Preliminary investigation into the occurrence of wave groups in seas and swell

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Verification of the Kimura model for the description of wave groups

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PRELIMINARY INVESTIGATION INTO  
THE CHARACTERISTICS OF  
WAVE GROUPS IN SEAS AND SWELL  

by  

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Preface

This report is the first of two parts of my master thesis prepared for Delft University of Technology. The first part gives the results of an investigation into the occurrence of wave groups in seas and swell. The second part contains the verification of the Kimura model for wave group statistics. The work for this thesis was carried out under the guidance of Prof. J.A. Battjes of Delft University of Technology.

This report was originally written in the Dutch language. However, in the past years it became clear that an English version of this thesis was desired. The contents of the English version is identical to the Dutch version, except for a few corrected misprints. Recent results from the literature have not been included.

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Gerbrant van Vledder
Appendices

A References

B List of symbols

C Statistical properties of group length distributions
1 Introduction

The occurrence of wave groups in seas and swell is a well known phenomenon for fishermen and seamen. This is reflected by an old Icelandic saying:

\[ Sjaldan \text{ er ein baran stök } \]

Which means: a large wave rarely comes alone. This saying also stresses the meaning of the concept wave groups.

Knowledge of wave groups is of importance for various aspects of coastal- and offshore engineering, e.g. resonance phenomena in constructions can occur when they are attacked by groups of high waves. The quantitative description of wave groups is in development, but it still shows many shortcomings. It should be noted that observed group lengths are longer than most theories predict.

The objectives of this study are:
- inventory of the importance of wave groups for coastal and offshore engineering,
- analysis of published theories and results about wave groups, and
- indications on which topics further research is needed.

The structure of this report is as follows. In chapter 2 of this report some qualitative effects of wave groups are presented. Chapter 3 gives a description of wave groups in terms of individual waves. A distinction will be made between theories which assume dependence or independence between succeeding wave heights. A description of wave groups in terms of envelope theories is presented in chapter 4. In chapter 5 the relation between groupiness and spectral shape will be dealt with. Also the role of the phase spectrum will be discussed. Chapter 6 gives an indication which topics deserve further study.

Three appendices are given at the end of this report:

A) list of references,
B) list of symbols,
C) statistical distribution of group lengths.
2 Effects of wave groups

2.1 Introduction

The occurrence of wave groups is of importance for coastal- and offshore engineering. Wave groups can cause variations in the mean water level, such as 'surf beats' and harbour oscillations. Further, wave groups can damage offshore structures and breakwaters and they can influence the stability of ships. These aspects will discussed in the following sections.

2.2 Set down

One of the effects of wave groups is that they cause local variations of the mean water level such as 'set-down' in wave groups. This phenomenon was first described by Longuet-Higgins and Stewart (1962) and can be explained with the concept of radiation stress. The local value of the radiation stress is approximately proportional to the square of the local wave height. This implies that within a group of high waves the radiation stress is higher than within a group of low waves. Variations of the radiation stress then cause local variations of the mean water level (See Fig 2.1). The radiation stress, which varies with the presence of wave groups, generates a long wave whose velocity is the same as the group velocity. So, this long wave is a bounded one. More remarks about the relation between set-down and wave groups can be found in Brevik (1979).

Fig. 2.1 Set down and second-order currents within a wave group
2.3 Second-order currents

The occurrence of wave groups may cause the generation of set-down and second-order currents (Longuet-Higgins and Stewart, 1962). These effects are also shown in Fig 2.1. The second order wave-system is in anti-phase with the envelope of the first order wave-system. A trough of the second-order wave-system coincides with a maximum of the group envelope and a crest of the second-order wave-system coincides with a minimum of the group envelope. As will be seen later, these second order currents are of importance for the computation of drag forces on structures. Further, these currents may hinder moored ships.

2.4 Surf-beats

The occurrence of set-down in the coastal zone may cause the generation of free long waves. Munk (1949) and Tucker (1950) described this phenomenon and called these long waves 'surf-beats'. Tucker (1950) found a significant correlation between long periodic fluctuations of the mean water level and the envelope of occurring wave groups. These long waves were observed 4-5 minutes later than the maxima of the amplitude envelope.

Longuet-Higgins and Stewart (1962) proposed that the observed time lag was equal to the time needed for the short waves to arrive at the breaker zone, added with the time needed for the associated reflected long wave to reach the measurement point. They assumed that the short waves were dissipated in the breaker zone.

2.5 Ships

Wave groups often lead to loss of stability of ships. Kjeldsen and Myrhaug (1979) analysed the asymmetry of waves within wave groups. It was assumed that this asymmetry influences the stability of ships. This is one of the reasons that for the Norwegian coastal waters wave forecasts are given with respect to the occurrence of wave groups and wave asymmetry.

2.6 Harbour resonances

Harbour resonances or seiches are free standing waves in harbour basins. They have a period which can vary from some minutes to tens of minutes. It is possible that these oscillations are generated by variations of the mean water level outside the harbour with periods similar to the seiches. See Gravesen et al. (1978a, 1978b).

The generation of seiches with a period of some minutes is often ascribed to the occurrence of surf-beats. Bowers (1977) showed that harbour oscillations can also be generated directly by set-down within wave groups. If the group period (i.e. the average duration between succeeding wave groups) is almost the same as the period of the seiches, the harbour basin
will start to oscillate. Harbour oscillations with periods in the order of some minutes can give problems to moored ships (e.g. Massie, 1976 and Bowers, 1980). Although the vertical amplitude of the water movement for seiches is rather small, the horizontal water movements can be of significant magnitude. For this case moored ships can get problems with their mooring if they are moored in the vicinity of a knot of the seiche. This may even lead to the breaking of mooring lines (Stammers et al. 1977).

2.7 Offshore structures

Floating offshore structures, e.g. semi-submersibles, may be subject to resonance phenomena when the eigenperiod of the whole construction is of the same order as the period of the maxima of the envelope of the wave groups.

The existence of a second-order wave-system, resulting from the presence of wave groups (section 2.3) may have consequences for the design of offshore structures if the drag forces are greater than the inertia forces. In certain conditions it is possible that the drag forces are significantly reduced as a consequence of the presence of this second-order wave-system (Dean, 1979, his Fig 1). A specific combination of the first and second-order wave system results in a reduction of the total drag force in the crest position of the wave group envelope.

Especially for shallow water conditions the amplitudes of this second-order wave-system are relatively large. Reductions of the drag forces in the order of 10%-40% are possible if the second-order amplitudes are about 10%-20% of the amplitudes of the first-order wave-system.

Remarks

1) Many field measurements are necessary to examine the influence of this second-order wave-system on the drag forces acting on an offshore platform.

2) The conditions for the generation of these reductions in the drag forces are strict, limiting the practical applicability of the theory of Dean (1979).
2.8 Breakwaters

The design of breakwaters is based on a long experience with structures on full scale. The use of wave flumes has made it possible to test breakwaters on model scale. Therefore, the knowledge of breakwaters with respect to the stability and the collapse mechanisms has increased significantly. The stability of breakwaters depends on a set of many wave parameters of which the groupiness of waves is one.

In wave flumes it is possible to generate regular and irregular waves. The irregular waves may have any possible wave spectrum. By varying the shape of the spectrum it is possible to influence the stability of breakwaters. This relation was observed by Carstens et al. (1966) when they were examining the stability of breakwaters. They found that the damage to breakwaters increases when the spectral width decreases while keeping the wave height distribution constant. Carstens et al. (1966) did not mention the relation between wave groups and damage to breakwaters. This relation was noticed by Johnson et al. (1978). They found that certain sequences of waves, such as occurring in wave groups, can do more damage to breakwaters than waves with the same wave heights but more evenly distributed over the whole wave train. Burcharth (1979, 1980) mentioned some wave patterns which are characteristic for wave groups and their effects on structures:

1) a big jump in wave height of succeeding waves can do more damage than a sequence of some high waves,
2) groups of high waves often show a large measure of regularity and may therefore cause damaging resonance phenomena in structures.

The collapse of a breakwater is possibly due to a mechanism which occurs when a structure is attacked by a group of high waves. Johnson et al. (1978) proposed the following mechanism:

*The first wave in a group loosens an element of the breakwater, the second wave pulls or presses this element out of the top layer, whereas the third wave is moving this element over the construction.*

This mechanism cannot work for a non-grouped wave train and therefore this is less dangerous for the breakwater.

2.9 Conclusions

The prediction of groupiness is of importance for coastal engineering purposes because of the effects of long periodic perturbations caused by wave groups. Knowledge of the statistical properties of wave groups is of importance for the design of onshore and offshore structures.
3 Wave group analysis in terms of individual waves

3.1 Introduction

3.1.1 Wave heights and wave periods

The analysis of wave groups is performed in terms of sequences of wave heights determined with the zero-up crossing method. Each wave has two characteristics, a wave height and a wave period. The wave height $H$ is defined as the difference between the greatest maximum and the lowest minimum of the surface elevation between two succeeding zero-up crossings. The wave period $T$ is defined as the time interval between two zero-up crossings, see Fig 3.1 for a definition sketch.

![Figure 3.1 Definition sketch of wave height and wave period.](image)

3.1.2 Wave height distribution

For the analysis of wave groups it is necessary to use the distribution function of wave heights. Longuet-Higgins (1952) showed that the wave height distribution for a narrow spectrum can be described by the Rayleigh distribution:

$$P(H) = \text{Prob}\{H \leq H\} = 1 - \exp\left(-\frac{\pi}{4} \frac{H^2}{H_m^2}\right)$$  \hspace{1cm} (3.1)

in which $H_m$ is the mean wave height.
Many investigations have shown that the Rayleigh distribution is a valid representation for the wave height distribution, even for relatively wide spectra. In the following we will make use of this fact.

### 3.1.3 Definition wave group

On the basis of a sequence of wave heights, a wave group is defined as a sequence of succeeding waves with heights that all exceed a certain level (Goda, 1970). This level is called the group level and is denoted by $H_c$. A common choice of $H_c$ is the mean wave height $H_m$ or the significant wave height $H_{1/3}$. The significant wave height is defined as the mean of the highest third part of all wave heights in a record. In the following high and low waves are defined as waves that are larger or lower than the group level respectively. The lengths of a wave group is defined as the number of waves in a sequence of high waves. The group length is denoted by $j_1$. A sequence of high waves is succeeded by a sequence of low waves. The length of this sequence is denoted by $j_2$ and is equal to the number of low waves in this sequence. Finally, the total length is defined as the number of waves in a group of high waves followed by a group of low waves. The length of this sequence is denoted by $j_3$. The minimum value of $j_3$ is 2 (See Fig 3.2 for a definition sketch).

