Stellingen

behorende bij het proefschrift

*Ground-based remote sensing of precipitation using a multi-polarized FM-CW Doppler radar*

doors

Herman Russchenberg

Delft, 14 december 1992
1 De drijfveer van wetenschappers moet niet zijn de wil tot het begrijpen, maar fascinatie voor de schoonheid waarmee de natuur in elkaar zit; deze fascinatie neemt toe als het begrip toeneemt.

2 Radarmetingen van de atmosfeer vergroten de betrouwbaarheid van grootschalige klimaatmodellen, omdat ze een betere parametrisatie van kleinschalige fenomenen mogelijk maken.

3 Met de kruispolaire radarreflectie kan, behalve vanaf de grond, ook vanuit een satelliet of vliegtuig, regen van sneeuw onderscheiden worden.

4 Alhoewel smeltende sneeuwvlokken een onregelmatige vorm hebben, kunnen ze om de radarmetingen te verklaren als sferoiden beschouwd worden.

5 De Maxwell Garnett theorie voor het berekenen van de permittiviteit van een homogezen mengsel is niet geldig voor grote volumefracties van het ingesloten materiaal, maar voldoet wel.

6 Als de maxima van zdr en ldr in de smeltlaag niet samenvallen, verandert de oriëntatie van de sneeuwvlokken tijdens het smelten.

7 Turbulentie veroorzaakt wanorde in de ruimtelijke verdeling van regendruppels, waardoor er een verband tussen het snelheidsspectrum en de kruispolaire radarreflectie ontstaat.
8 De parameters van de grootteverdeling van regendruppels zijn statistisch gerelateerd, maar deze relaties moeten van regenbui tot regenbui variëren.

9 Het is vaker bewolkt, dan dat het regent; de aanwezigheid van een smeltlaag in de bewolking kan een beperking opleggen aan het gebruik van satellietontvangers met kleine antennes, de zgn VSAT’s.

10 De combinatie van rass en radar maakt de meting van hoogteprofielen van de luchvochtigheid mogelijk.

11 Modellering van propagatie van radiosignalen door de smeltlaag behoeft geen complexe verstrooiingsmodellen: een uitbreiding van de Rayleigh-theorie voldoet.

12 De huidige beschrijving van de werkelijkheid is ingewikkeld geworden; dit duidt op een groter analytisch dan synthetisch vermogen van wetenschapsbeoefenaars.

13 Het regent nooit tijdens kantooruren.

14 De stichters van het stadje Nederland in de Rocky Mountains waren of ironisch, of Nederlanders.
Ground-based remote sensing of precipitation using a multi-polarized FM-CW Doppler radar
Ground-based remote sensing of precipitation using a multi-polarized FM-CW Doppler radar

Over het op afstand waarnemen van neerslag met een op de grond gestationeerde, meervoudig gepolariseerde, FM-CW Doppler radar,

een proefschrift ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus, prof. drs. P.A. Schenck, in het openbaar te verdedigen ten overstaan van een commissie, aangewezen door het College van Dekanen, op 14 december 1992 te 16.00 uur door

Herman Walter Johan Russchenberg

geboren te Valkenburg,
elektrotechnisch ingenieur

Delft University Press / 1992
Dit proefschrift is goedgekeurd door de promotor prof. dr. ir L.P. Ligthart
Ik draag dit proefschrift op aan mijn ouders
Een dag op het strand

de coclacant hapt lachend naar adem en sterft
de monnik schrikt de visser ook
want is dit rebellie?

de vis: de vis
versteent de theorie
wat zeker is
is poëzie

uit logica
niet na te
denken

dempt men de oceaan
men denkt: ik denk
           ik ben

dus waar kom ik vandaan?
slenteren de monnik en visser op het strand
de zee is stil en stil onthult de zee
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Voorwoord

Dit proefschrift is het resultaat van zes jaar onderzoek. In die zes jaar heb ik menigeen meewarig zien lachen, wanneer ik vertelde dat ik mij bezig hield met regen en smeltende sneeuw. Of nog niet alles bekend was, want het regende immers zo vaak in Nederland? Wanneer ik dan uitlegde dat het onderzoek van belang was voor het ontwerpen van goede radioverbindingen, of voor het waterbeheer in Nederland, werd het al iets duidelijker, maar als nog volgde dan vaak de vraag of ik het weer kon voorspellen, of dat het onderzoek soms te maken had met zure regen. Welnu, dit proefschrift stelt iedereen in de gelegenheid om zelf te ontdekken wat ik de afgelopen jaren heb gedaan (al ben ik bang dat het soms een beetje onbegrijpelijk is).


Ik ben hen allen zeer erkentelijk.
Mijn promotor, Leo Ligthart, moet ik speciaal bedanken. Hij heeft mij altijd zijn vertrouwen geschonken en in de gelegenheid gesteld mijzelf te ontplooien. Hij heeft mij de mogelijkheid gegeven om veel contacten te leggen met collega’s in binnen- en buitenland. Ik ben mij ervan bewust dat niet iedereen zulke kansen krijgt.

Tijdens het schrijven van een proefschrift verwaagt het privé-leven. De promovendus trekt zich terug in zijn studeerkamer en merkt af en toe verbaasd dat er nog iemand bij hem in huis woont. Nu het proefschrift af is, zie ik pas hoe belangrijk Coldette was. Haar geduld heeft het mogelijk gemaakt dat ik in alle rust kon werken. Haar aanwezigheid gaf mij de energie om dit proefschrift te schrijven.

H.R.
Samenvatting

Hydrometeoren, zoals regendruppels en sneeuwvlokken, kunnen electromagnetische golven verstrooien. Weerradars trekken profijt van deze eigenschap. Zij stralen een gecodeerd radiosignaal in een regenbui en meten de energie die wordt terugverstoord. De code van het signaal wordt door het verstrooingsproces veranderd. Uit deze verandering kan informatie over de hydrometeoren verkregen worden. De Delft Atmospheric Research Radar codeert de polarisatie en de fase van de signalen die het uizendt. De verandering van deze specifieke codes geeft informatie over het type hydrometeoor en de bijbehorende snelheid, vorm, grootte en oriëntatie. De verandering van de polarisatie wordt uitgedrukt in de radargrootheden differential reflectivity \( Z_{dr} \), de linear depolarisation ratio \( L_{dr} \) en de horizontal reflectivity \( Z_h \). De verandering van de fase uit zich in het Doppler-snelheidsspectrum en wordt uitgedrukt in de gemiddelde snelheid \( V_d \) en de breedte van het spectrum \( W_d \).

De Delftse onderzoekradar is een FM-CW systeem, dat werkt op 3.3 GHz. Het FM-CW principe wordt uitgelegd en alle relevante aspecten van het radarsysteem, inclusief de signaalbewerking, worden besproken. In tegenstelling tot wat gebruikelijk is, wordt \( L_{dr} \) bepaald uit copolaire metingen. Hierdoor neemt de gevoeligheid van \( L_{dr} \) voor ruis af.

De radargrootheden reageren verschillend op veranderingen van de microstructuur van regen. Hierdoor wordt het mogelijk om, door gebruik te maken van de correlatie van de verschillende radargrootheden, meteorologische processen in een regenbui te bestuderen. Uit de analyse van een langdurige regenbui bleek een sterk verband tussen \( L_{dr} \) en \( W_d \).

Dit is een gevolg van turbulentie: turbulentie veroorzaakt een wanordelijke verdeling van de regendruppels, wat kruispolarisatie tot gevolg heeft. Op basis van de meetgegevens wordt een eenvoudig model van het verband tussen turbulentie en de oriëntatie van regendruppels ontwikkeld. Met gebruikmaking van dit model worden regenmetingen gesimuleerd.

De druppelgrootteverdeling wordt beschreven door een gamma-functie met drie parameters: de mediaan \( D_o \), de dispersiefactor \( \mu \) en een schalingsfactor \( N_o \). Met conventionele radarsystemen kunnen er maar er één of twee parameters bepaald worden, maar de combinatie van \( Z_h \), \( Z_{dr} \) en \( W_d \) maakt het mogelijk om ze alle drie te meten. Hiertoe moet het Dopplerspectrum gecorrigeerd worden voor turbulentie. Dit
wordt bereikt door de oriëntatie van de regendruppels te schatten uit de combinatie van $Z_{dr}$ en $L_{dr}$. Uit de meetgegevens blijkt dat er statistische verbanden tussen de drie parameters bestaan. Hiermee wordt, voor een specifieke regenbui, een nieuwe relatie tussen de radarreflectie en de regenintensiteit afgeleid.

Een nieuw Doppler-polarimetrisch model van terugwaartse vestrooing door de smeltlaag wordt ontwikkeld, gebaseerd op metingen met de Delftse onderzoeksradar. Het model voorspelt de hoogte-afhankelijkheid van $Z_h$, $Z_{dr}$, $L_{dr}$ en $V_d$. Er worden twee nieuwe elementen geïntroduceerd: het verband tussen de vorm en grootte van de smeltende sneeuwvlokken en de invloed van het smeltproces op hun oriëntatie. Het verband tussen de vorm en grootte is gebaseerd op correlatiestudies van de radargrootheden in de smeltlaag. In de bovenste helft van de smeltlaag zijn kleine sneeuwvlokken platter dan grote, maar in de onderste helft is het andersom. Dit inzicht maakt het mogelijk om regenbuien met een kleine smeltlaag en grote $Z_{dr}$ te simuleren. Het feit dat de oriëntatie van de sneeuwvlokken afhanger van het smeltproces resulteert in een verschuiving van de maxima van $L_{dr}$ en $Z_{dr}$: ze vallen niet samen. Het model van de smeltlaag wordt vergeleken met radarmetingen, die zijn gedaan met een elevatie van 30° en met de radar gericht naar het zenit.
Summary

Hydrometeors, such as raindrops or snowflakes, scatter incident electromagnetic waves. Weather radars benefit from this property. They transmit a coded radio signal into a rain shower and measure the reflected energy. The code of the signal may have changed and, if so, information about the hydrometeors is hidden in the changed code. The Delft Atmospheric Research Radar codes its signal in polarization and phase. The change of these two codes gives information on the velocity, shape, size, orientation and type of the hydrometeors. The polarization change is described by the radar observables differential reflectivity $Z_{dr}$, linear depolarization ratio $L_{dr}$, and horizontal reflectivity $Z_h$. The phase-change manifests itself in the Doppler velocity spectrum, which is characterized by the mean velocity $V_d$ and the spectrum width $W_d$.

The Delft Atmospheric Research Radar is an S-band FM-CW system. The FM-CW principle is here explained, and all relevant aspects of the radar system, including the signal processing, are described. In contrast to conventional methods, $L_{dr}$ is derived from three copolar measurements. This decreases the sensitivity of $L_{dr}$ to noise.

The radar observables are differently sensitive to changes in the microstructure of precipitation. This enables the study of meteorological processes in a rain shower by analyzing the correlation of the different radar observables. Analysis of a single rain shower revealed a strong relationship between $L_{dr}$ and $W_d$. This is due to turbulence. Turbulence induces canting of the raindrops, which causes cross-polarization. Based on the data, a simple model that relates canting to turbulence is here developed. Using appropriate values of parameters that describe the microstructure of rain, scatter diagrams of the rain measurements are simulated.

The dropsize distribution is described by a gamma function with three parameters: the median dropsize $D_o$, the dispersion factor $\mu$, and a scaling factor $N_o$. Conventionally, only one or two are derived from radar measurements, but the combination of $Z_h$, $Z_{dr}$, and $W_d$ enables the derivation of all three. To this end, the Doppler spectrum has to be corrected for turbulence. This is achieved by estimating the canting-angle distribution from the combination of $L_{dr}$ and $Z_{dr}$. It is shown that the three parameters are statistically related. Using these, a new relationship between the radar reflectivity and the rain intensity for one specific event is derived.
A new Doppler-polarimetric model of backscattering by the melting layer is developed, based on measurements with the Delft Atmospheric Research Radar. It predicts height profiles of $Z_h$, $Z_{dr}$, $L_{dr}$, and $V_d$. In comparison with existing models, two new elements are introduced: the size-shape relationship, and the dependence of the orientation of melting snowflakes upon the melting rate. The size-shape relationship is based on correlation studies of the radar observables in the melting layer. It appears that in the upper half of the melting layer small snowflakes are more oblate than large ones, whereas in the lower half large ones are more oblate than small ones. This feature enables the prediction of weak bright bands with a large maximum of $Z_{dr}$. The dependence of particle orientation upon the melting rate results in a shift of the maxima of $Z_{dr}$ and $L_{dr}$; they do not coincide. The model is compared with radar observations, done with an elevation angle of 30° and with the radar pointed towards the zenith.
List of symbols

<table>
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<td>( \alpha )</td>
<td>azimuth angle of the orientation vector hydrometeor</td>
</tr>
<tr>
<td>( \beta )</td>
<td>inverse axial ratio spheroid</td>
</tr>
<tr>
<td>( \tilde{\gamma} )</td>
<td>polarizibility</td>
</tr>
<tr>
<td>( \delta )</td>
<td>elevation angle of the orientation vector hydrometeor</td>
</tr>
<tr>
<td>( \varepsilon_0 )</td>
<td>permittivity of vacuum</td>
</tr>
<tr>
<td>( \varepsilon_{\text{mix}} )</td>
<td>relative permittivity of a mixture</td>
</tr>
<tr>
<td>( \varepsilon_r )</td>
<td>relative permittivity</td>
</tr>
<tr>
<td>( \theta )</td>
<td>elevation angle radar antennas</td>
</tr>
<tr>
<td>( \theta_{b,\text{r}} )</td>
<td>(-3, \text{dB beamwidths})</td>
</tr>
<tr>
<td>( \theta_{c} )</td>
<td>(-3, \text{dB equivalent beamwidth})</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>wavelength</td>
</tr>
<tr>
<td>( \lambda_1, \lambda_2, \lambda_3 )</td>
<td>depolarization factors ellipsoid</td>
</tr>
<tr>
<td>( \mu )</td>
<td>dispersion factor of the dropsize distribution</td>
</tr>
<tr>
<td>( \xi )</td>
<td>axial ratio spheroid</td>
</tr>
<tr>
<td>( \rho )</td>
<td>correlation coefficient of two time samples</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>mass density of snow</td>
</tr>
<tr>
<td>( \sigma_{ij} )</td>
<td>radar cross-section measured with polarizations ( i ) and ( j )</td>
</tr>
<tr>
<td>( \sigma_\delta )</td>
<td>spread of the canting-angle distribution</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>standard deviation of ( x )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>pulse width</td>
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<tr>
<td>( \tilde{\gamma} )</td>
<td>shape dyadic ellipsoid</td>
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<tr>
<td>( \phi )</td>
<td>polarization angle radar wave</td>
</tr>
<tr>
<td>( \chi )</td>
<td>normalized particle size</td>
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<tr>
<td>( \psi(\ell) )</td>
<td>backscattered signal</td>
</tr>
<tr>
<td>( \Lambda_1, \Lambda_2, \Lambda_3 )</td>
<td>shape factors ellipsoid</td>
</tr>
<tr>
<td>( \Lambda_d )</td>
<td>exponent of the dropsize distribution</td>
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<td>( \Upsilon )</td>
<td>flux of an electric field</td>
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<td>( a_e )</td>
<td>equivalent volumetric radius</td>
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<tr>
<td>( B )</td>
<td>receiver bandwidth</td>
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<tr>
<td>( c )</td>
<td>speed of light</td>
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<tr>
<td>( C_d )</td>
<td>drag coefficient</td>
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<tr>
<td>( D )</td>
<td>diameter</td>
</tr>
<tr>
<td>( D_e )</td>
<td>equi-volumetric diameter</td>
</tr>
<tr>
<td>( D_0 )</td>
<td>diameter of median drop volume</td>
</tr>
<tr>
<td>( \tilde{\varepsilon}_{v,h} )</td>
<td>horizontal or vertical polarization vector</td>
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\( \bar{E} \)  
\( f_b \)  
\( f_m \)  
\( f_p \)  
\( f_s \)  
\( \Delta f \)  
\( F \)  
\( G \)  
\( hZ_{dr} \)  
\( hZ_h \)  
\( H_{max} \)  
\( k \)  
\( \hat{k}_0 \)  
\( \hat{k}_s \)  
\( k_t \)  
\( L_{dr} \)  
\( L_f \)  
\( L_v \)  
\( mZ_{dr} \)  
\( mZ_h \)  
\( N(D) \)  
\( N_o \)  
\( N_t \)  
\( \bar{\eta} \)  
\( P_{r,t} \)  
\( r \)  
\( r_d \)  
\( r_{\text{max}} \)  
\( r_t \)  
\( \Delta r \)  
\( R \)  
\( R_{xy} \)  
\( S \)  
\( S_{ij} \)  
\( S_{\alpha}^{\bar{\beta}} \)  
\( S(v) \)  
\( SNR \)  

electric field  
beat frequency  
melted mass fraction  
polarizer frequency  
sampling frequency  
Doppler frequency shift  
frequency excursion  
antenna gain  
relative height of peak differential reflectivity in the melting layer  
relative height of peak reflectivity in the melting layer  
width of melting layer  
Boltzmann constant  
propagation vector incident wave  
propagation vector scattered wave  
thermometric conductivity  
linear depolarization ratio  
latent heat of fusion  
latent heat of condensation  
mean differential reflectivity of the melting layer  
mean reflectivity of the melting layer  
dropsize distribution  
scaling factor of dropsize distribution  
number concentration  
dipole moment  
received and transmitted power  
range  
distance of target  
maximum unambiguous range  
traveled distance  
range resolution  
rain intensity  
cross-correlation coefficient  
scattering matrix  
complex scattering parameters  
sensitivity factor of \( \alpha(\beta) \)  
Doppler spectrum  
signal-to-noise ratio
$t$  
  
$T$  
  time transformation matrix  

$T_r$  
  pulse repetition time  

$T_s$  
  sweep time  

$T_{op}$  
  receiver noise temperature  

$u$  
  normalized complex voltage  

$U_v$  
  gradient horizontal wind speed  

$U_r$  
  received complex voltage  

$v$  
  velocity of a particle  

$v_{max}$  
  maximum unambiguous velocity  

$V$  
  volume  

$V_t$  
  total volume of precipitation  

$V_d$  
  mean Doppler velocity  

$wZ_h$  
  width of the reflectivity peak of the melting layer  

$wZ_{dr}$  
  width of the differential reflectivity peak of the melting layer  

$W_d$  
  spread Doppler spectrum due to rain  

$W_t$  
  spread Doppler spectrum due to turbulence  

$W_{d,m}$  
  spread of measured Doppler spectrum  

$Z$  
  reflectivity factor  

$Z_e$  
  equivalent reflectivity factor  

$Z_{dr}$  
  differential reflectivity  

$Z_h$  
  horizontal reflectivity  

$Z_v$  
  vertical reflectivity  

$Z_d$  
  mean Doppler reflectivity  

$<A>$  
  ensemble average of quantity A  

$\bar{A}$  
  time average of A  

$\vec{A}$  
  vector A  

$A^*$  
  complex conjugate of A
In the past decade, the advent of multi-polarization weather radars has given a strong impetus to developments towards the operational use of radar for the quantitative measurement of rainfall. It has also enabled radar meteorologists to study physical processes in precipitation in more detail; identification of different types of hydrometeors, such as snow and hail, has become possible. Apart from meteorology, weather radars are also used for studying radio wave propagation and water management.

1.1 Radar

Radar is the acronym of Radio Detection and Ranging. Radars operate on the simple principle that radio signals can be reflected. A radar transmits a radio signal and measures the amount of energy that is reflected by objects that obstruct the transmitted signal. These objects may be located far from the radar. The reflected signal contains information about the kind of object that has caused it. For instance, a large object will reflect more energy than a small object of the same composition. Thus, radars can be used to study remote objects.

The distance of the object can be estimated by measuring the time interval between the transmission and reception of the radio signal. However, when signals are transmitted and received continuously, coding is necessary to ensure that corresponding transmit and receive signals are compared. The coded signal is reflected by the object, and received back by the radar some time later. When during that interval the coding of the transmit signal has changed, the difference between the codes of the transmit and receive signals depends upon the elapsed time, and hence upon the distance. The radar that is used in the study described in this thesis codes its signal by varying the frequency of the transmit signal, which is referred to as the frequency-modulated continuous wave principle (FM-CW).
The reflecting object may be moving. Due to the Doppler effect a phase variation is imposed on the radar signal. A coherent radar is able to measure this phase variation, and hence the velocity of the object can be obtained. Information about the shape of the target is obtained by coding the polarization of the radar signal, which is done by varying the direction in which the radar signal oscillates. The shape of the object is recognized then in the polarization dependence of the reflected signal. When a radar is able to measure the Doppler spectrum and the polarization dependence, it is called a multi-polarized Doppler radar. The Delft Atmospheric Research Radar is one.

This thesis discusses reflections from precipitation that are measured using the Delft Atmospheric Research Radar. This radar is able to measure weak reflections from rain, snow, and ice. Even reflections from clear air, caused by turbulence and wind, can be seen with it.

1.2 Weather radar

Ever since the radar principle was discovered and the first operational radars were built, the ability to measure rainfall was recognized. When the appropriate radar frequency is used, raindrops reflect the radar waves with sufficient energy to enable their detection. Rain showers can be traced and the rain intensity quantified. The great advantage of a weather radar over the traditional rain gauge is the area coverage: an overview of rainfall in large areas can be obtained within a few seconds, whereas the rain gauge measures the rain intensity at only one location.

Conventional weather radars measure only one quantity: the radar reflectivity. Although the radar reflectivity and the rain intensity are both caused by the same rain, relating the two is not self-evident. Information about the statistical properties of raindrops is required, and the different impact they have on the radar reflectivity and on the rain intensity needs to be examined. Rain is a highly stochastic phenomenon, as raindrops have different sizes and shapes, and occur in varying numbers. Therefore, when the radar measures only one quantity, only one parameter of the statistical distribution of rain can be derived. Usually, the radar reflectivity is converted into the mean dropsize and the number of drops is assumed to be uniquely related to it. This condition is never truly satisfied and accurate measurements of rainfall with conventional radars are therefore not always possible. Conventional radars are not able
to distinguish rain from other types of precipitation. Snow and hail storms can cause radar reflections that are comparable to those caused by rain, but will not necessarily contain the same amount of water. Even in normal rain showers, errors in the radar-derived rain intensity may result, due to the reflection of the radar waves from the melting layer where snowflakes melt into raindrops.

The installation of a polarizer in the radar antennas enables the measurement of the radar reflectivity at different polarizations. Provided that the two reflections are sufficiently different, two parameters of the statistical distribution of rain can be obtained; the resulting information about the mean dropsize and the number of drops enables a more accurate estimation of the rain intensity [Seliga and Bringi, 1976]. The use of more than one polarization benefits from the fact that raindrops are oblate, rather than spherical particles and thus reflect horizontally polarized radio waves more effectively than vertically polarized ones. The difference between the two reflections yields information about the drop shape. It is exactly this property that enables the radar meteorologist to distinguish rain from other types of precipitation, because the shape of raindrops differs from the shape of, for instance, snowflakes or hailstones. A few radars are also equipped with the ability to measure the cross-polarized radar return; it contains information about the drop orientation, which in turn can be due to turbulence. It may also serve as an identifier of the melting layer: the irregular shape of the particles in the melting layer causes an enhanced cross-polarized radar reflection [Goddard et al, 1988].

Older than polarization diversity, but just as relevant, is the technique to measure the velocity of the hydrometeors through the Doppler effect; a falling raindrop induces a time varying phase upon the reflected radar signal, from which a component of the velocity of the raindrop can be estimated. When the radar is pointed towards the zenith, Doppler measurements can be used to estimate the drop sizes, because the fall speed of a raindrop is strongly related to its size [Klaassen, 1989]. Thus, as with polarization diversity, Doppler measurements may improve the accuracy of the radar-derived rain intensity. The fall speed of hydrometeors differs for each type: snowflakes fall much slower than raindrops or hailstones. Consequently, Doppler measurements enhance the distinctions between different hydrometeor types [Russchenberg, 1989].

Doppler measurements are most sensitive to precipitation, when the radar is pointed towards the zenith. At other elevation angles, not only is the fall speed of the raindrops measured, but also the wind by which the raindrops are carried. Polarization diversity is
most sensitive to precipitation when the radar is pointed towards the horizon: when a raindrop is observed from underneath, a circular cross-section is seen and no polarization dependence would occur. To combine polarization diversity and Doppler techniques, the radar must be pointed to a position in between the two extremes. The combination enables an improvement in estimates of the rain intensity and also better observations of physical processes in precipitation can be made.

1.3 Applications of weather radars

Traditionally, weather radars have been used in meteorology. Although initially designed for the quantitative measurement of rainfall, weather radars have become valuable instruments to measure physical processes in precipitation. The combination of Doppler and polarization measurements enables the detailed study of the melting layer [Russchenberg et al, 1990] and that of physical processes in clouds. For instance, the polarization diversity mode enables the study of ice clouds [Matrosova, 1991; Pazmany et al, 1991] due to the well-defined shape of ice crystals, while concurrent Doppler measurements supply information about their movements. The evolution of hydrometeors in clouds, i.e. the development of ice particles into raindrops, can be observed [Illingworth, 1988]. With respect to the increasing importance of climatology and the underexposed role of clouds in climate models, weather radars are becoming increasingly useful instruments in the observation of air motions in and outside clouds. The combination with other instruments, like lidar and rass, may lead to a sound insight into the energy transport mechanisms in the atmosphere. Related to the meteorological applications, is the use of radar for environmental studies. An often used instrument to measure air pollution is the lidar, which is based on the same principle as radar, but uses light instead of radio waves. By choosing wavelengths that correspond to the absorption lines of certain gases in the atmosphere, such as CO₂ and O₃, lidar is used to detect those gases. Aerosols are detected because they scatter the light waves. However, lidar can not penetrate into clouds because water absorbs too much energy. Radar is useful then because it indirectly measures the transport of aerosols by observing the air motion itself.

Weather radars are becoming cheaper. Consequently, they are also being applied in disciplines other than meteorology. Hydrology is a branch of science that studies water systems on and below the surface of the earth. Hydrological information is used for
water management, which implies the controlling and regulating of the quantity and quality of water, for instance in rivers, canals, or sewer systems. It is clear that accurate measurements of rainfall are important to ensure optimal management. Weather radars are used to measure rainfall quantitatively. They are also used for the short-term weather forecasting, somewhat exaggeratedly referred to as now casting, in order to respond rapidly to weather changes. Traditionally, rain gauges have been used to measure rainfall. However, the capability to predict rainfall is leading to a prevalent use of weather radars for water management. Hydrologists require accurate rainfall measurements, especially in urbanized areas, where rainfall rapidly flows into the natural water resources and a fast response from the water control system is necessary to prevent flooding. To obtain the required accuracy, the radar volume must be sufficiently small to ensure that it is uniformly filled with raindrops and the radar must be designed to minimize the chance of reflection from the melting layer. This can be achieved by short-range FM-CW radars with a high spatial resolution [Ligthart and Nieuwkerk, 1990].

Weather radars are important instruments in the study of radio wave propagation. The ability of a multi-polarized Doppler radar to identify hydrometeors, combined with the ability to quantify the drop size distribution, enables it to predict the distortion of radio signals that travel through precipitation. Unlike traditional propagation measurements that can only obtain path-integrated effects, radars are able to distinguish different sources of distortion. Range-gating permits the analysis of contributions from different locations in the rain region. The radar measures backscattered radio waves, whereas propagation studies deal with forward-scattered radio waves. To relate the two, electromagnetic scattering models are used. These theoretical manipulations of radar data yield predictions of the propagation characteristics of the radio path at frequencies other than the radar frequency; radars are therefore flexible tools for propagation studies. The increasing use of radars has led to increasing research into basic scattering mechanisms by raindrops, ice crystals, and melting particles [e.g., Hume and Auchterlonie, 1989; Oguchi, 1989; De Wolf et al, 1990a, 1990c].

1.4 Radar research into precipitation at Delft University of Technology

Radar research into the troposphere was initiated at Delft University of Technology in 1983. The S-band FM-CW Delft Atmospheric Research Radar, at that time a single-
polarization Doppler radar, was used to demonstrate the capability of measuring clear air turbulence: dynamic processes within the lower part of the troposphere, the planetary boundary layer, were monitored [Ligthart and Nieuwkerk, 1987]. In the course of time, the emphasis was shifted towards the study of precipitation, mainly to characterize hydrometeors and to predict their influence on radio signals. A study of the melting layer was started in close collaboration with the European Space Agency. In 1988, research funded by the Dutch Technology Foundation resulted in a meteorological model of the melting layer that was applied to predict Doppler radar measurements [Klaassen, 1988]. Concurrently, the potential of the radar was increased by the inclusion of polarizers which enabled the measurement of copolar and cross-polar reflections. The combination of Doppler and polarization techniques created the ability to carry out detailed measurements of physical processes in precipitation [Russchenberg et al, 1990]. Delft University of Technology participated in the COST 210 Project on the influence of the atmosphere upon interference between radio communications systems at frequencies above 1 GHz. In this project, measurements made using the Delft Atmospheric Research Radar were used to characterize scattering by the melting layer [COST 210, 1991].

In 1989, the Olympus satellite was launched. This satellite is equipped with radio beacons at 12, 20 and 30 GHz, which are frequencies allocated to future telecommunication services. Coordinated by the European Space Agency, international universities are collaborating with research institutes and the telecommunication industry to collect sufficient information about the effect of precipitation on satellite communication at the Olympus frequencies. Delft University of Technology is cooperating with the Dutch PTT in a site diversity experiment; the area coverage of an X-band weather surveillance radar is combined with data of two sets of 12/30 GHz satellite receivers and a 20 GHz radiometer, one set located 10 kilometers away from the other, to optimize reception of satellite signals. The underlying consideration is that severe distortion of radio signals usually results from heavy rain storms that, however, are small in size; when the signal of one satellite receiver is severely distorted, the other receiver may still receive an acceptable signal. In cooperation with the European Space Agency and the Dutch Technology Foundation, Delft University of Technology is modeling propagation of radio signals through the melting layer, theoretically and based on knowledge obtained using the Delft Atmospheric Research Radar.
1.5 About this thesis

In 1986, a two-year project in cooperation with the European Space Agency on 20/30 GHz radar propagation studies commenced. In this project, the first combined Doppler-polarimetric radar measurements of precipitation were performed and analyzed. Initially, only off-line signal processing was possible and only parts of rain events could be measured, but hardware developments in the course of 1989 enabled on-line processing with data reduction, which enhanced the data storage capacity: rain showers could be monitored throughout their whole lifetime now. The research has since then been directed towards the interpretation of radar data in terms of physical processes in precipitation, especially with regard to the melting layer. The study included the following subjects:

- measurement techniques,
- signal processing,
- electromagnetic properties of hydrometeors,
- electromagnetic scattering theory,
- particle identification,
- the correlation between Doppler and polarimetric radar measurements,
- measurement of the dropsize distribution,
- modeling backscattering by the melting layer.

In chapter 2 of this thesis, a short outline of meteorological aspects of precipitation is given. The microstructure, which describes the statistical distributions of the size, shape, fall speed, and orientation, of hydrometeors is discussed. Chapter 2 summarizes existing knowledge; in the course of this thesis the combination of Doppler and polarization measurements is used to describe the dropsize distribution of rain more accurately, by quantifying it with three parameters, instead of the usual two.

In chapter 3, the basics of Doppler and polarization measurements are discussed. The analysis of Doppler measurements is based on the assumption of Rayleigh scattering by spherical particles. The Doppler spectrum is characterized by its spectral moments: the mean reflectivity $Z_d$, the mean velocity $V_d$, and the standard deviation $W_d$. The spectral moments are related to parameters of the microstructure of precipitation. Polarization diversity is discussed, assuming Rayleigh scattering by spheroidal particles. The influence of the particle orientation on the radar measurements is discussed. The polarization measurements are described by the horizontal reflectivity $Z_h$, the
differential reflectivity $Z_{dr}$, and the linear depolarization ratio $L_{dr}$. Finally, a special case of polarization diversity with a vertically pointing radar is treated.

In chapter 4, the Delft Atmospheric Research Radar is discussed. The FM-CW principle is explained. A system description, including the signal processing, is given. Special attention is given to the technique of clutter suppression. The radar is equipped with two unorthodox low-loss polarizers that have a special impact on the radar signal; the necessary polarimetric processing is treated in detail. The Doppler processing is carried out with standard procedures and is therefore only briefly described.

In chapter 5, physical processes in precipitation are studied using the cross-correlation of the radar observables. Using a theoretical model of the microstructure of rain, the relationship between the different radar observables is described. By randomizing the parameters of the microstructure, scatter diagrams of the radar observables are simulated and compared with a radar measurement. Based on the measurements, a simple model that relates $L_{dr}$ measurements to turbulence is developed. The height profile of the cross-correlation coefficients of the radar observables is discussed and used to propose a new model of the axial ratio of melting snowflakes.

In chapter 6, the combination of $Z_{dr}$ and $W_d$ measurements is used to calculate the dispersion factor of the dropsize distribution of rain. Usually, dual-polarized radars convert the combined $Z_h-Z_{dr}$ measurements into two parameters of the dropsize distribution of rain, by using $W_d$ as well, three parameters can be obtained. In order to achieve these three parameters, the radar signal has to be corrected for turbulence. To this end, a correction procedure is here developed. Data analysis shows that the three parameters are statistically related.

In chapter 7, backscattering by the melting layer is discussed and illustrated by measurements. Three measurements are discussed: two with a radar elevation of 30° and one with the radar pointed towards the zenith. One of the 30° measurements was carried out during very weak and the other during moderate rain. Each measurement reveals a different dynamic behavior of the radar observables of the melting layer. The zenith measurement shows that $Z_{dr}$ loses its ability to differentiate between hydrometeor types, whereas $L_{dr}$ does not. A simple model of the melting layer, which predicts the height profiles of $Z_h$, $Z_{dr}$, $L_{dr}$, and $V_d$ as functions of the rain intensity and mass density of the snowflakes, is proposed.
Chapter 2
On the structure of precipitation

Within a rain shower, hydrometeors such as snowflakes, ice crystals, and raindrops can all occur at the same time, although not necessarily at the same place. The microstructure of precipitation comprises the physical and statistical properties of the individual particles. A general model of the structure of precipitation is discussed, beginning with a description of the spatial structure of a rain shower and concluding with the microphysical properties of different types of precipitation.

2.1 The spatial structure of precipitation

In this chapter, two categories of precipitation are discussed: widespread and convective rain. The rationale for this division is twofold: they are the most frequently occurring rainfall types in the Netherlands, and radar measurements of these basic categories produce data that differ significantly. A widespread rain event usually coincides with a passing cold front and typically produces low to moderate rainfall rates. Widespread rain events can extend over several hundreds of kilometers and last for many hours. Convective rain is caused by local instabilities of the atmosphere, due to the heating and cooling of the surface of the earth. This type of rain usually covers a limited area (a few kilometers) and has a short lifetime (up to one hour). Convective rain can exhibit a strong intensity. In the Netherlands, widespread rain occurs more often than convective, although small convective cells may occur in a widespread rain area. When radar data of rain showers is applied to telecommunications or hydrology, rain cell modeling may be necessary. To this end, the shape of a rain shower is considered as a vertical structure with a circular or elliptic horizontal cross-section. The size of the rain cell is related to the rain intensity [COST 210, 1991]. A more rigorous discussion of rainfall types is given by Battan [1973] and Collier [1989].

In this thesis, most attention is given to the vertical structure of rain cells. Figure 2.1 depicts a schematic vertical cross-section of widespread rain. Usually, the air temperature decreases when the height increases. The height where it becomes 0°C is referred to as the 0°C isotherm. Above the 0°C isotherm ice particles occur and beneath
it raindrops. Sublimation of water vapor causes ice crystals to form at the top of the cloud. These ice crystals are very small and float on air. However, ice crystals may grow when the temperature is low and the humidity large enough. While they are growing, they gain speed, and during their fall they may collide. Aggregation then produces snowflakes. These falling snowflakes pass the 0°C isotherm, melt and, eventually, turn into raindrops. The layer in which melting occurs may extend over several hundreds of meters. Because of the stratified structure of the rain shower, it is often referred to as stratiform rain. In the case of widespread rain, a radar measures an enhanced reflection from the region just below the 0°C isotherm caused by melting snowflakes and for this reason the melting layer is often called the bright band. Physical processes in the bright band have been modeled by Klaassen [1988].

Convective rain is not as stratified as widespread rain, because strong up- and downdrafts distort the layered structure. Raindrops may be transported to altitudes above the 0°C isotherm, freeze and eventually turn into hailstones; different types of hydrometeors may occur at the same altitude: the radar will not observe a bright band.
2.2 The microstructure of precipitation

The microstructure of precipitation is determined by the size, shape, fall speed, orientation and composition of the individual particles. It is obvious that these characteristics are different for each type of hydrometeor: raindrops, for instance, can have a greater fall speed than snowflakes, while snowflakes can be larger than raindrops. Reflections of the radar signal from precipitation are caused by many particles in the radar beam and are therefore representative of the statistical nature of the microstructure. To enable a full understanding of the radar measurements, knowledge of the statistical properties of the microstructure is necessary. In this section the microstructure of different types of hydrometeor is discussed.

