Influence of gun design on Coulomb interactions in a field emission gun

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The authors investigate by simulation the Coulomb effects on brightness and energy spread for cold field emitters. At first, we show that brightness is ultimately limited by Coulomb interactions. The authors analyze the maximum attainable brightness for tip radii ranging from 1 nm to 1 μm . Remarkably, the brightness of a tip of 1 nm is comparable to a tip of 1 μm . Higher brightness can be achieved by adding a plate at the base of the field emitter, and therefore decreasing the shank length. However, the brightness enhancement due to the protrusion is ultimately limited by large extractor voltages and increased energy spread. In particular, the energy spread for larger tip radii grows faster with brightness. The authors conclude that performance enhancement due to the protrusion works best for smaller tip radii. © 2011 American Vacuum Society.

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I. INTRODUCTION

Among the various parameters of an electron gun, the brightness and energy spread are key parameters. Brightness is usually conserved, but with electron–electron interactions included, stochasticity increases the size of a focused spot. We already demonstrated the importance of stochastic Coulomb interactions for the brightness.¹ This effect is calculated by projecting the displaced beam trajectories back to the virtual source plane. The reduction on the brightness is obtained by adding the broadening to the size of the virtual source,

$$B_{R(e-e)} = B_R \left(\frac{r_{VS}^2}{r_{VS}^2 + r_D^2} \right),$$

where the intrinsic brightness is defined as

$$B_R = \frac{I}{\pi \alpha^2 \pi r_{VS}^2 V},$$

where πr_{VS}^2 is the virtual source size. This is the area where the current appears to come from. The solid angle is defined by $\pi \alpha^2$ and V is the beam potential. For our purpose, it suffices to approximate the trajectory displacement by cutting the beam in slices and sum the individual contributions.² The Coulomb interactions are calculated by the extended two-particle approximation developed by Jansen and Kruit,³ which are in reasonable agreement with experiment.⁴ The extended two-particle approximation considers longitudinal and lateral interactions. In this article we investigate both effects. The angular displacement and energy spread are given as the full width containing 50% of the distribution. Jansen and Kruit make a distinction between different beam regimes. The relevant regimes for the simulations presented here are the pencil-beam regime and Holtmarkian regime. In the pencil-beam regime, the electrons are traveling in a line after each other, whereas in the Holtmarkian regime the electrons are traveling next to each other, which enhances

the lateral displacement. The slice method is not applicable in the case of Gaussian beams, which are characterized by high particle densities. We do not expect to enter the Gaussian regime until the brightness and energy spread are already fully determined by Coulomb interactions. Neither do we expect to enter the regime in which the emission is influenced by continuous space charge effects, such as space charge limited emission, which according to Ref. 5 only start at current densities exceeding $1 \times 10^8 \text{ A cm}^{-2}$.

II. CALCULATION METHOD

We model the gun as shown in Fig. 1, with a needle that ends in a spherical tip. The radius of the tip is variable and the cone angle is fixed at 7.5° as illustrated in the inset. The distance from the end of the tip to the extractor is $508 \mu\text{m}$. This design matches with the calculations we have done before.¹

There are four calculations involved in obtaining the brightness and energy spread: (1) the calculation of the electric field, (2) the quantum mechanical calculation of the emission current, (3) the tracing of the electron trajectories through the gun, and finally (4) the trajectory displacement as given by Jansen's equations.^{2,3} We have combined the four calculations in one simulator. This allows us to simulate, in one run, the effect on brightness and energy spread, simultaneously varying the parameters of the gun. All simulators involved are home-built and utilize the streaming parallelism offered by modern graphics cards (we have used NVIDIA's GeForce GTX 480).⁶ The calculation of the electric field is based on a graphics processing unit implementation of the charge density method. The emission current is calculated by the free-electron theory of metals as proposed by Sommerfeld.⁷ For the cold field emitter, we have used a work function of $W = 4.3 \text{ eV}$, Fermi level of $E_F = 4.5 \text{ eV}$ and a temperature of $T = 300 \text{ K}$. We solve Schrödinger's equation with Numerov's method,⁸ which is based on a finite difference scheme of sixth order per iteration step. The particle integrator is based on a fourth-order symplectic integrator⁹ with variable time step. The simulators have been benchmarked against previous calculations¹ and analytical solutions.¹⁰

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FIG. 1. Geometry design for simulating the cold field emitter. The geometry is rotational symmetric. The dashed lines identify the origin of coordinates. The geometry shows a needle and an extractor. The inset illustrates the spherical tip at the end of the needle. As a reference, the origin of coordinates is also shown in the inset. Parameters considered in this geometry are the tip radius and the extractor voltage.

III. RESULTS FOR THE BASIC GUN DESIGN

We start by investigating the effect of Coulomb interactions on the basic cold field-emitter design, as given in Fig. 1. The tip radius is fixed to 10 nm. We vary the extractor voltage and observe the effect on brightness with and without Coulomb interactions. The result is given in Fig. 2, which shows that the brightness is limited due to Coulomb interactions. The explanation for the decreasing brightness is as follows. As the extractor voltage is increased, the emission current increases and so does the virtual source size. The brightness starts to decrease when the virtual source contribution from the Coulomb interactions starts to grow faster than the current.

The maximum brightness of Fig. 2 can be identified for other tip radii as well. By repeating the procedure, we iden-

FIG. 2. Brightness vs electric field for a tip radius of 10 nm. This result is obtained by increasing the extractor voltage and observing the brightness. The figure shows that brightness is limited by Coulomb interactions. Note that there is a maximum brightness for this configuration.

