Stellingen

1. In tegenstelling tot deplacementsschepen waarbij het nodig is onderscheid te maken tussen rekenmethoden voor weerstand en voortstuwing, voor gedrag in golven en voor manoeuvreren, kan voor draagvleugelschepen worden volstaan met één rekenmethode.

2. Wanneer voor modelproeven voor draagvleugelschepen van vrijvarende modellen gebruik wordt gemaakt, kan bij de extrapolatie naar ware grootte slechts gedeeltelijk worden gecorrigeerd voor de visceuze effecten in de liftkracht werkend op de draagvleugels.

3. Naast toepassing voor snelle schepen bieden draagvleugels ongekende mogelijkheden voor recreatieve vaartuigen.

4. Recente gevallen van media hysterie vragen om striktere gedragsregels voor de media.

5. De huidige gehaastheid van de maatschappij zou wel eens op 1 januari 2000, in ieder geval tijdelijk, door één van de grootste veroorzakers ervan drastisch af kunnen nemen.

6. Bedrijven die gebruik maken van telefonische marketing dienen de door hen hiermee opgewekte ergernis financieel te compenseren.

7. De kans dat Europese samenwerkingsprojecten op technologisch gebied succesvol worden uitgevoerd, wordt verhoogd door zich bij het opzetten van het projektvoorstel minder te laten leiden door politieke overwegingen die het voorstel een grotere acceptatiekans geven bij de Europese Commissie.

8. De westerse maatschappij begint eenzelfde mate van decadentie te vertonen als het Romeinse Rijk in zijn nadagen.

Stellingen behorende bij het proefschrift van F. van Walree getiteld: 'Computational methods for hydrofoil craft in steady and unsteady flow'.
COMPUTATIONAL METHODS FOR HYDROFOIL CRAFT IN STEADY AND UNSTEADY FLOW

Frans van Walree
COMPUTATIONAL METHODS FOR HYDROFOIL CRAFT IN STEADY AND UNSTEADY FLOW

Proefschrift

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus prof. ir. K.F. Wakker, in het openbaar te verdedigen ten overstanaan van een commissie, door het College voor Promoties aangewezen.

op dinsdag 9 maart 1999 te 13.30 uur

door Frans VAN WALREE
scheepsbouwkundig ingenieur
geboren te Noordwijk
Dit proefschrift is goedgekeurd door de promotor:
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</table>
NOMENCLATURE

A  Area
AR Aspect ratio
B Hull beam, bias error
b Foil span
br Model basin width
Cd Two-dimensional viscous drag coefficient
Ci Interaction coefficient for foil→hull interaction
Cf Interaction coefficient for hull→foil interaction
Cp Pressure coefficient
Cl Three-dimensional lift coefficient
Ct Two-dimensional lift coefficient
Clx Three-dimensional lift curve slope
Cla Two-dimensional lift curve slope
Cd1 Induced drag coefficient, intake drag coefficient
Cdm Minimum sectional drag coefficient
Cln Lift coefficient at angle of attack where $C_d = C_{dm}$
$\Delta C_d$ Incremental sectional drag coefficient due to lift
Cf Frictional resistance coefficient
$\Delta C_f$ Roughness allowance on $C_f$
Cx Force coefficient vector (ship-fixed axis system)
Cm Moment coefficient vector (ship-fixed axis system)
Cp Hull draft coefficient Series 65
Cix Hull wetted chine length coefficient Series 65
Cpk Hull wetted keel coefficient Series 65
Cm Hull moment coefficient Series 65
Cr Hull resistance coefficient Series 65
Cs Hull wetted surface coefficient Series 65
Cra Hull wind resistance coefficient
C(k) Theodorsen function
c Foil chord
ci Foil chord at tip
cr Foil chord at root
cf/ct Flap chord to foil chord ratio
D Hull draft at stern, drag force, propeller diameter
d Diameter
Ei Complex exponential integral
F Force
$F_n, F_{nc}, F_{nh}, F_{nd}, F_{nv}$ Froude numbers based on hull length, foil chord, foil submergence, water depth and hull volume respectively
$f/c$ Camber to chord ratio
$fa$ Viscosity correction factor on lift curve slope
$fb$ Viscosity correction factor on flap efficiency
$fc$ Viscosity correction factor on camber
\( f_r \)  Thickness correction factor on lift curve slope
\( f(x) \)  Camber line function
\( G \)  Green's function
\( G' \)  Time domain free surface Green's function term
\( G'^{\circ} \)  Time domain vortex plus biplane image Green's function term
\( g \)  Gravitational constant
\( H \)  Step function, head rise
\( H_{0.1} \)  Hankel functions
\( h \)  Submergence, height
\( I \)  Integral functions, mass inertia moments
\( i \)  Imaginary unit
\( J \)  Advance coefficient
\( K_T \)  Thrust coefficient propeller
\( K_Q \)  Torque coefficient propeller
\( k \)  Reduced frequency
\( k_e \)  Reduced frequency based on frequency of encounter
\( k_r \)  Equivalent sand grain roughness height
\( k_n \)  Wave number for unsteady flow
\( k_0 \)  Wave number for steady flow
\( L \)  Lift force, length
\( l \)  Length
\( N \)  Number of chordwise vortex elements
\( M \)  Moment, number of spanwise vortex elements
\( m \)  Mass
\( n \)  Rate of revolutions
\( n \)  Normal vector
\( P_D \)  Delivered power
\( p \)  Pressure
\( Q_e \)  Engine torque
\( Q_p \)  Propeller torque
\( Q_f \)  Flow rate waterjet system
\( R \)  Distance between field and singularity points
\( R_0 \)  Distance between field point and biplane image of singularity point
\( R_h \)  Hull residuary resistance
\( R_T \)  Total resistance
\( R_s \)  Reynolds number
\( S \)  Reference area
\( S_e \)  Slenderness ratio
\( S_o(\omega) \)  Wave spectral density function
\( S_o(\omega_o, \omega) \)  Wind spectral density function
\( s \)  Spanwise coordinate
\( \Delta s \)  Vortex strip width
\( T \)  Thrust force
\( T_0 \)  Coordinate transformation matrix
\( t \)  Time, thrust deduction fraction
<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
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<tr>
<td>$t/c$</td>
<td>Thickness to chord ratio foil section</td>
</tr>
<tr>
<td>$\Delta t_d$</td>
<td>Default time step</td>
</tr>
<tr>
<td>$U$</td>
<td>Free stream velocity, speed of advance</td>
</tr>
<tr>
<td>$(u,v,w)$</td>
<td>Velocity vector components</td>
</tr>
<tr>
<td>$(u_w,v_w,w_w)$</td>
<td>Wave orbital velocity vector components</td>
</tr>
<tr>
<td>$(\Delta u,\Delta v,\Delta w)$</td>
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<tr>
<td>$V_u$</td>
<td>Wind velocity</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>$W$</td>
<td>Craft weight</td>
</tr>
<tr>
<td>$(X,Y,Z)$</td>
<td>Force components (ship-fixed axis system)</td>
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<td>$(x,y,z)$</td>
<td>General and ship-fixed coordinate axes</td>
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<td>$(x_0,y_0,z_0)$</td>
<td>Space-fixed coordinate axes</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>Gradient operator, submerged hull volume</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Hull displacement, Laplace operator</td>
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<tr>
<td>$\alpha$</td>
<td>Incidence angle</td>
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<tr>
<td>$\alpha_0$</td>
<td>Zero lift angle of attack</td>
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<td>$\alpha_0$</td>
<td>Flap efficiency</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Dihedral angle</td>
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<td>$\beta_h$</td>
<td>Hull deadrise angle</td>
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<td>$\Gamma$</td>
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<td>$\gamma$</td>
<td>Vortex strength</td>
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<td>$\epsilon$</td>
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<td>$\Phi, \phi$</td>
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<td>$\Phi_r, \Phi, \phi_w$</td>
<td>Total, perturbation and incident wave potentials for unsteady flow</td>
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<tr>
<td>$\lambda$</td>
<td>Wave length</td>
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<tr>
<td>$\mu$</td>
<td>Doublet strength</td>
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<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
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<tr>
<td>$\tau$</td>
<td>Trim angle</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>Taper ratio</td>
</tr>
<tr>
<td>$\tau_{wa}$</td>
<td>Critical frequency</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation, cavitation number</td>
</tr>
</tbody>
</table>
\( \psi_w \)  
Wind direction

\( \psi_w \)  
Wave direction

\( \omega \)  
Frequency

\( \omega_r \)  
Frequency of encounter

\( \omega_p \)  
Propeller rate of rotation

(\( \xi, \eta, \zeta \))  
Singularity coordinates

(\( \phi, \theta, \psi \))  
Angular orientation vector
1. INTRODUCTION

1.1 General

One way of discriminating between ship types is by the nature of the vertical force supporting the weight of the ship. Conventional displacement type ships rely on buoyancy or hydrostatic forces due to the displacement of water by their submerged volume. The terms fast and/or advanced ships are used for ships that fully or partially rely on the use of hydrodynamic lift. Hydrodynamic lift is associated with relatively high speeds. Planing craft use the hydrodynamic lift acting on the hull bottom at Froude numbers above one to reduce the buoyancy and thereby their resistance. The Froude number $F_n$ is defined as:

$$F_n = \frac{U}{\sqrt{gL}}$$  \hspace{1cm} (1.1)

where $U$ is the velocity and $L$ the waterline length of the ship and $g$ is the gravitational constant. At planing conditions, for $F_n > 1.5$, the induced or pressure resistance is independent of speed and proportional to the trim angle and the weight of the craft.

Craft equipped with hydrofoils use a lift force that is proportional to the speed squared. The lift force on a hydrofoil originates from the particular shape of the foil section which results in a high flow velocity over the section’s upper side, relative to the lower side. A high flow velocity is associated with a low pressure or suction force, which results in the hydrodynamic lift.

During the last decade research into and development of fast and advanced ships has increased significantly, as can for instance be observed from the biannual Conferences on Fast Sea Transportation: FAST-91 through FAST-97. This development is due to the growing demand for fast and comfortable transportation of passengers and time sensitive goods over water. Hydrofoils provide means to satisfy this demand, see Akagi (1991, 1993) and Gee and Dudson (1993). The hydrodynamic lift acting on hydrofoils may be used for three purposes. Firstly, hydrodynamic lift reduces the submerged hull volume whereby, at relatively high speeds, the resistance of the vessel decreases. Secondly, lifting the hull above water reduces the wave excitation on the hull. Thirdly, hydrodynamic lift is used as a control force to stabilize the ship’s motions in waves.

Apart from hydrofoil craft whereby the hull is completely lifted above the water surface, a series of hybrid ship types have appeared. Here the hull is lifted only slightly or partially above water as the main purpose of the foils is to provide hydrodynamic lift for stabilization. Hybrid types include hydrofoil supported monohulls, multihulls, surface effect ships and craft consisting of a fully submerged body of revolution, hydrofoils and an above water hull, see Meyer (1991). Although this thesis is restricted to pure hydrofoil craft, the developed computational methods are also applicable to foil systems applied on hybrid hydrofoil supported craft. Figure 1.1 shows a categorization of hydrofoil craft types.
A hydrofoil craft has two modes of operation: the low speed hullborne mode and, with increasing speed through take-off, the foilborne mode. The take-off speed is defined as the speed whereby the hull is just fully desubmerged. Typical attitude and resistance curves for hydrofoil craft are shown in Figure 1.2. The attitude defines the position of the craft relative to the watersurface by means of the hull draft and trim angle. As speed increases towards take-off, the craft experiences an increasing trim angle and a gradual rise of the centre of gravity due to hydrodynamic lift forces acting on the hull and on the foils. At or just before the take-off speed, typically at one half of the cruise speed, the maximum resistance or hump drag appears. After take-off the trim gradually decreases and the resistance initially reduces with speed as the lower foil lift coefficient and wetted area reduce induced and frictional drag respectively. At higher speeds the resistance curve rises again due to the relatively strong growth of the frictional drag.

A number of hydrofoil configurations may be used. Distinctions can be made with respect to the distribution of the lift over the forward and aft foil systems, the submergence of the foil system, the type of foil section used and the way the craft is stabilized. The description of hydrofoil types and configurations is based on reviews given by the Reports of the High Speed Marine Vehicle Committee of the 16th and 17th ITTC, see Savitsky et al. (1981) and ITTC (1984).
1.1 General

Figure 1.2a  Draft and trim versus speed

Figure 1.2b  Resistance and thrust versus speed
The lift may be divided evenly over the forward and aft foil in a tandem configuration, the lift may be derived mainly from the forward foil in a conventional or airplane configuration or the lift may be derived mainly from the aft foil in a canard configuration. Both the forward and aft foils may be split into two parts. The canard configuration has certain advantages with respect to seakeeping and the interaction between the two foils. Figure 1.3 shows a number of foil configurations.

![Foil configurations](image)

Figure 1.3 Foil configurations - From Ellsworth (1967)

Hydrofoils may be surface piercing or fully submerged, see Figure 1.1. Surface piercing hydrofoils have the advantage of lift control by means of lifting area variation so that these types of foils are inherently stable and do not necessarily require additional lift control mechanisms for safe operation.

Fully, but shallowly submerged hydrofoils obtain some lift control through the free surface effect which reduces lift as the hydrofoil approaches the water surface. These types of hydrofoils are used in inland waterways only, as the foils are vulnerable to broaching in waves.

Fully, relatively deeply submerged hydrofoils do need a lift control system since their lifting area does not vary with submergence. Trailling edge flaps linked to a digital ride control system are nowadays commonly used for lift control, not only for operation in waves but also to assist take-off and to enable manoeuvring. Due to the lower wave excitation and high effectiveness of digital ride control systems coupled to trailing edge flaps, the seakeeping behaviour of deeply submerged hydrofoils is superior to that of surface piercing hydrofoils.
1.1 General

Virtually all hydrofoil craft in service today use sub-cavitation foil sections. These foil section types can be used up to speeds of 40 to 45 knots in moderate sea conditions without serious performance degradation due to cavitation, see Acosta (1973). For the higher speed regions base vented and super-cavitating foil sections have been used on experimental and prototype hydrofoil craft, see Ellsworth (1967).

In the design of hydrofoil craft it is required to investigate a number of hydrodynamic aspects. These aspects relate to the powering requirements, seakeeping characteristics, manoeuvring capabilities and the susceptibility to cavitation. Traditionally, experiments with scale models are carried out to obtain information on the hydrodynamic characteristics of ships and maritime constructions, see Van Walree and Buccini (1990). For hydrofoil craft however, model testing is relatively expensive and experimental results have a limited value. Investigating for instance the cavitation characteristics in waves requires a depressurized towing tank with wave generators and with a high speed towing carriage. Such a facility does not exist at present. Investigating the seakeeping characteristics of a fully submerged hydrofoil craft with a ride control system requires a large wave basin with a high speed towing carriage. These facilities are rare and the costs of manufacturing a self-propelled scale model with a ride control system actuating trailing edge flaps are rather high. Furthermore, scale effects on the foil lift and drag are significant and can actually make an accurate extrapolation from model scale to prototype impossible.

For these reasons, there is a demand for computational methods which are practical in use and deliver design data with an acceptable level of uncertainty. Such methods should not require a huge computational effort for determining for instance the resistance curve for a single design alternative. Its results should be sufficiently accurate to distinguish between design alternatives and to optimize basic designs with respect to their hydrodynamic characteristics. For instance, predicted trends in resistance due variations in hull form, foil loading and take-off speed should be reliable.

The aim of this thesis is to develop such computational methods. These need to address the integrated hydrodynamic aspects of hydrofoil craft design, rather than to solve one specific hydrodynamic aspect in great detail. Both steady and unsteady flow conditions will be considered. The term steady flow is used here to describe the time independent flow around a hydrofoil craft operating in calm water at a constant speed, trim and flying height. The term unsteady flow is used here for flows which do depend on time, for instance for hydrofoil craft operating in waves or performing manoeuvres.

In the following sections a more detailed description of surface piercing and deeply submerged hydrofoil systems is given first. Next, hydrodynamic aspects of sub-cavitating foil sections are described and some consideration is given to typical hull form types. Furthermore, the major hydrodynamic aspects in hydrofoil craft design are described which pose a number of requirements on computational methods. Finally, a subdivision into computational methods for steady and unsteady flow is made.
1.2 Surface piercing hydrofoil systems

Hydrofoil craft with surface piercing foil systems operate in a speed range from 28 to 40 knots. In the foilborne mode the resistance of these vessel types is less than 50% of similar sized displacement vessels, see Savitsky et al. (1981). Additional merits of surface piercing hydrofoils relative to displacement vessels include good seakeeping qualities and a small speed loss in waves. Table 1.1 shows a review of overall dimensions of surface piercing hydrofoil craft for passenger transport.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td>9 - 40 m</td>
</tr>
<tr>
<td><strong>Beam (hull)</strong></td>
<td>3 - 7 m</td>
</tr>
<tr>
<td><strong>Beam (foils)</strong></td>
<td>3 - 16 m</td>
</tr>
<tr>
<td><strong>Displacement</strong></td>
<td>4 - 200 tons</td>
</tr>
<tr>
<td><strong>Speed (foilborne)</strong></td>
<td>28 - 40 knots</td>
</tr>
<tr>
<td><strong>Foil system aspect ratio</strong></td>
<td>6 - 10</td>
</tr>
</tbody>
</table>

Table 1.1 Main particulars of surface piercing hydrofoil craft

Surface piercing hydrofoil systems feature foils with an increasing camber towards the tips. The chord length of the foil also increases at the tip. This provides additional lift required for a relatively low take-off speed. Streamlined propeller nacelles and blunt based flap actuator housings may be mounted at the intersection of the strut and foil parts. Fences are positioned on the strut and foil parts to prevent ventilation of foil parts operating close to the water surface. The planform of surface piercing foil types is V- or W- shaped, see Figure 1.4.

Figure 1.4 Surface piercing foil system

*From Savitsky et al. (1981)*
1.2 Surface piercing hydrofoil systems

For both surface piercing and fully submerged hydrofoil craft, the craft velocity components introduce variations in the instantaneous angle of attack which provide damping forces and moments. Pitch motions for instance introduce vertical velocities at the foils which result in a stabilizing incidence variation which is generally much larger than the destabilizing geometrical angle of attack variation due to the pitch rotation itself. For surface piercing hydrofoil craft, in the foilborne mode the foil area is partially submerged thus providing a foil area portion for stabilization. A deviation from the equilibrium position results in a change in wetted surface which, in turn, results in restoring forces and moments. Surface piercing foil types are inherently stable in heave, pitch and roll.

When operating in waves the foil submergence varies due to the wave elevation while the flow velocity and angle of attack on a foil section are affected by the wave orbital velocities. These effects introduce excitation forces and moments which depend on the wave direction and wave frequency of encounter. In following seas for instance, the hydrofoil experiences downward wave orbital motions when climbing up in a wave and in the wave crest the orbital velocities reduce the flow velocity over the foil. These effects reduce the foil lift and the craft tends to 'sit down' when a rise would be needed. In order to improve the seakeeping capabilities of surface piercing foil types additional lift control is required. Lift control may be provided by air feeding, angle of attack control and the use of trailing edge flaps. This last option is nowadays the most commonly used. The flaps are of the plain, sealed type.
1.3 Fully submerged hydrofoil systems

Fully submerged hydrofoil craft are designed to operate at speeds above 30 knots, with maximum speeds of 40 to 50 knots. Experimental craft have reached 80 knots. The foils may be either deeply or shallowly submerged. Deeply submerged foil types are running deeper than two chord lengths while shallowly submerged types run at a submergence of less than one chord length. Table 1.2 shows main particulars of commercial and military operated craft.

<table>
<thead>
<tr>
<th>Table 1.2 Main particulars for fully submerged hydrofoil craft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Beam (hull)</td>
</tr>
<tr>
<td>Beam (foils)</td>
</tr>
<tr>
<td>Displacement</td>
</tr>
<tr>
<td>Speed (foilborne)</td>
</tr>
<tr>
<td>Foil system aspect ratio</td>
</tr>
</tbody>
</table>

At present fully submerged hydrofoils feature a canard foil area distribution. The downwash at the aft foil due to the lift generation of the forward foil is relatively low for such a foil arrangement, which is furthermore more stable in longitudinal direction than conventional arrangements. This offers advantages with respect to operation in calm water as well as in a seaway. The foil systems may be split in a starboard and port side portion so that they can be rotated about a longitudinal axis towards a position above water for mooring and low speed operation. Rotation about a transverse axis may also be applied. The foils may have a negative dihedral angle for reasons of stabilization: dihedral increases the lever arm of control forces relative to the centre of gravity. Foil planforms are often swept back and tapered at the tips for optimum lifting efficiency and minimizing structural loads. Streamlined pods are arranged at the intersection of the struts and lifting parts for providing room for flap actuators and waterjet intakes. A typical foil system is shown in Figure 1.5.

Deeply submerged foil systems do not have variable lifting area stabilization and require a continuous control of the lift force for stable operation, even in calm water. The lift control is in general obtained by using trailing edge flaps which are actuated by an automatic control system. These control systems are used for seakeeping as well as manoeuvring purposes. Such a system requires motion sensors, hydraulic systems and control system hardware and software and is therefore relatively complex and expensive. On the other hand, this type of control is more effective than lifting area stabilization due to the speed of actuation and the relatively large control forces that can be generated. Furthermore, as wave orbital velocities decay with submergence and the lifting area does not vary with wave elevation, the wave excitation is generally lower than for surface piercing foil types. Due to the absence of wave excitation forces on the hull, the low displaced volume and the effectiveness of control systems, deeply submerged hydrofoil craft with an automatic control system show superior manoeuvring and seakeeping characteristics relative to other high speed craft.
Figure 1.5  Fully submerged foil system - *From Johnston (1985)*
1.4 Hydrofoil section types

Hydrofoil craft may be classified, apart from their foil system configuration, according to their maximum cruise speed. The maximum cruise speed from a hydrodynamic point of view is mainly determined by the occurrence of cavitation. Cavitation is the formation of vapour bubbles in the flow over the foil area due to local pressures lower than the vapour pressure of the fluid. A low pressure is associated with a high flow velocity, a high lift coefficient and discontinuities in the foil section shape. Pronounced cavitation may result in lift breakdown and a sharp increase in drag. Unsteady formation of cavitation and the often unsteady and violent collapse of the cavities may cause noise, vibration and erosion problems.

Cavitation does not depend solely on the maximum speed of the craft. Variations of the sectional angle of attack due to fore-aft foil interaction, ship motions and wave orbital velocities induce lift force variations with accompanying low pressure regions. In the hydrodynamic design of high speed and/or heavily loaded hydrofoil systems a significant effort must be devoted to the prevention of detrimental cavitation effects. For speeds up to 40-45 knots sub-cavitating foil section types are used which are designed to operate free of cavitation. For higher speeds so-called super-cavitating foil section types are used.

In addition to cavitation, problems may be encountered due to ventilation. Ventilation may have a significant effect on the lift and drag forces acting on the foil and is often an unstable phenomenon. Ventilation can occur through surface pierced foil and strut parts at incidence and through tip vortex tubes that vent to the free surface. Ventilation problems are usually dealt with by the use of so-called fences blocking the passage of air from the free surface.

The hydrodynamic characteristics of deeply submerged, sub-cavitating hydrofoil systems are very similar to subsonic aerodynamic characteristics of aircraft wings. As a consequence most foil and strut sections have been derived from the NACA design data, for example NACA-16 and NACA-66 sections, see Abbott and Von Doenhoff (1958). The last decade has shown the use of so-called YS sections which delay cavitation inception towards higher speeds, see Shen (1985). The NACA sections mentioned and the YS section types have a characteristically flat suction side pressure distribution over a certain incidence range. In practice foil loadings of 80,000 N/m² can be used for operation in moderate sea conditions without risking pronounced cavitation.

Above a certain limiting speed detrimental cavitation effects can not be avoided and an opposite design approach must be taken which aims to encourage the formation of a stable and pronounced cavity. To this end super-cavitating foil sections are used. These foil sections feature a sharp leading edge and a blunt trailing edge. At sufficiently low cavitation numbers the sharp leading edge causes the formation of a fully developed cavity covering the entire suction side of the foil. Cavity collapse occurs well aft of the trailing edge and erosion problems are thereby avoided. Although a significant amount of research has been devoted to the use of super-cavitating foil sections their use has been limited to experimental craft. This is due to problems associated with the generation of a stable cavity form, the low lift to drag ratio at low speeds, free surface effects, structural problems with respect to the thin leading edge and difficulties in achieving a reliable and effective control of lift, see Ellsworth (1967).
1.4 Hydrofoil section types

For the speed region from 40 to 70 knots so-called base vented section types may be used. These section types have a rounded leading edge and a blunt trailing edge. By means of natural ventilation along the cavitating trailing edge of the strut the sensitivity of cavitation and ventilation with respect to the angle of attack is reduced and stable operating conditions are established. The use of these section types on practical hydrofoil craft is not yet established as the majority of the craft operate at speeds below 50 knots and the need for higher speeds is not large at present.
1.5 Hydrofoil craft hull forms

For hydrofoil craft the volume Froude number \( F_{nv} \) is often used to describe the flow conditions around the hull. \( F_{nv} \) is defined by:

\[
F_{nv} = \frac{U}{\sqrt{gV^{1/3}}}
\]  
(1.2)

Here \( V \) is the displaced hull volume, \( U \) is the velocity and \( g \) is the gravitation constant. In the hullborne mode, prior to take-off, the displaced volume reduces while speed increases, resulting in \( F_{nv} \) values in between 2 and 5. For the upper \( F_{nv} \) values the hull gets in a planing mode and experiences a hydrodynamic lift force which affects the draft and running trim. The hull resistance contributes significantly to the total resistance. As the resistance at or just prior to take-off, the so-called hump drag, is important with respect to the powering requirements, minimization of the hull resistance is a major item in hydrofoil craft hull form design.

Hydrofoil craft hull forms are derived from planing craft practice. Usually hard chine but occasionally round bilge hull forms are used. The hull shape depends largely on the lift distribution between the forward and aft foil and on the required combination of hull and foil lift during take-off. The hull must not only perform well in the hullborne mode, but also during take-off and foilborne operation where impacts with waves may occur. This latter aspect requires V-shaped sections with a high deadrise at the bow. Hull length to beam ratios range from 3 to 4.5 and from 5 to 8 for canard and airplane type foil configurations respectively.

For canard type hull forms, the stern part is relatively bluff with a low deadrise angle. The chine beam of the aft body is virtually constant up to one third of the length. Such hull forms get in a planing condition at relatively low volume Froude numbers, which facilitates take-off, but on the other hand also introduces a pronounced resistance hump, see Savitsky et al. (1981). Furthermore, low length to beam ratio hull forms experience a high trim angle prior to take off, in the order of 4 to 6 degrees.

In designing hull forms, use is often made of the Series 65 hydrofoil craft hull form series data, see Figure 1.6.
1.5 Hydrofoil craft hull forms

Figure 1.6a  Lines and body plans of Series 65-A hulls
*From Holling and Hubble (1974)*

Figure 1.6b  Lines and body plans of Series 65-B hulls
*From Holling and Hubble (1974)*
1.6 Hydrodynamic aspects in hydrofoil craft design

In this section the main hydrodynamic aspects in the design of hydrofoil craft are reviewed. These aspects may have been briefly touched upon in the preceding sections, but are described here in more detail as they lead to the requirements set for the computational methods to be developed.

Of primary importance with respect to hydrofoil craft design are the following four items:

- the maximum resistance at take-off,
- the hull clearance, trim angle and resistance at foilborne velocities,
- the seakeeping and manoeuvring characteristics of the craft,
- the cavitation characteristics at take-off and cruise speed conditions.

Maximum resistance at take-off

In the hullborne mode the craft’s resistance is made up of the hull resistance and the foil system resistance. The hull resistance consists of viscous resistance and resistance due to wavemaking. The viscous resistance increases approximately with the speed squared while the wave making resistance increases with a higher power of the speed. When the hull displacement reduces sufficiently as take-off is approached, the hull resistance reduces with increasing speed.

The foil system resistance consists of viscous resistance and induced resistance (drag due to lift). Prior to take-off these components continuously increase with speed as the foil lift coefficient increases due to the hull trimming and/or due to lift augmentation in order to lift the hull out of the water. As speed increases after take-off, the foil system lift remains approximately constant. Hereby the induced resistance reduces with the reducing lift coefficient. The viscous resistance may initially decrease due to the lower wetted surface as speed increases. At higher speeds the viscous resistance increases approximately with the speed squared. At cruise speed approximately 50% to 75% of the total resistance consists of viscous resistance, depending on the foil loading.

As a consequence, the total resistance curve shows a pronounced maximum at or just before take-off: the take-off hump. In order to take-off quickly against wind and waves the available thrust at take-off must be approximately 30 to 40% larger than the resistance hump value. Due to the relatively low propulsive efficiency of high speed propulsors at low speeds, the required power is often determined by the take-off hump rather than by the resistance at cruise speed. Van Walree (1986) and Zhong and Cheng (1992) provide more detailed information on this subject.

Take-off at a low speed results in a high resistance hump due to the induced drag of the foil system associated with the required high lift coefficient. Take-off at a relatively high speed results in a high resistance hump due to the hull resistance. In general, an optimum take-off speed exists for which the resistance hump will have a minimum height. The optimum take-off speed may be reached by controlling the lift of the foil system in an appropriate manner, for instance by using trailing edge flaps. Note that any optimization of the take-off speed should not only include resistance but also an assessment of the propulsor efficiency in order to determine the powering requirements.
1.6 Hydrodynamic aspects in hydrofoil craft design

For a given hull, hydrofoil configuration, craft weight, centre of gravity and for a certain speed, the hull resistance depends on the hull displacement and running trim, whereas the foil system resistance depends on the foil submergence and lift produced. Both the hull displacement and running trim depend to a large extent on the lift produced by the foil system. At the same time, the foil system lift production depends on the attitude of the craft, in terms of draft and trim, through the angle of attack and free surface effects.

The presence of the free surface affects the flow over the foil surface, while the pressure disturbance due to the foil lift in turn creates waves. Free surface effects are characterized by a decrease in lift and an increase in drag due to wave making as the foil submergence decreases. These effects are largest at intermediate chord Froude numbers (1<\(F_{nc}\)<5), or submergence Froude numbers (1<\(F_{nh}\)<2), experienced during take-off. The chord and submergence Froude numbers are defined as:

\[
\begin{align*}
F_{nc} &= \frac{U}{\sqrt{gc}} \\
F_{nh} &= \frac{U}{\sqrt{gh}}
\end{align*}
\]

(1.3)

where \(c\) and \(h\) are the chord length and submergence of the foil respectively.

As a further complication, the lift and resistance of the aft foil depend on the lift produced by the forward foil. Flow disturbances due to the forward foil lift are carried with the free stream towards the aft foil and result there in local variations of the inflow angle. This directly affects the lift and drag. Since the flow direction varies, the lift force generates an additional induced drag or thrust force. As the lift is an order of magnitude larger than the resistance, this additional force component is of importance for determining the overall lift to drag ratio of the craft. This phenomenon is called fore-aft foil interaction.

Finally, interaction between the hull and the foil systems may be of significance as well. The velocity disturbances due to the hull may affect the inflow conditions at the foils and struts. Vice versa, velocity disturbances due to the foil system lift introduce pressure disturbances on the hull and thereby additional hull lift and drag components.

In conclusion, the total resistance in the hullborne mode depends on the attitude of the craft as reflected by the hull draft or displacement and running trim. The attitude of the craft is determined by all resistance and lift force components acting on the hull and foil system whereby the resistance components depend to a large extent on the lift forces.

The dependence of the force components on the craft's attitude is non-linear. For instance, the free surface effect on lift and drag as a function of submergence and fore-aft foil interaction as a function of trim and draft are non-linear, see Van Walree (1986). The attitude of the craft must then be determined by an iterative procedure in which all force components acting on the hull and foil system are described as a function of speed, general parameters (weight, centre of gravity), geometrical parameters (hull form, foil system type and planform, foil section type, means for lift control) and the attitude of the craft. In order to find the optimum resistance curve the attitude and
the resulting force components must be determined for a matrix of trailing edge flap and/or foil incidence settings per speed.

**Foilborne attitude and resistance**

In the foilborne mode the problem is reduced to accounting for the foil system forces only. The principal problem is to determine the attitude of the hydrofoil craft as a function of speed and foil system flap angle or incidence setting. The attitude is reflected by the hull clearance and the running trim. The hull clearance is defined by the vertical distance at the transom between the keel line of the hull and the watersurface.

At cruise speed, between say 70% and 100% of the maximum speed, a certain hull clearance and trim are required for comfortable and safe operation in waves. Surface piercing hydrofoil craft without means for lift control can operate at a minimum hull clearance only at some minimum speed. Fully submerged hydrofoil craft are equipped with a ride control system, enabling an attitude more or less independent of speed.

At high speeds the viscous drag is generally larger than the induced drag since a high speed is associated with a low lift coefficient. Therefore, variations in the wetted area of surface piercing foil system parts due to attitude variations, affect resistance significantly. Free surface effects on drag mainly depend on the foil submergence and the developed lift. Therefore, the attitude of the craft determines the hydrodynamic resistance to a large extent. The attitude of hydrofoil craft is mainly governed by the forward and aft foil lift forces. In other words, given a certain foil system, the foil system lift forces determine the resistance to a large extent.

Furthermore, for fully submerged hydrofoil craft at a high speed the lift variation with foil submergence is small due to the low cross sectional area of the struts and the relatively small free surface effect on lift at higher Froude numbers. A small deviation in the lift force then introduces a large offset in the hull clearance. Therefore, the lift forces for the required attitude at the cruise speed need to be predicted accurately in the design phase. During trials, the required attitude may be obtained by adjusting the foil incidence or using trailing edge flaps but this may have unfavourable effects on the cavitation characteristics and the lift control potential for operation in waves.

In conclusion, for determining the foilborne behaviour of hydrofoil craft it is required to develop a computational method with the same functionality as specified for the take-off condition.

**Seakeeping and manoeuvring**

Operation in waves induces motions which may reduce the capability of the crew and may cause seasickness to passengers. Due to the relatively low wave excitation forces on a foil system in comparison to ship hulls, the performance of hydrofoil craft in waves in terms of speed loss and comfort is good. Nevertheless, more stringent requirements are set continuously with respect to seakeeping capabilities of high speed craft. In order to investigate this, knowledge is required of the
forces acting on the foil system in waves and the effectiveness of control surfaces in varying the lift force.

This latter aspect is also of interest in investigating the safety of operation. At high speed it is essential to have a good manoeuvrability. Surface piercing hydrofoil craft usually only have vertical control surfaces (rudders) for performing manoeuvres. Fully submerged hydrofoil craft can enlarge their manoeuvrability by also using their horizontal control surfaces, in analogy with aircraft practice.

Furthermore, the use of the control surfaces during touch-down and emergency manoeuvres has to be investigated. For instance, to investigate a safe way to perform a crash-stop during a coordinated turn with a speed of 45 knots and a roll angle of 7.5 degrees requires a detailed knowledge of the forces acting on the foil system under arbitrary inflow conditions and the impulsive use of control surfaces.

Finally, the performance of the craft in extreme conditions where hull slamming and foil broaching occurs may be investigated. Under such conditions extreme acceleration levels and structural loads can be expected.

The preceding paragraphs on the hump resistance and foilborne attitude relate to steady flow conditions. For operation in waves and manoeuvring this is no longer the case. During operation in waves the lift force acting on a foil system continuously varies due to wave orbital motions, craft motions and control surface actions.

The continuous variations in angle of attack, submergence and control surface deflection may not be properly described in a quasi-steady manner by accounting for instantaneous values of these quantities only. Two unsteady flow effects play a role. First, force components proportional to the time derivative of the lift occur. Second, memory effects on the instantaneous forces are present. The memory effect is the influence of the wake vorticity on the instantaneous foil forces. The strength and position of the vorticity in the vortex wake reflect the history of the motions of, and forces acting on, a foil.

Unsteady flow effects are generally characterized by the reduced frequency \( k \), which is defined as:

\[
k = \frac{\omega c}{2U}
\]  

(1.4)

where \( \omega \) is the frequency of the incidence or control surface deflection variations. At speed, the encounter frequency \( \omega_e \) is the governing frequency. The encounter frequency is largest for operation in head waves for which it is defined by:

\[
\omega_e = \omega (1 + \frac{\omega U}{g})
\]  

(1.5)
Increasing the speed $U$ increases $\omega_\ast$, but at the same time, decreases $k$. For a hydrofoil craft operating in head waves in between speeds of 30 and 50 knots, the reduced frequency varies from $k=0.26$ at 30 knots to $k=0.24$ at 50 knots for a typical chord length $c$ of one meter. Hereby a wave frequency $\omega$ of 2 rad/sec was used, which may be regarded as an upper limit.

For this reduced frequency range quasi-steady force components are dominant, nevertheless unsteady flow effects are of importance. This is shown by the so-called Theodorsen function for two-dimensional foil sections performing harmonic motions in an unbounded fluid. In Section 4.2 a more detailed description is given of this function. The Theodorsen function is shown in Figure 1.7, by means of its real and imaginary parts $F$ and $G$, respectively. Unsteady flow effects reduce the lift curve slope by a factor $(F^2 + G^2)^{0.5}$ and introduce a phase lag $\arctan(-G/F)$ with respect to the incidence at the ¾ chord point, when compared to a quasi-steady lift proportional to the instantaneous incidence and lift curve slope of $2\pi$. For a reduced frequency of $k=0.25$, the lift curve slope reduction is 27% while the phase angle is 15 degrees. This shows that unsteady flow effects are of importance, at least for two-dimensional sections in an unbounded fluid. For hydrofoil craft, it is expected that unsteady flow effects are of importance as well, considering the aspect ratio of foil systems: $4<AR<10$.

![Figure 1.7 Theodorsen function](image)

Manoeuvring results in a low frequency variation of the foil incidence, although control surfaces still may be operated more or less impulsively. As manoeuvres are usually performed while operating in waves, relatively low and high frequency variations of foil system forces will be present simultaneously. In this respect it is not possible to distinguish principal differences between unsteady flow effects for seakeeping and manoeuvring.
In conclusion: unsteady flow effects need to be taken into account for describing the dynamic behaviour of hydrofoil craft.

Cavitation characteristics

Cavitation may appear at take-off due to the high lift coefficients required for lifting the hull above water at a relatively low speed. A high lift coefficient due to a high incidence (running trim) and/or trailing edge flap angle results in a low-pressure peak at the leading edge of the foil section. Despite the relatively high cavitation number, cavitation may appear in the form of an attached sheet. Extensive sheet cavitation, covering a substantial portion of the foil suction side may lead to lift breakdown and an increase in drag. This may hinder take-off at the desired speed, see Van Walree (1985). Structural damage and/or noise and vibration problems due to this type of cavitation do not cause problems as its duration is relatively short.

For cruise speed conditions, at much lower cavitation numbers, sheet cavitation may appear due to incidence variations of only a few degrees. This type of cavitation cannot be prevented easily. On the other hand, sheet cavitation generally does not cause problems provided the cavity remains of limited length. The lift to drag ratio of a foil section may even increase somewhat for short cavity lengths. Due to the unsteady nature of cavitation, vibrational problems may in principle appear, but are hardly observed in practice.

Cavitation at cruise speed may also simply be caused by a too high speed for the foil section in question. For sufficiently low cavitation numbers, cavitation may appear at the mid-chord position of the section, due to a more or less flat pressure distribution on the section’s suction side at the ideal angle of attack. This bubble type of cavitation unfavourably affects the section lift to drag ratio and thus the performance of the craft, see Acosta (1973). If this type of cavitation is of an unsteady nature it can induce uncontrollable craft motions.

Fore-aft foil interaction may induce relatively large incidence variations along the span of the aft foil. This is especially the case for the region behind the tips of the forward foil. In order to prevent cavitation in that region a variable camber or incidence along the aft foil span may be required.

An analytical assessment of the susceptibility to cavitation requires a computational method to determine the three dimensional pressure distribution over the foil system surfaces including effects due to the free surface, unsteady flow, fore-aft foil interaction and effects due to partial span trailing edge flaps. Furthermore, assessing cavitation effects on lift and drag needs the inclusion of a cavity flow model.

Steady and unsteady flow methods

In the previous paragraphs a number of hydrodynamic features have been described which play an important role in the design process of hydrofoil craft. In order to investigate the seakeeping and manoeuvring performance of hydrofoil craft unsteady flow methods are needed. For determining the
attitude and resistance in calm water a steady flow method is required. In principle, results for steady conditions can be obtained from an unsteady flow method by determining the equilibrium position assumed by the craft starting from some initial position, in calm water. However, it is expected that this approach will require an impractical amount of computer time for the design problem of determining the lift and resistance characteristics of the craft. In steady flow computational methods there is no time dependence and the forces can be determined with a relatively small computational effort.

On the other hand, the take-off behaviour of hydrofoil craft is in essence an unsteady flow problem. The craft is accelerated through the take-off region by using a sufficient amount of thrust. Both the thrust developed by the propulsion system and the attitude and resistance of the craft are then depending on time. The problem of determining the required power should ideally be solved by using an unsteady flow calculation method which would be able to predict transient effects on all force components of interest, i.e. the propulsor, the foil system and the hull. Furthermore, for determining the effect of waves on the required take-off power, the prediction of wave induced loads on the foil system as well as on the hull must be included. The development of such a method would go beyond the ambition of the present study which concentrates on steady and unsteady flow computational methods for foil systems only.

In hydrofoil design practice a quasi-steady approach is used to determine the powering requirements. The required thrust is based on the maximum resistance in calm water plus an increment needed for a sufficient acceleration and accounting for wind and waves. A similar approach is followed for conventional ships in determining a sea margin on power, accounting for hull fouling and operation in wind and waves.

The required computational effort for determining the resistance for a number of discrete values of the velocity will be substantial already. This is due to the fact that for determining an optimized resistance curve a large number of conditions has to be analyzed. For each condition the attitude and resistance of the craft have to be determined iteratively for a matrix of forward and aft foil flap or incidence angle combinations.

It has been decided to make a major subdivision of the hydrofoil craft problem into steady and unsteady flow problems. The steady flow problem will deal with the lift and resistance characteristics for the take-off and foilborne speed regions. The unsteady flow problem will deal with the seakeeping and manoeuvring performance of hydrofoil craft at foilborne speeds.

Chapters 2 and 3 concentrate on the problem of determining the attitude and resistance of hydrofoil craft for steady flow conditions. In Chapters 4 and 5 the unsteady seakeeping and manoeuvring problem will be dealt with.
1.6 Hydrodynamic aspects in hydrofoil craft design

Summary Chapter 1

Hydrofoil craft are characterized by a planing hull form and by either surface piercing or fully submerged foil systems. The resistance at calm water is characterized by a pronounced hump prior to take-off. For attitude control, seakeeping and manoeuvring, use is made of ride control systems actuating trailing edge flaps on the foils.

In the design of hydrofoil craft it is required to investigate a number of hydrodynamic aspects relating to the powering requirements, seakeeping characteristics, manoeuvring capabilities and the susceptibility towards cavitation. By means of model tests these aspects can be investigated only partially and with a limited accuracy. Therefore, there is a need for computational methods.

A subdivision into steady and unsteady flow methods is made. The steady flow method will deal with the resistance and powering problem in calm water. The resistance depends on the attitude (trim, draft) of the craft for both the hullborne and foilborne speed ranges. The attitude must be determined by an iterative procedure in which all force components acting on hull and foil system are addressed.

For the seakeeping and manoeuvring problem quasi-steady forces are dominant, nevertheless unsteady flow effects need to be taken into account.
References Chapter 1


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2. COMPUTATIONAL METHOD FOR STEADY FLOW CONDITIONS

2.1 Review of steady hydrofoil theories

Hydrofoil craft methods

The problem of determining the attitude of hydrofoil craft with the associated lift and resistance force components has received little attention in literature. The hydrofoil problem is an extension of the basic problem of finding the equilibrium position and associated resistance force for planing craft. This problem has been dealt with by Savitsky (1964) and Hadler (1966). They formulate three non-linear equations which represent the force and moment equilibrium of the craft in the vertical plane, assuming starboard/port-side symmetry. Each force component in the equations is formulated as a function of three unknown quantities: the trim angle, the draft and the thrust force. The formulations for the force and moment components are of an empirical nature, primarily based on experimental data. The equations are solved iteratively until the equilibrium condition is found. All force components are then known. The method of Hadler (1966) is an extension of that of Savitsky (1964) in the sense that Hadler also takes into account propulsion aspects such as the propeller-hull interaction and the appendage lift and drag components.

For determining the hull force components acting on high speed hull forms, the method of Savitsky is often used. This method is based on experiments with prismatic planing hull forms at planing conditions, i.e. at speeds past the resistance hump. Hadler et al. (1974) show that the use of the Savitsky method for hydrofoil craft hull forms at take-off speeds leads to inaccurate results. Resistance errors up to 60% are found for the take-off speed range where the hull resistance is dominant. This is probably due to the fact that hydrofoil craft hull forms are not in a real planing condition during that part of the take-off speed range.

Hadler et al. (1974) based their comparison study on the Series 65 model test data. The Series 65 consists of 16 hydrofoil craft hull forms, see Holling and Hubble (1974). The resistance and lift characteristics of the hull were determined by means of captive model tests for a systematic variation in speed (zero to take-off), displacement (full load to almost zero) and trim angle (zero to six degrees). These data form a good basis for setting up a computational method for hull forces, similar to that of Savitsky, but better suited for hydrofoil craft purposes.

A recent computational method for planing hull forms is described by Zhao et al. (1997). They present a slender body theory for high speed hull forms. The theory is based on a two-dimensional Laplace equation with a non-linear, three-dimensional free surface condition. The Froude number is assumed to be infinitely high. Flow separation (spray) from chines is included. The location of the separation lines must be specified a priori. Calculation results for the hydrodynamic lift and trimming moment on a prismatic planing hull are compared with empirical data from Savitsky (1964). The comparison is reasonably good. The disadvantage of this approach is that, similar to Savitsky's method, a high Froude number is assumed while hydrofoil craft hull forms are not in a real planing mode during the largest part of the take-off speed range.
Chapter 2  Computational method for steady flow conditions

For hydrofoil craft an analogous approach must be followed as in the method of Hadler, whereby the foil system force and moment components are added. Sakic (1982) presents a calculation scheme for hydrofoil craft based on simple empirical relations for the hull and foil system force components. The accuracy of this method is rather limited and the method does not contain the amount of detail required for optimization purposes. A similar, more extended method is provided by Latorre and Teerasin (1992) and Latorre and Bourg (1993). Reasonable results are obtained for foilborne conditions. The computational method is however of limited use since the expressions for the hull and foil system force components are based on experiments on a particular hull form and foil system configuration. The take-off hump can not be predicted due to lack of experimental data. Furthermore, fore-aft foil interaction is not included in the computational method.

The importance of fore-aft foil interaction is shown by Bai-Qi (1981). In his calculation method for foilborne conditions, equilibrium equations as mentioned before are used. The force components of surface piercing foil systems are expressed by means of polynomials as a function of incidence, submergance and flap angle. These polynomials are based on experimental data for isolated foil systems. A calculation scheme for fore-aft foil interaction is provided, based on a lifting line description for a plain, rectangular foil with an elliptical circulation distribution. The surface piercing forward foil is represented by an equivalent foil at its mean submergence. The interaction effect on the aft foil is accounted for by means of the induced inflow angle averaged over the horizontal projection of its span. Hereby it is assumed that the trailing vortices originating from the forward foil run parallel to the free stream direction. The change in inflow angle results in a correction on the lift and induced drag of the aft foil. Inclusion of the fore-aft foil interaction clearly improves the correlation between calculated and experimental data.

The calculation methods described so far do not have a wide range of applicability due to the use of empirical relations based on one particular hull and foil system configuration. The accuracy of the calculation methods is reasonable for foilborne conditions only. Furthermore, validation of these methods is often based on the same experimental data as used for deriving the empirical expressions. Important aspects like free surface effects and fore-aft foil interaction are rather schematically included, if at all.

Van Walree (1988) describes a calculation method wherein a simplified lifting line method is used to describe the foil system forces underneath a free surface. Herein the basic three-dimensional lift curve slope, $C_{la}$ is obtained as follows:

$$C_{la} = \frac{2\pi AR}{AR + 2}$$  \hspace{1cm} (2.1)

where $AR = b^2/S$, the aspect ratio of the foil whereby $b$ and $S$ denote the span and planform area of the foil respectively. Empirical functions are used to correct the lift curve slope for planform variations such as sweep and taper, the presence of struts, propulsion pods and nacelles, etc. Free surface effects are taken into account by using a method described by Keldysh and Lavrentiev (1949) for infinite aspect ratio lifting surfaces. Herein the lifting surface is represented by a single lifting line with a constant circulation distribution over its span. The free surface effects on lift and induced drag are determined by considering the effect of a biplane image vortex line and a gravity
wave term on the flow around the lifting line. These corrections are applied to finite aspect ratio lifting surfaces as well. For cambered and flapped foil sections, an equivalent flat plate angle of attack is used, based on empirical data. Foil interaction is included in a similar way as the lifting line method described by Bai-Qi (1981). The Series 65 data are used to describe the hull forces. Empirical relations for viscous resistance, appendage forces and propulsor forces are included to obtain predictions for the required power.

For both surface piercing and fully submerged foil systems reasonably accurate lift and resistance results are obtained for some hydrofoil craft, see Van Waluwe (1992). However, for other hydrofoil craft, errors up to 30% have been found for resistance predictions for take-off conditions while for cruise speed conditions errors in lift and drag of 15% have been found, see Yamaguchi (1992). He finds that the major contributions to the resistance errors are caused by errors in the computation of effects on lift and drag due to planform variations, the free surface and foil interaction. This could be shown by comparing calculation and experimental results for several foil systems in isolation and in tandem arrangement for a range of submersiond and velocities.

Foil systems

Potential flow calculations for foil systems only have been developed by amongst others, Kaplan et al. (1960), Nishiyama (1965a/b), Hough and Moran (1969) and Morch (1992). These computational methods are based on a lifting line description of the hydrofoil planform. Linearized free surface boundary conditions at the undisturbed free surface are used to derive expressions for the velocity potential.

Kaplan et al. (1960) derived expressions for the flow field behind a plain rectangular, finite span hydrofoil. After introducing certain assumptions on the spanwise circulation distribution (elliptical), velocity (high Froude number) and distance from the forward hydrofoil (large), expressions are derived which enable the calculation of the downwash angle at the aft foil.

Hough and Moran (1969) describe a numerical method for two-dimensional hydrofoils. Thin foil sections are represented by a bound vortex distribution satisfying the linearized free surface conditions and a tangential flow condition on the section. The effect of gravity waves is included. Figure 2.1 shows the free surface effect on the lift coefficient for a two-dimensional flat plate as a function of the submergence to chord ratio h/c and the submergence Froude number $F_{nh}$. It is seen that for high submergence Froude numbers ($F_{nh}>10$) the speed dependence of the lift coefficient vanishes. For such conditions gravity wave effects are small and a biplane image vortex system is then sufficient to describe the flow problem. At low submergence Froude numbers, (1<$F_{nh}$<2) gravity waves are dominant and the reductions in lift are largest. At very low submergence Froude numbers ($F_{nh}$<1) and low submersion of the lift tends to get larger than the lift at infinite depth. Free surface effects on wave making drag are similar in the sense that the wave making drag is at a maximum when the lift decrease is at a maximum. This can be deduced from other calculation methods, see Morch (1992), and experimental data given by Wilson (1983).
Nishiyama (1965b) presents the derivation of the velocity potential for a single lifting line with an arbitrary dihedral angle. The dihedral angle is the angle between the lifting line and the horizontal plane. Nishiyama shows that free surface effects due to gravity waves on lift and drag become less significant when the aspect ratio reduces. However, for aspect ratio's used for practical foil systems, free surface effects are still large.

Morch (1992) presents a more modern computational method based on a so-called Kelvin-Havelock Green's function description of free surface effects. This approach is better suited for numerical treatment than the earlier expressions given by Kaplan et al. and Nishiyama. Morch uses a lifting line discretization presented by Blackwell (1969). Herein the foil planform is divided into a number of horseshoe vortex elements with a bound vortex filament at the quarter chord line and a single control point at the three-quarter chord position. At this control point flow tangency with the base line of the section is imposed. The lifting line is assumed to run parallel to the transverse axis, i.e. no swept planforms can be dealt with.

Nishiyama (1965a) presents an approximate lifting surface theory for fully submerged hydrofoils. A disturbance velocity potential is derived from linearized boundary conditions at the undisturbed free surface. The integral equation resulting from the tangential flow boundary condition at the foil surface is approximated by using the so-called Küchemann concept, in which a quasi lifting line solution is provided.

Bai-Qi (1986) describes a so-called vortex lattice method for isolated, horizontal hydrofoils with an
arbitrary planform underneath the free surface. Herein the circulation is introduced by a distribution of flat horseshoe vortex elements. For tapered and swept planforms the bound vortex line segments run parallel to the local sweep angle. The potential flow problem is solved by applying a tangential flow condition on the foil and a linearized free surface condition on the undisturbed free surface, yielding the circulation distribution. The perturbation potential includes contributions from the horseshoe vortex systems, their biplane images and from the gravity wave system. The lift curve slope as a function of submergence and Froude number is shown to be well predicted by this method. No data on the accuracy of the method for the induced and wave making drag are given. Also, no information on the method used for computing the complicated integrals representing the gravity wave effects is given.

Thiart (1997) describes the development of a similar vortex lattice method for single rectangular, horizontal hydrofoils below a free surface. Again, linearized free surface conditions are applied at the undisturbed free surface. A reasonably good agreement with experimental results is found for the lift curve slope reduction due to the presence of the free surface. No validation for wave making drag has been performed. The integrals representing gravity wave effects are computed by means of numerical integration, using standard methods like Simpson’s rule and Gaussian quadrature. Thiart mentions the following computing time on a Alpha 3000-800 workstation: 1 minute without taking the gravity wave components into account and 1 hour for taking the gravity wave terms into account. Herewith the rectangular hydrofoil was represented by a lattice of 10 chordwise and 80 spanwise vortex elements. This number of vortex elements was deemed to be required after a study on the sensitivity of lift and drag on the number of vortex elements. This number is rather high for such a simple foil, see for instance DeJarnette (1976). Some scatter in the results of Thiart is introduced by errors in the computation of the free surface integrals, which may be responsible for the high number of vortex elements required. For using a similar method in an iterative procedure for determining the equilibrium position of hydrofoil craft, obviously a much more efficient procedure for computing the free surface effects is required.

Panel methods

Panel methods may be used for determining the potential flow around the hull and foil system combination simultaneously. Interaction between the hull and foil system can then be taken into account. In panel methods available at MARIN the hull and foil surface geometries are represented by Rankine source panels. The free surface is represented by a number of Rankine source panels as well. By defining vortex line segments on the foil camber plane and on its wake, circulation is introduced. Suitable boundary conditions on the body and on the free surface are used to solve the potential flow problem.

In linear panel methods the position of the craft with respect to the undisturbed watersurface is fixed and linearized free surface conditions are applied at the undisturbed free surface. Both the equilibrium position of the craft and the watersurface disturbances are unknown a priori. The free surface conditions should however be applied at the disturbed water surface while the actual underwater geometry should be accounted for. Therefore, linear panel methods may be used iteratively in order to determine the equilibrium position of the craft. In non-linear panel methods
such an iterative scheme is applied while the non-linearized free surface conditions are applied on the actual disturbed free surface in each iteration. The solution is found when both the craft's position and the free surface disturbances converge to constant values.

Linear and non-linear panel methods which can treat lifting surfaces are available at MARIN. The linear Dawson method is available for more than a decade, see Raven (1988). This method uses a linearization of the free surface conditions with respect to a zero Froude number, or double body flow. Dawson has been replaced by the non-linear Rapid method, see Raven (1993), which has been extended with lifting surfaces only recently. Panel methods are however thought not to be suited for the present purposes due to a variety of reasons.

The Dawson method is not suited for high speed hulls and foils. Although the flow about sufficiently slender hull forms at high speeds can be reasonably well described by linear panel methods, practice at MARIN shows that numerical problems limit the maximum Froude number to 0.50 to 0.75, depending on the hull form. The Froude number, based on the actual waterline length at speed, for hydrofoil craft hulls during take-off ranges from 0.50 to 1.5. For foils the chord Froude number may be as high as 6 to 8. This points to the use of the non-linear Rapid method which is numerically more stable. However, the presence of hard chines in the hull form and lift carrying, surface piercing foil parts may result in convergence problems in the iterative process of non-linear solvers due to strong local watersurface disturbances. For more knowledge into this matter a detailed and separate investigation is required.

Furthermore, the success of using any panel method for hard chine planing hull forms with a low length to beam ratio is uncertain due to the assumption of a potential flow. Due to the neglect of viscosity effects flow separation around submerged chines can not be represented. Note that flow separation around chines at the free surface, i.e. spray generation, can be modelled separately in a potential flow, see Zhao et al. (1997). Partially separated transom stern flows can not be represented in a potential flow method either. Wave making resistance results from panel methods based on pressure integration are known to be inaccurate, although progress has been made recently in this respect by applying a transverse wave cut analysis to non-linear results, see Raven and Prins (1998). Viscous resistance still needs to be determined separately by using semi-empirical methods.

Finally, the panel method has to be used for a matrix of conditions for each velocity in order to determine the optimum take-off configuration as a function of flap angle or incidence setting. For determining an optimum resistance curve it is estimated that 250 conditions have to be analyzed. This will require an impractically large amount of computer time, even on a supercomputer. For determining the foilborne attitude and resistance a lower number of conditions, 50 say, needs to be analyzed for a matrix of hull clearance and/or trim conditions. Nevertheless, estimated computer costs remain high.

Fore-aft foil interaction

Fore-aft foil interaction is the effect on the forces acting on the aft foil caused by the waves and wake generated by the forward foil. The vorticity in the wake and the generated waves induce velocity disturbances at the aft foil. These disturbances result in inflow variations, usually termed
2.1 Review of steady hydrofoil theories

upwash or downwash and sidewash, which affect the magnitude and direction of the lift force and thereby the induced drag. The interaction effect due to wave formation is implicitly accounted for when the free surface effects on the foil system are determined. The interaction effect due to the forward foil wake depends strongly on its position relative to the aft foil.

In the computational methods described so far it is assumed that trailing vortices form a flat sheet in the plane behind the forward foil. In reality the wake sheet will roll-up due to self-induced velocities. Hereby, the centre part of the wake sheet will move downwards while the rolled-up edges will move upwards and inwards. This deformation of the wake sheet progresses with distance from the forward foil and is affected by the presence of the free surface and the vortex system belonging to the aft foil. The free surface introduces a wave like displacement in the wake sheet with a wave length proportional to the chord Froude number squared. At the same time, the vorticity will move towards the edge of the wake sheet. These edges will eventually form line vortices in which all the vorticity is concentrated. Note that at the same time the wake sheet position affects the wave formation which in turn affects the velocity disturbances at the aft foil.

In potential flows the vorticity in the wake sheet remains constant in time. In reality, due to viscous diffusion, the vortex strength in the wake sheet will decay. At practical Reynolds numbers, however, convection is much stronger than diffusion so that all vorticity remains in a thin sheet, up to several spans behind the trailing edge.

Spreiter and Sacks (1951) give the following estimate for the distance \( x_r \) behind an elliptically loaded wing in an unbounded flow at which the wake sheet is fully rolled-up into line vortices:

\[
x_r = 0.28 \frac{ARb}{C_L}
\]  

(2.2)

where \( AR \) is the aspect ratio, \( b \) is the span and \( C_L \) is the lift coefficient of the wing, defined by:

\[
C_L = \frac{L}{\frac{1}{2} \rho U^2 S}
\]  

(2.3)

here \( L \) is the lift force, \( \rho \) is the fluid density, \( U \) is the velocity and \( S \) is the planform area of the wing.

Applying this formula to practical hydrofoil configurations, a distance of 4 to 7 spanwidths is found. This distance is equal to or larger than the foil spacing for typical hydrofoil craft which is in between 3 to 5 spanwidths. This means that for the estimation of induced velocities at the aft foil, the wake sheet might be replaced by two vortex lines emanating from the tip of the forward foil. These vortex lines carry all the circulation present on the forward foil. Spreiter and Sacks (1951) also provide empirical formulations for the position in the vertical plane of these two vortex lines relative to the forward foil, assuming a certain planform and foil loading. Using these formulations, it would be quite straightforward to estimate the induced velocities at the aft foil. However, such a procedure does not include effects due to the forward foil geometry and the free surface on the position of the wake sheet.
Morch (1992) carried out a study on hydrofoil interaction, including free surface effects, for hydrofoil systems applied to foil catamarans. The wave-like variations in the induced velocity decay relatively slowly with distance from the forward foil. For practical hydrofoil craft, this effect may lead to significant variations in lift and drag of the aft foil, up to 40%, depending on the aspect ratio of the forward foil, the submergence Froude number and the foil spacing.

Morch compared experimental results with calculation results based on his lifting line model. Hereby longitudinal foil spacings up to 4.5 forward foil spanwidths were investigated for a forward foil with an aspect ratio $AR=8$. He found that using a non-rolled up vortex sheet to compute induced velocities at the aft foil results in fairly good predictions of foil interaction effects. The results were less good for a foil system with a lower aspect ratio ($AR=3.5$) forward foil, for longitudinal foil spacings between 3.75 and 11.5 forward foil spanwidths. Morch attributes this to the neglect of the vortex sheet roll-up, which is stronger for lower aspect ratio foils than for higher aspect ratio foils.

Morch developed a time domain simulation model with which the roll-up of the vortex sheet could be simulated. To reduce computer time, the problem is solved in a series of two-dimensional sections of the flow domain, which are convected in longitudinal direction with the free stream velocity. Longitudinal variations in the induced velocity components due to free surface effects are then neglected. This was deemed permitted for low aspect ratio forward foils as free surface effects are not of major importance for such foils. The method is rather demanding in terms of computing costs: about 100 vortex elements are needed to represent the two-dimensional wake sheet sections of a single rectangular foil while computations for about 250 time steps are needed before the wake sheet reaches the aft foil.

Morch shows realistically rolled-up wake sheet shapes but does not calculate the interaction effects on the aft foil. Therefore, improvements in calculated interaction effects in comparison to results obtained with the lifting line model can not be assessed. Morch does find that the induced velocities due to the rolled-up vortex sheet are rather well represented by two concentrated tip vortices located in the centre of the rolled-up wake sheet, at a distance of 7 spanwidths behind a foil. This is however not true for the region close to the concentrated vortex lines, where unrealistically high velocities are induced, and for conditions where the position of the wake sheet is affected by free surface effects and the circulation carried by the aft foil.

Conclusions

At present there do not exist computational methods that have a wide range of applicability and that predict at the same time the hull and foil system force components with an acceptable uncertainty and computational efficiency for use in hydrofoil craft design practice.

Relatively simple computational methods for hydrofoil craft rely too heavily on a limited set of empirical data to be used successfully for arbitrary hydrofoil craft. Methods for planing hull forms are based on assumptions valid for high Froude numbers which do not apply to the part of the take-off speed range where the hull resistance is high.
Potential flow methods for foil systems only, not relying on empirical data and including free surface effects in a more fundamental manner are available. These methods are based on a lifting line or on a vortex lattice approach. The inclusion of free surface effects and fore-aft foil interaction in such a method is essential for obtaining reliable results for the forces acting on the foil system. The application of such methods for foil systems consisting of a number of foil and strut parts with an arbitrary planform and dihedral is in principle possible. Their computational efficiency needs to be improved before such methods can be used in an iterative scheme to determine the equilibrium position of hydrofoil craft. Furthermore, validation of free surface effects on the wave making drag needs to be performed.

The use of panel methods is thought to be neither practical nor feasible for determining the required take-off power of hydrofoil craft. For determining the foilborne performance non-linear panel methods are suited for fully submerged configurations, but are still considered not to be practical for use in an iterative scheme.

For foil systems with a low aspect ratio forward foil it is probably required to determine the rolled-up position of the wake sheet in order to accurately predict interaction between foils. This will however be quite costly in terms of required computer time.
2.2 Simplification of the steady flow problem and adopted approach

General

The aim is to develop a practical computational method that has a wide range of applicability and that predicts the hull and foil system force components for steady flow conditions. The method should be suited for investigating a number of hydrodynamic aspects of hydrofoil craft, as mentioned in the introduction, Section 1.6. Preferably, the method should be suited for extension to an unsteady flow method for use in the second part of this study. To meet these demands within the framework of the present study, certain simplifications of the hydrodynamic problem are introduced and the adopted approach for the computational method for steady flow is specified.

Cavitation

Cavitation aspects will not be taken into account. The susceptibility with respect to cavitation may be investigated by using panel methods for the foil system only, for certain critical conditions, for instance at maximum speed. Also for detailed analysis of the flow around for instance foil-nacelle-strut combinations, panel methods may be used. For predicting cavitation effects on lift and drag separate studies have been and still are carried out. Furthermore, ventilation aspects will be neglected. As a consequence, the pressure distribution on the foil surface is not needed, only the resulting lift and drag force components are of interest.

Vortex lattice method

The fact that the pressure distribution is not needed allows for the use of relatively simple computational methods for obtaining the foil forces, in comparison to panel methods. For the purposes of the present study the vortex lattice method is well suited. The adequacy of vortex lattice methods for arbitrary planforms has been proven, see for instance Hough (1976) and Bai-Qi (1986). Trailing edge flaps may be accounted for as well. Furthermore, the vortex lattice method can be extended to unsteady flow conditions, see Katz and Plotkin (1991). Therefore it has been decided to use the vortex lattice method for determining the foil system forces.

By adopting a vortex lattice method the lifting line option is always available: by using only one chordwise vortex element a discretized lifting line approach is obtained. Obviously, the effects of camber and flap deflections need then to be accounted for separately.

Notwithstanding the widespread availability and use of the vortex lattice method for airfoils, the method must be extended so that important hydrodynamic phenomena for hydrofoil craft are included. These are described in the following paragraphs.
Free surface effects

A first phenomenon concerns free surface effects on lift and drag. Free surface effects may be incorporated in two ways. The first option is to use Rankine source panels on the free surface to represent gravity wave effects. The alternative is to use the Kelvin-Havelock formulation in which a more complex Green's function is used, replacing the Rankine source panels on the free surface. Both methods have their advantages and disadvantages as follows.