2.2.1 Rain

The fall speed of raindrops is related to their size. The experiments by Gunn and Kinzer [1949] show that in stagnant air the fall speed increases when the size increases, and eventually approaches an asymptotic value of 9.6 m/s for the large drops. Atlas et al [1973] derived a general relationship between fall speed $v$ and diameter $D$ ($D$ given in millimeters):

$$v = 9.65 - 10.3 \exp(-0.6D) \quad \text{[m/s]}$$  \hspace{1cm} 2.1

Relationship 2.1 is valid at sea level. At higher altitudes a correction factor to compensate for variations in the air pressure is required. Foote and du Toit [1969] suggest multiplying $v$ with $(\iota_0/\iota)^{0.4}$, where $\iota$ denotes the air density at the point of observation and $\iota_0$ the air density at sea level, to get the fall speed at other heights as well. Because the air density decreases when the height increases, the fall speed of raindrops is greater at higher altitudes than at sea level. Foote and du Toit [1969] report enhancements of 1% at 500 m, 13 % at 3 km and 23 % at 5 km above sea level. The falling raindrops experience air resistance. When a spherical raindrop is released and falls through a column of air, aerodynamic forces cause a flattening of the particle. To describe the shape of a raindrop mathematically, it is modeled by an ellipsoid with dimensions $a_1$, $a_2$ and $a_3$. Usually, $a_1$ is set equal to $a_2$, which results in a spheroid.
Figure 2.2 depicts the drop-shape model.

![Diagram of a drop with labeled axes](image)

Figure 2.2 The approximated shape of a raindrop.

Theoretical calculations of the shape of raindrops, based on wind tunnel measurements, have been performed by Pruppacher and Beard [1970], Pruppacher and Pitter [1971], and Beard and Chuang [1987]. The general conclusion is that large raindrops are more oblate than small ones. However, they report different quantitative relationships between oblateness and particle size. Pruppacher and Beard [1970] found a simple relationship between the axial ratio $\xi$ and the equi-volumetric radius $D_e$:

\[
\begin{align*}
\xi &= 1 & [D_e < 1.0 \text{ mm}] \\
\xi &= 1.030 - 0.062 D_e & [1.0 \leq D_e \leq 9.0 \text{ mm}]
\end{align*}
\]

2.2

$D_e$ is defined as the radius of a spherical particle with the same volume as the ellipsoid: $D_e = 2 \sqrt{a_1 a_2 a_3}$. The axial ratio $\xi$ is defined as the ratio of the shortest and longest dimension of the particle ($= a_9/a_1$, in the case of a spheroid).

Figure 2.3 gives the three drop-shape models as function of $D_e$. The Pruppacher-Beard and the Beard-Chuang model are in reasonable agreement. The Pruppacher-Pitter model deviates from the other two for large drops: it predicts raindrops that are less oblate. At dropsizes around 4 mm the Pruppacher-Pitter model predicts raindrops that are more oblate, in comparison with the other models.
Figure 2.3 The axial ratio of raindrops versus the equi-volumetric drop diameter. Three models: Pruppacher-Pitter (PP), Beard-Chuang (BC), and Pruppacher-Beard (PB).

The symmetry of the spheroid model of the drop shape only approximates reality: the bottom of a large raindrop tends to be flatter than the top. To analyze radar measurements of rain, the scattering of electromagnetic waves by raindrops has to be calculated. To make these calculations more tractable, the deviation of the raindrop shape from the spheroid is neglected.

In a column of stagnant air, raindrops will fall with their axis of symmetry ($a_3$ in figure 2.2) along the vertical. There is no reason for any tilting. However, the atmosphere is not a column of stagnant air. Local disturbances of the air density and wind variations may force the raindrop to cant. Brussaard [1976] developed a meteorological model of raindrop canting which relates the canting of an individual raindrop to the vertical variations in the horizontal wind field (known as wind shear). The model assumes a stable atmosphere; it does not deal with the influence of turbulence. The relationship between the canting angle $\delta$, which is defined relative to the vertical, and the height...
gradient $U_v$ of the horizontal wind speed is given by

$$\tan(\delta) = \frac{v U_v}{g}$$  \hspace{1cm} (2.3)

with $v$ denoting the fall speed of the raindrop, and $g$ as the gravitation constant. The gradient $U_v$ is small at altitudes above 100 m, and consequently $\delta$ is small ($< 3'$). Note that, because of the one-to-one relationship between fall speed and size, the canting angle implicitly depends on the particle size. In a stable atmosphere, no large canting angles seem to occur, but when a more realistic dynamic atmosphere is considered, one has to include the influence of turbulence as well. It is likely that, because of its statistical nature, turbulence will cause a random distribution of canting angles of an ensemble of raindrops. The exact relationship between turbulence and canting is not known, but it is often assumed to result in a Gaussian distribution of canting angles:

$$t(\delta) = \frac{1}{\sigma_\delta \sqrt{2\pi}} e^{-\frac{(\delta - \bar{\delta})^2}{2\sigma_\delta^2}}$$  \hspace{1cm} (2.4)

with $\bar{\delta}$ as the mean canting angle and $\sigma_\delta$ as the statistical spread. Hendry et al [1987] proposed a one-parameter canting-angle distribution that assumes that a fraction $\zeta$ of the raindrops is canted over an angle $\bar{\delta}$, and the remaining fraction $(1-\zeta)$ is completely randomly oriented. Although this model appears to explain their radar measurements reasonably well, the Gaussian distribution is assumed to be more realistic.

Raindrop size measurements have been, and still are, important issues in radar meteorology. The relevance of accurate knowledge of dropsize distributions is evident when remote sensing techniques are used to estimate the amount of rainfall. Marshall and Palmer [1948] derived a widely used expression from dyed filter paper measurements:

$$N(D) = N_0 e^{-\Lambda_d D}$$  \hspace{1cm} (2.5)

where $N(D)$ is the number of particles with an equivalent diameter between $D$ and
$D + dD$. $N_o$ is a constant scaling factor and equals 8000 mm$^{-1}$m$^{-3}$. $\Lambda_d$ depends on the rain intensity $R$ (in mm h$^{-1}$) as

$$\Lambda_d = 4.1 R^{-0.21} \quad \text{[mm}^{-1}]$$  

2.6

and

$$\Lambda_d = \frac{3.67}{D_o}$$  

2.7

with $D_o$ (in mm) as the diameter of the median drop volume. Marshall and Palmer's results are in close agreement with those of Laws and Parsons [1943]. Relationship 2.5 clearly simplifies reality. The negative-exponential form implies that the number of small drops is overestimated. Further, research by Waldvogel [1974] indicated that $N_o$ varies during rain showers. Ajayi and Olsen [1985] report that distributions with a negative-exponential shape are not suited to model tropical rainfall, but that a log-normal distribution should be used then. To correct for the overestimation of small drops, Ulbrich [1983] suggested representing the dropsize spectrum by a gamma distribution:

$$N(D) = N_o D^\mu e^{-\frac{3.67 + \mu}{D_o} D}$$  

2.8

with $D_o$ (as before) equal to the diameter of the median drop volume. The number of small drops is then dependent upon the value of $\mu$ then. Throughout this thesis, $D_o$ will be referred to as the median dropsize, although it is the size of the median drop volume.

Figure 2.4 shows the distribution of the drop volumes for $\mu = -1, 0, 2, 4, 6$, and $D_o = 1$ or 2 mm. The curves are calculated for the condition that the total amount of water is the same for varying $\mu$, but fixed $D_o$. The peak value of the dropsize distribution increases when $\mu$ increases and the distribution becomes narrower. For this reason, $\mu$ is referred to as the dispersion factor. The peak shifts slightly to larger values of $D$, but this shift decreases for large values of $\mu$. The distribution becomes broader when $D_o$ increases. In
In this thesis the shown range of values of $\mu$ will appear to be realistic.

![Graph showing distribution of drop volumes for different values of $\mu$; $D_o = 1$ or 2 mm.]

Figure 2.4 Distribution of drop volumes for different values of $\mu$; $D_o = 1$ or 2 mm.

2.2.2 Snow

A snowflake is a loose mixture of air and ice. The relationship between the mean mass density $< \rho_s >$ of a snowflake and its size $D_s$ [mm] is given by Klaassen [1988]:

$$< \rho_s > = \frac{0.7}{D_s} \quad [\text{gcm}^{-3}]$$

but Hobbs et al [1974] observed considerable scatter around the mean. The mass density can be as low as 0.03 gcm$^{-3}$ for large snowflakes of 25 mm. The fall speed of snowflakes depends on the air resistance (and consequently on the density of the air). Generally speaking, the fall velocity increases when the particle size increases, but the size-dependence is not as strong as in the case of rain, because the particle density acts as a
damping force. Simulations by Matsuo and Sasyo [1981] show that the fall velocity can vary between 0 and 2 m/s. When the snowflake melts, its velocity increases until it has reached the velocity of the resulting raindrop.

A snowflake consists of interconnected ice particles and it usually has a highly arbitrary shape. For the interpretation of radar measurements it is modeled as a spheroid. This may seem unrealistic, but in chapter 7 of this thesis it will appear to suffice for interpretation of the radar data; shape irregularities are relatively insignificant, because, for wavelengths much larger than the particle size, radars are not sensitive to small-scale effects: the radar 'sees' a smooth object. Little is known about the relationship between the size and the shape of snowflakes. Magono and Nakamura [1965] used dyed filter paper measurements to derive the ratio \(\xi_s\) of the shortest and longest dimension of falling snowflakes. They did not observe a trend that relates \(\xi_s\) to the size. However, their data shows that the axial ratio fluctuates around 0.9, suggesting that snowflakes are slightly oblate. This is in agreement with radar measurements, which reveal only a small polarization dependence of the radar signal. While the snowflake is melting, its shape is changing as well, and eventually it adopts the shape of a raindrop. One of the problems, concerning laboratory research of (melting) snowflakes, is that only relatively few snowflakes can be studied. Radar measurements of snowfall, however, inherently deal with a large number of particles. In chapter 7 of this thesis radar data is used to define a new shape-size relationship of (melting) snowflakes. A definition of the orientation of the irregularly shaped snowflakes is difficult to give, but when snowflakes are modeled as spheroids the canting angle of the particles can be used to account for the irregular movements of the snowflakes. This is explained in chapter 7.

Usually, the size of snowflakes is discussed in terms of the melted diameter, which is the diameter the particle would have if it were melted into a spherical raindrop. There is little experimental data on direct measurements of the size distribution of snowflakes. However, Sekhon and Srivastava [1970] found that the size distribution can also be approximated by the negative-exponential relationship, but with different expressions for \(N_o\) and \(\Lambda\):

\[
N_o = 2.5 \cdot 10^3 R^{-0.94} \quad [\text{mm}^{-1}\text{m}^{-3}] \quad 2.10
\]

\[
\Lambda = 229 R^{-0.45} \quad [\text{mm}^{-1}] \quad 2.11
\]
where $R \text{ [mmh}^{-1}]$ denotes the rain intensity that would result if all snowflakes were melted. Passarelli [1978] performed indirect measurements of the size distribution of snowflakes by using a vertically pointing Doppler radar, and concluded that the number of small particles is larger, while the number of large particles is smaller than reported by Sekhon and Srivastava. The same argument about the overestimation of small particles, as mentioned in the discussion about the dropsize distribution of rain, applies here. Klaassen [1988], therefore, used the gamma distribution with $\mu = 2$.

### 2.2.3 Ice crystals

Ice crystals can appear in a wide variety of shapes. Their behavior depends on the conditions under which they were formed; humidity and temperature are dominant factors in the formation process. Ice crystals are light and small and, consequently, convect with air. Typical ice clouds are the cirrus clouds, which appear as white, featherlike structures at high altitudes, and the related cirrocumulus and cirrostratus. However, rimed ice crystals also appear in cumulonimbus clouds at lower altitudes, as they are the beginning stage of a snowflake. In cirrus clouds, ice crystals can have a long lifetime, but in cumulonimbus clouds they may undergo a process of sublimation and aggregation, resulting in snowflakes that eventually melt into raindrops.

Magano and Lee [1966] classified the most frequently occurring shapes of ice crystals into several basic forms, varying from simple needles to irregular snow crystals. A further simplification only permits two shapes: rotational symmetric needles and discs, mathematically described as spheroids. Auer and Veal [1970] analyzed photographic data from ice crystals in clouds and derived relationships between the axial ratio $\xi_{\text{ice}}$ of the crystal and the longest dimension of the particle. In general, large crystals are more oblate than small ones, but the relationship between $\xi_{\text{ice}}$ and size differs for each crystal type:

$$
\xi_{\text{ice}} = \alpha D^\beta
$$

2.12

with $\alpha$ and $\beta$ depending on the crystal types. For instance, $\alpha = 1.099$ and $\beta = -0.389$ for needles, and $\alpha = 2.02$ and $\beta = -0.55$ for plate-like crystals [$D$ in mm]. Roughly speaking, plates occur in a temperature regime from $-25$ to $-10 \, ^\circ C$, and needles
predominate in regimes from $-10$ to $-5$ °C and below $-25$ °C [Gossard and Strauch, 1983].

There is little data on the orientation of ice crystals. However, from aero-dynamic principles it may be assumed that in stagnant air both the needles and plates will convect with wind, with their longest dimension in the horizontal plane. A stationary wind flow around the ice crystals will align the needles to some extent, and electrostatic forces inside clouds may cause even stronger alignment [Allnut, 1989]. Some evidence for these effects is given by Bostian and Allnut [1979], who observed the effect of particle alignment due to lightning bolts on satellite links.

The size of ice crystals is studied by Heymsfield and Knollenberg [1972]. They used optical-array particle-size spectrometers to measure the size distribution of ice crystals in cirrus clouds. Although different cloud types yield different size distributions, a gamma distribution with $\mu = 5$ describes their data adequately. The particle concentration may be as large as 25000 m$^{-3}$, while the mean crystal length is in the order of 0.8 mm. The density of the particle varied between 0.6 and 0.9 gcm$^{-3}$, which is much greater than the density of snow. In cumulonimbus clouds, which can produce rain showers, a much higher concentration was observed: 140000 m$^{-3}$, and the mean particle size was slightly larger: 1 mm.
The microstructure of precipitation is subject to changes: evaporation and condensation occurs, particles may collide and break, snowflakes may melt, and ice crystals may grow and aggregate into snowflakes. The changing microstructure can be expressed in terms of dropsize, particle shape, velocity, orientation angle, and number concentration. The microstructure is measured using the Doppler effect, which is induced by the movement of the hydrometeors, and by means of polarization diversity, which benefits from the non-spherical shape of hydrometeors. Doppler measurements are optimally performed when the radar is pointed towards the zenith, while polarization measurements are most effective when they are made in the horizontal plane. The combination of the two is most fruitful when the radar line of sight is at an elevation angle somewhere in between these two extremes.

3.1 Backscattering by dielectric spheres

A monostatic radar is characterized by the fact that transmitter and receiver are co-located; in that case backscattering is measured. Backscattering is commonly described by the radar cross-section $\sigma$, which is defined as the, fictional, area intercepting the amount of power that, when scattered equally in all directions, produces the power that is measured by the radar [Skolnik, 1980]. The radar cross-section should not be confused with the geometrical cross-section of the object. Modeling the radar cross-section of precipitation requires the application of electromagnetic scattering theories. First backscattering by a single particle is calculated and, secondly, the resultant reflection by an ensemble of particles is obtained by summation of the contributions of all individual particles. Scattering by homogeneous dielectric spheres has been analyzed by Mie [1908]. The Mie expression for the radar cross-section of spheres is given by

$$\sigma = \frac{\pi D^2}{\lambda^2} \left| \sum_{n=1}^{\infty} \frac{(-1)^n (2n-1)(c_n - d_n)}{n} \right|^2$$

3.1
$D$ is the particle diameter, $\chi = \frac{\pi D}{\lambda}$, with $\lambda$ as the radar wavelength, and $c_n$ and $d_n$ are coefficients that involve Bessel and Hankel functions with arguments dependent upon $\chi$ and $\epsilon_r$, the relative complex permittivity of the sphere. Deirmendjian [1969] developed iterative procedures to calculate the radar cross-section. The Mie theory is valid for a wide range of frequencies. However, implementation in computer software is tedious, because of computation-time consuming iterations and easily occurring instabilities. When the particles are small compared to the wavelength ($D \ll \lambda$ and $\chi \ll 1$), equation 3.1 reduces to the Rayleigh expression

$$\sigma = \frac{\pi^5}{\lambda^4} \left| \frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} \right|^2 D^6$$

abbreviated to

$$\sigma = \frac{\pi^5 |K|^2}{\lambda^4} D^6$$

When $\sigma$ is normalized to the geometric cross-section, it becomes

$$\sigma_{\text{norm}} = \frac{\sigma}{\pi D^2} = |K|^2 \chi^4$$

The accuracy of the Rayleigh approximation depends on the permittivity and particle size. For $|\sqrt{\epsilon_r}| \chi < 2$, the Rayleigh approximation gives results within 3% of the exact Mie solution [Ulaby et al, 1981]. The maximum value of $|\sqrt{\epsilon_r}|$ at weather radar wavelengths is approximately 9 for water [Ray, 1972] and the minimum approximates 1 in the case of very loose snow. At a radar wavelength of 10 cm, the Rayleigh approximation holds well for particles with diameters smaller than 7 mm. In most types of rain this requirement is met. Snowflakes, however, can be larger than 7 mm, but it is nature's saving grace that their smaller permittivity still permits the use of the Rayleigh-approximation. Equation 3.3 clearly expresses the fact that the radar cross-section depends on the hydrometeor type: it depends on the material and on the size of the hydrometeors involved. As an illustration: the factor $|K|^2$ equals 0.93 in the case of rain and 0.17 in the case of ice, which causes a 7.4 dB difference in backscattered power;
the mean size of ice crystals can be as large as 0.8 mm, while an average raindrop size of 1 mm is not uncommon, giving rise to 5.8 dB difference in backscattered power.

The radar cross-section of a volume $V$ filled with $N$ hydrometeors is, when interaction between the particles is neglected, given by the sum of the contributions of all individual particles:

$$\sigma_{tot} = \sum_{i=1}^{N} \sigma_i = \frac{\pi^5 |K|^2}{\lambda^4} \sum_{i=1}^{N} D_i^6$$  \hspace{1cm} (3.5)

The radar cross-section is frequency dependent. To enable comparison of measurements of backscattering from different volumes and at different frequencies, the reflectivity factor $Z$ is defined:

$$Z = \frac{\lambda^4}{\pi^5 |K|^2 \ V} \sigma_{tot} = \frac{1}{V} \sum_{i=1}^{N} D_i^6$$  \hspace{1cm} (3.6)

Usually, the unit of $Z$ is set to mm$^6$/m$^3$. When $Z$ is expressed in decibel, the unit dBZ is used, with 0 dBZ corresponding to 1 mm$^6$/m$^3$. It may not be known what type of precipitation causes the radar reflection or if the conditions for the Rayleigh approximation are fulfilled. In that case the equivalent reflectivity factor $Z_e$ is introduced [Battan, 1973]:

$$Z_e = \frac{\lambda^4}{\pi^5 |K_r|^2 \ V} \sigma_{tot}$$  \hspace{1cm} (3.7)

in which $K_r$ is the value $K$ would have if rain filled the volume.

3.2 The concept of Doppler measurements

A particle that moves into the direction of the radar with a velocity $v$ will induce a Doppler frequency shift $\Delta f$ on the radar signal [Battan, 1973]:

39
\[ \Delta f = \frac{2v}{\lambda} \]  

3.8

When there is precipitation, the velocity depends on the type of hydrometeor and on its size. Thus, when the Doppler shift is measured, information about the microstructure can be obtained. However, several complicating factors must be dealt with: a radar resolution cell is filled with \( N \) particles, which gives rise to a Doppler spectrum, and the velocity may not only be determined by the fall speed, but also by other atmospheric phenomena like wind and turbulence. It is important to realize that only the radial velocity, which is the projection of the true velocity vector on the main axis of the radar antenna, is measured. For most weather radar applications, the Doppler frequency shift is in the audio range, while the radar frequency is in the order of 3 GHz. It is obvious that a highly stable radar system is necessary to enable the accurate measurement of the Doppler shift.

Reflection by a single particle causes a signal \( \psi(t) \) at the radar receiver:

\[ \psi(t) = \alpha \psi_o \left( t - \frac{2R}{c} \right) e^{j2\pi \Delta ft} \]

3.9

with \( \alpha \) as a constant complex amplitude that is related to the radar cross-section of the particle. The incident wave \( \psi_o \) is delayed by the travel time \( \frac{2R}{c} \). The power \( P \) of \( \psi(t) \) is given by \( \frac{1}{2} |\psi(t)|^2 \). When \( N \) particles are involved, \( \psi(t) \) becomes

\[ \psi(t) = \sum_i^{N} \alpha_i \psi_{o,i} \left( t - \frac{2R_i}{c} \right) e^{j2\pi \Delta f_i t} \]

3.10

and the corresponding power:

\[ P = \frac{1}{2} \left| \sum_i^{N} \alpha_i \psi_{o,i} \left( t - \frac{2R_i}{c} \right) e^{j2\pi \Delta f_i t} \right|^2 \]

3.11

40
Equation 3.11 can be written as:

\[
P = \frac{1}{2} \left( \sum_{i}^{N} |\alpha_{i}|^2 + \Re \sum_{i}^{N} \sum_{j \neq i} \alpha_{i} \alpha_{j}^{*} \psi_{o,i}(t - \frac{2R}{c}) \psi_{o,i}^{*}(t - \frac{2R}{c}) e^{2\pi \Delta f_{ij} t} \right)
\]

where \( \Delta f_{ij} \) is the difference of the Doppler frequency of particles \( i \) and \( j \); the Doppler spectrum is coded in the second term. For large \( N \) and random velocity differences such that \( 2\Delta v/\lambda \geq 1 \), as is usually the case in precipitation and for wavelengths in the order of 10 cm, the second term becomes 0, and consequently the Doppler phases have disappeared. Thus, by measuring the power alone, the Doppler spectrum cannot be obtained. For small values of \( N \), the relative motion of the particles may cause a significant contribution of the second term, and fluctuations in the mean power may not only be caused by a changing microstructure of the precipitation, but also by relative motion of the particles. When the second term of equation 3.12 is negligible, the resulting summation of powers is called incoherent addition. When it still is significant, the summing process is called (partially) coherent addition. Note that the derivation of the radar cross-section in the previous section implicitly assumed incoherent addition.

To obtain the Doppler spectrum, the phase of the signal must be measured as well. The radar system itself should not introduce phase and amplitude fluctuations; it must be a coherent system. The formulation of the radar signal \( \psi(t) \) resembles the Fourier time-series. To get the Doppler frequency spectrum, a Fourier transform can be applied to the measured radar signal; this procedure will be discussed in more detail in chapter 4. By using equation 3.8, the Doppler frequency spectrum can be transformed into the Doppler velocity spectrum. A Doppler velocity spectrum \( S_{v}(v) \) can be regarded as a distribution of the radar cross-section over velocity:

\[
S_{v}(v) = N_{v}(v) \sigma_{v}(v)
\]

with \( N_{v}(v) \) as the number of particles with a velocity between \( v \) and \( v + dv \), with a radar cross-section of \( \sigma_{v}(v) \). \( N_{v}(v) \) is related to the size distribution of the hydrometeors. For each type of precipitation \( \sigma_{v}(v) \) can be theoretically calculated, which implies that when \( S(v) \) is measured, \( N_{v}(v) \) can be derived. The Doppler spectrum itself is not easy to use for analysis of the microstructure of precipitation: it may consist of many different
velocities, and many parameters of the microstructure may determine it, which makes curve-fitting of \( N(v) \) and \( \sigma_v(v) \) to the measured spectrum \( S(v) \) a laborious exercise. Usually, the spectrum is described by its statistical moments. Of these, the mean backscattered power \( Z_d \), the mean velocity \( V_d \) and the variance \( W_d^2 \) are used in this study:

\[
Z_d = \frac{V_{\text{max}}}{V_{\text{min}}} \int \frac{S_v(v)}{dv}
\]

\[
V_d = \frac{V_{\text{max}}}{V_{\text{min}}} \int \frac{vS_v(v)}{dv}
\]

\[
W_d^2 = \frac{V_{\text{max}}}{V_{\text{min}}} \int (v - V_d)^2 S_v(v)\ dv
\]

Figure 3.1 schematically depicts a Doppler spectrum. Also given are \( V_d \) and \( W_d \). \( Z_d \) is the area under the curve. The mean power is related to the number, size and type of hydrometeors. The mean velocity is related to the mean particle movement and is caused by the fall speed of the hydrometeors and wind. The wind influence depends on the elevation: it is only minimized when the radar is pointed towards the zenith or, more generally, when the radar axis is perpendicular to the mean wind direction. Therefore, the mean velocity is often unsuited for deriving information on the microstructure. The variance, however, is independent of the mean velocity and is, consequently, less sensitive to wind. It depends on the type of hydrometeor and the size-distribution. It also depends on turbulence, wind shear and the phase patterns of the radar antennas [Battan, 1973]. Turbulence affects the fall speed of all particles, but small particles respond to the rapid air motions more strongly than large particles do, which effectively broadens the Doppler spectrum. Wind shear causes spectral broadening: two equal particles experience a different wind speed when they are located at different heights in the radar beam, and therefore achieve a different velocity. A non-uniform phase pattern of the radar antennas induces an additional phase difference between signals from particles at different locations and consequently spectral broadening occurs. Assuming mutual independence of these four factors, Hitschfeld and Dennis [1956] showed that the total variance \( W_{d,m}^2 \) of the Doppler spectrum can be written as the sum of the variances of the individual components:
where the indices \( d, t, w, \) and \( b \) denote, respectively, precipitation, turbulence, wind shear and beam broadening. The effect of wind shear depends on the beamwidth. It can be neglected for beamwidths smaller than 1°, but for larger beamwidths it becomes significant [Battan, 1973] for reflections from targets at a distance of some tens of kilometers.

![Graph showing relative power versus velocity](image)

**Figure 3.1** A typical Doppler velocity spectrum.

### 3.3 On the relationship between precipitation and the Doppler spectrum

In stagnant air, the fall velocity and radar cross-section of raindrops depend only on the dropsize. The Doppler spectrum for Rayleigh scattering can then be expressed in terms of the dropsize \( D \) as
\[ S_D(D) = N(D) \sigma(D) V = \frac{\pi^5 |K|^2}{\lambda^4} N(D) D^6 V \] 3.18

and when expressed in terms of the reflectivity factor:

\[ Z(D) = N(D) D^6 \] 3.19

Figure 3.2 shows \( Z(D) \) in the case of a Marshall-Palmer dropsize distribution, after using the relationship between fall speed and dropsize that is given in chapter 2, section 2.2.1. The radar elevation is set to 30°. The median diameter \( D_o \) is set to 1, 1.5 and 2 mm, respectively. The influence of wind or turbulence is ignored.

![Graph showing reflectivity factor vs. velocity for different median diameters](image)

**Figure 3.2** A simulated Doppler spectrum, for different values of \( D_o \).

Three significant features appear: when the median diameter increases, the maximum value of the spectrum increases, the spectrum becomes skewed and suddenly decreases at \( v = \pm 4.8 \) m/s. When the median diameter increases, more large drops contribute to the Doppler spectrum. Since they have a large velocity and since they cause a strong
reflection, the peak value of the spectrum increases and occurs at larger velocities. The sudden drop at the end of the spectrum is caused by the asymptotic behavior of the relationship between dropsize and fall speed: the maximum possible fall speed of raindrops is approximately 9.6 m/s and, consequently, 4.8 m/s is observed with an elevation angle of 30°. The combination of maximum fall speed and increasing number of large drops causes the skewness; the spectrum becomes more skewed when the median increases towards the maximum dropsize.

![Graph showing the statistical moments of the Doppler spectrum as function of $D_o$.](image)

**Figure 3.3** The statistical moments of the Doppler spectrum as function of $D_o$.

Figure 3.3 shows the mean reflectivity, the mean velocity and the width of the Doppler spectrum as function of the median dropsize. The range of values of $D_o$ corresponds to a range of rain intensities from 0 to 50 mm/hr. The mean reflectivity and the mean velocity both increase monotonically when $D_o$ increases, but the width shows a different behavior: it increases for $D_o < 1$ mm, and decreases again for larger $D_o$. This behavior is attributable to the maximum possible fall speed of raindrops. It implies that the width alone is not sufficient to derive the median dropsize; it must be combined with other radar observables.
The Doppler spectra of other hydrometeor types are more difficult to simulate because there is no one-to-one relationship between the radar cross-section and the particle speed. In the case of snowflakes the highly stochastic mass density of the particles, which affects both the radar cross-section and the fall velocity of the particles, has to be taken into account. Klaassen [1988] simulated Doppler spectra of snow and melting snow. When these spectra are compared to those of rain, it appears that their mean velocity and width are smaller. The mean reflectivity of snow is usually of the same order as the reflectivity caused by rain. During melting \( V_d \) and \( W_d \) gradually change from those of snow, into those of rain. In chapter 7, on the melting layer, this process is discussed in more detail. Ice crystals fall only very slowly, and all of them have more or less the same speed. The Doppler spectrum of an ensemble of ice crystals will therefore be very narrow. Since ice crystals reflect the radar signal less efficiently than raindrops, the mean reflectivity will be very small.

3.4 Backscattering by a dielectric spheroid

The Doppler analysis implicitly assumes spherical hydrometeors. It is evident, however, that most types of precipitation consist of non-spherical particles. Raindrops are oblate, ice crystals oblate or prolate, and snowflakes are irregularly shaped. Backscattering by non-spherical particles depends on the polarization of the incoming electric field; by varying the polarization of the radar antennas information about the shape may be obtained. When a radar employs this polarization diversity, it improves its capability to identify hydrometeor types [Hall et al., 1984], and also a better estimate of the amount of rainfall can be given [Bringi et al., 1976]. To account for the fact that hydrometeors are not spherical, oblate or prolate spheroids are used to model their shape. In the Rayleigh regime a closed-form expression for the radar cross-section can be derived then. For high frequencies, or large particles, laborious numerical techniques must be applied. For weather radar wavelengths, however, the Rayleigh approximation still holds.

Figure 3.4 shows the geometric scattering configuration. When an electric field impinges on a small, homogeneous particle, an electric dipole is induced. The dipole moment \( \vec{p} \) is related to the incident field \( \vec{E}_i \) via the polarizibility tensor \( \vec{\gamma} \):
\[ \bar{p} = \bar{n} \hat{E}_i \]

\[ \bar{n} = V \varepsilon_0 (\varepsilon_r - 1) \bar{\tau} \]

with \( V \) as the volume of the particle, \( \varepsilon_r \) as the relative permittivity of the particle, and \( \bar{\tau} \) as the shape dyadic, which is a tensor that expresses the influence of the particle shape and geometry on the backscattering process.

Figure 3.4  The scattering configuration. The principal axes \( \hat{u}_i \) of the spheroid coincide with \( \hat{z}_i \). \( \hat{k}_o \) is the propagation vector of the radar wave. \( \hat{e}_e \) and \( \hat{e}_h \) are the polarization vectors of the incident field. \( \phi \) is the polarization, and \( \theta \) the elevation angle of the radar. \( \hat{k}_s \) is the propagation vector of the scattered radar wave.

When the principal axes \( \hat{u}_i \) of the spheroid coincide with the \( \hat{z}_i \) of the coordinate system, the shape dyadic is related to the dimensions of a spheroid through [De Wolf et al, 1990a]:
\[ \tilde{r} = \sum_{i=1}^{3} \Lambda_i \tilde{u}_i \tilde{u}_i = \sum_{i=1}^{3} \Lambda_i \tilde{v}_i \tilde{v}_i = \begin{bmatrix} \Lambda_1 & 0 & 0 \\ 0 & \Lambda_2 & 0 \\ 0 & 0 & \Lambda_3 \end{bmatrix} \]

\[ \Lambda_i = \frac{1}{1 + \lambda_i (\varepsilon_r - 1)} \]

\[ \lambda_i = \frac{1}{2} a_1 a_2 a_3 \int_0^\infty dt \frac{1}{(t + a_1^2) \sqrt{(t + a_2^2)(t + a_3^2)}} \]

\[ \lambda_1 + \lambda_2 + \lambda_3 = 1 \]

with \( a_i \) as the principal spheroid radii, and \( \lambda_i \) as depolarization factors. For spheroids \( a_1 = a_2, \lambda_3 = 1 - 2\lambda_1 = 1 - 2\lambda_2, \) and \( \Lambda_1 = \Lambda_2. \) Using the inverse axial ratio \( \beta = 1/\xi = a_1/a_3, \) the integral can be evaluated analytically [De Wolf et al, 1990b]:

\[ \lambda_3 = \frac{1 - \varepsilon^2}{\varepsilon^2} \left( -1 + \frac{1}{2} \ln \frac{1 + \varepsilon}{1 - \varepsilon} \right) \]

\[ \varepsilon^2 = 1 - \beta^2; \quad 0 < \beta < 1 \] (prolate spheroids)

\[ \lambda_3 = \frac{1 + f^2}{f^2} \left( 1 - \frac{1}{f} \arctan f \right) \]

\[ f^2 = \beta^2 - 1; \quad \beta > 1 \] (oblate spheroids)

<table>
<thead>
<tr>
<th></th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \Lambda_1 )</th>
<th>( \Lambda_2 )</th>
<th>( \Lambda_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Needles</strong> ( (a_3 \to \infty) )</td>
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<td>( \frac{1}{2} )</td>
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<td>( \frac{2}{1 + \varepsilon_r} )</td>
<td>( \frac{2}{1 + \varepsilon_r} )</td>
<td>1</td>
</tr>
<tr>
<td><strong>Plates</strong> ( (a_3 \to 0) )</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( \frac{1}{\varepsilon_r} )</td>
</tr>
<tr>
<td><strong>Spheres</strong> ( (a_1 = a_3) )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{3}{2 + \varepsilon_r} )</td>
<td>( \frac{3}{2 + \varepsilon_r} )</td>
<td>( \frac{3}{2 + \varepsilon_r} )</td>
</tr>
</tbody>
</table>

Table 3.1 Several representative \( \lambda \) and \( \Lambda \) factors for spheroids
Table 3.1 specifies $\lambda_i$ and $\Lambda_i$ in the case of scattering by a sphere, a needle or plate. The scattered field $\vec{E}_s$ is given by [Stratton, 1941]:

$$\vec{E}_s = \frac{k_o^2}{4\pi\varepsilon_o r} \left( \vec{p} - (\vec{p} \cdot \hat{k}_o) \hat{k}_o \right) e^{-jk_o r} \tag{3.28}$$

in which $k_o$ is the wave number and $\varepsilon_o$ the permittivity of vacuum. The distance dependence is denoted by $r$. The dipole moment $\vec{p}$ is assumed to follow the incoming field $\vec{E}_i$ instantaneously, which implies that its direction is perpendicular to $\hat{k}_o$. Consequently, the backscattered field becomes $(\hat{k}_s = -\hat{k}_o)$:

$$\vec{E}_s = \frac{k_o^2}{4\pi\varepsilon_o r} \vec{p} e^{-jk_o r} \tag{3.29}$$

which, when the axes $\hat{u}_i$ of the particle are assumed to coincide with the coordinate system axes $\hat{x}_i$, can be written as

$$\vec{E}_s = \frac{k_o^2}{4\pi r} V(\varepsilon_r - 1) \vec{E}_i e^{-jk_o r} = \frac{k_o^2}{4\pi r} V(\varepsilon_r - 1) \vec{\hat{r}}_i |\vec{E}_i| e^{-jk_o r} \tag{3.30}$$

To calculate the complex received voltage $U_r$, the polarization vector $\vec{e}_r$ of the receive antennas must be taken into account:

$$U_r = \vec{e}_r \cdot \vec{E}_s = \frac{k_o^2}{4\pi r_I} V(\varepsilon_r - 1) |\vec{E}_i| \vec{e}_r \cdot \vec{\hat{r}}_i e^{-jk_o r_I} \tag{3.31}$$

in which $r_I$ denotes the total distance the wave has traveled. The flux $\Upsilon$ of an electric field is proportional to $\frac{1}{2} \vec{E} \cdot \vec{E}^* = \frac{1}{2} |U_r|^2$. Since the radar cross-section is formally defined as [Skolnik, 1980]:

$$\sigma = 4\pi r^2 \frac{\Upsilon_r}{\Upsilon_i} \tag{3.32}$$
the radar cross-section of the spheroid becomes:

\[ \sigma_{ri} = \frac{k_0^4}{4\pi} V^2 (\varepsilon_r - 1)^2 \left| \hat{e}_r \cdot \vec{\pi} \hat{e}_i \right|^2 = \frac{k_0^4}{4\pi} V^2 (\varepsilon_r - 1)^2 q_{ri} \quad (3.33) \]

In the case of backscattering by a sphere, \( \vec{\pi} \) reduces to \( \Lambda \sum_i \hat{u}_i \hat{u}_i \) with \( \Lambda = \frac{3}{\varepsilon_r + 2} \). When \( \hat{e}_i = \hat{e}_r \), evaluation of the radar cross-section results in

\[ \sigma = \frac{\pi^5}{\lambda^4} \left| \frac{(\varepsilon_r - 1)}{\varepsilon_r + 2} \right|^2 D^6 \quad (3.34) \]

which is the (earlier used) expression for Rayleigh scattering by spheres.

