{2} \text{ GRF} \) and \( f = +\frac{1}{2} \text{ GRF} \), repeating itself every GRF. However:

1) Loran-C phase tracking loops are not simple linear Type-I or Type-II loops. The incoming signals are sampled and processed in micro-processors. Many non-linear computations can (and are) performed on the antenna signal. This
makes the calculation of an amplitude transfer function for a particular loop difficult.

2) Many loops will have an automatic adaptation mechanism for varying Signal-to-Noise Ratios. These mechanisms change the loop bandwidth $f_b$. Sequential Detection loops as described in [4.1], are the most important simple tracking loops with such properties.

3) The application of a weighting function as shown in fig. 4.1, implies that information about the CWI spectrum is available in continuous form. In some applications this might be true (as described in later chapters), but often spectrum information is available only as discrete numbers. The resolution of this spectrum information should be at least 10 times better than the tracking bandwidth in order to be able to apply accurate weighting functions. With bandwidths of 0.01 Hz to 0.1 Hz, this implies spectrum information with resolutions between 0.001 Hz and 0.01 Hz. In cases where such resolutions are not feasible, only very inaccurate weighting functions can be used.

Since establishing a generally valid weighting function is clearly impossible, no attempt will be made here. However, from the reasons listed above it will be clear that in any application of weighting functions, there is a risk of using a function that does not match the actual amplitude transfer of the tracking loop in the Loran-C receiver used. Fig. 4.2 shows what happens in a case where the weighting function has a larger tracking bandwidth than the tracking loop.

![Diagram](image)

**Fig. 4.2:** Weighting function bandwidth is too large.
It is assumed from now on that the weighting function principle is used for the selection of the most harmful CWI signals in a Loran-C receiver. These signals are then filtered with notch filters. It is also assumed that notch filtering is ideal, i.e. if a notch filter is put on a CWI frequency, the interfering signal disappears completely. In this case, applying an incorrect weighting function as shown in fig. 4.2 has the following effects:

1) After the two signals \( f_{int1} \) and \( f_{int2} \) in fig. 4.2 are multiplied with their respective weighting factors, the corrected amplitude of \( f_{int2} \) will be larger than the corrected amplitude of \( f_{int1} \), even though \( f_{int1} \) will cause a larger tracking error than \( f_{int2} \).

2) A selection mechanism used to determine which CWI signals should be filtered and which CWI signals not, will select \( f_{int2} \) before \( f_{int1} \) is selected. Then other signals (not drawn in fig. 4.2) might be selected, depending on the amount of harmful CWI signals present. Finally, if enough notch filters are available, \( f_{int1} \) will be selected.

So, if a weighting function is chosen with a bandwidth \( f_b \) that is larger than the bandwidth of the tracking loop used, then still all harmful signals will be found, as long as the amount of notch filters available exceeds the number of harmful CWI signals present.

Fig. 4.3 shows what happens in case a weighting function is chosen with a bandwidth \( f_b \) that is smaller than the tracking bandwidth of the loop used. In a case as shown in fig. 4.3, the effects will be very similar to those described for fig. 4.2:

1) After the two signals \( f_{int1} \) and \( f_{int2} \) in fig. 4.3 are multiplied with their respective weighting factors, the corrected amplitude of \( f_{int1} \) will be larger than the corrected amplitude of \( f_{int2} \), even though \( f_{int2} \) will cause a larger tracking error than \( f_{int1} \).

2) A selection mechanism used to determine which CWI signals should be filtered and which CWI signals not, will select \( f_{int1} \) before \( f_{int2} \) is selected. Because \( f_{int2} \) is located on the slope of the weighting function, its corrected amplitude will be (much) smaller than the corrected amplitude of \( f_{int1} \). The risk that the selection mechanism will not select \( f_{int2} \) for filtering, depends
Fig. 4.3: Weighting function bandwidth is too small.

again on the amount of CWI signals present and the amount of notch filters available. However, the risk of overlooking $f_{int2}$ will be considerable.

Generally, using a weighting function that has a larger bandwidth $f_b$ than the tracking bandwidth of the Loran-C receiver employed, is safer than using a smaller bandwidth $f_b$. This is certainly the case if the number of notch filters available is not of importance, as is the case with system design (this will be explained in more detail in chapter 5).

4.4 Final remarks

In the next chapters, two solutions to the CWI interference problem will be described: one is applied during Loran-C chain design, the other can be used in Loran-C receivers. The selection procedure for harmful CWI signals developed in this chapter, will be a very important element in these two solutions. A problem to be solved in both cases, is how to select a weighting function that introduces no risk of incorrect selection, but does not need spectrum information that cannot be obtained (in terms of accuracy or resolution). This problem will be dealt with in the coming chapters, for both system design and receiver implementation.
4.5 References

5 Reduction of CWI at system level

5.1 Introduction

In this chapter, a method for the reduction of CWI interference through proper chain design will be shown. This method was developed initially under contract for Rijkswaterstaat in The Netherlands and has been used later for the Loran-C expansion project in western and northern Europe. The aim was to select GRI's for several new West-European chains, which are set up to improve Loran-C coverage in western and northern Europe. These GRI's had to exhibit minimum sensitivity to (near-)synchronous CWI. The method developed for the GRI selection of these new chains, however, can be applied world-wide.

In several reports ([5.3], [5.4] and [5.5]) the status of the West-European Loran-C installation plans in 1988, 1989 and 1990 has been described. These plans called for a Loran-C navigation system (consisting of several chains) to be designed by the end of 1991. Unfortunately, major changes were necessary in the system design in summer 1991. No official document or publication describing these changes, has appeared until now, because planning has not yet been finished. One of the options currently under consideration is illustrated in fig. 5.1, as an example.

The Loran-C system shown in fig. 5.1, incorporates four new Loran-C chains, all named after their respective master stations:

1) the Sandur chain provides coverage for the eastern part of the Atlantic Ocean, with master Sandur and secondaries Angissoq, Jan Mayen, Ejde and West-Ireland;

2) the Boe chain enables navigation in the northern Atlantic Ocean, with master Boe and secondaries Jan Mayen, Gamvik and Fedje;

3) the Sylt chain covers the North Sea, with master Sylt and secondaries Fedje and Lessay;

Reduction of CWI at system level
Fig. 5.1: European Loran-C expansion option, 1991.

4) and the Lessay chain provides signals for the Gulf of Biscay and part of the southern Atlantic Ocean, with master Lessay and secondaries West-Ireland, Ejde, Sylt and Soustons.

This Loran-C system does meet the coverage requirements laid down by the countries participating in the European Loran-C installation plans, fixing the positions of the transmitters and their respective radiated powers. Thus the only parameter that can still be varied in order to minimise CWI, is the Group Repetition Interval. One GRI has to be selected for each chain in the system; so in the case of fig. 5.1 the algorithm described in this chapter has to be executed four times.

5.2 Strategy

First, it is assumed that for a given area (in the example of fig. 5.1: western Europe), a database is available with all transmissions in the spectrum from 50 to 150 kHz. In section 5.4, details will be supplied of such a database.
Next, the minimum permissible GRI \((\text{GRI}_{\text{min}})\) for the chain under consideration, has to be calculated. This minimum GRI is determined solely by the chain timing requirements laid down by the United States Coast Guard in [5.6]. All GRI’s between \(\text{GRI}_{\text{min}}\) and the maximum permissible GRI of 9999, can be used in the GRI selection process.

Then, for each GRI between \(\text{GRI}_{\text{min}}\) and 9999 an expression has to be found for the amount of harmful interference expected for that GRI. In the GRI selection algorithm described here, this expression consists of only one number, giving the average tracking error expected over the whole coverage area of the chain. This number is calculated using the following strategy:

- In the decision which CWI signals are harmful and which are not, a weighting function as defined in chapter 4 is best used. Since such a weighting function repeats itself every \(\text{GRF} = \frac{1}{2 \text{GRD}}\), the weighting factors with which the amplitudes of the CWI signals are multiplied, and therefore the weighted amplitudes themselves, depend heavily on the GRI.

- To be able to use a weighting function, amplitude information about the CWI signals is needed. The amplitude of CWI signals is (of course) position-dependent. It is therefore necessary to specify an area in which the analysis is valid. This area is ideally equal to the operational area of the Loran-C chain for which the GRI is selected.

- In the operational area of the Loran-C chain under analysis, a **grid** is defined. This grid contains **grid elements**: at each grid element the Loran-C and CWI field-strengths are calculated. This way, the field-strengths can be said to be sampled at each grid element, reducing the (continuous) field-strength information for each transmitter, to a discrete set of field-strength values, ready for computer processing. It is assumed that the distance between grid elements (the sampling distance) is small enough to catch all significant changes in field-strength values. In the selection of GRI’s for the new European chains, grid elements were chosen at points 0.5 degrees in longitude and 1 degree in latitude apart [5.2]. An example of a grid with this element size is shown in fig. 5.2, where each grid element is shown as a dot.
- On each grid element the field strength has to be calculated of all Loran-C stations in the chain and all CWI signals in the CWI database. The CWI field strengths are then multiplied with two functions:

1. receiver bandpass filter attenuation; this function represents the decreasing CWI susceptibility of Loran-C receivers for CWI signals that are located far away from the Loran-C band;

2. a weighting function as described in chapter 4; this function represents the distinction between (near-)synchronous and a-synchronous CWI signals.

- Then, for each grid element and each CWI signal the expected tracking error is calculated. The tracking errors are then summed, yielding one expected tracking error per grid element.

- Finally, the tracking errors are averaged over all grid elements belonging to the coverage area of the Loran-C chain, thus yielding one average tracking error estimation for one GRI (remember that the weighting function is GRI-dependent).
For a Loran-C chain with minimum CWI susceptibility the GRI with the lowest expected average tracking error is selected.

In section 5.3, the calculation of the minimum GRI (called GRI_{min}) for a chain will be dealt with. After GRI_{min} has been calculated, the area in which field-strength calculations are done, has to be defined. For the example discussed in section 5.6, the operational areas (including the grid definition) for the new European chains generated by Last et. al. in [5.2], have been used. Here, no further explanation of the generation of operational areas or grids will be given.

Then, in section 5.4 the construction of a database with CWI signals will be discussed, as well as methods to calculate CWI field strengths in grid elements from this database. Also, algorithms for the calculation of Loran-C field strengths, will be needed. With both the CWI and Loran-C field-strengths, the Signal-to-Interference Ratio (SIR) can be calculated; the SIR is then used to determine the worst-case tracking error. These formulas will be introduced in section 5.4 too.

Another aspect of the selection strategy is that receiver parameters as bandpass filter attenuation and tracking loop characteristics (weighting function!) play an important role in this selection process. These receiver parameters must be specified before a GRI is selected; such a specification should of course not favor any particular receiver type, but should help to find a GRI that is acceptable for a wide range of receivers. Proper selection of these parameters will be discussed in section 5.5.

As has been stated in the introduction, the GRI selection strategy described in this section has been used in the design of the new western European Loran-C system. Section 5.6 describes the results obtained for these chains.

5.3 Calculation of the minimum permissible GRI

Any GRI has to meet the specifications for chain timing defined by the USCG in [5.6]. These specifications are split into two parts:

1) minimum and maximum allowable GRI's: GRI's can be chosen between 40000 µs and 99990 µs;

*Reduction of CWI at system level*
2) minimum time differences between consecutive transmissions of two different stations in a chain. The USCG specifies minimum allowable time differences \( \Delta t_{n, n+1, \text{min}} \) between two stations \( n \) and \( n+1 \) anywhere in the operational area as follows:

- \( \Delta t_{n, n+1, \text{min}} \) between master and first secondary should be greater than 10.900 \( \mu \)s;
- \( \Delta t_{n, n+1, \text{min}} \) between any two consecutive secondaries should be greater than 9.900 \( \mu \)s;
- \( \Delta t_{n, n+1, \text{min}} \) between master and last secondary should be greater than \( (\text{GRI} - 9.900) \mu \)s.

These criteria are visualized in Fig. 5.3.

![Diagram](attachment:image.png)

*Fig. 5.3: Chain timing requirements set by the USCG.*

The smallest time difference \( \Delta t_{n, n+1, \text{min}} \) between the signals of two consecutively transmitting stations \( n \) and \( n+1 \) can be found at the position of transmitter \( n+1 \) itself. Hence the following condition should be satisfied:

\[
\Delta t_{n, n+1, \text{min}} = \Delta T_{E, n, n+1} - \frac{d_{n, n+1}}{V_{\text{prop}}} \geq T_{\text{min, USCG, } n, n+1} \tag{5.1}
\]

where:

- \( \Delta T_{E, n, n+1} \) is the difference in emission delay of the transmissions between stations \( n \) and \( n+1 \),
- \( d_{n, n+1} \) equals the length of the signal path between stations \( n \) and \( n+1 \),
- \( v_{prop} \) equals the propagation velocity for Loran-C radio waves (the propagation velocity over land should be used here to obtain worst case \( \Delta t \)'s), and

- \( T_{\text{min}, \text{USCG}, n, n+1} \) is defined along the USCG requirements. If we define \( n = 0 \) to identify the master and \( n \geq 1 \) for secondaries, then:

\[
\begin{align*}
T_{\text{min}, \text{USCG}, 0, 1} &= 10900 \ \mu s \\
T_{\text{min}, \text{USCG}, n, n+1} &= 9900 \ \mu s \\
T_{\text{min}, \text{USCG}, N, 0} &= 9900 \ \mu s
\end{align*}
\]

with \( N \) the total number of secondaries.

The minimum permissible GRI \( \text{GRI}_{\text{min}} \) can now be calculated:

\[
\text{GRI}_{\text{min}} = \sum_{n=0}^{N} \left( T_{\text{min}, \text{USCG}, n, n+1} + \frac{d_{n, n+1}}{v_{prop}} \right) \quad (5.2)
\]

\( \text{GRI}_{\text{min}} \) obviously depends on the geographical position of the different stations and on the sequence in which they transmit. For reasons still to be explained, it should be reduced as far as possible. Usually, the geographical positions of the stations is determined by coverage requirements etc., and cannot be used to reduce CWI. The master station has usually been selected already too. Therefore, the only remaining parameter useful for reducing \( \text{GRI}_{\text{min}} \) is the transmission sequence of the secondary stations.

Loran-C chains always have a master station located in the center of the operational area of the chain and several secondaries around the master. A polygon can be drawn through all secondaries. \( \text{GRI}_{\text{min}} \) is minimum for transmission sequences that fire the secondaries in the same sequence as their position on the polygon. This is illustrated in fig. 5.4.

Before the actual GRI selection process can be started, another important requirement must be fulfilled: a database with reliable data about all CWI signals received in the operational area, must be available. The setup of such a database is discussed in the next section.
Fig. 5.4: Transmitting sequence for minimum GRI.

5.4 From database to tracking error

For the GRI selection algorithm described in this chapter, a database with the following elements is needed:

1) frequencies of all transmissions between 50 and 150 kHz: necessary in order to calculate the appropriate weighting function value for a CWI signal;

2) Effective Radiated Power (ERP) of all transmissions between 50 and 150 kHz: necessary for field-strength calculations;

3) positions of all stations transmitting between 50 and 150 kHz: also necessary for field-strength calculations.

A database useful for CWI analysis should at least contain the elements listed above. For ease of use, some method for referencing transmitters should be added too (e.g. call signs issued or well-known transmitter names).

Once this database has been constructed, the next step is the calculation of the field strength $S_{CWI}$ for all grid elements and all CWI signals. First, the propagation distances $D_{CWI,prop}$ from the transmitter to the grid elements are calculated. This is a standard geodetic exercise and will not be discussed here.

The CWI field-strength $S_{CWI}$ is a function of the distance $D_{CWI,prop}$, the transmitter power and the propagation path properties. For GRI selection purposes, the worst-case field-strength is needed; this implies using groundwave
field-strength values close to the CWI transmitter, and fist- or second-hop skywave field-strength values otherwise.

In the GRI selection algorithm developed at the Delft University, results from existing field-strength research were used to generate a curve, showing the (approximate) field-strength $S_{\text{CWI}}$ as a function of distance $D_{\text{CWI, prop}}$. This curve is depicted in fig. 5.5 (where it is called the D.U.T. curve), together with field-strength curves for several different propagation modes: ground- and skywave, for different times of day and seasons. The D.U.T. curve is drawn using the worst-case CWI field-strength curves for all different propagation modes:

- For grid elements located close to the transmitter, ground-wave propagation determines $S_{\text{CWI}}$. For distances below 100 km, data from Loran-C field-strengths over an all-sea-water path, supplied by the University of Wales, were adapted for CWI field-strength calculation and called Bangor Ground in fig. 5.5. For ground-wave propagation up to 550 km (300 nautical miles), a curve published by Van Etten [5.1] for an all-sea-water path was used. This curve is called Van Etten Ground in fig. 5.5.

- For all grid elements where first-hop skywave CWI propagation is dominant (between 550 km and 2750 km / 1500 nm), again a curve supplied by the University of Bangor was used. This curve describes skywave field-strengths.

![Fig. 5.5: Field-strength as function of distance.](image)

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for winter nighttime propagation, which is a worst-case situation. This curve is called Bangor Sky in fig. 5.5.

- For all distances above 2750 km, second-hop skywave propagation determines the field-strength. Here again a curve supplied by Van Etten in [5.1] is used, appropriately called Van Etten Sky in fig. 5.5.

Finally, using the D.U.T. curve in fig. 5.5, a look-up table was developed, which is used by the GRI selection software to very quickly find the value for \( S_{CWf} \) belonging to \( D_{CWI,prop} \). Because of the rapid change in field-strength at distances closer than 500 km, the table contains a new value for \( S_{CWf} \) every 20 km; above 500 km a new value is supplied only every 100 km.

Note that this look-up table cannot be used for the calculation of Loran-C signal field strengths, since it represents a skywave propagation path. For the calculation of Loran-C signal levels, formulas used for coverage area prediction [5.2] can be used with success. In the example presented in section 5.6, Loran-C field-strength data was supplied by the University of Bangor; the methods used to obtain these data will therefore not be discussed here.

