Master thesis
Stability of a chinook with multiple underslung loads

R.J. Thijs - 1179179
Na vele jaren is het dan eindelijk zo ver: het afronden van mijn studie! Het heeft dan even geduurd en wat motivatie van derde nodig gehad. Vooral wil ik mijn vriendin Danielle en mijn afstudeer begeleidster Marilena bedanken voor de nodige "pepers" die zij hebben moeten inbrengen om dan eindelijk tot dit resultaat te komen. Mijn dank!

*R.J. Thijs - 1179179*

*Delft, november 2015*
SUMMARY

This thesis is a continuation of a series of theses treating various aspects of the Chinook helicopter flight with underslung loads. It is aimed at finding the effect that underslung loads have on the stability of a Chinook helicopter.

The results are obtained in three distinct steps. The first step is finding the trim condition, followed by linearisation around the trim conditions and finally calculating the eigenvalues by the linearized state matrix. The results found are compared to reference data when possible.

The trim is calculated with the aid of a Newton-Raphson variation. The results at hover did not converge, so a very small forward velocity was used to approximate hover conditions. The trim data seems to match the reference data closely, with the exception of the longitudinal cyclic stick data, which has a reversed sign compared to the reference graphs. This is most likely caused by a different sign convention for the forward stick deflection. All other trim curves where found to match the reference data close enough to validate the trim model.

The helicopter derivative graphs are the first results of this particular helicopter model. The derivatives are calculated by introducing a small perturbation in one of the helicopter states from the trimmed helicopter state. The effect this perturbation has on the helicopter forces is subtracted by the trimmed helicopter forces and divided by the perturbation step size.

The numerical linearisation curves show significant differences compared to the reference data produced by Ostroff and Davis. To determine the origin of these differences, the derivatives are segmented between the front and rear rotor forces and the aerodynamic forces. In most situations, the rotor forces are leading in the shape of the derivative graphs.

The eigenvalues are calculated numerically by using the results found with the linearisation routine. This is done by the method described in detail by Padfield. Surprisingly, the eigenvalues in longitudinal direction showed no imaginary eigenvalues. This means that there is no oscillating phugoid or short period motion present in the used model.

The eigenvalues found in the lateral derivatives do posses a non-zero imaginary part. Determining the matching eigenmotion was difficult since the regular method of approximating the eigenvalues was invalid due to the large deviations with some of the derivatives.

The direct effect that the underslung load has on the helicopters responds due to the perturbations is minimal. The main reason of this result, is the way the model is build. A small perturbation in velocity gives a small velocity change in the cable. Since the damping value of the cables is small, the effect of a rotational or translational velocity perturbation is too small to have a noticeable impact compared to the other helicopter derivatives.

The differences that are found between the bare Chinook and the Chinook with underslung loads, are mainly caused by the change in trim conditions. A load inside the helicopter that would create the same forces and moment as the underslung load during trimmed flight would yield almost identical results.
CONTENTS

Preface iii
Summary v
List of symbols ix
1 Introduction 1
2 Literature review on stability of a tandem rotor helicopter with an underslung load 3
3 Research questions and experimental setup 7
4 Deriving the Chinook model 9
  4.1 Defining the helicopter reference system and force locations . . . . . . . . . . . . . . . . . . . 9
  4.2 Equation of motion of the bare Chinook . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10
  4.3 Rotor forces . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10
    4.3.1 Defining the rotor hub reference systems and angles . . . . . . . . . . . . . . . . . . . 11
    4.3.2 Calculating the control angles from pilot input . . . . . . . . . . . . . . . . . . . . . . 11
    4.3.3 Rotor coefficients and angle corrections . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
  4.4 Helicopter fuselage aerodynamics . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 15
  4.5 Gravitational forces . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 16
5 Underslung load model derivation 17
  5.1 Defining the conditions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 17
  5.2 Load forces . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18
    5.2.1 Load aerodynamics . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18
    5.2.2 Gravity . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18
  5.3 Sling forces . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18
  5.4 Load equation of motion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 20
6 Derivative analysis 21
  6.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 21
  6.2 Calculating the derivatives for the bare Chinook . . . . . . . . . . . . . . . . . . . . . . . . . 22
  6.3 Results . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 22
  6.4 Longitudinal derivatives . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 23
    6.4.1 Chinook responses due to a wind gust in the negative $x$–direction . . . . . . . . . . . . 23
    6.4.2 Chinook responses due to a wind gust in the negative $z$–direction . . . . . . . . . . . . 26
    6.4.3 Chinook responses due to a pitch rate perturbation . . . . . . . . . . . . . . . . . . . . . 29
  6.5 Lateral derivatives . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 32
    6.5.1 Chinook responses due to a wind gust in the negative $y$–direction . . . . . . . . . . . . 32
    6.5.2 Chinook responses due to a roll rate perturbation . . . . . . . . . . . . . . . . . . . . . 35
    6.5.3 Chinook responses due to a yaw rate perturbation . . . . . . . . . . . . . . . . . . . . . 39
  6.6 Helicopter derivatives with various underslung loads . . . . . . . . . . . . . . . . . . . . . . . . 42
    6.6.1 Longitudinal load derivatives $X$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 43
    6.6.2 Longitudinal load derivatives $Z$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 45
    6.6.3 Longitudinal load derivatives $M$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . 47
    6.6.4 Lateral load derivatives $Y$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 48
    6.6.5 Lateral load derivatives $L$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 51
    6.6.6 Lateral load derivatives $N$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 53
7 Eigenvalues 57
  7.1 Eigenvalues of the single load configuration . . . . . . . . . . . . . . . . . . . . . . . . . . 58
  7.2 Eigenvalues of the double load configuration . . . . . . . . . . . . . . . . . . . . . . . . . 59
  7.3 Eigenvalues of the triple load configuration . . . . . . . . . . . . . . . . . . . . . . . . . . 61
# LIST OF SYMBOLS

**LATIN SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>translational acceleration of the helicopter or load from the center of gravity ((\dot{u}, \dot{v}, \dot{w}))</td>
</tr>
<tr>
<td>(a_0)</td>
<td>rotor blade coning angle</td>
</tr>
<tr>
<td>(a_1)</td>
<td>rotor blade longitudinal flapping angle</td>
</tr>
<tr>
<td>A</td>
<td>state matrix</td>
</tr>
<tr>
<td>(A_0, 1, \ldots, 6)</td>
<td>single load configuration (0,1,\ldots,6)</td>
</tr>
<tr>
<td>(b_1)</td>
<td>rotor blade lateral flapping angle</td>
</tr>
<tr>
<td>(b_f, r)</td>
<td>absolute distance from front/rear rotor to helicopter c.g. on the (y)-axis</td>
</tr>
<tr>
<td>B</td>
<td>control matrix</td>
</tr>
<tr>
<td>(B_0, 1, \ldots, 7)</td>
<td>double load configuration (0,1,\ldots,7)</td>
</tr>
<tr>
<td>(C_0, 1, \ldots, 8)</td>
<td>triple load configuration (0,1,\ldots,8)</td>
</tr>
<tr>
<td>c</td>
<td>inequality constraint value</td>
</tr>
<tr>
<td>(c_{\text{upper,lower}})</td>
<td>damping value of the upper or lower sling</td>
</tr>
<tr>
<td>(c_{\text{eq}})</td>
<td>equality constraint value</td>
</tr>
<tr>
<td>(C_{D,\text{eq}})</td>
<td>equivalent flat plate area vector</td>
</tr>
<tr>
<td>(C_{\text{FF}})</td>
<td>equivalent flat plate area (function of the angle of attack and the side slip angle)</td>
</tr>
<tr>
<td>(C_{\text{LA}})</td>
<td>fuselage lift coefficient as a function of the angle of attack</td>
</tr>
<tr>
<td>(C_{\text{Lp}})</td>
<td>fuselage roll moment coefficient as a function of the side slip angle</td>
</tr>
<tr>
<td>(C_{\text{Ma}})</td>
<td>fuselage yaw moment coefficient as a function of the angle of attack</td>
</tr>
<tr>
<td>(C_{\text{M,eq}})</td>
<td>equivalent flat plate area moment vector</td>
</tr>
<tr>
<td>(C_{\text{Na}})</td>
<td>fuselage yaw moment coefficient as a function of the angle of attack</td>
</tr>
<tr>
<td>(C_{\text{Yp}})</td>
<td>side force pressure coefficient with respect to the side slip angle</td>
</tr>
<tr>
<td>(C)</td>
<td>coefficient vector</td>
</tr>
<tr>
<td>(F)</td>
<td>force vector, (\langle F_x, F_y, F_z \rangle), for helicopter or load</td>
</tr>
<tr>
<td>(g)</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>(h_{f, r})</td>
<td>absolute distance from front/rear rotor to helicopter c.g. on the (z)-axis</td>
</tr>
<tr>
<td>(H_{xy})</td>
<td>rotor forces in (x) and (y) direction in the rotor disk plane</td>
</tr>
<tr>
<td>(i)</td>
<td>rotor incidence angle or counter value</td>
</tr>
<tr>
<td>(I_{CG})</td>
<td>helicopter mass moment of inertia matrix around the center of gravity</td>
</tr>
<tr>
<td>(I_{xy, xx, yz})</td>
<td>product of inertia around the axis of moment of inertia around (x), (y), and (z)-axes</td>
</tr>
<tr>
<td>(I_{xx, yy, zz})</td>
<td>mass moment of inertia around (x), (y), or (z)-axis</td>
</tr>
<tr>
<td>(j)</td>
<td>counter value</td>
</tr>
<tr>
<td>(J)</td>
<td>Jacobian</td>
</tr>
<tr>
<td>(k)</td>
<td>counter value</td>
</tr>
<tr>
<td>(k_{\text{upper,lower}})</td>
<td>spring constant of the upper or lower part of the cable</td>
</tr>
<tr>
<td>(l_{\text{upper,lower}})</td>
<td>length of the upper or lower part of the cable</td>
</tr>
<tr>
<td>(l_{f, r})</td>
<td>absolute distance from front/rear rotor to helicopter c.g. on the (x)-axis</td>
</tr>
<tr>
<td>(L_{v, p, r, \phi})</td>
<td>(\frac{1}{m s}, \frac{1}{rad s}, \frac{1}{rad^2 s^2})</td>
</tr>
<tr>
<td>(m_1)</td>
<td>(\frac{1}{m s}, \frac{1}{rad s}, \frac{1}{rad^2 s^2})</td>
</tr>
<tr>
<td>(m)</td>
<td>(\frac{1}{m s}, \frac{1}{rad s}, \frac{1}{rad^2 s^2})</td>
</tr>
<tr>
<td>(M_{l,m,n})</td>
<td>(N\ m)</td>
</tr>
<tr>
<td>(M_{\text{w, q, }\theta})</td>
<td>(\frac{1}{m s}, \frac{1}{rad s}, \frac{1}{rad^2 s^2})</td>
</tr>
<tr>
<td>(N_{v, p, r, \phi})</td>
<td>(\frac{1}{m s}, \frac{1}{rad s}, \frac{1}{rad^2 s^2})</td>
</tr>
<tr>
<td>Symbol</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----</td>
</tr>
<tr>
<td>$p$</td>
<td>$\text{rad} / \text{s}$</td>
</tr>
<tr>
<td>$q$</td>
<td>$\text{rad} / \text{s}$</td>
</tr>
<tr>
<td>$r$</td>
<td>$\text{rad} / \text{s}$</td>
</tr>
<tr>
<td>$R$</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$T$</td>
<td>$\text{N}$</td>
</tr>
<tr>
<td>$T_{\phi,\theta,\psi}$</td>
<td>$[-]$</td>
</tr>
<tr>
<td>$u$</td>
<td>$\text{m} / \text{s}$</td>
</tr>
<tr>
<td>$v$</td>
<td>$\text{m} / \text{s}$</td>
</tr>
<tr>
<td>$u$</td>
<td>$\text{m} / \text{s}$</td>
</tr>
<tr>
<td>$V$</td>
<td>$\text{m} / \text{s}$</td>
</tr>
<tr>
<td>$w$</td>
<td>$\text{m} / \text{s}$</td>
</tr>
<tr>
<td>$\mathbf{x}$</td>
<td>$[\text{m}], [\text{rad}], [\text{rad}], [\text{m}]$</td>
</tr>
<tr>
<td>$\mathbf{X}$</td>
<td>$[\text{m}], [\text{rad}], [\text{rad}], [\text{m}]$</td>
</tr>
<tr>
<td>$X_{u,v,q,\theta}$</td>
<td>$[\frac{1}{s}], [\frac{\text{m}}{\text{rad}}, \text{rad}], [\frac{\text{m}}{\text{rad}}, \text{rad}], [\frac{\text{m}}{\text{rad}}, \text{rad}]$</td>
</tr>
<tr>
<td>$Y_{v,p,r,\phi}$</td>
<td>$[\frac{1}{s}], [\frac{\text{m}}{\text{rad}}, \text{rad}], [\frac{\text{m}}{\text{rad}}, \text{rad}], [\frac{\text{m}}{\text{rad}}, \text{rad}]$</td>
</tr>
<tr>
<td>$Z_{u,w,q,\theta}$</td>
<td>$[\frac{1}{s}], [\frac{\text{m}}{\text{rad}}, \text{rad}], [\frac{\text{m}}{\text{rad}}, \text{rad}], [\frac{\text{m}}{\text{rad}}, \text{rad}]$</td>
</tr>
</tbody>
</table>

**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{\alpha}$</td>
<td>$\text{rad} / \text{s}^2$</td>
<td>rotational acceleration vector of the helicopter around respectively, $\langle \dot{p}, \dot{q}, \dot{r} \rangle$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\text{rad}$</td>
<td>angle of attack</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\text{rad}$</td>
<td>side slip angle</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$\text{m}$</td>
<td>control surface deflection</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\text{rad}$</td>
<td>pitch angle</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$[-]$</td>
<td>eigenvalue</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\frac{1}{\text{s}}$</td>
<td>damping</td>
</tr>
<tr>
<td>$\mu_{u,v,w}$</td>
<td>$[-]$</td>
<td>dimensionless inflow velocity</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$[-]$</td>
<td>the ratio of a circle's circumference to its diameter</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\text{kg} / \text{m}^3$</td>
<td>air density as experienced by the helicopter and load</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>$\text{kg} / \text{m}^3$</td>
<td>air density in normal atmosphere at sea level</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$[-]$</td>
<td>correction value or scale value</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\text{rad}$</td>
<td>roll angle</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$\text{rad}$</td>
<td>yaw angle</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$\text{rad} / \text{s}$</td>
<td>frequency of the eigenvalues</td>
</tr>
<tr>
<td>$\widetilde{\omega}$</td>
<td>$\text{rad} / \text{s}$</td>
<td>rotational velocity vector of the helicopter respectively $\langle p, q, r \rangle$</td>
</tr>
<tr>
<td>$\widetilde{\Omega}$</td>
<td>$\text{rad} / \text{s}$</td>
<td>rotational velocity of the rotor blades</td>
</tr>
</tbody>
</table>
SUBSCRIPT ANNOTATIONS

0 coning angle notation or initial value
1c lateral cyclic pitch angle
1s longitudinal cyclic pitch angle
\textit{aero} forces or moments caused by aerodynamics
\textit{center} indice used to signal the central point of the cables
\textit{CG} center of gravity
\textit{col} collective control value
\textit{cor} first disk angle correction
\textit{disk} in the rotor disk reference plane
\textit{D} drag notation
\textit{f} front rotor hub
\textit{fus} fuselage/aerodynamic contribution
\textit{heli} values assigned to the helicopter
\textit{i} denotes indice
\textit{lat} lateral control value/lateral motions
\textit{LB} lower boundary value of the optimization state vector
\textit{load} values assigned to the load
\textit{long} longitudinal control value/longitudinal values
\textit{lower} lower sling (four cables from load suspension points to central point)
\textit{max} maximal value
\textit{min} minimal value
\textit{M} moment notation
\textit{neut} deflection in neutral stance
\textit{Norm} normalization vector
\textit{ped} pedal control value
\textit{pr} progressive side
\textit{preLim} preliminary value of the disk angles before control system corrections
\textit{Q} moment around the \(z\)–axis
\textit{r} rear rotor hub
\textit{re} retreating side
\textit{Rfs} rotor rear/rotor front contribution
\textit{rotor} in the rotor reference plane (heli reference plane transposed to the respective rotor hub)
\textit{shaft} in the rotor shaft reference plane
\textit{trim} indicating a trimmed value
\textit{tot} total value
\textit{T} thrust
\textit{UB} upper boundary value of the optimization state vector
\textit{upper} upper part of the underslung load cable; cable connecting the central point to the helicopter hook
\textit{X} \(x\)–direction
\textit{Y} \(y\)–direction
\textit{Z} \(z\)–direction
Helicopters are unique multipurpose vehicles that have proven their worth in transportation, surveillance, scouting, combat, heavy lifting, and search and rescue. They are faster than trucks and can land at places inaccessible by aircraft.

When used for transport, even the biggest helicopters have a relatively small cargo space. If a helicopter is able to carry the mass of the load, but is not able to place it in its cargo hold; the cargo can be carried under the helicopter with the aid of slings. This loading style is normally referred to as underslung loads.

Underslung loads greatly increase the use of the helicopters cargo carrying capabilities, but an underslung load comes at a price. The characteristics of the helicopter are affected by the underslung load. Besides the added mass, also the relative motion of the load compared to the helicopter, play a dominant role in the stability of the helicopter. The alternating angles at which the load forces act on the helicopter suspension points, have a major impact on the stability and maneuverability of the helicopter; sometimes putting the cargo, helicopter, and the pilots at risk.

Manwaring, Conway and Garrett identified that at least 230 external load accidents occurred between 1980 and 1995 [1]. One of the causes for accidents are unstable loads.

Gabel and Wilson[2] and Asseo and Whitback [3] also acknowledged some problems concerning the stability of underslung loads and made a distinction between three instability phenomenon associated with an underslung load.

The first phenomenon is vertical bounce, which is caused by the spring value of the cable connecting the load and the helicopter. This motion is excited when the eigenfrequency of the helicopter and underslung system matches that of the rotor rotational frequency.

The second instability is aerodynamic instability of the underslung load. This instability is a yawing moment of the load caused by the aerodynamic center being at the center of gravity. Especially loads suspended from a single point are susceptible to this phenomenon.

The last instability is sling leg flapping. This is not direct an instability, but a source of large strain on the suspending cables. The rotor aerodynamics cause heavy movement on the cables which can grate the cable. The grating can causes damage to the cable, which in turn can lead to sling failure.

One of the most used helicopters known for using underslung loads is the Chinook, a tandem rotor helicopter created by Boeing. The Chinook was designed with three cargo hooks allowing for multiple options to carry an underslung load.

Various militaries use the Chinook helicopter (Boeing CH-47 series) for a wide variety of tasks. Among those militaries is the Royal NetherLands Air Force RNLAF[4]; which uses the Boeing CH-47D and Boeing CH-47F.

In this thesis, the effect that the underslung load has on the stability of the CH-47B is analyzed. This is done by using a helicopter model and calculating the helicopter derivatives and the helicopters eigenvalue. How these results are formed is discussed in eight chapters.

In chapter 1, an introduction is given to the subject matter.
In chapter 2, the results of a literature review are discussed. Each paragraph in chapter two summarizes one report and identifies the most important findings of the specified report. Special attention goes towards research done by TUDelft; as this thesis is a continuation of their research regarding the effect that underslung loads have on tandem rotor helicopters.

Chapter 3 formulates the research question and experimental setup. It gives the reader an idea on what methods are used, what underslung load configurations are tested and why.

Chapter 4, shows the derivation of the Chinook model. It starts by defining the forces and reference system of the Chinook, and is followed by how those different forces are calculated.

In chapter 5 the underslung load model is discussed. It comprises of two parts, the load itself and the cable suspension system. The model build up and reference systems used are explained and shown in figures.

Chapter 6 comprises of the derivative analysis in multiple parts. The first part handles the bare Chinook model and discusses the dominant influences of the derivative force and moment build-up in detail for the longitudinal Eigenmotions; in the second part the lateral Eigenmotions are discussed. Followed by this analysis the effect of the underslung load is discussed. Initially the single underslung load is discussed, followed by the double, and finally the triple underslung load.

Chapter 7 discusses the eigenvalues. In the introduction of this chapter the method is discussed on how the eigenvalues are calculated, directly followed by the results of the bare Chinook where the different expected eigenmotions are identified and the results are discussed. Followed by this discussion are the effects the load has on the helicopter model. This is done in the following three paragraphs, first treating the single load configuration, followed by the double and finally the triple load configuration.

