ABSTRACT

When pumping shells through a pipeline one must consider that shells are not spherical, but more disc-shaped. When shells settle they will settle like leaves where the biggest cross section is exposed to the drag. But they will settle in the same orientation, flat on the sediment, so the sides of the shells are exposed to the horizontal flow in the pipeline. Since the side cross section is much smaller than the horizontal cross section, a much higher velocity is required to make them erode and go back into suspension.

The settling velocity is much smaller because of the large area of the cross section. Even when the slurry velocity exceeds the settling velocity, some shells will reach the bottom of the pipe owing to the combination of settling velocity and turbulence. Once these shells are on top of the sediment they are hard to remove by erosion, because they lay flat on the surface and only a small cross section is exposed to the flow compared with the weight of the shell.

So although their settling velocity is much lower than equivalent sand particles, the erosion velocity is much higher. On a shell-covered beach, shells are always visible on top of the sand. In fact, these shells are shielding the sand from erosion. Bigger shells will also shield the smaller pieces, because smaller pieces settle faster. Shells settle more slowly than sand grains, so they will be on top of the bed (if there is a bed) just as on the beach.

These shells are hard to erode, in fact, they protect the bed from being eroded, even if the line speed is increased. The combination of high erosion velocity and the shells “protecting” the bed means that even a small amount of shells can lead to relatively thick bed in the pipeline. But there will always be a velocity above the bed that will make the shells erode.

This article describes the settling and erosion process of shells and the consequences of this on the critical velocity when pumping a sand/shell mixture through a pipeline. A mathematical model of the processes involved will be presented.

INTRODUCTION

When pumping shells through a pipeline one must consider that shells are not spherical, but more disc shaped. When shells settle they will settle like leaves where the biggest cross section is exposed to the drag. But when they settle, they will settle in the same orientation, flat on the sediment, so the side of the shells is exposed to the horizontal flow in the pipeline. Since the side cross section is much smaller than the horizontal cross section, a much higher velocity is required to make them erode and go back into suspension. The settling velocity is much smaller because of the large area of the cross section. Normally pipeline resistance is calculated based on the settling velocity, where the resistance is proportional to the settling velocity of the grains. The critical velocity is also proportional with the settling velocity. Since shells have a much lower settling velocity than sand grains with the same weight and much lower than sand grains with the same sieve diameter, one would expect a much lower resistance and a much lower critical velocity, matching the lower settling velocity.

This is only partly true. As long as the shells are in suspension, on average they want to stay in suspension because of the low settling
velocity. But settling and erosion are stochastic processes because of the turbulent character of the flow in the pipeline. Since we operate at Reynolds numbers above 1 million, the flow is always turbulent, meaning that eddies and vortices occur stochastically making the particles in the flow move up and down, resulting in some particles hitting the bottom of the pipe. Normally these particles will be picked up in the flow because of erosion, so there exists equilibrium between sedimentation and erosion, resulting in not having a bed at the bottom of the pipeline.

In fact the capacity of the flow to erode is bigger than the sedimentation. If the line speed decreases, the shear velocity at the bottom of the pipe also decreases and less particles will be eroded, so the erosion capacity is decreasing. This does not matter, because as long as the erosion capacity is bigger than the sedimentation, there will not be sediment at the bottom of the pipeline. As soon as the line speed decreases so much that the erosion capacity (erosion flux) is smaller than the sedimentation flux, not all the particles will be eroded, resulting in a bed to be formed at the bottom of the pipe. Having a bed at the bottom of the pipe also means that the cross section of the pipe decreases and the actual flow velocity above the bed increases. This will result in a new equilibrium between sedimentation flux and erosion flux for each bed height.

From the moment there is a bed, decreasing the flow will result in an almost constant flow velocity above the bed, resulting in equilibrium between erosion and sedimentation. This equilibrium however is sensitive for changes in the line speed and in the mixture density. Increasing the line speed will reduce the bed height, a decrease will increase the bed height. Having a small bed does not really matter, but a thick bed makes the system vulnerable for plugging the pipeline.

The critical velocity in most models is chosen in such a way that a thin bed is allowed. As said before, some shells will always reach the bottom of the pipe owing to the combination of settling velocity and turbulence. Once these shells are on top of the sediment they are hard to remove by erosion, because they lay flat on the surface and have a small cross section that is exposed to the flow compared with the weight of the shell. So although their settling velocity is much lower than equivalent sand particles, the erosion velocity is much higher. Looking at a beach in an area with many shells, there are always shells visible on top of the sand, covering the sand. In fact the shells are shielding the sand from erosion, because they are hard to erode. The bigger shells will also shield the smaller pieces, because the smaller pieces settle faster. Compare this with leaves falling from a tree, the bigger leaves, although heavier, will fall slower, because they are exposed to higher drag. The same process will happen in the pipeline. Shells settle more slowly than sand grains, so they will be on top of the bed (if there is a bed), just as on the beach. Since they are hard to erode, in fact, they protect the bed from being eroded, even if the line speed is increased. But there will always be velocities above the bed that will make the shells erode. Now the question is how to quantify this behaviour in order to get control over it.