With the interference field-strength \( S_{CWf} \) and the Loran-C field-strength \( S_{Loran-C} \), the Signal-to-Interference Ratio and the worst-case tracking error \( E_{track} \) (in seconds) can be calculated. This is done with formula 2.14, introduced in chapter 2:

\[
E_{track} = \frac{T_L}{2\pi} \sin^{-1}\left(\frac{1}{SIR}\right)
\]

(5.3)

with \( T_L \) the Loran-C carrier cycle time of 10 \( \mu s \).

At first sight, it might seem as if all data are now available for the calculation of actual tracking errors in an operational area due to CWI. However, the SIR in formula 5.3 is the SIR of the signals going into the receiver tracking loop; in order to calculate this SIR, first the attenuation due to bandpass filtering of Loran-C and CWI signals has to be taken into account. Also, formula 5.3 does not take into account the distinction between (near-)synchronous and a-synchronous signals. This distinction is made with the help of the weighting function mentioned in chapter 4 and section 5.2. So, before formula 5.3 can be
applied, first a definition has to be given of the receiver bandpass filter and tracking loop to be used in the analysis. This definition is dealt with in the next section.

5.5 Definition of receiver properties for GRI selection

A very important aspect of GRI selection for minimum CWI susceptibility, is the definition of a set of receiver characteristics. Depending on the chain requirements, different receiver specifications can be chosen; for the selection of GRI’s for the new western European chains, the following receiver properties were assumed:

1) The receiver is an ideal linear type, without any non-linear processing in the front-end. This property greatly simplifies calculations of tracking loop characteristics.

2) In the sampling process, one sample per burst is processed, using standard phase coding patterns. Assuming non-standard patterns would limit the validity of the analysis to receivers using such patterns. Since it is believed that the majority of receivers employs standard phase code sampling patterns, this cannot be acceptable.

3) The tracking loop itself was assumed to be a critically dampened second order loop with a -3 dB bandwidth of 0.05 Hz. Note that this bandwidth is much wider than the -3 dB point of 0.006 Hz specified in the Minimum Performance Standards [ 5.8 ]; it has been chosen to include receivers used for land-based and aeronautical operation. Receivers with smaller tracking loop bandwidths will always see less (near-)synchronous interference with a particular GRI than receivers with wide tracking loops; therefore GRI’s that are optimal for receivers with wide tracking loops are expected to be satisfactory for narrow tracking loops too.

4) For the receiver input bandpass filter attenuation, a transfer function as shown in fig. 5.6 was assumed. This figure shows the amplitude transfer resulting from a lowpass-to-bandpass transformation applied on a fifth order Butterworth low-pass filter, using a center frequency of 100 kHz and a -3 dB bandwidth of 20 kHz. This filter has two advantages for GRI selection purposes:
Fig. 5.6: Transfer of the selected bandpass filter.

- No built-in notches are present. Some other filter types do have deep nulls (notches) at certain frequencies. Choosing one of these filter types for GRI selection purposes, introduces a risk of improper operation of receivers using bandpass filters without nulls at the same frequencies.

- The filter slopes are not very steep; a filter as shown in fig. 5.6 will hardly ever be used in an actual Loran-C receiver due to insufficient CWI suppression. A GRI selected with the filter of fig. 5.6 and exhibiting little or no harmful CWI, can be expected to work well for receivers using filters with steeper slopes too.

Note that the selection of the above mentioned receiver properties is in fact part of a larger research problem: how to define a "standard Loran-C receiver" for system research purposes. Solutions to this problem can be found based on a much more theoretical approach, compared to the rather empirical selection methods described here. This thesis, however, will not deal with this interesting problem.

Now that the necessary receiver properties have been defined, the algorithm described in section 5.2, can be applied. As stated before, this has been done for four proposed European chains; the next section describes the application of the algorithm and the results for these chains.
5.6 Selecting GRI’s for four European chains - an example

The analysis method as described in paragraphs 5.2 to 5.5, was used to find optimum Group Repetition Intervals for the four proposed european chains described in section 5.1 and shown in fig. 5.1. First the minimum allowable GRI’s for the four chains were calculated. With the transmitter configuration as shown in fig. 5.1 and approximate transmitter positions, minimum GRI’s of 7322 for the Sandur chain, 5457 for the Boe chain, 5699 for the Sylt chain and 6722 for the Lessay chain were found.

Using the ITU International Frequency List [ 5.7 ], a computer database was constructed containing all transmissions between 50 and 150 kHz in an area between −60° and 60° longitude and between 30° and 90° latitude. This area contains all transmitters that are potentially harmful to the proposed European chains. Since the ITU list used in the generation of the database, was dated march 1990, letters were sent to the administrations of all European countries, responsible for frequency assignment. In these letters, information was requested concerning frequency assignments and actual transmissions on frequencies between 50 and 150 kHz. This information was then used to supplement the ITU list already stored in the database. This database now contains a total of 886 interference sources.

Before an average tracking error $E_{\text{track, average}}$ was calculated for every GRI between GRI$_{\text{min}}$ and 9999, a rough pre-selection of GRI’s was done. This reduced total processing time considerably. In this pre-selection, only GRI’s which are not multiples of 2, 3, or 5, were passed on for further examination. The reason for this pre-selection lies in the frequency allocation policy for non-Loran-C transmitters found in Europe, and the placement of synchronous CWI for GRI’s that are multiples of 2, 3, or 5:

- Formula 5.4 describes all frequencies $f_s$ on which a Loran-C receiver is sensitive to synchronous interference:

$$f_s = \frac{N}{2 \cdot \text{GRI}}$$  \hspace{1cm} (5.4)

where $N$ is an arbitrary integer larger than 0. GRI’s that are multiples of e.g. 50 µs can be written as:

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GRI = 5 \cdot 10^{-5} \cdot K \quad (800 \leq K \leq 1999) \quad (5.5)

and for those GRI's formula 5.4 can be rewritten as:

\[ f_s = \frac{N}{2 \cdot 5 \cdot 10^{-5} \cdot K} = \frac{N}{10^{-4} \cdot K} = 10000 \cdot \frac{N}{K} \quad (5.6) \]

For every K in formula 5.6 (considering a frequency range of e.g. 50 to 150 kHz) a number of N's can be found that are exact multiples of that K. This means that every GRI that is a multiple of 50 µs, will be sensitive to synchronous interference on multiples of 10 kHz (and on many other frequencies as well...). Similarly, it can be proven that all GRI's being multiples of 20 µs or 30 µs, are sensitive to interference on multiples of respectively 25 and 16.666666. Unfortunately, a fair number of stations in Europe has been assigned frequencies exactly on these multiples. GRI's that are not multiples of 2, 3, or 5 do generally suffer much less from (near-)synchronous interference.

After this first selection process, the strategy described in section 5.2 was applied to the four proposed European chains. This resulted in the following GRI's: Sandur chain 7307, Boe chain 6001, Sylt chain 7643, Lessay chain 7223.

As an illustration of the results obtained by careful GRI selection, all frequencies on which (near-)synchronous interference signals can be found, are shown in tables 5.1 to 5.4 for the four proposed chains. In these tables, a (near-)synchronous signal is defined as a signal to be found on frequencies closer than 0.1 Hz from the nearest multiple of GRF.

<table>
<thead>
<tr>
<th>CWI frequency (kHz)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0000</td>
<td>77.8500</td>
</tr>
<tr>
<td>56.3500</td>
<td>78.5000</td>
</tr>
<tr>
<td>65.8000</td>
<td>126.0230</td>
</tr>
<tr>
<td>71.2125</td>
<td>134.2000</td>
</tr>
<tr>
<td>71.3630</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: (Near-)synchronous CWI of Sandur chain.
<table>
<thead>
<tr>
<th>CWI frequency (kHz)</th>
<th>50.0000</th>
<th>50.2000</th>
<th>50.4500</th>
<th>88.5770</th>
<th>128.5870</th>
</tr>
</thead>
</table>

*Table 5.2: (Near-)synchronous CWI of Boe chain.*

<table>
<thead>
<tr>
<th>CWI frequency (kHz)</th>
<th>50.0000</th>
<th>73.2500</th>
<th>128.8500</th>
</tr>
</thead>
</table>

*Table 5.3: (Near-)synchronous CWI of Sylt chain.*

<table>
<thead>
<tr>
<th>CWI frequency (kHz)</th>
<th>50.0000</th>
<th>50.4500</th>
<th>57.4000</th>
<th>70.8500</th>
<th>72.2000</th>
</tr>
</thead>
</table>

*Table 5.4: (Near-)synchronous CWI of Lessay chain.*

These results should be compared with the results found for a GRI of 7970: the current Norwegian Sea Chain [5.6]. This list can be found in table 5.5.

<table>
<thead>
<tr>
<th>CWI frequency (kHz)</th>
<th>50.0000</th>
<th>60.0000</th>
<th>70.4580</th>
<th>75.0000</th>
<th>84.0150</th>
<th>85.0000</th>
<th>88.3500</th>
<th>88.8770</th>
<th>112.0200</th>
<th>112.9800</th>
<th>115.0000</th>
<th>115.5520</th>
<th>120.0000</th>
<th>125.0000</th>
<th>128.3520</th>
<th>140.0000</th>
<th>145.0000</th>
</tr>
</thead>
</table>

*Table 5.5: (Near-)synchronous CWI of 7970 chain.*
As a comparison of these tables shows, the number of frequencies on which a (near-)synchronous signal is found in the database, is considerably lower for the proposed European chains than for the existing Norwegian Sea Chain with GRI 7970. The reduction that can be reached with a proper GRI choice, is especially clear when considering the Boe chain, which is designed to operate in roughly the same area as the Norwegian Sea Chain [5.6]: it has only 5 frequencies on which (near-)synchronous interference is found, compared to 17 for the current Norwegian Sea Chain.

Even though a considerable reduction in harmful interference can be obtained by proper GRI selection, some CWI signals still remain. These signals will have to be filtered out by Loran-C receivers. In conventional receivers, this is done with hardware notch filters, tuned manually or automatically. The lists in tables 5.1 to 5.4 can be used very well to generate recommended notch filter settings for such receivers.

Note that not all signals transmitting on frequencies shown in tables 5.1 to 5.5, will cause real problems. The ITU list [5.7] contains frequencies assigned to a radio station; not all of these frequencies are actually used. This also means that the total number of 886 interfering transmitters stored in the database, is less dramatic (though still annoying) than may seem at first sight. Still, spectrum measurements as shown in e.g. fig. 2.6 show that there are still more than enough potentially harmful interfering signals around.

5.7 Some conclusions and recommendations

A general method has been presented, which can be used in the selection of GRI’s that have minimum harmful CWI interference. This method does not completely solve the CWI problem, but considerable reductions in the number of harmful CWI signals can be reached.

During the analysis carried out for the proposed European chains, some general guidelines were also found. First, it was found that GRI’s that are prime numbers, usually (though not always) have less synchronous interference than other GRI’s. The reason is that in Europe, many stations have been assigned frequencies that are exact multiples of 1 or 5 kHz. For GRI’s that are multiples of 100 μs, all these stations are synchronous. This is shown in formulas 5.4 to 5.6.
Formula 5.4 also shows that GRI's should be as low as possible: the distance between spectral lines is $GRF = \frac{1}{2GRI}$. This means that the number of such lines falling in a specific frequency range decreases with decreasing GRI's. Assuming randomly distributed interference signals over this frequency range, the risk of encountering (near-)synchronous interference with lower GRI thus also decreases. This was apparent when the number of CWI signals was counted for all GRI's between 4000 and 9999.

The algorithm described in this chapter, does produce useful results, but it is not perfect. In a next development step, two possible improvements are:

- Transmitter drift compensation. Some transmitters producing CWI signals, are controlled by atomic frequency standards, and can therefore be expected to transmit exactly on their assigned frequencies. The well-known standard time transmissions on 75 kHz and 77.5 kHz are good examples of this type of transmitter. However, most CWI signals will be generated using simple crystal-controlled transmitters, which exhibit a drift of at least $10^{-6}$ or 0.1 Hz at 100 kHz. This possible drift is not taken into account in the current algorithm.

- Calculation of the average tracking error. Currently, a tracking error is calculated for each CWI signal on every grid element. The tracking errors found for one grid element are then summed to calculate the worst-case total tracking error for that grid element. This is acceptable for (near-)synchronous signals, which cause errors that are completely deterministic. The effect of a-synchronous signals, however, can best be described as an increase in the noise level (even though a-synchronous signals, of course, are completely deterministic too). A root-sum-square addition of the errors caused by a-synchronous signals is probably a better approximation of the tracking error observed in a real Loran-C receiver affected by a multitude of such signals.

5.8 References


[5.7] ITU List of All Transmissions between 10 and 200 kHz, March 1990.

6 Detection of harmful CWI signals in Loran-C receivers

6.1 Introduction

As has been stated in chapter 2, the influence of CWI signals on Loran-C can be reduced by properly designing chains as well as by suitable provisions inside Loran-C receivers. The first possibility has been dealt with in the previous chapter; this chapter will describe new techniques to improve the CWI rejection capabilities of Loran-C receivers.

The problem of sine-wave interference to a desired signal is not new and not limited to Loran-C either. In Loran-C literature, several different cures for receivers have been proposed; these "conventional" solutions will be discussed in section 6.2. A common feature of these solutions is that they do need little or no computer processing power, since they were developed when large amounts of computing power was not readily available and expensive. This, however, means that CWI immunity with these methods is not optimal, as will be shown in section 6.2.

Other methods for detecting and rejecting sine-wave interference, not specifically developed for application in Loran-C receivers, are also available. Most important for application in Loran-C receivers are those methods which provide adaptive filtering of CWI signals, enabling the receiver to work in widely different CWI environments without any readjustments. Such methods will be discussed too in section 6.2; again, it will be shown that these methods do not provide an optimal detection of CWI signals in Loran-C receivers.

With the availability of today's powerful signal processing hardware, it becomes possible to develop much better detection methods for CWI signals in Loran-C receivers. Such detection methods should be able to determine the precise frequency and amplitude of a CWI signal. Using the weighting function principle as described in chapter 4, the most harmful interference signals are then detected and subsequently filtered. The hardware necessary for implementation of this method, must fit into a Loran-C receiver; this imposes
limits on the hardware size and cost. In this chapter, a new detection method for harmful CWI signals, based on digital signal processing and suitable for implementation in Loran-C receivers, will be developed.

6.2 Conventional interference suppression methods

Most conventional methods to get rid of CWI signals in Loran-C receivers, can be split into two parts: first the most harmful CWI signals must be detected, and then the detected signals are removed. This separation of detection and filter actions will appear regularly in the following discussion of published interference rejection methods. Note that not all CWI rejection methods use detection of interference signals in the receiver itself, as will be discussed in the next sub-section. In that case, the filtering system in the receiver is tuned to a pre-determined set of CWI signals. The interference signals detection can be said to be done by the person tuning the receiver.

6.2.1 Hardware solutions

In this sub-section, all CWI detection and rejection methods will be discussed which rely on analog (hardware) notch filters and optional analog CWI signal detection, or digitally implemented direct equivalents. Receivers employing such methods will have a general architecture as shown in fig. 6.1.

![Fig. 6.1: Conventional Loran-C receiver architecture.](image)

First the antenna signal is bandpass-filtered with an analog filter that is usually 15 to 20 kHz wide. This filter can be followed by any number of notch filters.
Next the signal is A/D-converted and processed in a micro-computer system. A/D conversion is performed either with a linear converter and gain control, or with a hard-limiting converter. Both methods offer potential dynamic ranges of more than 85 dB (the minimum dynamic range of a Loran-C receiver as required in the Minimum Performance Standards [6.16]). More recent receiver types can have digital filtering, but this does not change the principles behind the architecture of fig. 6.1.

In receivers with analog filtering usually four to eight notches are installed. Receivers with digital filtering sometimes contain many more notches. In both cases, two types of notch filters can be distinguished:

1) Fixed notch filters. These filters are pre-aligned in the company’s production plant, usually optimized for the local situation. With fixed notch filters, no CWI signal detection in the receiver is necessary. This approach has two disadvantages:

- The notch filters should have a -3 dB bandwidth in the order of 2.5 kHz or less and a peak attenuation of preferably 40 dB or more. Such filters have very steep slopes: on these slopes a variation in frequency of +50 Hz can cause a variation in attenuation of 10 dB. This shows that in order to guarantee proper attenuation of unwanted signals, these filters should be very temperature- and vibration-stable over long periods. Building fixed notch filters in the 100 kHz range that fulfill this requirement, is not easy and expensive.

- Even if the notches are properly aligned to suppress all harmful interference signals in one area, this alignment might perform poorly in other areas, where the local interference frequencies and levels are different. Therefore, reliable operation is limited to only one relatively small area.

2) Automatic-tuning notch filters, selecting the interference signals with highest amplitude (often called level-sensitive notch filters). While this type of notch filters can be very helpful in preventing a very strong local interference source from blocking the receiver, it should never be the only type of notch filter. If it were, then the risk of filtering out a moderately harmful a-synchronous signal, while passing on a much more harmful (near-)synchronous signal, would be high. The actual detection of the

Detection of harmful CWI signals in Loran-C receivers  Page 85
strongest CWI signals is basically a spectrum analysis problem. Hardware solutions have been proposed [6.15]; receivers can also use digital signal processing techniques based on the Fast Fourier Transform (FFT) for finding the strongest CWI signals.

The disadvantages of conventional hardware CWI suppression methods can be summarised easily as follows: interference suppression is done with selection criteria which are not correct (suppressing the strongest CWI signals is not the same as suppressing the most harmful CWI signals) and work only in limited areas (because of alignment problems).

