Green water along the side of an FPSO

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by

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Preface

This document contains the thesis work performed for the graduation on the specialisation Floating Offshore Structures of Offshore and Dredging Engineering at the Delft University of Technology. The effect of green water on FPSOs is very important for the safety and the structural strength of the vessel. In this perspective, Nevesbu b.v. has an interest to get an insight in the effect of green water along the side of an FPSO and therefore facilitated the research assignment.

For the results of the research assignment, the conclusion at the end of this report or the summary at the beginning can be read. For more detailed information, the chapters in between are relevant.

I would like to thank ir. Florin Golea and ir. Ana Maria Tocu from Nevesbu b.v. for supporting and providing feedback. furthermore, I would like to thank prof. dr. ir. René Huijsmans from the Delft University of Technology for his supervision during my graduation.
Summary

Green water has a significant effect on safety and on structural strength of an FPSO. When designing an FPSO for a benign environment, classification society rules are available to determine the measures which have to be considered against green water. When the probability of green water and the freeboard exceedance along the side are actually known, no potentially unnecessary measures, implied by classification society regulations, have to be implemented on the FPSO.

The probability of green water occurring along the side of the Petrojarl 1 is analysed with linear 3D diffraction theory. The analysis is performed for every sea state in the scatter diagram and for all extreme sea states resulting from the wave analysis. The linear roll damping in the diffraction analysis is improved by adding non-linear roll damping based on Ikeda’s method. By means of the diffraction theory with non-linear roll damping, the RAOs of the vessel motions and the free surface elevation RAO along the side of the hull are calculated. With these RAOs, the relative wave motion RAO is determined. In order to determine the probability of green water, the Rayleigh and the Forristall distributions are applied. The maximum freeboard exceedance during a three hours storm is determined, based on the Rayleigh distribution.

Green water along the side of the FPSO is likely to occur in two sea states. The 50 year return period sea state ($H_s = 8.16$ meters, $T_p = 15.34$ seconds) is the extreme wave sea state with the highest probability of green water along the side (4.1%). The relative wave motion, on which the probability of green water is based, is the result of a combination of the heave, pitch and roll motion of the vessel, while the free surface elevation reduces the relative wave motion. One of the sea states along the contour of the scatter diagram results in an even larger probability of green water (31.8%). In this sea state, the relative wave motion at the peak response frequency is for 97% caused by the free surface elevation (incoming, diffracting and radiating waves) and for only 3% caused by the vessel motions. The vessel is pitching a little, while the waves run up along the side of the hull.

Between 15% and 30% aft of midships, the probability of green water is the highest. At $x/L_{pp} = 0.23$, the probability is 31.8% and the most probable maximum freeboard exceedance is 5.8 meters. Another peak in the probability (4.1% in the extreme wave sea state) is located just aft of the forecastle. Between 15% and 30% forward of midships, the probability of green water shows a minimum for both analysed sea states.

The most probable maximum freeboard exceedance is compared with the height of the equipment on the Petrojarl 1. The green water barriers, which are placed on the main deck, are needed as protection against the green water. The freeboard exceedance of 5.8 meters, might even result in damage of the unprotected equipment on the process deck.

The green water along the side of the FPSO is for a major part caused by the wave run-up along the hull, the free surface elevation. Therefore, the free surface elevation is validated by means of model tests. The wave run-up along a fixed plate is measured during model tests and is calculated with a diffraction calculation. The measurements and calculations are compared and the free surface elevation calculation with linear diffraction theory proved to be reasonably accurate.

The relative wave angle is also of great importance for the probability of green water. Therefore, for both sea states, a diffraction analysis with varying relative wave angles has been performed. The result shows that the probability of green water along the side increases with increasing relative wave angle, for both sea states and for angles up to 45°. For larger relative wave angles, a calculation has to be performed to determine which angle results in the highest probability of green water.
Nomenclature

\( \alpha \)  Weibull scale parameter
\( \beta \)  Weibull shape parameter
\( \Delta S_m \)  Area of the m-th panel
\( \epsilon \)  Phase angle
\( \gamma \)  Peakedness factor of JONSWAP spectrum
\( \lambda \)  Wave length
\( C \)  Damping matrix
\( \mathbf{K}_{\text{hys}} \)  Hydrostatic stiffness matrix
\( \mathbf{M}_a \)  Added mass matrix
\( \mathbf{M}_s \)  Structural mass matrix
\( \mu \)  Incoming wave heading
\( \nabla \)  Displacement
\( \nu \)  Kinematic coefficient of viscosity
\( \Omega \)  Fluid domain
\( \omega \)  Wave frequency
\( \omega_p \)  Wave spectrum peak frequency
\( \omega_0,\phi \)  Natural roll period
\( \xi \)  Coordinates vector
\( \mathbf{n} \)  Unit normal vector
\( \mathbf{r} \)  Position vector
\( \mathbf{X}_g \)  Vector with location coordinates x, y and z of center of gravity
\( \mathbf{X} \)  Vector with location coordinates x, y and z
\( \Phi \)  Flow velocity potential
\( \phi \)  Roll motion
\( \phi \)  Space dependent flow potential
\( \phi_d \)  Diffraction wave potential
\( \phi_r \)  Radiation wave potential
\( \phi_w \)  Incident undisturbed wave potential
\( \rho \)  Density of water
\( \sigma \)  Source strength
\( \sigma \)  Step function for JONSWAP spectrum
\( \theta \)  Pitch motion
\( \xi \)  Wave steepness
\( \zeta \)  Wave elevation
\( \zeta_a \)  Wave amplitude
\( \zeta_p \)  Wave elevation at point p (combined incoming, diffracted and radiated wave)
\( A \)  Auxiliary parameter for JONSWAP spectrum
\( a_{44} \)  Added mass coefficient for roll motion
\( A_{pp} \)  Aft perpendicular of FPSO
\( A_{jk} \)  Added mass coefficient
\( B \)  Breadth of FPSO
\( B_{bk} \)  Width of bilge keel
\( B_{jk} \)  Damping coefficient
\( C \)  Wave phase velocity
\( c_b \)  Block coefficient
\( C_d \)  Drag coefficient
\( c_m \)  Amidships section coefficient
\( c_{\phi \phi} \)  Roll spring coefficient
\( \text{COG}_x \)  Distance from aft perpendicular to center of gravity of FPSO
\( \text{COG}_y \)  Distance from center line to center of gravity of FPSO
\( \text{COG}_z \)  Distance from base line to center of gravity of FPSO
\( D \)  Depth of FPSO
\( D \)  Slender structure diameter
\( d \)  Water depth
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>Total wave force</td>
</tr>
<tr>
<td>$f$</td>
<td>Freeboard</td>
</tr>
<tr>
<td>$F_{pp}$</td>
<td>Forward perpendicular of FPSO</td>
</tr>
<tr>
<td>$F_d$</td>
<td>Wave force due to diffracting wave</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Wave force due to radiating wave</td>
</tr>
<tr>
<td>$F_w$</td>
<td>Wave force due to incoming wave (Froude-Krylov force)</td>
</tr>
<tr>
<td>$G$</td>
<td>Green’s function</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$GM$</td>
<td>Metacentric height</td>
</tr>
<tr>
<td>$h_0$</td>
<td>Distance from the origin of the fixed reference axes to the sea bed</td>
</tr>
<tr>
<td>$H_s$</td>
<td>Significant wave height based on calculations</td>
</tr>
<tr>
<td>$H_{1/3}$</td>
<td>Significant wave height based on visual measurements</td>
</tr>
<tr>
<td>$H_{max}$</td>
<td>Maximum wave height</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>Moment of inertia with respect to x-axis</td>
</tr>
<tr>
<td>$J_0$</td>
<td>Bessel function of the first kind</td>
</tr>
<tr>
<td>$k$</td>
<td>Wave number</td>
</tr>
<tr>
<td>$k_{\phi\phi}$</td>
<td>Apparent radius of gyration for roll</td>
</tr>
<tr>
<td>$k_{xx}$</td>
<td>Radius of gyration for roll motion</td>
</tr>
<tr>
<td>$k_{yy}$</td>
<td>Radius of gyration for pitch motion</td>
</tr>
<tr>
<td>$k_{zz}$</td>
<td>Radius of gyration for yaw motion</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of FPSO</td>
</tr>
<tr>
<td>$L_{bk}$</td>
<td>Length of bilge keel</td>
</tr>
<tr>
<td>$L_{pp}$</td>
<td>Length between perpendiculars of FPSO</td>
</tr>
<tr>
<td>$m_{ns}$</td>
<td>$n^{th}$ order moment of area under the relative wave motion spectrum</td>
</tr>
<tr>
<td>$N$</td>
<td>Total number of measured waves</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of measured waves given a certain condition</td>
</tr>
<tr>
<td>$N_p$</td>
<td>Total number of panels over the mean wetted surface</td>
</tr>
<tr>
<td>$OG$</td>
<td>Distance from still water level to center of gravity with downwards positive</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$P_{LT}$</td>
<td>Long term probability of exceedance</td>
</tr>
<tr>
<td>$P_{ST}$</td>
<td>Short term probability of exceedance</td>
</tr>
<tr>
<td>$R$</td>
<td>Part of Bessel function of the first kind</td>
</tr>
<tr>
<td>$s$</td>
<td>Relative wave motion</td>
</tr>
<tr>
<td>$S_0$</td>
<td>Mean wetted surface of the vessel</td>
</tr>
<tr>
<td>$S_1$</td>
<td>Steepness parameter</td>
</tr>
<tr>
<td>$S_\zeta$</td>
<td>Wave spectral density</td>
</tr>
<tr>
<td>$S_s$</td>
<td>Relative wave motion spectral density</td>
</tr>
<tr>
<td>$T$</td>
<td>Draft of FPSO</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Mean wave period</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Peak period of spectrum</td>
</tr>
<tr>
<td>$T_z$</td>
<td>Zero-crossing period</td>
</tr>
<tr>
<td>$T_{0\phi}$</td>
<td>Natural roll period</td>
</tr>
<tr>
<td>$u$</td>
<td>Wave water particle velocity</td>
</tr>
<tr>
<td>$U_r$</td>
<td>Ursell number</td>
</tr>
<tr>
<td>$x_p$</td>
<td>Distance from center of gravity to point p in x direction</td>
</tr>
<tr>
<td>$y_p$</td>
<td>Distance from center of gravity to point p in y direction</td>
</tr>
<tr>
<td>$z$</td>
<td>Heave motion</td>
</tr>
</tbody>
</table>
1 Introduction

Green water is an important safety issue for oil production vessels. It can affect the structural safety as well as the safety of the people on board. First, this chapter explains the green water phenomenon. After that the research assignment for this report is explained and finally an overview of research on green water in the past is given.

1.1 Green water

Since 1891 oil has been extracted via offshore platforms. First starting with very shallow water platforms built on piles in a lake and gradually expanding to deeper waters in the previous century. In 1977, the first FPSO (Floating Production, Storage and Offloading vessel) began producing oil [www.offshore-technology.com, 2008]. Because the FPSOs are nearly always directly connected to the wells, they cannot be easily disconnected when bad weather is approaching. The FPSOs remain positioned at their location while the storm passes over them. While an FPSO is situated in such heavy weather conditions, it is hit by numerous waves. Some of these waves might flow onto the deck of the FPSO. An actual wave flowing onto the deck is called 'green water' or 'shipping of water' (Figure 1.1). Water spraying over the deck is not considered green water. A massive flow of water on deck can cause damage to structures on deck and can be a danger for the crew on the vessel. For example, in 2000, four of the five production ships on the Norwegian continental shelf have experienced damage due to green water on topside equipment [Ersdal and Kvitrud, 2000]. To ensure the safety on board the FPSO, the green water problem will have to be taken into account when designing or modifying an FPSO.

Figure 1.1: Green water incident on the MV Selkirk [Luf, 2002]
1.2 Objectives

Green water mostly occurs at the bow of an FPSO because of the weathervaning capability. When the waves, current and wind are not co-directional, it can occur that waves are coming in on the side of the FPSO. This can cause green water on the side.

When designing an FPSO, the classification societies state rules which account for green water along the side. The classification society rules for green water and green water barriers are very simplistic. The weather conditions are accounted for by defining two conditions, harsh environment and benign environment. For the harsh environment, calculations have to be made to determine the sea loads. For the benign environment, rules are given to determine the sea loads (Appendix A). When the environment is benign, the sea pressure loads, on which the green water barriers are based, are only based on the geometry of the vessel itself. The environmental conditions are not taken into account, unless direct calculations, based on the specified sea state or scatter diagram, are performed.

To get more insight in green water on the side of an FPSO and to determine if these green water barriers are necessary, the green water on the side of an FPSO is researched in this report. The research is focused on the the Petrojarl 1 FPSO. The following three objectives are the subject of this research assignment:

- Calculate the probability of green water occurrence and the most probable maximum freeboard exceedance along the side of the FPSO. The most probable maximum freeboard exceedance can be applied to determine the green water loads on the FPSO and on the structures on the FPSO.
- Determine the influence of the motions of the vessel and the free surface elevation on the probability of green water. The green water probability and most probable maximum freeboard exceedance can be reduced by reducing the vessel motions or the free surface elevation, depending on the influence of the phenomena.
- Determine the influence of the relative wave heading on the probability of green water. What is the effect on the probability of green water when the relative wave heading is increased or decreased?

1.3 Previous research on green water on an FPSO

Literature shows a difference between green water on the bow of a vessel and green water along the side of a vessel. Green water on the bow has been the object of research more often than green water on the side. The research on green water on the bow can be helpful for the phenomenon of green water on the side. Therefore, first the research on the bow of an FPSO will be discussed, after that the green water on the side. Finally, the green water flow and green water loads are discussed briefly.

1.3.1 Green water on the bow of an FPSO

One of the first to research green water on FPSOs is Buchner [1995]. The study defines four stages during the green water occurrence:

- The relative wave motions exceed the freeboard
- The water flows onto the deck
- The flow over the deck acts as a shallow water wave
The wave hits a structure on deck

By comparing model tests with responses calculated with linear diffraction theory, several observations were made. Large relative wave motions generally occur when the wave frequency is just above the peak frequency in the pitch response. It is also observed via model tests that the use of linear diffraction theory for the green water problem at the bow is limited.

The probability of green water at the bow of an FPSO is studied in 1998 by Hamoudi and Varyani [1998]. It was found that the probability of deck wetness depends mainly on the relative wave motions but also on, for example, environmental conditions. For the probability of green water, the formula given by Price and Bishop [Price and Bishop, 1974] was used, which depends on the variance of the relative bow motion. The relative bow motion was measured and applied to predict the probability of deck wetness. It was found that the probability increases as the significant wave height increases. The wave period does not affect the probability. Increasing the freeboard height decreases the probability. The wave velocity does not relate to the velocity of the water on deck because the bow heavily disturbs the flow.

Faltinsen et al. [2002] developed a two-dimensional method that satisfies the non-linear free surface conditions of the potential theory exactly. The method is partly validated by model tests. A trim angle of up to 5° was found not to affect the green water problem.

A new method for measuring the water velocity of green water to develop a green water velocity profile was tested by Ryu et al. [2007b]. The wave run-up on a fixed model was measured. For the wave run-up a maximum velocity of 2.9 times the phase velocity of the incoming wave was found. For the maximum horizontal velocity during the entire green water event, this was 1.13 times the phase velocity of the incoming wave. The maximum vertical velocity occurred before the water moved on the deck. The dominant velocity before the wave hits the model is horizontal, directly in front of the model, though when the wave hits the model, the dominant velocity is vertical and the dominant velocity of the water on deck is horizontal again.

1.3.2 Green water on the side of an FPSO

The first research on green water on the side of an FPSO is performed by Vestbøstad [1999]. A linear 3D diffraction analysis was performed on the Norne FPSO, to determine the 100 year return period relative wave motion along the side of an FPSO. The incoming wave angle was 15° relative to head waves. Sea states with wave periods between 9 and 18 seconds and significant wave heights between 9.5 and 16.8 meters were assessed. The largest relative wave motions occurred at the bow, stern and between 5 and 20% aft of midships. Approximately 20% - 30% forward of midships there was a pronounced decrease in the relative wave motion. The worst sea states for green water on the side of the FPSO have wave periods of 10 - 14 seconds and significant wave heights resulting in steep waves. The calculations of the relative wave motion were related to model tests for waves coming in at 10° relative to head waves. The maximum differences in relative wave motion were 20% - 30%.

Buchner [2002] carried out model tests with waves coming in at an angle of 15° and 30° relative to head waves. The wave height was measured at different locations along the hull and the tests were carried out with regular and irregular waves. It was observed that as the wave crests travel along the side of the hull, the waves crests tend to become more and more peaked. The fact that the vessel pitches stern down when the wave crests reach midships can cause a freeboard exceedance just aft of midships. Another observation was that at the moment when the waves exceed the freeboard a surprisingly fast transverse flow occurs on the deck. This flow has a dominant component, not in the direction of the incoming waves but in the direction perpendicular to the length of the ship.

Also a 3D linear diffraction analysis was performed by Buchner [2002]. The results of the analysis proved reasonably in line with the measurements during the model tests. The fact that it is a linear
analysis excludes the effect of the higher harmonics in the analysis. This causes an under prediction of the freeboard height exceedance. Buchner developed an empirical based correction of the probability of freeboard exceedance for the side of an FPSO hull which corrects for the non-linearities in the relative wave height.

In 2002 a green water evaluation for FPSOs in the Gulf of Mexico has been carried out [Buchner et al., 2002]. This evaluation was based on the method developed by Buchner [2002]. The evaluation showed that smaller waves than the 100 year return period wave can cause higher freeboard exceedance. Therefore, the 100 year return period wave does not have to be the wave determining the maximum freeboard exceedance and maximum green water load.

The long term distribution of wave heights and the identification of sea states in which green water occurs is assessed by Fyfe and Ballard [2003]. The methodology is based on the methodology of Buchner [2002]. The operations of the FPSO cause a variation in draft and trim. For the long term probability it is assumed that the vessel is at maximum draft 10% of the total time. The maximum angle between incoming waves and head waves was chosen to be 30°. The locations along the vessel analysed are the bow and 15 meters aft of midships. The analysis has shown that the largest wave crest heights rarely account for the most severe green water events. Green water events were found to occur in sea states with periods of 12 - 14 seconds. The largest freeboard exceedance at 15 meters aft of midships is predicted to be between 8 and 10 meters. The corresponding sea state has a peak period of 9 seconds and a significant wave height of 7 - 8 meters. The calculations for the bow of the vessel were related to operational experiences and were found to be in good agreement.

Model tests have been carried out to investigate the wave run-up along the broadside of an FPSO hull [Xiao et al., 2014]. The tests have been performed with irregular waves and current. The current causes the waves to come in at a certain angle with respect to head waves. It is shown that the angle between the incoming waves and the longitudinal axis of the ship has a significant effect on the relative wave motion along the hull. The range along the broadside where green water events occur, increases if the waves come in oblique instead of head on. The non-linearity of the relative wave motion is larger for oblique incoming waves than for head waves. Also a frequency domain analysis is performed for the relative wave motion RAOs (Response Amplitude Operator). This response is concentrated around the incoming wave frequency. From these RAOs it can be seen that the diffraction effects amplify the relative wave motions. By comparing the measured wave elevations and the measured motion RAOs, it can be seen that the motions have a small effect on the relative wave motion compared to the effect of the wave itself. This study also shows that a smaller gap between the bottom of the ship and the sea bed causes an increased wave run-up. A side note is the fact that these measurements were taken in extreme waves in shallow water conditions.

1.3.3 Green water flow and loads

The classification society rules request the calculation of the loads due to green water, in order to determine the capability of the vessel in withstanding the loads due to green water. The necessary strength of the structures and pipework on board of the FPSO will also be based on these green water loads. This section shows how the results of this study can be implemented to determine the green water loads.

**Green water flow on deck** Buchner [2002] observed from model tests that the green water flow on deck from the side of the FPSO does not travel with the orbital velocity of the wave but the flow has a dominant component perpendicular to the longitudinal axis of the vessel. The height as well as the velocity of the flow on deck has been measured. The measurements show clear similarities with the theoretical dam-breaking problem with shallow water assumptions. This means that the velocity
is assumed to be constant over the height of the flow. The dam-break flow is a common used model to simulate the green water flow on deck [Schoenberg and Rainey, 2002]. It seems to agree with experimental results. It however does not account for the following effects according to Schoenberg and Rainey [2002]. The green water flow is three-dimensional and not two-dimensional, like the dam-break flow. This effect seems more important at the bow region than along the side of the hull of a vessel. The dam-break flow assumes an unlimited amount of water flowing on the deck, while in reality the flow is cut-off after a finite time interval. The dam-break flow also ignores the flow effects like viscosity, turbulence and wave plunging. To account for the fact that the water flow is cut-off after some time interval, a moving shelf model is used. The results of the model tests show differences with the dam-break flow solution. A reduction in the damage is found, if using the shelf model. But it is known that the model predicts higher fluid velocities than in reality because of higher relative gravity on the vessel. The model is also not validated against experimental or real data of green water flow. Cox and Ortega [2002] did model tests with one extreme wave overtopping a fixed deck. It was concluded that the green water flow can be simulated with dam-break theory, but the horizontal velocity of the wall of water was found not to be zero. The leading edge of the flow has a uniform velocity over its height except for a boundary layer close to the deck. The velocity is always the largest at the front of the wave. The maximum horizontal velocity was similar to the maximum crest velocity measured without the presence of the deck. On the deck the wave collapses but not breaks and develops a velocity multiple times the velocity of the maximum crest velocity without the presence of the deck. Yilmaz et al. [2003] developed a non-linear solution for the dam-break flow problem. The method does not make use of the shallow water assumption and uses the Fourier Series Analysis and the Fourier Transformation technique to come up with a solution for the flow. This model however is not validated with measurements. The flow of green water over the deck is measured in laboratory tests by Ryu et al. [2007a]. They tried to validate the Ritter’s dam-break flow solution for the green water problem. The differences between the green water flow and the dam-break flow are that the green water flow has a small but not zero initial velocity, the green water flow is highly aerated and the green water flow is followed by the crest wave with horizontal momentum. Despite these differences, the conclusion was that the Ritter’s dam-break flow solution represents the green water front velocity surprisingly well. An important part of the dam-break flow solution is determining the correct initial water depth. It was concluded that matching the front velocity of the green water flow with the front velocity of the dam-break flow, which is dependent on the initial water depth, gives a better result than assuming the initial water depth is equal to the freeboard exceedance from the linear diffraction theory. All in all Ritter’s dam-break flow solution represents the front velocity of the green water flow reasonably well. Song et al., [2015] tested a model in a large wave basin and measured velocities and pressures on the deck. The results were compared to small scale measurements [Ryu et al., 2007b]. They found the horizontal velocity above deck gradually increases from 0.5C at the deck edge up to 1.5C at the wave front. C is the phase velocity of the incoming wave. The wave front velocity is the maximum velocity in the flow during the green water event. This observation is consistent with the small model tests [Ryu et al., 2007b]. In the direction perpendicular to the incoming wave, the mean velocity is approximately zero. The mean velocity of the front in this direction is not uniform because of turbulence effects but the flow velocity is believed to ought to be uniform. In case the wave length is much greater than the deck size, the size of the structure does not seem to affect the horizontal green water velocity.
**Green water loads** Buchner [2002] came up with a way to determine the loads on structures on deck, like pipes or support structures. When the green water flow on deck is simulated with the dam-breaking theory, the loads on slender structures can be determined. The structures affected by green water loads are usually slender structures. It concerns mostly pipework and support structures for equipment.