One must distinguish between sedimentation and erosion. First of all assume shells are disc shaped with a diameter $d$ and a thickness of $\alpha \cdot d$ and let's take $\alpha = 0.1$ this gives a cross section for the terminal settling velocity of $\pi \cdot 4 \cdot d^2$, a volume of $\pi / 40 \cdot d^3$, and a cross section for erosion of $d^2 / 10$. Two processes have to be analysed to determine the effect of shells on the critical velocity: the sedimentation process and the erosion process.

**THE SEDIMENTATION PROCESS**

The settling velocity of grains depends on the grain size, shape and specific density. It also depends on the density and the viscosity of the fluid the grains are settling in and upon whether the settling process is laminar or turbulent. Discrete particles do not change their size, shape or weight during the settling process (and thus do not form aggregates). A discrete particle in a fluid will settle under the influence of gravity. The particle will accelerate until the frictional drag force of the fluid equals the value of the gravitational force, after which the vertical (settling) velocity of the particle will be constant.

In general, the settling velocity $v_s$ can be determined with the following equation:

$$v_s = \sqrt{\frac{4 \cdot g \cdot (\rho_p - \rho_w) \cdot d \cdot \psi}{3 \cdot \rho_w \cdot C_d}}$$  \[1\]

The settling velocity is thus dependent on: the density of the particle and fluid, diameter (size) and shape (shape factor $\psi$) of the particle, and flow pattern around the particle. The Reynolds number of the settling process determines whether the flow pattern around the particle is laminar or turbulent. The Reynolds number can be determined by:

![The drag coefficient for different shape factors](Image)
Re_p = \frac{v_s \cdot d}{v} \tag{2}

The viscosity of the water is temperature dependent. If a temperature of 10° is used as a reference, then the viscosity increases by 27% at 0° and it decreases by 30% at 20° centigrade. Since the viscosity influences the Reynolds number, the settling velocity for laminar settling is also influenced by the viscosity. For turbulent settling the drag coefficient does not depend on the Reynolds number, so this settling process is not influenced by the viscosity.

The drag coefficient

The drag coefficient C_d depends upon the Reynolds number according to Turton and Levenspiel (1986), which is a 5 parameter fit function to the data:

\[ C_d = \frac{24}{Re_p} \left[ 1 + 0.173 Re_p^{0.647} \right] + \frac{0.413}{1 + 16300 \cdot Re_p^{0.849}} \tag{3} \]

It must be noted that in general the drag coefficients are determined based on the terminal settling velocity of the particles. Wu and Wang (2006) recently gave an overview of drag coefficients and terminal settling velocities for different particle Corey shape factors. The result of their research is reflected in Figure 1. Figure 1 shows the drag coefficients as a function of the Reynolds number and as a function of the Corey shape factor.

For shells settling the Corey shape factor is very small, like 0.1, resulting in high drag coefficients. According to Figure 2 the drag coefficient should be like:

\[ C_d = \frac{32}{Re_p} + 2 \text{ up to } C_d = \frac{36}{Re_p} + 3 \tag{4} \]

For shells lying flat on the bed, the drag coefficient will be similar to the drag coefficient of a streamlined half body (0.09), which is much smaller than the drag coefficient for settling (3).

So there is a large asymmetry between the settling process and the erosion process of shells, while for more or less spherical sand particles the drag coefficient is considered to be the same in each direction.

Terminal settling velocity equations from literature

The shape factor was introduced into equation (1) by multiplying the mass of a sand particle with a shape factor \( \psi \). For normal sands this shape factor has a value of 0.7. Zanke (1977) has derived an equation for the transitional region (in m and m/sec):

\[ v_s = \frac{10 \cdot v}{d} \left( 1 + \frac{R_d \cdot g \cdot d^3}{100 \cdot \nu^2} - 1 \right) \tag{5} \]

With the relative density \( R_d \) defined as:

\[ R_d = \frac{p_s - p_w}{p_w} \tag{6} \]

Figure 3 shows the settling velocity as a function of the particle diameter for the Stokes, Budryck, Rittinger and Zanke equations. Instead of using the shape factor in equation (1), it is better to use the actual drag coefficient according to equation (4) giving the shape factor a value of 1.