The Kelvin-Havelock Green's function results in rather complex integrals in the formulations for the influence coefficients, compared to the $1/R$ contributions from Rankine source panels. On the other hand, using Rankine source panels on the free surface increases the number of unknowns significantly, approximately by a factor 5, so that a much larger matrix needs to be inverted. Also, no discretisation of the free surface is needed in the Kelvin-Havelock approach. Provided that numerical methods are adopted, or developed, for an efficient computation of the aforementioned integrals, it is anticipated that the Kelvin-Havelock approach will be the most efficient one in terms of computing cost.

The Kelvin-Havelock Green's function satisfies the radiation condition, namely that there should be no upstream waves present. In the Rankine source approach the radiation condition must be imposed numerically. Furthermore, in the Rankine source approach, the infinite free surface domain must be truncated which may lead to undesirable wave reflections.

Finally, in the Rankine source approach the non-linear boundary conditions may be imposed on the actual free surface, while the Kelvin-Havelock Green's function satisfies the linearized boundary conditions on the undisturbed free surface. Although free surface effects on the hydrofoil forces are significant, the amplitudes of the waves created are small. It can be observed from experiments, see Section 3.3, that the wave amplitude is generally less that 1 percent of the wavelength, for chord Froude numbers above 3 and submergence to chord ratio's as low as 0.25. This indicates that the application of the free surface boundary conditions on the undisturbed free surface is allowed without introducing significant errors. Although local wave disturbances above the foil may introduce non-linear effects due to refraction, it is anticipated that the forces on the hydrofoils can be adequately predicted by using the Kelvin-Havelock approach.

In conclusion it is thought that the Kelvin-Havelock approach offers the best prospects for the present problem and is therefore adopted here.

Fore-aft foil interaction

For hydrofoil craft in general, free surface effects on foil interaction are of importance in view of the aspect ratio of the forward foil, the Froude number range and the foil submergence. This can be deduced from the experimental and calculation results of Morch (1992). From a practical point of view the determination of the roll-up of the wake sheet with a full account of free surface effects is not feasible due to the high computing costs. On the other hand, the use of two equivalent concentrated vortices is permitted only for special cases. Therefore, interaction will be based on the use of the complete trailing vortex system originating from the forward foil and assuming that its shape is given a priori, namely a flat surface behind the forward foil. The consequences of this
simplification will be investigated in Section 2.3.

Some consideration with respect to the roll-up of wake sheets will be given though in the unsteady flow method, see Section 5.2.

**Viscosity effects**

Using a vortex lattice method for the foil system forces implies that effects due to the viscosity of the flow and the thickness of the foil section are not accounted for. Viscosity not only introduces viscous drag, but also affects the lift curve slope and effective camber. The section thickness increases the viscous drag and increases the lift slope in a potential flow, relative to that of a thin wing section. In Section 2.6 it is shown that these effects are of importance, not only for model scale, when comparing calculation results with experimental data, but also for prediction of full scale hydrodynamics.

Computational fluid dynamics methods exist which can predict the three-dimensional characteristics of lifting surfaces with thickness in a viscous flow, but application of these methods to the hydrofoil problem is considered not practical, if possible at all. Therefore, viscosity and thickness effects on foil lift and drag will be obtained from empirical data and from results of two-dimensional viscous flow methods applied to foil sections.

**Hull characteristics**

Adopting a vortex lattice method for the foil system forces implies that interaction between the hull and the foil system can not be taken into account directly, in comparison to a panel method in which the hull and foil system are modelled simultaneously. The significance of this interaction is unknown. By means of some exploratory calculations this will be investigated and, if needed and feasible, taken into account. This is addressed in Section 2.3.

The hull characteristics will be obtained by interpolation on experimental data for hydrofoil craft hull forms. For this purpose the Series 65 hullform data are used, see Section 2.7. This approach allows for a fast and accurate calculation of the hull force components, provided the hull form characteristics are within the limits of the database used. The restriction to use Series 65 alike hull forms makes the method less general in use. This is however not considered as a major drawback since the last decade has shown that hull forms for newly designed hydrofoil craft were generally obtained from the Series 65 hull forms.

**Innovative aspects**

Only two hydrofoil craft calculation methods exist at present. The method by Sakic (1982) relies heavily on empirical formulations and is unsuited for optimization purposes. The method developed by Van Walree (1988) is more advanced but still has a number of shortcomings such as the use of
a simplified lifting line method for obtaining the foil system forces and the rather schematic account of free surface effects and fore-aft foil interaction.

In the present study the simplified lifting line method will be replaced by a vortex lattice method including free surface effects and foil interaction. A vortex lattice method is not exactly a new development and innovative aspects in this study are related to the use of this method in a wider framework. Innovative aspects can be summarized as follows.

Theoretical and semi-empirical methods are combined into a computational method with unique capabilities: the prediction and optimization of the hull- and foilborne performance of hydrofoil craft. The method will be suited for fully submerged as well as surface piercing foil systems. To this end a vortex lattice method will be developed that is suited for arbitrary foil configurations consisting of a number of foil and strut parts with an arbitrary planform and including free surface effects.

As the hydrofoil problem requires the use of an iterative scheme to determine the equilibrium position of the craft, the computational efficiency of the computational method must be optimized.

For accurate resistance predictions, appendage drag and viscosity effects on the foil system lift and drag are included in the method. Propulsor characteristics are included to obtain power predictions.

Furthermore, interaction between the forward and aft foil and between the foils and the hull is addressed.

Finally, the method will be validated for single foils, foil systems consisting of several foil and strut parts and complete hydrofoil craft.

**Adopted approach**

- no cavitation aspects will be taken into account,
- the hull characteristics will be based on the Series 65 model test data,
- the foil system characteristics will be based on a vortex lattice method. Free surface effects will be incorporated by means of a Kelvin-Havelock Green's function formulation whereby linearized free surface conditions are applied at the undisturbed free surface. The wake sheet is assumed to be a flat surface behind the foils,
- interaction effects between the hull and foil system and between the forward and aft foils will be investigated,
- viscosity effects on the foil system forces will be addressed,
- propulsor characteristics and appendage force components will be included,
- the computational method for hull, foil system, appendage and propulsor forces will be used in an iterative procedure to determine the equilibrium position of the craft.
2.3 Hull-foil, foil-hull and fore-aft foil interaction

Interaction between the hull and the foil system

Hull-foil interaction denotes the effect of the hull on the foil system forces, vice versa foil-hull interaction denotes the effect of the foil system on the hull forces. Qualitatively the following can be said about interaction between the hull and the foil system. Already in the hullborne speed range, prior to take-off, the hull becomes fully desubmerged at the forward foil position due to the trim and the foil system lift. The hull will then have no effect on the flow at the forward foil. Vice versa, the forward foil does affect the hull through its vortex system. This vortex system generates watersurface disturbances and induces velocity disturbances at the submerged hull surface. This affects the attitude and resistance of the hull which in turn affects the lift and drag of the foil system. Note that for trimmed conditions the trailing vortices of the forward foil travel close to the bottom part of the stern and the induced velocities may be relatively large. Finally, the presence of the hull affects the evolution of the forward foil wake sheet and thereby the interaction between the forward foil and the hull and between the forward foil and the aft foil.

The vertical spacing between the hull and aft foil system is typically 2 to 3 chords. Although this is not considered to be a particularly small spacing, induced velocities at the stern hull part due to the aft foil might be of significance. Vice versa, the presence of the hull may affect the flow at the aft foil. Not only through the displacement volume of the hull, but also through the violation of the free surface boundary conditions used in the vortex lattice method for the foils. These conditions are, see Section 2.5, that at the free surface the flow must be tangential to the free surface and that the pressure must be atmospheric. When a hull is present, the flow must be tangential to the hull surface while the pressure must equal the local hydrostatic pressure plus the unknown hydrodynamic pressure.

Investigation into interaction effects between hull and foil system

In order to investigate the significance of mutual interaction between the foil system and the hull a number of computations have been performed for a schematic hydrofoil craft in the take-off speed range. The computations have been performed by means of the Dawson code, see Raven (1988). The Dawson program is a panel method to compute the potential flow about ship hulls. Lifting surfaces can be included. In Dawson, linearized free surface boundary conditions are applied at the undisturbed free surface. This limits the accuracy of the calculation results, especially the wave pattern predictions, in comparison to non-linear panel methods. However, at the time this investigation was carried out, no non-linear panel method capable of handling lifting surfaces was available at MARIN while for the present purpose of investigating the significance of the interaction between the hull and the foils the Dawson results are adequate.

Using a panel method for hydrofoil craft in the take-off speed region proved not without difficulties, as was anticipated in Section 2.1. For a typical low length to beam ratio ($L/B=3.75$), hard-chine hull form used for hydrofoil craft, unrealistic results were obtained for speeds corresponding to Froude numbers above 0.50. In this Section Froude numbers are based on the waterline length of the hull
at rest. Unrealistic wave patterns and large velocity gradients along the chines were present in the results. It is then difficult to determine relatively small velocity components due to interaction with sufficient accuracy.

For the take-off speed range a Froude number between 0.4 and 0.8 is of interest. By using a schematic planing hull shape with a very fine bow shape, realistic results could be obtained up to Froude numbers of 0.65. This is still not sufficiently high to investigate the entire speed range but it will give a good impression of interaction effects.

The panelling arrangement on the hull and foils is shown in Figure 2.2. Two cases are considered here for one hull form in two combinations of submerged hull volume, trim and speed. Table 2.1 shows the main parameters of the craft for the two cases. Case 1 reflects conditions at a low speed, 12 kt, where the hull displacement is still large. For Case 2, the hull displacement is reduced, reflecting a condition closer to take-off at a speed of 19 kt. The reduction of hull displacement with speed was taken as follows, neglecting hydrodynamic lift forces on the hull and assuming a constant foil lift coefficient:

\[ \nabla = \nabla_0 [1 - (\frac{U}{U_t})^2] \]  

(2.4)

Here \( \nabla \) is the hull volume at speed \( U \), \( \nabla_0 \) is the hull volume at rest and \( U_t \) is the take-off speed, 24 kt.

For the two cases runs were made for the hull only, for the foil system only and for the hull plus foil system. For both cases flow detachment at the intersection of the transom with the bottom of the hull was specified in Dawson to simulate a fully ventilated transom.

Figure 2.2a  Panelling arrangement Case 1 - side view
Table 2.1 Main parameters of the schematic hydrofoil craft

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
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</thead>
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<tr>
<td>Froude number based on waterline length at rest (22.5 m)</td>
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<td>0.65</td>
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<tr>
<td>Hull waterline length (m)</td>
<td>15.00</td>
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<td>Waterline length to submerged beam ratio</td>
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<td>2.08</td>
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<tr>
<td>Waterline length to draft ratio</td>
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<td>19.23</td>
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<td>Hull displacement volume (m³)</td>
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<td>Hydrostatic force</td>
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<td>Hull trim angle (deg)</td>
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<td>3.0</td>
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<td>Foil aspect ratio</td>
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<td>6.0</td>
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<td>Foil lift coefficient</td>
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<td>Foil planform area (total, m²)</td>
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<td>Forward foil submergence (chords)</td>
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<tr>
<td>Aft foil submergence (chords)</td>
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<tr>
<td>Number of hull, foil and free surface panels</td>
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</tbody>
</table>
2.3 Hull-foil, foil-hull and fore-aft foil interaction

Figure 2.3 shows the wave pattern for the hull without foils for Case 2. Figure 2.4 shows that for both cases the wave pattern around the hull is slightly affected by the presence of the foils.

Figure 2.3  Wave pattern for hull without foils - Case 2

Figure 2.4a  Wave contours - Case 1
Figure 2.4b  Wave contours - Case 2

Table 2.2 shows the relative changes in the vertical hydrodynamic force (lift) and trimming moment acting on the hull, and the lift, moment and induced drag acting on the foil system, due to interaction. Since the reliability of the wavemaking resistance of the hull determined by means of Dawson is rather low, no interaction coefficients are given for this quantity.

Table 2.2  Interaction coefficients

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{dh}$</td>
<td>+0.018</td>
<td>-0.106</td>
</tr>
<tr>
<td>$C_{mh}$</td>
<td>-0.019</td>
<td>-0.029</td>
</tr>
<tr>
<td>$C_{sf}$</td>
<td>-0.027</td>
<td>-0.018</td>
</tr>
<tr>
<td>$C_{sf}$</td>
<td>-0.034</td>
<td>-0.022</td>
</tr>
<tr>
<td>$C_{mf}$</td>
<td>-0.033</td>
<td>-0.029</td>
</tr>
<tr>
<td>$C_{sl}$</td>
<td>-0.016</td>
<td>-0.128</td>
</tr>
<tr>
<td>$C_{ml}$</td>
<td>-0.052</td>
<td>-0.058</td>
</tr>
</tbody>
</table>
The interaction coefficients are defined as follows for the foil-hull interaction (foil effect on hull):

\[ C_{ih} = \frac{F_{ih} - F_{ih}}{F_{it}} \]  

(2.5)

and for the hull-foil interaction (hull effect on foils):

\[ C_{fi} = \frac{F_{fi} - F_{fi}}{F_{it}} \]  

(2.6)

where \( F_{ih} \) is the force or moment on the hull with the foil system present, \( F_{ih} \) is the force or moment on the foil system with the hull present, \( F_{i} \) is the force or moment on the hull without the foil system and \( F_{fi} \) is the force on the foil system without the presence of the hull. \( F_{i} \) denotes the total force or moment acting on hull plus foil system. The index \( i \) represents the indices for the horizontal and vertical forces, \( x \) and \( z \) respectively, and the trimming moment, \( m \). \( C_{ix} \) and \( C_{im} \) denote the total interaction coefficients for hull plus foils.

The total horizontal force contains all potential and viscous flow resistance components of the hull and foil system. The total vertical force and moment consist of the hydrodynamic force acting on the hull and foils plus the hydrostatic force acting on the hull and foil system. The total hull and viscous foil resistance components were estimated from Van Walree (1988).

It is seen in Table 2.2 that the total vertical force coefficient is significantly affected by interaction for Case 2, due to the effect of the foil system on the vertical force acting on the hull (\( C_{va} \)). The interaction effect on the total moments is about -5% for both cases. Figure 2.5 shows the pressure coefficient contours on the hull with and without the presence of the foil system. The pressure coefficient is defined as:

\[ C_p = 1 - (u^2 + v^2 + w^2) \]  

(2.7)

where \( u, v \) and \( w \) are the velocity components in longitudinal, transverse and vertical direction respectively, non-dimensionalized by means of the free stream velocity \( U \).

The reduction in pressure coefficient on the hull for the condition with foils for Case 2 is mainly due to the transverse velocity components induced by the foil system. This results in an additional suction force acting on the hull. From a calculation with the hull and forward foil only it is deduced that approximately 75% of this suction force is due to the trailing vortices from the forward foil, while the remaining percentage is due to the bound vortex system of the aft foil. Note that for the Dawson calculations the trailing vortices were positioned in the same horizontal plane as the foils. In reality the trailing vortex system will be displaced due to self-induced velocity components, free surface effects and hull induced velocity components. Although free surface effects may induce upward velocity components, in general the combined effects are such that the trailing vortex system moves downward, see Morch (1992). Therefore, assuming a horizontal trailing vortex system results in a conservative estimate of the foil-hull interaction.
Chapter 2  Computational method for steady flow conditions

Figure 2.5a  Pressure coefficient contours - Case 1

Figure 2.5b  Pressure coefficient contours - Case 2
For Case 1, due to the lower Froude number the length of the waves created by the forward foil is relatively low and a wave crest with longitudinal velocity components below unity is present at the hull position. These velocity components result in a small increase in pressure on the hull. Again, the largest part of the interaction effect (70%) is due to the forward foil.

The interaction coefficients for the hull-foil interaction ($C_{\theta}$) actually show the hull effect on the aft foil, because the forward foil is hardly affected by the hull. Figures 2.6 and 2.7 show the inflow angles along the aft foil semi span ($2y/b$) with and without the presence of the hull for both cases. The angles $\alpha$ and $\beta$ denote the inflow angles in the vertical and horizontal plane respectively, at the quarter chord position. The vertical inflow angles induced by the hull for Case 1 are larger than for Case 2 which is obviously due to the larger hull displacement. Since the contribution of the aft foil lift to the total vertical force is relatively small for Case 1, the total interaction effect remains limited. For Case 2, the vertical induced inflow angles for the forward foil only and hull plus forward foil conditions are approximately the same, when averaged over the span of the aft foil. For Case 1, the horizontal inflow angle induced by the hull is small, relative to that of the forward foil. For Case 2, the hull effect is larger. The effect of these horizontal inflow variations on the foil lift is small though. Although not shown in the Figures, the mean horizontal inflow angle on an imaginary strut positioned at the foil tip is limited to about 1 deg for both cases. This induces an additional induced drag component, which is estimated to be at maximum only 0.25% of the total resistance.

Figure 2.6a  Vertical inflow angle along aft foil span - Case 1
Figure 2.6b   Vertical inflow angle along aft foil span - Case 2

Figure 2.7a   Horizontal inflow angle along aft foil span - Case 1
2.3 Hull-foil, foil-hull and fore-aft foil interaction

The hull-foil interaction for the present configuration is neither very large nor very small. The reduction of the foil lift force is 3.4% and 2.2% for Case 1 and 2 respectively. Interaction coefficients for the trimming moments delivered by the foils are of similar magnitude. No means to estimate these interaction effects are available in the computational method to be developed and the errors due to the neglect of hull-foil interaction have to be accepted.

The foil-hull interaction coefficient for the vertical force for Case 2 (-10.6%) is relatively large and therefore worth to take into account. Also for other hull and foil configurations than considered here, it is probably useful to have an estimation method for this interaction component. For instance for surface piercing foil types, where the trailing vortices emanating from the forward foil may run relatively close to the hull due to the W-shape of the foil planform.

**Estimation of foil-hull interaction**

For determining the foil effect on the hull in the computational method for hydrofoil craft, it is assumed that the velocity components induced by the foil system at an imaginary hull surface can be determined by the vortex lattice method. Suppose that the non-dimensionalized (by $U$, the free stream velocity) perturbation velocity components at panel $j$ are given by $\Delta u_j$, $\Delta v_j$ and $\Delta w_j$ then the perturbation pressure coefficient $\Delta C_{p_j}$ and interaction force $\Delta F_j$ and moment $\Delta M$ components follow from:
\[ \Delta C_{pi} = -2 \Delta u_j - (\Delta u_j^2 + \Delta v_j^2 + \Delta w_j^2) \]
\[ \Delta F_z = \frac{1}{2} \rho U^2 \sum_{j=1}^{np} \Delta C_{pi} A_j n_{ij} \]
\[ \Delta M = \frac{1}{2} \rho U^2 \sum_{j=1}^{np} \Delta C_{pi} A_j x_j n_{ij} \]

Here, \( np \) denotes a number of hull surface panels, \( A_j \) is the area of panel \( j \), \( n_{ij} \) is the normal vector component in \( z \)-direction and \( x_j \) is the longitudinal position of the panel.

The velocity components induced by the foil system at the hull surface have been determined by means of Dawson, in order to assess the accuracy of the approximation method for the present cases. Besides for the actual flow, results are also shown for the so-called zero Froude number flow. The zero Froude number flow is a flow without waves and is obtained by computing the flow around a double-body model which consists of the actual body with its mirror image in upside down position on top. This flow is used in Dawson as the base flow about which the wavemaking flow is linearized. The Tables 2.3 and 2.4 show the estimated and actual interaction coefficients for the two cases. 'Estimated' denotes by use of eq. (2.8) while 'actual' denotes interaction coefficients based on Dawson results for hull plus foil system, as shown in Table 2.2.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Estimated</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F_n = 0 )</td>
<td>( F_n = 0.40 )</td>
</tr>
<tr>
<td>( C_{th} )</td>
<td>-0.039</td>
<td>+0.022</td>
</tr>
<tr>
<td>( C_{mph} )</td>
<td>-0.018</td>
<td>-0.025</td>
</tr>
</tbody>
</table>

| Table 2.4 Estimated and actual interaction coefficients for Case 2 |
|-----------------|-----------------|--------------|
| Case 2          | Estimated       | Actual       |
|                 | \( F_n = 0 \)   | \( F_n = 0.65 \) | \( F_n = 0 \) | \( F_n = 0.65 \) |
| \( C_{th} \)    | -0.102          | -0.124       | -0.100       | -0.106           |
| \( C_{mph} \)   | -0.032          | -0.036       | -0.033       | -0.029           |

These results show that for the zero-Froude number flow the estimation method yields fairly good results. Apparently the boundary condition at the hull, requiring a tangential velocity, is well approximated by using the double-body approach. With wavemaking the estimation method...
overestimates the interaction. The induced velocities at the imaginary hull due to the forward foil are apparently disturbed by the wavemaking of the hull when the hull is present.

The results obtained with the double-body approach can also be used for the conditions at speed, i.e. use the estimated coefficients for $F_a=0$ also for the cases with $F_a>0$. This results in a good approximation, except for the interaction coefficient for the vertical force for Case 1 which would be largely in error. An alternative way is to use the calculated perturbation velocities for the condition at speed whereby twice the $u$- and $v$-components are taken and the $w$-component is set to zero in eq. (2.8). This simulates the tangential flow condition at the hull surface. However, this approach results in interaction coefficients which are a factor two to three too large for all cases considered.

**Conclusions interaction effects between hull and foil system**

The mutual interaction between the hull and the foil system is neither very large nor very small, at least for the configuration considered here.

The effect of the foil system on the hull can be estimated by using the velocity components induced by the foil system at the hull surface.

The estimation method whereby the perturbation velocity components at speed are taken into account overestimates the foil-hull interaction, relative to the zero speed approach. However, it also results in the lowest remaining error since it compensates partially for the hull-foil interaction which can not be taken into account.

Therefore, the estimation method based on the induced velocity components at speed will be included in the computational method for hydrofoil craft.

The remaining errors due to interaction effects are limited to -2.7% for the total resistance and to -3.8% and -2.7% for the total vertical force and moment acting on hull plus foils.
Fore-aft foil interaction

An important part of fore-aft foil interaction is the effect of the wake, shed by the forward foil, on the aft foil. The vorticity in this wake induces velocity components at the aft foil. These lead to inflow variations which affect the magnitude and orientation of the lift force and thereby the induced drag. In the computational method it will be assumed that the wake sheet is a flat surface behind the reference plane of the foil, parallel to the free stream direction. As described in Section 2.1, wakes roll-up due to self induced velocities and are displaced due to free surface effects. This affects the position of the wake sheet relative to the aft foil and thereby the magnitude and orientation of induced velocities.

Another aspect of interest is the type of singularity elements used to discretize the wake sheet. In vortex lattice methods vortex lines enclosing a semi-infinite, horseshoe shaped strip are used. Very large, unrealistic inflow variations at the aft foil occur when induced velocities are determined at locations close to such a trailing vortex line. For such cases the use of vortex elements with less singular induced velocity components is to be preferred.

Investigation into vortex lines and distributed vortex strips

A flat wake sheet is represented by a set of parallel vortex lines or, alternatively, by a set of strips carrying a constant, distributed surface vorticity, see Figure 2.8. The vortex lines carry a circulation corresponding to an elliptically loaded foil. When the foil is discretized into \(nv\)-1 strips, the circulation around the \(nv\) vortex lines \(\Gamma_j\) is defined by the difference in bound circulation at the midpoints of the enclosed strips with width \(\Delta y\):

\[
y_j = \Delta y \left( j - \frac{1}{2} \right) \quad j = 1, nv - 1
\]

\[
\Delta y = \frac{b}{(nv - 1)}
\]

\[
\Gamma_j = \sqrt{1 - \left( \frac{2y_j}{b} \right)^2} \quad j = 1, nv - 1
\]

\[
\Gamma_y = \Gamma_j - \Gamma_{j-1} \quad j = 2, nv - 1
\]

\[
\Gamma_{\infty} = \Gamma_1
\]

\[
\Gamma_{nv} = -\Gamma_{nv-1}
\]

where \(y\) is the spanwise coordinate and \(b\) is the span of the foil. For the distributed vortex strip elements, the vortex strength is defined by:

\[
\gamma_j = \frac{d\Gamma_j}{dy} \quad j = 1, nv - 1
\]
The induced velocity components due to a vortex line are given by, see Katz and Plotkin (1991):

\[
\begin{align*}
\nu &= \sum_{j=1}^{j_{\text{rev}}} \frac{\Gamma_j z_{\eta_j}}{2\pi r_j^2} \\
\omega &= \sum_{j=1}^{j_{\text{rev}}} \frac{-\Gamma_j y_{\eta_j}}{2\pi r_j^2} \\
\rho &= \sqrt{(y_{\eta_j}^2 + z_{\eta_j}^2)}
\end{align*}
\]

(2.11)

where \(y_{\eta_j}\) and \(z_{\eta_j}\) are the distances between the field point and the \(j^{th}\) vortex line in horizontal and vertical directions respectively. For a piecewise constant vortex strip, the induced velocities are given by, see Katz and Plotkin (1991):

\[
\begin{align*}
\nu &= \sum_{j=1}^{j_{\text{rev}}} \gamma_j \left[ \frac{\arctan \left( z_{\eta_j} \right)}{y_{r_{2j}}} - \frac{\arctan \left( z_{\eta_j} \right)}{y_{r_{1j}}} \right] \\
\omega &= \sum_{j=1}^{j_{\text{rev}}} \frac{\gamma_j}{4\pi} \ln \left[ \frac{\left( y_{r_{2j}}^2 + z_{\eta_j}^2 \right)}{\left( y_{r_{1j}}^2 + z_{\eta_j}^2 \right)} \right]
\end{align*}
\]

(2.12)

where \(y_{r_{1j}}\) and \(y_{r_{2j}}\) denote the horizontal distances to the left and right edges of the vortex strip, respectively. On the centre of a distributed vortex strip, the horizontal velocity component \(\nu\) is
discontinuous and is taken to be zero.

Figure 2.9 shows a plot of induced velocity vectors due to the vortex line and vortex strip discretizations \((n_v=17)\), for one half of the symmetrical flow field. The position of the field points is such that the \(y\)-coordinates lie at the centre line of each vortex strip, i.e. half-way between the vortex lines.

It is seen that at some distance from the wake sheet the velocity vectors have the same direction but that the length of the vectors is somewhat smaller for the distributed vortex strips. On the wake sheet and especially near the outward vortex lines, the velocity vectors are differing in both direction and length.

Figure 2.10 shows a similar plot, but now the \(y\)-coordinates are such that the field points lie close to the edges of the vortex strips, at 90% of the strip width. The velocity vectors induced by the vortex lines are unrealistic near the edge of the wake sheet. Although the induced velocity due to a distributed vortex strip is singular at the edge of a strip, the Figure shows that these are much less sensitive to the position of the field point than those due to vortex lines.

![Figure 2.9 Induced velocity vectors for centred field points](image)
Figure 2.10 Induced velocity vectors for displaced field points

Figure 2.11 shows the mean values of the induced inflow angles determined from the data shown in Figure 2.9, over a width (y) of 1.5 times the wake sheet width. This width corresponds to that of an aft foil in a canard foil arrangement. The inflow angles $\alpha$ and $\beta$ are defined as $180\psi/\pi U$ and $180\omega/\pi U$ respectively, where the free stream velocity $U$ has been determined on basis of a lift coefficient of 0.50 and a mean circulation $\pi\Gamma/4$ from:

$$U = \frac{\pi \Gamma b}{2SC_l}$$  \hspace{1cm} (2.13)

where $S$ is the planform area of the foil.

Spreiter and Sacks (1951) give an estimation method for determining the vertical displacement of the rolled-up wake sheet cores, based on the lift coefficient and aspect ratio of the forward foil. Applying this method for a typical hydrofoil configuration, the vertical displacement is limited to approximately $2z/b=0.075$. Taking twice this distance to account for the vertical displacement of the centre of the wake sheet and free surface effects, a maximum displacement is estimated to be $2z/b = 0.15$. 

Figure 2.11  Mean induced inflow angles for canard arrangement

Figure 2.12  Mean induced inflow angles for tandem arrangement
It is seen in Figure 2.11 that the variation of the downwash angle $\alpha$ with height ($z$) is not very large for $2z/b<0.15$. The opposite contributions from the inward and outward span regions largely cancel the variation. Due to the discontinuity in the horizontal velocity component between positions just below and above the wake sheet, the variation in the sideward angle $\beta$ with height is large for the region of interest. The sideward affects the inflow angle for dihedral (inclined) foil parts.

The mean inflow angles induced by distributed vortex strips are much more sensitive to the number of vortex strips used to represent the wake than those induced by vortex lines. For using 16 vortex strips, the mean downwash induced by distributed vortex strips is 25% lower than that induced by vortex lines. When using 64 vortex strips, this difference reduces to 12%. Although not shown in the Figures, using 256 vortex strips reduces the difference to 1%. This is probably due to the singularity in the vortex strength $\gamma$ at the tips of the lifting surface which is not well represented when a low number of vortex strips is used. It has been verified that using, instead of the analytical derivative of the circulation in eq. (2.10) at the centre of the strip width, a difference scheme based on the circulation at the edges of the strips does not improve the convergence.

Figure 2.12 shows the mean inflow angles over a width equal to the wake sheet width, corresponding to a tandem foil arrangement. The downwash varies now more strongly with the height above the wake sheet. The downwash induced by distributed vortex strips is now 40 to 50% lower than that of vortex lines, for 16 vortex strips. For using 64 vortex strips, the difference reduces to 11 to 23%.

For airplane foil configurations the differences in induced velocity components between vortex lines and distributed vortex strips are smaller than for a tandem arrangement. In that case the aft foil does not operate behind the tip region of the forward foil.

Figure 2.13 shows a comparison of induced velocity vectors for a flat and a curved wake sheet, both represented by vortex lines. The curved wake sheet is a schematic representation of a rolled-up wake sheet. Appreciable differences for the velocity vectors occur for positions close to the outward vortex lines. However, Figures 2.14 and 2.15 show that the variation of the mean downwash angle $\alpha$ with height and the differences due to the curving of the wake sheet are small. The variation in sideward angle $\beta$ for $0<2z/b<0.15$, the region of interest, is again larger than for the downwash angle, but still not dramatic.

Figure 2.16 shows the sideward angle experienced by a vertical strut as a function of its transverse position. The sideward angle shown is the mean value over a strut inbetween $z=0$ and $2z/b=0.5$. The sideward is more affected by vertical wake sheet displacements than the downwash angle. Near the edge of the wake sheet the variation in sideward angle is appreciable and causes additional induced drag. This additional induced drag is estimated to be 0.5% of the total resistance for two vertical aft foil struts for a typical hydrofoil craft. The induced lift (side) force will be cancelled by the strut on the opposite side of the centre line. Therefore, the differences in induced sideward angle at struts due to variations in the wake sheet position do not significantly affect the total resistance. On the other hand, sideward variations at lift producing surface piercing foil parts may induce appreciable variations in lift and induced drag.
Figure 2.13  Induced velocity vectors for flat and curved wake sheets

Figure 2.14  Mean induced inflow angles for flat and curved wake sheets for canard arrangement
Figure 2.15 Mean induced inflow angles for flat and curved wake sheets for tandem arrangement

Figure 2.16 Mean induced sidewash angles
Conclusions fore-aft foil interaction

When the position of the field points is not half-way between vortex lines, vortex strips must be used in order to avoid unrealistic interaction effects.

When the field points are located half-way between the vortex lines, induced velocity components due to distributed vortex strips are more sensitive to the number of vortex strips used to discretize the wake sheet than induced velocity components due to vortex lines.

In general, the position of the aft foil control points, where the induced velocity due to the forward foil trailing vortices and the aft foil itself are evaluated, can be selected such that these lie half-way between the vortex lines. This can be achieved by varying the number of vortex strips on the forward and aft foils and varying the tip inset width, as explained in Section 2.5. Therefore it is decided to use vortex lines for the discretization of the wake sheet.

Downwash due to fore-aft foil interaction for canard foil arrangements is not very sensitive to position variations of a flat wake sheet relative to the aft foil. Downwash for tandem foil arrangements is much more sensitive to such position variations. However, for a curved wake sheet this sensitivity reduces significantly. Sidewash is more sensitive to vertical position variations of wake sheets, but the effects of sidewash variations at struts on lift and drag are small.
2.4 Equilibrium equations

In order to determine the attitude of hydrofoil craft for steady flow conditions a set of force and moment equations has to be solved. These equations contain all force and moment components acting on the craft in the vertical plane, and represent an equilibrium condition in terms of the trim angle and hull draft at the keel-stern intersection. A space-fixed axis system is used in the plane of symmetry with its origin at the intersection of the keelline and the transom, the x-axis pointing forward, the z-axis pointing upwards, see Figure 2.17. The equations read as follows:

\[ \Sigma F_x + T \cos(\epsilon + \tau) = 0 \]  \hspace{1cm} (2.14)

\[ \Sigma F_z + T \sin(\epsilon + \tau) = 0 \]  \hspace{1cm} (2.15)

\[ \Sigma M - T \cos(\epsilon + \tau) z_y + T \sin(\epsilon + \tau) x_y = 0 \]  \hspace{1cm} (2.16)

![Figure 2.17 Axis system for equilibrium equations](image)

Here \( F_x \) and \( F_z \) denote the horizontal and vertical force components respectively, \( M \) denotes the moment about the origin of the coordinate system due to the \( F_x \) and \( F_z \) components. A bow-up moment is positive. \( T \) denotes the thrust force, acting at position \( (x_T, z_T) \) and \( \epsilon \) and \( \tau \) denote the thrust inclination and trim angle, respectively.

It is understood that the moment lever arms incorporate the effect of the trim angle as follows:

\[ x = x_c \cos(\tau) - z_s \sin(\tau) \]  \hspace{1cm} (2.17)

\[ z = z_c \cos(\tau) + x_s \sin(\tau) \]

where \( x_s \) and \( z_s \) denote the location of a force centre of effort in a ship fixed axis system.
Chapter 2  Computational method for steady flow conditions

It is understood that the moment lever arms incorporate the effect of the trim angle as follows:

\[ x = x_i \cos(\tau) - z_i \sin(\tau) \]  
\[ z = z_i \cos(\tau) + x_i \sin(\tau) \]  

(2.17)

where \( x_i \) and \( z_i \) denote a force centre of effort location in a ship fixed axis system.

The force components \( F_x \) and \( F_z \) originate from the hull, the foil system and the appendages. Apart from the hydrodynamic forces acting on the hull and foil system, the craft's weight and the aerodynamic forces on the hull and superstructure have to be accounted for.

It is convenient to reduce the number of equations from three to two by elimination of the thrust force \( T \). The equations then read:

\[ \sum F_z - \tan(\varepsilon + \tau) \sum F_x = 0 \]  
\[ \sum M - x_i \sum F_z + z_i \sum F_x = 0 \]  

(2.18)  
(2.19)

For a given hull, foilsystem and appendage configuration at a given speed, all force components are in principle a function of four variables: trim angle \( \tau \), hull draft \( D \) (foilborne) or displacement \( \Delta \) (hullborne), fore foil flap angle \( \delta_i \) and aft foil flap angle \( \delta_a \). In case of incidence control, the variable incidence angle \( \alpha \) can be used instead of the flap angle \( \delta \).

To solve the equilibrium equations (2.18) and (2.19) it is necessary to assume a value for two of the four unknown variables \( (\tau, D \text{ or } \Delta, \delta_i, \delta_a) \). For hullborne conditions it is convenient to calculate the attitude of the craft for a matrix of given flap angle values for each speed. As will be shown in Section 2.7, the force components acting on the hull are described as a function of trim and displacement. For a given set of flap angle values, the equilibrium equations are solved for the trim and hull displacement. In this way a good insight in the take-off characteristics is obtained. For foilborne conditions, it is more convenient to use a matrix of given trim and draft values per speed in order to calculate the corresponding flap angles. The two non-linear equilibrium equations are solved by a hybrid Powell method, see Garbow et al. (1980).

Once the equilibrium equations are solved, the thrust force \( T \) follows from:

\[ T = \frac{-\sum F_x}{\cos(\varepsilon + \tau)} \]  

(2.20)

In the following sections the derivation of the various force components is described. Herein most attention is paid to the calculation method used for the foil system forces.
2.5 Foil system forces

General

A body-fixed, right-handed Cartesian coordinate system is defined in the undisturbed free surface with the z-axis pointing upward, the y-axis pointing to starboard and the x-axis pointing along the incoming flow direction, see Figure 2.18. The hydrofoil system moves at constant horizontal speed $U$. The hydrofoil parts are assumed to be thin and are represented by their reference planes. A vortex system is thought to be present in the reference plane of the foil and in the wake sheet lying in the flat plane behind the foil, extending towards infinity, see Figure 2.19. The hydrofoil system may consist of several parts, each consisting of a foil with an arbitrary dihedral angle $\beta$, sweep angle $\Lambda$, and a linearly varying chord along its span, see Figure 2.20.

![Figure 2.18 Foil axis system](image)

![Figure 2.19 Reference and wake planes of foil part](image)
Potential flow theory

The fluid is assumed to be incompressible and inviscid. The fluid domain is bounded only by the undisturbed watersurface \( z = 0 \) and the infinitesimal thin foil and wake surfaces. The normal vector is defined positive into the fluid domain. A velocity potential \( \Phi \) is used to describe the fluid velocity vector. A necessary condition for using a velocity potential is that the fluid be irrotational. Then the vorticity vector \( \omega \):

\[
\omega = \nabla \times \mathbf{V}
\]

is zero everywhere in the fluid domain. By using the incompressibility of the fluid:

\[
\nabla \cdot \mathbf{V} = 0
\]

it can be shown that the velocity potential has to satisfy the Laplace equation everywhere in the fluid domain:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]

(2.23)

The total velocity potential \( \Phi \) is written as:

\[
\Phi = \phi_\infty + \phi
\]

(2.24)

where \( \phi_\infty \) represents the undisturbed incoming flow and \( \phi \) is the unknown velocity potential associated with the circulation induced by the foil. The undisturbed velocity potential is given by:

\[
\phi_\infty = Ux
\]

(2.25)
The fluid velocity vector $\mathbf{V}$ associated with the disturbance potential $\phi$ is expressed as:

$$\mathbf{V}(x,y,z) = \nabla \phi = i \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} + k \frac{\partial \phi}{\partial z}$$  \hspace{1cm} (2.26)

at a point $(x,y,z)$ in the fluid.

**Boundary conditions**

In order to be able to determine the potential function $\phi$ and its gradient a number of conditions are imposed. For a rigid body moving with velocity $U$ relative to the fluid, the boundary condition:

$$U \cdot \mathbf{n} + \frac{\partial \phi}{\partial n} = 0 \quad \text{on the chord plane}$$  \hspace{1cm} (2.27)

specifies that the flow must be tangential to the chord plane. Here $\partial/\partial n$ denotes differentiation along the normal on the body surface.

On the watersurface two boundary conditions are imposed, see Faltinsen (1993). The kinematic condition:

$$\frac{\partial \phi}{\partial z} - \frac{\partial \phi}{\partial x} \frac{\partial \zeta}{\partial x} - \frac{\partial \phi}{\partial y} \frac{\partial \zeta}{\partial y} = 0 \quad \text{on } z = \zeta(x,y)$$  \hspace{1cm} (2.28)

reflects the tangential flow condition at the free surface. The dynamic condition:

$$\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2 + 2g \zeta = 0 \quad \text{on } z = \zeta(x,y)$$  \hspace{1cm} (2.29)

reflects that the pressure at the free surface equals the constant atmospheric pressure. Here $\zeta(x,y)$ denotes the wave elevation or free surface disturbance and $g$ denotes the gravitation constant.

These boundary conditions have to be applied at the actual free surface, which position is unknown a priori. In order to circumvent this problem the free surface conditions are linearized in the sense that they are imposed on the undisturbed free surface ($z=0$) instead on the disturbed free surface $\zeta(x,y)$. Furthermore, quadratic and cross product terms will be neglected. In doing so it is assumed that the disturbance velocities are small relative to the undisturbed flow, which also implies that the wave elevation is small relative to the wave length.

The kinematic condition then assumes the form:

$$\frac{\partial \phi}{\partial z} - U \frac{\partial \zeta}{\partial x} = 0 \quad \text{on } z = 0$$  \hspace{1cm} (2.30)

while the dynamic condition then reads:
\[ U \frac{\partial \phi}{\partial x} + g \zeta = 0 \quad \text{on} \ z = 0 \]  
(2.31)

The kinematic and dynamic free surface conditions may be combined into one condition as follows:

\[ \frac{\partial^2 \phi}{\partial x^2} + k_0 \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \ z = 0 \]  
(2.32)

where the wave number is defined as:

\[ k_0 = g/U^2 \]  
(2.33)

The last condition applied to the present problem is the radiation condition which states that there are no waves present upstream of the hydrofoil.

**Potential flow solution based on a Green's function**

Newman (1987) gives the Green's function for a unit strength source in steady motion underneath a planar free surface. Here we define an analogous Green's function for a unit strength vortex line segment underneath the free surface:

\[ G(x,y,z;\xi,\eta,\zeta) = \mathcal{R} \left[ \frac{1}{R} + \frac{1}{R_0} - \frac{i2k_0^{n/2}}{\pi} \int_{-\pi/2}^{\pi/2} \cos\theta e^{-iE_i(v)} d\theta - i4k_0H(k_0(x-\xi)) \int_{-\pi/2}^{\pi/2} \sec^2\theta e^{-iE_i(v)} d\theta \right] \]  
(2.34)

where

\[ v = k_0(z+\zeta)\cos\theta + k_0 |y-\eta| \cos\theta \sin\theta + ik_0 |x-\xi| \cos\theta \]

\[ u = k_0(z+\zeta)\sec^2\theta + ik_0 |y-\eta| \sec^2\theta \sin\theta - ik_0 |x-\xi| \sec\theta \]

\[ R = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2} \]

\[ R_0 = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2} \]

\[ E_i = \frac{\pi}{2} k_0 \log \left( |z-\xi| \right) \]

\[ k_0 = g/U^2 \]

\[ H(x) = \begin{cases} 1 & \text{for} \quad x \geq 0 \\ 0 & \text{for} \quad x < 0 \end{cases} \]  
(2.36)

The first integral in eq. (2.34) represents the formation of non-radiating local waves while the
second integral represents the downstream free wave system.

In contrast to the usual Green's function formulation for a submerged source, the biplane image term has a positive sign and the wave terms have a negative sign. Due to this approach the free surface conditions are satisfied for arbitrary Froude numbers as follows. It can be shown that at a Froude number approaching zero, the wave terms approach a value of $-2/R_0$ so that the Green's function value is $1/R-1/R_0$. For a vortex plus a negative biplane image the normal velocity at the free surface is zero. This is the free surface condition for zero speed, which follows from eq. (2.30). At an infinite Froude number the wave terms in eq. (2.34) approach zero so that the Green's function is $1/R+1/R_0$. For a vortex plus a positive biplane image the longitudinal velocity at the free surface is zero. This is the free surface condition for a very high speed, which follows from eq. (2.31).

It can be shown that the Green's function defined in eq. (2.34) satisfies the Laplace eq. (2.23), the radiation condition and the free surface condition (2.32).

The formulation for the velocity potential follows from using Green's second identity, see for instance Moran (1984), which states that the velocity potential of any irrotational flow can be represented by a distribution of singularities over its bounding surfaces:

$$4\pi \Phi(x,y,z) = \oint_{S_w} (\Phi^* - \Phi) \frac{\partial G}{\partial n} dS + \oint_{S_r} (\Phi \frac{\partial G}{\partial n} - G \frac{\partial \Phi}{\partial n}) dS$$

(2.37)

where $S_{bw}$ denotes the chord plane and the wake sheet behind the foil, $S_r$ denotes the free surface and $(\Phi^* - \Phi)$ denotes the discontinuity across the vortex surface $S_{bw}$ which equals the strength of a doublet distribution $\mu$. The fact that for a doublet distribution the normal derivative of $\Phi$ is continuous over the vortex surfaces has been used to omit the $\partial \Phi/\partial n$ term in the integral over $S_{bw}$.

The second term in eq. (2.37) can be shown to disappear as follows. On the undisturbed free surface the normal derivative equals the negative derivative in $z$ direction:

$$\oint_{S_r} (\Phi \frac{\partial G}{\partial n} - G \frac{\partial \Phi}{\partial z}) dS = \oint_{S_r} (G \frac{\partial \Phi}{\partial z} - \Phi \frac{\partial G}{\partial z}) dS$$

(2.38)

By using the linearized free surface boundary condition (2.32) and changing to the $\xi$ variable, the following expression is obtained:

$$\frac{1}{k_0} \oint_{S_r} (\Phi \frac{\partial^2 G}{\partial \xi^2} - G \frac{\partial^2 \Phi}{\partial \xi^2}) dS = \frac{1}{k_0} \oint_{S_r} (\Phi \frac{\partial^2 G}{\partial \xi^2} - G \frac{\partial^2 \Phi}{\partial \xi^2}) d\xi d\eta$$

(2.39)

By using the behaviour at infinity:
\[ \lim_{r \to R} G = \frac{1}{R_0} \]  

(2.40)

\[ \lim_{r \to R} \frac{\partial G}{\partial \xi} = \frac{x - \xi}{R_0^3} \]

one finally finds:

\[ \frac{1}{k_0} \int \int \left( \phi \frac{\partial^2 G}{\partial \xi^2} - G \frac{\partial^2 \phi}{\partial \xi^2} \right) d\xi d\eta = \frac{1}{k_0} \int \int \left[ \phi \frac{\partial G}{\partial \xi} - G \frac{\partial \phi}{\partial \xi} \right] d\eta = 0 \]  

(2.41)

The expression for the velocity potential (2.37) is now written as:

\[ 4\pi \phi(x,y,z) = \int \int \mu \frac{\partial G}{\partial n} dS = \int \int \mu \frac{\partial G}{\partial n} d\xi ds \]  

(2.42)

where \( \xi \) denotes the chordwise coordinate and \( s \) denotes an integration variable along the span of the foil parts. By using a coordinate transformation over the dihedral angle \( \beta \) the normal derivative in eq. (2.42) is written as:

\[ 4\pi \phi(x,y,z) = \int \int \mu \left( -\sin \beta \frac{\partial G}{\partial \eta} + \cos \beta \frac{\partial G}{\partial \xi} \right) d\xi ds \]  

(2.43)

For evaluation of this velocity potential function the following notation, derived from Morch (1992), is adopted. Define the integral \( I \) as:

\[ I = \int_{\xi} G d\xi \]  

(2.44)

This integral can be split into three parts as follows:

\[ I_1 = \int_{\xi} \mu \left[ \frac{1}{R} + \frac{1}{R_0} \right] d\xi \]  

(2.45)

\[ I_2 = \Re \left\{ \frac{-2ik_0}{\pi} \int_{-\pi/2}^{\pi/2} \cos \theta \int_{\xi} \mu e^{i\nu E_1(\nu) d\xi} d\theta \right\} \]  

(2.46)

\[ I_3 = \Re \left\{ \frac{-4ik_0}{\pi} \int_{-\pi/2}^{\pi/2} \sec^2 \theta \int_{\xi} \mu H(k_0(x - \xi)) e^{i\nu \xi} d\xi d\theta \right\} \]  

(2.47)

The final velocity potential expression is then:
\[ 4\pi \phi(x,y,z) = \Re \left\{ \left[ \left( \sum_{i=1}^{3} \frac{\partial l_i}{\partial \eta} \right) \sin \beta + \left( \sum_{i=1}^{3} \frac{\partial l_i}{\partial \zeta} \right) \cos \beta \right] ds \right\} \] (2.48)

This expression and its gradient, the disturbance velocity vector:

\[ \mathbf{V} = \nabla \phi \] (2.49)

are elaborated in Appendix A.

Discretization into vortex elements

The discretization of the doublet distribution \( \mu \) into horseshoe vortex elements and the determination of the circulation \( \Gamma \) by using the tangential flow boundary condition are described. In essence, the vortex lattice method is used here. The following discretization is applied. The planform of the foil is divided into trapezoidal elements. Each element consists of a swept horseshoe vortex system with unknown circulation strength \( \Gamma_h \) which equals the strength of the doublet distribution on the element. The vortices lie in the reference plane (zero camber and incidence) of the foils. Bound vortex filaments lie along the quarter chord line of each trapezoidal element. The control point, where the tangential flow boundary condition is imposed, is located at the three quarter chord point, half-way the element width. Trailing vortex filaments extend from the bound vortex edges towards infinity parallel to the \( x \)-axis, see Figure 2.21.

![Figure 2.21 Vortex lattice](image-url)
The local chord at each spanwise station is divided into $N$ vortex elements while the span is divided into $M$ vortex elements. By using only one chordwise element ($N=1$) a lifting line approach is obtained. The number of chordwise elements is constant for each foil part, the number of spanwise elements may vary per foil part. The locations of the bound vortex segments and the control point follow from the so-called lumped vortex method. In this method the distributed vorticity used in the thin airfoil theory is concentrated in a number of bound vortex lines. For parabolic camber line types the flow tangency condition is exactly satisfied at the three quarter chord point, for a single bound vortex filament. For other camber line types than parabolic, the actual camberline may be represented by parabolic parts, by using a series of chordwise vortex elements.

The Kutta condition for steady flow is that the velocity along the trailing edge of lifting surfaces remains finite. The thin airfoil equivalent of this condition is that the vorticity at the trailing edge be zero. This requirement is satisfied by the lumped vortex method for each vortex element at the trailing edge.

When the camber surface of a hydrofoil is discretized into uniformly spaced chordwise and spanwise vortex elements, the convergence of lift and drag with the number of elements is rather slow, see Hough (1976) and DeJarnette (1976). The convergence can be improved drastically by positioning the outermost trailing vortex line a bit inwards, relative to the tip. Hough (1976) shows that the optimum free tip width is a quarter of the vortex element width when the vortex elements have a constant width. This spanwise discretization is adopted in the present method.

A further improvement in convergence, especially when trailing edge flaps are used, can be obtained by using cosine spaced chordwise elements, whereby the midpoint of the forward edge of the vortex elements is positioned according to:

$$\frac{x}{c} = \frac{1 - \cos \theta}{2} \quad \text{(2.50)}$$

where

$$\theta_i = \frac{(i - 1) \pi}{N} \quad (i = 1, 2, ..., N) \quad \text{(2.51)}$$

In case of using trailing edge flaps, the number of elements on the flap $N_f$ is taken as:

$$N_f = N \frac{c_f}{c} \quad \text{(2.52)}$$

where $c_f$ and $c$ denote the flap chord and total foil chord respectively. For partial span flaps, the trailing vortex lines of the flap tip vortex elements are made to coincide with the flap edges.

**Determination of the circulation strength**

The tangential flow condition imposed at the control point is stated as follows:
\[ \frac{w_c}{U} = - \frac{\partial f_c(x=\frac{3}{4} c)}{\partial x} \]  

(2.53)

where \( w_c \) is the vertical induced velocity component normal to the chord plane of a hydrofoil part, \( x \) denotes the chordwise coordinate at some vortex element and \( c \) denotes the vortex element length. The function \( f_c(x) \) which includes incidence \( \alpha \) and flap angle \( \delta \) contributions as appropriate:

\[ \frac{\partial f_c(x)}{\partial x} = - \frac{\partial f(x)}{\partial x} + (\alpha + \delta) \]  

(2.54)

where \( f(x) \) represents the actual camber line relative to the chord line of the foil section. In case of a lifting line approach the effects of camber and flap deflections may be included in the angle of incidence by using the zero-lift angle of attack \( \alpha_0 \) as follows in the tangential flow condition:

\[ \frac{\partial f_c(x)}{\partial x} = (\alpha - \alpha_0) \]  

(2.55)

where, according to Glauert's (1948) method for thin airfoils:

\[ \alpha_0 = \frac{\int_0^c f(x) dx}{\pi (1-x) \sqrt{x(1-x)}} \]  

(2.56)

here \( f(x) \) may include the effect of the flap angle on the camber line by defining the section chord line from the leading edge to the deflected trailing edge.

A more practical way to obtain the effective camber for use in a lifting line approach is to use tabulated airfoil section data, see Abbott and Von Doenhoff (1958). Tabulated data on flap efficiencies may be obtained from Martin (1963a), based on thin wing theory. Here the flap is represented by a shift in zero lift angle of attack:

\[ \Delta \alpha_0 = -\alpha_0 \delta \]  

(2.57)

\[ \alpha_0 = f\left(\frac{c_f}{c}\right) \]

where \( \alpha_0 \) denotes the flap efficiency which is a function of the flap chord to foil chord ratio.

The induced velocity component normal to the chord line of the section results from:

\[ w_c = \frac{u_c \cdot n_c}{c} \]  

(2.58)

where \( u_c \) is the induced velocity vector \((u,v,w)\) at the control point and \( n_c \) is the normal vector on the chord plane. By using the foil dihedral angle \( \beta \) one finds:
\[ w_c = -v \sin \beta + w \cos \beta \]  
\[ \text{(2.59)} \]

The induced velocity vector \( \mathbf{u}_c \) consists of contributions from each vortex element. By substituting the expression for the derivatives of the velocity potential the following scheme may be set up, see eqs. (2.48), (2.49) and (2.53):

\[ -\frac{\partial \phi}{\partial y} \sin \beta + \frac{\partial \phi}{\partial z} \cos \beta = -\frac{\partial f_c(x)}{\partial x} U \]  
\[ \text{(2.60)} \]

or

\[ \Re \{ \int_s \left[ \sum_{k=1}^{3} \frac{\partial I_k}{\partial y \eta} \sin \beta_i + \sum_{k=1}^{3} \frac{\partial I_k}{\partial z \eta} \cos \beta_i \right] ds \} \sin \beta_j + \]
\[ \Re \{ \int_s \left[ \sum_{k=1}^{3} \frac{\partial I_k}{\partial z \eta} \sin \beta_i + \sum_{k=1}^{3} \frac{\partial I_k}{\partial z \zeta} \cos \beta_i \right] ds \} \cos \beta_j = \]
\[ -4\pi U \frac{\partial f_c(x)}{\partial x} \]  
\[ \text{(2.61)} \]

where the subscript \( j \) denotes the vortex element at which the control point is located and the subscript \( i \) denotes the vortex element containing the bound vortex line segment. The integration is carried out over the span \( s \) of each foil part. Within the assumptions of linearization it is consistent to rewrite eq. (2.61) as a system of linear equations in the unknown circulation strength \( \Gamma_i \) at vortex element \( j \):

\[ \sin \beta_j \sum_{i=1}^{n} \Gamma_i F_{wi} + \cos \beta_j \sum_{i=1}^{n} \Gamma_i F_{wi} = -4\pi U \frac{\partial f_c(x)}{\partial x} \]  
\[ \text{(2.62)} \]

or

\[ A_{ji} \Gamma_i = X_j \]

where

\[ A_{ji} = \sin \beta_j F_{wi} + \cos \beta_j F_{wi} \]  
\[ \text{(2.63)} \]

\[ X_j = -4\pi U \frac{\partial f_c(x)}{\partial x} \]

The induced velocity functions \( F_w \) and \( F_{wi} \) follow from eq. (2.61) as:
\[ F_w = \Re \{ \int \left( \sum_{k=1}^{3} \frac{\partial I_k}{\partial \eta} \sin \beta_i - \sum_{k=1}^{3} \frac{\partial I_k}{\partial \zeta} \cos \beta_i \right) ds \} \]
\[ F_{wi} = \Re \{ \int \left( -\sum_{k=1}^{3} \frac{\partial I_k}{\partial \eta} \sin \beta_i + \sum_{k=1}^{3} \frac{\partial I_k}{\partial \zeta} \cos \beta_i \right) ds \} \] (2.64)

**Calculation of hydrofoil forces**

Once the circulation strength $\Gamma_i$ at element $i$ is known, the force on each vortex element can be obtained by means of integration of the pressure difference and, alternatively, by applying the Kutta-Joukowski law. The disadvantage of pressure integration is that the drag is inaccurate due to the existence of the leading edge suction force which cannot be evaluated from a distribution of horseshoe vortex elements. The Kutta-Joukowski law implicitly accounts for the leading edge suction force as the resulting force is, for a two-dimensional flat plate at incidence, directed normal to the free stream direction. Thus, for the two-dimensional problem, the force in flow direction is zero. Pressure integration for a two-dimensional flat plate at incidence results in a force in flow direction which must be balanced by the leading edge suction force to have a zero total force. The disadvantage of applying the Kutta-Joukowski law is that additional evaluations of the velocity components at the bound vortex positions are required. Also, the velocity is singular at these positions. To reduce computer time requirements, these can be obtained from interpolation on the velocity components at the control points. This option is however not investigated in the present study.

The Kutta-Joukowski law is as follows:

\[ F_i = \rho U_{\text{ei}} \times \Gamma_i \Delta s_i \] (2.65)

where $F_i$ denotes the force vector, $U_{\text{ei}}$ denotes the effective velocity vector and $\Delta s_i$ denotes the vortex element width. The effective velocity vector contains the free stream velocity $U$ and the induced regularized velocity components due to all vortex elements:

\[ U_{\text{ei}} = (U + u_i, v_i, w_i) \] (2.66)

The induced velocity components are determined by using the induced velocity functions, eq. (2.64), and actual circulation strengths, at the centre of the bound vortex filament.

The circulation strength $\Gamma_i$ is a vector with components as follows:

\[ \Gamma_i = (\sin \lambda_i, \cos \beta_i, \cos \lambda_i, \sin \beta_i, \cos \lambda_i) \] (2.67)

which reflects the effect of the foil orientation angles $\beta$ (dihedral) and $\Lambda$ (sweep). For three-dimensional flows, the force vector $F_i$ is directed perpendicular to the local velocity vector $U_{\text{ei}}$, rather than perpendicular to the undisturbed velocity vector in a two-dimensional flow. This introduces a force component in the free stream direction, the induced drag. The non-dimensional
force coefficients are obtained from:

\[
C_{se} = \frac{F_i}{\frac{1}{2} \rho U^2 c_i \Delta s_i} \tag{2.68}
\]

for each vortex element \( i \), where \( c_i \) denotes the chord of the vortex element \( c/N \). Summation over the number of chordwise elements \( N \) results in stripwise force coefficients:

\[
C_s = \frac{1}{N} \sum_{i=1}^{N} C_{se} \quad \tag{2.69}
\]

**Numerical aspects**

For setting up a computer program containing the vortex lattice method several numerical aspects have to be investigated. These aspects relate to the required computer time for numerical solutions for integrals and the integration of functions with a singular behaviour.

Analytical integration of the \( I_i \) terms, see eq. (2.45), is possible. The resulting expressions are however quite laborious and time consuming both in programming and computer time. A more efficient and elegant approach is to use the Biot-Savart law directly for a horseshoe type vortex system in a local, vortex bound axis system and to transform the velocity components into the global \((x,y,z)\) axis system. The derived expressions for the velocity components by using this procedure are shown in Appendix A.

If a field point is located exactly on a vortex line segment the induced velocity due to that vortex segment is singular. For determining the forces according to the law of Kutta-Joukowski, for each vortex element the self-induced velocity at the bound vortex position \( r = \xi \) is set to zero.

As shown in Section 2.3, trailing vortices emanating from a forward foil will induce large velocity components when passing closely to an aft foil control point. Therefore, the forward and aft foil vortex element widths must be selected such that the trailing vortex lines from both foils coincide, when lying in the same plane. This can be obtained by selecting a suitable number of vortex elements of the forward and aft foils and by variations in the free tip width.

It should be noted that the maximum number of vortex elements is limited. Using a high number of vortex elements reduces the spacing between control points and the vortex lines, especially in chordwise direction due to the cosine element spacing. Calculation results become inaccurate due to loss of significant digits in combination with the singular behaviour of induced velocities. The vortex lattice method is programmed on a PC in double precision. For this configuration the maximum number of chordwise vortex elements that can be used is about 128, for each chordwise vortex strip.

The expressions for the integrals associated with \( I_2 \), see eq. (2.46) and Appendix A, are singular for \( \nu = 0 \) due to the complex exponential integral \( E_i(\nu) \) and the term \( (1/\nu) \). \( E_i(\nu) \) is defined by Abramowitz and Stegun (1970) as follows:
2.5 Foil system forces

\[ E_1(v) = \int_{v}^{\infty} \frac{dt}{t} e^{-t} \]  

(2.70)

where

\[ v = k_0(z + \zeta) \cos^2 \theta + k_0 \left| y - \eta \right| \cos \theta \sin \theta + ik_0 \left| x - \xi \right| \cos \theta \]  

(2.71)

The argument \( v \) becomes zero for:

\[ \theta = \pm \frac{\pi}{2} \]  

(2.72)

i.e. at the edges of the integration interval, and for:

\[ \theta = \arctan \left( \frac{\xi + \zeta}{|y - \eta|} \right) \]  

(2.73)

for \( x = \xi \), at the bound vortex position.

The singularity in \( E_1(v) \) is of a logarithmic type and can be numerically approximated by using systematic expansions, as shown by Newman (1987). He considers integrals of the type:

\[ \int_{-\pi/2}^{\pi/2} \cos \theta e \cdot E_1(v) \, d\theta \]  

(2.74)

which must be solved when a discretization into Rankine source panels is utilized. In the present problem partial derivatives of \( I_2 \) with respect to \( x, y, \zeta \) and \( z \) have to be evaluated. This leads to integrals of the type:

\[ \int_{-\pi/2}^{\pi/2} \cos^n \theta \sin^m \theta (e \cdot E_1(v) - \frac{1}{v}) \, d\theta \]  

(2.75)

where \( n \) and \( m \) are integers. Series expansions for integrals of this type have been developed, analogous to the work of Newman (1987), see Appendix A. The partial derivatives can then be determined by means of analytical differentiation.

The integrals involved in the \( I_j \) expressions, see eq. (2.47) and Appendix A, pose no singularity problems as the sec^\( n \theta \) terms for \( \theta = \pm \pi/2 \) are counteracted by the \( e^{-v} \) terms and the integrand is zero at the edges of the integration interval. Despite the absence of singularities, the integrands have a rather steep and oscillatory behaviour near edges of the integration interval, for field points close to the free surface. This requires a large number of function evaluations, approximately 1000 for a relative accuracy of 10^\(-4\), when these integrals are determined by means of standard numerical integration schemes.
A more efficient approach is obtained by transforming the integrals to a $t$-domain by using $t = \tan \theta$, following Hally (1994), see Figure 2.22.

**Figure 2.22a** Integrand of a partial derivative of $I_3$ in the $\theta$-domain

**Figure 2.22b** Integrand of a partial derivative of $I_3$ in the $t$-domain
This transformation reduces the relative steepness of the oscillations, but on the other hand it introduces an infinite integration interval. However, this interval can be truncated to a finite interval \( t \in [-T, T] \), by using the fact that the integrands are smaller than \( t \exp(\varepsilon T^2) \). \( T \) can be determined such that \( T \exp(\varepsilon T^2) = \varepsilon \), where \( \varepsilon \) is a small quantity, by using a suitable recursion relation. This approach reduces the number of function evaluations by a factor 5 to 10, relative to using the untransformed integrals. After testing a number of numerical integration methods an auto-adaptive subroutine based on an 8-point Newton-Cotes rule, see Stoer and Bulirsch (1980), was selected for reasons of computational efficiency and reliability.

The integration of the derivatives of the \( I_5 \) and \( I_3 \) integrals over the foil span is carried out by applying the trapezium rule on the vortex element widths \( s_r \).

### Computer time requirements

The computational method has been implemented in a Fortran 90 computer program. This stand-alone program is called Hydvlm and runs on a PC under DOS. Table 2.5 shows the CPU-time requirements for a 166 MHz Pentium PC (32 Mb RAM) for 16 spanwise (\( M \)) and 4 chordwise (\( N \)) vortex elements, with and without accounting for the wave integrals in the Green's function.

<table>
<thead>
<tr>
<th>Condition</th>
<th>CPU (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_5 ) terms only</td>
<td>&lt;1</td>
</tr>
<tr>
<td>( I_5, I_2 ) and ( I_3 ) terms</td>
<td>8</td>
</tr>
</tbody>
</table>

The required computer time on a Cray C98 mainframe is about five times lower, after some initial optimization of the code with respect to vectorization. Due to the dominance of scalar operations relative to vector operations in the subroutines for evaluating the integrals associated with the \( I_5 \) terms, it may take a relatively large effort to fully vectorize the code.
2.6 Viscosity effects on foil characteristics

General

The preceding section deals with a potential flow in which no effects of viscosity are present. In a real flow, viscosity is present and affects both the lift and drag forces. To describe the flow along a solid body in a potential flow the requirement of flow tangency along the body surface is a sufficient condition to determine the potential. However, in a viscous flow the no-slip condition must be applied: the velocity at the surface is zero. This results in the development of a boundary layer close to the body surface. In this boundary layer the mean velocity gradually increases from zero at the body surface to the potential flow velocity at some distance from the body surface. This distance is termed the boundary layer thickness. The displacement thickness is defined as the distance the foil surface must be displaced in order to have the same flow rate as the viscous flow, but with an inviscid velocity profile.

If a boundary layer flow experiences an increase in pressure in the direction of the flow, the flow velocity will reduce. If the pressure increase is strong, the flow may be forced to reverse and the streamlines no longer follow the body surface. The boundary layer is then said to separate from the body surface.

For an attached (non separated) flow, the effect of the boundary layer on the pressure distribution can be accounted for by calculating the potential flow past a body which shape differs from the physical one by the addition of the displacement thickness to its contour. This change in geometry, which is in general different for the suction and pressure sides affects the lift curve slope and the zero-lift angle of attack.

In a boundary layer flow, shear stresses are acting on the body surface, resulting in a skin friction drag force. Near the trailing edge, the displacement thickness prevents the inviscid flow streamlines to follow the foil contour. This results in a lower pressure than the stagnation pressure and hence in an additional drag component, termed form drag. The sum of the skin friction drag and form drag is called viscous drag.

Two types of boundary layer flow exist: laminar and turbulent, depending on the Reynolds number, the smoothness of the body surface, the turbulence in the onflow and the pressure distribution over the body. The Reynolds number is defined as:

\[ R_n = \frac{Uc}{v} \]  \hspace{1cm} (2.76)

where \( U \) is the free stream velocity, \( c \) is a characteristic length of the body, for foil sections the chord length, and \( v \) is the kinematic viscosity of the flow. At low Reynolds numbers, in a smooth, low-turbulent onflow and for a smooth body surface the boundary layer flow will initially be stable to small disturbances that are always present in practice. In such a laminar flow, the fluid flows in layers past one another at speeds varying from zero at the surface to the flow speed just outside the boundary layer.
At higher Reynolds numbers the boundary layer flow becomes time dependent and will appear chaotic. This is called a turbulent flow. Skin friction is much higher for a turbulent than for a laminar boundary layer flow. Besides skin friction, the displacement thickness, and thereby lift and form drag, are affected by the type of boundary layer flow.

