MEASUREMENT OF THE REYNOLDS STRESSES IN A CIRCULAR PIPE AS A MEANS OF TESTING A DISA CONSTANT-TEMPERATURE HOT WIRE-ANEMOMETER

by

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SUMMARY

Measurements of the turbulent stresses in fully-developed pipe flow have been made as an overall test of the DISA constant-temperature hot-wire anemometer. The measurements were made with single slanting wires and a normal wire.

The shearing stress was measured at two Reynolds numbers and compared with values computed from the pressure drop down the pipe: an accuracy of 10% or better was achieved. The computed longitudinal and transverse normal stresses were in good agreement with the measurements of Laufer. It was confirmed that the heat loss from the wire varied linearly with $U^{0.45}$ (Collis' law) rather than $U^{0.50}$ (King's law).
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**NOTATION**

- **a, b and c** - constants in the law describing the cooling of the wire (equation (1))
- **d** - diameter of pipe
- **e** - instantaneous bridge voltage fluctuation
- **$P_a$** - atmospheric pressure
- **$P_s$** - static pressure
- **$P_t$** - pitot total pressure
- **R** - operating resistance of hot wire
- **r** - radius of pipe
- **$R_a$** - wire resistance at still air temperature
- **$Re$** - Reynolds number ($= \frac{U_c d}{v}$)
- **$T_w$** - hot-wire operating temperature
- **$T_\infty$** - ambient temperature
- **U** - local mean flow velocity
- **u** - longitudinal fluctuating component of velocity
- **$U_c$** - mean flow velocity at centre of pipe
- **$U_f$** - skin friction velocity ($= \sqrt{\frac{10}{\rho}}$)
- **V** - bridge D.C. voltage
- **v** - transverse fluctuating component of velocity
- **$V_0$** - bridge D.C. voltage at zero air speed
- **x** - distance along the axis of pipe
- **y** - distance from wall of pipe
- **$\beta$** - a function of bridge D.C. voltage ($= \frac{2V}{V^2-V_0^2}$)
- **$\gamma = c \cot \psi$**
v - kinematic viscosity

ρ - density

τ - turbulent shear stress = -ρuv

τ₀ - shear stress at y = 0

ψ - angle of yaw of slanting wire.
1. INTRODUCTION

Following the pioneering work of King\(^{(1)}\) on the convection of heat from small cylinders in a stream of fluid, much use has been made of hot wires in the measurement of turbulence. The hot-wire turbulence-measuring techniques have now been developed to a stage that the hot-wire anemometer has become a useful tool in the routine investigation of turbulent shear flows. The most recent review (1955) of hot-wire techniques has been given by Cooper and Tulin.\(^{(2)}\)

The purpose of the present investigation was to measure the Reynolds stresses in fully developed turbulent pipe flow with a single slanting wire (for shearing stress and transverse turbulence) and a normal wire (for longitudinal turbulence) using a constant temperature anemometer\(^{(4)}\) (DISA. 55A01) and thereby to obtain an overall check of the instrument.

The results presented in this paper are for two single slanting hot-wires at pipe Reynolds numbers of \(2.74 \times 10^5\) and \(3.64 \times 10^5\). The measurements of normal Reynolds stress were made at the lower Reynolds number using a normal wire, and the results are compared with those of Laufer.\(^{(5)}\)
2. THEORETICAL ANALYSIS

Assuming that the convective heat loss from the wire varies linearly with $U^c$, where $U$ is the instantaneous velocity of the flow and $c$ is a constant, the working equations for a constant temperature anemometer can be readily derived.

In the DISA 55AOI anemometer the hot wire forms one arm of a bridge circuit as shown in Fig.1. The wire is maintained at constant temperature by means of the feed-back circuit to the bridge. The mean bridge voltage is measured and, being proportional to the wire current, is related to the mean velocity of the flow. Across the bridge terminals an additional circuit is connected to measure the bridge voltage fluctuations which correspond to the flow fluctuations or turbulence.

Assuming that the wire is cooled only by that component of the flow velocity which is perpendicular to the hot-wire then, for a wire yawed to the mean flow at an angle $\psi$, the governing equation can be written:

$$\frac{v^2}{R} = a + b(U \sin \psi)^c (R - R_a) \quad \ldots \ldots (1)$$

where $V$ is the bridge D.C. voltage,

$R$ is the hot-wire operating resistance,

$R_a$ is the wire resistance at still air temperature,

$a$ and $b$ are constants depending slightly on the wire temperature,

$U$ is the mean flow velocity, and

$c$ is a constant.
Equation (1) was originally proposed by King with \( c = 0.5 \). However, later and more accurate measurements by Collis indicated that \( c = 0.45 \) for the usual range of wire Reynolds numbers (less than 44).

