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EVALUATING THE ROBUSTNESS EFFECTS OF INFRASTRUCTURE PROJECTS BASED ON THEIR TOPOLOGICAL AND GEOMETRICAL ROADWAY DESIGNS

ABSTRACT

When infrastructures projects are evaluated, it is not only important to evaluate them with models that represent the average daily situation, but also to evaluate them in case of irregular situations like incidents. This becomes especially relevant when various project alternatives are expected to show significantly different scores in case of incidents. Project alternatives and their road sections have different topological and geometrical characteristics. The focus of this paper is on the following characteristics: hard shoulders, the number of lanes, parallel road structures and weaving sections. The main question that this paper addresses is how these network characteristics affect both the risk of different types of incidents occurring and the effects of those incidents on the network performance (robustness). In order to answer this question, analytical examples are presented for small theoretical networks that give insight into how the selected characteristics affect the total delay caused by incidents and its dependence on the traffic volume, capacity, severity and duration of incidents. A marginal simulation based method is presented that can be used to compute the robustness effects of project alternatives, given their geometrical and topological characteristics, on a network level. A case study for an infrastructure project in the Netherlands is presented that illustrates how the robustness effects of infrastructure projects can be computed given their topological and geometrical characteristics.

Keywords: robustness; incidents; number of lanes; hard shoulder; parallel road structure; weaving sections.

1 INTRODUCTION

In many urbanized areas, incidents and other disturbances can cause large delays. In the Netherlands incidents are responsible for 20-25% of the total travel time delay (Snelder, et al., 2013) and roadworks are responsible for 4% of the total travel time delay. In the United States of America, 39% of the total travel time delay is caused by traffic incidents, 18% by different weather conditions and 1% by work zones (TRB, 2013). Therefore, when infrastructures projects are designed and evaluated, it is not only important to perform this for an average daily situation, but also to do that for disturbances like incidents, roads works, bad weather conditions, demand fluctuations and special events. This becomes especially relevant when different project alternatives are expected to have significantly different scores when disturbances are considered. Project alternatives have different topological and geometrical characteristics, which affect the network performance when disturbances occur. Often decisions have to be made on the utilization of scarce available space. For instance, if a hard shoulder is replaced by a regular lane, the throughput is higher under regular conditions, but if an incident occurs more lanes have to be closed and more traffic is affected. Therefore, it is important to understand how different topological and geometrical characteristics affect the performance of road networks in case of disturbances.

The term robustness is directly related to the performance of road networks in case of disturbances. Robustness is defined as the extent to which, under pre-specified circumstances such as incidents, roadworks, bad weather conditions, demand fluctuations and special events, a network is able to maintain the function for which it was originally designed (Snelder, et al., 2012). An increasing amount of work has been performed on the assessment of the robustness of a network or the vulnerability of links (e.g. Murray-Tuite, and Mahmassani, 2004; Jenelius, 2010; Sullivan, et al., 2010; Knoop, et al., 2012; Snelder, et al., 2012; Calvert and Snelder, 2015). However, little is known on how topological and geometrical characteristics affect network performance and consequently the robustness of a road network.

The main question that this paper addresses is how topological and geometrical characteristics affect both the risk of different types of incidents occurring and the effects of those incidents on network performance (robustness). The focus of this paper is on the following characteristics: hard shoulders, the number of lanes, parallel road structures and weaving sections. With respect to disturbances, the focus of this paper is on incidents. However, the presented method can also be used for other disturbances that cause local capacity reductions (e.g. roadworks). This paper contributes to the existing literature by giving an overview of how hard shoulders, the number of lanes, parallel road structures and weaving sections affect the risk, duration and capacity reduction of incidents and the delays caused by these incidents and therewith the robustness of a network. This paper also shows how the robustness effects of infrastructure projects can be computed given their topological and geometrical characteristics.

Section 2 presents theoretical insights into how the number of lanes, hard shoulders, parallel road structures and weaving sections affect the delays that are caused by incidents on a single road stretch. Section 3 presents a method that can be used to compute the robustness of project alternatives, given their topological and geometrical characteristics, on a network level. Section 4 presents a case study in which the method is applied. Finally, in section 5 general conclusions are presented.

2 THEORETICAL INSIGHTS: DELAYS CAUSED BY INCIDENTS AND ROADWORKS

This section gives analytical insights into how the number of lanes, hard shoulders, parallel road structures and weaving sections affect the robustness of a single road stretch in case of incidents. These insights can be used to understand the effects that occur on a network level as explained in more detail in sections 3 and 4.

In Olmstead (1999) and Knoop (2009) it is shown that the total delay that is caused by an incident can be computed using Equation (1). Both assume homogeneous and stationary traffic, no spillback to other roads, and a constant capacity reduction during the duration of the incident.

$$D = \max\left(0, \frac{t^2(rC - C)(I - rC)}{2(I - C)}\right) = \max\left(0, \frac{t^2(r - 1)(rC - I)}{2\left(1 - \frac{I}{C}\right)}\right) \quad (1)$$

in which D = total delay for all vehicles caused by an incident [vehicle hours]; t = incident duration [hour]; r = capacity reduction factor [-](e.g. 0.9 implies a capacity reduction of 10%); I = traffic volume [vehicles/hour] and C = capacity [vehicles/hour].