6.2.2 Sampled data notch filters

Sampled data notch filters have been described by Jasper et al. in [6.8] for hard-limiting receivers and by Bregstone in [6.2] for linear receivers. In [6.10] a suitable interference detection method is introduced, which can be used for proper tuning of sampled data notch filters. The versions for linear and hard-limiting receivers operate on the same principle, which is illustrated in fig. 6.2. A wanted signal consisting of one pulse (e.g. a Loran-C pulse as used in fig. 6.2), and an interfering sine-wave signal are shown. Two samples of the composite signal are taken at a distance $T_{\text{notch}}$: one sample ($S_1$ taken at time $t_1$ in fig. 6.2) is located in an area where the wanted signal is not present, a second sample $S_2$ is taken at a moment $t_2$ in which the wanted signal is available. These

![Image](image_url)

**Fig. 6.2: Operation of sampled data notch filters.**
two samples are then subtracted, cancelling out all sine-waves at frequencies $f_{notch}$ that have an integer number of cycles in time interval $T_{notch}$ [6.2]:

$$f_{notch} = \frac{N}{T_{notch}}, \quad N = 1, 2, 3, \ldots \quad (6.1)$$

The bandwidth of this notch filter is inversely proportional to interval $T_{notch}$; with a minimum desirable bandwidth of 100 Hz (at 20 dB rejection) the maximum possible value for $T_{notch}$ then becomes approximately 300 $\mu$s, according to [6.2]. The minimum practical limit for $T_{notch}$ is 50 $\mu$s [6.2]; with these values $\frac{1}{T_{notch}}$ ranges from 3.3 kHz to 20 kHz.

Jasper et al. have shown in [6.10] that the sampling structure illustrated in fig. 6.2 can also be used for the detection of signals with frequencies $f_{notch}$. This detection method is based on calculating the autocorrelation function of the composite signal over time interval $T_{notch}$, with the samples taken as shown in fig. 6.2. It only provides an indication of the presence or absence of an interfering signal on a frequency $f_{notch}$, the actual interference signal frequency and amplitude are not measured. It also assumes that sample $S_2$ is taken at a zero-crossing of the pure Loran-C signal, i.e. the phase tracking mechanism is not disturbed by the CWI signal that is to be detected. These disadvantages severely limit the usefulness of this CWI detection mechanism.

Duym [6.4] has shown that sampled data notch filters cannot be cascaded; this means that in a Loran-C receiver employing this technique, only interference signals on multiples of one value for $\frac{1}{T_{notch}}$ can be detected and suppressed. This might be acceptable in situations where only few CWI signals are present. However, in areas with many interference signals one can hardly expect to find harmful CWI signals only on frequencies which are all a multiple of $\frac{1}{T_{notch}}$. In that case, not all harmful CWI signals can be filtered. Therefore sampled data notch filtering is not useful in heavily polluted areas [6.4].

### 6.2.3 Special phase coding used to detect and reject CWI signals

The use of special phase coding mechanisms for the rejection of CWI signals, has been discussed by Jasper et al. in [6.9] and by Van Etten in [6.6]. Both approaches manipulate the phase code used by the receiver to sample the incoming signal. Normally, a Loran-C receiver uses the standard sampling
pattern described in section 2.5, with 16 samples taken per period of 2 GRI and every sample being multiplied with the proper phase code (+1 or -1). However, Loran-C receiver designers are free to deviate from this sampling pattern by using a different phase code or ignoring bursts.

The approach described in [6.9] makes use of the fact that (near-)synchronous CWI affects either the uneven (first, third, fifth etc.) or the even (second, fourth, sixth etc.) Loran-C pulses in a period of 2 GRI. With a suitable detection mechanism which is also described in [6.9], it is possible to detect whether a (near-)synchronous CWI signal is present and which group of pulses (even or odd) is affected. By ignoring the samples taken by the receiver in the even or odd pulse sequence, the CWI signal can be effectively removed. This mechanism is very limited in the number of (near-)synchronous interference signals that can be handled; if different (near-)synchronous CWI signals are found which affect both the even and uneven pulses, it becomes useless. Also, by ignoring every second pulse in the Loran-C transmission sequence, an effective SNR degradation of 6 dB is introduced.

In [6.6], the Fourier Transform of the phase code function used in the receiver, is analysed. By ignoring some bursts and reversing the phase code of others, it is possible to introduce zeros into the Fourier Transform of the phase code function, thus in effect notching out all signals on the frequencies of these zeros. The exact position of the zeros is dependent on the GRI as well as the phase code function; with the GRI's chosen in [6.6] and a properly selected phase code function, all signals on frequencies that are multiples of 500 Hz, can be notched out. In this example, only 10 of the 16 bursts in a period of 2 GRI can still be used; this introduces a SNR degradation of 4 dB. Because of the changes in the phase code, protection against long-delayed skywaves is also degraded [6.6]. No detection mechanism for CWI signals is supplied; it is assumed that the frequencies of the signals to be removed, are known.

The combination of SNR degradation, the dependence of the notch frequencies on the GRI and the lack of a CWI signal detection method severely limit the usefulness of the proposal in [6.6]. As a stand-alone CWI suppression mechanism in an area with many CWI signals, it is virtually useless.
6.2.4 Adaptive Noise Cancelling applied to Loran-C

As stated in the introduction, the problem of interference to a desired signal is not limited to Loran-C. It is therefore not surprising that general methods have been developed to detect and reject interference signals. In this sub-section, only those methods will be discussed which adapt their filtering automatically to changes in the interference signal conditions. Non-adaptive filtering techniques do not differ fundamentally from hardware notch-filters; they have the same disadvantages.

General-purpose adaptive interference rejection is described in detail by Widrow et.al. in [6.14]. In the original form as introduced in [6.14], the interference signal is assumed to occupy a broad frequency band, and the rejection method is therefore appropriately called Adaptive Noise Cancelling (ANC). Two variations on this technique have been published: Glover [6.7] has described how Adaptive Noise Cancelling can be applied to sine-wave interference signals, and Elliot and Darlington [6.5] have adapted the ANC technique to suppress all signals that have a synchronous relation with the main sampling frequency used. All three variations can in principle be applied in Loran-C receivers:

1) The original Adaptive Noise Cancelling technique as described in [6.14], assumes that the interference $n_1$ can be modelled as (broad-band) noise. A reference noise signal $n_0$ is generated locally and filtered with an adaptive Finite Impulse Response (FIR) filter until it matches the interference signal as closely as possible. The filtered reference signal is then subtracted from the contaminated input signal, removing the interference. The power of the resulting output signal $e$ is minimised by adjusting the adaptive filter. A block diagram illustrating this mechanism is shown in fig. 6.3.

Adaptive Noise Cancelling can be implemented in Loran-C receivers if the CWI interference spectrum can be approximated and therefore cancelled with a filtered noise spectrum. However, this method has one severe disadvantage: the only criterion used in adjusting the canceller is the amplitude of the CWI signals; the interference frequency is not used at all. Therefore, ANC using a broad-band reference signal has the same disadvantages as level-sensitive notch filters. In a highly CWI-contaminated area as e.g. western Europe, this method will not give good results.
Fig. 6.3: The Adaptive Noise Cancelling concept.

2) The reference signal $n_1$ in fig. 6.3 does not necessarily have to be noise. In [6.7] a setup is described where the reference is a (pure) sine wave; in this case the adaptive noise canceller becomes a notch filter with a null at the reference signal frequency. This notch filter might be very useful in removing CWI signals in Loran-C receivers. However, since according to [6.7] it requires that the interference frequency is known, it is not able to detect CWI signals.

3) A third possible reference signal is a pulse train, which is synchronised to the sampling frequency used in the ANC signal. This case has been described by Elliott and Darlington in [6.5]; it enables filtering out all signals on multiples of the pulse train repetition frequency, thus in effect generating a comb filter. At first sight this approach looks very promising for application in Loran-C receivers: by generating a pulse train with a repetition frequency of $GRF = \frac{1}{2GR1}$, all (near-)synchronous signals could be filtered out.

A practical sampling frequency for a Loran-C receiver is 300 kHz [6.1]; the minimum value for GRF is 5 Hz. All signals on multiples of GRF must be removed, except those in the band between 90 and 110 kHz - these belong to the Loran-C signal itself. According to [6.5] an adaptive FIR filter with $3 \cdot 10^5$ taps is then needed in the ANC circuit. Such a filter, running at 300 kHz, cannot be implemented with current Digital Signal Processing technology. Also, if the technology were available, it would be
much more advantageous to use it to implement a FIR input bandpass filter with very steep slopes (FIR input bandpass filters are discussed in more detail in chapter 7). Such a filter would simply remove all CWI signals instead of only the (near-)synchronous ones. This shows that the adaptive canceller described in [ 6.5 ] should not be used to remove all (near-)synchronous CWI in a Loran-C receiver.

From the discussion of published CWI suppression methods, it has now become clear that several good methods exist for filtering CWI signals (hardware notch filters, Adaptive Noise Cancelling applied to sine-waves). Therefore, CWI filtering techniques will not be dealt with in detail in the remainder of this thesis. However, no really satisfactory interference detection method has yet been found; all published methods for detecting CWI signals suffer from at least one of the two following problems:

1) Only a very limited amount of (near-)synchronous CWI signals can be dealt with, leaving the method inoperable in areas with high CWI density as e.g. western Europe.

2) The proposed methods detect only the interference signal amplitude, which is not a good criterion for the potential harm done to Loran-C operation by the CWI signal.

Obviously, a good detection method for harmful CWI signals in Loran-C receivers is still lacking. The next sections and chapter 7 will describe a new method, which automatically detects the most harmful CWI signals according to the weighting function principle described in chapter 4. This method is suitable for implementation in Loran-C receivers.

6.3 Automatic CWI signal detection in Loran-C receivers - some basics

6.3.1 Digital CWI spectrum analysis - a first attempt

As stated in the previous section, current CWI detection techniques in Loran-C receivers do not provide optimal results. New interference detection methods are needed; such methods will certainly cause changes to Loran-C receiver architectures too.
For proper CWI detection, the weighting function of chapter 4 can be used very effectively. Such a weighting function needs information about two signal parameters of every CWI signal at the receiver antenna: its frequency and relative signal strength. This means that a receiver using weighting functions to adapt its filter system, must have a spectrum analyser on board. In order to keep receivers from becoming both unwieldy and expensive, the spectrum analysis and all other signal processing necessary, should be done digitally. This already implies using Fast Fourier Transform or similar techniques for the spectrum analysis.

In fig. 6.4 a receiver architecture is shown that is adequately equipped for a spectrum analysis as mentioned above. First, the antenna signal is bandfiltered between 50 and 150 kHz. It is then A/D converted with a resolution high enough to fulfill the receiver dynamic range requirements of the Minimum Performance Standards (MPS) [6.16]. Samples are stored in a RAM bank, which is used by a signal processor to estimate the spectrum (using FFT algorithms), filter the signals and track Loran-C pulses.

![FFT sampling channel](image)

**Fig. 6.4: Receiver with digital spectrum analysis.**

Practical considerations lead to the use of two separate channels in the simple architecture of fig. 6.4: one for tracking and another for spectrum analysis. For spectral analysis using FFT algorithms, samples should be taken at equal distances and therefore the sampling clock phase cannot be shifted during the sampling process. For Loran-C phase tracking, however, the sampling clock phase must be varied to be able to follow changes in the Loran-C signal phase. If the fixed spectrum analysis clock at 300 kHz would be used for tracking, then the minimum tracking phase shift would be equal to one clock period of 3.333 µs or 1 km range. Since such a low resolution in range measurements is not acceptable, the sampling clocks used for spectral analysis and for tracking
should be separated. Methods to combine the two channels are available, but will not be dealt with in this thesis.

The architecture of fig. 6.4 has one major disadvantage, which renders implementation impossible. This disadvantage is connected to the need for very accurate spectrum information: the phase tracking loop of a Loran-C receiver is very narrow (< 0.1 Hz). This small bandwidth means that in order to separate (near-)synchronous CWI from a-synchronous signals, the FFT resolution must be no more than 0.1 Hz. With the standard (complex) FFT algorithm, such a resolution can only be obtained with a very large RAM storage space, as formula 6.2 shows:

\[ N = \frac{f_s}{f_r} \]  

(6.2)

where:

\( N \) = number of samples  
\( f_s \) = sampling frequency  
\( f_r \) = frequency resolution of the FFT

With a sampling frequency of 300 kHz as selected in fig. 6.4 and a resolution of 0.1 Hz, \( 3 \cdot 10^6 \) samples are needed. Sample values will have to be stored in 8-byte floating point format to keep round-off errors down in the FFT. Samples are complex numbers, thus requiring \( 2 \cdot 8 = 16 \) bytes per sample. The total RAM space needed in fig. 6.4, is then 48 Mbyte. For a small, cheap and portable Loran-C receiver, this is clearly unacceptable.

A solution to this problem has to be found before spectrum analysis in Loran-C receivers can become even remotely viable. Before a solution is presented, however, first an explanation of the properties and limitations of the FFT will be given.

**6.3.2 FFT resolution limitations**

A Fast Fourier Transform is an algorithm for fast execution of the Discrete Fourier Transform (DFT). The DFT can be written as:

\[ \text{Detection of harmful CWI signals in Loran-C receivers} \]
\[ X(f) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi \frac{nf}{f_s}} \]  \hspace{1cm} (6.3)\]

where \( x(n) \) can be a complex number. Note that equation 6.3 differs from the DFT equation as it is usually presented:

\[ X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi \frac{nk}{N}} \]  \hspace{1cm} (6.4)\]

with \( k \) running from 0 to \( N - 1 \). The DFT as defined in equation 6.4 is used for the implementation of the Fast Fourier Transform algorithm. It (unnecessarily) restricts spectrum analysis to a pre-defined set of points \( f = \frac{k}{N} \cdot f_s \), with \( f_s \) the sampling frequency.

The theory behind the Discrete Fourier Transform is described in detail in [6.3]. There, the Discrete Fourier Transform is derived from the continuous Fourier Transform:

\[ X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \, dt \]  \hspace{1cm} (6.5)\]

A very important difference between the DFT and the continuous Fourier Transform, which can be seen clearly from a comparison of formulas 6.3 and 6.5, is the truncation in the time domain: while in the continuous Fourier Transform an integration is done over an interval \((-\infty, +\infty)\), the DFT contains a summation from \( n = 0 \) to \( N - 1 \). With a sampling frequency \( f_s \) this is a time interval of \( \frac{N}{f_s} \).

The finite summation interval of a DFT implies that its operation can be described in two steps:

1) First the incoming CWI signal is sampled only during an interval \( t_s \). This is equivalent to modulating the incoming signal with a rectangular function:

\[
\begin{align*}
    f_{\text{mod}}(t) &= 0 \quad t < 0 \\
    f_{\text{mod}}(t) &= 1 \quad t \geq 0 \text{ and } t \leq t_s \\
    f_{\text{mod}}(t) &= 0 \quad t > t_s
\end{align*}
\]  \hspace{1cm} (6.6)\]
with $t_s = \frac{N}{f_s}$ being the sampling interval.

Due to this modulation, the spectrum of the incoming CWI signal (here supposed to be a pure sine wave) will get sidebands. For the rectangular function of formula 6.6 these sidebands will be shaped according to a sinc-function. The CWI signal amplitude can then be expressed in the frequency domain as:

$$S_{CWI}(f) = A \cdot \left| \frac{\sin(\pi \cdot (f - f_{CWI}) \cdot t_s)}{\pi \cdot (f - f_{CWI}) \cdot t_s} \right| \quad (6.7)$$

with $f_{CWI}$ the frequency of the CWI signal. Formula 6.7 is the Fourier Transform of a carrier at frequency $f_{CWI}$, modulated with the function of formula 6.6. It is illustrated in fig. 6.5.

![Fig. 6.5: Spectrum of rectangularly modulated carrier.](image)

2) Then the modulated signal is matched in the time domain with a second signal:

$$s_{\text{match}}(t) = e^{-j2\pi f_{\text{match}} t} \quad (6.8)$$

with $0 \leq f_{\text{match}} \leq \frac{f_s}{2}$.
This matching operation is done by multiplying all samples of the incoming modulated signal with samples taken at the same time from $s_{\text{match}}$, and accumulating the result. A mathematical description of this operation yields again the DFT formula 6.3.

In this step, the spectrum analysis is actually performed. Because in any implementation this matching process can be done only for a limited set of values for $f_{\text{match}}$, it can be seen as a sampling process in the frequency domain: the incoming continuous spectrum is sampled at points $f_{\text{match}}$. This terminology will be used throughout the rest of this thesis: a DFT samples the spectrum at selected frequencies.

If the CWI frequency $f_{\text{CWI}}$ is exactly equal to $f_{\text{match}}$ in formula 6.8, then the DFT will yield a direct measure of the amplitude of the CWI signal. However, if $f_{\text{CWI}}$ is not exactly equal to $f_{\text{match}}$, the DFT formula will still yield a non-zero result. This is due to the modulation with a rectangular function as described in step 1 and the resulting sidebands of the CWI signal as shown in fig. 6.5. According to formula 6.7, the DFT formula will yield a result 0 only if:

$$\sin(\pi \cdot (f_{\text{match}} - f_{\text{CWI}}) \cdot t_s) = 0 \quad \rightarrow \quad \pi \cdot (f_{\text{match}} - f_{\text{CWI}}) \cdot t_s = n \cdot \pi \quad \rightarrow$$

$$f_{\text{match}} = f_{\text{CWI}} - n \cdot \frac{1}{t_s}$$  \hspace{1cm} (6.9)

with $n = (..., -2, -1, 1, 2, ...)$. Formula 6.9 explains why the DFT resolution $f_r$ is always set equal to:

$$f_r = \frac{1}{t_s} = \frac{f_s}{N}$$  \hspace{1cm} (6.10)

and formula 6.10 is equal again to formula 6.2.