In chapter 8 the results are discussed, conclusions are drawn and the limitations of this report are highlighted.
Early work on the stability of helicopters with an underslung load is produced by Lucassen and Sterk [5]. They constructed a three degree of freedom model where aerodynamic forces and moments where ignored. After their report, several other parties performed research on the matter. Padfield[6] wrote a book where the exact details of helicopter flight and underslung load systems are treated, allowing for more accurate simulations to be developed.

In Australia, the Department of Defense performed research regarding the underslung loads of a Chinook. R.A. Stuckey from the aeronautical and maritime research laboratory wrote two reports on the Chinook trying to identify the operational limits. In his first report[7], the author develops a simple model that covers the basics of an underslung tandem rotor helicopter.

For his model he used the Newton-Euler equations in terms of generalized coordinates and velocities. For the cable mechanics he used the explicit constrained method to separate the solid mechanics of the sling load and the helicopter, from that of the elongating cable. His most important conclusion during his simulations where that the lateral phugoid is greatly affected by the mass ratio; the higher the mass-to-helicopter weight ratio, the more unstable the lateral phugoid.

Afters Stuckey’s first report, he wrote a report on the dynamics during multiple slung loads [8], here the frequency of the slung load was tested by an open loop system. The longitudinal and lateral eigenvalues of the system for load-to-helicopter mass ratio ranging from 0.1 to 1.0 were simulated. The results show that the frequency response of the longitudinal and lateral pendulum goes up with increasing load to helicopter mass ratios, while pitch, heave and roll get lower dampening values. Yaw rate doesn’t seem to be influenced by the load to helicopter mass ratios.

Cao, Li and Yang[9] used two different mathematical models to calculate the trim of a tandem rotor helicopter, and compared them both to test data and reference data. Their first model was based on momentum theory, which portraits the rotor as an infinite thin disk accelerating the air. The second was based on the vortex model, or the Prandtl lifting line theory. This theory calculates the lift generated for each blade individually.

At the simulation range of -60 to 180 knots, it was found that the data of both the longitudinal stick and collective controls, were independently of the calculation model used.

The lateral stick shows that there is a large variation in the trim results. Especially the vortex model results showed a large unsteady path during increasing velocities. The momentum theory, reference and test flight results look very similar till 80 knots. Here the values of the test data have a wider spread than the reference data and the momentum theory, something that can be expected from test data.

The directional trim data have very similar results until a speed of 120 knots. From 120 knots the derivatives change, the vortex model and flight test data both turn from a negative to a positive derivative, while the
reference data and the momentum model both maintain their negative derivative. The pitch trim shows that all the values are close together, until the vortex model reaches the speed of 100 knots. Above 100 knots, the vortex model decreases the pitch angle a lot faster than the other models.

The last data evaluated is the trim data for the rolling angle. This graph shows the most variation between the different methods and references. Only the reference data and the model data have similar results. None of the models seem to get close to the test data. The vortex model shows a very unpredictable change in rolling angles compared to the relative steady line of the moment model and the test data.

One can conclude that the vortex model was only more accurate in predicting the test data in case of the directional control stick trim. In the three other graphs it was distinctively worse in the prediction of the trim than the momentum model. The main conclusion drawn was that the momentum model has the most accurate results when compared to the test flight data.

Marguerettaz and Guglieri\[10\] wrote a report where two traditional helicopter models are tested using two underslung load models.

Their first underslung load model is modeled as a pendulum, the second model is a rigid model that uses nine linear equations to calculate the velocities, angular rates and attitude angles. Three additional equations are used to determine the load center of mass in the reference frame. The damped natural frequency is plotted against the damping as a function of the advance ratio. From the plots, the authors concluded that the phugoid is the most critical eigenmotion, as it is the only eigenmotion that has poles in the unwanted level three stability domain.

They conclude that attitude dynamics of the suspended load has a minor role in the coupling of the systems. They mention that adding mass in the helicopter has a different responds than adding mass as an underslung load. It is also stated that cable damping provides a source of additional dynamic stability. It was also found that modeling the underslung load as a pendulum has clear limitations.

The authors state that for accurate results the six degrees of freedom (DOF) seems a necessity. It states that aerodynamic effects and cable length have little influence on the dynamic behavior of the helicopter in case of a high mass and low CDS of the system. Contrary, low density loads are effected by aerodynamics and cable length. They noted that the lack of main rotor down wash is a limiting factor of the range where their model is valid. Final conclusion is that the six DOF system for the suspended load may account for inertial and aerodynamics complexity of the slung load with very limited computation workload.

Fusato, Guglieri and Celi \[11\] made another contribution to the field of underslung load dynamics. They wrote a report on the flight dynamics of an articulated rotor helicopter with an external slung load. For the model they used rigid body dynamics, individual flap and lag blade dynamics and inflow dynamics.

They performed simulations with an underslung load till 2000 kg with helicopter mass ratios of up to 28% and advance ratios of up to 0.3. They concluded that the load affects the trim primarily through the overall increase in weight; influence of cable length was negligible. This might be due to the fact that Fusato e.o. used an inelastic cable and the aerodynamic forces were simplified by using a quasi steady-drag force which was only depended on the constant equivalent flat plate area and the absolute velocity of the underslung load. Higher order dynamics as rotor and inflow dynamics have a modest effect on the stability. A quasi steady rotor model is probably sufficiently accurate for simulating purposes.

Oktay and Suktan\[12\] also derived a simple helicopter model in which the load is modeled as a point mass with quasi steady drag force acting in the local airflow. The down wash from both the main rotor and tail rotor are taken into account in this model. The cable is assumed in-elastic and mass less without any aerodynamic forces acting on the helicopter. The system is trimmed using the procedures of Matlab’s "fsolve". Like the other researchers he concluded that cable length does not affect trim values. After the design and trimming of the helicopter different controllers where designed and tested.

The NLAF also got an interest in the Chinooks underslung load behavior; they started a cooperation with the TUDelft. Under the guidance of Pavel several students started their master thesis on the subject.

First on the list was Kamp. He did research on how the Chinook behaved in case of a cable failure\[13\]. Chinooks often carry underslung loads and some heavy loads are connected to two hooks. This lightens the tension on both the cables and the hooks and it increases the stability of the load and with it the helicopter. If one of the cables fail during flight the slinging motion of the underslung load can pull the helicopter in such an angle that recovery is impossible.
To prevent this from happening a set of redundant cables can be used. These cables are connected on all four corners of the load and to the central hook of the Chinook helicopter. When one of the cables fail, the load isn’t slung to the front or back. The redundant slings will take over from the failing sling, stopping the slinging motion the cargo container would make if one side collapses. This effectively prevents the mass from slinging out of control and decreasing the chance of a crash.

Though the redundant sling seems like a perfect idea, the extra workload and personnel needed to attach this extra line can be very undesirable under combat conditions, where time can be extremely important. In this report, research was done under what circumstances the redundant cable could be left out and still avoid life threatening situations that could occur in case of a cable failure.

The research done in this report, is a good foundation, but did not give an accurate view on actual Chinook response, as no Aircraft Flight Control System (AFCS) was implemented. Reijm[14] expanded the program written by Kamp with an AFCS. With the pilot reaction time modeled in the report of Kamp, it replaced the almost instantaneous responses of the Chinooks flight control system and the results drastically changed. Comparing the results of Reijm and Kamp it was confirmed that the AFCS enabled a larger flight envelope for recovering the Chinook helicopter from a possibly unstable responds.

Klaver[15] continued the work of the TUDelft on the subject of the Chinook Helicopter. This time, research was done regarding the handling qualities of the Chinook carrying multiple underslung loads. He used the model created by Kamp and expanded it to contain two underslung loads and performed simulation with the CH-47B. After computer simulations the program was converted to operate with the TUDelft simulator SIMONA and a RNLAF pilot performed tests with it.

The main conclusion is that the heaviest load should be carried by the front hook using a longer rope than the lighter rear load.

Besides helicopter simulation research, there are also some reports that are focused on expanding the knowledge on the load response models. Greenwell[16] wrote a report that focuses on flat plate aerodynamics techniques to find the aerodynamic forces acting on an underslung load sea container. He used the 8x6x6ft CONEX and the 8x8x20ft Milvan cargo containers. His research showed that the aerodynamics forces and moments can be calculated by making a distinction in two different flows: attached flows and detached flows. The universal nature of the attached and detached flow make the box relatively simple to model with just a few parameters, of which only few vary significantly with box geometry.

In a cooperative project between United States universities, American military, NASA and an Israeli university [17], a CFD (Computational Fluid Dynamics) method was used to simulate the flow around a 6x6x8 ft CONEX cargo container. To validate the simulator additional test flights with the helicopter have been flown.

They noted that the load of the swiveled sling spins at a rate that increases with airspeed to a maximum and then decreases. Variation on the spin speed were dependent on load mass and swivel friction.

Load swinging occurs at the natural frequency of the cable-load combination. The load was unstable with an empty container with the center of gravity on the symmetry lines. When a large off set in the center of gravity (C.G.) is introduced, the load stabilizes with the heavy part into the flight direction.

The authors constructed a model that predicts the unsteady moment around the z-axis and were able to reproduce the yaw angles of the flight test data. A general method for reliably predicting instability was not found.

The CFD simulation proved to be very successful in predicting the drag and moments. Poorer accuracy was obtained in the transverse forces. It was stated that a better accuracy might be found by reducing the time step size and increasing the resolution. Sufficient computer resources were not available at the time of writing. It can be concluded that CFD has the potential to predict and accurately simulate the movement of an underslung cargo container, but is not practical.

Another phenomenon researched is vertical vibration, it is caused by the helicopter rotor frequency nearing, or reaching the natural frequency of the sling-load combination. Sometimes this effect is enlarged by the pilot, causing so called pilot induced oscillations. Though normally just uncomfortable, it can cause the loss of the helicopter. Hoff[18] created a model specifically aimed to simulate the effect of this vertical bounce mode.

The sling system is modeled as a spring damper. Besides the sling system, aerodynamic damping is applied in the thrust vector. When damping is left out, all equilibrium conditions would be unstable, regardless
of system parameters.

The model connects the helicopter with the load using a spring; aerodynamic damping is applied to the thrust force of the helicopter. Instead of expressing the equations in the Newtonian model, used in most simulators, the Lagrangian approach is chosen. This is done due to the complexity of the system. With these data an initial Lagrangian model is build which acts as predicated, and will never become unstable.

To test the flight mechanics, the individual blades are added to the equations. Since this increases the amount of equations drastically, they limit the degrees of freedom to the system to longitudinal directions only i.e. all rotations except for rotor blade rotations are completely omitted. It was discovered that vertical bounce mode, or bifurcation, is unstable when vertical load frequency is within 20% of the high progressive blade frequency.

Ostroff, Downing and Rood[19] discuss a technique to determine the trims and the quasi-static derivatives of a non-linear vehicle model. The aim of their model is to assist in the development of a linear feedback controller. In this report a non linear model, referred to as HELICOP is used to simulate the responds of the Boeing CH-47B helicopter.

When starting to calculate the trim of the helicopter a start is made by defining the trim conditions, i.e. horizontal steady flight. An initial guess is made regarding the control deflections. Through Newton Raphson iterations and a step size correction the trim values are calculated.

With the trim conditions known, the stability derivatives are calculated by making a small change to the trimmed state. A perturbation is added to one helicopter state at the time. The influence that these perturbations have on the helicopter forces are calculated and represented in a graph.

Their results where compared with the derivatives calculated by Davis[20]. It was found that two third of the derivatives are in excellent agreement. The other derivatives have some differences in magnitude. Most of the graphs show some clear deviation at hover condition. Several derivative graphs deviate from the reference data. The authors conclude that this is most likely because of the difference between a linear and non-linear model.
Research Questions and Experimental Setup

In this thesis, research will be performed on the stability of the Boeing CH-47B helicopter [4] with multiple underslung loads. The goal is to describe the natural behavior of the CH-47B when underslung loads are attached to the helicopter.

This will be done by creating a Matlab program that can simulate the helicopter movements with the underslung load. The results will be presented in a graph that shows the damping against the natural frequency. To plot these graphs several steps need to be made. First a model of the helicopter needs to be programmed and verified in Matlab. After this step, the loads and sling will be modeled. To obtain valid results, both the helicopter and the load(s) need to be trimmed. Earlier experiments have shown that the trimming needs to be done in three stages. One starts with the trimming of the helicopter after which the loads are trimmed. When both the loads and the helicopter are trimmed at the same speed they need to be combined and trimmed again.

After the helicopter trim, the helicopter model is linearized. The linearisation is done by giving a small perturbation from the helicopter trimmed states; one states at the time. For the next step the lateral and longitudinal motions are separated and semi-normalized by dividing the forces by the helicopter mass, and the moments by their respective moment of inertia. Finally the results are plotted against the helicopter velocity and compared to results found in the literature.

With the linearized forces and moments known, the state matrices can be constructed. This is done separately for the longitudinal and lateral motions. The eigenvalues of these matrices can be calculated by calculating the determinants of the eigenvalue identity matrix subtracted by the state matrix. These eigenvalues can be plotted with their imaginary part versus their real part; showing the dampening and frequency of the helicopter response.

The following step is analyzing the underslung load configurations attached to the helicopter. The configurations are sorted by: single (A series) underslung load as shown in table 3.1, double (B series) underslung load as shown in table 3.2, and triple (C series) underslung load which are found in table 3.3. In each of these configurations the ‘0’ entry shows the bare Chinook to clearly see the effect of the underslung load.
3. RESEARCH QUESTIONS AND EXPERIMENTAL SETUP

### Table 3.1: Single underslung load configurations

<table>
<thead>
<tr>
<th>conf. #</th>
<th>Load mass [kg]</th>
<th>Sling length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A1</td>
<td>1000</td>
<td>9.14</td>
</tr>
<tr>
<td>A2</td>
<td>1000</td>
<td>15.1</td>
</tr>
<tr>
<td>A3</td>
<td>1000</td>
<td>17.9</td>
</tr>
<tr>
<td>A4</td>
<td>4000</td>
<td>9.14</td>
</tr>
<tr>
<td>A5</td>
<td>4000</td>
<td>15.1</td>
</tr>
<tr>
<td>A6</td>
<td>4000</td>
<td>17.9</td>
</tr>
</tbody>
</table>

### Table 3.2: Double underslung load configurations

<table>
<thead>
<tr>
<th>conf. #</th>
<th>Load mass front [kg]</th>
<th>Load mass rear [kg]</th>
<th>Sling length front [m]</th>
<th>Sling length center [m]</th>
<th>Sling length rear [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B1</td>
<td>1000</td>
<td>1000</td>
<td>9.14</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>1000</td>
<td>2000</td>
<td>9.14</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>1000</td>
<td>2800</td>
<td>9.14</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>2000</td>
<td>1000</td>
<td>9.14</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>B5</td>
<td>2000</td>
<td>2000</td>
<td>9.14</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>B6</td>
<td>3600</td>
<td>1000</td>
<td>9.14</td>
<td>17.9</td>
<td></td>
</tr>
<tr>
<td>B7</td>
<td>1000</td>
<td>3600</td>
<td>9.14</td>
<td>17.9</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3.3: Triple underslung load configurations

<table>
<thead>
<tr>
<th>conf. #</th>
<th>Load mass front [kg]</th>
<th>Load mass center [kg]</th>
<th>Load mass rear [kg]</th>
<th>Sling length front [m]</th>
<th>Sling length center [m]</th>
<th>Sling length rear [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>17.9</td>
<td>15.1</td>
<td>9.14</td>
</tr>
<tr>
<td>C2</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>9.14</td>
<td>15.1</td>
<td>17.9</td>
</tr>
<tr>
<td>C3</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>9.14</td>
<td>17.9</td>
<td>9.14</td>
</tr>
<tr>
<td>C4</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>17.9</td>
<td>9.14</td>
<td>17.9</td>
</tr>
<tr>
<td>C5</td>
<td>1000</td>
<td>2200</td>
<td>1000</td>
<td>17.9</td>
<td>15.1</td>
<td>9.14</td>
</tr>
<tr>
<td>C6</td>
<td>1000</td>
<td>2200</td>
<td>1000</td>
<td>9.14</td>
<td>15.1</td>
<td>17.9</td>
</tr>
<tr>
<td>C7</td>
<td>1000</td>
<td>2200</td>
<td>1000</td>
<td>9.14</td>
<td>17.9</td>
<td>9.14</td>
</tr>
<tr>
<td>C8</td>
<td>1000</td>
<td>2200</td>
<td>1000</td>
<td>17.9</td>
<td>9.14</td>
<td>17.9</td>
</tr>
</tbody>
</table>
Deriving the Chinook Model

The model used in this report is largely an extension based on the work of the TUDelft by: Kamp[13], Reijm[14] and Klaver[15]. In this chapter the physics of the model is treated. References are made to the appendices, in which the program structure is shown. Unless referred to otherwise, the equations are directly extracted from the program written by Kamp[13].

4.1. Defining the Helicopter Reference System and Force Locations

In order to write the model, the problem is split into multiple small problems. This is done by modeling the helicopter as a rigid body with several forces acting upon it. To do this one first has to define the coordinate system of this rigid body. The coordinate system of the helicopter is a body fixed reference system with its origin in the center of gravity, the $x$–axis pointing forward, the $y$–axis pointing right and the $z$–axis pointing down, as depicted in figure 4.1.

![Figure 4.1: Helicopter body fixed reference frame][13]

After the model reference frame is chosen, the forces and moments acting upon the helicopter are defined and placed with respect to the body fixed reference frame. The external forces and moments acting on the helicopter are grouped by origin and location.

- Front rotor - 3 forces, 3 moments
- Rear rotor - 3 forces, 3 moments
- Center of gravity - 3 forces
- Aerodynamic center - 3 forces, 3 moments
- Suspension point 1 - 3 forces
- Suspension point 2 - 3 forces
- Suspension point 3 - 3 forces
All the forces and moments that are acting on the helicopter are written in the same body fixed reference system, that is defined in figure 4.1. The distance at which the forces are acting are taken from the program and the report written by Kamp[13] and shown in figure 4.2. Please note, that all the distances are defined by the absolute distance between the force suspension points, and not by the relative distance from the positive body fixed reference frame.

Figure 4.2: distances from external forces on the rigid helicopter model body [13]

4.2. EQUATION OF MOTION OF THE BARE CHINOOK

Now that the rigid body coordinate system is defined, and the locations of the forces and moments that are acting on the helicopter are identified, the equations of motion valid for a rigid body can be applied. The equation of motion is calculated in matrix form and is shown in equation 4.1.

\[
\begin{bmatrix}
F_{heli} \\
M_{heli}
\end{bmatrix} =
\begin{bmatrix}
m I_0 & 0 & I_{CG} \\
0 & I_{CG} & \vec{a}_{CG} \\
\vec{\omega} \times m v_{CG} & \vec{\omega} \times I_{CG} \vec{\omega}
\end{bmatrix}
\]

Equation 4.1 \(F_{heli}\) shows the force vector in respectively, \(x\), \(y\) and \(z\) direction. \(M_{heli}\) represents the moment vector around respectively \(x\), \(y\) and \(z\)-axes. \(m_i\) is the 3x3 diagonal matrix in which each diagonal value is the mass of the helicopter.

The inertia matrix around the center of gravity, \(I_{CG}\), is simplified by assuming that the helicopter is mass symmetrical in the \(XZ\) plane, which sets the products of inertia \(I_{XY}\) and \(I_{YZ}\) to zero; leaving the only product of inertia \(I_{XZ}\). When one places the moments of inertia on the diagonal, one can form the inertia matrix as shown in equation 4.2.

\[I_{CG} = \begin{bmatrix}
I_{XX} & -I_{XY} & -I_{XZ} \\
-I_{XY} & I_{YY} & -I_{YZ} \\
-I_{XZ} & -I_{YZ} & I_{ZZ}
\end{bmatrix} = \begin{bmatrix}
I_{XX} & 0 & -I_{XZ} \\
0 & I_{YY} & 0 \\
-I_{XZ} & 0 & I_{ZZ}
\end{bmatrix}
\]

4.3. ROTOR FORCES

Rotor forces are the most complex calculations treated in the simulation and comprise of multiple steps. The first step is calculating the effect of the pilot inputs, transforming the pilot stick, pedal and lever deflections to the rotor hub angles. The second step is transforming the rotor hub angles to rotor coefficients and force angles. This in turn, enables the calculation of the forces and moments on the rotor hub. The final step is to transform the forces and moments from the rotor reference systems back to the helicopter reference system.
4.3. DEFINING THE ROTOR HUB REFERENCE SYSTEMS AND ANGLES

The rotor reference systems, shaft axes, have their origin in the front and rear rotor hubs and are orientated depending on the rotor rotational direction in such a way that a positive rotational velocity around its z-axis results in a force roughly upward with respect to the helicopter. The x-axis is perpendicular to the helicopter xz-plane pointing roughly backwards with respect to the helicopter. The xy-plane represents the area in which the blade root has its rotational velocity. The rotor reference system is depicted in figure 4.3; note that the coordinate systems of the front and the rear rotor are different, this is done in such a way that both rotors have a positive rotational velocity. From a top view perspective, the front rotor is turning counter-clockwise, while the rear rotor is turning clockwise.