Equation 1 represents the dashed surface in Figure 1a. The Equation can also be derived from Figure 1b by multiplying the number of vehicles in the queue (density * queue length) with the delay of those vehicles integrated over all time periods. Figure 1b shows a space time plot for a road section. The traffic drives from the bottom of the figure to the top. The dashed area shows the congestion that is caused by an incident. The queue length varies over time (vertical cross section of the dashed area) and the queue moves upstream over time.

Figure 1: a) cumulative link outflow without an incident and with an incident; b) space-time plot with congestion caused by an incident.

Knoop (2009) shows how the above mentioned theory can be extended to compute the total delay [vehicle hours] caused by incidents near convergent and divergent points by taking spillback effects and secondary bottlenecks into account that occur at junctions when the incident queues solve. Below we use Equation 1 and the extended theory from Knoop (2009) to give insights into the influence of the number of lanes, hard shoulders, parallel road

structures and weaving sections on the robustness of a road network. The assumptions on capacity reductions and durations of incidents that are used in the examples below are based on Table 2 and are presented in section 4.2.

Example number of lanes

In this example, we assume that an incident occurs on a road with 2, 3, 4, 5 or 6 lanes with a capacity of respectively 4200, 6300, 8200, 10000 and 11500 vehicles per hour. Two lanes are assumed to be closed and the capacity of the remaining lanes is assumed to be 80% of the original lane capacity. For the road with 2 lanes, we assume a capacity reduction of 90% instead of 100%. The duration of the incident is assumed to be 34 minutes. The total delay caused by an incident can be computed using Equation 1. Figure 2 shows the delay caused by the incident of different traffic volumes. From Figure 2 it can be concluded that for identical traffic volumes, having more lanes available results in a lower delay per incident, which can be explained by the fact that there is more spare capacity. However, motorways with more lanes can accommodate more traffic. Therefore, when severe incidents occur that block the entire road for instance, the total delay caused by that incident will be higher on a road with more lanes.

Figure 2: influence of the number of lanes and the traffic volume on delays caused by incidents.

Example hard shoulder

In this example, we assume that a car breakdown on a road with 3 lanes and a capacity of 6000 vehicles/hour reduces the capacity by 10% (factor 0.9) for 30 minutes. This implies that although the car is on the hard shoulder, the capacity of the other lanes is reduced by 10% due to rubbernecking where drivers slow down to look at the vehicle on the hard shoulder. If the hard shoulder is not present, the car blocks an entire lane and the traffic on the other lanes slows down as well, which results in a capacity reduction of 40% (factor 0.6). Furthermore, we assume that it takes 2.5 minutes longer for the towing services to get to the car breakdown. Figure 3 shows the total delay for the situation with and without a hard shoulder for different traffic volumes. Under the above mentioned assumptions, it can be seen that having a hard shoulder is especially important in case of a high volume to capacity ratio and high traffic volumes. If the traffic volume is 5800 vehicles/hour for instance, having a hard shoulder reduces the total delay caused by each car breakdown by about 3800 hours of total delay compared to the situation without a hard shoulder. Furthermore, the probability of having accidents is also higher if there is no hard shoulder, as will be shown in section 4.2.

These findings can be generalized for other incident types (e.g. car accidents, truck accidents) and roads with a different number of lanes: in general, having a hard shoulder reduces the total delay caused by incidents when the traffic volumes exceed the spare capacity. The impact is larger when the traffic volumes are higher.

Figure 3: example total delay caused by a car breakdown on a road with 3 lanes with and without a hard shoulder (under the assumption of an infinitely long road stretch with no route alternative).

Example parallel road structure

In this example, we compare a roadway with 6 lanes that has two parallel carriageways each with 3 lanes. The lane capacity is assumed to be 2000 vehicles/hour. We consider three different types of incidents: one lane blocked, two lanes blocked and three lanes blocked. When one lane is blocked we assume a capacity reduction of 30% ($r=0.7$) on the road with 6 lanes and 40% ($r=0.6$) on the road with 3 lanes (2400 vehicles/hour). When 2 lanes are blocked this is respectively 55% and 70% and when 3 lanes are blocked 80% and 90%. Furthermore, we assume that the parallel carriageway is not affected.

Figure 4 shows the total delay caused by three incidents (1 lane blocked, 2 lanes blocked, 3 or more lanes blocked) for different traffic volumes as can be computed by Equation 1. The traffic volume represents the volume on the road with six lanes and on the two parallel carriageways together. We assumed that the flow is equally spread over both parallel carriageways and remains equally spread during the incident (no rerouting over the other parallel carriageway occurs). Figure 4 illustrates that there is a switch point: when the traffic volumes are low, the road with 6 lanes has the lowest total delays, but when the flows are higher the total delay on the road with 6 lanes is higher than with the parallel carriageway structure. This can be explained by the fact that at low volumes a road with 6 lanes has a larger spare capacity which can be used to avoid the incident than a road with 3 lanes. However, when volumes increase, there is less spare capacity and larger traffic volumes are blocked, which results in higher total delays. The switch point is at lower volumes when the capacity reduction is larger.

Figure 4: example total delay caused by 3 different types of incidents on a road with 6 lanes and a parallel road structure with two times 3 lanes (no spillback effects and no alternative route choice).