On foil sections, two types of flow separation may occur. At high angles of attack a sharp low pressure peak exists at the leading edge. Just after this pressure peak a strong positive pressure gradient exists and the flow may separate. If this separation occurs in a laminar flow region, early transition to turbulent flow may follow. As a turbulent boundary layer has a greater resistance against separation, the flow usually reattaches to the foil surface and an increase in friction drag results. This phenomenon can be observed during model testing where the Reynolds number is relatively low.

Turbulent flow separation occurs at the trailing edge, where the pressure gradient is also positive and the boundary layer is thicker with less resistance against separation. At lower Reynolds numbers, the boundary layer thickness is relatively large and separation is more likely to occur. Furthermore, using trailing edge flaps results in steep pressure gradients and an increased risk of separation. Massive separation (stall) at the trailing edge limits the maximum lift coefficient and results in a sharp increase in form drag.

The computational method will be validated by means of comparisons with both model scale and full scale data in Chapter 3. At model scale Reynolds numbers, in the range $10^5 < Rn < 10^6$, the flow may be partially laminar, partially turbulent. At full scale Reynolds numbers, in the range $1 \times 10^7 < Rn < 3 \times 10^7$, the flow will be turbulent over almost the entire foil section. In the next paragraphs the effects of viscosity on lift and drag will be described for full scale Reynolds numbers as well as for model scale Reynolds numbers.

**Full scale Reynolds numbers**

At full scale Reynolds numbers the flow is turbulent over almost the entire foil section due to flow turbulence and surface roughness. Boundary layer transition due to laminar flow separation at the leading edge does not occur. Experimental data at full scale Reynolds numbers for typical hydrofoil sections indicate that incidence angles up to 8 degrees and flap angles up to 20 degrees can be applied without serious flow separation, see Abbott and Von Doenhoff (1958). For hydrofoil craft operational incidence and flap angle variations are within these limits, therefore it is assumed that no significant boundary layer separation occurs at the trailing edge.

The viscosity effect on resistance is accounted for by adding viscous drag to the induced drag, yielding the total resistance. For streamlined shapes like hydrofoil sections this can be performed by using relatively simple empirical formulations, provided the type of boundary layer flow and/or the appropriate frictional resistance coefficient is known.

Alternatively, one can use two-dimensional viscous flow analysis methods, for instance Xfoil developed by Drela (1989), for calculating the boundary layer characteristics and the associated lift
and drag coefficients. Xfoil uses a panel method for solving the inviscid flow problem. The solution for the viscous flow is described by an integral boundary layer formulation and a transition criterion. The entire viscous solution is strongly coupled with the potential flow via a surface transpiration model. This permits the calculation of the effects of limited separation regions on lift and drag.

The results of both approaches can be applied in a stripwise manner over the foil span. Each strip, with a width corresponding to that of the vortex elements, is then assumed to be a two-dimensional foil section. In Section 3.2 the validity of using two-dimensional results for three-dimensional foils will be investigated. In the present computational method empirical formulations will be used for obtaining the viscous drag in view of their ease of application to arbitrary foil sections, although results from viscous flow programs such as Xfoil can be incorporated as well.

The minimum viscous drag coefficient $C_{dm}$ of a streamlined foil section can be expressed in basic section parameters by using experimental data as provided for instance by Abbott and Von Doenhoff (1958):

$$C_{dm} = 2C_F[1 + 1.2(t/c) + 60(t/c)^4 + 120[(t/c) + 0.2C_{li} + 0.1C_{ig}^3]]$$  \hspace{1cm} (2.77)

where $C_F$ denotes the frictional drag coefficient, $t/c$ denotes the section thickness to chord ratio, $C_{li}$ denotes the ideal lift coefficient of the section and $C_{ig}$ denotes the sectional lift coefficient increment due to a trailing edge flap. These latter two quantities depend on the camber and on the flap chord to foil chord ratio respectively. The reference area to be used for this equation is the planform area of the foil. The term in between brackets accounts for form drag.

For a turbulent flow over a hydrodynamically smooth surface a turbulent friction line may be used to determine the frictional drag coefficient. A frequently used friction line formulation as given by the ITTC (1957) is:

$$C_F = \frac{0.075}{(\log_{10}R^2_{c})^2}$$  \hspace{1cm} (2.78)

In practice however, surfaces of hydrofoils are not hydrodynamically smooth, i.e. the surface roughness is large enough to affect the frictional resistance. For a certain roughness height the Reynolds number dependency of the frictional resistance coefficient vanishes. A suitable frictional resistance coefficient for these conditions is given by Schlichting (1968):

$$C_F = (1.14 - 0.86 \ln(k_{eq}/c))^{-2}$$  \hspace{1cm} (2.79)

where $k_{eq}$ denotes the equivalent sand grain roughness height of the foil surface. The equivalent sand grain roughness is an artificial roughness which definition is based on systematic experiments on frictional losses in tubes covered with closely spaced sand grains. Equivalent sand grain roughness values range from 1 to 60 $\mu$m for polished metal and antifouling paint, respectively. The problem in using this concept is that the roughness grains on ship and foil surfaces may significantly vary in size and density. For such irregularly roughened surfaces no equivalent sand grain roughness is known. Therefore one often applies increments to the frictional resistance coefficient based on a
turbulent friction line. Typical values are $0.0002 < \Delta C_f < 0.0008$, based on experience.

For sectional lift coefficients $C_l$ above or below the lift coefficient $C_{lm}$, associated with $C_{dn}$, the viscous resistance coefficient increases according to:

$$\Delta C_d = k C_{dn} (C_l - C_{lm})^2$$  \hfill (2.80)

where the constant $k$ depends on the Reynolds number and the foil section type. For full scale Reynolds numbers and commonly used foil section types the $k$-value is typically 0.50. The lift coefficient $C_{lm}$ where the drag coefficient has its minimum depends on the section type and the Reynolds number. Specific information for particular section types can be found in Abbott and Von Doenhoff (1958), or may be obtained from results of viscous flow calculations.

The total viscous resistance coefficient follows finally from:

$$C_d = C_{dn} + \Delta C_d$$  \hfill (2.81)

The boundary layer thickness and its distribution over the section’s surface affect the pressure distribution over the section and thereby the lift force. Martin (1963a) describes the viscosity effect on the lift curve slope as a function of the Reynolds number, the state of the boundary layer (laminar, turbulent) and the closure angle of the section at the trailing edge, based on a compilation of wind tunnel data from the Royal Aeronautical Society, see Figure 2.23. The curves represent a fairing through experimental data points. The maximum offset of data points from the faired curves is about 5%, according to Martin. The curves are valid for cases for which boundary layer transition is induced at the leading edge by using turbulence stimulation techniques. The lift curve slope reduction factor $f_a$ shown in Figure 2.23 is defined as:

$$f_a = \frac{C_{iav}}{C_{iav}}$$ \hfill (2.82)

$$C_{iav} = 2\pi f_i$$

$$f_i = (1 + 0.80(t/c))$$

where $C_{iav}$ is the sectional lift curve slope in a viscous flow. The two-dimensional lift curve slope in a potential flow, $C_{iav}$, is larger than $2\pi$ due to non-linear effects caused by the thickness of the foil section $(t)$, which is not accounted for by a vortex lattice method. Thus, for hydrofoil sections used in practice with a 10 percent thickness ratio, the potential flow lift curve slope may be 8% higher than predicted by thin wing theory, for which $C_{iav} = 2\pi$. This only partially compensates for the viscosity effect. On basis of Figure 2.23, for a Reynolds number of $10^5$ and a half closure angle of 0.15, $f_a = 0.83$. Taking $t/c = 0.10$, the sectional lift curve slope $C_{iav} = 0.90 \times 2\pi$. Such a reduction in lift relative to the vortex lattice method result would significantly affect the attitude of the craft and thereby the resistance.
The data shown in Figure 2.23 represent a general trend. In order to determine the uncertainty of these data, available experimental lift curve slope reductions for hydrofoil sections are plotted in Figure 2.23 as well. Unfortunately, suitable experimental data for typical hydrofoil sections at full scale Reynolds numbers are not available. For the experimental data points shown in Figure 2.23 the Reynolds numbers vary between $1 \times 10^6$ and $6 \times 10^6$. Experimental data are used for which the boundary layer transition was fixed at the leading edge. The identification of the cases is given in Table 2.6.

<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>$R_n$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NACA 16-309</td>
<td>$2.5 \times 10^6$</td>
<td>Jones and MacKay (1979)</td>
</tr>
<tr>
<td>2</td>
<td>YS 920</td>
<td>$2.5 \times 10^6$</td>
<td>Shen (1985)</td>
</tr>
<tr>
<td>3</td>
<td>NACA 66-mod</td>
<td>$2.5 \times 10^6$</td>
<td>Shen (1985)</td>
</tr>
<tr>
<td>4</td>
<td>NACA 66-2xx</td>
<td>$6.0 \times 10^6$</td>
<td>Abbott and Von Doenhoff (1958)</td>
</tr>
<tr>
<td>5</td>
<td>NACA 64-4xx</td>
<td>$6.0 \times 10^6$</td>
<td>Loftin and Smith (1949)</td>
</tr>
<tr>
<td>6</td>
<td>NACA 64-4xx</td>
<td>$1.0 \times 10^6$</td>
<td>Loftin and Smith (1949)</td>
</tr>
</tbody>
</table>

A 'xx' in the type description means that experimental data are available for a series of thickness ratio's. Spot-checking experimental and 'Martin' $f_a$ values learns that the general trend with Reynolds
number and trailing edge thickness is well predicted by the Martin curves. However, there also appears a tendency to overestimate the lift slope reduction due to viscosity, especially for the first three cases. For case 6, for which the Reynolds number is low, the Martin data are fairly good. The maximum deviation between experimental and Martin $f_a$ values is 9%, for case 1. The inaccuracy in the experimental data should also be appreciated, but is unknown. Inaccuracies may be introduced through uncorrected blockage and side-wall boundary layer effects and errors in the force recording devices. In general, inaccuracies for tests performed in water tunnels, cases 1, 2 and 3 are larger than wind tunnel test cases which have been corrected for blockage effects. For the water tunnel cases the deviations between the Martin and experimental cases are largest.

For some of the sections listed above the viscosity effect on the lift curve slope ($f_a$) was also determined by means of the Xfoil program. Hereby the boundary layer transition was fixed at a position $0.01c$ from the leading edge in order to simulate the effect of a turbulent boundary layer. The results are shown in Table 2.7. The thickness effect on the lift curve slope in a potential flow is given as well ($f_t$), as determined from Xfoil and from the Martin results, eq. (2.82).

<table>
<thead>
<tr>
<th>Type</th>
<th>$R_n$</th>
<th>$f_a^{\text{exp.}}$</th>
<th>$f_a^{\text{Xfoil}}$</th>
<th>$f_a^{\text{Martin}}$</th>
<th>$f_t^{\text{Xfoil}}$</th>
<th>$f_t^{\text{Martin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA 16-309</td>
<td>2.5x10^6</td>
<td>0.816</td>
<td>0.861</td>
<td>0.748</td>
<td>1.074</td>
<td>1.072</td>
</tr>
<tr>
<td></td>
<td>1.0x10^7</td>
<td>-</td>
<td>0.915</td>
<td>0.842</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NACA 64-409</td>
<td>1.0x10^6</td>
<td>0.832</td>
<td>0.867</td>
<td>0.844</td>
<td>1.066</td>
<td>1.072</td>
</tr>
<tr>
<td></td>
<td>6.0x10^6</td>
<td>0.934</td>
<td>0.926</td>
<td>0.870</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0x10^7</td>
<td>-</td>
<td>0.930</td>
<td>0.892</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YS 920</td>
<td>2.5x10^6</td>
<td>0.885</td>
<td>0.880</td>
<td>0.801</td>
<td>1.067</td>
<td>1.072</td>
</tr>
<tr>
<td></td>
<td>1.0x10^7</td>
<td>-</td>
<td>0.938</td>
<td>0.892</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NACA 66-209</td>
<td>6.0x10^6</td>
<td>0.912</td>
<td>0.893</td>
<td>0.862</td>
<td>1.065</td>
<td>1.072</td>
</tr>
</tbody>
</table>

It is seen that Xfoil predictions for the lift curve slope are in fairly good agreement with the experimental values. The maximum deviation is 5.5% for NACA 16 case. As mentioned before, the viscosity effect, $f_a$, is overpredicted by the Martin curves, except for the lower Reynolds number NACA 64 case. The lift curve slope increment factor due to thickness, $f_t$, is well represented by eq. (2.82). It is noted that the combination of $f_a$ and $f_t$ values in Table 2.7 for Reynolds numbers of 10^7 results in a lift curve slope close to 2\pi, when the Xfoil data are used. No lift curve slope corrections are then required in the vortex lattice method for full scale Reynolds numbers. However, for consistency with other viscosity correction factors and for use at model scale Reynolds numbers, the lift curve slope correction factor $f_a$ will be maintained in the following paragraphs.
Again based on data from the Royal Aeronautical Society, Martin (1963a) also provides the viscosity effect on the flap efficiency. A correction factor is defined as follows:

\[
f_b = \frac{\alpha_{5v}}{\alpha_{5p}}
\]  

(2.83)

where \( \alpha_{5v} \) is the flap efficiency in viscous flow and \( \alpha_{5p} \) is the flap efficiency in a potential flow. These flap efficiencies are defined by:

\[
\Delta C_{lv} = C_{la v} \alpha_{5v} \delta
\]

(2.84)

\[
\Delta C_{lp} = C_{lap} \alpha_{5p} \delta
\]

where \( \delta \) is the flap angle, \( \Delta C_l \) is the sectional lift due to the flap and \( C_{la} \) is the sectional lift curve slope. The subscripts \( v \) and \( p \) denote viscous and inviscid flow respectively. The data are valid for cases without significant flow separation and for flap deflections less than 15 degrees. The accuracy is specified to be within 5%. Table 2.8 provides a comparison between calculated viscosity correction factors on the flap efficiency by means of Xfoil and the data provided by Martin, for a NACA 16-309 section with a flap chord ratio of 0.25.

<table>
<thead>
<tr>
<th>( R_a )</th>
<th>( f_b ) - Xfoil</th>
<th>( f_b ) - Martin</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \times 10^5 )</td>
<td>0.670</td>
<td>0.296</td>
</tr>
<tr>
<td>( 1 \times 10^6 )</td>
<td>0.898</td>
<td>0.532</td>
</tr>
<tr>
<td>( 1 \times 10^7 )</td>
<td>0.941</td>
<td>0.725</td>
</tr>
</tbody>
</table>

The viscosity correction factor based on experimental data for this section at a Reynolds number of \( 2.5 \times 10^6 \) is \( f_b = 0.894 \), for a limited flap angle range without strong boundary layer separation at the trailing edge, see Jones and MacKay (1979). The Xfoil values are in much better agreement with this value than the Martin data which predict again a too strong viscosity effect: the \( f_b \) values are too low. At a Reynolds number of \( 10^7 \), the flap efficiency is about 5% lower than the potential flow value.

To investigate the viscosity effect on camber, or zero-lift angle of attack \( \alpha_0 \), the same set of experimental data is used as for the lift curve slope. Experimental \( \alpha_0 \) values are compared with Xfoil results in Table 2.9. The zero-lift angle according to thin wing theory (TWT), see Abbott and Von Doenhoff (1958), is also given.
Table 2.9 Comparison zero-lift angle of attack

<table>
<thead>
<tr>
<th>Type</th>
<th>$R_n$</th>
<th>$\alpha_{\text{exp}}$</th>
<th>$\alpha_{\text{Xfoil}}$</th>
<th>$\alpha_{\text{Twt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA16-309</td>
<td>2.5x10^6</td>
<td>-2.29</td>
<td>-2.27</td>
<td>-2.74</td>
</tr>
<tr>
<td></td>
<td>1.0x10^7</td>
<td>-</td>
<td>-2.64</td>
<td>-2.74</td>
</tr>
<tr>
<td>NACA64-409</td>
<td>1.0x10^6</td>
<td>-3.30</td>
<td>-2.83</td>
<td>-3.65</td>
</tr>
<tr>
<td></td>
<td>6.0x10^6</td>
<td>-3.01</td>
<td>-2.73</td>
<td>-3.65</td>
</tr>
<tr>
<td></td>
<td>1.0x10^7</td>
<td>-</td>
<td>-2.87</td>
<td>-3.65</td>
</tr>
<tr>
<td>YS 920</td>
<td>2.5x10^6</td>
<td>-2.15</td>
<td>-2.17</td>
<td>-2.65</td>
</tr>
<tr>
<td></td>
<td>1.0x10^7</td>
<td>-</td>
<td>-2.23</td>
<td>-2.65</td>
</tr>
<tr>
<td>NACA 66-209</td>
<td>6.0x10^6</td>
<td>-1.65</td>
<td>-1.70</td>
<td>-1.52</td>
</tr>
</tbody>
</table>

It is seen that viscosity effects on the zero lift angle are appreciable and that the Xfoil predictions are in fairly good agreement with the available experimental data. For the NACA64 section the difference is however appreciable for the lower Reynolds number. This may be partially due to the uncertainty in the experimental data which is estimated from the scatter in the data to be about 0.25 deg, see Abbott and Von Doenhoff (1958).

In conclusion it is thought that Xfoil results give the most accurate information on viscosity effects on lift, at least for Reynolds numbers approaching full scale conditions. At actual full scale Reynolds numbers above 10^7, viscosity effects will be smaller in magnitude and it may be expected that Xfoil results are even better for these conditions. Therefore, Xfoil results will be used in the computational method.

For the NACA16-309 section at a Reynolds number of 10^7 the total viscosity effect on the sectional lift coefficient is 5.5%, for an incidence of 3 deg, a zero-lift angle of -3 deg, a flap efficiency of 0.50 and a flap angle of 10 deg. This is considered as a significant effect and viscosity effects on lift are taken into account for full scale conditions.

The next paragraphs describe how viscosity effects are implemented in the vortex lattice method. Thus far the viscosity effects in lift have been described for two dimensional foil sections. It is assumed that these viscosity effects also apply to vortex element strips of a lifting surface with a sufficiently large aspect ratio. In Section 3.2 the validity of using two-dimensional results for three-dimensional foils will be investigated. In the vortex lattice method separate corrections for the lift due to flaps, due to incidence and due to camber must be applied. The sectional lift coefficient can be expressed as follows in terms of lift curve slope, incidence, zero-lift angle of attack and flap efficiency and flap angle:

$$ C_l = C_{l_{\alpha}}(\alpha - \alpha_{\alpha} + \alpha_{\delta} \delta) $$

(2.85)
In a viscous flow the sectional lift coefficient can be expressed as follows:

\[ C_{lv} = C_{lav} (\alpha - \alpha_{lw} + \alpha_{s}, \delta) \]  
\[ (2.86) \]

which can be written as follows to define an effective incidence \( \alpha_e \), camber ratio \( f/c_e \), and flap angle \( \delta_e \) for use in the vortex lattice method:

\[ C_{lv} = 2\pi f_a f_l (\alpha - f_a f_c \alpha_{op} + f_a f_c \alpha_{sp}, \delta) \]
\[ = 2\pi (f_a f_l \alpha - f_a f_c f_c \alpha_{op} + f_a f_c f_c \alpha_{sp}, \delta) \]  
\[ = 2\pi (\alpha_e - \alpha_{lw} + \alpha_{sp}, \delta_e) \]  
\[ (2.87) \]

where

\[ \alpha_e = f_a f_l \alpha \]
\[ \alpha_{lw} = f_a f_l f_c \alpha_{op} \]  
\[ (2.88) \]
\[ \delta_e = f_a f_l f_c \delta \]

and where \( f_l \) is the viscosity correction on camber \( \alpha_{op} / \alpha_{sp} \). In the vortex lattice method the camber ratio of the section \( f/c \) can be multiplied with \( f_e \) to get the appropriate zero-lift angle of attack.

**Model scale Reynolds numbers**

In the previous paragraphs the viscosity effects on lift and drag of foil sections were discussed for full scale Reynolds numbers. At these Reynolds numbers the boundary layer will be turbulent over almost the entire foil section. The viscosity effects on lift and drag can then be described by using results from two-dimensional viscous flow methods, whereby boundary layer transition is imposed at the leading edge, and empirical formulations.

For validation of the computational method for hydrofoil craft use will be made of experiments in a model basin, see Section 3.4. For conducting model tests on high speed craft, Froude scaling is applied, i.e. the inertia forces are represented correctly at model scale. Herein a number of restrictions with respect to the size of the model and the speed of the towing carriage have to be dealt with. This results typically in model scale ratios from 6 to 10. The corresponding Reynolds number based on the foil section chord ranges from \( 2 \times 10^5 \) at hullborne velocities to \( 8 \times 10^5 \) at foilborne velocities.

In this Reynolds number range the boundary layer on the foil section may range from fully laminar to partially laminar, partially turbulent. The transition from a laminar to a turbulent boundary layer depends on the Reynolds number, the pressure distribution over the foil section, the turbulence level in the free stream and the roughness of the foil surface. The Reynolds number varies with speed,
2.6 Viscosity effects on foil characteristics

the pressure distribution varies with incidence, flap angle, speed and submergence, the turbulence level in a towing basin is usually rather low but depends on the time interval between successive runs while the roughness of the foil system surface can be controlled to some extent. In practice this means that the state of the boundary layer during model testing is not precisely known.

As an example of scale effects, Figure 2.24 shows the sectional lift and drag curves versus incidence and flap angle for a typical laminar flow hydrofoil section (NACA 16-309). The data have been obtained from wind tunnel and water tunnel tests as described by Jones and MacKay (1979), at Reynolds numbers ranging from $2 \times 10^5$ to $4 \times 10^6$. Although this Reynolds number range is higher than that during hydrofoil craft model testing, the phenomena of interest also occur at lower Reynolds numbers. The term transition fixed in Figure 2.24 implies that turbulence stimulation was applied, by means of carborundum grains, to force transition from a laminar to a turbulent boundary layer at the section's leading edge.

Without forced transition, laminar separation occurs at the leading edge at the higher lift coefficients due to the steep pressure increase following a negative pressure peak. After flow attachment the boundary layer will be turbulent with an associated high skin friction. The drag-lift polar shows then a sudden increase in drag coefficient. The effect on lift is significant as well: a sharp discontinuity in the lift curve slope appears.

For the transition fixed cases, the irregularities in lift and drag are not present. These results show that forced boundary layer transition has to be applied to remove laminar flow effects on lift and drag. Laminar flow effects are undesirable because its occurrence and extent are difficult to predict and therefore difficult to correct in the extrapolation of model test results to full scale values.

Forced boundary layer transition is achieved by introducing turbulence in the boundary layer at the leading edge. This can be done by applying carborundum roughness grains. Ideally, the boundary layer transition from laminar to turbulent is thus fixed at the leading edge and does not depend any more on the Reynolds number, incidence, flow turbulence and surface roughness. At low Reynolds numbers the size of the roughness grains must be significant in order to be effective and may then distort the section shape whereby the lift and drag characteristics are affected unintentionally. A small increase in drag due to the roughness of the grains is unavoidable. Another side effect of using turbulence stimulation is that the lift curve slope and zero-lift angle of attack are affected. Turbulent boundary layers at low Reynolds numbers are relatively thick. This adversely affects the effective section shape which results in general in a reducing lift curve slope and zero-lift angle with a reducing Reynolds number. On the other hand, the resistance against separation at the trailing edge is larger in a turbulent boundary layer than in a laminar boundary layer.

The disadvantages of applying turbulence stimulation are less strong than the more uncontrollable laminar flow effects on lift and drag. Therefore, it is best to apply turbulence stimulation for hydrofoil craft model testing. Then, its effectiveness in inducing turbulence must be checked and its effect on the lift characteristics must be investigated.
Figure 2.24  Section characteristics for free and fixed boundary layer transition  From Jones and MacKay (1979)

For the hydrofoil craft model tests described in Section 3.4, carborundum grains were used for turbulence stimulation. At low Reynolds numbers, in the take-off speed range, the incidence of the foil sections is relatively large due to trim, typically 4 to 6 degrees. At the leading edge of the section, high negative pressure peaks are present with high local velocities which enhance the effectiveness of the carborundum grains. At foilborne conditions, the incidence is lower, but the Reynolds number is higher and transition is achieved as well. The effectiveness of the grains was
verified by means of paint smear tests. Hereby some paint is applied at the leading edge of the foil model in a separate test run. During the test the paint is smeared over the chord by the flow. The paint streaks may be inspected after the run. The character of the paint streaks gives an indication of the type of boundary layer flow present during the test. Figure 2.25 shows an example of a paint smear test result.

![Paint smear test result](image)

Figure 2.25  Paint smear test result

Besides laminar boundary layer separation at the leading edge, turbulent boundary layer separation may occur at the trailing edge due to the steep pressure recovery in that region. This type of separation, if occurring upstream of the trailing edge, leads to a significant reduction in lift and an increase in drag. At low Reynolds numbers the resistance of the boundary layer to separation is less than at high Reynolds numbers. The effects of trailing edge separation on for instance the flap efficiency is shown in Figure 2.26, for a NACA16-309 (NACA a=1.0) section at a Reynolds number of $2.5 \times 10^6$. For this mean line type the pressure increase at the trailing edge is very steep which makes the flow susceptible to separation, even at Reynolds numbers an order of magnitude higher than model scale Reynolds numbers.
For hydrofoil models at take-off conditions, at a low Reynolds number and at a relatively high incidence angle, due to trim, and at a high flap angle, turbulent flow separation at the trailing edge flap may readily occur. Mild flow separation at the trailing edge of a hydrofoil model is shown in Figure 2.25. It is seen that the paint collects at a certain distance in front of the trailing edge which indicates separation.

It is difficult to include the effects of strong separation on lift and drag in the computational method. Ideally, experimental data for the actual foil section at the actual Reynolds number in a flow with similar turbulence properties would be required to include separation effects in an empirical manner, similar to the procedure taken for full scale Reynolds numbers. However, such data are in general not available. Viscous flow calculation methods such as the Xfoil program can be used to detect the occurrence of separation and to predict the effects on lift and drag for mildly separated flows. Results of Xfoil calculations for a flapped NACA16-309 (NACA a=1.0) section were already shown in the paragraphs on full scale Reynolds number effects. For a not too strongly separated flow condition the Xfoil result appeared in good agreement with the experimentally observed flap efficiency. Table 2.10 shows flap efficiencies for other flap angles as well. The experimental data are obtained from Figure 2.26. For flap angles below -5 and beyond +5 degrees no sectional lift and drag coefficients could be determined by means of Xfoil due to too strong separation.
Table 2.10  Experimental and calculated flap efficiencies

<table>
<thead>
<tr>
<th>δ (deg)</th>
<th>$\alpha_6 \ - \ Xfoil$</th>
<th>$\alpha_6 \ - \ Exp.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10.0</td>
<td>-</td>
<td>0.201</td>
</tr>
<tr>
<td>-5.0</td>
<td>0.598</td>
<td>0.500</td>
</tr>
<tr>
<td>-2.5</td>
<td>0.615</td>
<td>0.603</td>
</tr>
<tr>
<td>0.0</td>
<td>0.640</td>
<td>0.625</td>
</tr>
<tr>
<td>2.5</td>
<td>0.636</td>
<td>0.603</td>
</tr>
<tr>
<td>5.0</td>
<td>0.625</td>
<td>0.510</td>
</tr>
<tr>
<td>10.0</td>
<td>-</td>
<td>0.243</td>
</tr>
</tbody>
</table>

It is seen that for non-zero flap angles the reduction in flap efficiency is underpredicted by Xfoil, but that the occurrence of strong separation is indicated approximately at the right flap angles.

It is clear that experimental data for flapped hydrofoil sections must be considered with care, as separation at the trailing edge is probable, at least for section types with NACA a=1.0 mean line types at Reynolds numbers at and below $2.5 \times 10^6$. For each experimental case the occurrence of separation should be verified by means of paint smear tests or calculations.

In Section 3.4 it will be shown that validation of computational methods on basis of model test results is only meaningful if viscosity effects on foil lift and drag are taken into account in the computational method itself. The next paragraphs describe how viscosity effects are taken into account.

For obtaining the viscous drag of foil sections with turbulence stimulation, eq's. (2.77-2.81) can be used together with the ITTC57 turbulent friction line formulation. An incremental drag factor of $k=1.50$ is used in eq. (2.80) for Reynolds numbers below $10^6$. This $k$ value is taken somewhat larger than the value found when analysing experimental results which shows that $k$ increases from approximately 0.50 at $R_n \geq 10^7$ to 1.0 at $R_n = 10^6$. Alternatively, Xfoil results may be used.

Figure 2.27 shows a comparison between experimental and calculated sectional drag coefficients for a NACA 64-409 section at two Reynolds numbers. The $k$ value used here is 1.0. An increment in frictional resistance coefficient of $\Delta C_f = 0.0002$ was used in eq. (2.77) to account for the additional drag due to the carborundum roughness used for turbulence stimulation in the experiments. Minimum sectional drag coefficients shown by Abbott and Von Doenhoff (1958) for a range of carborundum roughness heights show that such a value is reasonable. The comparison between the experimental and calculated values is seen to be satisfactory for lift coefficients in between 0.2 and 0.6. For a better fit at higher lift coefficients a $k$ value of 1.25 should be used. Xfoil results are also shown in Figure 2.27. The same roughness allowance $\Delta C_f = 0.0002$ has been added to these results. The minimum drag coefficient is about 15% too low for Xfoil.
Table 2.11 shows minimum drag coefficients for NACA16, YS920 and NACA66 section types.

<table>
<thead>
<tr>
<th>Section type</th>
<th>$R_n$</th>
<th>$C_{dm}^{-}$ eq. (2.77-78)</th>
<th>$C_{dm}^{-}$ Xfoil</th>
<th>$C_{dm}^{-}$ Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA 16-309</td>
<td>$4.0\times10^6$</td>
<td>0.00877</td>
<td>0.00853</td>
<td>0.00864</td>
</tr>
<tr>
<td>wind tunnel</td>
<td>$2.5\times10^6$</td>
<td>0.00956</td>
<td>0.00938</td>
<td>0.00842</td>
</tr>
<tr>
<td>water tunnel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>YS 920</td>
<td>$2.5\times10^6$</td>
<td>0.00944</td>
<td>0.00952</td>
<td>0.00875</td>
</tr>
<tr>
<td>NACA 66-209</td>
<td>$6.0\times10^6$</td>
<td>0.00807</td>
<td>0.00806</td>
<td>0.00850</td>
</tr>
</tbody>
</table>

Figure 2.27 Comparison calculated and experimental viscous drag coefficients

The comparison between the empirical and Xfoil results is quite good for these sections. The calculated values for the YS920 section are about 8% higher than the experimental values, for the NACA66 section the calculated values are 5% too low. It should be noted that the sectional drag is difficult to measure with a high accuracy due to the small magnitude of the drag force and side wall and blockage effects in a water tunnel. The data for the YS 920 and NACA66 sections were obtained in a water tunnel, while the experimental results shown in Figure 2.27 for the NACA64 section were obtained in a wind tunnel. Also for the NACA16 section, the data obtained in a wind tunnel are in better agreement with the calculated values than the values obtained in a water tunnel.
The empirical results for the sectional drag coefficient are at least as accurate as Xfoil results. Furthermore, in the computational method it is easiest to use the empirical relations. Therefore, these empirical relations will be used in the computational method to determine the viscous drag at model scale Reynolds numbers. It is noted that due to lack of experimental data no check on the accuracy of Xfoil and the empirical formulations can be made for actual model test Reynolds numbers in between $10^5$ and $10^6$.

As to the viscosity effect on the lift curve slope, it is assumed that no serious flow separation occurs at the trailing edge. The viscosity effects on the lift, in terms of $f_w, f_o, f_e$ and $f_{wo}$, are obtained by using calculation results from the Xfoil program. Xfoil results were shown to be in fairly good agreement with experimental data for Reynolds numbers in between $10^6$ and $10^7$. Again, for lower Reynolds numbers there are no suitable experimental data available to check Xfoil results. For the section types used during the model tests, viscosity correction factors have been generated for use in the potential flow computational method. These factors are reviewed in Appendix C. In the computational method the incidence, flap angle and camber ratio of the foils are adjusted as defined in eq. (2.88).

Using the data in Appendix C for a NACA16-309 section at a Reynolds number of $5\times10^5$, reductions in sectional lift coefficient may be expected in the order of 25% and 50% for take-off and cruise speed conditions respectively. It is noted that such reductions in lift are large and that these will introduce appreciable differences in the craft's attitude and thereby in resistance.

**Conclusions**

Viscosity effects on the sectional lift coefficient are significant for full scale Reynolds numbers, mainly due to a lower flap efficiency. For a ten degree flap deflection a lift reduction of about 5% occurs.

At model scale Reynolds numbers, the viscosity effect on lift is quite large, due to the combined viscosity effects on lift curve slope, zero-lift angle and flap efficiency.

For both full scale and model scale conditions, viscosity effects on lift are taken into account in the vortex lattice method by correcting the incidence, camber and flap angle.

The viscosity corrections factors for lift are best determined by means of the Xfoil program.

Xfoil results for the sectional viscous drag are reasonably accurate, empirical formulations are at least as accurate. In the computational method, viscous drag will be based on empirical formulations.

At model scale, laminar flow effects must be precluded by using turbulence stimulation techniques.
2.7 Hull, propulsor and appendage force components

In the computational method not only forces due to the foil system must be considered, but also force components acting on the hull and force components due to propulsors. Moreover, to determine the powering requirements, interaction between the foil system and the propulsor and the efficiency of the propulsor needs to be considered. Finally, additional lift and drag components due to foil and propulsion system appendages must be described.

This Section deals with such force components and the propulsor efficiency. The descriptions are relatively brief due to the empirical nature of the calculation methods involved. For more detailed descriptions of these force components one is referred to the appropriate references. The forces acting on the hull are described first, followed by the propulsors and appendages.

Hull force components

A computational method is required that provides the resistance, the hydrostatic and hydrodynamic lift force and the trimming moment for hydrofoil craft type hull forms. For a given hull, the forces must be described as a function of the trim angle and the hull displacement. In general, the trim angle ranges from zero to six degrees while the displacement ranges from full load to zero. Furthermore, since the computational method is to be used within an iterative procedure for determining the equilibrium attitude of the craft, its computer time consumption should be modest.

As discussed in Section 2.1, suitable analytical methods which fulfil these requirements do not yet exist. The use of panel methods for planing conditions is uncertain and would require a separate study. Besides, the CPU requirements will not be modest. It is estimated that approximately 1500 evaluations of hull forces are required for determining the resistance curve of hydrofoil craft. This would take hours of computer time on a fast CRAY-C98 mainframe.

Therefore, an empirical method is developed. The data most suited for the present purpose are the Series 65 model tests, see Holling and Hubble (1974). This series of 16 hydrofoil craft hull forms consists of 9 canard type and 7 airplane type hull forms. Figure 1.6 shows the Series 65 sections. During the tests the models were restrained in all modes of motion, except for heave. Holling and Hubble (1974) provide the following experimental data for a matrix of velocity $V$, trim $\tau$ and displacement $\Delta$ conditions for each model:

- resistance $R_f$,
- trimming moment $M_f$,
- draft $D_f$,
- wetted hull surface $S$,
- wetted chine length $L_c$,
- wetted keel length $L_k$.

In order to use these data for the present purpose the following non-dimensional coefficients have been determined from the model test data and stored in an interpolation database:

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- residuary resistance coefficient $C_R = R_R / \Delta_0$
- trimming moment coefficient $C_M = M_T / (L \Delta_0)$
- draft coefficient $C_D = D_1 / V_0^{1/3}$
- wetted surface coefficient $C_S = S / V_0^{2/3}$
- wetted chine length coefficient $C_{1c} = L_c / L$
- wetted keel length coefficient $C_{2k} = L_k / L$

Here $L$ is the total hull length at the chine, $\Delta_0$ is the hull displacement weight at rest for a hull with a weight consisting of the total craft weight minus the foil system lift at speed and $V_0$ is the corresponding submerged hull volume at rest: $V_0 = \Delta_0 / \rho g$. By allowing freedom in heave during the tests, the vertical forces are always in equilibrium and the draft varies with speed due to the hydrodynamic force acting on the hull bottom. The actual submerged hull volume at speed is unknown. Therefore non-dimensionalisation of the first four coefficients is based on the hull displacement weight and volume at rest.

The residuary resistance $R_R$ is obtained from the total experimental resistance by subtracting the frictional resistance at model scale according to:

$$R_R = R_T - R_F$$

$$R_F = \frac{1}{2} \rho U_m^2 SC_{fm}$$

where $U_m$ denotes the model velocity and $C_{fm}$ denotes a frictional resistance coefficient based on partially laminar, partially turbulent boundary layer flow:

$$C_{fm} = \frac{0.074}{R_n^{0.2}} - \frac{1050}{R_n}$$

The Reynolds number $R_n$ is herein based on the average wetted length as follows:

$$R_n = \frac{(L_c + L_k) U_m}{2 \nu}$$

where $\nu$ denotes the kinematic viscosity.

The experimental Reynolds numbers ranged from $6 \times 10^5$ to $8 \times 10^6$ while no turbulence stimulation devices were used. At the lower experimental Reynolds numbers the use of the ITTC’57 turbulent friction line for determining $C_{fm}$ resulted repeatedly in negative $C_R$ values. This indicates the existence of a partially laminar boundary layer. Therefore, a transition curve for the frictional resistance coefficient, eq. (2.90), was used rather than the ITTC’57 formulation. At higher experimental Reynolds numbers the difference between the transition and turbulent friction lines is small, see Figure 2.28. This Figure also shows experimentally derived friction coefficient values, taken from Savitsky and Ross (1952), which support the current approach for the Reynolds number range of interest.
The moment coefficient $C_M$ is corrected for the contribution of the frictional resistance in a similar way. Hereby it is assumed that the frictional resistance acts at the geometrical centre of the wetted surface. This appears to be a small correction, though.

The coefficients $C_R$ through $C_{1A}$ are stored in a database as a function of the hull length to beam ratio $L/B$, the hull deadrise angle $\beta_h$, the trim angle $\tau$, the volume Froude number $F_{\nuV}$ and the slenderness ratio $S_V$, where:

$$F_{\nuV} = \frac{U}{\sqrt{g \ N_0^{1/5}}}$$

$$S_V = \frac{L}{\sqrt{N_0^{1/3}}}$$

A five-dimensional, linear interpolation scheme is set up to determine the hull coefficients $C_R$ through $C_{1A}$. Table 2.12 shows the limits of the five independent variables. It is noted that the given limits for $F_{\nuV}$ and $S_V$ are indicative only as these depend on the hull form type, trim angle and displacement.
Table 2.12 Limit values of independent variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L/B$</td>
<td>2.32</td>
<td>9.40</td>
</tr>
<tr>
<td>$\beta_\theta$ (deg)</td>
<td>16</td>
<td>37</td>
</tr>
<tr>
<td>$F_{n\Sigma}$</td>
<td>0.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$S_\psi$</td>
<td>0.07</td>
<td>0.20</td>
</tr>
<tr>
<td>$\tau$ (deg)</td>
<td>0.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

For hullborne conditions and a given hull, the independent variables in the equilibrium equations are the trim angle and zero speed hull displacement $\Delta_0$. Interpolating in the database, the hull force components $F_{zh}$ and $F_{zh}$, the hull trimming moment $M_h$ and the hull draft $D_k$ are derived as follows:

$$F_{zh} = R_R + R_F$$

$$R_R = C_R \Delta_0$$

$$R_F = \frac{1}{2} \rho U^2 S(C_F + C_{FA})$$

$$S = C_S \nabla_0^{2/3}$$

$$C_F = \frac{0.075}{(\log R_n - 2)^3}$$

$$R_n = \frac{(L_k + L_c) V_m}{2 \nabla}$$

$$L_k = C_{1k} L$$

$$L_c = C_{Lc} L$$

$$F_{zh} = \Delta_0$$

$$M_h = C_M \Delta_0 L$$

$$D_k = C_D \nabla_0^{1/3}$$

(2.93)

Here $C_{FA}$ denotes an allowance on the frictional resistance coefficient accounting for hull surface roughness effects, typically $C_{FA} = 0.0004$. The hull force and moment components are used in the equilibrium equations as outlined in Section 2.4 while the hull draft and trim define the position of the craft relative to the watersurface.
For illustration, Figure 2.29 shows the hull characteristics of Series 65 Model 5240 for two trim angles as a function of the volume Froude number. In this Figure, the hull displacement depends on speed as follows:

$$\Delta_{0,U} = \Delta_{0,U=0} (1 - \frac{U}{U_t})^2$$ \hspace{1cm} (2.94)

where $U_t$ is the take-off speed, taken as 30 knots. The variation of the draft coefficient $C_D$ with speed is due to the hydrodynamic pressure acting on the hull bottom. The wetted length coefficient $C_{Lc}$ is seen to be speed dependent for the higher trim angle. The wetted surface coefficient $C_S$ varies significantly with speed for the higher volume Froude numbers. This affects the frictional resistance coefficient $C_{RF}$ ($R_f/\Delta_0$) proportionally. The residuary resistance coefficient $C_R$ shows a characteristic hump at a low speed and a high trim angle. The moment coefficient $C_m$ finally depends mainly on the trim angle.

Aerodynamic force components are of importance for high speed craft. At cruise speed conditions the aerodynamic resistance of hydrofoil craft may be as high as 10% to 15% of the total resistance.

![Diagram](image-url)
Figure 2.29b  Wetted length coefficient for a Series 65 hull vs speed

Figure 2.29c  Wetted surface coefficient for a Series 65 hull vs speed
Figure 2.29d  Frictional resistance coefficient for a Series 65 hull vs speed

Figure 2.29e  Residuary resistance coefficient for a Series 65 hull vs speed
2.7 Hull, propulsor and appendage force components

![Graph](image)

Figure 2.29f Trimming moment coefficient for a Series 65 hull vs speed

The aerodynamic resistance force is taken into account by means of a resistance coefficient which may for instance be obtained from wind tunnel experiments, see Von Wagner (1967) and Martin (1963b). The aerodynamic resistance force follows from:

\[
F_{xa} = \frac{1}{2} \rho_a U^2 S_a C_{xa}
\]  

(2.95)

where \( \rho_a \) denotes the mass density of air, \( S_a \) denotes a suitable reference area and \( C_{xa} \) denotes the resistance coefficient.

**Propulsor forces**

Two types of propulsors are considered here: propellers and waterjet systems. The propellers will be described first.

**Propellers**

The propeller forces are obtained from published results of open water characteristics for several propeller type series, see Van Walle (1988). The propeller types include:
- the Wageningen B series,
- the Gawn-Burrill series,
- the Newton-Rader series,
- the NSRDC supercavitating series.
The open water characteristics include cavitation effects on the propeller performance. The advance coefficient and the cavitation number are determined from the speed of advance \( U_a \), rate of rotation \( n \) and submergence \( z_p \) as follows:

\[
J = \frac{U_a}{nD}
\]

\[
\sigma = \frac{p_a + \rho g z_p}{\frac{1}{2} \rho U_a^2}
\]  
(2.96)

where \( D \) is the propeller diameter, \( p_a \) is the atmospheric pressure minus the vapour pressure, \( \rho \) is the water density and \( g \) is the acceleration due to gravity. The speed of advance is in general not equal to the speed of the craft, \( U \), due to induced velocities by the foil system, propeller axes and struts. Van Walree (1988) gives a description of methods to account for these velocity components.

By means of the known pitch-diameter ratio \( (P/D) \) and blade area ratio \( (A_p/A_o) \) of the propeller, the thrust and torque coefficients, \( K_T \) and \( K_Q \), are interpolated from the open water data on basis of \( J \) and \( \sigma \). The thrust \( T \), torque \( Q_p \) and delivered power \( P_D \) follow from:

\[
T = \rho n^2 D^4 K_T
\]

\[
Q_p = \rho n^2 D^5 K_Q
\]

\[
P_D = 2\pi n Q_p
\]  
(2.97)

The thrust \( T \) is decomposed into force and moment components \( F_{xp} \), \( F_{zp} \) and \( M_p \) as follows, for each propeller present:

\[
F_{xp} = T (1-t) \cos(\varepsilon + \tau)
\]

\[
F_{zp} = T (1-t) \sin(\varepsilon + \tau)
\]  
(2.98)

\[
M_p = F_{xp} x_p - F_{zp} z_p
\]

where \( \varepsilon \) is the propeller axis inclination in the vertical plane and \( x_p \) and \( z_p \) denote the location of the propeller in the vertical plane. The thrust deduction fraction \( t \) accounts for propeller-foil interaction, see Van Walree (1988).

**Waterjet systems**

For waterjet systems the net thrust force is given by:

\[
T = \rho Q_j (U_j - U_i) - \frac{1}{2} \rho U_i^2 A_i C_{Di}
\]  
(2.99)

where \( Q_j \) is the flow rate ingested by the waterjet, \( U_j \) is the jet exit velocity, \( U_i \) is the intake velocity of the ingested stream tube and the second term on the right accounts for the internal intake drag.
The intake velocity $U_i$, the intake area $A_i$ and the nozzle area $A_j$ are obtained from a waterjet system design procedure, see Van Walree and de Wit (1994). This design procedure also establishes the impeller properties such as type, diameter, pitch and rate of revolutions, by using published results on impeller characteristics, similar to the use of the propeller open water diagrams.

The flow rate and jet exit velocity then follow from the continuity relation:

$$Q_j = U_j A_j$$

$$U_j = \frac{Q_j}{A_j}$$  \hspace{1cm} (2.100)

The head rise $H$ to be delivered by the impeller is obtained from:

$$H = \frac{(U_j^2 - U_i^2)}{2g} + H_L + \Delta h$$  \hspace{1cm} (2.101)

where $H_L$ denotes the internal ducting losses, mainly due to friction, and $\Delta h$ is the elevation change from the watersurface to the jet exit. The internal ducting losses are obtained from empirical formulations relating the losses to the ducting geometry.

The power delivered to the impeller $P_D$ follows from:

$$P_D = \frac{\rho g Q_j H}{\eta_p \eta_r}$$  \hspace{1cm} (2.102)

where $\eta_p$ is the impeller efficiency and $\eta_r$ is the relative rotative efficiency of the impeller. These efficiencies are obtained from impeller characteristics and empirical relations respectively.

Force and moment components due to the waterjet thrust are finally obtained in a similar way as shown in eq. (2.98). The thrust deduction is already accounted for by means of the intake drag, see eq. (2.99).

**Appendage forces**

Hydrofoil craft appendages include inclined propeller shafts, control surfaces not included in the foil system (rudders), propeller nacelles and waterjet intakes (pods). Resistance forces on appendages of hydrofoil craft can be as much as 20% of the total resistance of the craft. Apart from resistance forces, lift forces may also be significant, for instance on inclined propeller shafts.

Traditionally, these force components are estimated by using empirical formulations derived from experimental data. Suitable empirical formulations are provided amongst others by Hoerner (1965), Hoerner and Borst (1975) and Kirkman and Kloetzli (1980). These formulations will be briefly reviewed here.

In the following formulations frictional drag coefficients $C_F$ are used which value depends on the
Reynolds number. Depending on the flow regime the following formulations are used:

\[ C_r = \frac{1.327}{\sqrt{R_n}} \]  
Blasius laminar flow formulation

\[ C_r = \frac{0.074}{R_n^{0.2}} - \frac{1700}{R_n} \]  
Prandtl – Schlichting transition formulation \hspace{1cm} (2.103)

\[ C_r = \frac{0.075}{(\log R_n - 2)^2} \]  
ITTC 57 turbulent flow formulation

Propeller shafts

The equations (2.104) to (2.108) define lift and drag coefficients \( C_L \) and \( C_D \) as follows:

\[ C_L = \frac{L}{\frac{1}{2} \rho U^2 S} \]  \hspace{1cm} (2.104)

\[ C_D = \frac{D}{\frac{1}{2} \rho U^2 S} \]

where \( U \) is the free stream velocity and \( S \) is a suitable reference area.

According to Kirkman and Kloetzli, the drag coefficient of a two-dimensional cylinder at an inclination angle \( \alpha \) relative to the flow direction is:

\[ C_D = 0.60 \sin^3(2.25\alpha) + \pi C_r \]  \hspace{1cm} (2.105)

This formulation is valid for supercritical Reynolds numbers \( R_n > 5 \times 10^5 \), while the reference area \( S \) is the cylinder length \( l \) times diameter \( d \). Small corrections may be added that account for the resistance of the cylinder ends.

All equations defined next are either given by or derived from data given by Hoerner. The lift coefficient of an inclined cylinder \( C_L \) is obtained from experimental data as follows:

\[ C_L = 1.1 \sin^2 \alpha \cos \alpha \]  \hspace{1cm} (2.106)

where the reference area is \( ld \). For surface piercing propeller shafts spray drag must be taken into account according to:

\[ C_{Ds} = 0.8 \sin \alpha \]  \hspace{1cm} (2.107)

based on reference area \( d^2 \).
Drag due to ventilation is estimated from:

\[ C_{Dv} = F_h^{-2} \]

\[ F_h = \frac{V}{\sqrt{gh}} \]  

(2.108)

based on reference area \( ld \) and \( h \) is the submergence of the shaft’s end.

The wave resistance of surface piercing shafts is given as follows:

\[ C_{Dw} = \sin\alpha F_h^{-2} \]  

(2.109)

based on reference area \( ld \).

**Rudders and struts**

The frictional resistance of rudders and propeller axis struts with symmetrical sections, at zero incidence is obtained from Section 2.6:

\[ C_D = 2C_F [1 + 1.2(t/c) + 180(t/c)^4] \]  

(2.110)

where \( t/c \) denotes the section thickness to chord ratio while the reference area is the planform area.

In case of surface piercing struts and foil parts a spray drag is added:

\[ C_{Dx} = 0.24t^2 \]  

(2.111)

based on the thickness \( t \) squared.

A wave drag is present for a chord Froude number between 0.50 and 2.0, based on a reference area \( t^2 \):

\[ C_{Dw} = 2.0 - F_{nc} \]

\[ F_{nc} = \frac{V}{\sqrt{gc}} \]  

(2.112)

At strut-hull connections interference drag is accounted for by:

\[ C_{Di} = 0.75(t/c) - 0.0003(t/c)^2 \]  

(2.113)

while at strut-strut or strut-foil connections an allowance is made according to:

\[ C_{Di} = 17(t/c)^2 - 0.050 \]  

(2.114)
Eq.'s (2.113) and (2.114) are based on $t^2$.

The decrease in foil lift at such connections due to interference and loss of foil area is:

$$\Delta C_L = 0.40 \frac{(t/c)}{AR}$$  \hspace{1cm} (2.115)

based on the planform area of the foil. $AR$ denotes the aspect ratio of the (lifting) foil part.

**Nacelles and pods**

Nacelles are streamlined bodies of revolution, used for housing transmission parts in case of propeller propulsion with a Z-drive transmission system. Nacelles are also used for housing flap actuator parts. Pods are formed by the rear part of nacelles with a forward facing opening for ingestion of water for waterjet propulsion. Nacelles and pods are usually located at the connection of foil and strut parts.

The drag coefficient of streamlined bodies of revolution is given by Hoerner (1965) as:

$$C_D = C_F[1 + 1.5(d/l)^{1.5} + 7(d/l)^3]$$  \hspace{1cm} (2.116)

based on the wetted surface area of the body and where $l$ and $d$ denote the body length and diameter respectively and where the frictional drag coefficient $C_F$ is taken as appropriate. For more bluntly shaped pods a constant drag coefficient is used:

$$C_D = 0.075$$  \hspace{1cm} (2.117)

based on the width times the height of the pod cross section.

Nacelles and pods not only contribute to the viscous drag, the foil lift and induced drag are also affected. Empirical formulations are extracted from wind tunnel data on airplanes as given by Hoerner (1965). The loss of lift and increase in induced drag relative to a foil where the nacelle or pod is not modelled are given by:

$$\Delta C_L = -0.01(\alpha_b - \alpha_0)$$

$$\Delta C_{Di} = 0.03 \Delta C_L^2$$  \hspace{1cm} (2.118)

where $\alpha_b - \alpha_0$ is the incidence of the nacelle or pod centreline relative to the zero lift angle of the section. The reference area to be used for these equations is the foil chord times the nacelle diameter or pod width. For hydrofoil configurations the changes in lift and induced drag are less than 0.5% of the lift and drag of the foil system without nacelle or pod.
2.8 Design and analysis program Hydres

In the preceding Sections a description is given of the components of a computational method for hydrofoil craft for steady flow conditions. These components have been integrated into the Hydres computer program. Hydres is used at MARIN for design and analysis work on foil systems.

Hydres includes a design option which enables the user to determine the foil system geometry on basis of a number of global input parameters such as foil system type (canard, tandem, airplane), foil planform type (inverted π, inverted T, double inverted T, surface piercing W) foil loading (lift per unit area), and aspect ratio. For more detailed geometric parameters, such as planform sweep and taper, default values may be used. The required camber of the forward and aft foil sections is determined by solving the equilibrium equations for a specified design speed, hull clearance and trim angle. Herein, the camber ratios are the unknown variables. In this procedure, the foil incidence is set equal to the section's ideal angle of attack. This design option enables the designer to quickly obtain a starting point for optimization work.

Hydres also contains design and analysis modules for propellers and waterjet systems based on the computational method described in the previous section.

In Hydres on average 6 iterations are required for determining the equilibrium condition of the craft. Per speed say 25 equilibrium conditions must be determined (a 5×5 flap setting matrix), so in total 300 evaluations of the forward plus aft foil system force components by means of the vortex lattice method are required. This requires a computer time on a 250 MHz Pentium PC of about 20 minutes per velocity. Hereby a sufficient number of vortex elements is used for numerically accurate results, see Section 3.1.

In this section an example of the use of Hydres is given. This example concerns the resistance characteristics for two hydrofoil craft, one craft with a relatively low foil loading, 50000 N/m², and one craft with a relatively high loading of 90000 N/m². The main parameters of the craft are given in Table 2.13.

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Canard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foil types forward, aft</td>
<td>Inverted T,</td>
</tr>
<tr>
<td></td>
<td>inverted π</td>
</tr>
<tr>
<td>Mass</td>
<td>50 tons</td>
</tr>
<tr>
<td>Hull length</td>
<td>25 m</td>
</tr>
<tr>
<td>Design speed</td>
<td>40 kt</td>
</tr>
<tr>
<td>Take-off speed</td>
<td>25 kt</td>
</tr>
<tr>
<td>Foil submergence to chord ratio</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Figure 2.30 shows the resistance, trim, draft and flap angles versus speed. Below the take-off speed, constant forward and aft flap angle settings were used. The resulting trim, draft and resistance follow then from the equilibrium equations. At speeds at and above the take-off speed, a constant hull clearance and trim angle, equal to the attitude at the 40 kt design speed, were imposed. The solution of the equilibrium equations then includes the flap angles required for the specified attitude.

At the design speed, the lower foil loading case has a 6% higher resistance than the higher foil loading case. Due to the larger wetted area for the lower loading case, the frictional resistance is about 55% higher. This is partially compensated by a 45% higher induced drag for the higher loading case, due to the higher lift coefficient. Also of importance is the interaction between the forward and aft foil systems. For the higher loading case this turns out to be more unfavourable than for the lower loading case. The higher foil loading case has more difficulties in taking off. At 25 knots, the required flap angles are over 30 deg. These are unrealistically high and result in a very high induced drag. This phenomenon also appears to a lesser extent for the lower foil loading case.

A better performance is obtained if the trim angle and draft are allowed to reduce more continuously from the hullborne values towards the design condition. The results are shown in Figure 2.30 with the 'alternative attitude' identification. It is seen that the flap angles now decrease much more gradual from the hullborne settings down to zero at the design speed. The resistance peak at take-off has disappeared and a typical resistance curve for hydrofoil craft remains.

At a constant attitude the foil lift is a linear function of the flap angle. Reducing the hull displacement by increasing the foil flap angles introduces non-linearities in the attitude of the craft through the behaviour of the hull. The hull trimming moment for instance is a non-linear function of trim angle and displacement. Also fore-aft foil interaction, foil-hull interaction and free surface effects introduce non-linearities in the craft’s attitude.

![Resistance-displacement ratio vs speed](image)
Figure 2.30b  Trim angle vs speed

Figure 2.30c  Draft vs speed
Chapter 2  Computational method for steady flow conditions

Figure 2.30d  Forward flap angle vs speed

Figure 2.30e  Aft flap angle vs speed
2.8 Design and analysis program Hydres

Figure 2.31 shows contour plots of the trim, draft, hull displacement and resistance versus the flap angles at a speed of 20 knots for the lower foil loading case. The solid contour lines ('non-simplified') denote normal Hydres results while the dashed lines denote results in which no gravity wave effects on the foil forces, fore-aft foil interaction and foil-hull interaction were taken into account.

Figure 2.31a  Trim contours vs flap angles

Figure 2.31b  Draft contours vs flap angles
Figure 2.31c Displacement ratio contours vs flap angles

Figure 2.31d Resistance contours vs flap angles
It is seen in Figure 2.31 that the trim is only a slightly non-linear function of the flap angles, at least for the present case. The slope of the trim contours varies due to the difference of forward and aft foil contributions to the trimming moment, per unit flap angle. Non-linearities in the stern draft and displacement ratio contours are stronger than for the trim angle. Note that the stern draft not only depends on the foil lift but also depends on the trim angle. The resistance is a strong non-linear function of the flap angles, it has a minimum for low flap angles and a maximum for high flap angles, indicating that the foil system induced drag is the main contributor to total drag. Neglecting gravity wave effects on the foils and interaction between the foils and the hull has a significant effect on the attitude and especially the resistance. The character of the contours is however not greatly affected which indicates that the hull characteristics are responsible for the main non-linearity of the craft's attitude with respect to the developed lift.

Conclusions

Hydres fulfils the requirements set to the steady flow method. It can be used for optimization of the take-off characteristics and to determine the foilborne performance for hydrofoil craft with arbitrary foil systems. In that sense it is a unique tool.

Interaction effects between the forward and aft foils, interaction effects between the foils and the hull and free surface effects on the forces acting on the hull and foils affect the attitude of the craft, and thereby the resistance significantly.

Summary Chapter 2

A review is given of available computational methods for hydrofoils in steady flow, ranging from relatively simple empirical methods to panel methods. Important hydrodynamic phenomena for hydrofoil craft are free surface and interaction effects on the forces acting on the foils.

The steady flow problem is simplified by discarding cavitation aspects. This enables the use of a vortex lattice method for the determination of the foil system forces. For determining the forces acting on the hull an empirical model is chosen.

The mutual interaction between the foil system and the hull is investigated. It is shown that it is possible to estimate the effects of the foil system on the hull. The sensitivity of interaction between the forward and the aft foil with respect to vertical displacements of the wake sheet is investigated.

A computational method is described whereby the equilibrium equations, the vortex lattice method and the hull, propulsor and appendage force components are addressed. Viscosity effects on foil lift and drag are investigated for both model and full scale conditions. The computational method is implemented in a computer program called Hydres. An example of application of Hydres is given.
References Chapter 2


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3. VALIDATION OF THE STEADY FLOW METHODS HYDVLML AND HYDRES

3.1 Introduction

For validating hydrodynamic computational methods, data from tests on scaled ship models in towing basins are often used. During model testing, the gravity and inertia forces are properly scaled by performing the model tests at velocities such that the full scale and model scale Froude numbers correspond. It is impossible to perform model tests at the right Reynolds number simultaneously, so that forces of viscous origin are not properly scaled. For Froude number scaling, the model scale Reynolds number is too low by a factor $\lambda^{15}$, where $\lambda$ denotes the scale ratio.

For conventional ships relying on hydrostatic lift forces, the experimental resistance can be extrapolated to full scale values by correcting for differences in viscous resistance only. For hydrofoils viscosity affects the hydrodynamic lift force. This influences the attitude and thereby the resistance of the craft. A straightforward extrapolation of model test values towards full scale values is not possible, then. By estimating the viscosity effect on lift and drag during model testing, full scale predictions are obtained, whereby the viscosity effects are only partially corrected, see Section 3.4. It is therefore best to perform validation calculations for the model scale condition. This still requires information on the viscosity effects on foil lift and drag. Suitable information may be obtained from viscous flow calculations, see Section 2.6, but some uncertainty with respect to viscosity effects remains present. Appendix D describes an uncertainty analysis for experimental results. The results are applied in Sections 3.3 and 3.4.

Data from full scale measurements will be used as well. At full scale, viscosity effects are still present, but can be estimated with less uncertainty than at model scale. The general disadvantages of using full scale trials data are that environmental conditions can affect the measurements which effects are difficult to include in the calculation results, and that the measurement accuracy of forces and velocities is usually less than in a model basin.

Prior to validating the Hydres program for hydrofoil craft, which includes the complete computational method described in Chapter 2, results obtained from the program Hydvlml for foil systems only will be presented. Hydvlml contains only the vortex lattice method for foil systems below a free surface. The sensitivity of Hydvlml with respect to the number of vortex elements used will be shown first. For examples of the usefulness and accuracy of vortex lattice methods, one is referred to Wang (1974), Hough (1976) and DeJarnette (1976) who compare basic vortex lattice method results with results from other lifting surface methods and experiments. In the present study lift and induced drag coefficients for swept and tapered planform hydrofoils underneath the free surface are compared with data from a panel method. Furthermore, the effects of trailing edge flaps on lift and drag are calculated and compared with thin wing theory results.

Next, free surface effects on lift and drag are shown as a function of the submergence and Froude number and are compared with experimental data. Lift and drag data are presented for captive tests with single and tandem foil systems. This provides knowledge on the validity of the Hydvlml program for arbitrary hydrofoil configurations and for assessing fore-aft foil interaction effects. Furthermore, wave patterns and wave profiles are compared with both experimental results and results from a panel method.
The Hydres program is validated by means of model test data for three hydrofoil craft configurations. The calculated and measured behaviour in terms of attitude and resistance of the hydrofoil craft is compared for the entire speed range.

Finally, comparisons with full scale powering data for a surface piercing and a fully submerged hydrofoil craft are presented.
3.2 Numerical validation of Hydvlm

In this Section the sensitivity of calculation results with respect to the number of vortex elements will be investigated. Some basic comparisons with calculation results from other methods are made to verify that the vortex lattice method has been correctly implemented in Hydvlm. Furthermore, lift increments due to trailing edge flaps will be calculated and the errors introduced by representing camber and flap deflections by an equivalent incidence angle will be determined.

No viscosity effects are taken into account here as comparisons are made with other potential flow results. The distribution of vortex elements over the planform area is performed in agreement with the description in Section 2.5: a cosine distribution of the vortex element length in chordwise direction, a constant vortex element width in spanwise direction and a tip inset of 0.25 times the vortex element width have been applied. For flapped sections, the cosine distribution is applied separately on both the flapped and non-flapped chord portions.

Planforms deeply submerged and below the free surface

In order to investigate the sensitivity of Hydvlm with respect to the number of spanwise \( M \) and chordwise \( N \) vortex elements a series of calculations have been carried out for a plain, uncambered, rectangular planform foil of aspect ratio 5.3 and an uncambered, 20 degrees swept back planform foil with an aspect ratio equal to 5.3 and a taper ratio equal to 0.333. The definitions of the aspect \( AR \) and taper \( (\tau_t) \) ratio’s are:

\[
AR = \frac{b^2}{S}
\]

\[
\tau_t = \frac{c_t}{c_r}
\]

where \( b \) denotes the span of the foil, \( S \) denotes the planform area and \( c_t \) and \( c_r \) denote the chord lengths at the tip and root of the planform respectively. Results are shown for two foil submergence to chord ratio’s: \( h/l = 10 \) and \( h/l = 1.0 \). For the larger submergence case, free surface effects are negligible. The velocity corresponds to a chord Froude number of \( F_{nc} = 2.0 \). The chord Froude number is defined as:

\[
F_{nc} = \frac{U}{\sqrt{g\ c}}
\]

where \( U \) is the speed of advance, \( g \) is the gravitation constant and \( c \) is the mean chord of the foil.

Figure 3.1 shows the ratio between the lift and induced drag coefficients at a certain \( N \) and \( M \) value and these for \( N = 16 \) and \( M = 32 \), i.e. the maximum number of vortex elements used. In Figure 3.1d the maximum number of chordwise vortex elements is \( N = 32 \). It is seen that for the rectangular foil, for\( N > 1 \), the spanwise number of vortex elements \( M \) is most important for the convergence towards lift and drag values that are nearly independent of the number of vortex elements.

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An estimate of the discretization errors can be obtained by using the extrapolated result for $M^{-1} \rightarrow 0$ as the true value. When doing so, for the lifting line approach ($N=1$) with $M=16$, the lift coefficient is within 1% of its value at $N=32$ and $M=16$ for all cases. A somewhat lower error is obtained for $N=4$ and $M=8$.

For the rectangular planform and a lifting line approach ($N=1$) with $M=16$, the induced drag coefficient is within 2% of its value at $N=32$ and $M=16$. A somewhat lower error is obtained for $N=4$ and $M=8$.

For the swept planform foil at a deep submergence, see Figure 3.1d, the convergence of the induced drag with the number of vortex elements is not nearly as fast as for the plain rectangular foil. This is due to the discontinuity in the bound vortex lines at the foil’s centre plane which induces a relatively large velocity at adjacent control points. Hough (1976) shows that the convergence can be improved by using non-swept bound vortex segments for determining the induced drag. This is confirmed in Figure 3.1d in the sense that the sensitivity with respect to the number of spanwise vortex elements is reduced. It is difficult to derive recommendations for a minimum number of vortex elements from Figure 3.1d.

Free surface effects do not affect the sensitivity with respect to the number of vortex elements much, except for the induced drag for the swept and tapered planform. The wavelength of free surface disturbances is large, relative to the dimensions of the vortex elements. For the present case with $a$, for hydrofoil craft, low chord Froude number the wavelength $\lambda$ is approximately 24 chords ($\lambda=2\pi F_{F}^{2}$). Figure 3.1h shows that at a submergence of one chord the sensitivity of the induced drag for the swept foil is much lower than at a deep submergence. Apparently, the free surface induces additional velocity components which reduce the relative importance of the bound vortex segments.

