CHAPTER 16

WIND EFFECT ON PRE-EXISTING WAVES

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ABSTRACT

Wind action on pre-existing waves is examined experimentally in a wind-wave channel, in which the pre-existing waves are generated mechanically by a paddle at the windward end of the channel. Air is blown over these waves, in the direction of wave travel, at velocities up to about 30 feet per second, with the formation of wind-generated waves essentially suppressed by addition of a wetting agent to the water. Wind shear forces on these regular wave surfaces are deduced for a variety of wave geometries.

INTRODUCTION

The ability to reliably determine the force exerted by a wind on a disturbed water surface is of extreme importance in a variety of oceanographic and coastal engineering applications. The prediction of storm tides and the effect of winds on swell arriving from a different storm area are but two examples of such problems. Many investigators have presented wind stress information from both field and laboratory studies. Wilson (1960) has very capably summarized the state of the art and has coordinated much of the significant available data, arriving at the following result. In the expression

$$\tau_s = Cd \rho V^2$$

where $\tau_s$ is the wind shear stress, $\rho$ is the density of air, and $V$ is the wind velocity at a height of 10 meters above the sea surface,

$$C_d = 0.0024 \pm 0.0005$$ for "strong" winds

and

$$C_d = 0.0015 \pm 0.0008$$ for "light" winds.

"Light" winds refer to velocities up to about 10 - 15 miles per hour. Wilson concluded that although the consensus of adjusted results appeared satisfactory for the case of strong winds, the light wind situation left much to be desired.

The purpose of the investigation reported here is to attempt to obtain some physical insight into wind action on disturbed water surfaces by fundamentalizing the experiment to the extent that only regular wave trains are considered. That is, to the extent possible, wind forces are examined only on smooth, mechanically generated waves which are uncluttered by additional wind-generated waves. Experiments have been reported, e.g.,
Motzfield (1937), on wind forces on solid surfaces formed into wave configurations. It is felt, however, that the wind-produced surface currents constitute an integral part of the picture and that the use of actual water waves under controlled conditions would be purposeful. Francis (1951) has investigated wind effect on mechanically produced waves, but his results included the effect of wind-generated wavelets.

**METHOD OF APPROACH**

**VELOCITY DISTRIBUTION**

Wind shear stresses are customarily determined by one of two methods. The first entails obtaining a wind velocity profile and employing appropriate Karman-Prandtl logarithmic velocity distribution equations to deduce the friction velocity $U_f$ and hence the surface shear stress. This procedure did not appear to be suitable in this work, and the reason is suggested in figure 3. Included here is a velocity profile over a surface of large waves. It is seen that the maximum velocity occurs fairly close to the surface. The more symmetrical distribution corresponding to a smooth water surface can also be seen in figure 3. Therefore the available region of logarithmic velocity distribution is, in many cases, quite small, often of approximately the same length as the wave height. This behavior, coupled with a degree of uncertainty as to the proper reference elevation for velocity measurements, appeared to create an uninviting climate for this type of analysis. For smoother surfaces, of course, this procedure would be satisfactory, provided that a sufficient number of vertical velocity traverses were made.

**SETUP MEASUREMENT**

A second method of determining shear stress involves the measurement of the wind setup, or wind tide. This is a procedure particularly adaptable to laboratory channels. It can be shown (see, for example, the derivation of Keulegan (1951)) that in a two-dimensional channel of depth, $d$, and finite length