In practice spillback effects may occur. If the queue on the affected road spills back upstream and blocks the section where the parallel roadways split, the flows to the parallel carriageway are also affected. Figure 5 shows the impact of spillback effects in case an incident blocks two lanes and occurs 1 kilometer downstream of the split point. The dotted grey line shows the delay that is caused by the incident in case of fixed route choice (nobody changes to the parallel road). The figure indicates that the total delay that is caused by an incident is higher when there are two parallel carriageways instead of one road with 6 lanes. This can be explained by the fact that the queue spills back upstream past the split point and therefore also partially blocks the flow to the parallel carriageway. The outflow of the upstream link is reduced to the remaining capacity on the roadway where the incident occurs divided by the split rate of 50%. This capacity reduction is larger than the capacity reduction caused by a similar incident on a road with 6 lanes.

The solid grey line shows the total delay caused by the incident in case a maximum number of vehicles takes the parallel carriageway as route alternative. The number of vehicles that takes the parallel carriageway is defined by Equation 2:

$$I_{alt} = \max(\min(I - rC, S), 0) \quad (2)$$

in which I_{alt} = number of vehicles that takes the parallel carriageway [vehicle/hours]; r = capacity reduction factor [-], C = capacity [vehicles/hour] and S = spare capacity parallel carriageway [vehicles/hour].

The figure indicates that if spillback effects and alternative route choice are taken into account, a switch point still remains. However, in this case the parallel carriageway structure scores better than the single road with 6 lanes when the traffic volumes are low. The explanation for this is that the spare capacity is higher when there are two roadways, because one road remains unaffected as long as the queue does not spill back upstream of the split point. At higher traffic volumes, all vehicles experience delay once the queue spills back upstream of the split point.

The above mentioned conclusions can be generalized to parallel carriageway structures with a different number of lanes and to incidents more or less than 1 kilometer away from the split point.

Figure 5: example total delay caused by an incident that blocks two lanes on a road with 6 lanes, a parallel road structure with two times 3 lanes (with spillback effects and alternative route choice).

Example weaving section

Weaving sections reduce the capacity under regular conditions (depending on the traffic volumes) and therewith the spare capacity in case of incidents. Furthermore, the probability of accidents occurring also could be higher depending on the complexity of the weaving section, as will be shown in section 4.2.

Below the example of the parallel carriageway structure is extended by including a weaving section before the split point. Instead of a capacity of 12,000 (6×2000) vehicles/hour, the capacity is now 10,000 vehicles/hour for 6 lanes. Figure 6 shows that due to the weaving section, higher delays already occur at lower traffic volumes. This can be explained by the fact that there is less spare capacity on the road section upstream of the split point.

Figure 6: example total delay caused by an incident that blocks two lanes on a road with 6 lanes, a parallel road structure with two times 3 lanes and a weaving section before the splitting point.

3 MODELLING APPROACH: DELAYS CAUSED BY INCIDENTS

The previous section computed the delays caused by incidents under the assumption of homogeneous and stationary traffic and a constant capacity reduction during the duration of the incident. In practice, this assumption doesn't hold, because the traffic volume and the capacity reduction are not constant for the duration of the incident. Furthermore, queues might spill back over a larger part of the network. Therefore, in order to be able to quantify the effect of incidents in realistic situations, more advanced methods that consider spillback effects, time dynamics and alternative route choice are required.

Furthermore, when different infrastructure project alternatives are to be evaluated on their robustness effects, different types of incidents need to be simulated on all the links and on different timestamps. If we for instance choose to model 6 incident types on 1000 links for 24 time steps (e.g. each hour of the day) and for 5 project alternatives, 720,000 incidents need to be simulated. Since dynamic traffic simulation models have large computation times, methods are required to reduce the computation time or reduce the number of simulation runs.

In this paper, we apply a method/framework that is described in (Snelder, et al., 2012) that belongs to the class of methods that reduce the computation time: marginal simulation models. As far as the authors are aware, this is the first time that such a method is applied in order to quantify the robustness effects of project alternatives and to give insights into the contribution of topological and geometrical characteristics to the robustness of a road network. The two step evaluation method is briefly summarized below:

For all project alternatives:

- Step 1: model reference situation without disturbances.
- Step 2: for all links and for all time intervals: model all incident types one by one.

In the first step, for all the project alternatives, a reference situation without disturbances is modelled with an extended version of the macroscopic dynamic equilibrium assignment model ‘Indy’ (Bliemer, 2007). This model has an accurate network loading model that models spillback effects according to the simplified kinematic wave theory of Newell (Yperman, 2007). Indy is a dynamic model, which makes the modelling of time dynamics possible. Among others, the output of this model consists of cumulative inflows and outflows per link to all other next links. This is used as an input for the second step. For future model runs, a fixed demand is assumed. Of course trip generation, distribution, modal split and departure time choices also have to be considered when infrastructure projects are being evaluated. However, these demand effects are outside the scope of this paper.

In the second step, for all the project alternatives, a marginal incident computation model (MIC) (Corthout, et al., 2009) is applied in order to model the effects of incidents. On each link of the network, different incident types are simulated on different time intervals. Each incident type has an incident risk, duration, capacity reduction and maximum percentage of people who choose to reroute. Section 4.2 explains which incident types are used in our case study.