For fluctuations in velocity about the mean, equation (1) gives for constant \( R \),

\[
\frac{2V}{R(R-R_a)} \, dV = b \, c \, \cos \psi \, \sin^{c-1} \psi \, U^c \, d\psi + b \, \sin^{c} \psi \, \cos^{c-1} \psi \, d\psi
\]

If the fluctuating components \( u \) and \( v \) are small compared with mean velocity \( U \)

\[
dU = u \quad \text{and} \quad Ud\psi = v
\]

Substituting equations (3) in equation (2),

\[
\frac{2V}{R(R-R_a)} \, dV = b \, \sin^{c} \psi \, U^{c-1} \left[ cu + \gamma v \right]
\]

where \( \gamma = c \, \cot \psi \)

If \( V_0 \) be the value of \( V \) corresponding to \( U = 0 \) in equation (1)

\[
a = \frac{V_0^2}{R(R-R_a)}
\]

Substituting equation (6) in equation (1),

\[
b \, \sin^{c} \psi \, U^{c} = \frac{1}{R(R-R_a)} \left( v^2 - V_0^2 \right)
\]

Equation (7) in equation (4) gives:

\[
\left( \frac{2V}{v^2 - V_0^2} \right) \, dv = \frac{1}{U} \left( cu + \gamma v \right)
\]

or \( \beta e = \frac{1}{U}(cu + \gamma v) \)

where \( \beta = \frac{2V}{v^2 - V_0^2} \) and \( e = dv \) - instantaneous fluctuation
in bridge voltage.

Squaring equation (9) and taking the time mean,

\[ \beta^2 e^2 = \frac{1}{u^2} \left( c^2 u^2 + \gamma^2 v^2 + 2\gamma c \overline{uv} \right) \quad \ldots(10) \]

Taking two readings of \( e^2 \) by re-orientating the single slanting wire in the plane defined by the wire and the mean velocity \( U \), from an angle of yaw of \( +\psi \) to an angle of yaw of \(-\psi \), equation (10) becomes:

\[
\beta^2_1 e^2_1 = \frac{1}{u^2} \left( c^2 u^2 + \gamma^2 v^2 + 2\gamma c \overline{uv} \right) \quad \ldots(11) \\
\beta^2_2 e^2_2 = \frac{1}{u^2} \left( c^2 u^2 + \gamma^2 v^2 - 2\gamma c \overline{uv} \right)
\]

where \( \beta_1 \) should equal \( \beta_2 \) if the wire is correctly orientated with respect to the mean velocity.

Subtracting equations (11):

\[
-\rho \overline{uv} = \frac{1}{2c} \frac{1}{\gamma} \frac{1}{2} \rho u^2 \left[ \beta^2_2 e^2_2 - \beta^2_1 e^2_1 \right] \quad \ldots(12)
\]

Thus the turbulent shearing stress can be measured with a single slanting wire in conjunction with the constant-temperature anemometer (DISA. 55A01) provided the constants \( c \) and \( \gamma \) are known. If the calibration curve for a hot-wire follows King's law then \( c = 0.5 \) and \( \gamma \) would then be \( 0.5 \cot\psi \) (equation (5)). Newman and Leary\(^3\) have used \( c = 0.5 \) and \( \gamma = 0.457 \cot\psi \) for a constant current anemometer and obtained good agreement in a similar pipe test. Collis\(^8\) has noted that this is consistent with his own findings since the fluctuations follow a King's law.
line for constant current operation due to the small variation of $a$ and $b$ with temperature. However for constant-temperature operation $c = 0.45$ and $\gamma = 0.45 \cot \psi$. The latter values are used in the present paper.

It is also interesting to note that equation (10) is directly applicable to a normal wire for which $\gamma = 0$.

$$\frac{v^2}{u^2} = \frac{\beta^2}{c^2} e^2$$

...(13)

The addition of equations (11) for a slanting wire, in conjunction with the normal-wire readings, gives an equation from which the transverse turbulence $\frac{v^2}{u^2}$ may be computed.

It is interesting to note that the assumption of King's law rather than Collis' law would lead to values of the computed turbulence stresses which were 19% too low.
3. EXPERIMENTAL INVESTIGATION

The general layout of the experimental apparatus is shown in Fig. 3. The apparatus was primarily designed to study radial channel flow (9) however for the present investigations the radial channel section at exit of the pipe was removed. The pipe consisted of 3 ins. I.D. precision brass tube approximately 14 feet in length. Static pressure taps (0.015 ins. diameter) were provided at various locations along the length of the pipe. The hot-wire traverses were made about 2 ins. upstream of the pipe exit. A double ended dial gauge with 0.001 inch graduations was used for traversing the flow.