As can be seen easily from formula 6.10, the only way to decrease the DFT resolution is to increase the sampling interval $t_s$. The sampling frequency $f_s$ is determined mainly by the Nyquist criterium and can therefore not be lowered arbitrarily; with a constant $f_s$ any increase in the sampling time means a proportional increase in the total amount of samples $N$ too. This is a basic property of the DFT.
6.4 A DFT algorithm with automatic CWI weighting

In the previous section, a basic problem of the Discrete Fourier Transform in a Loran-C receiver has surfaced: in order to obtain a high resolution, an impractical number of samples has to be collected. This is due to the time domain sampling process: it modulates the incoming CWI signals with a rectangular function, generating CWI sidebands. In this section, a method is introduced that uses the modulation sidebands for automatic weighting of CWI signals. This method is best introduced by taking a look again at the requirements for a spectrum analysis system in Loran-C receivers:

- The spectrum analysis system should distinguish between (near-) synchronous and a-synchronous CWI signals. This is done by multiplying their detected amplitudes with a frequency-dependent weighting factor. This weighting function requires highly accurate CWI frequency measurements: \( \leq 0.1 \text{ Hz} \).

- The strongest CWI signals after weighting, are selected for filtering with notch-filters. This is done by setting the notches to the detected frequencies of the highest peaks in the weighted spectrum. However, no reason exists for using notch filters with bandwidths less than 10 Hz (most notch-filter implementations will have much wider bandwidths anyway). This implies that the actual peak detection accuracy used for setting these notches, does not have to be better than approximately 10 Hz either.

Now assume that a DFT algorithm is used to detect CWI signals. It is also assumed that the spectrum is sampled at multiples of GRF. Fig. 6.6 shows what happens in case of a synchronous CWI signal.

![Figure 6.6: Synchronous CWI signal detected.](image)

\[ f_{\text{CWI}} = n \cdot \text{GRF}, \quad n = 1, 2, 3, \ldots \]
As fig. 6.6 illustrates, the peak of the spectrum of the modulated signal is located exactly on the point where the spectrum is sampled by the DFT algorithm. It can be said that this synchronous signal is multiplied with a weighting factor 1.

Fig. 6.7 shows what happens in case of a near-synchronous CWI signal. This CWI signal is (of course) not located on the multiple of GRF, where the spectrum is sampled. However, due to its modulation sidebands, a spectral line is found on the frequency where the spectrum is sampled, but the amplitude of this line is lower than the amplitude of the CWI signal. This can be used for automatic weighting, as Nieuwland [6.11] has first reported: the amplitude of the near-synchronous CWI signal is multiplied with the value of the sideband function at \( f = f_{CW} - n \cdot \text{GRF} \), which is smaller than 1. The weighting function is the Fourier Transform of the time-domain sampling modulation function; in the examples of figs. 6.6 and 6.7 the signals are weighted with a sinc function as given in formula 6.7. This weighting process is similar for a-synchronous signals, only here the distance between \( f_{CW} \) and the nearest multiple of GRF is much larger and the sideband at that distance much smaller. This yields a lower weighting function, as was desired.

![Diagram showing weighting factor](image)

Fig. 6.7: Near-synchronous CWI signal detected.

Several problems with this approach can be found:

- First, a near-synchronous or a-synchronous signal is not detected at its own frequency, but at the nearest multiple of GRF. This can lead to a maximum measurement error of \( \frac{1}{2} \text{GRF} = 6.25 \text{ Hz} \) (worst case) for a GRI of 4000. However, as stated earlier in this section, this is no problem considering the typical notch-filter bandwidth.
- A second and much more serious problem is that the resolution of the DFT still needs to be high, leading again to large numbers of samples. The DFT resolution formula 6.10 is not applicable here anymore, because it treats the modulation of the incoming CWI signals as a problem instead of a feature. Here, the resolution of a DFT with automatic weighting is defined to be the -3 dB bandwidth of the Fourier Transform of the modulation function. In the case of the rectangular function of formula 6.6 and the resultant sinc-function 6.7 in the frequency domain, the -3 dB bandwidth is approximately \(0.5 \cdot \frac{1}{t_s}\), with \(t_s\) again being the total sampling time. Assuming a -3 dB bandwidth of 0.1 Hz again, the total sampling time is now 5 seconds. Compared to the architecture of fig. 6.4, this means a reduction of the number of samples from \(3 \cdot 10^6\) to \(1.5 \cdot 10^6\) (with a sampling frequency of 300 kHz) and a corresponding reduction in RAM size from 48 Mb to 24 Mb. This reduction of 50 % may look impressive, but is far from enough. Methods to further drastically reduce the number of samples, are discussed in section 6.5.

- A third problem is that near-synchronous signals which are located on a distance \(f_r\) from the nearest multiple of GRF, are weighted with a factor 0! This phenomenon is shown in fig. 6.8.

![Fig. 6.8: Near-synchronous signal weighted with 0.](image)

The problem illustrated in fig 6.8 is due to the selection of a rectangular modulation function in the time domain, leading to a sinc-function in the frequency domain with its zeros at multiples of \(f_r\). The obvious solution here is to modulate the incoming CWI signals not with a rectangular function as in formula 6.6, but with a function that has a Fourier Transform without zeros. In practice this means that during the CWI signal sampling process
each incoming sample has to be multiplied with a modulation value between 0 and 1 before being stored in the RAM bank.

- A fourth problem is that until now, it has been assumed that in the DFT algorithm both the choice of the number of time samples $N$ and the points on which the spectrum is sampled, can be chosen freely. This assumption holds for a DFT transform implemented according to formula 6.3, but such an algorithm would be much too slow for practical use. A fast implementation of the DFT is the FFT, but this algorithm is neither in the number of time samples nor in the spectrum sampling points free. With a true FFT, the number of time samples can only be a power of two, while the spectrum is sampled automatically at frequencies:

$$f_{\text{sample}} = n \cdot \frac{f_s}{N}, \quad n = 0, 1, 2, 3, \ldots, N - 1$$  \hspace{1cm} (6.11)$$

with $N$ the number of samples and $f_s$ the sampling frequency. Since the frequencies $f_{\text{sample}}$ are automatically weighted with a factor $1$, these frequencies should coincide with the synchronous lines at multiples of GRF:

$$\text{GRF} = \frac{f_s}{N} \rightarrow N = \frac{f_s}{\text{GRF}} = 2 \cdot f_s \cdot \text{GRI}$$  \hspace{1cm} (6.12)$$

If $f_s$ is a multiple of 100 kHz, $N$ calculated with formula 6.12 will be an integer number, since GRI's are chosen as multiples of 10 μs. However, for an efficient implementation of the FFT algorithm $N$ should be a power or two, or at least be highly composite ($N = r_1 \cdot r_2 \ldots \cdot r_i$, with $r_i$ being an arbitrary integer). GRI's can be chosen freely between 0.04 s and 0.09999 s, and formula 6.12 shows that even with $f_s$ being a multiple of 100 kHz, for many GRI's a value for $N$ will be found which makes efficient FFT implementation impossible.

The above mentioned problems clearly rule out the direct use of FFT's in Loran-C receiver spectrum analysis. A better algorithm (which incidentally makes use of the FFT algorithm again) will be presented in the next section.
6.5 The Chirp Z-transform: a flexible implementation of the DFT

6.5.1 Introduction

Before digital spectrum analysis with high resolution becomes feasible in Loran-C receivers, first an implementation of the DFT formula 6.3 has to be found which:

- does allow large sample times $t_s$ without requiring a large RAM storage space;
- does not impose any limits on the exact number of time samples $N$;
- does allow sampling in the frequency domain at multiples of GRF, independent of the number of time samples $N$;
- preferably has a processing time that is proportional to $N \cdot \log(N)$, just like any FFT algorithm.

Such an algorithm is the Segmented Chirp Z-Transform (SCZT), which was introduced by Wang in [6.13]. In order to explain the operation of the SCZT, it can be split into two parts:

1) the basic Chirp Z-transform, and

2) the segmentation process.

The Chirp Z-transform, which is described in detail in [6.12], enables spectrum analysis at frequency points which are selected independent of the sampling frequency $f_s$ and the total number of signal samples $N$. This Chirp Z-transform is then used in a segmentation technique, which greatly reduces the total necessary RAM storage space [6.13]. Although the Chirp Z-transform and the segmentation technique are not new, they will be described briefly in the next two sections for completeness’ sake.
6.5.2 The Chirp Z-Transform

The Chirp Z-transform is best understood by rewriting the basic DFT formula 6.3:

\[
X(\omega) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j \frac{n \omega}{\omega_s}}
\]  

(6.13)

with \( \omega_s \) the sampling frequency. All frequencies are normalised by dividing them by \( \omega_s \). Next, the spectrum is sampled at normalised frequencies \( \omega = \omega_0 + k \cdot \omega_1 \), with \( k \) running from 0 to \( K - 1 \):

\[
X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j n (\omega_0 + k \omega_1)}
\]  

(6.14)

Equation 6.14 is then transformed into a convolution:

\[
X(k) = e^{-j \omega_1 k \cdot \nu_2} \sum_{k=0}^{N-1} y(n) \cdot v(k-n)
\]  

(6.15)

with

\[
y(n) = x(n) \cdot e^{-j \omega_0 n} \cdot e^{-j \omega_1 n \cdot \nu_2}
\]  

(6.16)

\[
v(n) = e^{j \omega_1 n \cdot \nu_2}
\]  

(6.17)

According to [6.3] this convolution can be calculated with three FFT's. For an efficient implementation of these FFT's, functions \( y(n) \) and \( v(n) \) padded with zeros until array sizes are obtained which are the smallest power of two larger than \( N + K - 1 \). Then they are FFT transformed separately and the two output arrays of these FFT's are multiplied. The result is transformed back with the third (inverse) FFT, and after multiplication with \( e^{-j \omega_1 k \cdot \nu_2} \) the first \( K \) elements of the output array contain the desired spectrum. While this algorithm consumes considerably more processing power than a simple FFT, the total execution time is still approximately proportional to \( N \cdot \log(N) \). Note that while a FFT uses only two arrays for its operation (one for in-place transformation, another for a sine /
cosine table), a Chirp Z-transform uses three: one for transforming \( y(n) \), a second for transforming \( \nu(n) \) and a third for the sine / cosine table needed in the FFT's. The sine / cosine tables are in principle not necessary, but speed up FFT operation considerably and have therefore been included in the array count.

### 6.5.3 Segmentation

In the previous sub-section, the Chirp Z-transform has been introduced. It enables digital spectrum analysis with free choice of the total amount of signal samples \( N \), and spectrum sampling at points \( \omega_0 + k \cdot \omega_1 \), with \( k \) running from 0 to \( K - 1 \) and \( K \) independent of \( N \). This algorithm still leaves the problem with large numbers of \( N \) and the corresponding RAM memory requirements, unsolved.

In order to reduce RAM size, \( N \) is written as:

\[
N = L \cdot M
\]  

(6.18)

with \( L \) and \( M \) integer numbers (note: this implies automatically that \( N \) cannot be a prime number). Formula 6.15 is then rewritten as:

\[
X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j n (\omega_0 + k \cdot \omega_1)}
\]

\[
= \sum_{n=0}^{N-1} x(n) \cdot z_k^n
\]

\[
= \sum_{n=0}^{LM-1} x(n) \cdot z_k^n
\]  

(6.19)

with \( z_k^n = e^{-j n (\omega_0 + k \cdot \omega_1)} \). Formula 6.19 is then split up into \( L \) segments:

\[
X(k) = \sum_{n=0}^{M-1} x_1(n) \cdot z_k^n + \sum_{n=2M-1}^{2M-1} x_2(n) \cdot z_k^n + \ldots + \sum_{n=LM-1}^{LM-1} x_L(n) \cdot z_k^n
\]  

(6.20)

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with each summation in formula 6.20 called a segment and \( x_i \) a sample taken in segment \( i \). Formula 6.20 can be rewritten as:

\[
X(k) = \sum_{n=0}^{M-1} x_1(n) \cdot z_k^n + \sum_{n=0}^{M-1} x_2(n) \cdot z_k^n + \ldots + \sum_{n=0}^{M-1} x_L(n) \cdot z_k^n \tag{6.21}
\]

Formula 6.21 shows that a Chirp-Z transform can be split up into segments. For each segment a total of \( M \) samples is collected. These samples are then Chirp-Z transformed (with array of size \( M + K - 1 \), which are much smaller than arrays of size \( N + K - 1 \)). Next the output array is multiplied with a factor \( z_k^M \), with \( i \) the number of the current segment running from 0 to \( L - 1 \). The array resulting from this operation, is then added to the accumulation array used to build up \( X(k) \). This means that for a Segmented Chirp-Z, a total of at least four arrays is needed: three for the Chirp-Z transform performed on one segment and one for the accumulation process. Extra arrays will be necessary in any real-world implementation for sample data buffers etc.

Note that the whole segmentation process does not put any restraints on the choice of \( M \) and \( L \) other than that \( N = M \cdot L \). This means that \( M \) can be reduced at will, until a power of two is found that is larger than \( M + K - 1 \), but does not require large amounts of RAM storage space. \( K \) cannot be reduced: this number is determined by the number of samples to be taken of the spectrum. In practice, \( M \) can best be chosen approximately equal to \( K \).

Chapter 7 contains more information about the actual use of the Segmented Chirp Z-Transform in Loran-C receivers. In section 7.5, actual values for \( M \) and \( K \) will be chosen, and from these values it will become clear that with properly selected \( M \) and \( K \), a drastic reduction in RAM storage space can be achieved.

### 6.6 Some concluding remarks

In this chapter, attention has been focused on the detection and rejection of harmful interference signals in Loran-C receivers. Several existing techniques have been discussed, and it has been shown that while CWI signal rejection methods can give good results, the conventional techniques for interference signal detection do all have severe disadvantages when applied in areas with many CWI signals as e.g. western Europe.
Next, a better interference detection method has been introduced. It is based on the Segmented Chirp Z-transform (SCZT) and provides for automatic weighting of all CWI signals according to the weighting function principle presented in chapter 4.

Now that a proper method for automatic weighting of CWI signals in Loran-C receivers has been established, a receiver architecture has to be developed incorporating this method. This receiver architecture then has to be implemented and tested. The next chapter will contain details of the development of a suitable architecture, as well as results of tests carried out on this architecture.

6.7 References


Detection of harmful CWI signals in Loran-C receivers
7 A Loran-C receiver design with digital spectrum analysis

7.1 Introduction

Chapter 6 contains a description of a suitable algorithm for real-time spectrum analysis in Loran-C receivers. This algorithm incorporates automatic weighting of CWI signals, distinguishing between a-synchronous and (near-)synchronous signals. Memory requirements, which were initially a problem, can be reduced to levels which can be accommodated in portable Loran-C receivers.

In this chapter, a description will be given of an actual Loran-C receiver architecture with automatic spectrum analysis using the algorithm of chapter 6. In section 7.2 a block diagram is given; this diagram determines the placement of filters, sampling gates and other building blocks. The proposed receiver structure makes use of sub-nyquist sampling techniques for its spectrum analysis. These techniques will be described in section 7.3.

After the Loran-C receiver architecture has been defined, the design parameters have to be chosen. In section 7.4 attention will be focused on the selection of a proper weighting function along the guidelines laid out in section 6.4. In section 6.5 the Segmented Chirp Z-Transform has been introduced; before this transform can be used, the number of samples in the time and frequency domains has to be established. This is done in section 7.5. Finally, the Loran-C receiver simulation program LOSP has been adapted for research into the proposed architecture; the results obtained with the simulations carried out with LOSP, will be discussed in section 7.6.

7.2 A new receiver architecture

Fig. 7.1 shows the proposed Loran-C receiver architecture with automatic spectrum analysis.
As shown in fig. 7.1, the antenna signal is first bandpass filtered, sampled and A/D-converted. The analog bandpass filter is included to prevent aliasing; it should not be designed for maximum CWI attenuation. This is due to a conflict in design criteria, which has been explained before in section 2.4: a bandpass filter with steep slopes, while attenuating the CWI signals, also distorts the Loran-C pulse to an unacceptable extent. Therefore, the analog bandpass filter should be designed to have just enough aliasing suppression at 200 kHz (the Nyquist rate), with minimum Loran-C pulse distortion.

The sampling clock $f_s$ is set to 400 kHz, mainly for two reasons:

- The absolute minimum sampling rate would be $2 \cdot 110 \text{ kHz} = 220 \text{ kHz}$ (upper edge of the Loran-C band). This sampling frequency is not practical because it would require a low-pass anti-aliasing filter with a rectangular transfer function; a better choice would be $f_s = 300 \text{ kHz}$. This would put the Nyquist frequency at 150 kHz; at this frequency the filter system has to provide enough attenuation to prevent aliasing. In practice this means an attenuation of at least 60 dB, while 90 dB would be better. This can be done, but such filters have an unfortunate tendency to distort the Loran-C pulse. A sampling frequency of 400 kHz puts much less stringent requirements on the anti-aliasing filter.
- For the sampling scheme used in the spectrum analysis system, and for other applications as well, it is very practical to have samples of the antenna signal spaced $90^\circ = 2.5 \, \mu s$ apart at the Loran-C carrier frequency. With $f_c$ set to 400 kHz, samples are available at this spacing automatically.

CWI rejection is obtained with a second filter in fig. 7.1: the Finite Impulse Response filter behind the A/D-converter. FIR filters can be designed to have linear-phase transfer characteristics [ 7.4 ]; this means that such a filter will not distort the Loran-C burst, enhancing the skywave rejection properties of the Loran-C receiver (see section 2.4). The disadvantage of FIR filters is the large amount of processing power that is needed if steep slopes are to be obtained. Keeping this disadvantage in mind, the FIR filter should be designed according to two criteria:

1) In the frequency bands that are not guarded by the real-spectrum spectrum analysis system, the FIR filter must provide enough CWI attenuation to allow no worst-case phase tracking and Cycle Identification errors of more than a predefined limit. Here, the maximum allowable phase tracking error has been set (rather arbitrarily) to 50 ns or 15 m; the maximum permitted apparent ECD shift (see chapter 3) is set to $\pm 2.4 \, \mu s$.