Hindered settling

The above equations calculate the settling velocities for individual grains. The grain moves downwards and the same volume of water has to move upwards. In a mixture, this means that when many grains are settling, an average upwards velocity of the water exists. This results in a decrease of the settling velocity, which is often referred to as hindered settling. However, at very low concentrations the settling velocity will increase because the grains settle in each other’s shadow. Richardson and Zaki (1954) determined an equation to calculate the influence of hindered settling for volume concentrations \( C_v \) between 0.05 and 0.65. Theoretically, the validity of the Richardson and Zaki equation is limited by the maximum solids concentration that permits solids particle settling in a particulate cloud. This maximum concentration corresponds with the concentration in an incipient fluidised bed \( C_v \) of about 0.57. Practically, the equation was experimentally verified for concentrations not far above 0.30. The exponent in this equation is dependent on the Reynolds number. The general equation yields:

\[ \frac{v_s}{v} = (1 - C_v) \beta \tag{7} \]

The following values for \( \beta \) should be used (using the following definition does give a continuous curve):

\[
\begin{align*}
&\beta = 4.65 \quad \text{for } \text{Re}_p < 0.1 \\
&\beta = 4.35 \cdot \text{Re}_p^{-0.03} \quad \text{for } \text{Re}_p > 0.1 \text{ and } \text{Re}_p < 1.0 \\
&\beta = 4.45 \cdot \text{Re}_p^{-0.1} \quad \text{for } \text{Re}_p > 1.0 \text{ and } \text{Re}_p < 400 \\
&\beta = 2.39 \quad \text{for } \text{Re}_p > 400
\end{align*}
\]

Other researchers found the same trend with sometimes different values for the power \( \beta \), as shown in Figure 4.

Erosion

In Miedema (2010), a model for the entrainment of particles as a result of fluid (or air) flow over a bed of particles has been developed. The model distinguishes sliding, rolling and lifting as the mechanisms of entrainment. Sliding is a mechanism that occurs when many particles are starting to move and it is based on the global soil mechanical parameter of internal friction. Both rolling and lifting are mechanisms of

<table>
<thead>
<tr>
<th>Shape</th>
<th>Drag Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>0.47</td>
</tr>
<tr>
<td>Half Sphere</td>
<td>0.42</td>
</tr>
<tr>
<td>Cone</td>
<td>0.50</td>
</tr>
<tr>
<td>Cube</td>
<td>1.05</td>
</tr>
<tr>
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<tr>
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<tr>
<td>Short Cylinder</td>
<td>1.15</td>
</tr>
<tr>
<td>Streamlined Body</td>
<td>0.04</td>
</tr>
<tr>
<td>Streamlined Half-Body</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Figure 2. Some drag coefficients (source Wikipedia).
individual particles and are based on local parameters such as the pivot angle and the exposure and protrusion rate. Equations (9), (10) and (12) give the Shields parameter for these 3 mechanisms.

**Sliding**

\[ \theta_{\text{sliding}} = \frac{u^2}{R_d \cdot \rho \cdot g \cdot d} = \frac{4}{3} \frac{1}{\alpha} \left( \frac{\text{s}}{\text{m} \cdot \text{s}} \right) \]  

**Rolling**

\[ \theta_{\text{rolling}} = \frac{u^2}{R_d \cdot \rho \cdot g \cdot d} = \frac{4}{3} \frac{1}{\alpha} \left( \frac{\text{s}}{\text{m} \cdot \text{s}} \right) \]  

With the effective rolling friction coefficient \( \mu_{\text{rolling}} \):

\[ \mu_{\text{rolling}} = \frac{\sin(\psi + \phi_{\text{RKL}})}{\text{Lever} - \text{d} \cdot \cos(\psi + \phi_{\text{RKL}})} \]  

**Lifting**

\[ \theta_{\text{lifting}} = \frac{u^2}{R_d \cdot \rho \cdot g \cdot d} = \frac{4}{3} \frac{1}{\alpha} \left( \frac{\text{s}}{\text{m} \cdot \text{s}} \right) \]  

**Non-uniform particle distributions**

In the model for uniform particle distributions, the roughness \( k_s \) was chosen equal to the particle diameter \( d \), but in the case of non-uniform particle distributions, the particle diameter \( d \) is a factor \( d^+ \) times the roughness \( k_s \), according to:

\[ d^+ = \frac{d}{k_s} \]  

The roughness \( k_s \) should be chosen equal to some characteristic diameter related to the non-uniform particle distribution, for example the \( d_{50} \).