The MIC-module is able to generate an estimate of the impact of local capacity reductions very quickly based on a numerical approximation of the queues that build up upstream of the bottleneck. In fact, the MIC-module extends Equation (1) by taking spillback effect into account. The MIC-module alters the cumulative vehicle numbers of the reference simulation given the capacity reduction that is imposed by a disturbance downstream of the link, reducing them to fit the constraints of the spillback wave on the affected link. First the downstream cumulative vehicle numbers are changed, then the changes in the downstream curve are copied to the upstream boundary according to Newell’s theory (Newell, 1993). If a spillback wave moving on an affected link reaches the upstream node, congestion spills back onto some or all of the incoming links. When the disturbance ends, the capacity at the location of the disturbance is restored. The acceleration wave proceeds through the affected links in a way similar to the spillback wave and finally catches up with the spillback wave. The total delay that is caused by the disturbance can now be computed based on the differences between the cumulative curves of the base run with Indy for the reference situation and the run with the MIC-model for the incident situation.

An approximation method for the use of alternative routes is added to the MIC-module (Snelder, et al., 2012). The approximation algorithm extends Equation 1 to Equation 3 in which RR is the absolute number of vehicles per hour that choose an alternative route.

$$TTT_i = \max \left(0, \frac{\Delta t^2 (rC - C)(I - RR - rC)}{2(I - RR - C)} \right) \quad (3)$$

The output of the method is the total delay per incident per link. Multiplying the total delay by the incident probabilities gives the expected total delay.

It should be noted that the MIC-module neglects lane effects, downstream effects and delayed spillback. Delayed spillback occurs if multiple spillback waves travel towards the same node via different routes. Because delayed spillback is not considered, multiple spillback waves over the same affected link are not simulated, which means that gridlock effects are implicitly neglected. The module also neglects up- and downstream bottlenecks during queue

dissipation. Corthout, et al. (2009) show that despite these simplifications results obtained with MIC vary only slightly from the outcome of a complete dynamic network loading.

4 CASE STUDY – MOTORWAY A27 - THE NETHERLANDS

The method that we presented in section 3 can be applied to all kinds of infrastructure projects all over the world. Based on a case study for the Netherlands, this section illustrates how the method can be used to evaluate the performance of different project alternatives in case of incidents. In order to do so the different network, elements of the project alternatives are considered in detail. For illustrative purposes, the project alternatives are chosen in such a way, that differences between the project alternatives give direct insight in the effects of having a hard shoulder, weaving sections and a parallel road structure.

Section 4.1 describes the project alternatives and their network elements. Section 4.2 presents the incident types that are used in this case study, Section 4.3 presents the model assumptions and calibration and Section 4.4 describes the results.

4.1 Project alternatives and their network elements

In this paper, the focus is on the road network of Utrecht and surroundings in the Netherlands as shown in Figure 7. Much congestion is expected in 2030 caused by a bottleneck on the A27 near Rijnsweerd. Furthermore, there is a large weaving section in the reference situation. The incident risk on that section is relatively high (see Table 2). In order to reduce the congestion and travel times and to improve the safety level, an infrastructure project is planned. Investment is focused around the road section between junction (Kp.) Rijnsweerd and Kp. Lunetten. In Figure 8, a schematic overview of this road section is presented. In the direction from South to North, there are 6 lanes and in the opposite direction there are 4 lanes.

Figure 7: Reference duration of congestion (minutes per day).

Figure 8: Schematic overview Reference between kp Rijnsweerd and Lunetten.

There is a preferred project alternative (alternative 1) and there are two fictive project alternatives, which require less space (in order to preserve nature) which illustrate what the effect would be if no hard shoulders are constructed or if less lanes are constructed and the

road is not split in two separate carriageways. The characteristics of the project alternatives are shown in Table 1. Figure 9 shows the alternatives schematically.

Table 1: Characteristics of the Reference network and project alternative between kp Rijnsweerd and Lunetten

Variant	Hard Shoulder	Parallel road structure	Number of lanes
Reference	south-north: no north-south: yes	No	south-north: 6 north-south: 4
Alternative 1	both directions: yes	Yes	south-north: 5+2 north-south: 3+4
Alternative 2	both directions: no	Yes	south-north: 5+2 north-south: 3+4
Alternative 3	both directions: no	No	both directions: 6

Figure 9: Schematic overview of the alternatives; a) Impression of the parallel road network structure of alternative 1 and 2; b),c) and d) number of lanes between kp. Rijnsweerd and kp. Lunetten in alternative 1, 2 and 3. Source figure a): (Min I&M/RWS, 2012)

The alternatives are chosen in such a way that they lead to the following insights:

- A comparison between the reference situation and alternative 1 shows if the proposed measures to split the roadway into a parallel carriageway structure and to increase the total number of lanes to 7 lanes in both directions with a hard shoulder, improve the network performance under normal conditions and improve the robustness and traffic safety.
- By comparing alternative 1 with alternative, 2 the impact of having hard shoulders is shown.
- A comparison between alternative 2 and 3 shows the impact of having a parallel road structure with one extra lane.

4.2 General incident types

In this paper, we distinguish between 6 different incident types: 1) Car breakdown, 2) Truck breakdown, 3) Truck accident, 4) Car accident with one lane closed, 5) Car accident with two lanes closed, 6) Car accident with three or more lanes closed. Only incidents that result in a closure of one or more lanes are considered, because these are the incidents that have the highest impact on the traffic flow. It must be noted that a car breakdown on the hard shoulder can also affect the capacity and therewith the traffic flow. However, these incidents are not considered.