$$
\tau_s + \tau_b = \rho gd \cdot \frac{S}{X}.
$$

Here $\tau_s$ is the shear stress on the surface due to wind, $\tau_b$ is the oppositely directed shear stress along the bottom, due to return flow, $\rho$ is density of the liquid, $g$ is gravitational acceleration and $S$ is the setup occurring over a distance $X$ along the channel. Equation (2) is derived for the condition that $\tau_s$ represents the only active force at the air-water interface, that is, there is no ambient pressure gradient. Also implicit in the solution is the absence of significant changes in kinetic energy along the length of the channel.
It is seen that the result of the setup measurement yields the total of \( \tau_w + \tau_h \), although it is \( \tau_h \) alone that is of prime interest. There exists apparently no analytical way of separating them, except for the case of laminar motion, and the question arises as to whether, in a particular channel, \( \tau_w \) (and side wall stresses) are negligible relative to \( \tau_h \). This question has been considered at some length by Keulegan (1951). Also, Francis (1951), working with a small channel (7.5 cm wide) concluded from internal velocity measurements, that the error in ignoring \( \tau_w \) is only about 3 percent. This order of error would certainly be acceptable, although the additional shear stresses in the present investigation, created by the wave motions, could possibly affect this error. A more important error, however, may be introduced by the fact that some fraction of the force exerted on the wavy surface can reasonably be expected to be manifested in an increase in wave height rather than in an increase in setup. Primarily for this last reason, the procedure described in the following paragraphs was selected.

**MOMENTUM ANALYSIS**

If a wind-wave channel is available which is sufficiently long for fully developed "pipe" flow to occur in a selected length of the air passage, then a momentum analysis of the air flow can yield the desired water surface shear force. Reference to figure 1 shows that this momentum equation between upstream and downstream sections 1 and 2, respectively, a distance \( X \) apart, can be written as

\[
(p_1 h_1 B - p_2 h_2 B) - \tau_0 X [B + 2h_{ave}] - f - \frac{(p_1 + p_2)}{2} BX \left( \frac{s}{h} \right) - \tau_w BX = \Delta \text{Momentum. (3)}
\]

Here \( B \) is the constant channel width; \( h_1 \) and \( h_2 \) are air passage heights, and \( p_1 \) and \( p_2 \) are static pressures at sections 1 and 2; \( \tau_0 \) is the average shear stress around the top and dry side walls; \( f \) is an additional, or correction, force directed upstream and associated with projecting instrumentation, misalignments in wall and top panels, etc.; \( \tau_w \) is the unknown stress exerted by the water surface on the air flow. The pressure terms in equation (3) may be rearranged and the equation written in the form below.

\[
(p_1 - p_2) h_1 B - \frac{1}{2} (p_1 - p_2) SB - \tau_0 X [B + 2h_{ave}] - f - \tau_w BX = \Delta \text{Momentum. (4)}
\]
The first bracketed term on the left side of equation (4) can be relatively routinely determined with pressure drop and setup measurements. In the second term on the left, the average dry wall shear stress can be obtained with a series of Preston tube measurements around the dry periphery. More will be said of this determination in the section on apparatus. The corrective force \( f \) can be estimated with the aid of a series of dry-channel runs which will also be described in a later section. Concerning the evaluation of the change of momentum, it was assumed that fully developed flow was already established at section 1, and that there is no change in the basic shape of the turbulent-flow velocity profiles between the two sections. Unfortunately, in order to provide as long a working section as possible, section 1 had to be located closer to the inlet—60 hydraulic diameters—than would be ideally desirable. However, work reported on flow through rectangular pipes by Eckert and Irvine (1957) suggests that this entrance length might be satisfactory in view of the many factors tending to stimulate rapid transition to turbulence. Also this assumption is essentially substantiated by available velocity profiles. The momentum change term then consists only of the change in momentum flux associated with the setup. The desired \( \tau_g \) term is obtained as a moderate difference between large quantities, and the result is consequently relatively sensitive to small errors in the major terms. For example, a 1 percent error in the pressure difference term can, in some cases, result in a 5 percent error in \( \tau_g \). This situation, although not greatly desirable, can be made tolerable with care in measurements.

APPARATUS

The major elements of the experimental setup are depicted in figure 2. The transparent plastic wind-wave channel is approximately 30 meters long, 28.5 cm wide and 58.5 cm deep. The still water depth for all runs was about 19.6 cm. Wind was provided by a 15 h.p. blower, which drew air through an inlet nozzle containing a honeycomb. Air flow control is provided by adjustable openings in a panel across the downstream end of the channel.