2) In order to minimise processing power requirements, the FIR filter should not provide full attenuation inside the frequency band guarded by the real-time spectrum analysis system, since in this band dedicated notch-filters are used to remove CWI signals. Note that this criterion is chosen purely on practical grounds: if much processing power is available at low cost, then the extra attenuation provided by a FIR filter with steep slopes, is of course always welcome.

Fig. 7.2 shows the amplitude transfer of a suitable FIR filter with 128 taps. This filter does fulfill the two selection criteria:

1) Below 75 kHz and above 125 kHz the CWI rejection is at least 80 dB. Together with the rejection of more than 10 dB provided by the Loran-C phase coding (see section 2.5), this provides a CWI attenuation of 90 dB. Both the MPS [ 7.6 ] and the IEC Loran-C receiver performance specifications [ 7.2 ] specify a worst-case SIR of 0 dB for (near-)synchronous CWI signals from 70 to 90 kHz and 110 to 130 kHz, and -60 dB for signals below 50 kHz or above 200 kHz. However, for european CWI conditions this
specification is rather optimistic. In the receiver design of fig. 7.1 the more realistic maximum field-strength difference of 60 dB between different antenna signals is used to derive a worst-case unfiltered SIR in fig. 7.1 of -60 dB for all signals below 75 kHz or above 125 kHz. The corresponding worst-case filtered SIR then becomes 30 dB. A quick calculation with formula 2.14 shows that the corresponding worst-case tracking error is then 50 ns or 15 m range; this error is within specification. Using the algorithm introduced in chapter 3, the requirement concerning the apparent ECD shift due to synchronous CWI, is checked too: with a SIR of 30 dB, the apparent ECD shift is lower than ±2.4 μs for all frequencies below 75 kHz and above 125 kHz.

2) The spectrum analysis system in fig. 7.1 will be designed to scan only the frequency band between 75 kHz and 125 kHz. Within this band, the proposed FIR filter does not provide full CWI attenuation (see fig. 7.2). This shows that its slopes are not steeper than strictly necessary for its task, reducing the necessary processing power as far as possible.
Another important aspect of the architecture in fig. 7.1 is the use of quadrature bandpass sampling in the spectrum analysis system. Quadrature bandpass sampling is not new; a good discussion of this technique can be found in [7.3]. There it is used to reduce sampling frequencies for A/D converters; after the incoming signal has been sampled with such a reduced sampling rate, it is reconstructed again. It can be applied in all cases where the spectrum of the signal being sampled, is located on high frequencies with a small bandwidth, as is illustrated in fig. 7.3.

![Fig. 7.3: Quadrature bandpass sampling.](image)

In a system using quadrature bandpass sampling, first a proper sampling frequency $f_s$ must be chosen:

$$f_s \geq f_h - f_l = B \quad (7.1)$$

with $f_h$ and $f_l$ being the upper resp. lower edges of the spectrum occupied by the signal (see fig. 7.3). Next, a center frequency $f_c$ is defined according to:

$$f_c = N \cdot f_s, \quad N = 2, 3, 4, 5, \ldots \quad (7.2)$$

and

$$f_l \leq f_c \leq f_h \quad (7.3)$$
For proper application of quadrature bandpass sampling, two sampling channels are used: the so-called In-phase (I) and Quadrature (Q) channels. Each channel samples the incoming signal with a rate $f_s$, with a fixed time shift $t_d$ between the sampling clocks of the two channels. This time shift must represent a 90° phase shift at frequency $f_c$:

$$t_d = \frac{1}{4 \cdot f_c} \quad (7.4)$$

If the sampling rate $f_s$ and time shift $t_d$ are chosen according to formulas 7.1 and 7.4, then complete reconstruction of the original signal is possible using the samples from the I- and Q-channels, according to [7.3].

In the Loran-C receiver architecture of fig. 7.1, it is assumed that the FIR filter removes all signals below 75 kHz and 125 kHz completely. According to formula 7.1, the minimum permissible sampling rate $f_s$ is then 50 kHz, with a center frequency $f_c$ of 100 kHz and a time shift $t_d$ of 2.5 μs. Samples are coming into the spectrum analysis system at a rate of 400 kHz, so quadrature bandpass sampling in fig. 7.1 is realised by first taking a sample for the I-channel, then one for the Q-channel, and then skipping 6 samples. This sampling scheme satisfies all requirements, and is shown in fig. 7.4.

![Diagram](attachment:image.png)

Fig. 7.4: Quadrature bandpass sampling scheme.

Note that in the case of the architecture in fig. 7.1, the incoming signal is not reconstructed after having been bandpass-sampled. Rather, the samples are used
in a Segmented Chirp Z-Transform (SCZT) to find harmful CWI signals. As will be shown in the next section, quadrature bandpass sampling introduces "ghost" signals into the spectrum information resulting from the SCZT. With a descrambling technique, also introduced in the next section, these "ghost" signals can be completely removed.

A last feature of the architecture in fig. 7.1 is that the samples coming out of the FIR filter, are used for tracking as well as spectrum analysis. This implies a potential conflict: samples for spectrum analysis should be taken at equal distances, while zero tracking algorithms usually require adaptive sampling clocks. Methods exist for phase measurements with fixed clocks, but will not be dealt with in this thesis.

### 7.3 Quadrature Bandpass Sampling

In the receiver architecture of fig. 7.1 a CWI signal coming out of the FIR filter, is sampled twice with a rate of 50 kHz and a fixed time difference between the samples of 2.5 µs. The two samples are then combined to form a complex value. The CWI signal (with amplitude normalised to 1) is written as:

\[ s_{cwI} = \sin(2\pi \cdot f_{cwI} \cdot t + \varphi_{cwI}) \]  
(7.5)

where \( t \) can be written as:

\[
t = \begin{cases} 
  n \cdot t_i & \text{for in-phase sampling} \\
  n \cdot t_i + 2.5 \, \mu s & \text{for quadrature sampling}
\end{cases}
\]  
(7.6)

with:

- \( n = 0, 1, 2, 3, ... \)

- \( t_i = \frac{1}{f_s} \) the sample interval

and with a sampling rate of 50 kHz formula 7.6 becomes:

---

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\[
\begin{aligned}
    t &= \begin{cases} 
        n \cdot \frac{1}{f_s} & \text{for in-phase sampling} \\
        (n + \frac{1}{8}) \cdot \frac{1}{f_s} & \text{for quadrature sampling} 
    \end{cases} \\
\end{aligned}
\]

(7.7)

The complex sample value \( x(n) \) going into the spectrum analysis system, is then:

\[
    x(n) = \sin(2\pi n \frac{f_{CWI}}{f_s} + \Phi_{CWI}) + j \cdot \sin(2\pi (n + \frac{1}{8}) \frac{f_{CWI}}{f_s} + \Phi_{CWI})
\]

(7.8)

Spectrum analysis is done with the basic DFT formula 6.3:

\[
    X(f) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j 2\pi n \frac{f}{f_s}}
\]

\[
    = \sum_{n=0}^{N-1} \left[ \sin(2\pi n \frac{f_{CWI}}{f_s} + \Phi_{CWI}) + j \cdot \sin(2\pi (n + \frac{1}{8}) \frac{f_{CWI}}{f_s} + \Phi_{CWI}) \right] \cdot e^{-j 2\pi n \frac{f}{f_s}}
\]

\[
    = \sum_{n=0}^{N-1} \sin(2\pi n \frac{f_{CWI}}{f_s} + \Phi_{CWI}) \cdot e^{-j 2\pi n \frac{f}{f_s}}
\]

\[
    \quad + j \cdot \sum_{n=0}^{N-1} \sin(2\pi (n + \frac{1}{8}) \frac{f_{CWI}}{f_s} + \Phi_{CWI}) \cdot e^{-j 2\pi n \frac{f}{f_s}}
\]

(7.9)

Remembering that \( 1.5 \cdot f_s < f_{CWI} < 2.5 \cdot f_s \), a new frequency \( f_{CWI}' \) is introduced:

\[
    f_{CWI}' = f_{CWI} - 2 \cdot f_s \rightarrow f_{CWI} = f_{CWI}' + 2 \cdot f_s
\]

(7.10)

and now \(-0.5 \cdot f_s < f_{CWI}' < 0.5 \cdot f_s \). With formula 7.10, formula 7.9 can now be written as:

\[
    X(f) = \sum_{n=0}^{N-1} \sin(2\pi n \frac{f_{CWI}'}{f_s} + 4\pi n + \Phi_{CWI}) \cdot e^{-j 2\pi n \frac{f}{f_s}}
\]

\[
    \quad + j \cdot \sum_{n=0}^{N-1} \sin(2\pi (n + \frac{1}{8}) \frac{f_{CWI}'}{f_s} + 4\pi (n + \frac{1}{8}) + \Phi_{CWI}) \cdot e^{-j 2\pi n \frac{f}{f_s}}
\]

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\[ X(f) = \sum_{n=0}^{N-1} \sin(2\pi n \frac{f_{cwi}'}{f_s}) + \phi_{cwi} \cdot e^{-j 2\pi n \frac{f}{f_s}} \]

\[ + j \cdot \sum_{n=0}^{N-1} \sin(2\pi (n + \frac{1}{8}) \frac{f_{cwi}'}{f_s} + \frac{1}{2} \pi + \phi_{cwi}) \cdot e^{-j 2\pi n \frac{f}{f_s}} \rightarrow \]

\[ X(f) = \sum_{n=0}^{N-1} \sin(2\pi n \frac{f_{cwi}'}{f_s} + \phi_{cwi}) \cdot e^{-j 2\pi n \frac{f}{f_s}} \]

\[ + j \cdot \sum_{n=0}^{N-1} \cos(2\pi (n + \frac{1}{8}) \frac{f_{cwi}'}{f_s} + \phi_{cwi}) \cdot e^{-j 2\pi n \frac{f}{f_s}} \] (7.11)

From formula 7.11 it can be seen that bandpass sampling and DFT transforming a sine wave at \( f_{cwi}' \), yields spectrum data which are a linear combination of the results that would be obtained if two separate signals were sampled normally:

1) a sine wave at \( f_{cwi}' \):

\[ s_{1, cwi} = \sin(2\pi n \frac{f_{cwi}'}{f_s} + \phi_{cwi}) \] (7.12)

and

2) a cosine wave with extra phase shift \( \frac{\pi}{4} \cdot \frac{f_{cwi}'}{f_s} \), also at \( f_{cwi}' \):

\[ s_{2, cwi} = \cos(2\pi n \frac{f_{cwi}'}{f_s} + \frac{\pi}{4} \frac{f_{cwi}'}{f_s} + \phi_{cwi}) \] (7.13)

With \(-0.5 \cdot f_s < f_{cwi}' < +0.5 \cdot f_s\), the extra phase shift of \( s_{2, cwi} \) can vary between \(-\frac{\pi}{8}\) and \(+\frac{\pi}{8}\).

In order to understand the effect of bandpass sampling, the spectrum output of \( s_{1, cwi} \) and \( s_{2, cwi} \) can be analysed separately. In both cases, it should be remembered that a Segmented Chirp Z-Transform is used to perform the actual
DFT operation. This means that the operation described in formula 7.11 is carried out for a limited set of frequencies $f$ as described in section 6.5:

$$f = f_0 + k \cdot f_1$$  \hspace{1cm} (7.14)

with $f_0$ and $f_1$ chosen arbitrarily, and $k$ an integer running from 0 to $K - 1$. The result from the SCZT will be presented in the form of an array containing complex values. Array element $i$ in this array gives amplitude- and phase-information about the signals found at frequency $f = f_0 + i \cdot f_1$.

The output array from a SCZT performed on signal $s_{1,CW}$ will contain two complex peak values, one at an array position $k_{CW}$, corresponding to frequency $f_{CW}'$ and another at array position $K - 1 - k_{CW}$, corresponding to frequency $f_{-1} - f_{CW}'$. The real and imaginary values of each peak are determined by the phase shift $\varphi_{CW}$ as well as by the sign of frequency $f_{CW}'$ (remember that $f_{CW}'$ can be negative). For each combination of $f_{CW}'$ and $\varphi_{CW}$ a unique combination of complex peaks at array positions $k_{CW}$ and $K - 1 - k_{CW}$ is found. An example is given in fig. 7.5, which shows the SCZT output of a signal $s_{1,CW}$ at a frequency $f = 0.2 \cdot f_s$ with $\varphi_{CW} = 0$. The horizontal axis shows the array index, the vertical axis the array output value relative to the highest peak found in the output array.

![Diagram](image)

Fig. 7.5: Result of DFT performed on $s_1$. 

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A similar result will be found after performing a SCZT on signal $s_{2, \text{CW}}$: again two complex peaks will be found in the output array on array positions $k_{\text{CW}}$ and $K - 1 - k_{\text{CW}}$. The peak values will be dependent on frequency $f_{\text{CW}}$ and phase $\phi_{\text{CW}}$, with unique peak value combinations found for each combination of $f_{\text{CW}}$ and $\phi_{\text{CW}}$. Fig. 7.6 shows the result of a SCZT on $s_{2, \text{CW}}$ under the same conditions as in fig. 7.5. Again, the horizontal axis contains the array index, and the vertical axis the array output value relative to the highest peak found.

Fig. 7.6: Result of DFT performed on $s_2$.

The result of a SCZT on the bandpass sampled signal 7.8 is obtained by adding the results obtained from performing an SCZT on signals $s_{1, \text{CW}}$ and $s_{2, \text{CW}}$. This will result again in four peaks in the output array, with different amplitudes. Calculating these amplitudes is straightforward mathematics; the results show again that for each combination of $f_{\text{CW}}$ and $\phi_{\text{CW}}$ there is a unique set of peak amplitudes.

With the SCZT implementation as described until now, signals on frequencies $+|f_{\text{CW}}|$ ($f_{\text{CW}} > 0$) as well as on $-|f_{\text{CW}}|$ ($f_{\text{CW}} < 0$), will both cause peaks to appear on array positions $k_{\text{CW}}$ and $K - 1 - k_{\text{CW}}$, though the peak values will be different. This means that from the SCZT output array, it is not possible to determine whether a signal is located on $+|f_{\text{CW}}|$ or $-|f_{\text{CW}}|$. Before this output array can be used for notch filter control, the number of complex peak values for
each signal has to be reduced from 2 to 1. This is done by writing the combination of peaks for any one value of \( f_{cw}' \), as a vector \( X \):

\[
X = \begin{pmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4
\end{pmatrix}
\]  

(7.15)

with

- \( X_1 \) the real peak at \( f_{cw}' \),

- \( X_2 \) the imaginary peak at \( f_{cw}' \),

- \( X_3 \) the real peak at \( f_s - f_{cw}' \), and

- \( X_4 \) the imaginary peak at \( f_s - f_{cw}' \).

This vector must be transformed into another vector \( Y \):

\[
Y = \begin{pmatrix}
\cos(\varphi_{cw}) \\
\sin(\varphi_{cw}) \\
0 \\
0
\end{pmatrix}
\]  

for \( f_{cw}' \geq 0 \) and \( Y = \begin{pmatrix}
0 \\
0 \\
\cos(\varphi_{cw}) \\
\sin(\varphi_{cw})
\end{pmatrix}
\)  

for \( f_{cw}' < 0 \).  

(7.16)

Calculating \( Y \) from \( X \) is done with a matrix equation:

\[
\begin{pmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
Y_4
\end{pmatrix} = \begin{pmatrix}
A_{11} & A_{21} & A_{31} & A_{41} \\
A_{12} & A_{22} & A_{32} & A_{42} \\
A_{13} & A_{23} & A_{33} & A_{43} \\
A_{14} & A_{24} & A_{34} & A_{44}
\end{pmatrix}
\begin{pmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4
\end{pmatrix}
\]  

or \( Y = A \cdot X \)  

(7.17)

Note that this matrix calculation (from now on called the output descrambling operation) has to be done for each complex value in the DFT output array and its mirror separately, with the matrix elements in \( A \) dependent on the array index! Note too, that equation 7.17 can only be used if the SCZT output array does actually contain peak value information at frequencies \( f_{cw}' \) and \( f_s - f_{cw}' \). As stated before, output array elements contain peak information on frequencies \( f = f_0 + k \cdot f_i \), with \( k \) running from 0 to \( K - 1 \). To be able to use equation 7.17, \( f_0 \) and \( f_i \) must be adhering to the following formula:
\[ f_0 = f_s - \left( f_0 + (K - 1) \cdot f_s \right) \rightarrow 2 \cdot f_0 + (K - 1) \cdot f_s = f_s \]  \hspace{1cm} (7.18)

In order to properly implement quadrature bandpass sampling, matrix \( A \) must be found. This is done as follows:

- Vector \( \mathbf{X} \) is calculated for four different cases:

  1. \( f_{CWI}' \geq 0 \) and \( \varphi_{CWI} = 0 \),

  2. \( f_{CWI}' \geq 0 \) and \( \varphi_{CWI} = \pi/2 \),

  3. \( f_{CWI}' < 0 \) and \( \varphi_{CWI} = 0 \), and

  4. \( f_{CWI}' < 0 \) and \( \varphi_{CWI} = \pi/2 \).

The four resultant different vectors are called \( \mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3 \) and \( \mathbf{X}^4 \).