Air was supplied from a centrifugal compressor, driven by a 10 H.P. constant-speed, three-phase motor. A fibre glass filter (approximately 0.002 ins. fibres) was provided at the compressor entry to reduce dust accumulation on the hot-wire and this was found to be extremely effective. The mass flow in the pipe was roughly controlled by a bleed valve situated at the compressor outlet.

The compressor was situated about 40 feet away from the supply station in the Aerodynamics Laboratory and thus the supply pipe was sufficiently long to damp out any large fluctuations emanating from the compressor. The bends in the supply pipe were gradual. The precision brass pipe was connected to the supply station by 12 feet of flexible tubing. An assembly of deep cell honeycomb and a symmetrical bleed valve was provided
at the junction of the flexible tubing and the brass pipe. The deep cell honeycomb was used to straighten the flow and the symmetrical bleed valve was used for fine speed control.

The hot-wire anemometer used for the present investigation was a commercial unit supplied by DISA Electronik A/S of Denmark. The operating procedures for this unit is described in their instruction manual(4).

The hot wires were made from platinum-coated tungsten of nominal diameter 0.0002 ins. They were operated at a resistance $R = 1.8 R_a$, corresponding to a temperature about 200°C above ambient. The measurements were made using two single slanting wires (to check repeatability) and one normal wire. Tests were made for two pipe Reynolds numbers $\frac{U_{cd}}{v} = 2.75 \times 10^5$ and $3.64 \times 10^5$.

The angle of yaw ($\psi$) between the wire and the axis of rotation for each slanting-wire probe was determined by mounting the probe in a photographic enlarger with a magnification of about 10.

The wires were examined under a microscope before and after each test to check for any accumulation of dust. The fibre glass filter at the compressor inlet was effective in preventing this for the present tests.

Correct alignment of the slanting wire probe with the axis of the pipe was assured when the mean voltage reading was independent of the rotation of the wire.
Pitot traverses of the pipe were made with a 0.030 ins. O.D. pitot tube with sharpened lips. Wall static pressures were measured on an inclined multitube manometer.
4. DISCUSSION OF RESULTS

Verification of Collis' Law:

It was considered desirable to confirm Collis' law before making any turbulence measurements. The normal wire was therefore mounted at the centre of the pipe and velocity there varied from 0 to approximately 336 ft./sec. The round pitot tube was used to measure this velocity, the static temperature being taken as ambient. Since the hot-wire readings are sensitive to the temperature of the air stream and this varied slightly, it was necessary to apply corrections. To do this Collis' law was assumed, the variations of the thermal conductivity and the kinematic viscosity of the air with the ambient temperature being neglected. Taking logarithmic differentials:

\[
\frac{\Delta V}{V} = \frac{\Delta T_\infty}{2} \left[ \frac{0.17}{T_w + T_\infty} \frac{0.17}{T_\infty} - \frac{1}{T_w - T_\infty} \right]
\]

(14)
gives the required correction to the bridge voltage.

The static temperature of the air was measured with a mercury thermometer and, where necessary, compressibility corrections were applied assuming a recovery factor of 0.90.

Fig.(2) shows the measured values of \( U^{0.45} \) plotted against \( V^2 \) the latter being corrected to an ambient temperature of 90°F. In all cases the corrections to bridge voltage were less that 1.5%. The linearity of the results substantially confirms Collis' law. Furthermore the zero-wind readings lies fairly well on the straight line so that \( V_0 \) in equations (7) and (8)
may be taken as the measured bridge voltage with wind off.

Fig. (4) shows the pressure drop along the pipe for the two Reynolds numbers of 2.74 x 10^5 and 3.64 x 10^5 and is seen to be satisfactorily linear. The slope gives the distribution of shearing stress across the pipe (3) and is shown in Figs. (7) and (9).

Pitot Traverses:

From equation (12) it is seen that the computation of turbulent shearing stress, \(-\rho \overline{u} \overline{v}\), requires a knowledge of \(\frac{1}{2} \rho \overline{U}^2\). This information could be obtained from the mean hot-wire readings. However in this case the variation of \(\frac{1}{2} \rho \overline{U}^2\) was determined more directly by traversing the flow with the round pitot tube. Fig. (5) shows the measured values of dynamic pressure across half the pipe at the section where hot-wire measurements were taken.