Waves were generated by a paddle covering the width of the channel and extending about 2 cm above the still water surface. The paddle was suspended from a single strut, which was pivoted on top of the channel and which was actuated, above the pivot, by a 3/4 h.p. variable-speed motor. The generator was located, as indicated in figure 2 at the extreme upwind end of the channel. The sloping bottom just upwind of the wave generator served as an absorber for the waves generated in that direction. A flatter beach was installed at the downwind end of the channel for dissipation of energy of the principal train of waves. Just downwind of the wave paddle, a solution of Aerosol was injected as necessary in order to inhibit the formation of wind-generated waves.
The average wind velocity was obtained from the static-pressure difference across the constriction of the inlet nozzle, which had been calibrated over the anticipated range of velocities. As was mentioned in an earlier section, shear stresses around the dry walls and top of the channel were deduced from Preston tube readings. The Preston tubes were fabricated of hypodermic tubing 2.5 mm in outer diameter and approximately 1.5 mm inside diameter. There appears to be little question of the validity of the Preston-tube concept, provided that it is used in a turbulent nonseparated flow. One of the Preston tubes was calibrated in place (with the channel dry) with the aid of the Karman-Prandtl logarithmic velocity distribution equation for smooth walls. Within the accuracy of measurement, the calibration curve matched Preston's (1954) original curve. This result is not surprising, since it has been shown by Hsu (1955) that differences in the inner-outer diameter ratio have only secondary effects on the calibration. Preston-tube readings were obtained at two stations within the momentum analysis section. When used away from the walls, the Preston tubes served as conventional pitot tubes for measurement of local velocities.

Static-pressure taps were located as needed along the channel. Three taps were used as a ring at each station. Preston tube, pressure drop, and inlet-nozzle readings were all obtained, for convenience, on commercial oil manometers. Because the flat slopes of these manometer tubes had to be accurately known, they were periodically checked by being placed in parallel with water micro-hook gages.

The setup was measured with a water micro-hook gage. It is important to note that, with the manometer arrangement shown in figure 2, the measured setup is corrected for the change in ambient air pressure; that is, the setup for use in equation (2). To obtain the actual water elevation difference within the channel, the air pressure difference must be added to the measured setup.

Wave heights along the channel were obtained simply by marking the crests and troughs on cardboard strips at 22 stations evenly spaced over the length. This seemingly inelegant procedure is quite adequate provided that the waves remain regular, which was the case here. The procedure was carried out fairly rapidly. In addition, the mechanical and electronic problems involved in providing a movable resistance (or other type of gage) over such a length of enclosed channel appeared formidable indeed.

**EXPERIMENTS**

Approximately sixty runs were made for a variety of wave geometries. This number included six runs which were made with no mechanically generated waves and with wind waves essentially suppressed. Each complete run...
Figure 1. Control volume for momentum balance.

Figure 2. Schematic sketch of apparatus.
Figure 3. Sample wind velocity distributions.

Note: Each point represents the average velocity at that elevation.

- ○ 900 cms. upwind of sta. 1.
- ● at station 2
  - $V_{ave} = 330 \text{ cm/sec}$
- △ at station 2, waves,
  - $V_{ave} = 750 \text{ cms/sec}$
consisted primarily of the measurements mentioned in the preceding section—namely, average flow rate; pressure drop over the working section; setup; Preston-tube readings at 0.1 ft intervals along the dry side walls and top at two stations within the working section; wave heights; and air and water temperatures. After the extent of the distortion of the wind profile was realized, wind velocities at a fixed height above the undisturbed water surface (7.6 cm) were obtained at the centerline and quarter-width points of the two Preston-tube stations. This elevation was selected because it was always below the location of the peak velocity.

In addition to these data runs, eight runs were made with the channel completely dry, in order to determine the value of the additional force term in the momentum equation. These runs required only average air flow rate, Preston-tube, pressure drop and air temperature measurements.