![Definition sketch of sequence lengths of wave groups](image)

**Remark**

A sequence of high waves will be called a wave group, unless stated otherwise.
3.2 Independence between succeeding wave heights

3.2.1 Introduction

A first model describing the statistical properties of wave groups was given by Goda (1970). His model is based on statistical independence between succeeding wave heights. This model is often called the Goda model.

3.2.2 Model of Goda

To describe the probability of occurrence of a certain group length use is made of elementary statistics. The probability for a wave height \( H \) to be greater than the group level \( H_e \) is denoted by \( p \) and the probability for a wave height to be lower than or equal to \( H_e \) is denoted by \( q \). So:

\[
p = \text{Prob}\{H > H_e\} \quad (3.2)
\]

and

\[
q = \text{Prob}\{H \leq H_e\}. \quad (3.3)
\]

By definition \( p + q = 1 \). For ease of notation only the parameter \( p \) is used in the following. If the wave height \( H_n > H_e \), then the probability of occurrence of a wave group with length \( j \) is equal to the occurrence of the following event:

\[
(H_{n+1} > H_e) \land \ldots \land (H_{n+j-1} > H_e) \land (H_{n+j} \leq H_e)
\]

Because it is assumed that succeeding wave heights are independent, the probability of this event is equal to the product of the probabilities of the separate events. Thus, the probability for a wave group to have length \( j \) is:

\[
P_1(j) = (1-p)p^{j-1} \quad (3.4)
\]

The mean group length \( \bar{j}_1 \) and standard deviation \( \sigma(j_1) \) are computed as (See appendix A):

\[
\bar{j}_1 = \frac{1}{1-p} \quad (3.5)
\]

and

\[
\sigma(j_1) = \frac{\sqrt{p}}{1-p}. \quad (3.6)
\]
Similarly, the probability for a group of low waves to have length \( j \) is given by:

\[
P_g(j) = (1 - p)^{j-1}p
\]

with mean

\[
\bar{j}_2 = \frac{1}{p}
\]

and standard deviation

\[
\sigma(j_2) = \frac{\sqrt{1-p}}{p}.
\]

The probability of occurrence for a total sequence of length \( j \) \((j \geq 2)\) is computed as:

\[
P_s(j) = \frac{p(1-p)}{2p-1}(p^{j-1} - (1-p)^{j-1})
\]

with mean

\[
\bar{j}_3 = \frac{1}{1-p} + \frac{1}{p}
\]

and standard deviation

\[
\sigma(j_3) = \sqrt{\frac{(1-p) + \frac{p}{(1-p)}}{p^2(1-p)^2}}.
\]

### 3.2.3 Theoretical value mean group length

The model of Goda can be used to compute theoretical values of the mean group lengths for different group levels. Table 3.1 gives a review for theoretical values of \( p \) and \( \bar{j}_1 \) for some group levels. The values of Table 3.1 are used as a reference for the groupiness of waves. The larger the mean group length compared to those of Table 3.1, the higher the amount of wave grouping.
Chapter 3

Wave group analysis in terms of individual waves

3.2.4 Groupiness

The number of grouped high waves is a measure for the groupiness. Expressed in terms of the Goda model, the groupiness can be related to the number of wave groups occurring in a measurement. This measure was used by Carstens et al. (1966) to specify the difference in groupiness between two measurements. These measurements should have the same number of wave heights. However, it is better to couple groupiness to the mean group length. This will be explained using the model of Goda.

Assume that in a registration (or a measurement) \( N_w \) waves and \( N_g \) wave groups occur. According to Eq (3.11) the number of groups is:

\[
N_g = \frac{N_w}{\bar{j}_1} = N_w p(1 - p). \tag{3.13}
\]

Consider \( N_g \) as a function of \( p \). It then follows that \( N_g \) has a maximum for \( p = 0.5 \). The mean group length \( \bar{j}_1 \) increases as \( p \) increases, see Eq (3.5). When the mean group length increases this doesn’t mean that the number of wave groups in a registration increases. This consideration is relevant for \( p \approx 0.5 \), e.g. when the group level is equal to \( H_{med} \) or \( H_m \).

3.2.5 Test of Goda’s model

Introduction

Goda (1970) used computer simulations to analyse the groupiness of waves with different spectral shapes. The phases of the wave components were uniformly distributed on the interval \( [0,2\pi] \). Goda found a relation between the spectral shape and the groupiness. This relation will be discussed in chapter 5.

To compare the model of Goda with field measurements the following publications will be used. Wilson and Baird (1972), Rye (1974), Goda (1976) and Dattatri et al. (1977).

<table>
<thead>
<tr>
<th>Group level</th>
<th>( p )</th>
<th>( \bar{j}_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{med} )</td>
<td>0.5000</td>
<td>2.00</td>
</tr>
<tr>
<td>( H_m )</td>
<td>0.4559</td>
<td>1.84</td>
</tr>
<tr>
<td>( H_{1/3} )</td>
<td>0.1348</td>
<td>1.16</td>
</tr>
<tr>
<td>( H_{1/10} )</td>
<td>0.0392</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 3.1: Group levels and mean group lengths according to Goda’s theory
Sources of data

Wilson and Baird (1972) analysed measurements from the Atlantic Ocean for the coast of Western Head, Nova Scotia, in the period May 1970 - July 1970. The water depth was 20 fathoms (= 36.5 m).

Rye (1974) used North Sea data from Utsira near the Norwegian coast. These measurements were collected during 3 storms in October, November and December 1970. The water depth was 100 m.

Goda (1976) used a great number of data from measurements collected from the seas around Japan.

Dattatri et al. (1977) analysed measurements near the coast of Western India at Mangalore Harbour obtained in August 1974. The water depth was 13 meters. Every year during August the west coast of India is under the influence of the Indian monsoon resulting in strong winds and high seas.

In Table 3.2 a summary is given for the measured mean group lengths for the group levels $H_{med}$ and $H_{1/3}$ together with the theoretical mean group lengths of Table 3.1.

<table>
<thead>
<tr>
<th>Group Level</th>
<th>$H_{med}$</th>
<th>$H_{1/3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goda's theory</td>
<td>2.00</td>
<td>1.16</td>
</tr>
<tr>
<td>Wilson and Baird (1972)</td>
<td>-</td>
<td>1.49</td>
</tr>
<tr>
<td>Rye (1974)</td>
<td>-</td>
<td>1.35</td>
</tr>
<tr>
<td>Goda (1976)</td>
<td>2.54</td>
<td>1.42</td>
</tr>
<tr>
<td>Dattatri et al. (1977)</td>
<td>2.23</td>
<td>1.34</td>
</tr>
</tbody>
</table>

Table 3.2: Measured mean group lengths

Results

All measurements indicate that the measured mean group lengths are greater than the values according to the theory of Goda. An explanation for these differences was not given by the authors. Rye (1974) noticed a significant dependence between succeeding wave heights. He assumed that wave heights have a 'memory', by which it is more likely that a high wave is followed by another high wave than by a low wave. This means that the heights of succeeding waves are correlated.
Conclusions

1) The model of Goda is not valid because it under predicts the expected mean group lengths.

2) Succeeding wave heights are statistically dependent causing higher mean group lengths than might be expected on the basis of independence between succeeding wave heights. This dependence is discussed in the next section.

Remark

The paper by Dattatri et al. (1977) contains a serious error. They mention the use of the group levels $H_m$ and $H_{1/3}$, whereas they actually use the group levels $H_{med}$ and $H_{1/3}$. 

3.3 Dependence between succeeding wave heights

3.3.1 Introduction

The dependence between succeeding wave heights was analysed by various authors by looking at their joint probability distribution. The correlations of succeeding wave heights were studied in detail by Rye (1974), Dattatri et al. (1977), Arhan en Ezraty (1978), Su et al. (1982) and Goda (1983). Other characteristics of this distribution were analysed by Arhan and Ezraty (1978) and also by Su et al. (1982). Arhan and Ezraty (1978) developed a theoretical description for this distribution which makes it possible to study its characteristics theoretically.

Sources of data

The data which were used by Rye (1974) and Dattatri et al. (1977) are described in section 3.2.5. Other data which were used by these authors are mentioned below.

Arhan and Ezraty (1978) processed a large number of measurements obtained in the North Sea, including about 26,000 waves.

Su et al. (1982) used measurements which were collected in the Gulf of Mexico. These measurements were performed at an offshore platform in a water depth of 340 feet ($\approx 110$ m). These registrations were selected using the criterion $H_{1/3} > 2$ m.

Goda (1983) analysed records of old long swell with a very narrow spectrum which were collected off the coast of Costa Rica in a water depth of about 15 meters. This swell was generated in the oceans near Antarctica and had travelled about 7,000-9,000 km.

3.3.2 Theory of Arhan and Ezraty

Arhan and Ezraty (1978) gave a theoretical solution for the joint probability distribution (joint p.d.f.) of the heights of two succeeding waves $H_i$ and $H_{i+1}$. This distribution will be denoted by $p(H_i, H_{i+1})$.

They used the theory developed by Rice (1944, 1945) for the joint probability distribution for two values of the envelope of a realisation of a Gaussian process at two different times. They also used the joint probability distribution for wave heights and wave periods given by Cavanie et al. (1976). The use of these theories made it possible to study the characteristics of a process with a certain spectrum. The solution of Arhan and Ezraty is rather complex and difficult to handle. The theory of Arhan and Ezraty will be commented upon in section 5.2.
3.3.3 Check of the theory of Arhan and Ezraty

Introduction

The properties of $p(H_i, H_{i+1})$ were studied by Arhan and Ezraty (1978) both theoretically and empirically. Su et al. (1982) limited themselves to an empirical study of $p(H_i, H_{i+1})$. In both studies the wave records were normalized in order to combine them with other records with different significant wave heights. This was achieved by dividing all wave heights in a record by the standard deviation of the surface elevation $\sqrt{\langle \eta^2(t) \rangle}$. This quantity is also referred to as the square of the zero's spectral moment $m_0$ (see section 4.4).