The method proposed by Buchner [2002] determines the load on slender structures due to drag. The drag on a slender structure, per meter length, can be determined with Equation (1.1).

\[ f = \frac{1}{2} C_d \rho D u^2 \] (1.1)

The velocity (approximately constant over the height) can be determined, based on the dam-breaking theory. The drag coefficient is known for a lot of shapes of structures. The drag coefficient for a circular pipe is approximately 1.1, for example. With all data known, the force excited on the structure can be determined. By multiplying the force with halve the height of the flow, the maximum moment in the structure can also be determined.

The green water loads can be calculated with the dam-break flow solution and the drag on slender structures. These green water loads are of direct use in determining the necessary strength of the structures on the FPSO.
2 Methodology

The probability of exceedance of the freeboard, resulting in green water, is calculated by a 3D linear diffraction analysis. With this analysis the RAOs (Response Amplitude Operator) for the different motions of the vessel and for the free surface elevation along the side of the vessel are computed. The RAOs could also be determined with a time domain analysis, in which the non-linearities in the equation of motion are also included. Since the time record of the waves at the actual location where the FPSO will be positioned is not available, the RAOs are not calculated with a time domain analysis. An RAO for the relative wave motion can be determined by combining the RAOs from the diffraction analysis. By assuming a certain probability distribution for the relative wave motion, the probability of exceedance can be determined. With this probability of exceedance, an estimation of the height of exceedance of the freeboard is calculated. The following steps are taken in the analysis:

1. Create a 3D geometrical model of the FPSO.
2. Perform a 3D linear diffraction analysis to determine the RAOs of the different vessel motions and the RAO of the free surface elevation.
3. Determine the short term probability of exceedance along the side of the FPSO for all sea states, based on the RAOs and the weather conditions.
4. Determine the long term probability of exceedance along the side of the FPSO by combining the short term probability of exceedance for every sea state.
5. Calculate the most probable maximum freeboard exceedance along the side of the FPSO, based on the probability of exceedance.
6. Validate the free surface elevation RAO calculated with the diffraction analysis by means of the free surface elevation RAO measured during model tests.

2.1 The hydrodynamic model

The entire analysis is carried out for the Petrojarl 1 FPSO. For any other FPSO, the analysis would be the same. To be able to perform a 3D diffraction analysis, a 3D geometric model of the vessel is needed. This model is created, based on a lines plan of the Petrojarl 1. The Petrojarl 1 is an FPSO owned by Teekay Offshore. The vessel was newly built as an FPSO and delivered in 1986. In 29 years the vessel is deployed on ten different fields in the North Sea. Currently it is being prepared for the Atlanta field in the Santos Basin, see Figure 2.1. The vessel is meant to start producing oil off the coast of Brazil in 2016. It has an overall length of 215 meters and is 32 meters wide. The vessel has a storage capacity of 180000 barrels of oil and will be able to produce 30000 barrels of oil per day [Teekay, 2013].

A 3D model is created based on the lines plan of the FPSO. Part of the lines plan is shown in Figure 2.2. A number of the frames on the lines plan are translated into sketches in SOLIDWORKS 2015. The sketches are converted into a surface which serves as input for the hydrodynamic model of the FPSO. This surface model is the basis for the diffraction analysis.
2.2 3D diffraction analysis

The diffraction analysis is performed in ANSYS Aqwa 16.1. ANSYS Aqwa is an engineering toolset for the investigation of floating and fixed offshore and marine structures like FPSOs and TLPs (Tension Leg Platform) [ANSYS, 2015]. The diffraction analysis is performed in the frequency domain. A panel model of the vessel is used to solve the velocity potentials. The equations are solved by means of Green’s function [ANSYS, 2015]. The analysis resulted in the vessel motion RAOs and the free surface elevation RAOs.

2.2.1 Conventions

The conventions for the analysis in ANSYS Aqwa are given in Figure 2.3. The vessel motions are defined in the local axes system. This system is a right handed system as can be seen in Figure 2.3b. For the calculation of the free surface elevation RAOs along the side of the vessel, nodes are defined along the side of the vessel at which these RAOs are calculated. The coordinates of these nodes are
based on an origin at the aft perpendicular and at the baseline. The location of the origin is the same as in the lines plan of the vessel, on which the model of the vessel is based.
The environmental parameters are defined with respect to the fixed reference axes. The wave heading is defined as the angle between the x-axis and the line in which the waves are travelling. A wave heading of 0° means the waves are travelling along the x-axis from negative to positive. A 90° wave heading means the waves are travelling along the y-axis from negative to positive.
The phase angle is defined as the time difference between an oscillatory parameter and the incoming wave. Zero phase angle corresponds to the incoming wave crest located at the center of gravity. A positive phase angle corresponds to the parameter having a lag on the incoming wave. A positive phase angle means the peak of the motion of this parameter comes after the peak of the incoming wave.

### 2.2.2 Assumptions

For the analysis, several assumptions are made. Of the following assumptions, the first five are assumptions for the diffraction analysis to be valid.

- The fluid is inviscid, incompressible and the flow is irrotational.
- The incident regular wave has a small amplitude compared to it’s length.
- The motions are first order harmonic and have to be of small amplitude.
- The vessel is assumed to be a rigid body.
- Only the immersed part of the hull is taken into account in the diffraction analysis. The shape of the hull above the still water line is assumed to continue straight upwards. Therefore, during the analysis, the water does not flow onto the deck.
- The vessel is assumed to be ship shaped. The radii of gyration of the vessel are calculated with rules of thumb which are based on ship shaped vessels.
- The body has zero or very small forward speed. This assumption is valid because the vessel will actually be located at the same location for multiple years.
- The relative wave motion spectrum is assumed to be a narrow banded spectrum. The Rayleigh probability distribution is based on the fact that the spectrum is a narrow banded spectrum.
- The vessel is assumed to be free floating. The effects of the mooring lines and risers on the motion of the vessel are not taken into account.
- The vessel is assumed to always be floating in fully loaded condition, which results in maximum draft. In reality the draft will change in a cyclic manner. The vessel will be filled with oil, which increases the draft, until the vessel is emptied by means of a shuttle tanker which decreases the draft again.

### 2.2.3 Analysis preparation

The first input for the analysis is a model of the FPSO itself. In Figure 2.4 the model is shown. The brown part is the above still water line part and the grey part is the immersed part.
The particulars of the FPSO are summarized in Table 2.1. The data in Table 2.1 is retrieved from the stability and longitudinal strength study of the Petrojarl 1 [Teekay, 2015].
The volumetric mass of the model for the diffraction analysis is 2.2% larger than the mass given in
the study. The displacement of the vessel proved to be slightly different from the displacement of the model of the vessel because the model is based on old drawings of the vessel. Reading dimensions from the lines plan is sensitive for small errors in the geometry of the FPSO. The particulars of the box keel are retrieved from the lines plan and added at the bottom of the table. The midship section coefficient has been determined from the lines plan. The radii of gyration of the vessel are approximated by rules of thumb given in Equation (2.1) [Journée and Massie, 2001]. These approximations are for ships and because the FPSO is a ship shaped vessel, it is assumed that these approximations are correct (Section 2.2.2). To calculate the radii of gyration,

Table 2.1: Particulars of Petrojarl 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{pp}$</td>
<td>194.20 m</td>
<td>$k_{xx}$</td>
</tr>
<tr>
<td>B</td>
<td>32.00 m</td>
<td>$k_{yy}$</td>
</tr>
<tr>
<td>D</td>
<td>16.00 m</td>
<td>$k_{zz}$</td>
</tr>
<tr>
<td>T (without box keel)</td>
<td>11.24 m</td>
<td>$\rho$</td>
</tr>
<tr>
<td>$c_b$</td>
<td>0.80 -</td>
<td>d</td>
</tr>
<tr>
<td>Mass</td>
<td>58702 mt</td>
<td>g</td>
</tr>
<tr>
<td>$COG_x$ (from $A_{pp}$)</td>
<td>88.88 m</td>
<td>Height box keel</td>
</tr>
<tr>
<td>$COG_y$ (from center line)</td>
<td>0.00 m</td>
<td>Width top of box keel</td>
</tr>
<tr>
<td>$COG_z$ (from baseline)</td>
<td>11.55 m</td>
<td>Width bottom of box keel</td>
</tr>
</tbody>
</table>
Table 2.2: Scatter diagram [TetraTech, 2014]

<table>
<thead>
<tr>
<th>Hs [m]</th>
<th>Tp [s]</th>
<th>0 - 2</th>
<th>2 - 4</th>
<th>4 - 6</th>
<th>6 - 8</th>
<th>8 - 10</th>
<th>10 - 12</th>
<th>12 - 14</th>
<th>14 - 16</th>
<th>16 - 18</th>
<th>18 - 20</th>
<th>Total</th>
</tr>
</thead>
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<td>0.0 - 0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5 - 1.0</td>
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<td>0</td>
<td>10</td>
<td>34</td>
<td>14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>2</td>
<td>51</td>
<td>708</td>
<td>392</td>
<td>180</td>
<td>83</td>
<td>35</td>
<td>4</td>
<td>1</td>
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<td>1.5 - 2.0</td>
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<td>0</td>
<td>367</td>
<td>1888</td>
<td>1574</td>
<td>690</td>
<td>250</td>
<td>74</td>
<td>32</td>
<td>4</td>
<td>4879</td>
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</tr>
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<td>2.0 - 2.5</td>
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<td>0</td>
<td>627</td>
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<td>2194</td>
<td>1375</td>
<td>523</td>
<td>127</td>
<td>18</td>
<td>3</td>
<td>6499</td>
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</tr>
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<td>432</td>
<td>80</td>
<td>9</td>
<td>1</td>
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</tr>
<tr>
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<td>1</td>
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<td>104</td>
<td>235</td>
<td>233</td>
<td>53</td>
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</tr>
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<td>4.5 - 5.0</td>
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<td>0</td>
<td>4</td>
<td>41</td>
<td>123</td>
<td>127</td>
<td>33</td>
<td>0</td>
<td>0</td>
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<td></td>
</tr>
<tr>
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<td>0</td>
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<td>35</td>
<td>2</td>
<td>0</td>
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</tr>
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<td>5.5 - 6.0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>21</td>
<td>40</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>6.0 - 6.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>18</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>6.5 - 7.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>7.0 - 7.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td></td>
</tr>
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<td>Total</td>
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<td>1179</td>
<td>6461</td>
<td>6588</td>
<td>5506</td>
<td>2697</td>
<td>600</td>
<td>83</td>
<td>12</td>
<td>23128</td>
<td></td>
</tr>
</tbody>
</table>

Besides these parameters, the wave heading and wave frequency have to be defined. The wave heading is defined by means of the rules of DNV. Table 2-2 of DNV-OS-C102 states the design basis for the survival condition for the wave load analysis for ultimate strength calculations based on the most severe storm condition in a 100 years [DNV, 2014b]. The environmental conditions to be assessed are including the survival conditions because they are the most extreme conditions the FPSO will encounter. By taking into account all conditions, including the most severe conditions, it can be determined which conditions result in the highest green water probability.

The wave environment has to be site specific. For 8 years the wave environment has been measured at the site where the Petrojarl 1 will be moored, off the coast of Brazil. With 1 hour intervals the significant wave height and the peak period has been measured. This results in a scatter diagram of the area where the FPSO will be moored (Table 2.2). In the scatter diagram the significant wave height and the peak period are given in ranges. For the analysis, the mean values of the ranges are used for the peak periods. For the significant wave height the upper limit of the ranges has been used. The upper limit is chosen in order to be more conservative with the calculations.

Besides the scatter diagram, the extreme sea states with different return periods are determined, based on the measurements of the waves. The sea states are given in Table 2.3.

The wave spectrum on which the waves in the diffraction analysis are based, is the JONSWAP wave spectrum. The actual spectrum at the location of the FPSO is not determined during the wave analysis, therefore the most commonly used wave spectrum, the JONSWAP spectrum, is used during the diffraction analysis [DNV, 2014b]. The wave spectrum is defined in Journé and Massie [2001] and given in Equation (2.2).

\[
S_c(\omega) = \frac{320 \cdot H^2}{T^4_p} \cdot \omega^{-5} \cdot \exp \left\{ -\frac{1950}{T^4_p} \cdot \omega^{-4} \right\} \cdot \gamma^A
\]  

For the middle of the given ranges has been used.

\[
k_{xx} \approx 0.30 \cdot B \text{ to } 0.40 \cdot B
\]
\[
k_{yy} \approx 0.22 \cdot L \text{ to } 0.28 \cdot L
\]
\[
k_{zz} \approx 0.22 \cdot L \text{ to } 0.28 \cdot L
\] (2.1)
Table 2.3: Extreme sea states diagram [TetraTech, 2014]

<table>
<thead>
<tr>
<th>Heading</th>
<th>Parameter</th>
<th>1 year</th>
<th>5 years</th>
<th>10 years</th>
<th>50 years</th>
<th>100 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE</td>
<td>H_s [m]</td>
<td>3.59</td>
<td>4.06</td>
<td>4.21</td>
<td>4.60</td>
<td>4.80</td>
</tr>
<tr>
<td></td>
<td>T_p [s]</td>
<td>10.49</td>
<td>11.00</td>
<td>11.16</td>
<td>11.56</td>
<td>11.78</td>
</tr>
<tr>
<td>E</td>
<td>H_s [m]</td>
<td>3.47</td>
<td>4.71</td>
<td>5.09</td>
<td>6.04</td>
<td>6.53</td>
</tr>
<tr>
<td></td>
<td>T_p [s]</td>
<td>10.37</td>
<td>11.68</td>
<td>12.09</td>
<td>13.09</td>
<td>13.61</td>
</tr>
<tr>
<td>SE</td>
<td>H_s [m]</td>
<td>3.90</td>
<td>5.84</td>
<td>6.44</td>
<td>7.94</td>
<td>8.72</td>
</tr>
<tr>
<td></td>
<td>T_p [s]</td>
<td>10.83</td>
<td>12.88</td>
<td>13.51</td>
<td>15.10</td>
<td>15.93</td>
</tr>
<tr>
<td>S</td>
<td>H_s [m]</td>
<td>5.10</td>
<td>6.52</td>
<td>6.98</td>
<td>8.16</td>
<td>8.77</td>
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<tr>
<td></td>
<td>T_p [s]</td>
<td>12.09</td>
<td>13.61</td>
<td>14.09</td>
<td>15.34</td>
<td>15.99</td>
</tr>
<tr>
<td>SW</td>
<td>H_s [m]</td>
<td>5.12</td>
<td>6.11</td>
<td>6.43</td>
<td>7.26</td>
<td>7.69</td>
</tr>
<tr>
<td>W</td>
<td>H_s [m]</td>
<td>1.80</td>
<td>3.13</td>
<td>3.52</td>
<td>6.18</td>
<td>6.93</td>
</tr>
<tr>
<td></td>
<td>T_p [s]</td>
<td>8.60</td>
<td>10.01</td>
<td>10.42</td>
<td>13.24</td>
<td>14.03</td>
</tr>
</tbody>
</table>

with:

\[
\gamma = 3.3 \quad \text{(peakedness factor)}
\]

\[
A = \exp \left\{ - \left( \frac{\omega}{\omega_p} - 1 \right)^2 \frac{1}{\sigma \sqrt{2}} \right\}
\]

\[
\omega_p = \frac{2\pi}{T_p} \quad \text{(circular frequency at spectral peak)}
\]

\[
\sigma = \begin{cases} 
0.07 & \text{if } \omega < \omega_p \\
0.09 & \text{if } \omega > \omega_p
\end{cases}
\]

The peakedness factor is chosen as 3.3 because it is not defined in the metocean data and it is the average value for the JONSWAP data [DNV, 2010].

The wave frequency at which the spectra are defined is a range from 0.20 rad/s upto 2.18 rad/s. This range is based on the wave scatter diagram (Table 2.2). The spectra with the highest significant wave height per peak period show that the spectral density is nearly zero outside the range (Figure 2.5).

The extreme sea state wave spectra are also within this range of frequencies because the peak periods are well within the range of peak periods defined in the scatter diagram. Therefore all the assessed sea states are covered by the range of frequencies.

The wave headings, based on the DNV regulations [DNV, 2014b], for which the responses are calculated in the diffraction analysis are 150°, 165° and 180°. The responses in the headings -150° and -165° are not calculated in the diffraction analysis because the RAOs corresponding to these headings are the same as for 150° and 165°. The vessel is symmetrical in y-direction with respect to the y-axis, therefore the RAOs on the starboard and port side of the vessel are symmetrical with respect to the y-axis when comparing two wave headings that are mirrored in the y-axis.

With all the data necessary for the diffraction calculation known, the diffraction calculation is performed for every defined sea state. All the sea states result in different RAOs for the vessel motions and for the free surface elevation. These RAOs are input for calculating the most probable maxima of the responses.
The mathematical model of the potential theory, which is implemented in ANSYS Aqwa, is elaborated on in Appendix B. The calculation of the motion RAOs is explained, as well as the calculation of the free surface elevation RAOs.

2.2.4 RAO comparison

DNV has also calculated the motion RAOs of the Petrojarl 1 with linear 3D diffraction theory [Bakksjo, 2015]. It is expected that some differences appear between the RAOs calculated with Aqwa and calculated by DNV, because the input parameters differ slightly. The differences in the input of the two analyses are shown in Table 2.4. The DNV model has a higher draft and a lower mass than the model used in Aqwa. The reason for this is the fact that the DNV model has a moonpool in the hull, at the location where the turret is supposed to be. The model in Aqwa has no moonpool in it. The reason to not include the moonpool is the fact that the moonpool in reality will hold the turret. This means the moonpool will not be empty. The moonpool causes damping of the motions,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DNV</th>
<th>Ansys Aqwa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>57343</td>
<td>58702</td>
</tr>
<tr>
<td>$T (A_{pp})$</td>
<td>11.28</td>
<td>11.24</td>
</tr>
<tr>
<td>$T (F_{pp})$</td>
<td>11.42</td>
<td>11.24</td>
</tr>
<tr>
<td>COG$_x$</td>
<td>88.65</td>
<td>88.88</td>
</tr>
<tr>
<td>COG$_z$</td>
<td>11.16</td>
<td>11.55</td>
</tr>
<tr>
<td>$GM_t$</td>
<td>3.40</td>
<td>2.63</td>
</tr>
</tbody>
</table>
which in reality will be different because the turret will be in the moonpool. The model in Aqwa is more conservative than the model of DNV because the moonpool adds damping to the motions of the vessel. The calculation in Aqwa results in higher motion RAOs, than the DNV calculation.

2.3 Probability and freeboard exceedance

The motion RAOs and free surface elevation RAOs calculated with the diffraction analysis are used to determine the probability of green water and the most probable maximum freeboard exceedance. The probability of freeboard exceedance is calculated for the short term and for the long term. Two different probability distributions for the short term probability of exceedance are considered. The probability distribution is then used to calculate the most probable maximum relative wave motion and most probable maximum freeboard exceedance.

2.3.1 Short term probability of exceedance

The diffraction analysis results in the motion RAOs for the vessel in the center of gravity and the free surface elevation RAOs around the vessel. With these RAOs a relative wave motion RAO can be determined at any point (in this case point p) along the hull (Equation (2.3)).

\[
s_p(\omega, t) = \zeta_p - z + x_p \theta - y_p \phi
\]

(2.3)

In Equation (2.3), all the motions are still harmonic motions in the form of \( s_p = s_{p,a} \cos(\omega t + \epsilon_{\zeta}) \).

For the probability of freeboard exceedance, the amplitude of the relative wave motion is of interest. To calculate the relative wave motion RAO, Equation (2.3) can be split into two equations:

\[
\begin{align*}
  s_{p,a} \cos(\epsilon_{\zeta}) &= \zeta_{p,a} \cos(\epsilon_{\zeta}) - z_a \cos(\epsilon_{\zeta}) + x_p \theta_a \cos(\epsilon_{\theta}) - y_p \phi_a \cos(\epsilon_{\phi}) \\
  s_{p,a} \sin(\epsilon_{\zeta}) &= \zeta_{p,a} \sin(\epsilon_{\zeta}) - z_a \sin(\epsilon_{\zeta}) + x_p \theta_a \sin(\epsilon_{\theta}) - y_p \phi_a \sin(\epsilon_{\phi})
\end{align*}
\]

The variables on the right hand sides of these two equations are calculated in the diffraction analysis. The two equations can be solved for the amplitude and phase angle of the relative wave motion of a certain point along the hull of the vessel:

\[
\begin{align*}
  s_{p,a} &= \sqrt{(s_{p,a} \sin(\epsilon_{\zeta}))^2 + (s_{p,a} \cos(\epsilon_{\zeta}))^2} \\
  \epsilon_{\zeta} &= \arctan \left( \frac{s_{p,a} \sin(\epsilon_{\zeta})}{s_{p,a} \cos(\epsilon_{\zeta})} \right) \quad \text{with: } 0 \leq \epsilon_{\zeta} \leq 2\pi
\end{align*}
\]

The motions and wave elevation are calculated as RAOs. The equations above, to determine the relative wave motion, can be applied exactly the same for the RAOs. This results in the RAO for the relative wave motion at a certain location along the hull of the FPSO.