Three measurement schemes are considered:

1- transmitter and receiver horizontally polarized: \( \hat{e}_r = \hat{e}_i = \hat{e}_h \),
2- transmitter and receiver vertically polarized: \( \hat{e}_r = \hat{e}_i = \hat{e}_v \),
3- transmitter horizontally, receiver vertically polarized: \( \hat{e}_r = \hat{e}_v \wedge \hat{e}_i = \hat{e}_h \).

Cases 1 and 2 are copolar measurements. Case 3 is a cross-polar measurement. The horizontal and vertical field vectors \( \hat{e}_h \) and \( \hat{e}_v \) can be written as

\[ \hat{e}_h = -\hat{z}_1 \sin \theta \sin \phi + \hat{z}_2 \cos \phi - \hat{z}_3 \cos \theta \sin \phi \quad (3.35) \]
\[ \hat{e}_v = \hat{z}_1 \sin \theta \cos \phi + \hat{z}_2 \sin \phi + \hat{z}_3 \cos \theta \cos \phi \quad (3.36) \]

The horizontal and vertical radar cross-sections, \( \sigma_{hh} \) and \( \sigma_{vv} \), are copolarly measured with respectively \( \hat{e}_r = \hat{e}_i = \hat{e}_h \) and \( \hat{e}_r = \hat{e}_i = \hat{e}_v \). The cross-polar radar cross-section \( \sigma_{hv} \) is measured with \( \hat{e}_r = \hat{e}_h \) and \( \hat{e}_i = \hat{e}_v \). They are given by:

\[ \sigma_{hh} = \frac{k_0^4}{4\pi} |\varepsilon_r - 1|^2 q_{hh} = \frac{\pi^5}{9\lambda^4} |\varepsilon_r - 1|^2 q_{hv} \quad (3.37) \]
\[ \sigma_{vv} = \frac{k_0^4 V^2}{4\pi} |e_r - 1|^2 q_{vv} = \frac{\pi^5 D_e^6}{9\lambda^4} |e_r - 1|^2 q_{vv} \]  
3.38

\[ \sigma_{hv} = \frac{k_0^4 V^2}{4\pi} |e_r - 1|^2 q_{hv} = \frac{\pi^5 D_e^6}{9\lambda^4} |e_r - 1|^2 q_{hv} \]  
3.39

with \( D_e \) expressing the equivalent drop diameter, and \( q_{hh} \), \( q_{vv} \) and \( q_{hv} \) for vertically oriented spheroids equal to

\[ q_{hh} = \left( (\Lambda_3 - \Lambda_1) \cos^2 \theta \sin^2 \phi + \Lambda_1 \right)^2 \]  
3.40

\[ q_{vv} = \left( (\Lambda_3 - \Lambda_1) \cos^2 \theta \cos^2 \phi + \Lambda_1 \right)^2 \]  
3.41

\[ q_{hv} = \left( \frac{1}{2} \sin 2\phi \cos^2 \theta (\Lambda_3 - \Lambda_1) \right)^2 \]  
3.42

When \( \phi = 0 \) or \( \phi = 90 \), no cross-polarization is measured, irrespective of the elevation angle. When \( \theta = 90 \), the radar is pointed towards the zenith. The cross-section of the spheroid in the plane of incidence, which is spanned by \( \hat{e}_h \) and \( \hat{e}_v \), is then a circle. Note that in this case \( q_{hh} = q_{vv} = \Lambda^2_1 \) and \( q_{hv} = 0 \). The radar will not observe any polarization dependence.

So far, the particle was assumed to have it principal axes along the axes \( \hat{z}_i \). To account for the orientation of the particle, the azimuth and elevation angle, \( \alpha \) and \( \delta \) respectively, are introduced (see figure 3.5). The axes \( \hat{u}_i \) of the particle can then be expressed in the \( \hat{z}_i \) as:

\[ \hat{u}_1 = \hat{z}_1 \cos \delta \cos \alpha - \hat{z}_2 \cos \delta \sin \alpha + \hat{z}_3 \sin \delta \]  
3.43

\[ \hat{u}_2 = \hat{z}_1 \sin \alpha + \hat{z}_2 \cos \alpha \]  
3.44

\[ \hat{u}_3 = -\hat{z}_1 \sin \delta \cos \alpha + \hat{z}_2 \sin \delta \sin \alpha + \hat{z}_3 \cos \delta \]  
3.45
Figure 3.5 The scattering configuration with particle canting. $\delta$ is the elevation angle of the particle, $\alpha$ is the azimuth angle of the particle. $\hat{u}_i$ are the principal axes of the spheroid. $\hat{k}_o$, $\hat{k}_s$, $\hat{e}_n$, and $\hat{e}_h$ are the same as those shown in figure 3.4.

The new tensor $\tilde{\tau}_x$ in the coordinate system $\hat{x}_i$ is given by:

$$\tilde{\tau}_x = T^t \tilde{\tau} T$$

with

$$T = \begin{bmatrix}
\cos \delta \cos \alpha & -\cos \delta \sin \alpha & -\sin \delta \\
\sin \alpha & \cos \alpha & 0 \\
-\sin \delta \cos \alpha & \sin \delta \sin \alpha & \cos \delta
\end{bmatrix}$$

which yields
\[ \vec{r}_x = \begin{bmatrix} A & B & C \\ B & D & E \\ C & E & F \end{bmatrix} \]

with elements

\[ A = (\Lambda_3 - \Lambda_1) \sin^2 \delta \cos^2 \alpha + \Lambda_1 \]

\[ B = \frac{1}{2} \sin 2\alpha \sin^2 \delta \left( \Lambda_1 - \Lambda_3 \right) \]

\[ C = \frac{1}{2} \sin 2\delta \cos \alpha \left( \Lambda_1 - \Lambda_3 \right) \]

\[ D = (\Lambda_3 - \Lambda_1) \sin^2 \delta \sin^2 \alpha + \Lambda_1 \]

\[ E = -\frac{1}{2} \sin 2\delta \sin \alpha \left( \Lambda_1 - \Lambda_3 \right) \]

\[ F = (\Lambda_3 - \Lambda_1) \cos^2 \delta + \Lambda_1 \]

When \( \delta = 0 \), \( \vec{r}_x \) reduces to \( \vec{r} \), irrespective of the value of \( \alpha \). The expressions for the \( q_{ri} \) are somewhat more complicated now,

\[ q_{hh} = (\Lambda_1 + (\Lambda_3 - \Lambda_1) \cdot \Phi_{hh})^2 \]

\[ q_{vv} = (\Lambda_1 + (\Lambda_3 - \Lambda_1) \cdot \Phi_{vv})^2 \]

\[ q_{hv} = ((\Lambda_3 - \Lambda_1) \cdot \Phi_{hv})^2 \]

with

\[ \Phi_{hh} = \sin^2 \delta \cos^2 \alpha \sin^2 \phi \sin^2 \theta + \sin^2 \delta \sin^2 \alpha \cos^2 \phi + \cos^2 \delta \sin^2 \phi \cos^2 \theta - \frac{1}{2} \sin 2\delta \cos \alpha \sin^2 \phi \sin 2\theta - \frac{1}{2} \sin 2\delta \sin \alpha \sin 2\phi \cos \theta + \frac{1}{2} \sin^2 \delta \sin 2\alpha \sin 2\phi \sin \theta \]
\[ \Phi_{vv} = \sin^2 \delta \cos^2 \alpha \cos^2 \phi \sin^2 \theta + \sin^2 \delta \sin^2 \alpha \sin^2 \phi + \cos^2 \delta \cos^2 \phi \cos^2 \theta - \frac{1}{2} \sin 2\delta \cos \alpha \cos^2 \phi \sin 2\theta + \frac{1}{2} \sin 2\delta \sin \alpha \sin 2\phi \cos \theta - \frac{1}{2} \sin^2 \delta \sin 2\alpha \sin 2\phi \sin \theta \]  
3.59

\[ \Phi_{hv} = \frac{1}{2} \sin 2\phi \left( \sin^2 \delta \sin^2 \alpha - \sin^2 \delta \cos^2 \alpha \sin^2 \theta - \cos^2 \theta + \frac{1}{2} \sin 2\delta \cos \alpha \sin 2\theta \right) - \frac{1}{2} \cos 2\phi \left( \sin 2\alpha \sin^2 \delta \sin \theta + \frac{1}{2} \sin 2\delta \sin \alpha \cos \theta \right) \]  
3.60

but they result in the same \( q_{ri} \) as before when \( \delta = 0 \) and \( \alpha = 0 \). When again a vertically pointing radar is considered, \( \theta = 90 \), and for clarity \( \phi = 0 \) as well, the \( q_{ri} \) become

\[ q_{hh} = \left( \Lambda_3 - \Lambda_1 \right) \sin^2 \delta \sin^2 \alpha + \Lambda_1 \right)^2 \]  
3.61

\[ q_{vv} = \left( \Lambda_3 - \Lambda_1 \right) \sin^2 \delta \cos^2 \alpha + \Lambda_1 \right)^2 \]  
3.62

\[ q_{hv} = \left( \frac{1}{2} \sin 2\alpha \sin^2 \delta \left( \Lambda_3 - \Lambda_1 \right) \right)^2 \]  
3.63

When these expressions of the radar cross-sections are compared to those for the case without canting, it clearly appears that \( \alpha \) and \( \delta \) are angles similar to \( \phi \) and \( 90 - \theta \), respectively. When the particles are cantied with respect to the vertical, the radar still receives a polarization dependent signal.

### 3.5 The scattering matrix

The scattering process is often described by means of the scattering matrix \( S \), which relates the polarization vectors of \( \vec{E}_s \) and \( \vec{E}_r \) as

\[ \begin{pmatrix} \vec{e}_{sh} \\ \vec{e}_{sv} \end{pmatrix} = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix} \begin{pmatrix} \vec{e}_{sh} \\ \vec{e}_{sv} \end{pmatrix} e^{-jk_or} \]  
3.64

The scattering matrix is defined to describe the polarization dependence of the scattered field in the coordinate system of the plane of incidence of the radar waves, instead of the Cartesian system \( \vec{z}_i \). The relationship with Cartesian coordinate system is implicit in the elements of the matrix. The scattering matrix consists of complex scattering
parameters that each represent one scatter-mode. \( S_{hh} \) and \( S_{vv} \) represent the complex amplitudes of the copolar scattered fields with respectively horizontal and vertical polarization. \( S_{hv} \) is the complex amplitude of the cross-polarly scattered field. Because of antenna-reciprocity \( S_{th} \) equals \( S_{hv} \). The complex received voltage \( U_r \) is equal to

\[
U_r = \hat{e}_r \cdot \hat{e}_s = \left( \frac{\hat{e}_{rh}}{\hat{e}_{rv}} \right) \left( \begin{array}{cc} S_{hh} & S_{hv} \\ S_{hv} & S_{vv} \end{array} \right) \left( \begin{array}{c} \hat{e}_{ih} \\ \hat{e}_{iv} \end{array} \right) e^{-jk_0 r_t} r_t
\]

When the three earlier mentioned measurement schemes are combined with equation 3.31, the polarization dependent voltages result:

\[
U_{hh} = S_{hh} \frac{e^{-jk_0 r_t}}{r_t} = \frac{k_0^2}{4\pi} V(\epsilon_r - 1) \hat{e}_{rh} \cdot \hat{e}_h e^{-jk_0 r_t} r_t
\]

\[
U_{hv} = S_{hv} \frac{e^{-jk_0 r_t}}{r_t} = \frac{k_0^2}{4\pi} V(\epsilon_r - 1) \hat{e}_{rv} \cdot \hat{e}_h e^{-jk_0 r_t} r_t
\]

\[
U_{vv} = S_{vv} \frac{e^{-jk_0 r_t}}{r_t} = \frac{k_0^2}{4\pi} V(\epsilon_r - 1) \hat{e}_{iv} \cdot \hat{e}_h e^{-jk_0 r_t} r_t
\]

The amplitude of the incident field is set to 1. The radar cross-section is related to the scattering parameters by

\[
\sigma_r = 4\pi |S_{ri}|^2
\]

The scattering matrix can be written as

\[
S = \frac{k_0^2}{4\pi} V(\epsilon_r - 1) \left[ \begin{array}{ccc} \hat{e}_{rh} \cdot \hat{e}_h & \hat{e}_{rv} \cdot \hat{e}_h \\ \hat{e}_{rv} \cdot \hat{e}_h & \hat{e}_{iv} \cdot \hat{e}_h \end{array} \right]
\]

Initially, the scattering matrix is defined with respect to a basis of linearly polarized \( \hat{e} \)-vectors, but basis transformations can be applied to include elliptic or circular polarization [Huymen, 1970].

55
3.6 Backscattering by an ensemble of spheroids

In the previous section, only backscattering by a single spheroid was discussed. Precipitation, however, involves backscattering by many particles, and consequently the radar signal contains contributions from all individual particles. Each particle may differ in size, shape and orientation, thus when the radar cross-sections of all individual particles are integrated, probability density functions of these particle properties are required. When interaction between the particles is ignored, the radar cross-section plants becomes

\[
\sigma_{ri}^{tot} = \int \sigma_{ri}(D_e, \delta, \alpha, \Lambda_3, \Lambda_1) p(D_e, \delta, \alpha, \Lambda_3, \Lambda_1) \, dD_e \, d\delta \, d\alpha \, d\Lambda_3 \, d\Lambda_1
\]  

where \( p(D_e, \delta, \alpha, \Lambda_3, \Lambda_1) \) denotes the joint probability density function of particle size, orientation and shape. Strictly speaking, the particle properties are not statistically independent, but usually the particle orientation is assumed to be uncorrelated to size and shape. The azimuth and elevation angle of the particle are assumed to be uncorrelated as well. In the case of rain, size and shape are strongly correlated; the size distribution implies the shape distribution. The probability density function now becomes

\[
p(D_e, \delta, \alpha, \Lambda_3) = p_D(D_e) \, p_\delta(\delta) \, p_\alpha(\alpha)
\]  

and the corresponding radar cross-section

\[
\sigma_{ri}^{tot} = \int \sigma_{ri}(D_e, \delta, \alpha) \, p_D(D_e) \, p_\delta(\delta) \, p_\alpha(\alpha) \, dD_e \, d\delta \, d\alpha
\]  

The probability density function of the size is given by the dropsize distribution as introduced in chapter 2. To enhance readability, \( \alpha \) is set to \( \pi/2 \) and \( p_\alpha(\alpha) = \delta(\alpha - \pi/2) \), the delta-function; the particles are canted in the plane spanned by \( \hat{z}_2 \) and \( \hat{z}_3 \). When \( \alpha \) is set to a fixed value, \( \delta \) is also referred to as the canting angle.
Polarimetric radar measurements of precipitation are expressed by several radar observables. The most familiar is the horizontal reflectivity $Z_h$, which is copolarly measured using horizontal polarization. Seliga and Bringi [1976] proposed the use of the differential reflectivity $Z_{dr}$:

$$Z_{dr} = \frac{Z_h}{Z_v}$$

3.74

to estimate the median dropsize of rain. Further evaluation of $Z_{dr}$ gives:

$$Z_{dr} = \frac{\int \sigma_{hh}(D_e, \delta) p_P(D_e) p_\delta(\delta) \, dD_e \, d\delta}{\int \sigma_{vv}(D_e, \delta) p_P(D_e) p_\delta(\delta) \, dD_e \, d\delta}$$

3.75

and recalling that the dropsize distribution is modeled with a gamma distribution, $Z_{dr}$ becomes

$$Z_{dr} = \frac{\int \sigma_{hh}(D_e, \delta) D_e^{\mu} e^{-\frac{3.67 + \mu D_e}{D_o}} p_\delta(\delta) \, dD_e \, d\delta}{\int \sigma_{vv}(D_e, \delta) D_e^{\mu} e^{-\frac{3.67 + \mu D_e}{D_o}} p_\delta(\delta) \, dD_e \, d\delta}$$

3.76

The radar cross-section $\sigma_{r_1}(D_e, \delta)$ of raindrops is calculated using the Rayleigh theory. When the distribution of canting angles is assumed to be known and the dispersion factor $\mu$ is set to a fixed value, $Z_{dr}$ only depends on the median dropsize $D_o$. After $D_o$ is derived, $Z_h$ is used to obtain the scaling factor $N_o$ of the dropsize distribution, since

$$Z_h = N_o \int \sigma_{hh}(D_e, \delta) D_e^{\mu} e^{-\frac{3.67 + \mu D_e}{D_o}} p_\delta(\delta) \, dD_e \, d\delta$$

3.77

Apart from its use to quantify the dropsize distribution, $Z_{dr}$ is also suitable for identifying hydrometeor types [Hall et al, 1984], because it depends on the shape of the particle.
Figure 3.6  The differential reflectivity $Z_{dr}$ and the linear depolarization ratio $L_{dr}$ as a function of the median dropsize $D_o$ for different values of $\sigma_\delta$. 

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The cross-polarized radar return is used to define the linear depolarization ratio $L_{dr}$:

$$L_{dr} = \frac{\sigma_{hv}(D_e, \delta) \, p_D(D_e) \, p_\delta(\delta) \, dD_e \, d\delta}{\sigma_{hh}(D_e, \delta) \, p_D(D_e) \, p_\delta(\delta) \, dD_e \, d\delta}$$  \hspace{1cm} 3.78

$L_{dr}$ is sensitive to particle shape and to the orientation of the particle: a spherical particle will not produce a cross-polar radar return and a spheroidal particle has to be canted in order to reflect a cross-polar signal. To illustrate the effect of dropsize and orientation, $Z_{dr}$ and $L_{dr}$ are plotted in figure 3.6 as function of the median dropsize $D_o$ of rain for different values of $\sigma_\delta$, the spread of the Gaussian canting-angle distribution. The mean canting angle equals 0°, the radar elevation 30°, the dispersion factor 0, and the maximum diameter 8 mm. The Pruppacher-Pitter drop shapes are used.

$L_{dr}$ and $Z_{dr}$ both increase when $D_o$ increases. When $\sigma_\delta$ increases, however, $Z_{dr}$ decreases and $L_{dr}$ increases. For large $D_o$, $L_{dr}$ mainly reacts to canting, while for small $D_o$ particle sizes are important as well. $Z_{dr}$ becomes less dependent on $D_o$ when $\sigma_\delta$ increases. The sensitivity of $Z_h$, $Z_{dr}$, and $L_{dr}$ to changes of the microstructure of precipitation is discussed in chapter 5.

### 3.7 Polarization measurements with a vertically pointing radar

Polarization diversity benefits from the apparent particle shape, which is the cross-section of the particle shape in the plane of incidence: when the radar is pointed towards the zenith, the apparent shape of the hydrometeors will approximate a circle. The differential reflectivity and linear depolarization ratio of a single particle are then

$$Z_{dr} = \frac{q_{hh}}{q_{vv}} = \frac{\left(\Lambda_3 - \Lambda_1\right) \sin^2 \delta \sin^2 \alpha + \Lambda_1}{\left(\Lambda_3 - \Lambda_1\right) \sin^2 \delta \cos^2 \alpha + \Lambda_1}$$  \hspace{1cm} 3.79

$$L_{dr} = \frac{q_{hv}}{q_{hh}} = \frac{\left(\frac{1}{2} \sin 2 \alpha \sin^2 \delta \left(\Lambda_3 - \Lambda_1\right)\right)^2}{\left(\Lambda_3 - \Lambda_1\right) \sin^2 \delta \sin^2 \alpha + \Lambda_1}$$  \hspace{1cm} 3.80

59
When $\delta = 0$, then $Z_{dr} = 1$ and $L_{dr} = 0$, irrespective of the azimuth angle $\alpha$. When $\alpha = 0$ or $\alpha = 90$, then $L_{dr} = 0$, irrespective of the canting angle $\delta$. $Z_{dr}$, however, may deviate from 1, because the canting angle $\delta$ causes an elliptic apparent shape.

When there is backscattering by an ensemble of particles, appropriate averaging of the orientation angles and shape factors occurs. When $\alpha$ and $\delta$ are uncorrelated and $\alpha$ is uniformly distributed between 0 and $2\pi$, then $Z_{dr}$ and $L_{dr}$ become

$$Z_{dr} = 1, \text{ for all } \delta$$

$$L_{dr} = \frac{\frac{1}{2} \langle \sin^2 2\alpha \rangle \langle \sin^4 \delta \rangle \langle (\Lambda_1 - \Lambda_3)^2 \rangle}{\langle \Lambda_1 \rangle^2 + \langle (\Lambda_3 - \Lambda_1)^2 \rangle \langle \sin^4 \alpha \rangle \langle \sin^4 \delta \rangle + 2 \langle \Lambda_1 (\Lambda_3 - \Lambda_1) \rangle \langle \sin^2 \alpha \rangle \langle \sin^2 \delta \rangle}$$

The probability-density function $p_r(\mathbf{r})$ of the orientation vector of the spheroid can be written as $\sin(\delta) p_{\alpha \delta}(\alpha, \delta) = \sin(\delta) p_{\alpha}(\alpha) \cdot p_{\delta}(\delta)$ and hence

$$\int_{\alpha=0}^{2\pi} \int_{\delta=0}^{\pi} \sin \delta p_{\alpha}(\alpha) p_{\delta}(\delta) \, d\alpha \, d\delta = 1$$

which implies that

$$p_{\delta}(\delta) = \frac{1}{1 - \cos \sigma_\delta}$$

The appropriate averages now become

$$\langle \sin^2 \alpha \rangle = \langle \sin^2 2\alpha \rangle = \langle \cos^2 \alpha \rangle = \frac{1}{2}$$

$$\langle \sin^4 \alpha \rangle = \langle \cos^4 \alpha \rangle = \frac{3}{8}$$

$$\langle \sin^2 \delta \rangle = \frac{1}{1 - \cos \sigma_\delta} \left( \frac{2}{3} - \frac{3}{4} \cos \sigma_\delta + \frac{1}{12} \cos 3\sigma_\delta \right)$$
\[
<\sin^4 \delta> = \frac{1}{1 - \cos \sigma_\delta} \left( \frac{8}{15} \cos \sigma_\delta + \frac{5}{48} \cos 3\sigma_\delta - \frac{1}{80} \cos 5\sigma_\delta \right)
\]

3.88

In chapter 7, on the melting layer, these expressions are used to simulate radar measurements of the bright band. When \( \sigma_\delta = \pi \), implying a completely random distribution of the orientation, \( L_{dr} \) becomes

\[
L_{dr} = \frac{<(\Lambda_3 - \Lambda_1)^2>}{8 <\Lambda_1^2> + 4 <\Lambda_1 \Lambda_3> + 3 <\Lambda_3^2>}
\]

3.89

**Figure 3.7** \( Z_{dr} \) and \( L_{dr} \) as function of the radar elevation angle \( \theta \).

Figure 3.7 shows \( Z_{dr} \) and \( L_{dr} \) for backscattering by a collection of raindrops as function of the elevation angle \( \theta \). Each raindrop is assumed to have an axial ratio of 0.7. The elevation angle \( \delta \) of the raindrops is uniformly distributed between 0 and 40°. The azimuth angle is uniformly distributed between 0 and 2\( \pi \). It clearly shows that \( Z_{dr} \) becomes 0 dB when the radar is pointed towards the zenith, while \( L_{dr} \), although it becomes smaller, can still be measured.
3.8 The combination of Doppler and polarization measurements

Both the Doppler and polarization measurements depend on the microstructure of precipitation. However, their optimal modes of operation differ: Doppler analysis is most efficient when measurements are performed with the radar pointed towards the zenith, while polarization measurements produce useful data optimally when the radar is pointed towards the horizon. To combine the Doppler and the polarization measurements, the radar should have an elevation angle somewhere in between. Although both the Doppler and the polarization measurements lose some sensitivity then, the combination offers more possibilities than the two apart. It may improve the particle identification procedures, because not only is shape information gathered, but also the velocity of the hydrometeors. The dropsize distribution can be estimated more accurately, because fixing the dispersion factor $\mu$, which is done in the case $Z_{dr}$ is used to estimate the mean dropsize, is not necessary when the Doppler measurements are used as well. This option is described in more detail in chapter 6. Another application of combined Doppler and polarization measurements is the study of physical processes within precipitation. For instance, turbulence-induced particle canting can be observed using the width of the Doppler spectrum and the linear depolarization ratio. These topics are discussed in this thesis when appropriate.
Chapter 4
The Delft Atmospheric Research Radar

The Delft Atmospheric Research Radar DARR has been used for precipitation studies since 1983. It is an S-band multi-parameter radar in that it is able to measure the scattering matrix and the Doppler spectrum simultaneously. Unlike most other weather radars, DARR operates according to the FM-CW principle, which permits the use of low transmit power and yet achieves a sensitivity comparable to, or higher than, the traditional pulse radar. DARR is equipped with polarizers that can change the polarization angle of the radar antennas. The FM-CW principle and the use of polarizers require dedicated signal processing in order to be able to perform real time measurements.

4.1 The FM-CW principle

An FM-CW radar transmits a continuous wave, the frequency of which is modulated in time. When the wave is reflected by a target, the radar measures the returned signal some time later. During this time lapse, the frequency of the transmit signal changes, implying that at the moment of reception the received and the transmitted signal have different frequencies. The frequency difference is called the beat frequency. As the beat frequency depends on the travel time of the wave, it also depends on the distance of the target.

The FM-CW principle is illustrated in figure 4.1. The frequencies of the transmit and receive signal during one sweep are depicted in figure 4.1a. The frequency of the signals increases linearly in time. The received signal, caused by reflection at a single stationary target at a distance $r_d$ from the radar, is delayed by $\Delta t$. When the transmitted and received RF signals are supplied to a mixer and low-pass filter, a sinusoidal beat signal with frequency $f_b$ is obtained; this beat frequency is proportional to $r_d$. The frequency of the beat signal, and hence the distance of the target, is derived by means of a Fourier transform. When there is precipitation, the beat signal contains a wide spectrum of beat frequencies. Application of the Fourier transform then results in a beat frequency spectrum, which is equivalent to a range spectrum. Figure 4.1c shows the frequency
spectrum, due to reflection from a single target. The sinc-like envelope is a product of the finite time frame that is used for the Fourier transform. Thus, a single target initiates a spectrum of frequencies, which erroneously may be interpreted as originating from a number of targets at different distances. Usually, the beat signal is digitized and a Fast Fourier Transform (FFT) is used to extract the beat frequency. In this case the cross-talk only appears when the beat signal shows a discontinuity in the time frame that is used to calculate the discrete Fourier transform [Brigham, 1974].

Figure 4.1 Principle of the FM-CW radar, using a linear frequency modulation. The solid line in figure a represents the frequency of the transmitted signal, the broken line the received signal.

Only a finite number of discrete beat frequencies can be calculated. Consequently, the total range of the radar is divided into distinct range cells. When an FFT is applied to a real signal, a symmetrical spectrum is obtained; half of the spectrum suffices to get the range information. When the digitized beat signal contains $N$ samples, $N/2$ range cells
remain. With reference to figure 4.1, the relationship between the beat frequency and the time lapse is given by

\[ f_b = \frac{F}{f_s} \Delta t \]  \hspace{1cm} 4.1

with \( F \) as the frequency excursion and \( T_s \) as the sweep time. The distance \( r_d \) of the target is related to the beat frequency as

\[ r_d = \frac{c \Delta t}{2} = \frac{c T_s}{2 F} f_b \]  \hspace{1cm} 4.2

with \( c \) denoting the speed of light. The maximum distinguishable beat frequency \( f_{max} \) is obtained from the Nyquist criterion:

\[ f_{max} = \frac{1}{2} f_s \]  \hspace{1cm} 4.3

in which \( f_s \) is the sampling frequency. The maximum range \( r_{max} \) now becomes

\[ r_{max} = \frac{c T_s}{2 F} f_{max} = \frac{N c}{4 F} \]  \hspace{1cm} 4.4

with \( N \) as the number of samples within one sweep. Recalling that if a sweep contains \( N \) samples, \( N/2 \) range cells remain, it follows that the range resolution \( \Delta r \) is given by

\[ \Delta r = \frac{c}{2 F} \]  \hspace{1cm} 4.5

The range resolution is easy to change by adjusting the frequency excursion \( F \). The maximum range can be altered by changing the sampling frequency, the frequency excursion, or the sweep time. With DARR, \( F \) can be set to values between

65
1 and 50 MHz, corresponding to values of $\Delta r$ between 150 m and 3 m. The sampling frequency can be set to 100 kHz, 200 kHz, or 2 MHz. The sweep time can be altered between 0.625 ms and 640 ms. In table 4.1 some typical values of the radar parameters during precipitation measurements are given.

<table>
<thead>
<tr>
<th>Sampling frequency</th>
<th>100 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweep time</td>
<td>1.25 ms</td>
</tr>
<tr>
<td>Frequency excursion</td>
<td>1, 2, 5 MHz</td>
</tr>
<tr>
<td>Maximum range</td>
<td>9600, 4800, 1920 m</td>
</tr>
<tr>
<td>Range resolution</td>
<td>150, 75, 30 m</td>
</tr>
</tbody>
</table>

Table 4.1 Typical radar parameters for precipitation measurements

4.2 Comparison of FM-CW and pulse radar

In general, the minimum detectable signal is determined by the thermal noise $P_n$ of the radar receiver. Consequently, the receiver bandwidth $B$ plays an important role, because

$$P_n = k T_{op} B$$  \hspace{1cm} (4.6)

with $k$ as the Boltzmann constant, and $T_{op}$ as the noise temperature of the receiver. The frequency bandwidth that corresponds to the range resolution of FM-CW radars is given by

$$B = \frac{f_s}{N} = \frac{1}{T_s}$$  \hspace{1cm} (4.7)

The bandwidth of pulse radars can be approximated by the same expression, with $T_s$ replaced by $\tau$, the pulse width [Skolnik, 1980]. Comparison of the noise levels of a pulse and FM-CW radar leads to
\[ \Delta P_n = 10 \log \left[ \frac{T_r}{T_s} \right] \]

The received power \( P_r \) is given by the radar equation [Skolnik, 1980]:

\[ P_r = \frac{G_t G_r \lambda^2}{(4\pi)^3 r_d^4} P_t \sigma = C P_t \sigma \]

in which \( G_t \) and \( G_r \) are the antenna gains of, respectively, the transmit and the receive antenna, \( \lambda \) is the radar wavelength, \( P_t \) is the transmit peak power, and \( \sigma \) is the radar cross-section of the target. The peak power of a pulse radar is related to the mean power \( \bar{P} \) by

\[ P_t = \bar{P} \frac{T_r}{\tau} \]

in which \( T_r \) is pulse repetition time. In the case of an FM-CW radar the peak transmit power equals the mean transmit power because \( T_r = \tau = T_s \). The signal-to-noise ratio \( SNR \) of both radars is given by

\[ SNR_{FM-CW} = \frac{C \sigma}{k T_{op}} \frac{\bar{P}}{B} = \frac{C \sigma}{k T_{op}} \bar{P} T_s \]

\[ SNR_{PULSE} = \frac{C \sigma}{k T_{op}} \frac{T_r \bar{P}}{\tau B} = \frac{C \sigma}{k T_{op}} \bar{P} T_r \]

For many pulse weather radars \( \tau \) is in the order of 1 \( \mu s \), while for FM-CW radars \( T_s \) is in the order of 1 ms. This implies that the noise level of a typical FM-CW radar is 30 dB lower than the noise level of a typical pulse radar. The pulse repetition time and the sweep time are usually of the same order of magnitude, and thus the signal-to-noise ratio is comparable for both radar types. The advantage of FM-CW radar is, however, that only a low transmit power needs to be generated to obtain the same sensitivity as a pulse radar. The range resolution \( \Delta r_p \) of pulse radars is given by [Skolnik, 1980]
where shows that the range resolution can only be changed by varying the pulse width \( \tau \). This implies that, in order to maintain the sensitivity, the transmit power has to be changed as well. With an FM-CW radar the range resolution can be changed by changing the frequency excursion, which does not affect the transmit power. Consequently, an FM-CW radar is more flexible than a pulse radar.

FM-CW radars require two antennas, because they transmit and receive a continuous signal. A pulse radar takes advantage of the time between two transmit pulses to receive the reflected signal, and consequently can use the same antenna for transmitting and receiving. The earlier-mentioned cross-talk attributable to the FFT is a drawback of FM-CW radars. Simulations, assuming backscattering by a volume that is uniformly filled with moving point targets, have shown that when there is precipitation the cross talk to adjacent range cells equals approximately \(-13\) dB.

The weather radar equation is discussed in many radar textbooks [e.g., Battan, 1973; Skolnik, 1980; Doviak, 1984]. For a circularly symmetric Gaussian antenna pattern it is given by

\[
P_r = \frac{P_t G_t G_r \theta_k^2 \pi^3 |K|^2 \Delta r Z}{512(\ln 2) \lambda^2} \frac{Z}{r_d^2}
\]

in which \( \theta_b \) is the \(-3\) dB power beamwidth of the antenna. Note that in the case of scattering by precipitation the received power decreases with \( r_d^2 \), instead of \( r_d^4 \). The weather radar equation does not differ for pulse or FM-CW radars. It is derived assuming that one antenna is used. However, when two antennas with (different) beamwidths are employed, the equivalent beamwidth \( \theta_e \) is used. It is defined as

\[
\theta_e = \sqrt{\frac{2 \theta_i^2 \theta_r^2}{\theta_i^2 + \theta_r^2}}
\]
in which $\theta_t$ and $\theta_r$ are the beamwidths of the transmit and receive antenna, respectively.

4.3 A short system description

DARR is an S-band system, operating at a frequency of 3.315 GHz. Figure 4.2 shows a block diagram of the radar system. The received signal is amplified by a low-noise amplifier LNA, converted to a lower frequency by mixing with the transmit signal, and after low-pass filtering, amplified by a 6 dB/octave amplifier. The 6 dB/octave amplifier is used to correct the signal for spatial expansion of the radar waves, since the $r_d^2$ decrease of the received power implies a $f^{-2}$ dependence. Also, the receiver requires less dynamic range. Unfortunately, the noise level of the receiver is amplified by 6 dB/octave as well, so the amplifier does not enhance the sensitivity of the receiver. The resultant low-frequency signal is digitized and transferred to a combination of an Analog A-500 array processor and an HP-1000 host computer system, where real time data processing is done. Each 3.2 seconds a complete data set, consisting of the Doppler and polarimetric radar observables of 64 range cells, is obtained. The processed data is stored in the computer memory, while selected raw data is recorded on a magnetic tape for the development and the analysis of new techniques. The internal stability of the radar system is monitored by means of a delay line, which is an optimally controlled 750 m transmission line between the transmitter and receiver. The receiver output of the 'delay line signal' is treated by the same signal processing routines as signals coming from precipitation. However, the delay line itself does not introduce signal fluctuations; the output power of the delay line serves as a means of monitoring the internal stability.

DARR is equipped with two parabolic reflector antennas; both antennas are illuminated by a focal feed. Apart from a small antenna to illuminate the reflector, the feed also contains the polarizer. The two reflectors are not equal in size: the diameter of the transmit antenna equals 4.28 m, while the diameter of the receive antenna equals 2.12 m. The beamwidths of the antennas are given by [Skolnik, 1980]

$$\theta_{t,r} = 1.22 \frac{\lambda}{D_{a,tr}}$$  \hspace{1cm} (4.16)
with $D_{a, tr}$ as the diameter of the transmit and receive reflector. In the case of DARR, $	heta_l=1.5\degree$, and $\theta_r=3\degree$. The equivalent beamwidth equals 1.9\degree. The antennas can be steered using an azimuth, and an *inclined* axis. The latter enables the positioning in elevation [Ligthart, 1980].

[Diagram of the Delft Atmospheric Research Radar]

Figure 4.2 A simplified block diagram of the Delft Atmospheric Research Radar.

The polarizers are able to rotate the polarization of the transmit and receive antennas continuously and independently. The polarization angle $\phi$ (using the same coordinate system as that described in chapter 3) is a sinusoidal function of time:

$$\phi(t) = \phi_r + \phi_{max} \sin(2\pi f_p t) \tag{4.17}$$

with $f_p$ as the polarizer frequency, $\phi_{max}$ as the maximum deviation, and $\phi_r$ as the start position of the polarizer. The polarizer frequency is 25 Hz, corresponding with a polarizer period of 40 ms. The initial polarization $\phi_r$ depends on the angular position of the inclined axis, and may obtain values between 0\degree and 90\degree [Sinttruyen, 1990]. However, the relationship between $\phi_r$ and the inclination angle is completely deterministic, and so
standard rotation routines can be applied to express all measurements in the same frame of reference.