In practice, the wind-generated disturbances could not be completely suppressed; and, although a wind "sea" was not developed, the water surface did become roughened by small three-dimensional ripples. Below an average velocity (defined as the measured volumetric flow rate divided by the cross-sectional air-flow area, based on the undisturbed water depth) of about 500 cm per second the surface remains almost mirror-smooth for both the no-wave and mechanical-wave case. This is pictured in figure 4(a). When the average velocity exceeds about 600 cm per second, surface roughnesses begin to develop. This situation is depicted in the other photographs of figure 4. Unfortunately the intensity of the surface roughness depends in general, for the same wind velocity, on the shape of the primary, or mechanical waves. For example, in figure 4(c), the mechanical wave is on the verge of severe distortion. However, in figure 4(d), for the same wind velocity but with a flatter wave, the roughness does not appear ready to deteriorate into wind-created gravity waves, and is in fact approaching the appearance of the no-(mechanical) wave surface. Cases were noted in which the generation of the primary waves appeared to have a damping effect on the ripples. This area could not be delved into in these experiments; but it would undoubtedly prove an interesting study.

On this subject, it is worthy of note that Keulegan (1951), working with a channel of similar length and height, but much narrower (width 10 cm) than the present facility, was able, with the addition of detergents, to maintain virtually mirror-smooth surfaces at wind velocities higher than those employed in this study. This goal could not be achieved for these experiments, and the physical explanation of this width effect (if it is indeed a width effect) is not known to the writer. The surface roughness is not, however, associated with disturbances produced by the projecting edge of the wave flap, since the same behavior was observed when the wave generator was removed.
WIND EFFECT

NO MECHANICAL WAVES

MECHANICAL WAVES (H=2.3, L=80 cms.)

\[ V_{\text{AVE}} = 330 \text{ cms/sec.} \]

Figure 4a.

\[ V_{\text{AVE}} = 755 \text{ cms/sec.} \]

Figure 4b.
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Figure 4c.

\[ V_{\text{AVE.}} = 895 \text{ CMS./SEC.} \]
NO MECHANICAL WAVES

Figure 4d.

\[ V_{\text{AVE.}} = 895 \text{ CMS./SEC.} \]
MECHANICAL WAVES (H=3.5, L=80CMS.)
As has been previously mentioned, the dry-channel runs were made in order to evaluate an additional, or corrective force term in the momentum equation. For each run, the total peripheral shear force determined from Preston tube measurements was subtracted from the force associated with the measured pressure drop. The resulting difference, assuming no change in momentum flux, is the desired corrective force. It was necessary to make two separate groups of runs, because, part way through the experimental program, a set of pressure holes came to be regarded with suspicion, and was replaced. The corrective forces were expressed in the form \( f = K_p \cdot V_{ave}^2 \), and the values of the coefficient \( K_p \) are shown below.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{ave}, \text{ cm/sec} )</td>
<td>( K_p )</td>
</tr>
<tr>
<td>619</td>
<td>0.104</td>
</tr>
<tr>
<td>744</td>
<td>0.082</td>
</tr>
<tr>
<td>370</td>
<td>0.092</td>
</tr>
<tr>
<td>462</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Ave. \( K_p = 0.089 \) Ave. \( K_p = 0.123 \)

Because the scatter does not appear to be ordered, it was considered that \( K_p \) is not a function of Reynolds number, and a constant average value was used for each group as shown.

There remains the question of what fraction of these force coefficients is to be assigned to the dry periphery for use in the regular runs, when there is 20 centimeters of water in the channel. It was found that, by using 72 percent of the cited values of \( K_p \), the momentum-analysis determination of shear stress was brought into agreement with the setup determination of shear stress for the no-wave runs at low wind speeds. Actually, the air-flow portion of the periphery accounts for only 61 percent of the total channel periphery. The determined proportioning is not unreasonable, however, because the instrumentation protuberances and the worst cases of channel misalinement are found at the top.

NO-WAVE RUNS

The results of momentum-analysis treatment of runs with no mechanically produced waves are shown in figure 5, in the form of shear stress plotted against velocity squared. Henceforth, unless otherwise noted, velocity \( V \) refers to air velocity 7.6 cm above the undisturbed water surface, determined as described in the section on experiments. Also shown
Figure 5. Surface shear stress – no waves.