- For each vector \( \mathbf{X}^i \) a vector \( \mathbf{Y}^i \) is found, containing one 1 and three 0's:

\[
\begin{align*}
\mathbf{Y}^1 &= A \cdot \mathbf{X}^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \\
\mathbf{Y}^2 &= A \cdot \mathbf{X}^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \\
\mathbf{Y}^3 &= A \cdot \mathbf{X}^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \\
\mathbf{Y}^4 &= A \cdot \mathbf{X}^4 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\end{align*}
\]  \hspace{1cm} (7.19)

- Vectors \( \mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3 \) and \( \mathbf{X}^4 \) are combined into one matrix \( \mathbf{X}_m \). \( \mathbf{Y}^1, \mathbf{Y}^2, \mathbf{Y}^3 \) and \( \mathbf{Y}^4 \) are combined into matrix \( \mathbf{Y}_m \), which is identical to the unity matrix \( \mathbf{I} \). Matrix equations 7.19 can now be written as:

\[
\mathbf{Y}_m = \mathbf{I} = A \cdot \mathbf{X}_m \rightarrow A = \mathbf{X}_m^{-1}
\]  \hspace{1cm} (7.20)

In other words, matrix \( A \) can be found simply by combining vectors \( \mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3 \) and \( \mathbf{X}^4 \) into one matrix and inverting this matrix. Using this simple algorithm, matrix \( A \) is found to be:

\[
A = \begin{pmatrix}
1+\cos(\alpha) & \sin(\alpha) & 1-\cos(\alpha) & \sin(\alpha) \\
-\sin(\alpha) & 1+\cos(\alpha) & \sin(\alpha) & -1+\cos(\alpha) \\
1-\cos(\alpha) & -\sin(\alpha) & 1+\cos(\alpha) & -\sin(\alpha) \\
-\sin(\alpha) & -1+\cos(\alpha) & \sin(\alpha) & 1+\cos(\alpha)
\end{pmatrix}
\]  \hspace{1cm} (7.21)

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with:

\[
\alpha = \frac{\pi}{4} \cdot \frac{f_k}{f_1}
\]  

(7.22)

and \( f_k \) the frequency belonging to output array index \( k \) according to equation 7.14: \( f_k = f_0 + k \cdot f_1 \). Index \( k \) must run from 0 to \( \frac{1}{2} \cdot (K - 1) \) for this descrambling operation, since descrambling is always done for array positions \( k \) and \( K - 1 - k \) simultaneously.

Note that the whole descrambling operation as described in formula 7.17, is a linear operation. This means that it will work for any combination of CWI signals. Also, the modulation effect due to the finite sampling interval as described in chapter 6, will not be changed.

### 7.4 Choosing a proper CWI weighting function

In section 6.4 the concept of digital spectrum analysis with automatic weighting has been introduced. In this concept, the weighting function is the Fourier Transform of the function used to describe the sampling process. In its simplest form, this function is 1 during the sampling interval \( t_s \) and 0 outside this interval. As formula 6.7 and fig. 6.6 show, the Fourier Transform of this function yields a sinc-characteristic. Such a weighting function has the severe disadvantage of weighting some CWI signals with a factor zero (see fig. 6.8). Clearly, a better weighting function is desired.

At first sight, generating the proper weighting function seems easy:

1) First establish the amplitude transfer function \( H_{\text{track}}(f) \) of the phase-tracking loop of the Loran-C receiver.

2) Then calculate the inverse Fourier Transform \( h_{\text{track}}(t) \) of \( H_{\text{track}}(f) \):

\[
h_{\text{track}}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_{\text{track}}(f) \cdot e^{j2\pi ft} \, dt
\]

(7.23)
3) Use $h_{\text{track}}(t)$ as a weighting function: multiply all incoming samples with $h_{\text{track}}(t)$.

This approach has two problems:

- The real weighting function generated with this method, is not equal to $H_{\text{track}}(f)$, but must be written as:

$$H_{\text{track}}'(f) = \int_{0}^{t_s} h_{\text{track}}(t) \cdot e^{-j2\pi ft} \, dt \neq \int_{-\infty}^{+\infty} h_{\text{track}}(t) \cdot e^{-j2\pi ft} \, dt$$

(7.24)

Due to the finite integration interval in formula 7.24, the weighting function $H_{\text{track}}'$ will contain a certain amount of ripple. This can be understood by realising that the finite integration interval can be represented by a multiplication of $h_{\text{track}}(t)$ with a rectangular function with value 1 during the interval $t_s$ and 0 outside that interval. In the frequency domain this multiplication should be translated into a convolution of a sinc-function and $H_{\text{track}}(f)$ in order to obtain $H_{\text{track}}'(f)$. This convolution with a sinc-function introduces ripple in $H_{\text{track}}'(f)$.

- The amplitude transfer of the receiver phase-tracking loop is usually difficult to establish. This is due to the often highly non-linear nature of the tracking algorithms employed. Also, several tracking algorithms adapt the loop bandwidth automatically to the amount of noise present [7.5]. Finding a usable description of the transfer function of such an adaptive loop, is impossible.

The first problem is solved here by trial-and-error. Once the wanted weighting function $H_{\text{track}}(f)$ has been established, calculating the error $|H_{\text{track}}(f) - H_{\text{track}}'(f)|$ is easy. If this error is too large, $H_{\text{track}}(f)$ is changed until the error is acceptable.

The second problem has been dealt with in detail in section 4.3. There, it has been shown that the best method to avoid this problem, is to assume a weighting function that is always wider than the actual loop transfer function. This introduces a risk of marking some signals near-synchronous signals that are really asynchronous. However, as long as all signals marked harmful by the spectrum analysis system, can be notched out, no unacceptable tracking errors need be expected.
For a first implementation of the receiver architecture of fig 7.1, a simple weighting function has been chosen. This weighting function has a low-pass characteristic with a first-order, -6 dB per octave roll off:

\[ H_{track}(f) = \frac{p}{p - j 2\pi f} \]  \hfill (7.25)

The inverse fourier transform of this function is:

\[ h_{track}(t) = \begin{cases} \quad p \cdot e^{p t} & \text{for } t \leq 0 \\ \quad 0 & \text{for } t > 0 \end{cases} \]  \hfill (7.26)

According to this formula, automatic weighting of CWI signals is therefore achieved when all incoming samples are multiplied with the exponential function 7.26. Before this function can be used, it must be rewritten as:

\[ h_{track}(t) = \begin{cases} \quad p \cdot e^{p (t-t_s)} & \text{for } t \leq t_s \\ \quad 0 & \text{for } t > t_s \end{cases} \]  \hfill (7.27)

This is necessary because sampling takes place between \( t = 0 \) and \( t = t_s \). The original weighting formula 7.28 is zero during this interval. Now the effect of the finite sampling interval should be analysed:

\[
H'_{track}(f) = p \cdot \int_{0}^{t_s} e^{p (t-t_s)} \cdot e^{-j 2\pi f t} \, dt
\]

\[
= p \cdot e^{-j 2\pi f t_s} \cdot \int_{-t_s}^{0} e^{p t} \cdot e^{-j 2\pi f t} \, dt
\]

\[
= \frac{p}{p - j 2\pi f} \cdot \left[ e^{-j 2\pi f t_s} - e^{-p t_s} \right] \]  \hfill (7.28)

and the corresponding amplitude transfer function is:

\[
|H'_{track}(f)| = \frac{p}{\sqrt{p^2 + (2\pi f)^2}} \cdot \left| e^{-j 2\pi f t_s} - e^{-p t_s} \right| \]  \hfill (7.29)
If \( t_s \) goes to infinity, formula 7.29 becomes the wanted weighting function again, with a -6 dB per octave roll-off. The error due to the finite sampling interval, manifesting itself as ripple, is determined by the term \( e^{-p t_s} \) (the amplitude of the term \( e^{-2\pi f t_s} \) is always 1). This ripple is visible in the weighting function chosen for the simulations in section 7.5, which is shown in fig. 7.8. The -3 dB point of formula 7.29 is calculated with:

\[
\text{f-3 dB} = \frac{p}{2 \pi}
\]  

(7.30)

Note that this is an approximation, based on the assumption that the term \( e^{-p t_s} \) in formula 7.29 is negligible. Note too, that the spectrum analysis resolution is now determined primarily by \( p \), while \( t_s \) merely controls the error in the weighting due to the finite sampling interval.

### 7.5 Choosing sample intervals, segment length and other parameters

In this section, actual implementation values will be given for the following parameters: the sampling interval \( t_s \), the number of segments \( L \) and the segment length \( M \) for the Segmented Chirp Z-Transform, the weighting function coefficient \( p \) and the SCZT parameters \( \omega_0 \), \( \omega_1 \) and \( K \). These values will be used in the simulations described in section 7.6. Note that they describe one possible implementation only; other values for these parameters are certainly feasible.

The tracking loop used in the simulations of section 7.6 has a bandwidth that is inversely proportional to the GRI currently being tracked. Such loops have the advantage of providing a tracking noise suppression that is independent of the GRI. This feature means that the -3 dB bandwidth of the weighting function must also be GRI-dependent.

In order to get parameter \( p \) GRI-dependent, the error term \( e^{-p t_s} \) in formula 7.29 has been set to 0.05, independent of the GRI:

\[
e^{-p t_s} = 0.05 \rightarrow p \cdot t_s \approx 3 \rightarrow p \approx \frac{3}{t_s}
\]

(7.31)
Next, the sampling time $t_s$ is chosen directly proportional to GRI:

$$t_s = c_s \cdot \text{GRI} \Rightarrow c_s = \frac{t_s}{\text{GRI}} \quad (7.32)$$

For a GRI of $5000 = 0.05$ s, a sampling time $t_s$ of 10 s has been chosen. This yields a value for $c_s$ of 200. Now combine 7.31 and 7.32:

$$p = \frac{3}{c_s \cdot \text{GRI}} = \frac{3}{200 \cdot \text{GRI}} = \frac{0.015}{\text{GRI}} \quad (7.33)$$

and the -3 dB bandwidth $f_{-3\,dB}$ is then calculated according to formula 7.30:

$$f_{-3\,dB} = \frac{0.015}{2\pi \, \text{GRI}} = \frac{2.4 \cdot 10^{-3}}{\text{GRI}} = 0.05 \text{ Hz for a GRI of } 5000 = 0.05 \text{ s} \quad (7.34)$$

Figure 7.7 shows the weighting function to be used for a GRI of 5000, fig 7.8 shows the corresponding Fourier Transformed function.

![Graph](image)

*Fig. 7.7: Weighting function used during simulations.*

A sampling interval that is directly proportional to GRI, can best be implemented by setting the segment length $M$ for the SCZT to GRI, with GRI written as a multiple of 10 $\mu$s. With a GRI of 5000 and a sampling frequency of
Fig. 7.8: Fourier transform of function in fig. 7.7.

50 kHz, this yields a sampling time per segment of 0.1 s. The total sampling time $t_s$ is 10 s; this means that the number of segments $L$ should be set to 100.

Note that the selected -3 dB bandwidth $f_{-3\, \text{dB}}$ must always be equal to or larger than the -3 dB bandwidth of the phase tracking loop used, as explained in section 4.3. With such a choice for $f_{-3\, \text{dB}}$, the set of signals marked as being harmful will automatically include all signals damaging to the Cycle Identification mechanism, since such a mechanism uses a much smaller bandwidth than any phase tracking loop. In the simulations described in section 7.6, no phase tracking or Cycle Identification has been employed; therefore, for these simulations any reasonable choice for $f_{-3\, \text{dB}}$ can be used.

The last parameters still to be determined, are the SCZT parameters $\omega_0$ and $\omega_1$. The corresponding frequencies are called $f_0$ and $f_1$. These two frequencies determine the points on which the frequency spectrum is sampled, as explained in paragraphs 6.3 and 6.5:

$$f = f_0 + k \cdot f_1$$  \hspace{1cm} (7.35)

with $k$ running from 0 to $K - 1$. The values for $f_0$ and $f_1$ should be chosen along the following guidelines:
The spectrum between 75 kHz and 125 kHz must be sampled on multiples of \( GRF = \frac{1}{2} \text{GRI} \).

Values chosen for \( f_0 \) and \( f_1 \) must conform to equation 7.18, enabling proper descrambling of the SCZT output array.

The Loran-C carrier at 100 kHz is always a multiple of GRF. Any signal on this frequency is converted into a DC value by the bandpass sampling system, due to the fact that 100 kHz is exactly twice the sampling frequency. This means that the SCZT algorithm must start sampling the frequency spectrum at \( f = 0 \), so:

\[
f_0 = 0
\]

(7.36)

The frequency step \( f_1 \) is set to:

\[
f_1 = \frac{1}{2 \text{GRI}}
\]

(7.37)

with GRI in seconds. Now equation 7.18 becomes:

\[(K - 1) \cdot f_1 = f_s \rightarrow (K - 1) \cdot \frac{1}{2 \text{GRI}} = f_s \rightarrow K = 2 \cdot f_s \cdot \text{GRI} + 1
\]

(7.38)

and with a sampling frequency \( f_s \) of 50 kHz we can write:

\[K = 10^5 \cdot \text{GRI} + 1
\]

(7.39)

and since GRI is always a multiple of 10 μs, \( K \) is guaranteed to have an integer value.

Looking at the requirements set up for the selection of \( f_1 \), a potential problem can be seen: \( f_1 \) is directly related to the GRI, which is controlled by the Loran-C chain timing clocks, but \( f_1 \) is also directly locked to the sampling frequency \( f_s \). If \( f_s \) is generated by the receiver frequency standard, and if this frequency standard drifts with respect to the chain timing (as it usually does), \( f_1 \) is not equal to GRF anymore and the spectrum will be sampled at the wrong frequencies. This problem has already been dealt with in detail in [7.1]; there the proposed
solution is to lock the sampling clock \( f_s \) to the zero-crossing of the Loran-C signal currently being tracked. This clock generation method has been employed in the simulations described in the next section, too.

### 7.6 Simulation results

The Loran-C receiver simulation program LOSP (described in detail in appendix A) has been adapted for simulation of the receiver architecture of 7.1. In this section, some results of simulations carried out on the spectrum analysis system, will be shown.

First, for all simulations a constant set of external conditions has been defined:

1) No skywaves or ECD simulated (these are not part of the CWI detection problem).

2) A GRI of 5000.

3) A set of six CWI signals, all with Signal-to-Interference Ratios of 0 dB according to MPS definitions [7.6]. Table 7.1 contains a list of the CWI frequencies, their distance from the nearest multiple of GRF, plus a number \( N \), calculated according to:

\[
N = f_{cw1} \cdot 2 \cdot \text{GRI} \quad (7.40)
\]

with GRI in seconds. The fractional part of \( N \) indicates whether the signal is synchronous (fraction = 0), near-synchronous (fraction almost 0 or almost 1) or a-synchronous (anything else).

<table>
<thead>
<tr>
<th>( f_{cw1} ) (Hz)</th>
<th>( \Delta f ) (Hz)</th>
<th>( N )</th>
<th>( f_{cw1} ) (Hz)</th>
<th>( \Delta f ) (Hz)</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>76334.86</td>
<td>4.86</td>
<td>7633.486</td>
<td>111556.85</td>
<td>3.15</td>
<td>11155.685</td>
</tr>
<tr>
<td>79642.64</td>
<td>2.64</td>
<td>7964.264</td>
<td>115000</td>
<td>0</td>
<td>11500</td>
</tr>
<tr>
<td>85000</td>
<td>0</td>
<td>8500</td>
<td>123419.98</td>
<td>0.02</td>
<td>12341.998</td>
</tr>
</tbody>
</table>

*Table 7.1: Six simulated CWI signals.*

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4) No analog bandpass (anti-aliasing) filter. This is not a completely realistic condition, since such a filter will always be necessary. However:

- Such a filter, if designed along the guidelines laid out in section 7.2, will attenuate CWI signals only modestly. The operation of the spectrum analysis system will not be changed anyway.

- The Loran-C pulse will also be affected (if only slightly) by such a filter, but in the operation of the spectrum analysis system this pulse plays a rather unimportant role.

- The effect of filtering on noise is similar to that on CWI signals: noise components outside the Loran-C band will be attenuated, but this does not change the spectrum analysis principles.

5) Several different values for the SNR have been used: \( \text{SNR} = \infty \) (no noise present), \( \text{SNR} = 0 \, \text{dB} \) and \( \text{SNR} = -20 \, \text{dB} \).

6) Simulations have been carried out with and without the FIR filter of fig. 7.2.

7) All other relevant receiver parameters (sampling time, weighting function bandwidth etc.) have the values chosen in section 7.5.

First, a simulation has been carried out of the Loran-C receiver operating without noise. The result is shown in fig. 7.9. The upper window contains the results of the spectrum analysis, with all peaks detected on multiples of \( \sqrt{2 \, \text{GRF}} \) shown. The interference signals can be detected easily as single lines. The Loran-C spectrum, with its typical bell-like shape can also be seen. The weighting function effect is clearly visible too: while the SIR at the antenna input is 0 dB for all signals, the a-synchronous CWI produces much lower peaks in the spectrum of fig. 7.9 than the (near-)synchronous signals.

The lower window contains a table showing the result of an experimental spectrum scan algorithm, which detects the 16 highest peaks in the spectrum found by the Segmented Chirp Z-Transform. All CWI signals are listed in order of weighted signal-strength. Also included in the table are some "ghost" frequencies close to (near-)synchronous CWI. These "ghosts" are the result of the application of weighting functions: at a distance of GRF from a synchronous CWI signal, the weighting function is not completely zero yet, and therefore the
synchronous signal will cause a small peak to appear on its neighboring multiples of GRF too. The appearance of these ghosts in the table is a deficiency of the scan algorithm, which has not yet been solved.

Note that the spectrum in the upper window does not give any absolute signal strength reading. The y-axis is a relative axis, referenced to the strongest peak (0 dB). No absolute readings are necessary anyway for notch filter control.

With fig. 7.9 a quick check can be made of whether the weighting function is operating correctly. This is done as follows:

- A synchronous CWI signal is not attenuated by the weighting function. Such a signal can be found on 85 kHz. The detected peak of this signal is taken as an amplitude reference point.

- Next, an a-synchronous signal is taken. Fig. 7.9 shows that the a-synchronous signal at 76334.86 Hz is attenuated by ≈ 40 dB, compared to
the synchronous peak at 85 kHz. This attenuation should be equal to the amplitude transfer of the weighting function given by formula 7.29:

\[ |H_{\text{track'}}(f)| = \frac{p}{\sqrt{p^2 + (2\pi f)^2}} \cdot \left| e^{-j2\pi ft_1} - e^{-p ft_1} \right| \approx \frac{p}{\sqrt{p^2 + (2\pi f)^2}} \]

with \( f \) being the distance from the CWI frequency of 76334.86 Hz to the nearest multiple of GRF. In the case of fig. 7.9 this multiple is found on 76330 Hz; the corresponding distance is then 4.86 Hz and with a value of 0.1 \( \cdot \pi \) for \( p \) as calculated in the previous section, the attenuation of this a-synchronous CWI signal is indeed \( \approx 40 \) dB.