4.4 Clutter suppression

Weather radars do not only receive reflections from precipitation but also from objects such as trees and buildings. This ground clutter must be removed for the reflected radar signal to represent precipitation only. Clutter filtering takes advantage of the different decorrelation times of the complex clutter and precipitation signals. The method itself is simple: two samples of the complex beat signal, corresponding with one range cell, but measured at different moments, are subtracted. The clutter signal disappears when the time interval between the two samples exceeds the decorrelation time of rain, while the fixed target reflections are still correlated. The samples must be measured with the same polarization.

The complex signal \( U(t) \) can be regarded as the sum of a stochastic component \( U_r(t) \) that is caused by rain and a deterministic component \( U_c(t) \) that is caused by ground clutter:

\[
U(t) = U_r(t) + U_c(t)
\]

After taking two samples of the measured signal, separated by time \( T \), and subtracting them, the signal \( U_c(t) \) results:

\[
U_c(t) = U(t) - U(t+T) = U_r(t) - U_r(t + T)
\]

\( T \) is assumed to be small enough for \( U_c(t) \) to equal \( U_c(t+T) \) and it is assumed that the radar does not change. The time averaged power \( P_c(t) \) of \( U_c(t) \) is given by

\[
P_c(t) = \frac{1}{T} \left| U_c(t) \right|^2 = \left| U_r(t) \right|^2 + \left| U_r(t+T) \right|^2 - \overline{U_r(t) U_r^*(t+T)} - \overline{U_r(t+T) U_r^*(t)}
\]

71
When $U_r(t)$ and $U_r(t+T)$ are uncorrelated, the cross-terms disappear and the resultant power $P_r(t)$ follows from

$$P_r(t) = \frac{1}{2} P_e(t)$$

On a clear day, the radar was set up to measure ground clutter under the same experimental conditions as used during rain measurements. The radar elevation angle was 30°, and the azimuth angle was fixed. Figure 4.3 shows the range spectrum of the measurement, integrated over 32 seconds: one plot shows clutter and the other the spectrum after clutter suppression. $T$ is set to 40 ms.

![Graph showing range spectrum of clutter, before and after suppression.](image)

Figure 4.3 A typical range spectrum of clutter, before and after suppression.

The clutter reflection is strong when it comes from nearby targets, but reduces approximately to 10 dBZ after 1500 meter. Since the radar elevation angle is 30°, most of the clutter is seen through the side lobes of the antennas. The nearby clutter is suppressed with approximately 20-25 dB. The suppressed-clutter signal increases linearly with the distance, which for distances greater than 1500 meter is caused by the
earlier mentioned 6 dB/octave amplifier: the clutter is suppressed down to the noise level of the radar receiver.

4.5 Signal processing

The beat signal is composed of contributions from all individual particles within a range cell. Each particle can be described by its position \( r_i \) relative to a central position \( r_d \) in the range cell, its Doppler radial frequency \( \omega_i \) and its polarization-dependent scattering parameters \( S_{kl}^i \). The copolarly received voltage \( U(t) \) is given then by

\[
U(t) = \sum_i S_i^i(\phi(t)) e^{j\omega_i t} e^{-j2k_0(r_d + r_i)}
\]

\[
S_i^i(\phi(t)) = S_{hh}^i \cos^2 \phi(t) + S_{hv}^i \sin 2\phi(t) + S_{vv}^i \sin^2 \phi(t)
\]

\[
\phi(t) = \phi_r + \phi_{max} \sin(\omega_p t)
\]

The Doppler and polarization components of the beat signal must be separated before the parameters of the scattering matrix can be obtained. This can be achieved by sufficiently long incoherent integration (see section 3.2) of samples that correspond to the same polarization angle \( \phi \): the Doppler and range phasors will disappear, only polarization information is left.

The extraction of Doppler information from the measured beat signal is based on the time dependence of the complex signal. In general, the application of a complex FFT over a number of succeeding time samples suffices to obtain the Doppler spectrum [Doviak and Zrnic, 1984]. However, in the case of DARR the time dependence may not only be determined by Doppler effects, but also by the varying polarization. During the time that is needed to obtain the Doppler spectrum the signal may experience an additional amplitude modulation, induced by the polarizers. This modulation reveals itself in the Doppler spectrum through cross-talk from one velocity to another. This effect is analyzed in the section on Doppler processing.
4.5.1 Polarimetric processing

The polarimetric processing of DARR is based on the power of the received signal. The polarization dependence of the radar signal is determined by the scattering function of the range cell,

$$P_r(\phi) = \sum_i |S^i(\phi)|^2$$

or fully written,

$$P_r(\phi) = \sum_i |S_{hh}^i|^2 \cos^4 \phi + \sum_i |S_{vv}^i|^2 \sin^4 \phi + \sum_i |S_{hv}^i|^2 \sin^2 2\phi +$$

$$\sum_i \left( S_{hh}^i S_{hv}^i + S_{hv}^i S_{hh}^i \right) \sin^2 \phi \cos^2 \phi +$$

$$\sum_i \left( S_{hh}^i S_{hh}^i + S_{hv}^i S_{hv}^i \right) \sin 2\phi \cos^2 \phi +$$

$$\sum_i \left( S_{hv}^i S_{hv}^i + S_{vv}^i S_{hv}^i \right) \sin 2\phi \sin^2 \phi$$

The backscattered signals from individual hydrometeors are statistically independent, implying that

$$\sum_i S_{kk}^i S_{kl}^{i\ast} = N^2 < S_{kk} > < S_{kl} > \ast \text{ (} k \text{ and } l \text{ represent } h \text{ or } v \text{ when appropriate})$$

with $N$ as the number of particles. Time integration over a period $T$ reduces the variance: for $T$ sufficiently long, $\frac{< S_{kl} >}{2} \approx \frac{< S_{kl}^2 >}{2}$. Consequently, the time averaged power $\bar{P}_r(\phi)$ for real $S_{kl}$ can be approximated by
\[ P_r(\phi) = \left| \overline{S}_{hh} \cos^2 \phi + \overline{S}_{hv} \sin 2\phi + \overline{S}_{vv} \sin^2 \phi \right|^2 \] 4.28

with

\[ \overline{S}_{kl} = \sqrt{\sum_i |S_{kl}^i|^2} = \sqrt{N \langle |S_{kl}|^2 \rangle} \] 4.29

The differential reflectivity \( Z_{dr} \) and linear depolarization ratio \( L_{dr} \) are related to the scattering parameters like

\[ Z_{dr} = 20 \log \frac{|\overline{S}_{hh}|}{|\overline{S}_{vv}|} \] 4.30

\[ L_{dr} = 20 \log \frac{|\overline{S}_{hv}|}{|\overline{S}_{hh}|} \] 4.31

The estimator \( \overline{S}_{hv} \) is positive by definition, although individual \( S_{hv}^i \) may be negative, because of their proportionality to \( \Lambda_\phi^i - \Lambda_\phi^i \) (see chapter 3). This discrepancy, however, does not affect the determination of \( L_{dr} \), because then only absolute values are relevant. The time-integrated scattering function contains three unknown variables. To solve these, three power measurements at different polarizations are sufficient.

The question as to how long the integration time should be can only be dealt with after a discussion of the theoretically expected signal. Figure 4.4 shows the normalized, simulated power of a radar signal throughout one polarizer cycle. The initial position of the polarizer is set to 57\(^\circ\), corresponding to an elevation angle of 30\(^\circ\). \( Z_{dr} \) varies between 0 and 3 dB. \( L_{dr} \) is set to -30 dB, and when \( Z_{dr} = 0 \) dB, the result of \( \overline{S}_{hv} = 0 \) is shown as well.

Figure 4.4 can be understood by considering an elliptic particle as depicted in figure 4.5. At \( t=0 \) the polarizer starts from its initial position. First it will rotate towards the vertical position, at which point minimum power is received. The vertical position, however, does not correspond to the extreme position of the polarizer and the power will increase again until the polarizer reaches its extreme. After this point the
Figure 4.4  Scattering function during one polarizer period for different values of $Z_{dr}$. $L_{dr} = -30$ dB and when $Z_{dr} = 0$ dB, $L_{dr}$ is also set to $-\infty$ dB.

Figure 4.5  Scanning of an oblate spheroid by an electric field $E(t)$ during one polarizer period.
polarizer will return to its starting point and thus pass through the position with vertical polarization again. When the polarizer rotates towards the other extreme, a maximum is measured twice at horizontal polarization. Note that for $Z_{dr}$ equal to 0 dB, but $L_{dr}$ not infinitely small, the received signal still exhibits a polarization dependence. This situation may occur when there is backscattering by a collection of randomly oriented particles or during a measurement with the radar pointed towards the zenith, as was discussed in chapter 3.

![Graph](image)

**Figure 4.6** The uncalibrated received power from one rain-filled range cell as function of time. The range resolution is 75 m. A running average over 640 ms with a step size of one polarizer period is applied.

To determine the necessary integration time, a measurement during a rain event of approximately 3 mmh$^{-1}$ is analyzed. Figure 4.6 gives the uncalibrated received power as function of time. A running average over 640 ms is applied. As can be seen, the number of maxima gradually changes from three to one per polarizer period. This is caused by interference between signals that are reflected by (clusters of) particles with a different fall speed. Apparently, some coherent addition still occurs and the integration time has to be increased. The result of integration over 3.2 seconds is shown in figure 4.7. Two
maxima and two minima per polarizer period occur, just as theoretically predicted. The fact that the power at readings 1 and 2 do not match was found to be caused by undersampling of the signal and an asymmetry of the polarizers: the extreme positions are not exactly 90 degree deviations from the start position, but differ by some 4 degrees from each other. For precipitation measurements three sweeps, centered around the starting position of the polarizer, are selected to calculate the scattering parameters.

![Diagram of power over time]

Figure 4.7  As figure 4.6 but after a running average of 3.2 seconds.

$Z_{dr}$ is usually obtained after integrating time series of $Z_h$ and $Z_v$ measurements. Bringi et al [1983] argued that the optimum estimator of $Z_{dr}$ is given by the ratio of the mean $Z_h$ and the mean $Z_v$, rather than the mean of the $Z_{dr}$ itself. The standard deviation of $Z_{dr}$ depends on the cross-correlation coefficient of $Z_h$ and $Z_v$, and, of course, on the number of samples. The time interval between sampling $Z_h$ or $Z_v$ is a dominant term in the cross-correlation coefficient; it must be sufficiently small to assure good correlation. A thorough discussion of $Z_{dr}$ is given by Bringi et al [1983], and by Cherry and Goddard [1983], leading to the conclusion that an accuracy of 0.1 dB can be obtained, when more than 90 samples are used, the signal-to-noise ratio exceeds 15 dB and a $Z_h$-$Z_v$ pair is measured within an interval of 8 ms. These requirements are all met during the DARR measurements.
4.5.2  Effect of the differential phase

The vertically and horizontally polarized waves may have different phases. This differential phase can be caused by the backscattering process itself and by propagation of the waves through the medium intervening the range cell under consideration and the radar. Both mechanisms reveal themselves differently in the equations that describe the measured scattering parameters. In the case of backscattering by similar particles, with an azimuth angle of 90° and an elevation angle δ, and considering a radar elevation θ and polarization angle of 0°, the normalized scattering parameters can be written as (see chapter 3)

\[ S_{hh} = \Lambda_1 + (\Lambda_3 - \Lambda_1) \sin^2 \delta \]  \hspace{1cm} 4.32

\[ S_{vv} = \Lambda_1 + (\Lambda_3 - \Lambda_1) \cos^2 \delta \cos^2 \theta \]  \hspace{1cm} 4.33

\[ S_{hv} = \frac{1}{4}(\Lambda_3 - \Lambda_1) \sin 2\delta \cos \theta \]  \hspace{1cm} 4.34

The differential backscatter phase is the phase difference between \( \Lambda_3 \) and \( \Lambda_1 \). In the Rayleigh regime \( \Lambda_3 \) and \( \Lambda_1 \) are real, and hence no phase difference occurs, but at higher frequencies they may become complex. To estimate the effect of the differential backscatter phase, the real \( \Lambda_3 \) is modified into \( \Lambda_3 e^{i\phi_{bs}} \). To include the differential propagation phase, the scattering parameters \( S_{kl} \) are transformed into \( \tilde{S}_{kl} \) with

\[ \tilde{S}_{hh} = \Lambda_1 + (\Lambda_3 - \Lambda_1) \sin^2 \delta \]  \hspace{1cm} 4.35

\[ \tilde{S}_{vv} = (\Lambda_1 + (\Lambda_3 - \Lambda_1) \cos^2 \delta \cos^2 \theta)e^{i2\phi_{fs}} \]  \hspace{1cm} 4.36

\[ \tilde{S}_{hv} = \left(\frac{1}{4}(\Lambda_3 - \Lambda_1) \sin 2\delta \cos \theta\right)e^{i\phi_{fs}} \]  \hspace{1cm} 4.37

The method to obtain the three scattering parameters from copolar measurements assumed real scattering parameters. Consequently, an error is made when they are not. Figure 4.8 shows the error of \( L_{dr} \) versus the true \( L_{dr} \) for different values of \( \phi_{bs} \) and \( \phi_{fs} \), calculated with \( \theta = 30° \) and \( \delta = 25° \). When \( \phi_{bs} = 0, \phi_{fs} = 5, 10 \) or \( 15° \), or vice versa.
The differential propagation phase has a much larger impact on the accuracy of $L_{dr}$ than the differential backscatter phase. The latter is smaller than 2 dB, whereas the former can be as large as 9 dB. Both $\phi_{bs}$ and $\phi_{fs}$ result in an overestimation of $L_{dr}$, although the error decreases when $L_{dr}$ increases. The error of $Z_{d_r}$ was found to be less than 0.1 dB for the same range of values of $\phi_{bs}$ and $\phi_{fs}$ and $Z_{d_r}$ varying between 0 and 3 dB.

![Graph showing the error of $L_{dr}$ due to the differential propagation and backscatter phase.](image)

**Figure 4.8** The error of $L_{dr}$ due to the differential propagation and backscatter phase.

How realistic are the values of $\phi_{bs}$ and $\phi_{fs}$? Bringi et al [1991] simulated the differential propagation phase at C-band and S-band as function of the reflectivity factor, and found that at S-band it varies between $1^\circ$ and $2^\circ$ per kilometer for heavy rain of 50 mmh$^{-1}$. It is negligible for small and moderate rain intensities. At C-band, however, $\phi_{fs}$ is larger and should be taken into account for range cells not nearby the radar. At S-band the differential backscatter phase can be neglected as well [Uzunoglu et al, 1977]. The impact of the differential phase is large for small values of $L_{dr}$, but since these correspond to small rain intensities and small differential phase, the error of the actually measured $L_{dr}$ is small.
Performing Doppler measurements using an FM-CW radar may seem paradoxical, because the frequency shift is used for range extraction. Thus, how can the velocity be measured? For meteorological targets the beat frequency is approximately given by [Doviak and Zrnic, 1984]

\[ f_b \approx \frac{2r_d F}{c T_s} + \Delta f + \frac{2vt F}{c T_s} \]  

4.38

in which \( \Delta f \) is the Doppler frequency shift and \( t \) symbolizes the time dependence of the beat frequency. The first term is used for range extraction and the two last terms are produced by the velocity of the target. To investigate the influence of Doppler movement on the beat frequency, the radar specifications given in Table 4.1 are used, and \( v \) is set to 5 m/s. Range extraction is done per sweep, thus \( t = T_s \). The Doppler frequency \( \Delta f \) is in the order of 100 Hz at S-band and the third term of Equation 4.38 is negligible. The range term becomes \( 5.3\cdot r_d F \), with \( F \) in MHz. The influence of target motion on the range extraction is significant only for very short ranges. When \( F \geq 1 \) MHz, and \( r_d > 500 \) m, the error in the calculated range is less than 4%. The influence of target motion on the beat frequency is small and so \( f_b \) is not used for velocity measurements. The phase difference \( \phi_D(t) \) of two succeeding sweeps is used instead. It is approximately given by

\[ \phi_D(t) \approx 2k \cdot r_d + 2\pi \Delta f t \]  

4.39

The first term is caused by the distance of the target. The second term contains the velocity. The Doppler spectrum is obtained by applying an FFT to \( N \) successive sweeps of the complex time signal; the resultant spectrum consists of \( N \) velocity cells. Since the Doppler velocity is based on the phase difference of two samples, and only phase differences between 0 and \( 2\pi \) can be uniquely measured, a maximum unambiguous obtainable velocity \( v_{\text{max}} \) exists. It is given by [Doviak and Zrnic, 1984]:

\[ v_{\text{max}} = \pm \frac{\lambda}{4F_0} \]  

4.40

81
with $\lambda$ as the radar wavelength and $T_0$ as the time between two succeeding samples. With the radar setup shown in table 4.1, $v_{\text{max}}$ equals $\pm 9 \text{ m/s}$; by convention a positive velocity is directed towards the radar. The number of velocity cells $N$ is set to 64, implying a velocity-cell size of 28 cm/s.

The Doppler spectrum is obtained within 160 ms. When the Doppler spectrum is interpreted in terms of fall velocity, phase variations must be caused by Doppler effects alone. However, the Doppler spectrum is obtained using rotating polarizers, and hence an additional amplitude modulation is induced. Applying the FFT to the amplitude modulated signal may result in a broadening of the Doppler spectrum [Brigham, 1974]. In order to quantify the effect, backscattering by an ensemble of 200 particles was analyzed. Each particle was attached with a velocity, $Z_{dr}$, $L_{dr}$, and differential phase. The velocity varied as function of the dropsize, following equation 2.1. $Z_{dr}$ was assumed to depend linearly on the dropsize and varied between 0 and 4 dB. $L_{dr}$ was set to $-20$ dB for all drops. The differential phase increased linearly with velocity, up onto a maximum value of 90° for the largest drops. A Gaussian velocity spectrum was assumed.

The Doppler spectrum of the ensemble of particles was calculated for two situations: the polarization was fixed at horizontal, or it varied sinusoidally with time between horizontal and vertical. Finally, the two spectra were compared. The varying polarization resulted in a decrease of the maximum power of the spectrum of approximately 1.5 dB: the polarization-dependent backscattering due to the oblate particles is averaged. The spectrum width did not undergo any significant changes. To confirm the simulation, a 5-minute test measurement was done during a severe rain storm. The polarizers were alternately turned on and off every 15 s, but no change in the spectrum width that could be attributed to the polarizers was observed, even though the $Z_{dr}$ approximated 2 dB throughout the whole event.

4.6 $L_{dr}$ from copolar measurements?

Precipitation may cause $L_{dr}$ values between $-20$ and $-35$ dB, which, when $L_{dr}$ is obtained from actual cross-polar measurements, makes high demands on the cross-polar isolation of the radar antennas and on the signal-to-noise ratio. Deriving $L_{dr}$ from three copolar measurements, as is done in this study, makes these requirements less crucial. The received signal, at any polarization between horizontal or vertical, contains
contributions from all three parameters of the scattering matrix. If the measurements are done at horizontal, vertical and an additional polarization, a system of three equations has to be solved for the scattering parameters. The measurement at the additional polarization has approximately the same signal-to-noise ratio as the measurements at horizontal and vertical polarization.

Bringi et al [1983] have shown that, to measure Z_{dr} with optimum accuracy, a fast polarization switch is necessary. Evidently, the same holds true for L_{dr}. The achieved accuracy is based on the correlation between consecutive samples with different polarization and depends on the time lag between them. However, one of the decorrelating phenomena is noise. The correlation coefficient ρ of the two consecutive samples is modified by noise into

\[
ρ' = \frac{ρ}{\sqrt{(1 + \frac{1}{SNR})(1 + \frac{X}{SNR})}}
\]

where SNR is the signal-to-noise ratio for horizontal polarization and X a quantity that depends on the mode of measurement (copolar or cross-polar). The degradation of ρ during Z_{dr} measurements (X=Z_{dr} then) is described by Cherry and Goddard [1982] and Bringi et al [1983]. In this section emphasis is put on the L_{dr} measurement.

When L_{dr} is measured directly through a cross-polar measurement, X=1/L_{dr}. L_{dr} can be as low as −35 dB. When in that case SNR=35 dB, ρ is degraded by a factor 0.7, necessitating long integration times to obtain the accuracy of the "non-degraded" measurement. The decorrelation is less severe when L_{dr} is derived from three copolar measurements. For example, let three measurements be done at the polarization angles 0°, 45° and 90°, respectively. The decorrelation between samples at 0° and 90° is described by equation 4.41 with X=Z_{dr}, while the decorrelation between 0° and 45° or 45° and 90° samples can be obtained with a value of X smaller than Z_{dr}. When Z_{dr}=3 dB and SNR=35 dB, the degradation is negligible. Consequently, L_{dr} is derived from samples with the same degree of correlation as the samples that are used to estimate Z_{dr}.

Another effect of noise is that it increases the measured signal power. Assuming that noise is uncorrelated to precipitation, the effect can be evaluated by adding the noise power N to equation 4.28. The real scattering parameters obtained from copolar measurements at 0°, 45° and 90° are affected as
\[ S_{hh}^n = S_{hh} \sqrt{1 + \frac{N}{P_r(0)}} \equiv C_0 S_{hh} = S_{hh} \sqrt{1 + \frac{1}{SNR}} \]

\[ S_{vv}^n = S_{vv} \sqrt{1 + \frac{N}{P_r(90)}} \equiv C_{90} S_{vv} = S_{vv} \sqrt{1 + \frac{Z_{dr}}{SNR}} \]

\[ S_{hv}^n = \frac{1}{2} S_{hh} (C_{45} - C_0) + C_{45} S_{hv} + \frac{1}{2} S_{vv} (C_{45} - C_{90}) \]

with

\[ C_{45} = \sqrt{1 + \frac{N}{P_r(45)}} = \sqrt{1 + \frac{1}{SNR P_r(45)}} \]

When \( S_{hv} \) is measured directly through a cross-polar measurement, the noise influence is given by

\[ S_{hv}^n = S_{hv} \sqrt{1 + \frac{N}{P_{hv}}} = S_{hv} \sqrt{1 + \frac{1}{SNR L_{dr}}} \]

where \( P_{hv} \) denotes the cross-polarly received power without noise. Figure 4.9 shows the error of \( Z_{dr} \) as function of the signal-to-noise ratio with the true \( Z_{dr} \) as parameter. \( L_{dr} \) is set to \(-30\) dB. \( Z_{dr} \) is underestimated. The figure represents the average, the decorrelation is not taken into account. In the case of precipitation \((-3 < Z_{dr} < 3\) dB) the influence of noise is small. A signal-to-noise ratio of 15 dB suffices to measure the \( Z_{dr} \) with an accuracy of 0.1 dB.
Figure 4.9 The difference between the estimated $Z_{dr}$ and the true $Z_{dr}$ as a function of the signal-to-noise ratio, for different values of the true $Z_{dr}$. $L_{dr} = -30$ dB.

Figure 4.10a shows the error in the copolarly estimated $L_{dr}$ as function of the signal-to-noise ratio with the true $L_{dr}$ as parameter. $Z_{dr}$ is set to 0 dB. The error is weakly dependent on the true $L_{dr}$. $L_{dr}$ is underestimated, but the effect is negligible for a signal-to-noise ratio better than 20 dB. Although not shown here, the effect of high $Z_{dr}$ values on the estimation of $L_{dr}$ was calculated, and it was found to be very small for a signal-to-noise ratio better than 20 dB. The calculations were done for positive and for negative $\overline{S}_{hv}$, but the differences were negligible.

Figure 4.10b is similar to figure 4.10a, except that $L_{dr}$ is measured directly through a cross-polar measurement. Note the strong influence of noise. $L_{dr}$ is overestimated. To obtain the same accuracy as the copolarly derived $L_{dr}$, the signal-to-noise ratio must be greater than 35 dB.
Figure 4.10 The difference between the estimated $L_{dr}$ and the true $L_{dr}$ as a function of the signal-to-noise ratio, for different values of the true $L_{dr}$. $Z_{dr} = 0$ dB. $L_{dr}$ is obtained from a copolar (a) or cross-polar (b) measurement.
Overview of the Delft Atmospheric Research Radar

Table 4.2 summarizes the specifications of the Delft Atmospheric Research Radar \cite{Ligthart and Nieuwkerk, 1989}. The FM-CW principle was illustrated by means of a sawtooth signal that modulates the frequency, but other signals are suited as well; the precipitation measurements are done with a triangular signal to smooth the effect of an abrupt frequency change. However, only the up-going sweep is used for further analysis. The sweep time is set to 1.25 ms, which implies that the Doppler velocities are derived from samples that are 2.5 ms apart. The minimum detectable signal is based on a signal-to-noise ratio of 10 dB.

<table>
<thead>
<tr>
<th>DARR specifications</th>
<th>DARR specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar type</td>
<td>linear FM</td>
</tr>
<tr>
<td></td>
<td>triangular, sawtooth</td>
</tr>
<tr>
<td>Transmitted power</td>
<td>50 dBm [max]</td>
</tr>
<tr>
<td></td>
<td>30 dBm [for rain]</td>
</tr>
<tr>
<td>Center frequency</td>
<td>3.315 GHz</td>
</tr>
<tr>
<td>Frequency excursion</td>
<td>1 – 50 MHz</td>
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<tr>
<td>Sweep time $T_s$</td>
<td>0.625 – 640 ms</td>
</tr>
<tr>
<td>Receiver noise figure</td>
<td>2.5 dB</td>
</tr>
<tr>
<td>Beat frequencies</td>
<td>0.4 - 1000 kHz</td>
</tr>
<tr>
<td>Antenna gain</td>
<td>32.7 dB receiver</td>
</tr>
<tr>
<td></td>
<td>40.0 dB transmitter</td>
</tr>
<tr>
<td>Antenna beamwidth</td>
<td>3.0° receiver</td>
</tr>
<tr>
<td></td>
<td>1.5° transmitter</td>
</tr>
<tr>
<td>Antenna isolation</td>
<td>&gt; 90 dB</td>
</tr>
<tr>
<td>Range $r_d$</td>
<td>0.5 – 30 km</td>
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<tr>
<td>Analyzer bandwidth B</td>
<td>$1/T_s$</td>
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<tr>
<td>Smallest signal</td>
<td>$-160 + 10 \log(B)$ dBm</td>
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<tr>
<td>Fixed target suppression</td>
<td>&gt; 20 dB</td>
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<tr>
<td>Maximum Doppler speed</td>
<td>± 18 m/s</td>
</tr>
<tr>
<td>polarization angles</td>
<td>± 90°</td>
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<tr>
<td>polarizer period</td>
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</tr>
<tr>
<td>accuracy</td>
<td>1.5°</td>
</tr>
</tbody>
</table>

Table 4.2 Characteristics of the Delft Atmospheric Research Radar
Chapter 5
Variations in the microstructure of precipitation: analysis by correlation

Each radar observable depends on the microstructure of precipitation. How an observable depends on it, however, may differ for each. For instance, the horizontal reflectivity responds differently to a variation in the particle orientation than the differential reflectivity or linear depolarization ratio. Or, the number of particles affects the horizontal reflectivity, but not the variance of the Doppler spectrum. To identify the variations within rain, the interrelationships between the radar observables are analyzed, and quantified by the cross-correlation coefficient. Measurements at a 30° elevation of the radar antennas are used to show the behavior of the radar observables, in rain and in the melting layer. A simple model that relates canting to turbulence is introduced.

5.1 Rain recapitulated

The microstructure of rain is determined by the statistical distributions of the size, shape, orientation and fall speed of the drops. Of these, the fall speed and particle shape are uniquely related to the dropsize, leaving only the dropsize and orientation as independent parameters. The dropsize distribution $N(D)$ is given by the gamma distribution, while the orientation follows a Gaussian distribution $t(\delta)$. Both are repeated here for convenience:

\[ N(D) = N_o D^\mu e^{-\frac{3.67 + \mu}{D_o} D} \]

\[ t(\delta) = \frac{1}{\sigma_\delta \sqrt{2\pi}} e^{-\frac{(\delta - \bar{\delta})^2}{2\sigma_\delta^2}} \]

The orientation angle $\delta$ is defined relative to the vertical. The particles are assumed to be canted in a plane perpendicular to the plane of incidence of the radar wave ($\alpha=90^\circ$, 89
see chapter 3). This may not be realistic but it suffices for the purpose of this chapter, which is to show the effect of variations of the microstructure on the radar observables. The mean orientation angle \( \bar{\theta} \) is usually small [Brussaard, 1976] and in this study set to 0°. The particle concentration \( N_t \) is given by

\[
N_t = \int_0^{D_{max}} N(D) dD = N_o \int_0^{D_{max}} D^\mu e^{-\frac{3.67+\mu}{D_o} D} dD
\]

5.3

which, when \( D_{max} \to \infty \), results in

\[
N_t = N_o \left( \frac{D_o}{3.67+\mu} \right)^{\mu+1} \mu!
\]

5.4

A similar expression can be derived for the volume of the amount of water per m\(^3\), \( V_t \):

\[
V_t = \frac{\pi}{6} N_o \left( \frac{D_o}{3.67+\mu} \right)^{\mu+4} (\mu+3)!
\]

5.5

The rain intensity \( R \) is defined as

\[
R = \int_0^{D_{max}} V(D) v(D) N(D) dD
\]

5.6

with \( V(D) \) as the volume and \( v(D) \) as the fall speed of a single raindrop. Assuming the relationship between the fall speed and the diameter of equation 2.1 and \( D_{max} \to \infty \), \( R \) [mmh\(^{-1}\)] is written as

\[
R = 600 \cdot 10^{-6} \pi N_o (\mu+3) \left\{ 9.65 \left( \frac{D_o}{3.67+\mu} \right)^{\mu+4} - 10.3 \left( \frac{D_o}{3.67+\mu+\alpha D_o} \right)^{\mu+4} \right\}
\]

5.7

with \( \alpha = 0.6 \) mm\(^{-1}\). The units of \( N_o \) and \( D_o \) are mm\(^{-1}\cdot\mu\) m\(^{-3}\) and mm, respectively. Note that \( N_o \) depends on \( \mu \). The integrals in equations 5.3 and 5.6 are only weakly dependent
upon $D_{\text{max}}$ if $D_{\text{max}} > 2.5 D_o$ [Ulbrich, 1986], which is usually the case, and hence $D_{\text{max}}$ is set to $\infty$. For rain intensities less than 50 mmh$^{-1}$ equations 5.6 and 5.7 differ less than 1%.

The calculation of the radar observables involves numerical integrations of the dropsize distribution. $D_{\text{max}}$ is set to 8 mm then, because larger drops are unstable and break up into smaller drops [Pruppacher and Pitter, 1971]. The calculations given in this chapter are also based on the following assumptions:

- the Marshall-Palmer dropsize distribution ($\mu = 0$, $N_o = 8000 \text{ mm}^{-1} \text{ m}^{-3}$) is valid,
- the spread of the canting-angle distribution is 20°,
- the radar elevation is 30°,

unless explicitly stated otherwise. The 30° elevation angle has been chosen, because it allows the combination of Doppler and polarization measurements. Where $\mu \neq 0$, $N_o$ is derived from the Marshall-Palmer distribution by keeping the total volume $V_t$ constant for a given $D_o$.

5.2 $Z_h$, $Z_{dr}$, $L_{dr}$, and $W_d$ as function of $D_o$, $\mu$ and $\sigma_\delta$

Figure 5.1 shows the radar observables as function of $D_o$. $Z_{dr}$ is smaller than 2 dB and will often be smaller than 1 dB, because median dropsizes larger than 2 mm correspond to rarely occurring rain intensities larger than 50 mmh$^{-1}$. If a large $Z_{dr}$ is measured at a low rain intensity, the radar cell is probably filled with a small number of drops in which the large drops predominate. Usually $L_{dr}$ is smaller than −20 dB. Most operational radars would only be able to measure it when $D_o > 1.5$ mm and consequently the rain intensity greater than approximately 15 mm/h: lower values of $L_{dr}$ would require antennas with a high cross-polar isolation. $Z_{dr}$ and $L_{dr}$ increase when $D_o$ increases, because of the increased oblateness of the raindrops. $Z_h$ increases then as well, but this is primarily caused by the increased size. $W_d$ shows a different behavior: it increases when $D_o < 0.8$ mm and decreases again when $D_o > 0.8$ mm. As explained in chapter 3, this is caused by the asymptotic velocity-size relationship. The figures clearly show that each radar observable reacts differently to variations of $D_o$. 

91
Figure 5.1  $Z_{dr}$, $L_{dr}$, $Z_h$, and $W_d$ as function of $D_\phi; \mu = 0, \sigma_\delta = 20^\circ$. 
Figure 5.2  \( Z_{dr}, L_{dr}, Z_h, \) and \( W_d \) as function of \( \mu; D_o = 1 \) mm, \( \sigma_\delta = 20^\circ. \)
In chapter 2, figure 2.4, the distribution of drop volumes is shown for different values of \( \mu \). For a fixed \( V_t \) the distribution becomes narrower when \( \mu \) increases, and hence the numbers of large and small drops decrease. The radar observables are high-order moments of the drop size distribution: their dependence upon \( D \) is stronger for large drops than for small ones. Consequently, the radar observables decrease when \( \mu \) increases, as shown in figure 5.2 for \( D_o=1 \). Calculations for other values of \( D_o \) did not reveal significant deviations from the qualitative trends, although the actual values differed. The figures clearly show that each radar observable reacts differently to variations of \( \mu \).

Figure 5.3 shows \( Z_h \), \( Z_{dr} \) and \( L_{dr} \) as function of \( \sigma_\delta \); \( D_o=1 \) mm. \( W_d \) has been omitted, because it does not take canting into account. \( Z_h \) equals approximately 23 dBZ, and does not show a significant dependence upon \( \sigma_\delta \). \( Z_{dr} \) decreases from \( \pm 0.6 \) dB to \( \pm 0.15 \) dB. With increasing randomness of the microstructure, \( Z_{dr} \) becomes less suited for determination of the mean particle shape. \( L_{dr} \) increases when \( \sigma_\delta \) increases. Only \( D_o=1 \) mm is shown, but similar tendencies occur for other values of \( D_o \). Figure 5.3 clearly shows that each radar observable reacts differently to variations of \( \sigma_\delta \).

![Graph showing the relationship between spread canting angle and radar observables](image)

Figure 5.3 \( Z_h \), \( Z_{dr} \) and \( L_{dr} \) as function of \( \sigma_\delta \); \( D_o=1 \) mm, \( \mu=0 \). Note that \( -L_{dr} \) is depicted.
5.3 Sensitivity and cross-correlation; definition and meaning

$N_o$ affects only $Z_h$. The other parameters of the microstructure, $\mu$, $D_o$, and $\sigma_h$, also affect $Z_{dr}$, $L_{dr}$, and $W_d$. To investigate the influence of the microstructure on the radar observables, the sensitivity factor $S_\alpha^\beta$ is used. It is defined as

$$S_\alpha^\beta = \frac{\partial \alpha(\beta)}{\partial \beta} \frac{\beta}{\alpha(\beta)}$$

in which $\alpha$ symbolizes the radar observable under consideration and $\beta$ a parameter of the dropsize distribution. The sensitivity factor quantifies the effect of a change $\Delta \beta$ in $\beta$ on $\alpha$:

$$\frac{\alpha(\beta + \Delta \beta)}{\alpha(\beta)} = \left( \frac{S_\alpha^\beta \Delta \beta + \beta}{\beta} \right)$$

If $\alpha(\beta)$ is insensitive to changes of $\beta$, $S_\alpha^\beta = 0$. For linear relationships, $S_\alpha^\beta = 1$, and hence the relative change of $\alpha(\beta)$ equals the relative change of $\beta$. For a non-linear relationship, $S_\alpha^\beta$ is a function of $\beta$. When $S_\alpha^\beta > 1$, the relative change of $\alpha(\beta)$ is greater than that of $\beta$: $\alpha(\beta)$ is more than proportionally sensitive. When $S_\alpha^\beta < 1$, the relative change of $\alpha(\beta)$ is less than that of $\beta$: $\alpha(\beta)$ is less than proportionally sensitive. $S_\alpha^\beta$ is dimensionless, which makes it suitable for the comparison of different radar observables.

To investigate which parameters are involved when the microstructure varies, the cross-correlation coefficient $R_{xy}$ is used. $R_{xy}$ is defined as

$$R_{xy} = \frac{E[(x - \bar{x})(y - \bar{y})]}{\sigma_x \sigma_y}$$

in which $\sigma_x$ and $\sigma_y$ are the standard deviations of the stochastic variables $x$ and $y$. $E[\cdot]$ is the statistically expected value. $R_{xy}$ estimates the degree of linear dependence between two variables. $R_{xy} = 0$ implies that no linear relationship between $x$ and $y$ exists. A positive $R_{xy}$ implies that when $x$ increases, $y$ increases as well. A negative $R_{xy}$ implies that when $x$ increases, $y$ decreases, or vice versa. $R_{xy} = \pm 1$ implies a direct one-
to-one relationship between $z$ and $y$. However, low correlation figures do not necessarily mean that no relationship exists at all: $z$ could be non-linearly related to $y$. Now, by calculating the cross-correlation coefficient of the radar observables, and interpreting it in terms of the sensitivity of the radar observables, dynamic processes in precipitation may be recognized. For instance, when the canting-angle distribution becomes broader, $Z_{dr}$ decreases and $L_{dr}$ increases, which results in a negative correlation coefficient between the two. Or, when the number of drops increases, $Z_h$ increases, but $Z_{dr}$, $L_{dr}$ and $W_d$ do not necessarily change. This may result in no correlation at all between $Z_h$ and the other three radar observables.