The control angles of the helicopter are defined in Bramwell [21] and are adapted for the Chinook by Kamp [13]. Figure 4.4 and 4.5 show three axes, the first axis is the shaft axis, followed by the control axis and finally the disc axis.

- The shaft axis is the axis that connects the rotor plane to the helicopter and is completely fixed. It is in symmetry with the helicopter xz-plane, but has a slight forward incidence angle denoted by \( i_f \) and \( i_r \).
- The control axis is the axis over which the pilot, (together with the AFCS), has direct control. They are defined from the shaft axis by two angles: \( \theta_{1c} \) defines the angle from the shaft axis to the control axis, from z to y direction. \( \theta_{1s} \) defines the control angle from shaft axis to control axis from the z to the x direction.
- The disc axis is perpendicular to the disc plane, which is defined as the path that the rotor tip travels. The axes have an offset of angles \( a_1 \) and \( b_1 \) with respect to the control axis. Where \( a_1 \) is used in the longitudinal plane, and \( b_1 \) is used in the lateral plane.

A last angle that is controllable by the pilot, but not depicted in any figure, is the collective pitch, \( \theta_0 \). This angle rotates the rotor blades in its longitudinal axis, changing its local angle of attack, mainly controlling the thrust of the helicopter.

4.3.2. CALCULATING THE CONTROL ANGLES FROM PILOT INPUT

As stated in the previous paragraph, the control angles are solely defined by the pilot input and the automated flight control system (AFCS). The pilot of a helicopter has three main instruments that manipulate the helicopter in four main ways. The first control instrument is the stick, which has two inputs denoted by subscripts \( \text{lat} \) and \( \text{long} \). The second is the collective lever which has the subscript \( \text{coll} \). The last are the pedals, which are denoted with the subscript \( \text{ped} \).

The influence these inputs have are defined by the control input deflection \( \Delta \text{control} \), the control derivative \( \frac{\Delta \theta}{\Delta \text{control}} \), and the neutral position \( \theta_{\text{neut}} \). Written as an expression results in equations 4.3 till 4.5. Control deflections are set indirectly by the swash plate limits. The controls are assumed to be able to reach these
Figure 4.4: Rotor angles front view [13]

Figure 4.5: Rotor angles side view [13]
With the initial disk angles calculated, the configuration parameters come into play. In this program longitudinal slip. This is calculated using equations 4.8 till 4.10, in which subscript $i$ denotes the front and rear rotor location.

\[
\begin{align*}
\Delta \theta_{\text{prelim}} &= \theta_{\text{neut}} + \frac{\Delta \theta}{\Delta \text{control}} \\
\left[ \begin{array}{c}
\theta_{0,f} \\
\theta_{0,r}
\end{array} \right] &= \left[ \begin{array}{c}
\theta_{0,\text{neut},f} \\
\theta_{0,\text{neut},r}
\end{array} \right] + \left[ \begin{array}{c}
\frac{\Delta \theta_{0,\text{clean}}}{\Delta \text{control}_{\text{long}}} \\
\frac{\Delta \theta_{0,\text{clean}}}{\Delta \text{control}_{\text{long}}}
\end{array} \right] \left[ \begin{array}{c}
\Delta \theta_{0,f} \\
\Delta \theta_{0,r}
\end{array} \right] \left[ \begin{array}{c}
\Delta \text{control}_{\text{long}} \\
\Delta \text{control}_{\text{long}}
\end{array} \right]
\end{align*}
\]

\[
\left[ \begin{array}{c}
\theta_{1c,f} \\
\theta_{1c,r,\text{clean}}
\end{array} \right] = \left[ \begin{array}{c}
\frac{\Delta \theta_{1c,f}}{\Delta \text{control}_{\text{lat}}} \\
\frac{\Delta \theta_{1c,r,\text{clean}}}{\Delta \text{control}_{\text{lat}}}
\end{array} \right] \left[ \begin{array}{c}
\Delta \text{control}_{\text{lat}} \\
\Delta \text{control}_{\text{lat}}
\end{array} \right]
\]

(4.5)

With the initial disk angles calculated, the configuration parameters come into play. In this program longitudinal pitch correction and beta reduction can be turned on and off. For the longitudinal pitch correction a switch is introduced. This switch introduces the first disk angle correction, which is described in equation 4.6.

\[
\theta_{1s,\text{cor}} = \begin{cases} 
\theta_{1s,\text{min}} & \text{u} \leq 100\text{kts} \\
\theta_{1s,\text{max}} & \text{u} \geq 140\text{kts} \\
(\theta_{1s,\text{max}} - \theta_{1s,\text{min}}) \frac{u - 100}{40} + \theta_{1s,\text{min}} & \text{otherwise}
\end{cases}
\]

\[
\theta_{1s,\text{cor}} = \begin{cases} 
\theta_{1s,\text{cor},\text{front}} & \text{LPC = 'on'} \\
\theta_{1s,\text{cor},\text{rear}} & \text{LPC = 'off'}
\end{cases}
\]

(4.6)

In these equations the subscripts min and max represent the minimum and the maximum deflections possible. The values of these limits are shown in appendix $F$. The last correction made is the side slip correction, $\beta$. Side slip reduction is depended on the local rotor side slip. This is calculated using equations 4.8 till 4.10, in which subscript $i$ denotes the front and rear rotor location.

\[
\mu_{R,\text{prec},i} = \frac{1}{\Omega_i R_i} \left[ \begin{array}{c}
u_{R,i} \\
v_{R,i}
\end{array} \right] = \frac{1}{\Omega_i R_i} \left[ \begin{array}{c}
u_{\text{heli}} \\
v_{\text{heli}}
\end{array} \right] + \left[ \begin{array}{c}0 \\
-\frac{1}{2}
\end{array} \right] \left[ \begin{array}{c}z_{R,i} \\
x_{R,i}
\end{array} \right] - \left[ \begin{array}{c}0 \\
-\frac{1}{2}
\end{array} \right] \left[ \begin{array}{c}z_{R,i} \\
x_{R,i}
\end{array} \right]
\]

(4.7)

\[
\mu_{R,i} = \begin{pmatrix} \mu_{R,\text{front}} \\
\mu_{R,\text{rear}} \end{pmatrix} = \begin{pmatrix} \cos i_i & \sin i_i \\
-\sin i_i & \cos i_i \end{pmatrix} \mu_{R,\text{prec},i}
\]

(4.8)

\[
\beta_R = \begin{pmatrix} \beta_{R,\text{front}} \\
\beta_{R,\text{rear}} \end{pmatrix} = \begin{pmatrix} -\pi/2 & \text{if } \mu_x = 0, \mu_y < 0, \\
0 & \text{if } \mu_x = 0, \mu_y = 0, \\
\pi/2 & \text{if } \mu_x = 0, \mu_y > 0, \\
\arctan \frac{\mu_y}{\mu_x} & \text{if } \mu_x \neq 0 \end{pmatrix}
\]

With all AFCS settings known, the final control angles can be calculated. This is done using equations 4.11 and 4.12.

\[
\theta_{1s,i} = \begin{pmatrix} \theta_{1s,\text{front}} \\
\theta_{1s,\text{rear}} \end{pmatrix} = \begin{pmatrix} \cos \beta_{R,i} \cdot \theta_{1s,\text{clean},i} - \sin \beta_{R,i} \cdot \theta_{0,\text{cor},i} \\
\theta_{1s,\text{clean},i} \end{pmatrix} \begin{pmatrix} \theta_{1s,\text{cor},i} \end{pmatrix}
\]

if $\beta$-reduction = on

(4.11)

\[
\theta_{0,i} = \begin{pmatrix} \theta_{0,\text{front}} \\
\theta_{0,\text{rear}} \end{pmatrix} = \begin{pmatrix} \sin \beta_{R,i} \cdot \theta_{1s,\text{clean},i} + \cos \beta_{R,i} \cdot \theta_{0,\text{cor},i} \\
\theta_{0,\text{cor},i} \end{pmatrix} \begin{pmatrix} \theta_{0,\text{cor},i} \end{pmatrix}
\]

if $\beta$-reduction = on

(4.12)
4.3.3. Rotor Coefficients and Angle Corrections

Next step in calculating the rotor forces, is making angle corrections such that all the coefficients are written in the shaft, rotor, reference frame, followed by the calculation of the forces and writing these forces in the helicopter reference frame. With the angles and direction of flow known, the helicopter rotor coefficients can be calculated. The exact calculations are beyond the scope of the report, but can be found in Kamp [13].

The forces and moment coefficient data is a set of eight values that are calculated in the rotor disk plane and the shaft plane. The first correction executed in this data is to ensure that all data is in the same reference system. This is done in equation 4.13.

\[
C_{\text{rotor, shaft}} = \begin{bmatrix}
C_{T, \text{shaft}, f} & C_{T, \text{shaft}, r} \\
C_{X, \text{disk}, f} & C_{X, \text{disk}, r} \\
C_{Y, \text{disk}, f} & C_{Y, \text{disk}, r} \\
C_{Q, \text{shaft}, f} & C_{Q, \text{shaft}, r}
\end{bmatrix}
\begin{bmatrix}
1 & 1 \\
C_{T, \text{shaft}, f} \cdot \alpha_{f} & C_{T, \text{shaft}, r} \cdot \alpha_{r} \\
C_{T, \text{shaft}, f} \cdot b_{f} & C_{T, \text{shaft}, r} \cdot b_{r} \\
1 & 1
\end{bmatrix}
\] (4.13)

With the rotor coefficients in the proper plane now the rotor forces in the rotor plane can be calculated, as is shown in equation 4.14. The rotor torque coefficient is calculated in equation 4.15.

\[
\begin{bmatrix}
T_{f} & T_{r} \\
H_{X, r} & H_{X, f} \\
H_{Y, r} & H_{Y, f}
\end{bmatrix}
= \begin{bmatrix}
C_{T, \text{shaft}, f} & C_{T, \text{shaft}, r} \\
C_{X, \text{disk}, f} & C_{X, \text{disk}, r} \\
C_{Y, \text{disk}, f} & C_{Y, \text{disk}, r}
\end{bmatrix}
\begin{bmatrix}
\sin i_{f} & 0 & \Omega R \sin \pi R^{2} \\
0 & \cos \alpha_{f} & \Omega R \cos \pi R^{3}
\end{bmatrix}
\] (4.14)

\[
M_{\text{rotor}, f} = \begin{bmatrix}
M_{l, f} & M_{m, f} \\
M_{m, f} & M_{l, f}
\end{bmatrix}
\begin{bmatrix}
\sin \alpha_{f} & -\cos \alpha_{f} & -\sin \alpha_{f} \sin \beta_{f} & T_{f} \\
\cos \alpha_{f} & 0 & \cos \beta_{f} & H_{X, f} \\
0 & \cos \beta_{f} & \sin \beta_{f} & H_{Y, f}
\end{bmatrix}
\] (4.15)

The next step is to transform the rotor forces to the helicopter reference frame. For the forces this is a simple transformation, which is shown in equation 4.17. For easy readability, angles \( \alpha_{\text{cor}} \) and \( \beta_{\text{cor}} \) are introduced to substitute the angles shown in figure 4.4 and 4.5. The corresponding angles are shown in equation 4.16.

\[
\begin{align*}
\alpha_{\text{cor}, f} &= \theta_{1, f} - \alpha_{1, f} + \theta_{f} \\
\beta_{\text{cor}, r} &= \theta_{1, r} + \beta_{1, r} + \theta_{r}
\end{align*}
\] (4.16)

\[
F_{\text{rotor, heli}, f} = \begin{bmatrix}
\sin \alpha_{\text{cor}, f} \cos \beta_{\text{cor}, f} & -\cos \alpha_{\text{cor}, f} & -\sin \alpha_{\text{cor}, f} \sin \beta_{\text{cor}, f} & T_{f} \\
\cos \alpha_{\text{cor}, f} \sin \beta_{\text{cor}, f} & \cos \alpha_{\text{cor}, f} & \sin \alpha_{\text{cor}, f} \sin \beta_{\text{cor}, f} & H_{X, f} \\
-\cos \alpha_{\text{cor}, f} \sin \beta_{\text{cor}, f} & -\sin \alpha_{\text{cor}, f} & \sin \alpha_{\text{cor}, f} \sin \beta_{\text{cor}, f} & H_{Y, f}
\end{bmatrix}
\] (4.17)

\[
F_{\text{rotor, heli}, r} = \begin{bmatrix}
\sin \alpha_{\text{cor}, r} \cos \beta_{\text{cor}, r} & -\cos \alpha_{\text{cor}, r} & \sin \alpha_{\text{cor}, r} \sin \beta_{\text{cor}, r} & T_{f} \\
\cos \alpha_{\text{cor}, r} \sin \beta_{\text{cor}, r} & \cos \alpha_{\text{cor}, r} & -\cos \beta_{\text{cor}, r} & H_{X, r} \\
-\cos \alpha_{\text{cor}, r} \sin \beta_{\text{cor}, r} & -\sin \alpha_{\text{cor}, r} & \sin \beta_{\text{cor}, r} & H_{Y, r}
\end{bmatrix}
\]

The moments created by the rotor consist of two parts, the moments created by the torque, and the moments created by the forces and their distance to the center of gravity. The moments created by the torque are already calculated in equation 4.15. The moments created by the forces are shown in equation 4.18. The definition of the distances can be found in figure 4.2 and the values in appendix F.

\[
M_{\text{RF, heli}, f} = \begin{bmatrix}
0 & -h_{f} & b_{f} \\
h_{f} & 0 & -l_{f} \\
-b_{f} & l_{f} & 0
\end{bmatrix}
\]

\[
M_{\text{RF, heli}, r} = \begin{bmatrix}
0 & -h_{r} & b_{r} \\
h_{r} & 0 & -l_{r} \\
-b_{r} & l_{r} & 0
\end{bmatrix}
\] (4.18)
Finally, the total forces and moments generated by the rotors are a simple addition of forces and moments, as is shown in equation 4.19 and 4.20 respectively.

\[
\begin{bmatrix}
F_R \\
F_y \\
F_z
\end{bmatrix} = \begin{bmatrix}
F_{R,fus} \\
F_{R,fus} + F_{R, heli}
\end{bmatrix} \tag{4.19}
\]

\[
\begin{bmatrix}
M_R \\
M_y \\
M_z
\end{bmatrix} = \begin{bmatrix}
M_{R,fus} \\
M_{R,fus} + M_{R, heli}
\end{bmatrix} \tag{4.20}
\]

4.4. Helicopter Fuselage Aerodynamics

Helicopter aerodynamics are due to rotor down wash and alternating free stream inflow angles extremely complex. As exact aerodynamics are extremely difficult to calculate a simplification is made where both y and -y, z and -z have the same drag coefficient. This simplification is based on the work of Ostroff et al. [19].

For forward flight, empirical flight data is used where the angle of attack, \(\alpha\), and the side slip angle, \(\beta\), is used to determine the fuselage drag in \(x\)-direction. The empirical data consists of three polynomials, each polynomial is a representation of the equivalent flat plate drag coefficient as a function of the side slip angle with a static angle of attack. The values for the polynomial are found in appendix F. The static angle of attack has the value of -20, 0 and 20 degrees. Depending on the angle of attack, one or two of the polynomials are chosen, subsequently an interpolation is performed giving an estimate on the drag coefficient of the helicopter in forward flight. If the absolute side slip angle becomes larger than 40 degrees, the forward fuselage equivalent drag coefficient \(C_{FE}\) is set to zero, as is shown in equation 4.21.

\[
C_{FE} = \begin{bmatrix}
p_0 (\beta_{fus}) (1 - \beta_{fus}) (1 - \beta_{fus}) & \text{if } 0 \leq \alpha_{fus} \leq \frac{\pi}{2}, \text{ and } |\beta_{fus}| \leq \frac{\pi}{2} \\
p_0 (\beta_{fus}) (1 + \beta_{fus}^2) - p_{20} (\beta_{fus}) \beta_{fus} \frac{3\alpha_{fus}}{\pi} & \text{if } \frac{\pi}{2} \leq \alpha_{fus} < 0, \text{ and } |\beta_{fus}| \leq \frac{\pi}{2} \\
p_{20} (\beta_{fus}) (1 + \beta_{fus}^2) & \text{if } \frac{\pi}{2} < \alpha_{fus} \leq \pi, \text{ and } |\beta_{fus}| \leq \frac{\pi}{2} \\
p_{-20} (\beta_{fus}) (1 + \beta_{fus}^2) & \text{if } \frac{\pi}{2} \leq \alpha_{fus} < \frac{\pi}{2}, \text{ and } |\beta_{fus}| \leq \frac{\pi}{2} \\
0 & \text{if } |\alpha_{fus}| > \frac{\pi}{2}, \text{ and/or } |\beta_{fus}| > \frac{\pi}{2}
\end{bmatrix} \tag{4.21}
\]

With the forward drag coefficient known, the remaining formula to calculate the drag forces and moments is shown in equation 4.22, in which \(C_{D,eq}\) is the equivalent flat plate drag coefficient, shown in equation 4.23.

\[
F_{aero} = -\frac{1}{2} C_{D,eq} \theta(z) V \cdot |V| \cdot \begin{bmatrix}
\cos \alpha \cos \beta \\
\sin \beta \\
\sin \alpha
\end{bmatrix} \tag{4.22}
\]

\[
C_{D,eq} = \begin{bmatrix}
C_{FE} \\
C_{Y\beta} \\
C_{L\alpha}
\end{bmatrix} \tag{4.23}
\]

For the moment coefficient the equivalent moment coefficients are considered to be constant. Herefor the formula for the aerodynamic moments created by the fuselage are shown by 4.24, in which the \(C_{M,eq}\) is given by 4.25.

\[
M_{aero} = \frac{1}{2} \theta V \cdot |V| \cdot C_{M} \begin{bmatrix}
\sin \beta (\cos \beta) (1 - |\sin \alpha|) \\
\sin \alpha \cos \alpha \\
\sin \beta \cos \beta (0.94 \sin \alpha + 0.342 \cos \alpha)
\end{bmatrix} \tag{4.24}
\]

\[
C_{M,eq} = \begin{bmatrix}
-C_{L\beta} \\
C_{M\alpha} \\
-C_{N\beta}
\end{bmatrix} \tag{4.25}
\]
4.5. **GRAVITATIONAL FORCES**

The gravitational forces are acting on the center of gravity of the helicopter. The gravity acceleration is considered to be constant. The gravitational force in the helicopter body fixed reference frame is shown in equation 4.26.

\[
F_{CG} = mg \begin{bmatrix}
-\sin \theta \\
\cos \theta \sin \phi \\
\cos \theta \cos \phi
\end{bmatrix}
\]  

(4.26)
In this chapter, the six degrees of freedom underslung load model is derived. Similar to the helicopter derivation, the first step is to define the coordinate system of the load. Next, the different forces and moments that are acting on the load are quantified. Finally, the load model forces are transformed to the helicopter reference system so they can be integrated in the model.

5.1. Defining the Conditions

The first step in creating the model of the underslung load is simplifying the problem. This is done by dividing the underslung load into two parts: the cargo container and the slings. The mass is assumed to be uniformly distributed, which results in a center of gravity that is the crossing of the three symmetry planes of the box. The coordinate system for the underslung load is a body fixed reference frame with its origin in the center of gravity. The $x$-coordinate is aimed towards the normal flying direction, the $y$-coordinate to the right hand side and the $z$-coordinate to the ground, as depicted in figure 5.1.

![Load reference frame](image)

Figure 5.1: Load reference frame [13]

The dimensions of the box are defined as follows: length, $l_{load}$, in the $x$-direction; width, $w_{load}$, in the $y$-direction and height, $h_{load}$, in the $z$-direction.
5.2. **LOAD FORCES**

As with the helicopter model the first step is to find all the forces and moments acting on the helicopter during specific conditions. When all the forces are known, the equation of motion can be formed which will be used to determine the derivatives.