With the relative wave motion RAO, the relative wave motion spectrum for a certain significant wave height and peak period can be determined, Equation (2.4).

\[
S_{s}(\omega) = \left( \frac{s_{p,a}}{\zeta_a} \right)^2 \cdot S_{\zeta}(\omega)
\]

(2.4)
Rayleigh distribution

For waves exceeding the freeboard and causing green water on deck, the relative wave motion must exceed the freeboard. In case of a narrow band response spectrum, the Rayleigh distribution can be used to determine the probability of exceeding a certain value [Buchner, 2002; Journée and Massie, 2001]. However, model experiments show that the Rayleigh distribution underestimates the freeboard exceedance. The underestimation due to the Rayleigh distribution increases with increasing wave height [Buchner, 2002; Buchner and van den Berg, 2013]. However, the Rayleigh distribution is used for the calculation of the probability of exceedance because it is tested and validated with model experiments. The probability of exceedance of the freeboard is given in Equation (2.5).

\[
P_{ST}\{s > f\} = \exp\left\{\frac{-f^2}{2m_{0s}}\right\}
\]

with:

\[
m_{ns} = \int_{0}^{\infty} \omega^n \cdot S_s(\omega) \cdot d\omega \quad \text{with } n=0,1,2,...
\]

This results in the probability of freeboard exceedance for a certain location along the hull, for a certain wave spectrum and for a certain wave heading. The different spectra are defined by the scatter diagram and the extreme sea states. The different headings are defined by DNV. The probability of freeboard exceedance is determined along the side of the hull with 5 meter intervals. In Figure 2.6 the locations (nodes) are given as green points along the hull. These points are exactly at the waterline and at the side of the hull.
The freeboard in still water conditions is the following:

\[
f = \begin{cases} 
8.26 \text{ m} & \text{for } -0.04 \leq \frac{x}{L} < 0.18 \\
4.76 \text{ m} & \text{for } 0.18 \leq \frac{x}{L} < 0.85 \\
7.76 \text{ m} & \text{for } 0.85 \leq \frac{x}{L} < 0.93 \\
9.16 \text{ m} & \text{for } 0.93 \leq \frac{x}{L} \leq 1.00 
\end{cases}
\] (2.6)

Forristall distribution

The Rayleigh distribution is a linear distribution which does not account for the non-linear effects in the green water phenomenon and therefore underestimates the probability of exceedance [Buchner and van den Berg, 2013]. In Buchner [2002] model experiments are assessed to come up with a modified Rayleigh distribution to fit the relative wave motion more accurately. The parameters needed for this distribution are only determined for sea states with a peak period of 12, 14 and 16 seconds. The analysis performed for the Petrojarl 1 shows that these are not the peak periods which are of most interest for the green water along the side of the Petrojarl 1. Therefore, the modified Rayleigh distribution is not used to determine the probability of exceedance.

Additional model tests are carried out to investigate green water along the side of ships and FPSOs [Buchner and van den Berg, 2013]. The results show that the 2nd order Forristall crest height distribution [Forristall, 2000] predicts the probability of exceedance more accurately than the Rayleigh distribution. The Forristall distribution accounts for the 2nd non-linearities in the waves, like the fact that the wave crests are more peaked and the wave troughs are more shallow than a sinusoidal wave. For this reason, the probability of exceedance is also determined with the 2nd order Forristall crest height distribution. This distribution is given in Equation (2.7).

\[
P_{ST}\{s > f\} = \exp \left\{ - \left( \frac{f}{4\alpha \sqrt{m_0}} \right)^\beta \right\}
\] (2.7)

with:

\[
\alpha = \sqrt{\frac{1}{8}} + 0.2568S_1 + 0.0800Ur \\
\beta = 2 - 1.7912S_1 - 0.5302Ur + 0.2824Ur^2 \\
S_1 = \frac{2\pi}{g} \frac{4\sqrt{m_0}}{T_1^2} \\
Ur = \frac{4\sqrt{m_0}}{k_1^2d^3} \\
T_1 = 2\pi \frac{m_0}{m_1} \\
k_1 = \omega^2 \frac{g}{g} = \left( \frac{2\pi}{T_1} \right)^2 \\
d = \text{waterdepth}
\]

This distribution reduces to the Rayleigh distribution for \(\alpha = \sqrt{\frac{1}{8}}\) and \(\beta = 2\).

The probability of green water with both the Forristall and the Rayleigh distribution is calculated to be able to compare the distributions. The differences between the distributions are a result of the 2nd order effects in the incoming waves.
2.3.2 Long term probability of exceedance

The long term probability of exceedance can be determined by combining the short term probability of exceedance with the scatter diagram (Table 2.2). The entire scatter diagram has a probability of 1. Each combined significant wave height and peak period, sea state, has a certain probability:

\[ P_{\text{seastate}} = \frac{n(H_s, T_p)}{N} \quad (2.8) \]

With:

\[ N = \text{Total number of measured waves} \]
\[ n(H_s, T_p) = \text{Number of measured waves given a certain } H_s \text{ and } T_p \]

For every sea state, the probability distribution for the wave heading is the same. By combining the probability of the sea state, the probability of the heading and the probability of freeboard exceedance, the long term probability of freeboard exceedance in a certain node can be determined. The long term probability of a single wave exceeding the freeboard at a certain node is then given by Equation (2.9).

\[ P_{\text{LT}\{s>f\}} = \frac{\sum_{i=1}^{n_{H_s}} \sum_{j=1}^{n_{T_p}} P_{\text{heading}} \cdot P_{\text{seastate}} \cdot P_{\text{ST}\{s>f\}}}{\sum_{i=1}^{n_{H_s}} \sum_{j=1}^{n_{T_p}} P_{\text{heading}} \cdot P_{\text{seastate}}} \quad (2.9) \]

By calculating \( P_{\text{LT}} \) for every node along the hull, a distribution of probability of freeboard exceedance along the hull can be determined.

2.3.3 Number of waves exceeding freeboard per time interval

By multiplying the probability of freeboard exceedance with the number of waves occurring in a certain time interval, the number of waves exceeding the freeboard during this time interval can be determined. The number of waves occurring in a time interval is calculated with the zero-crossing period. The zero-crossing period can be calculated with Equation (2.10).

\[ T_z = 2\pi \cdot \sqrt{\frac{m_{0s}}{m_{2s}}} \quad (2.10) \]

The number of waves exceeding the freeboard in a given time interval can be calculated with Equation (2.11).

\[ n_{\text{time}}\{s>f\} = \frac{\text{time}}{T_z} \cdot P_{\text{LT}\{s>f\}} \quad (2.11) \]

2.3.4 Most probable maximum freeboard exceedance

To determine the maximum water height of green water at the deck edge, the most probable maximum value is calculated. The loads due to the most probable maximum freeboard exceedance are the values for which the structures have to be designed [DNV, 2014b]. According to Journée and Massie [2001] the most probable maximum is determined by Equation (2.12). This is the most probable maximum considering an extreme sea state with a certain return period (for example 100 years), lasting 3 hours. The formula is based on a long term three parameter Weibull distribution and a short term Rayleigh distribution. The long term cumulative distribution function (Equation (2.13)), resulting
from the Weibull distribution, combined with the short term Rayleigh distribution (Equation (2.5)) for a certain sea state, results in the formula for the most probable maximum value.

\[
s_{mpm} = \sqrt{2 \cdot m_0 \cdot \ln(N)}
\] (2.12)

\[
F = 1 - \frac{1}{N}
\] (2.13)

According to DNV [2014b] the most probable maximum is the value that corresponds to the 37th percentile, in other words, a probability of exceedance of 63%. The cumulative distribution function for this probability of exceedance is given by Equation (2.13). This results in the same formula for the most probable maximum freeboard exceedance.

### 2.3.5 Sea states

An FPSO remains on location for a number of years, for this reason, DNV [DNV, 2014b] states that the FPSO strength calculations have to be based on the most severe 100 year storm conditions and all sea states along the contour line of the scatter diagram. To get more insight in the green water problem along the side of an FPSO, the probability of green water and the most probable maximum freeboard exceedance are determined both for the entire scatter diagram and for all extreme sea states. The sea state resulting in the highest probability of exceedance is analysed more thorough, to determine what the cause for the high probability of exceedance is.

### 2.4 Roll damping

The diffraction analysis is a linear diffraction analysis. This means the only damping on the vessel’s motions calculated by the linear diffraction theory is the wave making damping. For all the motions, the viscous effects on the damping are negligible. The wave making damping is the major part of the motion damping [Journée and Massie, 2001]. Only for roll motions, this is not the case. A cylinder rolling in water does not produce any waves, the wave making damping is zero. At a breadth to draft ratio of 2.5 the vessel has a shape coming close to a cylinder, causing a low roll damping, resulting in very large roll motions [Journée and Massie, 2001]. The breadth to draft ratio of the Petrojarl 1 is \( \frac{B}{T} = \frac{32}{11.24} = 2.85 \), resulting in a small wave making roll damping. A box keel is fitted under the Petrojarl 1. This box keel increases the wave making damping significantly. The wave making damping due to the box keel is also calculated by the diffraction analysis. Besides wave making roll damping there are four other types of roll damping [Ikeda et al., 1978].

The natural roll period is calculated to have a comparison for the roll period determined from the diffraction analysis. The roll period is determined with Equation (2.14) [Pinkster, 2006]. The roll radius of gyration is determined with an approximation given in Equation (2.15) [Pinkster, 2006]. The range is applicable for freight and passenger ships, which have a similar geometry compared to an FPSO. The middle of the range is used to calculate the roll radius of gyration.

\[
T_{0\phi} = \frac{2k_{\phi\phi}}{\sqrt{GM}}
\] (2.14)

\[
k_{\phi\phi} \approx 0.35 \text{ to } 0.45 \cdot B
\] (2.15)
2.4.1 Types of roll damping

The same five types of roll damping for ships are defined by Kawahara et al. [2009] and [Ikeda et al., 1978]. The first type is wave making roll damping. The other four types of roll damping are explained in the following paragraphs.

Friction damping The friction damping is caused by the friction between the hull and the water. This friction depends on the viscosity of water and because the linear diffraction theory assumes the water to be inviscid (Section 2.2.2), it does not account for the friction between the hull and the water.

Eddy making damping The eddy making damping is caused by the small separation of bubbles or small shedding vortices occurring at the bilge section amidships and the relatively sharp sections of the hull at the bow and stern. This is not taken into account by the linear diffraction theory because the fluid is assumed irrotational and inviscid.

Bilge keel damping The bilge keel damping is caused by a keel fitted at the bilge area. A pressure difference around the keel, on the hull, and a normal force on this keel result in roll damping. This is usually the largest component of the roll damping in case there is no forward speed and the vessel does have bilge keels. This is not taken into account by the linear diffraction theory because the fluid is assumed irrotational and inviscid.

Lift damping The lift damping is caused by the forward speed of a vessel. Because in this study an FPSO is considered, the forward speed is zero. At zero forward speed, the lift damping part of the roll damping is zero.

2.4.2 Roll damping coefficient

The roll damping is determined by means of the Ikeda method [Kawahara et al., 2009; Journée, 2001]. This method predicts the roll damping reasonably well, based on the shape of the vessel. It is a widely accepted method and model tests have been carried out to check the method. The mathematical model of this method is given in Appendix C.

The method does not account for the box keel under the FPSO. This keel causes more roll damping because the sharp edges of the box keel cause vortices in the water. The wave roll damping accounts for the box keel because it is added in the hydrodynamic model used for the diffraction analysis. The roll damping including the box keel is higher in reality than the roll damping calculated with the Ikeda method. However, it is assumed that the effect on the probability of green water is negligible.

Based on certain particulars of the vessel, the formulas predict the roll damping coefficient depending on the roll amplitude and the incoming wave frequency. In Table 2.5 the input parameters for the roll damping coefficient calculation are given. These are the additional particulars besides the particulars given in Table 2.1.

The dimensions of the bilge keels are given in Table 2.5. The keels are fitted from frame number 14 up to frame number 120. These frames correspond to \( \frac{x}{L_{pp}} = 0.05 \) and \( \frac{x}{L_{pp}} = 0.54 \).

To calculate the roll damping coefficient, the roll amplitude is needed. However, the amplitude of the roll motion depends on the roll damping coefficient itself. To solve this problem, the ship motions are calculated iteratively. First the vessel motions without roll damping are calculated, after that
Table 2.5: Additional particulars of the Petrojarl 1 for the non-linear roll damping calculation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>1.004</td>
</tr>
<tr>
<td>$10^{-6}$ m$^2$s</td>
<td>10^{-6} m$^2$s</td>
</tr>
<tr>
<td>OG</td>
<td>-0.31</td>
</tr>
<tr>
<td>m</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Vessel section dependent</td>
</tr>
<tr>
<td>m</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Vessel section dependent</td>
</tr>
<tr>
<td>m</td>
<td></td>
</tr>
<tr>
<td>$L_{bk}$</td>
<td>95 m</td>
</tr>
<tr>
<td>m</td>
<td></td>
</tr>
<tr>
<td>$B_{bk}$</td>
<td>0.90 m</td>
</tr>
<tr>
<td>m</td>
<td></td>
</tr>
</tbody>
</table>

the roll amplitude is used to determine the roll damping. With the roll damping, the vessel motions are calculated again. By repeating this step several times, the motions of the vessel and the roll damping coefficient converge, resulting in the solution of the vessel motions including the roll damping approximated by Ikeda’s method. The wave heading for which the roll amplitude and roll damping coefficient are calculated is 150°, but can be any angle except 180° (a wave heading of 180° does not cause a roll motion). The damping coefficient is the same for every roll angle. A different wave heading might cause a different roll angle, but the damping coefficient will still be the same.

2.5 Free surface elevation validation

The relative wave motion RAO along the side of the FPSO that is calculated with the linear 3D diffraction theory does not take into account non-linearities, for example an actual wave is more peaked than a sinusoidal wave or when the wave exceeds the freeboard, the water can flow onto the deck. To determine if the linear diffraction calculation is valid, the free surface elevation calculated with the diffraction theory is compared with the free surface elevation from model experiments performed at MARIN [Buchner and van den Berg, 2013]. A long thin plate representing a ship of 263 meters length and with a draft of 16.5 meters was placed in a towing tank. Irregular waves were generated to measure the relative wave motion at 4 points along the length of the plate. The plate itself was fixed in its place. The results of the experiment are compared with the results of the diffraction analysis. The measurements are taken at frequencies of 0.31 rad/s upto 1.09 rad/s. This range is a lot smaller than the range in the diffraction calculation of the FPSO. If the peak of the response spectrum of the relative wave motion along the side of the FPSO is within the measured range during the experiment, the measurements can still be compared with the calculations.

To validate the calculations of the Petrojarl 1, several 3D diffraction analyses are performed. First an analysis of the plate itself is performed. A plate with the same dimensions is modelled and assessed in a diffraction analysis. By analysing this plate, the difference between the diffraction calculations and the measurements can be determined.

The plate from the model tests has a draft of 16.5 meters. The FPSO has a draft of 11.24 meters. The influence of the difference in draft on the free surface elevation has to be taken into account. To determine the effect on the free surface elevation, both a plate with a draft of 16.5 and a plate with a draft of 11.24 meters is analysed. The free surface elevation measured during the experiments is compared with the free surface elevation of the diffraction analyses of the two plates.

The plate has a length of 263 meters, while the FPSO has a length of 201.49 meters at the still water line. This longitudinal difference of the plate and the FPSO also has to be taken into account. The free surface elevation of the measurements and the diffraction analysis is linked to each other at only three locations. The fourth location at which the free surface elevation RAO was measured during the experiments is at the aft end of the plate, 263 meters aft of the most forward location. The FPSO is only 201.49 meters long at the waterline, resulting in the last location being positioned behind the vessel. The first three locations are at the front of the plate (RELM1), 87.67 meters behind the
front (RELM6) and 175.33 meters behind the front of the plate (RELM11), as shown in Figure 2.7. These locations are linked to the following on the FPSO: The bow (RELM1, $\frac{x}{L_{pp}} = 1.00$), 106.70 meters forward of the aft perpendicular (RELM6, $\frac{x}{L_{pp}} = 0.55$) and 19.04 meters forward of the aft perpendicular (RELM11, $\frac{x}{L_{pp}} = 0.10$). This means that the distance in x-direction of the measured points is the same for the FPSO and the plate and the first point is in both cases the most forward point of the vessel respectively plate at the waterline.

The depth in the tank during the experiments was 5 meters, which corresponds to 250 meters in full scale. This is a large difference with the 1550 meters water depth in the diffraction analysis. 1550 meters is considered deep water, therefore it has to be determined if 250 meters of water depth also can be considered deep water. Deep water is water deep enough for the waves to not 'feel' the bottom. A criterion for deep water is $\frac{d}{\lambda} > \frac{1}{2}$ [Journée and Massie, 2001]. The criterion is $2 \cdot d = 500$ meters for waves in the tank, waves longer than 500 meters start to 'feel' the bottom of the tank. A wave length of 500 meters corresponds to a wave frequency of 0.35 rad/s. Therefore, a wave with a frequency of 0.35 rad/s and lower might act differently compared to a deep water wave. Because 0.35 rad/s is at the bottom part of the frequency range used in this analysis, the waves may be considered deep water waves. To be certain that the waves during the measurements act as deep water waves, the RAOs for the free surface elevation around the FPSO for a depth of 1550 meters and 250 meters are compared. When the RAOs are the same, the water is in both cases deep water.

By comparing the free surface elevation RAO, calculated with diffraction theory, with the free surface elevation RAO measured during the model tests, it can be concluded if the diffraction analysis is a suitable method for calculating the free surface elevation.
3 Results

To determine the probability of green water along the side of the FPSO, the vessel motion RAOs and free surface elevation RAOs are calculated with linear diffraction theory. The probability of green water and the most probable maximum freeboard exceedance are calculated based on these RAOs. The effect of the relative wave direction on the probability of green water is assessed by calculating the probability of green water for a number of different relative wave headings. Finally, different aspects of the analysis itself are assessed.

3.1 Response amplitude operators

The response of the vessel in the waves is determined by means of the RAOs. These RAOs are calculated with a diffraction analysis. The wave frequency ranges from 0.20 rad/s upto 2.18 rad/s with in between steps of 0.02 rad/s. The RAOs are independent of the incoming wave height because the value is normalized by the incoming wave amplitude. The wave heading and frequency of the incoming wave and the location on the vessel where the RAOs are assessed, are of influence on the RAOs. The RAOs for the vessel in the center of gravity are given in Figure 3.1.

In this section the RAOs are elaborated on and the RAO calculations with Aqwa are compared with the DNV calculations. To assess the RAOs, the wave length is related to the wave frequency. This is achieved by means of the dispersion relationship, Equation (3.1).

$$\omega^2 = k \cdot g \quad \text{for deep water}$$  (3.1)

With:

$$k = \frac{2\pi}{\lambda}$$

This results in the following relation between the wave frequency and the wave length:

$$\lambda = \frac{2\pi \cdot g}{\omega^2}$$

$$\omega = \sqrt{\frac{2\pi \cdot g}{\lambda}}$$

**RAO comparison**  For the heave, roll and pitch motion, the RAOs calculated by DNV are also added in Figure 3.1. The RAOs of the heave and pitch motion are reasonably similar to the RAOs calculated in ANSYS Aqwa. However, the RAO of the roll motion shows a large difference. The peak of the roll motion RAO is 4.6 deg/m in the Aqwa calculation, while it is 1.3 deg/m in the DNV calculation. Since the input of the analyses is comparable it is expected that the output should also be similar. The method used for the non-linear roll damping in the DNV calculations is not known, therefore the roll damping coefficient could be the source of the differences. The presence of a moonpool could also be a source of the differences. The moonpool causes roll damping which would decrease the peak in the roll motion RAO. The reason for the difference is not completely known, but the RAO as calculated in Aqwa, is used for the green water calculation, because a higher roll RAO amplitude results in a higher probability of green water. It is more conservative to apply a larger roll RAO.
Figure 3.1: The response amplitude operators in the center of gravity for a wave heading of $150^\circ$ and $180^\circ$
Surge RAO  In Figure 3.1, the surge RAO tends to one for very small frequencies or very long waves. In very long waves, much longer than the length of the vessel, it moves exactly with the wave. The water particles move along a circular path. The vessel follows this rotational motion, which results in a RAO of one for long waves. For shorter waves, the RAO tends to zero because the waves are much shorter than the vessel. Several waves affect the vessel simultaneously and the rotational motions of several waves combined result in no surge motion at all. Therefore, the RAO tends to zero for very short waves. The difference for the different wave headings is small. For waves not coming in at 180°, the vessel still follows the motion of the water particles, resulting in the RAOs for the different wave headings to be similar.

Sway RAO  For a wave heading of 180° the sway RAO is zero. The wave is exactly similar on both sides of the vessel. Therefore, the force due to the waves on both sides of the vessel is the same. There is no force which moves the vessel in sway direction. For 150° the vessel again follows the water particles in the wave. Because the rotational motion of the water particles is in the direction of the wave, the vessel also moves in sway direction when the waves are not coming in head on. For very short waves the RAO tends to zero for the same reason as for the surge motion.

Heave RAO  The RAO for heave tends to one for very long waves. The wave is much longer than the vessel and the vessel simply floats on top of the wave and moves up and down as the wave surface moves up and down. For shorter waves, the RAO drops and shows a peak at 0.70 rad/s for a heading of 180° and at 0.74 rad/s for a heading of 150°. The peak is caused by the resonance in heave. The waves appear longer with respect to the vessel, which results in a slightly higher frequency for the peak in a wave heading of 150°. For a wave to appear the same length as for the waves with a heading of 180°, the wave has to be a bit shorter. For very short waves, the heave response of the vessel tends to zero. The waves are much shorter than the vessel. Therefore multiple waves excite force on the ship at the same time. The combination of several waves exciting force on the ship results in a total heave of zero.

Roll RAO  The roll motion of the vessel is zero for a wave heading of 180°. The wave is exactly the same at both sides of the vessel, which results in equal forces on both sides of the vessel. Equal forces on both sides of the vessel result in a roll motion RAO of zero. For a wave heading of 150°, the RAO tends to zero for very long waves. For very long waves the vessel only moves up and down on top of the wave, the vessel experiences it like an up and down movement of the water surface. The slope of the wave is very small and the roll angle will tend to zero. Therefore, the vessel will not roll in very long waves. At a wave frequency of 0.42 rad/s there is a large peak in the roll motion of the ship. 0.42 rad/s is approximately the natural roll frequency of the vessel. At this frequency the vessel’s roll motion is larger, because this frequency is dominated by the roll damping of the vessel and not by the inertia or the change in displacement of the vessel [Journée and Massie, 2001]. The inertia part and the spring part in the equation of motion for roll of the vessel counteract other, resulting in the movement being dominated by the roll damping of the vessel. In very short waves, the response of the vessel tends to zero again because the roll motion of the vessel caused by the different waves counteract each other. The waves are very short, which means that the average effect on the vessel is zero.
Pitch RAO  For small frequencies, the RAO of the pitch motion of the vessel tends to zero. For very long waves the vessel just moves up and down on the wave and does not pitch. The angle of the wave surface tends to zero, which results in a pitch motion tending to zero.