Table 2 shows an average risk, duration and capacity reduction factor for each incident type. The incident risk (risk per million vehicle kilometers), the duration (hours) and the capacity reduction factor of each incident type is determined for regular road sections, for road sections without a hard shoulder, for weaving sections and for weaving sections without a hard shoulder based on a literature review (see below). Other important influencing factors like the number of lanes of a road and the volume/capacity ratio are also considered. It must be noted that the incident risk, the duration and capacity reduction factor can vary per country or even per road section (Calvert, et al., 2016). Therefore, we combined the international literature with the results of an extensive data-analysis for the Netherlands and specifically Utrecht and surroundings.

For regular road sections with a hard shoulder the incident risk, duration and capacity reduction of the different incident types are based on an extensive data-analyses for the motorway network of the Netherlands for 2007-2009 (Snelder, and Drolenga, 2012). Since the data analysis did not distinguish between road sections with 4, 5 and 6 lanes, we refined the table based on literature about the effect of the number of lanes on the incident risk (Milton, and Mannering, 1998; Martin, 2002; Noland, and Oh, 2004; Kononov, et al., 2008; Wang, et al., 2009). The literature indicates that the incident risk increases with the number of lanes. Kononov et al. (2008) explain this by the fact that having more lanes results in more lane changes with a higher incident risk as a result. Furthermore, the number of lanes that are closed in case of an incident increases with the number of lanes. The extent to which the risk increases varies between 10%-100%. In this paper, we assume that the risk on a lane in the middle of two other lanes is twice as high as the risk on a lane that has only one neighboring lane (based on Kononov et al., 2008).

Effect of hard shoulders on the incident risk, duration, capacity reduction

The numbers for road sections without a hard shoulder are based on literature and expert judgment. The literature about the effect of hard shoulders mainly focuses on the incident risk. Aron et al. (2013), In 't Veld (2009), Van Veluwen and de Vries (2013), Halbert and Orme (2008), Hoornaert et al. (2012) and Lemke (2010) point out that the incident risk does not increase if a peak hour lane is opened (in that case the hard shoulder is used as a lane). It

should be noted that most literature does not correct for a change in congestion patterns. Secondly, the results can be affected by advanced safety systems on peak hour lanes. Bauer et al. (2004) found that the incident risk increases by 10% – 11% when the capacity is extended from 4 to 5 lanes and the hard shoulder is removed. For a capacity extension from 5 to 6 lanes and removing the hard shoulder, they found a non-significant increase of 3-7%. In (Agency, 2012) it is explained that the number of accidents increases by 9% if there is no hard shoulder. Finally, Abdel-Aty and Radwan (2000), Milton and Mannering (1998) and Noland and Oh (2004) show that the width of the hard shoulder affects the incident risk as well: the smaller the width, the bigger the risk. Based on the previously mentioned literature, we made two assumptions on the incident risk and tested them both. First we assumed that if there is no hard shoulder, the incident risk is equal to that of a regular road section. In a sensitivity analysis, we assume that the incident risk increases by 9% when there is no hard shoulder. Furthermore, the risk of a car breakdown in which one or more lanes have to be closed increases because there is no hard shoulder. The probability that more lanes need to be closed in case of an accident increases slightly, because cars can't deviate to a hard shoulder and emergency services can't use the hard shoulder to arrive at the accident location. We also assume that the duration of the accidents increases by 2.5 minutes, because the damaged cars cannot be moved to the hard shoulder and because it takes longer for emergency services to reach the accident. This assumption is based on a data-analysis for Utrecht (Snelder, et al., 2014).

Effect of weaving sections on the incident risk, duration, capacity reduction

The literature on traffic safety on weaving sections indicates that the incident risk increases on weaving sections (Golob, et al., 2004b; Snelder, et al., 2010). In line with this, we assume that the incident risk increases by 14% on weaving sections and that for 4% of the incidents, an additional lane has to be closed due to an increase in side-on collisions.

Effect of the level of congestion on the incident risk

Congestion levels affect the risk of incidents occurring. This is not included in the table, but is taken into consideration in the model simulations. Literature on the relation between congestion level and incident risk is not unanimous. Lord et al. (2005), Golob et al. (2004a), Golob and Recker (2004), Zhou and Sisiopiku (1997) and Martin (2002) found a U-shaped relation whereas Lord et al. (2005) and Kononov et al. (2008) found a relation that initially increases with the traffic volume and then decreases near capacity. Finally, Garber and Ehrhart (2000), Snelder and Drolenga (2012) and Kononov et al. (2012) found an increasing incident risk as the congestion level increases. An explanation for different results can be found in the fact that different methodologies are used and in the fact the studies are carried out in different countries with different traffic regulations. In this research, we based our assumptions on Snelder, et al., (2010) and Snelder and Drolenga (2012). In a sensitivity analysis, we also applied the assumption of Kononov et al. (2008) in which the incident risk first increases and then decreases as the congestion level increases.