5.2.1. **LOAD AERODYNAMICS**

The aerodynamics of the load is calculated using flat plate theory. Since the load is modeled as a container, this method is accurate enough for the purpose of stability analysis. The equation used by flat plate theory is shown in equation 5.1 till 5.4.

\[
\begin{bmatrix}
    F_{x,\text{load}} \\
    F_{y,\text{load}} \\
    F_{z,\text{load}}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
    C_{F,\text{load}} \\
    C_{Y,\text{load}} \\
    C_{L,\text{load}}
\end{bmatrix} \rho V_{\text{load}} |V_{\text{load}}| \begin{bmatrix}
    \cos \alpha_{\text{load}} \cos \beta_{\text{load}} \\
    \sin \beta_{\text{load}} \\
    \sin \alpha_{\text{load}}
\end{bmatrix}
\]

\[
F_{\text{load,aero}} = \frac{1}{2} C_{D,\text{eq,load}} \rho V_{\text{load}} |V_{\text{load}}|
\]

\[
\begin{bmatrix}
    M_{l,\text{load}} \\
    M_{m,\text{load}} \\
    M_{n,\text{load}}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
    -C_{L,\text{load}} \\
    C_{M,\text{load}} \\
    -C_{N,\text{load}}
\end{bmatrix} \rho V_{\text{load}} |V_{\text{load}}| \begin{bmatrix}
    \sin \beta_{\text{load}} \cos \beta_{\text{load}} (1 - |\sin \alpha_{\text{load}}|) \\
    -\sin \alpha_{\text{load}} \cos \alpha_{\text{load}} \\
    \sin \beta_{\text{load}} \cos \beta_{\text{load}} (0.94 \sin \alpha_{\text{load}} + 0.342 \cos \alpha_{\text{load}})
\end{bmatrix}
\]

\[
M_{\text{load,aero}} = \frac{1}{2} C_{M,\text{eq,load}} \rho V_{\text{load}} |V_{\text{load}}|
\]

5.2.2. **GRAVITY**

The gravitational acceleration is considered to be constant and the gravitational forces are thus only depended on the load angles. The forces that are acting on the load due to gravity are shown in equation 5.5.

\[
F_{\text{load,grav}} = m_{\text{load}} g \begin{bmatrix}
    -\sin \theta_{\text{load}} \\
    \cos \theta_{\text{load}} \sin \phi_{\text{load}} \\
    \cos \theta_{\text{load}} \cos \phi_{\text{load}}
\end{bmatrix}
\]

5.3. **SLING FORCES**

The sling forces are the connecting elements between the external cargo and the helicopter. The forces that are acting on the cable are due to the elongation and relative velocity of the cable. As elongation of the rope and the velocity cause the absorption of the forces, it is important that the begin point of the cable (the helicopter suspension point) and the end point (the underslung cargo container) are written in the same coordinate system. This is done with aid of Euler transformation matrices.

The following assumptions are made in the cable calculations:

- Cables are unable to carry moments.
- The cable is not able to carry any force with a negative elongation.
- The connection between the helicopter cable and cable cargo container are assumed to be incapable to carry any moments.
- The cable is modeled as a spring dampener system.
- The cable is assumed to be massless.
- Cable dampening and cable spring stiffness is assumed to be constant.
- Helicopter and load are considered to pose infinite stiffness.

The state and state flux vectors both consist of global coordinates, therefore it is most convenient to write the locations of the helicopter hooks and the locations of the load suspension points in global coordinates. This is done using the Euler transformation matrices which are shown in equation 5.6. The Euler angles are visualized in figure 5.2.
5.3. SLING FORCES

\[
T_\psi = \begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
T_\theta = \begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]

\[
T_\phi = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\]

\[T_{bodyToGlobal} = T_\psi T_\theta T_\phi\]

\[T_{globalToBody} = T_\phi T_\theta T_\psi = T_{bodyToGlobal}^{-1}\]

With these transformation matrices, the positions and velocities of the cargo container suspension points
and the helicopter suspension points can be calculated and placed in the earth fixed reference system.

The next step is to calculate the elongation of the cables and the elongation velocity (velocity in direction
of the rope). The cable consists out of five pieces; one upper part and four lower parts. These parts are all
connected to the central point. The velocity and location of the central point is unknown and needs to be
calculated. This is done with the following steps:

1. Find an estimate for the central position.
2. Use the central position estimate to calculate the elongation of the cables.
3. Calculate the velocity of the central point.
4. Calculate the velocity of the rope elongation by changing the coordinate system basis to that which
   contains the direction of the rope.
5. Calculate the forces created by elongation and damping.
6. Check if the error between top and bottom forces is acceptable, if not, use Newton-Raphson to find a
   new estimate.

In order to solve the problem of the location and velocity, a routine is used that uses an estimated initial
location. With the location known, an estimate of the central velocity is made using equation 5.7. This routine
calculates the forces that are acting on the central point. Since the rope is assumed massless, changes in
location are instantaneous and force balance must exist.

\[
V_{center} = V_{susp, heli} + \left( \sum_i \frac{V_{susp, load, i}}{l_{upper}} - V_{susp, heli} \right) \frac{l_{upper} + \Delta l_{upper}}{l_{upper} + \Delta l_{upper} + l_{lower} + \Delta l_{lower}}
\]

(5.7)

With the velocity of the center estimated and the location put into an optimizer, the forces acting on the cable
can be calculated. The sum of the forces acting on the midpoint of the sling should be equal to zero. If this is
not the case, the estimated location of the central point is not located properly and has to be relocated.

The forces in the optimizer are calculated by creating a new reference system for each sling. This is done
to ensure that the cables do not carry a moment. The first step is creating an orthogonal basis of which one

---

Figure 5.2: Euler angles
entry is pointing in the direction of the cable.

When this is done the elongation of the cable is calculated as is shown in equation 5.9. For each cable the elongation is calculated. If the elongation is smaller than or equal to zero, the forces are set to zero, else calculations are performed.

The forces are calculated by creating a new reference system for each cable with one arm in the direction of the cable. The velocity in perpendicular to the cable is calculated, the tangential velocity components are discarded and multiplied by the damping constant. The elongation is multiplied by the spring constant and multiplied by the the base. This is shown in equation 5.10 and equation 5.11 for respectively the upper and lower cables.

\[
\Delta l_{upper} = \sqrt{(\mathbf{x}_{center} - \mathbf{x}_{upper}) (\mathbf{x}_{center} - \mathbf{x}_{upper})' - l_{upper}} \quad (5.8)
\]

\[
\Delta l_{lower,i} = \sqrt{(\mathbf{x}_{center} - \mathbf{x}_{load,i}) (\mathbf{x}_{center} - \mathbf{x}_{load,i})' - l_{lower}} \quad (5.9)
\]

\[
F_{upper} = \begin{cases} 
\text{base}^{-1} (\mathbf{V}_{center} - \mathbf{V}_{hook}) \mathbf{c}_{upper} & \text{if } \Delta l_{upper} > 0 \\
0 & \text{if } \Delta l_{upper} \leq 0
\end{cases} 
\]

\[
F_{lower,i} = \begin{cases} 
\text{base}_i^{-1} (\mathbf{V}_{center} - \mathbf{V}_{load,i}) \mathbf{c}_{lower} & \text{if } \Delta l_{lower,i} > 0 \\
0 & \text{if } \Delta l_{lower,i} \leq 0
\end{cases} 
\]

If there is a force equilibrium at the central point, as shown in equation 5.12, the central point is found successfully. If not, the Newton-Raphson routine continues until the central point has been found successfully.

\[
\mathbf{F}_{cp} = F_{upper} + \sum_{i=1}^{4} F_{lower,i} \approx 0 \quad (5.12)
\]

The forces that are acting on the load are rewritten in the load body reference system. A special note is that the forces work in opposite direction than in the central point as is shown in equation 5.13.

\[
\mathbf{F}_{cable,load,i} = -T_{globalToBody} \mathbf{F}_{lower,i} \quad (5.13)
\]

The moments that are created by the cables are calculated by taking the forces acting on the suspension points and multiplying them by their respective arms, which is done in equation 5.14.

\[
\mathbf{M}_{cable,load,i} = \begin{bmatrix} 
0 & -h_{sp,i} & b_{sp,i} \\
h_{sp,i} & 0 & -l_{sp,i} \\
-b_{sp,i} & l_{sp,i} & 0
\end{bmatrix} \mathbf{F}_{cable,load,i} \quad (5.14)
\]

5.4. LOAD EQUATION OF MOTION

The load equation of motion is identical to that of the helicopter, with off course the exception of the force and moment entries. The forces and moments that are acting on the load are added, as shown in equations 5.15 and 5.16. The equation of motion is shown in equation 5.17.

\[
\mathbf{F}_{load} = \mathbf{F}_{load,aero} + \mathbf{F}_{load,grav} + \sum_{i=1}^{4} \mathbf{F}_{cable,load,i} \quad (5.15)
\]

\[
\mathbf{M}_{load} = \mathbf{M}_{load,aero} + \sum_{i=1}^{4} \mathbf{M}_{cable,load,i} \quad (5.16)
\]

\[
\begin{bmatrix} 
\mathbf{F}_{load} \\
\mathbf{M}_{load}
\end{bmatrix} = \begin{bmatrix} 
m_{load} & 0 \\
0 & I_{CG,load}
\end{bmatrix} \mathbf{a}_{CG,load} + \begin{bmatrix} 
\ddot{\omega}_{load} \times m_{load} \mathbf{v}_{CG} \\
\ddot{\omega}_{load} \times I_{CG,load} \ddot{\omega}_{load}
\end{bmatrix} \quad (5.17)
\]
In this chapter the Chinook stability derivatives will be discussed. First a small introduction is given into what the helicopter derivatives are, how they are created and why they are important for the stability analysis.

The entire set of helicopter forces and moments are linearized. After the linearisation the helicopter state matrix is divided by the longitudinal and lateral motions. First the longitudinal forces and its perturbations are discussed; followed by the lateral. The results of the lateral derivatives excited by the longitudinal excitations, and vice versa, are omitted.

When discussing the derivatives, the first item treated is the definition of the perturbation. The helicopter rotors play a dominant role in the creation of the derivative graphs, therefore the rotor forces and moments are plotted to get a rough idea of the forces that are working on the helicopter. These graphs are analyzed and together with the estimated behavior of the aerodynamics, a prediction can be made on how the perturbation affects the derivatives.

Followed by the general perturbation discussion are the force and moment discussions. Each force and moment is discussed in detail. A figure is given that shows two graphs. The left side graph, gives the dissected derivatives of the calculated values, in which the contribution of the front rotor, rear rotor fuselage and center of gravity form the total derivative. The right side graph, shows both the total derivative and two reference values obtained from the reports of Ostroff[19] and Davis[20].

When all the major derivatives have been discussed, the underslung load is introduced. From all the configurations, two selections have been made which are compared to each other and to the bare Chinook derivatives. The derivatives results for all the configurations can be found in appendix E.

6.1. INTRODUCTION
The helicopter derivatives show the change in force or moment that the helicopter experiences due to a change in the helicopter state. The helicopter model is numerically linearized by giving a perturbation, one state at the time, and checking the force or moment responds compared to the trimmed responds.

The sign of the derivative can determine whether it is a stabilizing or destabilizing derivative. If, for example, the derivative $X_u$ or $M_q$ are negative, the introduction of the perturbation produces a force or moment that is counteracting the perturbation, and thus has a stabilizing effect.

As will be seen from the derivatives graphs in this chapter, a change in state can have multiple effects on forces and moments in different directions. After the analysis of the derivatives it is hard to see what the final result is on the stability. For this reason, stability plots are made and discussed in chapter 7, which shows the Eigenmotions of the helicopter and give a clear graph of the helicopters in terms of dampening and oscillatory behavior.
6.2. Calculating the Derivatives for the Bare Chinook

The used model of the bare Chinook has six degrees of freedom and will have 54 derivatives, three translational velocity perturbations, three angular perturbations, and three angular velocities perturbations. The $\psi$ perturbation is ignored because it has no effect on the bare Chinook and lack of reference data; this report will thus discuss a total of 48 derivatives.

Adding an underslung load will add a total of 54 derivatives to the helicopter body. For these analyses the load will remain under the trimmed conditions at $t = 0$. Perturbations in the underslung loads are thus not applied.

To calculate the derivatives, the first step is determining the trimmed forces. Trimming is done with a Newton Raphson variation and is explained in detail in appendix B. The helicopter is trimmed for all the configurations ranging from a speed of 0 to 80 m/s, with an interval of 2 m/s.

Linearisation is done by giving a perturbation to all translational velocities, all the rotational velocities and the rotations around the $x$ and $y$ axes. The resulting derivative vector is shown in equation 6.1, in which $f(x)$ is the result of equation 4.1.

\[
x = \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \\ \phi \\ \theta \end{bmatrix}, \quad f(x) = \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_p \\ M_q \\ M_r \end{bmatrix} \tag{6.1}
\]

\[
\begin{bmatrix} X \\ Y \\ Z \\ L \\ M \\ N \end{bmatrix} = \frac{f(x + \Delta x_i) - f(x)}{\Delta x_i} \cdot \begin{bmatrix} 1/m \\ 1/m \\ 1/m \\ 1/I_{xx} \\ 1/I_{yy} \\ 1/I_{zz} \end{bmatrix} \tag{6.2}
\]

The forces and moments that are calculated are semi-normalized as is suggested by Padfield [6]. For the forces this is done by dividing the force by the helicopter mass, the moments are divided by their respective moments of inertia. The semi-normalization is shown in equation 6.2. The derivative value is checked by reducing the value of $\Delta x_i$, which represents the i-th perturbation. The perturbations start with a magnitude of $10^{-7}$ and are followed by a perturbation of $10^{-8}$. If the maximum absolute change of the derivative value is smaller than $10^{-5}$, the value is accepted. Else, the step size is reduced by multiplying the previous perturbation step size by 0.1. This procedure is repeated until a suitable value has been found.

This procedure is repeated until all eight states had their perturbation. The schematics used for these calculations can be found in appendix D.

6.3. Results

The found derivatives, that are calculated with the coupled equation of motion, are split into two parts: the first part for the longitudinal motions and perturbations and the second part for the lateral motions and perturbations. This will result in a total of 24 derivatives.

The effect the fuselage aerodynamics have on the force buildup is easy to explain, as the equations are based on the flat plate theory.

The effect that perturbations have on the rotor derivatives is more complex. One can state, that there are three variables in the blade mechanics that are dominant and can be used to determine the effects perturbations have on the rotors responds on the perturbations [6]. These are the flapping angles $a_0$, $a_1$ and $b_1$.

In order to compare the rotor derivatives, not the angles are plotted, but the resultant forces and torque. This allows one to plot the lines in the same graph and make it easier to identify the influence that the rotors
have on the complete derivative. Since the rotor derivatives are fixed for each perturbation, they are shown at the beginning of each perturbation section and used for explaining the behavior of the full derivative set.

6.4. LONGITUDINAL DERIVATIVES

This section discusses the helicopter derivatives that are created due to perturbations in the longitudinal direction: $u, w, q$ and $\theta$. Each of these perturbations will be used to linearize the forces $X$, and $Z$; and the moment $M$.

6.4.1. CHINOOK RESPONSES DUE TO A WIND GUST IN THE NEGATIVE X–DIRECTION

The first helicopter perturbation looked at, is a wind gust in the negative $x$–direction i.e. a wind gust originating from the front of the helicopter moving towards the rear. This is simulated by an increase in the forward velocity as described in equation 6.3.

\[ u' = u + \Delta u \]  \hspace{1cm} (6.3)

From the aerodynamics perspective it is safe to assume that a higher velocity will increase the drag acting on the helicopter, resulting in a higher force acting towards the negative $x$–axis.

The effect that an increased velocity has on the rotor blades can be seen in figure 6.1. The perturbation seems to have the largest effect on the $Z$ force at low velocities.

An increase in velocity will in general increase the lift. A positive derivative in $z$–direction indicates a decrease in lift, when looking at the solid yellow line for the front rotor and the dashed blue line for the rear rotor, it can be seen that for the majority of the velocities the rotors have a lift reduction with a small increase in velocity. This is thus opposite of the expected behavior. The force in negative $x$–direction is acting as expected. An increase in velocity will increase the drag, causing a larger force in the negative $x$–direction.

The right hand side of figure 6.1 shows the resultant moments that are created. Here it is clear that the $Z$–forces create a large, but opposite effect on the moment around the $y$–axis, depicted by $M_{uF}$ and $M_{uR}$.

\[ \Delta \alpha_{pr} \propto \tan^{-1} \frac{\Omega R + u + \Delta u}{w} \]  \hspace{1cm} (6.4)

\[ \Delta \alpha_{re} \propto \tan^{-1} \frac{\Omega R - u - \Delta u}{w} \]  \hspace{1cm} (6.5)

![Figure 6.1: Rotor hub forces and derivatives due to a perturbation in the forward velocity](image-url)
The change in lift between the progressing and retreating blade will cause a twisting moment that will be introduced around the $x$–axis of the helicopter’s body. This effect can be seen on the right hand side of figure 6.1 in the yellow and blue dotted line, representing respectively $N_uF$ and $N_uR$. These lines are in opposite direction, as is expected, since the rotors turn in opposite directions.

**Helicopter derivative $X_u$**

![Graph](image)

**Figure 6.2:** Change in force in the $x$–direction due to an increase in the forward velocity

The change in forward force, due to an increase in forward velocity, is shown in figure 6.2. As is expected from the drag forces on the helicopter fuselage, the initial change in the velocity only causes a small change in the increase of drag. The drag increases approximately linearly with the increase of velocity.

The linear behavior of the fuselage drag derivative can be explained with the method used for calculating drag, which is shown in equation 6.6. When one assumes that the change in equivalent flat plate area, $C_D S$, is negligible when calculating the drag with changing velocities; one could state that the derivative is approximately proportional to the velocity, as is shown in equation 6.7.

$$D = \frac{1}{2} \rho V^2 C_D S$$  \hspace{1cm} (6.6)

$$\Delta D \propto \frac{(u + \Delta u)^2 - (u - \Delta u)^2}{2\Delta u} = 2u$$  \hspace{1cm} (6.7)

Besides the increase in fuselage drag, one can see that at low velocities the effect of the rotor blades are leading the shape of the derivative curve, while at higher velocities the fuselage drag becomes dominant.
Helicopter derivative $Z_u$

For the force in the $z$–direction, the thrust coefficients of the front and rear rotor blades are leading. The fuselage lift generated by helicopter bodies are in general negligible, this is also the case with the Chinook. Only at velocities nearing the maximum velocities of the Chinook, an effect on the helicopter body can be seen. At this velocity the nose downward attitude creates a force pushing the helicopter down; resulting in a small positive force derivative, which is shown on the left hand side of figure 6.3, with the purple dotted line.

![Figure 6.3: Change in force in the $z$–direction due to an increase in the forward velocity](image)

The right hand side of figure 6.3 shows the comparison of $Z_u$ with the reference data. There are two major differences between the graphs, the first part is the value at hover velocity. This difference between the calculated data and reference can be explained by the trim conditions and lack of calculated points in the reference data. The reference data trimmed the helicopter at hover, 0 m/s. The calculated data has trim values just above zero due to trimming issues. Since there are no calculated data points just above zero of the reference data it is hard to determine if there is a difference between the lines.

The second major difference is the opposite sign of the reference data and the calculated data. As is stated in the introduction paragraph, the positive change in force in $z$–direction is very counter intuitive and the reference data clearly shows that something strange is indeed happening with this derivative.

When examining the other fuselage derivative on the left hand side of figure 6.3, it can be seen that the fuselage does show the expected behavior; higher velocities trim conditions give a pitch down attitude, which would logically lead to a downward force when the forward velocity is increased. It is thus clear that the strange behavior of the $Z_u$ derivative is caused by the calculation of the rotor forces.

The cause of these strange values at this point is speculation, but could be caused by an overdetermined progressive blade flapping value. If the progressive blade loses more lift due to the flapping than it gains by the increased velocity, the $Z_u$ derivative will be positive.

Helicopter derivative $M_u$

The helicopter derivative $M_u$ discusses the pitch up moment caused by a wind gust that increases the effective forward velocity. This pitching moment is a key parameter for any aircraft as it shows static stability. An increase in the forward velocity will, generally, increase the lift over the lifting surfaces.

In case of a single rotor helicopter, the tail plane will increase the pitch down moment and thus causes a negative moment around the $y$–axis. The effect of the rotor forces will be relatively small because of the small arm the rotor forces have with respect of the center of gravity.

Since the effective arms of the Chinook rotor hubs, with respect to the center of gravity, are a lot larger than that of a single rotor helicopter; one can expect the rotor derivatives to be leading in the $M_u$ derivative. The left hand side of Figure 6.4 shows that the rotors are indeed the leading parameters in the $M_u$ derivative. The
moment derivative created by the fuselage is negligible most of the times, and only seems to contribute above 60 m/s.

The effect of the front and rear rotor are similar, but opposite in shape. This is not surprising since the forces act in the same direction, but the rear rotor has a negative arm, while the front rotor arm is positive.

When the derivative values are compared to the reference data, shown on the right hand side of figure 6.4, one can see that the reference data shows a clear pitch up moment at lower velocities ranging from 0 to just under 20 m/s. At 30 m/s and higher, the lines seem to match up and become comparable.