Results and conclusions

Arhan and Ezraty (1978) studied the properties of $p(H_i, H_{i+1})$ by analysing the conditional expected value of $H_{i+1}$ given $H_i$. This conditional expectation is denoted by $E\{H_{i+1} \mid H_i\}$. Theoretical values of $E\{H_{i+1} \mid H_i\}$ were computed using a JONSWAP spectrum with a peakedness factor $\gamma = 3.3$. This functional relationship is indicated in Fig 3.3 with the curved line. The dots indicate the empirical values. If succeeding wave heights are uncorrelated and Rayleigh distributed then $E\{H_{i+1} \mid H_i\}$ must be constant and equal to $\sqrt{2\pi m_0}$. However, in this figure it is seen that $E\{H_{i+1} \mid H_i\}$ increases as a function of $H_i$. This means that succeeding wave heights are correlated.

In addition it is seen in Fig 3.3 that theoretical and empirical values are in good agreement with each other. Su et al. (1982) also studied the behaviour of $E\{H_{i+1} \mid H_i\}$ as a function of $H_i$. They found results corresponding to those of Arhan and Ezraty (1978). After Rye (1974) they also made a distinction between wave growth and wave decay without finding significant differences between both cases.

Arhan en Ezraty (1978) also studied the properties of the ratios of the expected values of $H_{i+1}$ and $H_{i+2}$ for a given value of $H_i$. These ratios are denoted by $H_i E\{H_{i+1} \mid H_i\}$ and $H_i E\{H_{i+2} \mid H_i\}$ respectively. Arhan and Ezraty (1978) computed these ratios using spectra from measurements and a theoretical JONSWAP spectrum ($\gamma = 3.3$). The results of these computations are indicated in Fig 3.4. The empirical values are denoted by small dots. The theoretical values (based on a JONSWAP spectrum) are indicated by triangles and crosses. If succeeding wave heights are uncorrelated and Rayleigh distributed then both the expected values of $H_{i+1}/\sqrt{m_0}$ and $H_{i+2}/\sqrt{m_0}$ must be equal to $\sqrt{2\pi} = 2.51$. This is indicated by straight lines in Fig 3.4. From Fig 3.4 it follows that a next wave height $H_{i+1}$ with $H_{i+1}/\sqrt{m_0} > 3$ is more dependent on a previous wave height $H_i$ than a lower wave. This finding is in agreement with the Icelandic saying 'a large waves rarely comes alone'. This saying indicates that especially high waves are dependent on each other, implying groupiness of high waves.

The results of Fig 3.4 also show that the dependence between the succeeding wave heights $H_i$ and $H_{i+2}$ is insignificant.
Joint probability density function \( p(H, H_{i+1}) \).

![Graph showing the joint probability density function](image)

**Figure 3.3** \( E(H_{i+1} | H_i) \) as a function of \( H_i/\sqrt{m_0} \)

![Graphs showing the ratios](image)

**Figure 3.4** \( H_i/\text{E}(H_{i+1} | H_i) \) and \( H_i/\text{E}(H_{i+2} | H_i) \) as a function of \( H_i/\sqrt{m_0} \)
3.3.4 Correlation between succeeding wave heights

Introduction

The dependence between succeeding wave heights is expressed by their correlation coefficient (Rye 1974). This correlation coefficient is computed as:

\[ r_{hh,k} = \frac{1}{\sigma_H^2} \frac{1}{N_w-k} \sum_{i=1}^{N_w-k} (H_i - \bar{H}_m)(H_{i+k} - H_m) \]  

(3.14)

where \( N_w \) is number of wave heights in a record, \( \bar{H}_m \) is the mean wave height, \( \sigma_H \) is the standard deviation of the wave heights, and \( k \), in discrete counting, is the lag between succeeding wave heights in a record.

Measured correlation coefficients

Rye (1974) was the first author who computed the correlation coefficient between successive wave heights. For \( r_{hh,1} \) Rye (1974) found a mean value of 0.24. He also made a distinction between wave growth and wave decay. To make this distinction he used the trend of the mean wave height \( H_m \). \( H_m \) increases for wave growth and decreases for wave decay. The values for \( r_{hh,1} \) for wave growth and wave decay were respectively about 0.30 and about 0.20. The results of Rye (1974) are based on records including 5,400 individual waves.

Dattatri et al. (1977) found a mean value of 0.236 for \( r_{hh,1} \) and a standard deviation of 0.104.

Arhan and Ezraty (1978) computed \( r_{hh,1} \) on the basis of 26,000 waves. They found a mean value of 0.297.

Following Rye (1974), Su et al. (1982) made a distinction between wave growth and wave decay by looking at the trend in \( H_{l3/3} \). For both cases they also analysed the correlation of wave heights separated by a lag up to 4. Their analysis was based on 50,000 waves.

Goda (1983) analysed old long swell with a very narrow spectrum and found surprisingly high values for correlations between succeeding waves with a lag up to 4.

The measured correlation coefficients are listed in Table 3.3.

Computed correlation coefficients

On the basis of a certain theoretical spectrum Arhan and Ezraty (1978) gave values for the correlation coefficients \( r_{hh,1}, r_{hh,2} \) and \( r_{hh,3} \). For this purpose they used their theoretical distribution for \( p(H_i,H_{i+1}) \). See also sections 3.3.2 and 5.2. They used two JONSWAP spectra with \( \gamma = 3.3 \) and 1.0 representing the mean JONSWAP spectrum and the Pierson-
Moskowitz (PM) spectrum. For $r_{hh,1}$ they found respectively 0.298 and 0.163 (See Table 3.3). These results also indicate that $r_{hh,1}$ depends on the spectral shape (See also chapter 5 of this report).

**Remarks**

Comparison of the results of Rye (1974) to those of Arhan and Ezraty (1978) shows a difference in the mean value of $r_{hh,1}$ for wave growth and a JONSWAP spectrum on the one hand and wave decay and a Pierson-Moskowitz spectrum on the other hand. These differences are likely considering the fact that a JONSWAP-spectrum is characteristic for wave growth and a PM-spectrum for a fully developed sea.

Rye (1974) and Su et al. (1982) are the only authors who made a distinction between wave growth and wave decay by looking at the trends in $H_m$ and $H_{1/3}$. Rye (1974) found a clear difference in the measured values for $r_{hh,1}$, whereas Su et al. (1982) didn’t find such a difference. Su et al. (1982) remarked that these different results are possibly caused by the number of waves analyzed.

Su et al. (1982) also remarked that it could be meaningful to make a distinction between wave growth and wave decay in relation with groupiness of wind waves. It is possible that variations in the trend of $H_m$ or $H_{1/3}$ are not sufficient to distinguish between wave growth and wave decay but that the spectral shape is also important.

<table>
<thead>
<tr>
<th></th>
<th>$r_{hh,1}$</th>
<th>$r_{hh,2}$</th>
<th>$r_{hh,3}$</th>
<th>$r_{hh,4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rye (1974) wave growth</strong></td>
<td>0.30</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>wave decay</td>
<td>0.20</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>total</td>
<td>0.24</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
<td>-</td>
</tr>
<tr>
<td><strong>Dattatri et al. (1977)</strong></td>
<td>0.236</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Arhan and Ezraty (1978)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North Sea measurements</td>
<td>0.297</td>
<td>0.051</td>
<td>0.036</td>
<td>-</td>
</tr>
<tr>
<td>JONSWAP spectrum</td>
<td>0.298</td>
<td>0.113</td>
<td>$&lt; 0.01$</td>
<td>-</td>
</tr>
<tr>
<td>PM spectrum</td>
<td>0.163</td>
<td>0.043</td>
<td>$&lt; 0.01$</td>
<td>-</td>
</tr>
<tr>
<td><strong>Su et al. (1982)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>wave growth</td>
<td>0.374</td>
<td>0.066</td>
<td>-0.000</td>
<td>-0.021</td>
</tr>
<tr>
<td>wave decay</td>
<td>0.340</td>
<td>0.070</td>
<td>0.021</td>
<td>0.013</td>
</tr>
<tr>
<td>total</td>
<td>0.329</td>
<td>0.070</td>
<td>0.003</td>
<td>-0.006</td>
</tr>
<tr>
<td><strong>Goda (1983)</strong></td>
<td>0.649</td>
<td>0.351</td>
<td>0.178</td>
<td>0.070</td>
</tr>
</tbody>
</table>

Table 3.3 Measured and computed correlation coefficients between successive wave heights.
3.3.5 Conclusions

1) Succeeding wave heights depend on each other. This dependence is especially clear for relatively high waves \((H_i/\sqrt{m_0}) > 3\).

2) The dependence between succeeding wave heights is decreasing rapidly as the waves become more separated in terms of their index number in a wave record.

3) The value of \(r_{h1} \) depends on the spectral shape, with higher correlations as the spectrum becomes more narrower.
3.4 Model of Kimura

3.4.1 Introduction

Kimura (1980) derived a model for the mean group lengths for the case that succeeding wave heights are correlated. For this he used the concept of a Markov-chain. Correlations of non-succeeding wave heights are not taken into account in this model. Kimura assumed that the joint probability of two succeeding wave heights is given by the 2-dimensional Rayleigh distribution.

3.4.2 The 2-dimensional Rayleigh distribution

The joint p.d.f. of two wave heights $H_1$ and $H_2$ is given by Kimura (without proof or reference) as:

$$p(H_1, H_2) = \frac{\pi^2}{4} \frac{H_1 H_2}{H_m^4 (1 - \kappa^2)} \exp \left[ -\frac{\pi}{4} \frac{H_1^2 + H_2^2}{H_m^2 (1 - \kappa^2)} \right] I_0 \left( \frac{\pi}{2} \frac{\kappa}{1 - \kappa^2} \frac{H_1 H_2}{H_m^2} \right)$$  \hspace{1cm} (3.15)

in which $\kappa$ is a correlation parameter, $H_m$ the mean wave height and $I_0$ the modified Bessel function of order zero. The relationship between $r_{hh,1}$ and $\kappa$ may be computed using Eq (3.15) as a function of $\kappa$:

$$r_{hh,1} = \frac{E(\kappa) - \frac{1}{2} (1 - \kappa^2) K(\kappa) - \frac{\pi}{4}}{1 - \frac{\pi}{4}}$$  \hspace{1cm} (3.16)

in which $E$ and $K$ are the complete elliptic integrals of the second and first kind respectively.