In fig. 7.10 the result of a simulation with SNR = 0 dB, is shown.

\[ \text{Fig. 7.10: Simulation results with SNR = 0 dB.} \]

Compared to fig. 7.9, the Loran-C bell-shaped spectrum is now much less distinguishable, as can be expected with such a SNR. However, all CWI peaks, including the a-synchronous ones, are still clearly visible above the noise floor! This has the following reason:
- A DFT, with or without special weighting functions, samples the spectrum at a pre-determined set of frequencies. On one such frequency, peaks are found only from signals on or very close to this frequency. This means that the DFT in fact implements a set of narrow bandpass filters around the set of frequencies. One such filter has an amplitude transfer as given by formula 7.29.

- The noise encountered in a Loran-C antenna signal, is almost by definition wide-band noise. This noise will be filtered in the set of narrow bandpass filters that is formed by the DFT. Because of their small bandwidth, the output noise level of any single bandpass filter is much smaller than the input level. A CWI signal that happens to be at the center of such a narrow bandpass filter (i.e. a synchronous signal) will, however, not be attenuated at all. This means that the Signal-to-Noise Level of the CWI signal is increased by the DFT operation, and that even signals with an antenna SNR of 0 dB are easily detected.

This effect is demonstrated even more dramatically in fig. 7.11, where a simulation result is shown obtained with a SNR of -20 dB. Now, the

![Simulation results with SNR = -20 dB.](image)

Fig. 7.11: Simulation results with SNR = -20 dB.
a-synchronous signals have disappeared into the noise. However, the (near-)synchronous signals at 85 kHz, 115.00009 kHz and 123.41998 kHz are still clearly visible. Remember that the antenna SIR (ratio between CWI and Loran-C signals) was 0 dB. The CWI signals are detected in fig. 7.11 with a SNR of approximately +30 dB. This means that synchronous signals that are up to 30 dB below Loran-C level, can still be detected under very high noise levels. Such signals do not pose a threat to phase tracking anymore. Therefore, from fig. 7.11 it can be concluded that the proposed receiver architecture enables detection of harmful CWI signals even under highly unfavourable noise conditions.

Until now, all simulations were done without any filtering. Fig. 7.12 shows a simulation result with the FIR filter shown in fig. 7.2, and a SNR of 0 dB. Not surprisingly, the only effect of the filter is to reduce the amplitudes of the CWI signals and the noise outside the Loran-C band. The ratio between the noise floor and the CWI peaks is not changed. The filter used here was a FIR filter; an equal effect can be expected when using an analog bandpass filter system.

![Image of a graph and a table]

**Fig. 7.12: Simulation with SNR 0 dB and FIR filter.**
7.7 References


A Loran-C receiver design with digital spectrum analysis Page 135
8 Some concluding remarks

8.1 Introduction

In this thesis, two methods have been presented for reducing the deterioration of Loran-C operation due to Carrier Wave Interference. The first method is applied during chain (system) design. It cannot solve the CWI problem on its own and is of a supplementary nature. Therefore, attention has been focused mainly on the second method, which is useful for making Loran-C receivers immune against CWI.

The CWI reduction methods presented in this thesis, should not be seen as recipes, ready for application. More research (though maybe of a less fundamental nature) will be needed. Some topics requiring attention, will be identified in this concluding chapter.

8.2 Loran-C chain design for minimum CWI susceptibility

A method useful for designing Loran-C chains with minimum CWI susceptibility, has been presented in chapter 5. It makes use of the fact that the influence of a CWI signal on Loran-C, depends on its frequency in relation to the Group Repetition Interval of the Loran-C chain. The basic method has been developed to a satisfactory level; it can, however, certainly be expanded. Two main issues seem to be worth investigating:

1) Standard receiver definition. This issue has been dealt with in section 5.5. There, one receiver was defined for which a GRI was selected with minimum CWI. The methods used to get to such a definition, can certainly be improved. Very important here is the selection of a weighting function that resembles actual tracking loops as closely as possible. Also, the receiver input bandpass filter is an element worth investigating. Finally, methods can be developed to take into account several different receiver types simultaneously.
2) Inclusion of GRI selection software into coverage area prediction models. Coverage area prediction algorithms have already been described in [8.1]. There all limiting factors for Loran-C coverage except CWI, have been analysed: Signal-to-Noise Ratio, Envelope-to-Cycle Difference, skywave interference and station geometry. With the inclusion of the GRI selection algorithm of chapter 5, an integrated Loran-C chain design program can be developed - an interesting prospect.

8.3 Loran-C receiver spectrum analysis

This thesis has focused mainly on methods that can be applied to Loran-C receiver design. Most important was the detection of harmful CWI signals, since this was traditionally a weak point. The proposed method, embedded in a special receiver architecture, has been proven to work with computer simulations. However, not all aspects of this architecture have yet been analysed:

- Noise limitations. The receiver architecture presented in chapter 7 must be able to detect harmful CWI signals even in high-noise conditions, with Loran-C and CWI signals far below the noise level. Simulation results (as shown in fig. 7.11) show that this requirement can be fulfilled, but no theoretical analysis of the noise rejection properties of the proposed architecture has yet been presented. This gap in the receiver description should be filled before the architecture is actually implemented.

- Analog-to-Digital conversion of Loran-C antenna signals. In the architecture presented in chapter 7, it has been assumed that A/D conversion of Loran-C signals with all possible interference and all dynamic-range requirements, is not a problem. However, preliminary investigations show that in order to avoid complicated analog gain-control blocks, full 16-bit conversion might well be needed. At a sample rate of 400 kHz such a converter, though feasible, is not yet a low-cost item.

- Processing power requirements. The proposed receiver architecture makes use of FIR filters and FFT algorithms. FIR filters are notorious for their processing power requirements. The FFT algorithms will have to run in real-time too, with many indications that high-performance hardware will be necessary. Again, preliminary investigations show that the processing power
needed for these parts of the receiver, is available. However, in order to keep down cost, a very clever choice of hardware for these algorithms will have to be made.

- Phase tracking. In the architecture of fig. 7.1, one sampling channel is used for both phase tracking and spectrum analysis. The requirements put on the sampling clock by these two parts of the receiver, are contradictory: tracking requires a clock that can be shifted in small time steps, while spectrum analysis needs a clock that is absolutely fixed. One possible solution might be to use a CORDIC algorithm [8.2] to do the phase tracking, and keep the actual sampling clock fixed for spectrum analysis.

- CWI signal filtering. As has been said before, this thesis concentrates on harmful interference signal detection in Loran-C receivers. However, detecting CWI signals is of course not enough: these signals have to be filtered before reliable Loran-C operation can be obtained. In fig. 7.1 digital notch filters have been included; however, no choices have been made as to what type of digital notch filter should be used here, or how it should be implemented.

8.4 References


Appendix A  

LOS\textsuperscript{P}: an analysis tool for Loran-C receivers

A.1 Introduction

In chapters 6 and 7, results of computer simulations of Loran-C receivers have been shown. These results were obtained with the simulation program LOS\textsuperscript{P}, of which a previous version has been described in [ A.1 ]. In this chapter, a complete description of LOS\textsuperscript{P} will be given, including the necessary provisions for Digital Signal Processing techniques. First, however, a short overview of the development history of LOS\textsuperscript{P} will be presented.

Development of the first version of LOS\textsuperscript{P} [ A.4 ] was started in fall 1986 by two students of the Electrical Engineering department of the Delft University of Technology, working in the laboratory for pulse and digital electronics: A. Duym and L.P. Remmerswaal. At the time, these two students were analysing the effects of CWI on Loran-C phase tracking; results from this work have been published in [ A.7 ] and [ A.3 ]. They needed a program simulating a Loran-C phase tracking loop to help them in their analysis. Such a loop of course needs input signals (a Loran-C burst, noise and interference), so these had to be simulated as well. Thus LOS\textsuperscript{P} was born.

LOS\textsuperscript{P} version 1.0 turned out to be very useful for analysing many different problems with Loran-C receiver behaviour, and several special versions were developed. However, the internal program structure was not very flexible, making modifications increasingly difficult. Therefore, in late summer 1987 development of a completely new version was started by A. Duym: LOS\textsuperscript{P} v.2.0.

LOS\textsuperscript{P} v.2.0 contains many improvements over its previous versions:

- In LOS\textsuperscript{P} v.1.0, the behaviour of the tracking loop is monitored by continuously plotting the Loran-C signal phase as estimated by the loop, on screen. This is done in all later versions of LOS\textsuperscript{P} as well, but starting with LOS\textsuperscript{P} v.2.0 two windows can be used for plotting, so that two loop parameters can be monitored simultaneously.
In LOSP v.2.0 a complete set of signals at a Loran-C receiver antenna can be simulated: a Loran-C signal with ECD and skywaves, any number of CWI signals, and of course noise. LOSP v.1.0 is limited to simulating a pure Loran-C signal (without ECD, skywaves or modulation) and one CWI signal.

LOSP v.2.0 simulates the effects of an analog bandpass filter as found in all conventional Loran-C receivers, including a user-definable number of notch filters. This part of LOSP v.2.0 was developed by H.J.A. Linclaen Arriëns, also of the laboratory of pulse and digital electronics, and was a completely new capability of LOSP.

Because of the much larger number of signal parameters to be specified by the user, a good user-interface was needed. In LOSP v.2.0, the user-interface was completely redesigned.

In winter 1988, the on-going development of LOSP was taken over by the writer of this thesis. Since then, again many additions and improvements have been made. Improvements have been aimed mostly at making the program easily adaptable to new simulation task. Additions include:

- simulation of long-delayed skywaves;

- pulse place modulation of the Loran-C signal for data transmission purposes;

- an experimental Cycle Identification algorithm for hard-limiting receivers, described in [ A.8 ];

- statistical operations on tracking data coming from the phase tracking loop (average estimated phase, estimated phase standard deviation etc.); and

- the algorithm for analysing the influence of (near-)synchronous CWI described in chapter 3.

The additions listed above, do already give an indication of the many different applications in which LOSP has been used: research into methods to use Loran-C as a data-transmission channel [ A.5 ], experiments with Cycle Identification methods, and many other topics.
In 1990, a major effort was started to verify LOSP, in order to make sure that the program does actually generate simulated signals as specified. This verification process is described in detail in section A.5. H.J.A. Lincklaen Arriens of the laboratory for pulse and digital electronics, was the main contributor to this verification effort.

Currently, two main versions of LOSP are in use: LOSP v.5.1 is the latest version for the simulation of conventional Loran-C receivers, while LOSP v.6.0/DSP has been developed specifically for research into new receiver architectures based on Digital Signal Processing, as described in chapters 6 and 7. This latest version is the most important one for this thesis, since it has been used to generate the simulation results described in section 7.6. Therefore, the remainder of this appendix will be focused on LOSP v.6.0/DSP. Keep in mind, though, that the basic simulation methods described in sections A.2 and A.3 have been used for all LOSP versions from LOSP v.2.0 on.

A.2 What is simulated by LOSP?

Fig. A.1 shows a complete navigation system, from a transmitter over a radio path to the receiver.

![Diagram of Loran-C navigation system]

*Fig. A.1: Loran-C navigation system.*

The Loran-C transmitter generates a pure Loran-C burst, which will be contaminated on its way to the receiver by noise, interferences etc. The task of the receiver is to measure the Time Of Arrival (TOA) of the incoming Loran-C burst even under unfavorable circumstances. It consists of several "building
blocks", as the example of fig. A.1 shows. In order to be able to write a simulation program, one has to define first which "building blocks" are to be simulated.

In LOSP a general choice has been made to simulate all those receivers which have the commonly used architecture of fig. A.1: analog bandpass- and notch-filters in the front end, some kind of A/D conversion (hard-limiter or linear) of the filtered signal and a (micro-)processor system used for the actual Time Of Arrival or Time Difference measurement. Note that all selectivity is assumed to be concentrated in the filter system; therefore amplifiers present in the receiver will not change any amplitude ratio's (e.g. SNR) and can therefore be neglected. It is assumed too that the receiver does not actually calculate a position in longitudes and latitudes; this is more a geodetic than an electronic problem and is therefore not implemented in LOSP.

As shown in fig. A.1, simulation of the analog part of the receiver yields a signal \( S(t) \) going into the signal processor. This processor will take care of (optional) digital filtering of the incoming signal, as well as all algorithms necessary for locking onto the Loran-C transmission frame, identifying the proper cycle and measuring TOA's. Algorithms for signal analysis (including CWI spectrum determination) are also included at this point.

For proper receiver simulation LOSP must be able to calculate the signal \( S(t) \) going into the signal processor at any time \( t \), as well as the intermediate signals of which \( S(t) \) is composed. These are:

- A signal as transmitted by a standard Loran-C transmitter, with phase coding. Note that for the purpose of this program (receiver analysis), it is enough to simulate one transmitter signal. Simulating more than one transmitter signal would not be useful, as the extra data would be used only in position calculations. These calculations are not be simulated by LOSP, for the reason mentioned above.

- The influence of the radio path between transmitter and receiver on the Loran-C signal: Envelope-to-Cycle-Discrepancy (ECD), noise, interfering signals and skywaves.

- The influence of an analog bandpass filter system in the front end of a receiver on all incoming signals. There should be a choice of as many
bandpass filters as possible and a possibility to add more than one notch filter.

Note that wherever suitable, LOSP keeps to the definitions of Loran-C and other signals as published in the Minimum Performance Standards (MPS) [A.10] of the Radio Technical Commission for Marine Services, Special Committee no. 70.

After the signal $S(t)$ has been calculated, LOSP will execute the algorithms implemented in the signal processor of fig. A.1. Because these algorithms can differ widely, the user of LOSP is not offered a choice at run-time, as this choice would have to be very limited. Instead, a well-defined interface between the analog "receiver" part and the signal processing section in LOSP, is established. This interface can be used to implement widely different processing algorithms.

A.3 Simulation methods: generating the necessary signals

A.3.1 Introduction

In the previous section it has been shown that a very important task of LOSP is to calculate the signal $S(t)$ going into the signal processor conform the USCG standards, at any time $t$. This signal consists of several parts: groundwave, skywave, interferences and noise. In a real receiver these signals are summed in the antenna and then filtered and processed. In LOSP, the signals are first filtered separately and then summed. Because the filter system is linear, this is permissible, and filtering separately first and summing afterwards is easier and faster than summing first and filtering later.

So in generating $S(t)$, the parts which together form $S(t)$ have to be calculated first and then summed. In order to be able to properly sum these "sub"-signals, a reference amplitude has to be established. In LOSP, the amplitude at the peak of the Loran-C groundwave burst at the receiver antenna is set to 1; the amplitudes of all other signals are referenced to this amplitude (incidentally, as LOSP does not simulate groundwave attenuation, the amplitude at the peak of the transmitted burst is 1 too in LOSP). Note however, that most Loran-C specifications are not referenced to the peak of the Loran-C burst, but to a continuous sine wave with the same amplitude as the envelope of the Loran-C
burst at 25 μs [ A.10 ]. This value will be called \( Env_{25} \) in all following formulas, and for a peak amplitude of 1 it is about 0.506.

The four signal types of which \( S(t) \) is built up, are then:

- A filtered groundwave, optionally with ECD. The method used to calculate a filtered Loran-C burst with ECD is explained in section A.3.2.

- A filtered skywave, again optionally with ECD. The filtered skywave is calculated in the same way as the filtered ground wave, and then multiplied with the sky-to-ground-wave ratio (this will not be changed by the filter system) and delayed with the skywave delay.

- The filtered interference signals. The version of LOSP described here, is designed to simulate Carrier-Wave Interferences (CWI) only, but provisions have been made to simulate Cross Rate Interferences (CRI) too. CWI signals can be written as:

\[
S_{i, \text{unfiltered}}(t) = A \cdot \sin (\omega_I \cdot t + \varphi_I) \tag{A.1}
\]

In this formula, \( \omega_I \) and \( \varphi_I \) are specified directly by the user, but amplitude \( A \) has to be calculated from the specified Signal-to-Interference Ratio (SIR). For this calculation, the USCG definition of the SIR is used. This definition makes use of the value of the envelope of the burst at 25 μs:

\[
A = \frac{Env_{25}}{10^{(SIR_{20}/20)}} \tag{A.2}
\]

with the SIR specified in decibels. The effect of filtering such a signal can be expressed as:

\[
S_{i, \text{filtered}}(t) = A \cdot |H(j\omega_I)| \cdot \sin(\omega_I \cdot t + \varphi_I + \Phi(\omega_I)) \tag{A.3}
\]

with \( |H(j\omega_I)| \) and \( \Phi(\omega_I) \) resp. the amplitude and phase transfer of the filter system at \( \omega_I \). The calculation of these filter parameters will be explained in section A.3.2.
- Filtered noise, specified and simulated according to the USCG specifications. The methods used to simulate noise in LOSP are explained in section A.3.3.

A.3.2 Simulation of analog bandpass filter systems

In LOSP, three types of analog bandpass filter calculations have to be done:

1) Calculation of amplitude- and phase-transfer of the filter at any frequency \( \omega \): \(|H(j\omega)|\) and \(\Phi(j\omega)\), for e.g. simulation of CWI-signals.

2) Calculation of the noise bandwidth of the filter: this is the bandwidth of a filter with a square amplitude transfer and the same noise power at its output as the filter selected in LOSP.

3) Calculation of the filtered Loran-C burst: phase transfer at the Loran-C transmitting frequency of 100 kHz and influence of the group delay on the Loran-C envelope.