**Laminar region**

For the laminar region (the viscous sub layer) the velocity profile of Reichardt (1951) is chosen. This velocity profile gives a smooth transition going from the viscous sub layer to the smooth turbulent layer.

\[ u_{\text{top}} = \frac{u_y}{u_+} \]  

\[ \frac{u_y}{u_+} = \frac{\ln(1 + \kappa \cdot y_+)}{\ln(1/9) + \ln(\kappa)} \approx \frac{\ln(1 + \kappa \cdot y_+)}{\ln(1/9) + \ln(\kappa)} \]  

For small values of the boundary Reynolds number and thus the height of a particle,
the velocity profile can be made linear to:

\[ u_{\text{top}}^* = y_{\text{top}}^* = d^+ \cdot \varepsilon \cdot \Re = d^+ \cdot E \cdot k_+^* \]  \[ 15 \]

Adding the effective turbulent velocity to the time averaged velocity, gives for the velocity function \( \alpha_{\text{lam}} \):

\[ \alpha_{\text{lam}} = y_{\text{top}}^* + u_{\text{eff}}^* \]  \[ 16 \]

**Turbulent region**

Particles that extend much higher into the flow will be subject to the turbulent velocity profile. This turbulent velocity profile can be the result of either a smooth boundary or a rough boundary. Normally it is assumed that for boundary Reynolds numbers less than 5 a smooth boundary exists, while for boundary Reynolds numbers larger than 70 a rough boundary exists. In between in the transition zone the probability of having a smooth boundary is:

\[ P = e^{-0.95 \frac{\Re}{\delta} = e^{-0.95 \frac{k^*_+}{\delta}} } \]  \[ 17 \]

This probability is not influenced by the diameter of individual particles, only by the roughness \( k_+ \) which is determined by the non-uniform particle distribution as a whole. This gives for the velocity function \( \alpha_{\text{turb}} \):

\[ \alpha_{\text{Turb}} = \frac{1}{k^*_+} \ln \left( 95 \cdot \frac{y_{\text{top}}^*}{k^*_+} + 1 \right) P + \frac{1}{k^*_+} \]  \[ 18 \]

The velocity profile function has been modified slightly by adding 1 to the argument of the logarithm. Effectively this means that the velocity profile starts \( y \) lower, meaning that the virtual bed level is chosen \( y \) lower for the turbulent region. This does not have much effect on large exposure levels (just a few percent), but it does on exposure levels of 0.1 and 0.2. Not applying this would result in high (not realistic) shear stresses at very low exposure levels.

**The exposure level**

Effectively, the exposure level \( E \) is represented in the equations (9), (10) and (12) for the Shields parameter by means of the velocity distribution according to equations (16) and (18) and the friction coefficient \( k \) or the pivot angle \( \psi \). A particle with a diameter bigger than the roughness \( k_+ \) will be exposed to higher velocities, while a smaller particle will be exposed to lower velocities. So it is important to find a relation between the non-dimensional particle diameter \( d^+ \) and the exposure level \( E \).

**The angle of repose and the friction coefficient**

Miller and Byrne (1966) found the following relation between the pivot angle \( \psi \) and the non-dimensional particle diameter \( d^+ \) with \( c_\psi = 61.5^\circ \) for natural sand, \( c_\psi = 70^\circ \) for crushed quartzite and \( c_\psi = 50^\circ \) for glass spheres.

\[ \psi = c_\psi \cdot (d^+) - 0.3 \]  \[ 19 \]

Wiberg and Smith (1987A) re-analysed the data of Miller and Byrne (1966) and fitted the following equation:

\[ \psi = \cos^{-1} \left( \frac{d^+ + 2 \cdot z^*}{d^+ + 1} \right) \]  \[ 20 \]

The average level of the bottom of the almost moving grain \( z \), depends on the particle sphericity and roundness. The best agreement is found for natural sand with \( z = 0.045 \), for crushed quartzite with \( z = 0.320 \) and for glass spheres with \( z = 0.285 \). Wiberg and Smith (1987A) used for natural sand with \( z = 0.16 \), for crushed quartzite with \( z = 0.16 \) and for glass spheres with \( z = 0.14 \).

The values found here are roughly 2 times the values as published by Wiberg and Smith (1987A). It is obvious that equation (20) underestimates the angle of repose for \( d^+ \) values smaller than 1.