![Graph](image_url)  
**Figure 3.1a** Sensitivity of lift to the number of vortex elements  
Rectangular foil, $h/c=\infty$
3.2 Numerical validation of Hydvlm

Figure 3.1b  Sensitivity of induced drag to the number of vortex elements - Rectangular foil, $h/c=\infty$

Figure 3.1c  Sensitivity of lift to the number of vortex elements
Swept and tapered foil, $h/c=\infty$
Figure 3.1d  Sensitivity of induced drag to the number of vortex elements - Swept and tapered foil, $h/c=\infty$

Figure 3.1e  Sensitivity of lift to the number of vortex elements
Rectangular foil, $h/c=1$
3.2 Numerical validation of Hydylm

Figure 3.1f  Sensitivity of induced drag to the number of vortex elements - Rectangular foil, $h/c=1$

Figure 3.1g  Sensitivity of lift to the number of vortex elements
Swept and tapered foil, $h/c=1$
Results from Hydvlm are compared with results from the Dawson panel method in Table 3.1. In Dawson, see Raven (1988), a similar system of vortex elements is used as in Hydvlm, to introduce lift due to circulation. The tangential flow boundary conditions are imposed at surface source panels representing the actual body shape. The lift is determined from the integration of the pressure over the body surface panels. The induced drag follows from a Trefftz plane analysis on the trailing vortex system at infinity, while the wave making resistance is determined from applying Lagally's law on the free surface source panels.

As mentioned in Section 2.6, the lift curve slope in a panel method is somewhat higher than in a vortex lattice method due to the non-linear effect of the section thickness. This effect is approximately taken into account in Hydvlm by an increase in incidence, proportional to the lift curve slope increment due to thickness given by Martin (1963), eq. (2.82). For the section considered here (NACA16-310) an increase in incidence of 8.0% has been used.

In Hydvlm, 16 vortex elements in spanwise and 32 elements in chordwise direction were used, while in Dawson the number of foil surface panels used was 16 in spanwise direction while 81 panels over the chord were used. The number of free surface panels used was 1874. The number of vortex elements used in the camber plane of the foils was 32 for Dawson. It has been verified that these panel numbers are sufficient for numerically accurate results, by comparing results from runs with a lower number of panels and a less extended free surface domain. The results are valid for a chord Froude number of 2.0. In Table 3.1, the coefficients are based on the foil planform area while $C_{D_i}$ includes induced and wave making drag.
### Table 3.1  Comparison between Hydvlm and Dawson results

<table>
<thead>
<tr>
<th>Planform</th>
<th>Submergence ( h/c )</th>
<th>( C_L ) - Dawson</th>
<th>( C_L ) - Hydvlm</th>
<th>( C_D ) ( /C_L^2 ) - Dawson</th>
<th>( C_D ) ( /C_L^2 ) - Hydvlm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rect.</td>
<td>10.0</td>
<td>0.382</td>
<td>0.376</td>
<td>0.059</td>
<td>0.060</td>
</tr>
<tr>
<td>Rect.</td>
<td>1.0</td>
<td>0.308</td>
<td>0.310</td>
<td>0.106</td>
<td>0.108</td>
</tr>
<tr>
<td>Swept</td>
<td>10.0</td>
<td>0.380</td>
<td>0.360</td>
<td>0.055</td>
<td>0.058</td>
</tr>
<tr>
<td>Swept</td>
<td>1.0</td>
<td>0.304</td>
<td>0.296</td>
<td>0.106</td>
<td>0.106</td>
</tr>
</tbody>
</table>

The agreement between the Dawson and Hydvlm results is fairly good although there is a substantial difference in lift and induced drag ratio for the swept planform for the deep submergence case: 5.5%. This is probably caused by the dominance of induced velocity components due to swept bound vortex segments in combination with the different locations of the control points in Dawson and Hydvlm. Free surface effects on lift and drag itself are described in Section 3.3.

The distribution of the circulation and the sectional lift and drag coefficient over the span are given in Figure 3.2, for the swept and tapered planform foil at a submergence of one chord. The agreement between Hydvlm and Dawson results is fairly good. Near the centre plane, the circulation is somewhat higher for the Dawson result, the sectional lift coefficient is however the same there for the two methods. Near the tip, the circulation values are close while the lift deviates somewhat. These differences are probably due to the different methods used for the calculation of the lift coefficient. In Hydvlm the lift is directly related to the circulation while in Dawson the lift follows from a pressure integration over the surface panels.

The difference in sectional induced drag between Hydvlm and Dawson is constant over almost the entire span width. Note that the sectional induced drag coefficient for Dawson is based on pressure integration. This is generally less accurate than the result for the total drag coefficient which is based on a Trefftz plane analysis and Lagally's law.

Figure 3.3 shows the convergence of the chordwise lift distribution versus \( N \) for a very large aspect ratio (\( AR=100 \)) rectangular foil with a NACA \( a=0.8 \) camber line, at the ideal incidence angle. Two dimensional results can not be approximated more closely in Hydvlm as larger aspect ratios lead to numerical problems due to truncation errors in the chordwise coordinates. The number of spanwise vortex elements used is \( M=32 \). Free surface effects are not present in the results as the biplane image and Green function terms were discarded in the calculations. The lift distribution is in good agreement with results from thin wing theory, as specified by Abbott and Von Doenhoff (1958), for \( N \geq 16 \).
Figure 3.2a  Lift distribution over span for swept and tapered planform

Figure 3.2b  Induced drag distribution over span for swept and tapered planform
3.2 Numerical validation of Hydvlm

Figure 3.2c  Circulation distribution over span for swept and tapered planform

Figure 3.3  Lift distribution over chord versus number of vortex elements
The lift coefficient according to thin wing theory for a NACA a=0.8 mean line at it's ideal incidence is: $C_l=0.30$. Lifting line theory predicts a lift reduction of 2% due to the finite aspect ratio, see Abbott and Von Doenhoff (1958), leading to a three-dimensional lift coefficient of 0.294. Table 3.2 shows the calculated three-dimensional lift coefficient versus the number of chordwise vortex elements. It is seen that the calculated lift coefficient becomes constant for $N \geq 24$ where the difference with the corrected thin wing theory value is 1.4%.

Table 3.2  Lift coefficient versus $N$ for cambered foil

<table>
<thead>
<tr>
<th>$N$</th>
<th>$C_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.286</td>
</tr>
<tr>
<td>8</td>
<td>0.295</td>
</tr>
<tr>
<td>16</td>
<td>0.289</td>
</tr>
<tr>
<td>24</td>
<td>0.290</td>
</tr>
<tr>
<td>32</td>
<td>0.290</td>
</tr>
</tbody>
</table>

Flapped hydrofoil

Figure 3.4 shows the variation of the lift and induced drag coefficients with the number of chordwise vortex elements $N$, for a large aspect ratio ($AR=25$), flapped foil at a submergence of 10 chords. The flap is of the plain, sealed type. The number of spanwise vortex elements used for this case is $M=16$.

It is seen that a relatively large number of chordwise vortex elements is required to achieve lift and induced drag values that are virtually independent of the number of vortex elements. The discontinuity in the circulation strength for vortex elements at the flap hinge position has a marked effect on the local lift. This is shown in Figure 3.5 where the lift coefficient based on the vortex element length is shown for various numbers of chordwise vortex elements.

For an error in lift and induced drag coefficients less than 1% and 2% respectively, 32 vortex elements are required.
3.2 Numerical validation of Hydvlm

Figure 3.4  Sensitivity of lift and induced drag to number of vortex elements for flapped foil

Figure 3.5  Lift distribution over chord vs number of vortex elements for flapped foil
Figure 3.6 shows the calculated flap efficiency versus the flap chord to foil chord ratio for a very large aspect ratio foil ($AR=100$). The flap efficiency $\alpha_6$ is here defined as:

$$\alpha_6 = \frac{\Delta C_f}{2\pi \delta}$$

(3.3)

where $\Delta C_f$ is the incremental lift coefficient due to the flap deflection. The number of chordwise and spanwise vortex elements used are $N=64$ and $M=16$. In order to approximate two dimensional results, the flap efficiency from Hydvlm was multiplied by 1.02, again using the lifting line result of Abbott and Von Doenhoff (1958) for the aspect ratio effect on the lift curve slope. The flap efficiency in Figure 3.6 is compared with a result from thin wing theory for foil sections, see Glaucert (1927):

$$\alpha_6 = \frac{4}{\pi} \sqrt{\frac{c_f}{c}}$$

(3.4)

where $c_f$ is the flap chord. The dependency of the flap efficiency on the flap chord ratio is seen to have a similar character, but the Hydvlm result is progressively lower than the thin wing theory result for increasing flap chord ratios. Figure 3.4 indicates that an increase in $N$ beyond 64 might result in a slight increase in flap efficiency, of 0.5% say, but this would not explain the difference shown here. Glaucert's result is however incorrect for the larger flap chord ratios: for $c_f/c=1.0$ a lift curve slope of $4/\pi$ results according to eq. (3.4). This should be just one, which is better approximated by the Hydvlm result.
Effective incidence approach

The number of chordwise vortex elements needed to accurately resolve flap deflections and camber is eight times higher than the number needed for sections without camber and flaps. It is tempting to represent camber and flaps by an equivalent incidence angle and to treat the section as a flat plate in order to reduce computer time. For two-dimensional foils this approach is consistent with the linearization applied in thin wing theory where the tangential flow boundary conditions are applied at the chord line. The present vortex lattice method uses the same boundary condition linearization, but the question is whether this approach introduces errors for finite aspect ratio foils. This is investigated in the next paragraphs. The results can also be used to obtain an indication for errors due to the use of two-dimensional viscosity correction factors in finite aspect ratio foils, as discussed in Section 2.6.

The first comparison concerns a foil with a NACA a=0.8 mean line with an ideal lift coefficient \( C_{l0} = 0.30 \). The equivalent incidence is set equal to minus the zero-lift angle of attack for an aspect ratio 100 foil. The lift curve for the aspect ratio 100 foil is determined by means of Hydvlm, by using 32 and 16 vortex elements in chordwise and spanwise directions respectively. For two foils with aspect ratio's of 8 and 4, Hydvlm runs were made for a cambered section at zero incidence and without camber at the equivalent incidence. The relative errors in lift and induced drag are shown in Table 3.3, for an infinite submergence. The induced drag to lift coefficient squared ratio is used rather than the induced drag coefficient itself; if resistance comparisons are made for a certain attitude of the craft, lift coefficients are constant. The relative errors are defined as follows:

\[
\Delta C_L = 100 \frac{C_{Le} - C_{Lc}}{C_{Lc}} \\
\Delta \frac{C_{Di}}{C_L^2} = 100 \frac{C_{Di e} - C_{Di c}}{C_{Di c}^2} \\
\text{(3.5)}
\]

where subscripts \( e \) and \( c \) denote the foil at the equivalent incidence and the cambered foil respectively.

Table 3.3  Lift and drag errors due to equivalent incidence for a cambered foil at an infinite submergence

<table>
<thead>
<tr>
<th>AR</th>
<th>( \Delta C_L ) (%)</th>
<th>( \Delta \frac{C_{Di}}{C_L^2} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-0.4</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>-3.1</td>
<td>-1.9</td>
</tr>
<tr>
<td>4</td>
<td>-6.6</td>
<td>-1.4</td>
</tr>
</tbody>
</table>
It is seen that for the aspect ratio 100 foil the results are practically identical, as could be expected. As the aspect ratio is reduced, appreciable differences appear for the lift coefficient. This is due to the different chordwise lift distributions for the cambered section and the uncambered section at incidence. These are affected differently by the induced downwash for a finite aspect ratio. The induced drag ratio is less affected.

Similar calculations have been performed for a flapped section. The equivalent incidence was set equal to \( \alpha_\theta \delta \), where the flap efficiency \( \alpha_\theta \) was determined by means of Hydvlm for an aspect ratio 100 foil. The flap chord to foil chord ratio was 0.25 while an incidence of 5 deg was used. The calculation results are shown in Table 3.4. Although the lift coefficient errors for the flapped foil are smaller than for the cambered foil, they are still appreciable for the lower aspect ratio.

<table>
<thead>
<tr>
<th>Table 3.4</th>
<th>Lift and drag errors due to equivalent incidence for a flapped foil at an infinite submergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>( \Delta C_L ) (%)</td>
</tr>
<tr>
<td>100</td>
<td>-0.2</td>
</tr>
<tr>
<td>8</td>
<td>-2.2</td>
</tr>
<tr>
<td>4</td>
<td>-5.1</td>
</tr>
</tbody>
</table>

The Tables 3.5 and 3.6 show results for the two cases at a submergence of one chord and a very high chord Froude number, i.e. no gravity wave effects are accounted for. The errors are seen to be a bit reduced in comparison to the infinite submergence cases.

<table>
<thead>
<tr>
<th>Table 3.5</th>
<th>Lift and drag errors due to equivalent incidence for a cambered foil at a submergence of one chord.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>( \Delta C_L ) (%)</td>
</tr>
<tr>
<td>100</td>
<td>-0.4</td>
</tr>
<tr>
<td>8</td>
<td>-2.3</td>
</tr>
<tr>
<td>4</td>
<td>-5.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3.6</th>
<th>Lift and drag errors due to equivalent incidence for flapped foil, at a submergence of one chord.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>( \Delta C_L ) (%)</td>
</tr>
<tr>
<td>100</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>-1.7</td>
</tr>
<tr>
<td>4</td>
<td>-3.9</td>
</tr>
</tbody>
</table>
3.2 Numerical validation of Hydvlm

Conclusions

The vortex lattice method is correctly implemented in the Hydvlm program. This conclusion is based on comparisons with numerical results from a panel method for a rectangular and a swept, tapered planform and on results for cambered and flapped sections.

The numerical validation shows that for rectangular planforms with uncambered sections a lifting line approach needs at least 16 spanwise \((M)\) vortex elements for results with a discretization error less than 1\% in lift and 2\% in induced drag. If 4 chordwise \((N)\) vortex elements are selected, 8 spanwise vortex elements are required for the same accuracy, resulting in a larger number of vortex elements than for the lifting line approach.

For the uncambered, swept and tapered planform foil at a deep submergence, the sensitivity of the induced drag with respect to the number of vortex elements is relatively high. This sensitivity can be reduced by using non-swept bound vortex segments for the determination of induced drag. Also, with strong free surface effects present the sensitivity reduces.

For all uncambered planform cases considered in this Section, \(N=4\) and \(M=16\) are required for errors less than 1\% in lift and 2\% in induced drag. This is also true for using the actual, swept bound vortex geometry to determine the induced drag. For foil systems consisting of parts with a swept planform the sensitivity of especially the induced drag with respect to the number of vortex elements should be investigated for each case.

For cambered or flapped sections at least 32 chordwise vortex elements and 16 spanwise vortex elements are needed for results with an error less than 1\% in lift and drag.

For finite aspect ratio foils, camber and flap angle deflections may be represented by an equivalent incidence angle. This results in an error in lift of about 5\% for an aspect ratio four foil at a practical submergence, in comparison to calculation results for the foil whereby camber and flap deflections are actually present. For an aspect ratio eight foil the difference in lift reduces to about 2\%. The induced drag to lift squared ratio is less affected than the lift: errors are limited to 2\% for both aspect ratio’s.
3.3 Validation of Hydvlm on free surface effects and foil interaction

The results presented in this Section are used to validate Hydvlm with respect to the calculation of free surface effects and foil interaction. It has been verified that the number of chordwise and spanwise vortex elements used in Hydvlm is such that the errors in lift and drag are less than 1%, based on the results shown in Section 3.2. Additional errors due to representing camber and flap deflections by an equivalent incidence are less than 2%. The total discretization error is then lower than 2.3% for lift and drag. This is judged acceptable in view of the relatively large uncertainty in experimental results due to viscosity effects.

Free surface effects on lift and drag

Free surface effects are taken into account in the calculation model partly by considering a biplane image vortex system and fully by the wave part of the Green’s function. These two phenomena induce velocity disturbances which curve the streamlines about the hydrofoil downwards and cause additional downwash behind the foil. The first effect affects the zero-lift angle while the second effect reduces the lift curve slope of the foil, $C_{l_{0}}$, and increases the drag. The increase in drag is often termed wave making drag, but will be considered here as an addition to the induced drag. Finally, the flow disturbance due to the foil lift affects the pressure at the free surface which results in the formation of waves. Figure 3.7 shows these effects for a cambered foil with an aspect ratio of six. The lumped vortex discretization is exact for parabolic camber lines, therefore only one chordwise vortex element is used here.

![Figure 3.7a Free surface effect on zero-lift angle](image_url)
3.3 Validation of Hydvlm on free surface effects and foil interaction

Figure 3.7b  Free surface effect on lift curve slope

Figure 3.7c  Free surface effect on induced drag ratio
Experimental results provided by amongst others Wilson (1983) show that the lift remains a linear function of incidence down to submergence to chord ratio ratios of $h/c = 0.25$. Figure 3.8, taken from Wilson, shows the experimentally determined increase in zero-lift angle for a symmetrical hydrofoil of aspect ratio six and a six percent thickness ratio ($t/c = 0.06$) at submergence to chord ratio's of 0.25 and 1.0. The zero-lift angle is seen to increase quite drastically at the lower submergence, for low chord Froude numbers. This is caused by the asymmetry in the flow above and below sections with thickness due to the free surface boundary conditions. Such a change in zero-lift angle can not be determined in a vortex lattice method. For hydrofoil craft the submergence to chord ratio is in general larger than 1.0 and the chord Froude number for conditions with a relatively low submergence, i.e. when foilborne, is relatively high, above $F_{nc} = 4$. For these conditions the change in zero lift angle is negligible.

The free surface effect on the lift curve slope $C_{Lm}$ and induced drag $C_D$ versus the submergence Froude number $F_n$ and submergence to chord ratio $h/c$ is shown in Figure 3.9. The submergence Froude number is defined as:

$$F_{nh} = \frac{U}{\sqrt{gh}}$$

(3.6)

The results shown are valid for a plain rectangular hydrofoil with an aspect ratio of six. Experimental results are again obtained from Wilson (1983). The free surface effect on lift is shown in the form of the ratio of the lift curve slope at the actual submergence to that at a submergence of four chords. The free surface effect on drag is shown as the ratio between the induced drag
3.3 Validation of Hydvlm on free surface effects and foil interaction

coefficient and the lift coefficient squared. By representing the free surface effects in this way it is assumed that viscosity effects are negligible. The experimental induced drag was obtained by Wilson by subtracting an estimated viscous drag from the total measured drag, by using similar formulations as the ones shown in Section 2.6.

Figure 3.9a  Comparison experimental and calculated free surface effect on lift curve slope

Figure 3.9b  Comparison experimental and calculated free surface effect on induced drag ratio
The free surface effects on the lift curve slope are seen to be adequately predicted by Hydvlm. The agreement is also satisfactory for the induced drag ratio, taking into account the scatter in the experimental induced drag ratio's. The scatter in induced drag values is obtained from measurements for one condition at various incidence angles. At lower speeds and submergences the drag is a relatively small quantity with an inherent large uncertainty when measured by means of strain gauges. The neglect of foil thickness in Hydvlm may also contribute to the differences in induced drag for low submergences and low speeds.

**Wave formation**

Although not of primary interest for the prediction of the performance of hydrofoil craft it is interesting to compare the free surface disturbances, or waves, calculated by means of Hydvlm with experimental results and with calculation results obtained with the Dawson panel method.

Results suited for validation purposes were obtained from model tests at MARIN as described by Van Walree and Yamaguchi (1993). Measurements were performed at a low submergence to chord ratio \((h/c=0.25)\) and at low submergence Froude numbers in order to check the validity of the linearization with respect to the free surface conditions. For a rectangular, inverted T and uncambered hydrofoil \((AR=5, \text{NACA16-012 section type, turbulence stimulation, } 3\times10^5<Rn<8\times10^5)\), wave profiles are given in Figure 3.10, at two transverse cuts behind the foil and one longitudinal cut over the foil. For the calculations, the incidence angle was reduced so that the experimental lift coefficient was obtained. This is mainly necessary due to the viscosity effect on the lift slope and to a lesser extend due to the free surface effect on the zero-lift angle. The incidence reduction was as much as 15\%, which is in agreement with the viscosity effects shown in Appendix C.

**Figure 3.10a** Comparison experimental and calculated transverse wave cut, \(x/c=4\)
3.3 Validation of Hydvlm on free surface effects and foil interaction

Figure 3.10b Comparison experimental and calculated transverse wave cut, $x/c=10$

Figure 3.10c Comparison experimental and calculated longitudinal wave cut, $y/c=0.7$
Although the experimental results are somewhat affected by the generation of spray from the strut, the comparison with calculated results is reasonably good for both transverse cuts. For the longitudinal wave cut, the wave elevation close to the foil is underpredicted. This is probably due to the neglect of the foil thickness. Apparently thickness does not affect the wave elevation at some distance behind the hydrofoil. For predicting wave elevations, it is allowed to use the linearized free surface boundary conditions down to submergence to chord ratios of 0.25, at least at low submergence Froude numbers and at a distance of two chords behind the foil.

Figure 3.11 compares wave patterns above and behind a swept back, tapered hydrofoil calculated by means of Hydvlm and Dawson. The hydrofoil configuration and panelizations used here are identical to the ones used in Section 3.2. The Dawson results were verified to contain no significant discretization errors due to inadequate panel distributions on the foil and the free surface. The grids on the free surface at which wave elevations were calculated were identical: 75x25 points in longitudinal and transverse directions respectively. The incidence used in the Hydvlm calculations was adjusted to account for the thickness effect on the lift curve slope. The incidence correction used was 8.0%, corresponding to the increase in lift curve slope according to eq. (2.82).

Results are shown for three Froude numbers \( F_{w} = F_{a} = 2, 4 \) and 8) for a submergence to chord ratio \( h/c = 1.0 \). At the lower speed the resemblance is good. At higher speeds the wave patterns are still similar but differences in wave height appear.

![Figure 3.11a Comparison calculated wave contours, h/c=1.0, F_w=2](image-url)
3.3 Validation of Hydvlm on free surface effects and foil interaction

Figure 3.11b  Comparison calculated wave contours, $h/c=1.0$, $F_m=4$

Figure 3.11c  Comparison calculated wave contours, $h/c=1.0$, $F_m=8$
The lift coefficients and induced drag over lift squared ratio's for Hydvlm and Dawson are given in Table 3.7. The lift is in good agreement, the difference in induced drag is 10% at the highest Froude number.

<table>
<thead>
<tr>
<th>Table 3.7</th>
<th>Comparison free surface effects on lift and drag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dawson</td>
</tr>
<tr>
<td>$F_{nc}$</td>
<td>$C_L$</td>
</tr>
<tr>
<td>2.0</td>
<td>0.304</td>
</tr>
<tr>
<td>4.0</td>
<td>0.330</td>
</tr>
<tr>
<td>8.0</td>
<td>0.324</td>
</tr>
</tbody>
</table>

At a lower submergence ($h/c=0.25$) the comparison between Dawson and Hydvlm results is not good. Figure 3.11d shows that for a chord Froude number of $F_{nc}=4.0$ ($F_{nc}=8.0$) the Dawson result is unrealistic. The wave trough at the foil is three times the submergence of the foil. The Hydvlm result appears more realistic.

This erroneous behaviour of linear (with respect to free surface conditions) panel methods at high Froude numbers was also noted by Nakatake et al. (1992). In Dawson the free surface condition is linearized about the zero Froude number flow. Apparently, for high Froude numbers the linearization about this base flow introduces large errors in the solution. Better results could be obtained when
the free surface boundary condition as used in HydSim (the so-called Kelvin boundary condition) is implemented in Dawson and the free stream is selected as the base flow.

**Single foil systems**

The cases described here concern the comparison of lift and drag forces on foil systems used for actual hydrofoil craft designs. The aim of these comparisons is to investigate the suitability of HydVlm for practical hydrofoil systems. Such foil systems consist of a number of foil and strut parts with a varying planform. For this purpose results are used from an experimental program aimed at obtaining hydrodynamic characteristics of a hydrofoil system for the development of a ride control system.

The experiments, see Kapsenberg and Keukens (1991), were performed on a 1:8 scale model of a hydrofoil system consisting of a forward and an aft foil. The foils were of the inverted $\pi$ type with an aspect ratio of approximately eight. The foil configuration is schematically shown in Figure 3.12. Each foil was connected to a six-component measurement frame which was mounted on a stiff beam. This beam was subsequently mounted on the towing carriage, see Figure 3.13. The experiments were carried out in the High Speed Basin of MARIN, measuring 200 x 4.0 x 4.0 m in length, width and depth, respectively. By varying the velocity of the towing carriage and the vertical position of the beam, the lift and drag forces could be measured for different speeds and submergences. Furthermore, the beam could be rotated about a transverse axis so that the trim angle and submergence of the foil system were varied simultaneously. If required, flap angle settings were adjusted before each measurement run.

![Figure 3.12 Schematic foil system geometry](image-url)
The Reynolds number for these tests ranged from $3.8 \times 10^3$ to $6.2 \times 10^3$. Turbulence stimulation was applied on the foil models by using carborundum roughness with a grain size of 80 μm and a density of 50%, on a strip at the leading edge with a width of 10% of the chord length. The effectiveness of this configuration was verified by means of paint tests. The flap angle deflections used here (up to 4 deg) did not result in significant flow separation. This has been verified by means of paint tests as well. Xfoil results also indicate that no significant flow separation occurs for flap angles below 4 deg.

The viscosity effects on lift and drag are accounted for in Hydvlm by means of eq. (2.88), i.e. the foil section was represented by an uncambered and non-flapped section at an equivalent incidence angle corrected for camber and flap deflection including viscosity effects. The viscosity effects on the zero lift angle, lift curve slope and flap efficiency were determined from Xfoil results for the appropriate section at a range of experimental Reynolds numbers. The foil sections were similar to the YS-920 type, but with an increased camber ratio, resulting in a lift coefficient of 0.43 at zero incidence. Using the uncertainty due to viscosity effects on model scale, blockage effects, foil geometry errors as given in Appendix D, the total uncertainties in lift and drag are estimated to be 16.1% and 6.8% respectively. Per foil, 4 chordwise and 48 spanwise vortex elements were used.

Figure 3.13 Foil system arrangement in model basin
Figure 3.14 shows the calculated and experimental lift and drag to craft weight ratios for the forward and aft foil systems. These quantities are denoted by \( L_{f} \), \( D_{f} \), \( L_{a} \) and \( D_{a} \) respectively. These results show the forces on the foil systems in isolation, i.e. no fore-aft foil interaction is present. The Froude number based on the main (aft) foil chord is 6.9 for all cases, except for the high submergence case of \( h/c=1.8 \) for which \( F_{mc}=4.2 \). The high Froude number case represents cruise speed conditions, the lower one represents conditions just after take-off.

The effect of the foil submergence on lift and drag is adequately predicted. The differences between experimental and calculated results are within the experimental uncertainty, except for the forward foil drag at \( h/c=0.6 \) and the aft foil drag at \( h/c=1.8 \).

Changing the trim angle of the foil system about the craft's centre of gravity primarily results in a change in angle of attack. The foil submergence is also affected, for the forward foil by \( \Delta h/c=0.30 \) and for the aft foil by \( \Delta h/c=0.14 \) per unit of trim angle. The mean submergence to chord ratio for the foil systems was 1.6 and 1.2 for the forward and aft foil respectively. The effect of trim is well predicted for both lift and drag, the differences are within the experimental uncertainty.

Finally, for the flap angle variations, the trends in lift and drag due to a flap deflection are well predicted again, while the differences between experimental and calculated results are within the experimental uncertainty.

Figure 3.14a Comparison experimental and calculated submergence effect on forward and aft foil lift force, \( F_{mc}=6.9 \) and 4.2
Figure 3.14b Comparison experimental and calculated submergence effect on aft foil drag force, $F_{nc}=6.9$ and 4.2)

Figure 3.14c Comparison experimental and calculated trim effect on forward and aft foil lift force, $F_{nc}=6.9$
3.3 Validation of Hydvlm on free surface effects and foil interaction

Figure 3.14d Comparison experimental and calculated trim effect on forward and aft foil drag force, $F_{nc} = 6.9$

Figure 3.14e Comparison experimental and calculated flap angle effect on forward and aft foil lift force, $F_{nc} = 6.9$
The same experimental program has also been carried out with both foils present, i.e. the tandem configuration. In these results the fore-aft foil interaction is present. However, it appeared that for this case the interaction was not very large and within the experimental uncertainty for lift and drag. Therefore, for investigating foil interaction it is more useful to use experiments which were specifically aimed at investigation foil interaction. This is described in the next section.

**Tandem foil systems**

By performing model tests on two foil systems in a tandem arrangement data can be obtained on the interaction effects between the forward and aft positioned foil systems. Interaction effects are defined here by means of the ratio between lift and drag forces on the aft foil with and without the presence of the forward foil:

\[
C_{Lr} = \frac{C_{Laf}}{C_{La}} \\
C_{Dr} = \frac{C_{Daf}}{C_{Da}}
\] (3.7)
3.3 Validation of Hydvlm on free surface effects and foil interaction

where $C_{L_d}$ and $C_{D_d}$ are the aft foil lift and drag coefficients with the presence of the forward foil respectively and where $C_{L_a}$ and $C_{D_a}$ are the aft foil lift and drag coefficient without the presence of the forward foil. These lift and drag ratios only show the global interaction effects. These effects result from inflow angle variations over the span of the aft foil. The inflow variations are caused by induced velocities due to trailing vortices running from the forward foil towards infinity, thereby passing the aft foil, and by the wave system created by the forward foil.

Data for investigating interaction effects are given by Morch (1992). Morch performed experiments for two sets of foil systems. The first foil system consisted of two identical rectangular planform foils with three struts, positioned at the centre line and at the foil tips. The foil section was of the type NACA 16-408 with a modified NACA $a=0.8$ mean line. The aspect ratio of the foils was $AR=8.1$. The aft foil tip is positioned in the same longitudinal plane as the tip of the forward foil. In Section 2.3 it was shown that interaction is then sensitive to forward foil wake sheet displacements in the vertical plane due to roll-up and free surface effects.

The second foil system consisted of two inverted T forward foils with an aspect ratio $AR=3.5$ and a rectangular aft foil with two struts positioned at the foil tips. The forward foil planform was tapered ($\tau=0.43$). The transverse position of the forward foils was such that the strut spacing equalled the span of the aft foil. The same camber and thickness distribution was used as for the first foil system. For this foil arrangement, the aft foil tip was not positioned in the same longitudinal plane as the tips of the forward foil. Therefore, interaction is expected not to be very sensitive to forward foil wake sheet displacements in the vertical plane.

Apparently no turbulence stimulation was applied while the Reynolds numbers ranged between $2.0\times10^5$ and $4.5\times10^5$. Analysis of the measured drag coefficients by means of Hydvlm showed that a reasonable agreement with the experimental drag could be obtained if a laminar to turbulent transition curve was used for the frictional resistance coefficient. This points to the existence of a partially laminar boundary layer on the foil models. Therefore, in Hydvlm the viscosity effect on the lift curve slope and zero-lift angle of attack for a free boundary layer transition was used, as predicted by the Xfoil program. These viscosity effects were accounted for in Hydvlm according to eq. (2.88). For the higher aspect ratio foils, the equivalent incidence approach was used. The number of chordwise and spanwise vortex elements used were 4 and 8 respectively, per foil part. For the first foil configuration in total 80 spanwise vortex elements were used, for the second foil configuration 72 spanwise vortex elements were used.

Figure 3.15a shows a comparison between the experimental and calculated lift curves for a single high aspect ratio forward foil.
The zero lift angle of attack is reasonably well predicted, the lift curve slope is however too low in Hydvlm. Since planform and aspect ratio effects as well as free surface effects on the lift curve slope have been shown to be adequately predicted by Hydvlm, it is most likely that the viscosity effect on the lift curve slope is too strong in Hydvlm. The reduction factor on the lift curve slope used in Hydvlm is $f_\alpha=0.91$. However, to match the experimental lift curve slope, a factor $f_\alpha=0.97$ would be needed, i.e. hardly any viscosity effect would be present. For NACA16 section types (NACA a=1.0 mean line) at Reynolds numbers of $9\times10^5$ and without the use of turbulence stimulation, experimental data described by Lindsey et al. (1948) show that the lift curve slope is discontinuous due to laminar separation. For the incidence range of interest the viscosity effect on the lift curve slope $f_\alpha$ varies between 0.70 and 0.95, depending on the incidence, thickness ratio and camber. This illustrates the uncertainty with respect to viscosity effects on lift for low Reynolds numbers experiments without the use of turbulence stimulation.

Figure 3.15b shows a comparison between the experimental and calculated drag curves. The minimum drag coefficient is well predicted. At larger positive and negative incidence angles the drag is underestimated, presumably due to the start of the boundary layer transition and/or the too low calculated lift in Hydvlm.
In order to have a fair comparison between calculated and experimental interaction effects, the viscosity effect on the lift curve slope in Hydvlm was set to unity so to have the same lift characteristics as for the foils during the model testing. This is of importance as the strength of the trailing vortices, and thereby the inflow variations at the aft foil, are proportional to the lift of the forward foil. The changes in lift and drag of the aft foil depend on these inflow variations and the lift curve slope of the aft foil itself.

Comparisons between measured and calculated data are shown in the Figures 3.16 and 3.17. For the first foil configuration, comparisons are shown for four foil spacings, two submergences and two velocities. The interaction on lift and drag is seen to be appreciable. The lift ratio has a maximum value just above unity when the foil spacing is a quarter of the principal transverse wavelength of the foil, $2\pi c F_{nc}^2$. When the lift ratio is above unity, the foil experiences upwash and the drag ratio correspondingly decreases below unity. At a low submergence the favourable interaction effects are more extreme. The trends with foil spacing, velocity and submergence are well predicted by Hydvlm.

The actual differences in calculated and experimental drag ratio's are in general larger than these for the lift ratio. This is probably caused by the high sensitivity of the drag on the inflow variations. In general, the induced drag is proportional to the lift coefficient squared and thus the incidence squared. Furthermore, due to inflow variations the lift vector rotates, thereby creating an additional force in the free stream direction. For the present case, an inflow variation of 1 degree changes the lift some 17% while the induced drag is altered by as much as 70%.
It is further seen in Figures 3.16 that the predictions for the lift and drag ratio's do not deteriorate with increasing foil spacing. This indicates that the roll-up of the wake sheet probably does not have a large effect on the interaction for this case. This is in contrast to what was expected on basis of the location of the aft foil tip relative to the tip of the forward foil. The displacement of the tip vortex in the horizontal plane may be responsible for this. In Section 2.3 only displacements in the vertical plane have been considered.

For the second, low aspect ratio foil configuration similar interaction effects and tendencies are found as for the first, high aspect ratio configuration, see Figure 3.17. However, differences between calculated and experimental results are somewhat larger. Again, the error does not increase with distance from the forward foil. This distance extends up to 11 foil spanwidths for which the wake sheet will be fully rolled-up. This suggests that wake sheet roll-up is not an important factor in foil interaction. On the other hand, the minimum foil spacing value is 4 forward foil spanwidths for which the wake sheet may already be rolled up to a large extent.

![Graph showing lift and drag ratio comparisons](image)

Figure 3.16a Comparison experimental and calculated interaction effect
Foil configuration 1, $F_a=2.78$, $h/c=1.575$
3.3 Validation of Hydvlm on free surface effects and foil interaction

Figure 3.16b Comparison experimental and calculated interaction effect
Foil configuration 1, \( F_n = 4.15, \frac{h}{c} = 1.575 \)

Figure 3.16c Comparison experimental and calculated interaction effect
Foil configuration 1, \( F_n = 2.78, \frac{h}{c} = 0.75 \)
Figure 3.16d  Comparison experimental and calculated interaction effect
Foil configuration 1, $F_n=4.15$, $h/c=0.75$

Figure 3.17a  Comparison experimental and calculated interaction effect
Foil configuration 2, $F_n=2.94$, $h/c=0.86$
Conclusions

Free surface effects on the lift curve slope and induced drag are well predicted by Hydvlm, at least for practical submergences and chord Froude numbers. Based on two transverse and one longitudinal wave cuts, it is concluded that the formation of waves at some distance behind the foil is also well predicted by Hydvlm. For the purpose of determining free surface effects on lift and drag, linearization of free surface effects is permitted for conditions with strong free surface effects.

The effects on lift and drag due to variations in foil submergence, trim and flap angle are qualitatively well predicted by Hydvlm. The quantitative differences between experimental and calculated results are generally within the experimental uncertainty.

Effects of fore-aft foil interaction on the aft foil lift and drag are qualitatively well predicted by Hydvlm. This is also true for large foil spacings and low aspect ratio forward foils for which the roll-up of the wake sheet is strong. For a low aspect ratio forward foil the quantitative agreement between experimental and calculated results deteriorates somewhat. The maximum differences between calculated and experimental interaction effects are 12.5% and 8% for the drag and lift ratio's respectively, at a relatively low submergence and Froude number.
3.4 Validation of Hydres

In this Section results obtained with the Hydres computer program are compared with model test and full scale data. Hydres includes Hydvlm for determining the foil system force components.

The number of chordwise and spanwise vortex elements used is such that the discretization errors in lift and total drag are within 1%. This has been verified by means of calculations with a varying number of vortex elements. The results from the numerical validation described in Section 3.2 are not directly applicable here since, especially for surface piercing foil systems, the foil system consists of a number of parts (up to 8) with a varying planform, incidence, camber and with and without trailing edge flaps. Per foil part, 4 chordwise (N=4) and 8 spanwise (M=8) vortex elements were used.

Foil sectional camber and flap deflections are represented by an equivalent incidence. For the relatively large aspect ratios of the foils used here, see Table 3.8, this leads to errors in lift and induced drag ratio less than 2%.

Viscosity effects in lift curve slope, zero-lift angle and flap efficiency are taken into account by using Xfoil results as outlined in Section 2.6, eq. (2.88), for the appropriate Reynolds number. The correction factors are reviewed in Appendix C. In Appendix D an uncertainty analysis is applied to the model tests used in this section. The uncertainty in the viscosity correction factors is hereby attributed to the model tests instead of the computational method. This is further discussed in Appendix D.

The equilibrium equations were considered to be solved successfully when the error in the force and moment equilibrium was less than 0.1%. It has been verified that this is a sufficient requirement for obtaining accurate results for the trim, draft and force components.

Model test data

Model tests have been performed with scaled models of hydrofoil craft consisting of hull and forward and aft foil systems. Experimental data are available for three models denoted as model A, B and C. The main characteristics of these hydrofoil craft are shown in Table 3.8 while in Figure 3.18 general outlines of the models A and C are shown; model B has a similar foil arrangement as model A. All three hull forms were derived from the Series 65 hull form series data, see Section 2.7. The purpose of the tests was to determine the trim, draft and resistance curves versus speed.

<table>
<thead>
<tr>
<th>Model</th>
<th>Length (m)</th>
<th>Mass (ton)</th>
<th>Model scale</th>
<th>Foil type</th>
<th>Aspect ratio forward foil</th>
<th>Aspect ratio aft foil</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>27.5</td>
<td>75</td>
<td>10</td>
<td>surface piercing</td>
<td>12.9</td>
<td>9.3</td>
</tr>
<tr>
<td>B</td>
<td>21.5</td>
<td>50</td>
<td>8</td>
<td>surface piercing</td>
<td>9.7</td>
<td>9.8</td>
</tr>
<tr>
<td>C</td>
<td>21.5</td>
<td>55</td>
<td>8</td>
<td>fully submerged</td>
<td>7.8</td>
<td>8.3</td>
</tr>
</tbody>
</table>
The model tests were performed in MARIN's High Speed Basin for model A and in the Deep Water Basin for models B and C. The High Speed Basin measures 200 x 4.0 x 4.0 m while the Deep Water Basin measures 250 x 10.5 x 5.5 m in length, width and depth respectively. The models were attached to a special pushing rod arrangement attached at the position of the propulsor on the aft foil system, see Figure 3.19. The pushing rod was connected via an air lubricated cylinder to the towing carriage. The weights of the cylinder and pushing rod arrangement were balanced externally. In this way the propulsive force acted at the right position in order not to affect the running trim unintentionally. The cylinder gave the model freedom in heave and trim. The other modes of motion were restrained. Strain gauges were positioned in the hinge links between the pushing rod and the aft foil for measuring the resistance force. The trim and draft were recorded by measuring the
vertical displacement of the model at the bow and stern. Some photographs of the model tests are presented in Figure 3.20.
Figure 3.19  Pushing rod arrangement
Figure 3.20a  Above water part model B at 22 kt.

Figure 3.20b  Submerged part model B at 22 kt.
3.4 Validation of Hydres

The chord Reynolds number for the tests ranged from $2.7\times10^5$ to $7.5\times10^5$. Turbulence stimulation by means of carborundum grains was applied on the foil models as described in the paragraphs on single foil systems, Section 3.3. The effectiveness of the turbulence stimulation was verified by means of paint tests.

The calculated and experimental results for model A are shown in Figure 3.21. The resistance to displacement ratio is based on the model test conditions, so including the viscosity effects on lift and drag at the model test Reynolds number. Calculation results with and without viscosity effects on lift are shown. The foil-hull interaction is estimated as outlined in Section 2.3. Without viscosity effects on lift means that all viscosity correction factors have a value equal to one.

The viscosity effect on lift is seen to be very significant, as could be expected from the large correction factors used for the NACA16 section types, see Appendix C, and the sensitivity of the craft towards lift changes, see Appendix D. Without viscosity effects the additional lift of the foil system reduces the trim angle as speed increases in the hullborne speed range. At foilborne speeds the hull clearance is much too high (or the draft is too negative), while at hullborne speeds the hull draft is much too low. The resistance is much too low for the entire speed range. Apparently, the lower trim angle at hullborne speeds results in a lower hull wave making resistance, the higher hull clearance at foilborne speeds results in a decrease in resistance due to the lower wetted surface, despite the extra induced drag due to the increase in foil system lift coefficient.
Figure 3.21a  Comparison experimental and calculated trim angle, model A

Figure 3.21b  Comparison experimental and calculated draft-length ratio, model A
In Section 2.3 an estimation method for foil-hull interaction is described. The method is based on induced velocities due to the foil systems at the submerged hull surface. It was shown that interaction results in a suction force on the hull. For the present case, foil-hull interaction is more significant than for the example shown in Section 2.3 due to the relatively large magnitude of the transverse velocity components induced by the trailing vortex lines from the forward foil. For surface piercing foil types these run relatively close to the aft part of the hull.

Figure 3.21 shows calculation results with and without accounting for foil-hull interaction. Without interaction, the calculated trim and draft are too low for speeds below take-off, while the resistance hump is underpredicted by 12%. With interaction, the additional suction force acting on the stern part of the hull bottom clearly improves the prediction. The error in the resistance hump reduces to 5%. The hull draft is in good agreement with the experiments while the trim remains somewhat too low.

At foilborne speeds, the comparison with the experimental data is reasonably good. The resistance tends to be too high, though.

The uncertainties for the model tests for model A are determined in Appendix D. They are repeated in Table 3.9.
Table 3.9 Uncertainties in model test results model A.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Hump speed</th>
<th>Cruise speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trim uncertainty (%)</td>
<td>5.5</td>
<td>22.2</td>
</tr>
<tr>
<td>Draft uncertainty (%)</td>
<td>8.0</td>
<td>23.5</td>
</tr>
<tr>
<td>Resistance uncertainty (%)</td>
<td>6.2</td>
<td>14.0</td>
</tr>
</tbody>
</table>

The errors between calculation results and experiments for the hump and cruise speed conditions are given in Table 3.10.

Table 3.10 Errors in calculation results model A.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Hump speed</th>
<th>Cruise speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trim error (%)</td>
<td>7.2</td>
<td>9.3</td>
</tr>
<tr>
<td>Draft error (%)</td>
<td>8.3</td>
<td>16.7</td>
</tr>
<tr>
<td>Resistance error (%)</td>
<td>4.1</td>
<td>4.5</td>
</tr>
</tbody>
</table>

The trim and draft errors for the hump speed are somewhat larger than the experimental uncertainty. The errors for the other quantities are within the experimental uncertainty.

Apart from the error sources considered in Appendix D, models A and B present some additional difficulties in determining the experimental resistance. The production of spray from highly cambered surface piercing foil and strut parts is rather strong, see Figure 3.20a. In Hydvlm, spray resistance is estimated for vertical strut parts with uncambered sections, which generate generally less spray. Furthermore, the tip parts of the foil system are highly cambered with ideal lift coefficients $C_l$ up to 2.0 while the upper parts of the struts have circular arc sections with a thickness ratio as high as 25%. For these section types flow separation is highly probable at model scale. Note that these foil parts are positioned above the watersurface at foilborne velocities. Finally, underwater photographs, see Figure 3.20b, show that the thick circular arc strut parts ventilate to the free surface for the take-off speed region. Although ventilation of the lifting parts of the aft foil cannot be clearly discerned in Figure 3.20b, if this occurs, the lift and drag may be affected significantly. No attempts have been made to correct for these effects in Hydvlm. Despite these additional error sources, the resistance is predicted quite well. In fact, taking all error sources into consideration, the resistance predictions are surprisingly close to the experimental values.

For model B the results are shown in Figure 3.22. Similar trends as shown for model A are observed. Including foil-hull interaction improves the predictions, but the calculated foil-hull interaction seems somewhat too strong: the draft is too large and the resistance hump is at a too high speed. The resistance at low speeds is significantly underpredicted. This may be due to ventilation on the aft struts which increases resistance. At foilborne speeds the resistance is underpredicted. The trim is reasonably well predicted apart from the too low trim at low speeds. The hull clearance is somewhat too high at foilborne speeds.
3.4 Validation of Hydres

Figure 3.22a  Comparison experimental and calculated trim angle, model B

Figure 3.22b  Comparison experimental and calculated draft-length ratio, model B
The errors between calculation results and experiments for the hump and cruise speed conditions are given in Table 3.11.

Table 3.11 Errors in calculation results for model B.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Hump speed</th>
<th>Cruise speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trim error (%)</td>
<td>7.5</td>
<td>25.2</td>
</tr>
<tr>
<td>Draft error (%)</td>
<td>10.1</td>
<td>15.6</td>
</tr>
<tr>
<td>Resistance error (%)</td>
<td>2.2</td>
<td>2.0</td>
</tr>
</tbody>
</table>

If the experimental uncertainty determined for model A is also used for model B, Tables 3.9 and 3.11 show that again the error in trim angle and draft are somewhat larger than the experimental uncertainty. For the cruise speed condition, all prediction errors are within the experimental uncertainty. Again, resistance errors are surprisingly small.
For model C results are shown in Figure 3.23 for a flap angle setting on the forward and aft foils of 10 and 7.5 degrees respectively. This craft has the same hull form as hydrofoil craft B. Results are available up to a velocity of 32 knots as this hydrofoil model with fully submerged foils was unstable in heave and pitch at speeds around and beyond 32 knots. For the actual craft, the ride control system actuating trailing edge flaps is used to assure stability. For the model tests only passive flap settings were used. Therefore, for the higher velocities captive tests were carried out as described in Section 3.3 in the paragraphs on tandem foil systems. Calculation results are shown including foil-hull interaction for consistency, but for this case the interaction was found to be much smaller than for cases A and B. The trim angle and draft are well predicted, although a somewhat too early take-off is predicted by the calculations. For speeds above 26 knots the Hydres program had difficulty in solving the equilibrium conditions due to the instability of the configuration. The resistance is predicted satisfactorily as well. The effect of the too early take-off on the calculated resistance is clearly present at a speed of 30 knots.

The uncertainties for the model tests for model C are determined in Appendix D. They are repeated in Table 3.12.

Table 3.12 Uncertainties in model test results model C.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Hump speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trim uncertainty (%)</td>
<td>8.0</td>
</tr>
<tr>
<td>Draft uncertainty (%)</td>
<td>17.1</td>
</tr>
<tr>
<td>Resistance uncertainty (%)</td>
<td>7.2</td>
</tr>
</tbody>
</table>

The errors between calculation results and experiments for the hump speed are given in Table 3.13.

Table 3.13 Errors in calculation results model C.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Hump speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trim error (%)</td>
<td>6.0</td>
</tr>
<tr>
<td>Draft error (%)</td>
<td>11.1</td>
</tr>
<tr>
<td>Resistance error (%)</td>
<td>4.4</td>
</tr>
</tbody>
</table>

All errors are within the experimental uncertainty.
Figure 3.23a Comparison experimental and calculated trim angle, model C, $\delta_f=10.0$ deg, $\delta_u=7.5$ deg

Figure 3.23b Comparison experimental and calculated draft-length ratio, model C, $\delta_f=10.0$ deg, $\delta_u=7.5$ deg
In view of the good correlation of calculated and experimental data, apparently no significant flow separation on the flaps was present during the model tests, at least for this first flap setting. The YS1040 type foil section used for the hydrofoil system for this model features a gradual increase in pressure near the trailing edge, similar to the NACA 60.8 mean line.

However, if a comparison is made with model test data for larger flap angles, 12.5 degrees on both foils, the correlation between experimental and calculated data is not good, especially for the trim angle, see Figure 3.24a. The experimental trim angle is significantly larger than predicted. Also in comparison to the trim angle for the first flap angle setting, see Fig 3.23a, the trim angle shown here is larger.

This is in contradiction to what could be expected. Firstly, in comparison to the flap setting $\delta=10.0/7.50$ deg for the first case, for a flap setting of $\delta=12.5/12.5$ deg a lower trim angle may be expected as the flap angle increase for the aft foil is largest. Secondly, the lift force increases for larger flap angles, the hull displacement reduces and so will, in general, the hull trim angle. This can be observed from the Series 65 data. The calculated trim angle is indeed lower for the second flap angle setting than for the first one. The experimental increase in trim for the second case indicates a loss of lift on the aft foil due to flow separation.
Figure 3.24a Comparison experimental and calculated trim angle, model C, $\delta_f=12.5$ deg, $\delta_r=12.5$ deg

Figure 3.24b Comparison experimental and calculated draft-length ratio, model C, $\delta_f=12.5$ deg, $\delta_r=12.5$ deg
3.4 Validation of Hydres

On the other hand, the draft values for the second flap setting case are lower than for the first one, and in good agreement with the calculation results. This indicates that the lift force does increase for the second case, then again this increase in lift may also be due to the higher trim angle of the hull. Furthermore, the resistance is not much larger for the second flap setting case than for the first one. One would expect a significant resistance increase if flow separation occurs. On the other hand, loss of lift due to separation also leads to loss of induced drag and the increase in trim may result in a lower hull resistance. In conclusion, there is a strong suspicion of flow separation for the second flap angle setting. This is in agreement with Xfoil results which indicate a strong flow separation at the trailing edge for flap angles above 10 degrees.

General remarks on the extrapolation of model test results

It was shown for model A that viscosity has a strong effect on the lift production of the foils. This affects the position of the craft relative to the watersurface and thereby the resistance. This raises questions about the use of experimental data for full scale conditions. Obviously, experimental results can not be extrapolated to full scale conditions in a straightforward manner, similar to the Froude scaling applied to conventional ships. What can be done is to estimate a reduction on foil incidence relative to the experimentally determined incidence for which the required hull clearance at the design speed is obtained. This incidence reduction must be applied at full scale so that for the cruise speed condition the design hull clearance will be obtained. The incidence reduction may be determined from the viscosity effects on the lift curve slope and zero-lift
angle of attack as outlined in Section 2.6. Assuming furthermore that, due to the incidence reduction, at lower speeds the experimental attitude of the craft is obtained at full scale as well, the usual extrapolation of the frictional drag can be performed. However, due to the higher lift curve slope at full scale, for trimmed conditions in the hullborne speed region, the lift at full scale will be larger than at model scale and differences in attitude and resistance will be introduced. What cannot be solved either is that due to viscosity effects, the full lifting potential of trailing edge flaps can not be exploited at model scale for purposes of resistance optimization for the hump speed region, even when no extensive flow separation occurs on the flaps.

Full scale measurement data

Some data are available from full scale trials. The main advantage of using these data is that the Reynolds numbers are relatively high so that due to the inevitable surface roughness of the foils the boundary layer will be turbulent over almost the entire foil section. For these conditions, frictional resistance coefficients for turbulent flow may safely be used and flow separation effects will be small provided the incidence and flap angles are within normal operational limits. The disadvantage of using full scale trials data is that the measurements are not performed under laboratory conditions which may introduce uncertainties in the measurements due to for instance uncontrollable environmental conditions.

Data from three hydrofoil craft will be used here. The first two concern the well known Jetfoil developed by Boeing Marine Systems. The Jetfoil features a fully submerged, canard foil system, see Figure 3.25. The foil system is equipped with a number of trailing edge flaps coupled to a ride control system. The craft uses a waterjet system for propulsion, with a ram type inlet mounted at the aft foil. The foil system geometry has been obtained with sufficient detail from publications available in the open literature, see Feipel (1981), Noreen et al. (1979) and Dixon et al. (1980).

Feipel (1981) provides thrust curves for the Jetfoil with two alternative forward foil geometries. The thrust measurements are based on monitoring the propulsion system performance during the trials. Test stand thrust data were used to calibrate the waterjet propulsion pumps. No details on the procedures followed are available. For ram type waterjet inlet systems the thrust deduction is assumed to consist entirely of the drag of the inlet pod and strut. These drag components are taken into account in Hydres as appendage drag components, as outlined in Section 2.7. Therefore, the calculated resistance is here directly compared with the measured thrust.

The trials have been performed at calm water with no significant wind. In the calculations a frictional resistance coefficient is used corresponding to the ITTC-57 line at the appropriate Reynolds number, plus an increment of $\Delta C_F = 0.00040$, accounting for the roughness of the foil system surface. Such a $\Delta C_F$ value is used as common practice in hydrofoil craft design.
In the configuration indicated with 110, a low aspect ratio fore foil is used with a rectangular planform while in the 115 configuration a higher aspect ratio, tapered forward foil is used. Furthermore, the 115 configuration has a strut with an increased chord length, a larger nacelle and a 1.8% increased craft weight.

The Figures 3.26 and 3.27 show a comparison between the calculated resistance and the measured thrust for the two types. For the speed range shown, the craft is in a foilborne condition at a prescribed trim and hull clearance. The required trim and hull clearance are attained by the ride control system which deflects the trailing edge flaps as required. The agreement is satisfactory, although for the 115 type the thrust is underpredicted at speeds above 45 knots. This is probably due to the occurrence of extensive cavitation on the outer span part of the aft foil due to the upwash induced by the fore foil. Feifel (1981) shows the occurrence of such a cavitation pattern during the trials.

The difference in the craft's resistance for the two forward foil types is predicted well. Despite the increase in wetted surface and weight, the 115 configuration has a 3% lower resistance which must be attributed to the higher aspect ratio of the forward foil, which reduces induced drag.
Figure 3.26 Comparison full scale and calculated resistance for Jetfoil-115

Figure 3.27 Comparison full scale and calculated resistance for Jetfoil-110
The second case concerns a surface piercing hydrofoil craft (type MEC1-SP) developed by Rodriguez Cantieri Navale. This craft corresponds to model B described in the previous paragraphs on the comparison with the model tests. A comparison with extrapolated model tests results is unfortunately not possible due to differences in the displacement and the location of the centre of gravity.

The propulsion system consists of two tractor propellers mounted at the forward end of two nacelles at the aft foil-strut intersections. The propellers are driven by a hydrostatic power transmission system which enables the monitoring of rpm and torque at the propeller from within the craft.

During the craft’s trials, the following quantities were monitored: speed (GPS), propeller rate of revolutions (RPM) and torque. Via ride control system sensors indications of the flap angles, the running trim and hull clearance could be obtained. The trials were performed at deep water in the Strait of Messina. A light wind (Bf 2.5) was blowing during the trials, but no significant waves and current were present. Runs were performed with and against the wind. The variations in velocity between these runs ranged from 0.1 knots to 1.9 knots, in RPM from 10 to 20 revolutions per minute and in torque from 0.15 to 0.30 kNm at take-off and cruise speeds respectively.

For calculating the RPM and torque the following procedure was used in the Hydres program. The required thrust was based on the calculated resistance and on the experimentally determined thrust deduction factors for the actual propulsion configuration, see Surace (1991). Next the propeller RPM was determined from the thrust identity, using an open water diagram for an equivalent B-Series propeller, see Van Lammeren et al. (1969). The thrust identity expresses the thrust force as a function of the propeller RPM. The equivalent B-Series propeller was based on the characteristics of the actual propeller for which the detailed geometry was available. The propeller is assumed to operate in an undisturbed flow, which seems reasonable for tractor propellers. The torque was finally determined from the open water diagram, at the calculated RPM.

For the comparisons with calculation results the mean values of the runs with and against the wind are used here. The running trim and hull clearance data from the trials are indicative only and were found in general agreement with the calculation results and are not further discussed here. The Figures 3.28 and 3.29 show the comparisons between the trial data and the calculation results. The differences in predicted and measured absorbed power are about 4% for both the low and high speeds. Differences of similar magnitude were found for the model test resistance. It should be noted that especially for the cruise speed region significant variations in speed, RPM and torque are present in the trial measurements with a magnitude equal to the difference between the mean measurement values and calculation results.
Figure 3.28a Comparison full scale and calculated propeller rate of revolutions for craft B, take-off speed region

Figure 3.28b Comparison full scale and calculated propeller torque for craft B, take-off speed region
3.4 Validation of Hydres

Figure 3.28c  Comparison full scale and calculated delivered propeller power for craft B, take-off speed region

Figure 3.29a  Comparison full scale and calculated propeller rate of revolutions for craft B, cruise speed region
Figure 3.29b  Comparison full scale and calculated propeller torque for craft B, cruise speed region

Figure 3.29c  Comparison full scale and calculated delivered power for craft B, cruise speed region
3.4 Validation of Hydres

Conclusions

Hydres predictions for trim and draft for surface piercing hydrofoils (models A and B) are somewhat outside the experimental uncertainty for hump speed conditions. For cruise speed conditions the trim and draft are within the experimental uncertainty. The resistance is well within the experimental uncertainty for both speed regimes. Considering the uncertainty in the viscosity effects on foil characteristics, the occurrence of spray and ventilation, it is thought that the attitude and resistance of surface piercing hydrofoil craft are surprisingly well predicted by the Hydres program.

For the fully submerged hydrofoil (model C) the Hydres predictions are within the experimental uncertainty, provided there is no significant flow separation from the flaps.

In addition to the strong viscosity effects on the foil characteristics, the differences between calculation results and experimental results are probably also due to the simplifications in the computation of fore-aft foil interaction and the neglect of hull-foil interaction. It is shown that including a rather crude estimation of foil-hull interaction clearly improves the predictions of Hydres for the cases A and B.

The full scale predictions are satisfactory for the two Jetfoil configurations. Also, effects due to foil system geometry variations are well predicted. For the MEC1-SP trial data for the hullborne speed region the agreement between calculations and measurements is satisfactory as well. For the foilborne speed region the agreement is somewhat less, but this may be caused by scatter in the trials data.

The Hydres method is considered as a useful tool for resistance predictions for the entire speed region.

Summary Chapter 3

The vortex lattice method is numerically validated for rectangular and swept planform foils at deep submergence and below the free surface. Also, comparisons with calculation results obtained with a panel method are made for the swept planform foil.

A further numerical validation study is performed for cambered and flapped sections. Calculation results for the efficiency of trailing edge flaps are compared with results from thin wing theory. An equivalent incidence approach is tested for cambered and flapped sections.

Free surface effects on lift and drag for basic foils are validated on basis of model test data. The generation of waves is validated on basis of experimental wave cuts. Also, wave pattern comparisons are made with results obtained with a panel method. Interaction effects between the forward and the aft foil are validated on basis of model test data for two tandem foil arrangements underneath the free surface.
The calculated attitude and resistance for three hydrofoil craft models are compared with experimental results. Hereby, both the hullborne and foilborne speed regions are considered. The significance of viscosity effects on lift and foil-hull interaction is shown.

Finally, comparisons are made with full scale trials data for a fully submerged and a surface piercing hydrofoil craft.
References Chapter 3


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Lindsey W.F. et al (1948), 'Aerodynamic Characteristics of 24 NACA 16 Series Airfoils at Mach Numbers between 0.3 and 0.8', NACA Technical Note 1546, Washington.


4. COMPUTATIONAL METHOD FOR UNSTEADY FLOW CONDITIONS

4.1 Introduction

In the Introduction in Chapter 1 it was noted that the use of high speed craft for transport of passengers and time sensitive goods has considerably increased during the last decade. Transport of passengers needs ferries with a high degree of comfort, also under less favourable weather conditions. Not only from the viewpoint of passengers, which can do without seasickness, but also from an economic viewpoint of the ferry operator. Competitiveness with other means of transport, by air and by road, requires that the operational time schedule must be maintained as long as possible. Consequently, the instant that a speed reduction is necessary due to too violent motions must be postponed to ever higher sea states.

Furthermore, the use of high speed craft on confined waterways and/or with a high traffic density sets requirements to the manoeuvrability of these craft types. Manoeuvrability concerns amongst others the ability to change course and to perform an emergency stop within a specified distance. Especially for hydrofoil craft relying on the use of a ride control system for seakeeping and manoeuvring, stringent requirements are set to the safety of operation during normal and emergency conditions. Emergency conditions include for instance ride control system failures while performing a turning manoeuvre whereby the craft assumes a certain roll angle to counteract the centrifugal forces. If one or more flaps become disabled, the craft must not capsize but must be able to resume a safe attitude.

In the Netherlands examples of the use of high speed craft on confined and heavily used waterways are given by the recent introduction of hydrofoil craft on the Rhine and Waal rivers, in the Rotterdam harbour, on the IJsselmeer and on the Noordzeekanaal between Amsterdam and IJmuiden. The Rhine and Waal rivers are heavily used by inland water vessels and certain manoeuvring requirements are set to the hydrofoil craft. The introduction of hydrofoils on the IJsselmeer has raised protests from the owners of sailing and motor yachts. They point at the risk of collisions with the high speed (32 knots) hydrofoils when the IJsselmeer in certain areas is crowded with pleasure craft.

The High Speed Marine Vehicle Committee of the 21st International Towing Tank Conference (1996) describes new guidelines set to safety of operation for high speed craft as recently issued by the International Maritime Organisation: the IMO Code of Safety for High Speed Craft (1994). This Code was issued in view of the growth in both the use of high speed craft and the number of accidents involving high speed craft. The Code outlines requirements for verification of craft performance in normal operation conditions, in the worst intended conditions during manoeuvring and in failure conditions.

For hydrofoil craft in particular, the intact transverse stability in the take-off and foilborne modes must be investigated. This must be done by the use of validated computer simulations, including effects of failures in systems or operational procedures. The performance should be verified by full scale tests. The ITTC Committee stresses the need for the IMO guidelines and notes that currently available computational methods need to be enhanced in order to investigate the seakeeping and manoeuvrability aspects in a preliminary design phase.
Computational methods are needed especially for hydrofoil craft since performing model tests for these crafts is complicated, if not impossible, and thus costly. This is due to the high speed which needs large towing tanks with high speed carriages and oblique wave generators, the use of ride control systems which needs expensive and complicated actuation mechanisms for high frequency flap control and the need for model scale propulsion units for self propelled tests.

For such methods time domain simulations in six degrees of freedom are required. In contrast to frequency domain methods, in time domain methods non-linearities and transient effects can be included, which might perhaps be needed for the seakeeping analysis on a straight course, but certainly is needed in the analysis of the manoeuvring behaviour and the description of transient phenomena.

The traditional development of separate computational methods for one of the three main items in hydrodynamics: powering, seakeeping and manoeuvring, cannot be maintained for analysis of a turning manoeuvre in waves or a crash stop procedure. Within a time domain simulation method for hydrofoil craft an accurate description of all force components acting on hydrofoils is required. Although motions in the vertical plane are in general governed by lift forces, the forward speed is not necessarily constant and resistance and thrust forces must be included as well since the velocity has a large effect on lift forces. A ride control system may be used to have the lift and thereby hull clearance more or less independent of speed for the foilborne speed range, but speed variations reduce the remaining control power for seakeeping and manoeuvring and thus affect the performance of the craft.

The following requirements are set to the unsteady computational method for hydrofoil system forces:

- from a hydrodynamic point of view, free surface and foil interaction effects need to be included to obtain an accurate description of forces acting on foil systems. As shown in the previous Chapters, these are of importance for steady flow conditions. For a non-linear computational method for unsteady flow conditions, steady or quasi-steady forces are of importance as well, at least for low frequency manoeuvring.
- the method has to be suited for handling transient and oscillatory motions simultaneously: manoeuvres are usually performed while waves are present.
- the method must be suited to take into account finite aspect ratio hydrofoil configurations with a varying planform due to taper, sweep, and dihedral angles, and the presence of supporting struts and partial span trailing edge flaps. Furthermore, a ride control system, actuating trailing edge flaps, needs to be included.
- the influence of irregular waves (non-constant frequency and amplitude) needs to be considered.
- the method must allow a non-constant forward speed.

This computational method forms the basis of a time domain simulation method for hydrofoil craft. First, in Section 4.2 a review is given of existing unsteady flow hydrofoil theories. In Section 4.3 a basic unsteady computational method is selected. Section 4.4 describes the general outlines of the simulation method. Finally, the unsteady flow method for foils or lifting surfaces is described in Section 4.5.
4.2 Review of unsteady hydrofoil theories

The term linearization is used in this and following Sections in various contexts. The equations of motions may be linearized, but also boundary conditions on the free surface and on a lifting surface may be linearized. In the following, the term linearization is used with respect to the equations of motion, if not stated otherwise.

Unsteadiness in the flow domain around a lifting surface may be introduced by the motions of a lifting surface, it may also already be present in the flow in which the lifting surface advances. A stationary lifting surface operating in a steady but non-uniform inflow also experiences unsteadiness. In all cases, unsteadiness is manifest by a time dependent velocity distribution on the lifting surface which leads to a time dependent circulation around the lifting surface. The circulation about a contour surrounding the lifting surface and its wake must be zero, according to Kelvin's theorem. Any change in the circulation about the lifting surface is then balanced by an equal but opposite change in the vorticity shed into the wake. Consequently, the vortex strength of the wake sheet varies in time, relative to the advancing lifting surface. The wake vorticity then induces time varying velocity components at the lifting surface. This is often termed as the memory effect: the forces on the lifting surface depend not only on the instantaneous motion and inflow but also on the history of the motion. Note that also the position of the wake relative to the lifting surface may vary in time.