Table 2: Incident types and their characteristics for the Netherlands

ID	Incident type	With hard shoulder			without hard shoulder			Weaving section			Weaving section without hard shoulder		
		risk	duration	capacity reduction factor	risk	duration	capacity reduction factor	risk	duration	capacity reduction factor	risk	duration	capacity reduction factor
Road with 1 lane	1 car break down on the road	0.18	0.28	0.75	0.31	0.33	0.75	0.18	0.28	0.75	0.31	0.33	0.75
	2 truck break down	0.53	0.53	0.80	0.70	0.58	0.80	0.53	0.53	0.80	0.70	0.58	0.80
	3 truck accident	0.18	1.80	0.80	0.23	1.84	0.80	0.21	1.80	0.80	0.26	1.84	0.80
	4 car accident with one lane closed	0.13	0.44	0.60	0.19	0.49	0.60	0.14	0.44	0.60	0.21	0.49	0.60
	5 road closure	0.03	0.89	0.90	0.05	0.93	0.90	0.04	0.89	0.90	0.06	0.93	0.90
Road with 2 lanes	6 car break down on the road	0.18	0.28	0.53	0.31	0.33	0.53	0.18	0.28	0.53	0.31	0.33	0.53
	7 truck break down	0.53	0.53	0.53	0.70	0.58	0.53	0.53	0.53	0.53	0.70	0.58	0.53
	8 truck accident	0.18	1.80	0.76	0.23	1.84	0.76	0.21	1.80	0.76	0.26	1.84	0.76
	9 car accident with one lane closed	0.13	0.44	0.60	0.19	0.49	0.60	0.14	0.44	0.60	0.21	0.49	0.60
	10 car accident with two lanes closed	0.03	0.89	0.90	0.05	0.93	0.90	0.04	0.89	0.90	0.06	0.93	0.90
Road with 3 lanes	11 car break down on the road	0.24	0.28	0.36	0.37	0.33	0.36	0.24	0.28	0.36	0.37	0.33	0.36
	12 truck break down	0.70	0.53	0.38	0.87	0.58	0.38	0.70	0.53	0.38	0.87	0.58	0.38
	13 truck accident	0.24	1.80	0.59	0.29	1.84	0.59	0.27	1.80	0.59	0.33	1.84	0.59
	14 car accident with one lane closed	0.10	0.41	0.40	0.16	0.46	0.40	0.11	0.41	0.40	0.17	0.46	0.40
	15 car accident with two lanes closed	0.03	0.65	0.70	0.05	0.69	0.70	0.04	0.65	0.70	0.06	0.69	0.70
Road with 4 lanes	16 car accident with three or more lanes closed	0.00	1.93	0.90	0.01	1.98	0.90	0.01	1.93	0.90	0.01	1.98	0.90
	17 car break down on the road	0.27	0.28	0.28	0.40	0.33	0.28	0.27	0.28	0.28	0.40	0.33	0.28
	18 truck break down	0.79	0.53	0.31	0.96	0.58	0.31	0.79	0.53	0.31	0.96	0.58	0.31
	19 truck accident	0.27	1.80	0.46	0.32	1.84	0.46	0.31	1.80	0.46	0.36	1.84	0.46
	20 car accident with one lane closed	0.18	0.38	0.30	0.24	0.42	0.30	0.19	0.38	0.30	0.26	0.42	0.30
Road with 5 lanes	21 car accident with two lanes closed	0.05	0.57	0.55	0.08	0.61	0.55	0.06	0.57	0.55	0.09	0.61	0.55
	22 car accident with three or more lanes closed	0.01	1.22	0.80	0.01	1.26	0.80	0.01	1.22	0.80	0.02	1.26	0.80
	23 car break down on the road	0.29	0.28	0.28	0.42	0.33	0.28	0.29	0.28	0.28	0.42	0.33	0.28
	24 truck break down	0.84	0.53	0.31	1.02	0.58	0.31	0.84	0.53	0.31	1.02	0.58	0.31
	25 truck accident	0.29	1.80	0.46	0.34	1.84	0.46	0.33	1.80	0.46	0.38	1.84	0.46
Road with 6 lanes	26 car accident with one lane closed	0.19	0.38	0.30	0.25	0.42	0.30	0.21	0.38	0.30	0.27	0.42	0.30
	27 car accident with two lanes closed	0.05	0.57	0.55	0.08	0.61	0.55	0.07	0.57	0.55	0.10	0.61	0.55
	28 car accident with three or more lanes closed	0.01	1.22	0.80	0.02	1.26	0.80	0.01	1.22	0.80	0.02	1.26	0.80
	29 car break down on the road	0.30	0.28	0.28	0.43	0.33	0.28	0.30	0.28	0.28	0.43	0.33	0.28
	30 truck break down	0.88	0.53	0.31	1.05	0.58	0.31	0.88	0.53	0.31	1.05	0.58	0.31
Road with 6 lanes	31 truck accident	0.30	1.80	0.46	0.35	1.84	0.46	0.34	1.80	0.46	0.40	1.84	0.46
	32 car accident with one lane closed	0.20	0.38	0.30	0.26	0.42	0.30	0.22	0.38	0.30	0.28	0.42	0.30
	33 car accident with two lanes closed	0.06	0.57	0.55	0.09	0.61	0.55	0.07	0.57	0.55	0.10	0.61	0.55
	34 car accident with three or more lanes closed	0.01	1.22	0.80	0.02	1.26	0.80	0.01	1.22	0.80	0.02	1.26	0.80

4.3 Model assumptions and calibration

Dynamic traffic assignment model Indy is calibrated for the base year 2012. The model is first calibrated for regular conditions without incidents and roadworks based on traffic counts. Thereafter, the marginal incident module is calibrated based on the results of a data-analysis of incidents that occurred in 2010 and 2011 on the A12 and the A27. With the method that is described in Snelder, et al. (2013) the total delay caused by each of those incidents was computed. This was compared with the results of the marginal incident module. The model was calibrated by fine-tuning the assumptions that were made on the percentage of drivers that are willing to take an alternative route.