**The equal mobility criterion**

Two different cases have to be distinguished. Particles with a certain diameter can lie on a bed with a different roughness diameter. The bed roughness diameter may be larger or smaller than the particle diameter. Figure 5 shows the Shields curves for this case (which

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**LIST OF SYMBOLS USED**

- \( A \): Surface or cross section
- \( C \): Pivot angle at \( d^+ = 1 \)
- \( C_{\alpha}, C_d \): Drag coefficient
- \( C_c \): Lift coefficient
- \( C_v \): Volumetric concentration
- \( d \): Diameter of particle or sphere
- \( d^+ \): Dimensionless particle diameter
- \( f_d, f_l \): Drag and lift surface factor
- \( F_{\text{down}} \): Submerged gravity force (downwards)
- \( F_{\text{drag}} \): Drag force (upwards)
- \( g \): Gravitational constant
- \( k \): Bed roughness
- \( P \): Probability related to transition smooth/rough
- \( R_g \): Relative submerged density (1.65 for sand)
- \( \Re, \Re_s \): Reynolds number
- \( T \): Temperature
- \( u, u^* \): Friction velocity
- \( u \): Velocity
- \( u_{\text{top}} \): Dimensionless velocity at top of particle
- \( u_{\text{eff}}^* \): Dimensionless effective turbulent added velocity
- \( U \): Average velocity above the bed.
- \( m^2 \): Terminal settling velocity including hindered settling
- \( \nu \): Terminal settling velocity
- \( \nu \): Volume of particle or sphere
- \( \nu \): Dimensionless height of particle
- \( \zeta \): Coefficient
- \( \alpha \): Velocity coefficient Shields parameter
- \( \beta \): Hindered settling coefficient
- \( \delta \): Thickness of the viscous sub-layer
- \( \kappa \): Dimensionless thickness of the viscous sub-layer
- \( \lambda \): Von Karman constant
- \( \phi \): Friction coefficient (see Moody diagram)
- \( \rho_s \): Density of quartz
- \( \rho_w \): Water density
- \( \psi \): Shape factor particle
- \( \psi \): Pivot angle
- \( \phi \): Friction angle
- \( \theta \): Kinematic viscosity
- \( \phi \): Shields parameter
- \( \mu \): Friction coefficient
- \( \lambda \): Drag arm factor
- \( l_{\text{add}} \): Additional lever arm for drag

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\[ \ln \left( 30 \cdot \frac{y_{\text{top}}^*}{k^*_+} + 1 \right) \left( 1 - \frac{1}{k^*_+} \right) \]  \[ 18 \]

\[ \psi = c_\psi \cdot (d^+) - 0.3 \]  \[ 19 \]

\[ \psi = \cos^{-1} \left( \frac{d^+ + 2 \cdot z^*}{d^+ + 1} \right) \]  \[ 20 \]
are different from the graph as published by Wiberg and Smith (1987A)), combined with the data of Fisher et al. (1983), and based on the velocity distributions for non-uniform particle size distributions. Fisher et al. carried out experiments used to extend the application of the Shields entrainment function to both organic and inorganic sediments over passing a bed composed of particles of different size.

Figure 5 shows a good correlation between the theoretical curves and the data, especially for the cases where the particles considered are bigger than the roughness diameter \(\frac{d}{k_s}\). It should be noted that most of the experiments were carried out in the transition zone and in the turbulent regime. Figure 5 is very important for determining the effect of shells on a bed, because with this figure the critical Shields parameter of a particle with a certain diameter, lying on a bed with a roughness of a different diameter, can be determined. In the case of the shells the bed roughness diameter will be much smaller than the shell diameter (dimensions). To interpret Figure 5 one should first determine the bed roughness diameter and the roughness Reynolds number and take the vertical through this roughness Reynolds number (also called the boundary Reynolds number). Now determine the ratio \(\frac{d}{k_s}\) and read the Shields parameter from the graph. From this it appears that the bigger this ratio, the smaller the Shields value found. This is caused by the fact that the Shields parameter contains a division by the particle diameter, while the boundary shear stress is only influenced slightly by the changed velocity distribution. Egiazaroff (1965) was one of the first to investigate non-uniform particle size distributions with respect to initiation of motion. He defined a hiding factor or exposure factor as a multiplication factor according to:

\[
\theta_{cr,ij} = \theta_{cr,0} \left( \frac{\log(19)}{\log\left(\frac{19d_i}{d_{50}}\right)} \right)^2
\]

The tendency following from this equation is the same as in Figure 5, the bigger the particle, the smaller the Shields value, while in equation (21) the \(d_{50}\) is taken equation to the roughness diameter \(k_s\). The equal mobility criterion is the criterion stating that all the particles in the top layer of the bed start moving at the same bed shear stress, which matches the conclusion of Miedema (2010) that sliding is the main mechanism of entrainment of particles. Figure 6 shows that the results of the experiments are close to the equal mobility criterion, although not 100%, and the results from coarse sand from the theory as shown in Figure 5, matches the equal mobility criterion up to a ratio of around 10. Since shells on sand have a \(\frac{d}{k_s}\) ratio bigger than 1, the equal mobility criterion will be used for the interpretation of the shell experiments as also shown in Figure 5.