Relation (3.16) between $\kappa$ and $r_{hh,1}$ can be simplified using a series expansion of $E(\kappa)$ and $K(\kappa)$ (Battjes 1974):

$$r_{hh,1} \approx \frac{\pi}{16 - 4\pi} \left( \kappa^2 + \frac{\kappa^4}{16} + \frac{\kappa^6}{64} \right)$$  \hspace{1cm} (3.17)

To compute $\kappa$ explicitly from $r_{hh,1}$, this series is inverted, giving:

$$\kappa^2 \approx r^* - \frac{r^{*2}}{16} - \frac{r^{*3}}{128}$$  \hspace{1cm} (3.18)

in which
Chapter 3 Wave group analysis in terms of individual waves

\[ r^* = \frac{16 - 4\pi}{\pi} r_{mh,1} \]

The relation between \( r_{mh,1} \) and \( \kappa \), according to Eqs (3.16) through (3.18), is shown in Fig 3.5. Assuming that \( r_{mh,1} \) is seldom higher than 0.5 and allowing a maximum error of 1\% in the values of \( r_{hh,1} \) and \( \kappa \), then the approximations (3.17) and (3.18) are useful.

![Graph showing the relation between \( r_{hh,1} \) and \( \kappa \)](image)

Eqs.

\[ r_{mh,1} = \frac{E(\kappa) - \frac{1}{2}(1 - \kappa^2)K(\kappa) - \frac{\pi}{4}}{1 - \frac{\pi}{4}} \]  (3.16)

\[ \kappa^2 = \frac{r^* - r_{hh}^2}{16} - \frac{r^* - r_{hh}^2}{128} \text{ with } r^* = \frac{16 - 4\pi}{\pi} r_{mh,1} \]  (3.18)

\[ r_{bh,1} = \frac{\pi}{16 - 4\pi} \left( \kappa^4 + \frac{\kappa^4}{16} + \frac{\kappa^4}{64} \right) \]  (3.17)

Figure 3.5: Relations between \( r_{bh,1} \) and \( \kappa \)
3.4.3 Group length distribution

To compute the probability of a sequence of high or low waves Kimura (1980) used the following conditional probabilities:

\[
p_{11} = \text{Prob}\{ H_{i+1} \leq H_c \mid H_i \leq H_c \} \tag{3.19}
\]

\[
p_{22} = \text{Prob}\{ H_{i+1} > H_c \mid H_i > H_c \} \tag{3.20}
\]

The probabilities \( p_{11} \) and \( p_{22} \) are computed from the joint p.d.f. of \( H_1 \) and \( H_2 \):

\[
p_{11} = \int \int_{H_1 = 0, H_2 = 0}^{H_1 = H_c, H_2 = H_c} p(H_1, H_2) \, dH_1 \, dH_2 \tag{3.21}
\]

\[
and \quad p_{22} = \int \int_{H_1 = H_1, H_2 = H_c}^{H_1 = 0, H_2 = H_c} p(H_1, H_2) \, dH_1 \, dH_2 \tag{3.22}
\]

The probability of occurrence of a sequence of \( j \) high waves is given by:

\[
P(j) = p_{22}^{j-1} (1 - p_{22}) \tag{3.23}
\]

with mean

\[
\bar{j} = \frac{1}{1 - p_{22}} \tag{3.24}
\]

and standard deviation
\[ \sigma(j_i) = \frac{\sqrt{p_{22}}}{1 - p_{22}}. \]  

(3.25)

Analogous to this, it follows that the probability of occurrence of a sequence of low waves of length \( j \) is equal to:

\[ P_2(j) = (1 - p_{11}) p_{11}^{j-1} \]  

(3.26)

with mean

\[ j_2 = \frac{1}{1 - p_{11}} \]  

(3.27)

and standard deviation:

\[ \sigma(j_2) = \frac{\sqrt{p_{11}}}{1 - p_{11}}. \]  

(3.28)

The probability of occurrence of a total sequence with length \( j \) is equal to

\[ P_3(j) = \frac{(1 - p_{11})(1 - p_{22})}{p_{11} - p_{22}} \left( p_{11}^{j-1} - p_{22}^{j-1} \right) \]  

(3.29)

with mean

\[ j_3 = \frac{1}{1 + p_{11}} + \frac{1}{1 + p_{22}} \]  

(3.30)

and standard deviation

\[ \sigma(j_3) = \left( \frac{p_{22}}{(1 - p_{22})^2} + \frac{p_{11}}{(1 - p_{11})^2} \right)^{1/2}. \]  

(3.31)

Remarks

1) To apply the theory of Kimura, it is necessary to know the correlation parameter \( \kappa \). If \( r_{kk,1} \) is known then \( \kappa \) can be computed using Eq (3.16).

2) For \( r_{kk,1} = 0 \) the model of Kimura is identical to the model of Goda.
3.4.4 Check of the Kimura model

Computer simulations

Kimura (1980) made computer simulations to generate time series of the surface elevation. These sequences were based on spectra with different peakedness. The phases of the wave components were uniformly distributed on the interval \((0, 2\pi]\). His next step was to generate time series of wave heights. These series of wave heights served as a basis for wave group analysis. The correlation coefficients of succeeding wave heights were computed using Eq (3.14). The results of Kimura (1980) are summarized in the Tables 3.4 and 3.5.

Under the heading "sim" the results of the computer simulations of Kimura (1980) are given. The theoretical values for the mean group lengths based on the models of Goda and Kimura are given under the headings "Goda" and "Kimura". In Fig 3.6 some group length distributions based on computer simulations are given. In this figure the dotted line and the solid line represent the theories of Goda and Kimura respectively. There is a good agreement between theory and experiment.

Field measurements

Goda (1983) analysed data of old long swell record collected off the coast of Costa Rica. This swell was characterized by a narrow spectrum and an accordingly high correlation coefficient between succeeding wave heights, see Table 3.3. The measurement results of Goda (1983) are given in Table 3.6 under the heading "data" and are compared with theoretical values based on the theory of Kimura which are given under the heading "Kimura". From Table 3.6 it follows that the mean group length for the group level \(H_{1/3}\) are even higher than the values based on the theory of Kimura. For the group level \(H_{1/3}\) there is a reasonable agreement between the measurements and the theory.

<table>
<thead>
<tr>
<th>(r_{hh,1})</th>
<th>(H_c = H_m)</th>
<th>(H_c = H_{1/3})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sim</td>
<td>Kimura</td>
</tr>
<tr>
<td>0.19</td>
<td>2.20</td>
<td>2.08</td>
</tr>
<tr>
<td>0.23</td>
<td>2.29</td>
<td>2.15</td>
</tr>
<tr>
<td>0.29</td>
<td>2.34</td>
<td>2.28</td>
</tr>
<tr>
<td>0.33</td>
<td>2.42</td>
<td>2.37</td>
</tr>
<tr>
<td>0.38</td>
<td>2.45</td>
<td>2.46</td>
</tr>
</tbody>
</table>

Table 3.4: Mean group lengths for high waves, Kimura (1980)
### Chapter 3: Wave Group Analysis in Terms of Individual Waves

<table>
<thead>
<tr>
<th>$r_{hk1}$</th>
<th>$H_e = H_m$</th>
<th></th>
<th>$H_e = H_{1/3}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sim</td>
<td>Kimura</td>
<td>Goda</td>
<td>sim</td>
</tr>
<tr>
<td>0.19</td>
<td>4.66</td>
<td>4.55</td>
<td>4.03</td>
<td>9.33</td>
</tr>
<tr>
<td>0.23</td>
<td>4.67</td>
<td>4.67</td>
<td>4.03</td>
<td>9.47</td>
</tr>
<tr>
<td>0.29</td>
<td>4.94</td>
<td>4.90</td>
<td>4.03</td>
<td>10.00</td>
</tr>
<tr>
<td>0.33</td>
<td>5.17</td>
<td>5.10</td>
<td>4.03</td>
<td>9.95</td>
</tr>
<tr>
<td>0.38</td>
<td>5.36</td>
<td>5.32</td>
<td>4.03</td>
<td>10.71</td>
</tr>
</tbody>
</table>

Table 3.5: Mean lengths of a total sequence, Kimura (1980)

<table>
<thead>
<tr>
<th>$r_{hk1}$</th>
<th>$H_e = H_m$</th>
<th></th>
<th>$H_e = H_{1/3}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>data</td>
<td>Kimura</td>
<td>Goda</td>
<td>data</td>
</tr>
<tr>
<td>0.630</td>
<td>3.77</td>
<td>3.50</td>
<td>1.84</td>
<td>2.02</td>
</tr>
<tr>
<td>0.688</td>
<td>4.15</td>
<td>3.84</td>
<td>1.84</td>
<td>2.49</td>
</tr>
<tr>
<td>0.694</td>
<td>4.42</td>
<td>3.89</td>
<td>1.84</td>
<td>2.21</td>
</tr>
</tbody>
</table>

Table 3.6: Mean group length high waves, Goda (1983)

Fig. 3.6: Group length distributions, Kimura (1980). Solid line: Kimura model, dashed line: Goda model.
3.5 Conclusions

The results of this chapter are summarized as:

1) Succeeding wave heights are dependent on each other.
2) This dependence increases with wave height.
3) The measure of dependence between non-succeeding wave heights decreases very rapidly as the waves are more separated.
4) The correlation coefficient $r_{hh,1}$ is the governing parameter for the measure of groupiness.
5) The model of Kimura is more realistic than the model of Goda.
4 Envelope theories

4.1 Introduction

This chapter gives a discussion of envelope theories that are used for the description of wave groups. One of these theories was developed by Rice (1944, 1945). This theory was used by Nolte and Hsu (1973) and Ewing (1973) for wave group analysis. More recently a theory was developed by Funke and Mansard (1979, 1980) to construct a wave envelope by using the concept of the "Smoothed Instantaneous Wave Energy History", abbreviated to SIWEH. Both theories will be discussed, where Rice’s theory will receive the most attention.

4.2 Description envelope

The wave crest and wave trough envelopes are constructed by connecting succeeding wave crests and succeeding wave troughs with a smooth curve respectively. The distance between crest envelope and trough envelope is called the height envelope. The distance between the crest envelope and the mean water level is called the amplitude envelope. In Fig 4.1 the amplitude envelope is shown as a smooth curve. The height- and amplitude envelope indicate respectively the variations of the wave height and wave amplitude. However, in reality wind waves don’t always have a symmetrical shape as is suggested by the term 'amplitude' envelope. However, to describe the envelope mathematically, it is assumed that the amplitude envelope is half as great as the height envelope.

Figure 4.1 Envelopes and wave groups
4.3 Definition wave group

Wave groups can be defined in terms of the crest- and through envelopes (Nolte and Hsu, 1973). A wave group is now defined as the part of the amplitude envelope that exceeds a certain level. This level is called the group level and is denoted by $R_e$. The length of a wave group is defined as the time period of the exceedance of the group level. This time period is denoted by $L_1$.