A peak is expected approximately at the frequency for which the wave has the same length as the ship or a bit longer. In that case the vessel pitches approximately similar to the angle of the wave surface. When the waves are even longer, the vessel also pitches similar to the angle of the wave surface but the angle is in that case smaller. For a wave heading of 180°, the wave has to be 205.8 meters to be the same length as the vessel. This results in a wave frequency of 0.55 rad/s. The peak is found at 0.48 rad/s, which corresponds to a wave length of 267.4 meter. For this wave length, the pitch angle is at its largest.

For the incoming wave with a heading of 150°, a wave appears longer when looking in the longitudinal direction of the vessel. The peak in the pitch RAO is at 0.52 rad/s. This corresponds to a wave length of 227.9 meters. When taking into account the 30° angle between the incoming waves and the longitudinal axis of the vessel, the wave appears to be \( \frac{227.9}{\cos(30)} = 263.2 \) meters. This is approximately the same wave length as for waves coming in at 180°.

For very short waves the RAO tends to zero again because very short waves mean that multiple waves affect the ships motions simultaneously and the effect of the waves together sums up to zero.

Yaw RAO  The yaw motion RAO of the vessel for a wave heading of 180° is zero. The wave is exactly the same at both sides of the vessel, which results in equal forces on both sides of the vessel. Equal forces on both sides of the vessel result in a yaw motion RAO of zero.

For a wave heading of 150° the RAO is not zero. For very small frequencies, the RAO tends to zero because the wave becomes very long. The surface elevation at the front and at the aft of the vessel is nearly the same, resulting in the force, which causes a yaw motion, tending to zero.

A very sharp peak is located at 0.42 rad/s, this is exactly the natural roll frequency. It appears that the roll motion also results in a local peak in the yaw motion.

For very high frequencies, the yaw motion again tends to zero because the waves are that small that the effects of individual waves counteract each other and the entire motion sums up to zero.

Relative wave motion RAO  The relative wave motion RAOs at two locations along the vessel are given in Figure 3.2. The RAOs at \( \frac{x}{Lpp} = 0.23 \) are given because this proved to be the location where the probability of exceedance was the highest. For small frequencies, all relative wave motions tend to zero. The vessel exactly follows the wave and the relative wave motion is zero when the vessel exactly follows the wave.

For the bow, the relative wave motion tends to one for very high frequencies. The vessel does not move in very high frequencies because all the motion RAOs are approximately zero. If the vessel does not move the wave surface moves up and down along the side of the vessel, resulting in a relative wave motion RAO of one. This is the case for both wave headings considered here.

The relative wave motion RAO at \( \frac{x}{Lpp} = 0.23 \) and with a wave heading of 180° tends to one for very high frequencies because the diffraction and radiation at this location are very small. The radiation and diffraction are small because the wave flows in line with the surface of the hull.

For a wave heading of 150°, for very high frequencies, the relative wave motion RAO increases to approximately 2.5 due to diffraction and radiation.
3.2 Long term probability of green water

The long term probability of green water, based on the probability of exceedance during every sea state in the scatter diagram, is presented in Figure 3.3. The number of waves exceeding the freeboard per year and the probability of a single wave exceeding the freeboard are calculated. On average, per year, 20 waves will cause green water along the side of the FPSO, which is at a location of $\frac{x}{L_{pp}} = 0.23$. This is the location with the highest probability of exceedance, which is $8 \times 10^{-4}$. The probability of green water shows a peak at the bow of the vessel, which is $2 \times 10^{-4}$. Further aft, the probability decreases but the decreasing freeboard results in an increase in the probability. Along the side of the vessel where the freeboard is the lowest (4.76 meters), the probability shows a minimum of $2 \times 10^{-5}$ and further aft the probability of green water increases again.

The probability is based on the entire scatter diagram, taking into account all the sea states and taking into account the probability a certain sea state occurs. The reason why the probability of green water peaks just behind midships, is assessed by studying the short term probability of green water in every sea state (Section 3.3).

Operating limit The operating limit due to the probability of deck wetness is 0.05 [Nielsen, 1987]. For the long term probability of exceedance, for every sea state in the scatter diagram, the probability of green water is determined (Figure 3.4). If the probability during a sea state is above 0.05, it is above the operating limit. 16 of the registered 81 different sea states result in a probability above 0.05. The long term probability of the weather conditions being above the operating limit can be determined with the probability of the different sea states in the scatter diagram occurring. This results in a long term probability of 0.062 of weather conditions in which the deck wetness probability is above the operating limit. It has to be taken into consideration that the assumption was made that the vessel is always floating at maximum draft (Section 2.2.2). In reality, the probability of operating above the operating limit is smaller because the vessel is not always operating at maximum draft.
3.3 Short term probability of green water

For every sea state in the scatter diagram (Table 2.2) and for every extreme sea state (Table 2.3), the probability of green water along the side of the hull is determined. All these sea states are assessed with a wave heading of 150° because this is the heading with the largest offset to head waves the FPSO will encounter. The largest offset to head waves will cause the highest relative wave motions along the side of the hull (Section 3.4). For every sea state, the probability of freeboard exceedance in a 3 hours storm with that specific sea state is determined (Figure 3.4, Figure 3.5). The sea state with $H_s = 5$ meters and $T_p = 7$ seconds (scatter diagram sea state) was the sea state which resulted in the highest probability of exceedance. The probability of green water and the most probable maximum

<table>
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<th>$H_s$ [m]</th>
<th>$T_p$ [s]</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
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<th>17</th>
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</tbody>
</table>

Figure 3.4: Probability of green water for every sea state in the scatter diagram
Figure 3.5: Probability of green water for every extreme sea state

freeboard exceedance in this sea state are compared with the probability of green water and the most probable maximum freeboard exceedance in the extreme wave sea state. The extreme wave sea state is defined as the sea state with the significant wave height and peak period with a return period of 50 years, because the probability of exceedance of the 50 year return period sea state is the highest probability of all the extreme sea states. This sea state has a significant wave height of 8.16 meters and a peak period of 15.34 seconds.

The sea states are both assessed more in depth because the conditions of the sea states are very different, but still both sea states result in a high probability of green water.

3.3.1 Green water analysis of the extreme wave sea state

First, the green water analysis of the 50 year return period sea state is discussed. The relative wave motion RAO is split up into several RAOs. After that the relative wave motion itself is analysed. For this sea state, the diffraction analysis is recalculated with a different range of frequencies. Instead of a frequency range with a constant step of 0.02 rad/s, the steps are smaller close to the peak of the response spectrum. A gradually changing frequency range results in a more accurate result.

**FPSO motion RAOs**

To be able to explain why the probability and most probable maximum wave height vary along the ships length, the relative wave motion RAO is analysed more thorough. The relative wave motion RAO is based on 4 underlying RAOs (Equation (2.3)), on which the probability and most probable maximum of green water on deck is based:

- The free surface elevation RAO
- The heave RAO
- The RAO of the heave as a result of roll
- The RAO of the heave as a result of pitch

By assessing these RAOs separately, the contribution of each motion is determined (Figure 3.6). By assessing the influence of every motion, the motions with the largest influence on the relative wave motion are determined.
Figure 3.6: The RAOs contributing to the relative wave motion and the relative wave motion RAO itself for a wave heading of 150°
Free surface elevation RAO  The free surface elevation RAO is the result of all the wave elements. It is the elevation due to the diffraction and radiation of the wave as well as the incoming wave itself along the side of the hull of the vessel.

The RAO is one for small frequencies. Small frequencies are long waves and in long waves the vessel moves on top of the waves with very small to zero diffraction and radiation of the wave. The wave is much longer than the length of the vessel.

Due to the diffraction and radiation of the waves, the RAO starts to differ from one with increasing frequency in different ways at different positions along the vessel. At the stern, the free surface elevation RAO is decreasing with increasing frequency. The stern is shielded by the vessel itself, which results in a lower free surface elevation. At the bow, the RAO is slightly larger than one for large frequencies due to radiation of the wave. Along the side of the FPSO, the free surface elevation RAO is larger than one, with peaks up to 2.4.

Heave RAO  The heave motions at different locations along the vessel are all the same as the heave motion at the center of gravity because the vessel moves as a rigid body. The rigid body motion is an assumption in the linear diffraction theory (Section 2.2.2). The explanation for the heave motion RAO of the center of gravity is also valid for the heave motion RAO at other locations on the vessel.

The RAO of the heave due to roll  The heave of the vessel caused by rolling of the ship is zero for very long waves (Figure 3.6d). For very long waves, the vessel only moves up and down on top of the wave, the vessel experiences it like an up and down movement of the water surface. The slope of the wave is small and the roll angle will tend to zero. The vessel will not roll in very long waves.

At a wave frequency of 0.42 rad/s, there is a large peak in the heave motion due to rolling of the ship. This is because 0.42 rad/s is the natural roll frequency of the vessel, as seen earlier. The vessel is rolling at its natural roll frequency.

In very short waves, the response of the vessel tends to zero again because the roll motion of the vessel is affected by multiple waves. The waves are very short, which means that the average effect on the vessel is zero.

For the bow and for the stern, the heave as a result of roll RAO is zero. The nodes which are used to determine the RAO at the stern and at the bow are exactly at the center line of the vessel. The distance in y-direction of the center line to the node is zero, therefore the amplitude of the RAO is also zero.

The RAO of the heave due to pitch  The heave due to pitch is explained by the pitch motion RAO of the vessel at the center of gravity. The pitch motion RAO of the vessel at the center of gravity is the same for the rest of the vessel because it is a rigid body motion.

The heave motion due to pitch RAO differs linearly along the length of the vessel. The heave motion is calculated by multiplying the pitch motion RAO (in rad/m) with the distance from the considered point to the center of gravity. This results in the linear change of the heave motion due to pitch along the length of the vessel. At the center of gravity the heave due to pitch is zero and at the bow and stern, the motion is the largest because the distance to the center of gravity is the largest.

Relative wave motion RAO  The relative wave motion RAO is a summation of the previously mentioned RAOs, taking into account the phase angle differences between the RAOs (Equation (2.3)). In Figure 3.6a it can be clearly seen that the relative wave motion is formed this way. The peak caused by the roll motion is present at 0.42 rad/s. The peaks resulting from the pitch motion can also be seen in the relative wave motion RAO. The RAO of the free surface elevation above approximately 1
rad/s is also clearly visible. All in all, depending on the location along the side of the FPSO, the relative wave motion RAO differs between 0 around the stern for high frequencies and 2.5 along the side for high frequencies.

**FPSO motion response spectra**

The RAOs do not completely explain the probability and most probable maximum relative wave motion. To determine these values, the relative wave motion spectrum is analysed. The relative wave motion spectrum (Figures 3.7a and 3.7f) is calculated by squaring the RAO for the relative wave motion and multiplying it with the wave spectrum (Equation (2.4)). The peak period of the wave spectrum corresponds to a frequency of 0.41 rad/s. The spectral energy is for the majority focused around this frequency. If this is combined with the RAOs for the different motions as given in Figure 3.6, the response spectra for the different motions can be calculated separately (Figure 3.7). It is shown that the RAO around a frequency of 0.41 rad/s is the part of the spectrum which affects the relative wave motion of the vessel the most.

By comparing the spectra of the different motions, it can be seen that for the peak response frequency of 0.42 rad/s, the free surface elevation, the heave due to pitch and the heave due to roll result in comparable spectral densities, while the heave motion has a relatively low spectral density.

The effect of the free surface elevation on the relative wave motion is spread over a wider range of frequencies because the free surface elevation RAO increases for higher frequencies. This effect is decreased because the wave spectral density decreases with increasing frequency. But it results in the fact that for higher frequencies the free surface elevation is the most important parameter influencing the relative wave motion.

The heave motion spectrum and heave due to roll motion spectrum indicate that these motions affect the relative wave motion only at and around the wave peak frequency, which nearly coincides with the roll RAO peak frequency. Especially the heave due to roll spectrum is very narrow around a peak frequency of 0.42 rad/s. The peak of the heave spectrum is lower but has a wider frequency spreading. The heave motion due to pitching of the vessel affects the relative wave motion mostly at the bow and at the stern of the vessel. This is caused by a linear increasing RAO with respect to the distance from the center of gravity to the point where the RAO is assessed. The pitch motion has less effect around the middle of the vessel, where the center of gravity is located.

All in all, the wave spectrum results in the fact that only the frequencies at and around the peak wave frequency are affecting the relative wave motion.

**Probability and most probable maximum of green water**

The probability of freeboard exceedance along the length of the vessel peaks at the location where the freeboard drops to 4.76 meters ($\frac{x}{L_{pp}} = 0.82$) with a probability of 4.1% (Figure 3.8). A second peak in the probability, also 4.1%, is located at $\frac{x}{L_{pp}} = 0.36$. The peak aft of midships is also what has been found in earlier research [Buchner, 2002; Vestbostad, 1999]. A probability of 4.1% means that during this storm 37 waves will exceed the deck and cause green water (Figure 3.8b).

The most probable maximum relative wave motion has also been calculated and related to the freeboard along the side of the vessel (Figure 3.8c and Figure 3.8d). The most probable maximum freeboard exceedance, located at $\frac{x}{L_{pp}} = 0.82$, is 1.8 meters. At $\frac{x}{L_{pp}} = 0.36$, the most probable maximum freeboard exceedance is 1.7 meters.
Figure 3.7: Response spectra for wave heading 150° contributing to the relative wave motion and the response spectrum of the relative wave motion itself
Figure 3.8: Probability of green water and most probable maximum height of green water on deck along the side of the FPSO during a 3 hours storm in a 50 year return period sea state

**Freeboard** A parameter with a large effect on the probability of green water is the freeboard. To be able to assess the effect of the differing freeboard along the length of the vessel on the probability, another calculation with a constant freeboard of 4.76 meters over the entire length of the vessel has been performed (Figure 3.9). The effect of the jumps in the freeboard are clearly visible when Figure 3.8 and Figure 3.9 are compared. A jump in the freeboard also causes a jump in the probability. A higher freeboard decreases the probability of exceedance.

The probability of green water along the side of the vessel from the bow to the stern (Figure 3.8a) shows a peak in the probability at the location where the still water freeboard level drops to 4.76 meters. In front of this peak, the probability is lower because the freeboard is higher. Further aft, the probability increases, with again a maximum aft of midships. Even further aft, the probability decreases again because of the freeboard level.

Especially at the bow and just behind the bow, the increased freeboard due to the forecastle decreases the probability significantly. At the bow, the probability of green water is 26.2% with a freeboard of 4.76 meters. The probability at the bow is 1.0% with a freeboard of 9.16 meters.
Vessel motions and free surface elevation

In order to determine the influence of the different vessel motions and the free surface elevation on the relative wave motion, the areas of the different spectra are calculated. After that, the phase angles are assessed and finally a combination of the RAO amplitude and RAO phase angles is assessed to determine the influence of every motion and the free surface elevation on the relative wave motion.

Response spectral area  To clarify the link between the different motion spectra along the length of the vessel and the probability along the length of the vessel, in Figure 3.10 the area of the spectra for every node along the length of the vessel is shown. In the calculation of the relative wave motion spectrum, the RAOs are squared and the phase angles are taken into account. Therefore, the values of the area do not simply add up to the area of the relative wave motion. But the area of each motion separately indicates the potential contribution of each motion to the relative wave motion.

The effect of roll is constant along the length of the vessel, as long as the breadth of the vessel is constant. The vessel is a rigid body, therefore the roll angle is the same for the entire vessel. The
heave due to roll is calculated by multiplying the roll angle and the distance in y-direction of a certain point and the center of gravity. This results in a linear decrease in the heave due to roll amplitude with respect to the distance in y-direction to the center of gravity. From $x_{L_{pp}} = 0.77$ forward, the breadth of the vessel decreases and therefore, the area of the heave due to roll spectrum decreases. This effect is visible in the relative wave motion as a small kink in the line at $x_{L_{pp}} = 0.77$. At the stern the area of the heave due to roll spectrum decreases because the distance in y-direction decreases. Exactly at the stern and at the bow, the RAO amplitude is zero because the nodes where the RAO is assessed for the stern and bow are located at the center line of the vessel. At the stern and at the bow, the area of heave due to pitch is the largest because the RAO depends on the distance from the location to the center of gravity which is located at 88.88 meters forward of the aft perpendicular, $x_{L_{pp}} = 0.46$. This is also the location where the contribution of the heave due to pitch is zero. The effect on the relative wave motion at the bow is larger than at the stern because the distance from the center of gravity to the bow is larger than the distance from the center of gravity to the stern. The relative wave motion spectral area at the front of the vessel increases similar to the increase in the area of the heave due to pitch spectrum.

The free surface elevation is fairly constant along the side of the vessel. Towards the bow, the free surface elevation RAO is smaller than at midships and further aft. The larger free surface elevation further aft of the vessel is caused by the radiating and diffracting wave. The radiating and diffracting wave increase when proceeding aft, resulting in a higher free surface elevation RAO amplitude. At the stern, the free surface elevation RAO decreases because the stern area is shielded by the vessel itself.

**Phase angles** Not only the area of the different spectra influence the relative wave motion, the phase angle of the RAO of the different vessel motions and of the free surface elevation also influence the relative wave motion (Figure 3.11). The phase angles of the different motion RAOs of the vessel are constant over the length of the vessel. The motion is exactly the same for the entire vessel because it is modelled as a rigid body. The phase angles of the heave due to pitch and the heave due to roll change over the length or width of the vessel because the roll and pitch RAO are multiplied with the distance with respect to the center of gravity. The phase angle of the free surface elevation is dependent on both location along the vessel as well
Relative wave motion contribution

The areas of the response spectra, which are based on the response RAO amplitudes, of the different motions and the free surface elevation and the RAO phase angles combined result in the relative wave motion spectrum. To account for the RAO phase angles and RAO amplitudes simultaneously, the contribution of each motion to the relative wave motion is calculated, with the phase angles taken into account. For every motion and the free surface elevation, for every location and every frequency, the amplitude is calculated as a percentage of the amplitude of the relative wave motion RAO. The locations for which these percentages are calculated are the locations where the probability of...
green water is the largest, \( \frac{x}{L_{pp}} = 0.36 \) and \( \frac{x}{L_{pp}} = 0.82 \).

To determine which frequencies are influencing the relative wave motion the most, the relative wave motion spectrum is assessed, along with the phase angles of all the separate motions. At a frequency of 0.42 rad/s, a jump in the phase angle RAO of the relative wave motion and the roll motion is located (Figure 3.11). This is also the peak frequency of the relative wave motion spectrum (Figure 3.12). A frequency below the phase angle jump, at the phase angle jump and above the phase angle jump shows the effect of the phase angle jump. The frequencies, 0.41, 0.42 and 0.43 rad/s, are also governing the relative wave motion (Figure 3.12) and therefore these frequencies are assessed to determine the effect of the vessel motions and the free surface elevation on the relative wave motion and probability of green water.

Table 3.1 illustrates the amplitude of the motions and the free surface elevation as a percentage of the amplitude of the relative wave motion for a frequency of 0.41, 0.42 and 0.43 rad/s. The phase angles between the different motions and the relative wave motion are taken into account, therefore these percentages are the actual contributions to the relative wave motion. The frequency at which the influence on the probability of green water is the largest is 0.42 rad/s.

### Table 3.1: Contribution of the free surface elevation, heave, heave due to roll, heave due to pitch in percentages of the relative wave motion RAO amplitude, taking into account both the RAO amplitude and the RAO phase angle

<table>
<thead>
<tr>
<th>( \frac{x}{L_{pp}} = 0.36 )</th>
<th>( \frac{x}{L_{pp}} = 0.82 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frequency</strong></td>
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<tr>
<td>Relative wave motion</td>
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<tr>
<td>Free surface elevation</td>
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<td>Heave</td>
<td>29</td>
</tr>
<tr>
<td>Heave due to pitch</td>
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</tr>
<tr>
<td>Heave due to roll</td>
<td>120</td>
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<td>Total motions</td>
<td>121</td>
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</table>
At this frequency and at $\frac{x}{L_{pp}} = 0.36$, the heave due to roll causes 107% of the relative wave motion, the heave causes 54%, the heave due to pitch causes 8%, resulting in a combined contribution of the vessel motions of 169%. The relative wave motion is reduced by the free surface elevation by 69%.

At $\frac{x}{L_{pp}} = 0.82$, the vessel motions combined result in 130% of the relative wave motion. The heave due to roll is 76%, the heave motion is 47% and the heave due to pitch is 7%. The relative wave motion is reduced by the free surface elevation with 30%.

All in all, the vessel motions cause the relative wave motion and this motion is reduced by the free surface elevation.

First of all, the roll motion is at both locations and at all frequencies a large contribution to the relative wave motion because 0.42 rad/s is the natural roll frequency. The vessel roll motions are large at and around 0.42 rad/s.

The free surface elevation is decreasing the relative wave motion at all frequencies at both locations except for 0.43 rad/s at $\frac{x}{L_{pp}} = 0.82$. The contribution of the three vessel motions combined is also given in Table 3.1 and the free surface elevation reduces the relative wave motion caused by the vessel motions.

The heave motion is contributing to the relative wave motion for all cases except for 0.43 rad/s at $\frac{x}{L_{pp}} = 0.36$, at which the relative wave motion is slightly reduced by the heave motion.

The heave due to pitch is enlarging or reducing the relative wave motion, depending on the frequency. The effect of the heave due to pitch is the smallest of all motions and the free surface elevation.

The main reason why this sea state results in a high probability of green water is the roll motion. The effect of the roll motion is large compared with the effect of the other motions and the free surface elevation.

The RAO amplitude and phase angles at these frequencies and locations are given in Table 3.2. The heave due to roll motion RAO amplitude and phase angle are varying significantly for different frequencies. 0.42 rad/s is approximately the natural roll period of the FPSO, which is the peak in the RAO. A slightly higher or lower frequency results in the RAO amplitude below or above the peak, which is smaller.

The other motions and the free surface elevation differ less significantly between the frequencies, but the differences result in a change in relative wave motion. The motions and free surface elevation combine into the relative wave motion and this combination is different for every frequency. Therefore the percentages differ significantly between frequencies.

All in all, all three motions and the free surface elevation are influencing the relative wave motion in this sea state, but the roll motion is the main reason for the large relative wave motion.

