Developing mass appraisal with fuzzy systems

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Why do we need new tools to property appraisal?

What’s the problem with traditional hedonic approach, based on regression analysis?
Property mass appraisal

- Problems in hedonic models using regression analysis:
  - Property are different, performing submarkets by size, age, location, and another characteristics
  - Submarkets are not clearly divided in crisp and homogeneous parts
  - Hedonic prices may change among submarkets and thus there are abrupt transition between contiguous regions or property type (“neighbour” models)

- Alternative tool: fuzzy systems
  - They’re able to consider vagueness and imprecision, generating systems with soft transitions
What are fuzzy sets and fuzzy rules?
Fuzzy sets

- Fuzzy sets are based on fuzzy logic
- There is a great difference among fuzzy and classical sets:
  - Classical, crisp, sets: \([0,1]\) – zero or one values
    - Membership function: yes/no
  - Fuzzy sets: \([0,1]\) – zero to one values
    - The membership function gives a continuous range of relationship – for example, two fuzzy sets, identifying relationship with Black and White colors:

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_{\text{Black}}(x))</td>
<td>0,00</td>
<td>0,05</td>
<td>0,10</td>
<td>0,15</td>
<td>0,20</td>
<td>0,25</td>
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<td>0,70</td>
<td>0,75</td>
<td>0,80</td>
<td>0,90</td>
<td>1,00</td>
</tr>
<tr>
<td>(\mu_{\text{White}}(x))</td>
<td>1,00</td>
<td>0,95</td>
<td>0,90</td>
<td>0,85</td>
<td>0,80</td>
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</tbody>
</table>
Fuzzy rule-based systems

- A fuzzy TSK rule is such as:
  \[ \text{IF } x_1 \text{ is } A_1 \text{ and... } x_k \text{ is } A_k \text{ THEN } y_i = p_1.x_1 + ... + p_k.x_k + p_0 \]

  - where \( x_i \) are input variables, \( A_i \) are fuzzy sets and \( p_i \) are coefficients

- In fuzzy systems, the result to a particular case is determinate by a weighting mechanism – for example, in a 3-rules system:
  
  - A) determinate the memberships: \( m_i = A_i(x_i) \), with \( m_1 + m_2 + m_3 = 1 \)
  - B) determinate partial outputs: \( y_1, y_2, y_3 \)
  - C) final system output: \( Y = m_1*y_1 + m_2*y_2 + m_3*y_3 \)
Fuzzy systems

- Each rule can be seen as an hedonic model, but working in group with the other rules (there is a weighted result, using two or more rules)

- However, fuzzy systems do not learn alone the rules

- There are several methods to extracting rules from data, in general neuro-fuzzy or genetic-fuzzy systems
Neuro-fuzzy systems (NFS)
Artificial neural networks (ANNs)

- Neural networks are multiple-connected systems
- The neurons are simple units, composed by an addition (+) and an active part ($f_A$):
Artificial neural networks (ANNs)

an example

bias

0

District

1

GBA

2

Quality

3

Age

4

Month

5

input layer

hidden layer

output layer

Sale Price

11
Artificial neural networks (ANNs)

- In the last years, there are several studies using ANN, but with a same problem: there are not explanation about the inner functioning or about the relationship between inputs and outputs (the “black box” nature of neural networks);

- The explanation may be developed using neural networks and fuzzy rules jointly, in a NFS
Fuzzy rules are a convenient way to extract knowledge from neural networks – or – neural networks are a convenient way to generate fuzzy systems from data.

There are several methods of rule extraction.

FAGNIS it’s a simple and direct method to extract fuzzy rules.
The fundaments of FAGNIS

- Neurons have two parts: + and $f_A$
- Activation functions of hidden neurons often have a small working range
- A curve can be approximated by a set of linear segments:
  $$f_A(a_j) \sim \sum_i [F_i(a_j) \ast (p_i \ast a_j + q_i)]$$
- The segments can be combined:
  $$G_r(a_j) = F_1(a_j) \ast F_2(a_j) \ast \ldots \ast F_n(a_j)$$
  $$y_r = (p_1 + \ldots + p_n) \cdot a_j + (q_1 + \ldots + q_n)$$
Genetic-fuzzy systems (GFS)
Genetic fuzzy rule-based systems

- The rules may be adjusted by genetic algorithms
  - GA are search procedures, using a random search to choose among potential alternative solutions
- There are two approaches in generating these systems:
  - Pittsburgh: a set of rules is extracted simultaneously
  - Michigan: each rule is obtained individually
1) Fuzzy rules based in one-dimensional characteristics

- Fuzzy system based on size, age or another one-dimensional characteristic
- The fuzzy sets are such as (GBA):
2) Rule system based on location

- We can apply a penalty (B) in the fitness function, forcing the adjust to location measure:

  Fitness function: \( F_i = \frac{1}{1 + \text{mMAPE}_i} \),

  with \( \text{mMAPE}_i = \sum_{i,j} \left( \frac{|Y_j - Y_{h,i,j}|}{Y_j} \times 100 / B_{i,j} \right) \)

  Coordinates of the units: \((X_j, Y_j)\)

  Coordinates of the rules: \((X_i, Y_i)\)

  \( B_{i,j} = \frac{1}{1 + \left( (X_i - X_j)^2 + (Y_i - Y_j)^2 \right)^{0.5}} \)

- It is a “conical”, spatial membership
Concluding remarks

- It’s important to use individual training and test samples
- Fuzzy TSK rules are a kind of hedonic models, which have a crossing or soft trespass between two models
- Fuzzy rules perform an explicit model, different from neural networks alone
- The analyst can include empirical rules, based on your experience or knowledge about specific situations in the market (a new rule with $Y = $cte)
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