The wave height level $H_e$ and the group level $R_e$ are related by:

$$H_e = 2R_e. \quad (4.1)$$

Wave groups are defined here on the basis of a continuously varying amplitude envelope. This method assumes a narrow spectrum. For this case the amplitude envelope is connected to almost all wave crests and wave troughs.

**Remark**

The length of a wave group has the dimension time. In chapter 3 the length of a wave group was defined in terms of a dimensionless number, viz. the number of wave heights in a wave group. To make $L_1$ dimensionless it is possible to divide $L_1$ by the mean wave period $T_m$. However, as will be explained later, this method has certain disadvantages. Therefore, this method is not adopted here.
4.4 Mathematical description amplitude envelope

Let \( I(t) \) be a Gaussian signal with a narrow energy spectrum \( S(f) \) with a representative middle frequency \( f_m \). Decomposing the signal in Fourier components gives:

\[
I(t) = \sum_{n=1}^{\infty} c_n \cos\left(2\pi(f_n - f_m)t - \phi_n\right)
\]

in which \( \phi_n \) is uniformly distributed on the interval \((0, 2\pi] \), \( c_n = \sqrt{2S(f_n)f} \) and \( f_n = n\Delta f \). Next \( I(t) \) is rewritten as:

\[
\begin{align*}
I_c(t) &= \sum_{n=1}^{\infty} c_n \cos\left(2\pi(f_m - f_n)t - \phi_n\right) \\
I_s(t) &= \sum_{n=1}^{\infty} c_n \sin\left(2\pi(f_m - f_n)t - \phi_n\right)
\end{align*}
\]

The amplitude envelope is now defined as:

\[
R(t) = \left(I_c(t)^2 + I_s(t)^2\right)^{1/2}.
\]

Rice (1944) gave a number of statistical properties of the amplitude envelope. He gave expressions for the distribution function \( P(R) \) and for the probability density function \( p(R) \):

\[
P(R) = 1 - \exp\left(-\frac{R^2}{2m_0}\right)
\]

and

\[
p(R) = \frac{R}{m_0} \exp\left(-\frac{R^2}{2m_0}\right).
\]

Longuet-Higgins (1957) gave an expression for the expected number of up-crossings of the amplitude envelope through the level \( R_c \) per unit of time:

\[
N^*(R_c) = \left(\frac{2\pi\mu_2}{m_0}\right)^{1/2} \frac{R_c}{m_0^{1/2}} \exp\left(-\frac{R_c^2}{2m_0}\right)
\]

in which

\[
m_n = \int f^n S(f) df
\]
and
\[ \mu_2 = m_2 - \frac{m_1^2}{m_0}. \] (4.9)

The mean duration between an up-crossing of the amplitude envelope through the level \( R_c \) and the first down-crossing through this level is given by:
\[ \overline{L}_1 = \frac{P(R_c)}{N'(R_c)} = \left( \frac{1}{2\pi\mu_2} \right)^{1/2} \frac{m_0}{R_c}. \] (4.10)

An approximation of the mean group length \( \bar{j}_1 \) of a wave group follows by multiplying \( \overline{L}_1 \) with \( N_0 \), i.e. the mean number of wave heights per unit of time.

Taking \( N_0 = \sqrt{m_2/m_0} \) gives:
\[ \bar{j}_1 = N_0 \overline{L}_1 = \left( \frac{m_2}{2\pi\mu_2} \right)^{1/2} \frac{m_0^{1/2}}{R_c}. \] (4.11)

**Remarks**

1) Equation (4.11) was given by Ewing (1973). However, this formula is a bad approximation. By using \( \bar{j}_1 = \overline{L}_1 N_0 \), a large discretisation error is introduced. Computations indicate that \( \bar{j}_1 \) can be lower than 1, whereas \( \bar{j}_1 \) must be equal to or greater than 1. (A wave group has always a length greater than or equal to 1.)

2) A difficult problem is the unknown distribution function of the stochastic variable \( L_1 \). Only the mean of \( L_1 \) is known, see Eq (4.10). Nolte and Hsu (1973) assumed that subsequent level crossings \( L_1 \) of the amplitude envelope form a Poisson process, from which it follows that \( L_1 \) is exponentially distributed. This distribution has the mean of \( L_1 \) as free parameter.

3) Goda (1976) discussed the discretisation error introduced by Eq (4.11). He developed a discretisation method to minimize this error. Goda assumed that a wave group with length \( j \) must have a corresponding time period \( L_1 \) satisfies:
\[ (j-1)T_m < L_1 < jT_m \] (4.12)

in which \( T_m \) the mean wave period. The discretisation method of Goda is discussed in section 4.6.

The Poisson model of Nolte and Hsu (1973) and the discretisation method will be discussed in more detail in the next sections.
4.5 Poisson model of Nolte and Hsu

4.5.1 Description of the model

Nolte and Hsu (1973) assumed that the distribution function of the exceedance of the amplitude envelope between two subsequent level crossings of a certain level is given by an exponential distribution. The time period of exceedance is denoted by $L_1$ and the level that is crossed is denoted by $R_e$. Conditions for applying the Poisson-model are:

1) The probability of the envelopes crossing the level $R_e$ in an non-overlapping time interval is independent and invariant of time,

2) the probability of crossing the level $R_e$ in a small time interval is proportional to the size of this interval,

3) the probability of multiple crossings in a small time interval is negligible compared to the probability of a single crossing.

The Poisson-model gives the probability that the time period $L_1$ of the exceedance of the level $R_e$ by the amplitude envelope is smaller than a certain time $t$:

$$P_p(t) = \text{Prob}(L_1 \leq t) = 1 - \exp(-t/L_1)$$ (4.13)

in which $L_1$ is the mean period of level exceedance of level $R_e$ (index $p$ of Poisson). The mean of the stochastic variable $L_1$ may be expressed in the moments of the energy spectrum according to Eq (4.10).

Remark

Goda (1976) remarked that the first condition implies that subsequent wave heights are independent. This condition is not satisfied for a narrow spectrum.

Nolte and Hsu (1973) also gave some other statistical properties of wave groups on the basis of the Poisson model:

1) The mean number of wave groups per unit of time with group level $H_e$ is given by:

$$\nu(H_e) = \frac{\exp(-2H_e^2)}{L_1}.$$ (4.14)
2) Denoting the mean wave period as $T_m$, it follows that the mean number of waves $N_0$ in a time interval $\Delta t$ is equal to $N_0 = \Delta t/T_m$. This implies that the probability of a wave group with more than $N_0$ wave heights that are all greater than $H_e$ is equal to:

$$ p(H_e, N_0) = \exp \left( -\frac{N_0 \Delta t}{L_1} \right). \quad (4.15) $$

3) The probability that $M$ wave groups with group level $H_e$ occur in a time interval with length $L$ is equal to:

$$ P_H(H_e) = \frac{[v(H_e)L]^M}{M!} \exp(-v(H_e)L). \quad (4.16) $$

4) Consider a wave group of $N_0$ waves with wave heights greater than $H_e$, then the probability for a wave height to be lower than a certain height $H$ is given by:

$$ P(H_e, N_0) = \left( 1 - \exp(-2[H^2 - H_e^2]) \right)^{N_0}. \quad (4.17) $$

### 4.5.2 Check of the Poisson model

Nolte and Hsu (1973) used data of measurements in the Gulf of Mexico. On the basis of 900 individual waves they found a good agreement between theory and experiment. This is shown in Fig 4.2 in which the theoretical group length distributions and are compared with their experimental results.
Figure 4.2  Group length distribution, Nolte and Hsu (1973)

Remarks

1) Nolte and Hsu checked their model on the basis of only 900 waves. In their data set 59 wave groups with group level $H_{1/3}$ occurred. This number is too small to draw any firm conclusions.

2) The parameter $L_1$ from the Poisson model of Nolte and Hsu is computed from the energy spectrum according to Eq (4.11) Rye (1977) showed that computations of $L_1$ for a JONSWAP spectrum indicated a strong sensitivity of the value of $L_1$ on the upper integration limit. This implies that $L_1$ is not a good parameter to compute mean group lengths.
4.6 Discretisation method of Goda

4.6.1 The method

Goda (1976) proposed a relation between the length of a wave group in terms of individual waves and the length of a wave group in terms of a time period of a certain level crossing by the amplitude envelope.

Goda assumed that the waves in a wave group have a wave period which is close to the mean wave period $T_m$ of the waves in a record. In addition, he stated that a wave group has length $j$, counting the number of waves higher than $H_e$, if the following relation holds:

$$(j-1)T_m < L_1 < jT_m.$$  \hspace{1cm} (4.18)

In this model, the probability of occurrence of a wave group with length $j$ of a wave group is denoted by $P_d(j)$ ($d$ of discretisation). In terms of the Poisson model $P_p(t)$ this gives:

$$P_d(j) = P_p(T_m) - P_p((j-1)T_m)$$

$$= \exp\left(-(j-1) \frac{T_m}{L_1}\right) - \exp\left(-j \frac{T_m}{L_1}\right)$$

$$= \left(1 - \exp\left(- \frac{T_m}{L_1}\right)\right)\exp^{j-1}\left(- \frac{T_m}{L_1}\right)$$  \hspace{1cm} (4.19)

and for the mean group length $\bar{j}_1$:

$$\bar{j}_1 = \frac{1}{1 - \exp\left(- \frac{T_m}{L_1}\right)}$$  \hspace{1cm} (4.20)

For $L_1 > T_m$, $\bar{j}_1$ is approximated by a series expansion:

$$\bar{j}_1 = \frac{1}{1 - \frac{T_m}{L_1}} = \frac{1}{1 - \frac{1}{2} \left(\frac{T_m}{L_1}\right)^2 + \frac{1}{3!} \left(\frac{T_m}{L_1}\right)^3 + ...}$$

$$\approx \frac{L_1}{T_m} + \frac{1}{2} \left(\frac{T_m}{L_1}\right) + \frac{1}{12} \left(\frac{T_m}{L_1}\right)^3.$$  \hspace{1cm} (4.21)
4.6.2 Remarks

1) Computer simulations (Goda, 1976) show that application of Eq (4.21) to narrow spectra yields too high values for the mean group length.

2) The discretisation method of Goda has some disadvantages concerning the choice of the wave period and the method of discretisation. Both aspects are discussed in the next section.