### 3.3.2 Green water analysis of the scatter diagram sea state

The sea state resulting in the highest probability of green water along the side is the sea state with a significant wave height of 5 meters and a peak period of 7 seconds. For this sea state, the diffraction analysis is recalculated with a different range of frequencies. Instead of a frequency range with a constant step of 0.02 rad/s, the steps are smaller close to the peak of the response spectrum. A gradually changing frequency range results in a more accurate result.
Table 3.2: RAO amplitude and phase angle at two different locations for the free surface elevation, heave, heave due to roll, heave due to pitch and the relative wave motion

<table>
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<th>Frequency</th>
<th>Amp</th>
<th>ϵ</th>
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<th>Location</th>
<th>Frequency</th>
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<th>ϵ</th>
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<td>1.03 m</td>
<td>-67 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_heave</td>
<td>0.72 m</td>
<td>-1 degrees</td>
<td>Amp_heave</td>
<td>0.72 m</td>
<td>-1 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hroll</td>
<td>0.98 m</td>
<td>-60 degrees</td>
<td>Amp_hroll</td>
<td>0.87 m</td>
<td>-60 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hpitch</td>
<td>0.24 m</td>
<td>-87 degrees</td>
<td>Amp_hpitch</td>
<td>0.89 m</td>
<td>93 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x/Lpp = 0.82</td>
<td>Amp_rwm</td>
<td>1.16 m</td>
<td>-156 degrees</td>
<td>Amp_rwm</td>
<td>1.47 m</td>
<td>177 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_fse</td>
<td>0.91 m</td>
<td>-3 degrees</td>
<td>Amp_fse</td>
<td>0.78 m</td>
<td>-59 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_heave</td>
<td>0.70 m</td>
<td>-1 degrees</td>
<td>Amp_heave</td>
<td>0.70 m</td>
<td>-1 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hroll</td>
<td>1.30 m</td>
<td>9 degrees</td>
<td>Amp_hroll</td>
<td>1.15 m</td>
<td>9 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hpitch</td>
<td>0.24 m</td>
<td>-87 degrees</td>
<td>Amp_hpitch</td>
<td>0.91 m</td>
<td>93 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Frequency = 0.41 rad/s</td>
<td>Amp_rwm</td>
<td>0.80 m</td>
<td>108 degrees</td>
<td>Amp_rwm</td>
<td>1.07 m</td>
<td>139 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_fse</td>
<td>0.98 m</td>
<td>8 degrees</td>
<td>Amp_fse</td>
<td>1.03 m</td>
<td>-67 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_heave</td>
<td>0.72 m</td>
<td>-1 degrees</td>
<td>Amp_heave</td>
<td>0.72 m</td>
<td>-1 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hroll</td>
<td>0.98 m</td>
<td>-60 degrees</td>
<td>Amp_hroll</td>
<td>0.87 m</td>
<td>-60 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hpitch</td>
<td>0.24 m</td>
<td>-87 degrees</td>
<td>Amp_hpitch</td>
<td>0.89 m</td>
<td>93 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Frequency = 0.42 rad/s</td>
<td>Amp_rwm</td>
<td>1.16 m</td>
<td>-156 degrees</td>
<td>Amp_rwm</td>
<td>1.47 m</td>
<td>177 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_fse</td>
<td>0.91 m</td>
<td>-3 degrees</td>
<td>Amp_fse</td>
<td>0.78 m</td>
<td>-59 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_heave</td>
<td>0.70 m</td>
<td>-1 degrees</td>
<td>Amp_heave</td>
<td>0.70 m</td>
<td>-1 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hroll</td>
<td>1.30 m</td>
<td>9 degrees</td>
<td>Amp_hroll</td>
<td>1.15 m</td>
<td>9 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hpitch</td>
<td>0.24 m</td>
<td>-87 degrees</td>
<td>Amp_hpitch</td>
<td>0.91 m</td>
<td>93 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Frequency = 0.43 rad/s</td>
<td>Amp_rwm</td>
<td>0.74 m</td>
<td>-90 degrees</td>
<td>Amp_rwm</td>
<td>0.83 m</td>
<td>-144 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_fse</td>
<td>0.95 m</td>
<td>5 degrees</td>
<td>Amp_fse</td>
<td>0.95 m</td>
<td>-71 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_heave</td>
<td>0.67 m</td>
<td>-1 degrees</td>
<td>Amp_heave</td>
<td>0.67 m</td>
<td>-1 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hroll</td>
<td>0.67 m</td>
<td>64 degrees</td>
<td>Amp_hroll</td>
<td>0.59 m</td>
<td>64 degrees</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Amp_hpitch</td>
<td>0.25 m</td>
<td>-87 degrees</td>
<td>Amp_hpitch</td>
<td>0.94 m</td>
<td>93 degrees</td>
<td></td>
</tr>
</tbody>
</table>

The probability and most probable maximum freeboard exceedance

The maximum probability of waves exceeding the freeboard is 31.8% and located at x/Lpp = 0.23 (Figure 3.13a). A peak located aft of the middle of the vessel is confirmed by research performed earlier [Buchner, 2002]. A probability of 31.8% corresponds to 603 waves exceeding the freeboard during a 3 hours storm with these sea state conditions. A second peak of 8.4% is located at x/Lpp = 0.82. A probability of 8.4% corresponds to 158 waves exceeding the freeboard in a 3 hours storm.

Figure 3.13c and Figure 3.13d show the most probable maximum freeboard exceedance and the most probable maximum relative wave motion. At x/Lpp = 0.23, the most probable maximum freeboard exceedance is 5.8 meters and the most probable maximum relative wave motion is 10.6 meters. At x/Lpp = 0.82, the most probable maximum freeboard exceedance is 2.3 meters and the most probable maximum relative wave motion is 7.0 meters.

Freeboard In Figure 3.14 the probability and most probable maximum freeboard exceedance are shown with an equal freeboard along the length of the vessel. Again the effect of the change in freeboard is clearly visible. An increase in freeboard results in a large decrease in the probability of green water. Towards the bow, at x/Lpp = 0.95, the probability of green water with a freeboard of 9.16 meters is 0.4%. At the same location, with a freeboard of 4.76 meters, the probability of green water is 17.3%.

Near the stern of the vessel, the same effect is visible. The probability along the side with an equal freeboard along the length, indicates the shielding effect of the vessel on the free surface elevation near
the stern of the vessel. Especially at the stern, this effect is visible because the probability decreases to 0.0%, even for a freeboard of 4.76 meters.

The increase in freeboard also affects the most probable maximum freeboard exceedance. The increase in freeboard is equal to the decreases of the most probable maximum freeboard exceedance. The diffraction analysis does not account for the change in freeboard above the still water line. The most probable maximum freeboard exceedance is affected by the water flowing onto the deck when it exceeds the deck edge. This decreases the most probable maximum freeboard exceedance. The effect of the difference in freeboard on the most probable maximum freeboard exceedance is in reality smaller. When the freeboard exceedance is larger, more water can flow onto the deck, resulting in a larger decrease in the freeboard exceedance. When the freeboard exceedance is smaller, the effect of the water flowing onto the deck is also smaller. Therefore, the difference in freeboard exceedance, when lowering the freeboard, is smaller than calculated.
Vessel motions and free surface elevation

The probability of green water behind midships is very high. To determine what is causing this high probability of exceedance, the response spectra of the different motions and the free surface elevation are calculated (Figure 3.15).

The free surface elevation spectrum (Figure 3.15b) shows a peak around 0.90 rad/s. The peak period of the wave spectrum is 7 seconds, which is 0.898 rad/s. The peak frequency of the free surface elevation is approximately the same frequency as the peak of the wave spectrum.

The spectral density of the heave, heave due to roll and heave due to pitch motions are all very small. The peaks have a spectral density of respectively 0.14, 0.02 and 1.30 m²/s. While the spectrum of the free surface elevation has a peak of 24.97 m²/s. The relative wave motion in this sea state is almost completely the result of the free surface elevation. The only major difference is the spectral density at the bow. The spectral density of the relative wave motion spectrum at the bow is higher than the spectral density of the free surface elevation at the bow. At the bow, the spectral density of the heave due to pitch motion is the largest. The heave due to pitch motion amplifies the relative wave motion at the bow because the distance between the center of gravity and the bow is large.
Figure 3.15: Response spectra for a wave heading 150° contributing to the relative wave motion and the response spectrum of the relative wave motion itself.
Response spectral area  To clarify the potential contribution of the different motions and the free surface elevation to the probability of green water, the spectral area of the different motions and the free surface elevation is determined (Figure 3.16). The free surface elevation is nearly the same as the relative wave motion, while the spectral areas of the motions of the vessel are nearly zero. Near the stern and near the bow, the pitch motion is also influencing the green water probability slightly, but this influence is very small because the spectral area is small compared with the spectral area of the relative wave motion.

Relative wave motion contribution  For this sea state, the influence of the different vessel motion RAOs and the free surface elevation RAO are expressed in a percentage of the relative wave motion, taking into account the phase angles (Table 3.3). The calculation is again performed for the peak frequencies and a higher and lower frequency. The locations assessed are the locations where a peak in the probability of green water occurred, \( x_{L_{pp}} = 0.23 \) and \( x_{L_{pp}} = 0.82 \).

At \( x_{L_{pp}} = 0.23 \), for all assessed frequencies, the relative wave motion is almost entirely caused by the free surface elevation. For the peak frequency of 0.90 rad/s, the relative wave motion is for 97.2% the result of the free surface elevation. The vessel motions combined result in 2.8% of the relative wave

Table 3.3: Contribution of the free surface elevation, heave, heave due to roll, heave due to pitch in percentages of the relative wave motion RAO amplitude, taking into account both the RAO amplitude and the RAO phase angle

<table>
<thead>
<tr>
<th>( \frac{x}{L_{pp}} ) = 0.23</th>
<th>( \frac{x}{L_{pp}} ) = 0.82</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Frequency</td>
</tr>
<tr>
<td>0.89 0.90 0.91 rad/s</td>
<td>0.89 0.90 0.91 rad/s</td>
</tr>
<tr>
<td>Relative wave motion</td>
<td>Relative wave motion</td>
</tr>
<tr>
<td>100.0 100.0 100.0 %</td>
<td>100.0 100.0 100.0 %</td>
</tr>
<tr>
<td>Free surface elevation</td>
<td>Free surface elevation</td>
</tr>
<tr>
<td>96.2 97.2 98.1 %</td>
<td>75.7 77.3 79.0 %</td>
</tr>
<tr>
<td>Heave</td>
<td>Heave</td>
</tr>
<tr>
<td>0.4 -0.4 -1.0 %</td>
<td>2.4 2.3 2.3 %</td>
</tr>
<tr>
<td>Heave due to pitch</td>
<td>Heave due to pitch</td>
</tr>
<tr>
<td>4.0 4.1 4.1 %</td>
<td>21.2 19.3 17.0 %</td>
</tr>
<tr>
<td>Heave due to roll</td>
<td>Heave due to roll</td>
</tr>
<tr>
<td>-0.5 -0.9 -1.2 %</td>
<td>0.7 1.2 1.7 %</td>
</tr>
<tr>
<td>Total motions</td>
<td>Total motions</td>
</tr>
<tr>
<td>3.9 2.8 1.9 %</td>
<td>24.3 22.8 21.0 %</td>
</tr>
</tbody>
</table>
Table 3.4: RAO amplitude and phase angle at two different locations for the free surface elevation, heave, heave due to roll, heave due to pitch and the relative wave motion

<table>
<thead>
<tr>
<th>( \frac{x}{L_{pp}} ) = 0.23</th>
<th>( \frac{x}{L_{pp}} ) = 0.82</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency = 0.89 rad/s</td>
<td>Frequency = 0.90 rad/s</td>
</tr>
<tr>
<td>( Amp_{rwm} )</td>
<td>2.22 m</td>
</tr>
<tr>
<td>( \epsilon_{rwm} )</td>
<td>109 degrees</td>
</tr>
<tr>
<td>( Amp_{fse} )</td>
<td>2.14 m</td>
</tr>
<tr>
<td>( \epsilon_{fse} )</td>
<td>106 degrees</td>
</tr>
<tr>
<td>( Amp_{heave} )</td>
<td>0.04 m</td>
</tr>
<tr>
<td>( \epsilon_{heave} )</td>
<td>-150 degrees</td>
</tr>
<tr>
<td>( Amp_{hroll} )</td>
<td>0.01 m</td>
</tr>
<tr>
<td>( \epsilon_{hroll} )</td>
<td>141 degrees</td>
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<tr>
<td>( Amp_{hpitch} )</td>
<td>0.19 m</td>
</tr>
<tr>
<td>( \epsilon_{hpitch} )</td>
<td>170 degrees</td>
</tr>
<tr>
<td>Frequency = 0.91 rad/s</td>
<td></td>
</tr>
<tr>
<td>( Amp_{rwm} )</td>
<td>2.19 m</td>
</tr>
<tr>
<td>( \epsilon_{rwm} )</td>
<td>116 degrees</td>
</tr>
<tr>
<td>( Amp_{fse} )</td>
<td>2.15 m</td>
</tr>
<tr>
<td>( \epsilon_{fse} )</td>
<td>114 degrees</td>
</tr>
<tr>
<td>( Amp_{heave} )</td>
<td>0.03 m</td>
</tr>
<tr>
<td>( \epsilon_{heave} )</td>
<td>164 degrees</td>
</tr>
<tr>
<td>( Amp_{hroll} )</td>
<td>0.03 m</td>
</tr>
<tr>
<td>( \epsilon_{hroll} )</td>
<td>145 degrees</td>
</tr>
<tr>
<td>( Amp_{hpitch} )</td>
<td>0.16 m</td>
</tr>
<tr>
<td>( \epsilon_{hpitch} )</td>
<td>170 degrees</td>
</tr>
</tbody>
</table>

motion. Of the vessel motions, the heave due to pitch influences the relative wave motion the most with 4% of the amplitude of the relative wave motion.

At \( \frac{x}{L_{pp}} = 0.82 \), the free surface elevation is still the major cause for the relative wave motion for all frequencies. However, the heave due to pitch also has a significant effect on the relative wave motion as it is approximately 20% of the relative wave motion RAO for all three frequencies. For 0.90 rad/s, the free surface elevation is 77.3% of the relative wave motion RAO amplitude, the heave is 2.3%, the heave due to pitch is 19.3% and the heave due to roll is 1.2%. At this location, the wave runs up along the side of the hull, while a small pitch motion adds to the relative wave motion.

The RAOS at 0.89, 0.90 and 0.91 rad/s and at both frequencies are shown in Table 3.4. At \( \frac{x}{L_{pp}} = 0.23 \), for all three frequencies the free surface elevation RAO amplitude and RAO phase angle are nearly the same as the amplitude and phase angle of the relative wave motion RAO. For the response peak frequency of 0.90 rad/s, the free surface elevation RAO amplitude is 2.14 meters, while the relative wave motion RAO amplitude is 2.21 meters. The amplitudes of the vessel motions are relatively small and therefore barely affect the relative wave motion. Of the motions, the heave due to pitch affects the relative wave motions the most with an RAO amplitude of 0.16 up to 0.19.

At \( \frac{x}{L_{pp}} = 0.82 \), for all three frequencies the free surface elevation RAO phase angle and the relative wave motion RAO phase angle are nearly the same, but at this location the heave due to pitch RAO phase angle is also nearly the same. Therefore, the pitch motion has a larger contribution to the relative wave motion at this location.

All in all, the vessel moves with a small pitch motion during this sea state while the waves run up along the side of the hull, potentially causing green water. The heave and roll motion are nearly zero.
In contrast to the extreme wave sea state, where the green water is caused by a combination of vessel motions and the free surface elevation, the green water in this sea state is mainly the result of the waves running up against the side of the hull.

**Vessel shape**

The free surface elevation has a large influence on the probability of exceedance along the side of the vessel. To assess the effect of the shape of the vessel on the free surface elevation, another diffraction analysis has been performed, but this time with a barge with the particulars similar to the FPSO. The length, the width and the draft of the midship section are kept the same and are constant over the length of the barge. This results in a different mass of the vessel. The mass changes from 58702 mt to 74600 mt. The center of gravity is assumed to be still at the same location.

The difference in free surface elevation RAO between the Petrojarl 1 and a barge with the same particulars is shown in Figure 3.17. For the barge, the natural roll frequency is 0.30 rad/s. At this frequency, there is a peak in the free surface elevation RAO. This peak is the natural roll frequency of the barge. But unlike the free surface elevation RAO of the Petrojarl 1 at its natural roll frequency, the free surface elevation RAO for the barge is nearly constant over the length of the vessel. It is varying along the length between 0.94 and 1.03 m/m. The large variance of the free surface elevation RAO along the length of the Petrojarl 1 at the natural roll frequency is caused by the varying vessel shape along the length. This is the only major difference between the free surface elevation RAO of the FPSO and of the barge. The shape of the vessel is mainly important for the free surface elevation along the side of the FPSO at the natural roll frequency.

### 3.3.3 Comparison of the two sea states

The two sea states ($H_s = 8.16$ m, $T_p = 15.34$ s and $H_s = 5$ m, $T_p = 7$ s), both result in a high probability of green water. The reasons behind the high probabilities are very different.

The sea state with $H_s = 8.16$ m, $T_p = 15.34$ s results in a high probability of green water along the side of the FPSO because the vessel motions are large in this sea state, especially the roll motion. The free surface elevation lowers the relative wave motions a little, but it still results in a high probability of exceedance.

![Figure 3.17: Free surface elevation RAO](image-url)
The sea state with $H_s = 5$ m and $T_p = 7$ s results in an even higher probability of green water (31.8%). This is mostly caused by the free surface elevation. For the response peak frequency, 97% of the relative wave motion is caused by the free surface elevation and only 3% is caused by the vessel motions. Of the vessel motions, the pitch motion has the most influence on the relative wave motion. All in all, there are two different situations which can lead to large relative wave motions. On one hand due to large vessel motions, slightly lowered by the free surface elevation, on the other hand due to the large free surface elevation while the vessel pitches a little.

**Green water effect on Petrojarl 1**

The most probable maximum freeboard exceedance along the side of the Petrojarl 1 is between 5.8 meters at $x_L = 0.23$ and 2.0 meters at $x_L = 0.77$ (Figure 3.13). The exceedance of 2.0 up to 5.8 meters is relative to the main deck of the FPSO on which all equipment is protected with green water barriers. The process deck, 2.0 meters above the main deck, is where all other equipment is located. The equipment on the process deck is not protected against green water. The green water barriers on main deck are needed along the entire side of the vessel, where the freeboard is 4.76 meters ($0.18 \leq \frac{x_L}{L_{pp}} < 0.85$). Aft of $x_L = 0.77$, the freeboard exceedance is more than 2.0 meters, therefore the process deck equipment which is located near the edge of the deck should also be protected against green water. During this analysis, the effect of the water flowing onto the deck, when it exceeds the deck, which results in a lower most probable maximum freeboard exceedance, is not taken into account. Therefore, the 5.8 meters of freeboard exceedance is a little conservative.

**Draft variation**

It was assumed that the largest draft of the vessel will result in the largest probability of green water along the side. But the extreme wave sea state results in a large probability of green water, which is also caused by the roll motion of the vessel. The effect of the draft on the roll motion is assessed to determine if the assumption is reasonable. The influence of the roll motion depends on the roll RAO itself and on the wave conditions. In the extreme wave sea state, the roll natural period is nearly equal to the peak period of the wave spectrum. Therefore, the change in natural roll period is assessed, compared with the environmental conditions. The natural roll period is calculated with Equation (3.2).

$$T_{00} = \frac{1}{\omega_{00}} = \sqrt{\frac{I_{xx} + a_{44}}{c_{\phi\phi}}}$$

In which:

$$I_{xx} = \rho \nabla k_{xx}^2$$
$$k_{xx} = 0.35 \cdot B$$
$$c_{\phi\phi} = \rho g \nabla \cdot GM$$
$$a_{44} = \text{added mass for roll motion}$$

In the stability and longitudinal strength study of the Petrojarl 1 [Teekay, 2015], 13 different loadings conditions are defined, of which three conditions are assessed here. The GM value of the Petrojarl 1 is calculated for these loading conditions, which means for different drafts of the vessel. The following conditions contain the highest and lowest GM value and the highest and lowest draft of the vessel:
1. LS1.00  GM = 3.34 m  T = 9.16 m  $f_{\min} = 6.84$ m
2. LS1.06  GM = 1.71 m  T = 10.48 m  $f_{\min} = 5.52$ m
3. LS1.12  GM = 2.63 m  T = 11.24 m  $f_{\min} = 4.76$ m

The natural roll period is calculated with Equation (3.2) for these three sea states, which results in the following wave periods and frequencies:

1. LS1.00  $T_0 = 13.80$ s  $\omega_0 = 0.455$ rad/s
2. LS1.06  $T_0 = 19.00$ s  $\omega_0 = 0.331$ rad/s
3. LS1.12  $T_0 = 15.21$ s  $\omega_0 = 0.413$ rad/s

Small draft  Loading condition LS1.00 results in a slightly lower natural roll period than LS1.12. The sea state with the peak period close to 13.80 seconds and the highest significant wave height, taking into account the scatter diagram and the extreme state diagram, is the sea state with a peak period of 13 seconds and a significant wave height of 7.5 meters. This significant wave height is smaller than during the extreme wave sea state. This is an indication that the probability of green water might be lower than during the extreme wave sea state.

The decrease in draft between LS1.12 and LS1.00 results in an increase in freeboard along the side of the FPSO by 2.08 meters. The freeboard analysis in Section 3.3.1 shows that an increase in freeboard significantly decreases the probability of green water. The increase in freeboard is also an indication that the probability of green water is smaller than for LS1.12.

The resonance peak in the roll motion RAO, which is governing for the effect of the roll motion on the probability of green water, is dominated by the roll damping. The wave making roll damping in LS1.12 is relatively small because the breadth to draft ratio is 2.85. For a ratio of 2.5, the vessel midship section is the closest to a circular section, which results in very low wave making damping. In LS1.00, the breadth to draft ratio is 3.5, which is a lot less similar to a circular cross section. Therefore, the wave making damping in LS1.00 is larger than the wave making damping in LS1.12. A larger damping results in a smaller roll motion RAO, which decreases the probability of green water. All in all, the probability of green water for a draft of 9.16 meters is most likely lower than the probability for a draft of 11.24 meters.

Small GM  For LS1.06, the GM value and the draft is smaller than for LS1.12. This results in a natural roll period of approximately 19.00 seconds. Table 2.2 shows that a sea state with this period occurs very rarely and the maximum significant wave height with this peak period is 3.5 meters. A significant wave height of 3.5 meters is very low compared with the significant wave height of the extreme wave sea state (8.16 meters). Therefore, the probability of green water for this draft is likely to be smaller than the probability for the extreme wave sea state.