6.4.2. **CHINOOK RESPONSES DUE TO A WIND GUST IN THE NEGATIVE $z$–DIRECTION**

The effect of a wind-gust in the negative $z$–plane is simulated by an increase of the downward velocity, as shown in equation 6.8.

$$w' = w + \Delta w$$

With an additional upward velocity component acting on the helicopter blades, one can state that the angle of attack on all sections of the helicopter blades will be increased. In order to quantify the effect of this perturbation, a quick estimate can be made showing the effects on the rotor blades. This is done using equation 6.9 for the progressive blade and equation 6.10 for the retreating blade.

$$\Delta \alpha_{pr} \propto \tan^{-1} \frac{\Omega R + v}{w + \Delta w}$$

$$\Delta \alpha_{re} \propto \tan^{-1} \frac{\Omega R - v}{w + \Delta w}$$

As is shown in the above equation, the wind-gust will always increase the angle of attack; it will thus increase the lift for all velocities until a point where stall occurs. When one takes a look at the left side of figure 6.5, this condition is reached at around 70 m/s for the front rotor and around 75 m/s for the rear rotor.

The front rotor has a larger increase of force in the $x$–direction, this is caused because the front rotor incidence angle is larger than that of rear rotor. The largest derivatives are in the negative $z$–direction, this is exactly as expected due to the increased angle of attack. At higher velocities this effect is slightly reduced, this behavior is most likely caused by the tip stall characteristics of the progressing blades.

This theory is supported by checking the forces in the $y$–axis, shown on the left hand side of figure 6.5 where the red line and green dotted line (respectively $Y_{wF}$ and $Y_{wR}$) are of opposite sign.
The right graph in figure 6.5 shows the moment derivatives. As expected, the moments around the y−axis are the largest. Since the force derivatives were in the same direction, and the arms are in opposite direction, the moments counteract each other and are thus of opposite sign. The slightly larger arm of the front rotor with respect to the center of gravity causes the magnitude of $M_{wF}$ to be slightly larger than that of the rear rotor, $M_{wR}$.

Besides the obvious difference between the moments around the y−axis, one can also see a difference of the moments around the z−axis. Although the $Y_v$ derivatives would indicate that the moments around the z−axis would be in the same direction, the graph clearly shows that $N_{wF}$ and $N_{wR}$ are in opposite direction. This is caused by the moment induced by the rotational drag. An increased angle of attack also increases the drag on the rotor blades, therefor the moments around the z−axis are also increased. Since the rotors turn in opposite direction, the moments are in opposite direction.

**Helicopter derivative $X_w$**

When looking at the left graph in figure 6.6, there is a distinct difference between all the force derivatives acting on the helicopter. When looking at the rotor blades, the effect the wind gust has on the front rotor is larger than that of the rear rotor. This is mainly caused by the higher incidence of the front rotor, which rotates the thrust vector further forward than that of the rear rotor.

The effect of the fuselage aerodynamics show an interesting bump at 50 m/s, which is caused by the longitudinal cyclic pitch correction. This correction is inserted to keep the Chinook nose pointing a little upward during higher forward velocity, reducing the fuselage drag.

The right hand side of figure 6.6 shows the reference values compared to the calculated values. Again a clear difference can be seen between the reference data and the calculated data. The reference data lies roughly between 0.02 and 0.04 1/s and is almost a horizontal line compared to the calculated data, which runs from over 0.12 to almost 0.00. When looking at the left hand side, one can see that the negative trend line is a direct result from both the front and the rear rotor derivatives.

**Helicopter derivative $Z_w$**

The helicopter responds to the wind gust will be an upwards force with respect to the helicopter. When taking a look at the left graph in figure 6.7, it can be seen that the aerodynamic effect of the gust is small compared to the effect of the wind gust on the rotor blades. The effect on the fuselage upward derivative increases with increasing forward velocity.

The effect the gust has on the rotor blades is nearly constant, right until tip stall condition at 70 m/s and higher.
When comparing the $Z_w$ with the reference data, the right graph in figure 6.7, one can see a clear distinction between the lines. All graphs show a clear negative trend line, but the reference data trend lines are steeper, and their magnitude is smaller than that of the calculated data.

HELIКОТЕР DЕРИВАТIVE $M_w$

The effect that the wind gust has on the pitching moment is mostly dependent on the rotor forces in the $z$–direction. This can clearly be seen when comparing the left graphs in figure 6.5 and figure 6.8. The moments shown in the latter figure show the exact same shape as the forces in figure 6.5, with the exception of the front rotor moment, which is mirrored around the zero axis. The slightly larger magnitude of the front rotor is due to the larger arm the front rotor has with respect to the center of gravity.

The fuselage moment derivative has a small magnitude compared to the front and the rear rotor moment derivative. It does have a significant impact on the final shape of the derivative due to the opposite direction of the front and rear rotor derivatives.

![Figure 6.6: Change in force in the $x$–direction due to a wind gust from below](image)

![Figure 6.7: Change in force in the $z$–direction due to a wind gust from below](image)
When one compares the pitch moment derivative with the reference data, as is done in the right side of figure 6.8, one can see that there is a clear difference between the reference data and the calculated data. The reference data has a steep inclination at the beginning of the graph until a velocity of 20 m/s. From here on, the line flattens and continues between 0.05 and 0.06 1/m·s. In case of the calculated data, the trend line is almost a straight line coming up from 0.01 at hover till 0.05 1/m·s at 75 m/s.

6.4.3. CHINOOK RESPONSES DUE TO A PITCH RATE PERTURBATION
The pitch rate perturbation is a nose up rotational velocity perturbation around the y-axis, as is shown in equation 6.11.

\[ q' = q + \Delta q \]

(6.11)

This movement causes the front rotor to move up, which increases the inflow velocity of the blades and reduces the angle of attack experienced. For the rear rotor the opposite happens; the rear rotor moves down,
increasing the angle of attack. A secondary effect of this perturbation, is a reduced forward velocity for both the front and the rear rotor.

The effect of this perturbation on the forces and moments can be seen in figure 6.9. The front rotor gets a large positive derivative in the \( z \)-direction, while the rear rotor has a negative derivative in the \( z \)-direction.

The change in angle of attack will also have an impact on the \( x \)-direction force derivative. The front rotor thrust will decrease, giving a negative derivative in the \( x \)-direction. The rear rotor thrust will increase, giving a positive \( x \)-direction force derivative for the rear rotor. One could thus state, that the rear rotor force derivative will always be positive, while the front rotor derivatives sign is dependent on the contribution of the angle of attack with respect to the contribution of the reduced velocity.

When taking a look at figure 6.9, it clearly shows that the \( X_{wF} \) derivative has a negative value; showing that the reduced thrust of the front rotor is leading in the forward force derivative. Strangely enough the rear rotor derivative, \( X_{wR} \), starts slightly positive and becomes negative around 20 m/s. This happens despite the decreased forward velocity and the increased angle of attack; this would logically lead to a positive derivative.

When one takes a look at the moments that are created by the pitch rate perturbation, shown on the right side of figure 6.9; it is clearly seen that the moments around the \( y \)-axis for both the front and the rear rotor are negative. The negative derivative will cause stabilizing effect on a pitch rate perturbation.

HELICOPTER DERIVATIVE \( X_q \)
The left side of figure 6.10 shows the force derivative composition of the rotational velocity perturbation around the \( y \)-axis. As explained in the previous paragraph, the behavior of the \( X_q \) derivative is unexpected. The front rotor derivative could be explained by stating that the effect of reduced thrust outweighs the effect of the decreased forward velocity. At the rear rotor one would expect a positive derivative in both cases. If the thrust is increased, and the thrust vector faces forward, the derivative would be positive. If the forward velocity of the rotor is reduced, one could also assume that the force in the negative \( x \)-direction is reduced; which would also lead to a positive derivative.

When the calculated data is compared to the reference data, shown at the right side of figure 6.10, one can see that there is again a conflict in sign. Both the Davis and Ostroff curves show a positive value for all the calculated derivatives. Since it was deducted that the logical sign for the rear rotor would be positive, and this is not the case for the calculated data, one can conclude that there is an anomaly in the calculation of the rotor forces.

HELICOPTER DERIVATIVE \( Z_q \)
The change in angle of attack is what is leading the derivatives in \( z \)-direction on the left hand side of figure 6.11. The increased angle of attack of the rear rotor increases the lift, resulting in a negative derivative. The
opposite happens at the front rotor, where the reduced lift causes a positive derivative. The pitch rate perturbation has a larger effect on the front rotor than on the rear rotor, leading to a small positive derivative in the $z$–direction.

Since the aerodynamics module calculates the velocities at the center of gravity, the effect of a rotational perturbation has no effect on the fuselage aerodynamics.

The right hand side of figure 6.11 shows how the calculated data is comparable to the reference data. The majority of the reference points are on the negative side of the derivative line. A negative derivative means that the lift increases under a positive pitch rate perturbation. In case of the calculated data, all the derivative points are in the positive side, which results in a reduced lift.

**HELICOPTER DERIVATIVE $M_q$**

The introduction of a rotational velocity around the $y$–axis decreases the lift on the front rotor, while it increases at the rear rotor. Both effects create a negative moment around the $y$–axis.

The forces in the $x$–direction, that are introduced to the decreased forward velocity at the rotor hubs, will also cause a negative moment around the $y$–axis. The right side of figure 6.12 shows that the magnitude of the front rotor derivative is slightly larger than that of the rear rotor derivative, both are clearly negative.

When the values are compared to the reference derivative, shown on the right side of figure 6.12, it shows that the reference data has the same sign as the calculated data. The front of the reference graphs start of with a negative slope, while in the end it has a small inclination. The calculated data shows, in comparison, a more stable line; while having a large peak around 75 m/s.
6.5. LATERAL DERIVATIVES

This section discusses the responses of the helicopter due to perturbations in the lateral directions. First it will discuss the side gust of \( \Delta v \), followed by the rotational velocity perturbations \( \Delta p \) and \( \Delta r \). Each of these perturbations will have its own dedicated subsection which will discuss the \( Y \) force and the moments \( L \) and \( N \).

6.5.1. CHINOOK RESPONSES DUE TO A WIND GUST IN THE NEGATIVE \( y \)-DIRECTION

The first lateral perturbation that is discussed is a wind gust in the negative \( y \)-direction. It is a wind gust originating from the right side of the helicopter, moving towards the left side. This is simulated by an increase of the sideways velocity with \( \Delta v \), as described in equation 6.12.

\[
v' = v + \Delta v
\]

For regular configuration the tailplane would be influenced by the side wind primary by a yawing moment. This happens because of the fuselage drag and the aerodynamic center being behind the center of gravity and because of the influence of alternating angle of attack on the tail rotor. Depending on the tail rotor configuration this will increase or decrease the yawing moment.

For a tandem rotor helicopter like the Chinook, the moments created due to the fuselage drag will be significantly lower than standard single rotor helicopters. This is because it has no tail and it will have its aerodynamic center closer to the center of gravity than a conventional configured helicopter. Figure 6.13 shows the rotor forces on the right side and the rotor moments on the left side. As can be expected, the largest forces are created in the \( y \)-direction of the rotor blades resisting the increased sideways velocity.

The forces in \( z \)-direction, represented by the yellow line for the front rotor and the blue dotted line for the rear rotor, are roughly mirrored around 20 \( \frac{1}{s} \) line. For the forward rotor, an increase of force derivative in the \( z \)-direction (decrease of lift) is seen, until it crosses the neutral line at 45 \( \frac{m}{s} \).

The mirrored effect is due to the opposite rotation of the blades in combination with the rotor coning angle. The front rotor has counter clockwise rotating blades, when a side gust hits the rotor, the right, progressive side of the rotor disk has an increased angle of attack, while the retreating side has a reduced angle of attack. The effect is, that the increased angle of attack at low forward velocities increases the lift more on the progressive side than reducing it on the retreating side, see equation 6.14. This behavior on increased lift remains until stall conditions occur; where the increased angle of attack reduces the lift on the progressive side faster due to earlier stall.

When taking a closer look at figure 6.13, one can see that the exact opposite happens of what is expected, the front rotor creates less lift at low velocities and the rear rotor produces more lift.
6.5. **Lateral Derivatives**

When looking at the left hand side of figure 6.13, one can see that the forces in \( y \)-direction are creating a negative moment around the \( x \)-axis for both the front and the rear rotor. The opposite sign of the rotor forces in the \( z \)-direction creates a pitch down moment around the \( y \)-axis at low velocities, while velocities above 40 m/s create a pitch up responds.

The yawing moments, mainly created by the forces in \( y \)-direction, have the largest magnitudes, but since they work in opposite direction they have a small effective contribution.

**HELICOPTER DERIVATIVE \( Y_r \)**

The detailed \( Y_r \) derivative is plotted in the left side of figure 6.14. The graph shows, that the fuselage aerodynamics are a dominant factor in the sideways responds to a velocity perturbation from the side. The linear line is expected for the drag lines as is shown in equation 6.7.

When the calculated data is compared to that of the reference data, see right graph in figure 6.14, the lines closely match the reference data of Davis. Ostroff’s data points for velocities under 20 m/s are an exception. The difference is hard to explain, since the fuselage drag derivation seems to have a logical shape in the calculated data and the data from Davis.

**HELICOPTER DERIVATIVE \( L_v \)**

The detailed effect of the side gust perturbation on the roll rate is shown in the right side of figure 6.15. The effect of the side gust on the roll rate of the helicopter is, predominantly dependent on the rotors. The rolling moment by the rotors are created by two factors. The first factor is the drag force created by side gush in the negative \( y \)-direction. The second factor is the difference in lift distribution due to the increased angle of attack in the positive \( y \)-axis, increasing the lift on the right hand side of the helicopter, while reducing it on the left hand side. Both effects create a negative rolling moment around the \( y \)-axis.

Fuselage aerodynamics also create a rolling moment, as can be seen by the purple dotted line. The fuselage contribution to forces and moments has the expected linear shape.

The reference data, shown on the right of figure 6.15, show a roughly similar shape. Especially Ostroff and the calculated data have matching lines with a small inclination at the start, reaching its peak around 30 m/s, followed by an increased magnitude with increasing velocity.
HELICOPTER DERIVATIVE $N_y$

The detailed derivative $N_y$ is shown in 6.16. The right side shows that the yawing moment around the rear rotor axis is positive, while that of the front rotor axis is negative. The yawing moment created by the fuselage is at start linear, but seems to have reached its highest magnitude, just over 60 $m/s$ where the magnitude decreases slightly.

When comparing the derivatives with the reference data, done in the right graph in figure 6.16, one can see that there is a large difference between the reference values.

The Davis reference seems to have the same starting position as the calculated data, starting slightly negative around hover, but quickly reaches a positive value, while continuing in a linear path. The Ostroff reference starts with the lowest value at the beginning, but gets a large positive peak around 10 $m/s$, followed by an increasingly negative slope at a velocity between 20 and 30 $m/s$. Although the magnitude of the yawing moment derivative is very small, it is strange that the shapes of the graphs are very different.
6.5.2. CHINOOK RESPONSES DUE TO A ROLL RATE PERTURBATION

The rotational perturbation around the $x$–axis is treated in this section. The roll rate perturbation is simulated by introducing a rotational velocity, $\Delta \rho$, as described in equation 6.12.

$$\rho' = \rho + \Delta \rho$$ (6.15)

The aerodynamic effects on the fuselage body are none existent, as only translational velocities are used for the calculation of the fuselage aerodynamics. Since the velocities are calculated at the center of gravity of the helicopter, there is no change in translational velocity due to a change in roll rate.

The rotors experience two effects, primarily a rotational effect directly related by the introduced rotation, and a secondary translational velocity due to the distance of the rotors with respect to the center of gravity. The roll rate causes an increased angle of attack on the right side of the helicopter and a reduced angle of attack on the left side.

A secondary effect is a translational velocity moving from the right to the left. The effect of a translational velocity on the rotor blades will be a force moving from the right to the left on the helicopter. The rotation of the rotor around the $x$–axis increases the angle of attack on the right side of the rotor while reducing it on the left side.

The expected behavior of the translation component would thus be a negative derivative at both the front and the rear rotor in the $y$–direction. The effect of the rotation is very similar, though coming from a different direction, the increase in angle of attack on the right will increase the lift. When there is a positive coning angle, there will be a resultant force in the negative $y$–direction. When checking the forces in this direction, it becomes clear that both the front and the rear rotor derivatives are positive, which contradict the expected value.

For the forces in $z$–direction it can be assumed that an increase in angle of attack will have the most influence at the progressive side of the rotor. In case of the front rotor, the progressive side of the rotor has its angle of attack increased. Assuming that an increase in angle of attack has a linear effect on the increase in lift, the lift increase on the advancing side has a larger influence than the decreasing lift on the retreating side. This would thus mean that the front rotor would see an increase in lift, while the rear rotor has a reduction in lift.

When looking at the left hand side of the graph this is indeed what is happening. The negative force in $z$–direction indicates a reduction in downward force, thus an increase in lift.

The moments created by the perturbation are shown on the right side of figure 6.17. The rolling moment around the $x$–axis, presented by the blue line and dotted purple line, show a small arc that resemble the shape found at the $Y_{pF}$ and $Y_{pR}$. The small difference in shapes, is caused by the increased moment the front
rotor experiences due to the difference in $Z$ force between the left and right half of the rotors. The $Y$ forces also have a large impact on the yawing moment. The arm of the front rotor is positive and that of the rear rotor is negative, causing a positive and negative moment around the $z$–axis.

**HELICOPTER DERIVATIVE $V_p^\gamma$**

The detailed force derivative $V_p^\gamma$, is shown on the right side of figure 6.18. As can be expected on a rotational velocity perturbation, there are no changes in the aerodynamic fuselage forces, and also no changes in the gravity. The changes are thus solely depended on the force derivatives created by the rotor. As discussed in the general perturbation discussion, the derivatives found here are exactly opposite of what is expected. The introduced sideways velocity in combination with the rotation would logically create a force in the negative $y$–direction.

When looking at the right figure in 6.18, it is seen that the derivatives of the reference data does have the expected negative value for the side force derivative. When comparing the magnitudes of the reference data
with the calculated data, it can be seen that the magnitude of the Davis reference data is closely matched by the calculated data. The Ostroff reference data shows values approximately 40% higher than the calculated data. As in some earlier cases, the negative sign clearly seems to point to a problem in the rotor force calculations used in this program.

**HELICOPTER DERIVATIVE $L_p$**

The roll moment derivative, shown on the left side of figure 6.19, consists of two effects acting on the rotor. The first effect is the vertical component created by the translation of the role rate from the center of gravity to the rotor hubs, used to calculate the inflow velocity. The second effect is the actual roll rate. The roll rate perturbation creates an increased angle of attack on the right side, while it reduces the angle of attack on the left side. This would logically result in a negative moment around the $x$-axis, however figure 6.19 shows that this is not the case.

When the reference data is compared to the calculated data, on the right side of figure 6.19, it can be seen that the shapes are fairly similar, but the sign is wrong. Another striking difference is the magnitude. The magnitude of the calculated data starts at a value around 0.15 and reaches a maximum that is just under 0.4. In case of the reference data, the starting magnitude is above 0.65 $\frac{1}{\text{rad/}}$ and even goes above 0.8 $\frac{1}{\text{rad/}}$. It can thus be concluded that besides the difference in sign, there is also a difference in magnitude, this despite the matching shapes.
Since the derivative $L_p$ is a roll moment excited by a roll rate perturbation, its sign will have a large impact on the roll stability of the helicopter. The positive value of this derivative will have a negative impact on the stability of the helicopter. An excitation in the roll rate will result in a force that increases the moment in the rolling direction. This is thus an unstable responds, as a small excitation from trimmed state results in a larger excitation.

**HELICOPTER DERIVATIVE $N_p$**

The effect of roll rate perturbation on yaw is plotted in detail in figure 6.20. The yaw moment is created by two effects, one is the rotational drag created by the rotors spinning, the other effect is caused by the sideways forces of the front and rear rotor.

For the yawing derivative moment, $N_p$, it is shown that the aerodynamic forces on the helicopter fuselage does not have a noticeable effect on the roll moment derivative. The effect on the rotor blades is clearly visible in the plot. As can be predicted, the change in angle of attack is causing a force in the $y$–direction to be created at both rotor blades. This force is creating a moment around the $z$–axis, a positive moment at the front rotor and a negative moment at the rear rotor. As discussed at the previous paragraph, those forces are dependent on the forward velocity of the helicopter and thus the moments are not perfectly symmetrical. This can be seen by the resulting moment derivative, shown by the blue circled line. Where the front rotor has a higher derivative at the low and high velocities, while the rear rotor is leading between 38 and 65 m/s.
6.5.3. Chinook responses due to a yaw rate perturbation

The final helicopter perturbation treated, is a rotational perturbation around the \( z \)-axis. This yaw rate perturbation is simulated by introducing a rotational velocity, \( \Delta r \), as described in equation 6.16.

\[
r' = r + \Delta r
\]  

(6.16)

The yaw rate has two main effects on the rotor. The first one, is the rotation around the \( z \)-axis of the rotor. This clockwise motion will increase the rotational velocity of the front rotor and decrease it on the rear rotor. A secondary effect is the change in sideways velocity due to the offset of the rotor with respect to the position of the center of gravity. For the front rotor this will mean an additional negative sideways inflow velocity component, while the rear gets a positive sideways inflow velocity. As is the case with the previous rotation, there is no impact on the fuselage aerodynamics.