Wave period

Goda assumed that the mean period of all waves in wave groups, here denoted by \( T_{gm} \), is approximately equal to the mean wave period of all waves in a registration. This assumption, however, is incorrect. If wave heights and wave periods are dependent on each other, it is necessary to account for the joint probability distribution of wave heights and wave periods \( p(H,T) \). To compute the mean wave period \( T_{gm} \), only waves that are higher than the group level \( H_c \) may be used. Using \( p(H,T) \), this gives for \( T_m \) and \( T_{gm} \):

\[
T_m = E\{T\} = \int_0^\infty \int_0^\infty T p(H,T) dHdT
\]

(4.22)

and

\[
T_{gm} = E\{T\} = \int_0^{H_c} \int_0^{T_{gm}} T p(H,T) dHdT
\]

(4.23)

Thompson and Sedivy (1980) analysed the mean wave period of waves in wave groups. This period was denoted by \( T_g \). The values for \( T_g \) were compared to the peak period \( T_p \) of the energy spectrum and to the corresponding group length \( j \). Their results are summarized as:

1) \( T_g \) has a large spread,

2) the expected value of \( T_g \), \( E\{T_g\} = T_{gm} \), satisfies \( T_{gm} = T_p \),

3) especially for long wave groups: \( T_g = T_p \).

Assuming that \( T_m < T_{gm} \) and using \( T_{gm} = T_p \), application of Eq (4.21) yields lower values for the mean group length, instead of using \( T_{gm} = T_m \). This directly follows from the following equation when using \( T_m < T_{gm} \):

\[
\int_0^{H_c} \int_0^{T_{gm}} T p(H,T) dHdT < \int_0^{H_c} \int_0^{T_m} T p(H,T) dHdT
\]
\[
\frac{1}{1 - \exp\left(\frac{T_m}{L_1}\right)} < \frac{1}{1 - \exp\left(\frac{T_m}{L_1}\right)}.
\]

(4.24)

**Discretisation**

Goda (1976) gave a relation between the group length \( j \) (in discreet counting) and the group length \( L_1 \) (in the time domain):

\[(j-1)T_m < L_1 \leq jT_m.
\]

(4.25)

It is easily shown that this relation is incorrect. Taking \( j=1 \), then Eq (4.25) yields \( 0 < L_1 \leq T_m \), whereas \( j \) can also be zero when \( 0 < L_1 \leq T_m \). This is shown in Fig 4.3. This figure shows that there is no wave height \( H_i > 2R_c \) in case wave heights are defined with the zero-up crossing method. For larger values of \( j \) it can be shown that relation (4.25) is at least doubtful.

It is possible that the following relation holds:

\[
\begin{align*}
j = 0 & \quad 0 < L_1 \leq T_m \\
j = 1 & \quad 0 < L_1 \leq 2T_m \\
j = 2 & \quad T_m < L_1 \leq 3T_m \\
j = 3 & \quad 2T_m < L_1 \leq 4T_m \\
& \quad \vdots \\
j = n & \quad (n-2)T_m < L_1 \leq nT_m
\end{align*}
\]

However, this introduces a problem: in case \( L_1 = 0.5T_m \), the corresponding group length \( j \) is undefined, is it equal to, zero or one? How often \( j = 0 \) or \( j = 1 \) occurs is then a statistical problem. The solution of this problem is not the purpose of this study and will therefore not be discussed.
Figure 4.3  Illustration of discretisation error
4.7 Group analysis in terms of the SIWEH

4.7.1 Description of method

As a result of laboratory experiments to simulate wave groups in wind waves, Funke and Mansard (1979, 1980) introduced the concept of the 'Smoothed Instantaneous Wave Energy History', abbreviated to SIWEH. The SIWEH is computed by averaging the square of the surface elevation $\eta(t)$ over a time interval with a length of twice the peak period of the spectrum. The SIWEH is a measure of the distribution of the wave energy as a function of time and is computed by:

$$ E(t) = \frac{1}{T_p} \int_{-\infty}^{\infty} \eta^2(t+\tau)Q_k(\tau) d\tau $$

(4.26)

in which $Q_k$ is a window function. For this Funke and Mansard (1979) used the so-named Bartlet-window:

$$ Q_k(\tau) = \begin{cases} 
1 - \frac{|\tau|}{T_p} & \text{for } |\tau| \leq T_p \\
0 & \text{for } |\tau| > T_p 
\end{cases} $$

(4.27)

in which $T_p$ the peak period of the spectrum.

In addition, they also introduced the Groupiness Factor ($GF$) to quantify the measure of wave grouping:

$$ GF = \frac{1}{m_0} \sqrt{\frac{1}{t_0} \int_{t_0}^{t_0} [E(t) - E_m]^2 dt} $$

(4.28)

in which $m_0$ is the variance of $\eta(t)$, $t_0$ the length of the measurement and $E_m$ the mean of $E(t)$. The Groupiness Factor $GF$ is equal to the standard deviation of $E(t)$ normalized with the variance of $\eta(t)$.

Field measurements in seas and swell gave values of 0.46-1.0 for $GF$ (Funke and Mansard (1980), Rye (1982) and Goda (1983)).
4.7.2 Definition of a wave group

The energy function $E(t)$ approximates the shape of the amplitude envelope reasonably well and may also be used to recognize wave groups by observing the places where the function $E(t)$ exceeds a certain level. Thompson and Sedivy (1980) used the function $E(t)$ to localize the beginning and end of wave groups. However, the actual wave group analysis was performed in terms of individual waves.

Usefulness of the SIWEH

The practical importance of the 'Groupiness Factor' was analysed by Goda (1983). Goda gave the following remarks:

1) $GF$ shows a large spread for field measurements,

2) there is a weak correlation between $GF$ and the peakedness parameter $Q_p$ of the energy spectrum defined by Goda (1983), see also section 5.1,

3) there is no correlation between $GF$ and $r_{hk,1}$,

4) $GF$ is not sensitive enough to distinguish the measure of wave grouping in seas and in long-travelled swell,

5) $GF$ is useful to describe the measure of wave grouping in simulated wave trains, but not in field measurements.
4.8 Concluding remarks

The following concluding remarks can be given with respect to envelope theories of wave groups.

1) The Poisson-model of Nolte and Hsu (1973) is historically seen the first model in which the group length distribution is coupled to the spectral shape.

2) The group length distributions addressed in this report all have the same algebraical structure. All distributions are described by one parameter. The probability of a group length $j$ is denoted by $P_j(j)$:

$$P_j(j) = B^{j-1}(1-B)$$  \hspace{1cm} (4.29)

Goda (1970) assumed independence between succeeding wave heights and found:

$$B = p = \text{Prob}\{H>H_c\}$$  \hspace{1cm} (4.30)

Goda (1976) gave a group length distribution based on the Poisson-model and his discretisation method:

$$B = \exp\left(\frac{T_m}{L_1}\right) = \text{Prob}\{L_1>T_m\}.$$  \hspace{1cm} (4.31)

Kimura (1980) gave a distribution based on a dependence between succeeding wave heights:

$$B = p_{22} = \text{Prob}\{H_{i+1}>H_c \mid H_i > H_c \}.$$  \hspace{1cm} (4.32)

3) The Groupiness Factor $GF$ is not suitable for wave group analysis of wind waves.
5 Wave grouping and spectral shape

5.1 Introduction

As expected, a relation exists between the spectral shape and the amount of wave grouping. The envelope varies slower for a narrow spectrum than for a broad spectrum, implying longer wave groups. This relationship was noticed already by Carstens et al. (1966) when they were analysing the stability of breakwaters. They used two different spectra to simulate wave trains in a laboratory. The spectra were a measured spectrum near Berlevåg in the Barents Sea (B-spectrum) and a theoretical Neumann spectrum (N-spectrum). The B-spectrum was narrower and more peaked than the N-spectrum (see Fig. 5.1).

![Figure 5.1 Berlevåg and Neumann spectrum](image)

Carstens et al. (1966) noticed that the B-waves contained more wave groups than the N-waves. Referring to the remarks made in section 3.2.4 it is clear that the mean group length of B-waves is higher than the mean group length of the N-waves. Goda (1983) described the occurrence of wave groups in old long-travelled swell with a very narrow spectrum. The mean group lengths were very large. Goda also found high values for the correlation coefficient between succeeding wave heights and even for non-succeeding wave heights.

In order to quantify the peakedness of the spectrum Goda (1970), introduced the peakedness factor $Q_p$:

$$Q_p = \frac{2}{m_0^2} \int_0^\infty s^2(f) df.$$  \hspace{1cm} (5.1)
Although $Q_p$ has no mathematical foundation in relation to wave grouping it is frequently used by many authors. Computer simulations of Goda (1970) and Kimura (1980) showed that the measure of wave grouping and the peakedness factor $Q_p$ are related. The mean group length increases with increasing $Q_p$. From these findings Goda (1976) concluded that the measure of wave grouping depends on the spectral shape.

Goda (1976) and Dattatri et al. (1977) computed the mean group length $\bar{J}_1$ and the peakedness factor $Q_p$ from field measurements. They found a large scatter for related values of $\bar{J}_1$ and $Q_p$. However, Goda (1980) found a smaller spread when using computer simulations.

Yamaguchi (1979) used a large number of measurements from which he computed the group length distribution and the peakedness factor $Q_p$. Each measurement was classified by looking at the value of $Q_p$. The classes used by Yamaguchi are given in Table 5.1.

The group length distributions of all measurements within the same class of $Q_p$ were combined into a new distribution. For each class of $Q_p$, the mean group lengths were computed. The wave group levels $H_m$, $H_{med}$ and $H_{1/3}$. His results are presented in Table 5.1. This table shows that the mean group length increases with increasing $Q_p$.

<table>
<thead>
<tr>
<th>Group level</th>
<th>$H_{med}$</th>
<th>$H_m$</th>
<th>$H_{1/3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.75 &lt; Q_p \leq 2.25$</td>
<td>2.475</td>
<td>2.400</td>
<td>1.372</td>
</tr>
<tr>
<td>$2.25 &lt; Q_p \leq 2.75$</td>
<td>2.717</td>
<td>2.633</td>
<td>1.498</td>
</tr>
<tr>
<td>$2.75 &lt; Q_p \leq 3.25$</td>
<td>2.929</td>
<td>2.848</td>
<td>1.629</td>
</tr>
<tr>
<td>$3.25 &lt; Q_p \leq 3.75$</td>
<td>2.964</td>
<td>3.006</td>
<td>1.651</td>
</tr>
</tbody>
</table>

Table 5.1 Mean group lengths as a function of group level and $Q_p$ (after Yamaguchi 1979).