Classical theories

For a two dimensional lifting surface in an infinite fluid, performing harmonic oscillations with a small amplitude, the memory effects of the wake can be expressed as frequency dependent force coefficients proportional to the Theodorsen function $C(k)$, see Newman (1977):

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + i H_0^{(2)}(k)}$$  (4.1)

where $H_0^{(2)}(k)$ and $H_1^{(2)}(k)$ are Hankel functions, see Abramowitz and Stegun (1970), with argument:

$$k = \frac{\omega c}{2U}$$  (4.2)

The argument of the Hankel functions $k$ is termed the reduced frequency. This non-dimensional parameter is $\pi$ times the ratio of the chord length of the lifting surface to the wavelength of the wake vortices and is generally used as a measure of unsteadiness.

Assuming harmonic heave and pitch motions $z = z_0 e^{i\omega t}$ and $\alpha = \alpha_0 e^{i\omega t}$ respectively, about the mid chord position, the lift can be calculated from:

$$L = -2\pi \rho U^2 Re\{C(k) [ikz - (1 + ik)\alpha]\} - \pi \rho (\bar{z} - U\bar{\alpha})$$  (4.3)
where $U$ is the advance velocity, $i$ is the imaginary unit and $\rho$ is the fluid density and an overdot denotes differentiation with respect to time.

The last term in this equation is proportional to the acceleration of the lifting surface and is often termed added mass. The Theodorsen function $C(k)$ was already discussed in Section 1.6 and is shown in Figure 1.7, by means of its real and imaginary parts $F$ and $G$ respectively. It can be shown that the Theodorsen function in eq. (4.3) reduces the lift curve slope by a factor $(F^2 + G^2)^{0.5}$ and introduces a phase lag $\tan^{-1}(-G/F)$ with respect to the incidence at the $\frac{1}{4}$ chord point, when compared to a quasi-steady lift proportional to the instantaneous incidence and lift curve slope of $2\pi$.

Generally speaking, $k$ varies between 0.0 and 0.25 for current hydrofoil craft. Unsteady flow effects are clearly present for this reduced frequency range, at least for two-dimensional lifting surfaces. However, quasi-steady force components remain dominant in magnitude.

In non-uniform or unsteady inflow the interaction between the fluid and the lifting surface will be unsteady as well. This type of unsteady lifting surface problem is known as the gust problem. The lift on a two dimensional lifting surface in a sinusoidal gust in the direction normal to the direction of motion is given by, see Newman (1977):

$$L = -2\pi \rho U^2 \text{Re} \left\{ \frac{2i\alpha / \pi k}{H_1^{(2)}(k) + iH_0^{(2)}(k)} \right\}$$

(4.4)

where $\alpha = \alpha_0 e^{i\omega t}$ denotes the incidence variation. The term in brackets in eq. (4.4) is known as the Sears function.

The general case of arbitrary time dependence can be treated by Laplace transform techniques.

Early work for finite aspect ratio lifting surfaces was performed by Reissner (1949). The potential flow problem is formulated with the following boundary conditions imposed: tangential flow at the lifting surface, the Kutta condition of smooth flow at the trailing edge, a continuous pressure in a direction normal to the wake sheet and a zero perturbation velocity at infinity. The resulting equation is an integral equation for the surface vorticity. An approximate solution of the integral equation for 'large aspect ratio' lifting surfaces performing harmonic oscillations is given. Numerical examples are given by Reissner and Stevens (1947) while Lawrence and Gerber (1952) show examples for a lifting surface method applied to 'low' aspect ratio wings. These latter examples show that for the reduced frequency range of interest of hydrofoil craft, unsteady flow effects are significant for low aspect ratio lifting surfaces as well.

**Hydrofoil craft methods**

The classical theories were developed for aeronautical applications. Two-dimensional theories were introduced in hydrofoil computational methods in the 1950's. In that period an upsurge in hydrofoil research started in the USA, and the first analog and later digital computers could be used for
computational methods that were, at least in those days, considered to be complex. A number of computational methods were developed for investigating the performance of hydrofoil craft in waves. Examples of such methods are given by Ogilvie (1958), Kaplan (1955) and Henry and Ali (1963).

In these methods the equations of motion are set up for heave and pitch for a hydrofoil craft advancing with a constant velocity in head and following seas. The basic problem is to determine the instantaneous lift forces acting on the foils. Once this problem is solved, the motions can be determined in a straightforward manner by integration of the equations of motion. The lift force is based on the product of dynamic pressure, wetted surface, angle of attack and lift curve slope. The angle of attack is calculated from the constant forward speed and the wave orbital and craft motion components. The lift curve slope is based on experimental values or estimated from simplified lifting line theory. No free surface effects are accounted for. Corrections on foil lift to account for unsteady flow effects are sometimes applied.

In the 1950's and 60's it was desirable to reduce the required computer time by linearizing the motions of the craft. In a linear simulation method the craft is assumed to perform small-amplitude motions about a certain mean position. The main consequence is that the mean lifting surface area instead of the actual lifting surface area is used to determine the lift forces. Furthermore, it is assumed that the forces and moments vary linearly with a set of disturbance variables and that products of motion components are neglected. It is noted that variations in lifting surface area due to the craft motions and wave elevation may be significant for surface piercing foil parts. Such foil parts have typically a dihedral angle in the order of 30 degrees, relative to the horizontal plane. A certain height variation then results in a variation of the wetted foil length twice as large.

In linear methods the forces are related to motion, velocity and acceleration components by means of a set of so-called hydrodynamic derivatives. The hydrodynamic derivatives are in turn related to the lift curve slope and drag coefficient of the foils and geometrical parameters like dihedral angle, wetted surface, etc., by using semi-empirical relations. The hydrodynamic derivatives are constant and are determined for the mean position of the craft. The linearized equations of motion can be derived from Newton's second law, see Martin (1962), and have a general appearance as follows:

\[(m - Z_q)\ddot{z} = Z_q\dot{z} + Z_q\ddot{z} + Z_q\dot{\theta} + (Z_q + mU)\dot{\theta} + Z_q\theta + Z_e\]

\[(I_{yy} - M_q)\ddot{\theta} = M_q\dot{z} + M_q\ddot{z} + M_q\dot{z} + M_q\dot{\theta} + M_q\theta + M_e\]

(4.5)

Here \(Z\) and \(M\) are the heave force and pitch moment respectively, \(w, \theta\) and \(q\) are the heave velocity, pitch angle and pitch velocity respectively, \(Z_e\) is the hydrodynamic derivative of the heave force with respect to the heave velocity, etc., and \(Z_e\) and \(M_e\) are the wave excitation forces and moment respectively.

The hydrodynamic derivatives are typically calculated from formulations as follows:
\[ Z_w = -\frac{1}{2} \rho S U (C_{la} \cos^2 \beta + C_D) \]  

(4.6)

where \( S \) denotes a suitable reference area, \( C_{la} \) is the lift curve slope, \( \beta \) is the foil dihedral angle and \( C_D \) is the equilibrium drag coefficient. The lift curve slope and drag coefficient depend mainly on the aspect ratio, see for instance eq. (4.7). Martin (1963) gives a detailed description of hydrodynamic derivatives.

In a non-linear method no constant relations between foil system parameters and forces are used. The hydrodynamic forces are determined at each time step, based on actual geometry parameters like aspect ratio, submergence, wetted surface, etc. The non-linear equations of motions are described in Section 4.4.

Ogilvie (1958) compares calculation results obtained from linear and non-linear, quasi-steady, computational methods with experimental results for a basic tandem surface piercing hydrofoil system. Motion amplitudes are shown to be in good agreement with experimental data for head seas and to give fair predictions in following seas. Phase angles between motions and the wave elevation may however deviate by as much as 50 degrees. Using linear or non-linear equations of motion does not appreciably affect the motion amplitudes and phase angles. However, non-linear calculation results include significant mean motion components in following seas, the so-called sit down behaviour, which may cause the craft to crash in the waves.

Ogilvie also investigated the use of unsteady flow corrections. The two-dimensional Theodorsen function is applied to correct the angle of attack induced by the craft motions. The variation in angle of attack due to unsteadiness of the inflow, due to wave orbital motion components, is corrected by means of the Sears function. Note that in time domain methods where these functions are incorporated directly from the frequency domain, only simulations for regular waves can be performed, since then the reduced frequency, which is the argument for the two functions, can be determined. In irregular waves the instantaneous frequency of encounter is unknown and impulse response techniques must be employed.

The effects of the corrections for unsteadiness are found to be limited for head sea conditions, despite the fact that the lift forces acting on the foils can be significantly reduced due to unsteadiness. As the unsteadiness reductions apply to both the wave excitation and the damping forces, the nett effect on the motion amplitude is apparently small. In following seas the motion amplitudes are reduced by about 15% and are in better agreement with the experiments than quasi-steady calculation results. The predictions of the phase angles are also in better agreement with the experimental values for both head and following seas, but appreciable differences remain.

Note that the term damping is used to describe the restoring forces and moments due to the translational and rotational velocity components of the craft. These velocity components introduce incidence variations on the foils which result in lift forces and moments acting against the motion.

Wetzel (1960) describes a similar study as Ogilvie. More experimental data is given for the same hydrofoil method as used by Ogilvie, including a more accurate assessment of mean heave and pitch
motions in head and following seas. Also, responses to initial disturbances are given. Wetzel and Schiebe (1960) give experimental data on foil interaction, for a captive tandem foil system with and without incident waves. The conclusion drawn by Wetzel is that Ogilvie's method gives qualitative agreement with the experimental data on heave and pitch motions in head and following seas. The corrections for unsteadiness in some cases slightly improve the correlation. The most significant effect of non-linearities is to show the existence of mean components of heave and pitch motions. Interaction for steady conditions is found to be very important by Wetzel and Schiebe. The lift to drag ratio of the aft foil could be improved by a factor two, by varying the longitudinal spacing between the two foils. However, some tests in waves at foil spacings for which the steady flow interaction was large, show only small variations in aft foil lift and drag amplitudes due to interaction, i.e. with and without the presence of the forward foil. This suggests that interaction affects only the mean lift and drag of the aft foil.

Kaplan (1951) gives an analytical treatment of free surface effects on fully submerged hydrofoils in unsteady motion in an unsteady or non-uniform flow. The hydrofoil is represented in the time domain by a two-dimensional vortex line with a time dependent circulation. A biplane vortex system is used to describe free surface effects, together with a Green's function term accounting for gravity wave effects. The theory is approximative in the sense that the circulation about the foil is obtained, by satisfying the Kutta and tangential flow conditions, without taking into account the biplane vortex system. Induced velocities due to the biplane image vortex system are based on the foil circulation strength and then used to correct the foil incidence and thereby lift and drag forces. Kaplan's derivation of wave induced forces is based on calculating the steady state forces, including free surface effects, and then using the Sears function to account for basic unsteadiness and finally to include correction terms for unsteady free surface effects. In this analytical method lengthy integrals are present which were difficult to compute in the 1950's, and probably still are. At present, the use of a fully numerical scheme for solving this class of hydrodynamic problems is likely to be more efficient.

Besides validating lift and drag forces for steady conditions, Kaplan only validates wave induced lift forces on steady hydrofoils. Presumably, his theory for unsteady motions was too complex to apply. The agreement between calculated and measured results is reasonably good. His main conclusion is that the wave excitation at high reduced frequencies (\(k=0.50\), in the 1950's expectations on hydrofoil speeds were rather high) is much lower (up to 60%) than predicted by quasi-steady theory.

Kaplan (1955) investigates the heave and pitch motions of a tandem hydrofoil craft in waves. For this purpose Kaplan uses his theory as described above and a quasi-steady method with no unsteadiness corrections to calculate the excitation forces in waves. The quasi-steady method results in too large motion amplitudes, when compared to experimental results at high reduced frequency values (\(k=0.50\)). The use of his own method results in better predictions for the motion amplitudes, although differences of about 30% could still be found. No information on phase angles is given.

In a subsequent analysis of the tandem hydrofoil problem, Kaplan (1955) investigates the effect of unsteady interaction between foils. The downwash at the aft foil due to the lift of the forward foil is obtained from quasi-steady theory and depends on the Froude number, submergence and aspect
ratio of the forward foil. A flat wake sheet behind the forward foil is assumed. The downwash field reaches the aft foil later than the instant of shedding due to the fore-aft foil spacing $\Delta x$. Kaplan introduces a phase angle of $2\pi \Delta x/\lambda$ due to the different spatial locations in a wave field with wavelength $\lambda$. The lift of the aft foil due to the downwash velocity is obtained from the Kussner function, a special case of the Sears function. Some experimental results for two velocities, one submergence and one foil spacing correlate reasonably well with calculated results for the lift amplitude for an aft foil of a tandem foil system in waves. The significance of the interaction for these cases is however not given.

Henry and Ali (1963) investigate the use of gust response operators for the wave excitation forces on hydrofoils. The gust response operators are obtained by transforming two existing theories, by Lawrence and Gerber (1952) and Reissner and Stevens (1947), for oscillating, finite aspect ratio lifting surfaces, to a theory for steady lifting surfaces in oscillating flow. For aspect ratio two and four lifting surfaces the response operators are found to be in fair agreement with experimental data, but the phase angles are not. Phase angle errors increase with increasing frequency and with increasing aspect ratio. A comparison between the infinite aspect ratio Theodorsen function, transformed in a similar way to account for oscillating flow, and the exact Sears function shows comparable response amplitudes but significant phase differences. It is concluded that for a good phase prediction a three dimensional unsteady theory is needed, accounting for unsteadiness in the lifting surface motions as well as in the inflow.

Keuning (1979) describes a more recent, non-linear computational method for the surge, heave and pitch motions of hydrofoils in head and following seas. The method used by Keuning is basically identical to the one developed by Ogilvie (1958). Extensions are the inclusion of drag forces, added mass and the more detailed estimation of the steady state lift curve slope of the foils. This latter item is based on a fit of lifting surface results for rectangular wings:

$$C_L = \frac{2\pi \alpha AR}{AR + 3}$$

$$C_{D_h} = \frac{C_L^2}{\pi AR}$$

(4.7)

where $AR$ denotes the aspect ratio and $\alpha$ is the incidence relative to the zero-lift angle.

Free surface effects are accounted for by using empirical corrections on the lift coefficient and induced drag. A downwash formulation derived from empirical relations is used to determine foil interaction. These empirical relations include contributions due to the biplane image and gravity wave effects. Time domain simulations are carried out for regular wave conditions.

The calculation results are verified by means of model test results for a foil system consisting of a surface piercing forward foil and a fully submerged aft foil, in head and following seas. The calculation results are found to be in a good qualitative agreement with the experimental data. Quantitatively, motion amplitudes are correctly predicted as well, with a maximum error of about 10%, while phase angles in head seas may differ by about 35 degrees. Especially for following seas, the predictions are in better agreement with experimental data than the results obtained by Ogilvie (1958). Non-zero mean values for sinkage and trim appeared especially in following seas but were
difficult to measure accurately. By systematically varying the downwash relation in the computational method, the motion response appears to be quite sensitive to foil interaction.

Bose and McGregor (1983) describe both linear and non-linear time domain methods for the simulation of hydrofoil motions in waves, in two degrees of freedom: heave and pitch. Experimental data for a surface piercing hydrofoil model are compared with calculation results. In head seas, the heave response is clearly better predicted by the non-linear method than the linear method. The same holds for the pitch response, but non-linear calculation results are nevertheless in error by as much as 300%. In following seas the linear method gives the best predictions. Possible reasons for the discrepancies mentioned are: strong non-linear effects like foil ventilation and hull and/or foil slamming, unsteady flow effects (not included in the simulation methods) and errors in the manufacturing of foil sections on model scale.

Van Walree et al. (1991) describe a non-linear time domain simulation method for hydrofoil craft in six degrees of freedom. The method is based on a lifting line description of the forces on the foils. Effects of planform variations, dihedral and struts are derived from empirical relations based on experimental data. Free surface effects are based on a quasi-steady approach, accounting for biplane image and gravity effects, see Keldysh and Lavrentiev (1946), while foil interaction is based on a quasi-steady lifting line approach with free surface effects, assuming elliptical foil loading, see Bai Qi (1981). Motion damping and wave excitation are based on determining the instantaneous velocity components at a number of spanwise foil strips. Unsteady flow effects are approximated by using the two-dimensional Theodorsen and Sears functions at the mean frequency of motion, for regular and irregular wave conditions. A ride control system actuating trailing edge flaps is included. Besides calculating the performance in waves, the powering performance and manoeuvring in waves can be predicted. The computational method includes further a quasi-steady description of hull forces and thrust forces due to waterjets and propellers, for simulation of take-off manoeuvres. Wind excitation is included as well in the simulation method. A limited number of comparisons between experimental and calculated results show a good qualitative agreement. The quantitative agreement is reasonable (error less than 15%) only for cases for which the empirical relations for the lift curve slope as function of planform variations are valid.

Saito et al. (1991) use a non-linear, six degree of freedom simulation method for the prediction of seakeeping and manoeuvring for the well known Jetfoil hydrofoil craft. A ride control system is included in the simulation method. The foil system forces as a function of submergence are based on extensive experimental results for the actual foil system of the Jetfoil in an open cavitation tunnel. Effects due to cavitation and ventilation are included. These forces are used in a quasi-steady way in the simulation method. Unsteadiness is included by using the Theodorsen and Sears functions, similar to Keuning (1979). In simulations with irregular wave conditions, convolution integrals are used to describe the unsteadiness as a higher order time lag system. Simulation results are compared with full scale data. Simulation predictions for seakeeping appear to be in good agreement with the measurement data. The craft response is fairly linear with the wave height, up to the point where cavitation becomes significant and foil lift breakdown occurs. Due to the resulting high vertical acceleration levels, this wave height is the operational limit at high speed. Non-linearities in the craft response at higher sea states are also introduced due to manual control of the foil submergence.
Ohtsubo and Kubota (1993) present a computational method for the vertical motions of high speed foil-assisted craft. The forces on the foils are obtained from basic two-dimensional section theory and the use of the Sears function to account for unsteadiness in the inflow. Foil lift variations due to incidence variations caused by ship motions are treated in a quasi-steady manner. An empirical correction factor is applied to the total lift to account for free surface effects. Experimental results for large wave amplitudes show a strong non-linearity of the motion response with respect to the wave height caused by ventilation and slamming of the hydrofoils. Calculated and experimentally obtained response functions for head sea conditions agree qualitatively for the entire frequency range. For short wavelengths the quantitative agreement is not satisfactory, probably due to strong unsteady flow effects in the lift components related to the foil motions.

Hamamoto et al. (1993) describe the linearized equations of motion for investigating the directional stability of surface piercing hydrofoil craft. The equations of motion have a similar appearance as shown in eq. (4.5) and use hydrodynamic derivatives as shown in eq. (4.6), but now six degrees of freedom are considered. Unsteady flow effects are not taken into account. The authors acknowledge a principal difficulty in their approach: the hydrodynamic derivatives depend on the equilibrium lift coefficients of the foil system which must be obtained from either model testing or theoretical analysis.

Recent hydrofoil methods

In the 1970-80’s a number of developments based on the classical lifting line theory were published. Unsteady, frequency domain lifting line methods were developed by using the method of matched asymptotic expansions. Ahmadi and Widnall (1985) and Van Holten (1976) study the low frequency motion regime for which the wake wave lengths are comparable to the span. Sclavounos (1987) presents an unsteady lifting line theory valid for the entire frequency range $0 < k < \infty$. In the latter method two approximate solutions valid far from and close to the lifting line are matched. The far field velocity potential is expressed as a distribution of normal dipoles on the wake and an expression for the oscillatory downwash is derived at the foil. The near field flow is two-dimensional and based on a strip theory with a correction for aspect ratio. The matching of the far and near field solutions leads to an integral equation for the distribution of the circulation along the wing span. Sclavounos only considers oscillatory motions of the lifting surface in the frequency domain.

Falch (1991) uses the computational method of Sclavounos (1987) in a linear frequency domain program for foil assisted craft. Free surface effects are included by using a biplane image vortex system.

The growth in development and use of Computational Fluid Dynamics methods during the last decades has also extended to unsteady flow problems for lifting surfaces. A typical example of such a method is given by Djordjihardjo and Widnall (1969). They describe a numerical method for finite aspect ratio lifting surfaces performing arbitrary motions in an unbounded potential flow. The tangential flow boundary condition is formulated on the actual lifting surface and the time dependent wake shape is predicted. The problem is governed by an integral equation which relates a doublet
distribution to the normal velocity on the lifting surface. Herein the Kutta condition of smooth flow at the trailing edge and the requirement of a zero pressure difference on the wake are imposed. The solution of the problem is obtained in the time domain by solving a linear set of equations, the discretized integral equation, after the wake shape has been determined by integration of induced velocities at the edges of wake singularity elements. Application of the computational method is limited to basic two-dimensional cases, like the initial lift and growth of circulation for an impulsively started two-dimensional flat plate and a two-dimensional flat plate in a sharp edged gust. Calculation results show a good agreement with analytical results using the Wagner and Kussner functions. Two-dimensional wake shapes are predicted which show a good qualitative agreement with experimental data. For application of the method to three-dimensional cases CPU requirements were probably prohibitive.

Leclerc and Salaun (1987) developed a lifting surface theory for the computation of hydrodynamic pressures on thin, three-dimensional bodies in the presence of the free surface. Free surface effects are included with the motivation that three-dimensional gravity wave effects are important at low Froude numbers and at low reduced frequencies. The method is derived in the frequency domain with harmonic time dependence. The boundary conditions at the free surface are linearized. The thin lifting surface and the stationary, flat wake sheet are represented by sheets of doublets. The problem is expressed in integral equations relating the doublet strength to the boundary condition of tangential flow at a number of collocation points. Much attention is given to the numerical method for determining the free surface terms in the integral equations.

Vortex lattice methods

Unsteady vortex lattice methods are a generalization of the vortex lattice methods for steady lifting flows. In the 1970's and 1980's unsteady vortex lattice methods became popular in the field of aerodynamics due to their relative simplicity and applicability to arbitrary lifting surface configurations, ranging from delta wings to helicopter rotor blades. The unsteady vortex lattice method is a simplification of the unsteady lifting surface theory of Djojodihardjo and Widnall (1969) in the sense that the section is represented by its camber line only. Vortex ring elements are generally used as singularity elements. The method is well suited for solving transient problems in the time domain. The circulation about the lifting surface is determined by satisfying the tangential flow condition at a number of locations on the base plane of the lifting surface. Each time step, the circulation at the trailing edge of the lifting surface is transferred into the wake sheet, in order to satisfy the spatial conservation of circulation. The wake sheet position may be determined by time integration of the induced velocity components due to all vortex ring elements, at all element corner points. Once the position and circulation of the wake sheet elements is known, its influence on the induced velocity on the lifting surface can be determined and used in the tangential flow condition, the next time step. This process is repeated each time step.

In the method of Kerwin and Lee (1978), the propeller blades are represented by discrete vortex lines and line sources in the camber plane. The vortex distribution is a function of both time and space. The source strength however is independent of time and determined from applying two-dimensional wing theory for thin sections in a strip wise manner for a number of radial stations. The trailing vortex wake sheet is allowed to contract and to roll-up. The wake vorticity is determined from the requirement of spatial conservation of circulation. The unsteady time domain solutions are obtained by rotating the blades through a sequence of discrete angular intervals until steady state oscillations of blade loadings are achieved and transient effects have disappeared. Transient effects are due to the evolution of the wake sheet towards its final rolled-up form at some distance behind the propeller plane.

Foil interaction

As described in Section 2.1, Morch (1992) studied the problem of hydrofoil interaction for steady conditions. In order to study the effect of the roll-up of the forward foil wake sheet on foil interaction, Morch describes the development of an initial value problem in which the actual wake sheet position is calculated. Longitudinal wave system components are not accounted for, free surface effects are approximated by a two-dimensional approach at each transverse plane behind the forward foil. The free vortex sheet is modelled with a distribution of dipole elements and, alternatively, by point vortices. The dipole approach did not result in a stable and convergent roll-up of the wake sheet. The point vortex approach was successful in predicting realistic vortex sheet shapes. In this latter approach it was required to use a so-called viscous vortex core to avoid singularities in the induced velocity components. The inflow variations on the aft foil appear sensitive to the viscous vortex core diameter used.

Morch finds that the free surface has a marked effect on the shape of the wake sheet for low aspect ratio's \((AR=3)\) and low submergences \((h/c<1)\) and large distances behind the forward foil. The spacing between the inner sides of the rolled-up wake sheet reduces significantly and its centre region displaces downwards. In Section 3.3 it was shown that for a low aspect ratio forward foil in steady flow the interaction can be qualitatively well described by neglecting wake sheet roll-up. However, it was also shown that quantitative differences between experimental and calculated interaction effects on foil forces remain.
4.3 Choice of computational method

From the review of the existing simulation methods and unsteady hydrofoil theories given in the previous Section, the following may be observed.

Existing computational methods

No unsteady hydrofoil theory exists that can be used in a non-linear time domain simulation method for arbitrary craft motions and inflow variations, that includes free surface effects in a non-empirical way, that offers possibilities to investigate foil interaction and that can be applied to arbitrary three-dimensional foil configurations.

Almost all simulation methods were developed for seakeeping analysis in two degrees of freedom (heave and pitch), only two simulation methods, Saito et al. (1991) and Van Walree et al. (1991), were developed in six degrees of freedom and can also be used for manoeuvring analysis. All methods lack, with a varying degree of seriousness, the applicability to arbitrary foil configurations. Experimental, quasi-steady force coefficients are used with two-dimensional and/or empirical corrections for unsteadiness, free surface effects and foil interaction (Saito et al.). Alternatively, a mixture of empirical formulations and lifting line theory is used to account for basic planform variations (Van Walree et al.).

Linear and non-linear methods

Simulation methods based on linearized equations of motion and the use of hydrodynamic derivatives result in reasonably good predictions of motion response amplitudes in waves, but phase angles are in general less well predicted. Non-linear methods for seakeeping simulations, taking the actual foil submergence and wetted surface into account, result in motion amplitudes which are slightly more accurate than linear methods. Phase angle predictions from non-linear methods are in better agreement with experimental data than phase angle predictions from linear methods. Furthermore, non-linear methods are required to properly take into account phenomena like mean sinkage and pitch. It is noted that these observations are not valid for the results presented by Bose and McGregor (1983).

Although linearization assumes small-amplitude motions, for simulations of hydrofoil craft manoeuvres linearization can possibly be applied as well since the forces on hydrofoils depend on velocity rather than on motion components. As the disturbance velocity components during manoeuvring are small relative to the forward speed, linearization is then probably allowed as well, although no evidence for this assumption has been found. For transient manoeuvres, for instance a crash stop, motion and velocity disturbances are large and linearization is not allowed.
Unsteady flow effects

Several authors point out that it is necessary to include unsteady flow phenomena in the computational methods. The use of quasi-steady theories leads to too large wave excitation forces, especially at high reduced frequency values. Unsteadiness reduces both excitation and damping so that the total effect on the motion amplitude may be limited, however. The classical theories for unsteady flow are essentially developed for two-dimensional foil configurations for frequency domain applications. Results from such methods can be transferred to the time domain and used in a strip theory approach to include three-dimensional effects. However, no free surface effects would be included and the suitability of applying such an approach to arbitrary foil configurations is uncertain.

Free surface effects

Several authors state that the inclusion of free surface effects does not affect the motion response to waves significantly. This again is probably due to the fact that free surface effects reduce the lift curve slope of the foils and thereby reduce excitation and damping to a similar extent. However, this is only true for small-amplitude motions about an equilibrium position. For a more general simulation program that can handle large and transient motions during manoeuvring with active motion control systems, excitation and damping need not be affected in a similar way and it is necessary to include free surface effects. Moreover, free surface effects are important for the accurate prediction of drag forces.

Foil interaction

Regarding fore-aft foil interaction, Wetzel (1960) finds that foil interaction is not important, that is for wave excitation forces on a captive foil system. Keuning (1979) finds that motion responses in waves are sensitive to the magnitude of foil interaction. Anyway, foil interaction is of importance for conditions in which quasi-steady foil forces are significant, for instance for manoeuvring, so that this must be addressed in the computational method to be developed.

Choice of computational method

The lifting surface theory of Reissner (1949) fulfils the requirement of using a three-dimensional unsteady flow theory. The integral equations are most efficiently solved by a numerical method, so that the strip theory approach of Reissner for including finite aspect ratio effects is no longer required. This is shown in the lifting surface method of Djojodihardjo and Widnall (1969) and the unsteady vortex lattice methods, see for instance Kostadinopoulos et al. (1985). These methods are used in the time domain which is a principal demand for our purposes in order to include non-linear effects due to large and transient motions. Furthermore, one of the great advantages of numerical lifting surface methods is their applicability to arbitrary hydrofoil configurations. The present interest lies in obtaining lift and drag forces acting on foil systems only, no detailed pressure distribution
on the foil surface is required. Therefore, an unsteady vortex lattice method is well suited for the present purposes.

In vortex lattice methods, arbitrary flow disturbances due to foil motions and wave orbital velocities can be incorporated quite easily. With respect to ride control systems, in Chapters 2 and 3 it was shown that trailing edge flap effects can be included as well. The wake sheet may be in a stationary position behind the lifting surface. Alternatively, its actual position can be determined by integrating induced velocities, at corner points of the vortex elements of the wake sheet, in time. This is of importance for fore-aft foil interaction which is described best by using the actual forward foil wake sheet shape and the actual vorticity distribution on that wake sheet, at the position of the aft foil.

It is however known, see for instance Hoeijmakers (1989), that using a time domain vortex lattice method is not a good approach to determine rolled-up wake sheet shapes accurately. Hoeijmakers describes a more advanced panel method to compute the wake sheet roll-up. This method consists of a time domain approach for a quasi two-dimensional panel method with a given circulation distribution on the wing. The wake sheet is discretized by means of panels carrying a doublet distribution and single vortex lines at the sheet edges. The development of a similar method for arbitrary lifting surfaces, including a full account of free surface effects would however be a study in itself.

Furthermore, the results shown in Section 3.3 do not indicate, at least for steady flow conditions, that it is necessary to determine the actual rolled-up wake sheet shape to approximately account for foil interaction, even for low aspect ratio forward foils. However, within a vortex lattice method the effects of a schematic rolled-up wake sheet on foil interaction can be determined without much modelling effort although the required computer time is probably quite substantial. Therefore, some attention will be given on the effect of wake sheet roll-up on fore-aft foil interaction in order to investigate its significance, but no attention will be paid to accurately determining rolled-up wake sheet shapes.

In boundary integral or panel methods, free surface effects are introduced in the solution via the free surface boundary conditions. These boundary conditions may be applied at the undisturbed free surface position in a linearized form. Alternatively, the actual boundary conditions may be applied at the disturbed free surface position in a non-linear approach. Note that the linearization of free surface effects can be performed independently of a possible linearization of the equations of motion as described earlier.

Water surface disturbances by hydrofoils are relatively small and linearization is permissible down to submergence to chord ratio's of $h/c=0.25$, at least for steady flow conditions at relatively high Froude numbers, see Section 3.3. It is expected that unsteady wave making can also be adequately described by means of linearized free surface conditions at the undisturbed free surface, at least for non-extreme flow conditions and at sufficiently high chord Froude numbers ($F_{nc}>3$). In extreme conditions, the combined effects of the hydrofoil motions and wave elevations can be such that the foil submergence may become quite small, in the order of $h/c=0.25$.

Water surface disturbances created by incident waves may be much larger than those created by
hydrofoils themselves. To determine the potential function of incident waves, linearization is often applied. Linear wave theory uses the same linearized free surface conditions at the undisturbed free surface as panel methods with linear free surface effects. The effects of incident waves on foil forces are accounted for by means of the orbital velocity components, which, in linear wave theory, are not well defined for positions close to the watersurface. A further disadvantage of linearization of the free surface conditions is that wetted area variations due to the wave elevations must be neglected. Wetted area variations are then limited to the displacements of the craft, relative to the undisturbed water surface. For surface piercing foil parts an important part of the wave excitation and damping forces is then neglected.

Experimental data on a fully submerged foil, see Van Walree and Yamaguchi (1993), shows that ventilation in head waves occurs when the wave amplitude equals or exceeds the foil submergence. When ventilation occurs, it has a dramatic effect on the forces acting on a hydrofoil. Up to the instant of ventilation, satisfactory results for lift variations are obtained when using linear wave theory for determining incidence variations due to the wave orbital velocity components. Much more than using linearized free surface conditions, the occurrence of ventilation is the limiting factor for using potential flow computational methods.

With respect to the description of free surface effects on lift and drag, in a linearized approach a Green’s function representation of free surface effects can be used, see Section 2.5. Alternatively, a Rankine source panel distribution on the actual or undisturbed free surface can be applied. Both approaches have their merits as described in the following.

The contributions of Rankine source panels to the flow are more simple to compute than the time domain Green’s function. The latter is however easier to compute than its steady flow equivalent, although in the time domain the computation of a convolution integral is required. The Rankine source approach results in a large system of linear equations to be solved each time step, as a relatively large number of panels on the free surface is needed. The matrix of equations in the Green’s function approach is an order of magnitude smaller, as the number of vortex elements used in vortex lattice methods is relatively small. Other advantages of the Green’s function formulation are the implicit compliance with the radiation condition, the non-truncated free surface domain and the absence of free surface discretizations.

In a fully non-linear approach a time stepping procedure must be employed to determine the exact free surface location where the kinematic and dynamic free surface boundary conditions are implied. A non-linear description of the wave orbital velocities for positions just below the actual wave elevation is then obtained as well. One must use Rankine source panels on the free surface in a non-linear method. Applications of such non-linear methods for conventional hull forms at normal speeds have shown stability problems.

It is expected that a non-linear approach for free surface effects will require a separate study which may well result in an unpractical computational method. Furthermore, the probable adequacy of linear wave theory to describe wave excitation up to the instant of ventilation does not require a non-linear description of incident waves, at least for fully submerged foils. For surface piercing foils, a correction on the foil forces due to wetted surface variations in waves may be required. When linear wave theory is used, it is consistent to use linearized free surface conditions as well.
4.3 Choice of computational method

A Green's function description of free surface effects then offers advantages in modelling and computer time requirements.

Saito et al. (1991) show that the inclusion of cavitation effects is necessary for determining the limits of operability in waves. In this respect, impact forces due to hull-wave contacts and foil broaching may be of importance as well. Furthermore, ventilation may also be a limiting factor. To incorporate all these effects in a computational method is a challenging task which is beyond the limits of the present study. However, in the simulation method of Van Walsem et al. (1991) the occurrence of cavitation and ventilation is detected using empirical criteria while hull-wave contact can be easily detected from the instantaneous position of the craft and local wave elevations. Some indication of the limits of operability can thus be obtained while in future extensions a more thorough description of cavitation, ventilation and hull slamming may be developed.

Conclusions

At present a simulation method does not exist that meets the demands specified in Section 4.1. Linear and non-linear (with respect to motions) simulation methods can both be applied to determine the motions of hydrofoil craft in waves. Non-linear methods can be expected to give better predictions for surface piercing foil systems. For simulating transient effects, a non-linear simulation method is required. The ability to perform simulations for transient motions is one of the requirements set to the computational method, therefore a non-linear simulation method must be developed.

The computational method for foil system forces must take into account unsteady flow effects, free surface effects and fore-aft foil interaction. It must also be suited for use in the time domain and should be able to include large motions, control system actions and incident wave effects. An unsteady vortex lattice method is well suited for these purposes and is therefore selected as the basic computational method within the simulation program.

No attention will be paid to accurately determine rolled-up wake sheet shapes, however effects on fore-aft foil interaction due to a schematically rolled-up wake sheet will be investigated.

Experimental data suggest that the use of linearized free surface conditions at the undisturbed free surface to determine free surface effects on foil forces, and the use of linear wave theory to determine wave excitation forces is allowed for fully submerged foil systems, up to wave elevations where ventilation occurs. The use of non-linear free surface conditions and non-linear wave theory is probably not practical in a time domain simulation program. Therefore, these linearizations will be applied to the present problem. Free surface effects will be described by means of a time domain Green's function which is expected to offer computational advantages relative to the use of Rankine source panels on the free surface. For surface piercing foil systems probably a correction is required which accounts for the wetted surface variations due to incident waves.
4.4 Simulation method Hydsim

In this section the fundamentals of the simulation method are described. This description is limited to the equations of motion and the various force components acting on the hydrofoil craft. The description of the computational method for the hydrofoil system itself, the unsteady vortex lattice method, is given in Section 4.5. The simulation method described here forms the time domain environment in which the hydrofoil method is used. The simulation method is implemented in the computer program designated Hydsim.

Equations of motion

In order to determine the motion of a hydrofoil craft it is required to solve a set of equations which relate the physical properties of the craft to the forces acting on it. Two Cartesian axis systems are used, see Figure 4.1. A ship-fixed axis system (x,y,z) with its origin in the ship’s centre of gravity and a space-fixed axis system (x₀,y₀,z₀) with its origin in the undisturbed water surface and the z₀ axis pointing upwards. The ship-fixed axis system is fixed to the centre of gravity and rotates with the ship. The x-axis points to the bow, the y-axis points to port and the z-axis points upwards.

![Figure 4.1 Axis systems](image)

The hydrofoil craft is considered to be a rigid body. By using Newton’s second law, and a suitable angular orientation vector (ϕ,θ,ψ), expressing the craft’s orientation relative to the space-fixed axis system, it can be shown that the basic so-called Euler equations of motion are as follows, in a ship-fixed axis system, see Synge and Griffith (1959):
\[ m(\dot{u} + qw - rv) = X + mg \sin \theta \]
\[ m(\dot{v} + ru - pw) = Y - mg \cos \theta \sin \phi \]
\[ m(\dot{w} + pv - qu) = Z - mg \cos \theta \cos \phi \]
\[
\dot{\theta} \left( I_x - I_y \right) + qr(l_z - I_y) = K
\]
\[
\dot{\phi} \left( I_y - I_z \right) + rp(l_x - I_z) = N
\]
\[
\dot{\psi} \left( I_z - I_x \right) + pq(l_y - I_x) = M
\]

(4.8)

where \( X, Y \) and \( Z \) denote the force components and \( K, M \) and \( N \) denote the moment components acting on the craft. The translational velocity vector is given by \((u, v, w)\) and the angular velocity vector is \((p, q, r)\). The \( \text{mg} \) terms represent the gravity forces acting on the craft. The mass moments of inertia \( I_x, I_y \) and \( I_z \) are constant in the ship-fixed axis system. An overdot denotes differentiation with respect to time.

The time derivative of the angular orientation vector \((\phi, \theta, \psi)\) may be expressed in the angular velocity vector \((p, q, r)\) defined in the ship-fixed axis system:

\[
\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta
\]
\[
\dot{\theta} = q \cos \phi - r \sin \phi
\]
\[
\dot{\psi} = (q \sin \phi + r \cos \phi) \sec \theta
\]

(4.9)

The position and velocity vectors of the craft's centre of gravity, \( x_0 \) and \( \dot{x}_0 \) respectively, in the space-fixed axis system are obtained by using a transformation matrix \( T_0 \) based on the angular orientation of the craft:

\[
\dot{x}_0 = T_0 U
\]
\[
x_0 = \int \dot{x}_0 \, dt
\]

(4.10)

where \( U \) denotes the ship-fixed velocity vector \((u, v, w)\) and \( t \) is time.

The transformation matrix \( T_0 \) is given by:

\[
\begin{bmatrix}
\cos \psi \cos \theta & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \psi \sin \theta \cos \phi - \sin \psi \sin \phi \\
\sin \psi \cos \theta & \cos \psi \cos \theta + \sin \psi \sin \theta \sin \phi & \sin \phi \sin \theta \cos \phi - \cos \psi \sin \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{bmatrix}
\]

(4.11)

The sequence of rotation is first \( \psi \), then \( \theta \) and finally \( \phi \).
The rotation rate of propulsors $\omega_p$ is described by a separate equation of motion for each propulsor:

$$ I_p \dot{\omega}_p = Q_e - Q_p $$

(4.12)

where $I_p$ is the propulsor mass moment of inertia, $Q_e$ is the engine torque and $Q_p$ is the absorbed torque by the propulsor.

These equations of motion form the basis of the simulation program. The differential equations can be solved by means of standard numerical integration techniques. The challenge is to determine the various force components contributing to the right hand side of eq. (4.8), in particular the force components acting on the foil system. The force and moment vector $X$ consists of the following components:

$$ X = X_p + X_{ap} + X_a + X_f $$

(4.13)

where the indices $p$, $ap$, $a$ and $f$ denote the propulsors, appendages, aerodynamic and foil system, respectively. It is assumed that the craft is foilborne so that no hull contact with the watersurface exists.

Although mainly of empirical nature, a description of the force components due to propulsors and wind is given in the next sections for completeness.

**Propulsor forces**

Two types of propulsors are considered in the simulation method: propellers and waterjet systems. The forces originating from the propulsors are obtained as described in Section 2.7, and are used here in a quasi-steady manner. It is not the intention to accurately predict the dynamic behaviour of the propulsors. Propulsor dynamics are included to get insight in speed variations during manoeuvres.

The open water characteristics include cavitation effects on the propeller performance. At each time step the advance coefficient and the cavitation number are determined from the instantaneous speed of advance $U_a$, rate of rotation $n=\omega/2\pi$ and submergence $z_{op}$ as follows:

$$ J = \frac{U_a}{nD} $$

$$ \sigma = \frac{p_0 + \rho g z_{op}}{\frac{1}{2}\rho U_a^2} $$

(4.14)

where $D$ is the propeller diameter, $p_0$ is the atmospheric pressure minus the vapour pressure, $\rho$ is the water density and $g$ is the acceleration due to gravity. The speed of advance $U_a$ is set equal to the instantaneous speed of the craft, $u$. 
4.4 Simulation method Hydsim

By means of the known pitch-diameter ratio \((P/D)\) and blade area ratio \((A_b/A_o)\) of the propeller, the thrust and torque coefficients, \(K_T\) and \(K_Q\), are interpolated from the open water data on basis of \(J\) and \(\sigma\). The thrust \(T\), torque \(Q_p\) and delivered power \(P_D\) follow from:

\[
T = \rho n^2 D^4 K_T
\]

\[
Q_p = \rho n^2 D^5 K_Q
\]

\[
P_D = 2 \pi n Q_p
\]

(4.15)

The torque \(Q_p\) is used in the equations of motion, eq. (4.12), the absorbed power \(P_D\) is merely an output number and the thrust \(T\) is decomposed into force and moment components as follows, for each propeller present:

\[
X_p = T (1 - t) \cos \theta_p
\]

\[
Z_p = -T (1 - t) \sin \theta_p
\]

\[
M_p = X_p z_p - Z_p x_p
\]

(4.16)

where \(\theta_p\) is the propeller axis inclination in the vertical plane and \(x_p\) and \(z_p\) denote the location of the propeller in the vertical plane. The thrust deduction fraction \(t\) accounts for propeller-foil interaction, see Van Walree (1988).

For waterjet systems the nett thrust force is given by:

\[
T = \rho Q_j (V_j - V_i) - \frac{1}{2} \rho V_i^2 A_i C_{D_h}
\]

(4.17)

where \(Q_j\) is the flow rate ingested by the waterjet, \(V_j\) is the jet exit velocity, \(V_i\) is the intake velocity of the ingested stream tube and the second term on the right accounts for the intake drag. The determination of the velocity components and flow rate is identical as described in Section 2.7. Force and moment components due to the waterjet thrust are finally obtained in a similar way as shown in eq. (4.16).

**Appendage forces**

Appendage forces are obtained from using the empirical expressions described in Section 2.7 and the instantaneous velocity in a quasi-steady manner. These forces are merely of interest for obtaining the total drag forces acting on the craft.

**Aerodynamic forces**

Aerodynamic forces on the non-submerged parts of the craft contribute significantly to the total resistance of the craft. Furthermore, the velocity vector of the craft may be combined with that of the wind to determine an effective wind velocity at a certain relative direction. Wind gusts resulting
in low frequency excitation forces and moments may then be taken into account. Experience shows that these are sufficiently large to include in the simulation program. Again a quasi-steady approach is taken here, at each instant the instantaneous ship and wind velocities are used.

The aerodynamic force vector \( \mathbf{X}_a \) is obtained from a force coefficient vector \( \mathbf{C}_a(\Psi_a) \) as follows:

\[
\mathbf{X}_a = \frac{1}{2} \rho_a V_{ar}^2 \mathbf{C}_a(\Psi_a) \tag{4.18}
\]

where \( \rho_a \) is the density of air, \( V_{ar} \) is the relative wind velocity and \( \Psi_a \) is the relative wind angle in the horizontal plane. Moment components are obtained similarly by using aerodynamic moment coefficients. The force and moment coefficients are based on wind tunnel tests on typical hydrofoil craft, see for instance Von Wagner (1967).

The relative wind velocity and direction are obtained from combining the ship speed and heading with the wind velocity and direction:

\[
\begin{align*}
V_{ax} &= V_{ae}(t) \cos \Psi_a(t) - x_0 \\
V_{ay} &= V_{ae}(t) \sin \Psi_a(t) - y_0 \\
V_{ar} &= \sqrt{V_{ax}^2 + V_{ay}^2} \\
\Psi_{ar} &= \arctan \left( \frac{V_{ay}}{V_{ax}} \right) - \Psi
\end{align*}
\tag{4.19}
\]

where \( \Psi \) is the ship's heading, \( x_0 \) and \( y_0 \) are the space-fixed velocity components of the ship, \( V_{ae}(t) \) is the effective time varying wind velocity at a reference height \( h_r \) and \( \Psi_a(t) \) is the time varying wind direction.

The environmental wind velocity \( V_a(t,h_r) \) is obtained from a mean wind velocity and a time varying component as follows:

\[
\begin{align*}
V_a(t,h_r) &= V_{am}(h_r) + V_{av}(t,h_r) \\
V_{av}(t,h_r) &= \int_0^\infty \sqrt{2} S_a(\omega_a,h_r) d\omega_a \cos(\omega_a t + \varepsilon_\omega)
\end{align*}
\tag{4.20}
\]

where \( S_a(\omega_a,h_r) \) is the spectral density function at frequency \( \omega_a \) and reference height \( h_r \), \( \varepsilon_\omega \) is a random phase angle and \( V_{am}(h_r) \) is the mean wind velocity at reference height \( h_r \). Suitable formulations for wind spectra may be obtained from literature, see for instance Ochi and Shin (1988). The effective wind velocity finally follows from the integration of the height dependent velocity over the height of the ship by means of:
\[ V_{ae}(t) = \int_{z_i}^{z_f} V_a(t, h_r) \left( \frac{z}{h_r} \right)^p dz \]  

(4.21)

where an exponential velocity variation over the height coordinate \( z \) is assumed, with exponent \( p \).

A time varying wind direction \( \psi_a(t) \) is obtained from the following formulation:

\[ \psi_a(t) = \psi_{am} + \text{sign}(r - \frac{1}{2}) \sigma_\psi \sqrt{-\ln(r)} \]  

(4.22)

where \( \psi_{am} \) is the mean wind direction, \( r \) is a random number on the interval \((0,1)\) and \( \sigma_\psi \) is the standard deviation of the wind direction variation. The logarithmic term in combination with the random numbers results in an exponentially distributed wind direction variation, with a zero mean value and a user specified standard deviation \( \sigma_\psi \). The random numbers \( r \) are updated at time intervals much larger than the simulation time step to obtain low frequency variations of the wind direction. This procedure is obtained from simulation programs for dynamic positioning of offshore structures.

Irregular wave kinematics

In the description of the unsteady vortex lattice method in Section 4.5, only a description of the potential function for a regular wave is given. Here a more detailed description of irregular wave kinematics is given. This description is based on a linear superposition of regular wave components satisfying a certain spectral density function \( S_\omega(\omega) \) which relates the energy content (wave height) of a seaway to the wave frequency \( \omega \). The waves are assumed to be short crested and uni-directional.

A time series of the wave elevation \( \zeta_w(t) \) is generated by using a number \( n \) of irregularly distributed frequency intervals \( \Delta \omega \), as follows:

\[ \zeta_w(t) = \sum_{i=1}^{n} \sqrt{2S_\omega(\omega_i) \Delta \omega_i} \cos(k_{wi} x_r - \omega_i t + \phi_i) \]  

(4.23)

where \( \phi_i \) is a random phase angle, \( k_{wi} \) is the wave number \( \omega_i^2/g \) and \( x_r \) is a coordinate in an axis system fixed to the wave direction, see eq. (4.25). Note that eq. (4.23) is a discretized form of the analogous spectral formulation eq. (4.20).

The wave orbital velocity components \( (u_w, v_w, w_w) \), defined in the space-fixed axis system at a location \((x_r, z_0)\), are determined from:
\[ u_w = \sum_{i=1}^{n} \xi_{ai} \omega_i e^{k_w \xi} \cos \psi_w \cos (k_w x_r - \omega_i t + \epsilon_i) \]

\[ v_w = \sum_{i=1}^{n} \xi_{ai} \omega_i e^{k_w \xi} \sin \psi_w \cos (k_w x_r - \omega_i t + \epsilon_i) \]

\[ w_w = \sum_{i=1}^{n} \xi_{ai} \omega_i e^{k_w \xi} \sin (k_w x_r - \omega_i t + \epsilon_i) \]

(4.24)

where

\[ \xi_{swai} = \sqrt{2S(\omega_i) \Delta \omega_i} \]

\[ x_r = x_0 \cos \psi_w + y_0 \sin \psi_w \]

(4.25)

\[ \psi_r = \psi_w - \psi \]

here \( \xi_{swai} \) is the amplitude of wave component \( i \), \( \psi_w \) is the wave direction, \( \psi \) is the ship's heading and \( (x_0, y_0, z_0) \) is the space-fixed position vector. This position vector follows the travelled distance in time and implicitly incorporates the velocity effect on the frequency of encounter.
4.5 Unsteady hydrofoil method

Mathematical formulation

The problem is described in a space-fixed Cartesian coordinate system, see Figure 4.2. The \( z_0 \)-axis is pointing upwards and the \( x_0-y_0 \) plane is positioned in the undisturbed watersurface. A thin, finite aspect ratio lifting surface of arbitrary planform is considered that performs arbitrary motions in six degrees of freedom, below the free surface. Parts of the lifting surface may be surface piercing. Only the submerged parts are considered here. The submergence may vary in time.

![Vortex sheets, free surface and motion components](image)

Figure 4.2  Vortex sheets, free surface and motion components

The lifting surface is advancing through a homogenous and incompressible fluid. Surface tension is not included and the water depth is infinite. Incident waves from arbitrary directions are present. During the motion of the lifting surface, the surrounding fluid is set into motion and some vorticity is created in the boundary layers on the upper and lower surfaces. Most vorticity is created along the sharp edges and is convected away from the lifting surface as the lifting surface advances. In the method all vorticity in the flow is restricted to the thin region of the lifting surface and its wake. The flow outside this infinitesimal region is considered to be irrotational and inviscid.

A bound vortex sheet represents the lifting surface and a free vortex sheet represents the wake, see Figure 4.2. The position and velocity of the bound vortex sheet are assumed to be known from the equations of motions of the hydrofoil craft. A finite pressure jump exists across it. The position of the wake sheet is not necessarily specified a priori. It may be allowed to deform freely in time whereby it assumes a force-free position so that no pressure jump exists across it. Both the bound
and wake vortex sheets are streamsurfaces.

The formulation of the problem is based on a mathematical formulation for large amplitude ship motions by Lain and Yue (1990). Their formulation is based on the use of impulsive sources to represent the unsteady flow about ship hulls, here a formulation based on impulsive strength vortex elements is derived.

The motions in the fluid are described by a velocity potential $\Phi_f$:

$$\Phi_f(x_0, t) = \Phi(x_0, t) + \Phi_w(x_0, t)$$  \hspace{1cm} (4.26)

where $x_0$ is the space-fixed position vector, $\Phi$ is the disturbance potential associated with the vortex sheets and $\Phi_w$ is the incident wave potential. The incident wave potential is known a priori and can be shown to satisfy the conditions (4.28) and (4.29) specified in the following paragraphs. The definition of the potential function for sinusoidal waves is as follows, see Gerritsma (1979):

$$\Phi_w = \frac{\zeta_{wa}}{\omega} e^{i \omega t} \sin(k_w x_0 - \omega t)$$  \hspace{1cm} (4.27)

where $\zeta_{wa}$ is the wave amplitude, $\omega$ is the wave frequency and $k_w$ is the wave number ($k_w = \omega^2 / g$).

The disturbance potential $\Phi$ satisfies the Laplace equation:

$$\nabla^2 \Phi = 0$$  \hspace{1cm} (4.28)

On the undisturbed free surface $S_f(t)$, the following linearized condition is imposed (for $r > 0$):

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial z_0} = 0$$  \hspace{1cm} (4.29)

where $g$ is the gravitational constant. This condition follows from the combination of the time derivative of the linearized dynamic free surface condition:

$$\frac{\partial \Phi}{\partial t} + g \zeta = 0$$  \hspace{1cm} (4.30)

and the linearized kinematic free surface condition:

$$\frac{\partial \Phi}{\partial z_0} = \frac{\partial \zeta}{\partial t}$$  \hspace{1cm} (4.31)

On the instantaneous lifting surface $S_l(t)$ the tangential flow condition is imposed (for $r > 0$):

$$V_n = \frac{\partial \Phi}{\partial n} + \frac{\partial \Phi_w}{\partial n}$$  \hspace{1cm} (4.32)

where $V_n$ is the instantaneous velocity of the lifting surface in a direction normal to its camber
4.5 Unsteady hydrofoil method

surface. The tangential flow condition is applied at the reference plane of the lifting surface. The \( \partial / \partial n \) operator denotes the derivative in normal direction, \( \partial / \partial n = n \cdot V \). The unit normal vector \( n \) is positive into the fluid domain. The term \( \partial \Phi_w / \partial n \) denotes the wave orbital velocity component normal to the camber surface.

The conditions at infinity (\( S_\infty \)) are (for \( t > 0 \)):

\[
\Phi \to 0
\]
\[
\frac{\partial \Phi}{\partial t} \to 0
\]

(4.33)

Apart from incoming waves, the fluid is at rest at the start of the process, the initial conditions on the free surface \( S_0(t) \) are then (for \( t=0 \)):

\[
\Phi = \frac{\partial \Phi}{\partial t} = 0
\]

(4.34)

The transient Green's function is now introduced for a submerged vortex with an impulsive strength:

\[
G(p,t,q,\tau) = G^0 + G^f = \frac{1}{R} + \frac{1}{R_0} - 2 \int_0^\infty \left[ 1 - \cos \left( \sqrt{gk_w(t-\tau)} \right) \right] e^{k(t-\tau)} J_0(k_w r) \, dk_w
\]

(4.35)

for \( p \neq q, \, t \geq \tau \)

where \( p(x_0,y_0,z_0) \) and \( q(\xi,\eta,\zeta) \) are the field and singularity point coordinates respectively, \( \tau \) is a past time variable, \( G^0 \) is the vortex plus biplane image part and \( G^f \) is the free surface memory part of the Green's function, \( J_0 \) is the Bessel function of order zero, and

\[
R = \sqrt{(x_0-\xi)^2 + (y_0-\eta)^2 + (z_0-\zeta)^2}
\]
\[
R_0 = \sqrt{(x_0-\xi)^2 + (y_0-\eta)^2 + (z_0+\zeta)^2}
\]

(4.36)

\[
r = \sqrt{(x_0-\xi)^2 + (y_0-\eta)^2}
\]

This Green's function formulation is analogous to the one given by Newman (1985) for an impulsive source. Here, for a vortex element the biplane image term is \( +1/R_0 \) instead of \( -1/R_0 \) for sources and a minus sign for the free surface part of the Green's function is used.

It can be shown, see Wehausen (1959) and Pinkster (1998), that the Green's function, eq. (4.35) satisfies the following conditions, where \( V(t) \) is the fluid domain:
\[ \nabla^2 G = 0 \text{ in } V(t), \quad t > \tau \]
\[ \frac{\partial^2 G}{\partial t^2} + g \frac{\partial G}{\partial z_0} = 0 \text{ on } S_p(t), \quad t > \tau \]
\[ G, \quad \frac{\partial G}{\partial t} \rightarrow 0 \text{ on } S_\infty, \quad t > \tau \]
\[ \frac{\partial G}{\partial t} = 0 \text{ on } S_p(t), \quad t = \tau \]
\[ G = \frac{1}{R} + \frac{1}{R_0} \text{ on } S_p(t), \quad t = \tau \]

A boundary integral formulation for the problem is derived by applying Green's second identity to the potential \( \Phi(q, \tau) \) and \( \partial G/\partial t(p, t; q, \tau) \) for \( \tau < t \), in a fluid domain \( V(\tau) \), bounded by \( S(\tau) \) which consists of \( S_\infty(\tau), S_\theta(\tau), S_w(\tau), S_p \) at infinity, and a small surface \( S_p \), excluding field point \( p \) to the fluid domain \( V(\tau) \), see Figure 4.2:

\[ \int \int \left[ \Phi \nabla^2 \frac{\partial G}{\partial t} - \frac{\partial G}{\partial t} \nabla^2 \Phi \right] dV = - \int \int \left[ \Phi \frac{\partial^2 G}{\partial t \partial n} - \frac{\partial G}{\partial t} \frac{\partial \Phi}{\partial n} \right] dS \]  
(4.38)

The left-hand side of eq. (4.38) is zero by virtue of the Laplace equation. The integrals over \( S_\infty \) and \( S_p \) in the right-hand side are also zero, see Pinkster (1998). It is understood here that gradients and normal derivatives are to be taken at the singularity point \( q \) and that \( S_\infty(\tau), S_\theta(\tau) \) and \( S_w(\tau) \) do not include point \( p \).

It is desirable to remove the integral over the free surface since the free surface domain is unbounded. In order to do so, first integration from \( \tau = 0 \) to \( \tau = t \) is carried out on the remains of eq. (4.38):

\[ \int_0^t \left[ \int_{S_p(t)} \Phi \frac{\partial^2 G}{\partial t \partial n} - \frac{\partial G}{\partial t} \frac{\partial \Phi}{\partial n} \right] dS = 0 \]
(4.39)

The free surface condition (4.29) is applied to replace \( \partial/\partial n \) with \( \partial^2 /g \partial \tau^2 \). Using the transport theorem, see Newman (1977), the integral over \( S_p(\tau) \) is written as:

\[ \int \int \left[ \Phi \frac{\partial^2 G}{\partial t \partial n} - \frac{\partial G}{\partial t} \frac{\partial \Phi}{\partial n} \right] dS = \frac{1}{g} \frac{\partial}{\partial t} \int \int \left[ \Phi \frac{\partial^2 G}{\partial t^2} - \frac{\partial G}{\partial t} \frac{\partial \Phi}{\partial t} \right] dS + \]

\[ \int \left[ \Phi \frac{\partial^2 G}{\partial t^2} - \frac{\partial G}{\partial t} \frac{\partial \Phi}{\partial t} \right] V_n \ dL \]
(4.40)

where \( L_v(\tau) \) is the instantaneous intersection of the lifting surface with the free surface and \( V_n \) is the normal velocity at \( L_v \). In the current discretization, lifting surface parts are represented by singularity
elements with a zero thickness for which the integral over \( L_w \) yields a zero contribution and is therefore omitted in the following.

Integrating the remaining term of the right hand side of eq. (4.40) in time, from \( \tau = 0 \) to \( \tau = t \), results in:

\[
\int_0^t \int_{S_{\infty}(\tau)} \left[ \Phi \frac{\partial G}{\partial \tau} - \frac{\partial G}{\partial \tau} \frac{\partial \Phi}{\partial n} \right] dS = \int_0^t \int_{S_{\infty}(\tau)} \Phi \frac{\partial G}{\partial n} dS
\]  

(4.41)

whereby the free surface condition on \( G \) and the initial conditions on \( \Phi \), eq.’s (4.34) and (4.39), have been applied. By combining eq. (4.41) and eq. (4.39) the following equation results:

\[
\int_0^t \int_{S_{\infty}(\tau)} \Phi \frac{\partial G}{\partial n} dS + \int_0^t \int_{S_{\infty}(\tau)} \left[ \Phi \frac{\partial G}{\partial \tau} - \frac{\partial G}{\partial \tau} \frac{\partial \Phi}{\partial n} \right] dS = 0
\]  

(4.42)

The integral over the free surface can be further reduced by applying Green’s second identity again to \( \Phi(q,t) \) and \( G^0(p,q) \) in the fluid domain \( V(t) \), bounded by \( S(t) \) which consists of \( S_A(t) \), \( S_B(t) \), \( S_w(t) \), \( S_{\infty} \), and a small surface \( S_p \), excluding field point \( p \) to the fluid domain \( V(t) \). Green’s second identity results in:

\[
\int_{S(\tau)} \left[ \Phi \frac{\partial G^0}{\partial n} - G^0 \frac{\partial \Phi}{\partial n} \right] dS = 0
\]  

(4.43)

The contribution over \( S_{\infty} \) is zero, the contribution over \( S_p \) is however not zero because of the singularity of \( G^0 \) if \( p \) and \( q \) coincide. This contribution can be shown to be \( 4\pi T(p)\Phi(p,t) \), where

\[
T(p) = \begin{cases} 
-1 & p \in V(t) \\
-\frac{1}{2} & p \in S_{\infty}(t) \\
0 & \text{otherwise}
\end{cases}
\]  

(4.44)

Substituting this in eq. (4.43), and the result in eq. (4.42) leads to:

\[
4\pi T(p)\Phi(p,t) = \int_0^t \int_{S_A(\tau)} \left[ \Phi \frac{\partial G}{\partial \tau} - \frac{\partial G}{\partial \tau} \frac{\partial \Phi}{\partial n} \right] dS + \int_0^t \int_{S_{\infty}(\tau)} G^0 \frac{\partial \Phi}{\partial n} dS - \\
\int_0^t \int_{S_{\infty}(\tau)} \left[ \Phi \frac{\partial G'}{\partial \tau} - \frac{\partial G'}{\partial \tau} \frac{\partial \Phi}{\partial n} \right] dS
\]  

(4.45)

The first term on the right hand side is the vortex plus biplane image contribution, the second term is a free surface contribution which will be removed in the following while the third term contains the memory effect by means of the time integral.
In order to obtain an equation for the distribution of singularity elements, the discontinuity in velocity potentials between the lower (-) and upper (+) sides of the lifting surface and wake sheet is set equal to the strength of a doublet distribution on these surfaces. The normal derivative of the potential function $\partial \Phi / \partial n$ is then identical for the lower and upper surfaces:

$$\Phi^+ - \Phi^- = \mu$$

$$\frac{\partial \Phi^+}{\partial n} = \frac{\partial \Phi^-}{\partial n}$$  \hspace{1cm} (4.46)

Here $\mu$ is the doublet strength. The following equation results for the potential at field point $p$, located on $S_{byw}(t)$:

$$4\pi \Phi(p,t) = \int_{S_{byw}(t)} \mu(q,t) \frac{\partial G^0}{\partial n_q} dS - \int_0^t \int_{S_{byw}(t)} \mu(q,\tau) \frac{\partial^2 G^f}{\partial \tau \partial n_q} dS$$  \hspace{1cm} (4.47)

where $\mu(q,t)$ is the doublet strength at a position $q$, at time $t$, and $\partial / \partial n_q$ is the normal derivative at the singularity point $q$. By applying the $\nabla \cdot n_p$ operator to eq. (4.47) and by using the tangential flow condition (4.32) on the lifting surface $S_{by}(t)$, the following formulation is obtained:

$$V_n - \frac{\partial \Phi^w}{\partial n_p} = \frac{1}{4\pi} \left( \int_{S_{byw}(t)} \mu(q,t) \frac{\partial G^0}{\partial n_p} dS - \int_0^t \int_{S_{byw}(t)} \mu(q,\tau) \frac{\partial^2 G^f}{\partial \tau \partial n_p} dS \right)$$  \hspace{1cm} (4.48)

where $\partial / \partial n_p$ is the normal derivative at field point $p$ and $\partial G / \partial \tau$ has been replaced by $-\partial G / \partial t$.

Equation (4.48) is the principal equation to be solved for the unknown doublet strength $\mu(q,t)$. It does not have an unique solution for the conditions implied so far. A wake model needs to be established where conditions are specified which relate to the vortex strength at the trailing edge and the location and shape of the wake sheet. The vortex strength at the trailing edge is related to the unsteady Kutta condition, which will be described first in the following section.

**Wake model**

The Kutta condition for steady flow is that the velocity along the trailing edge of lifting surfaces remains finite. The thin wing theory equivalent is that the vortex strength at the trailing edge be zero. This is satisfied in a vortex lattice method by representing the continuous vortex distribution by a set of vortex elements with a bound vortex segment at the quarter chord position and requiring tangential flow at the three-quarter chord position of the element. Thus the vortex strength distribution on a two-dimensional flat plate, with a zero bound vortex strength at the trailing edge, is simulated along each vortex strip.

In unsteady flow the same approach can be used. At the same time vorticity must be shed from the lifting surface on the wake sheet in order to satisfy the first wake condition: in a potential flow the circulation $\Gamma$ around a contour enclosing the lifting surface and its wake must be conserved. This condition is also termed the Kelvin condition and can be mathematically expressed by:
\[
\frac{d\Gamma}{dt} = 0
\] (4.49)

By using so-called vortex ring elements on the lifting surface and on the wake sheet as a discretization of a continuous vortex sheet and by transferring each time step the circulation at the trailing edge elements into the wake elements, this requirement is satisfied: the sum of the circulations strengths along each individual vortex line segment is always zero. Once shed, the circulation strength of wake sheet elements remains constant.

Katz and Plotkin (1991) advocate the use of the steady Kutta condition for low reduced frequency conditions where quasi-steady force components are dominant and where no significant flow separation is present. Flow separation may occur at high incidence angles, at high reduced frequencies and large motion amplitudes. Flow separation violates the Kutta condition and may lead to changes in lift and drag. Katz and Plotkin point at experimental evidence showing that for small amplitude oscillations at reduced frequencies \( k > 0.60 \) some flow separation could be observed from flow visualizations but that the lift and drag were not appreciably affected. Katz and Plotkin suggest the following criteria for using the steady flow Kutta condition:

- the angle of attack must not exceed the stall angle,
- the oscillation amplitude perpendicular to the direction of motion is restricted to 10% of the chord length at \( k = 1.0 \), while an unspecified larger value is allowed at lower reduced frequency values,
- the velocity at the trailing edge normal to the lifting surface must be small relative to the speed of advance.

It is decided to use this Kutta condition for the present purpose. For hydrofoil craft in general quasi-steady forces are dominating the unsteady forces. The reduced frequency will not exceed \( k = 0.25 \), while incidence variations are limited to about 5 deg. Oscillation amplitudes may be in the order of one chord length, but at low reduced frequency values. The normal velocity of the trailing edge is usually an order of magnitude smaller than the speed of advance, even when a ride control system is used that actuates trailing edge flaps. Some experimental data on high speed trailing edge flap deflections are available and will be used to check the use of the steady flow Kutta condition, in Section 5.3.