5.4 The sensitivity of the radar observables

The polarimetric radar observables are usually expressed in the logarithmic units dB or dBZ. However, it is elucidating to see how their sensitivity changes when they are expressed in linear units. A straightforward calculation shows that the two sensitivity factors can be related as

$$S_\alpha^\beta = \ln(\alpha(\beta)) S_{10\log(\alpha)}^\beta$$  \hspace{1cm} (5.11)

$W_d$ and $Z_{dr}$ are usually smaller than $e=(2.7..)$, implying that they are most sensitive to changes in $\beta$ when they are expressed in logarithmic units. $Z_h$ and $L_{dr}$ are most sensitive to changes in $\beta$ when they are expressed in linear units, because usually $Z_h$ is larger than $e$ and $L_{dr}$ smaller than $e^{-1}$. Calculations in this section are based on the linear expressions of the radar observables, because the sensitivity factor of $Z_h$ does not depend on $N_o$ then. The absolute values of the sensitivity factors are presented graphically for ease of comparison.

Figure 5.4 shows the sensitivity factor of the radar observables with respect to variations of $D_o$. $Z_h$ is by far the most sensitive to changes of $D_o$. When $D_o \rightarrow 0$, $S_\alpha^Z \rightarrow -\infty$, because $Z_h$ becomes 0. The sensitivity factor of $L_{dr}$ decreases rapidly when $D_o$ increases. On average its sensitivity factor is larger than the one of $Z_{dr}$. The sensitivity factor of $W_d$ becomes 0 at $D_o \approx 0.8$ mm, because $W_d$ is at its maximum there and hence its gradient 0. Only $\mu=0$ is shown, but other values of $\mu$ were investigated too.
when $\mu$ increases the individual sensitivity factors become smaller, but the general tendencies remain unchanged. When $Z_h$, $Z_{dr}$, and $L_{dr}$ are expressed in dB $Z_{dr}$ is the most sensitive radar observable.

![Graph showing sensitivity factors](image)

Figure 5.4  The sensitivity factors of $Z_h$, $Z_{dr}$, $L_{dr}$ and $W_d$ with respect to changes of $D_o$; $\mu=0$ and $\sigma_\delta=20^\circ$. Calculations based on linear units.

Figure 5.5 shows the sensitivity factors of the radar observables with respect to changes of the dispersion factor $\mu$; $D_o=1$ mm. All sensitivity factors are smaller than 100%. $L_{dr}$ is the most sensitive observable, and $Z_{dr}$ is hardly sensitive to changes in $\mu$. Note that the sensitivity factors of all radar observables increase when $\mu$ increases. The radar observables are less sensitive to changes of $\mu$ than they are to changes of $D_o$. This implies a larger effect of $D_o$ than of $\mu$, in case they change simultaneously. Only $D_o=1$ mm is shown, but further analysis has revealed that the sensitivity factors of $L_{dr}$ and $W_d$ decrease when $D_o$ increases. However, the sensitivity factor of $Z_{dr}$ increases then. When $Z_h$, $Z_{dr}$, and $L_{dr}$ are expressed in dB, $Z_{dr}$ becomes the most sensitive to changes of $\mu$ then.
Figure 5.5 The sensitivity factors of $Z_h$, $Z_{dr}$, $L_{dr}$ and $W_d$ with respect to changes of $\mu$; $\sigma_\delta = 20^\circ$, $D_o = 1$ mm. Calculations are based on linear units.

Figure 5.6 shows the sensitivity factors for variations of the spread $\sigma_\delta$ of the canting-angle distribution; $D_o = 1$ mm. Again, $W_d$ has been omitted, because of the absence of particle canting in the Doppler analysis. $L_{dr}$ is the most sensitive to changes of the orientation of the particle. However, its sensitivity factor decreases rapidly when $\sigma_\delta$ increases. When linearly expressed, $Z_{dr}$ and $Z_h$ are comparably sensitive. However, when they are expressed in dB, $Z_{dr}$ gains sensitivity, while $Z_h$ and $L_{dr}$ lose theirs. For large $\sigma_\delta$, it appears that $Z_{dr}$ is the only sensitive parameter: $Z_h$ and $L_{dr}$ are hardly sensitive to variations of the particle orientation. Again, the tendencies remain unchanged for other values of $D_o$, although the values of the sensitivity factors change somewhat.
Figure 5.6 The sensitivity factors of $Z_h$, $Z_{dr}$, and $L_{dr}$ with respect to changes of $\sigma_\delta$; $\mu = 0$ and $D_o = 1$ mm. Calculations are based on linear units.

5.5 The interrelationships between the radar observables

The sensitivity of the radar observables was discussed in the preceding section. It was found that the radar observables respond differently to changes of $D_o$, $\mu$, and $\sigma_\delta$. When these change simultaneously, the combined effect is seen in the radar observables. To identify the parameters of the microstructure that have changed, the interrelationships between the radar observables are required. For instance, when $D_o$ and $\sigma_\delta$ increase at the same time, $Z_{dr}$ increases less than it would in case there were an increase in $D_o$ alone. On $L_{dr}$, however, canting has the opposite effect and $L_{dr}$ increases even more. The exact impact of the changing microstructure on the radar observables depends on the sensitivity factors. However, since four parameters are used to describe the microstructure, analysis of the interrelationships of measured radar observables is complicated. In subsequent sections, a measurement with the Delft Atmospheric Research Radar will be discussed and the statistical relationships between the radar
observables will be simulated by means of randomizing \( N_o, D_o, \mu, \) or \( \sigma_\delta \). In the process a simple model that relates canting to turbulence will be derived. Here, the interrelationships between \( Z_h, Z_{dr}, L_{dr}, \) and \( W_d \) are described.

Figure 5.7 shows \( Z_{dr} \) versus \( Z_h \), for \( \mu = 0 \) or \( \mu = 6 \). Either \( D_o \) varies and \( \sigma_\delta \) is constant, or \( \sigma_\delta \) varies and \( D_o \) is constant. For a fixed \( \sigma_\delta \), \( Z_h \) and \( Z_{dr} \) both increase when \( D_o \) increases. For a given \( D_o \), \( Z_h \) hardly changes when \( \sigma_\delta \) increases, but \( Z_{dr} \) decreases then. The change of \( \mu \) from 0 into 6 results in a decrease of \( Z_h \) and \( Z_{dr} \). Qualitatively, \( D_o \) and \( \mu \) have the same effect on \( Z_h \) and \( Z_{dr} \). Quantitatively, their influence differs because the corresponding sensitivity factors differ. When only \( \sigma_\delta \) changes, \( Z_h \) and \( Z_{dr} \) are hardly correlated. When \( D_o \) or \( \mu \) changes, the correlation is positive. When all change, the cross-correlation coefficient can have any value, dependent on the degree of variation of \( D_o, \mu \) and \( \sigma_\delta \).

![Diagram showing \( Z_{dr} \) versus \( Z_h \) with different curves for \( D_o \), \( \sigma_\delta \), and \( \mu \).](image)

**Figure 5.7** \( Z_{dr} \) versus \( Z_h \). Curves with \( D_o = 1, 1.5 \) or \( 2 \) \( \text{mm} \), or \( \sigma_\delta = 1, 10, 20, 30 \) or \( 40^\circ \); \( \mu = 0 \) or \( 6 \).

Figure 5.8 shows \( Z_h \) versus \( L_{dr} \). The same conditions apply as shown in figure 5.7. Again, variations of \( \sigma_\delta \) alone result in a small cross-correlation coefficient, while variations of \( D_o \) or \( \mu \) may cause a large correlation between \( Z_h \) and \( L_{dr} \).

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Figure 5.8 $L_{dr}$ versus $Z_h$. Curves of $D_0 = 1, 1.5$ or $2$ mm, or $\sigma_\delta = 1, 10, 20, 30$ or $40$; $\mu = 0$ or $6$.

Figure 5.9 $W_d$ versus $Z_h$ for different values of $\mu$. No canting.
Figure 5.10 $Z_{dr}$ versus $L_{dr}$. Curves of $D_o=1$, 1.5 and 2 mm, or $\sigma_\delta=1$, 10, 20, 30, and 40$^\circ$; $\mu$ is set to 0 and 6.

Figure 5.11 $Z_{dr}$ versus $W_d$ for different values of $\mu$. No canting.
Figure 5.12 $L_{dr}$ versus $W_d$ for different values of $\mu$; $\sigma_\delta=40^\circ$.

Figure 5.9 shows $W_d$ versus $Z_h$, for $\mu=0, 2, 4, 6$. The maximum of $W_d$ occurs at values of $Z_h$ that increase when $\mu$ increases. The gradient of the $W_d$-$Z_h$ relationship is positive for small $Z_h$, and negative for large $Z_h$. This indicates that the cross-correlation coefficient can be positive as well as negative when it is caused by $D_o$ variations alone. However, during most rain events $Z_h$ will be larger than 20 dBZ and hence a negative correlation may be expected. When $\mu$ increases, both $W_d$ and $Z_h$ decrease and so variations of $\mu$ may cause a positive correlation.

Figure 5.10 shows $Z_{dr}$ versus $L_{dr}$, derived under the same conditions as the previous plots. For a fixed $\sigma_\delta$, $Z_{dr}$ and $L_{dr}$ both increase when $D_o$ increases. For a fixed $D_o$, $Z_{dr}$ becomes smaller and $L_{dr}$ becomes larger when $\sigma_\delta$ increases. When $D_o$ and $\sigma_\delta$ both change at random, the cross-correlation between $Z_{dr}$ and $L_{dr}$ may be small, because of the opposing influences of $D_o$ and $\sigma_\delta$.

Figure 5.11 shows $Z_{dr}$ versus $W_d$. No canting is assumed, and $\mu=0, 2, 4, \text{and } 6$. For $Z_{dr} > 0.5 \text{ dB}, W_d$ and $Z_{dr}$ are negatively correlated: $Z_{dr}$ increases when $W_d$ decreases. When $\mu$ increases, $W_d$ and $Z_{dr}$ decrease. Therefore, when for a fixed $D_o$, $\mu$ changes, a
positive cross-correlation can be expected. And when both change simultaneously, decorrelation occurs.

Figure 5.12 shows \( W_d \) versus \( L_{dr} \) for \( \sigma_\delta = 40' \). For \( L_{dr} > -30 \) dB, \( W_d \) and \( L_{dr} \) are negatively correlated when \( D_o \) changes. When only \( \mu \) changes, a positive correlation coefficient may be expected.

The plots that involve \( W_d \) do not take turbulence into account although turbulence may cause spectral broadening. Turbulence induces particle canting, which reduces \( Z_{dr} \) and causes a larger \( L_{dr} \), but hardly affects \( Z_h \). An exact relationship between canting and turbulence is not yet known and so the effect of turbulence on the given plots can only be described qualitatively. In chapter 3, it was stated that the total variance of the Doppler spectrum is the sum of the individual variances of contributing variables. When only turbulence and precipitation are considered, the measured spectrum width \( W_{d,m} \) is

\[
W_{d,m} = \sqrt{W_i^2 + W_d^2}
\]

When the turbulent air motions are increasing in strength, small values of \( W_d \) are more affected than large ones. The effect of canting is most significant for large values of \( Z_{dr} \) and small values of \( L_{dr} \). Consequently, the \( Z_{dr} - W_d \) and \( L_{dr} - W_d \) plots become narrower.

<table>
<thead>
<tr>
<th></th>
<th>( N_o )</th>
<th>( D_o )</th>
<th>( \mu )</th>
<th>( \sigma_\delta )</th>
<th>( W_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_h )</td>
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<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Z_{dr} )</td>
<td>0</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( L_{dr} )</td>
<td>0</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( W_{d,m} )</td>
<td>0</td>
<td>+/-</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 5.1  The response of \( Z_h \), \( Z_{dr} \), \( L_{dr} \), and \( W_{d,m} \) to an increase of \( N_o \), \( D_o \), \( \mu \), \( \sigma_\delta \) and \( W_t \). Increase +; decrease: –; no change: 0.
Quantifying the effect of turbulence is difficult. However, when during a rain event canting is induced by turbulence and no other factors are involved, a positive correlation may be expected between $W_d$ and $L_{dr}$. The correlation between $Z_{dr}$ and $W_d$ is then negative. One important parameter of the dropsize distribution has not yet been considered: the scaling factor $N_o$. Of the radar observables discussed, only $Z_h$ depends on $N_o$. Consequently, a variation of $N_o$ will decorrelate $Z_h$ from the other radar observables. Table 5.1 summarizes the response of $Z_h$, $Z_{dr}$, $L_{dr}$, and $W_{d,m}$ on changes of $N_o$, $D_o$, $\mu$, $\sigma_\delta$, and $W_t$.

5.6 Measurement of a rain event; one range cell

The theory of the preceding sections is here compared with a series of measurements with the Delft Atmospheric Research Radar, which were performed during the passage of a cold front on December 20, 1989. One range cell is selected for analysis. The scatter diagrams of $Z_h$, $Z_{dr}$, $L_{dr}$ and $W_{d,m}$ are discussed. Note that $W_{d,m}$ is used instead of $W_d$ to distinguish the measured Doppler-spectrum width from the theoretical one. In the following section scatter diagrams are simulated, by allowing a certain degree of random variation of $N_o$, $D_o$, $\mu$ and $\sigma_\delta$. The measurements will be used to introduce a simple model of the relationship between turbulence and the canting-angle distribution.

<table>
<thead>
<tr>
<th>Ground temperature 10 °C</th>
<th>Maximum radar range</th>
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</thead>
<tbody>
<tr>
<td>0 °C isotherm</td>
<td>2000 m</td>
<td>Range resolution</td>
</tr>
<tr>
<td>Mean wind speed</td>
<td>20 m/s</td>
<td>Elevation</td>
</tr>
<tr>
<td>Mean wind direction</td>
<td>South-west</td>
<td>Maximum Doppler velocity</td>
</tr>
<tr>
<td>Weather type</td>
<td>Cold front</td>
<td>Signal-to-noise ratio</td>
</tr>
</tbody>
</table>

Table 5.2 The meteorological and experimental conditions during the radar measurements

The meteorological and experimental conditions during the measurements are given in table 5.2. The measurements lasted for approximately 1.5 hours. Concurrent rain gauge measurements indicated a mean rain intensity of approximately 3 mmh$^{-1}$. Initially, the measurements were carried out with the radar pointed perpendicular to the mean wind direction in order to minimize the influence of wind on the Doppler spectrum. Halfway
through the measurements the azimuth angle of the radar was changed 10°, which, as will be discussed in the next chapter, revealed the presence of turbulence on particle canting.

Figure 5.13 shows the mean height profile of $Z_h$, $Z_{dr}$, $L_{dr}$, $V_d$, and $W_{d,m}$, integrated over 42 minutes and 300 meter. Around an altitude of 2000 meter, a region of enhanced reflectivity occurs; this phenomenon is caused by melting snowflakes that eventually turn into raindrops. In this bright band, $W_{d,m}$ increases gradually and $Z_{dr}$ and $L_{dr}$ exhibit a peak value. In the rain region below the bright band $Z_{dr}$ and $Z_h$ no longer show a significant variation with height, whereas $L_{dr}$ and $W_{d,m}$ still do. On average, $Z_{dr}$, $L_{dr}$, and $W_{d,m}$ are larger in the rain region than in the snow region above the bright band. Note the relatively large values of $L_{dr}$ and $W_{d,m}$. In rain, $W_{d,m}$ almost always exceeds 1 m/s, which is larger than could have been caused by rain alone. Clearly other factors are involved. $V_d$ is not used for analysis, because it is distorted by horizontal wind. In chapter 7, on backscattering by the melting layer, the radar measurements of the bright band are discussed and a new model describing the behavior of the radar observables is introduced. In this chapter only the rain region is discussed, although the analysis will lead to some important remarks concerning the melting layer.
Figure 5.13 The height profile of the radar observables, integrated over 42 minutes and 300 meter.
Figure 5.14 Scatter diagrams of the radar observables, belonging to range cell 10 at a height of 850 meter. Integrated over 64 seconds and 300 meter. The cross-correlation coefficients have been calculated from linear data.
Figure 5.14 shows the scatter diagrams of the radar observables taken from data that belongs to range cell 10 at a height of 850 meter. The individual measurements are integrated over 64 seconds and 300 meter. On top of each plot the corresponding cross-correlation coefficient $R_{xy}$, calculated with linear data, is given. These calculations were also carried out on logarithmic data, but no significant difference between the two results appeared. If the variations of the radar observables were the product of one common cause, taking the logarithm of the radar observables would indeed have made a difference. Since no difference was observed, at least two parameters of the microstructure must have changed simultaneously, thereby canceling the effect of taking the logarithm.

$Z_h$ and $Z_{dr}$ are positively correlated, with $R_{xy} \approx 60\%$. With reference to table 5.1, positively correlating phenomena can be attributable to a varying $\mu$ and $D_o$. Since $R_{xy}$ is only 60\%, variations of $N_o$ and $\sigma_{\delta}$ may have caused some decorrelation. Suppose that $\sigma_{\delta}$ varies, then $Z_{dr}$ is less affected than $L_{dr}$ due to the smaller sensitivity factor of $Z_{dr}$. Consequently, $Z_h$ is more decorrelated from $L_{dr}$ than from $Z_{dr}$. This is indeed what is observed: $L_{dr}$ and $Z_h$ are not correlated at all.

The correlation between $Z_h$ and $W_{d,m}$ is very small: only $-15\%$. As can be seen in figure 5.9, a negative correlation can be expected due to variations of $D_o$. If $\mu$ changes, a positive correlation results and turbulence would result in no correlation at all. Considering the small correlation coefficient, all variables may have changed simultaneously.

The correlation coefficient of $L_{dr}$ and $W_{d,m}$ is approximately 64\%. Due to variations of $D_o$ a negative correlation coefficient would be expected, while variations of $\mu$ would have caused a positive correlation. However, since $W_{d,m}$ is larger than could be caused by dropsizes alone, variations of $\mu$ are not sufficient to explain the observed correlation: turbulence must be considered as a major common cause as well. Since $L_{dr}$ and $W_{d,m}$ are small quantities and therefore sensitive to noise, the calculation was repeated with a minimum signal-to-noise ratio of 30 dB, but no significant changes were observed.

$L_{dr}$ and $Z_{dr}$ are not strongly correlated. A varying $D_o$ and $\mu$ would result in a positive correlation, and a varying $\sigma_{\delta}$ in a negative one. When all parameters change simultaneously, a small correlation coefficient can be expected. The same holds true for the correlation between $Z_{dr}$ and $W_{d,m}$: variation of $\mu$ would result in a positive correlation, while variation of $D_o$ would result in a negative correlation for $Z_{dr} > 0.5$.
dB, and a positive one for $Z_{dr} < 0.5$ dB. A simultaneous variation of $\mu$ and $D_0$ may therefore lead to a small correlation coefficient. Turbulence also decorrelates $Z_{dr}$ and $W_{d,m}$; while the width of the Doppler spectrum increases, $Z_{dr}$ decreases due to canting.

5.7 Simulation of a rain event: one range cell

Is it possible to simulate the scatter diagrams of the previous section? To find out, the parameters of the microstructure are randomized; $V_t$, $D_0$, $\mu$, and $\sigma_\delta$ were artificially varied following a uniform probability density function. $N_o$ is then indirectly randomized because of its relationship with $V_t$. Table 5.3 gives the intervals of change, when appropriate, of the different parameters. $V_t^{MP}$ is the water volume for a given $D_0$, assuming the Marshall-Palmer dropsize distribution. For other values of $\mu$, $V_t^{MP}$ is used to calculate $N_o$.

<table>
<thead>
<tr>
<th>When variable</th>
<th>When constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0$</td>
<td>[1.1, 1.3] mm</td>
</tr>
<tr>
<td>$\mu$</td>
<td>[-1, 1]</td>
</tr>
<tr>
<td>$\sigma_\delta$</td>
<td>[25°, 40°]</td>
</tr>
<tr>
<td>$V_t$</td>
<td>$V_t^{MP} \cdot C$</td>
</tr>
<tr>
<td>$C = [0.5, 2]$</td>
<td>$V_t^{MP}$</td>
</tr>
</tbody>
</table>

Table 5.3 The intervals of change of $D_0$, $\mu$, $\sigma_\delta$, and $V_t$ during the simulations.

Figure 5.15 shows the scatter diagrams in case only $D_0$ is changing. Adding a randomly varying $\sigma_\delta$ results in the scatter diagrams of figure 5.16. Finally, figure 5.17 shows the effect of variations of $N_o$ and $\mu$. When only $D_0$ changes, the correlation between all radar observables is high, but not always perfect: the non-linear nature of the interrelationships between the radar observables slightly decorrelates. Figures 5.15 and 5.16 clearly show that a varying $\sigma_\delta$ decorrelates the radar observables. However, the model does not depend on canting yet, which makes the plots involving $W_d$ less realistic. The variation of $N_o$ and $\mu$ decorrelate most of the radar observables even more. Only the correlation between $Z_h$ and $Z_{dr}$ is slightly increased. $N_o$ only affects $Z_h$; the decorrelation of $Z_{dr}$, $L_{dr}$ and $W_d$ is caused by variations of $\mu$.

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Figure 5.15 Simulated scatter diagrams of the radar observables. $D_o \in [1.1, 1.3]$ mm, $\mu = 0$, $\sigma_{\delta} = 30^\circ$, $V_t = V_t^{MP}$. 
Figure 5.16 Simulated scatter diagrams of the radar observables. \( D_o \in [1.1, 1.3] \) mm, \( \sigma_\delta \in [25^\circ, 40^\circ] \), \( \mu = 0 \), \( V_i = V_i^{MP} \).
Figure 5.17 Simulated scatter diagrams of the radar observables. $D_o \in [1.1, 1.3] \text{ mm}, \sigma_\delta \in [25^\circ, 40^\circ], \mu \in [-1, 1], V_t \in [\frac{1}{2}, 2], V_t^{MP}$
Referring to figure 5.14, where measured data is shown, it appears that the $Z_{h^*}Z_{d^r}$ and $Z_{d^r}L_{d^r}$ scatter diagrams are simulated well. The simulated scatter diagram of $Z_{h^*}$ and $L_{d^r}$ still differs from the measured one: during the measurement no correlation was found, while the simulations still suggest a significant correlation. Extensive simulations, under many different conditions of parameters of the microstructure, did not result in a fit of the two without affecting the other diagrams too much.

The measured and the simulated scatter diagrams suggest a relationship between $\sigma_\delta$ and $D_o$. If small drops are more sensitive to turbulence, then $\sigma_\delta$ increases when $D_o$ decreases. The effect this would have on the simulated scatter diagram is that $L_{d^r}$ increases at small $Z_{h^*}$. The regression line would be tilted towards the horizontal as it is in the measured scatter diagram. However, to date no model relating $D_o$ to $\sigma_\delta$ is known to verify this hypothesis.

The measured and simulated scatter diagrams that involve $W_d$ do not look alike, because turbulence has not yet been considered. In the following section a simple model relating turbulence and canting is proposed and compared with the measured data.

5.8 A simple turbulence-canting model

The relationship between turbulence and particle canting is modeled, using the data shown in figure 5.14. $L_{d^r}$ is significantly correlated to $W_{d,m}$ and the figure suggests that a linear relationship between the two exists:

$$W_{d,m} = \kappa_1 L_{d^r} + \kappa_2$$  \[L_{d^r} \text{ in dB}]  \tag{5.13}$$

To extract the contribution of the turbulent spectrum, equation 5.13 is re-written as

$$\sqrt{W^2_d + W^2_t} = \kappa_1 L_{d^r} + \kappa_2$$  \tag{5.14}$$

which yields

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\[ W_t = \sqrt{\left(\kappa_1 L_{dr} + \kappa_2\right)^2 - W_d^2} \]  

5.15

With reference to figure 5.3, the relationship between \( L_{dr} \) and \( \sigma_\delta \) [deg] can be written in the form

\[ L_{dr} = L_{dr, max} \left\{ 1 + e^{-\frac{(\sigma_\delta + 2)}{10}} \right\} \pm 0.25 \text{ [dB]} \]  

5.16

which, when combined with equation 5.15, relates the width of the turbulence spectrum to canting. The model is complicated by several factors: \( W_d \) may change during the event, \( L_{dr, max} \) depends on \( D_o \), which may vary during the event, and the constants \( \kappa_1 \) and \( \kappa_2 \) are regression constants and may therefore differ from event to event.

Considering that a measurable \( L_{dr} \) is larger than \(-35\) dB and that for most types of rain \( 1 < D_o < 2 \) mm, it can be assumed that an observed \( L_{dr} \) is caused by values of \( \sigma_\delta \) larger than 10°. In the given range of \( D_o \) and for \( \sigma_\delta \in [10^\circ, 40^\circ] \), \( L_{dr, max} \) may vary between \(-35\) dB and \(-20\) dB. Since the sensitivity factor of \( L_{dr} \) decreases when \( \sigma_\delta \) increases, the effect of canting saturates for large values of \( \sigma_\delta \). Consequently, the probability density function of \( L_{dr, max} \) is skewed towards high values of \( \sigma_\delta \). When \( D_o \) is set to 1, 1.5, or 2 mm and \( \mu = 0 \), \( L_{dr, max} \) equals respectively \(-28\), \(-24\) and \(-21\) dB. Curve fitting \( L_{dr, max} \) to \( D_o \text{[mm]} \) results in

\[ L_{dr, max} = -21 \cdot \left\{ 1 + e^{-\left(\frac{D_o - 0.4}{0.55}\right)} \right\} \pm 0.25 \text{ [dB]} \]  

5.17

Previous experiments made using the Delft Atmospheric Research radar during calm events with low rain intensities, showed that \( W_d \) is normally distributed between 0.4 m/s and 0.9 m/s with a mean of 0.6 m/s and a standard deviation of approximately 0.2 m/s [Russchenberg and Ligthart, 1989]. To account for the influence of \( W_d \) on \( W_t \), the mean of equation 5.15 is calculated by integrating over \( W_d \), assuming the normal distribution of \( W_d \).
\( \kappa_1 \) and \( \kappa_2 \) are obtained through regression analysis of the data, and are equal to 0.104 and 4.13, respectively. Too little data has been analyzed to extend the validity of the constants to other rain events; one should be cautious when using the model on other data: \( \kappa_1 \) and \( \kappa_2 \) may be different.

![Graph](image)

Figure 5.18 The spread of the turbulence spectrum versus the spread of the canting-angle distribution for different values of \( D_o \).

Figure 5.18 shows \( W_t \) as function of \( \sigma_\delta \) for \( D_o = 1, 1.5 \) and 2 mm. The model clearly shows that small drops need less turbulence than large ones to account for a certain spread of the canting-angle distribution. In a real rain event this mean that, given a certain degree of turbulence, small drops are more canted than large ones, as was already assumed in the previous section. Figure 5.19 shows \( W_t \) versus \( L_{dr} \). Also given are the regression line of the measured data and a 3rd order polynomial fit to the model, given by

\[
W_t = 4.29 + 0.162 L_{dr} + 0.00372 L_{dr}^2 + 0.000074 L_{dr}^3 \quad [\text{m/s}]
\]

5.18
with $L_{dr}$ in dB. The polynome coincides with the model-curve for $L_{dr} > -32$ dB. The model curve is based on calculations made with $D_o = 1, 1.5$ and $2$ mm. The measurement was done at an elevation angle of $30^\circ$; other angles have not been considered.

Figure 5.19 The spread $W_t$ of the turbulence spectrum versus $L_{dr}$.

Figure 5.20 shows the scatter diagrams when the turbulence model is incorporated. Because turbulence is now included in the model, the width of the Doppler spectrum is referred to as $W_{d,m}$. All $W_{d,m}$ values are larger than in the previous scatter diagrams. The correlation coefficient of $L_{dr}$ and $W_{d,m}$ has been increased from $-0.38$ to $0.98$, as it should because of the relationship between $L_{dr}$ and $W_t$. The scatter in the $L_{dr}$-$W_{d,m}$ plot is less than measured, which is attributable to the fact that regression lines represent mean relationships. The scatter diagram of $Z_{dr}$ and $W_{d,m}$ is in good agreement with the measurements: $R_{xy}=0.34$. A positive correlation between $Z_h$ and $W_{d,m}$ has been simulated, whereas a small negative correlation is measured. Again, this suggests that there is some relationship between the dropsize and canting. The other scatter diagrams are hardly affected.
Figure 5.20 Simulated scatter diagrams of the radar observables. $D_o \in [1.1, 1.3]$ mm, $\sigma_\delta \in [25^\circ, 40^\circ]$, $\mu \in [-1, 1]$, $V_t \in \left[\frac{1}{2}, 2\right] \cdot V_{tIP}$. $W_{d,m}$ is based on the turbulence model.
5.9 The correlation at other heights

So far only one range cell has been considered. To investigate the consistency of the results of the previous sections, the height dependence is analyzed. Figure 5.21 shows the height profiles of the cross-correlation coefficients, based on 42 minutes worth of data. The rain region lies below 2000 meter. Most $R_{xy}$ values of the radar observables caused by rain are consistent. The correlation coefficient of $Z_{dr}$ and $Z_h$ fluctuates around 70%. $Z_h$ and $L_{dr}$ are uncorrelated throughout the whole rain region. $Z_h$ and $W_d$ are negatively correlated, as are $W_{d,m}$ and $Z_{dr}$. In the rain region $Z_{dr}$ and $L_{dr}$ are not correlated: the correlation coefficient fluctuates around 0. $L_{dr}$ and $W_{d,m}$ are positively correlated with a correlation coefficient of approximately 50%. Clearly, turbulence is dominantly present; it induces cross-polarization by canting and broadens the Doppler spectrum.

As far as the rain region is concerned, the height profiles confirm the findings in the previous section. In the melting layer around 2000 meter however, the cross-correlation between the polarimetric radar observables shows a peculiar behavior. $Z_h$ is negatively correlated to $Z_{dr}$ and $L_{dr}$ in the upper half of the melting layer. In the lower half they are positively correlated. $Z_{dr}$ and $L_{dr}$ are positively correlated throughout the whole melting layer. Trying to explain the observations through the use of a model similar to the rain model is difficult; a negative correlation between $Z_h$ and $L_{dr}$ can then only be caused by canting, but would also imply a positive correlation between $Z_h$ and $Z_{dr}$, which is not observed.

The density of a snowflake depends on its size (see chapter 2). It is likely that the shape of the snowflakes is also related to the density; it is well known that loose snowflakes are irregularly structured, while ice crystals, having a high density, can be shaped like needles or plates. The measurements indicate that, for a fixed height, an increase of $Z_h$ is caused by particles that are less oblate because $Z_{dr}$ decreases. Combined radar and aircraft measurements in the melting layer by Meischner et al [1991] indicated that wetted ice needles are often present just above the melting layer, and that a more pronounced $Z_{dr}$ is observed during very low reflectivities. The cross-correlation analysis of this section shows that the phenomenon is more general: also during events with a strong bright band $Z_{dr}$ increases when $Z_h$ decreases. In chapter 7, on the melting layer, a new relationship between the axial ratio of snowflakes and their size is proposed, and the effects on radar measurements is simulated. The main feature of the new relationship is that small snowflakes are more oblate than large ones.

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Figure 5.21 Height profiles of the cross-correlation coefficients. The radar observables are integrated over 32 seconds and 300 meter. 42 minutes worth of data.
The radar observables \( Z_h \), \( Z_{dr} \), \( L_{dr} \), and \( W_d \) have different sensitivities to changes in the microstructure of precipitation. When the polarimetric observables are expressed in linear units, the following conclusions hold. \( Z_h \) is most sensitive to variations in the median dropsize; the other radar observables change as well, but less than \( Z_h \). The sensitivity of all observables to changes in the dispersion factor is less than it is for changes in the median dropsize. \( L_{dr} \) is the most sensitive to changes in the dispersion factor. \( L_{dr} \) is also the most sensitive to changes in the orientation-angle distribution. When the radar observables are expressed in logarithmic units, quantities that are smaller than \( e (\approx 2.7..) \), like \( Z_{dr} \) and \( W_d \), will achieve a greater sensitivity factor; quantities that are larger than \( e \) become less sensitive. To simulate measured scatter diagrams of the radar observables, all parameters of the microstructure had to be randomized: all may simultaneously change during the event.

A strong correlation between \( L_{dr} \) and \( W_{d,m} \) is measured. This is interpreted as being caused by turbulence: it broadens the Doppler spectrum and it causes a random canting angle of the raindrops. The regression line of the scatter diagram is used to derive a simple model between \( L_{dr} \) and the spread of the turbulence spectrum. However, the model is based on one event only and further study is necessary in order to generalize it.

The height dependence of the cross-correlation coefficients shows a consistent behavior in the rain region: no large deviations occur. In the melting layer another tendency is observed: the correlation between \( Z_h \) on the one hand and \( Z_{dr} \) and \( L_{dr} \) on the other is negative in the upper part of the melting layer, but positive in the lower part of the melting layer. This implies that in the upper part of the melting layer, small particles are more oblate than large ones and that in the lower part of the melting layer large particles are more oblate than small ones.
Chapter 6
The gamma dropsize distribution; observation and analysis

The conversion of radar reflections into rain intensities is dependent upon assumptions regarding the dropsize distribution. The gamma dropsize distribution contains three unknown parameters; the number of parameters that can be obtained depends on the number of radar observables. When only the reflectivity is measured, one parameter is derived, combining it with the differential reflectivity results in the retrieval of two parameters, and when Doppler measurements are done as well, a third parameter is obtained. In this chapter the combination of $Z_{dr}$ and $W_d$ is used to measure the dispersion factor $\mu$ of the gamma distribution. The corresponding median dropsize $D_o$ and the scaling factor $N_o$ are derived using the combination of $Z_{dr}$ and $Z_h$.

6.1 Existing methods for radar-based prediction of the rain intensity

When one-parameter radar systems are used to obtain the rain intensity $R$, a power law that relates the measured reflectivity $Z$ to it is applied:

$$Z = \alpha R^\beta$$

in which $\alpha$ and $\beta$ are constants that depend on the dropsize distribution. The polarization dependence, and hence the effect of particle shape, is ignored. Usually, $\alpha$ and $\beta$ are derived from long-term rainfall statistics, while their instantaneous values may differ from event to event, and from place to place. $Z$ and $R$ are sensitive to the dropsize distribution and therefore the $Z$-$R$ relationship depends on the type of rain. Battan [1973] lists 69 $Z$-$R$ relationships derived at various locations in the world. To illustrate the dependence on the type of rain, three relationships are highlighted,
with $Z$ given in mm$^6$m$^{-3}$ and $R$ in mmh$^{-1}$. The relationships assume Rayleigh backscattering by raindrops. In the European COST 210 Project on interference of radio links, $\alpha = 400$ and $\beta = 1.4$ were found to optimize the procedures to predict hydrometeor scatter [COST210, 1991]. Equations of the form of 6.1 suggest a constant $N_o$ and $\mu$; $D_o$ is the running parameter. However, as shown in chapter 5, $N_o$ may vary several dB. Collier [1989] reviews a number of error sources that, when a fixed $Z$-$R$ relationship is employed, can affect the accuracy of the rain intensity; some are mentioned here. Because of the dominance of large drops in convective types of rain, the reflectivity may have been increased by approximately 2 dB while in warm frontal rain, the relatively large number of small drops may lead to a decrease in the reflectivity by 3-4 dB. Of course, a simple solution would be to use different values of $\alpha$ and $\beta$ for each type of rain, but that would also require a procedure for recognizing the rain type and hence a multi-parameter radar would be required. Single-parameter radars are not able to identify the hydrometeor types uniquely and therefore misinterpretations of reflections by hail and melting snow may result in an overestimate of the rain intensity by more than 5 dB. The concentration of raindrops may be less than predicted by the Marshall-Palmer distribution when strong downdrafts occur; the associated decrease in reflectivity can be of the order of 5 dB.