### Shells

Dey (2003) has presented a model to determine the critical shear stress for the incipient motion of bivalve shells on a horizontal sand bed, under a unidirectional flow of water. Hydrodynamic forces on a solitary bivalve shell, resting over a sand bed, are analysed for the condition of incipient motion including the effect of turbulent fluctuations. Three types of bivalve shells, namely Coquina Clam, Cross-barred Chione and Ponderous Ark, were tested experimentally for the condition of incipient motion. The shape parameter of bivalve shells is defined appropriately.

Although the model for determining the Shields parameter of shells is given, the experiments of Dey (2003) were not translated into Shields parameters. It is interesting however to quantify these experiments into Shields parameters and to see how this relates to the corresponding Shields parameters of sand grains. In fact, if the average drag coefficient of the shells is known, the shear stress and thus the friction velocity, required for incipient motion, is known, the flow velocity required to erode the shells can be determined. Figure 7 and Figure 8 give an impression of the shells used in the experiments of Dey (2003). From Figure 7 it is clear that the shape of the shells match the shape of a streamlined half body lying on a surface and thus a drag coefficient is expected of about 0.1, while sand grains have a drag coefficient of about 0.45 at very high Reynolds numbers in a full turbulent flow. The case considered here is the case of a full turbulent flow, in order to try to relate the incipient motion of shells to the critical velocity.

Equation (9) shows the importance of the drag coefficient in the calculation of the incipient motion, while the lift coefficient is often related to the drag coefficient. Whether the latter is true for shells is the question. For sand grains at high Reynolds numbers of then the lift coefficient is chosen to be 0.85 times the drag coefficient or at least a factor between 0.5 and 1, shells are aerodynamically shaped and also asymmetrical. There will be

![Non-uniform particle distribution](image-url)
shell and not the size of the shell. Using this definition, results in useful Shields values. Since convex upwards is important for the critical velocity analysis, this case will be analysed and discussed. It is clear however from these figures that the convex downwards case results in much smaller Shields values than the convex upwards case as was expected. Smaller Shields values in this respect means smaller shear stresses and thus smaller velocities above the bed causing erosion. In other words, convex downwards shells erode much easier than convex upwards.

Although the resulting Shields values seem to be rather stochastic, it is clear that the mean values of the Chione and the Coquina are close to the Shields curve for $d / k_s = 1$. The values for the Ponderous Ark are close to the Shields curve for $d / k_s = 3$. In other words, the Ponderous Ark shells are easier to erode than the Chione and the Coquina shells. Looking at the shells in Figure 8 it is visible that the Ponderous Ark shells have ripples on the outside and will thus be subject to a higher drag. On the other hand, the Ponderous Ark shells have an average thickness of 2.69 mm (1.95-3.98 mm) as used in the equation of the Shields parameter, while the Coquina clam has a thickness of 1.6 mm (0.73-3.57 mm) and the Chione 1.13 mm (0.53-2.09 mm). This also explains part of the smaller Shields values of the Ponderous Ark. The average results of the tests are shown in Table I.

A closer look at the data, based on Table I, shows the following: For the shells on the 0.8 mm sand the $d / k_s$ values vary from 1.41-3.36. The average Shields values found do not match the corresponding curves, but lead to slightly lower $d / k_s$ values. For example, the Cross Barred Chione had a Shields value of 0.0378, but based on the $d / k_s$ value of 1.41, a Shields value of about 0.02 would be expected, a ratio of 1.89. The Coquina Clam had an average Shields value of 0.0278, but based on the $d / k_s$ value of 2.00 a Shields value of about 0.015 would be expected, a ratio of 1.84. The Ponderous Ark had an average Shields value of 0.0129, but based on the $d / k_s$ value of 3.36 a Shields value of about 0.008 would be expected, a ratio of 1.61. For the 0.3 mm sand the average ratio is about 5.5. In other words, the shells require larger Shields values than corresponding sand
How can this critical velocity be combined with the erosion behavior of shells?

As mentioned above, there are different models in literature for the critical velocity and there is also a difference between the critical velocity and the minimum friction velocity. However, whatever model is chosen, the real critical velocity is the result of an equilibrium of erosion and deposition resulting in a stationary bed. This equilibrium depends on the particle size distribution, the slurry density and the flow velocity. At very low concentrations it is often assumed that the critical velocity is zero, but based on the theory of incipient motion, a certain minimum velocity is always required to erode an existing bed.