Shear Stress Distribution:

Fig. (6) shows the single slanting wire readings for two different orientations. The measurements were made at Reynolds number of 2.74 x 10^5. The single slanting wire had a value of \(\psi = 43^\circ - 40^\circ\). From the smoothed curves of Fig. (6) together with Fig. (5), the value of \(-\rho \overline{u} \overline{v}\) was computed. This is shown in Fig. (7). In this figure the straight line shear-stress distribution obtained from the pressure drop is also plotted for
comparison. The shear stress variation obtained from the single slanting wire is in good agreement with that obtained from the pressure drop.

To establish further confidence in this method an alternative probe ($\psi = 45^0$) was chosen to traverse the flow at the higher Reynolds number of $3.64 \times 10^5$. The measurements with this probe are presented in Figs.(8) and (9). Once again it is seen that the hot-wire measurements are in substantial agreement with those obtained from the pressure drop.

**Longitudinal and Lateral Turbulence Measurements:**

A normal wire was used to measure the longitudinal turbulence in the pipe at $Re = 2.74 \times 10^5$. In Fig.(10) the longitudinal turbulence made non-dimensional in terms of the skin friction velocity $\frac{v^2}{U_T}$, is plotted against $\frac{\psi}{\tau}$. Similar measurements were made by Laufer(5) at a Reynolds number of $5.0 \times 10^5$ and these are also shown for comparison. The agreement is good.

The transverse turbulence $\frac{v^2}{U_T}$ was computed from both the slanting and normal wire readings, and is shown in comparison with Laufer's measurements in Fig.(11). Once again the agreement is satisfactory.
CONCLUSIONS

The measurement of the Reynolds stresses in fully developed turbulent flow down a circular pipe is commended as a reliable means of checking a hot-wire anemometer. It is demonstrated that, for the level of turbulence encountered in a pipe, the turbulent shearing stress can be measured with a single slanting wire controlled by the DISA anemometer to an accuracy within 10%. Furthermore the measurements of longitudinal and transverse turbulence are in good agreement with those of Laufer(5).

It is noted that Collis' law rather than the conventional King's law must be used to determine the turbulent stresses when a constant-temperature hot-wire anemometer is used.
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FIG. 1

BLOCK-DIAGRAM OF DISA CONSTANT-TEMPERATURE HOT-WIRE ANEMOMETER
VERIFICATION OF COLLIS' LAW

FIG. 2

\[ (V^2) \]

\[ (\text{Ft}^2/\text{Sec}^2)^{0.45} \]

\[ 150 \]
VALVE FOR ADJUSTING THE FLOW.

DEEP CELL HONEYCOMB.

PRECISION BRASS PIPE 3" I.D.

STATIC PRESSURE TAPS.

DIAL GAUGE

SLANTING HOT WIRE PROBE

SYMMETRICAL BLEED VALVE FOR ADJUSTING THE FLOW.

DISA CONSTANT TEMPERATURE ANEMOMETER 55 AO1.

GENERAL LAYOUT OF APPARATUS
FIG. 4

PRESSURE DROP ALONG THE PIPE

\[ \triangle R_e = 3.64 \times 10^5 \]

\[ \odot R_e = 2.74 \times 10^5 \]

\[ \frac{6 \left(\frac{p_s - p_a}{\text{ins}}\right) \text{ins} \cdot \text{ft} \cdot \text{sec}^2}{(\text{sp. gr. 0.80})} \]
FIG. 5

PITOT TRAVERSE ACROSS HALF THE PIPE

\[ \triangle R_e = 3.64 \times 10^5 \]

\[ \bigcirc R_e = 2.74 \times 10^5 \]
SLANTING WIRE READINGS ACROSS HALF THE PIPE

\[ R_e = 2.74 \times 10^5 \]

Angle of yaw \( \psi = 43^\circ - 40' \)
FIG. 7

SHEAR STRESS DISTRIBUTION ACROSS HALF THE PIPE

\[ R_e = 2.74 \times 10^5 \]
FIG. 8
SLANTING WIRE READINGS ACROSS HALF THE PIPE

\[ R_e = 3.64 \times 10^5 \]

Angle of yaw \( \psi = 45^\circ \)
FIG. 9

SHEAR STRESS DISTRIBUTION ACROSS HALF THE PIPE

\[ R_e = 3.64 \times 10^5 \]
LONGITUDINAL TURBULENCE DISTRIBUTION ACROSS HALF THE PIPE

$$R_e = 2.74 \times 10^5$$
TRANSVERSE TURBULENCE DISTRIBUTION

\[ R_e = 2.74 \times 10^5 \]