A significant error source is the assumption of fixed $N_o$ and $\mu$. With the advent of multi-parameter radar the accuracy of the radar-derived rain intensity may be increased, because more parameters of the drop size distribution can be estimated. Klaassen [1989] used a vertically pointed Doppler radar to derive $N_o$ and $D_o$ of the drop size distribution with $\mu = 0$ and calculated the rain intensity. To this end three Doppler spectrum parameters were defined, corresponding to the minimum, mean and maximum velocity of the raindrops. The method relies heavily on the presence of large drops and can therefore only be used during events with a high rain intensity. While Klaassen parametrized the Doppler spectrum using three velocities, Hauser and Amayenc [1981] used curve-fitting techniques during similar experiments to obtain $N_o$ and $D_o$ from a best fit of the theoretical to the measured Doppler spectrum. Both
methods are very sensitive to the vertical wind speed. Therefore, they lose their applicability in convective storms, where strong up and down drafts disturb the Doppler spectrum too much. To overcome the dependence on the mean wind speed, the variance of the Doppler spectrum is used in this study to characterize the dropsize distribution. The vertical air velocity can be measured directly by wind profilers. These are UHF and VHF radars that measure wind fields, but an increasing effort is put into using them to estimate the rain intensity as well [Chu et al, 1990]. Steiner [1991] used \( Z_{dr} \) to estimate the fall speed of raindrops, which, when combined with Doppler radar measurements, may improve radar measurements of the rain intensity. Fujita et al [1989] report on the use of a dual-wavelength radar to estimate the rain intensity, employing both a long wavelength that does not suffer from attenuation by rain and one that does. The ratio of the two return powers depends only on the attenuation and is converted into the rain intensity using well-known power laws. The method is favorable for deriving \( R \) from a fixed \( Z-R \) relationship, but in the case of strong rain cells the short-wavelength radar signal may be attenuated too much to allow a correct conversion. Also, long time and long range integration is necessary to obtain sufficient accuracy. Seliga and Bringi [1976] gave a strong impulse to the use of multi-parameter radar systems by suggesting the use of orthogonal polarizations for measuring precipitation: combining the differential reflectivity with the horizontal reflectivity significantly increases the accuracy of rainfall measurements. This method is used in this study. Steinhorn and Zrnic [1988] suggest the use of the differential propagation phase shift \( K_{DP} \) to improve the estimate of the dropsize distribution. \( K_{DP} \) is defined as the difference between the propagation-phase shifts at horizontal and vertical polarization of the radar wave; since \( K_{DP} \) depends on the particle shape, it also depends on the dropsize distribution. By combining \( K_{DP} \) with \( Z_h \) and \( Z_{dr} \), three parameters of the dropsize distribution can be obtained, but the method imposes strong demands on the accuracy of the measured differential phase shift (which may be achieved after long time and range integration). For operational radars with sufficient spatial resolution the \( Z-R \) relationship may be replaced by a \( Z_h-K_{DP} \) relationship to improve rainfall estimations and to discriminate between rain and hail. Chandra et al [1991] used \( K_{DP} \) to demonstrate its potential to measure the dispersion factor.

This chapter deals with an alternative method to characterize the three parameters of the dropsize distribution. The combination of \( Z_{dr} \) and \( W_d \) is used to obtain \( \mu \). The corresponding \( D_o \) is derived from \( Z_{dr} \), and in the last step \( Z_h \) is used to obtain \( N_o \). The last two steps follow the approach of Seliga and Bringi [1976], which is discussed in the following section.
6.2 Deriving $N_o$ and $D_o$ from $Z_h$ and $Z_{dr}$

In chapter 3, $Z_h$ and $Z_{dr}$ were related to the dropsize distribution as

$$Z_h = N_o \int \sigma_{hh}(D, \delta) D^{\mu} e^{-\frac{3.67 + \mu}{D_o}} p_\delta(\delta) dD d\delta$$

$$Z_{dr} = \frac{\int \sigma_{hh}(D, \delta) D^{\mu} e^{-\frac{3.67 + \mu}{D_o}} p_\delta(\delta) dD d\delta}{\int \sigma_{vv}(D, \delta) D^{\mu} e^{-\frac{3.67 + \mu}{D_o}} p_\delta(\delta) dD d\delta}$$

in which $p_\delta(\delta)$ is the independent probability-density function of the particle orientation. The radar cross-sections $\sigma_{hh}(D, \delta)$ and $\sigma_{vv}(D, \delta)$ of the individual drops are obtained from the Rayleigh theory. $Z_{dr}$ is independent of $N_o$; when $p_\delta(\delta)$ and $\mu$ are known, measurement of $Z_{dr}$ directly results in $D_o$. The obtained $D_o$ is incorporated into equation 6.5 to derive $N_o$ from the measured $Z_h$. Finally, $N_o$ and $D_o$ are used to calculate the rain intensity.

The relationship between dropsize and shape is important, for it is the basis of the conversion of $Z_{dr}$ into $D_o$. In this study the results of Pruppacher and Pitter [1971] are used, although Cherry and Goddard [1983] modified the relationship by making small drops less oblate to improve the correlation between their radar data and distrometer measurements. Recently, to explain the large $Z_{dr}$ values they measured at C-band Kubista et al [1991] suggested that large drops are more oblate than predicted by the Pruppacher-Pitter model; this would be in agreement with the drop shape model of Beard and Chuang [1987]. The choice for the Pruppacher-Pitter shape model is based on the consideration that it was obtained in a controllable wind tunnel experiment, whereas the others are results of fitting radar data, in which many other factors may have been involved, or are derived theoretically.

The $Z_h$-$Z_{dr}$ method to parametrize the dropsize distribution assumes a fixed $\mu$ and known particle orientation. However, as demonstrated in chapter 5, both may change during the event, thereby affecting the accuracy of the radar-derived rain intensity. To simplify the calculation of the effect of a varying $\mu$ on the derived rain intensity, the particles are considered to be spheres.
When $D_{\text{max}} \to \infty$, then

$$Z = Z_h = N_o \left( \frac{D_o}{3.67 + \mu} \right)^{\mu + 7} (\mu + 6)!$$

The ratio of $Z$ and $R$ is given by

$$\frac{Z}{R} = \frac{(\mu + 6)! \left( \frac{D_o}{3.67 + \mu} \right)^{\mu + 3}}{(\mu + 3)! \left[ 9.65 - 10.3 \left( \frac{3.67 + \mu + \alpha D_o}{3.67 + \mu} \right)^{\mu + 4} \right]}$$

in which the expression of $R$ that was introduced in chapter 5, equation 5.7, is implicitly used. After $D_o$ is obtained from $Z_{d_r}$ for some fixed value of $\mu$, equation 6.8 can be used to derive $R$ from the measured $Z$. To estimate the standard deviation of $Z/R$ one needs to know the range of values $\mu$ that may occur. Ulbrich [1983], Bringi et al [1983], and Goddard and Cherry [1984] mainly reported values of $\mu$ between 0 and 5, although negative values may occur as well. Figure 6.1 shows the relative error, defined as the standard deviation normalized to the mean, of $Z/R$ as function of the reflectivity that corresponds to the $Z$-$R$ relationship of the Marshall-Palmer distribution, $Z = 200 R^{1.6}$. The range of variation of $\mu$ is either set to [-2, 6] or [0, 6]. In the former case the error varies between 35-55%, while in the latter it is confined to 15-20%. It is clear that when $\mu$ is obtained, the accuracy of remotely measuring the rain intensity is significantly increased.

6.3 Deriving $\mu$ from $Z_{d_r}$ and $W_d$

In chapter 5, figure 5.11, $Z_{d_r}$ is plotted versus $W_d$ for $\mu$ varying between 0 and 6. If there is sufficient accuracy, a combined measurement of $Z_{d_r}$ and $W_d$ can be converted into a value of $\mu$. An analytical expression of $W_d$ as function of $Z_{d_r}$ and $\mu$ is difficult to derive, which necessitates numerical techniques to investigate the accuracy of the method to determine $\mu$. The accuracy depends on several factors related to the rain model used and to the statistical nature of rain. To begin with the model-related factors, the accuracy can be affected by the choice of the maximum diameter.
Figure 6.1 The relative error of $Z/R$ due to variation of $\mu$ versus $Z = 200R^{1.6}$; $\mu$ varies between $[0, 6]$, or $[-2, 6]$.

$D_{\text{max}}$ and the spread of the canting-angle distribution. $D_{\text{max}}$ affects $W_d$ as well as $Z_{dr}$, turbulence directly affects $W_d$, and the turbulence-induced canting influences $Z_{dr}$. As previously mentioned, the influence of $D_{\text{max}}$ is insignificant as long as $D_{\text{max}}/D_o > 2.5$. This requirement is often satisfied, because $D_o < 2$ mm and $D_{\text{max}} \approx 8$ mm. It is also likely that $D_{\text{max}}$ increases when $D_o$ increases, thereby decreasing the probability of $D_{\text{max}}/D_o < 2.5$.

To achieve the required measurement accuracy of $W_d$, the gradient of $W_d$ due to variable $\mu$ is calculated. When $D_{\text{max}} \to \infty$, $W_d^2$ can be written as

$$W_d^2 = 10.3^2 \left\{ \left( \frac{3.67 + \mu}{3.67 + \mu + 1.2D_o} \right)^{\mu + 7} - \left( \frac{3.67 + \mu}{3.67 + \mu + 0.6D_o} \right)^{2(\mu + 7)} \right\}$$

which follows from manipulating
\[ Z = \int N(D) D^6 dD = N_o \left( \frac{D_o}{3.67 + \mu} \right)^{\mu+7} (\mu+6)! \quad 6.10 \]

\[ V_d = \frac{1}{2} \int N(D) D^6 v(D) dD = 9.65 - 10.3 \left( \frac{3.67 + \mu}{3.67 + \mu + 0.6D_o} \right)^{\mu+7} \quad 6.11 \]

and

\[ W_d^2 = \frac{1}{2} \int N(D) D^6 (v(D) - V_d)^2 dD = \frac{1}{2} \int N(D) D^6 v^2(D) dD - V_d^2 \quad 6.12 \]

The change of \( W_d \) due to a change \( \Delta \mu \) of \( \mu \) is given by

\[ \Delta W_d = \frac{1}{2W_d} \frac{\partial W_d^2}{\partial \mu} \Delta \mu \quad 6.13 \]

and after recognition of

\[ \pi(\mu) g(\mu) = e g(\mu) \ln \pi(\mu) \quad 6.14 \]

it becomes

\[ \Delta W_d = \frac{1}{2W_d} (\pi_1 - \pi_2) \Delta \mu \quad 6.15 \]

with \( \pi_1 \) and \( \pi_2 \) equal to

\[ \pi_1 = e^{(\mu+7)} g_1 \left\{ g_1 + \left( \frac{1.2D_o (\mu+7)}{3.67 + \mu + 1.2D_o (3.67 + \mu)} \right) \right\} \quad 6.16 \]
\[ \sigma_2 = e^{(\mu + 7)} \mathcal{G}_2 \cdot \left\{ \mathcal{G}_2 + \left( \frac{1.2D_0(\mu + 7)}{3.67 + \mu + 0.6D_0(3.67 + \mu)} \right) \right\} \]  

6.17

and

\[ \mathcal{G}_1 = \ln\left( \frac{3.67 + \mu}{3.67 + \mu + 1.2D_0} \right) \]  

6.18

\[ \mathcal{G}_2 = 2\ln\left( \frac{3.67 + \mu}{3.67 + \mu + 0.6D_0} \right) \]  

6.19

The relative change \( \Delta x_{rel}(\mu) \) of a variable \( x \) is related to \( \Delta x(\mu) \) by

\[ \Delta x_{rel}(\mu) = 10\log\left( 1 + \frac{\Delta x(\mu)}{x(\mu)} \right) \]  

6.20

Figure 6.2 shows the relative change of \( Z_{dr} \) and \( W_d \), referred to as \( \Delta Z_{dr,rel} \) and \( \Delta W_{d,rel} \) respectively, due to \( \Delta \mu = +1 \) as function of \( \mu \); \( D_0 = 1, 1.5 \) or \( 2 \) mm. The change of \( Z_{dr} \) with respect to changes of \( \mu \) was derived numerically with \( Z_{dr} \) expressed in dB, because \( Z_{dr} \) is most sensitive to \( \mu \) then. \( W_d \) is expressed in m/s. The elevation angle of the radar was set to 30°.

\( \Delta Z_{dr,rel} \) varies between -1.8 and -0.1 dB; it decreases for larger values of \( \mu \). \( \Delta Z_{dr,rel} \) is negative, which is due to the fact that \( Z_{dr} \) decreases when \( \mu \) increases. \( \Delta W_{d,rel} \) is smaller: -0.1 < \( \Delta W_{d,rel}(\mu) \) < 0.4 dB. It is negative for small, and positive for large \( \mu \). When \( D_0 \) increases the relative changes decrease.
Figure 6.2 The relative change of $Z_{dr}$ and $W_d$ due to $\Delta \mu = 1$ as function of $\mu$ for different values of $D_o$. 
Now the stochastic aspects of rain are taken into consideration. The measured radar observables are realizations of a stochastic process. They are *estimators*, whose mean and variance indicate the accuracy of the measurement. The accuracy of $Z_{dr}$ measurements is described by Bringi et al. [1983]. When $Z_{dr}$ is expressed as a ratio of the mean powers, as discussed in chapter 4, then its variance is given by

$$\text{Var}(Z_{dr}) = \left\{ \frac{(mp-1)(mp+1)}{mp(mp-2)} \left( 1 - \frac{\rho^2}{mp} \right)^{-2} \text{F}(2, -2; mp; \rho) - 1 \right\} \cdot E[Z_{dr}^2]$$  \hspace{1cm} (6.21)

in which $mp$ is the number of $Z_h-Z_v$ sample pairs that are used for integration, $\rho$ the correlation coefficient of the $Z_h$ and $Z_v$ samples. $\text{F}(2, -2; mp; \rho)$ a hypergeometric function. The correlation between $Z_h$ and $Z_v$ samples depends on the time lag between them and on the signal-to-noise ratio (see also chapter 4). It is clear that to obtain a small variance in $Z_{dr}$, $Z_h$ and $Z_v$ samples should be highly correlated and that a large signal-to-noise ratio is required. For the sample schemes of the Delft Atmospheric Research Radar the time lag between $Z_h$ and $Z_v$ is 8 ms. $\rho$ is of the order of 0.8 then [Cherry and Goddard, 1983] which value is hardly affected by noise when the signal-to-noise ratio is larger than 20 dB (see section 4.5).

When the Doppler spectrum is calculated by means of a Fourier transform over $md$ samples, the variance of the $W_d$ estimator is given by [Doviak and Zrnic, 1984]:

$$\text{Var}(W_d) = \frac{\lambda^2}{4md T_s} \left\{ \frac{3\sigma_{vn}}{32\sqrt{\pi}} + \sigma_{vn}^2 \frac{1}{\text{SNR}}^2 + \left( \frac{\text{SNR}}{320\sigma_{vn}} - \frac{1}{4} \right) \frac{1}{\text{SNR}^2} \right\}$$  \hspace{1cm} (6.22)

$$\sigma_{vn} = \frac{2T_s}{\lambda} W_d$$  \hspace{1cm} (6.23)

in which $\lambda$ is the radar wavelength, $T_s$ the time between two samples and $\text{SNR}$ the signal-to-noise ratio. For large $\text{SNR}$, $\text{Var}(W_d)$ can written as

$$\text{Var}(W_d) = \frac{3\lambda}{64\sqrt{\pi} md} W_d$$  \hspace{1cm} (6.24)

The accuracy of Doppler measurements increases when the wavelength decreases or the
number of samples increases.

In chapter 4, on the Delft Atmospheric Research Radar, the signal processing was discussed; each 3.2 seconds a complete set of Doppler and polarimetric radar observables is obtained. The number of samples $m_p$ to get $Z_{dr}$ is equal to 160. The number of samples $m_d$ to get the Doppler spectrum is 64 and 20 Doppler spectra are measured within 3.2 seconds. Figure 6.3 shows the relative error, defined analogously to the relative change, of $Z_{dr}$ [dB] and $W_d$ [m/s]. Two cases are shown: $N=1$, meaning no additional averaging of the 3.2 s samples, and $N=200$, meaning that $W_d$ and $Z_{dr}$ are integrated over time and/or range 200 samples. When $N=1$ the relative error of $Z_{dr}$ equals 1.1 dB, and that of $W_d$ varies between 0.1 and 0.2 dB. The error in $W_d$ decreases when $W_d$ increases. After the integration over 200 samples, the relative errors have become much smaller: that of $Z_{dr}$ is less than 0.1 dB, and that of $W_d$ is even smaller. For the data used in this chapter, $N$ can be set to 200 by integrating over 20 time samples and 10 range cells; the integration time and range equal 64 seconds and 1500 meter then.

![Figure 6.3](image_url)  

Figure 6.3  The accuracy of $Z_{dr}$ and $W_d$ measurements. $N$ is the number of samples for the post-integration of the 3.2 seconds samples of $W_d$ and $Z_{dr}$.  

133
Turbulence increases the variance of the Doppler spectrum, and the induced canting decreases $Z_{d_r}$. Instantaneous correction of the Doppler spectrum for turbulence is difficult, because of the uncertainty of the turbulence-canting model. An alternative correction is done by means of statistical integration of the radar observables over a long period. Assuming a fixed value of $\mu$, the mean $Z_{d_r}$ is used to obtain $D_o$, which is related to $W_d$. $W_d$ is compared with $W_{d,m}$, and finally $W_t$ results. The thus obtained long-term averaged $W_t$ is used for sample-wise correction of the measured signal. The derivation of $D_o$ from $Z_{d_r}$ relies on a priori knowledge of $\sigma_\delta$, the spread of the canting-angle distribution. To obtain the long-term averaged $\sigma_\delta$, $Z_{d_r}$ and $L_{d_r}$ are combined, again for a fixed value of $\mu$.

6.4 Measurements

The same data as that given in chapter 5 is used to illustrate the method; the experimental setup of the radar needs no further description. In chapter 5, scatter diagrams and height profiles were used to discuss physical phenomena in precipitation, whereas in this chapter time profiles are used to elucidate the method to quantify the dropsize distribution. Figure 6.4 shows the time dependence of the radar observables during approximately 1.5 hour. The data is integrated over 64 seconds and 300 meter. Initially, the antennas were pointed approximately perpendicular to the mean wind direction; halfway through the measurement the antennas were rotated 10° in the horizontal plane, implying a horizontal shift of the range cell under consideration of about 130 m. In the data, the antenna rotation reveals itself through a change of $L_{d_r}$, $W_d$ and $V_d$. $V_d$ is very small before the rotation, as expected, because only the radial component of the wind speed is measured, but after the rotation $V_d$ is increased to approximately +6 m/s (the plus sign indicating that the velocity is pointed towards the radar). $W_d$ is increased as well; on average from 1.1 m/s to 1.4 m/s. Since no significant change of $Z_h$ and $Z_{d_r}$ is observed, it is unlikely that rain has caused the change in $W_d$. Apparently, turbulence is not homogeneous: the variance of its velocity distribution depends on the location of the range cell. Inhomogeneous atmospheric turbulence has often been observed under clear air conditions by wind profilers, and is considered to be caused by atmospheric layers in which turbulent wind flows propagate [Woodman, 1989]. Most wind profiler data is representative for high altitudes, but the measurements described in this chapter show similar phenomena in the lower troposphere. $L_{d_r}$ increases approximately 2 dB, again indicating the relationship
between $L_{dr}$ and turbulence. $Z_h$ fluctuates around 30 dBZ, corresponding to approximately 3 mm/h. $Z_{dr}$ fluctuates around 0.4 dB. After approximately 55 minutes a large peak value of $Z_{dr}$ occurs, accompanied by a peak in $Z_h$.

![Graphs](image)

Figure 6.4  Time dependence of the radar observables, averaged over 64 s and 300 m. Altitude: 850 m.

As previously explained, the needed accuracy requires substantial integration of the radar signals. Figure 6.5 is the result of integrating over 1500 m and 64 s. It is slightly smoother than the previous figure; the spatial structure of the rain region appears to be reasonably homogeneous.
Figure 6.5  Time dependence of the radar observables, averaged over 64 s and 1500 m. Altitude: 850 m.

The first step in the procedure to derive the dispersion factor is the correction for turbulence: the data has to be integrated over a sufficiently long period. Due to the rotation of the antennas, and the resulting different turbulence spectra, the first and second half of the measurement have to be treated separately. The integration of the second part of the measurement is done with exclusion of the peak value of $Z_{dr}$, to avoid biasing by one dominant measurement. The first part is integrated over 42 and the second part over 22 minutes. The long-term integrated values of $Z_{dr}$ and $L_{dr}$ are used to estimate the value of $\sigma_\delta$. To this end $Z_{dr}$ and $L_{dr}$ are tabulated for $\mu=0$ and different values of $D_o$ and $\sigma_\delta$, with $D_o$ changing in steps of 0.08 mm and $\sigma_\delta$ in steps of 5°. The choice of $\mu = 0$ is based on work by Joss and Gori [1978], who showed that it represents long-term integrated drops size distributions. Table 6.1 gives the integrated
radar observables and derived $\sigma_\delta$. Note that the spread of the canting-angle distribution is the same for both parts, which is solely due to the 5' interval of the table entries. The actual values of $\sigma_\delta$ will differ somewhat: it is smaller in the first part of the measurement than it is in the second part.

The selected $D_o$ is used to calculate $W_d$ and finally the spread $W_t$ of the turbulence spectrum is estimated via

$$W_t^2 = \overline{W}_{d,m}^2 - W_d^2$$

in which $\overline{W}_{d,m}^2$ is the long-term integrated variance of the Doppler spectrum. The results are also given in table 6.1. In the first half of the measurement $W_t$ equals 0.9 m/s and in the second half 1.3 m/s. After removal of turbulence $W_d$ equals 0.6 m/s. Note that this value was used earlier, in chapter 5, to obtain the mean relationship between turbulence and canting. There, it was based on the statistical distribution of measured values of $W_d$ during other, non-turbulent, events whereas during this analysis it results after correction of a turbulent event.

The estimate of $W_t$ is based on long-term integrated data, but is used to correct short-term averaged data. The error $\Delta W_t$ of $W_t$ is estimated by taking the statistical spread of $Z_{dr}$ and $W_{d,m}$ into account, according to the following scheme:

$$\overline{Z}_{dr} + \frac{1}{2} \sigma_{Z_{dr}} |_{\overline{W}_{d,m}} \rightarrow W_{tt1}$$  \hspace{1cm} \text{(6.26)}

$$\overline{W}_{d,m} + \frac{1}{2} \sigma_{W_{d,m}} |_{\overline{Z}_{dr}} \rightarrow W_{tt2}$$  \hspace{1cm} \text{(6.27)}

$$\Delta W_t = 2 \sqrt{(W_t - W_{tt1})^2 + (W_t - W_{tt2})^2}$$  \hspace{1cm} \text{(6.28)}

$W_{tt1}$ is the width of the turbulence spectrum that would be calculated if the standard deviation $\sigma_{Z_{dr}}$ of $Z_{dr}$ were taken into account. $W_{tt2}$ would be calculated if the standard deviation $\sigma_{W_{d,m}}$ of $W_{d,m}$ were taken into account. Finally, the total error $\Delta W_t$ is taken
as an rms value to get the worst case estimation. Table 6.1 shows the results. The effect of $Z_{dr}$ variations is very small. $\Delta W_t$ is almost entirely caused by $W_{tt2}$.

<table>
<thead>
<tr>
<th></th>
<th>First part</th>
<th>Second part</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{dr}$</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$\bar{Z}_{dr}$</td>
<td>-28.4</td>
<td>-26.0</td>
</tr>
<tr>
<td>$D_o$</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>$\sigma_\delta$</td>
<td>30°</td>
<td>30°</td>
</tr>
<tr>
<td>$W_{d,m}$</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>$W_d$</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$W_t$</td>
<td>0.9</td>
<td>1.3</td>
</tr>
<tr>
<td>$W_t - W_{tt1}$</td>
<td>0.008</td>
<td>0.012</td>
</tr>
<tr>
<td>$W_t - W_{tt2}$</td>
<td>0.025</td>
<td>0.077</td>
</tr>
<tr>
<td>$\Delta W_t$</td>
<td>0.052</td>
<td>0.156</td>
</tr>
</tbody>
</table>

Table 6.1 Results of the long-term integration.

Finally, $Z_{dr}$ and $W_d$ are used to derive the dropsize distribution for the short term integrated data. To this end, $Z_{dr}$ and $W_d$ are tabulated for $\sigma_\delta = 30^\circ$, with $D_o$ changing from 0 to 4 mm in steps of 0.08 mm, and $\mu$ changing from $-3$ to $10$ in steps of 1. Figure 6.6 shows the resulting dropsize distribution as function of time. The dispersion factor $\mu$ is not constant: it fluctuates between $-2$ and 9 (excluding the extreme values of the tables). $D_o$ fluctuates around 1 mm, while $N_o$ varies between 35 and 50 dB. In the $N_o$-plot an additional line is drawn at $N_o=39$ dB, which is the value of $N_o$ of the Marshall-Palmer distribution. Note that most of the time $N_o$ is larger than that. In the time frame between 40 and 60 minutes $\mu$ and $D_o$ become large, while $N_o$ decreases. To exclude the influence of $\mu$ during that time interval, the calculation was repeated with $\mu$ fixed to 0. It revealed a similar behavior: $D_o$ increases and $N_o$ decreases. This effect is due to the combination of a large $Z_{dr}$ and moderate $Z_h$; the large value of $D_o$ that results from $Z_{dr}$ would normally correspond to a large $Z_h$, but since that is not observed, a small $N_o$ is necessary in order to the data.
Figure 6.6 Time dependence of the dropsize distribution, corresponding to data shown in figure 6.5.

6.5 Statistics of \( N_o, D_o \) and \( \mu \)

The dispersion factor was found to vary during the event and most of the time \( \mu \) was larger than 0. To illustrate the statistical distribution of the parameters of the dropsize distribution, histograms of \( N_o, D_o \) and \( \mu \) are given in figure 6.7. Again, data with \( \mu = -3 \) or \( \mu = 10 \) is ignored.
Figure 6.7  Histograms of $\mu$, $D_o$, and $N_o$, corresponding to the data of figure 6.5.

The distribution of $\mu$ is peaked around $\mu=0.6$, but the spread around the peak is considerable. The median diameter $D_o$ is distributed between 0.5 mm and 1.5 mm. The scaling factor $N_o$ is distributed between approximately 39 dB and 54 dB, with a peak at 45 dB. Table 6.2 gives the mean and standard deviation of $\mu$, $D_o$, and $N_o$. Also given are the mean and standard deviation of $N_o$ and $D_o$ in case $\mu=0$.

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$D_o$ [mm]</th>
<th>$N_o$ [dB]</th>
<th></th>
<th>$\mu$</th>
<th>$D_o$ [mm]</th>
<th>$N_o$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.6</td>
<td>0.9</td>
<td>45.6</td>
<td>0</td>
<td>1.2</td>
<td>36.4</td>
<td></td>
</tr>
<tr>
<td>standard deviation</td>
<td>2.3</td>
<td>0.3</td>
<td>9.0</td>
<td>0</td>
<td>0.2</td>
<td>5.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2  Mean and standard deviation of the dropsize distribution
Note that keeping $\mu$ constant reduces the scatter of $N_o$ by 4 dB. This is due to the dimensional relationship of $N_o$ and $\mu$. It appears that the mean value of $D_o$ is larger when $\mu$ is kept to 0; the standard deviation is hardly affected. Table 6.3 shows the mean drop concentration $N_t$ and total drop volume $V_t$, calculated with the mean values of $N_o$, $D_o$, and $\mu$. Note that, although the estimated number of drops differs substantially, the total drop volume does not differ much: approximately 3%.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$N_t$</th>
<th>$V_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1.5 \cdot 10^3$</td>
<td>156.5</td>
</tr>
<tr>
<td>varying</td>
<td>$2.4 \cdot 10^3$</td>
<td>160.3</td>
</tr>
</tbody>
</table>

Table 6.3 Drop concentration and total drop volume for fixed $\mu$, and for varying $\mu$. $N_t$ in m$^{-3}$, $V_t$ in mm$^3$m$^{-3}$.

The error $\Delta W_t$ of the standard deviation of the turbulence spectrum, due to variations of the radar observables, also results in an error of the derived dropsize distribution. To estimate the error, the procedure to calculate $N_o$, $D_o$, and $\mu$ was repeated with $W'_t = W_t + \frac{1}{2} \Delta W_t$, and $W'_t = W_t - \frac{1}{2} \Delta W_t$. The results are given in table 6.4, where for ease of comparison the original values are given as well.

<table>
<thead>
<tr>
<th>Mean parameters</th>
<th>$\mu$</th>
<th>$D_o$ [mm]</th>
<th>$N_o$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W'_t = W_t + \frac{1}{2} \Delta W_t$</td>
<td>1.9</td>
<td>1.0</td>
<td>46.1</td>
</tr>
<tr>
<td>$W'_t = W_t$</td>
<td>0.6</td>
<td>0.9</td>
<td>45.6</td>
</tr>
<tr>
<td>$W'_t = W_t - \frac{1}{2} \Delta W_t$</td>
<td>-0.3</td>
<td>0.8</td>
<td>43.7</td>
</tr>
<tr>
<td>Worst case error</td>
<td>$\pm 1.3$</td>
<td>$\pm 0.1$</td>
<td>$\pm 1.9$</td>
</tr>
</tbody>
</table>

Table 6.4 Estimated error of the mean parameters of the dropsize distribution

The errors of $\mu$ and $N_o$ are not symmetric around the mean: overestimation of $W_t$ results in a larger error than underestimation. The worst-case error neglects this asymmetry, because it is defined as twice the largest error.
6.6 On the relationship between $N_\circ$, $D_\circ$, and $\mu$

The accuracy of single-parameter radars could be enhanced if possible relationships between the different parameters of the dropsize distribution were taken into account. For instance, if $N_\circ$ and $\mu$ were related, a measurement of $\mu$ would imply $N_\circ$. When these two are incorporated into the $Z/R$ relationship, the estimation of the rain intensity is improved. Ulbrich [1983] suggested such a relationship:

$$N_\circ = C_N e^{\beta \cdot 10^{-(1+\mu)}} \quad [m^{-3} mm^{-1} \mu] \quad 6.29$$

Initially, equation 6.29 was derived from data that spanned different rainfall types and then $\beta$ and $C_N$ were found to be 3.2 and $6 \cdot 10^4$ respectively. However, when one particular event was considered they became 3.14 and $1.52 \cdot 10^4$. Clearly, $\beta$ and $C_N$ are subject to changes that may differ per event. Figure 6.8 shows the scatter diagram of $N_\circ$ and $\mu$, the regression line of the data, and equation 6.29 with $C_N=1.52 \cdot 10^4$ and $\beta=3.14$.

The correlation coefficient of $N_\circ$ and $\mu$ is 0.84, which implies a significant relationship between the two. The regression line is given by

$$N_\circ = 44.8 + 2.51 \cdot \mu \pm 1.7 \quad [dB] \quad 6.30$$

It appears that the data is not in agreement with equation 6.29: although the two regression lines are almost parallel, a mean offset of $N_\circ$ of approximately 10 dB occurs. The reason for this is difficult to give, because no details are available about the experiment from which equation 6.29 was derived. Note that equation 6.30 gives $N_\circ=44.8$ dB when $\mu=0$, which is 5.8 dB larger than the value of $N_\circ$ that corresponds to the Marshall-Palmer distribution, whereas relationship 6.29 results in 31.8 dB, which is 7.2 dB smaller. The larger value of $N_\circ$ indicates that more small drops are measured than predicted by Ulbrich's equation. Maybe the difference is caused by the moderate, maritime climate in The Netherlands, but further study is necessary to confirm this.
Figure 6.8  Scatter diagram of $N_o$ and $\mu$. Also given is the $N_o$-$\mu$ relationship of Ulbrich [1983], with $C_N=1.52 \cdot 10^4$ and $\beta=3.14$.

The scatter diagram of $D_o$ and $\mu$ is given in figure 6.9. For $\mu<3$, $\mu$ increases when $D_o$ increases. With reference to figure 2.4, the width of the dropsize distribution increases less with $D_o$ than it would in the case of a fixed $\mu$. For $\mu \geq 3$, the data points are spread around $D_o = 1$mm. The observations are in agreement with those of Wessels [1972], who performed long-term measurements of dropsize distributions using filter paper. In that experiment it was shown that, although a different dropsize distribution was used, a shape factor similar to $\mu$ increased for rain intensities smaller than approximately 4 mmh$^{-1}$, and decreased again for larger rain intensities. Also given in the figure is a 3$^{rd}$ order polynome that relates $\mu$ to $D_o$:

$$D_o = 0.97 + 0.14\mu - 0.043\mu^2 + 0.0033\mu^3 \pm 0.1 \text{ mm}$$

6.31

The measured $N_o$ and $D_o$ data were investigated for the presence of any correlation as well, but none was found. However, when the calculation of the dropsize distribution
was repeated with the assumption of $\mu = 0$, they were found to be negatively correlated with a coefficient of $-0.68$: $N_o$ tends to decrease when the rain intensity increases. Clearly, a varying $\mu$ decorrelates $N_o$ and $D_o$, because of the dimensional relationship of $N_o$ and $\mu$.

![Graph of $D_o$ vs. Dispersion factor](image.png)

Figure 6.9 Scatter diagram of $D_o$ and $\mu$.

6.7 The relationship between $Z$ and $R$

Equations 6.30 and 6.31 that relate $\mu$ to $N_o$ and $D_o$ can be used to calculate the $Z$-$R$ relationship by incorporating them into the expressions for $R$ and $Z$, given in previous sections. The resulting relationship can be expressed as a power law:

$$Z = 220 R^{1.2}$$  \[ \text{[mm}^6\text{m}^{-3}] \]  \hspace{1cm} 6.32

with $R$ in mmh$^{-1}$. The accuracy of the power law is better than 0.1 dB, when $Z$ is given
in dBZ. Figure 6.10 shows the new $Z-R$ relationship as well as the Marshall-Palmer relationship $Z = 200R^{1.6}$. Only $R < 10 \text{ mm h}^{-1}$ is shown, because the new $Z-R$ relationship was derived from one particular, moderate, event. In order to arrive at equation 6.32, $\mu$ had to be varied from 0 to 6, with small values of $\mu$ causing the larger rain intensities. For $\mu < 0$ instabilities occurred and extrapolation of the results of $\mu > 0$ was necessary.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.10.png}
\caption{The Z-R relationships derived from radar data and the Marshall-Palmer relationship.}
\end{figure}

The shapes of the curves are similar. For $R > 1 \text{ mm h}^{-1}$, the new relationship predicts higher rain intensities than the Marshall-Palmer $Z-R$ relationship. Chandrasekar and Bringi [1987] argued that using a mean relationship between $N_o$ and $\mu$ will not increase the accuracy of weather radars, because of the large sensitivity of $Z$ and $R$ to the, expected, variability of the $N_o-\mu$ relationship. The method described in this chapter, however, enables the derivation of the $N_o-\mu$ relationship per event; the relationships between the parameters of the dropsize distribution may differ per event, leading to different $Z-R$ relationships per event.
6.8 Comparison with rain gauge measurements

To test the method of measuring the dropsize distribution, the radar-derived rain intensity is compared with rain gauge data. Figure 6.11 shows the measurement configuration. The radar is located on a 100 m high building. The rain gauge is positioned at a distance of approximately 800 m from the radar site. The radar beam is not pointed above the rain gauge; it is shifted by approximately 50°. In the first half of the measurement the mean wind direction is perpendicular to the azimuth-angle of the radar beam; in the second half the radar beam is slightly shifted. The rain gauge measures rainfall at one location. The radar data is integrated over 1500 meter before the rain intensity is derived. The first range cell of the radar beam used for analysis is located at an altitude of 475 meter, the last one at 1225 meter. The average altitude is 850 meter.

Figure 6.11 The measurement configuration. The integration range, which is used for analysis of the radar data, is symbolized by a bold line. The dashed arrow represents the radar beam in the second half of the measurement.

Initially, the rain gauge data is integrated over 10 seconds. To match the integration time of rain gauge data with that of the radar data, 6 samples are integrated; the 64 seconds averaged value of the rain gauge data is estimated by means of linear interpolation of the 60 seconds data. Due to the different positions of the radar beam and rain gauge, the time lapse of the radar-derived rain intensity and the rain gauge result do not have to match; the rain gauge registration can be delayed with respect to
the radar signal. The data has been corrected for this delay.

Figure 6.12 shows the time dependence of the rain intensity. The radar-derived rain intensity is calculated in two ways: 1) only $Z_{dr}$ is used with $\mu=0$, and 2) $W_d$ and $Z_{dr}$ are used to incorporate variations of $\mu$ as well. The rain gauge did not register properly between 45 and 60 minutes. This is the same time interval as that excluded in the analysis of the previous sections, because of the risk of biasing the statistics due to the large $Z_{dr}$. In retrospect one could argue that the large $Z_{dr}$ peak during this interval was caused by hydrometeors other than raindrops.

![Graph showing the time dependence of radar-derived rain intensities](image)

**Figure 6.12** The time dependence of the radar-derived rain intensities $R_{W_dZ_{dr}}$ and $R_{Z_{dr}}$. Also shown is the rain gauge data. Integration time is 64 seconds. The rain gauge did not work properly between 45 and 60 minutes.