The second requirement for the wake model is that the wake sheet should be force free as it is not a solid surface; no pressure difference must be present between the upper and lower sides of the sheet. From the Kutta-Joukowsky law, the force \( F \) on a vortex sheet is given by:

\[
F = \rho V \times \gamma
\] (4.50)

so that for a zero force the vorticity vector \( \gamma \) should be directed parallel to the velocity vector \( V \). This can be accomplished by displacing the vortex element corner points with the local fluid velocity.

It is noted that calculating the position of each wake element corner point each time step needs a relatively large amount of computer time. The induced velocities due to all other vortex elements
need to be computed and the relative positions between the wake element corner points change continuously so that it is not possible to use so-called influence coefficient matrices.

Discretization

Instead of using doublet elements for discretization of the vortex sheets, so-called vortex ring elements carrying a circulation strength $\Gamma$ are used. These vortex elements consist of four discrete, straight vortex lines of constant strength which enclose the quadrilateral element area, see Figure 4.3. The induced velocity due to a vortex ring element is identical to that of a constant strength doublet element if $\Gamma = \mu$, see Katz and Plotkin (1991). The use of vortex ring elements satisfies the Kelvin condition while the induced velocities can be computed with a relatively small effort.

The lifting surface is divided into $N$ chordwise vortex elements and $M$ spanwise vortex elements. The vortex elements are uniformly distributed over both the chord and the span. A tip inset width as used in the steady problem, see Section 2.5, is also applied here for a faster convergence of results with the number of spanwise vortex elements used. A cosine distribution for the element length in chordwise direction can be applied. However, as will be explained in Section 5.2, this requires a very small simulation time step in order to have a wake element spacing equal to the length of the vortex element at the trailing edge. Therefore, constant length vortex elements are used in chordwise direction.

Figure 4.3  Bound and wake vortex lattices
The vortex elements are located on the base plane of the lifting surface. Following the lumped vortex method also applied for the steady problem, the leading segment of the vortex ring is placed on the element's quarter chord line. The control point where the tangential flow condition is applied is at the three-quarter length point of the vortex element, at the spanwise centre. The vortex elements located at the trailing edge of the lifting surface extend to a position $\frac{1}{4} c / N$ behind the trailing edge.

**Time stepping process**

At the start of the simulation ($t=0$) the lifting surface is impulsively set into motion. At this instant, the circulation on the lifting surface is determined for the condition without wake vortex elements. The last spanwise vortex line just behind the trailing edge represents the start vortex. At each subsequent time step, the lifting surface is advanced to a new position with an instantaneous velocity. Both the position and velocity are known from the equations of motion. The gap between the instantaneous trailing edge vortex element on the lifting surface and the wake vortex element shed in the previous time step is filled with a new wake vortex element. In this way, the wake vortex element at the trailing edge has the same orientation as the flow leaving the trailing edge, to a first order approximation. A discrete vortex line placed at the middle of the interval travelled would underestimate the induced velocity due to a continuous vortex sheet. The location of the last spanwise vortex element on the lifting surface, at one quarter of the element length behind the trailing edge, implicitly corrects for the discretization error.

The circulation strength of the new wake vortex elements is set equal to that of the trailing vortex element of the previous time step. With a known wake vortex position and circulation, lifting surface position and velocity, the tangential flow condition, eq. (4.48) can be solved for the unknown circulation on the lifting surface. Then, all vortex element particulars for the present time step are known and the induced velocity $\mathbf{V}_i$ at each wake vortex element corner point $p$ may be determined by means of:

$$
\mathbf{V}_i(p, t) = \nabla \Phi_i(p, t) = \frac{1}{4\pi} \int_{\Gamma(q, t)} \Gamma(q, t) \nabla \frac{\partial G^0}{\partial n_q} dS + \frac{1}{4\pi} \int_{\Gamma(q, t)} \int_{\Gamma(q, t)} \Gamma(q, \tau) \nabla \frac{\partial^2 G^f}{\partial t \partial n_q} dS + \nabla \Phi_w(p, t)
$$

(4.51)

where the wave orbital velocity components are added.

The position of each wake vortex element corner point is then updated by using:

$$
x_i(p, t) = x_i(p, t - \Delta t) + \mathbf{V}_i(p, t) \Delta t
$$

(4.52)

This procedure is repeated each time step $\Delta t$. 
Solution of the integral equation

The discretized form of the integral eq. (4.48) is given by:

\[
\sum_{j=1}^{NM(t)} \Gamma_j^{(i)} \int_{S_j} \frac{\partial^2 G_0}{\partial n_p \partial n_q} (p_i, q_j) dS = 4 \pi V(p_i, t) \cdot n_{p_i} - \\
\sum_{j=N M(t)}^{N (t)} \int_{S_j} \frac{\partial^2 G_0}{\partial n_p \partial n_q} (p_i, q_j) dS - \Delta t \sum_{k=0}^{l-1} \varepsilon \sum_{j=1}^{NM(t)} \Gamma_j^{(i)} \int_{S_j} \frac{\partial^2 G_0}{\partial t \partial n_p \partial n_q} (p_i, t; q_j, \tau) dS
\]

(4.53)

where \(NM\) is the number of vortex elements on the lifting surface, \(NT\) is the total number of vortex elements, \(t\) is the present time, \(\tau\) is the past time, \(i\) and \(j\) are the element indices for control point \(p\) and vortex element point \(q\) respectively and \(\varepsilon\) is an integration constant. The term \(\partial / \partial n_p\) denotes the normal derivative to the camber surface at the control point, defined in the ship-fixed axis system. This is convenient for the subsequent determination of force components which are defined in the ship-fixed axis system. The term on the left hand side denotes the normal induced velocity due to the vortex elements on the lifting surface, the first term on the right hand side denotes the normal velocity components due to the hydrofoil motions and the incident waves, the second term on the right hand side accounts for the normal induced velocity components due to the vortex elements in the wake while the last term contains the memory effect.

The vector \(V\) denotes the velocity components due to the hydrofoil translational velocities, \(V_b = (u, v, w)\) and angular velocities, \(\Omega_b = (p, q, r)\), and the orbital \((V_w)\) motion components due to incident waves:

\[
V(p_i, t) = V_b(t) + \Omega_b(t) \times r(p_i) + V_w(p_i, t)
\]

(4.54)

Here \(r\) denotes the position vector of the field point \(p\) and all velocity components are defined in the ship-fixed axis system.

Eq. (4.54) can be cast in a linear set of equations in the unknown circulation \(\Gamma_j^{(i)}:\n\[
\sum_{j=1}^{NM(t)} A_{ij} \Gamma_j^{(i)} = B_i \quad i = 1, 2, \ldots, NM(t)
\]

(4.55)

where \(A_{ij}\) contains the integral term of the left hand side of eq. (4.53) and \(B_i\) contains the entire right hand side of this equation. This set of equations is solved by using common linear algebra techniques.

The spatial derivatives of the vortex element and biplane image Green's function parts \(1/R\) and \(1/R_0\) are obtained from applying the Biot-Savart law to the four lines of each vortex element and its biplane image above the watersurface as follows:
\[
\frac{1}{4\pi} \int_S \frac{\partial^2 G^0}{\partial n_p \cdot \partial n_{q_j}} (p, q_j) \, dS = \frac{1}{4\pi} \int_S \left[ n_p \cdot \nabla \left( \frac{1}{R(p, q_j)} \right) \right] dS = \frac{1}{4\pi} \int_S \left[ \frac{\nabla}{\nabla} \left( \frac{1}{R(p, q_j)} \right) \right] dS = \frac{1}{4\pi} \int_S \left( \nabla (V(p, q_j)) + V_0(p, q_j) \right) dS
\]

where the velocity vectors \( V \) and \( V_0 \) are defined by:

\[
V = \frac{1}{4\pi} \oint \frac{dl \times R}{R^3}
\]

and

\[
V_0 = \frac{1}{4\pi} \oint \frac{dl \times R_0}{R_0^3}
\]

Here, \( R \) is the scalar distance between the field point \( p_i (x, y, z) \) and a point on the vortex element \( q_j (\xi, \eta, \zeta) \), \( R \) is the corresponding distance vector, \( R_0 \) is the scalar distance between the field point \( p_i \) and the biplane image point of the vortex element \( q_j (\xi, \eta, \zeta) \), \( R_0 \) is the corresponding distance vector and \( dl \) is the vortex element vector.

The evaluation of a \( G^j \) term by numerical integration requires a large amount of computer time. These terms must be evaluated at each control point for the entire time history of all vortex elements. Furthermore, due to the wake shedding process, the number of vortex elements increases in time. The total number of Green's function evaluations is given by:

\[
N_G = \sum_{i=1}^{N_i} i N_f (N M)^2 + \sum_{i=1}^{N_i} i^2 N_f N M^2 + \sum_{i=1}^{N_i} 4 i^3 N_f M^2
\]

if a steady wake is assumed and where \( N_i \) is the number of time steps, \( N_f \) is the number of function evaluations in the numerical integration of the equations of motion, \( N \) is the number of chordwise elements on the lifting surface and \( M \) is the number of spanwise elements on the lifting surface. For a practical application with \( N_f = 90000 \) (15 min), \( N_f = 4 \), \( N = 16 \) and \( M = 64 \), the number of Green's function evaluations is large: \( N_G = 10^{24} \).

An efficient evaluation of the \( G^j \) terms is therefore of importance. For low (t-\( \tau \)) values use is made of interpolation on predetermined values for \( G^j \) and its derivatives. For large (t-\( \tau \)) values use is made of polynomial expansions provided by Newman (1985, 1992). Appendix B describes the treatment of the Green's function terms. Despite the efficient evaluation of the \( G^j \) terms, the required computer time is still significant, typically 20 hours on a Cray C98 (at 500 Mflop's) for a seakeeping simulation of 15 minutes real time with stationary wake sheets, for a foil discretization with \( N = 4 \) and \( M = 64 \).
As a first step towards reduction of computer time and memory, the number of wake vortex elements is set to a maximum. Once this maximum number is reached, a new row of wake elements is still generated at the trailing edge, but the row of wake elements at maximum distance from the lifting surface is removed. The maximum number of wake vortex elements must be set such that the wake has a sufficient length so that the effect of removing a row of wake elements on the hydrofoil forces is negligible.

Next, the wake sheet position and form may be prescribed a priori. The prescription of the wake sheet position is simply that a wake sheet vortex element remains stationary, once it is shed. This saves the determination of the actual position of the wake vortex element corner points at each time step, for which the calculation of induced velocities at each wake vortex element corner point due to all vortex elements is required. A prescribed wake sheet position violates the requirement of a force free wake sheet, but experience has shown, see Katz and Plotkin (1991) for instance, that this has little effect on the forces acting on the lifting surface for practical conditions.

For unsteady conditions, the wake sheet is not a flat surface behind the lifting surface. Since the lifting surface, shedding the wake vortex elements, may perform arbitrary motions, the position of the wake sheet relative to the lifting surface varies and the induced velocities due all wake elements at the lifting surface have to be determined again each time step.

However, the computer time can be further reduced for seakeeping problems for which the speed and heading are assumed constant, if the motions of the craft are assumed to be small. In the integral eq. (4.48) the displacements of the craft around its mean position are not taken into account then. This will be termed the linear approach in the following.

Prescribing the wake sheet shape then results in a flat wake sheet in the \( x_0-y_0 \) plane behind the lifting surface. Due to the constant speed, the relative position between a row of wake elements and the lifting surface elements is constant. The concept of influence coefficients can then be used, when determining the induced velocities due to wake sheet elements at the lifting surface control points. Only for the first row of wake sheet elements, representing the start vortex, the influence coefficients need to be calculated each time step until the maximum wake sheet length is reached. For all the other rows of wake sheet elements, the induced velocities can be obtained by multiplying the influence coefficients with the actual circulation at the wake sheet elements. Furthermore, the integration of the \( \partial G/\partial t \) terms with respect to time can be performed analytically for wake sheet vortex elements since the distance between lifting surface control points and wake vortex elements is constant for each \((t-\tau)\) value. Appendix B describes this time integral of the free surface Green's function. The number of Green's function evaluations is now:

\[
N_G = N_r N_f N M^2 (1 + N) \tag{4.60}
\]

which is much lower than required for non-linear simulations. For the same example as given for eq. (4.59), here \( N_G = 10^{11} \).

Furthermore, a constant submerged geometry implies that the number of vortex elements on the hydrofoil configuration is constant. The influence coefficient matrix \( A \) in the linear system of eq.
(4.55) is then constant and needs to be inverted only once. A linear seakeeping simulation of 15 minutes now takes about 30 mins on a 200 MHz Pentium PC and 3 mins on a Cray C98, for a foil discretization with $N=4$ and $M=64$.

For the linear case, the system of equations can be set up as follows:

$$
\sum_{j=1}^{NM} A_{ij} \Gamma_j^{(i)} = -\{C_{ij}^{(i)} \Gamma_j^{(i)} + D_{ij}^{(i)} \Gamma_j^{(i)} + E_{ij}^{(i)} \Gamma_j^{(i)} - V_{ni}\} \quad i=1,2,\ldots,NM
$$

(4.61)

where $C_{ij}^{(i)}$ is the influence coefficient matrix for the lifting surface vortex element $G^i$ terms, $D_{ij}^{(i)}$ is the influence coefficient matrix for the wake sheet vortex element $G^i$ terms, $E_{ij}^{(i)}$ is the influence coefficient matrix for the wake sheet element $G^0$ terms and $V_{ni}$ denotes the lifting surface and wave orbital velocity components.

The effects of the simplifications mentioned above on the calculation results will be investigated in Section 5.2.

Limiting the number of wake elements has an additional advantage. A theoretical difficulty in linear seakeeping analysis is the singularity in the Green's function at the critical relative frequency $\tau_{\alpha} = \frac{1}{4}$. At this relative frequency, $\tau_{\alpha} = \omega U/g$, the group velocity of the waves generated by the oscillatory motion matches the ship speed $U$ and resonance occurs. In the time domain this resonance is manifest as a slow unbounded growth of the Green's function components with time. Assuming a typical hydrofoil speed of 40 kt, the encounter frequency $\omega_c$ must be 0.12 rad/sec in order to have $\tau_{\alpha} = \frac{1}{4}$. This is a rather low value, but for stern wave directions, the frequency of encounter may actually have such a low value; in the present example for $\omega_c = 0.50$ rad/sec, or a wave period of about 12 seconds. Although such a situation can be identified beforehand while performing seakeeping simulations in regular waves, it is useful to limit the growth of the Green's function terms for more general simulation cases with a relative frequency close to $\tau_{\alpha} = \frac{1}{4}$. This can be achieved by truncating the duration of the memory effect, see Lain and Yue (1990). The memory effect is effectively truncated by limiting the maximum number of wake vortex elements and relating the minimum ($t-t$) value to the maximum number of wake elements for the memory effect due to the lifting surface vortex elements.

It should be noted that also for impulsively started but otherwise steady conditions the effect of the $\tau_{\alpha} = \frac{1}{4}$ singularity is present. The Green's function $G^i$ consists of an integral over the wave number $k_w = \omega^2/g$ from $k_w = 0$ to $k_w = \infty$, see eq. (4.35). Therefore, a contribution due to frequencies corresponding to $\tau_{\alpha} = \frac{1}{4}$ is always present in the solution. This is manifest as a slowly decaying oscillation in the forces acting on the lifting surface. The frequency and amplitude of this oscillation depend on the Froude number. At low chord Froude numbers, $F_n < 1$ say, the amplitude and frequency of the oscillation are relatively large. At higher, more practical chord Froude numbers, the amplitude and frequency of the oscillation are relatively low. This behaviour is further discussed in Section 5.2.
Chapter 4  Computational method for unsteady flow conditions

Force evaluation

The forces acting on the lifting surface can be obtained from integrating the pressure difference over the vortex elements. The pressure difference follows from the unsteady flow Bernoulli equation, in a space-fixed axis system, see Katz and Plotkin (1991):

$$\frac{p_{ref} - p}{p} = \frac{1}{2} (\nabla \Phi_T)^2 + \frac{\partial \Phi_T}{\partial t}$$  \hspace{1cm} (4.62)

where $p$ denotes the pressure in the fluid, $p_{ref}$ is a reference pressure, and $\Phi_T$ is the total potential. The pressure difference can be integrated over the lifting surface in order to obtain the force acting on the lifting surface. This results in reasonable predictions for the lift force, but the induced drag will not be accurate due to the existence of the leading edge suction force, see Section 2.5.

An alternative approach is taken here for the force components, involving the $\nabla \Phi_T$ term, by using the Kutta-Joukowski law which implicitly accounts for the leading edge suction force by defining both the magnitude and direction of the force vector. For the component related to $\partial \Phi_T/\partial t$ no alternative method is available and pressure integration is necessary. This will presumably not lead to large drag errors since for practical hydrofoil applications unsteady effects are limited in magnitude. Furthermore, it is convenient to determine the forces in the ship-fixed axis system.

The Kutta-Joukowski force components are given by:

$$X_{bij} = \rho V \times \Gamma \Delta s$$  \hspace{1cm} (4.63)

where $\Delta s$ is the vortex element width, and $\Gamma$ is the circulation vector defined as:

$$\Gamma = (\Gamma_{ij} - \Gamma_{i,j}) \cdot n_i \quad \text{for } i > 1$$

$$\Gamma = \Gamma_{ij} \cdot n_i \quad \text{for } i = 1$$  \hspace{1cm} (4.64)

where $i$ and $j$ are chordwise and spanwise vortex element indices respectively and $n_i$ is the unit tangential vector of the vortex element. $V$ is the total velocity vector at the bound vortex position of the element, due to the kinematic velocity of the lifting surface and the disturbance and wave orbital motion components:

$$V = V_b + \Omega_b \times r + \nabla \Phi + \nabla \Phi_w$$  \hspace{1cm} (4.65)

where $V_b$ is the ship-fixed translational velocity vector $(u,v,w)$, $\Omega_b$ is the angular velocity vector $(p,q,r)$ and $r$ is the position vector of the point where the velocity is evaluated. The gradients of the velocity potentials are resolved in the ship-fixed axis system.

The kinematic velocity components of the lifting surface are known in the process of solving the equations of motion of the hydrofoil craft. The disturbance potential velocity components are obtained from eq. (4.51), at the bound vortex locations on the lifting surface. The wave induced velocity components follow from applying the gradient operator to the wave potential. A
disadvantage of applying the Kutta-Joukowski law is that the velocity components need to be known not only at the control points, for determining the circulation distribution, but also at the bound vortex position. This requires extra evaluations of the Green’s functions and the Biot-Savart terms. As a compromise, the Biot-Savart terms are actually evaluated at each bound vortex position, but the Green’s function terms are obtained from interpolation on the Green’s function term values at the control points, except for the first bound vortex on a vortex element strip.

The time derivative term in eq. (4.62) is expressed in a pressure difference as follows:

\[
\Delta p_t = p^+ - p^- = \rho \left[ \frac{\partial \Phi^+}{\partial t} - \frac{\partial \Phi^-}{\partial t} \right]
\]  
(4.66)

and can be expressed in terms of the circulation by means of using:

\[
\Gamma = \Delta \Phi = \Phi^+ - \Phi^-
\]  
(4.67)

where + and - denote the upper and lower side of the lifting surface respectively. The incident wave potential does not contribute to the jump in velocity potential. The force vector follows from the product of pressure difference and vortex element area:

\[
X = \rho \frac{\partial \Gamma}{\partial t} \Delta s \Delta c \cdot \hat{n}_b
\]  
(4.68)

where \(\Delta c\) is the mean length of the vortex element in chordwise direction. The time derivative is determined by means of a first order difference:

\[
\frac{\partial \Gamma}{\partial t} \approx \frac{\Gamma(t) - \Gamma(t - \Delta t)}{\Delta t}
\]  
(4.69)

The total force consists of the Kutta-Joukowski and the time derivative components:

\[
X_f = X_{fj} + X_{jt}
\]  
(4.70)

Integration of equations of motion

Equations of motion as described in Section 4.4 are usually integrated most efficiently by means of a 5th order Runge-Kutta method. In this numerical method four evaluations of the forces, in the right hand side of eq. (4.8), are required within each time interval: one at the initial time, two at an intermediate time and one at the end of the time interval. For each evaluation different values of the position and velocity vectors are used for which appropriate forces must be determined. The wake shedding process can however only be activated once for each time step, at the instant of the first force evaluation. The computation of induced velocities associated with the disturbance potential at each control point and the memory effect should be repeated for each force evaluation since the position of the lifting surface varies. This makes the Runge-Kutta integration scheme rather demanding in terms of computer time.
However, the induced velocities may be kept constant during one time step since the displacement variations are relatively small and, moreover, the forces acting on the lifting surface are determined to a large extent by the translational and rotational velocities of the lifting surface. The force variations within one time step are then entirely due to variations in the lifting surface velocity vector. This approach tends towards using an Euler integration scheme with only one force evaluation per time step. With regard to discretization errors in the integration of the equations of motion, an Euler integration scheme can often be used since the simulation time step needs to be small anyway when ride control systems are incorporated in the simulation. Comparisons between calculation results with the Runge-Kutta and Euler schemes in moderate sea states do not show significant differences in the results: for instance the extreme values of the accelerations are within 0.5%. Therefore, the Euler scheme is used in the time domain simulations described in Section 5.4.

**Force correction due to variations in wetted surface**

In the computational method linearized free surface conditions are used at the undisturbed free surface, enabling the use of the free surface Green's function terms $G^f$. This enables the use of the actual submerged foil parts, relative to the undisturbed water surface, in the non-linear (with respect to motions) simulation method, but wave elevation effects on the wetted surface can not be taken into account. The linear computational method can not account for wetted surface variations due to the motions of the craft nor due to the wave elevation.

For surface piercing hydrofoil systems wetted surface variations due to the wave elevation may be of importance though. Surface piercing foil systems often feature highly cambered sections with a relatively large chord length at the tips of the foils to achieve an early take-off. Furthermore, due to the low dihedral angle significant wetted area and thereby lift variations may result from the combined effect of submergence and wave elevation variations.

A simple procedure to correct for this latter phenomena is to multiply the forces acting on the surface piercing foil parts with the ratio between the actual wetted area and the wetted area based on the submergence relative to the undisturbed water surface. The actual wetted area can be obtained from determining the wave elevation as given by eq. (4.23). In principle, this procedure can be applied to both the linear and non-linear methods. However, in order to keep a clear distinction between the linear and non-linear methods, the correction accounting for the wave elevation is only applied in the non-linear method.

Such a force correction for surface piercing foil parts is used for conditions where the craft is in a so-called platforming mode of ride control. This type of ride control intends to keep the craft in a stationary position in the vertical plane and is the usual approach taken for operation in head waves where the frequency of encounter is high. The maximum operational wave height is then related to the spacing between the foils and the hull. In following seas, the encounter frequency is much lower and the craft can operate in waves much higher than the foil-hull spacing if a contouring mode of ride control is chosen. Hereby, the craft follows the wave elevation, i.e. the control system intends to keep the spacing between the hull and the wave surface constant, at the forward and aft foil positions. The displacement in the vertical plane may then be larger than the submergence of the foil system. For the linear simulation method, contouring does not impose problems since the
relatively large motions of the foil system are not taken into account. For the non-linear method however, displacements are accounted for and the foils may get in a position above the undisturbed water surface. The application of free surface conditions on the undisturbed water surface is then strongly violated and the simulation results are in error. An engineering approach to circumvent this problem is to position the foils below the undisturbed free surface at their actual submergence relative to the wave surface. Such a procedure simulates the application of the free surface boundary conditions on the actual water surface. Note that for contouring in high following seas the frequency of encounter is low and the wave length is large relative to the foil spacing. The slope of the wave surface in the vertical plane is typically lower than 1%. The virtual foil position is only used for determining the forces acting on the foils, it does not directly affect the position of the craft in the equations of motion.

**Summary Chapter 4**

The Chapter describes the computational method for determining the seakeeping and manoeuvring characteristics of hydrofoil craft operating in the foilborne speed region.

The demands set to the computational method are discussed. A review of unsteady hydrofoil theories is given and the computational approach is discussed.

The approach consists of a, with respect to the motion of the craft, non-linear simulation method in which a time domain Green's function formulation is used to determine linearized free surface effects. For computing the forces acting on the foil system an unsteady vortex lattice method is selected.

The simulation method Hydsim is based on the equations of motions in six degrees of freedom. Besides foil system forces, propellers and aerodynamic forces are included.

In the final Section the mathematical formulation of the unsteady vortex lattice method, the wake model and the discretization by means of vortex ring elements are described. In order to reduce computer time a linear method is derived which may be applied to the seakeeping problem if a constant speed and heading are assumed.
References Chapter 4


Kaplan P. (1951), 'A Hydrodynamic Theory for the Forces on Hydrofoils in Unsteady Motion', Dissertation submitted to the Faculty of the Stevens Institute of Technology, Hoboken, New Jersey.


Wetzel J.M. and Schiebe F.R. (1960), 'Lift and Drag on Surface-Piercing Dihedral Hydrofoils in Regular Waves', St. Anthony Falls Hydraulic Laboratory, Report No. 64.
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5. VALIDATION OF THE UNSTEADY FLOW METHODS UNVLML AND HYDSIM

5.1 Introduction

In this Chapter numerical results and basic validation cases are discussed for the unsteady vortex lattice method. The program Unvlml contains the unsteady vortex lattice method described in Section 4.5. The program Hydsim contains the simulation method described in Section 4.4 in which Unvlml is used to calculate foil system forces. Calculation results identified by Unvlml-nl denote the non-linear method while Unvlml-In denotes results obtained by the linear computational method. The term linear used here refers to the linearization of the motions of the craft only, as outlined in Section 4.5. In both the linear and non-linear methods, free surface conditions and the tangential flow condition on the lifting surface are linearized.

The reduced frequency used as a measure of unsteadiness is defined as \( k = \omega c_m / 2U \), where \( \omega \) is the frequency of oscillation or wave frequency of encounter, \( c_m \) is the mean foil chord and \( U \) is the speed of advance. The wake was in a stationary position, i.e. no wake roll-up was taken into account, unless mentioned otherwise. Force components are defined in an axis system fixed in the lifting surface or hydrofoil craft as follows:

\[
C_x = \frac{X}{\frac{1}{2} \rho U^2 S}, \quad C_y = \frac{Y}{\frac{1}{2} \rho U^2 S l}, \quad C_z = \frac{Z}{\frac{1}{2} \rho U^2 S l}, \quad C_M = \frac{M}{\frac{1}{2} \rho U^2 S l}, \quad C_N = \frac{N}{\frac{1}{2} \rho U^2 S l},
\]

(5.1)

where \( X \) and \( K \) denote the force and moment components \((X,Y,Z)\) and \((M,N)\) respectively, \( S \) is a suitable reference area, usually the projection of the planform area on the horizontal plane, unless stated otherwise and \( l \) denotes a suitable reference length, either the foil mean chord \( c_m \) if moments on a single hydrofoil are considered, or the distance from a foil to the centre of gravity if a hydrofoil craft is considered. Although the forces are defined in an axis system fixed to the lifting surface rather than in a flow axis system, the terms lift and drag will be used for the \( Z \) and \( X \) force components respectively in the following sections. Figure 5.1 shows the definitions of the force and moment components as well as the motion components. Amplitudes of the force and moment coefficient values are denoted by \( C_{xA} \) through \( C_{NA} \).

For each case, unless mentioned otherwise, a default time step \( \Delta t_d \) has been used. This time step depends on the number of chordwise vortex panels \( N \), the speed of advance and the mean chord via:

\[
\Delta t_d = \frac{c_m}{UN}
\]

(5.2)

By using such a time step, the streamwise length of vortex elements in the wake approximately (depending on the taper ratio) equals the chordwise length of vortex elements on the lifting surface.
In the paragraphs on the time step dependence it will be shown that this time step minimizes discretization errors.

It has further been verified for each time domain simulation for hydrofoil craft, as described in Section 5.4, that the default time step is sufficiently small for use in integrating the equations of motion, i.e. no significant errors are introduced due to a too large time step for the Euler integration scheme. Also, the default time step used was sufficiently small to use for ride control systems.

In case of oscillatory motions of the hydrofoil, the oscillations are sinusoidal and defined by:

\[ x(t) = F(t) x_a \sin(\omega t) \]

\[ \phi(t) = F(t) \phi_a \sin(\omega t) \]  \hspace{1cm} (5.3)

where \( x_a \) and \( \phi_a \) denote the amplitudes of the translational and rotational motions respectively and \( F(t) \) denotes a start-up function used for obtaining a smooth onset of the motion. This function is used up to the start-up time \( T_s \) and is defined by

\[ F(t) = \sin^2\left(\frac{\pi t}{2T_s}\right) \quad \text{for} \quad t \leq T_s \]  \hspace{1cm} (5.4)
5.1 Introduction

The definition of phase angles is given in Figure 5.2. In Section 5.2 the numerical validation of the unsteady vortex lattice method UnvIm will be described. Furthermore, the effects of linearization of motions, wake roll-up and truncation, time step and fore-aft foil interaction are discussed. Also, comparisons with analytical results are given for oscillating and impulsively started lifting surfaces. In Section 5.3 basic calculation results obtained with UnvIm are compared with experimental results, while in Section 5.4 time domain results obtained with HydSim are described and compared with full scale measurements.

Figure 5.2 Definition of phase angle
Convergence study with respect to number of vortex elements

The effect of the number of chordwise ($N$) and spanwise ($M$) vortex elements on the force coefficients is shown first for steady flow conditions. Time domain simulations for an impulsively started rectangular hydrofoil of aspect ratio ($AR$) six, at a submergence ($h$) of one chord ($c$) were carried out by means of the Unvlm-nl program. The simulation was ended when the force coefficients reached steady values. The foil section had no camber. Free surface effects are known to be relatively strong for the selected combination of Froude number and submergence, see Section 3.3. It is seen in Figure 5.3 that already for $N=4$ and $M=12$ the error in lift and drag coefficients is less than one percent, if extrapolation towards $M^{-1} \rightarrow 0$ is applied. Results for a lifting line approach ($N=1$) differ less than one percent from lifting surface discretizations.

It is noted that similar results were shown for the steady program Hydvlm in Section 3.2. In Hydvlm cosine spaced vortex elements are used in chordwise direction. In Unvlm, equidistant vortex elements are used in chordwise direction. Therefore, the use of cosine spaced vortex elements in chordwise direction does not improve the convergence with the number of chordwise vortex elements, at least if no camber is present.

Figure 5.3 Sensitivity of lift and induced drag to the number of vortex elements for steady conditions
For the same lifting surface, oscillating at a reduced frequency of \( k=0.20 \), it is expected that the sensitivity towards the number of vortex elements in chordwise direction is larger, especially for pitch oscillations. It is noted that for hydrofoil craft, pitch oscillations about a spanwise axis through the forward or aft foil only occur for specific combinations of the heave and pitch oscillations about the centre of gravity of the craft. Figure 5.4a shows that for an error less than 1\%, \( N=8 \) is required, for \( M=16 \). This is slightly higher than found for steady end conditions (\( N=8, M=12 \)). Figure 5.4b shows that the effect of an \( M \) variation for a constant \( N \) is less strong than vice versa, as was expected.

For a cambered foil section a larger sensitivity with respect to the number of chordwise elements is expected as well. The same lifting surface, now with a parabolic camber line with a maximum camber of \( f/c=0.025 \), performing heave oscillations is used. Figure 5.4c shows the dependency of the force amplitude coefficients on the \( N \) value. It is seen that indeed more chordwise elements (\( N \geq 16 \)) are needed than for the previous cases to achieve an inaccuracy lower than one percent. A similar result is found for cambered sections in steady flow conditions.

![Figure 5.4a](image-url)  
Figure 5.4a Sensitivity of lift and induced drag to the number of chordwise vortex elements for heave and pitch oscillations
Figure 5.4b  Sensitivity of lift and induced drag to the number of spanwise vortex elements for heave and pitch oscillations.

Figure 5.4c  Sensitivity of lift and induced drag to the number of chordwise vortex elements for a cambered foil performing heave oscillations.
In order to reduce the number of chordwise vortex elements sectional camber may be represented by an effective incidence. The consequences of such a simplification are shown in Table 5.1. Table 5.1 shows the relative differences between amplitudes (subscript A) and mean values (subscript M) for the force coefficients in X and Z direction for a cambered lifting surface and an uncambered lifting surface. The cambered lifting surface has a zero incidence while the uncambered lifting surface has an incidence equal to minus the zero-lift angle of the cambered section in two-dimensional flow. The lifting surface is performing an oscillatory pitch motion about the quarter chord axis. The reduced frequency is \( k = 0.20 \). The camberline is parabolic with a maximum camber to chord ratio of \( \beta/c = 0.025 \). The number of vortex elements used are: \( N=8 \) and \( M=16 \).

<table>
<thead>
<tr>
<th>Aspect ratio ( AR )</th>
<th>( \Delta C_{ZA} ) (%)</th>
<th>( \Delta C_{ZM} ) (%)</th>
<th>( \Delta C_{XM} ) (%)</th>
<th>( \Delta C_{ZM} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>25.9</td>
<td>0.7</td>
<td>11.1</td>
<td>8.3</td>
</tr>
<tr>
<td>4</td>
<td>47.5</td>
<td>0.8</td>
<td>17.4</td>
<td>11.4</td>
</tr>
</tbody>
</table>

The equivalent incidence approach is only allowed for the amplitude of the lift force. For the other quantities large errors are introduced. This implies that the equivalent incidence approach can only be used in the linear method, where the mean lift force and the amplitude and mean drag force are not taken into account.

The following is concluded. For comparing numerical results for uncambered sections, discretization errors are sufficiently small, less than 1% for the induced drag and less than 2.5% for the lift force, when \( N=4 \) and \( M=16 \) are used in the calculations. For comparisons with analytical and experimental data and applications for practical purposes, \( N=8 \) and \( M=16 \) should be used for errors less than 1%. For cambered sections, \( N=32 \) and \( M=16 \) should be used. The equivalent incidence approach for representing camber can be used in the linearized method only.

The Green's function terms \( G' \)

Figure 5.5 shows the induced drag and lift coefficients versus the chord Froude number \( F_{nc} \) for a rectangular hydrofoil of aspect ratio six at a submergence of one chord. The steady lift and drag ratios resulting from Unvlm-in simulations for an impulsively started lifting surface at otherwise steady flow conditions, are compared with the steady flow Hydvlm results as a check on the computation of the Green function terms \( G' \). In Hydvlm, a Green's function formulation for steady flow is used, while in Unvlm a time domain Green's function description is used. It has already been verified that the Biot-Savart terms \( 1/R \) and \( 1/R_{o} \) yield identical results for the steady and unsteady flow programs. The number and distribution of the vortex elements over the lifting surfaces was identical for both cases. A comparison between the steady flow Hydvlm results and experimental data has been given in Section 3.3. The differences between the Unvlm-nl and Hydvlm results for lift and drag are at maximum 0.5% which is sufficiently small for the present purposes. A smaller error may be achieved by reducing the approximation errors in both the steady and time domain.
Green function terms. This is however not considered useful as several other sources for inaccuracies are present which may have a larger magnitude, such as viscosity effects. It is concluded that the time domain Green’s function is correctly implemented in Unvlm, at least for steady flow conditions.

Figure 5.5a  Comparison Unvlm and Hydvlm result for the induced drag ratio vs the Froude number

Figure 5.5b  Comparison Unvlm and Hydvlm results for the lift coefficient vs the Froude number
The significance of the Green's function $G'$ terms

Before performing further numerical validations it is of interest to show the significance of the Green's function terms $G'$. The $G'$ terms account for the formation of gravity waves. Figure 5.6 shows for three chord Froude numbers the force coefficients for an impulsively started, rectangular and uncambered lifting surface of aspect ratio six, at a practical submergence of one chord. At a low chord Froude number ($F_w = 0.32$) the force coefficients show an oscillating behaviour with a decaying amplitude as time increases. The $G'$ terms are relatively large for this low Froude number; they reach a value of minus two times the biplane image terms at zero Froude number. The frequency of the oscillations corresponds to the critical frequency $\omega = g \tau / U$ with $\tau = 0.25$. It takes a relatively large amount of time before the oscillations have decayed sufficiently. At the intermediate chord Froude number ($F_w = 1.60$) the oscillations are already significantly reduced, while at the highest chord Froude number ($F_w = 6.40$) no oscillations are present and steady flow force coefficients are reached shortly upon the start of the motion. At infinitely high Froude numbers the $G'$ terms are zero. It should be noted that a steady state is also reached earlier in time at a higher speed due to the faster increase in wave length.

![Figure 5.6 Force coefficients vs time for three Froude numbers](image)

Although the oscillating nature disappears at the higher Froude numbers, the $G'$ terms may still be significant since these terms also affect the steady force coefficient values. Table 5.2 shows the ratio's between the steady force coefficients with and without taking the $G'$ terms into account. In the latter case only the terms $1/R$ and $1/R_e$ are taken into account and the force coefficients are then independent of the Froude number. The calculations were performed by means of Unvlm-nil. The force coefficient ratio's are defined as follows:
\[ C_{xr} = \frac{C_{xG}}{C_{xG}} \]

\[ C_{zr} = \frac{C_{zG}}{C_{zG}} \]

Table 5.2  Effect of \( G' \) terms on steady force coefficients

<table>
<thead>
<tr>
<th>( F_{mc} )</th>
<th>( C_{xr} )</th>
<th>( C_{zr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.60</td>
<td>-1.184</td>
<td>0.885</td>
</tr>
<tr>
<td>3.20</td>
<td>-1.066</td>
<td>0.957</td>
</tr>
<tr>
<td>6.40</td>
<td>-1.013</td>
<td>0.991</td>
</tr>
</tbody>
</table>

At a chord Froude number of 6.40, corresponding to cruise speed conditions for hydrofoil craft, it seems permissible to neglect the \( G' \) terms, at least for steady flow conditions. For take-off and low speed cruising the chord Froude number is typically 3, for which the error is about 6 and 4 percent in \( C_x \) and \( C_z \) respectively.

Table 5.3 shows the effect of the \( G' \) terms on the amplitude and phase angle for a rectangular, uncambered lifting surface with an aspect ratio six, performing heave oscillations at a mean submergence of one chord. The reduced frequency \( k \) amounts to 0.25 and the oscillation amplitude is 0.75 chords. The Table shows the amplitude ratios \( C_{xA} \) and \( C_{zA} \) and the phase angles differences \( \Delta \varepsilon_x \) and \( \Delta \varepsilon_z \), defined as follows:

\[ C_{xA} = \frac{C_{xG}}{C_{xG}} \]

\[ C_{zA} = \frac{C_{zG}}{C_{zG}} \]

\[ \Delta \varepsilon_x = \varepsilon_{xG} - \varepsilon_{xG} \]

\[ \Delta \varepsilon_z = \varepsilon_{zG} - \varepsilon_{zG} \]

where the subscripts \( G \) and \( nG \) denote with and without accounting for the \( G' \) terms respectively and the subscript \( A \) denotes the amplitude of the force coefficient.
Table 5.3 Effect of $G^f$ terms on force amplitude and phase for a heaving lifting surface

<table>
<thead>
<tr>
<th>$F_{nc}$</th>
<th>$C_{XAr}$</th>
<th>$C_{ZAr}$</th>
<th>$\Delta \varepsilon_x$ (deg)</th>
<th>$\Delta \varepsilon_z$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.60</td>
<td>2.498</td>
<td>1.600</td>
<td>-9.7</td>
<td>-0.2</td>
</tr>
<tr>
<td>3.20</td>
<td>0.762</td>
<td>1.121</td>
<td>-5.4</td>
<td>-0.1</td>
</tr>
<tr>
<td>6.40</td>
<td>0.920</td>
<td>1.037</td>
<td>-4.6</td>
<td>-4.7</td>
</tr>
</tbody>
</table>

For the lower and intermediate chord Froude numbers the error in the force coefficient amplitude is significant. The error is less than 10% only at highest chord Froude number. The relatively strong $G^f$ term effect is likely due to the oscillatory motion of the lifting surface which reduces the submergence to 0.25 chords at the top position. The phase angle errors are less significant.

Table 5.4 shows the force amplitude ratio and phase angle differences for the conditions with and without $G^f$ terms in an oscillating flow, i.e. in incident waves. The submergence of the lifting surface is again one chord, as for the steady flow case. The error due to the neglect of the Green's function is not large, for the highest chord Froude number. At lower chord Froude numbers the drag force is significantly affected by the $G^f$ terms.

Table 5.4 Effect of $G^f$ terms on force amplitudes and phase for an oscillating flow

<table>
<thead>
<tr>
<th>$F_{nc}$</th>
<th>$C_{XAr}$</th>
<th>$C_{ZAr}$</th>
<th>$\Delta \varepsilon_x$ (deg)</th>
<th>$\Delta \varepsilon_z$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.60</td>
<td>0.824</td>
<td>0.989</td>
<td>-4.0</td>
<td>-7.4</td>
</tr>
<tr>
<td>3.20</td>
<td>0.891</td>
<td>0.987</td>
<td>3.7</td>
<td>-1.8</td>
</tr>
<tr>
<td>6.40</td>
<td>1.027</td>
<td>0.994</td>
<td>-0.6</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Whether it is permissible to neglect the Green's function $G^f$ terms for hydrofoil craft applications, depends on the purpose of the simulations. For steady flow resistance estimates at an intermediate Froude number, the error is fairly large, not only due to the error in drag itself but also due to the lift error which has an additional effect on the drag. For linear seakeeping simulations, i.e. without accounting for hydrofoil motions other than the constant speed of advance, it seems permissible to neglect the relatively small lift errors. Drag errors do not matter for linear seakeeping. For non-linear seakeeping and manoeuvring, i.e. accounting for hydrofoil motions, in which both lift and drag forces are of importance, it is difficult to say beforehand. It depends on the speed, foil submergence and the motion amplitudes of the craft. In Section 5.4 non-linear simulation results for hydrofoil craft, with and without taking into account the Green's function terms will be discussed, which will shed more light upon this matter.

For more slow speed craft, using foils which are not deeply submerged, chord Froude numbers vary roughly from 1 to 4. For this Froude number region it is clear that the Green's function terms need to be taken into account in order to predict the forces acting on the lifting surfaces accurately.
Time step dependence

The time step used in the computational method may affect the results in four ways. Firstly, the time step affects the wake shedding process. This process is a zeroth order process in the sense that the vortex strength shed in the wake is taken from the trailing edge vortex elements from the previous time step. This shed vortex strength has a strong effect on the forces acting on the lifting surface at the actual point in time. Secondly, the time derivative of the circulation, $\partial \Gamma / \partial t$, is approximated by a first order difference formula. Hence a variation in time step will affect the $\partial \Gamma / \partial t$ value, and thereby the force acting on the lifting surface, see eq. (4.68). Thirdly, the integration of the Green's function terms over the wake element area is performed by means of numerical integration with a fixed number of function evaluations. Since the wake element length depends on the time step, the error in the numerical approximation will depend on the time step as well. Fourthly, the wake element length affects the induced velocities on the lifting surface due to the $1/R$ and $1/R_0$ terms. These time step dependencies will be investigated in the next paragraphs. It is noted that for unsteady flow conditions these dependencies can not be investigated separately.

First, the effect of the time step on the wake induced velocities due to the $1/R$ and $1/R_0$ terms will be investigated. Figure 5.7 shows a vortex strength distribution on a two-dimensional lifting surface and its wake. The wake sheet has a length of one chord. The vortex strength on the lifting surface is that for a flat plate at a certain angle of attack. The vortex strength on the wake sheet ($\gamma_w$) increases linearly with the distance from the trailing edge. At the trailing edge the vortex strength is zero. The flat plate vortex strength distribution $\gamma_b$ is given by

$$\gamma_b(\theta) = 2 \left[ \frac{1 + \cos \theta}{\sin \theta} \right]$$

(5.7)

$$\theta = \arccos \left( 1 - 2 \frac{x_i}{c} \right)$$

where $x_i$ is the local chordwise coordinate relative to the leading edge. The vortex distribution is discretized by means of constant strength vortex elements (strips) and vortex lines. For both discretizations the vertical induced velocity at the trailing edge element has been calculated for a variation in wake element length $\Delta x_w$ for three numbers of vortex elements ($N$) on the lifting surface. The wake element length reflects the time step by means of $\Delta x_w = U \Delta t$, while the element length on the lifting surface follows from $\Delta x_i = c / N$. Since the trailing edge region is most affected by the wake, the induced velocity is evaluated at the centre of the last element on the lifting surface. The induced velocity is calculated by using the equations given in Section 2.3, eq. (2.11) and (2.12).
Figure 5.7 Vortex strength distribution on two-dimensional section and wake sheet.

Figure 5.8 shows the induced velocity component (w) normal to the lifting surface, normalized by means of the integral of the vortex strength over the chord and wake length. It should be noted that the position where the velocity is evaluated varies with the number of vortex elements N on the lifting surface. Therefore, the induced velocity varies with N.

The induced velocity due to the wake only is discussed first, see Figure 5.8a. For small wake element lengths the induced velocities due to the vortex strips and lines converge to identical values. The velocity induced by the vortex strips converge more quickly with a reducing element length than the vortex lines. The converged results for the vortex strips approach the exact result for a linear vortex strength distribution, so it can be concluded that a reduction in vortex element length, or in time step, reduces the discretization error made by using vortex ring elements to represent the continuous vortex strength in the wake.

The total induced velocity at the trailing edge, due to the vortex elements on the wake and on the lifting surface, converges to different values for the vortex line and strip discretizations when the wake element length reduces, see Figure 5.8b. This is due to the difference in induced velocity by the vortex lines and strips on the lifting surface. The difference reduces with increasing N. For \( \Delta x_w / \Delta x_r = 1.0 \) however, the induced velocity by the vortex lines and strips are approximately equal. The discretization errors in the induced velocity due to the vortex elements on the lifting surface and on the wake cancel. It can be shown that when the vortex strength on the wake has an opposite sign, relative to that on the lifting surface, the discretization errors do not cancel but add up. However, provided the time step is sufficiently small relative to the period of oscillation, the vortex strength on the wake immediately behind the trailing edge has the same sign as on the lifting surface. For practical conditions for hydrofoil craft, at least 50 time steps are used within one oscillation period which is more than sufficient for equal signed vortex strengths on the lifting surface and on the wake immediately behind the lifting surface.
Figure 5.8a  Wake induced vertical velocity at the trailing edge for a variation in wake vortex element length

Figure 5.8b  Total induced velocity at the trailing edge for a variation in wake vortex element length
It is concluded that the time step should be such that $\Delta t = c/UN$, especially when a low number of vortex elements is used on the lifting surface. For higher numbers of vortex elements the sensitivity with respect to the time step reduces.

This conclusion will be checked for a two-dimensional lifting surface performing sinusoidal heave ($z_0/c = 0.25$) or pitch ($\theta_0 = 5$ deg) oscillations in an unbounded flow. Calculation results for a variation in time step are compared with results from the Theodorsen function, given in eq's. (4.1) through (4.3), in Figure 5.9. The Theodorsen function is based on a small motion amplitude assumption and the linearization of the tangential flow boundary condition on the lifting surface. Calculation results were obtained from Unvlm-ln which uses the same linearization. In the calculations, two-dimensionality was approximated by discarding trailing vortex segments and by using a lifting surface with an aspect ratio of 100. The biplane image terms and the free surface Green's function terms were not taken into account either. Figure 5.9a shows that already after one oscillation, start-up effects have decayed. Start-up effects are due to the impulsive start of the motion and the build-up of the wake sheet. The calculation results for the default time step, $\Delta t = \Delta t_0$, approach the Theodorsen results closest. When the number of chordwise vortex elements $N$ is increased, the Theodorsen results are closely approximated, see Figure 5.9b. Similar results are shown for the pitch motion in Figures 5.9c and 5.9d. Using the default time step indeed gives the best agreement with analytical results for a given number of chordwise vortex elements. Also, analytical results are closely approximated when the number of chordwise vortex elements is increased.
Figure 5.9b  Lift force coefficient for a two-dimensional lifting surface performing an oscillatory heave motion for a variation in number of vortex elements

Figure 5.9c  Lift force coefficient for a two-dimensional lifting surface performing an oscillatory pitch motion for a variation in time step
5.2 Numerical validation, linearization effects and foil interaction

The Wagner function is a special case of the Sears function which describes the growth of the lift force on a two-dimensional lifting surface in an unbounded fluid, after an impulsive start from rest. Herein the same linearization is applied as for the Theodorsen function. The Wagner function, as given by Newman (1977), is shown in Figure 5.10. Calculation results are shown for a variation in time step. In the Unvlm-In calculations, two-dimensionality was again approximated by discarding trailing vortex segments and by using a lifting surface with an aspect ratio of 100. The biplane image terms and the free surface Green's function terms were not taken into account either. Due to the force component proportional to \( \partial \Gamma / \partial t \) and discretization errors, the lift has a larger initial value, for small time steps and a lower initial value for large time steps. The lift force in the Wagner function is actually infinite at \( t=0^\circ \) due to the zero acceleration time. The initial differences reduce with time and the Wagner function is more closely approximated. Also, the results tend to become independent of the time step. This is to be expected since the net strength of the bound vortex segments in the wake reduces to zero. Using the default time step gives a fairly good resemblance with the Wagner function for \( Ut/c>0.5 \).

For a finite aspect ratio lifting surface below the free surface Figure 5.11 shows the effect of the time step on the development of the forces after an impulsive start. Here the free surface Green's function terms are included, so a discretization error in the integration of these terms over the wake element area may be expected. Figure 5.11 shows that this is indeed the case: for the larger time steps an appreciable deviation in the force coefficients appears. The chord Froude number for the cases shown here is \( F_{nc}=1.60 \) for which the Green's function terms \( G_f \) are significant. For the
default time step, the results are less than one percent different from the results for the smallest time step.

Figure 5.10  Lift development after an impulsive start

Figure 5.11a  Induced drag development for a finite aspect ratio lifting surface after an impulsive start, for a variation in time step
Wake roll-up effect

The wake sheet position may be determined by calculating the velocities at each wake element corner point and integrating these in time. This process requires a large amount of computer time. It is therefore desirable not to compute the actual wake sheet position but to assume that once the wake elements are shed from the lifting surface these remain in a stationary planar sheet. A further simplification is to neglect lifting surface displacements other than due to the (constant) speed of advance, so that the wake sheet is a plane surface behind the lifting surface. Influence coefficient matrices can then be used for determining the induced velocities on the lifting surface. This latter simplification is termed linearization, and is implemented in UnvIm-In. The effects of linearization are described later in this Section, here the effect of wake roll-up is described.

The velocity induced by vortex elements is highly singular at locations close to the vortex element. This may introduce numerical instabilities in the displacement of the wake element corner points and the wake sheet shape may become chaotic. If this happens, it will happen at a relatively large distance from the lifting surface, due to the high speed of the craft and the time needed for the wake sheet to roll-up and to become chaotic, so that the effect on the lifting surface itself will be small. Therefore, no desingularization is applied in determining the rolled-up wake sheet shape. Some initial calculations on wake sheet roll-up show that numerical instabilities occur at such a distance behind the forward foil that even the aft foil of a tandem hydrofoil system is not affected by this phenomenon, see Figure 5.30.
Wake sheet roll-up is strongest for a low aspect ratio lifting surface. Figure 5.12 shows the calculated wake sheet shape for a lifting surface with an aspect ratio of three, in steady motion below the free surface. In order to reduce the required computer time, the $G'$ terms were neglected. The roll-up is clearly visible. The similarity of the calculated wake sheet shape with wake sheet shapes which are determined experimentally or by means of a more advanced simulation method is not discussed here. It is sufficient to note that the wake sheet shape looks more or less realistic. The effect on the steady forces by assuming a flat, non-rolled-up wake sheet is found to be negligible, as shown in Table 5.5.

![Figure 5.12 Rolled-up wake shape](image)

**Table 5.5**  Effect of wake roll-up on force coefficients

<table>
<thead>
<tr>
<th></th>
<th>Roll-up</th>
<th>No roll-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_x$</td>
<td>-0.005151</td>
<td>-0.005150</td>
</tr>
<tr>
<td>$C_z$</td>
<td>0.2056</td>
<td>0.2059</td>
</tr>
</tbody>
</table>

Figure 5.13 shows the force coefficients for the same lifting surface performing vertical oscillations with an amplitude of 0.75 chords, with and without accounting for roll-up. Without accounting for roll-up, the wake has a sinusoidal form as shown in Figure 5.14a. The rolled-up wake sheet shape is shown in Figure 5.14b. The wake sheet tends to become chaotic at strongly deformed edges. Virtually no effect due to wake roll-up on the force coefficients is discernable in Figure 5.13.
5.2 Numerical validation, linearization effects and foil interaction

Figure 5.13 Effect of wake roll-up on forces for a heaving lifting surface

Figure 5.14a Wake sheet shapes for linear and non-linear simulations for a heaving lifting surface
Figure 5.14b  Rolled-up wake sheet shape for a heaving lifting surface

Figure 5.15  Force coefficients at a low reduced frequency and deep submergence at zero incidence, heave oscillation
It is noted in Figure 5.13 that the $C_x$-force coefficient shows four maxima per oscillation period while the $C_z$-force coefficient shows only two maxima, proportional to the incidence variations. This can be explained as follows. In a quasi-steady analysis, the longitudinal force $X$, defined in a body-fixed axis system, consists of components due to the induced drag and due to the lift force. The induced drag varies with the lift force squared, or incidence squared, while the lift force component in longitudinal direction is also a function of the incidence squared. Therefore, the longitudinal force will have the character of a squared sine function. The fact that the maxima have a different magnitude is due to the mean incidence of the section and due to free surface and unsteady effects. Figure 5.15 shows that for a zero mean incidence, at a low reduced frequency and deep submergence the longitudinal force coefficient $C_x$ has maxima with an equal magnitude. The character of the lift force coefficient is the same for the upwards and downwards parts of the oscillation.

The wake shape for a lifting surface advancing in waves can be determined by adding the wave orbital velocity components to the velocity components induced at the wake element corner points. For a regular wave with a frequency of $o=1.5$ rad/sec ($T=4.2$ sec; a short wave), the wave length is approximately 27 m which is large relative to the chord of a hydrofoil. The wake sheet will then be almost flat and the effect on the forces will be small. This is confirmed in Table 5.6 which shows the differences between the force coefficient amplitudes for a steady foil advancing in beam waves with a free, rolled up wake and a flat wake. This case concerns the same lifting surface as used in the previous paragraphs. The wave frequency is 1.5 rad/sec and the wave amplitude is 0.50 chords.

<table>
<thead>
<tr>
<th></th>
<th>Free</th>
<th>Flat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{xa}$</td>
<td>$-1.1424x10^4$</td>
<td>$-1.1420x10^4$</td>
</tr>
<tr>
<td>$C_{zh}$</td>
<td>0.032362</td>
<td>0.032367</td>
</tr>
</tbody>
</table>

**Linearization**

Figure 5.16 shows the effect of linearization on the force components acting on a lifting surface performing oscillatory pitch motions about its quarter chord axis. The aspect ratio is six, the submergence is one chord and the chord Froude number is 3.19. Linearization ignores the actual lifting surface displacements, but oscillatory velocities are taken into account. The force coefficients are normalized by means of the amplitude $r$ of the pitch motion. In this way, linear results should be independent of the oscillation amplitude. Furthermore, the magnitude of unsteady force components can be assessed since normalized quasi-steady force components are independent of the frequency of oscillation. The amplitudes $r$ are defined as:

$$r_x = \theta^2$$
$$r_z = \theta$$

(5.8)
For a pitching motion the effects of linearization will be limited since the vertical displacement of the lifting surface due to pitching is small relative to both the submergence and the wake sheet. With increasing frequency, and thereby \( k \), the difference between linear and non-linear results is expected to increase since the ratio between the wake wave length and the foil chord decreases, so that the displacement of the wake in vertical direction, in the area adjacent to the trailing edge, will have a larger effect on the induced velocities on the foil. This appears only true for the \( C_X \) coefficient in Figure 5.16. Furthermore, the effect of linearization is not very large: less than 6\% for \( C_X \) and less than 1\% for \( C_Z \) at \( k=0.50 \). The effect of the oscillation amplitude on the force coefficient values for the non-linear case is also small, less than 1.5\% for both \( C_X \) and \( C_Z \). The linear results are seen to be virtually independent of the oscillation amplitude. The small discrepancies present are most likely due to discretization errors. The phase angles between forces and motions were found to be hardly affected by linearization.

![Graph](image)

Figure 5.16a Linearization effect on \( C_X \) for a pitching lifting surface
5.2 Numerical validation, linearization effects and foil interaction

![Graph showing linearization effect on $C_Z$ for a pitching lifting surface](image)

**Figure 5.16b** Linearization effect on $C_Z$ for a pitching lifting surface

Figure 5.17 shows the effect of linearization for a lifting surface performing oscillatory heave motions below the free surface. The same geometry, discretization and speed were used as for the pitching case. The force coefficients are again normalized by means of the amplitude $r$ of the incidence variation due to the heave motion. The amplitudes $r$ are now defined as:

$$r_X = (z_a \omega / U)^2$$
$$r_Z = (z_a \omega / U)$$

(5.9)

For a heaving motion with a relatively large oscillation amplitude, the vertical displacement of the lifting surface is no longer small relative to both the wake sheet and free surface position, so that significant non-linear effects may be expected. This is confirmed in Figure 5.17. The linear results are again virtually independent of the amplitude of oscillation while the non-linear effects increase both with reduced frequency and oscillation amplitude. For the highest oscillation amplitude and the maximum reduced frequency the error in drag and lift amplitude due to linearization amounts to approximately 10%. Also the phase angles between forces and motions are affected by non-linear effects, the maximum errors due to linearization in phase angle for lift and drag are 8% and 28%, respectively. At reduced frequency values of $k=0.25$, which is a practical limit for hydrofoil craft, the linearization errors are about half of those for the maximum reduced frequency.
Figure 5.17a  Linearization effect on \( C_X \) for a heaving lifting surface

Figure 5.17b  Linearization effect on \( \varepsilon_x \) for a heaving lifting surface
5.2 Numerical validation, linearization effects and foil interaction

Figure 5.17c  Linearization effect on $C_Z$ for a heaving lifting surface

Figure 5.17d  Linearization effect on $\varepsilon_z$ for a heaving lifting surface
Figure 5.18 shows the free surface effect on linearization for the same lifting surface, by comparing linear and non-linear results for a deeply submerged and a one chord submerged lifting surface, oscillating with an amplitude of 0.75 chords. For the $C_x$-coefficient the linearization effect at deep submergence is somewhat smaller than at the submergence of one chord. For the $C_x$-coefficient the linearization effect is nearly the same. Similar trends are seen for the phase angles. The Figures show only the amplitudes of the force coefficients. The mean values are also affected by linearization, for the case where the submergence is one chord. The difference between non-linear and linear results for $C_x$ and $C_Z$ are: +7% and -1% of their amplitude, respectively. It can be concluded that the difference in lift force due to linearization is predominantly due to the neglect of displacements of the lifting surface relative to the wake sheet and to a lesser extent due to the neglect of submergence variations, i.e. variations in the free surface effects. For the drag force free surface effects on linearization are larger than for the lift force.

![Figure 5.18a Linearization effect on $C_x$ for a heaving lifting surface at a varying submergence](image)

AR=6, N=4, M=16, $F_s=3.19$, $\alpha=0.0$, $f/c=0.0$, $x_f=0.75$

- $\bigcirc$ Unvlm-nl
- $\square$ Unvlm-ln
- $-$ $h/c=1.0$
- $-\cdash- h/c=\infty$
5.2 Numerical validation, linearization effects and foil interaction

Figure 5.18b Linearization effect on $\varepsilon_x$ for a heaving lifting surface at a varying submergence

Figure 5.18c Linearization effect on $C_z$ for a heaving lifting surface at a varying submergence
Wake length

The number of wake elements may be in order to reduce CPU and memory requirements. The effects of the wake length on the lift and drag forces on an impulsively started, rectangular lifting surface are shown in Figure 5.19. The aspect ratio is six and the submergence is one chord. The number of wake elements used for the two Froude number cases is the same, ranging from 1200 to 4800 elements respectively for the lower and upper value for the wake length to chord length ratio. As the time step used equals $\Delta t = c/NU$, the length of a wake sheet element $\Delta t U$ is the same for the two cases. The total wake length is proportional to the Froude number for equal simulation times.

The relatively strong oscillations in lift and drag for the lower Froude number case are due to the $\tau_e = \frac{1}{4}$ singularity in the Green's function term $G'$. Limiting the wake length effectively dampens the oscillations. The truncation results in an error for the steady force coefficient values. It should be noted that the low Froude number case is only of academic interest, at least for hydrofoil craft. At a higher Froude number both the frequency and the amplitude of the oscillations are much smaller and the errors in steady force coefficients due to the truncation of the wake sheet are much smaller as well. For a wake length of 75 chords the truncation error in the force coefficients is less than 0.1%.
5.2 Numerical validation, linearization effects and foil interaction

Figure 5.19a  Effect of wake sheet length on forces for a low Froude number

Figure 5.19b  Effect of wake sheet length on forces for an intermediate Froude number
Fore-aft foil interaction

Foil interaction occurs for hydrofoils in a tandem arrangement and is caused by the velocity components induced at the aft foil by the wake sheet emanating from the forward foil and by the waves generated by the forward foil. In the following paragraphs only the first phenomenon is dealt with. Under normal operating conditions the aft foil is continuously positioned in or close to the forward foil wake sheet. Very high velocities are then induced at control points located close to vortex segments in the wake sheet. This may result in unrealistically large force fluctuations and an instable dynamic behaviour of the hydrofoil craft during time domain simulations.

For steady conditions this problem may be prevented by using a suitable vortex element discretization on the foils such that the aft foil control points are suitably located in between the trailing vortex line segments of the forward foil wake sheet, assuming flat, non rolled-up wake sheets are used. For steady conditions, the bound vortex line segments on the wake vortex lattice have a zero strength so that these do not contribute to the induced velocity. A problem may occur at the start of the simulation however, due to the first rows of vortex elements shed by the forward foil which do carry a nett vortex strength, including the start vortex. When the aft foil reaches these rows of wake sheet elements, a discontinuity in the induced velocities and resulting forces will occur.

For unsteady conditions the interaction problem is complicated by the non-zero bound vortex strength and the fact that the shape of the forward foil wake sheet and the position of the aft foil control points, are unknown a priori (in the non-linear method). As a result, aft foil control points are irregularly distributed on the forward foil wake sheet. This causes large discretization errors and possibly singular induced velocity components.

In order to prevent such a behaviour, it is required to limit induced velocities in magnitude for control points located close to or exactly on a vortex line segment. At the same time, some kind of smoothing must be applied to reduce discretization errors. A possible procedure is to position aft foil control points in the centre of the wake sheet vortex elements when a certain critical distance between the aft foil control point and the wake sheet vortex element is reached. This procedure does not lead to satisfactory results since the relocation of aft foil control points leads to irregularities in the induced velocities and hence in the forces acting on the aft foil.

Better results are obtained by using a so-called solid-body vortex core, see Figure 5.20, to desingularize the induced velocities and to reduce discretization errors. The desingularization is performed as follows. When a control point is positioned within a distance $\frac{1}{2}\delta_s\Delta s$ perpendicular to a vortex line segment, its distance perpendicular to that vortex line segment is set at $\frac{1}{2}\delta_s\Delta s$ while the velocity induced by the vortex line segment is reduced by a factor $2r_e/(\delta_s\Delta s)$. Herein $r_e$ is the actual distance from the control point perpendicular to the vortex line segment and $\Delta s$ is the length or width of the vortex segment, as appropriate.

It should be noted that by using this procedure, the disturbance potential no longer satisfies the Laplace equation. However, this is not considered as a serious shortcoming of the procedure. The induced velocities at the aft foil due to the forward foil wake sheet can be considered as an external disturbance which can not be modelled in full agreement with the physical reality anyway. In reality
the wake sheet can not pierce the aft foil, in the present method it can. Furthermore, the procedure used here in essence results in an interpolation on the induced velocities obtained for control points located in the centre of wake sheet vortex elements, which remain unaffected by the procedure.

In the following paragraphs interaction effects will be investigated numerically, whereby the effect of the vortex core diameter parameter $\delta_c$ will be investigated for flat, curved and rolled-up wake sheets.

![Tangential velocity around a two-dimensional vortex line with solid-body vortex core](image)

Figure 5.20 Tangential velocity around a two-dimensional vortex line with solid-body vortex core

Figure 5.21a shows the vertical induced velocity at a control point travelling on a flat vortex lattice, with a linearly varying circulation per vortex element. A large discontinuity in velocity appears on entry and exit of the vortex lattice. The magnitude of the discontinuity is obviously affected by the $\delta_c$-value: increasing $\delta_c$ reduces the peak value. It is seen that for $\delta_c=1.0$ a smooth variation of the induced velocity as a function of its position appears when the control point is within the lattice. The velocity varies smoothly between the positions $x_c=0.875$ to $x_c=1.125$, as shown in detail in Figure 5.21b. At these positions no correction on the induced velocity is applied, for $\delta_c=1.0$. For a larger $\delta_c$ value the induced velocity reduces too much. Therefore, taking $\delta_c=1.0$ seems to be a sensible approach. Figure 5.21c shows the induced velocity when the point is travelling at various distances below the lattice. Some waviness is present in the induced velocity for distances smaller than the dimension of the vortex elements in the lattice. However, this does not lead to waviness in the simulation results as the control point is each time step positioned below a successive vortex element of the lattice, when the default time step is used.
Figure 5.21a  Vertical velocity at point travelling on a horizontal vortex lattice

Figure 5.21b  Vertical velocity at point travelling on a horizontal vortex lattice, detail
Figure 5.21c Vertical velocity at point travelling at various distances below a horizontal vortex lattice

Figure 5.22 shows the discontinuity in force coefficients for an aft foil entering the forward foil wake sheet, after an impulsive start-up of the simulation. The geometries of the forward and aft foils are identical. The subscripts \( a \) and \( f \) denote the aft and forward foil respectively. When the foil spacing \( \Delta x \) is such that \( \Delta x/U\Delta t \) is an integer value, the aft foil control points lie exactly in between the bound vortex line segments of the forward foil lattice. With \( \delta_t = 1.0 \) set for this case, the desingularisation has no effect. Only on entry of the aft foil on the forward foil wake sheet a discontinuity is present. Hereafter, a steady value is quickly established as the circulation on the forward foil wake sheet approximates a constant value in the motion direction and the bound vortex strength on its wake sheet reaches a zero value. Note that interaction reduces the aft foil lift by a factor two.