The problem can be looked at in two ways: one can compare the Shields values of the shells with the Shields values of sand particles with a diameter equal to the thickness of the shells, resulting in the factors as mentioned in the previous paragraph or one can compare the shear stresses occurring to erode the shells with the shear stresses required for the sand beds used. The latter seems more appropriate because the shear stresses are directly related to the average velocity above the bed with the following relation:

$$\rho_w \cdot u_2^2 = \frac{1}{8} \rho_w \cdot U^2$$  \[22\]

Pumping velocity or lowering the concentration will be enough to start the bed sliding, then erode the bed and return to stable operation.

With a sand-shell mixture, as described above, the critical velocity and minimum friction velocities become time-dependent parameters. The stochastic nature of the process means that some fraction of the shells will fall to the bottom of the pipe. The asymmetry between deposition and erosion velocity means that these shells will stay on the bottom, forming a bed that grows over time, increasing the critical velocity and minimum friction velocity. Unless the system is operated with very high margins of velocity, the new critical velocity and Vimin eventually fall within the operating range of the system, leading to flow instability and possible plugging.

Implicit in most models of slurry transport is the idea that the system can transition smoothly in both directions along the system resistance curves. So if the dredge operator inadvertently feeds too high of a concentration, dropping the velocity close to the minimum friction or even the critical velocity, he can recover by slowly lowering the mixture concentration, which in turn lowers the density in the pipeline and allows the velocity to recover. Alternatively the operator can increase the pressure by turning up the pumps to raise the velocity. In a sand-sized material this works because the critical and minimum friction velocities are fairly stable, so raising the pumping velocity or lowering the concentration will be enough to start the bed sliding, then erode the bed and return to stable operation.

A related concept is that of the minimum friction velocity, $V_{min}$, at which the friction in the pipeline is minimised. At low concentrations the $V_{min}$ may be equal or just above the critical velocity, but as concentration increases the critical velocity starts to decrease while the $V_{min}$ continues to rise. In operational terms, the $V_{min}$ represents a point of instability, so pumping systems are generally designed to maintain sufficiently high velocities that the system velocity never falls below (or close to) $V_{min}$ during the operational cycle.

A familiar phenomenon in the transport of sand slurries is critical velocity, the velocity at which the mixture forms a stationary bed in the pipeline. As the velocity increases from the critical, the mixture bed starts to slide along the bottom of the pipe. As the velocity increases further the bed begins to erode with the particles either rolling or saltating along the top of the bed, or fully suspended in the fluid.

Critical Velocity

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Where the left hand side equals the bed shear stress, $\lambda$ the friction coefficient following from the Moody diagram and $U$ the average flow velocity above the bed. The average shear stresses are shown in Table II.

The Shields values for both sands are about 0.035, resulting in shear stresses of 0.45 Pa for the 0.8 mm sand and 0.17 Pa for the 0.3 mm sand. The ratios between the shear stresses required eroding the shells and the shear stresses required to erode the beds are also shown in Table II. For the shells laying convex upwards on the 0.8 mm sand bed these ratio’s vary from 1.24-1.60, while this is a range from 2.18-3.41 for the 0.3 mm sand bed. These results make sense, the shear stress required for incipient motion of the shells does not change much because of the sand bed, although there will be some reduction for sand beds of smaller particles owing to the influence of the bed roughness on the velocity profile according to equation (14). Smaller sand particles with a smaller roughness allow a faster development of the velocity profile and thus a bigger drag force on the shells at the same shear stress. The main influence on the ratios is the size of the sand particles, because smaller particles require a smaller shear stress for the initiation of motion.

This is also known from the different models for the critical velocity, the finer the sand grains, the smaller the critical velocity. In other words, the smaller the velocity to bring the particles in a bed back into suspension. It also makes sense that the ratio between shell erosion shear stress and sand erosion shear stress will approach 1 if the sand particles will have a size matching the thickness of the shells and even may become smaller than 1 if the sand particles are bigger than the shells. Since the velocities are squared in the shear stress equation, the square root of the ratios has to be taken to get the ratios between velocities. This leads to velocity ratios from 1.11-1.26 for the 0.8 mm sand and ratios from 1.48-1.89 for the 0.3 mm sand.

Translating this to the critical velocity can be carried out under the assumption that the critical velocity is proportional to the average flow velocity resulting in incipient motion.

Although the critical velocity results from an equilibrium between erosion and deposition of particles and thus is more complicated, the here derived ratios can be used as a first attempt to determine the critical velocities for a sand bed covered with convex upwards shells. For the coarser sands (around 0.8 mm) this will increase the critical velocity by 11%-26%, while this increase is 48%-89% for the finer 0.3 mm sand. Even finer sands will have a bigger increase, while coarser sands will have a smaller increase.