The long-term averaged rain gauge data was converted to radar reflectivity using the Marshall-Palmer relationship and compared with the long-term averaged measured radar reflectivity; a small difference of 0.5 dB was found. In the first half of the measurement the temporal correlation between the rain gauge data and the radar data is good: all peaks coincide. In the second half of the measurement, after the rotation of the antennas, the rain gauge data is slightly delayed again; the radar beam was rotated
away from the rain gauge. The rain intensity that is derived with \( W_d \) and \( Z_{dr} \) is referred to as \( R_{W_dZ_{dr}} \). On average it gives larger values than \( R_{Z_{dr}} \) and \( R_{gauge} \).

Figure 6.13 Scatter diagrams of the radar-derived rain intensities versus the rain gauge measurements.

Figure 6.13 shows the scatter diagrams of \( R_{W_dZ_{dr}} \), \( R_{Z_{dr}} \), and \( R_{fit} \), which results from equation 6.32, versus \( R_{gauge} \). Also given are the regression lines and the cross-correlation coefficients. It appears that \( R_{fit} \) and \( R_{Z_{dr}} \) are in good agreement with the rain gauge data but also that \( R_{W_dZ_{dr}} \) is slightly larger than \( R_{gauge} \). The relative error \( \Delta_R \) is estimated by
\[ \Delta_R = \frac{\sum_{i=1}^{N} |R_{\text{radar},i} - R_{\text{gauge}}|}{\sum_{i=1}^{N} R_{\text{gauge}}} \] 6.33

The cross-correlation coefficients of the three scatter diagrams are in the same order of magnitude: 70 - 75 %. The relative errors are 52 % for \( R_{W_d, Z_{dr}} \), 23 % for \( R_{Z_{dr}} \), and 21 % for \( R_{fit} \). The accuracy of \( R_{Z_{dr}} \) and \( R_{fit} \) is comparable; the inclusion of a variable \( \mu \) does not seem to improve the radar estimation of the rain intensity. However, for larger rain intensities the scatter around the regression line is smaller in the case of \( R_{fit} \) than in the case of \( R_{Z_{dr}} \).

6.9 Conclusions

The combination of \( Z_h \), \( Z_{dr} \), and \( W_d \) can be used to calculate the three parameters of the gamma dropsize distribution: \( \mu \), \( N_o \), and \( D_o \). The combination of \( Z_{dr} \) and \( L_{dr} \) is required to correct \( W_{d,m} \) for turbulence. It is found that \( N_o \) and \( D_o \) are strongly related to \( \mu \), although \( D_o \) and \( N_o \) are not related to each other. The measured relationship between \( N_o \) and \( \mu \) qualitatively agreed with the result of Ulbrich [1983]. Quantitatively, however, an offset of approximately 10 dB appeared. Incorporating the measured relationship between the parameters of the dropsize distribution results in a \( Z-R \) relationship that differs from the Marshall-Palmer relationship. Comparison with rain gauge measurements revealed that the accuracy of the radar-derived rain intensity was not significantly enhanced by the inclusion of \( \mu \). However, only a limited amount of data was analyzed and the measurement configuration was not ideal: the rain gauge was not positioned underneath the radar beam. To study the accuracy of the method properly, a long-term measurement campaign is necessary.

Operational weather radars usually are not equipped with the capability of measuring \( W_d \) and \( Z_{dr} \) and are therefore not able to measure \( \mu \). Thus the major conclusion of this chapter is that relationships between the parameters of the dropsize distribution exist, which may improve the accuracy of the single-parameter radars. The limited amount of data that was analyzed in this chapter spanned only rain intensities lower than 10 \( \text{mmh}^{-1} \) and was obtained during one event. More data is necessary to extend the operational use of this chapter for different rain situations.
Chapter 7
Backscattering by the melting layer

In stratiform precipitation raindrops result from melting snowflakes. On their way down in rain-bearing clouds, dry snowflakes pass the 0°C isotherm and start to melt. The region in which they melt is called the melting layer or, referring to the enhanced radar reflectivity it causes, bright band. Radar measurements are useful in studying meteorological processes in the melting layer; Doppler and polarization techniques enable the observation of the changing properties of melting snowflakes. Measurements of the melting layer using the Delft Atmospheric Research Radar are discussed and a simple Doppler-polarimetric model of the melting layer is proposed to explain them.

7.1 Doppler-polarimetric radar measurements of the melting layer

In this section, radar measurements of the melting layer are discussed. Figure 7.1 shows a typical height profile of $Z_h, Z_{dr}, L_{dr}, W_d$ and $V_d$ in the melting layer, measured during the event that was discussed also in chapters 5 and 6. The data is integrated over 32 seconds and 300 meter. The elevation angle of the radar antennas was set to 30°, to enable the combination of Doppler and polarization measurements. The 0°C isotherm was located at an altitude of approximately 2200 meter.

From the onset of melting, $Z_h$ increases until it reaches a peak of approximately 15 dB relative to its value before melting. Then it decreases again to a, more or less, constant value in the rain region underneath the melting layer. The physical mechanism that causes this behavior is explained by, among others, Ekpenyong and Srivastava [1970], Jain [1984], and Klaassen [1988]. During melting the water content and mass density of the snowflakes increase. They experience less air resistance and as a result their fall speed increases. Hence, the concentration of snowflakes in the radar volume decreases during melting. In the upper half of the melting layer the water content is a dominant factor in the scattering process and consequently the reflectivity increases. However, in the lower half of the melting layer, where most of the snow is melted, the particle concentration dominates, resulting in a decrease of the reflectivity. In the following section this process is discussed in more detail.
The height profile of $Z_{dr}$ is somewhat different than the one of $Z_h$. $Z_{dr}$ increases as well during melting, but its maximum is shifted with respect to the peak of $Z_h$. $Z_{dr}$ is small until in the lower half of the melting layer, where it shows a sudden peak of approximately 1 dB with respect to its value in the snow region. Underneath the melting layer $Z_{dr}$ is more or less stable around 0.5 dB. $Z_{dr}$ is sensitive to the shape and orientation of the reflecting particles. The peak value of $Z_{dr}$ at the bottom of the melting layer indicates an oblate mean shape at that height. The small $Z_{dr}$ above and in the upper part of the melting layer indicates a spherical mean shape. However, this does not necessarily imply that melting snowflakes are spherical: they can also be very irregular, randomly oriented structures.

The height profile of $L_{dr}$ is similar to that of $Z_{dr}$. However, radar measurements with a high angular resolution by Illingworth and Caylor [1989] showed that the $Z_{dr}$ and $L_{dr}$ peaks do not necessarily coincide: the $L_{dr}$ peak is then found above the $Z_{dr}$ peak. As will appear later, this shift can be explained by a melting-induced motion of the particles to which $Z_{dr}$ and $L_{dr}$ have different sensitivities. Measurements performed using the Delft Atmospheric Research Radar have less resolution and are therefore less sensitive to this shift, although it has been observed occasionally.

The distribution of fall velocities of dry snowflakes is narrow: a small value of $W_d$ has been observed. During melting the fall velocity of the individual particles increases, and eventually reaches the value of the resulting raindrops. The statistical spread of the fall velocities of raindrops is larger than it is for snowflakes and so $W_d$ increases during melting. At the onset of melting, $W_d$ decreases approximately 0.3 m/s before it starts to increase. This can be explained by considering that in the upper part of the melting layer small snowflakes are melted further than large ones. When a small snowflake melts into a small raindrop, it can achieve approximately the same fall velocity as a large snowflake. Consequently, a narrowing of the velocity spectrum occurs in the upper part of the melting layer and $W_d$ decreases. This effect was theoretically predicted by Ekpenyong and Srivastava [1970]. The measured height profile of $V_d$ is not representative for the hydrometeors involved. It was mainly caused by wind, because the measurement was done with a 30° elevation angle. The increase of $V_d$ in the melting layer could indicate a down draft.
Figure 7.1  Height profile of the radar observables during an event with a strong bright band. Integrated over 32 seconds and 300 meter in range. Elevation: 30°.
Figure 7.2  Height profile of the radar observables during an event with a weak bright band. Integrated over 32 seconds and 300 meter. Elevation: 30°.
The event just discussed is one with a strong bright band. The radar observables can be somewhat different in events with a low rain intensity and weak bright band. Figure 7.2 depicts such a situation. The radar observables were obtained during the passage of a cold front on November 2, 1990. The 0°C isotherm was located at approximately 1200 meter and the ground temperature was approximately 8°C. The peak value of $Z_h$ is approximately 12 dB relative to the value of $Z_h$ underneath the melting layer. However, the absolute level of $Z_h$ is approximately 13 dB lower than in the event first discussed. $Z_{dr}$ starts to increase at the onset of melting, rather than halfway through the melting layer as shown in figure 7.1. Further, the peak value of $Z_{dr}$ is larger than shown in figure 7.1. $L_{dr}$ does not show a peak and is very small in the melting layer. $W_d$ increases during melting again, although it takes on different values than those in figure 7.1.

As with the description of the Doppler spectrum, statistical moments can be used to characterize the radar observables in the melting layer. The consistency of the radar observables can be studied then by inspecting the time dependence of these moments. The mean reflectivity $mZ_h$ of the melting layer is defined as

$$mZ_h = \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} Z_h(h) \, dh$$  \hspace{1cm} (7.1)$$

in which $h$ is the height, $h_1$ the bottom of the melting layer and $h_2$ the top. The position $hZ_m$ of the maximum value of $Z_h$, relative to the bottom of the melting layer, is estimated by

$$hZ_h = \frac{1}{h_2 - h_1} \int_{h_1}^{h_2} Z_h(h) \cdot h \, dh$$  \hspace{1cm} (7.2)$$

and the width $wZ_h$ of the bright band by

$$wZ_h = \sqrt{\frac{1}{h_2 - h_1} \int_{h_1}^{h_2} Z_h(h) \cdot (h - hZ_h)^2 \, dh}$$  \hspace{1cm} (7.3)$$

Similar expressions are used for the height profile of $Z_{dr}$, producing $mZ_{dr}$, $hZ_{dr}$ and
$wZ_{dr}$. This has not been done for $L_{dr}$ and $W_{dr}$, because they do not always show a distinct maximum in the melting layer; 'peak height' and 'peak width' are meaningless then.

Figure 7.3 shows the statistical moments of $Z_h$ and $Z_{dr}$ of the event shown in figure 7.1 as function of time. The top of the melting layer is somewhat arbitrarily set at the height where $Z_h$ starts to increase, and the bottom of the melting layer is set to the height where $Z_h$ and $Z_{dr}$ have more or less become stationary with respect to height. Since figure 7.3 is only used as an illustration and not to quantify the melting layer exactly, these definitions of $h1$ and $h2$ suffice. A rigorous quantification of the moments requires better estimates of the melting layer boundaries. The statistical moments are calculated with $Z_h$ and $Z_{dr}$ expressed in dB.

A positive correlation between $mZ_h$ and $mZ_{dr}$ is found. This indicates that, when the bright band increases in strength, the particles become more oblate. The heights at which the peaks occur are different for $Z_h$ and $Z_{dr}$. On average $hZ_{dr}$ is smaller than $hZ_h$, which implies that the peak of $Z_{dr}$ occurs in the lower half of the melting layer. Note that when $mZ_h$ increases, $hZ_{dr}$ decreases: the $Z_{dr}$ peak shifts downwards. The mean height $hZ_h$ does not vary much, even though the peak reflectivity itself varies over more than 15 dB. The width of the bright band is stable: $wZ_h$ does not vary much. The $Z_{dr}$ peak is consistently narrower than the $Z_h$ peak.

The correlation between the radar observables of this event was discussed in chapter 5. $Z_h$ was negatively correlated to $Z_{dr}$ and $L_{dr}$, in the upper half, and positively in the lower half of the melting layer. Mitra et al [1990], who observed melting snowflakes in a wind tunnel, describe four stages in the melting process. In the first stage small drops are formed at the tips of the ice branches with melting occurring on the entire periphery, but mainly at the bottom of the snowflake. In the second stage the water is sucked into the inside of the snowflake, where it is accumulated. In the third stage the branches inside the snowflake start to melt and consequently the structure of the snowflake is changed. In the fourth stage the snowflake collapses into a water drop. Because melting is most intense at the lower side of the snowflake, the 'electromagnetic' shape of the particle becomes oblate because the permittivity of water is much larger than the permittivity of the remaining unmelted snowflake. The negative correlation in the upper half of the melting layer can be understood by considering the melting process more closely. Large snowflakes fall faster than small ones, and therefore travel a longer distance during melting than small ones. This implies that at a given height in the
upper half of the melting layer small snowflakes are melted further than large ones, and hence are also more oblate than large ones. Thus, $Z_h$ is negatively correlated to $Z_{dr}$ and $L_{dr}$. As melting proceeds, small snowflakes turn into more or less spherical raindrops, and large snowflakes now become oblate. Consequently, large particles are more oblate than small ones, and $Z_h$ is positively correlated to $Z_{dr}$ and $L_{dr}$. In section 7.2.5, the shape of melting snowflakes is quantified as function of the melting rate.

Figure 7.3 Statistical moments of the melting layer in the event shown in figure 7.1.

Figure 7.4 shows the statistical moments of $Z_h$ and $Z_{dr}$ of the event shown in figure 7.2. In comparison with the previous event three significant differences appear:
- The mean reflectivity is small.
- $mZ_{dr}$ is on average approximately 1 dB larger than in figure 7.1.
- When $mZ_h$ increases $mZ_{dr}$ decreases.
- The peak of $Z_{dr}$ is located in the upper half of the melting layer.
Figure 7.4  Statistical moments of the melting layer in the event shown in figure 7.2.

These phenomena may be explained by the presence of oblate, wetted ice crystals that co-exist with aggregated snowflakes. Ice crystals are small and flat. They melt in the upper half of the melting layer. The mean reflectivity is small when ice crystals and small snowflakes predominate in the melting layer and ice crystals contribute significantly to $Z_{dr}$. Consequently, the $Z_{dr}$ peak does not occur at the bottom of the melting layer, but in the region where the ice crystals melt. Evidence for the presence of wetted ice crystals at the top of the melting layer is found by Meischner et al [1991], who combined radar measurements of the melting layer with in situ measurements from an aircraft. They also confirm the observations of more pronounced $Z_{dr}$ values in areas of the melting layer with a low reflectivity.
Figure 7.5 The height profiles of the cross-correlation coefficients of the radar observables during the event shown in figure 7.2. Integration: 300 meter, 32 minutes.
Figure 7.5 shows the height profile of the cross-correlation coefficients of the radar observables. The melting layer is located between 900 and 1300 meter. In the upper half of the melting layer, $Z_h$ is negatively correlated to $Z_{dr}$ and $L_{dr}$. However, only in the case of $Z_h$ and $Z_{dr}$ is a reversal observed in the lower half of the melting layer. In the case of $Z_h$ and $L_{dr}$, the correlation remains negative, although it becomes smaller. There is no correlation between $Z_{dr}$ and $W_d$ in rain. In the melting layer, however, they are positively correlated, as they were not in in the event shown in figure 7.1. Apparently, the Doppler spectrum is not significantly distorted by turbulence. $W_d$ is negatively correlated to $Z_h$, which means that with increasing mean particle size, the spectrum of fall velocities becomes narrower. However, the correlation coefficients are small and only reveal tendencies. In the rain region too little data satisfies the requirement of a signal-to-noise ratio larger than 20 dB: $R_{xy}$ is unreliable.

So far measurements with an elevation angle of 30° have been discussed. In chapter 3, however, it was shown that in case of sufficient particle canting, radar measurements towards the zenith may still exhibit a significant $L_{dr}$, event though then $Z_{dr}$ approximates 0 dB. Figure 7.6 shows the radar observables during such a measurement, performed on September 17, 1990. The ground temperature was 15°, the 0°C isotherm was located at approximately 2000 meter.

The bright band is located between 1600 and 2100 meter. The $Z_h$ peak is approximately 12 dB with respect to $Z_h$ in the rain region. $Z_{dr}$ is very small and independent of the height; it does not contain any information about the melting process. $L_{dr}$, however, shows a distinct peak at the lower bound of the melting layer: in the snow region and upper half of the melting layer it approximates $-30$ dB, suddenly increases to $-23$ dB, and finally drops to $-27$ dB in the rain region. $W_d$ has the same height profile as $L_{dr}$: it is small in the snow region and the upper part of the melting layer, shows a peak in the lower part of the melting layer, and drops again in rain. The peak of $W_d$ is caused by the co-existence of melting snowflakes and raindrops [Ekpenyong and Srivastava, 1970; Dyer, 1970]. They cause two separate Doppler spectra, because the fall velocities of raindrops and melting snowflakes differ. The width of the enveloping spectrum is larger than the widths of the two individual ones and appears as a peak at the lower bound of the melting layer. As soon as all particles are melted, the bi-modal spectrum disappears and the width decreases again. The peak of $W_d$ is not always observed: often melting is a gradual process throughout the melting layer, during which bi-modalities do not occur. The fall velocity $V_d$ increases from 1 to 6.5 m/s during melting.
Figure 7.6  Height profile of radar observables during a measurement with the radar pointed towards the zenith. Integrated over 32 seconds and 300 meter.
The scatter diagrams of figure 7.7 show $Z_h$ versus $Z_{dr}$, $L_{dr}$, and $W_d$. The data is taken from range cells just above and below the melting layer. It is clear that $Z_{dr}$ is small throughout the whole event and also that it does not reveal the difference between rain and snow. The same holds true for $Z_h$: rain and snow may cause reflectivity levels that are in the same order of magnitude. Rain produces $L_{dr}$ values between $-25$ and $-28$ dB, whereas snow consistently produces smaller values. However, the difference between rain and snow is not as distinct as in the case of Doppler measurements: $W_d$ equals approximately 0.5 m/s in snow and 1.2 m/s in rain.

Figure 7.7 Scatter diagrams of the radar observables, taken from range cells just above and just below the melting layer. Integrated over 32 seconds and 300 meter. Total observation time: 32 minutes.
Figure 7.8 The height profile of the cross-correlation coefficients of the radar observables during the event shown in figure 7.6.
Figure 7.8 shows the height profile of the cross-correlation coefficients of the radar observables of this event. $Z_{dr}$ is not correlated to any of the other radar observables. $Z_h$ is positively correlated to $W_d$ in the rain and snow region, but a sudden decrease occurs in the melting layer. This decrease coincides with the beginning enhancement of $W_d$, indicating that it is caused by particles that while they melt, gain speed and collapse into smaller ones. The cross-correlation between $Z_h$ and $L_{dr}$ follows the same trend as during the other events: it starts negative but as melting proceeds it gradually becomes positive and decreases again in the rain region. Note that in the rain region $L_{dr}$ and $W_d$ are positively correlated again.

7.2 Modeling the melting layer

With the increasing use of high frequencies at satellite links and the wider use of radar for hydrological purposes, knowledge of the influence of the melting layer is becoming increasingly important. The development of an easy-to-use model that relates the radar measurements to the characteristics of the melting layer and only requires knowledge of, for instance, the rain intensity to predict the influence of the melting layer, is receiving much attention [Klaassen, 1990], [Zhang et al, 1990, 1991], [Hardaker et al, 1991]. Reports on measurements of the melting layer date back to the early fifties, e.g. [Austin and Bemis, 1950], [Hooper and Kippaz, 1950], but a model that explained Doppler radar measurements [Ekpenyong and Srivastava, 1970] was not developed until 1970. This model explained the vertical profiles of the reflectivity, the mean velocity and the variance of the velocity distribution, and it was used by others [Dissanayaka and McEwan, 1978], [Jain, 1984] as a starting point to model attenuation of radio signals by the melting layer. Jain also used it to model the $Z_h$ and $Z_{dr}$ profiles. However, the models more or less represented a 'steady-state' melting layer, whereas radar measurements often showed large deviations from the models due to the stochastic nature of the physical processes in the melting layer. Klaassen [1988] developed an advanced meteorological model of the melting layer that was better able to adapt to statistical variations of the melting layer and also predicted Doppler radar measurements.

This section describes the physical phenomena in the melting layer and results in a radar-derived Doppler-polarimetric model that is able to calculate profiles of $Z_h$, $Z_{dr}$, $L_{dr}$, and $V_d$ in the melting layer. The model has two input parameters: the rain
intensity and the mass density of the snowflakes above the melting layer. As will appear later, the mass density of the snowflakes is of importance to account for the differences between the earlier-discussed measurements. During melting the following changes occur to the snowflakes:

- the water content and consequently the permittivity, increases,
- the mass density increases and consequently the particle size decreases,
- the shape changes: the crystal-like snowflake turns into a spheroidal raindrop,
- the fall speed increases, which affects the number of snowflakes in the radar volume.

These items are here discussed, following a description of the melting process itself. The melting layer model here introduced is based on the following assumptions:

- the shape of melting snowflakes can be modeled as a spheroid, which will shown to be a sufficient approximation for Rayleigh scatterers,
- each snowflake melts into one raindrop: no break-up or coalescence occurs,
- the mass flow is constant in the melting layer,
- all raindrops have the same size.

7.2.1 The melting process

The melted mass fraction $f_m$ of a spherical melting snowflake with diameter $D_{ms}$ is given by the differential equation [Ekpenyong and Srivastava, 1970]

$$L_f m_{ms} \frac{df_m}{dt} = 2\pi D_{ms} F [k_i \Delta T + KL_v \Delta \rho]$$

with $L_f$ and $L_v$ as, respectively, the latent heats of fusion and the condensation of water, $m_{ms}$ as the particle mass, $F$ as the so-called ventilation factor, $k_i$ as the thermometric conductivity of air and $K$ as the diffusivity of water vapor in air. The temperature difference between the surface of the snowflake and the surrounding air is given by $\Delta T$, and $\Delta \rho$ denotes the difference between the ambient water vapor density and that near the surface of the snowflake. $L_v$, $L_f$, $K$, and $k_i$ are material constants and are given in table 7.1 [Klaassen, 1988]
\[
\frac{df_m}{dt} = \frac{df_m}{dh} \frac{dh}{dt} = \frac{df_m}{dh} v_{ns}
\]

with \(v_{ns}\) as the fall velocity of the particle and \(h\) as the depth in the melting layer. Equation 7.4 is used by Ekpenyong and Srivastava [1970], Dissanayaka and McEwan [1978], Zhang et al [1990] and Hardaker et al [1991] to calculate the distance it takes for a snowflake to melt completely, thereby assuming a constant lapse rate of \(\Delta T\). However, heat is extracted from the surrounding air during melting, and so \(\Delta T\) is decreased. This process results in an isothermal layer, as observed by Willis and Heymsfield [1989]. The width of this layer depends on the rain intensity and the vertical air velocity [Klaassen, 1988] and can be in the order of 100 m. When the rain intensity increases the layer becomes thicker, because more heat is extracted during a longer time. When, on the other hand, the up-going vertical air velocity increases, warm air is transported to the melting zone and the layer becomes narrower. Using a wind tunnel, the melt-distance, which is equivalent to the depth in the melting layer, was experimentally related to the melted mass fraction by Mitra et al [1990], who found good agreement with the theoretical prediction. Their results, as well as the results of Klaassen [1988], can be approximated by

\[
f_m = \frac{1}{2} \left\{ \sin \left( \frac{h}{H_{max}} \pi - \frac{\pi}{2} \right) + 1 \right\}
\]

\(H_{max}\) is the distance at which the snowflake is completely melted and can be considered as the width of the melting layer. Klaassen [1988] found that the width of the melting layer is statistically related to the radar reflectivity of the rain just underneath it:
\[ H_{\text{max}} = 100 Z^{0.17} \quad [\text{m}] \]

with \( Z \) given in \( \text{mm}^6 \text{m}^{-3} \) and a spread of \( \pm 50 \text{ m} \). Assuming a Marshall-Palmer dropsize distribution with \( Z = 200 R^{1.6} \), \( H_{\text{max}} \) is written as

\[ H_{\text{max}} = 246 R^{0.272} \]

Figure 7.9 gives the depth a melting snowflake descends in the melting layer as function of the melted mass fraction of the particle for three rain intensities: \( R = 1, 10 \) and \( 50 \text{ mmh}^{-1} \). As expected, the width of the melting layer increases when the rain intensity increases. Apart from at the beginning and the end of the melting process, the depth in the melting layer is almost linearly related to the melted mass fraction.

![Figure 7.9](image)

Figure 7.9  The depth in the melting layer as function of the melted mass fraction for different rain intensities.
7.2.2 The mass density

The mass $m_{ms}$ of a melting snowflake may be considered as the sum of the mass $m_w$ of the melted and $m_s$ of the unmelted snow:

$$m_{ms} = m_s + m_w$$  \hspace{1cm} 7.9

It can be written in terms of the mass densities $\rho_{ms}$, $\rho_s$ and $\rho_w$ of the constituents as

$$\rho_{ms} V_{ms} = \rho_s V_s + \rho_w V_w$$  \hspace{1cm} 7.10

in which $V_{ms}$, $V_s$, $V_w$ are respectively the total volume of the particle, the volume of the unmelted and of the melted parts. Consequently, the total mass density is related to the volume fractions $f_s$ and $f_w$ of the snow and water parts as

$$\rho_{ms} = \rho_s f_s + \rho_w f_w = \rho_s (1 - f_w) + \rho_w f_w$$  \hspace{1cm} 7.11

or in terms of the melted mass fraction $f_m$,

$$\rho_{ms} = \frac{\rho_s}{1 - (1 - \rho_s)f_m}$$  \hspace{1cm} 7.12

in which it is implicitly assumed that $\rho_w = 1$ g cm$^{-3}$. Figure 7.10 shows the mass density of the melting particle as function of the melted mass fraction for three values of the mass density of the initial snowflake: $\rho_s = 0.01$, 0.1 and 0.5 g cm$^{-3}$.

The mass density of loose snowflakes remains small throughout most of the melting process and suddenly increases at the end of the melting process. The mass density of particles with a large initial mass density, e.g. graupel, increases more gradually. The mass density of the particles will turn out to be an important factor in modeling the
radar reflectivity from the melting layer. Particles with a small density may be large in size and since the Rayleigh scattering cross-section is proportional to the square of the particle volume, these particles may cause a large backscatter at low radar frequencies.

![Graph showing mass density vs. melted mass fraction for different values of mass density before melting.](image_url)

Figure 7.10 The mass density of a melting snowflake as function of the melted mass fraction for different values of the mass density of the snowflake before melting.

7.2.3 The particle size and the concentration

The size of a melting snowflake is closely related to its mass density. When a snowflake melts into a raindrop, the size is given by

\[ D_{ms} = \frac{1}{\rho_{ms}} D_r \]  

7.13

in which \( D_r \) is the diameter of the raindrop. Collisional break-up or coalescence of the snowflakes, as well as particle growth due to condensation of humidity, is ignored, since
*Klaassen* [1988] showed that their influence is of minor importance. Equation 7.13 enables the calculation of the particle size at any stage of melting, provided the mass density of the particle is known. To this end only the mass density of the snowflake before melting is required. The mean mass density \(<\rho_s>\) and the size \(D_s\) of a dry snowflake are related as *Klaassen*, 1988

\[
<\rho_s> = \frac{0.7}{D_s} \text{ [g cm}^{-3}, D_s \text{ in mm]}
\]  

7.14

As expected, small snowflakes are more dense than large ones. The combination of equations 7.13 and 7.14 could enable the derivation from \(\rho_s\) from \(D_r\). However, equation 7.14 only represents dry aggregated snow, while in reality other particle types, like graupel or wetted ice crystals, can occur as well. In that case the mass density of the unmelted particles is larger than predicted by equation 7.14.

The size of the raindrops is regarded as the mean dropsize of the Marshall-Palmer dropsize distribution, which results in

\[
D_r = 0.495 D_o
\]

7.15

\[
D_o = \frac{3.67}{4.1} R^{0.21}
\]

7.16

\[
D_r = 0.443 R^{0.21}
\]

7.17

The relationship between \(D_o\) and \(D_r\) results from averaging drop volumes, because \(D_o\) is defined as the diameter of the median drop volume. Conservation of mass implies *stationarity*:

\[
N_{ms} v_{ms} \rho_{ms} D_{ms}^3 = N_r \nu_r D_r^3
\]

7.18
\( N_{ms} \) and \( N_r \) are the number of, respectively, melting snowflakes with diameter \( D_{ms} \) and raindrops with diameter \( D_r \). \( v_r \) is the velocity of the raindrop. Equation 7.18 can be rewritten as

\[
N_{ms} = \frac{v_r}{v_{ms}} N_r \tag{7.19}
\]

The concentration of melting snowflakes is larger than the concentration of raindrops, because \( v_{ms} < v_r \). During melting \( v_{ms} \) increases and hence the number concentration of melting snowflakes decreases.

### 7.2.4 The fall speed of melting snowflakes

The fall speed of melting snowflakes is obtained from balancing the gravitational forces, the drag forces and the buoyancy of the snowflake [Matsuo and Sasyo, 1981],

\[
v_{ms} = \sqrt{\frac{4gD_{ms}\rho_{ms}}{3\rho_a C_d}} \tag{7.20}
\]

in which \( g \) is the acceleration of gravity, \( \rho_a \) the mass density of air and \( C_d \) the drag coefficient. The equation is derived for spherical particles. Spheroidal particles may have a somewhat different fall speed, because \( C_d \) depends on the shape of the particle, but in this study no attempt is made to model it. For dry snowflakes \( C_d \) equals approximately 1.2, but for melting snowflakes it shows a large scatter between 0.6 and 1.2 [Matsuo and Sasyo, 1981]. Unfortunately, little is known about the relationship between \( C_d \) and the melting process. Following the approach of Matsuo and Sasyo [1981], the drag coefficient of melting snowflakes is obtained through a linear interpolation between the \( C_d \) values of the initial snowflake and the resulting raindrop,

\[
C_d = \frac{1.2 - C_{dr}(D_{ms} - D_s)}{D_s - D_r} + 1.2 \tag{7.21}
\]
in which $C_{dr}$ is the drag coefficient of the raindrop, $D_s$ the diameter of the initial snowflake and $D_r$ the diameter of the raindrop. $C_{dr}$ is obtained from

$$
C_{dr} = \frac{4gD_r\rho_w}{3\rho_a v_r^2}
$$

in which the fall speed of the raindrop is calculated from the well-known relationship

$$
v_r = 9.65 - 10.3e^{-0.6D_r}
$$

Note that the fall velocity of melting snowflakes, due to its dependence upon $D_r$, is uniquely related to the rain intensity. Figure 7.11 gives $v_{ms}$ as function of the melted mass fraction for three rain intensities: $R=1, 10$ and $50$ mmh$^{-1}$. The mass density of the initial snowflake is set to 0.1 g cm$^{-3}$. The air density is taken as $1.3 \cdot 10^{-3}$ g cm$^{-3}$.

Figure 7.11 The fall speed of melting snowflakes for different rain intensities. The mass density of the initial snowflakes is set to 0.1 g cm$^{-3}$. 

172
The fall speed of a melting snowflake increases exponentially during melting, finally to reach the fall speed of the resulting raindrop. The enhancement is strongest in the last stage of melting. The fall speed increases when the rain intensity increases and, although not shown in the figure, also when the mass density increases.

The mean Doppler velocity \( V_{dms} \) of a melting snowflake is the mean, power-weighted, fall velocity. The reflectivity factor \( Z_{ms} \) of melting snow is by definition related to the radar cross-section \( \sigma_{ms} \) as

\[
Z_{ms} = \frac{\lambda^4}{|K_r|^2 \pi^5} \sigma_{ms} = \frac{\lambda^4}{|K_r|^2 \pi^5} \frac{|K_{ms}|^2 \pi^5}{\lambda^4} \int N_{ms}(D_{ms}) D_{ms}^6 dD_{ms},
\]

and consequently \( V_{dms} \) is related to the mean Doppler velocity \( V_{dr} \) of rain as

\[
V_{dms} = \frac{1}{\sigma_{ms}} \int V_{ms} N_{ms}(D_{ms}) D_{ms}^6 dD_{ms} \frac{|K_{ms}|^2 \pi^5}{\lambda^4} 7.25
\]

\[
= \frac{1}{\sigma_{ms}} \int V_r N_r(D_r) D_r^6 dD_r \rho_{ms} \frac{\pi^5}{\lambda^4} \frac{|K_{ms}|^2}{\lambda^4} 7.26
\]

\[
= \frac{|K_{ms}|^2}{|K_r|^2} \frac{Z_r}{Z_{ms}} \rho_{ms} \frac{\pi^5}{\lambda^4} V_{dr} 7.27
\]

The mean Doppler velocity depends on the density of the snowflakes and on the permittivity. Implicitly, it has been assumed that the mass density of melting snowflakes is independent on particle size. Nevertheless, in subsequent sections of this chapter it will be shown that equation 7.27 gives realistic results.

### 7.2.5 The shape and orientation of melting snowflakes

Modeling the shape of a melting snowflake with such a regular and symmetrical shape as a spheroid seems to be an over-simplification. However, Fujiyoshi [1986], who studied the melting of snowflakes after catching them in viscous silicone oil, found that snowflakes displayed lens-like shapes at the end of melting, rather than spherical shapes; the shape changed during melting. Quantifying the axial ratio of a melting snowflake is
difficult because little data is available. Magono and Nakamura [1965] performed measurements on dry snowflakes and concluded that, largely independent of the size, snowflakes may have an axial ratio between 0.5 and 1. Mitra et al [1990] used a value of 0.3 for dry snowflakes to explain their wind tunnel measurements of melting snowflakes, but they explicitly stated that this value was chosen arbitrarily. The change of the axial ratio during melting is even more difficult to model. Only one thing is sure: after melting the axial ratio equals that of the corresponding raindrop.

The earlier-discussed height profiles of the cross-correlation coefficient suggest that small melting snowflakes are more oblate than large ones in the upper half of the melting layer. In the lower half of the melting layer large snowflakes melt as well and become more oblate than the small ones. This process suggests modeling the axial ratio of snowflakes as a function of size, with the gradient of this function changing from positive to negative during melting. For reasons of simplicity, and considering that the size-shape relationship is highly stochastic, a linear function between the axial ratio $\xi_s$ and the equivalent drop diameter $D_e$ is proposed:

$$\xi_s(D_e) = \xi_1 + \frac{(\xi_2 - \xi_1)}{(D_{\text{max}} - D_{\text{min}})}D_e - \frac{(\xi_2 - \xi_1)}{(D_{\text{max}} - D_{\text{min}})}D_{\text{min}}$$  \hspace{1cm} 7.28$$

At the onset of melting $\xi_1$ is smaller than $\xi_2$ but during melting both parameters change. After melting $\xi_s(D_e) = 1 - 0.05D_e$ which is the Morrison-Cross relationship [Morrison and Cross, 1974] of the axial ratio of raindrops. For dry snow $\xi_2$ is set to 0.9, which approximates the mean result of Magono and Nakamura [1965]. Later in this study $\xi_1$ is selected to give realistic profiles of the polarimetric radar observables of the melting layer.

The shape of a melting snowflake results from internal, e.g. capillary, forces as well as external forces, like gravity, drag and buoyancy. It depends on both the mass and volume of the particle. However, it is intuitively felt that the shape of a melting snowflake is more strongly related to its volume than to its mass, because the mass does not change during melting, whereas the volume and shape do. It is therefore assumed that the axial ratio $\xi_{ms}$ changes as function of the melted volume fraction:
\[ \xi_{ms} = \xi_s + (\xi_r - \xi_s) f_w \]

in which \( \xi_r \) is the axial ratio of the resulting raindrop. \( D_{max} \) and \( D_{min} \) are related to the mean raindrop size \( D_r \) by

\[ D_{max} = \rho_{\min}^{\frac{1}{3}} D_r \]
\[ D_{min} = \rho_{\max}^{\frac{1}{3}} D_r \]

with \( \rho_{\max} = 0.5 \) and \( \rho_{\min} = 0.01 \text{ gcm}^{-3} \), which is a realistic range of values. The axial ratio \( \xi_r \) of the raindrop is derived as the reflectivity-weighted mean of the Morrison-Cross relationship:

\[ \xi_r = 1. - 0.19 D_r \]

with \( D_r \) in mm. When a melting snowflake is modeled as an oblate spheroid, its orientation has to be taken into account as well. The orientation of a snowflake is difficult to define, because it assumes a certain axis of symmetry, which may not be necessarily. The assumed orientation of the spheroid is not necessarily that of the snowflake, but may also represent effects caused by the irregular shape and motion of the particle.

Raindrop canting is caused by wind shear and turbulence. It is likely that the same holds for melting snowflakes. But there is more, as was found by Mitra et al [1990]. Before melting, the motion of a snowflake can be correlated to its physical properties, like shape, size and mass. However, as soon as melting starts, the particle starts to move in a completely random order: helical loci are interrupted by linear movements, swinging and rotation. This lasts until the ice frame of the particle collapses, from which moment on the particle starts to accelerate in a downwards direction. No tumbling is observed. The observations were carried out in a wind tunnel with constant air stream and temperature gradient, and so no external forces would have caused this random motion. Probably internal forces, caused by a random distribution of melt
water inside the snowflake, force the particle into this behavior.