Asymptotic formulas of Ewing

Ewing (1973) gave asymptotic expressions for $\bar{J}_1$ and $\bar{J}_3$ for a very narrow spectrum with large $Q_p$ and for a very broad spectrum. For a very narrow spectrum (assuming a Gaussian shape) he found:

$$\bar{J}_1 = \frac{Q_p \sqrt{m_0}}{\sqrt{2} \, R_c} \quad (5.2)$$
and

\[ \bar{j}_3 = \frac{Q_p \sqrt{m_0}}{\sqrt{2} R_c} \exp \left( \frac{R_c^2}{2m_0} \right) \]  \hspace{1cm} (5.3)

and for a very broad spectrum:

\[ \bar{j}_1 = 1 \]  \hspace{1cm} (5.4)

and

\[ \bar{j}_3 = \exp \left( \frac{R_c^2}{2m_0} \right) \]  \hspace{1cm} (5.5)
5.2 Spectral computation of correlation coefficients

5.2.1 Introduction

As was remarked in section 3.3.2, Arhan en Ezraty (1978) developed a theory to compute the correlation coefficient between succeeding wave heights from a given spectrum. They used some theories developed by Rice (1944, 1945) concerning the statistical properties of the amplitude envelope of a Gaussian process with a narrow spectrum. An important part of this theory is the joint p.d.f. of two values of the amplitude envelope separated by a time interval $\tau$. This distribution is also known as the 2-dimensional Rayleigh distribution. The method given by Arhan and Ezraty for the computation of the correlation coefficient $r_{AA,1}$ is unnecessarily complicated. As was remarked by Battjes (1974) it is possible to compute the correlation coefficient $r_{AA,1}$ rather easy from a given spectrum. This is achieved by using the theory of Rice (1943), results given by Uhlenbeck (1943) for this distribution and an approximation given by Battjes (1974). The theories mentioned above will be reviewed separately. Finally it will be shown how these theories can be combined to compute $r_{AA,1}$ from the spectrum.

5.2.2 Theories of Rice and Uhlenbeck

Rice (1944,1945) and Uhlenbeck (1943) computed independently of each other the joint probability density function (p.d.f.) of values of the amplitude envelope of a Gaussian process at two different times, separated by a time lag $\tau$. These two values are denoted by $R(t)$ and $R(t+\tau)$. In the following these two are denoted by $R(t) = R_1$ and $R(t+\tau) = R_2$. The joint p.d.f. of $R_1$ and $R_2$ is given by:

$$p(R_1, R_2) = \frac{\pi^2}{4} \frac{R_1 R_2}{R_m^4(1 - \kappa^2)} \exp\left[-\frac{\pi}{4} \frac{R_1^2 + R_2^2}{R_m^2(1 - \kappa^2)}\right] I_0\left(\frac{\pi \kappa R_1 R_2}{2(1 - \kappa^2) R_m^2}\right)$$

(5.6)

in which $\kappa$ is a correlation parameter, $R_m$ is the mean value of the amplitude envelope and $I_0$ is the modified Bessel function of order zero. $\kappa^2$ is the coefficient of linear correlation between $R_1$ and $R_2$. The parameter $\kappa$ satisfies:

$$0 \leq \kappa^2 \leq 1$$

(5.7)

The parameter $\kappa$ may be computed from the spectrum according to:

$$m_0^2 \kappa^2 = \left[\int_0^{\infty} S(f) \cos(2\pi(f_m - f)\tau) df\right]^2 + \left[\int_0^{\infty} S(f) \sin(2\pi(f_m - f)\tau) df\right]^2$$

(5.8)
in which $f_m$ is some representative mean frequency. Battjes (1974) showed that Eq (5.8) can also be written as:

$$m_0^2 \kappa^2 = \int_0^\infty \int_0^\infty S(f_1)S(f_2) \cos\left(2\pi(f_1 - f_2)\tau\right) df_1 df_2. \quad (5.9)$$

It then follows that the integrand is independent of $f_m$. Rewriting Eq (5.8) with $f_m = 0$ gives:

$$m_0^2 \kappa^2 = \left[ \int_0^\infty S(f) \cos(2\pi f \tau) df \right]^2 + \left[ \int_0^\infty S(f) \sin(2\pi f \tau) df \right]^2. \quad (5.10)$$

Uhlenbeck (1943) showed that the correlation coefficient $r$ between $R_1$ and $R_2$ can be computed from Eq (5.6) as:

$$r = \frac{E(\kappa) - \frac{1}{2}(1 - \kappa^2)K(\kappa) - \frac{\pi}{4}}{1 - \frac{\pi}{4}} \quad (5.11)$$

in which $E$ and $K$ are the complete elliptic integrals of the second and first kind respectively. As was described in section 3.4.2, this relation may be approximated by a series expansion (Battjes, 1974):

$$r = \frac{\pi}{16 - 4\pi} \left( \kappa^2 + \frac{\kappa^4}{16} + \frac{\kappa^2}{64} \right), \quad (5.12)$$

which greatly simplifies the computation of $r$.

### 5.2.3 Theory of Arhan and Ezraty

Arhan and Ezraty (1978) computed correlation coefficients between succeeding wave heights for a given energy spectrum. They used the joint p.d.f. of two values of the amplitude envelope separated by a time lag $\tau$ as given by Rice (1944, 1945), see Eq (5.6). Subsequently, they computed the auto-correlation function of the amplitude envelope from the following integral expression:

$$r(\tau) = \int_0^\infty \int_0^\infty (R_1 - R_m)(R_2 - R_m)p(R_1, R_2) dR_1 dR_2 \quad (5.13)$$
in which $R_m$ is the mean value of the amplitude envelope $R(t)$. The auto-correlation function was made dimensionless by dividing it by the variance of $R(t)$:

$$\rho(\tau) = \frac{r(\tau)}{\sigma_R^2}. \quad (5.14)$$

For a Gaussian process with a narrow spectrum $R(t)$ has a Rayleigh distribution:

$$p(R) = \frac{R}{m_0} \exp \left( -\frac{R^2}{2m_0} \right). \quad (5.15)$$

For the values of $R_m$ and $\sigma_R^2$ it follows:

$$R_m = \sqrt{\frac{\pi}{2} m_0} \quad (5.16)$$

and

$$\sigma_R^2 = 2m_0 \left( 1 - \frac{\pi}{4} \right). \quad (5.17)$$

To compute the correlation coefficient $r_{hh,1}$ between two succeeding wave heights they substituted $\tau = T_m$ (the mean zero-crossing period) into the normalized auto-correlation function. Similarly, it is also possible to compute the correlation coefficients $r_{hh,2}$ and $r_{hh,3}$ from Eq (5.14) with $\tau = 2T_m$ and $\tau = 3T_m$ respectively. The auto-correlation function is a monotonous decreasing function of $\tau$. 


5.2.4 Synthesis

The correlation coefficient $r_{hh,1}$ may be computed from the energy spectrum in the following way. First, compute $\kappa$ from the energy spectrum with:

$$m_0^2 \kappa^2 = \left[ \int_0^\infty S(f) \cos(2\pi f\tau) \, df \right]^2 + \left[ \int_0^\infty S(f) \sin(2\pi f\tau) \, df \right]^2$$  \hspace{1cm} (5.18)

in which $\tau = T_m = \sqrt{(m_0/m_2)}$ (the mean wave period). Next, $r_{hh,1}$ is computed from $\kappa$ with:

$$r_{hh,1} = \frac{\pi}{16 - 4\pi} \left( \kappa^2 + \frac{\kappa^4}{16} + \frac{\kappa^6}{64} \right)$$  \hspace{1cm} (5.19)

So, the correlation coefficient $r_{hh,1}$ can easily be computed from the spectrum.

To conclude this section it is remarked that Kimura mentioned the 2-dimensional Rayleigh distribution for $p(H_1,H_2)$ and Eq (5.11) for $r_{hh,1}$, however, without referring to the work of Rice or Uhlenbeck. Neither does he state that $r_{hh,1}$ can be computed from the energy spectrum via $\kappa$. Instead of this he used numerical simulations in the time domain.
5.3 The role of the phase spectrum

In the description of the sea surface often use is made of the Gaussian model. In this description it is assumed that the sea surface consists of the sum of a large number of sinusoidal wave components with different frequencies, phases and amplitudes. The wave spectrum is used to indicate the distribution of energy over the frequencies. The phases of these components are assumed to be distributed uniformly on the interval \((0, 2\pi]\) and to be independent of each other.

However, some authors have the opinion that a phase controlled spectrum is necessary for a good description of wave groups. (e.g. Johnson et al., 1978 and Burcharth, 1978). In order to investigate this, they generated various wave trains having the same energy spectrum but with different phase spectra. The generated wave trains were different with respect to the amount of groupiness.

Such experiments may indicate the role of the phase spectrum but they give no answer to the question which type of phase spectrum occurs at wind generated waves in nature.

Rye and Lervik (1981) investigated the role of the phase spectrum in relation with wave grouping for wind waves. For 120 field measurements they computed the energy- and phase spectra. The phase spectra were replaced by spectra in which the phases were independent and uniformly distributed on the interval \((0, 2\pi]\). The phase- and energy spectra were then back-transformed to the time domain. The time series thus obtained were investigated on their group statistics. They found no difference in the mean value and standard deviation of the group lengths between the original and re-computed time series although the individual group parameters per time series were different.

On the basis of these results Rye and Lervik (1981) concluded that the phases of the spectral components for wind waves can be considered as independent and uniformly distributed on the interval \((0, 2\pi]\). Similar conclusions were drawn by Goda (1983).
5.4 Conclusions

1) The measure of wave grouping depends on the spectral shape. Narrow spectra have a larger amount of wave grouping than broad spectra.

2) There is a theoretical basis for a quantitative spectral shape parameter ($\kappa$) which is a measure for wave grouping. Values of the correlation coefficient $r_{M,1}$, computed from the spectrum, are realistic for wind waves (Arhan and Ezraty, 1978).

3) It has been shown empirically that the spectral peakedness factor $Q_p$ of the energy spectrum is useful to quantify the measure of wave grouping. Field measurements show that the measure of wave grouping increases with increasing $Q_p$ although with a large spread. (See Goda, 1970; Goda, 1983; Dattatri et al., 1977; Yamaguchi, 1977 and Su et al., 1982). However, the peakedness factor $Q_p$ has no theoretical foundation in relation to wave grouping.

4) For wind waves the phases of the spectral components can be considered as mutually independent and uniformly distributed on the interval $(0, 2\pi]$. 
6 Conclusions and advice

6.1 Conclusions

The results of this preliminary investigation into the characteristics of wave groups in seas and swell can be summarized in the following conclusions:

1) Knowledge of wave groups is important for coastal- and offshore- engineering.