Similar to LS1.00, the smaller draft in LS1.06, compared with LS1.12, results in an increase in freeboard. The increase in the minimum freeboard up to 5.52 meters indicates a smaller probability of green water along the side of the FPSO. The breadth to draft ratio for LS1.06 is 3.05, while for LS1.12 the breadth to draft ratio is 2.85. Therefore, the wave making damping is higher because the similarity of the ship section to a cylindrical cross section is smaller for LS1.06 than for LS1.12.

All in all, the probability of green water along the side of the FPSO for LS1.06 is most likely to be smaller than the probability of green water for the loading condition LS1.12.
3.4 Incoming wave heading

It was assumed that the largest relative wave heading (angle between the incoming waves and head waves) would produce the largest relative wave motions along the side of the FPSO. To verify this assumption, a diffraction analysis has been performed with wave headings of 90° up to 180° with steps of 5°. The analysis has been performed for both sea states assessed before because the analysis of the two sea states showed two different phenomena causing the green water.

3.4.1 Scatter diagram sea state

The probability of green water along the side of the Petrojarl 1 for different relative wave headings is given in Figure 3.18. First of all, the probability of green water definitely has an effect on the relative wave heading. Waves with a heading of 165° result in a maximum probability of 13.4%, while waves coming in at 150° result in a probability of exceedance of 31.8% (Table 3.5). The freeboard varies along the length of the vessel. Where the freeboard is the lowest, the probability along the side is the highest. It depends on the incoming wave heading, what the location along the side with the highest probability is. Except for the wave headings of 95°, 175° and 180°, the maximum probability of green water is located between 14% and 32% behind midships. For the forward half of the vessel the probability becomes higher when the waves turn from head waves to beam waves. Aft of midships that is not the case. For example, just aft of midships, the probability of exceedance due to waves with heading 140° is larger than the probability due to waves with heading 105°. In the sea state assessed here (H_s = 7 meters, T_p = 5 seconds), the green water just aft of the middle of the vessel is almost entirely governed by the free surface elevation. This was concluded earlier for the wave heading of 150° and from Figure 3.19 can be seen that it is the same case for the other wave headings. The response spectral area of the free surface elevation is significantly larger than the response spectral area of the different vessel motions. The peak of the spectral area of the
Figure 3.19: Spectral area of the response split up in the different motions resulting in the relative wave motion and the relative wave motion itself.
free surface elevation is 8.2 m²/s.
The contribution of the heave motion to the relative wave motion is small. The maximum spectral area of the heave motion is 0.5 for a wave heading of 90°. The peak of the spectral area of the heave due to pitch is 1.3 m²/s for a wave heading of 110°. This peak is at the bow. The peak at the stern is a little lower because the distance from the center of gravity is smaller.
The roll motions hardly affect the relative wave motion in this sea state. The largest spectral density for the heave due to roll is 0.013 m²/s. Therefore, the heave due to roll is negligible in this sea state, for all assessed wave headings.
As concluded earlier, for the wave heading of 150°, the ship pitches a little, which affects the relative wave motion. This conclusion is also valid for all the other wave headings. Still there is some difference in wave headings. If the wave heading is closer to head waves, the vessel motions affect the relative wave motions less. For this sea state the free surface elevation is the most important parameter in determining the relative wave motions and the probability of exceedance, while the pitch motion has a little influence on the relative wave motions.

### 3.4.2 Extreme wave sea state

The probability of exceedance for the extreme wave sea state and different wave headings is given in Figure 3.20. An increasing angle between the waves and head waves, causes an increasing probability of green water along the side. The largest probability of exceedance is caused by beam waves, resulting in a probability of 66.5%.
The increase in probability, with increasing angle between the incoming waves and head waves, can be explained by Figure 3.21. The vessel motions increase with increasing relative wave angle. The heave due to roll spectrum has a peak at 20.2 m²/s for a wave heading of 90°. When the incoming wave angle increases, the heave due to roll spectral area decreases. The influence of the heave motion is similar. For a wave heading of 90°, the heave spectral area is 4.7 m²/s. With increasing wave heading, up to 180°, the heave spectral area decreases. For the heave due to pitch motion, the opposite is the case.
For a wave heading of 180° the spectral area is 7.0 m²/s. The spectral area decreases with increasing relative wave angle. For a wave heading of 90°, the spectral area is 0.0.
For small relative wave angles, the heave due to pitch has the largest influence towards the bow and stern of the vessel. Towards the center of the vessel, the heave due to pitch is small and the free surface elevation is 8.2 m²/s.

Table 3.5: The maximum probability of exceedance per wave heading during a 3 hours storm with $H_s = 7$ meters and $T_p = 5$ seconds

<table>
<thead>
<tr>
<th>Heading [°]</th>
<th>$\frac{\sigma}{L_{pp}}$ [-]</th>
<th>Probability [%]</th>
<th>Heading [°]</th>
<th>$\frac{\sigma}{L_{pp}}$ [-]</th>
<th>Probability [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>0.62</td>
<td>41.4</td>
<td>110</td>
<td>0.21</td>
<td>31.6</td>
</tr>
<tr>
<td>90</td>
<td>0.18</td>
<td>40.4</td>
<td>120</td>
<td>0.21</td>
<td>31.5</td>
</tr>
<tr>
<td>100</td>
<td>0.18</td>
<td>40.3</td>
<td>115</td>
<td>0.21</td>
<td>31.0</td>
</tr>
<tr>
<td>140</td>
<td>0.23</td>
<td>35.7</td>
<td>155</td>
<td>0.23</td>
<td>27.5</td>
</tr>
<tr>
<td>105</td>
<td>0.31</td>
<td>35.4</td>
<td>160</td>
<td>0.23</td>
<td>21.4</td>
</tr>
<tr>
<td>135</td>
<td>0.33</td>
<td>35.2</td>
<td>165</td>
<td>0.23</td>
<td>13.4</td>
</tr>
<tr>
<td>145</td>
<td>0.36</td>
<td>34.6</td>
<td>170</td>
<td>0.26</td>
<td>5.2</td>
</tr>
<tr>
<td>130</td>
<td>0.36</td>
<td>32.9</td>
<td>175</td>
<td>0.49</td>
<td>1.2</td>
</tr>
<tr>
<td>125</td>
<td>0.31</td>
<td>32.2</td>
<td>180</td>
<td>0.52</td>
<td>0.2</td>
</tr>
<tr>
<td>150</td>
<td>0.21</td>
<td>31.8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
elevation has a larger influence. For larger relative wave angles, the heave motion and especially the heave due to roll motion have a larger influence. This results in the increase in probability of green water with increasing angle between head waves and the wave heading. It depends on the location along the vessel and on the wave heading, which motion has the largest influence on the relative wave motion.

3.4.3 Varying incoming wave heading

The assumption that a wave heading of 150° produces a higher probability of exceedance than 165° or 180° is valid. For an angle between head waves and incoming waves up to 45°, the assumption that a larger angle results in a larger probability is valid. For angles even higher, an analysis will have to be performed to determine which wave heading results in the highest probability of green water along the side.

The maximum occurring angle between incoming waves and head waves is 30° according to DNV [DNV, 2014b]. This angle results in a maximum probability of green water of 31.8%. An increase of 5° would result in a probability of green water of 34.6%. This is an increase of 2.8%.

An angle between head waves and incoming waves of 25° would result in a decrease of the probability of green water of 4.3%. The angle at which the waves come in at the vessel is very important in determining the probability of exceedance of the freeboard. A slight change in the angle between the incoming waves and head waves can cause large differences in the probability of green water.
Figure 3.21: Spectral area of the of the response split up in the different motions resulting in the relative wave motion and the relative wave motion itself.
To determine the difference between these distributions, the probability of exceedance of the freeboard in a three hours storm with the two different sea states is calculated for both distributions. The probability is shown in Figure 3.22. The probability calculated with the Forristall distribution is 48% higher than the probability calculated with the Rayleigh distribution, at $x_{Lpp} = 0.23$.

For the extreme wave sea state, the distributions are also compared (Figure 3.22b). The probability calculated with the Forristall distribution is 41% higher than the probability calculated with the Rayleigh distribution at the peak of the probability, at $x_{Lpp} = 0.82$.

The Forristall distribution is defined by performing a large amount of second order wave simulations [Forristall, 2000]. The second order waves account for the sharpening of the wave crests. Surface ocean waves are not perfect sinusoidal waves, which is assumed in linear waves, but the wave crests are more peaked and the wave troughs are more flattened. This results in the peak of the wave to be higher than the peak of a sinusoidal wave.

Gibson et al. [2014] compared the Rayleigh and Forristall distribution with measured wave statistics during the December 2012 storm on the North Sea. The measured wave height and crest elevations were found to be in good agreement with the Forristall distribution. The Rayleigh distribution proved to underestimate the crest elevation during the storm. When the wind speed was larger than 25 m/s, the Forristall distribution was also found to underestimate the crest elevation. The measured crest elevation lies approximately 5% - 20% above the Forristall distribution.

The Forristall distribution is also compared with measurements from the North Sea in the period between 2004 and 2015 [Lian and Haver, 2015]. The measurements were proven to be very well
described by the second order Forristall distribution. Only for storms lasting longer than 24 hours, the measured crests were significantly higher than the second order wave crests. The deviation was found to be similar to the deviation found by Gibson et al. [2014].

The peakedness of the waves should also be accounted for in the probability of green water along the side of an FPSO. Due to the peakedness, the probability of green water shall be larger, because the wave crests are higher than the wave crests of a sinusoidal wave. This effect is accounted for by the Forristall distribution.

An effect which is not accounted for by the Forristall distribution is the water flowing onto the deck when the wave exceeds the freeboard. The second order wave is more peaked than the sinusoidal wave, which means the effect of the water flowing onto the deck might be more significant than with a sinusoidal wave. The water flows onto the deck when exceeding the freeboard because the water can flow freely onto the deck. In case of a peaked wave, the water around the peak can flow more easily onto the deck and disperse more easily because less water is located in the peak which has to be dispersed, compared with a sinusoidal wave. This means the probability in second order waves is higher than in linear waves, but the most probable maximum freeboard exceedance might be even lower because the effect of the water flowing onto the deck, on the most probable maximum freeboard exceedance, is larger.

To validate the Forristall distribution of the probability of green water and to determine of it is representable for the most probable maximum relative wave motion, the most probable maximum freeboard exceedance and the green water probability have to be determined with model tests with FPSOs. With the model tests, the effect of the water flowing onto the deck, on the most probable maximum freeboard exceedance can be determined.

### 3.5.2 Mesh

For the diffraction analysis, a 3D model of the FPSO was made and a mesh is defined to be able to perform the 3D panel method diffraction analysis. Aqwa requires the panel size to be maximum $1/7^{th}$ of the shortest wave length. This assures the element size is relatively small with respect to the wave length. The model used for all calculations had a mesh with 16075 diffracting elements and had a maximum element size to $1/7$ wave length ratio of 1.355. To determine the effect of smaller panels with respect to the wave length, a model has been analysed with panels with a maximum element size.
Figure 3.24: The roll damping coefficient and the roll motion RAO in a sea state with $H_s = 8.16$ meters and $T_p = 15.34$ seconds.

The difference in the probability of green water and in the freeboard exceedance for the two meshes is shown in Figure 3.23. The maximum probability along the length of the vessel is 31.8% for the model with a mesh of 16075 elements. The model with a mesh of 18801 elements results in a probability of 31.9%. The most probable maximum freeboard exceedance is 10.551 meters for the mesh of 16075 elements and 10.558 meters for the mesh of 18801 elements. The difference in probability of the mesh of 16075 elements and the mesh of 18801 elements is 0.1%. Therefore, the mesh of 16075 elements is valid for the diffraction analysis of the FPSO.

### 3.5.3 Non-linear roll damping

To determine the non-linear roll damping, the Ikeda method (Section 2.4) has been used. Roll damping does not depend on the short or long term analysis. It depends on the wave frequency, amplitude and heading of the incoming wave. To assess the effect of the non-linear roll damping, the roll damping coefficient and the roll motion RAO are shown in Figure 3.24. The roll motion RAO peak with only wave making damping is 22.6 deg/m. The roll motion RAO peak with non-linear damping is 4.6 deg/m.

The first step in determining the probability of exceedance was calculating the responses without eddy making, friction and bilge keel roll damping. Then the roll damping coefficient has been iterated multiple times, up until the difference in roll damping coefficient was less than 1% between successive iterations. The different roll damping types, the friction, eddy making and bilge keel damping are given in Figure 3.24a. In this figure only the damping coefficients after the final iteration step are given.

The peak in the damping coefficient at a frequency of 0.42 rad/s is the result of roll resonance. The natural roll period is 15.79 seconds (Equation (2.14)), the natural roll frequency is $\frac{2\pi}{T_{roll}} = 0.40$ rad/s. The difference in these frequencies can have multiple causes. First of all, the difference in this frequency is caused by the formula to determine the natural roll period. This formula is an approximation of the roll period. The natural roll period is also determined in Section 3.3.3, which resulted in a natural
roll period of 0.41 rad/s. Both are approximations of the natural roll period.

Secondly, the natural roll period is without the influence of the waves on the vessel, it is the roll period in still water conditions. The coupling effects of the motions also affect this frequency. The roll amplitude is calculated with Aqwa. From this calculation, it appears that 0.42 rad/s is the natural roll frequency because at this frequency the roll RAO amplitude shows a peak.

The damping increases with wave frequency because the roll damping also linearly depends on the wave frequency. Experiments [Ikeda et al., 1978] show that with increasing frequency, the bilge keel and friction damping also increases. This is visible for the bilge keel and friction damping coefficient in Figure 3.24a.

For very small frequencies, the roll damping coefficient tends to zero because the vessel motion with respect to the wave tends to zero, the vessel exactly follows the wave surface. Up to 1.16 rad/s the wave making component increases to nearly $2.4 \cdot 10^8$ kg/s and for even higher frequencies the wave making component decreases again. The eddy making damping tends to zero for high frequencies because the vessel motions for high frequencies also tend to zero. The frictional roll damping increases approximately linear with wave frequency.

3.6 Free surface elevation validation

To validate the free surface elevation RAO, calculated with diffraction theory, the measurements during model tests at MARIN are assessed [Buchner and van den Berg, 2013]. The plate used in the model experiments is modelled in ANSYS Aqwa and a diffraction analysis is carried out to determine the free surface elevation RAO along the side of the plate. The RAO, calculated with an irregular wave with $H_s = 5$ meters and $T_p = 7$ seconds, is compared with the measurements from the model tests. The wave conditions during the measurements are slightly different, the significant wave height was 6.0 meters and the peak period was 8.0 seconds.

The frequency range of the measurements is smaller than the frequency range during the diffraction calculation. To determine if the range during the measurements is large enough, the calculated free surface elevation spectra are related to the frequency range in Figure 3.25. The peak frequency of the relative wave motion spectrum with the highest probability of green water is 0.89 rad/s. The peak of the response spectrum of the extreme wave sea state is at 0.42 rad/s. These peak frequencies are within the range measured during the experiments. At the tail of the spectrum, the range measured does not cover the entire response spectrum of the free surface elevation. But the major part of the spectrum is within the range.

To determine if the measurements were performed in deep water, diffraction analyses with different water depths are performed. A depth of 250 meters and 1550 meters results in exactly the same free surface elevation RAOs. Therefore, the deep water assumption is valid.

Two diffraction analyses are carried out to validate the free surface elevation RAO calculation. One diffraction analysis is performed for a plate with the same dimensions as the plate of the measurements. Another diffraction analysis is performed for a plate with the same length, but with the draft equal to the draft of the FPSO. The free surface elevation RAOs, calculated with these diffraction analyses, are in Figure 3.26, along with the RAOs measured during the experiments.
Comparison of measurements and the plate with a draft of 11.24 meters

The RAOs measured during the tests are divided by the RAOs calculated with diffraction theory, in order to clarify the difference between the measurements and the diffraction calculations (Figure 3.27a). First, the measurements and RAOs calculated for the plate with a draft of 11.24 meters are assessed (Figure 3.27a). For low frequencies, the measurements and calculations are approximately equal because the ratio of the RAOs is approximately 1. For frequencies up to 0.7 rad/s, the ratio of the RAOs tends to 1.3 with peaks up to 1.39 (Table 3.6). For even higher frequencies, at $\frac{1}{3}$ and $\frac{2}{3}$ of the length of the plate (RELM6 and RELM11), the ratio tends towards 0.8. The ratio is lower than 1 for the points aft of the forward edge of the plate, which means the calculated free surface elevation is higher than the measured free surface elevation. During the experiments the waves became steeper while travelling along the plate and started to break. The breaking of the waves reduces the free surface elevation. This is an effect which is not included in the diffraction analysis. Therefore the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Average</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
</table>
| RAO
| measurement | RELM1 | 1.13 | 1.28 | 0.90 |
| RAO
| plate, $T = 11.24$ m | RELM6 | 1.05 | 1.32 | 0.86 |
| RAO
| plate, $T = 16.5$ m | RELM11 | 1.06 | 1.39 | 0.79 |
| RAO
| plate, $T = 11.24$ m | RELM1 | 1.00 | 1.01 | 0.99 |
| RAO
| plate, $T = 11.24$ m | RELM6 | 1.11 | 1.23 | 1.01 |
| RAO
| plate, $T = 11.24$ m | RELM11 | 1.14 | 1.34 | 1.00 |
| RAO
| measurement, $T$ compensated | RELM1 | 1.13 | 1.26 | 0.90 |
| RAO
| plate, $T = 11.24$ m | RELM6 | 0.94 | 1.08 | 0.77 |
| RAO
| plate, $T = 11.24$ m | RELM11 | 0.93 | 1.13 | 0.70 |

Figure 3.25: Free surface elevation spectra for the two assessed sea states
Figure 3.26: Free surface elevation RAOs of the calculations and measurements at three locations along the side of the FPSO

calculations result in a higher free surface elevation than the measurements. The calculated free surface elevation is higher because the waves start to break while travelling aft due to the steepness of the waves. The wave steepness for the three measured locations is given in Table 3.7. The steepness is calculated for the maximum wave height and for the significant wave height (Equation (3.3), [Laing et al., 1998]). The wave heights are including the free surface elevation due to the presence of the plate. The maximum wave height during a 3 hours storm, based on a Rayleigh distribution, is given in Equation (3.4).

\[
\xi = \frac{2\pi H_{\text{max}}}{gT_p^2} \quad (3.3)
\]

\[
P(H > H_{\text{max}}) = \frac{1}{1000} \quad \Rightarrow \quad e^{-2(\frac{H_{\text{max}}}{H_s})^2} = \frac{1}{1000} \quad \Rightarrow \quad H_{\text{max}} = 1.86 \cdot H_s \quad (3.4)
\]
The theoretical maximum wave steepness before waves start to break is 0.142 [Holthuijsen, 2007]. At RELM1 the wave steepness of the incoming wave is 0.130 at its maximum. This means the waves do not brake at the forward end of the plate, which is in line with the visual observations during the measurements.

The measurement at RELM1 of the free surface elevation RAO is higher than the calculations because the linear diffraction theory assumes perfectly sinusoidal waves, while in reality waves are a little steeper around the wave peak and more shallow in a wave through. This results in a higher free surface elevation.

Further aft, at RELM6 and RELM11, the steepness of the waves calculated with diffraction theory becomes higher than 0.142 for the plate with a draft of 16.5 meters. The steepness of the measured waves is lower than the steepness of the calculated waves, but still higher than 0.142. This is the case for both RELM6 and RELM11. Because the steepness of the waves is higher than 0.142 for the maximum occurring wave, the waves start to break, which is in line with the visual observations.
Table 3.7: Steepness of the measured waves

<table>
<thead>
<tr>
<th>Model</th>
<th>Point</th>
<th>Steepness ($H_s$)</th>
<th>Steepness ($H_{\text{max}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate measurements</td>
<td>RELM1</td>
<td>0.070</td>
<td>0.130</td>
</tr>
<tr>
<td>Plate measurements</td>
<td>RELM6</td>
<td>0.085</td>
<td>0.159</td>
</tr>
<tr>
<td>Plate measurements</td>
<td>RELM11</td>
<td>0.093</td>
<td>0.174</td>
</tr>
<tr>
<td>Plate ($T = 16.5$ m)</td>
<td>RELM1</td>
<td>0.064</td>
<td>0.118</td>
</tr>
<tr>
<td>Plate ($T = 16.5$ m)</td>
<td>RELM6</td>
<td>0.099</td>
<td>0.183</td>
</tr>
<tr>
<td>Plate ($T = 16.5$ m)</td>
<td>RELM11</td>
<td>0.116</td>
<td>0.216</td>
</tr>
<tr>
<td>Plate ($T = 11.24$ m)</td>
<td>RELM1</td>
<td>0.063</td>
<td>0.117</td>
</tr>
<tr>
<td>Plate ($T = 11.24$ m)</td>
<td>RELM6</td>
<td>0.082</td>
<td>0.151</td>
</tr>
<tr>
<td>Plate ($T = 11.24$ m)</td>
<td>RELM11</td>
<td>0.088</td>
<td>0.164</td>
</tr>
</tbody>
</table>

during the measurements.
For comparison, the wave steepness is also calculated for the significant wave height (Table 3.7). This calculation results in the fact that for none of the measurements or calculations, the wave steepness is larger than 0.142. The significant wave height is the average height of the highest $1/3$ of all the waves. On average, these are not high enough to break, therefore only the highest waves start breaking.