The increase of rotational velocity should directly effect the force in the \( z \)-direction. For the front rotor, one would expect a reduction in lift, while for the rear rotor one would expect an increase in lift. The effect of the sideways velocity will be an increase of lift in the rear sections of both the front and the rear rotor, while a reduction should be seen at the front. This would logically result in a pitch down moment.

Figure 6.21 shows that for both the front and the rear \( z \)-axis contribution, the force increases, due to a yaw rate perturbation. This effect reverts to an increase of lift between 40 and 45 \( m/s \). The prediction of opposite signs for the lift component is thus not visible in the graph. Although the rear rotor does have a higher lift curve than the front, it is not with the expected sign.

The right side of figure 6.21 shows the moment values. The additional pitch down moment is created by both the forces in \( z \)-direction and the local moments in the rotor hub. The effect of the pitch down moment on the rotor hubs can be seen by the offset the front and rear rotor have with respect to the zero line. The effective shape is mostly represented by the \( Z \) forces, shown in the left figure. The roll moment derivatives, all though similar in shape, are in opposite directions, counteracting each other. The yawing moments are, both for the front and the rear rotor, negative. The negative value indicates that the yaw rate perturbation is stable, as an excitation in this direction is countered by a moment in the opposite direction.

**Helicopter derivative \( Y_r \)**

The left side of figure 6.22 shows the force derivative in \( y \)-direction created by the yaw rate perturbation. The force created in the \( y \)-direction is mainly created by the offset of the rotor hubs with respect to the center of gravity. The front rotor will have an additional inflow velocity originating from the right, while the rear rotor has one coming from the left. Logically one would assume that the front rotor would thus have an increase in force in the negative \( y \)-direction, and the rear rotor will have a positive force derivative. The left side of the
The figure shows that this is indeed happening, the forces in the front and the rear rotor are opposite in direction. Since they have very similar magnitudes, but have slightly different shapes, the effective resultant derivative leads to a negative derivative at very low speeds and above 50 m/s, while it has a positive derivative between 5 and 50 m/s.

When the derivative is compared to the reference data, shown on the right in figure 6.23, one can see a distinct difference. The calculated data is averaging around the 0 line, while the Ostroff and Davis references have a clear negative value.

When analyzing figure 6.23, the effect of the height difference is clear. Although the force derivatives are very similar in size, the higher placed rear rotor creates a larger moment and is thus leading in the side force derivative.

**HELICOPTER DERIVATIVE \( L_r \)**

The roll moment derivative \( L_r \) shows a large resemblance with the side force derivative \( Y_r \). This is logical, since the sideways rotor force play a major roll in the moment around the center of gravity; as it is multiplied by the height of the rotor hubs with respect to the center of gravity. This also explains the offset between the front and the rear rotor derivative. The sideways forces are about equal, but the front rotor is located below the rear rotor and thus creates a smaller moment.
When comparing the calculated data with the reference data, on the right side of figure 6.23, there is a difference in sign. The Ostroff reference data shows a negative sign for all the calculated derivatives, while the Davis reference data only shows a positive derivative at a velocity above 80 $\text{m/s}$. The calculated reference data is positive and almost constant compared to the shapes of the reference curves.

**HELMICOPTER DERIVATIVE $N_r$**

Similar to the roll derivative, the yaw derivative is greatly dependent on the sideways forces. The front rotor is located approximately 6.4 meters in front of the center of gravity, and the rear rotor 5.5 meters behind it. The created moments can thus be expected to be in the same directions, with the front rotor having a slightly larger influence than the rear rotor. This theory coincides with the graph, shown in figure 6.24. The negative value of the derivative is an important stability derivative, as it shows that a disturbance in the yaw rate creates a moment in the opposite direction, stabilizing the helicopter.

The calculated data and the Ostroff reference data have a similar magnitude, as is shown on the right of figure 6.24. The curvature of the Ostroff data is a lot less noticeable than that of the calculated data. The magnitude
of the Davis reference, represents the shape of the calculated data closely, but also has a magnitude that is almost three times larger than the calculated data.

6.6. Helicopter derivatives with various underslung loads

In this section the helicopter derivatives are discussed with various underslung loads. For each main configuration mode, two setups are discussed and compared to the bare Chinook. This has been done to reduce the amount of raw data; for the interested reader appendix E shows all the major configurations graphs with all the sub configurations.

For the A series, configuration A1 and A4 are discussed, the B series is represented by B6 and B7 and for the C series, C1 and C5 are highlighted. In all graphs the 0 configuration represents the bare Chinook. The configurations are defined in tables 3.1 till 3.3.

The single load configurations are both suspended from the central hook and have a mass of 1000kg and 4000kg.

The double load configurations are suspended from the front hook and the rear hook. In case of the B6 configuration, the front hook carries a mass of 3600kg and the rear hook 1000kg. The B7 configuration has its loads switched, giving the front hook a mass of 1000 kg while the rear hook has a mass of 3600 kg.

In case of the triple underslung load configurations, every hook carries a mass of 1000kg with the exception of the C5 configuration; in which the central hook carries a mass of 2200kg.

The derivative routine works by giving a perturbation to the helicopter states, one state at the time. The underslung loads do not receive this perturbation. Since the cable has a very low damping, the effect the load has during changes in velocity will be very hard to observe and are most of the times negligible compared to other influences. The angular perturbations will have a significantly larger effect than the angular velocity perturbations, mainly because the value of the spring constant is large in comparison to the damping value. This leads to the expectation that the effects the underslung load will have on the derivative will most likely come from the effect the load has on the helicopter trim condition.
6.6.1. **LONGITUDINAL LOAD DERIVATIVES X**

The effect of the underslung load on the derivatives in the $x$-direction, is mostly caused by the change in the trim conditions. It is important to note that the effect of both translational and rotational velocity perturbation generate a much smaller change of force in the underslung cable in comparison to the positional change that occurs due to the angular perturbations. This is due to the fact that the dampening constant of the cable is very small compared to the spring constant of the cable. The direct effect, the effect caused by cable interaction between helicopter and load, of the underslung load is negligible.

![Figure 6.25: Comparison of the single load configurations A1 and A4 compared to the bare Chinook A0](image)

Figure 6.25 shows the effect of the single underslung load. The mass of the $A_1$ configuration load is 1000kg and that of the $A_4$ configured load is 4000kg. The effect on the top left graph, showing the $X_u$ derivative, shows that the load increases the resistance to movement in the forward direction.

The top right graph shows the $X_w$ derivative, where the effect of the load is advancing the curve of the bare helicopter. This can clearly be seen by the first bump of the bare Chinook at 50 m/s which can be seen on the $A_4$ configuration just over 40 m/s.

The $X_q$ derivative, shown on the bottom left, also indicates that the additional load leads to steeper derivative slopes.

The graph on the bottom right shows the direct effect the underslung load has on the helicopter, as the bare Chinook derivative is zero. It also shows that, although angular perturbations have the biggest direct influence, the direct influence of the load on the derivative is extremely small, $4 \cdot 10^{-8} \, \text{m rad}^{-1}$. The rough pattern is caused by the programs sensitivity. With the sensitivity of a correct value set at $10^{-5}$ (enough to create smooths lines at every other graph) it is not sensitive enough to create smooth lines for the small forces created by rotational perturbations.
Figure 6.26: Comparison of the single load configurations B6 and B7 compared to the bare Chinook B0

Figure 6.26 shows the effect of the location of the underslung loads. Configurations B6 and B7 both carry a 1000kg and 3600kg load. The B6 has the heavy load on the front while the B7 has it at the rear hook. All the derivatives in this figure seem about similar with the exception of the $X_q$ derivative, where the B6 configuration has a slightly higher value than the B7 derivative.

Figure 6.27: Comparison of the single load configurations C1 and C5 compared to the bare Chinook C0

The figure depicted by the triple load configurations, shown in figure 6.27, barely differs from the figures shown by the single load derivatives in figure 6.25. This is not so strange when one concludes that the front and rear hook roughly counteract the longitudinal moments.
6.6.2. Longitudinal load derivatives $Z$

In figure 6.28 the effect of the load on the ‘$Z$’ derivatives are shown. The effect the load has on a frontal wind gust seems negligible. The load moves the interesting line characteristics to a lower velocity than the bare Chinook. This effect is especially clear at the top right graph, $Z_w$. Without load, the steep inclination occurs at 73 m/s, while for the A1 and A4 configurations this is respectively 68 m/s and 58 m/s.

The bottom right graph shows the effect the underslung load has on the downward facing force. In contradiction to the very ‘noisy’ line shown in figure 6.25, this curve is a lot smoother and seems to be less of a trim variation than the force in $x$–direction. The force derivatives in the $z$–direction, with respect to the pitch angle $\theta$, is most likely caused by the combination of the difference between load and helicopter trim attitude and the cable force. This explains why the A4 derivative is not a factor four bigger than the A1 derivative.

The top right figure is an interesting figure, showing the responds of the $Z$-derivatives to a downward velocity perturbation.

In this figure the red dotted line shows the front loaded Chinook, and the yellow dot-dashed line the rear loaded Chinook. An obvious difference is the gap between the 'bumps'. The front loaded red line clearly shows that the front rotor is showing the first sign of tip stall at: 50 m/s for the front rotor, and at 65 m/s for the rear rotor. The rear loaded configuration has these effects around 60 m/s for both the front and the rear rotor.

![Figure 6.28: Comparison of the single load configurations A1 and A4 compared to the bare Chinook A0](image)

Figure 6.29 shows graphs on how the positioning of the load effects the helicopter. For a better understanding one can look at the trimmed states shown in appendix C, figure C.6 and C.8. In these figures can be seen, that the helicopter trimmed states of both loads are almost identical; a very minute difference in pitch angle can be found, with the corresponding changes in $u$ and $w$.

The largest difference in trim control can be seen in the longitudinal cyclic pitch required for stable flight. There are some corrections in the lateral stick and pedal, while the collective is again nearly identical between both configurations.

It is thus clear that the effect the perturbation has on the double underslung loads are entirely caused by their influence on the front and the rear rotor and not on the load. One can see that the front loaded B6 configuration creates a higher strain on the front rotor, pushing this rotor faster to its operational limits.
In the $Z_w$ derivative graph, where the added downward velocity increases the angle of attack on both rotor blades, the performance reduction between the front and rear rotor can clearly be detected. The front rotor of the B6 configuration stalls at a lower velocity, while the rear rotor stalls at a higher velocity. This can also be seen on the bottom left $Z_q$ derivative of figure 6.30. The moment around the $y$–axis increases the angle of attack on the rear rotor while decreasing it on the front rotor. Since the front rotor is operating closer to its limit in the B6 configuration, the stalling behavior is stretched over a broader range of velocities.

Figure 6.29: Comparison of the single load configurations B6 and B7 compared to the bare Chinook B0

Figure 6.30: Comparison of the single load configurations C1 and C5 compared to the bare Chinook C0
As is the case with the previous figures with three underslung loads; figure 6.30 values do not differ that much compared to the single underslung load configuration. The main effect is the change in trim values caused by the added mass.

### 6.6.3. **Longitudinal Load Derivatives M**

Figure 6.31 shows the effect that the perturbations have on the moment around the $y$–axis. Again the graphs show that the added load causes the anomalies to occur at lower velocities than without load. The most interesting graph is the bottom right graph, showing the effect of a pitch angle perturbation. The lines show that a nose up perturbation was neutrally stable in case of the bare Chinook. When the loads are added, they have a stabilizing effect on the helicopters pitch angle perturbation. One can state that the Chinook combination is pitch stable due to the negative sign of both the $M_q$ and the $M_\theta$ perturbation, creating a moment in the opposite direction.

![Figure 6.31: Comparison of the single load configurations A1 and A4 compared to the bare Chinook A0](image)

As with the previous graphs, the double configuration shows the most exciting graph. If one is looking at the bottom right graph of figure 6.32 one can see that the B6 configuration shows a positive, destabilizing moment to a perturbation, while the B7 has a stabilizing negative moment. The magnitude of the heavy load aft configuration is lower due to the smaller distance between the rear hook to the CG than that of the front hook to the CG.

Figure 6.33 holds little surprises, due to the added weight, the interesting points of the curves occur at a lower velocity. As with the previous C configurations, they are not that different compared to the single load configuration shown in figure 6.31.
6. DERIVATIVE ANALYSIS

6.6.4. LATERAL LOAD DERIVATIVES Y

Figure 6.34 shows the first of the lateral derivatives. As discussed earlier the effect that the load has on a rotational velocity perturbation is minimal due to the effect of the very low cable dampening value. The major influence of the load derivatives are mainly caused by the effect the load has on the trim conditions.
Figure 6.34: Comparison of the single load configurations A1 and A4 compared to the bare Chinook A0

Figure 6.35: Comparison of the single load configurations B6 and B7 compared to the bare Chinook B0

Figure 6.35 shows the double configuration load. The placement of the load seems to have no effect on the $Y_v$, $Y_p$ and $Y_\phi$ derivative. As expected, only the derivative roll rate perturbation is influenced by the placement of the heavy load, giving a higher value to the B7 configuration, and a lower value for the B6.
When comparing figure 6.34 till 6.36, one can conclude that the addition of the mass is the most dominant factor in the shape of the derivative graphs of the force in $y$–direction. The positioning of the load only has a distinct contribution in the yaw rate perturbation.
6.6.5. **LATERAL LOAD DERIVATIVES \( L \)**

The moments derivatives around the \( x \)-axis are shown in figure 6.37 till 6.39.

![Comparison of the single load configurations A1 and A4 compared to the bare Chinook A0](image)

Figure 6.37 shows the effect that the mass of the underslung load has on the helicopter. The top left clearly shows that the magnitude of the derivative increases due to a side gush. The higher the added weight, the larger the change in magnitude.

The top right shows the responds to a roll rate perturbation. The load initially make the helicopter more unstable, while at higher velocity it decreases the instability. For the 1000kg load this point is reached just under 40 m/s, while the 4000kg reaches this point at 70 m/s.

The roll moment derivative, created by a yaw perturbation, is plotted in the bottom left graph of figure 6.37. Similar to the \( L_v \) derivative, the \( L_r \) derivative increases the magnitude of the curves when the underslung load is added. In this graph, the effect that the mass has on the helicopter performance can be seen at the higher velocities. The heaviest \( A_4 \) configuration reaches a significantly larger magnitude when compared to the lighter \( A_1 \) configuration and the bare Chinook.

The bottom right graph represents the rolling moment due to a bank angle perturbation. The value shows a negative derivative, this is as expected. A positive perturbation generates a negative moment around the \( x \)-axis. Since the rotation to the right would move the helicopter hook slightly to the left in the global reference system, the force that cables exert on the helicopter will automatically generate a negative moment around the \( x \)-axis. The mass is an important parameter in the shape of this figure. The heavier \( A_4 \) configuration clearly shows that the stabilizing effect of the load decreases at higher velocities.
Figure 6.38: Comparison of the single load configurations B6 and B7 compared to the bare Chinook B0

Figure 6.38 shows rolling moment derivatives for the double load configuration graphs. The first two graphs show that the position of the load has a minor influence in both the side wind gust and roll rate perturbation. The bottom left graph shows that during a yaw rate perturbation the heavy load on the rear hook, generates a significant larger moment compared to the heavy load at the front hook. This is caused because the rear hook is located 1.8m behind the $z$–axis, while the front hook is 2.3m in front of the $z$–axis. The difference in arm in combination with the difference in mass is causing the large difference in curves.
6.6. HELICOPTER DERIVATIVES WITH VARIOUS UNDERSLUNG LOADS

Figure 6.39: Comparison of the single load configurations C1 and C5 compared to the bare Chinook C0

Figure 6.39 shows the triple load configuration moments around the $x$-axis. As was the case with the other underslung load derivatives, the difference between single and triple configurations are negligible.

6.6.6. LATERAL LOAD DERIVATIVES $N$

The yaw moment derivatives are plotted in figure 6.40 till 6.42.

Figure 6.40: Comparison of the single load configurations A1 and A4 compared to the bare Chinook A0

The single underslung load configuration, plotted in figure 6.40, shows the expected behavior for three of the graphs; an increase in magnitude. The $N_\phi$ derivative directly gives a more interesting graph. The bare
Chinook has a zero derivative under a bank angle perturbation. The added loads generate an offset in this graph; the roll angle perturbation moves the helicopter hooks in negative $y$-direction, generating a spring force on the hooks in the positive $y$-direction. Since the central hook has a position slightly forward of the helicopters CG, it generates a positive moment. This is seen in the bottom right graph in figure 6.40.

The $N_\phi$ derivative, shown in the bottom right of figure 6.41, shows the effect that the position of the mass has on the yawing moment during a bank angle perturbation. As one would expect, the heavy front load results in a positive derivative and a negative moment for the B7 configuration. This is indeed what is happening at low velocities. At higher velocities this behavior changes, the B6 configuration obtains a negative derivative, while the B7 value becomes positive.
Figure 6.42: Comparison of the single load configurations C1 and C5 compared to the bare Chinook C0.

Figure 6.42 shows the yaw moment responds due to lateral perturbations. As one can see, the effect of the heavier loads has very little influence on all of the lines. The heavier load seems to have a slightly larger deviation from the bare Chinook as can be expected.
In this chapter the eigenvalues of the Chinook with underslung loads are discussed. The first step in calculating the eigenvalues is the construction of the state matrix $A$, as shown in equation 7.1.

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (7.1)

To make it easier to identify the eigenvalues, the linearized state matrix $A$ is split into two parts: the longitudinal and the lateral eigenvalues. In this case the assumption is made that there is no interaction between the longitudinal and lateral motions of the helicopter. In which the longitudinal state matrix, $A_{\text{long}}$, is defined in equation 7.2; and the lateral state matrix, $A_{\text{lat}}$, is defined in equation 7.3. As can be seen from these equations, the gravity component is added to the equation, as well as the components created by the cross product of rotational and translational velocities.

$$A_{\text{long}} = \begin{bmatrix}
X_u & X_w & X_q - W_0 & -g \cos \theta_0 \\
Z_u & Z_w & Z_q + U_0 & -g \sin \theta_0 \\
M_u & M_w & M_q & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \hspace{1cm} (7.2)$$

$$A_{\text{lat}} = \begin{bmatrix}
Y_v & Y_p + W_0 & Y_r - U_0 & g \cos \theta_0 \\
L_v & L_p & L_r & 0 \\
N_v & N_p & N_r & 0 \\
0 & 1 & \tan \theta_0 & 0
\end{bmatrix} \hspace{1cm} (7.3)$$

The forces and moments are written in semi-normalized form by dividing the forces by the mass of the helicopter and the moments by their respective mass moment of inertia, as shown in equation 7.4 till 7.9.

$$X_{u,w,q} = \frac{\Delta F_x}{\Delta u, \Delta w, \Delta q} \frac{1}{m_{\text{heli}}} \hspace{1cm} (7.4)$$

$$Z_{u,w,q} = \frac{\Delta F_z}{\Delta u, \Delta w, \Delta q} \frac{1}{m_{\text{heli}}} \hspace{1cm} (7.5)$$

$$M_{u,w,q} = \frac{\Delta M_y}{\Delta u, \Delta w, \Delta q} \frac{1}{I_{yy}} \hspace{1cm} (7.6)$$

$$Y_{v,p,r} = \frac{\Delta F_y}{\Delta v, \Delta p, \Delta r} \frac{1}{m_{\text{heli}}} \hspace{1cm} (7.7)$$

$$L_{v,p,r} = \frac{\Delta M_z}{\Delta u, \Delta p, \Delta r} \frac{1}{I_{xx}} \hspace{1cm} (7.8)$$

$$N_{v,p,r} = \frac{\Delta M_z}{\Delta v, \Delta p, \Delta r} \frac{1}{I_{zz}} \hspace{1cm} (7.9)$$

The eigenvalues of the helicopter are calculated by equation 7.10, where the values of lambda represent all the solutions for which the determinants of the diagonal lambda matrix, subtracted by the state matrix, are equal to zero.

$$|\lambda I - A| = 0$$  \hspace{1cm} (7.10)
The eigenvalues of the helicopter give a representation on the stability of the helicopter. The real part of the
eigenvalues represents the damping of a motion. A negative damping means that a certain motion is
converging to an equilibrium, a positive damping means it is diverging and thus an unstable motion.