Future scenario

In the Netherlands, before 2016, four future scenarios are used in project appraisal. These future scenarios are described in (Janssen, et al., 2006). The scenarios give a quantitative description of long-term trends, such as the decreasing household size, the ageing population, international migration, economic growth and increasing personal welfare. In this case study, we considered the Global Economy (GE) scenario, which assumes the highest growth of traffic. We chose this scenario, because the relative difference in road network performance between the project alternatives is illustrated the best in this scenario.

4.4 Results and discussion

This section describes the results of incidents simulated on two levels: the project area level and the road section between highway junction Lunetten and highway junction Rijnsweerd. These two levels are shown in Figure 10. Of course, the alternatives can also have effects on the roads outside the project area, however the largest effects occur in the project area.

Figure 10: a) Model network, b) project area – red links; c) road section between kp. Lunetten and kp. Rijnsweerd.

Table 3 summarizes the results for the regular situation. This is the situation without incidents. The results for alternative 1 and 2 are identical, because these alternatives only differ in the presence of a hard shoulder, which doesn't affect the regular situation. The table shows that the regular total travel time delay reduces by 53% in alternative 1 and 2 and by 10% in alternative 3. Figure 11 shows the congestion locations and more specifically the duration of congestion per day. The congestion is expressed in minutes per day that the speed is below 50 km/hour and below 70% of the speed limit. A comparison between Figure 11 and Figure 7 shows that in all alternatives the bottleneck near Rijnsweerd in the southern direction is relieved. In all 3 alternatives new bottlenecks occur south of the original bottleneck.

Table 3: Results regular situation without incidents per workday

Variant	Vehicle kilometers in the project area x 1,000,000 (km)	Total travel time delay in the project area x 10,000 (hours)
Reference 2012	3.61	0.59
Reference 2030	4.29	1.65
Alternative 1 +2	4.33 (+1%)	0.77 (-53%)
Alternative 3	4.34 (+1%)	1.49 (-10%)

Figure 11: duration of congestion (minutes per day) in a) alternative 1 and 2 and b) alternative 3

The most important results of the incident analyses are summarized in Table 4. The relative differences between the alternatives and the reference situation for 2030 are shown in brackets. Secondly, the relative difference between alternative 2 and 1, and between alternative 3 and 2 are shown.

Table 4: Expected number of incidents and the delay caused by the incidents

	Expected number of incidents per workday		Average total delay per incident (vehicle hours)		Total delay caused by all incidents per workday (vehicle hours)	
	Project area	Kp. Lun. – kp. Rijnsd.	Project area	Kp. Lun. – kp. Rijnsd.	Project area	Kp. Lun. – kp. Rijnsd.
Ref. 2012	1.62	0.34	227	82	368	28
Ref. 2030	1.99	0.42	462	512	919	215
Alt. 1	1.91 (-4%)	0.31 (-27%)	226 (-51%)	148 (-71%)	433 (-53%)	46 (-78%)
Alt. 2	2.05 (+3%;+7%)	0.41 (+2%;+32%)	226 (-51%; 0%)	174 (-66%; +18%)	465 (-49%;+7%)	75 (-65%;+63%)
Alt 3	2.12 (+7%;+3%)	0.51 (+20%; +24%)	282 (-39%; +25%)	154 (-70%;- 11%)	600 (-35%; +29%)	78 (-64%;+4%)

Table 4 shows that alternative 1 reduces the total expected delay that is caused by incidents by 53% in the project area and by 78% on the road section kp. Lunetten-kp. Rijnsweerd. The

reduction is larger on this road section than in the project area because the measures that are taken focus on this road section. The reductions are smaller for the other alternatives, but they are still substantial. The explanation for the large reduction can be found in the change of the expected number of incidents and a reduction in the delay per incident as explained below.

Expected number of incidents: Alternative 1 reduces the expected number of incidents compared to the reference situation for 2030 by 4% in the project area and by 27% on the road section between kp. Lunetten and kp. Rijnsweerd. This can be explained by the fact that there are less lanes per roadway in alternative 1 (5 and 2 in the direction South – North and 3 and 4 in the direction North-South) than in the reference situation, which has 6 and 4 lanes respectively. As explained in section 2, the incident risk is lower for roadways with a lower number of lanes. Furthermore, the weaving section on the road section between kp. Lunetten and kp. Rijnsweerd is less complex in alternative 1. Finally, alternative 1 has a hard shoulder in the Southerly direction, which reduces the risk of incidents that may block one or more lanes.

If there are no hard shoulders, the positive effect of the parallel carriageway structure is overcompensated by the negative effect of missing hard shoulders as can be concluded from the fact that expected number of incidents increase in alternative 2 compared to the reference situation for 2030. The increase between alternative 1 and 2 is 7% in the project area and 32% on the road section between kp. Lunetten and kp. Rijnsweerd. In alternative 3, the expected number of incidents is even higher, because there is no parallel carriageway structure present and no hard shoulders.

Delay per incident: Table 4 shows that the average travel time per incident decreases in all project alternatives (39% to 51%). On the road section between kp. Lunetten and kp. Rijnsweerd, this decrease is even larger: 66% to 71%. Alternative 1 has the largest reduction, because this alternative has more spare capacity than alternative 3 since it has 7 instead of 6 lanes per direction. The traffic volume per roadway is lower in alternative 1 compared to alternative 3, because there are fewer lanes per roadway. For an incident, this results in a lower total delay per incident. Finally, the duration of the incidents is lower in alternative 1 than in alternative 3, because there is a hard shoulder which allows emergency service to reach the incident location faster.