For the second case, for which \( \Delta x = 20.0 + 0.5t \), the foil spacing is such that the aft foil control points always fall exactly on the bound vortex segments of the forward foil wake sheet. Here, \( t \) denotes the length of a wake vortex element in streamwise direction. The induced velocity due to these bound vortex elements is zero, then. Some effect on the lift and a larger effect on the drag are found for the discontinuity in the force coefficients when entering the forward foil wake sheet.

In the third case, a small offset is applied to the foil spacing, \( \Delta x = 20.0 + 0.6\delta_t \), so that the control points and bound vortex positions of aft and forward foil vary just a bit more than the limiting radius \( 0.5\delta_t \). This has a large effect on the discontinuity for especially the drag force, which is sensitive to the vertical induced velocity.

Unfavourable effects due to these discontinuities in the aft foil forces can be easily avoided in the time domain by keeping the hydrofoil craft captive for a brief period after the start of the
simulation. Once the aft foil has passed the forward foil start vortex, a steady condition is reached quickly, within a distance of twice the foil spacing. Therefore, the discontinuity present when the aft foil enters the forward foil wake sheet does not form a problem for the time domain simulations.

Figure 5.22a Interaction effect on $C_X$ when the aft foil reaches the forward foil wake sheet

Figure 5.22b Interaction effect on $C_Z$ when the aft foil reaches the forward foil wake sheet
Figure 5.23 shows the lift and drag force coefficients for both the forward and aft foils, for the foil configurations used earlier in Figure 5.22. Both foils perform oscillatory heave motions in phase. The wake sheets are still flat, i.e. linear simulations were performed. When the foil spacing is varied, the position of the aft foil relative to the circulation in the forward foil wake sheet varies, and so will the induced velocities relative to the aft foil motions. Interaction effects on the aft foil forces are again strong. Furthermore, the lift force on the aft foil is not very sensitive to the foil spacing. The drag force is more sensitive to the foil spacing, but the differences are still small relative to the total interaction effect.

Figure 5.24 shows the effect of varying the number of chordwise vortex elements on both the forward and aft foils. This affects the dimension of the wake sheet vortex elements as the default time step used is related to the number of chordwise vortex elements. Otherwise, the conditions are identical to those used in Figure 5.23. The effect of increasing the number of chordwise vortex elements on the interaction is seen to be very small, i.e. it is hardly discernable in the results. Figure 5.24 also shows the effect of reducing the time step, without reducing the chordwise number of vortex elements. The time step is taken as one third of the default one so that the aft foil control points are more irregularly positioned on the forward foil wake sheet. It is seen that the initial discontinuity is obviously much sharper, but that the effect on the aft foil forces is not much larger than the effect on the forward foil itself.

Figure 5.23a Unsteady interaction effect on $C_x$ for a linear simulation, for a heaving lifting surface
Figure 5.23b Unsteady interaction effect on $C_Z$ for a linear simulation, for a heaving lifting surface

Figure 5.24a Unsteady interaction on $C_X$ for a linear simulation and varying time step, for a heaving lifting surface
5.2 Numerical validation, linearization effects and foil interaction

For non-flat wake sheets, present in non-linear simulations, the aft foil control points may enter and leave the forward foil wake sheet at arbitrary positions and at oblique directions relative to the wake sheet of the forward foil. Figure 5.25 shows the time traces of the aft foil force coefficients in unsteady interaction, for the linear and non-linear cases. The conditions are identical as used for Figure 5.23. The forward foil wake sheet is curved, but not rolled-up. The curved wake sheet shapes of forward and aft foils are shown in Figure 5.26. For this case the interaction is strong as the ratio between the wake wavelength and the foil spacing, $\pi c/k\Delta x$, is close to one. For heave oscillations, the aft foil operates then continuously close to the forward foil wake sheet. Figure 5.25 shows that the use of a curved wake sheet, in the non-linear case, affects the interaction effect on the forces, relative to that for a flat wake sheet. Note that some difference in force coefficients is also due to the linearization itself, as can be observed from the force coefficients acting on the forward foil.

Figures 5.27 and 5.28 show the force coefficients and wake sheet shapes for the same case, but here the foils oscillate at a lower frequency. The ratio between the wake wave length and the foil spacing is approximately two, which makes the aft foil to cross the forward foil wake sheet each half oscillation period ($U/t=21j$ sec where $j=1,2,3,...$). Interaction effects on the aft foil are much smaller than for the case shown in Fig 5.24, where the aft foil is continuously located in the forward foil wake sheet. The differences in the aft foil force coefficients for the linear and non-linear cases are not larger at instants when the distance between the aft foil and the forward foil wake sheet is large, than at instants when this distance is small. The time traces do not show either that the interaction becomes very strong at instants when the aft foil crosses the forward foil wake sheet. Finally, the differences in lift force for the linear and non-linear cases are not much larger than due...
to the linearization itself while the drag force is somewhat more affected.

Figure 5.25a Unsteady interaction effect in $C_Y$ and $C_Z$ for linear and non-linear simulations, for a heaving lifting surface.

Figure 5.25b Linearization effect on forward foil force coefficients, for a heaving lifting surface.
Figure 5.26 Curved wake sheet shapes, side view

Figure 5.27 Unsteady interaction effect on $C_X$ and $C_Z$ for linear and non-linear simulations, for a heaving lifting surface
The effect of wake sheet roll-up for unsteady conditions is shown in Figure 5.29 for an infinite submergence. An infinite submergence was selected in order to reduce the required computer time for the computation of the wake roll-up to reasonable amounts. At an infinite submergence the biplane image and Green’s function terms $G'$ may be neglected.

Figure 5.29 again shows that the effect of wake sheet roll-up is negligible on the forces acting on the forward foil, i.e. without interaction: there appear no discernable differences between the curves for the non rolled-up and rolled-up wake sheets.

Furthermore, using a rolled-up forward foil wake sheet leads to irregularities in the aft foil force coefficients when a small $\delta_e$ value is used. When the standard value $\delta_e = 1.0$ is used, the irregularities reduce considerably in magnitude. Also, differences appear for the force amplitudes for the rolled-up and non rolled-up cases, especially for the lift force. Note that for this case the aft foil operates continuously in the partially rolled-up wake sheet of the forward foil, see Figure 5.30, for which interaction effects are strong.

Figure 5.31 shows that the effects of wake sheet roll-up are small for the lower frequency simulation, for which the aft foil does not operate continuously in the forward foil wake sheet, similar to the conditions shown in Figure 5.27.
Figure 5.29a Wake sheet roll-up effect on forward foil, for a heaving lifting surface

Figure 5.29b Wake sheet roll-up effect on interaction, for a heaving lifting surface
Figure 5.29c  Wake sheet roll-up effect on interaction, detail

Figure 5.30  Rolled-up wake sheet shape
5.2 Numerical validation, linearization effects and foil interaction

Figure 5.31 Wake sheet roll-up effect on interaction, for a heaving lifting surface

Conclusions

For unsteady flow conditions, calculation results converge to constant values if an increase in the number of chordwise vortex elements is accompanied by a reducing time step, according to $\Delta t = c/NU$. For a sufficient numerical accuracy, i.e., an error less than 1%, at least 8 chordwise and 16 spanwise vortex elements need to be used for an uncambered lifting surface in unsteady motion. For lifting surfaces with cambered sections, 32 chordwise and 16 spanwise vortex elements are required. These requirements are similar as found for steady flow conditions which indicates that the errors introduced by the time stepping process are small. Representing camber by an effective incidence is only allowed in the linear simulation method.

For steady flow and for oscillatory flow, i.e., incident waves, the significance of the free surface Green's function term $G^f$ is small for chord Froude numbers experienced at cruise speed. For oscillatory heave motions below the free surface the $G^f$ terms are significant when the oscillation amplitude is sufficiently large ($z_h/h \geq 0.75$).

When the default time step is used, $\Delta t = c/NU$, discretization errors in the induced velocity at the trailing edge, due to the use of discrete vortex elements to represent the continuous vortex distribution on the lifting surface and in the wake, approximately cancel. Therefore, this default time step should be used in the time stepping process. If a smaller time step is required due to other requirements, for instance the ride control system, the number of chordwise vortex elements must
requirements, for instance the ride control system, the number of chordwise vortex elements must be increased accordingly.

Results for oscillating and impulsively started two-dimensional lifting surfaces, derived from analytical methods developed by Theodorsen, are well approximated by the present method.

The rolling-up of the wake sheet has no significant effect on the forces acting on the lifting surface from which it emanates.

Effects due to linearization, i.e. disregarding the motions of the lifting surface, increase with increasing reduced frequency. The differences with non-linear results are mostly due to the neglect of wake sheet displacements, relative to the lifting surface, and not due to variations in the foil submergence. Errors in forces due to linearization for practical reduced frequencies, i.e. \( k \leq 0.25 \), are limited to 5%.

The truncation of the wake length is permitted at medium and higher chord Froude numbers without causing significant errors. A wake sheet length of 75 chords is more than sufficient for practical purposes.

The induced velocities due to a wake sheet need to be desingularized and smoothened in order to prevent irregularities in the aft foil force coefficients due to fore-aft foil interaction. A simple but effective vortex core model is used for this purpose. A practical value for the vortex core diameter or desingularization parameter is \( \delta_c = 1.0 \). For this value smooth time traces for the aft foil forces are obtained for flat, curved and rolled-up forward foil wake sheets. Furthermore, at the start of time domain simulations it is required to keep the hydrofoil craft in a captive mode until the forward foil start vortex is at a sufficiently large distance behind the aft foil. Interaction effects are sensitive to curving and rolling-up of the wake sheet if the combination of foil spacing and oscillation frequency is such that the aft foil continuously operates in or close to the forward foil wake sheet. Interaction effects are then much stronger than for conditions where the foil spacing is much smaller than the wake wavelength. For these latter conditions, it is permitted to use flat wake sheets, i.e. a linear method, without significantly affecting the interaction effect on lift.
5.3 Validation of Unvlm

Prior to showing and validating time domain simulation results for hydrofoil craft it is useful to validate calculation results for various components relevant for the dynamic behaviour of hydrofoil craft. To this purpose comparisons will be made with experimental data in this Section. The number of vortex elements used is sufficient for having numerical results with a convergence error less than 1%.

Scale effects

In Section 2.6 viscosity effects for steady flow conditions were described and found to be significant. For unsteady flow conditions only a few experimental data on viscosity effects for non-separated flow conditions were found in literature. For hydrofoil craft quasi-steady forces are dominant so that it may be expected that scale effects similar to those for steady flow conditions are present during model testing. As to full scale conditions, the increase in lift slope due to thickness will approximately balance the viscosity effect on the lift slope. When validating calculation results with model tests data obtained at Reynolds numbers below 10⁶, it is expected that the calculated potential flow lift will be too large. For validating calculation results on basis of full scale data, scale effects are assumed to be small.

Forces on oscillating lifting surfaces

In Section 5.2 it was shown that the forces predicted by Unvlm on a two-dimensional lifting surface oscillating in an infinite fluid are close to results from an analytical method. Here the Unvlm method will be validated on basis of experimental data for a finite aspect ratio foil oscillating at a large submergence and for a semi-infinite aspect ratio foil oscillating below the free surface. Although these cases are of limited practical interest for hydrofoil craft, they form a useful contribution to the validation of the Unvlm method for, for instance, stabilizing fins.

For the first case experimental data is provided by Kyozuka et al. (1990). The experiments were conducted in a circulating water tunnel with a free surface. Lifting surfaces with a varying planform and aspect ratio were oscillated in a direction normal to the chord line (heave) at various combinations of frequency and free stream velocity. The lifting surfaces were supported by a centreline strut giving a submergence of 3 chords. The combination of this submergence with the low aspect ratio (AR≤2) and the low chord Froude numbers (Fr<1) was to preclude any significant free surface effect on the lift force acting on the lifting surface. This has been checked by Unvlm calculations. The Reynolds numbers varied between 1.2x10⁵ and 2.3x10⁵. For the present case a flat plate lifting surface with a rectangular planform and with an aspect ratio of two was selected. The oscillation amplitude was a quarter chord.

Figures 5.32a and 5.32b show the normalized lift force amplitude C_L/r, r=ω_n/U, and its phase angle θ with respect to the oscillatory motion. It is seen that at low reduced frequencies the experimental force amplitude is lower than the calculated one. However, experimental results tend to get closer to the calculation results with increasing Reynolds number. At a zero reduced
frequency the normalized force amplitude represents the lift curve slope in steady flow. These were determined experimentally as well and show an increase in lift curve slope with the relatively small increase in Reynolds number. Therefore, the differences between calculated and experimental force amplitudes are probably due to viscosity effects. For the higher reduced frequency values the experimental results tend to increase faster with the reduced frequency than the calculation results. Otherwise, the comparison is fairly good. The phase angle is predicted quite well by Unvlm-nl. Increasing the Reynolds number does not appreciably affect the phase angle.

For the second case experimental data is provided by Kyozuka (1992). The experiments were conducted in a circulating water tunnel with a free surface. A rectangular lifting surface with a NACA-0012 section and aspect ratio $AR = 4.5$ was oscillated in a direction normal to the chord line (heave) at a constant free stream velocity ($F_{nc} = 0.71$). The centre section of the lifting surface (55% of the span) was mounted in between two dummy span parts. The loads were measured on the centre section only so that results for a two-dimensional flow were approximated. In Unvlm-nl the forces acting on the appropriate part of the span were taken. The Reynolds numbers was $1.7 \times 10^5$. The oscillation amplitude was 10% of the chord. The mean submergence to chord ratio was $h/c = 0.9$.

Figures 5.32c and 5.32d show the normalized force amplitude, $C_{zU}/r$, $r = \omega z / U$, and its phase angle $\varepsilon$, with respect to the oscillatory motion. The variation in force amplitude with reduced frequency is well predicted. The dip in force amplitude at $k = 0.25$ is due to wavemaking effects. This reduced frequency corresponds to the critical frequency $\tau_c = 0.25$. A shift in phase relative to $\varepsilon_c = 270$ deg. also appears at this frequency. The phase angle is well predicted at low reduced frequencies but deviates at higher reduced frequencies.

![Figure 5.32a Comparison experimental and calculated force amplitudes for a low aspect ratio lifting surface oscillating at a large submergence](image-url)
Figure 5.32b  Comparison experimental and calculated phase angles for a low aspect ratio lifting surface oscillating at a large submergence

Figure 5.32c  Comparison experimental and calculated force amplitudes for an infinite aspect ratio lifting surface oscillating below the free surface
Wave induced forces on a plain hydrofoil

Figure 5.33 shows experimental and calculated wave induced forces on a hydrofoil versus the reduced frequency of encounter $k_e$ for two chord Froude numbers. The frequency of encounter $\omega_e$ in head waves and reduced frequency of encounter $k_e$ are defined as:

$$\omega_e = \omega \left(1 + \frac{\omega U}{g}\right)$$

$$k_e = \frac{\omega_e c}{2U}$$

(5.10)

where $U$ is the speed of advance, $\omega$ is the wave frequency, $c$ is the foil chord and $g$ is the gravitational constant. The experimental data are obtained from Wilson (1983). The foil had an aspect ratio of six, a submergence of half a chord, and advanced in head waves. The foil had a NACA 64A010 symmetrical section. The chord Reynolds number was larger than $1 \times 10^6$ so that the steady lift curve slope will be close to $2\pi$. The regular waves had an amplitude of approximately 0.22 foil chords.
5.3 Validation of Unvlm

Figure 5.33a Comparison experimental and calculated wave induced drag coefficients

Figure 5.33b Comparison experimental and calculated wave induced lift coefficients
The drag force is in good agreement with experimental data, see Figure 5.33a. No reliable phase angle information for the drag could be obtained from the experiments. The drag force was determined from the experimental data by Wilson by subtracting an assumed viscous drag from the total measured resistance. The normalized amplitude for the lift force \( C_{l_{\infty}}/\rho \), \( r=2\zeta/c \), is well predicted in Figure 5.33b. No explanation is found for the large deviation of the phase angle of the lift force for the higher Froude number case in Figure 5.33c.

It is further noted that the Froude number has a large effect on the amplitude of the lift force coefficients. For a higher Froude number, the amplitude of the force coefficients decreases due to the higher velocity which reduces the incidence variation. Note that the amplitudes of the excitation forces do increase.

**Forces on an oscillating tandem foil system**

An extensive series of measurements has been conducted at MARIN for a fully submerged, tandem foil system, see Kapsenberg and Keukens (1991). The measurements consisted of calm water resistance tests, described in Section 3.3, and oscillation and wave excitation tests. These latter tests were performed to obtain information on the hydrodynamic characteristics of the craft which were needed for the development of a ride control system.

The foils were each connected to a six-component measurement frame which were in turn connected to a stiff beam mounted underneath a hydraulic oscillator fixed to the towing carriage. This set up
enabled the measurement of the lift, drag and side force on each foil advancing at a constant horizontal speed while performing harmonic oscillations about a certain mean position. The phase of each separate oscillator leg could be selected arbitrarily so that heave and pitch motions could be performed. By mounting an oscillator with legs moving in the horizontal plane, a roll motion could be imposed. The centre of rotation for pitch and roll oscillations was the centre of gravity of the craft. The force components on each measuring frame have been combined into moment components about the centre of gravity of the craft. The tests were performed in the High Speed basin of MARIN. This basin measures 220 x 4.0 x 3.75 metres in length, width and water depth respectively.

The foil system consists of a forward and an aft foil, both of the inverted π-type. The aft foil is designed to carry about 66% of the craft weight. Figure 3.12 shows the foil systems involved, while Figure 3.13 shows the experimental set-up.

The measurement data were recorded on digital tape. The sampling rate was 100 Hz. The experimental and calculated time series were analyzed by MARIN's computer program Harman, see Papoulis (1962). In Harman a harmonic analysis is performed, yielding the mean value of the signals and the amplitude and phase of the signals up to the fifth harmonic. The definition of the phase angle is given in Figure 5.2. For all signals, only the first harmonic was of significance. No drag forces have been analyzed, as these were of minor interest for the development of a ride control system.

In the following paragraphs comparisons are given between calculated and experimental results. The calculated results are based on results obtained with the linear calculation model. The foil submergence, motion amplitude and reduced frequency range are such that, based on the results of the numerical validation described in Section 5.2, it was expected that the differences between the linear and non-linear methods would be much smaller than the scatter in the experimental data. A number of comparisons between results for non-linear and linear versions of Unvlm showed that differences were within 1.5% for the force amplitudes and within 2.5 degrees for phase angles. These differences were also obtained for the aft foil, so that for the cases considered here the foil interaction seems to be adequately described by the linear model. Some calculations with the non-linear model with wake roll-up effects included show that the results for the aft foil were not significantly affected by wake roll-up effects. This is in agreement with the findings in Section 5.2, where it was shown that wake roll-up can be neglected for values of the ratio between the wave length in the wake and the foil spacing above one. For the cases considered here this ratio varies between 1.6 and 12.6 which indicates that the aft foil is not continuously located close to the forward foil wake sheet.

Each foil was represented by 8 (N) chordwise and 48 (M) spanwise vortex elements. The camber of the foils was represented by an incidence equal to the zero-lift angle of attack at the experimental Reynolds number. The Reynolds number for these tests was 5x10^5. No corrections for scale effects have been applied on the experimental or calculation data.

Repeatability tests showed that the scatter in lift force of the individual forward and aft foils is about 2% and that the scatter in phase angle is about 5 degrees. The scatter in the total experimental force
amplitude and phase angle for forward plus aft foil may however be larger when the forward and aft foil contributions have a different sign and, consequently, the magnitude of the sum is relatively small.

All force coefficients shown are normalized by the ratio between the amplitude of the oscillation velocity and the forward speed. Unsteady (interaction) effects not explained by a quasi-steady approach are then present if the force coefficient varies with the reduced frequency $k$. The velocity amplitude ratio $r$ is defined as follows:

\[
    r = \frac{\omega z_a}{U} \quad \text{heave oscillation}
\]

\[
    r = \frac{\omega \theta_a l_r}{U} \quad \text{pitch oscillation}
\]

\[
    r = \frac{\omega \phi_a l_r}{U} \quad \text{roll oscillation}
\]

where $\omega$ is the frequency of oscillation, $z_a$, $\theta_a$ and $\phi_a$ denote the heave, pitch and roll motion amplitudes respectively and $l_r$ denotes the foil spacing or main foil span for pitch and roll motions respectively.

Figure 5.34 shows the lift force and pitch moment coefficients and their phase angles for an oscillatory heave motion. These quantities are given for the forward foil, the aft foil and the forward plus aft foils. It is seen that the lift force and pitching moment amplitudes are reasonably well predicted. There exists a tendency to overpredict the damping which is presumably due to scale effects. The force and moment coefficients for the forward foil are independent of the reduced frequency. For the aft foil some dependency on the reduced frequency is observed, which is most likely due to foil interaction. This is shown in Figure 5.35 where calculated aft foil results are given with and without foil interaction. Without foil interaction the heave force amplitude is only weakly dependent on the frequency, with interaction the experimentally observed dependency is also shown by the calculation results.

The phase angles differences for the forces lie within the experimental scatter. For the pitching moment, the contributions due to the forward and aft foil lift forces have almost equal amplitudes but also an opposite phase (180 deg difference). Therefore, the forward plus aft foil pitching moment has a small magnitude and a phase angle which can be offset by 90 degrees, relative to the phase angles for the individual pitching moments. This offset in phase angle is well predicted.
5.3 Validation of Unvlm

Figure 5.34a Comparison experimental and calculated $C_{z'}$-coefficient for heave oscillations

Figure 5.34b Comparison experimental and calculated $\xi_{z'}$-phase angle for heave oscillations
Figure 5.34c  Comparison experimental and calculated $C_M$-coefficient for heave oscillations

Figure 5.34d  Comparison experimental and calculated $\epsilon_M$-phase angle for heave oscillations
5.3 Validation of Unvlm

Figure 5.35 Unsteady fore-aft foil interaction effect on aft foil

Figure 5.36 shows the same quantities for a pitch oscillation. Now the forward plus aft foil heave force is small while the forward plus aft foil pitch moment is large. The force amplitudes are again reasonably well predicted. Again probably due to scale effects the calculated damping is larger than the experimental one.

The offset in phase angle for the forward plus aft foil heave force is overpredicted, although the trend is properly indicated. Presumably, small errors in the amplitude of the forward and aft foil heave forces introduce a relatively large phase error when the signals are summed, due to the relatively small magnitude of this sum.

Figure 5.37 shows the side force, roll moment and yaw moment amplitudes and phase angles for an oscillatory roll motion. A reasonably good comparison exists between calculation results and experimental data. The calculation results for the forward foil systematically overpredict the damping forces and moments. However, such a systematic overprediction is less clear for the aft foil.

Finally, both the forward and aft foil forces and moments are virtually independent of the reduced frequency which indicates a weak fore-aft foil interaction.
Figure 5.36a Comparison experimental and calculated $C_{z'}$-coefficient for pitch oscillations

Figure 5.36b Comparison experimental and calculated $\varepsilon_{z'}$-phase angles for pitch oscillations
5.3 Validation of Unvlm

Figure 5.36c  Comparison experimental and calculated $C_M$-coefficient for pitch oscillations

Figure 5.36d  Comparison experimental and calculated $\varepsilon_M$-phase angle for pitch oscillations
Figure 5.37a  Comparison experimental and calculated $C_r$-coefficient for roll oscillations

Figure 5.37b  Comparison experimental and calculated $\epsilon_r$-phase angle for roll oscillations
5.3 Validation of Unvlm

Figure 5.37c Comparison experimental and calculated $C_K$-coefficient for roll oscillations

Figure 5.37d Comparison experimental and calculated $\varepsilon_k$-phase angle for roll oscillations
Figure 5.37e  Comparison experimental and calculated $C_N$-coefficient for roll oscillations

Figure 5.37f  Comparison experimental and calculated $\epsilon_N$-phase angle for roll oscillations
5.3 Validation of Unvlm

Wave Induced Loads

For the same tandem foil system as described in the previous paragraphs experiments were carried out in regular waves. During these experiments the foil system was stationary relative to the towing carriage. The experiments were carried out in the High Speed basin (head waves) and the Seakeeping basin (oblique waves) of MARIN. This latter basin measures 100 x 24 x 2.50 metres in length, width and water depth respectively.

The experimental and calculated signals were again analyzed by means of the Harman program, yielding the amplitude and phase of the harmonic components. For all signals, only the first harmonic component was significant. Again, no drag forces have been analyzed.

A similar scatter for the force amplitude and phase angles as found for the oscillation tests was found from repeatability tests. The Seakeeping basin is relatively shallow, but the speed for the tests in oblique waves was also relatively low so that the Froude number based on water depth amounted here to $F_{nh}=0.90$. Therefore, the remarks in Appendix D on blockage effects are also valid for these tests.

No significant differences between linear and non-linear calculation results have been found. As shown in the following Figures there appears again a tendency that the wave excitation is overestimated in Unvlm, probably due to scale effects. Furthermore, the experimental results for the lowest frequencies may be a bit inaccurate. Due to the high speed of the model and the limited model basin length, the wave induced loads could only be measured for a duration of one (beam waves) to two (head waves) wave periods.

Figure 5.38 shows the lift force and pitch moment in head seas ($\psi_w=180$ deg). The comparison between experimental and calculation data is generally satisfactory. When the wave length equals twice the distance between the foils ($k_r=0.08$), the forward plus aft foil lift force is minimal due to the 180 degree phase difference between the forward and aft foil components. The total lift force is at a maximum when the wave length equals the foil distance ($k_r=0.15$). The pitch moment shows an opposite behaviour: it increases when the phases of forward and aft foils are opposite.

The wave amplitude for the cases described here was approximately 30% of the foil submergence. In view of the generally good resemblance between experiments and calculations, the linearization of the free surface conditions, which does not account for variations in submergence due to wave elevations, seems permissible for the present fully submerged foil configuration.

Figure 5.39 shows the side force, lift force, roll and yaw moments for beam wave conditions ($\psi_w=270$ deg). For beam wave conditions, not only the horizontal foil parts but also the struts experience incidence variations and significant side forces, roll and yaw moments are generated. The heave force originates from foil incidence variations along the span.

For the cases shown here, the wave amplitude amounted to about 30% of the submerged strut length, therefore significant wetted strut length variations were experienced. This wetted area variation was not taken into account. As the resemblance between measurements and calculations is within the measurement accuracy for most cases this seems to be allowed for the present case.
Figure 5.38a  Comparison experimental and calculated wave induced $C_{z}$ coefficient in head waves

Figure 5.38b  Comparison experimental and calculated $\varepsilon_z$-phase angle in head waves
Figure 5.38c  Comparison experimental and calculated wave induced $C_{M}$ coefficient in head waves

Figure 5.38d  Comparison experimental and calculated $\epsilon_{M}$ phase angle in head waves
Figure 5.39a  Comparison experimental and calculated wave induced $C_r$ coefficient in beam waves

Figure 5.39b  Comparison experimental and calculated $\varepsilon_r$ phase angle in beam waves
Figure 5.39c  Comparison and calculated wave induced $C_x$ coefficient in beam waves

Figure 5.39d  Comparison experimental and calculated $\epsilon_x$ phase angle in beam waves
**Figure 5.39e** Comparison experimental and calculated wave induced $C_{K'}$ coefficient in beam waves

**Figure 5.39f** Comparison experimental and calculated $\epsilon_{K}$ phase angle in beam waves
Figure 5.39g  Comparison experimental and calculated wave induced $C_N$-coefficient in beam waves

Figure 5.39h  Comparison experimental and calculated $\varepsilon_N$-phase angle in beam waves
The dip in experimental force ($Y$) and moment ($K, N$) amplitudes in Figure 5.39 at $k_r=0.036$ is not present in the calculation results. The wave excitation is a function of the wave amplitude and wave frequency, or wave length. During the experiments, the excitation was found to be linearly dependent on the wave amplitude, therefore the normalized force and moment amplitudes depend only on the ratio between the wave length and the strut spacing and foil span. The wavelength varies between 70 and 8 m for the lowest and highest frequencies shown here. As the strut spacing and foil span vary between 3 and 7 m respectively, the wave length at the lower frequencies is an order of magnitude larger than the strut spacing and foil span. Therefore, no sudden variation in the force and moment amplitudes is expected for the lower frequencies. The dips are probably due to the relatively low accuracy of the experimental force and moment amplitudes. Also, reflections from the model basin side walls for low frequency waves may play a role here.

**Impulsive flap deflections**

A number of impulsive flap deflection tests were carried out for the same foil system as described in the previous paragraphs. The aft foil centre flap or the two tip flaps were hereby suddenly rotated by means of a special excitation mechanism, see Kapsenberg and Keukens (1991). The flaps could be rotated either upwards or downwards at a speed of 113 degrees per second (full scale value). This rotation speed corresponds to a reduced frequency value $k=0.05$. Two experimental results are compared with calculation results here: a centre flap deflection of 8 degrees and a tip flap deflection were the starboard and port flaps were rotated over 8 degrees in opposite directions. The aft foil was modelled in Unvlm-In by means of a lattice of $N=8$ and $M=28$ vortex elements. As the flap chord to foil chord ratio is 0.25, the last two chordwise panels represent the flap. The effect of flap angle and angular velocity was incorporated in the discretized integral equation (4.53) by adjusting the normal velocity of and normal direction at the control points as appropriate. From steady flow experiments it was observed that the flap efficiency was lower than indicated by potential flow theory. This is discussed in Section 2.6 and is due to viscosity effects. Furthermore, the experimental results showed that shortly after the instant that the flap reached its full deflection angle, the lift increment was equal to its steady state value. This indicates that no significant flow separation due to the sudden flap deflection itself was present during the experiments. The flap span used in the calculations has been reduced by as much as 38% as to have the same steady flow lift increment as experimentally observed.

It is seen in Figure 5.40 that the initial lift increase is well predicted. When the flap has reached its maximum deflection angle the calculation results for the lift force show some overshoot due to the abrupt change in circulation which results in a discontinuity in the unsteady $\partial \Gamma / \partial t$ term. After this overshoot the calculated lift coefficient $C_l$ gradually reaches a constant value, equal to its steady state value. The experimental lift coefficient also shows a gradual increase, but with a somewhat lower slope.

The calculated roll moment coefficient $C_r$ does not show the overshoot when the maximum flap deflection is reached. This is probably due to the cancellation of the discontinuities in the $\partial \Gamma / \partial t$ terms for the starboard and port tip flaps. The experimental roll moment coefficient does show however a gradual overshoot for the time instant $18 \leq U/c \leq 21$ (0.15 seconds) which is not present in the calculated result.
5.3 Validation of Unvlm

Figure 5.40a  Lift force due to impulsive centre flap deflection

Figure 5.40b  Roll moment due to impulsive tip flap deflection
Conclusions

Damping forces on a two-dimensional lifting surface below the free surface are in good agreement with experimental results. For a low aspect ratio lifting surface at a deep submergence, experimental damping forces tend to deviate from calculation results for higher reduced frequencies \( k > 0.30 \).

Wave excitation forces on a basic lifting surface are in good agreement with experimental data as well.

Wave excitation and damping forces and moments for a practical fully submerged foil system are found to be in fairly good agreement with experimental data.

Interaction effects on the aft foil of a tandem foil arrangement performing oscillatory motions are well predicted.

The lift force and roll moment due to impulsive flap deflections are in good agreement with experimental data.

It is concluded that Unvlm may be used with some confidence to predict the force components on a fully submerged hydrofoil system operating in waves.
5.4 Simulation examples and validation of Hydsim

The basics of Hydsim are described in Section 4.4. The Unvlm-In and Unvlm-nl computational methods have been incorporated in Hydsim to determine foil system forces. For the cases described in this Section, a ride control system has been used in Hydsim for stabilizing the motions of the craft. Hereby the position, velocities and accelerations are used as input signals to activate control surfaces. Control surfaces include trailing edge flaps and foil parts with a variable incidence.

Prior to validating Hydsim results on basis of full scale data, a number of time domain simulation results will be shown which illustrate the applicability of Hydsim and show the significance of linearization of motions, fore-aft foil interaction, the Green's function terms $G^f$ and unsteady flow effects.

In Hydsim a ride control system has been implemented for testing purposes. This ride control system is of the PLD-type with Proportional ($P$), Integral ($I$) and Differential ($D$) term coefficients. A control force vector $X_c$ is determined as follows:

$$X_c(i) = P(i)[x_r(i) - x_a(i)] + I(i) \int_0^t [x_r(i) - x_a(i)] dt + D(i) [\dot{x}_r(i) - \dot{x}_a(i)]$$

(5.12)

where $x_r$ is the required position vector, $x_a$ is the actual position vector, $\dot{x}_r$ is the required velocity vector and $\dot{x}_a$ is the actual velocity vector. Note that the surge $x$ and sway $y$ modes of motion ($i=1,2$) are not controlled. The control force vector $X_c$ is transformed into control surface deflections $\delta$ as follows:

$$\delta(j) = \sum_{i=1}^6 X_c(i) \frac{W(j,i)}{R(i)}$$

(5.13)

where $W(j,i)$ is a weight function expressing the efficiency of flap $j$ used for each mode of motion and $R(i)$ is the sum of control forces in mode $i$ of each flap with a unit deflection angle. The flap efficiency is obtained from empirical data provided by Martin (1963) as a function of the flap geometry relative to the foil geometry. In case of incidence control a flap with a unit efficiency is assumed. The user of Hydsim must specify appropriate $P$, $I$ and $D$ values.

For the cases for which full scale data are available for validation, the actual control system used on the craft during the trials has been implemented in Hydsim.

The example cases are based on simulations for a hydrofoil craft with fully submerged foils. The main dimensions of this craft are shown in Table 5.7. Each foil system was equipped with trailing edge flaps, see Figure 5.41. The default ride control system was used whereby the control coefficients were determined such that the craft was stable in all modes of motions. However, the control coefficients were not optimized to give the best possible performance, especially with respect to the damping of transient motions.

For propulsion, tractor propellers mounted at the aft foil were used, driven by engines delivering a thrust force sufficient for a speed of 36 knots.
Figure 5.41 Foil system of example hydrofoil craft

Table 5.7 Main particulars of example craft

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<td>Hull length (m)</td>
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<td>Hull beam (m)</td>
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<td>Displacement (ton)</td>
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<td>Cruise speed (kt)</td>
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<td>Foil submergence (m)</td>
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<tr>
<td>Hull clearance (m)</td>
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<td>Chord Froude number</td>
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<td>Forward foil type</td>
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<td>Aft foil type</td>
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Example 1: Start of a turning manoeuvre

This simulation was performed for calm water conditions. During a short, initial time interval the craft velocity was held constant until a sufficient wake sheet length was formed and constant forces were acting on the foils. Two and a half seconds after the start of the simulation a so-called coordinated turn was initiated. Hereby the craft assumes a certain roll angle during the turn which results in a transverse weight force component for balancing the centrifugal force acting on the craft. This has three advantages: no transverse force component is required from the struts which minimizes the bending loads and the susceptibility towards ventilation, a coordinated turn is more comfortable for passengers and crew and a higher turn rate can be achieved than for a flat turn. For the present case a yaw rate of 3.0 deg/s and a roll angle of -5.7 deg were commanded to the ride control system.

Simulations have been performed with and without taking fore-aft foil interaction and the $G^f$ terms into account in the non-linear method UnvLM-nl. Figure 5.42 shows the simulation results in terms of roll angle $\phi$, yaw rate $\frac{d\psi}{dt}$, forward speed $U$, propeller rate of revolutions $RPM$, forward foil flap angle $\delta_3$, used for heave, roll and pitch control, forward strut flap angle $\delta_4$ used for yaw control and the trajectory of the craft in terms of its space-fixed $x_0$ and $y_0$ coordinates.

It is seen that the craft quickly assumes the required yaw rate and roll angle. Hereafter, these motions show an oscillatory behaviour which decays only slowly with time. This is due to non-optimum control coefficient values used. The propeller rate of revolutions ($RPM$) required for a steady velocity of 36 knots was unknown a priori, therefore an estimated initial value was set. It is seen that the rate of revolutions decreases quickly with time until an equilibrium is reached at the start of the turning manoeuvre. When turning, the variations in $RPM$ and velocity are due to changes in foil resistance. The strut flap angle $\delta_3$ shows a peak at the start of the turn in order to achieve the required yaw rate. Hereafter a constant strut flap angle should ideally be maintained to counteract the strut incidence due to the yaw rate of the craft.

Foil interaction hardly affects the trajectory of the craft. The yaw rate is slightly affected. Interaction increases the variations in roll angle and introduces local peak values in the time trace. Furthermore, the oscillations decay more slowly in time when interaction is taken into account. The flap angles are significantly affected by interaction. This behaviour results from the use of the control system which aims to maintain the required roll angle and yaw rate so that the effect of interaction, if present, will be mainly reflected by the flap angles. Finally, the velocity is reduced due to interaction which, in this case, increases the induced drag of the aft foil.

Taking into account the $G^f$ terms does not result in large changes in the quantities plotted in Figure 5.42, although the local peaks in the time traces due to interaction are smoothened.

Taking interaction and the $G^f$ terms into account is not required for a first assessment of the dynamic behaviour of the example craft. However, fore-aft foil interaction does affect the required flap deflections and thereby the required control power and the susceptibility towards cavitation. Therefore, it is useful to take interaction into account for more detailed assessments of the dynamic behaviour of hydrofoil craft.
Figure 5.42a Roll angle and yaw rate for start of turning manoeuvre, non-linear simulations

Figure 5.42b Speed and RPM for start of turning manoeuvre, non-linear simulations
Figure 5.42c Flap angles for start of turning manoeuvre, non-linear simulations

Figure 5.42d Trajectory for start of turning manoeuvre, non-linear simulations
To investigate the need for a non-linear and an unsteady flow method, simulations have also been performed by using the linear and a linear, quasi-steady flow method, without taking interaction and the $G'$ terms into account. Note that the requirements for using the linear method, a constant forward speed and course, are violated then. However, for the present case the forward speed is approximately constant and lift force variations are expected to be mainly due to velocity disturbances rather than position variations. In the quasi-steady flow method the circulation of a stream wise row of wake vortex elements was set equal to that of the vortex element at the trailing edge of the foil, at each time step. Furthermore, the forces due to the time derivative of the circulation were not taken into account.

Figures 5.42e and 5.42f show the time traces for the roll angle, the yaw rate and the forward foil flap angles $\delta_1$ and $\delta_2$. The roll angle and yaw rate from the linear and quasi-steady flow methods are closer to their required values than those of the non-linear method. Apparently, in the non-linear method it is more difficult for the ride control system to control the motions due to the existence of additional force components not present in the linear and quasi-steady flow methods. Although the differences in roll and yaw rate are not very large, the flap angle settings are quite different for the linear and quasi-steady flow results when compared to the non-linear results. Therefore, for manoeuvring simulations with impulsive control system actions, a non-linear and unsteady flow method is required for a detailed assessment of the craft performance, at least for the present case.

![Figure 5.42e Roll angle and yaw rate for start of turning manoeuvre, linear and quasi-steady simulations](image-url)
5.4 Simulation examples and validation of Hydsim

![Figure 5.42f](image_url)  
*Flap angles for start of turning manoeuvre, linear and quasi-steady simulations*

**Example 2: Turning manoeuvre in waves**

This second example concerns the same turning manoeuvre as described above whereby now a regular incident wave is added. The wave direction is 180 deg, corresponding to initial head waves. The wave amplitude is 0.5 m, while the spacing between the hull and the foils is 2.0 m. The wave period is 6 sec. The non-linear simulation method was used for this case. Since the only purpose of this example is to illustrate the applicability of Hydsim, fore-aft foil interaction and the $G^f$ terms were not taken into account in the simulation.

Figure 5.43a shows the track of the craft by means of an $x_0$-$y_0$ plot, for the entire manoeuvre. The track is an almost perfect circle. During the turn, the wave direction and thereby the frequency of encounter varies which is clearly reflected in the time traces for the yaw rate and roll angle. The resistance of the foil system slowly increases during the turn, resulting in a slowly reducing mean forward speed. Due to the increasing propeller thrust with reducing speed, the mean velocity becomes approximately constant at the end of the turning manoeuvre. The velocity variations are small but clearly show the varying encounter frequency. The variations in the propeller rate of revolutions have a similar character. The flap angles again reflect the variations in the encounter frequency of the waves.
Figure 5.43a Trajectory for turning manoeuvre in waves

Figure 5.43b Roll angle and yaw rate for turning manoeuvre in waves
Figure 5.43c  Speed and RPM for turning manoeuvre in waves

Figure 5.43d  Flap angles for turning manoeuvre in waves
Example 3: Performance in head and following waves

The seakeeping behaviour of the example hydrofoil craft in head and following waves for regular waves with a varying wave period is shown next. The first wave period for head waves is selected to have a high pitch excitation while the second wave period is selected to have a high heave excitation. For these head wave conditions, a platforming mode of ride control was selected which aims to keep the craft in a fixed position relative to the undisturbed water surface. For following waves a relatively high wave period was selected to enable a contouring mode of ride control. Hereby the craft follows the wave profile by maintaining a constant hull clearance at the forward and aft sensor positions. The wave amplitudes are 0.5 m and 1.0 m for head and following seas respectively. The simulations were based on the linear Unvlm-In method. Simulations with and without interaction and the $G'$ terms were performed. Also, simulations were performed for head waves with a linear, quasi-steady method for the foil forces. Table 5.8 shows the identification of simulation cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Fore-aft foil interaction</th>
<th>Green function terms $G'$</th>
<th>Quasi-steady</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>4</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 5.9 shows the craft performance in head waves in terms of standard deviations of the:
- heave motion at the centre of gravity $\sigma_r$,
- pitch angle $\sigma_p$,
- vertical acceleration at the bow $\sigma_{socB}$,
- relative hull clearance at the bow $\sigma_{hb}$,
- aft foil flap angle $\sigma_b$,

and the mean value for the aft foil flap angle $\delta_m$. The relative hull clearance is the vertical distance between the sensor position at the hull bottom and the water surface.

<table>
<thead>
<tr>
<th>$T_w$ (sec)</th>
<th>$\sigma_r$ (m)</th>
<th>$\sigma_p$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 3</td>
</tr>
<tr>
<td>4.1</td>
<td>0.043</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 3</td>
</tr>
<tr>
<td>7.1</td>
<td>0.040</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 5.9b  Performance in head waves

<table>
<thead>
<tr>
<th>$T_w$ (sec)</th>
<th>$\sigma_{accB}$ (g)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>$\sigma_{nb}$ (m)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td></td>
<td>0.220</td>
<td>0.223</td>
<td>0.215</td>
<td>0.078</td>
<td>0.38</td>
<td>0.38</td>
<td>0.37</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td></td>
<td>0.048</td>
<td>0.047</td>
<td>0.048</td>
<td>0.034</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
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</table>

### Table 5.9c  Performance in head waves

<table>
<thead>
<tr>
<th>$T_w$ (sec)</th>
<th>$\sigma_8$ (deg)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>$\delta_m$ (deg)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td></td>
<td>2.13</td>
<td>2.14</td>
<td>2.20</td>
<td>2.17</td>
<td>-5.03</td>
<td>0.03</td>
<td>-2.15</td>
<td>-5.07</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td></td>
<td>1.69</td>
<td>1.69</td>
<td>1.75</td>
<td>1.71</td>
<td>-5.15</td>
<td>0.02</td>
<td>-2.17</td>
<td>-5.15</td>
<td></td>
</tr>
</tbody>
</table>

The heave and pitch motions in head waves are quite small while the vertical acceleration at the bow is substantial in short waves. The standard deviation for the relative bow height is almost equal to that of the wave elevation, since the heave and pitch motions are small. Fore-aft foil interaction and the free surface Green's function terms do not have a large effect on the motions and the variation in the aft foil flap angle, but do affect the mean value of the aft foil flap angle. The Green's function terms reduce the downwash at the aft foil due to interaction. The quasi-steady approach reduces the heave, pitch and vertical acceleration amplitudes significantly for the lower wave period. The reduced frequency of encounter $k_c$ ranges from 0.06 at $T_w=7.1$ sec to 0.16 at $T_w=4.1$ sec.

### Table 5.10a  Performance in following waves

<table>
<thead>
<tr>
<th>$T_w$ (sec)</th>
<th>$\sigma_z$ (m)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>$\sigma_\theta$ (deg)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.0</td>
<td></td>
<td>0.631</td>
<td>0.632</td>
<td>0.632</td>
<td>2.09</td>
<td>2.10</td>
<td>2.09</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.10b  Performance in following waves

<table>
<thead>
<tr>
<th>$T_w$ (sec)</th>
<th>$\sigma_{accB}$ (g)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>$\sigma_{nb}$ (m)</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.0</td>
<td></td>
<td>0.053</td>
<td>0.055</td>
<td>0.055</td>
<td>0.065</td>
<td>0.068</td>
<td>0.062</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.10c  Performance in following waves

<table>
<thead>
<tr>
<th>$T_w$ (sec)</th>
<th>$\sigma_s$ (deg)</th>
<th>$\delta_m$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>9.0</td>
<td>6.07</td>
<td>6.58</td>
</tr>
</tbody>
</table>

Table 5.10 shows the craft performance in following waves in terms of the same quantities as used for head waves. The standard deviations of heave, pitch and vertical acceleration reflect the standard deviations of the wave elevation, wave slope and the vertical acceleration of the wave profile which have values of 0.71 m, 1.8 deg and 0.49 m/s² respectively. Also, the standard deviation of the relative bow height is small. The contouring mode of ride control therefore performs as required. The effect of fore-aft foil interaction and the Green’s function terms on the motion quantities are small, however the effect on the variation and mean value of the aft foil flap angle is again significant.

It is concluded that using a quasi-steady flow method in short head waves underestimates the motions and especially the accelerations at the bow. The Green’s function terms and fore-aft foil interaction affect only the aft foil flap angle.

Validation case 1: Seakeeping of a surface piercing hydrofoil

Sferrazza et al. (1992) provide full scale measurement data of the performance of two Rodriguez RHS-160F surface piercing hydrofoil craft in waves. These craft feature two surface piercing hydrofoil systems of the airplane type, see Figure 5.44. Table 5.11 shows the main particulars of the craft.

Table 5.11  Main particulars of RHS 160F hydrofoil

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall (m)</td>
<td>31.2</td>
</tr>
<tr>
<td>Moulded beam (m)</td>
<td>6.7</td>
</tr>
<tr>
<td>Full load displacement (tons)</td>
<td>106</td>
</tr>
<tr>
<td>Cruising speed (knots)</td>
<td>36</td>
</tr>
<tr>
<td>Accommodation (passengers)</td>
<td>210</td>
</tr>
</tbody>
</table>

Both foil systems are equipped with trailing edge flaps for controlling the roll and pitch motions. The two crafts, named "Moretto" and "Aldebaran" were identical, except for small differences in the ride control systems.
The measurements were performed at deep water (Gulf of Naples) in head, beam and following seas. A wave buoy equipped with accelerometers was used to obtain time registrations of the wave elevations in the measurement area. Accelerometers were installed at the craft to measure the vertical accelerations at the centre of gravity, at the bow and at the stern. Furthermore, the roll and pitch angles were measured by means of rate gyro's. The speed of the craft during the measurements was 35 and 34.5 knots for the "Moretto" and "Aldebaran" respectively. The measurements had an average duration of 4 minutes, corresponding to approximately 300, 150 and 70 wave encounters for head, following and beam sea directions respectively. The measurements were performed at a sample rate of 20 Hz. Experience at MARIN shows that for obtaining statistically reliable results, about 180 wave encounters are needed. Therefore, the results for especially beam wave conditions may not be fully reliable.

The purpose of the trials was to investigate the comfort in various wave conditions, in comparison to a catamaran. For this purpose the vertical accelerations and their frequency are the most interesting quantities.
By means of statistical analysis the significant wave height and the zero up-crossing period were determined for each wave height registration based on the wave buoy measurements by Serrazza et al. (1992). As the time series of the wave elevations were not available anymore at the time of the present investigation, the spectral shapes were assumed to be of the mean Jonswap type. This spectrum type is often used for the type of waves encountered during the trials: wind waves in a sheltered area. The spectra were used in Hydsim to generate time series of the wave elevation and wave orbital velocity components as outlined in Section 4.4. The spectra were represented by 40 frequency intervals. The Hydsim simulations had a duration sufficient for 200 wave encounters. The actual ride control system used during the trials was incorporated in Hydsim. Foil interaction and the free surface Green function $G'$ terms were taken into account in all simulations.

Before focusing on the comparison between the trials and simulation data, some remarks on the trials data are made. The trials data are shown in Table 5.12 on the left side of each column. The wave periods and amplitudes are quite small and are typical for wind generated waves in sheltered regions. In such conditions motion amplitudes of hydrofoil craft are small, which is reflected by the trials data. The motion amplitudes may be contaminated with relatively large noise components. However, the standard deviations of the accelerations are considered to be significant and affect the comfort.

It is furthermore noted that for the two beam wave conditions there appears a considerable variation in the trials data. Such differences may be attributed to the randomness of the sea and variations in the wave direction, wind velocity and wind direction, in combination with the relatively low number of wave encounters.

During the trials course keeping was performed by a helmsman. In combination with the directional spreading of wind and waves, course deviations occur. Course deviations require a rudder action and thereby introduce roll angle variations, even for head and following sea conditions. In Hydsm, for head and following sea conditions the standard deviation of the roll angle is zero as there are no disturbances acting in the horizontal plane which would require a course keeping action. For the Hydsm simulations for beam sea conditions the course keeping was simulated by an auto pilot actuating trailing edge flaps on the aft foil struts.

For these reasons, validating the roll motions is difficult. The emphasis should be on the vertical accelerations and to a lesser extend on the motions in the vertical plane.

Table 5.12 show a comparison between the trials data for the “Aldebaran” and the Hydsm simulation results based on the non-linear simulation method Unvlm-nl. In the non-linear method wetted area variations are taken into account as outlined in Section 4.5. In the Table columns, the numbers on the left are the trials data while the numbers on the right are the calculated values. Furthermore, $\psi_w$ is the wave direction ($\psi_w = 180$ denotes a head sea condition), $H_s$ is the significant wave height, $\sigma$ is the standard deviation and the subscripts $\theta, \phi, accG, accB$ and accS denote the pitch and roll motions and vertical acceleration at the centre of gravity, bow and stern respectively. $T_2$ denotes the zero-upcrossing period which is the average value for the period interbetween the upward zero crossings of the signals.
### Table 5.12a  Comparison non-linear calculation results for "Aldebaran"

<table>
<thead>
<tr>
<th>$\psi_w$</th>
<th>$H_w$ (m)</th>
<th>$\sigma_\theta$ (deg)</th>
<th>$\sigma_\phi$ (deg)</th>
<th>$\sigma_{accG}$ (g)</th>
<th>$\sigma_{accB}$ (g)</th>
<th>$\sigma_{accS}$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>.84 .83</td>
<td>.39 .36</td>
<td>.58 .00</td>
<td>.065 .051</td>
<td>.110 .117</td>
<td>.110 .092</td>
</tr>
<tr>
<td>0</td>
<td>.89 .87</td>
<td>.38 .55</td>
<td>.98 .00</td>
<td>.093 .077</td>
<td>.110 .120</td>
<td>.120 .125</td>
</tr>
<tr>
<td>270</td>
<td>.70 .72</td>
<td>.30 .23</td>
<td>.50 .45</td>
<td>.040 .039</td>
<td>.052 .053</td>
<td>.056 .048</td>
</tr>
<tr>
<td>90</td>
<td>.70 .71</td>
<td>.24 .23</td>
<td>.24 .45</td>
<td>.046 .039</td>
<td>.057 .053</td>
<td>.066 .048</td>
</tr>
</tbody>
</table>

### Table 5.12b  Comparison non-linear calculation results for "Aldebaran"

<table>
<thead>
<tr>
<th>$\psi_w$</th>
<th>$T_z$ (sec)</th>
<th>$T_\theta$ (sec)</th>
<th>$T_\phi$ (sec)</th>
<th>$T_{accG}$ (sec)</th>
<th>$T_{accB}$ (sec)</th>
<th>$T_{accS}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>3.50 3.57</td>
<td>3.82 2.70</td>
<td>3.80 0.00</td>
<td>0.82 0.97</td>
<td>0.83 0.72</td>
<td>0.74 0.56</td>
</tr>
<tr>
<td>0</td>
<td>3.60 3.78</td>
<td>3.01 2.87</td>
<td>3.30 0.00</td>
<td>1.54 1.25</td>
<td>1.30 1.05</td>
<td>1.23 1.12</td>
</tr>
<tr>
<td>270</td>
<td>3.70 3.68</td>
<td>4.60 3.75</td>
<td>3.40 3.25</td>
<td>1.33 1.53</td>
<td>1.33 0.92</td>
<td>0.85 0.77</td>
</tr>
<tr>
<td>90</td>
<td>3.90 3.81</td>
<td>5.10 3.75</td>
<td>3.06 3.25</td>
<td>1.45 1.53</td>
<td>1.45 0.92</td>
<td>1.09 0.77</td>
</tr>
</tbody>
</table>

It is seen that with the non-linear method a good agreement with the trials data is obtained. The dependence of the acceleration levels on the wave direction is well predicted. The magnitude of the differences between the zero up-crossing periods for the angular motions and the accelerations are generally well predicted although differences remain, for instance the pitch period in beam wave conditions.

Tables 5.13 show a comparison between the "Aldebaran" measurements and calculation results based on the linear method Unvlm-In. Force variations due to changes in the wetted surface of the foil system, caused by the craft motions and wave elevation, are not taken into account in the linear method.

### Table 5.13a  Comparison linear calculation results for "Aldebaran"

<table>
<thead>
<tr>
<th>$\psi_w$</th>
<th>$H_w$ (m)</th>
<th>$\sigma_\theta$ (deg)</th>
<th>$\sigma_\phi$ (deg)</th>
<th>$\sigma_{accG}$ (g)</th>
<th>$\sigma_{accB}$ (g)</th>
<th>$\sigma_{accS}$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>.84 .83</td>
<td>.39 .20</td>
<td>.58 .00</td>
<td>.065 .032</td>
<td>.110 .081</td>
<td>.110 .088</td>
</tr>
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<td>0</td>
<td>.89 .87</td>
<td>.38 .61</td>
<td>.98 .00</td>
<td>.093 .023</td>
<td>.110 .042</td>
<td>.120 .044</td>
</tr>
<tr>
<td>270</td>
<td>.70 .72</td>
<td>.30 .03</td>
<td>.50 .48</td>
<td>.040 .027</td>
<td>.052 .028</td>
<td>.056 .028</td>
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<td>90</td>
<td>.70 .71</td>
<td>.24 .03</td>
<td>.24 .48</td>
<td>.046 .027</td>
<td>.057 .028</td>
<td>.066 .028</td>
</tr>
</tbody>
</table>
Table 5.13b Comparison linear calculation results for "Aldebaran"

<table>
<thead>
<tr>
<th>$\psi_w$</th>
<th>$T_2$ (sec)</th>
<th>$T_0$ (sec)</th>
<th>$T_\phi$ (sec)</th>
<th>$T_{accG}$ (sec)</th>
<th>$T_{accB}$ (sec)</th>
<th>$T_{accS}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>3.50 3.57</td>
<td>3.82 1.71</td>
<td>3.80 0.00</td>
<td>0.82 0.96</td>
<td>0.83 0.98</td>
<td>0.74 1.05</td>
</tr>
<tr>
<td>0</td>
<td>3.60 3.78</td>
<td>3.01 5.40</td>
<td>3.30 0.00</td>
<td>1.54 1.70</td>
<td>1.30 1.50</td>
<td>1.23 1.39</td>
</tr>
<tr>
<td>270</td>
<td>3.70 3.68</td>
<td>4.60 4.47</td>
<td>3.40 5.09</td>
<td>1.33 4.23</td>
<td>1.33 4.27</td>
<td>0.85 5.13</td>
</tr>
<tr>
<td>90</td>
<td>3.90 3.81</td>
<td>5.10 4.47</td>
<td>3.06 5.09</td>
<td>1.45 4.23</td>
<td>1.45 4.27</td>
<td>1.09 5.13</td>
</tr>
</tbody>
</table>

It is seen that the accelerations are significantly underpredicted by the linear method. The calculated and measured zero-upcrossing period for the accelerations in beam seas are not in good agreement either.

In Section 5.2 it was shown that for a fully submerged foil the use of a linear method does not result in large errors provided the reduced frequency and the (heave) motion amplitude are limited. For the head wave condition described here, the reduced frequency $k$ equals 0.17 while the heave amplitude is lower than 0.25 chords. For such motions, linearization errors are less than 1% in lift amplitude for a fully submerged foil. For the foil system used on the RHS-160F craft, the dihedral angle of the tip foil parts is about 30 deg which means that a certain submergence variation in the vertical plane results in a wetted length variation which is twice as large. Furthermore, the chord of the tip foil parts is relatively large for the forward foil. These facts suggest that the neglect of wetted area variations in the linearized method might be responsible for the poor predictions.

Tables 5.14 show a comparison between the trials data and non-linear simulation results for the Moretto hydrofoil. Here the wave height is even lower than for the Aldebaran data. Nevertheless, there is again a reasonably good correlation between the trials and simulation results.

Table 5.14a Comparison non-linear calculation results for "Moretto"

<table>
<thead>
<tr>
<th>$\psi_w$</th>
<th>$H_m$ (m)</th>
<th>$\sigma_\alpha$ (deg)</th>
<th>$\sigma_\phi$ (deg)</th>
<th>$\sigma_{accG}$ (g)</th>
<th>$\sigma_{accB}$ (g)</th>
<th>$\sigma_{accS}$ (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>.54 .53</td>
<td>.22 .18</td>
<td>.31 .00</td>
<td>.037 .043</td>
<td>.067 .072</td>
<td>- .070</td>
</tr>
<tr>
<td>0</td>
<td>.56 .58</td>
<td>.22 .24</td>
<td>.41 .00</td>
<td>.039 .035</td>
<td>.062 .058</td>
<td>.081 .065</td>
</tr>
<tr>
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<td>.60 .62</td>
<td>.25 .17</td>
<td>.67 .53</td>
<td>.018 .024</td>
<td>.025 .030</td>
<td>.030 .025</td>
</tr>
</tbody>
</table>

Table 5.14b Comparison non-linear calculation results for "Moretto"

<table>
<thead>
<tr>
<th>$\psi_w$</th>
<th>$T_2$ (sec)</th>
<th>$T_0$ (sec)</th>
<th>$T_\phi$ (sec)</th>
<th>$T_{accG}$ (sec)</th>
<th>$T_{accB}$ (sec)</th>
<th>$T_{accS}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>2.80 2.85</td>
<td>3.30 2.85</td>
<td>2.90 0.00</td>
<td>0.74 0.81</td>
<td>0.68 0.76</td>
<td>- 0.56</td>
</tr>
<tr>
<td>0</td>
<td>2.90 2.85</td>
<td>1.80 2.04</td>
<td>2.30 0.00</td>
<td>1.10 1.11</td>
<td>1.10 0.92</td>
<td>1.00 0.85</td>
</tr>
<tr>
<td>90</td>
<td>2.80 2.85</td>
<td>4.30 3.75</td>
<td>2.70 2.74</td>
<td>1.30 1.50</td>
<td>1.20 1.14</td>
<td>1.50 1.25</td>
</tr>
</tbody>
</table>
It is concluded that the non-linear method can be used successfully for investigating the comfort of surface piercing hydrofoil craft under normal operating conditions. The main reason for the linear method to give inadequate predictions is the neglect of wetted surface variations. These variations are due to the craft motions and the wave elevations. For the present case, and hydrofoil craft in general, wave amplitudes are an order of magnitude larger than the craft motion amplitudes. Including the correction on the foil lift force accounting for the wetted surface variation due to the wave elevation in the linear method can therefore be expected to significantly improve its predictions for surface piercing hydrofoil craft.

Validation case 2: Seakeeping and manoeuvring of a fully submerged hydrofoil craft

Saito et al. (1991) provide seakeeping data for the Jetfoil-115 type hydrofoil craft. This hydrofoil craft has a fully submerged foil system with a canard foil area distribution, see Figure 5.45a. Table 5.15 shows the main particulars of the craft. The craft is equipped with a ride control system actuating trailing edge flaps at the forward and aft foil systems, see Figure 5.45b. Directional stability is obtained from the forward strut which acts as a rudder. The relations between input and output signals have been obtained from Saito et al. (1991) on basis of calculated values for the hydrodynamic derivatives of the craft. These relations include corrections for the actuation of flaps.

Figure 5.45a Jetfoil-115 configuration - from Saito et al. (1991)
The relative hull clearance at the bow, the vertical acceleration at the bow and the forward foil flap angle were recorded during one month of commercial operation in the Japan Sea. The wave height could be determined from the measured hull clearance, the heave and pitch motions as recorded by the ride control system sensors and the sensor positions. Figure 5.46 shows plots of the measurement data versus the significant wave height. Predictions by Saito et al. based on their simulation method are included in this Figure. This simulation method uses the same basic equations of motions as Hydsim, see Section 4.4. For obtaining the foil system forces, a semi-empirical method is used, based on theoretical and experimental data on the Jetfoil foil systems in steady flow. These data were obtained from Feifel (1981) and Meldahl (1981) who used panel methods to obtain the basic lift characteristics of the foil system and experimental data to determine effects due to the free surface, cavitation and ventilation. These results are used in a quasi-steady manner, however unsteady flow effects are included as outlined by Ogilvie (1958) and Keuning (1979), based on the two-dimensional Theodorsen and Sears functions.
It is seen that the predictions by Saito et al. are in good agreement with the measurement data for the lower wave height range. Up to a wave height of 1.5 m the craft response is linearly depending on wave height. For wave heights above 1.5 to 2.0 m deviations are present which are mainly caused by a special mode of ride control, namely a correction of the selected operating depth of the forward foil by the helmsman. This manual correction is based on visual observations of the encountered waves and aims to reduce the risk on hull slamming and foil broaching.

Figure 5.46 Measurement data Jetfoil trials - from Saito et al. (1991)
The scatter in the measured data for a constant wave height is likely due to the variation in the wave periods encountered during the trials period. This variation in wave conditions is often expressed in wave scatter diagrams which show the probability of occurrence of a certain combination of wave height and wave period for a certain location on sea in a certain season.

For the Hydsim simulations such a scatter diagram for the Japan Sea was used to determine the two mean wave period ranges with the highest probability, for a series of significant wave height values. For the mean value in each wave period range simulations with Hydsim were carried out, both on basis of the linear and non-linear methods. The conditions thus described cover approximately 50% of the wave height-wave period combinations present in the scatter diagram, for wave heights up to 2.0 m. In the Hydsim simulations foil interaction was taken into account. Accounting for the free surface Green's function terms $G'$ showed a negligible effect on the simulation results.

The simulation results are shown in Figure 5.47. Non-linear results are somewhat closer to the mean value of the trials data than linear simulation results. The differences between the linear and non-linear results are however much smaller than for the earlier described surface piercing hydrofoil craft. Above a wave height of 1.5 m the simulation results start to deviate from the trials mean line. This may be due to cavitation, but also the manual depth control is of influence here.

![Figure 5.47a Comparison measured and calculated relative bow clearance for the Jetfoil craft](image-url)
5.4 Simulation examples and validation of Hydsim

Figure 5.47b Comparison measured and calculated vertical acceleration at the bow for the Jetfoil craft

Figure 5.47c Comparison measured and calculated forward flap angle for the Jetfoil craft
Figure 5.48 finally shows a comparison for the manoeuvring behaviour of the Jetfoil. The trials data are obtained from Saito et al. (1990). The Figure shows a coordinated turn manoeuvre whereby the helmsman suddenly sets a certain 'helm command' proportional to the required yaw rate. At this command, the entire forward foil is rotated (strut angle) and the craft assumes a certain roll angle and yaw rate. All flaps are active during this manoeuvre for controlling heave, pitch and roll. It is seen that there exists a fair agreement between the trials data and the non-linear simulation data. The roll angle during the turn is reasonably well predicted, but in the simulation the roll response of the craft is somewhat too fast. The same holds for the yaw rate and the lateral acceleration at the bow. This discrepancy may perhaps be caused by wave disturbances during the trials. These are present at least before the start of the manoeuvre where the strut angle is not equal to zero.

![Figure 5.48](image.png)

**Figure 5.48** Comparison measured and calculated turning manoeuvre for the Jetfoil craft

**Conclusions**

Calculation examples for a fully submerged hydrofoil craft show that at cruise speed fore-aft foil interaction and the Green's function terms $G'$ do not have a large effect on calculation results for motions and accelerations in waves.

For manoeuvring simulations, the effects of especially fore-aft foil interaction are relevant. Furthermore, fore-aft foil interaction and the $G'$ terms do affect the flap angles settings during both manoeuvring and seakeeping simulations.

Neglecting unsteady flow effects is not permitted for seakeeping simulations in head waves.
Using a linear or a quasi-steady flow method for manoeuvring simulations with impulsive control system actions results in large differences in flap angles. Therefore, the non-linear simulation method UnvIm-nl should be used for such simulations.

The simulation method, linearized with respect to motions, does not predict the seakeeping of surface piercing hydrofoils correctly. The main reason for this is the neglect of wetted area variations due to the wave elevation. The non-linear simulation method incorporates a simple correction to account for the wave elevation effect on the wetted surface, and predicts the seakeeping behaviour of surface piercing hydrofoil craft much better.

For a fully submerged hydrofoil craft, both the linear and non-linear methods yield good predictions for the seakeeping behaviour. The dynamic behaviour of a fully submerged hydrofoil craft during a turning manoeuvre is fairly well predicted by the non-linear simulation method.

It is concluded that the Hydsim simulation program is a useful tool for predicting the dynamic behaviour of hydrofoil craft.

**Summary Chapter 5**

The Chapter deals with the validation of Hydsim and its components. A numerical validation study is performed in which the sensitivity of results with respect to the number of vortex elements, the effects of time step variations and the significance of the free surface Green's function terms are investigated.

Next, effects due to the roll-up of the wake sheet, linearization of motions, truncation of the wake length and fore-aft foil interaction are described.

The unsteady vortex lattice method is validated on basis of experimental data for basic lifting surfaces performing oscillatory heave motions at deep submergence and underneath the free surface. Wave induced loads on a stationary lifting surface are compared with model test data as well.