As stated, the shear stress required to erode the shells is almost constant, but decreasing a little bit with decreasing sand particle diameters, an almost constant critical velocity for the shells is expected. From the measurements it is also clear, that very smooth shells (Coquina Clam and Cross Barred Chione) are harder to erode and will have a higher critical velocity than the rough shells (Ponderous Ark) (Figure 10).

![Critical shear stress versus grain diameter for Coquina shells](image)

**Table I. Average Shields values.**

<table>
<thead>
<tr>
<th>$d_{50}=0.8$ mm</th>
<th>$d_{50}=0.3$ mm</th>
<th>$d_{50}=0.8$ mm</th>
<th>$d_{50}=0.3$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_\theta$</td>
<td>$\theta$</td>
<td>$Re_\theta$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Coquina Clam</td>
<td>19.78</td>
<td>0.0277</td>
<td>6.71</td>
</tr>
<tr>
<td>Cross Barred Chione</td>
<td>17.51</td>
<td>0.0378</td>
<td>6.24</td>
</tr>
<tr>
<td>Ponderous Ark</td>
<td>18.46</td>
<td>0.0129</td>
<td>5.76</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$d_{50}=0.8$ mm</th>
<th>$d_{50}=0.3$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho w \cdot u^2$</td>
<td>$\rho w \cdot u^2$ ratio</td>
</tr>
<tr>
<td>Coquina Clam</td>
<td>19.78</td>
</tr>
<tr>
<td>Cross Barred Chione</td>
<td>17.51</td>
</tr>
<tr>
<td>Ponderous Ark</td>
<td>18.46</td>
</tr>
</tbody>
</table>

**Table II. Average shear stresses.**

<table>
<thead>
<tr>
<th>$d_{50}=0.8$ mm</th>
<th>$d_{50}=0.3$ mm</th>
</tr>
</thead>
<tbody>
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<td>Ponderous Ark</td>
<td>18.46</td>
</tr>
</tbody>
</table>

**Figure 10. The critical shear stresses of the shells compared with sand.**
CONCLUSIONS

The critical velocity for the hydraulic transport of a sand-water mixture depends on a number of physical processes and material properties. The critical velocity is the result of equilibrium between the deposition of sand particles and the erosion of sand particles. The deposition of sand particles depends on the settling velocity, including the phenomenon of hindered settling as described in this article. The erosion or incipient motion of particles depends on equilibrium of driving forces, like the drag force, and frictional forces on the particles at the top of the bed. This results in the so-called friction velocity and bottom shear stress. Particles are also subject to lift forces and so-called Magnus forces, owing to the rotation of the particles. Particles that are subject to rotation may stay in suspension owing to the Magnus forces and they do not contribute to the deposition. From this it is clear that an increasing flow velocity will result in more erosion, finally resulting in hydraulic transport without a bed. A decreasing flow velocity will result in less erosion and an increasing bed thickness, resulting in the danger of plugging the pipeline.

Shells lying convex upwards on the bed in general are more difficult to erode than sand particles, as long as the sand particles are much smaller than the thickness of the shells. The shells used in the research had a thickness varying from 1.13 to 2.69 mm. So the shells armour the bed and require a higher flow velocity than the original sand bed. Now as long as the bed thickness is not increasing, there is no problem, but since hydraulic transport is not a simple stationary process, there will be moments where the flow may decrease and moments where the density may increase, resulting in an increase of the bed thickness. Since the shells are armouring the bed, there will not be a decrease of the bed thickness at moments where the flow is higher or the density is lower, which would be the case if the bed consists of just sand particles.

This means a danger of a bed thickness increasing all the time and finally plugging the pipeline. The question arises, how much does one have to increase the flow or flow velocity in order to erode the top layer of the bed where the shells are armouring the bed.

From the research of Dey (2003) it appears that the bottom shear stress to erode the shells varies from 0.56-0.72 Pa for a bed with 0.8 mm sand and from 0.37-0.61 Pa for a bed with 0.3 mm sand. It should be noted that these are shear stresses averaged over a large number of observations and that individual experiments have led to smaller and bigger shear stresses. So the average shear stresses decrease slightly with a decreasing sand particle size owing to the change in velocity distribution. These shear stresses require average flow velocities that are 11%-26% higher than the flow velocities required to erode the 0.8 mm sand bed and 48%-89% higher to erode the 0.3 mm sand bed. From these numbers it can be expected that the shear stresses required to erode the shells, match the shear stresses required to erode a bed with sand grains of 1-1.5 mm and it is thus advised to apply the critical velocity of 1-1.5 mm sand grains in the case of dredging a sand containing a high percentage of shells, in the case the shells are not too fragmented.


REFERENCES