FIG. 3. Maximum brightness vs extractor voltage. The maximum brightness for each tip radius is obtained by varying the extractor voltage. The maxima are identified and shown with square markers. Note the low extractor voltage for the brightness curves at the left.

tify the maximum brightness for tip radii ranging from 1 nm to 1 μm . The result is given in Figs. 3 and 4. In Fig. 4, the maximum brightness is displayed as a function of tip radius. In addition, Fig. 4 also shows the total energy spread at the maximum brightness.

A remarkable feature of Fig. 4 is that, as far as brightness is concerned, a field emitter with a tip radius of 1 μm performs nearly as good as one with 1 nm. The general trend is explained as a competition between the pencil-beam and Holtsmarkian regime.¹ We note that Fig. 4 appears not to agree with our previous publication.¹ The reason is that the plotted brightness in Ref. 1 is calculated as $B_{R(e-e)} = 0.9B_R$. Here we plot the maximum brightness. In principle, we must be careful as a Gaussian beam may develop. However, the precaution measure for the development of a Gaussian beam does not affect the qualitative result. As Fig. 4 shows, the shape remains only shifted.

FIG. 4. Maximum brightness and energy spread at that maximum vs tip radius. The maximum brightness is derived from Fig. 3.

Fig. 5. Geometry design for simulating the cold field emitter with a protrusion. The geometry is rotationally symmetric. The dashed lines identify the origin of coordinates. The geometry shows a needle with a protrusion and an extractor, where the protrusion length is labeled by Δx . Parameters considered in this geometry are the tip radius, extractor voltage, and protrusion length.

IV. SIMULATION OF DESIGN CHANGES

We focus on the small extractor voltages in Fig. 3 corresponding to small tip radii ($r < 15$ nm). As the extractor voltages are getting lower, the flight time of the electrons increases. The consequence is that there is more time for Coulomb interactions to affect the brightness and energy spread. Consider the geometry in Fig. 5, where a protrusion is introduced in the geometry of the cold field emitter. In comparison to the basic gun design (Fig. 1), we need a larger extractor voltage for a fixed field strength at the tip. As a consequence of the increased extractor voltage, the electrons are accelerated to a higher velocity. This limits the flight time of the electrons and hence limits the Coulomb effects. Therefore, we expect that the geometry with a protrusion results in a higher brightness.

Fig. 6. Brightness vs extractor voltage for a tip radius of 10 nm. Each curve corresponds to a particular protrusion length. The arrow indicates the direction of decreasing protrusion lengths. The left-most curve has the largest protrusion and approximates the basic cold field emitter. The right-most curve has a protrusion of $\Delta x \approx 20$ μm .

Fig. 7. Maximum brightness vs protrusion length for a tip radius of 10 nm. For each protrusion, the maximum brightness is identified and shown with square markers. Also shown is the extractor voltage at the maximum brightness. On the right we have the largest protrusion, which approximates the basic cold field emitter.

This idea is tested by analyzing protrusion lengths in the range of 1 μm to 1 cm. The results for a fixed tip radius of 10 nm are given in Figs. 6 and 7. From Fig. 6, we observe that the larger the protrusion, the lower the maximum brightness. Also, the brightness peak is narrower. When the protrusion is large, the extractor voltage at a given electric field, and with that the intrinsic brightness, is lower. Due to the lower extraction voltage, the Coulomb effects are more significant. This explains the sharper curve with a lower maximum. Choosing a smaller protrusion results in a higher maximum brightness and less sensitivity to extractor voltage.

In Fig. 7 we plot the maximum brightness as a function of protrusion length. Also shown is the extractor voltage at the maximum brightness. One would suggest choosing the

Fig. 8. Maximum brightness and energy spread at that maximum as a function of tip radius and protrusion length. Each curve corresponds to a particular tip radius. From left-to-right we have decreasing protrusion lengths, where the left-most point on each curve approximates the basic cold field emitter. Note that the increase in energy spread is limited for smaller tip radii as the maximum brightness grows.

smallest possible protrusion (perhaps smaller than $1\text{ }\mu\text{m}$). However, as we decrease the protrusion, the extractor voltage ultimately increases to unrealistic values.

The energy spread is equally important to the performance of the emitter. In Fig. 8 we have illustrated the energy spread at the maximum brightness for various tip radii. As we increase the brightness, so does the energy spread. However, for larger tip radii the energy spread grows faster with increasing brightness.

V. CONCLUSIONS

The simulations show that brightness is limited due to Coulomb interactions. The Coulomb effects add up significantly for large flight times of the electrons through the gun. The addition of a plate to the base of the field emitter allows electrons to accelerate faster toward the extractor. This reduces the Coulomb effects and enhances the brightness. However, smaller protrusions imply larger extractor voltages, with the danger of reaching unrealistic and unpractical voltages. As the maximum brightness increases with protrusion length, so does the total energy spread. For larger tip radii, the energy spread grows faster with brightness. We conclude that, protrusion as a performance enhancement—high brightness, low energy spread—works best for smaller

tip radii. It must be noted that our quantitative results are valid only for the configurations given here and will change for other configurations. For example, a different cone angle, or cone shape, may slightly change the extractor voltages at which the described effects occur. Our present model assumes a perfect surface with a single-valued work function. Real emitters often show nonuniform emission from the surface, resulting in local concentrations of electron density in the beam. We do not know the effects of such behavior. In any case, the qualitative results are expected for other configurations as well: The effects of stochastic Coulomb interactions should be taken into consideration in electron gun design.

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