The imaginary part shows the frequency of the eigenmotions. Not all the eigenvalues have an imaginary
part. This means that the motion is non-oscillatory. If the motions have an imaginary eigenvalue, than it has
an equal, but opposite in sign, counter part; with the exact same real part eigenvalue.

Since not all eigenvalues have an imaginary part, the plots presented here are split up in the real eigen-
values and imaginary eigenvalues as follows: at the top left, the real eigenvalue versus the velocity; at the
top right, the imaginary eigenvalue versus the velocity; at the bottom left, the imaginary eigenvalue versus
the real eigenvalue; and at the bottom right a zoomed in picture of the imaginary eigenvalue versus the real
eigenvalue.

In the plots that show the imaginary versus the real eigenvalue, the begin and endpoints are given. At
the velocity \( V=0 \text{ m/s} \) the eigenvalue is marked with a ‘+’; and at the velocity \( V=80 \text{ m/s} \) it is marked with a ‘•’.

### 7.1. Eigenvalues of the Single Load Configuration

In figure 7.1 and 7.2 the eigenvalues are plotted for the single load configurations. The A0 configuration shows
the bare Chinook; the A1 configuration has a load with a mass of 1000kg while the A4 configuration has load
with a mass of 4000kg. In both cases the load has been attached to the central hook.

![Figure 7.1: Longitudinal eigenvalues of the single underslung load configurations](image)

In the top left of figure 7.1 one can see the real part of the eigenvalue plotted versus the velocity. The top right
of the figure shows the imaginary values of the longitudinal eigenmotions.

The top left shows that there is one set of values with a positive eigenvalue. A positive eigenvalue shows
that there is an eigenmotion that, if excited, increases in magnitude.

The top right shows a graph with only zero values. The imaginary part of the eigenvalues are an indica-
tion of the frequency responds. Since this value is zero in all cases, one can state that the behavior of the
longitudinal eigenmotions are non-oscillatory.
Figure 7.2: Lateral eigenvalues of the single underslung load configurations

Figure 7.2 shows the responses for the lateral motions. The top left graph is hard to analyze since the lines are plotted quite close to each other. It consists of four lines, of which three are situated at the proximity of the zero line. Two lines have a relatively small variation, with values ranging from 0 till $2.5 \cdot 10^{-3}$ s, which can be seen on the bottom right graph. The other line resides at zero. The bottom line represents a damped line ranging from -0.02 till -0.2 s.

The top right graph shows the imaginary eigenvalues. From 0 to 30 m/s there is a small reduction in frequency with an increase in velocity. From 30 m/s the oscillatory frequency becomes stable. The other two lines possess no imaginary eigenvalues, and are thus presented by a 0 line.

In the bottom left there are three distinct patterns visible. The first is a non-oscillatory eigenmotion, that experiences increased damping with increasing velocity. The second pattern presents a low frequency oscillatory motion, shown by the almost vertical line near the zero axis. The last line is a zero point, a motion that possesses damping nor oscillatory behavior.

The bottom right shows a zoomed in view of the imaginary eigenvalues. This reveals that the motion is lightly unstable at low velocities and becomes less unstable with increasing velocity. It also shows that the damping decreases with an increase in velocity.

7.2. Eigenvalues of the Double Load Configuration

For the double load configurations the B6 and B7 configurations are highlighted. The B6 configurations represents a mass of 3600kg suspended from the front hook and 1000kg from the rear. The B7 configuration has the 1000kg suspended from the front and 3600kg from the rear. The total mass of the underslung loads is thus identical and the difference between the two graphs are caused by the location of the suspended loads.

In Figure 7.3 shows the longitudinal eigenvalues. At the top left, one can see a distinction between four lines. Since there are also four eigenvalues one can automatically conclude that there are no imaginary components. The top line presents a positive value, which thus means that there is one value that shows unstable behavior. The other lines all possess a negative real part and are thus stable.

The frequency responds, shown on the top right, shows that the 'bare Chinook' (B0), and the B7 configuration have no imaginary eigenvalues and are non-oscillatory. The B6 configuration has, at high velocities, some imaginary eigenvalues. These are most likely caused by the end of the flight envelope of the front loaded configuration.
The bottom left plot shows the damping versus the oscillation. Since most of the imaginary eigenvalues are zero they are represented by flat lines. The exception can be found at the high velocity B6 configuration. For the directions of the line, the top left graph is more valuable and accurate.

The lateral eigenvalues are plotted in figure 7.4. One line shows a motion with increased damping with increasing velocity. Using the other graphs, one can conclude from the top right graph together with the bottom right graph that two lines are identical and run from approximately -0.02 till -0.2. The other line is a line that posses damping nor oscillations, which can be deduced from the combination of the graphs.

In comparison with the other graphs there is relatively little difference in the eigenvalues. The addition of a load increases the frequency of the oscillation. The damping value increases for the most noticeable -0.02 till -0.2 line.

The effect of the location of the hooks on the lateral stability is barely noticeable. The different lines are perfectly overlapping each other, with the exception of the bottom right graph. In this graph, one can conclude that the B7 configuration makes the movement slightly less unstable than the B6 configuration.
7.3. Eigenvalues of the triple load configuration

The triple load configurations, shown in figures 7.5 and 7.6, consists of the C1 and C5 configurations and the bare Chinook C0. The C1 configuration consists of three loads of 1000kg each, evenly distributed amongst the helicopter hooks. The C5 configuration is almost identical with the exception of the central hook load, which has a mass of 2200kg instead of 1000kg.

The effect of the underslung load in the top left graph of figure 7.5 is a decrease in damping at higher velocities.

The top right graph shows that the frequency line is equal to zero, indicating that there is no oscillatory behavior in the uncoupled lateral responds of the Chinook.

As a result, the plotted bottom left graph only shows the zero values in the frequency responds. The damping is best read from the top left figure, giving a better indication of how the damping progresses with velocity.

Figure 7.5 shows the lateral eigenvalues. In the top left figure, one can see that there is very little difference between the different configurations at velocities until 50 m/s. Here one can see that the magnitude of all the eigenmotions decrease with the addition of the underslung load.

The eigenvalues plotted in the top right of the figure show that the added mass increases the frequency of the oscillations; the heavier the mass, the larger the frequency with respect to the bare Chinook.

The bottom right graph shows that an addition in underslung mass results in a slight reduction of the amplification, while it increases the frequency of the oscillations.
Figure 7.5: Longitudinal eigenvalues of the triple underslung load configurations

Figure 7.6: Lateral eigenvalues of the triple underslung load configurations
The initial Chinook model, used in the TUDelft reports, is validated by comparing the trim states of the helicopter. The trimmed states show a remarkable resemblance with the reference data, but pilot input shows two values with deviating data.

The longitudinal cyclic pitch data in all the TUDelft reports is mirrored around the zero axis, when compared to the reference data. This is most likely caused by a different sign convention for longitudinal stick deflection between the reference data and the used model.

The other deviation, is the lateral cyclic pitch. Compared to the reference data it has a very similar shape, but has approximately half the magnitude. Since the lateral cyclic pitch values are close to neutral, there was no special attention payed to this deviation.

As can be seen in the results of chapter 6, there are some derivatives that show resemblance with the reference data. The majority of the derivatives have either a different shape, a different sign, or both.

The most likely cause of these results, is the calculation of the rotor forces. The derivatives have some counter intuitive data, followed by the intuitive data of the references; leading to believe that the reference data results are valid. When looking at the segmented graphs, one can see that the rotor derivatives have a dominant contribution to the final shape of the derivative.

One of the most striking examples of counter intuitive data, is the roll derivative of the helicopter, \( L_p \). With the location of the rotors, and the effect of rotor coning angles, a roll unstable helicopter would be very unlikely. This particular issue can be caused by a reversed coning angle or a reversed handling of the coning angle. This would result in correct trim results, but would also cause a change in the stability. Another option for the error is the reversion of the roll rotational velocity at the blades. Since the derivatives caused by a roll rate perturbation results in similar shapes and magnitudes, but are opposite in sign compared to the reference data, a simple sign conflict could result in the obtained results.

Another strange derivative is the \( Z_u \) derivative; in general an increase in forward velocity will result in an increase in lift. In the results of this report, an increase in velocity results in a positive derivative curve, indicating a reduction in lift.

The most dominant effect that the loads have on the derivative, is the influence they have on the trim condition. The direct effect that the load has on the derivative is solely dependent on the sling parameters: length, damping, and spring constant; the mass and shape of the load have thus no influence. The velocity perturbation, both rotational and translational have little effect on the load due to the small value of the cable damping. The spring constant is slightly higher, but still yields a relative small contribution to the derivatives.

The derivatives are the key values needed for the calculation of the eigenmotions, because of this the results of the eigenmotions need to be treated with caution. Effects as phugoid, known to exist for at least the low velocity range of the Chinook, do not occur in the results.

One of the ways used to calculate the damping and frequency of a helicopter, is by using approximations as found in Padfield [6]. These equations are in general simplified versions of finding the eigenvalues of the full linearized equation of motion. Sadly enough these simplifications used to derive these equations are not
valid with the obtained results.

The best example of this is the phugoid. The phugoid is normally the interaction between the pitch angle, pitch rate and forward velocity. In case of the simplification of the phugoid the value of $X_q$ is in general considered negligible. In case of this Chinook model this 'negligible' value is the largest value by a factor of three.

When the simplification is omitted, but the method stated in Padfield [6] is sustained, the results yield three real eigenvalues with no imaginary part, close to the obtained results found in the report.

Summarizing, one can conclude that numeric linearisation is an effective model to find the derivative of single body entity. When there are multiple bodies, numerical linearisation is less effective. The interaction of the helicopter and its underslung load is negligible in the derivatives. The most dominant effect that is seen from the underslung load is the effect the load has on the trim conditions.

In general, an added load reduces the magnitude of the damping value for the longitudinal motions, while it increases the magnitude lateral damping. The lateral frequency is increased by the added load.

In case of the dual suspended loads, the heavy load in the front has a slightly less unstable responds and is thus the favorable configuration.


Figure A.1: Program flowchart
This appendix is written in order to explain in detail how the trim conditions were found. First a discussion is made on what trim is and how it is reached. This is followed by the primary and secondary trim routine of the underslung load, and finally the helicopter trim.

B.1. Introduction

The trimming of the helicopter can be defined as a minimization problem. The derivative of any state should be zero with exception of $\dot{x}$; which should be equal to the desired trim velocity. Since this problem will be solved with the aid of an optimization routine, a zero value would most likely be unobtainable. The optimization routine finds a value close enough to zero so trim conditions can be assumed.

The helicopter and underslung load trim routine are split into two different sections: the helicopter and underslung load. This reduces the amount of optimization values per problem, which increases the calculation speed and reducing the amount of diverging solutions.

The load interacts with the helicopter through forces acting on the cable. Because the hook velocity is equal to the trim velocity, and the hook position can be relocated; it is most convenient to trim the load first, followed by the helicopter.

B.2. Trimming the Underslung Load

The trimming of the underslung load is performed by assuming the central part of the sling system\(^1\) is at position \(0, 0, 0\) and traveling at a velocity of \(\langle \dot{V}_{\text{trim}}, 0, 0 \rangle\).

With the initial conditions defined, the next step is finding the position, velocity and angles of the load at which trim can be assumed.

During the trimming of the helicopter it was found that the initial position of the load was extremely important in finding a solution. At certain starting conditions the solving algorithm would diverge. Some measures were installed that tweaked the program while running. Eventually the algorithm found a solution in 90% of the cases.

The 10% of the cases where the solution did not converge where when the starting position was too far from the trimmed solution. This problem was overcome by introducing a build an optimization routine which is build in Matlab: ’fmincon’. The ’fmincon’ has several algorithms it is able to utilize, it was found that the linesearch algorithm holds the best results. Ironically, this algorithm is also based on Newton-Raphson optimization.

The secondary trim routine is very time consuming and not very accurate. However, it is more effective at finding solutions, even with less favorable starting conditions. This is the main reason that the secondary trim routine is only used when the primary trim routine fails to find a solution.

\(^1\)The central part of the sling is the part of the sling at which all five cables are connected; four cables are connected to the load, and one is connected to the helicopter.
B.2.1. Main Load Trim Routine

Before an attempt is made to trim the underslung load, one needs to identify what trim is mathematically. Trim in case of the underslung load is defined as a state when there is no change in translational velocities or change in rotation and rotational velocity. Which thus can be defined as all translational accelerations, rotational velocities and rotational accelerations are equal to zero with an accuracy of $\pm 10^{-10}$.

\[ \dot{X}_{t+1} = X_t - \tau_{cor,f}J^{-1} \mathbf{f}(X_t) \]  

In equation B.1 $X_t$ denotes the state vector. Note that this is not exactly equal to the load states, as the terms $p$, $q$ and $r$ are missing. This is because they are proportional to the respective derivative calculation and is also exempt of 3 values; the values of $\phi$, $\dot{\theta}$ and $\psi$. This is because they are set to 0 for trim condition. The term $J$ denotes the Jacobian and is obtained by numerical differentiation, one column at the time. It uses the additional term $\tau$ which is introduced to influence the step size to the new state. Smaller values increase trim success rate, but also increase the required calculation time. If the method fails to find a solution within a certain amount of iterations the step size is reduced, and the program restarts at the, nearest to trim, starting conditions.

The primary trim routine utilized the Newton-Raphson optimization, which is an iterative process which updates the state with the aid of equation B.1. The additional term $\tau_{cor}$ is introduced to influence the step size to the new state. Smaller values increase trim success rate, but also increase the required calculation time. If the method fails to find a solution within a certain amount of iterations the step size is reduced, and the program restarts at the, nearest to trim, starting conditions.

B.2.2. Primary Load Trim Routine

The primary trim routine utilized the Newton-Raphson optimization, which is an iterative process which updates the state with the aid of equation B.1. The additional term $\tau_{cor}$ is introduced to influence the step size to the new state. Smaller values increase trim success rate, but also increase the required calculation time. If the method fails to find a solution within a certain amount of iterations the step size is reduced, and the program restarts at the, nearest to trim, starting conditions.

Before an attempt is made to trim the underslung load, one needs to identify what trim is mathematically. Trim in case of the underslung load is defined as a state when there is no change in translational velocities or change in rotation and rotational velocity. Which thus can be defined as all translational accelerations, rotational velocities and rotational accelerations are equal to zero with an accuracy of $\pm 10^{-10}$.

\[ \dot{X}_{t+1} = X_t - \tau_{cor,f}J^{-1} \mathbf{f}(X_t) \]  

In equation B.1 $X_t$ denotes the state vector. Note that this is not exactly equal to the load states, as the terms $p$, $q$ and $r$ are missing. This is because they are proportional to the respective derivative calculation and is also exempt of 3 values; the values of $\phi$, $\dot{\theta}$ and $\psi$. This is because they are proportional to the values of $p$, $q$ and $r$ and they are all zero.

$J$ represents the Jacobian and is obtained by numerical differentiation, one column at the time. It uses the main states $X_t$ and gives a perturbation to one of the state values. This process is repeated until all state values had a perturbation and the square matrix is generated.

The $\dot{x}$ velocity is subtracted by the desired trim velocity. During simulations it was found that setting $\nu$, $\phi$ and $\psi$ to zero greatly improved the calculation time and reduced the amount of failures. Their respective derivatives are still used for the error function to ensure no faulty trim conditions.

\[
\begin{bmatrix}
u_{t+1} \\
v_{t+1} \\
w_{t+1} \\
\phi_{t+1} \\
\theta_{t+1} \\
\psi_{t+1} \\
x_{t+1} \\
y_{t+1} \\
z_{t+1}
\end{bmatrix} =
\begin{bmatrix}
\dot{u}_t \\
\dot{v}_t \\
\dot{w}_t \\
\dot{\phi}_t \\
\dot{\theta}_t \\
\dot{\psi}_t \\
\dot{x}_t \\
\dot{y}_t \\
\dot{z}_t
\end{bmatrix}
- \tau_{cor,f}
\begin{bmatrix}
\frac{\dot{u}(u+\Delta u) - \dot{u}(u)}{\Delta u} & \cdots & \frac{\dot{u}(z+\Delta z) - \dot{u}(z)}{\Delta z} \\
\frac{\dot{v}(u+\Delta u) - \dot{v}(u)}{\Delta u} & \cdots & \frac{\dot{v}(z+\Delta z) - \dot{v}(z)}{\Delta z} \\
\frac{\dot{w}(u+\Delta u) - \dot{w}(u)}{\Delta u} & \cdots & \frac{\dot{w}(z+\Delta z) - \dot{w}(z)}{\Delta z}
\end{bmatrix}^{-1}
\begin{bmatrix}
\ddot{u}_t \\
\ddot{v}_t \\
\ddot{w}_t \\
\ddot{\phi}_t \\
\ddot{\theta}_t \\
\ddot{\psi}_t \\
\ddot{x}_t - V_{trim} \\
\ddot{y}_t \\
\ddot{z}_t
\end{bmatrix}
\]  

B.2.3. Secondary Load Trim Routine

The definition of trimmed flight is defined differently for the primary trim routine than for the secondary trim routine. This is done because this trim routine is less accurate and will fail to find a solution if the maximal derivative deviation is smaller than $10^{-5}$. The trimmed state definition for the secondary routine is stated in equation B.3. As one can see in this equation; the values for the attitude derivatives, $\phi$, $\dot{\theta}$ and $\psi$ are neglected. This is because they are directly related to the rotational velocities which are in a trimmed state per definition zero.

\[
\begin{bmatrix}
-0.0001 \\
-0.0001 \\
-0.0001 \\
-0.00005 \\
-0.00005 \\
-0.00005 \\
0.00001 \\
0.00001 \\
0.00001 \\
0.00005 \\
0.00005 \\
0.00005 \\
0.00005 \\
0.00005
\end{bmatrix}
\leq
\begin{bmatrix}
\dot{u}_{load} \\
\dot{v}_{load} \\
\dot{w}_{load} \\
\dot{p}_{load} \\
\dot{q}_{load} \\
\dot{r}_{load}
\end{bmatrix}
\leq
\begin{bmatrix}
0.0001 \\
0.0001 \\
0.0001 \\
0.00005 \\
0.00005 \\
0.00005
\end{bmatrix}
\]  

The global velocity of the helicopter hook and the load must be identical for trim condition. When symmetric flight is assumed, it leaves three variables that can be altered to find the trim condition of the underslung load. These values are:

\(^2\text{in \( m/s \) for velocities, \( m/s^2 \) for acceleration and \( rad/s^2 \) for angular accelerations. No magnitude corrections are performed.}\)
B.2. TRIMMING THE UNDERSLUNG LOAD

Figure B.1: Schematic representation of the load trim optimization
The roll angle and yaw angle are assumed zero because of the symmetric flight condition. The aerodynamic properties of the flat plate theory used in this model require a state in which the underslung load must have a zero bank angle and a zero heading angle. This results in an equilibrium of the forces in the $y$-axis.

In order to skip needless calculations, the load is trimmed with respect to the central point, which is assumed to be at global coordinates $(0, 0, 0)$ with a global velocity of $(V_{trim}, 0, 0)$. In this way the central point does not need to be calculated. Once the load is trimmed, the elongation of the master link is added to the load link to find the position of the load with respect to the helicopter hook. When the helicopter trim is known, the entire system will be translated, so it suspension point has its origin at its assigned helicopter hook.

The initial guess of the state trim variables is chosen as the length of the helicopter cable, with a small elongation for the position of the initial guess. For the following trim variables the trim value of the previous trim velocity are chosen as start state.

The boundary conditions of the systems are made to ensure only logical possibilities are tested; reducing the search area. For the attitude of the helicopter this is between $-\pi/2$ rad and 0.001 rad. Besides the $x$ and $z$-coordinate upper and lower bound, there is also an restriction set for the minimum sling length expressed in the $x$ and $z$-coordinates. This is done, to ensure the sling never has a negative elongation. A negative elongation causes no forces to act as sling forces, creating a ‘dead spot’ in which the optimizer gets stuck; greatly increasing calculation time. The restrictions that are imposed on the system are shown in equation B.4 till B.9.

$$\frac{\pi}{2} \leq \theta_{load} \leq 0.001$$  \hspace{1cm} (B.4)

$$x_{hel} - 0.1 \leq x_{load} \leq x_{hel} + l_{cable}$$  \hspace{1cm} (B.5)

$$y_{hel} - 0.1 \leq y_{load} \leq y_{hel} + 0.1$$  \hspace{1cm} (B.6)

$$z_{hel} - 0 \leq z_{load} \leq z_{hel} + 5l_{cable}$$  \hspace{1cm} (B.7)

$$c \leq x^2 + z^2 - \frac{1}{2} h_{load} - l_{sling}$$  \hspace{1cm} (B.8)

$$c_{eq} = \sum \dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r} = 0$$  \hspace{1cm} (B.9)

The most important prerequisite for the trim and optimization module, is the optimization function. The optimization function, is the difference between the trimmed state and the derivative of the translational velocity and the rotational velocity. Since Matlab does not allow vector minimization, an objective function is designed that needs to be minimized. As stated in the trim definition, shown in equation B.3, the trim comprises of three parts: translational acceleration, rotational velocity and rotational acceleration. The rotational velocity is directly related to the input and set to zero, this result does not have to be taken into account.