Comparison of a plate with a draft of 16.5 meters and a plate with a draft of 11.24 meters

Second, the ratio between the plates with different drafts is assessed because the FPSO has a different draft compared to the plate tested during the model tests. The effect of the additional draft is compensated in order to determine the difference between the calculations and the measurements for a plate with a draft of 11.24 meters.
For low frequencies, the ratio between the two plates is tending to 1. For increasing frequency, the ratio increases for RELM6 and RELM11 and decreases again to one for even higher frequencies. The ratio is decreasing to 1 again because the waves do not affect the water pressure at the depth between 11.24 and 16.5 meters for short waves. The water particles move in circles in regular waves. The radius of this circle is a measure for the energy of the wave. At the water surface, the radius of the circle is equal to the wave amplitude. The energy at the water surface can be calculated with Equation (3.5) [Journée and Massie, 2001].

$$E = \frac{1}{2} \rho g \zeta^2$$

(3.5)

The upper limit of the frequency range measured during the experiments is 1.09 rad/s. The radius of the circle of the particle motion at a depth of 11.24 meters is $0.20 \zeta_a$ (Equation (3.6) [Journée and Massie, 2001].

$$r = \zeta_a \cdot e^{kz}$$

(3.6)

Because the energy is dependent on the square of the wave amplitude, the wave energy at a depth of 11.24 meters for a wave with frequency 1.09 rad/s is decreased with 93%. Nearly all the energy of the wave is located above a depth of 11.24 meters at a frequency of 1.09 rad/s. For higher frequencies the energy is located even more shallow and the ratio of the free surface elevation RAO of the two plates becomes 1.
The non-linear effects on the free surface elevation

The minimum, maximum and average ratios of the free surface elevation RAOs are given in Table 3.6. The ratio \( \frac{\text{RAO}_{\text{measurement}}}{\text{RAO}_{\text{plate, } T = 11.24 \text{ m}}} \) divided by \( \frac{\text{RAO}_{\text{plate, } T = 16.5 \text{ m}}}{\text{RAO}_{\text{plate, } T = 11.24 \text{ m}}} \), which results in a single ratio which gives the difference caused by non-linearities between the calculations of a plate with a draft of 11.24 meters and the measurements. These are the non-linearities which are also affecting the free surface elevation along the side of the FPSO. The measurements are at the most 26% higher and 30% lower than the calculations. The average is actually maximum 13% higher, which shows the diffraction calculations gives a good estimate of the free surface elevation RAO along the side of the FPSO.

During the model tests, the waves started to break when travelling aft along the side of the plate. This effect is not accounted for by the linear diffraction theory. This results in a lower free surface elevation than predicted by linear theory.

The waves were also found to be steepening when travelling aft along the side of the plate. This effect is caused by the higher order harmonics in the waves [Buchner and van den Berg, 2013], which is also not accounted for by linear diffraction theory. The steeper wave crests result in a higher free surface elevation than calculated by linear diffraction theory.

The waves tested during the model tests performed by Buchner and van den Berg [2013], were relatively steep waves. Therefore the effect of the breaking of the waves has a larger impact on the free surface elevation than the effect of the steepening of the waves, resulting in an on average smaller free surface elevation than predicted with linear diffraction theory. Longer, less steep waves, which do not break when travelling aft along the side of an FPSO will result in a higher the free surface elevation than predicted with linear diffraction theory because the steepening of the waves will increase the free surface elevation and the waves do not break. Therefore, to get a more accurate answer for the free surface elevation along the side of the FPSO, the steepening and breaking of the waves also has to be taken into account. These effects can be taken into account by performing CFD calculations for the vessel in waves. The CFD calculations account for the steepening of the waves and the breaking of the waves and the effect of the water flowing onto the deck when exceeding the deck edge can also be taken into account.
4 Conclusion

All sea states in the scatter diagram and the extreme sea state diagram are assessed for the probability of green water. Two different phenomena were found to be causing a high probability of green water along the side of the FPSO. The sea state from the scatter diagram with the highest probability of exceedance (31.8%) has a significant wave height of 5 meters and a peak period of 7 seconds. The relative wave motion, causing the green water, is in this case for 97% the result of the free surface elevation for the response peak frequency. The radiating and diffracting wave result in a free surface elevation RAO amplitude of 2.14 m/m, while the relative wave motion RAO amplitude is 2.21 m/m.

A second peak in the probability (8.4%) is located at \( \frac{x}{L_{pp}} = 0.82 \). The relative wave motion RAO amplitude for the peak frequency is for 77.3% caused by the free surface elevation, for 2.3% by heave, for 19.3% by heave due to pitch and for 1.7% by heave due to roll. During this sea state, the vessel pitches a little while the wave runs up along the side of the hull.

The extreme wave sea state with the highest probability of exceedance (4.1%), has a significant wave height of 8.16 meters and a peak period of 15.34 meters (50 year return period). The reason for the probability of exceedance in this case is the result of a combination of the heave (54%), heave due to pitch (8%), heave due to roll (107%) and the free surface elevation (-69%) for the peak response frequency at \( \frac{x}{L_{pp}} = 0.36 \). At \( \frac{x}{L_{pp}} = 0.82 \), the relative wave motion RAO amplitude is a combination of the heave (47%), heave due to pitch (7%), heave due to roll (76%) and the free surface elevation (-30%). The heave due to roll has the largest influence on the relative wave motion because the peak period of the wave spectrum is nearly equal to the natural roll frequency of the FPSO. Large roll motions combined with large heave motions cause the large relative wave motion, on which the probability of green water is based, while the free surface elevation counteracts the vessel motions in terms of relative wave motion.

Both sea states result in a high probability of exceedance behind midships. The sea state from the scatter diagram results in the highest probability at \( \frac{x}{L_{pp}} = 0.23 \) and another peak at \( \frac{x}{L_{pp}} = 0.82 \). The 50 year return period sea state results in the highest probability at \( \frac{x}{L_{pp}} = 0.36 \) and at 0.82. Between 15% and 30% aft of midships, the probability of exceedance is the highest. The part forward of midships, between 15% and 30% forward, shows for both sea states a decrease in the probability of exceedance of the freeboard. In both sea states the probability increases further forward, up until the increase of the freeboard due to the forecastle.

The most probable maximum freeboard exceedance of 5.8 meters is located at \( \frac{x}{L_{pp}} = 0.23 \) for the scatter diagram sea state. At \( \frac{x}{L_{pp}} = 0.82 \), the most probable maximum freeboard exceedance is 2.3 meters. For the extreme wave sea state, the most probable maximum freeboard exceedance is 1.7 meters at \( \frac{x}{L_{pp}} = 0.36 \) and 1.8 meters at \( \frac{x}{L_{pp}} = 0.82 \). The locations along the side of the FPSO which are most vulnerable to green water damage are just behind the forecastle and 15% to 30% behind midships.

The equipment on the main deck, where the most probable maximum freeboard exceedance is 5.8 meters, is protected with green water barriers and these are necessary for protection. The process deck, which is 2.0 meters above the main deck and at which all equipment is unprotected against green water, has a most probable maximum freeboard exceedance of 3.8 meters. The equipment on this deck might be damaged by green water during the time the FPSO is operating on this oil field.

The most important phenomenon causing green water is the free surface elevation. The probability of green water in the scatter diagram sea state (31.8%), is almost entirely caused by the free surface elevation. Therefore, the free surface elevation RAO is validated with model tests in which the free
surface elevation along a plate is measured. The measurements of the free surface elevation are at the most 26% higher along the plate than for the calculations of the same plate with diffraction theory. On average the calculations with diffraction theory are 13% higher at the forward end of the plate. At $\frac{1}{4}$ and $\frac{2}{3}$ of the length of the plate, the calculated free surface elevation RAO is on average respectively 6% and 7% higher than the measured free surface elevation RAO. The reason for the calculation to be higher is the fact that the waves start to break because they become too steep. All in all, the diffraction calculation is a good estimate of the probability of green water along the side of an FPSO.

The angle of the incoming waves relative to the FPSO has a large effect on the probability of green water. For both assessed sea states, for waves up to 45° relative to head waves, the probability of green water along the side increases with increasing relative wave heading. The maximum relative wave heading occurring during the time the FPSO is exposed to the waves is very important. Differing the maximum relative wave heading with 5° from the assumed 30° can cause differences of up to 4.3% in probability of green water.

**Recommendations** First of all, the mooring lines and risers could be added to the diffraction analysis. In the current diffraction analysis, the vessel is free floating and not affected by the mooring lines and risers. When these are added in the analysis, the RAOs will be influenced by them. It will result in a more realistic and more accurate calculation of the RAOs and in turn of the probability of green water.

The free surface elevation is calculated with linear diffraction theory and validated with wave run-up measurements along a plate. This could be improved by applying a more accurate method for the calculation of the free surface elevation. Computational fluid dynamics could be applied to calculate the free surface elevation. An alternative would be performing model tests with a model from the actual FPSO along with the mooring lines. This would give a more accurate value of the free surface elevation. An example of a set up for a model test is given in Appendix E. Both the CFD calculations and the model tests also account for the fact that the water flows onto the deck when it exceeds the deck edge, resulting in a lower most probable maximum freeboard exceedance.

With the model tests, the Forristall distribution could also be validated for the probability of green water along the side of the FPSO. The Forristall distribution results in more accurate free surface elevation along the side of the plate. But along the side of a plate, the non-linear effects like water flowing onto the deck, are not taken into account. To determine the accuracy of the Forristall distribution for the probability of green water and the most probable maximum freeboard exceedance along the side of an FPSO, model tests should be performed.

In the analysis, it is assumed that the maximum angle between incoming waves and head waves is 30°. When all meteorological data of the area where the FPSO will be positioned is available, a long term probability analysis of the maximum angle could be performed to determine the maximum relative wave heading. It was shown that this angle has a large influence on the probability of green water and the maximum relative wave motion along the side of the FPSO. This angle could decrease or increase the probability of green water substantially, potentially decreasing or increasing the sea loads due to green water.

To improve the long term probability of green water along the side of the FPSO, the operational profile of the FPSO should also be taken into account. In the current analysis, it is assumed that the FPSO is always floating at maximum draft. In reality this is not the case. The draft will change periodically with the filling and emptying of the vessel with oil. Other processes, like pumping in or pumping out of ballast water or taking on supplies from a supply vessel, also affect the draft of the vessel. Most of the time, the draft of the FPSO will be smaller, which results in a lower probability of green water along the side of the FPSO.

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Bibliography


DNV (2010). Environmental conditions and environmental loads. DNV-RP-C205.


A DNV green water regulations

The green water sea loads for an FPSO are determined with requirements especially for FPSOs (Figure A.1). These requirements are given in Figure A.2. The design condition of interest is the survival condition, the transit condition is not taken into account in this study. The requirements depend on the environmental conditions of the area where the FPSO will be located. A difference is made between harsh environmental areas and benign waters. Figure A.3 shows the criteria for benign waters and harsh environmental areas. The 100 year return period significant wave height is 8.77 meters (Table 2.3). When the ship is longer than $\frac{8.77 - 8}{10} \cdot 100 + 100 = 138.5$ meters, the environment can be considered benign waters. The length between perpendiculars is 194.20 meters, therefore the environment may be considered benign. The environmental conditions are benign waters, therefore the rules in Figure A.4 may be applied for the green water loads calculation.

3.4 Green sea

3.4.1 The green sea is the overtopping by sea in severe wave conditions. The forward part of the deck and areas aft of midship will be particularly exposed to green sea. Short wave periods are normally the most critical.

3.4.2 Appropriate measures should be considered to avoid or minimise the green sea effects on the hull structure, accommodation, deckhouses, topside modules and equipment. These measures include bow shape design, bow flare, bulwarks and other protective structure. Adequate drainage arrangements shall be provided.

3.4.3 Structural members exposed to green sea shall be designed to withstand the induced loads. Green sea loads are considered as local loads.

3.4.4 When lacking more exact information, e.g. from model testing, green sea loads specified in unit specific provisions Sec.10 and Sec.11 shall be used.

3.4.5 Shadow effects from either green water protection panel or other structure may be accounted for.

Figure A.1: Green water sea load requirements [DNV, 2014b]
SECTION 11 SPECIAL PROVISIONS FOR FLOATING PRODUCTION, STORAGE AND OFFLOADING UNITS

1 Introduction
This section contains specific requirements and guidance applicable for floating production, storage and offloading unit which are intended to operate at site specific location.

2 Design principles

2.1 General

2.1.1 If the unit is defined for “benign waters operation”, see Sec.2 [2.6], the requirements to the midship section modulus are by definition more stringent than the design principles based on the direct calculations applied to “benign waters”. In this case, hull structures complying with the minimum midship section modulus and moment of inertia given in the Rules for Ships Pt.3 Ch.1 Sec.5 do not require additional calculations of the hull girder strength.

2.1.2 The design principles for transit and operating conditions are given in Table 11-1.

<table>
<thead>
<tr>
<th>Table 11-1 Design principles for floating production and storage units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design condition</strong></td>
</tr>
<tr>
<td>Transit</td>
</tr>
<tr>
<td>Worldwide transit</td>
</tr>
<tr>
<td>Harsh environmental areas</td>
</tr>
<tr>
<td>Survival condition</td>
</tr>
</tbody>
</table>

**Guidance note:**
Operation condition needs normally not to be considered for FPSO's as the weight distribution of the topside structure in the operation and in the survival condition is assumed to be similar.

Figure A.2: Wave load design requirements for FPSOs [DNV, 2014b]
2.6 Benign waters or harsh environmental areas

2.6.1 If the unit is restricted to operate in benign waters, the strength requirements given in [2.4] for the survival condition are not required as the transit condition given in [2.2] will be governing. Operation conditions given in [2.3] may still be relevant design conditions and shall be considered.

2.6.2 The Benign waters criteria are defined in Table 2-1.

a) If $H_{S_{100\text{ year}}} < 8.0 \text{ m} \text{ or } 10.0 \text{ m}$ depending on the ship length is specified and documented for the actual site specific location, no wave load analysis is required to demonstrate that the actual area is benign waters.

b) If $H_{S_{100\text{ year}}} > 8.0 \text{ m} \text{ or } 10.0 \text{ m}$ depending on the ship length or if no sufficient information of actual $H_{S_{100\text{ year}}}$ for the actual location is available, the $M_{W_{\text{Rule-20 year}}} > M_{W_{\text{Site-100 year}}}$ need to be determined by using a wave load analysis for the survival condition as described in Table 2-2 in order to demonstrate that $M_{W_{\text{Rule-20 year}}} > M_{W_{\text{Site-100 year}}}$, i.e. Benign waters.

If neither the a) or the b) requirements given above are satisfied, the unit is defined to operate in harsh environmental area.

<table>
<thead>
<tr>
<th>Table 2-1 Benign waters criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule ship length</td>
</tr>
<tr>
<td>100 &lt; L &lt; 200 m</td>
</tr>
<tr>
<td>$H_{S_{100\text{ year}}} \leq 8.0\text{ m} \text{ or when } M_{W_{\text{Rule-20 year}}} &gt; M_{W_{\text{Site-100 year}}}$</td>
</tr>
</tbody>
</table>

Between $L=100\text{ m}$ and $L=200\text{ m}$, linear interpolation may be used.

$H_{S_{100\text{ year}}}$: Significant wave height at site specific with a 100 year return period

$M_{W_{\text{Rule-20 year}}}$: Rule wave bending moment based on a 20 year return period in the North Atlantic

$M_{W_{\text{Site-100 year}}}$: 100 years linear wave bending moment at the specified location

Figure A.3: Requirements for benign or harsh environmental areas [DNV, 2014b]
Sea pressures

The pressure acting on the ship’s side, bottom and weather deck shall be taken as the sum of the static and the dynamic pressure as:

— for load point below summer load waterline:
  \[ p_1 = 10 h_0 + p_{dp} \text{ (kN/m}^2\text{)} \]

— for load point above summer load waterline:

\[ p_2 = a (p_{dp} - (4 + 0.2 k_2) h_0)^1 \text{ (kN/m}^2\text{)} \]

- minimum 6.25 + 0.025 L \(_1\) for sides
- minimum 5 for weather decks.

The pressure \( p_{dp} \) is taken as:

\[ p_{dp} = p_I + 135 \frac{y}{B + 75} - 1.2 (T - z) \text{ (kN/m}^2\text{)} \]

\[ p_I = k_s C_W + k_f \]
\[ = (k_s C_W + k_f) \left( 0.8 + 0.15 \frac{V}{\sqrt{L}} \right) \text{ if } \frac{V}{\sqrt{L}} > 1.5 \]

\[ k_s = \begin{cases} 
3 C_B + \frac{2.5}{\sqrt{C_B}} & \text{at AP and aft} \\
2 \text{ between } 0.2 \text{ L and } 0.7 \text{ L from AP} \\
3 C_B + \frac{4.0}{C_B} & \text{at FP and forward.}
\end{cases} \]

Between specified areas \( k_s \) shall be varied linearly.

\( a = 1.0 \) for ship’s sides and for weather decks forward of 0.15 L from FP, or forward of deckhouse front, whichever is the foremost position

- \( 0.8 \) for weather decks elsewhere

\( h_0 = \) vertical distance from the waterline at draught T to the load point (m)

\( z = \) vertical distance from the baseline to the load point, maximum T (m)

\( y = \) horizontal distance from the centre line to the load point, minimum B/4 (m)

\( C_W = \) as given in B200

\( k_f = \) the smallest of T and f

\( f = \) vertical distance from the waterline to the top of the ship’s side at transverse section considered, maximum 0.8 \( C_W \) (m)

\( L_1 = \) ship length, need not be taken greater than 300 (m).

1) For ships with service restrictions, \( p_2 \) and the last term in \( p_1 \) may be reduced by the percentages given in B202. \( C_W \) should not be reduced.

Figure A.4: Static and dynamic sea loads [DNV, 2014a]
B Mathematical model

The diffraction analysis is carried out in the frequency domain. This means it is a linear potential analysis in which the principle of superposition is applicable. It is based on the three-dimensional radiation and diffraction theory in regular waves. The theory in this chapter is based on Journée and Massie [2001] and ANSYS [2015].

A single wave is considered to be harmonic and can be represented by Equation (B.1).

\[ \zeta = \zeta_0 \cos(-\omega t + k(x \cos(\mu) + y \sin(\mu))) \]  

(B.1)

The potential of the wave is given by Equation (B.2).

\[ \Phi(\vec{X}, t) = \zeta_0 \phi(\vec{X}) e^{-i\omega t} \]  

(B.2)

The space dependent potential \( \phi \) can be separated into different components. These components are the incoming wave potential (\( \phi_w \)), the diffracted wave potential (\( \phi_d \)) and the radiated wave potentials due to the vessel motions (\( \phi_{r1-r6} \)). The motions 1 to 6 are defined as follows (Figure 2.3b):

1. Surge
2. Sway
3. Heave
4. Roll
5. Pitch
6. Yaw

This results in the velocity potential as given in Equation (B.3).

\[ \Phi(\vec{X}, t) = \zeta_0 \phi(\vec{X}) e^{-i\omega t} = \zeta_0 \left( \phi_w + \phi_d + \sum_{j=1}^{6} \phi_{rj} x_j \right) e^{-i\omega t} \]  

(B.3)

with:

\[ \phi_w(\vec{X}) e^{-i\omega t} = -ig\zeta_0 \frac{\cosh(k(z + d))}{\omega \cosh(kd)} e^{i(-\omega t + k(x \cos(\mu) + y \sin(\mu)) + \epsilon)} \]  

(B.4)

With the wave velocity potentials, the first order hydrodynamic pressure distribution can be obtained by means of the linearised Bernoulli’s equation (Equation (B.5)).

\[ p^{(1)} = -\frac{\partial \Phi(\vec{X}, t)}{\partial t} = i\omega \rho \phi(\vec{X}) e^{-i\omega t} \]  

(B.5)

The fluid forces on the vessel can be calculated by integrating the pressure distribution over the wetted surface of the vessel (Equation (B.6)).

\[ F_j e^{-i\omega t} = -\int_{S_0} p^{(1)} n_j dS = \left( -i\omega \rho \int_{S_0} \phi(\vec{X}) n_j dS \right) e^{-i\omega t} \]  

(B.6)

With:

\[
\begin{bmatrix}
  n_1 \\
  n_2 \\
  n_3 \\
  n_4 \\
  n_5 \\
  n_6 \\
\end{bmatrix} = \vec{n} \\
\begin{bmatrix}
  \vec{n}_1 \\
  \vec{n}_2 \\
  \vec{n}_3 \\
\end{bmatrix} = \vec{n} \times \vec{r} \\
\begin{bmatrix}
  \vec{r} \\
\end{bmatrix} = \vec{X} - \vec{X}_g
\]
Combining Equation (B.3) with Equation (B.6) results in Equation (B.7).

\[ F_j = \left( F_{wj} + F_{dj} + \sum_{k=1}^{6} F_{rjk} x_k \right) \quad \text{for} \quad j = 1 - 6 \]  

(B.7)

With:

\[ F_{wj} = -i\omega \rho \int_{S_0} \phi_w(\vec{X}) n_j dS \quad \text{Froude-Krylov force due to incoming wave} \]

\[ F_{dj} = -i\omega \rho \int_{S_0} \phi_d(\vec{X}) n_j dS \quad \text{diffraction force due to diffracting wave} \]

\[ F_{rjk} = -i\omega \rho \int_{S_0} \phi_{rk}(\vec{X}) n_j dS \quad \text{radiation force due to radiation wave induced by} \ k^{th} \ \text{body motion} \]

The radiation wave potential can also be described complex and substituted in the radiation force formula, Equation (B.8). By this means, the added mass and wave damping coefficients can be calculated (Equation (B.9)).

\[ F_{rjk} = -i\omega \rho \int_{S_0} \left( Re(\phi_{rk}(\vec{X})) + i Im(\phi_{rk}(\vec{X})) \right) n_j dS \]

\[ = \omega \rho \int_{S_0} Im(\phi_{rk}(\vec{X})) n_j dS - i\omega \rho \int_{S_0} Re(\phi_{rk}(\vec{X})) n_j dS \]  

(B.8)

\[ = \omega^2 A_{jk} + i\omega B_{jk} \]

Where the added mass and damping coefficients are:

\[ A_{jk} = \frac{\rho}{\omega} \int_{S_0} Im(\phi_{rk}(\vec{X})) n_j dS \]  

(B.9)

\[ B_{jk} = -\rho \int_{S_0} Re(\phi_{rk}(\vec{X})) n_j dS \]

With these coefficients the equations of motion of the vessel are given by Equation (B.10).

\[ [-\omega^2 (M_s + M_a) - i\omega C + K_hys][x_j] = [F_j] \]  

(B.10)

The encounter frequency (\(\omega_e\)) is in this case the same as the wave frequency because the vessel itself has a forward speed of zero.

The matrix \(M_a\) is the added mass matrix with the added mass coefficients. The matrix \(C\) is the matrix with the damping coefficients.

To determine the RAOs of the different motions of the vessel in the center of gravity, the six equations of motion are first divided by the wave amplitude, \(\zeta_w\). The six equations of motion then contain an imaginary part and a real part. The parts can be separated which results into two times six equations. The unknowns in these 12 equations are the six vessel motion RAO amplitudes and the six vessel motion RAO phase angles. Solving these 12 equations for these 12 unknowns, results in the RAOs of the vessel motions.