Damping and wave excitation forces on a tandem hydrofoil system are compared with experimental data. The time-varying lift force and roll moment due to impulsive flap deflections are compared with experimentally determined time traces.

A number of examples of Hydsim simulations are given for a hydrofoil craft performing a turning manoeuvre and operating in head and following seas. The significance of fore-aft foil interaction, the free surface Green's function terms and unsteady flow effects are assessed. Finally, aspects of the seakeeping behaviour of a surface piercing and a fully submerged hydrofoil craft are validated on basis of full scale measurement data.
References Chapter 5


Saito Y. et al. (1990), Fully Submerged Hydrofoil Craft', 7th Marine Dynamics Symposium, Society of Naval Architects of Japan, Japan.


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6. CONCLUSIONS AND RECOMMENDATIONS

In the following paragraphs the main conclusions drawn from the development and validation of the steady and unsteady flow methods are reviewed and a number of recommendations for future work are formulated.

Steady flow method Hydres

In order to determine the characteristics of hydrofoil craft in calm water it is required to determine the attitude of the craft, in terms of draft and trim. The attitude is mainly governed by lift forces acting on the hull and on the foil system and determines to a large extent the resistance of the craft. For determining the hull forces, an empirical method has been developed while for the foil system forces, a vortex lattice method has been developed. Interaction effects between the foil system and the hull and between the forward and aft foils system themselves are significant and are taken into account, the first interaction effect only partially, however. Also, free surface effects on the foil lift and drag forces are of importance and are addressed.

The vortex lattice method Hydvlm for foil systems below a free surface forms the basis of the Hydres method and is validated for basic and more practical hydrofoil systems. Satisfactory results are found for free surface effects on foil lift and drag forces, for the Froude number range of interest. Fore-aft foil interaction is well described in a qualitative way, but quantitatively differences between experimental and calculated results for especially the induced drag are found. A better quantitative description of fore-aft foil interaction might be obtained by taking the roll-up of the forward foil wake sheet into account. However, it will not be an easy task to develop a practical computational method for arbitrary foil systems with a full account of free surface effects.

Viscosity effects on lift are found to be large for model test Reynolds numbers. Especially for cruise speed conditions, the attitude of hydrofoil craft models is significantly affected by viscosity effects on the zero-lift angle of attack and the lift curve slope. Correction factors for viscosity effects may best be determined on basis of a viscous flow method for foil sections. The viscous drag is accounted for by using empirical formulations.

Hydres results show in general a fair agreement with experimental results for the attitude and resistance. Hydres predictions for the attitude of surface piercing hydrofoil craft are somewhat outside the experimental uncertainty range for hump speed conditions. For cruise speed conditions the trim and draft are within the experimental uncertainty. The resistance is well within the experimental uncertainty for both speed regimes. The experimental uncertainty is however quite large and is mainly due to viscosity effects on lift and errors in the foil geometry. If additional experimental phenomena like ventilation and excessive spray generation are considered, it is thought that the attitude and resistance of surface piercing hydrofoil craft are surprisingly well predicted by the Hydres method.

For a fully submerged hydrofoil model, Hydres predictions are within the experimental uncertainty, provided there is no significant flow separation from trailing edge flaps.
Notwithstanding the experimental uncertainty, the differences between calculation and experimental results are probably also due to inadequacies in the calculation of fore-aft foil interaction and the neglect of hull-foil interaction. Including a rather crude estimation of foil-hull interaction clearly improves the predictions of Hydres for surface piercing hydrofoils.

Predictions for full scale conditions are satisfactory for two fully submerged hydrofoils. For a surface piercing craft, calculation data for the hullborne speed region are in fair agreement with trial results as well. For the foilborne speed region the agreement is somewhat less, but this may be caused by scatter in the trials data.

Hydres fulfils the requirements set to the steady flow method, i.e., it can be used to predict the hullborne and foilborne performance of hydrofoil craft with surface piercing and fully submerged foil systems with an accuracy sufficient for design purposes.

The inadequacies and limitations of Hydres can be largely overcome if a viscous flow solver were available, capable of handling free surface effects, arbitrary hard chine hull forms at planing speeds and fully submerged as well as surface piercing foil systems. In such a solver, the mutual interaction between the foil system and the hull could be addressed. Also, the restriction to use Series 65 alike hull forms in Hydres would then be removed. However, the development of such a solver is perhaps not feasible. Furthermore, if such a development were feasible, it would require a considerable amount of research which might not be justified by the use of such a tool for the limited number of hydrofoil cases, relative to the consultancy work carried out for more conventional ships.

A computational method consisting of a panel method for hull and foils, taking the separation of the free surface at chines into account and with a boundary layer method for the foils could be an intermediate step requiring less development effort.

Important aspects in hydrofoil craft design not considered in the present study are the inception and effects of foil cavitation. The spanwise lift distribution obtained from the present vortex lattice method could be used in combination with a panel method for foil sections to obtain the pressure distributions for investigating cavitation inception. A panel method could be used in succession, for certain critical conditions thus established, for obtaining more accurate pressure distributions. To assess cavitation effects, two-dimensional methods are available that could be used in a similar stripwise method.

Unsteady flow method HydSim

The unsteady flow method in HydSim was developed to simulate the seakeeping, manoeuvring and transient motions of hydrofoil craft in the time domain. Linear and a non-linear versions for the method for determining the forces acting on the foil system (UnvIm) have been developed. Linearization is in this respect performed with respect of the motions of the craft and offers substantial savings in computer time. Similar to the steady flow case, free surface effects are taken into account and a vortex lattice method is used to determine the foil system forces. Linearization is also applied to the tangential flow condition on the lifting surface and the free surface conditions
while linear wave theory is used to determine the wave orbital velocity components.

Linearization of the motions for an oscillatory heave motion introduces lift and drag errors of about 5%, for the practical frequency range for hydrofoil craft, i.e. a reduced frequency below \( k = 0.25 \).

It is necessary to desingularize the induced velocity due to vortex line segments for determining foil interaction. A simple but effective solid-body vortex core method is used for this purpose. Interaction effects on the aft foil for oscillating lifting surfaces are sensitive to curving and rolling-up of the wake sheet if the combination of foil spacing and oscillation frequency is such that the aft foil continuously operates in or close to the forward foil wake sheet. Interaction effects are then much stronger than for conditions where the foil spacing is much smaller than the wake wavelength. For these latter conditions, it is permitted to use flat wake sheets, i.e. a linear method, without significantly affecting the interaction effect on lift.

Damping and wave excitation forces on basic and practical hydrofoils at deep submergence and below the free surface are in good agreement with experimental results. Interaction effects on the aft foil of a tandem foil arrangement performing oscillatory motions are well predicted. The lift force and roll moment due to high speed flap deflections are in good agreement with experimental data, provided large viscosity corrections are taken into account.

Hydsim simulation results for a typical fully submerged hydrofoil craft show that at cruise speed fore-aft foil interaction and the Green's function term \( G^f \) do not significantly affect calculation results for motions and accelerations in waves. However, for manoeuvring simulations, the effects of fore-aft foil interaction are relevant. Furthermore, fore-aft foil interaction and the \( G^f \) term affect the flap angles settings during manoeuvring and seakeeping simulations. Flap angle settings are of importance for determining the ride control system requirements and cavitation inception.

The results of a validation study on basis of full scale data show that the linear simulation method does not predict the seakeeping of surface piercing hydrofoils correctly. The main reason for this is the neglect of wetted area variations due to the wave elevation. The non-linear simulation method incorporates a simple correction to account for the wave elevation effect on the wetted surface, and predicts the seakeeping behaviour of surface piercing hydrofoil craft much better. A similar correction can be introduced in the linear method so that much better predictions from this method can be obtained. For a fully submerged hydrofoil craft, both the linear and non-linear methods yield good predictions for the seakeeping behaviour. The dynamic behaviour of a fully submerged hydrofoil craft during a turning manoeuvre is fairly well predicted by the non-linear simulation method.

It is concluded that the Hydsim simulation method is a useful tool for predicting the dynamic behaviour of hydrofoil craft.

For investigating the performance of hydrofoil craft in extreme environmental conditions, necessary extensions of Hydsim would be a model for wave impact forces on hull forms and a model that predicts dynamic ventilation and cavitation inception and effects of foils. At the same time, the necessity of using non-linear free surface conditions and non-linear wave theory for extreme
conditions should be investigated more thoroughly.

At present, the powering requirements for hydrofoil craft are determined on basis of steady flow results and an allowance for accelerating to a foilborne condition in waves and wind. For investigating the powering requirements during take-off on a more fundamental basis, an unsteady flow method for the forces acting on hull forms and propulsors accelerating in waves would be needed. This is quite a challenging task which will probably need more research effort than can be justified from the limited use and benefits. In this respect, the use of a quasi-steady flow method for the resistance and lift forces acting on hulls in calm water should be investigated first. The same holds for the propulsor characteristics.
APPENDIX A  STEADY FLOW VELOCITY POTENTIAL EXPRESSIONS

In this Appendix analytical expressions and numerical approximations for the integral terms involved in the partial derivatives of the steady flow velocity potential are described. The circulation \( \Gamma \) is assumed to have a unit strength.

**Infinite fluid and biplane image terms \( I_1 \),**

\[
I_1 = \int \left( \frac{1}{R} + \frac{1}{R_0} \right) d\xi
\]  

(A.1)

This integral represents the contribution of the potentials for a swept vortex system in infinite fluid \((R^t \, \text{term})\) and its biplane image \((R_0^t \, \text{term})\). Analytical expressions for the partial derivatives of \( I_1 \) with respect to \( x, y, z, \xi, \eta \) and \( \zeta \) can be derived relatively easily, see for example Morch (1992) for non-swept planforms. The resulting expressions can be integrated analytically, but rather lengthy expressions result which require a relatively large amount of computer time. A more efficient approach is to use the law of Biot-Savart to obtain the induced velocity components due to a semi-infinite, swept back horseshoe vortex element.

Consider the horseshoe vortex system shown in Figure A1. For convenience the induced velocity components will be derived in a local axis system such that the vortex system lies in the \(xy\)-plane. The induced velocity components in the general axis system may be obtained by means of a simple coordinate transformation. The induced velocity components due to the biplane image vortex elements may be obtained analogously.

According to the law of Biot-Savart, the induced velocity vector due to a vortex line segment is given by:

\[
u = \frac{1}{4\pi} \frac{\phi d\xi d\eta \, d\zeta}{r^3}
\]  

(A.2)

whereby \( r \) is the scalar distance between the field point \((x, y, z)\) and a point on the vortex line segment \((\xi, \eta, \zeta)\), \( r \) is the distance vector and \( d\ell \) is the line segment vector:

\[
r = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}
\]  

(A.3)

\[
r = (x - \xi)\hat{i} + (y - \eta)\hat{j} + (z - \zeta)\hat{k}
\]  

(A.4)

\[
d\ell = d\xi \hat{i} \quad \text{for} \quad \ell \to a
\]
\[
d\ell = d\eta \hat{j} \quad \text{for} \quad a \to b
\]
\[
d\ell = d\zeta \hat{k} \quad \text{for} \quad b \to \infty
\]  

(A.5)

where \( \hat{i}, \hat{j} \) and \( \hat{k} \) are unit vectors in \( \xi, \eta \) and \( \zeta \) direction respectively.

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Longitudinal component \( u \)

\[
u = u \cdot i = \frac{1}{4\pi} \oint \frac{(dl \times r) \cdot i}{r^3}
\]

(A.6)

The integration contour consists of three parts, \( \infty \to a, a \to b \) and \( b \to \infty \), with corresponding velocity components \( u_x, u_y \) and \( u_z \). The integration yields only a contribution due to line segment \( ab \), with \( u_1 = u_2 = 0 \):

\[
u_3 = \frac{1}{4\pi} \int_a^b \frac{(z - \xi) \, d\eta}{r^3}
\]

\[
= \frac{1}{4\pi} \left( z - \xi \right) \int_a^b \frac{dl \cos \Lambda}{[c + 2bl + l^2]^{3/2}}
\]

(A.7)

\[
= \frac{1}{4\pi} \left( z - \xi \right) \cos \Lambda \left[ \frac{l + b}{R_b} - \frac{b}{R_a} \right]
\]

where
\[ R_a = \sqrt{(x-x_a)^2 + (y-y_a)^2 + (z-\zeta)^2} \]
\[ R_b = \sqrt{(x-x_b)^2 + (y-y_b)^2 + (z-\zeta)^2} \]
\[ l = \sqrt{(x_b-x_a)^2 + (y_b-y_a)^2} \]
\[ b = (x-x_a) \sin\Lambda - (y-y_a) \cos\Lambda \]
\[ c = R_a^2 \]
\[ \sin\Lambda = \frac{x_a-x_b}{l} \]
\[ \cos\Lambda = \frac{y_b-y_a}{l} \]

(Note: \( u_x \equiv 0 \) if the field point is located on \( ab \))

Transverse component \( v \)

\[ v = u_x \cdot \hat{n} = \frac{1}{4\pi} \oint \frac{(dl \times r) \cdot \hat{j}}{r^3} \]

For \( \infty \rightarrow a \):
\[ v_1 = \frac{1}{4\pi} \int_{\zeta=0}^{\zeta_{\infty}} -d\zeta \frac{(z-\zeta)}{r^3} \]
\[ = \frac{\Gamma(z-\zeta)}{4\pi[(y-y_a)^2 + (z-\zeta)^2]} \left[ 1 + \frac{(x-x_a)}{R_a} \right] \]

For \( b \rightarrow \infty \):
\[ v_2 = \frac{-(z-\zeta)}{4\pi[(y-y_b)^2 + (z-\zeta)^2]} \left[ 1 + \frac{(x-x_b)}{R_b} \right] \]
For $a \rightarrow b$:

$$v_3 = \frac{1}{4\pi} \int_{\xi_a}^{\xi_b} \frac{-d\xi(y-\eta)}{r^3}$$  \hspace{1cm} (A.12)

$$v_3 = -\frac{1}{4\pi} \frac{(y-y_0)}{(b^2-c)} \left[ \frac{\sin\Lambda}{R_b} - \frac{b}{R_a} \right] - \frac{1}{4\pi} \frac{\sin\Lambda \cos\Lambda}{(b^2-c)} \left[ \frac{bl + R_a^2 - R_a R_b}{R_b} \right]$$  \hspace{1cm} (A.13)

(Note: $v_3 = 0$ if the field point is located on $ab$)

**Vertical component $w$**

$$w = u \cdot k = \frac{1}{4\pi} \oint (dl \times r) \cdot k \frac{k}{r^3}$$  \hspace{1cm} (A.14)

For $\infty \rightarrow a$:

$$w_1 = \frac{1}{4\pi} \int_{\xi_a}^{\xi_b} \frac{-d\xi(y-\eta)}{r^3}$$  \hspace{1cm} (A.15)

$$w_1 = -\frac{(y-y_a)}{4\pi[(y-y_a)^2 + (z-\xi)^2]} \left[ 1 + \frac{(x-x_a)}{R_a} \right]$$  \hspace{1cm} (A.16)

For $b \rightarrow \infty$:

$$w_2 = \frac{(y-y_b)}{4\pi[(y-y_b)^2 + (z-\xi)^2]} \left[ 1 + \frac{(x-x_b)}{R_b} \right]$$  \hspace{1cm} (A.17)

For $a \rightarrow b$:

$$w_3 = \frac{1}{4\pi} \int_{\xi_a}^{\xi_b} \frac{d\xi(y-\eta)}{r^3} - \frac{d\eta(x-\xi)}{r^3}$$  \hspace{1cm} (A.18)

$$w_3 = \frac{1}{4\pi} \frac{(y-y_a)\sin\Lambda + (x-x_a)\cos\Lambda}{(b^2-c)} \left[ \frac{l+b}{R_b} - \frac{b}{R_a} \right]$$  \hspace{1cm} (A.19)

(Note: $w_3 = 0$ if the field point is located on $ab$)
Wave terms \( I_2 \),

\[
I_2 = \Re \left\{ \frac{-i2k_0}{n} \pi \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos \theta \int_{-\xi}^{\xi} e^{\nu E_1(\nu)} d\xi d\theta \right\}
\]

where

\[
\nu = k_0(z + \xi) \cos^2 \theta + k_0 \left| y - \eta \right| \cos \theta \sin \theta + ik_0 \left| x - \xi \right| \cos \theta
\]

In view of handling the \( \left| x - \xi \right| \) term in \( \nu \), two regions are identified: \( x > \xi \) and \( x < \xi \), i.e. behind or in front of the bound vortex line segment \( \xi = \xi \).

\( x > \xi \)

For \( \xi < x \) : \( \frac{\partial \nu}{\partial \xi} = -ik_0 \cos \theta \)

\( \xi = \xi : \nu = k_0(z + \xi) \cos^2 \theta + k_0 \left| y - \eta \right| \cos \theta \sin \theta + ik_0 \left| x + \eta \tan \theta \right| \cos \theta \)

\( \xi = x : \nu = k_0(z + \xi) \cos^2 \theta + k_0 \left| y - \eta \right| \cos \theta \sin \theta \)

For \( \xi > x \) : \( \frac{\partial \nu}{\partial \xi} = ik_0 \cos \theta \)

\( \xi = x : \nu = \nu_x \)

\( \xi = \infty : \nu_{\infty} = k_0(z + \xi) \cos^2 \theta + k_0 \left| y - \eta \right| \cos \theta \sin \theta + i \infty \)

Thus:

\[
I_2 = \Re \left\{ \frac{-2}{\pi} \pi \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_{-\xi}^{\xi} e^{\nu E_1(\nu)} d\xi d\theta \right\}
\]

Before integrating with respect to \( \nu \), \( I_2 \) is differentiated with respect to \( \eta \) and \( \zeta \).

By using:
\[
\int_{\nu_i}^{\nu_f} \frac{\partial}{\partial \eta} (e^\nu \Phi_i(\nu)) \, d\nu = \frac{\partial}{\partial \eta} \left[ \frac{\partial}{\partial \nu} (e^\nu \Phi_i(\nu)) \right]_{\nu_i}^{\nu_f}
\]

(A.22)

and

\[ e^\nu \Phi_i(\nu_a) = 0 \]

it follows that:

\[
\frac{\partial I_2}{\partial \eta} = \Re \left\{ \frac{2k_0 \text{sign}(\eta - \eta_c)}{\pi} \int_{-\pi/2}^{\pi/2} \cos \theta \sin \theta \left[ e^\nu \Phi_i(\nu_i) - 2e^\nu \Phi_i(\nu_a) \right] \, d\theta \right\}
\]

(A.23)

\[
\frac{\partial I_2}{\partial \zeta} = \Re \left\{ \frac{-2k_0}{\pi} \int_{-\pi/2}^{\pi/2} \cos^2 \theta \left[ e^\nu \Phi_i(\nu_i) - 2e^\nu \Phi_i(\nu_a) \right] \, d\theta \right\}
\]

\[ x < \xi_a \]

A similar procedure yields for this case:

\[
\frac{\partial I_2}{\partial \eta} = \Re \left\{ \frac{2k_0 \text{sign}(\eta - \eta_c)}{\pi} \int_{-\pi/2}^{\pi/2} \cos \theta \sin \theta e^\nu \Phi_i(\nu_i) \, d\theta \right\}
\]

(A.24)

\[
\frac{\partial I_2}{\partial \zeta} = \Re \left\{ \frac{2k_0}{\pi} \int_{-\pi/2}^{\pi/2} \cos^2 \theta e^\nu \Phi_i(\nu_i) \, d\theta \right\}
\]

The partial derivatives of these terms with respect to \( x, y, \) and \( z \) are needed to determine the gradient of the potential. These can be obtained by means of analytical differentiation of the integrands. The resulting integrals must then be determined by means of numerical integration. Due to the singularity in the integrands for \( \nu = 0 \), a relatively large amount of computer time is needed. A more efficient approach to compute the integrals is obtained by extending an approach given by Newman (1987). Newman describes a method for analytical removal of the singularity for \( \nu = 0 \) and a series representation for the \( I_2 \) term used in the Green's function for a steady source under the free surface:
\[ I_2 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \theta e^\phi E_1(v) d\theta \]  
(A.25)

Here a similar method is used for the \( I_2 \) terms defined in eq.'s (A.23) and (A.24). An ascending series representation for the exponential integral is given by:

\[ E_1(v) = -\ln(v) - \gamma - \sum_{n=1}^{\infty} (-1)^n \frac{v^n}{nn!} \]  
(A.26)

The singularity for \( v \rightarrow 0 \) is seen to be logarithmic. Spherical coordinates \((r,\phi,\varepsilon)\) are introduced such that:

\[ x = r \sin \phi \]
\[ z + iy = r \cos \phi e^{i\varepsilon} = \rho e^{i\varepsilon} \]  
(A.27)

Thus:

\[ v(\theta) = [i |x| - \rho \cos(\theta + \varepsilon)] \cos \theta \]  
(A.28)

For small values of \( v \) the contribution to for instance \( \partial I_2 / \partial \xi \) in eq. (A.24) from the logarithmic singularity in the exponential integral can be represented by the integral \( S \) by using a series expansion for \( e^v \):

\[ S = \frac{2i}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 \theta \ln(1 + v + \frac{v^2}{2} + \frac{v^3}{6} + \ldots) d\theta \]  
(A.29)

By using trigonometric relations, the integral \( S \) in (A.29) can be expressed in terms of two types of generic integrals:

\[ U_m = \frac{2i}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^m \theta \ln(v) d\theta \]  
(A.30)

\[ V_m = \frac{2i}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^m \theta \sin \theta \ln(v) d\theta \]

Newman (1987) gives series expansions for these integrals. The accuracy of the integral expression for \( S \) is six decimals.
Except for the singularity for \( v=0 \), the \( I_2 \) integrals are regular functions of the space coordinates. Denoting the singular component for small \( r \) values by \( S \), Newman gives the following series expansions based on Tchebychev polynomials \( T \):

\[
D = S + \sum_{i=0}^{16} \sum_{j=0}^{16} \sum_{k=0}^{8} C_{ijk} T_i[f(r)] T_j(\frac{4\phi}{\pi} - 1) T_k(\frac{2\varepsilon}{\pi})
\]  
(A.31)

The argument \( f(r) \) is defined separately in four \( r \) domains. The coefficient values \( C_{ijk} \) have been determined by the procedure given by Newman, applied to the present \( I_2 \) integral forms.

Both the \( S \) and \( D \) terms are evaluated in Hydvlm with an accuracy of six decimals. The partial derivatives of \( \partial I_2/\partial \eta \) and \( \partial I_2/\partial \zeta \) with respect to \( x, y \) and \( z \) are determined by analytical differentiation on the series expansions. The partial derivatives have an accuracy of four to five decimals.

**Wave terms I**

\[
I_3 = \Re \{-4ik_0 \int_{\kappa}^{\pi} \sec^2 \theta \int_{\zeta}^{\pi} \frac{1}{\kappa} H(k_0(x - \xi)) e^* d\xi d\theta\}
\]  
(A.32)

where

\[
u = k_0(z + \zeta) \sec^2 \theta + ik_0 \left| y - \eta \right| \sec^2 \theta \sin \theta - ik_0 \left| x - \xi \right| \sec \theta
\]

In view of the definition of the step function \( H \), two integration intervals are considered: \( x, \zeta \) and \( x, \zeta \). For \( x, \zeta \), in front of the lifting line, \( I_3 \) is zero. For \( x, \zeta \) it follows:

\[
I_3 = \Re \{-4ik_0 \int_{\kappa}^{\pi} \sec^2 \theta \int_{\zeta}^{\pi} e^* d\xi d\theta\}
\]  
(A.33)

By using partial integration one finds for \( I_3 \):

\[
I_3 = \Re \{-4 \int_{\pi}^{\kappa} \sec \theta e^u(1 - e^{-u}) d\theta\}
\]  
(A.34)

where
$u_0 = k_0(z + \zeta) \sec^2 \theta + ik_0 |y - \eta| \sec^2 \theta \sin \theta$  \hspace{1cm} (A.35)

$u_i = -ik_0(x + \eta \tan \Lambda) \sec \theta$

while the derivatives with respect to $\eta$ and $\zeta$ are:

$$\frac{\partial I_3}{\partial \eta} = \Re \{-4ik_0 \text{sign}(\eta - y) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^3 \theta \sin \theta e^{it}(1 - e^{-it}) d\theta - 4ik_0 \tan \Lambda \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^2 \theta e^{(it, \eta)} d\theta\} \hspace{1cm} (A.36)$$

$$\frac{\partial I_3}{\partial \zeta} = \Re \{-4ik_0 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^3 \theta e^{it}(1 - e^{-it}) d\theta\} \hspace{1cm} (A.37)$$

The derivatives of $\partial I_3/\partial \eta$ and $\partial I_3/\partial \zeta$ are zero for $x < \xi_c$. For $x \geq \xi_c$, the following derivatives are derived by analytical differentiation:

$$\frac{\partial^2 I_3}{\partial x \partial \eta} = \Re \{-4k_0^2 \text{sign}(\eta - y) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^4 \theta \sin \theta e^{(it, \eta)} d\theta - 4k_0^2 \tan \Lambda \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^3 \theta e^{(it, \eta)} d\theta\} \hspace{1cm} (A.38)$$

$$\frac{\partial^2 I_3}{\partial x \partial \zeta} = \Re \{-4ik_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^4 \theta e^{(it, \eta)} d\theta\} \hspace{1cm} (A.39)$$

$$\frac{\partial^2 I_3}{\partial y \partial \eta} = \Re \{-4k_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^3 \theta \sin^2 \theta e^{it}(1 - e^{-it}) d\theta + 4k_0^2 \text{sign}(y - \eta) \tan \Lambda \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^3 \theta \sin \theta e^{(it, \eta)} d\theta\} \hspace{1cm} (A.40)$$
\[ \frac{\partial^2 I_2}{\partial y \partial \xi} = \mathcal{R} \left\{ -4ik_0^2 \text{sign}(y - \eta) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^2 \theta \sin \theta e^{ik_0(r - e^{j\theta})} d\theta \right\} \tag{A.41} \]

\[ \frac{\partial^2 I_3}{\partial z \partial \eta} = \mathcal{R} \left\{ -4ik_0^2 \text{sign}(\eta - y) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^2 \theta \sin \theta e^{ik_0(r - e^{j\theta})} d\theta - 4ik_0^2 \tan \Lambda \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^4 \theta e^{ik_0(r - e^{j\theta})} d\theta \right\} \tag{A.42} \]

\[ \frac{\partial^2 I_3}{\partial z \partial \xi} = \mathcal{R} \left\{ -4k_0^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^2 \theta e^{ik_0(r - e^{j\theta})} d\theta \right\} \tag{A.43} \]

These integrals are solved numerically, as outlined in Section 2.5.

References


APPENDIX B NUMERICAL EVALUATION OF THE TIME DOMAIN GREEN'S FUNCTION

In the computational approach it is required to evaluate the following integral containing the memory part of the Green function \( G^f \):

\[
\int_0^t \int_{S_{sv}} \Gamma(q, \tau) \nabla \frac{\partial G^f}{\partial \tau} dS
\]

where \( t \) denotes the time, \( \tau \) is the past time parameter, \( \Gamma \) is the circulation at vortex point \( q \), \( \nabla \) is the gradient operator \( \frac{\partial}{\partial x, \partial y, \partial z} \) at the field point \( p(x,y,z) \), \( n_q \) is the normal derivative at vortex point \( q(\xi, \eta, \zeta) \) and \( S \) is the vortex element area. \( S_{sv} \) denotes the lifting surface and wake vortex element area's. The normal derivative is obtained by using \( \partial / \partial n = n \cdot \nabla \), so that the second partial spatial derivatives of \( \partial G^f / \partial t \) must be evaluated. The time derivative of \( G^f \), denoted here by \( G_f^f \), is needed for non-stationary lifting surface and wake vortex elements. For stationary wake vortex elements, the position of the wake vortex elements in space is fixed so that at any time step the distance between control points on the lifting surface and the history of the position of wake vortex elements is constant. The time integration of the Green function can then be carried out analytically. The two forms of the Green function are then, see Section 4.4:

\[
G^f(p,t; \tau, q) = -2 \int_0^\infty [1 - \cos(\sqrt{gk_w(t-\tau)})] e^{k(z+\zeta)} J_0(k_w r) dk_w
\]

and

\[
G_f^f(p,t; \tau, q) = -2 \int_0^\infty \sin(\sqrt{gk_w(t-\tau)}) \sqrt{gk_w} e^{k(z+\zeta)} J_0(k_w r) dk_w
\]

where \( k_w \) is the wave number, \( J_0 \) is the Bessel function of zero order and \( g \) is the gravitational constant. The horizontal distance between field and vortex point is given by:

\[
r = \sqrt{(x-\xi)^2 + (y-\eta)^2}
\]

Direct numerical evaluation of the integrals for \( G^f \) and \( G_f^f \) requires a relatively large amount of computer time due to the oscillating nature of the integrands and the semi-infinite integration interval. Especially for \((z+\zeta)\rightarrow0\) the oscillations decay slowly with increasing \( k_w \) value and accurate numerical integration is difficult. Due to the large number of Green function evaluations in the time domain simulation the required computer time becomes excessive. In order to develop a more efficient approach, a treatment suggested by Beck and Magee (1992) is adopted and extended here. In this treatment tables for principal partial derivatives of \( G^f \) and \( G_f^f \) are generated. During the time domain simulation these principal derivatives are obtained by means of interpolation on the tabulated values.
By means of the following substitutions:

\[ \lambda = k_w R_0 \]
\[ \mu = \frac{(z + \zeta)}{R_0} \]
\[ \beta = \sqrt{\frac{g}{R_0}} (t - \tau) \]

the following two-parameter formulations are obtained:

\[ G^f(\mu, \beta) = -\frac{2}{R_0} \int_0^\infty \left[ 1 - \cos(\sqrt{\lambda} \beta) \right] e^{-\mu \lambda} J_0(\lambda \sqrt{1 - \mu^2}) d\lambda \]  \hspace{1cm} (B.6)

\[ G_i^f(\mu, \beta) = -2 \sqrt{\frac{g}{R_0^3}} \int_0^\infty \sin(\sqrt{\lambda} \beta) e^{-\mu \lambda} J_0(\lambda \sqrt{1 - \mu^2}) \sqrt{\lambda} d\lambda \]  \hspace{1cm} (B.7)

with 0 \leq \mu \leq 1 and \beta \geq 0. The parameter \( \mu \) is the ratio between the horizontal and vertical distance between vortex and field points while \( \beta \) is a time parameter related to the phase of the generated waves.

A plot of \( \hat{G}^f \) and \( \hat{G}_i^f \) defined by

\[ \hat{G}^f(\mu, \beta) = -\frac{R_0}{2} G^f(\mu, \beta) \]  \hspace{1cm} (B.8)

\[ \hat{G}_i^f(\mu, \beta) = -\sqrt{\frac{R_0^3}{4g}} G_i^f(\mu, \beta) \]

is shown in the Figures B1a and B2a. The functions are highly oscillating, especially for small \( \mu \) values, i.e. near the free surface.
Figure B1a  Plot of Green's function $\hat{G}^f$

Figure B1b  Plot of Green's function $\hat{G}^f$
Figure B1c  Plot of Green’s function $\partial^2 \hat{G} / \partial \mu^2$

Figure B1d  Plot of Green’s function $\partial^2 \hat{G} / \partial \beta^2$
In view of these oscillations, Green functions $\hat{G}_r'$ and $\hat{G}_{\nu}'$ are defined here as:

$$\hat{G}_r' (\mu, \beta) = \hat{G}_r (\mu, \beta) - [1 + e^{i(\beta_0\nu)}\hat{G}_r'(0, \beta)]$$  \hspace{1cm} (B.9)

where, see Wehausen (1959):

$$\hat{G}_r'(0, \beta) = \frac{\pi \beta^2}{8 \sqrt{2}} (J_\nu (\nu \beta^2) \cdot J_\nu (\nu \beta^2) - J_{-\nu} (\nu \beta^2) \cdot J_{-\nu} (\nu \beta^2))$$  \hspace{1cm} (B.10)

and

$$\hat{G}_{\nu}' (\mu, \beta) = \hat{G}_r' (\mu, \beta) - e^{i(\beta_0\nu)} \hat{G}_r'(0, \beta)$$  \hspace{1cm} (B.11)

where

$$\hat{G}_r'(0, \beta) = \frac{\pi \beta^3}{16 \sqrt{2}} (J_\nu (\nu \beta^2) \cdot J_{-\nu} (\nu \beta^2) + J_\nu (\nu \beta^2) \cdot J_{-\nu} (\nu \beta^2))$$  \hspace{1cm} (B.12)

here $J_\nu$ is the Bessel function of real order $\nu$.

The $\hat{G}_r'$ and $\hat{G}_{\nu}'$ functions are much smoother than $\hat{G}_r$ and $\hat{G}_r'$, see Figures B1b and B2b. The use of these modified functions is beneficial for the accuracy of the numerical differentiation used to determine the first and second partial derivatives of $\hat{G}_r'$ and $\hat{G}_{\nu}'$ with respect to $\mu$ and $\beta$.

For $\beta$ values between 0.0 and 20, interpolation tables are generated for $\hat{G}_r'$ and $\hat{G}_{\nu}'$ and their first and second partial derivatives with respect to $\mu$ and $\beta$. The tables for $\hat{G}_r'$ and $\hat{G}_{\nu}'$ are determined by means of numerical integration with a relative error of $10^{-6}$. The grid consists of 101x1001 points in $\mu$ and $\beta$ directions respectively. The grid in $\mu$ direction is cosine spaced with the higher density near the $\mu=0$ axis, while in $\beta$ direction an equidistant spacing is used. The partial derivatives with respect to $\mu$ and $\beta$ are obtained by using numerical differentiation of cubic spline representations of $\hat{G}_r'$ and $\hat{G}_{\nu}'$. Finally, a bi-linear interpolation scheme is used on the tabulated values of $\hat{G}_r'$ and $\hat{G}_{\nu}'$ and their first and second partial derivatives.

The partial derivatives of the $\hat{G}_r'(0, \beta)$ and $\hat{G}_{\nu}'(0, \beta)$ functions are computed analytically and expressed in terms of Bessel functions which are computed with a relative accuracy of $10^{-6}$.

The relative errors in the partial derivatives of $\hat{G}_r'$ and $\hat{G}_{\nu}'$ with respect to $\mu$ and $\beta$, due to the numerical differentiation and subsequent linear interpolation from the tables range between $10^{-7}$ and $10^{-4}$. These errors have been obtained by comparing the results of the procedure described above with direct numerical integration of the analytical expressions for the partial derivatives of $\hat{G}_r'$ and $\hat{G}_{\nu}'$ with respect to $\mu$ and $\beta$. The larger relative errors appear when the partial derivative terms are relatively small, due to loss of significant digits. The total memory effect contributions to the induced velocity at the lifting surface have a relative inaccuracy smaller than $5 \times 10^{-4}$. Figures B1c, B1d, B2c and B2d show the second partial derivatives of $\hat{G}_r'$ and $\hat{G}_r'$ with respect to $\mu$ and $\beta$. The large oscillations for small $\mu$ and large $\beta$ values are distinct.
Figure B2a  Plot of Green's function $\hat{G}_t$

Figure B2b  Plot of Green's function $\hat{G}_n$
Appendix B  Numerical evaluation of the time domain Green's function

Figure B2c  Plot of Green's function $\partial \hat{G} / \partial \mu$

Figure B2d  Plot of Green's function $\partial^2 \hat{G} / \partial \beta^2$
If a higher accuracy, with a relative error less than $10^{-6}$ is required for relatively large $\beta$ values, use can be made of spherical-harmonic expansions and the large-time asymptotic expansions derived by Newman (1985, 1992). Newman derives series in Legendre polynomials, powers of $e$ and ordinary polynomials, for both the $\hat{G}'$ and $\hat{G}''$ functions. The partial derivatives with respect to $\mu$ and $\beta$ are obtained from analytical differentiation on the series.

Although the use of these series descriptions is much more efficient than direct numerical integration, the use of the interpolation table remains the most efficient method. In the current numerical method the interpolation tables are used for $\beta$ values below 20, for larger $\beta$ values the Newman series are used.

The partial derivatives of $G'$ and $G''$ with respect to $x$ through $\zeta$ are obtained by using the chain rule as follows:

$$\frac{\partial G'}{\partial \zeta}(p,t,q,x) = \frac{\partial}{\partial \zeta} \left( -\frac{2}{R_0} \right) \hat{G}'(\mu,\beta) + \left( -\frac{2}{R_0} \right) \frac{\partial \hat{G}'}{\partial \zeta}(\mu,\beta)$$

$$\frac{\partial \hat{G}'}{\partial \zeta}(\mu,\beta) = \frac{\partial \hat{G}'}{\partial \mu} \frac{\partial \mu}{\partial \zeta} + \frac{\partial \hat{G}'}{\partial \beta} \frac{\partial \beta}{\partial \zeta}$$

$$\frac{\partial \mu}{\partial \xi} = \mu \frac{(x-\xi)}{R_0^2}$$

$$\frac{\partial \mu}{\partial \beta} = \beta \frac{(x-\xi)}{R_0^2}$$

with similar differentiations for the other derivatives involved.

The integration of the partial derivatives over the vortex element area is performed by using a 4 or 9 point two-dimensional integration rule after mapping the function values on a unit square.

References


APPENDIX C  VISCOSITY EFFECTS ON FOIL SECTIONS USED IN EXPERIMENTS

The following Tables show viscosity effect parameters as defined in Section 2.6. These data are based on Xfoil results for Reynolds numbers of $10^5$, $10^6$ and $10^7$. Flap efficiencies were determined for a plain, sealed flap with a chord ratio of 25% and a deflection angle of 5 degrees.

Table C1  Viscosity effect parameters for a NACA 16-309 (a=0.8) section

<table>
<thead>
<tr>
<th>$R_n$</th>
<th>$f_a$</th>
<th>$f_c$</th>
<th>$f_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^5$</td>
<td>0.845</td>
<td>0.716</td>
<td>0.607</td>
</tr>
<tr>
<td>$1 \times 10^6$</td>
<td>0.871</td>
<td>0.904</td>
<td>0.789</td>
</tr>
<tr>
<td>$1 \times 10^7$</td>
<td>0.912</td>
<td>0.948</td>
<td>0.917</td>
</tr>
</tbody>
</table>

Table C2  Viscosity effect parameters for a NACA 16-309 (a=1.0) section

<table>
<thead>
<tr>
<th>$R_n$</th>
<th>$f_a$</th>
<th>$f_c$</th>
<th>$f_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^5$</td>
<td>0.861</td>
<td>0.584</td>
<td>0.637</td>
</tr>
<tr>
<td>$1 \times 10^6$</td>
<td>0.878</td>
<td>0.785</td>
<td>0.814</td>
</tr>
<tr>
<td>$1 \times 10^7$</td>
<td>0.912</td>
<td>0.861</td>
<td>0.919</td>
</tr>
</tbody>
</table>

Table C3  Viscosity effect parameters for a YS 1040 section

<table>
<thead>
<tr>
<th>$R_n$</th>
<th>$f_a$</th>
<th>$f_c$</th>
<th>$f_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^5$</td>
<td>0.846</td>
<td>0.746</td>
<td>0.691</td>
</tr>
<tr>
<td>$1 \times 10^6$</td>
<td>0.889</td>
<td>0.852</td>
<td>0.912</td>
</tr>
<tr>
<td>$1 \times 10^7$</td>
<td>0.935</td>
<td>0.911</td>
<td>0.938</td>
</tr>
</tbody>
</table>

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APPENDIX D  UNCERTAINTY ANALYSIS FOR RESISTANCE MODEL TESTS

Introduction

In Section 3.4 the Hydres program is validated on basis of model test and full scale results. When a validation is performed it is of interest to estimate the uncertainty of the reference quantities. The uncertainty in resistance model tests is the subject of this Appendix. The uncertainty of full scale measurements can not be assessed due to lack of data.

The uncertainty analysis involves three quantities. Bias errors are systematic or fixed errors, precision errors have a random character and the sensitivity describes the propagation of a single error in the measurement results. The objective of the analysis is to obtain an uncertainty interval with a chosen confidence for the true value of the measurement.

The High Speed Marine Vehicle committee of the 20th ITTC provides a compilation of errors which may occur during a resistance test for a high speed displacement hull, see the ITTC (1993). This assessment of errors is based on standard test practice in the Deep Water Towing Basin of MARIN. Error data have been gathered for two model speeds. For the model tests used for validation of Hydres, use has been made of the same facility for two of the three models. One model was towed in a facility with smaller dimensions, the High Speed Basin, see Section 3.4. The data provided by the ITTC are assumed to be valid for the smaller basin as well, apart from blockage effects, and are used here as a basis for assessing the uncertainty in resistance for hydrofoil craft hull forms.

The following error sources identified by the ITTC are applicable to the hydrofoil hull resistance problem:
- instrumentation and data analysis: force transducers, filtering of measurement signals,
- residual flow effects: flow disturbances in the basin created by a previous test run,
- model preparation: model weight and location of centre of gravity,
- aerodynamic effects: overspeed due to the towing carriage.

For hydrofoil craft, an additional analysis is needed for the following items:
- viscosity effects on foil lift and drag,
- inaccuracies in the foil geometry,
- blockage effects.

Possible error sources not investigated by the ITTC nor in this Appendix, due to lack of data, are:
- hull model geometry,
- scale effects in spray generation,
- flow turbulence level and water contamination in the basin.

Three measurement results are considered: the trim, draft and resistance of the model. The hullborne and foilborne conditions need to be investigated separately since the errors and the propagation of errors have a different character for these two conditions. It is then straightforward to assess the errors for the hump speed and the cruise speed conditions. The ITTC data are given for two speeds, 5 and 9 m/s. For the hydrofoil craft model tests the hump and cruise model speeds are in between
3-4 m/s and 7-8 m/s respectively. As the ITTC errors tend to increase with speed, somewhat pessimistic error estimates are obtained when the ITTC data are used for hump and cruise speed conditions respectively. This will not affect the total uncertainty significantly since the errors identified by the ITTC are relatively small in comparison with the other errors involved.

Measurement errors

The total errors in hull resistance due to the four aforementioned items identified by the ITTC (1993) are given in Table D1. Hereby the contributions due to blockage have been subtracted.

<table>
<thead>
<tr>
<th>Table D1</th>
<th>Measurement errors in hull resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Speed (m/s)</td>
</tr>
<tr>
<td></td>
<td>Hump speed</td>
</tr>
<tr>
<td></td>
<td>Cruise speed</td>
</tr>
</tbody>
</table>

At cruise speed, the hull is desubmerged and only the foil system contributes to the total resistance. From repeatability tests for a captive foil system at cruise speed, a precision error of 3% in foil resistance has been determined. This error corresponds to the ITTC precision error given in Table D1. The foil system used was the one described in Section 3.3, Figure 3.13.

The measurement of the trim and draft for hydrofoil craft model testing is performed by measuring the vertical displacement at two positions (fore and aft) by means of thin steel wires and potentiometers. The measurement errors involved are estimated to be at least one order of magnitude smaller than viscosity and geometry inaccuracy effects on the foil lift. Therefore, no attempts have been made to estimate these measurement errors.

Viscosity effects

Viscosity effects on foil characteristics at model scale can be classified as an error source that must be attributed to the calculation model since viscosity effects are taken into account in the calculation model. However, including viscosity effects in the calculation model is only required for validation purposes. Without doing so, validation would be pointless due to the large viscosity effect on lift and thereby on drag. The difference between the actual viscosity effect on model scale and the viscosity effect used in the calculation model is therefore considered as a model test error.

In Section 2.6 comparisons are made between experimental and calculated (Xfoil) viscosity correction factors at Reynolds numbers between 1x10^6 and 1x10^7. The Reynolds number range for model tests is 2x10^4<Re<8x10^5, so that data for deriving error estimates for the proper Reynolds number range is lacking. The lower the Reynolds number is, the greater is the chance that separation occurs which effects are only to some extent predicted by Xfoil. Therefore, a pessimistic estimate
of the error is made, based on the data given in Section 2.6. Table D2 shows these bias error estimates due to the uncertainty in viscosity correction factors and the sensitivities with respect to the total lift coefficient.

<table>
<thead>
<tr>
<th>Item</th>
<th>Bias error (%)</th>
<th>Sensitivity Hump speed</th>
<th>Sensitivity Cruise speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift slope correction $f_a$</td>
<td>5</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Zero lift angle correction $f_c$</td>
<td>15</td>
<td>0.3</td>
<td>0.8</td>
</tr>
<tr>
<td>Flap efficiency correction $f_b$</td>
<td>20</td>
<td>0.3</td>
<td>0.0</td>
</tr>
</tbody>
</table>

For the cruise speed condition it is assumed that no flaps are used. The bias error in lift coefficient is obtained from:

$$B(C_l) = \sqrt{\sum_{k=1}^{3} (\theta_k B_k)^2}$$  \hspace{1cm} (D.1)

where $\theta_k$ and $B_k$ denote the sensitivity and bias error of component $k$ respectively. This results in bias errors in the lift coefficient of 9.8% and 16.0% for hump and cruise speed conditions respectively. If no flaps are used at hump speed either (model A), the bias error is 7.2%.

The bias error in viscous drag is estimated in a similar way as 8.5% and 2.5% for high (hump speed) and low (cruise speed) lift coefficients respectively. The sensitivity with respect to the total drag is 0.2 and 0.4 for hump and cruise speed respectively. This results in bias errors for the drag of 1.7% and 1.0%.

Errors in lift affect the trim, the draft and the resistance. The following sensitivities for lift errors have been determined, based on Hydres results for the surface piercing hydrofoil craft model A and fully submerged hydrofoil model C, at model scale Reynolds numbers.

<table>
<thead>
<tr>
<th>Item</th>
<th>Hump speed</th>
<th>Cruise speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trim</td>
<td>0.72</td>
<td>1.89</td>
</tr>
<tr>
<td>Draft</td>
<td>1.02</td>
<td>1.36</td>
</tr>
<tr>
<td>Resistance</td>
<td>0.49</td>
<td>0.71</td>
</tr>
</tbody>
</table>
Table D4  Sensitivities of lift errors for hydrofoil model C

<table>
<thead>
<tr>
<th>Item</th>
<th>Hump speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trim</td>
<td>0.80</td>
</tr>
<tr>
<td>Draft</td>
<td>1.72</td>
</tr>
<tr>
<td>Resistance</td>
<td>0.63</td>
</tr>
</tbody>
</table>

For the fully submerged hydrofoil model at the hump speed the sensitivity of the draft is relatively large as it does not have the inherent stability of a surface piercing foil system.

Foil geometry

The surface piercing foil models for hydrofoil craft A and B, see Section 3.4, were assembled of casted foil parts which were welded together. Such a process is susceptible to geometry errors due to errors in the casting mould and relative incidence errors due to the welding of the foil parts. For model C the foil parts were milled on a numerically controlled machine with a very low tolerance so that geometry errors are assumed to be negligible. The different foil parts were assembled with high accuracy so that relative incidence errors are assumed to be negligible as well. Besides geometry errors on the foil models, the absolute incidence setting on the models during testing is also subject to errors.

The geometry of model B was checked. Errors in the sectional geometry of at maximum 9% of the thickness were found, over a length of 20% of the foil chord (0.1<\(c/c<0.3\)). Such deviations affect the camber of the section and thereby the lift and drag. Lift and drag deviations were determined by means of Xfoil for model scale Reynolds numbers. The bias errors are set equal to these lift and drag deviations and are shown in Table D5.

Table D5  Bias errors due to geometry inaccuracies

<table>
<thead>
<tr>
<th>Condition</th>
<th>Error in (C_l) (%)</th>
<th>Error in (C_d) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hump speed</td>
<td>1.2</td>
<td>2.5</td>
</tr>
<tr>
<td>Cruise speed</td>
<td>2.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The error in incidence was found from measurements to be 0.2 and 0.3 deg for the dihedral parts, relative to the centre part of the foil. The absolute incidence error due to the incidence adjustment on the model is estimated to be 0.10 deg. A total bias error in incidence follows from:
where \(B(\alpha_r)\) is the absolute incidence adjustment error, \(B(\alpha_t)\) is the relative incidence error of the dihedral parts and \(\theta\) is the sensitivity of the lift contribution of these parts relative to the total lift which is taken as 0.25. A bias error \(B(\alpha)\) of 0.19 deg results from eq. (D.2), which in turn leads to lift coefficient errors of 3.1% and 6.4% for the hump and cruise speed conditions respectively, for model A. Bias errors for the total drag are 0.5% and 0.6% for hump and cruise speeds respectively, for model A. Hereby the sensitivities shown in Table D3 are used.

For model C, the incidence errors consist of the absolute incidence adjustment error only. This leads to lift and resistance errors of 1.6% and 0.3% respectively, for the hump speed condition.

### Blockage effects

The ITTC, see Savitsky et al. (1981), specifies the following criteria for the absence of blockage effects during resistance testing of fast displacement hull forms:

\[
\frac{d_r}{L_m} > 0.8 \tag{D.3}
\]

\[
\frac{b_r}{L_m} > 2.0
\]

where \(d_r\) is the depth of the basin, \(b_r\) is the width of the basin and \(L_m\) is the length of the model. The first criterion excludes shallow water effects while the second excludes blockage effects due to the side walls of the basin.

When for the hump speed condition a model length of one half of the total model length is used then \(d_r/L_m=2.7\) and \(b_r/L_m=2.7\) which satisfies both criteria. These numbers are valid for the smallest basin used for the model tests, the High Speed Basin, see Section 3.4.

Tamura (1981) gives the following equation for the mean speed increase around the model due to blockage:

\[
\frac{\Delta U_m}{U_m} = 0.67 M \left( \frac{L_m}{b_r} \right)^{0.4} \left( 1 - F_{nd}^2 \right)^{-1} \tag{D.4}
\]

At the hump speed the blockage \(M\) in terms of the ratio between the cross sectional areas of the model and channel is 0.0035 while the Froude number based on the depth of the towing basin \(F_{nd}\) is 0.48. This results in an overspeed of 0.14%, which is not significant.

At cruise speed, the blockage ratio is very small \((M=0.0008)\) while the water depth Froude number is 0.98. When the critical speed with respect to the water depth is approached, a strong increase in wavemaking resistance occurs, at least for ship models. Obviously, eq. (D.4) is not suited for such
conditions.

In order to check the empirical relations, eq. (D.3) and (D.4), and to determine blockage effects at high speeds, the non-linear panel code Rapid, see Raven (1993), was used. In Rapid the potential flow about a ship hull with lifting surfaces advancing in a channel can be determined. A schematic hydrofoil craft was used: the hard chine hull form with foil system as defined in Section 2.3, for conditions corresponding to the model tests performed for model A, see Section 3.4. The corresponding conditions include speed, submerged hull dimensions, foil submergence, foil system lift and channel width and depth. Rapid runs were performed for unrestricted water, shallow water and channel conditions, for the hump speed and for the cruise speed. At hump speed, the hull was partially submerged, at cruise speed only the foils were submerged. During the iterations in Rapid for obtaining a steady wave pattern, the position of the craft was fixed. Table D6 specifies the calculation conditions.

<table>
<thead>
<tr>
<th>Item</th>
<th>Hump speed</th>
<th>Cruise speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_n$ (based on hull length and foil chord respectively)</td>
<td>0.88</td>
<td>6.20</td>
</tr>
<tr>
<td>$F_{nl}$</td>
<td>0.48</td>
<td>0.98</td>
</tr>
<tr>
<td>Hull waterline length (m)</td>
<td>12.5</td>
<td>-</td>
</tr>
<tr>
<td>Trim angle (deg)</td>
<td>3.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Transom submergence (m)</td>
<td>0.65</td>
<td>-</td>
</tr>
<tr>
<td>Foil span (m)</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Foil chord (m)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Foil submergence forward/aft (m)</td>
<td>1.50/2.50</td>
<td>1.00/1.25</td>
</tr>
<tr>
<td>Foil lift coefficient</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>Channel width</td>
<td>40.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Channel depth</td>
<td>40.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Number of hull panels</td>
<td>2370</td>
<td>-</td>
</tr>
<tr>
<td>Number of foil panels</td>
<td>730</td>
<td>730</td>
</tr>
<tr>
<td>Number of free surface panels</td>
<td>2225</td>
<td>2080</td>
</tr>
<tr>
<td>Number of panels on channel side walls</td>
<td>2220</td>
<td>2220</td>
</tr>
<tr>
<td>Blockage ratio</td>
<td>0.0035</td>
<td>0.0008</td>
</tr>
</tbody>
</table>
Appendix D  Uncertainty analysis for resistance model tests

In Section 2.2, it is argued that panel methods are not well suited for hard chine hull forms at relatively high Froude numbers. This turned out to be true for the present case: for the hump speed condition no convergence could be achieved. Therefore, the empirical formulations for hull blockage can not be checked and it must be assumed that for this condition blockage effects are negligible indeed.

For the foilborne condition, the restricted water effects are shown in Table D7 in terms of the changes in foil lift and induced drag at shallow water and in a channel, relative to the lift and drag at unrestricted water. The induced drag includes wave making drag due to the finite submergence of the foils for both restricted and unrestricted water. The lift and drag forces are based on pressure integration over the foil surfaces.

<table>
<thead>
<tr>
<th>Item</th>
<th>Shallow water</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induced drag</td>
<td>0.0%</td>
<td>-0.8%</td>
</tr>
<tr>
<td>Lift</td>
<td>0.1%</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

Table D7  Shallow water and blockage effects on foil forces

Shallow water effects appear very small. Figure D1 shows the wave patterns for unrestricted water and in the channel. The wave pattern at shallow water is virtually the same as on unrestricted water. Hydrofoils at a high chord Froude number create, apart from a wave trough directly behind the foil, mainly divergent waves with a relatively short wavelength which are insensitive to shallow water effects, i.e. when the wavelength is less than twice the water depth.

Blockage effects are limited to about one percent. Similar results are observed from a study into blockage effects on hydrofoils presented by Koushan and Kruppa (1997). Figure D2 shows longitudinal wave cuts for the three conditions, at the centreline and near the tip of the foils. The forward and aft foil position is at x/c=17.5 and 2.5 respectively at which position a local wave crest is present. The depth of the wave trough behind the foils is about 1% of the channel depth. In a channel the watersurface is located above that for unrestricted and shallow water, for almost the entire region shown in the Figures. At larger distances aft of the foils, the watersurface tends to be lower than in unrestricted water.
Figure D1  Wave contours at unrestricted water and in a channel

Figure D2a  Longitudinal wave cuts, $2y/b=0.93$
Total uncertainty

Based on the errors determined in the previous paragraphs, the following total uncertainty is derived. The total uncertainty $U_i$ follows from:

$$U_i = \sqrt{B_i^2 + (t_{95}S_i)^2}$$  \hspace{1cm} (D.5)

where $B_i$ is the total bias error, $S_i$ is the total precision error and $t_{95}$ is Student's confidence interval parameter. Here, a 95% confidence interval is selected for which $t_{95}=2.0$. The total bias and precision errors are obtained from the individual errors as follows:

$$B_i = \sqrt{\sum_{k=1}^{n} B_k^2}$$  \hspace{1cm} (D.6)

$$S_i = \sqrt{\sum_{k=1}^{n} S_k^2}$$
The individual bias errors $B_i$ are obtained from the errors in lift and drag and the sensitivities for the lift force on trim, draft and resistance as shown in Tables D3 and D4 for models A and C respectively.

For model A at hump and cruise speed respectively, Tables D8 and D9 show the total uncertainty, while Table D10 shows that for model C at the hump speed.

### Table D8: Total uncertainty for model A at hump speed

<table>
<thead>
<tr>
<th>Item</th>
<th>Trim</th>
<th>Draft</th>
<th>Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement bias error (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Viscosity bias error (%)</td>
<td>5.1</td>
<td>7.3</td>
<td>5.1</td>
</tr>
<tr>
<td>Geometry bias error (%)</td>
<td>2.1</td>
<td>3.2</td>
<td>1.6</td>
</tr>
<tr>
<td>Blockage bias error (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Total bias error $B_i$ (%)</td>
<td>5.5</td>
<td>8.0</td>
<td>5.4</td>
</tr>
<tr>
<td>Total precision error $S_i$ (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Total Uncertainty $U_i$ (%)</td>
<td>5.5</td>
<td>8.0</td>
<td>6.2</td>
</tr>
</tbody>
</table>

### Table D9: Total uncertainty for model A at cruise speed

<table>
<thead>
<tr>
<th>Item</th>
<th>Trim</th>
<th>Draft</th>
<th>Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement bias error (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>2.7</td>
</tr>
<tr>
<td>Viscosity bias error (%)</td>
<td>18.5</td>
<td>21.8</td>
<td>11.4</td>
</tr>
<tr>
<td>Geometry bias error (%)</td>
<td>12.1</td>
<td>8.7</td>
<td>4.6</td>
</tr>
<tr>
<td>Blockage bias error (%)</td>
<td>2.3</td>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td>Total bias error $B_i$ (%)</td>
<td>22.2</td>
<td>23.5</td>
<td>12.6</td>
</tr>
<tr>
<td>Total precision error $S_i$ (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Total Uncertainty $U_i$ (%)</td>
<td>22.2</td>
<td>23.5</td>
<td>14.0</td>
</tr>
</tbody>
</table>
Table D10  Total uncertainty for model C at hump speed

<table>
<thead>
<tr>
<th>Item</th>
<th>Trim</th>
<th>Draft</th>
<th>Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement bias error (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Viscosity bias error (%)</td>
<td>7.8</td>
<td>16.9</td>
<td>6.4</td>
</tr>
<tr>
<td>Geometry bias error (%)</td>
<td>1.4</td>
<td>2.8</td>
<td>1.1</td>
</tr>
<tr>
<td>Blockage bias error (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Total bias error $B_i$ (%)</td>
<td>8.0</td>
<td>17.1</td>
<td>6.6</td>
</tr>
<tr>
<td>Total precision error $S_i$ (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Total Uncertainty $U_i$ (%)</td>
<td>8.0</td>
<td>17.1</td>
<td>7.2</td>
</tr>
</tbody>
</table>

It can be concluded that the uncertainty in trim and draft is quite large for cruise speed conditions for model A. This is mainly due to the large sensitivity of trim and draft to the lift force of the foils, and due to the error in viscosity effect on the zero-lift angle which significantly affects the foil lift for cruise speed conditions, i.e. when the lift coefficient is relatively low. For model C, the uncertainty in draft is relatively large due to the large sensitivity of the draft to errors in foil lift forces.

References


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COMPUTATIONAL METHODS FOR HYDROFOIL CRAFT IN STEADY AND UNSTEADY FLOW

by
Frans van Walree

SUMMARY

The objective of the study is to develop tools for hydrodynamic analysis of hydrofoil craft. Three main subjects in hydrodynamics are addressed: powering, seakeeping and manoeuvring. For the powering problem a steady flow method is developed while for the seakeeping and manoeuvring problem an unsteady flow method is developed.

Despite the fact that a substantial amount of knowledge on the characteristics of lifting surfaces and ship hull forms exists, no calculation method exists for predicting the powering performance of hydrofoil craft for the entire speed range, that has a wide range of applicability and that predicts at the same time the forces acting on the hull and foil system with an acceptable accuracy and computational efficiency. In order to develop a suitable tool the following approach is taken whereby most attention is paid to foil systems.

The lift and drag characteristics of the foil system are derived from a vortex lattice method. A Green's function formulation is used that accounts for linearized free surface conditions at the undisturbed water surface. Interaction between the forward and aft foil and between the foils and the hull is addressed. Hull force components are derived from experimental data on series of hydrofoil craft hull forms. Appendage force components are based on empirical formulations. Propulsor characteristics are based on open water diagrams of propeller series and on an existing calculation method for waterjet systems. The computational method is used within an iterative scheme to determine the equilibrium position of the craft relative to the water surface. An example of the application of the computer program Hydres, containing the steady flow computational method, shows that its results can be used to minimize the required propulsion power to take-off.

Hydres is validated on basis of model test and full scale data for surface piercing and fully submerged hydrofoil craft. The uncertainty in model test results are assessed and found to be significant, mainly due to viscosity effects on the lift characteristics and foil system geometry errors. Nevertheless, the predictions are found to be in fair agreement with the model test data. The agreement with full scale data is satisfactory as well.

Similar to the steady flow method, no unsteady flow method exists that can be applied to arbitrary foil systems, that takes unsteady flow effects into account properly and that can be used in the time domain in six degrees of freedom. Therefore, such a time domain simulation method is developed for seakeeping and manoeuvring at cruise speed conditions. It is assumed that the hull has no contact with the water surface. The simulation method is based on an unsteady vortex lattice method. A time domain Green's function formulation is used to account for linearized free surface effects at the undisturbed free surface. Unsteady interaction between the forward and aft foil is addressed. A linear simulation method is developed that assumes small motions about a certain equilibrium position, a constant speed and a constant course. A non-linear simulation method that
can handle large and transient motions is developed as well.

The unsteady vortex lattice method is validated numerically by means of comparisons with analytical results for basic foils. A validation study into damping and wave excitation forces on a tandem foil system is based on experimental data. A good agreement is found. Examples of the application of the time domain simulation method HydSim, containing the unsteady vortex lattice method, are given for seakeeping and manoeuvring. These examples show the effects of linearization of the motions, unsteady flow effects and the significance of foil interaction and the wave part of the Green's function. Finally, HydSim is validated on basis of full scale data for seakeeping and manoeuvring for surface piercing and fully submerged foil hydrofoil craft. A satisfactory agreement between simulation results and the full scale data is found.

The results of the study are thought to be valuable for use for investigations into the powering performance and the comfort and safety of operation of hydrofoil craft. The second part of the work may also serve as a basis for calculation tools for more conventional, fin stabilized craft types and propsors utilizing hydrofoils.
REKENMETHODEN VOOR DRAAGVLEUGELSCHEPEN IN
STATIONAIRE EN INSTATIONAIRE STROMING

door
Frans van Walree

SAMENVATTING

Het doel van de promotie studie is de ontwikkeling van gereedschappen voor het analyseren van de hydrodynamische eigenschappen van draagvleugelschepen. In de studie worden drie hoofdonderwerpen in de scheepshydrodynamica behandeld: weerstand en voortstuwing, zeegang en manoeuvreren. Voor het weerstandsprobleem is een rekenmodel ontwikkeld voor een stationaire stroming, voor het zeegangs- en manoeuvrereerproubleem is een rekenmodel ontwikkeld voor instationaire stroming.

Ondanks de omvangrijke en gedegen kennis van stromingen rond schepen en draagvlakken bestaat er voor het bepalen van de weerstand van draagvleugelschepen geen rekenmodel, dat breed toepasbaar is en dat de krachten op zowel scheepsromp als draagvleugels met een aanvaardbare nauwkeurigheid en doelmatigheid voorspelt. Teneinde een geschikt rekenmodel te ontwikkelen is de volgende aanpak gekozen, waarbij de meeste aandacht uitgaat naar de draagvleugels.


Hydres wordt vervolgens gevalideerd op basis van modelproef resultaten en ware grootte metingen. De onzekerheidsmarge van modelproef resultaten blijkt vrij groot, met name door visceuze effecten op de liftkracht van draagvleugels en instelfouten in de invalshoek van draagvleugels. Indien rekening wordt gehouden met deze onzekerheid is de overeenkomst tussen experimentele en berekende resultaten vrij goed. Ook is de overeenkomst met ware grootte metingen bevredigend.

Evenals voor het stationaire model, bestaat er geen geschikt rekenmodel voor instationaire condities. In het tweede deel van de studie wordt een dergelijk rekenmodel ontwikkeld. Dit rekenmodel kan worden toegepast voor het bepalen van het gedrag van draagvleugelschepen in golven, waarbij het
schip al dan niet manoeuvres uitvoert. Hierbij wordt aangenomen dat het schip op kruissnelheid vaart, dat wil zeggen dat de romp zich boven water bevindt. Het rekenmodel is gebaseerd op een instationaire vortex lattice methode. Er wordt een tijdsdomein Greense functie formulering gebruikt welke wederom gebaseerd is op gelineariseerde randvoorwaarden op het ongestoorde vrije oppervlak. De instationaire interaktie tussen draagvleugels wordt in rekening gebracht. Met betrekking tot de bewegingen van de draagvleugels is een gelineariseerde versie van het rekenmodel ontwikkeld, dat uitgaat van kleine bewegingen rond een bepaalde gemiddelde positie en een constante snelheid en koers. Het niet gelineariseerde rekenmodel kan worden gebruikt voor willekeurige en grote bewegingen.

Resultaten verkregen met het instationaire vortex lattice model worden vergeleken met analytische resultaten voor eenvoudige draagvleugels. Voor een meer praktische draagvleugel configuratie worden berekeningsresultaten voor dempingskrachten en excitatiekrachten in golven ge valideerd op basis van modelproeven. De berekende resultaten blijken goed overeen te komen met de analytische en de experimentele resultaten. Het instationaire vortex lattice model is ondergebracht in het tijdsdomein simulatie programma Hydsim. Middels enkele simulatie voorbeelden wordt de toepasbaarheid van Hydsim voor onder andere het manoeuvreren in golven geillustreerd. Het rekenmodel wordt tenslotte ge valideerd aan de hand van enkele ware grootte metingen voor een draagvleugelschip in golven en een manoeuvrerend draagvleugelschip. De berekende resultaten komen wederom goed overeen met de meetresultaten.

De resultaten van het onderzoek zijn van waarde voor het bepalen van het benodigde voorstuwingvermogen en het gedrag in golven van draagvleugelschepen. De resultaten van het tweede deel van het onderzoek kunnen ook als basis dienen bij het ontwikkelen van rekenmodellen voor schepen met stabilisatie vinnen alsmede voortstuwers gebaseerd op roterende draagvleugels.
ACKNOWLEDGEMENT

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CURRICULUM VITAE

The author was born on August 9, 1960, in Noordwijk, Netherlands. He studied Naval Architecture at the 'Hogere Technische School Dordrecht' from 1977 to 1981. His graduation work consisted of a design study into push boats and barges, for which he received an award from the 'Nederlandse Vereniging van Technici op Scheepvaartgebied' (NVTS). He subsequently studied Naval Architecture at Delft University of Technology. He graduated in 1985 with honour on cavitation effects on the lift and drag characteristics of hydrofoil craft, for which he again received an award from the NVTS. In 1985 he assumed a position at the Maritime Research Institute Netherlands (MARIN) as a project manager in the software engineering department. He has been working there on a wide variety of subjects, ranging from simulation models for dynamic positioning of offshore structures to powering predictions for high speed craft. Hydrofoil craft have had his special interest ever since graduating from Delft University of Technology.