The translational acceleration and rotational acceleration are added with a weight factor so that the magnitudes of rotational accelerations and translational acceleration are in approximately the same order of magnitude. These values are then added resulting in the objective function as defined in equation B.10. The weight vector $W_{rot}$ is set at 20.

$$f_{obj, load} = \ddot{u}^2_{load} + \ddot{v}^2_{load} + \ddot{w}^2_{load} + W_{rot}^2 \left( \dot{p}^2_{load} + \dot{q}^2_{load} + \dot{r}^2_{load} \right)$$  \hspace{1cm} (B.10)

Now that all items are known the trim module is build. The program exists out of four modules:

- trimloadMain
- trimLoadOptFunction
- constraintsLoad
- loadDerivativeCalculation

The trimLoadMain module is the first module called upon. It defines the starting conditions, the upper and lower bounds, and normalizes these bounds. Normalization is done by dividing the bounds and the optimization vector by the normalization vector. The normalization vector is the upper bound subtracted by the
lowerbound; as shown in equation B.11.

$$\vec{X}_{\text{norm}} = \vec{X}_{UB} - \vec{X}_{LB}$$ \hspace{1cm} (B.11)

The \textit{trimLoadMainFunction} is the function that runs the optimization algorithm. It contains the algorithm itself, the constraints of the optimizer and the normalization vector.

The algorithm that is used in the load optimization is the interior point method. With trial and error it was found that this optimizer yielded the fastest results to find an optimum that satisfied all the constraints. The predominant exit criteria was found in the \textit{Tolcon}, the deviation from the desired constraints, which was set at $10^{-5}$.

\textit{trimLoadObjFunction} is the program that contains the objective function. Which has as goal to minimize the function shown in B.10. It rewrites the normalized values into the regular values after which it uses the \textit{loadDerivativeCalculation} to calculate the load state derivative. Which is in turn used to calculate the value of the objective function.

After the first optimization calculation, the constraint function is called upon. This function checks two things; first it checks the inequality constraint. Making sure that the $x$ and $z$ constraints as shown in equation B.9 are not breached. The next step is to determine if the equality constraint is reached which is determined by equation B.3.

This cycle is repeated until the exit criteria of the optimization routine are met. In case of the ‘interior-point’ algorithm chosen in the load trim module the exit criteria are defined as followed:

- TolX - $10^{-10}$, sets a limit to the step-size of the normalized state space.
- TolFun$^3$ - $10^{10}$, stop criteria of the change in the objective function.
- TolCon - $10^{-5}$, constraint that is set to the deviation from the constraints.

![Figure B.2: Secondary load trim routine trim optimizer schematic program structure](image)

\(^3\text{The objective function is set as a guide, the constraints are the stopping criteria}\)
B.3. TRIMMING THE HELICOPTER

The helicopter trim is more complex than the load trim routine, as it has four more control variables. The solving routine however is similar to that of the primary load trim routine, the Newton-Raphson approach is fairly effective in finding trimmed conditions.

The conditions of trim in case of the helicopter are identical to the load, but with an addition of the dimensionless rotor inflow velocities denoted as $\lambda_f$ and $\lambda_r$. The translational and rotational accelerations and angular velocities are equal to zero. The Newton-Raphson equation, as shown in equation B.1, is rewritten in equation B.12. As can be seen, the derivatives of the angles derivatives $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$ are lacking from the $f(X)$. This is because they are directly related to the rotational velocities of $p$, $q$ and $r$ which are set to zero. As is the case with the load functions, the desired trim velocity is subtracted from the derivative in $x$-direction to ensure the desired trim velocity is reached.

$$
\begin{bmatrix}
     u_{i+1} \\
     v_{i+1} \\
     w_{i+1} \\
     \phi_{i+1} \\
     \theta_{i+1} \\
     \lambda_{f,i+1} \\
     \lambda_{r,i+1} \\
     \delta_{\text{col},i+1} \\
     \delta_{\text{long},i+1} \\
     \delta_{\text{lat},i+1} \\
     \delta_{\text{ped},i+1}
\end{bmatrix}
= 
\begin{bmatrix}
     u_i \\
     v_i \\
     w_i \\
     \phi_i \\
     \theta_i \\
     \lambda_{f,i} \\
     \lambda_{r,i} \\
     \delta_{\text{col},i} \\
     \delta_{\text{long},i} \\
     \delta_{\text{lat},i} \\
     \delta_{\text{ped},i}
\end{bmatrix}
- 
\tau
\begin{bmatrix}
\frac{\ddot{u}(u_i+\Delta u)-\ddot{u}(u_i)}{\Delta u} & \cdots & \frac{\ddot{u}(\delta_{\text{ped},i}+\Delta \delta_{\text{ped}})-\ddot{u}(\delta_{\text{ped},i})}{\Delta \delta_{\text{ped}}} \\
\vdots & \ddots & \vdots \\
\frac{\ddot{\lambda}_f(\delta_{\text{ped},i}+\Delta \delta_{\text{ped}})-\ddot{\lambda}_f(\delta_{\text{ped},i})}{\Delta \delta_{\text{ped}}} & \cdots & \frac{\ddot{\lambda}_r(\delta_{\text{ped},i}+\Delta \delta_{\text{ped}})-\ddot{\lambda}_r(\delta_{\text{ped},i})}{\Delta \delta_{\text{ped}}}
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w} \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \\
\dot{p} \\
\dot{q} \\
\dot{r} \\
\dot{x} - V_{\text{trim}} \\
\dot{y} \\
\dot{z} \\
\dot{\lambda}_f \\
\dot{\lambda}_r
\end{bmatrix}
$$

(B.12)

The schematic of this trim routine is shown in figure B.3. The schematic flowchart of this problem is given in figure B.2. The error function for this schematic is slightly different than that of the load. The error function is the maximum absolute value found in the optimization function $f(X_i)$ in equation B.13

$$
\epsilon = \max |f(X_i)|
$$

(B.13)
Figure B.3: Flowchart of the helicopter trim routine
HELICOPTER TRIM DATA OF THREE GROUPS OF CONFIGURATIONS

This appendix is divided in four parts, in the first part the trim results are validated and compared to reference data. The other three parts are dedicated to the configuration groups: single, double and triple underslung loads.

C.1. REFERENCE TRIM DATA

The reference data used in this report, are obtained from the non-linear model used by Ostroff [19] and the linear model obtained from Davis [20].

The trim data is plotted figure C.1. In the top left one can see the longitudinal stick deflection. The trim data seems to be exactly mirrored around the zero axis to that of the trim values found by Ostroff and Davis. The lateral stick deflection, shown on the top right, shows a very similar shape but about 0.5 cm lower deflection.

The collective stick deflection shows the expected saddle shape and matches the reference data fairly close as is shown in the left center graph. The pedal deflection, shown in the center right, seems to match the curves accurately.

The roll angle roughly follows a similar curve as shown in the reference data, but as is the case with the lateral stick deflection, it is closer to the zero line than the reference data, which can be seen on the bottom left. The bottom right shows the pitch angle. The pitch angle shows the pitch angle correction setting being active at 100kts, ±50 m/s.
Figure C.1: Bare Chinook trim values for: pilot control input, bank angle and pitch angle
C.2. SINGLE UNDERSLUNG LOAD TRIMMED STATES

Figure C.2: Trim states for a Chinook with a single underslung load: A0, A1 and A4

Figure C.3: Trim states for a Chinook with a single underslung load
C.3. SINGLE UNDERSLUNG LOAD TRIMMED CONTROL VALUES

Figure C.4: Trim control input for a Chinook with a single underslung load: A0, A1 and A4

Figure C.5: Trim control input for a Chinook with a single underslung load
C.4. **Double Underslung Load Trimmed States**

Figure C.6: Trim states for a Chinook with a double underslung load: B0, B6 and B7

Figure C.7: Trim states for a Chinook with a double underslung load
C.5. **Double Underslung Load Trimmed Control Values**

![Trim control input for a Chinook with a double underslung load: B0, B6, and B7](image)

![Trim control input for a Chinook with a double underslung load](image)

Figure C.8: Trim control input for a Chinook with a double underslung load: B0, B6, and B7

Figure C.9: Trim control input for a Chinook with a double underslung load
**C.6. TRIPLE UNDERSLUNG LOAD TRIMMED STATES**

Figure C.10: Trim states for a Chinook with a triple underslung load: C0, C1 and C5

Figure C.11: Trim states for a Chinook with a triple underslung load
C.7. TRIPLE UNDERSLUNG LOAD TRIMMED CONTROL VALUES

![Diagram showing trim control input for a Chinook with a triple underslung load: C0, C1, and C5](image)

Figure C.12: Trim control input for a Chinook with a triple underslung load: C0, C1, and C5

![Diagram showing trim control input for a Chinook with a triple underslung load: C0, C1, C2, C3, C4, C5, C6, C7, and C8](image)

Figure C.13: Trim control input for a Chinook with a triple underslung load
Figure D.1: Flow chart for finding the state matrix
This appendix is divided in three parts, each part is dedicated to a single configuration group. Single, double and triple underslung loads. Each part shows six graphs, the X,Y,Z forces and the L,M,N moments; each with either symmetric or asymmetric deflections. As is the case in the derivatives here are semi normalized. The forces are normalized by dividing the Force by the helicopters mass. The moments L, M and N are normalized by division around their moments of inertia, $I_{xx}$, $I_{yy}$, $I_{zz}$.

A second note is on how the helicopter model works and how this effects the results. As is described in the helicopter modeling section the load and the helicopter are 2 separate bodies, connected by a cable. This cable is elastic, and has a spring constant and a very limited dampening constant. The perturbations that are acting on the model; are acting on the helicopter alone. The effect the load has from the perturbations on the helicopter can thus only be perceived by changes in the helicopters hook position or velocity.

Since the dampening of the cable is negligible, the derivatives that have an effect on the velocity of the helicopter suspension hooks will have no noticeable effect on the results. The effect of the cables spring constant might have a contribution, as the cable has a fairly high spring constant. One can thus conclude that the major differences that are contributing to the helicopter derivatives come from the effect of the trim. If the underslung load has a direct effect on the helicopter derivatives, than this will most likely come from the the perturbations that directly effect the location of the underslung hook location; the perturbations of the angles $\theta$ and $\phi$.

For easy reading the configuration tables treated in chapter 3 are repeated here at the beginning of each their respective section.

### E.1. Helicopter Derivatives with the Single Underslung Load

In this section the single underslung loads are treated. The configurations used are shown in table E.1.

<table>
<thead>
<tr>
<th>conf. #</th>
<th>Load mass [kg]</th>
<th>Sling length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>1000</td>
<td>9.14</td>
</tr>
<tr>
<td>A2</td>
<td>1000</td>
<td>15.1</td>
</tr>
<tr>
<td>A3</td>
<td>1000</td>
<td>17.9</td>
</tr>
<tr>
<td>A4</td>
<td>4000</td>
<td>9.14</td>
</tr>
<tr>
<td>A5</td>
<td>4000</td>
<td>15.1</td>
</tr>
<tr>
<td>A6</td>
<td>4000</td>
<td>17.9</td>
</tr>
</tbody>
</table>

Table E.1: Single underslung load configurations
E.1.1. X-DERIVATIVES IN SINGLE LOAD CONFIGURATIONS

In figure E.1 the force in the positive $x$-direction is plotted against the symmetrical perturbations in $u, w, q$ and $\theta$. Although there are seven lines plotted only three are clearly visible. The bare Chinook configuration, $A_0$, can clearly be distinguished at the top. In the middle one can detect $A_1$, $A_2$ and $A_3$ and at the bottom are $A_4$, $A_5$ and $A_6$. When comparing those lines in table E.1 it can be seen that common characteristic is the mass. Cable length does not seem to have a significant effect on the helicopter in any of the figures plotted.

![Figure E.1: Helicopter derivatives with single load configuration: Force in direction of the $x$-axis](image-url)
E.1.2. Y-DERIVATIVES SINGLE UNDERSLUNG LOAD

Figure E.2: Helicopter derivatives with single load configuration: Force in direction of the y–axis
E.1.3. Z-DERIVATIVES IN SINGLE LOAD CONFIGURATIONS

Figure E.3: Helicopter derivatives with single load configuration: Force in direction of the $z$–axis.

Figure E.4: Helicopter derivatives with single load configuration: Moment around the $x$–axis.
Figure E.5: Helicopter derivatives with single load configuration: Moment around the $y$–axis

Figure E.6: Helicopter derivatives with single load configuration: Moment around the $z$–axis
E.2. Helicopter Derivatives with a Double Underslung Load

Figure E.7: Helicopter derivatives with double load configuration: Force in direction of the $x$–axis

Figure E.8: Helicopter derivatives with double load configuration: Force in direction of the $y$–axis
Figure E.9: Helicopter derivatives with double load configuration: Force in direction of the \( z \)-axis

Figure E.10: Helicopter derivatives with double load configuration: Moment around the \( x \)-axis
Figure E.11: Helicopter derivatives with double load configuration: Moment around the $y$–axis.

Figure E.12: Helicopter derivatives with double load configuration: Moment around the $z$–axis.
E.3. HELICOPTER DERIVATIVES WITH A TRIPLE UNDERSLUNG LOAD

Figure E.13: Helicopter derivatives with triple load configuration: Force in direction of the $x$–axis

Figure E.14: Helicopter derivatives with triple load configuration: Force in direction of the $y$–axis
Figure E.15: Helicopter derivatives with triple load configuration: Force in direction of the $z$–axis

Figure E.16: Helicopter derivatives with triple load configuration: Moment around the $x$–axis
Figure E.17: Helicopter derivatives with triple load configuration: Moment around the $y$–axis

Figure E.18: Helicopter derivatives with triple load configuration: Moment around the $z$–axis
PROGRAM INPUT VALUES

HELICOPTER CONSTANTS

\[ I_{xx} = 50386.3 \text{ kg} \cdot m^2 \]
\[ I_{yy} = 273536 \text{ kg} \cdot m^2 \]
\[ I_{zz} = 257685 \text{ kg} \cdot m^2 \]
\[ I_{xz} = 19838.3 \text{ kg} \cdot m^2 \]
\[ h_f = 2.093 \text{ m} \]
\[ h_r = 3.527 \text{ m} \]
\[ b_f = 0 \text{ m} \]
\[ b_r = 0 \text{ m} \]
\[ l_f = 6.425 \text{ m} \]
\[ l_r = 5.45 \text{ m} \]
\[ \Omega_B = 23.562 \text{ rad/s} \]
\[ R = 9.1444 \text{ m} \]
\[ m_{heli} = 14968.6 \text{ kg} \]
\[ C_Y\beta = 43.3 \text{ m}^2 \]
\[ C_{La} = 32.5 \text{ m}^2 \]
\[ C_{Lg} = 6.57 \text{ m}^2 \]
\[ C_{Ma} = 142 \text{ m}^2 \]
\[ C_{N\beta} = 51.5 \text{ m}^2 \]
\[ \alpha_{rot} = 0.81 \text{ m} \]
\[ n = 3 \text{ } - \]
\[ C_{la} = 5.75 \text{ } - \]
\[ C_{i0} = 0.126 \text{ } - \]
\[ C_{d0} = 0.0098 \text{ } - \]
\[ C_{dt} = 38.66 \text{ } - \]
\[ i_f = 9 \frac{\pi}{180} \text{ rad} \]
\[ i_r = 4 \frac{\pi}{180} \text{ rad} \]
\[ M_W = 510.2 \text{ kg} \cdot m \]

Hook\text{location}_x_1 = 2.28092 \text{ m} \\
Hook\text{location}_y_1 = 0 \text{ m} \\
Hook\text{location}_z_1 = 1.309 \text{ m} \\
Hook\text{location}_x_2 = 0.19812 \text{ m} \\
Hook\text{location}_y_2 = 0 \text{ m} \\
Hook\text{location}_z_2 = 1.309 \text{ m} \\
Hook\text{location}_x_3 = -1.78308 \text{ m} \\
Hook\text{location}_y_3 = 0 \text{ m} \\
Hook\text{location}_z_3 = 1.309 \text{ m} \\

99
**Drag polar**

\[
\begin{align*}
\text{DragPolar}_{-20} &= \begin{bmatrix}
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0001 & 0.0056 & -0.0065 & 3.7236 \\
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0001 & 0.0089 & -0.0131 & 3.9503 \\
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0002 & 0.0084 & -0.0199 & 2.3593 
\end{bmatrix} \\
\text{DragPolar}_0 &= \begin{bmatrix}
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0001 & 0.0056 & -0.0065 & 3.7236 \\
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0001 & 0.0089 & -0.0131 & 3.9503 \\
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0002 & 0.0084 & -0.0199 & 2.3593 
\end{bmatrix} \\
\text{DragPolar}_{20} &= \begin{bmatrix}
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0001 & 0.0056 & -0.0065 & 3.7236 \\
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0001 & 0.0089 & -0.0131 & 3.9503 \\
-0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0001 & 0.0002 & 0.0084 & -0.0199 & 2.3593 
\end{bmatrix}
\end{align*}
\]

**Load constants**

\[
\begin{align*}
\epsilon_{\text{cable}} &= 10 \quad \text{Ns/m} \\
k_{\text{cable}} &= 7.25 \cdot 10^5 \quad \text{N/m} \\
C_{\text{FE load}} &= 4 \quad \text{m}^2 \\
C_{\text{LF load}} &= 4 \quad \text{m}^3/\text{rad} \\
C_{\text{MF load}} &= 3 \quad \text{m}^3/\text{rad} \\
C_{\text{UL load}} &= 4 \quad \text{m}^3/\text{rad} \\
C_{\text{UW load}} &= 4 \quad \text{m}^3/\text{rad}
\end{align*}
\]

**Atmospheric constants**

\[
\begin{align*}
g &= 9.81 \quad \text{m/s}^2 \\
R &= 287.05 \quad - \\
\lambda &= -0.0065 \quad \text{K/m} \\
T_0 &= 288.15 \quad \text{K} \\
h_{\text{strat}} &= 11000 \quad \text{m} \\
\rho_0 &= 1.225 \quad \text{kg/m}^3
\end{align*}
\]

**Stick to blade values**

\[
\begin{align*}
\theta_{0,f,\text{min}} &= 0 \quad \text{rad} \\
\theta_{0,f,\text{max}} &= 16 \frac{\pi}{180} \quad \text{rad} \\
\theta_{0,r,\text{min}} &= 0 \quad \text{rad} \\
\theta_{0,r,\text{max}} &= 16 \frac{\pi}{180} \quad \text{rad} \\
\theta_{1c,f,\text{min}} &= -10 \frac{\pi}{180} \quad \text{rad} \\
\theta_{1c,f,\text{max}} &= 10 \frac{\pi}{180} \quad \text{rad} \\
\theta_{1c,r,\text{min}} &= -10 \frac{\pi}{180} \quad \text{rad} \\
\theta_{1c,r,\text{max}} &= 10 \frac{\pi}{180} \quad \text{rad} \\
\theta_{0,\text{neut},f} &= 7.85 \frac{\pi}{180} \quad \text{rad} \\
\theta_{0,\text{neut},r} &= 7.85 \frac{\pi}{180} \quad \text{rad} \\
\Delta \theta_{\text{col},r} &= 73.4 \frac{\pi}{180} \quad \text{rad/m} \\
\Delta \theta_{\text{col},f} &= 73.4 \frac{\pi}{180} \quad \text{rad/m} \\
\Delta \theta_{\text{long},r} &= -24.2 \frac{\pi}{180} \quad \text{rad/m} \\
\Delta \theta_{\text{long},f} &= 24.2 \frac{\pi}{180} \quad \text{rad/m} \\
\Delta \theta_{\text{lat},f} &= 75.2 \frac{\pi}{180} \quad \text{rad/m} \\
\Delta \theta_{\text{lat},r} &= 75.2 \frac{\pi}{180} \quad \text{rad/m} \\
\Delta \theta_{\text{ped},f} &= 125 \frac{\pi}{180} \quad \text{rad/m} \\
\Delta \theta_{\text{ped},r} &= -125 \frac{\pi}{180} \quad \text{rad/m}
\end{align*}
\]