The free surface elevation RAO is also calculated in ANSYS Aqwa. The free surface elevation is derived from the pressure in the fluid domain. At predefined locations along the side of the FPSO hull, points are defined at which the pressure head amplitude is calculated. If the points along the hull are located at the still water surface, this pressure head amplitude is equal to the free surface elevation.
B.1 Potentials

The velocity potential for the incoming wave is given by Equation (B.4). The velocity potentials for the radiated and diffracted waves still have to be determined. These velocity potentials (Equation (B.3)) are solved with the source distribution method. The velocity potentials have to satisfy a number of boundary conditions, which are used to solve the potentials. The boundary conditions are:

1. The continuity condition and Laplace equation. The Laplace equation is based on the continuity equation, which in turn is a condition for the preservation of mass.

\[
\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \quad (B.11)
\]

2. Linear free surface equation. The pressure at the surface of the fluid is equal to the atmospheric pressure.

\[-\omega^2 \phi + g \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = 0 \quad (B.12)\]

3. Body surface condition. This condition states that the velocity of a water particle at the surface of the body is equal to the velocity of the body point itself. This ensures the surface of the body is watertight.

\[
\frac{\partial \phi}{\partial n} = \begin{cases} 
- i \omega n_j & \text{for radiation potential} \\
- \frac{\partial \phi}{\partial n} & \text{for diffraction potential} 
\end{cases} \quad (B.13)
\]

4. Sea bed boundary condition. This equation ensures that the water velocity through the sea bed is zero. No water will flow through the seabed.

\[
\frac{\partial \Phi}{\partial z} = 0 \quad \text{with:} \quad z = -h_0 \quad (B.14)
\]

5. A suitable radiation condition which ensures that as \( \sqrt{x^2 + y^2} \to \infty \) the generalized wave disturbance dies out.

To solve the velocity potentials governed by the boundary conditions, the source distribution method is used. This method is based on the fact that the potentials \((\phi_r, \phi_d)\) at a certain point on the mean wetted body surface can be given by a continuous distribution of single sources on the body surface [Journée and Massie, 2001], Equation (B.15) [ANSYS, 2015].

\[
\phi(\overrightarrow{X}) = \frac{1}{4\pi} \int_{S_0} \sigma(\overrightarrow{\xi})G(\overrightarrow{X}, \overrightarrow{\xi}, \omega) dS \quad \text{where} \quad \overrightarrow{X} \in \Omega \cup S_0 \quad (B.15)
\]

In Equation (B.15), \(G(\overrightarrow{X}, \overrightarrow{\xi}, \omega)\) is the pulsating Green’s function in finite depth (Equation (B.16)). This Green’s function is bound by the same boundary conditions, except the condition in the fluid field is given by Equation (B.17).

\[
\Delta G(\overrightarrow{X}, \overrightarrow{\xi}, \omega) = \frac{1}{r_1} + \frac{1}{r_2} + \int_0^{\infty} \frac{2(k + \nu)e^{-kd}\cosh(k(Z + d))\cosh(k(\zeta + d))}{k \sinh(k_0d) + k_0d \cosh(k_0d) - \nu d \sinh(k_0d)} J_0(kR) dk + i2\pi \frac{(k_0 + \nu)e^{-k_0d}\cosh(k_0(Z + d))\cosh(k_0(\zeta + d))}{\sinh(k_0d) + k_0d \cosh(k_0d) - \nu d \sinh(k_0d)} J_0(k_0R) \quad (B.16)
\]
\[ \delta(\vec{X} - \vec{\xi}) = \begin{cases} 0 & \text{where } \vec{X} - \vec{\xi} \neq 0 \\ \infty & \text{where } \vec{X} - \vec{\xi} = 0 \end{cases} \]  

\[ (B.17) \]

\( \sigma(\vec{\xi}) \) are the unknown source strengths. These can be determined by the hull surface boundary condition, Equation (B.18), combined with Equation (B.15).

\[
\frac{\partial \phi(\vec{X})}{\partial n(\vec{X})} = -\frac{1}{2} \sigma(\vec{X}) + \frac{1}{4\pi} \int_{S_0} \sigma(\vec{\xi}) \frac{\partial G(\vec{X}, \vec{\xi}, \omega)}{\partial n(\vec{X})} dS \quad \text{where } \vec{X} \in S_0
\]

\[ (B.18) \]

Equation (B.18) is solved with the Hess-Smith constant panel method. The mean wetted surface is divided into triangular and quadrilateral panels. The potential and source strength are assumed to be constant over each panel. This results in Equation (B.19).

\[
\phi(\vec{X}) = \frac{1}{4\pi} \sum_{m=1}^{m=N_p} \sigma_m G(\vec{X}, \vec{\xi}_m, \omega) \Delta S_m \quad \text{where } \vec{X} \in \Omega \cup S_0
\]

\[
-\frac{1}{2} \sigma_k + \frac{1}{4\pi} \sum_{N_p}^{m=1} \sigma_m \frac{\partial G(\vec{X}_k, \vec{\xi}_m, \omega)}{\partial n(\vec{X}_k)} \Delta S_m = \frac{\partial \phi(\vec{X}_k)}{\partial n(\vec{X}_k)} \quad \text{where } \vec{X} \in S_0, k = 1, N_p
\]

\[ (B.19) \]

With:

- \( N_p \) = Total number of panels over mean wetted surface
- \( \Delta S_m \) = Area of the \( m^{th} \) panel
- \( \vec{\xi}_m \) = Coordinates of the \( m^{th} \) panel geometric center
- \( \vec{X}_k \) = Coordinates of the \( k^{th} \) panel geometric center

When the velocity potentials of the radiated and diffracted waves are known, the diffraction analysis is solved. With the vessel motions RAOs and the free surface elevation RAOs from the diffraction analysis, the probability of green water can be determined.
C Ikeda’s method mathematical model

The different types of roll damping are calculated separately and added to form a total roll damping coefficient.

Frictional roll damping

\[ B_{44F} = \frac{8}{3\pi} \phi_a \omega \frac{1}{2} \rho r_f^3 S_f C_f \]  

(C.1)

With:

\[ r_f = \frac{0.887 + 0.145 \cdot C_B}{\pi} \frac{S_f}{T_f} + 2OG \]

\[ C_f = 1.328 \cdot Rn^{-0.5} + 0.014 \cdot Rn^{-0.114} \]

\[ Rn = \frac{0.512 \cdot (r_f \phi_a)^2 \omega}{\nu} \]

\[ \nu = \frac{1.063 + 0.1039 \cdot (\rho - 1025) + 0.02602 \cdot (\rho - 1025)^2}{1 \cdot 10^6} \]

Eddy making roll damping

\[ B_{44E} = \frac{8}{3\pi} \phi_a \omega B_{44E_0}^{(2)} \]  

(C.2)

With:

\[ B_{44E_0}^{(2)} = \int_{b}^{L} B_{44E_0}^{(2)'} dx_b \]

\[ B_{44E_0}^{(2)'} = \frac{1}{2} \rho D_s^4 \left( \frac{r_{max}}{D_s} \right)^2 C_p \cdot \left( 1 - \frac{f_1 r_b}{D_s} \right) \left( 1 + \frac{OG}{D_s} - \frac{f_1 r_b}{D_s} \right) + f_2 \left( H_0 - \frac{f_1 r_b}{D_s} \right)^2 \]

\[ r_{max}(\psi) = M_s \sqrt{(1 + a_1 \sin(\psi) - a_3 \sin(3\psi))^2 + ((1 - a_1) \cos(\psi) + a_3 \cos(3\psi))^2} \]

\[ M_s = \frac{B_s}{2(1 + a_1 + a_3)} \]

\[ a_1 = \frac{H_0 - 1}{H_0 + 1} (a_3 + 1) \]

\[ H_0 = \frac{B_s}{D_s} \]

\[ a_3 = -c_1 + 3 + \sqrt{9 - 2c_1} \]

\[ c_1 = 3 + \frac{4\sigma_s}{\pi} + \left( 1 - \frac{4\sigma_s}{\pi} \right) \cdot \left( \frac{H_0 - 1}{H_0 + 1} \right)^2 \]

\[ \sigma_s = \frac{A_s}{B_s D_s} \]

\[ \psi = \begin{cases} \psi_1 = 0.0 \\ \psi_2 = \frac{0.5}{\cos\left(\frac{a_1 + a_3}{4a_3}\right)} \end{cases} \]
\[ r_{\max} = r_{\max}(\psi_1), \quad \psi = \psi_1 \quad \text{for} \quad r_{\max}(\psi_1) > r_{\max}(\psi_2) \]
\[ r_{\max} = r_{\max}(\psi_2), \quad \psi = \psi_2 \quad \text{for} \quad r_{\max}(\psi_1) < r_{\max}(\psi_2) \]
\[ C_p = 0.35e^{-\gamma} - 2.0e^{-0.187\gamma} + 1.5 \]
\[ \gamma = \frac{f_3 \sqrt{\pi}}{2D_s (\sigma_s + \frac{O_e}{D_s}) \sqrt{\rho_0 \sigma_s}} \left( r_{\max} + \frac{2M_s}{H} \sqrt{a^2 + b^2} \right) \]
\[ f_3 = 1 + 4e^{-1.65 \times 10^7 (1 - \sigma_s)^2} \]
\[ H = 1 + a_1^2 + 9a_3^2 + 2a_1(1 - 3a_3) \cos(2\psi) - 6a_3 \cos(4\psi) \]
\[ a = -2a_3 \cos(5\psi) + a_1(1 - a_3) \cos(3\psi) + (6 - 3a_1)a_3^2 + (a_1^2 - 3a_1)a_3 + a_1^3) \cos(\psi) \]
\[ b = -2a_3 \sin(5\psi) + a_1(1 - a_3) \sin(3\psi) + (6 + 3a_1)a_3^2 + (a_1^2 + 3a_1)a_3 + a_1^3) \sin(\psi) \]
\[ f_1 = 0.5(1 + \tanh(20\sigma_s - 14)) \]
\[ f_2 = 0.5(-\cos(\pi\sigma_s)) - 1.5(1 - e^{5-5\sigma_s} \sin^2(\pi\sigma_s) \]
\[ r_b = 2D_s \sqrt{\frac{H_0(\sigma_s - 1)}{\pi - 4}} \quad \text{for} \quad r_b < D_s \quad \text{and} \quad r_b < \frac{B_s}{2} \]
\[ r_b = D_s \quad \text{for} \quad H_0 > 1 \quad \text{and} \quad r_b > D_s \]
\[ r_b = \frac{B_s}{2} \quad \text{for} \quad H_0 < 1 \quad \text{and} \quad r_b > H_0 D_s \]

Bilge keel damping

\[ B_{44K} = \frac{8}{3\pi} \phi_0 \omega \int_L \left( B_{44K_N}^{(2)} + B_{44K_s}^{(2)} \right) dx_b \quad \text{(C.3)} \]

\[ B_{44K_N}^{(2)} = \rho r_k^3 h_k f_k^2 C_D \]
\[ r_k = D_s \left( H_0 - 0.293 \frac{r_b}{D_s} \right)^2 + \left( 1.0 + \frac{O_G}{D_s} - 0.293 \frac{r_b}{D_s} \right)^2 \]
\[ f_k = 1.0 + 0.3e^{-160(1.0 - \sigma_s)} \]
\[ C_D = 22.5 \frac{h_k}{\pi r_k \phi_f f_k} + 2.40 \]
\[ B_{44K_N}^{(2)} = \frac{1}{2} \rho r_k^2 f_k D_s \left( -A \cdot C_p^- + B \cdot C_p^+ \right) \]
\[ C_p^+ = 1.20 \]
\[ C_p^- = -22.5 \frac{h_k}{\pi f_k r_k \phi_a} - 1.20 \]
\[ A = (m_3 + m_4)m_5 - M_f^2 \]
\[ B = \frac{m_3^2}{3(H_0 - 0.215m_1)} + \frac{(1 - m_1)^2(2m_3 - m_2)}{6(1 - 0.215m_1)} + m_1(m_3m_5 + m_4m_6) \]
\[ S_0 = 0.3\pi f_k r_k \phi_a + 1.95h_k \]
\[ m_1 = \frac{r_b}{D_s} \]
\[ m_2 = -\frac{OG}{D_s} \]
\[ m_3 = 1.0 - m_1 - m_2 \]
\[ m_4 = H_0 - m_1 \]
\[ m_5 = \frac{0.414H_0 + 0.0651m_1^2 - (0.382H_0 + 0.0106)m_1}{(H_0 - 0.215m_1)(1 - 0.215m_1)} \]
\[ m_6 = \frac{0.414H_0 + 0.0651m_1^2 - (0.382 + 0.0106H_0)m_1}{(H_0 - 0.215m_1)(1 - 0.215m_1)} \]
\[ m_7 = \frac{S_0}{D_s} - 0.25\pi m_1 \]
\[ = 0.0 \quad \text{for } S_0 < 0.25\pi r_b \]
\[ m_8 = m_7 + 0.414m_1 \]
\[ = m_7 + 1.414m_1 \left( 1 - \cos \left( \frac{S_0}{r_b} \right) \right) \quad \text{for } S_0 > 0.25\pi r_b \]
D Modelling effects

D.1 Response frequencies

In the diffraction analysis, the results are calculated based on a wave spectrum. This wave spectrum is defined at a number of frequencies, at which ANSYS Aqwa calculates the response. The maximum number of different frequencies at which the response can be calculated is 100. The step size between succeeding frequencies can have an effect on the response because large changes in the response between the frequencies can be passed over.

The frequencies defined for the long term probability calculations were with a step size of 0.02 rad/s. This means that between 0.2 and 2.18 rad/s at every 0.02 rad/s, the response is determined.

The short term probability calculations are performed with a gradually changing step size between the frequencies. For the spectrum with a peak period of 15.34 seconds, the frequencies at which the results are calculated are given in Table D.1. Around the frequency where a peak in the response spectrum is located (0.42 rad/s), the frequency step is made smaller, to account accurately for the change in the response.

To determine the effect of the chosen frequencies on the result, the calculations with frequency steps of 0.02 and with gradually changing frequency steps are compared. The response spectrum for a wave spectrum with a peak period of 15.34 seconds is determined with the two different frequency ranges (Figure D.1).

The difference in response spectrum is small, but around the peak of the response spectrum it has a significant effect. The probability of green water is based on the area underneath the response spectrum. The maximum probability of green water for this sea state is 5.0% when calculating it with the step size of 0.02 rad/s (Figure 3.4). When calculating it with the gradually changing step size, the probability decreases with 0.9% to 4.1% (Figure 3.8a). The results depend significantly on

![Graph](a) Relative wave motion spectrum calculated with different frequency step sizes

![Graph](b) Relative wave motion spectrum calculated with different frequency step sizes, at the frequencies around the peak

Figure D.1: The response spectra at $\frac{\pi}{\t_{pp}} = 0.23$, calculated with different frequency step sizes.
Table D.1: Frequencies used for diffraction analysis with changing frequency step size

<table>
<thead>
<tr>
<th>Frequency [rad/s]</th>
<th>Frequency [rad/s]</th>
<th>Frequency [rad/s]</th>
<th>Frequency [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.100</td>
<td>23</td>
<td>0.440</td>
</tr>
<tr>
<td>2</td>
<td>0.150</td>
<td>24</td>
<td>0.445</td>
</tr>
<tr>
<td>3</td>
<td>0.200</td>
<td>25</td>
<td>0.450</td>
</tr>
<tr>
<td>4</td>
<td>0.250</td>
<td>26</td>
<td>0.455</td>
</tr>
<tr>
<td>5</td>
<td>0.260</td>
<td>27</td>
<td>0.460</td>
</tr>
<tr>
<td>6</td>
<td>0.280</td>
<td>28</td>
<td>0.465</td>
</tr>
<tr>
<td>7</td>
<td>0.300</td>
<td>29</td>
<td>0.470</td>
</tr>
<tr>
<td>8</td>
<td>0.320</td>
<td>30</td>
<td>0.480</td>
</tr>
<tr>
<td>9</td>
<td>0.340</td>
<td>31</td>
<td>0.500</td>
</tr>
<tr>
<td>10</td>
<td>0.360</td>
<td>32</td>
<td>0.520</td>
</tr>
<tr>
<td>11</td>
<td>0.380</td>
<td>33</td>
<td>0.540</td>
</tr>
<tr>
<td>12</td>
<td>0.385</td>
<td>34</td>
<td>0.560</td>
</tr>
<tr>
<td>13</td>
<td>0.390</td>
<td>35</td>
<td>0.580</td>
</tr>
<tr>
<td>14</td>
<td>0.395</td>
<td>36</td>
<td>0.600</td>
</tr>
<tr>
<td>15</td>
<td>0.400</td>
<td>37</td>
<td>0.620</td>
</tr>
<tr>
<td>16</td>
<td>0.405</td>
<td>38</td>
<td>0.640</td>
</tr>
<tr>
<td>17</td>
<td>0.410</td>
<td>39</td>
<td>0.660</td>
</tr>
<tr>
<td>18</td>
<td>0.415</td>
<td>40</td>
<td>0.680</td>
</tr>
<tr>
<td>19</td>
<td>0.420</td>
<td>41</td>
<td>0.700</td>
</tr>
<tr>
<td>20</td>
<td>0.425</td>
<td>42</td>
<td>0.720</td>
</tr>
<tr>
<td>21</td>
<td>0.430</td>
<td>43</td>
<td>0.740</td>
</tr>
<tr>
<td>22</td>
<td>0.435</td>
<td>44</td>
<td>0.760</td>
</tr>
</tbody>
</table>

the frequencies at which the response is calculated. For an accurate answer, the frequency step has to be small at the frequencies where the response spectrum changes significantly between succeeding frequencies.

D.2 Irregular frequencies

The free surface elevation RAO showed abrupt peaks at the frequencies above 1 rad/s (Figure D.2a). These peaks are caused by a phenomenon called irregular frequencies [Journé and Massie, 2001]. The discretization of the mean wetted surface into N panels, in the diffraction calculation in ANSYS Aqwa, results in the discretized Equation (B.19). The determinant of the matrix \( \frac{1}{\pi} \frac{\partial G}{\partial n} \Delta S_m \) becomes zero for certain discrete wave frequencies. This results in unrealistic values, the abrupt peaks in Figure 3.6b. This phenomenon is a numerical issue which is related to the internal and non-physical flow of the body.

The non-physical effects can be reduced by 'closing' the body by discretization of the free surface inside the body during the diffraction analysis. It is similar like putting a 'lid' on the free surface inside the body.

In Figure D.2 the free surface elevation RAO and response spectrum at different locations along the
vessel are shown. This short term analysis was based on a wave spectrum with a significant wave height of 5 meters and a peak period of 7 seconds. The calculations are performed twice, once with a 'lid' and once without a 'lid'. For higher frequencies, the effect of the lid is clear. The peaks caused by the irregular frequencies are prevented by the lid. The effect on the response spectrum of the free surface elevation is small. The high frequencies at which the irregular frequencies effect is an issue, do not have a large effect on the free surface elevation spectrum.

Figure D.2: Free surface elevation RAOs with and without the irregular frequencies effect
E Model tests

To study the free surface elevation RAO along the side of the FPSO, model tests can be performed. By measuring the free surface elevation along a model of the FPSO, the free surface elevation RAOs can be determined. The information in this appendix is based on Journée and Massie [2001] and Steen [2014].

The objective of the model tests is to verify the calculations of the free surface elevation RAO and the vessel motion RAOs. To measure the free surface elevation, the relative wave motion along the side of the FPSO and the vessel motion will be measured. The free surface elevation can then be calculated by subtracting the vessel motion in vertical direction from the relative wave motion. The relative wave motion will be measured at the same location as where the free surface elevation is calculated with the diffraction analysis. By adding two measurement point at the locations of RELM6 and RELM11 (Figure 2.7), the results can also be compared with the measurements performed by Buchner and van den Berg [2013].

Besides the RAOs, the roll damping coefficient can be verified by performing a decay test. The most probable maximum freeboard exceedance can be verified by measuring the green water height on the edge of the deck of the model.

The model The model will be a 1:50 scale model. By using the same scale as during the model tests with the wave run-up along a plate [Buchner and van den Berg, 2013], the results of the two model tests can be compared to determine the difference between a fixed plate and a floating FPSO. The main particulars of the model are given in Table E.1.

To study the effect of wave run-up along the side of the FPSO, the model has to be under an angle with the incoming waves. The wave run-up is also affected by the vessel motions, therefore the vessel has to be fixed in a weathervaning position. The vessel will be under a certain angle with the waves by using waves and current combined. By letting the waves come in along the x axis and the current along the y axis, the vessel can be positioned under a certain angle. The mooring system should be a scale model of the mooring system which will be installed on the actual FPSO. The most important for the mooring system is the fact that the mooring stiffness has to be correct, because the stiffness

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{pp} )</td>
<td>3.88  m</td>
</tr>
<tr>
<td>B</td>
<td>0.64 m</td>
</tr>
<tr>
<td>D</td>
<td>0.32 m</td>
</tr>
<tr>
<td>T (without box keel)</td>
<td>0.22 m</td>
</tr>
<tr>
<td>( c_b )</td>
<td>0.80</td>
</tr>
<tr>
<td>Mass</td>
<td>470 kg</td>
</tr>
<tr>
<td>F</td>
<td>Varies along length</td>
</tr>
<tr>
<td>Height box keel</td>
<td>0.04 m</td>
</tr>
<tr>
<td>Width top of box keel</td>
<td>0.08 m</td>
</tr>
</tbody>
</table>

Table E.1: Particulars of Petrojarl 1 scale model
affects the vessel motions.
A scale of 1:50 results in a scale water depth of 31 meters. No towing tanks with a water depth of 31 meters are available. Therefore the mooring system has to be adjusted in such a way that the stiffness is still the same but the needed water depth is less. Most suitable towing tanks for these tests have a depth of 10 meters [Steen, 2014], therefore a mooring system with a height of 10 meters would be convenient.

### The tests

First of all several still water tests will have to be performed to verify the characteristics of the model.

- The mooring stiffness can be verified with a static offset test.
- Decay tests have to be performed for the pitch, heave and roll natural periods and for the roll damping coefficient.

To determine for which sea states the diffraction calculation of the free surface elevation is valid, a range of sea states will be produced in which the free surface elevation will be measured. The different test cases are in Table E.2. Before taking data in the sea states, the relative wave angle, significant wave height and peak period of the sea state has to be verified. First of all, the sea states assessed in the analysis will be tested. Also the test case from Buchner and van den Berg [2013], used for the validation, will be a test case. Further more, several more severe sea states with similar steepness will be tested.

The time to test each test case is 1527 seconds. This corresponds to a 3 hours storm on full scale. Between the tests has to be sufficient time to not have any influence on a certain test from a previous test.