Polarimetric radar measurements are sensitive to the shape and orientation of the particles as projected on the plane of incidence of the radar wave. Consequently, the random motion of melting snowflakes causes the radar to see particles of varying shape and orientation, even if the particle remains the same. In the model the orientation angle is used to account for this. The azimuth angle of the axis of symmetry of the snowflake is assumed to be uniformly distributed between 0 and 2π. The elevation angle is also uniformly distributed up to a certain (variable) width. Since the randomness of particle motion decreases towards the end of melting, the width of the distribution decreases as well. It is assumed to depend linearly on the melted volume fraction:

\[
\sigma_{ms} = \sigma_s + (\sigma_r - \sigma_s) f_v
\]

7.33

in which \(\sigma_s\) and \(\sigma_r\) are the widths corresponding to the orientation of respectively unmelted snow and rain. It will appear that by letting the width depend on the melting rate, the peaks of \(Z_{dr}\) and \(L_{dr}\) no longer coincide: the \(L_{dr}\) peak occurs at a higher altitude than the \(Z_{dr}\) peak.

7.2.6 The permittivity of melting snowflakes

Melting snow is a mixture of air, ice and water. The calculation of the permittivity of melting snow therefore requires a mixing formula. The form of the mixing formula depends on the constitution of the melting particle: is it a water-coated snow particle, is it a homogeneous mixture, or perhaps something in between? The melting process is studied by Fujiiyoshi [1986] and Mitra et al [1990]. They concluded that melting occurs at the periphery of the snowflake, and that the water is rapidly sucked inside. The structure of ice branches remains intact throughout almost the whole melting process, indicating that the melt water forms a film around the ice branches. A mixing formula that is able to calculate the permittivity of such a mixture is the Maxwell Garnet theory [1904], which treats a mixture as matrix of homogeneous material that surrounds spherical inclusions of another material. Bohren and Battan [1982] extended the theory to let it allow ellipsoidal inclusions as well. To prevent their method from becoming
unstable, the permittivity of the matrix and of the inclusions should not differ too much: the electromagnetic contrast must be small. De Wolf et al [1990] restricted the theory to the case of spheroidal inclusions, in which case large contrasts are allowed as well. A general treatment of mixing formulas is given by Sihvola and Kong [1988], who showed that the Maxwell Garnet formula is a special case of a general formulation of mixing formulas. The Maxwell Garnet theory deals with a two-component mixture. In the case of melting snow, which consists of three components, the mixing rules must be applied twice. The use of mixing formulas requires some knowledge of the structure of the melting snowflakes; what constituent should be taken as the matrix and what as the inclusion?

In this study the formulation of De Wolf et al [1990] is used. The permittivity $\varepsilon_{\text{mix}}$ of a mixture is given by

$$
\varepsilon_{\text{mix}} = \varepsilon_m \frac{1 + \frac{1}{3} f_i \left\{ \varepsilon_i \left( <\Lambda_1> + <\Lambda_2> + <\Lambda_3> \right) - 3 \right\}}{1 + \frac{1}{3} f_i \left\{ <\Lambda_1> + <\Lambda_2> + <\Lambda_3> \right\} - 3}
$$

7.34

in which $f_i$ is the volume fraction of spheroids with shape factors $\Lambda_{1,2,3}$ (see chapter 3) and permittivity $\varepsilon_i$, embedded in a matrix with permittivity $\varepsilon_m$. Since the Rayleigh theory is used, only situations with wavelengths much longer than the spheroids involved are allowed. In practice this requirement is satisfied, because the spheroids are small droplets inside a snowflake. The snowflake is considered to consist of $N$ spheroids in a uniform matrix, and so ensemble averaging occurs. The particle distribution of these spheroids is not known and it is not likely that they are distributed according to a specific function. The shape factors are related to the depolarization factors $\lambda_{1,2,3}$ and in the case of spheroids $\lambda_1 = \lambda_2 = \frac{1}{2}(1 - \lambda_3) \in [0,1]$. A uniform distribution of $\lambda_3$ between $a$ and $b$ is used, which yields in

$$<\Lambda_1> = <\Lambda_2> = \frac{2\varepsilon_m}{(b - a)(\varepsilon_i - \varepsilon_m)} \ln \left\{ \frac{\varepsilon_i + \varepsilon_m - a(\varepsilon_i - \varepsilon_m)}{\varepsilon_i + \varepsilon_m - b(\varepsilon_i - \varepsilon_m)} \right\}$$

7.35

$$<\Lambda_3> = \frac{\varepsilon_m}{(b - a)(\varepsilon_i - \varepsilon_m)} \ln \left\{ \frac{\varepsilon_m + b(\varepsilon_i - \varepsilon_m)}{\varepsilon_m + a(\varepsilon_i - \varepsilon_m)} \right\}$$

7.36
De Wolf et al [1990] suggest using $a=0.2$ and $b=0.5$, since they found that $\lambda_3 > 0.6$, which corresponds to plate-like spheroids, strongly influences $\varepsilon_{mix}$, while melting snowflakes conserve their needle-like branches throughout almost the entire melting process. Since water forms a film around the ice branches, it seems plausible to use water as the matrix around ice inclusions and in a second stage to calculate the permittivity of the entire melting snowflake by taking the ice-water mixture as the matrix of air inclusions, since melting also occurs at the periphery of the particle. However, one could also argue that, just because melting occurs at the outside of the snowflake, water should be treated as the matrix of air-ice inclusions. Both options are investigated.

![Graph showing permittivity as a function of melted volume fraction](image)

Figure 7.12a The permittivity as function of the melted volume fraction for different values of the mass density of the dry snowflakes. Wet ice is taken as the matrix around air inclusions.

The calculation of the permittivity of air-ice mixtures was found to be hardly dependent on whether ice or air was taken as the matrix [De Wolf et al, 1990]. Figures 7.12a and b show the permittivity as function of the melted volume fraction in both cases: figure a represents air in wet ice, and figure b snow in water. Since the volume fractions of air and ice depend on the mass density of the initial snowflakes, the graphs are given for
\( \rho_s = 0.01, 0.1, \) and \( 0.5 \text{ gcm}^{-3} \). The permittivity of water and ice is taken from Ray[1972]: \( \varepsilon_w = 81.4 \) and \( \varepsilon_{\text{ice}} = 3.15 \) at 3.315 Ghz, the frequency of the Delft Atmospheric Research Radar. The imaginary part of the permittivity is neglected because of its minor significance at S-band.

Figure 7.12b The permittivity of melting snowflakes as function of the melted volume fraction, for different values of the mass density of dry snowflakes. Water is taken as the matrix around snow inclusions.

The 'wet ice around air' option is more sensitive to \( \rho_s \) than the 'water around snow' option. Also, the permittivity of 'snow in water' is systematically larger than the permittivity of 'air in wet ice'. However, too little is known about the melting process to make a well founded choice between the two options. Simulations of the height profiles of the radar observables in the melting layer are used to choose the most realistic option, or perhaps a combination of the two...
7.3 Simulations of the bright band

The rain intensity and the mass density of the dry snowflake are the major input parameters of the melting layer model. However, the axial ratio $\xi_1$ of the smallest dry snowflakes and the width of the orientation-angle distribution still have to be specified. As was seen in chapter 5, the width of the orientation-angle distribution may vary during an event. It is therefore unlikely that fixed values of $\sigma_s$ and $\sigma_r$ will suit all measurements. Two methods to calculate the permittivity were discussed in the previous section and no choice between them was made yet.

A parametric analysis of the melting layer model is performed with the following initial values of the relevant parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_1$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>50°</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>45°</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.12 gcm$^{-3}$</td>
</tr>
<tr>
<td>$R$</td>
<td>5 mmh$^{-1}$</td>
</tr>
</tbody>
</table>

Table 7.2 Input parameters for simulation of backscattering by the melting layer

The simulations will eventually yield parameter values that enable predictions of typical bright band observations.

Figure 7.13 shows $Z_h$, $Z_{dr}$, and $-L_{dr}$ as function of the depth in the melting layer, with $h=0$ corresponding to the top of the melting layer. The radar elevation is set to 30°. The dashed, horizontal line indicates a typical value of the $L_{dr}$ antenna limit. The thick line represent backscattering by water-coated snowflakes (denoted by /s), whereas the thin lines are based on modeling a melting snowflake as a wet-ice matrix that surrounds air (denoted by /a). The snow-in-water model gives a much stronger bright band than the air-in-wet-ice model. Also, the latter produces its peak below that of the snow-in-water model. The peak that is produced by the snow-in-water model is large, approximately 15 dB. The strength of the peak depends heavily on the mass density of the initial snowflake: the smaller the mass density, the larger the peak, as was also found by Klaassen [1988]. A large value of the mass density $\rho_s$ is used. Decreasing it to generally occurring values smaller than 0.1 gcm$^{-3}$, would result in large, unrealistic peak
reflectivities. It is concluded that the snow-in-water model produces a reflectivity value that is too large.

Figure 7.13 Simulated height profiles of $Z_h$, $Z_{dr}$, and $L_{dr}$. Two models to calculate the permittivity of melting snowflakes are used: snow in water (thick lines, /s) and air in wet ice [thin lines, /a]. The dashed horizontal line represents the antenna limit for $L_{dr}$ measurements. The radar elevation is set to 30°.

The air-in-wet ice model produces a narrow bright band with a small peak reflectivity of approximately 7 dB. A smaller mass density would enhance the peak, but the bright band would also become narrower, because, with reference to figure 7.12a, the increase of the permittivity starts at a later stage in the melting process. For both calculation methods, $Z_{dr}$ exhibits a peak at the lower bound of the melting layer. The snow-in-water model produces a larger $Z_{dr}$ peak than the other model and the peak is also slightly shifted upwards. As was seen in section 7.1, the height and strength of the $Z_{dr}$ peaks can vary during the event. However, the $Z_{dr}$ profile calculated with the air-in-wet ice model appears to follow a realistic trend: it is small in the upper half of the melting layer and only starts to increase at the bottom of the melting layer, whereas the snow-in-water model produces a $Z_{dr}$ that increases right from the onset of melting. The $L_{dr}$
profiles follow the same tendencies: a peak occurs at the bottom of the melting layer, and again the snow-in-water model gives a larger peak than the air-in-wet ice model.

Both methods of calculating the permittivity produce realistic height profiles of $Z_{dr}$ and $L_{dr}$, but $Z_h$ profiles that do not correspond to usually observed bright bands, and so the question arises as to which method to use. The snow-in-water model does not apply to the observation of water films around the ice branches in melting snowflakes but it does represent melting at the periphery of the snowflakes. On the other hand, the air-in-wet ice models represents the formation of water films around the ice branches but it does not take into account the melting at the periphery of the snowflake. It seems that the methods complement each other and therefore need to be combined. Figure 7.14 shows the result in case the mean of the two permittivities is used to calculate the height profiles. Also shown is the mean Doppler velocity.

![Graph showing profiles of radar observables](image)

**Figure 7.14** Simulation of the height profiles of the radar observables, with the methods to calculate the permittivity averaged. Same conditions as in figure 7.13.

The resultant profiles are in agreement with typically observed radar measurements of the melting layer. A symmetrical $Z_h$ profile is accompanied by $Z_{dr}$ and $L_{dr}$ profiles that are small in the upper half of the melting layer and show a peak at the bottom.
Note that in the upper half of the melting $L_{dr}$ lies below the antenna limit and is therefore hard to measure. In the following subsection the effect of the input parameters of the model is discussed.

7.3.1 Effect of the mass density of the initial snowflake

For a given melted diameter $D_r$, the size of the snowflake is determined by the mass density. The larger the mass density, the smaller the particle is. This implies that low-density particles are less oblate than high-density particles. Figures 7.15a and b give the height profiles of the radar observables, calculated with $\rho_s=0.05$, 0.1 and 0.5 g cm$^{-3}$. The remaining input parameters are given in table 7.2.

![Graph showing $Z_h$ and $Z_{dr}$ vs depth in melting layer](image)

Figure 7.15a Simulation of $Z_h$ and $Z_{dr}$ in the melting layer for different values of the mass density of the dry snowflake. Elevation = 30°.

When the mass density of the initial snowflakes increases $Z_h$ decreases, because the particles become smaller. The $Z_h$ peak shifts upwards because the permittivity of high-density particles changes gradually during melting, whereas the permittivity of low-
density particles increases abruptly towards the end of melting. Note that the strength of the bright band decreases rapidly for high mass densities. The $Z_{dr}$ and $L_{dr}$ profiles exhibit a peak in all three cases. These, co-located, peaks shift upwards, broaden and increase when the mass density increases. The fall speed $V_d$ increases monotonically during melting. The velocity at the top of the melting layer increases when the mass density increases, since the particles encounter less air resistance.

![Graph showing $L_{dr}$ and $V_d$ vs depth in the melting layer](image)

Figure 7.15b Simulation of $L_{dr}$ and $V_d$ in the melting layer, for different values of the mass density of the dry snowflake. Elevation = 30°.

Only the mass density of the initial snowflakes was varied during the simulations; the rain intensity was kept constant. One might argue that the rain intensity is correlated to the mass density of the snowflakes, for instance by saying that large snowflakes cause large rain intensities, and that therefore the simulation is not realistic. However, Klaassen [1988] has shown that no correlation exists between the radar reflectivity just below the melting layer and the strength of the bright band; the mass density is highly stochastic, and causes a large scatter in the $Z_h$-$Z_{peak}$ diagram.
7.3.2 Effect of the axial ratio of the particles

The apparent oblateness of melting snowflakes was justified by considering that melting occurred at the bottom of snowflakes. The fact that small particles are more oblate than large ones at the onset of melting was justified by considering that small particles fall slower than large ones. The effect of the axial ratio was investigated by setting the axial ratio $\xi_1$ of the smallest snowflake to 0.3, 0.6 and 0.9, and $\xi_2$ to 0.9. The axial ratio converges to the value of the corresponding raindrop during melting. Figures 7.16 show the results of the simulations.

$Z_h$ is hardly dependent upon the axial ratio: it varies less than 1 dB when $\xi_1$ varies. The $Z_{dr}$ and $L_{dr}$ profiles, however, change significantly. When $\xi_1 = 0.9$, implying more or less spherical snowflakes, no peak is observed at all. For $\xi_1 < 0.9$ the peak appears, and increases when $\xi_1$ decreases. The $Z_{dr}$ and $L_{dr}$ peaks coincide and do not shift up or down when the axial ratio is varied. The fall speed does not depend on the axial ratio, but is given here for the sake of completeness.

![Graph showing the variation of $Z_h$, $Z_{dr}$, and $L_{dr}$ with depth in the melting layer for different axial ratios](image)

**Figure 7.16**: Simulation of $Z_h$, $Z_{dr}$ in the melting layer for different values of the axial ratio of the smallest snowflake. Elevation = 30°.
Figure 7.16b Simulation of $L_{dr}$, and $V_d$ in the melting layer for different values of the axial ratio of the smallest snowflake. Elevation = 30°.

7.3.3 Effect of the orientation of melting snowflakes

The orientation of the melting snowflakes is varied by setting the spread $\sigma_s$ of the uniform orientation distribution of dry snowflakes to 75°, 50° and 45°, and keeping $\sigma_r$ at 45° for the resulting raindrops. Figures 7.17 give the results of the simulation.

$Z_h$ is hardly affected by the orientation of the snowflakes: it varies a few tenths of a dB when $\sigma_s$ varies. $L_{dr}$ increases when the particle canting increases, but $Z_{dr}$ decreases then. $Z_{dr}$ is more sensitive to canting variations than $L_{dr}$; the increase of $L_{dr}$ is small compared to the decrease of $Z_{dr}$. The $Z_{dr}$ peak moves downwards when the particle canting increases, but the location of the $L_{dr}$ peak does not change: the two peaks no longer coincide. The orientation angle was defined to account for the irregular motions of melting snowflakes; it does not necessarily represent real particle canting. However, the concept describes the typically observed behavior of $Z_{dr}$ and $L_{dr}$ well.

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Figure 7.17 Simulation of $Z_h$, $V_d$, $Z_{dr}$, and $L_{dr}$ in the melting layer for different values of the spread $\sigma_s$ of the orientation-angle distribution of dry snowflakes. Elevation = 30°.
7.3.4 Effect of the rain intensity

Raindrops become larger and more oblate when the rain intensity increases. The dry snowflakes at the top of the melting layer also become larger then, but less oblate. Figures 7.18 show the height profiles of the radar observables for $R=1$, 10, and 50 mm$h^{-1}$, corresponding to light, moderate and heavy rain. Note that the scaling differs for the three plots. The $R=50$ mm$h^{-1}$ plot will not be observed frequently, because in these heavy rain storms convective air motion may disturb the stratiform structure. The width of the bright band increases when the rain intensity increases, but the relative peak reflectivity remains more or less constant. This is in agreement with the earlier-mentioned lack of correlation between the reflectivity of rain and the strength of the bright band. The values of $Z_{dr}$ and $L_{dr}$ in the melting layer increase when the rain intensity increases, but the peak values become smaller relative to the underlying rain; the location of the peaks is constant. The mean fall speed of the dry snowflakes does not change much when the rain intensity increases, but it increases significantly in rain.

![Graph showing radar observables in the melting layer for $R=1$ mm$h^{-1}$. Elevation = 30°.](image)

Figure 7.18a The radar observables in the melting layer for $R=1$ mm$h^{-1}$. Elevation = 30°.
Figure 7.186 The radar observables in the melting layer for $R=10 \text{ mm h}^{-1}$ and for $R=50 \text{ mm h}^{-1}$. Elevation = $30^\circ$. 
7.3.5 Summary of the simulations

The model has been tested for its sensitivity to the mass density of the melting snowflakes, their axial ratio and orientation and to the rain intensity. The variations that the radar observables undergo are summarized in table 7.3. Of each radar observable the qualitative response to an increase of one of the input parameters $\rho_s$, $\xi_1$, $\sigma_s$ and $R$ is given. Val denotes the effect on the value and Loc on the location of the peak, if relevant. A '+' sign indicates an increase, a '-' sign a decrease, and 0 means no significant change. Table 7.3 deals with changes in the relative values of the radar observables. Changes in, for instance, the width of the bright band due to a variation of the rain intensity are not reflected by it.

The mass density of the dry snowflakes is a major factor in modeling the radar reflections. It affects all radar observables, whereas the other input parameters influence only $Z_{dr}$ and $L_{dr}$. In order to minimize the effect of variations of $\rho_s$, long-term measurements of the melting layer are required. Comparison of the data with the model may lead to a fixed value of $\rho_s$ that represents a large number of naturally occurring bright bands.

<table>
<thead>
<tr>
<th></th>
<th>$Z_h$</th>
<th>$Z_{dr}$</th>
<th>$L_{dr}$</th>
<th>$V_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Val</td>
<td>Loc</td>
<td>Val</td>
<td>Loc</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$R$</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7.3 Qualitative response of the radar observables to an increase of the mass density $\rho_s$, the axial ratio $\xi_1$, the spread $\sigma_s$, and to the rain intensity $R$.

It is worthwhile to recall the effect of the orientation of the melting particles. If the spread of the orientation-angle distribution depends on the melted fraction, the peaks of $Z_{dr}$ and $L_{dr}$ no longer coincide; the $Z_{dr}$ peak shifts downwards, whereas the $L_{dr}$ peak remains at the same location.
7.4 Comparison with measurements

In the first section of this chapter, three different measurements were discussed. Two were done at an elevation of 30°, and one was done with the radar pointed towards the zenith. In this section, the input parameters of the melting layer model are adjusted in order to fit to the data. It is shown that only the rain intensity and the mass density of the particles have to be varied to account for the difference between the three events.

First the event of figure 7.1 is discussed. The reflectivity underneath the melting layer corresponds to a rain intensity of approximately 2.5 mmh⁻¹. The peak reflectivity of the melting layer is approximately 15 dB with respect to the reflectivity in rain, and \( Z_{dr} \) exhibits a peak of approximately 1 dB. In rain, \( Z_{dr} \) equals approximately 0.5 dB. The peak value of \( L_{dr} \) is approximately -26 dB. In rain, \( L_{dr} \) equals approximately -28 dB. The relatively large peak of \( Z_h \) suggests a small mass density of the dry snowflakes. The peaks of \( Z_{dr} \) and \( L_{dr} \) almost coincide. This suggests that the orientation distribution of the melting snowflakes hardly depends on the melted fraction. Also, the peak value of \( Z_{dr} \) suggests that the axial ratio of the smallest snowflakes should be made small. \( V_d \) is too much affected by wind and is no longer representative of the fall speed of the hydrometeors. Figure 7.19 shows the radar observables, when calculated with the input parameters of table 7.4.

\[
R = 2.5 \text{ mmh}^{-1} \\
\rho_s = 0.05 \text{ gcm}^{-3} \\
\xi_1 = 0.3 \\
\sigma_s = 50° \\
\sigma_r = 45°
\]

Table 7.4 The input parameters of the simulation of backscattering by the melting layer in the case of the event shown in figure 7.1.

The simulated profiles are in agreement with the measurements. The \( Z_h \) profile is symmetrical and the peak value of \( Z_h \) is approximately 15 dB. In the rain region \( Z_h \) converges to 29 dBZ. The \( Z_{dr} \) peak occurs at the bottom of the melting layer and equals approximately 0.9 dB. In rain, it equals 0.5 dB. The \( L_{dr} \) peak also occurs at the bottom of the melting layer and equals approximately -21 dB. In rain, it equals -30 dB, which is less than measured. The simulated profile of \( L_{dr} \) in the upper half of the
melting layer is too small to be measured.

![Graph showing Zh, Zdr, Ldr, and Vd vs Depth in melting layer (m)]

**Figure 7.19** Simulation of the event shown in figure 7.1

The model predicts a width of the melting layer that is much smaller than the observed one: the difference is approximately 150 m. This is not a defect in the model but rather an experimental difficulty. The earlier-used relationship between $H_{max}$ and $R$ is derived from measurements using the Delft Atmospheric Research Radar with a range resolution of 30 meter, whereas the measurements discussed in this study were carried out with a range resolution of 150 meter. Recalling that cross-talk from one range cell into another is inherent to FM-CW radar systems, it is evident that the width of the melting layer is estimated less well with a range resolution of 150 meter, than with a range resolution of 30 meter. Unfortunately, the making of combined Doppler and polarimetric measurements with a high spatial resolution was not feasible during this study. The model also predicts a peak value of $L_{dr}$ that is larger than observed with DARR. Again, this is likely to be caused by the large range cell: measurements with a high angular resolution [*Illingworth and Caylor, 1989*] show that $L_{dr}$ can indeed be as large as predicted by the model.

The event shown in figure 7.6 is comparable to the one depicted in figure 7.1, although
the rain intensity is larger and the elevation angle equals 90°. The reflectivity above and below the bright band are in the same order of magnitude and the peak approximates 15 dB. $Z_{dr}$ is very small and does not show a significant height dependence. $L_{dr}$ equals $-30$ dB in the upper half of the melting layer and exhibits a peak value of $-23$ dB in the lower half. $V_d$ is small at the onset of melting and gradually increases to 6.5 m/s. It is no longer severely affected by wind and so indicates the melting layer. The event is simulated with the rain intensity $R$ set to 7.5 mm h$^{-1}$, and the remaining parameters from table 7.4. The result is given in figure 7.20.

![Figure 7.20 Simulation of the event shown in figure 7.6.](image)

The peak reflectivity and the mean fall speed are in good agreement with the measurements. The simulated $Z_{dr}$ equals 0 dB and does not depend on the height. The simulated $L_{dr}$ exhibits a peak at the lower bound of the melting layer. However, the $L_{dr}$ values do not agree with the data. The peak is too small and $L_{dr}$ does not converge to $-28$ dB in rain. In order to fit $L_{dr}$ to the data, the orientation-angle distribution has to be changed; the width must be set 90°. However, little is known about the true value of the orientation angles, especially because these do not represent the orientation of the snowflakes, but rather deal with the movements of the snowflakes. Fitting the model to the data by varying the orientation-angle distribution is therefore feasible but is not
(yet) convincing.

The event shown in figure 7.2 is different from the other events in that the rain intensity is very low and the $Z_{dr}$ peak in the melting layer is larger and broader than in the event shown in figure 7.1. The $Z_{dr}$ peak equals approximately 1.3 dB relative to $Z_{dr}$ in rain and is located in the middle of the melting layer. The reflectivity peak equals approximately 10 dB. The combination of a relatively small peak of $Z_h$ and large, broad peak of $Z_{dr}$ indicates a large mass density of the dry initial snowflakes. No accurate $L_{dr}$ could be measured, because the signal-to-noise ratio during the measurements was approximately 10 dB. The measurements are simulated with $R = 0.4$ mm$h^{-1}$ and $\rho_s = 0.2$ gcm$^{-3}$, and the other input parameters from table 7.4. The result is given in figure 7.21.

![Figure 7.21](image.png)

Figure 7.21 Simulation of the event shown in figure 7.2.

A narrow bright band is simulated with a $Z_h$ peak of approximately 11 dB. The peak of $Z_{dr}$ is approximately 1.5 dB with respect to $Z_{dr}$ in rain and occurs slightly below the peak of $Z_h$. Note that the $Z_{dr}$ profile is broader than the profile of $Z_h$, as it also is during the measurements. The model predicts a large $L_{dr}$ peak but the data does not show one. Recall that during the event $W_d$ and $Z_{dr}$ were positively correlated in the
melting layer, which could mean that little turbulence occurred, making canting less significant and hence \( L_{dr} \) small. However, further study is necessary to confirm this.

7.4 Conclusions

Doppler-polarimetric radar measurements of the melting layer enable the detailed study of meteorological processes in the melting layer. During melting the reflectivity increases and decreases again, due to changing dropsizes, permittivity, and number concentration. When the radar is pointed well away from zenith, the differential reflectivity manifests itself somewhere in the melting layer. The strength and value of the \( Z_{dr} \) peak depends on the circumstances: during events with a low rain intensity it can be large and occurs in the upper half of the melting layer, during moderate events it is located at the bottom of the melting layer and has a moderate value. During moderate events the linear depolarization ratio increases during melting, irrespective of the elevation angle of the radar. Even with the radar pointed towards the zenith, a significant enhancement of \( L_{dr} \) is observed, whereas \( Z_{dr} \) does not exhibit a height dependence at all.

The cross-correlation coefficients of the radar observables change during melting. \( Z_h \) is negatively correlated to \( Z_{dr} \) and \( L_{dr} \) in the upper half of the melting layer, and positively in the lower half. These correlations are used to model the shape of dry snowflakes as oblate spheroids that become less oblate when the particle size increases. During melting, the size-shape relationship changes and eventually adopts that of rain, which means that particles become more oblate when the particles size increases. This feature has been incorporated into a model of the melting layer that is able to predict \( Z_h, Z_{dr}, L_{dr}, \) and \( V_d \). The results of the model agree with experimental data. The frequently observed shift between the locations of the \( Z_{dr} \) and \( L_{dr} \) peaks can be attributed to a changing orientation-angle distribution; when particle canting is modeled as a function of the melting rate, the peaks no longer coincide. An event with a low rain intensity can be modeled, assuming small snowflakes with a large mass density. The model predicts a large \( Z_{dr} \) peak then that occurs in the middle of the melting layer.
Chapter 8
Radar remote sensing of precipitation; conclusions

Doppler-polarimetric weather radars are powerful tools to study physical processes in precipitation. Polarization diversity enables the measurement of the mean shape and orientation of the hydrometeors and Doppler measurements give their velocities. The interpretation of Doppler and polarization measurements requires a model of precipitation: the statistical distributions of the size, shape, velocity and orientation of the hydrometeors must be defined in advance. The radar measurements are used to quantify this model of the microstructure. To this end the Rayleigh theory of electromagnetic scattering by dipoles is used.

The shape of a raindrop is modeled as an oblate spheroid, the axial ratio of which depends on the size of the raindrop. The velocity depends uniquely on the size of the raindrop and increases when the raindrop becomes larger. A three-parameter gamma dropsize distribution and a Gaussian canting-angle distribution are used. The mean canting angle is assumed to be 0°. Simulations of the Doppler spectrum due to rain showed that the mean reflectivity and the mean velocity increase when the rain intensity increases. The width of the Doppler spectrum increases for small rain intensities, but decreases again when the rain intensity becomes large. This is because the raindrops attain the maximum terminal fall speed. The Doppler spectrum is symmetrical for moderate rain intensities but becomes skewed for large rain intensities. The differential reflectivity and the linear depolarization ratio increase when the rain intensity increases, because large raindrops are more oblate than small ones.

The Delft Atmospheric Research Radar DARR is an FM-CW radar that is able to measure the scattering matrix as well as the Doppler spectrum. An FM-CW radar uses less transmit power than a pulse radar and yet achieves the same sensitivity as a pulse radar. Because the range resolution does not depend on the transmit power, an FM-CW is more flexible than a pulse radar. However, the need for two separate antennas is a disadvantage. DARR achieves a 20-25 dB clutter suppression because of the subtraction of two sweeps with equal polarization. Mechanical polarizers change the polarization of the radar waves sinusoidally with a maximum deviation of 90°. By performing three copolar measurements at different polarizations, the cross-polar components of the scattering matrix can be obtained. This lightens the restrictions on the signal-to-noise
ratio for obtaining $L_{dr}$ measurements.

The radar observables $Z_h$, $Z_{dr}$, $L_{dr}$, and $W_d$ have different sensitivities to changes in the microstructure of precipitation. When the polarimetric observables are expressed in linear units, the following conclusions hold. $Z_h$ is most sensitive to variations of the median dropsize; the other radar observables change as well but less than $Z_h$. All observables are less sensitive to changes in the dispersion factor than to changes in the median dropsize. $L_{dr}$ is the radar observable that is most sensitive to changes in the dispersion factor and the orientation-angle distribution. However, when the radar observables are expressed in logarithmic units, quantities that are smaller than the base of the natural logarithm $e$, like $Z_{dr}$ and $W_d$, achieve a larger sensitivity factor; quantities that are larger than $e$ will become less sensitive. In order to simulate measured scatter diagrams of the radar observables, all parameters of the microstructure had to be randomized, indicating that all parameters could have been varied during the event.

A strong correlation between $L_{dr}$ and $W_{d,m}$ is measured. This is interpreted as being caused by turbulence: it broadens the Doppler spectrum and induces raindrop canting. Based on the measurements a simple model between $L_{dr}$ and the spread of the turbulence spectrum is derived. However, the model is based on one event only, and further study is necessary to generalize it. In rain, the cross-correlation coefficients of all radar observables are, more or less, consistent with height.

The combination of $Z_h$, $Z_{dr}$, and $W_d$ can be used to calculate the three parameters of the gamma dropsize distribution: $\mu$, $N_o$, and $D_o$. The combination of $Z_{dr}$ and $L_{dr}$ is necessary to correct $W_{d,m}$ for turbulence. It is found that $N_o$ and $D_o$ are statistically related to $\mu$, but that $D_o$ and $N_o$ are not related. The measured relationship between $N_o$ and $\mu$ qualitatively agreed with the result obtained by Ulbrich [1983]. Quantitatively, however, an offset of approximately 10 dB appeared. The measured relationships between $\mu$, $N_o$ and $D_o$ result in a different $Z$-$R$ relationship than $Z = 200 R^{1.6}$. Comparison with rain gauge measurements showed that the accuracy of the radar-derived rain intensity was not significantly enhanced by the inclusion of $\mu$. However, only a limited amount of data, with $R < 10$ mmh$^{-1}$, was analyzed and the measurement configuration was not ideal: the rain gauge was not positioned underneath the radar beam. To study the accuracy of the method properly, a long series of measurements are necessary.
Most operational weather radars are not equipped with the capability of measuring $W_d$ and $Z_{dr}$ and are therefore not able to measure $\mu$. In this light, the major conclusion of chapter 6 is that interrelationships between the parameters of the drop size distribution exist. The use of these relationships may improve the accuracy of the single-parameter radars.

In the melting layer the reflectivity increases and decreases again, due to changing dropsizes, permittivity, and number concentration. When the radar is pointed well away from zenith, the differential reflectivity shows a peak in the melting layer. The location and value of the peak depend on the circumstances: during events with a low rain intensity the $Z_{dr}$ peak can be large and occur in the upper half of the melting layer, during moderate events it is located at the bottom of the melting layer and has a moderate value. $L_{dr}$ increases during melting, irrespective of the elevation angle of the radar. Even with the radar pointed towards the zenith, a significant enhancement of $L_{dr}$ is observed, whereas $Z_{dr}$ remains constant during melting.

The cross-correlation coefficients of the radar observables change during melting. $Z_h$ is negatively correlated to $Z_{dr}$ and $L_{dr}$ in the upper half of the melting layer, and positively in the lower half. These correlations are used to model the shape of dry snowflakes as oblate spheroids that become less oblate when the particle size increases. The size-shape relationship changes during melting and eventually adopts that of rain. In rain, the particles become more oblate when the particle size increases. This feature has been incorporated into a model of the melting layer that is able to predict $Z_h$, $Z_{dr}$, $L_{dr}$, and $V_d$. The results of the model agree with radar observations. The frequently observed shift between the locations of the peaks of $Z_{dr}$ and $L_{dr}$ can be attributed to a changing canting-angle distribution; when particle canting is modeled as a function of the degree of melting, the peaks do not coincide anymore. Weak events with a small bright band and large $Z_{dr}$ peak can be modeled by assuming small snowflakes with a large mass density.
References

Allnutt, J.E., 'Satellite-to-ground radiowave propagation', Peter Peregrinus Ltd, London, United Kingdom, 1989

Atlas, D., Srivastava, R.C., Sekhon, R.S., 'Doppler radar characteristics of precipitation at vertical incidence', Reviews of Geophysics and Space Physics, Vol 11 No 1, 1973


Austin, P.M., Bernis, H.C., 'A quantitative study of the bright band in radar precipitation echoes', Journal of Applied Meteorology, Vol 7, 1950


Bostian, C.W., Allnutt, J.E., 'Ice-crystal depolarization on satellite-earth microwave paths', Proceedings IEE, Vol 126, No 10, 1979


Chandra, M., 'Prediction of propagation effects and rain-intensity using radar determined three-parameter raindrop-size distribution', Proceedings of the Seventh
Collier, C.G., 'Applications of weather radar systems', Wiley and Sons Ltd., Chichester, United Kingdom, 1989
COST 210 Final Report, 'Influence of the atmosphere on interference between radio communications at frequencies above 1 GHz', EUR 13407 en, Commission of the European Communities, 1991
Dyer, R.M, 'Particle fall speeds within the melting layer', Proceedings of the 11th Radar Meteorology Conference of the American Meteorological Society, 1970

202


Gossard, E. E., Strauch, R.G., 'Radar observations of clear air and clouds', Elsevier Amsterdam, 1983


203
Jain, Y.M., 'Microwave scattering from melting zone particles and oscillating raindrops', PhD thesis University of Bradford, United Kingdom, 1984
Klaassen, W., 'Radar observations and simulation of the melting layer of precipitation', Journal of Atmospheric Sciences, Vol 45, No 24, 1988
Klaassen, W., 'Determination of rain intensity from Doppler spectra of vertically scanning radar', Journal of Atmospheric Sciences, Vol 6, No 4, 1989
Klaassen, W., 'Attenuation and reflection of radio waves by a melting layer of precipitation', IEE Proceedings, Vol 137, Pt H, No 1, 1990
Ligthart, L.P., Nieuwkerk, L.R., 'Observations of clear air turbulent plumes with FM-CW Doppler radar' Proceedings of IGARRS'87, Ann Arbor, USA, 1987
Magono, C., Lee, C.W., 'Meteorological classification of natural snow crystals', Journal of the Faculty of Science, Series VII, Vol III, No 4, Hokkaido University, Japan, 1966
Matrosov, S.Y., 'Prospects for the measurement of ice clouds particle shape and orientation with elliptically polarized radar signals', Radio Science, Vol 26, No 4, 1991

204

Mie, G., 'Beitrag zur Optik trüber Medien, speziell Kolloidaler Metalasungen', Annalen Physik, Vol 25, 1908


Oguchi, T., Ito, S., 'Multiple scattering effects on the transmission and reflection of millimeter pulse waves in rain', Proceedings of the 5th URSI Commission F Symposium, La-Londe-les-Maures, France, 1989


Ray, R.S., 'Broadband complex refractive index of ice and water', Applied Optics, Vol 2, No 8, 1972


Sekhon, R.C., Srivastava, R.C., 'Snow size spectra and radar reflectivity', Journal of the
Atmospheric Sciences, Vol 27, 1970
Sintruyen, J.S., Memorandum Delft University of Technology
Ulbrich, C.W., 'Natural variations in the analytical form of the raindrop size distribution', Journal of Climate and Applied Meteorology, Vol 22, No 10, 1983
Uzunoglu, N.K., Evans, B.G., Holt, A.R., 'Scattering of electromagnetic radiation by precipitation particles and propagation characteristics of terrestrial and space communications systems', Radio Science, Vol 19, No 1, 1977
Wessels, H.R.A., 'Measurements of raindrops (Metingen van regengdruppels, in Dutch)', Scientific report WR 72-6, KNMI, De Bilt, 1972

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