An analog bandpass filter in LOSP is specified by the poles and zeros of the corresponding normalized low-pass filter. Data of these normalized low-pass filters can be found in all filter handbooks and therefore other filter types can be introduced relatively easily. A low-pass to bandpass transformation then yields complex conjugate pole- and zero-pairs \((p_i, \bar{p}_i)\) and \((z_i, \bar{z}_i)\). Such a transformation is described in [ A.2 ]. Next, for each (optional) notch, one pole pair and one zero pair are added on the proper places. Finding the general filter transfer function of the complete filter system is then a standard exercise in filter theory:

\[
H(s) = H \cdot \frac{\prod_{i=1}^{m}(s - z_i) \cdot (s - \bar{z}_i)}{\prod_{i=1}^{n}(s - p_i) \cdot (s - \bar{p}_i)} \quad \text{(A.4)}
\]

For a frequency \( \omega \), the amplitude- and phase-transfer functions are simply:
\[
|H(\omega)| = H \cdot \prod_{i=1}^{m} \left| j\omega - z_i \right| \cdot \left| j\omega - \bar{z}_i \right| \prod_{i=1}^{n} \left| j\omega - p_i \right| \cdot \left| j\omega - \bar{p}_i \right|
\]

(A.5)

\[
\Phi(\omega) = (n-m) \cdot \frac{\pi}{2} + \sum_{i=1}^{m} [\varphi(j\omega - z_i) + \varphi(j\omega - \bar{z}_i)] - \sum_{i=1}^{n} [\varphi(j\omega - p_i) + \varphi(j\omega - \bar{p}_i)]
\]

(A.6)

These two formulas are used for the calculation of amplitude and phase transfer for CWI-signals.

Calculation of the noise bandwidth of the selected analog filter system, is also a standard exercise in filter theory. The noise bandwidth \( B_n \) is easily calculated with:

\[
B_n = \int_{0}^{\infty} |H(\omega)|^2 d\omega
\]

(A.7)

For the calculation of a filtered Loran-C burst first the standard Loran-C burst is Laplace-transformed, then it is multiplied with the filter transfer of formula A.4 and the result of this multiplication is transformed back from the Laplace- into the time-domain. This calculation is complicated but contains only standard mathematics, and therefore here only the results will be shown.

The received Loran-C burst, including ECD, is given by:

\[
f_{\text{Loran, unfiltered}}(t) = [k \cdot (t - t_{\text{ECD}})]^2 \cdot e^{-2 \cdot k \cdot (t - t_{\text{ECD}})} \cdot \sin(\omega_0 t)
\]

(A.8)

with \( k = \frac{1}{65 \mu s} \), \( \omega_0 = 2\pi \cdot 100 \text{ kHz} \) and \( t_{\text{ECD}} \) the ECD shift. The Laplace-transform of this burst is:

\[
F_{\text{Loran, unfiltered}}(s) = A \cdot (e \cdot k)^2 \cdot \left\{ \frac{1}{(s - p_L)^3} - \frac{1}{(s - \bar{p}_L)^3} \right\}
\]

(A.9)

with \( p_L = - (2k \pm j\omega_0) \) and \( A = \sin(\omega_0 t_{\text{ECD}}) + j \cos(\omega_0 t_{\text{ECD}}) \).
The filtered Loran-C burst can be calculated by multiplying A.4 with A.9 and transforming the resulting formula back to the time-domain. The result from this (complicated) mathematical operation is:

\[
\begin{align*}
    f_{\text{Loran,filtered}}(t) &= 2H(e \cdot k)^2 \\
    &\left[ e^{-2kt} \sum_{x=1}^{3} \left\{ (-1)^{x-1} \frac{A_x}{(x-1)!} t^{x-1} \sin(\omega_n t - \varphi_x) \right\} \\
    &- \sum_{i=1}^{n} \left\{ e^{-\alpha_i t} A_i \sin(\beta_i t - \varphi_i) \right\} \right]
\end{align*}
\] (A.10)

with \( n \) the number of pole pairs in the filter system and parameters \( A_x, \varphi_x, A_i \) and \( \varphi_i \) functions of the filter poles and zeros, as well as the ECD. Parameters \( \alpha_i \) and \( \beta_i \) in formula A.10 are simply the real and imaginary parts of the filter poles \( p_i : p_i = \alpha_i + j\beta_i \).

### A.3.3 Simulation of noise

According to the Minimum Performance Standards, two different noise sources can be used for simulation of Loran-C signals:

1) White noise. This noise is filtered with a single-resonator LC filter with a bandwidth of 30 kHz and the level of the filter output signal is used as a reference for SNR calculations.

2) Atmospheric noise, simulated by:

- a noise source as described in point 1;
- bursts of 100 kHz sine waves, lasting 30 \( \mu \)s each and distributed randomly in time according to a Poisson distribution.

The two sources are added with a ratio of \( \frac{0.1585}{0.8416} \) to simulate atmospheric noise.

It will be clear that the first method mentioned above, will be executed much faster by the computer.
The noise source is considered to be present at the antenna input, and should therefore be filtered in LOSP with the selected analog bandpass filter before being added to the Loran-C and CWI signals. Such a filter operation will have two effects on a noise signal:

1) Samples of pure white noise, transmitted through a filter, will have a correlation coefficient which is not zero. This means: when taking noise samples at regular distances, the value of a filtered sample will not only depend on the momentary value of the unfiltered noise signal, but also on the values of the previous noise samples.

2) The total noise power will be limited: ideal white noise has a constant power density from 0 Hz to infinity, while the filter will transmit only a part of this spectrum.

In a conventional Loran-C receiver, samples of the antenna signal will be taken at distances around 1 ms. Filter theory states that the correlation coefficient between signals coming out of the filter, will be approximately zero at sampling intervals of more than $\frac{1}{B}$, with $B$ the -3 dB bandwidth of the filter. This would mean that even with a 1 kHz bandpass filter in the Loran-C receiver, samples taken at the Loran-C repetition rates will not be correlated. With the normal bandwidths in Loran-C receivers of 15 to 30 kHz, this will be even more so. So for samples taken at the burst repetition rates, one can safely take uncorrelated gaussian noise to simulate the noise coming out of the filter in a real Loran-C receiver.

For non-conventional Loran-C signal processing, however, a different picture emerges. In such applications (e.g. as described in chapter 7), many samples are taken within one burst. Due to the resultant small sampling interval, the noise component of these samples cannot be considered uncorrelated anymore. However, in order to minimise the processing power needed for noise generation, it was decided to use uncorrelated noise in the simulation of these applications too.

The noise power $\sigma_{LC30}^2$ of the antenna noise simulated according to the MPS, can be calculated easily (remember: in the MPS all amplitudes are referenced to the burst amplitude at 25 $\mu$s, which is equal to $Env_{25}$ in LOSP):
\[ \sigma_{LC30}^2 = \frac{Env_{25}^2}{2} \cdot 10^{-(\text{SNR})\frac{10}{10}} \]  \hspace{1cm} (A.11)

This is the power of the noise simulated in accordance with the MPS at the antenna input. By dividing this power through the noise bandwidth of a 30 kHz LC filter, the power density \( N_0 \) of the antenna noise can be found:

\[ N_0 = \frac{\sigma_{LC30}^2}{B_{n,LC30}} \]  \hspace{1cm} (A.12)

with \( B_{n,LC30} \) the noise bandwidth of the filter used in the MPS specifications. This noise bandwidth can be calculated relatively easily and is found to be 47.124 kHz.

Once the noise power density of the antenna noise is found, the power of the noise after the filter system can be calculated:

\[ \sigma_{\text{filtered}}^2 = B_{n,\text{filter}} \cdot N_0 \]  \hspace{1cm} (A.13)

with \( B_{n,\text{filter}} \) the noise bandwidth of the filter system (bandpass- and notch-filters) chosen by the user of LOSP. \( B_{n,\text{filter}} \) is calculated with formula A.7. Filtered noise in LOSP is now simulated using uncorrelated gaussian noise with power \( \sigma_{\text{filtered}}^2 \) (A.11 + A.12 + A.13):

\[ \sigma_{\text{filtered}}^2 = \frac{B_{n,\text{filter}}}{B_{n,LC30}} \cdot \frac{Env_{25}^2}{2} \cdot 10^{-(\text{SNR})\frac{10}{10}} \]  \hspace{1cm} (A.14)

As formula A.14 does indicate, every time the filter configuration is changed the noise bandwidth of the system has to be recalculated, because otherwise no proper noise simulation can be done.

The only problem still to be solved is how to generate uncorrelated gaussian noise. This is done in LOSP by generating and adding 12 random numbers with a pseudo-random noise generator. This technique has been described in among others \([\text{A.6}]\) and yields a good approximation of a gaussian distribution with average \( \mu = 6 \) and standard deviation \( \sigma = 1 \). Both \( \mu \) and \( \sigma \) can be scaled to the values wanted by the program designer.
A.4 Signal Processing

In the previous section, a description has been given of the methods used to generate a signal $S$ as function of the time $t$ (see also fig. A.1). As has been mentioned in section A.2, this signal is used in the processing algorithms built into LOSP. These algorithms have two tasks:

1) The sampling time $t$ has to be generated; this time $t$ is fed back into the "analog" section of LOSP in order to generate the signal $S(t)$.

2) The generated signal $S(t)$ has to be processed.

Obviously, the algorithms for generating time $t$ and processing signal $S(t)$ depend heavily on the simulation aim. In this section, an overview will be given of the implementation of the tracking and CWI analysis algorithms as described in chapters 6 and 7. Note that this overview serves two aims: first it is an example of the signal processing capabilities that can be incorporated into LOSP, and secondly it explains the methods used to obtain the simulation results described in chapters 6 and 7.

The receiver architecture described in chapters 6 and 7, uses a main sampling clock of 400 kHz. This clock is locked to the Loran-C carrier frequency at 100 kHz, to provide for automatic compensation of receiver clock errors and doppler shift (see also section 6.6). This means that the 400 kHz signal is generated in two steps (see fig. A.2):

1) Basically, a sampling pulse is generated every 2.5 μs. The value of the main time base $t_{main}$ is increased with 2.5 μs at every sampling pulse.

2) Also, every 2.5 μs the current doppler / clock error estimation (called speed in fig. A.2) is added to an accumulator register. Whenever the value stored in the accumulator register exceeds a certain maximum - or minimum in case of a negative doppler / clock error estimate - a time step (called speedstep in fig. A.2) is added to or subtracted from the basic sampling interval of 2.5 μs and therefore from $t_{main}$ too. This so-called rate-multiplier algorithm is described in more detail in [ A.9 ].
Fig. A.2: 400 kHz sampling clock generation.

The value of \( t_{\text{main}} \) is fed back into the analog signal generation part of LOSP, which then produces a value for the signal \( S(t_{\text{main}}) \). As explained in chapter 7, this signal is first entered into a FIR filter. The filtered signal is then used twice:

- If the current sample falls within the proper part of a Loran-C burst, it is used to track the selected Loran-C zero-crossing. It is assumed here that only a simple phase tracking algorithm is implemented: one sample per burst is taken and the sampling moment for this sample is adjusted until a zero-crossing of the Loran-C signal is found. Such algorithms are described in more detail in [ A.9 ]; fig. A.3 illustrates the simple phase tracker used for the simulations described in chapters 6 and 7. The sampling rate is 400 kHz; with a distance of 1 ms between consecutive bursts in one GRI, every 400th sample is used for this tracking algorithm.

- The FIR-filtered sample is also used for spectrum analysis. However, before the sample is actually stored, first the signals coming out of the FIR filter are bandpass sampled, as explained in section 7.2. The sampling rate for this bandpass sampling process is 50 kHz, with two samples being taken at this rate, spaced 2.5 µs apart. This means, simply, that only the first two samples out of eight coming out of the FIR filter, are used: the first one is designated the in-phase sample (see section 7.2), the second one the quadrature sample. These two samples together form a complex value, which is entered into an
Fig. A.3: A simple phase-tracking algorithm.

input array for a segmented Chirp-Z transform. This sampling pattern is shown in fig. A.4.

Fig. A.4: Pattern used for bandpass sampling.

In the input array for the segmented Chirp-Z transform, a total of GRI complex samples is collected. If the array is filled, a Chirp-Z transform is executed on it. Then, the output of this transform is added in a summation array to the output of previous segments (see section 6.5.3). If the current segment is the last, the summation array contains valid spectrum
This spectrum information is then scanned and the worst CWI signals are detected. This spectrum analysis algorithm is shown in fig. A.5.

Fig. A.5: Spectrum analysis algorithm in LOSP.

It should be noted that other, different signal processing algorithms can be incorporated into LOSP. The algorithms described above, should be seen as an
example and as an illustration of the results obtained with simulation in chapter 7.

A.5 Program verification: making sure that simulation results are valid

A.5.1 Introduction

Computer simulations can be very powerful tools in today’s electrical engineering practice, but meaningful simulation includes a verification of the simulation tool. In the case of LOSP, the signal $S(t)$ (see fig. A.1) has been verified to represent a Loran-C receiver antenna signal with the properties described in sections A.3.1 and A.3.3. This verification process included:

- Bandpass filter transfer functions: are all bandpass filter transfer functions (amplitude transfer, phase transfer etc.) and the filter noise bandwidth calculated correctly?

- Bandpass filtered signals: as described in section A.3.2, LOSP generates filtered signals directly, instead of first calculating unfiltered signals and then filtering them. Are these filtered signals calculated correctly?

- Noise: does the generated white noise have the proper gaussian distribution and power level?

The methods used to check LOSP on these points, will be described briefly in the next sections.

A.5.2 Verification of bandpass filter calculations

A step-by-step approach of the bandpass filter calculations has been used:

- First the transformation of normalised low-pass to bandpass filter has been checked. This transformation (which is described in section A.3.2) is done on the poles and zeros of the selected filter. The formulas used for this transformation can be found in [A.2]; these formulas have been implemented in the numerical solving program MathCAD. For several filters
the low-pass to bandpass transformation has been calculated with MathCAD, and the results were compared with the poles and zeros calculated by LOSP.

- With the MathCAD implementation of the amplitude transfer, checking the noise bandwidth calculations is easy: according to formula A.14 the amplitude transfer must be integrated from 0 to (approximately) infinity. The value for \( B_{\text{filter}} \) found with MathCAD is compared again with the noise bandwidth calculated by LOSP, for several filter systems.

- Next the formulas for calculation of amplitude transfer and group delay of a filter, have been obtained from [A.2] and implemented in MathCAD. With the bandpass filter poles and zeros calculated in the previous step, MathCAD is then able to calculate values for amplitude transfer and group delay. These values are again compared with the results calculated with LOSP for the same filter. Note that LOSP is also able to calculate phase transfer and phase delay functions of a bandpass filter. These functions have not been verified.

- Then, the Laplace-transformed Loran-C burst formula (A.9) has been checked. First, the inverse Laplace-transform formula of A.9 is obtained:

\[
\int_{0}^{\infty} \frac{1}{1} \left( e^{s} t \right) ds
\]

with \( A = \sin(\omega_{0} t_{ECD}) + j \cos(\omega_{0} t_{ECD}) \)

This integration formula has again been implemented in MathCAD and numerically calculated for several values of \( t \). The results have been compared to the values found with the formula of the unfiltered Loran-C burst (formula A.8). This step ensures that formula A.9 is correct.

- Finally, formulas A.9 and A.4 are multiplied, and the result is again implemented in MathCAD:

\[
F_{\text{Loran,filtered}}(s) = H(s) \cdot F_{\text{Loran,unfiltered}}(s)
\]

This yields a description of the filtered Loran-C burst in the s-domain; this description is then transformed back into the time domain:

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\[ f_{\text{Loran, filtered}}(t) = \int_{0}^{\infty} F_{\text{Loran, unfiltered}}(s) \cdot H(s) \cdot e^{st} \, ds \] (A.17)

For two different filter types this transformation has been done numerically with MathCAD, again for different values of \( t \). The values for \( f_{\text{Loran, filtered}} \) calculated by LOSP have been compared with these results, and were found to be correct.

A.5.3 Noise generator verification

The noise generator in LOSP has been checked by collecting 100000 noise samples on disk, for two different bandpass filters. With a separate program, the mean and the standard deviation of these samples is checked. The mean should be (almost) zero, since the noise generator should not introduce a DC term into the total simulated signal. The standard deviation is linked directly to the selected Signal-to-Noise Ratio:

\[ \sigma_{\text{filtered}} = \sqrt{\frac{B_{n, \text{filter}}}{B_{n, \text{LC30}}} \cdot \frac{\text{Env}_{25}}{2} \cdot 10^{-\frac{\text{SNR}_{20}}{2}}} \] (A.18)

with:

- \( B_{n, \text{filter}} \) the noise bandwidth of the selected filter system;

- \( B_{n, \text{LC30}} \) the noise bandwidth of the standard single-pole LC filter defined by the MPS [ A.10 ];

- \( \text{Env}_{25} \) the relative Loran-C envelope value at 25 \( \mu \text{s} \).

The standard deviation found with the 100000 noise samples of LOSP, is of course compared to the theoretical value found with formula A.18.

Finally, the noise sample distribution has to be checked (a gaussian distribution function is required). This is done simply by calculating and plotting the distribution function of the 100000 collected noise samples. Fig. A.6 shows the results of 100000 samples collected with a Cauer bandpass filter with noise bandwidth 15.6 kHz, and an SNR of -10 dB. Also plotted (as a dotted line) is a gaussian distribution curve with the standard deviation calculated from formula
A.18. This dotted line is hardly visible, indicating that it overlaps well with the distribution function obtained from the 100000 samples. While not a thorough scientific test, this indicates that the noise generator produces samples that are reasonably gaussian and have the proper power level.

![Graph showing noise distribution with a Cauer filter.](image)

Fig. A.6: Noise distribution with Cauer filter.

A.6 References


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About the author


On September 1st, 1981, he started studying mathematics at the Delft University of Technology. In January 1982 he switched to electrical engineering at the same university. He finished his studies there in November 1987, getting a Dutch engineering degree.

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