The relative changes in the project area show that the changes in average travel time per incident are much higher than the changes in the expected number of incidents. This indicates that having spare capacity is extremely important. On the road section between junction Lunetten and junction Rijnsweerd, a large decrease in the number of incidents can also be seen in alternative 1 compared to the references situation for 2030. This indicates that having a parallel carriageway structure and having hard shoulder available is important as well.

Finally, a sensitivity analysis was carried out on the assumptions that were made in section 4.2. The results indicate that in all alternative assumptions the delay caused by incidents is the highest in Reference 2030, followed by alternative 3, alternative 2 and alternative 1 which is identical to the results described above. The relative differences are also quite stable. The relative difference between alternative 3 and Reference 2030 is between -37% and -29%. The relative difference between alternative 2 and Reference 2030 is between -55% and -52% and the relative difference between alternative 1 and Reference 2030 is between -52% and -43%.

Discussion

For the case study, the following can be concluded: alternative 3, in which 2 additional lanes are constructed reduces the congestion under regular conditions with 10% and improves the robustness by 35%. Adding an additional lane in combination with creating a parallel road structure reduces the congestion under regular conditions with 53% and improves the robustness by 53% if a hard shoulder is present and by 49% if hard shoulder is not present. Because of the high congestion level under regular conditions, the relative impact of delays caused by incidents is low (3.9% to 5.9%) compared to other roads in the Netherlands since the average share of incidents in the total travel time delays is 20-25% in the Netherlands (Snelder, et al., 2013). Nevertheless, this can make a difference between positive and negative benefit-cost ratios of the project alternatives and the ranking of the project alternatives.

In the next step of the project, appraisal of the safety effects need to be computed. The incident analysis can be used as a starting point for safety analysis, since an expected number of different incident types is computed for each project alternative. An additional effort is required to compute the share of fatal accidents and accidents with (serious) injuries. Furthermore, a similar approach, as used for the incidents, can be used for small scale roadworks. Large scale roadworks that are announced in advanced require an extra effort in order to take changes in the number of trips, destinations, mode choices and departure times into account. Finally, the costs of the project alternatives and other benefits like environmental benefits need to be analyzed. This is outside the scope of this paper.

5 CONCLUSIONS AND RECOMMENDATIONS

This paper showed how hard shoulders, the number of lanes, parallel road structures and weaving sections contribute to the robustness of a road network and how, based on these network characteristics, the robustness effects of project alternatives can be computed with a marginal simulation approach.

Based on the previous sections, we conclude that literature is not unanimous about the effect of hard shoulders on the incident risk. If a hard shoulder is missing, the incident risk stays equal or increases by up to 9%. However, the risk that at least one lane is blocked increases, because car and truck breakdowns block at least one lane when there is no hard shoulder. Furthermore, the duration of incidents can increase as well when there is no hard shoulder due to spatial restriction for recovery of damaged vehicles and because it takes longer for emergency services to reach the accident location. The analytical example showed that having a hard shoulder becomes especially relevant in case of a high volume to capacity ratio and high traffic volumes.

The number of lanes, parallel carriageway structure and weaving section: the literature review showed that the incident risk increases with the number of lanes. Furthermore, the number of lanes that are closed in case of an incident increases with the number of lanes of a roadway. When the traffic volume is sustained, having extra lanes increases the spare capacity and therewith reduces the delay caused by incidents. However, extra capacity can attract extra traffic. When a road with more lanes is heavily used, incidents cause more delay. The question whether or not it is advisable to split a road with many lanes in a parallel road structure cannot be answered unambiguously. The combined incident risk of the two roadways of a parallel road structure is lower than for a single roadway. With respect to the delay caused by incidents, it appears that there is a switch point. When vehicles can switch to the parallel roadway in case of an incident, a parallel structure is more robust at low traffic

volumes, because the spare capacity of the parallel roadway can be used. When traffic volumes are high, a single road performs better than a parallel carriageway structure, because as soon as the queue spills back upstream of the split point of a parallel carriageway structure the traffic to the parallel roadway also becomes partially blocked. Also the mixture of incidents is important. The switch point occurs at lower volumes when the capacity reduction is larger. Finally, the length of parallel carriageway structure and the complexity of the weaving sections that are required are important. Therefore, the question whether or not it is advisable to split a roadway into two roadways needs to be answered on a network level and requires an additional analysis of the safety benefits, impact of roadworks and costs. The modelling approach presented in this paper can be used for that purpose as demonstrated and discussed in this paper.

With respect to the modelling approach, it can be concluded that the approach can be used to simulate hundred thousands of incidents. This makes the method suitable for evaluating robustness benefits of different project alternatives. The method considers spillback effects, but neglects lane effects, downstream effects, delayed spillback and up- and downstream bottlenecks during queue dissipation. Although Corthout, et al. (2009) showed that these simplifications don't have a large impact, we recommend analyzing the impact of the simplifications on the robustness effects of project alternatives in more detail, because small deviations in a single model run might add to larger deviations when thousands of model runs are combined. We also recommend to determine confidence intervals for incident risks, durations and capacity reductions of different incident types for regular road sections, for road sections without a hard shoulder, for weaving sections and for weaving sections without a hard shoulder. Finally, we recommend validating the results in more detail based on a data analysis of delays caused by incidents on road sections with different topological and geometrical characteristics.

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