2) From a historical viewpoint the Poisson model of Nolte and Hsu is the first model relating the group length distribution to the spectral shape.

3) The concept of the SIWEH is not important for wave group analysis of wind waves.

4) Succeeding wave heights cannot be regarded as independent, implying that the model of Goda is not useful.

5) The model of Kimura, which assumes dependence between succeeding wave heights, is promising. Kimura's theory gives better results than Goda's theory. The correlation coefficient between succeeding wave heights determines the group length distribution.

6) The dependency between succeeding wave heights decreases when the wave heights become more separated. The influence of correlations between non-succeeding wave heights on the group length distribution is not clear.

7) The relation spectral shape-wave grouping can be expressed via the spectral shape parameter $\kappa$. This relation has a theoretical basis. The frequently used spectral peakedness factor $Q_p$ lacks this basis.

8) The correlation coefficient between succeeding wave heights can easily be computed from the energy spectrum.

9) For the analysis of wind waves with respect to wave group statistics, the phases of the spectral components can be viewed as mutually independent and uniformly distributed on the interval $[0, 2\pi]$. 
6.2 Advice for future work

It is found that the model of Kimura gives the best results to describe wave grouping of wind waves. It is therefore logical to study this model in more detail. An empirical check of Kimura’s model is desirable, since Kimura only used computer simulations.

Many authors analysed a limited number of characteristics of the group length deviation. They often studied the mean group length and the standard distribution of the mean group length. Only a few authors studied the shape of the group length distribution (See for instance Goda, 1976). The shape of the group length distributions was judged visually.

The usage of the terms "good" or "very good" should be omitted if it is not clear which criterion is used. It is therefore recommended to qualify the characteristics of the group length distributions with (objective) statistical methods.

Finally, is it suggested that a study of Kimura’s model should consist of the following parts:

1) An empirical check of Kimura’s model against field data. The measured and theoretical mean group lengths and group length distributions should be compared using statistical tests.

2) Comparison of the two methods to compute the correlation coefficient, from the spectrum or from a sequence of wave heights.

3) Investigation of the influence between correlations of non-succeeding wave heights on the group length distribution.

Remark

A few months before this work was finished the publication of Goda (1983) appeared. In this report the model of Kimura was checked against field measurements. However, these measurements are very special because they cover a narrow and high range of correlation coefficients. The measurements which were analyzed by Goda consist of old long-travelled swell with a very narrow spectrum and high correlation coefficients.

Some remarks of Goda (1983) have been adopted in this report whenever possible and necessary.
Appendix A

References


Appendix A

References


Thompson, W.C. and D.G. Sedivy, 1980: Statistical characteristics of ocean wave groups, Preprint 17th. Int. Conf. on Coastal Eng., 163-164.


Appendix B

List of symbols

c_g  group velocity of short waves
E(t)  energy function used in SIWEH
E( )  complete elliptical integral of the second kind
E(x)  expected value of stochastic variable x
E_m  mean value of SIWEH energy function
f    frequency
f_m  characteristic mean frequency
GF   Groupiness Factor
H    wave height
H_c  group level in terms of wave height
H_m  mean wave height
H_{med} median of wave height distribution
H_{1/3} significant wave height, mean of highest third part
I_0()  modified Bessel function of order zero
j_1   length of a sequence of high waves
\bar{j}_1 mean value of j_1
j_2   length of a sequence of low waves
\bar{j}_2 mean value of j_2
j_3   length of a total sequence
\bar{j}_3 mean value of j_3
k    lag between succeeding wave heights in discrete counting
K( )  complete elliptical integral of the first kind
L_1   time duration exceedance amplitude envelope
\bar{L}_1 mean value of L_1
m_n   n-th moment of S(f) relative to f = 0
N^*   mean number of upward crossings through level \( R_e \) of the amplitude envelope per unit of time.
N_g   number of wave groups in a record
N_w   number of individual waves in a record.
p    probability \( \{H>H_c\} \)
p(x)  probability density function of x
P(x)  distribution function of x
Appendix B

List of symbols

\( p(x,y) \) joint probability density function of \( x \) and \( y \)

\( P_{11} \) conditional probability

\( P_{22} \) conditional probability

\( q \) probability \( \{H \leq H_c\} \)

\( Q_p \) spectral peakedness factor

\( Q_k() \) time function of Bartlet-window

\( r(t) \) auto-correlation function of amplitude envelope

\( R(t) \) amplitude envelope as a function of \( t \)

\( R_c \) group level in terms of the amplitude envelope

\( r_{hh,k} \) correlation coefficient of succeeding wave heights separated by a lag \( k \)

\( S(f) \) spectral density

\( t \) time

\( T \) wave period

\( T_{gm} \) mean wave period of all waves inside wave groups

\( T_m \) mean wave period of all waves

\( T_p \) peak period energy spectrum

\( \chi \) stochastic variable

\( \Delta f \) frequency interval

\( \gamma \) peak enhancement parameter of JONSWAP spectrum

\( \phi_n \) random phase

\( \kappa \) correlation parameter of 2-dimensional Rayleigh distribution

\( \eta \) surface elevation

\( \rho(\tau) \) normalised auto-correlation function of amplitude envelope

\( \mu_n \) n-th moment energy spectrum relative to \( f_m \)

\( \pi \) circular constant

\( \sigma \) standard deviation

\( \tau \) time interval
Appendix C

Statistical properties of group length distributions

The probability on a sequence of high waves with length \( j \) is given by:

\[
P_1(j) = p^{j-1}(1-p).
\]  \( \text{(C1)} \)

The mean group length \( \bar{j}_1 \) is computed with:

\[
E(j) = \bar{j}_1 = \sum_{j=1}^{\infty} jP_1(j) = \sum_{j=1}^{\infty} j p^{j-1}(1-p)
\]  \( \text{(C2)} \)

Elaboration of the summations yields:

\[
\bar{j}_1 = (1-p)[1 + p + p^2 + ...]
\]

\[
 pj_1 = (1-p)[p + 2p^2 + ...]
\]

\[
(1-p)\bar{j}_1 = (1-p)[1 + p + p^2 + ...]
\]

which simplifies to:

\[
\bar{j}_1 = 1 + p + p^2 + p^3 + ...
\]

\[
 pj_1 = p + p^2 + p^3 + ...
\]

\[
(1-p)\bar{j}_1 = 1
\]

The mean group length is given by:

\[
\bar{j}_1 = \frac{1}{1-p}.
\]  \( \text{(C3)} \)

The variance of the group length is defined as:

\[
\sigma^2(j_1) = E\{j^2\} - E(j)^2.
\]  \( \text{(C4)} \)

The term \( E\{j^2\} \) is defined as:

\[
E\{j^2\} = \sum_{j=1}^{\infty} j^2 p^{j-1}(1-p).
\]  \( \text{(C5)} \)
Elaboration of the summations gives:

\[
\begin{align*}
E\{j^2\} &= (1-p)[1 + 4p + 9p^2 + 16p^3 + \ldots] \\
pE\{j^2\} &= (1-p)[p + 4p^2 + 9p^3 + \ldots]
\end{align*}
\]

which simplifies to:

\[
\begin{align*}
E\{j^2\} &= 1 + 3p + 5p^2 + 7p^3 + \ldots \\
pE\{j^2\} &= p + 3p^2 + 5p^3 + \ldots \\
(1-p)E\{j^2\} &= 2(1 + p + p^2 + \ldots) - 1 \\
&= \frac{2}{1-p} - 1
\end{align*}
\]

yielding:

\[
E\{j^2\} = \frac{1+p}{(1-p)^2}.
\] (C6)

Subsequently, the variance is computed as:

\[
\sigma^2(j_i) = \frac{p}{(1-p)^2}
\] (C7)

and the standard deviation as:

\[
\sigma(j_i) = \frac{\sqrt{p}}{1-p}.
\] (C8)

The probability of occurrence of a total run with length \(j\) was given in section 3.2.2 as:

\[
P_3(j) = \frac{pq}{p-q} (p^{j-1} - q^{j-1})
\] (C9)

This expression will be proved using inductive reasoning. A total sequence with length \(j\) consists of \(k\) high wave heights followed by \((j-k)\) low wave heights. The value of \(k\) varies from 1 to \((j-1)\). A total run may consist of \((j-1)\) different sequences. The probability of a total sequence with length \(j\) is the sum of the individual probabilities of a sequence of \(k\) high and \((j-k)\) low wave heights for \(k = 1, \ldots, (j-1)\). The probability \(P_3(j)\) may be computed as:

\[
P_3(j) = \sum_{k=1}^{j-1} p^k q^{j-k}.
\] (C10)
Appendix C

Statistical properties of group length distributions

It must be proved that:

\[ P_3(j) = \sum_{k=1}^{j-1} p^k q^{j-k} = \frac{pq}{p-q} (p^{j-1} - q^{j-1}) \]  

(C11)

This statement is valid for \( j=2 \):

\[ P_3(j) = pq = \frac{pq}{p-q} (p-q). \]  

(C12)

It must be shown that Eq (C11) is also valid for every \( j \) with \( j \geq 2 \). That is possible by elaboration of Eq (C10) for \( j+1 \). This yields:

\[
P_3(j+1) = \sum_{k=1}^{j+1} p^k q^{j+1-k} \\
= q \sum_{k=1}^{j} p^k q^{j-k} + p^{j+1} \\
= q \frac{pq}{p-q} (p^{j-1} - q^{j-1}) + p^{j+1} \\
= \frac{pq}{p-q} (p^{j+1} - q^{j+1}) + \frac{pq}{p-q} p^{j-1} (p-q) \\
= \frac{pq}{p-q} (p^j - q^j). 
\]

This finishes the proof.

For \( p = q = 0.5 \), \( P_3(j) \) satisfies:

\[
P_3(j) = \sum_{k=1}^{j-1} p^k q^{j-k} = \sum_{k=1}^{j-1} p^j = (j-1)p^j 
\]

(C13)

The mean total run length is the sum of the mean sequence length of a sequence of high and a sequence of low wave heights:

\[ \bar{j}_3 = \bar{j}_1 + \bar{j}_2 = \frac{1}{1-p} + \frac{1}{p}. \]  

(C14)

The variance of the total sequence lengths is the sum of the individual variances:

\[ \sigma^2(j_3) = \sigma^2(j_1) + \sigma^2(j_2) = \frac{p}{(1-p)^2} + \frac{1-p}{p^2}. \]  

(C15)