Figure 6. Surface shear stress – waves.
on the figure are the surface shear stresses deduced from the setup, i.e., from equation (2). At high velocities, the momentum results are seen to exceed those derived from setup by an increasing amount. Although this effect could be ascribed to a breakdown in the procedure, a more likely explanation is that a considerable portion of the surface shear stress is necessary to maintain the surface roughness ripples (which would otherwise damp rapidly); and this portion of the stress is not manifested in the setup.

For three runs the surface shear stress was determined from near-surface air-velocity profiles. These points are also shown on figure 5. It can be concluded from these results that (1) the momentum analysis procedure, employing Preston-tube shear stress determinations, is a feasible one, with figure 5 suggesting that a certain measure of confidence is justified, and that (2) for moderate air velocities a linear relationship exists between $\tau_s$ and $V^2$.

WAVE RUNS

The momentum-analysis results for runs with mechanically generated waves are shown in figures 6, 7, and 8. For clarity, the stresses deduced from setup measurements have not been included in these curves and will be discussed separately in a later section. In spite of some scatter, it is clear that, for the same wind speed and wave length, the surface shear increases noticeably with increasing wave height. Some of the curves exhibit pronounced upward curvature at high wind speeds. It is felt that at least a portion of this effect is due to the presence of the previously discussed wind-generated ripples.

If we consider the portion of the surface shear stress due to waves alone, we can write

$$\Delta \tau_s = f (\rho, \mu, V, \gamma, H, L).$$

(5)

Here $\Delta \tau_s$ is the incremental shear stress due to the presence of waves, $\rho$ and $\mu$ are the density and dynamic viscosity of the air, $V$ is the wind speed at some reference elevation $\gamma$, and $H$ and $L$ are wave height and length, respectively. These variables can be combined into dimensionless numbers in the following manner.

$$\frac{\Delta \tau_s}{\rho V^2} = C_d = f (\frac{\rho V}{\mu}, \frac{H}{\gamma}, \frac{H}{L}).$$

(6)

The wave drag coefficient is thus a function of a Reynolds number and two geometry parameters. In the consideration of these experimental data, we assume that the wave drag is governed primarily by separation of the air.
Figure 7. Surface shear stress - waves.

Figure 8. Surface shear stress - waves.
flow behind the wave crests, that is,

\[ C_d = f_2 \left( \frac{H}{y_0}, \frac{H}{L} \right). \]  

(7)

There is no question that, for any given wave shape, there is a wind velocity sufficiently low so that drag will be a function of Reynolds number rather than of protuberance shape. Likewise, for any given wind velocity, there unquestionably exists a wave which is so flat that the Reynolds number becomes the significant parameter. (These statements can be made in analogy with the curves for head loss through sand-roughened pipes.) However, it is realized that the experimental techniques employed here cannot be sufficiently sensitive to detect such subtle transitions. Although the maximum Reynolds obtained was only about 25,000, the waves were always moderate in height and steepness, and the general appearance of the data seems to provide some measure of justification for the assumption that we are in a region of shape-dominated resistance.

For each data point, \( \Delta \tau_g \) was determined by subtracting the no-wave shear stress for the same wind velocity. It is not implied by this subtraction procedure that the tangential, or "skin friction" portion of the total shear stress is the same for the wave and no-wave cases. It is, rather, an arbitrary statement of the difference between an existing stress and the stress that would have existed had the waves not been present.

In figure 9 the average value of the computed drag coefficients for each series of runs is plotted against average wave steepness. The relative velocity, \( \bar{V} \), i.e., \( V - \text{wave celerity} \), was used in these computations. (The celerity ranged from about 100 to 125 centimeters per second in these experiments.) The scatter appears to be random and not correlated with \( H/y_0 \). It is therefore concluded that, at least within this range of variables, \( H/y_0 \) is not a sensitive parameter in comparison with the wave steepness. The line drawn through the data points has a 2:1 slope on the log-log plot, indicating a dependence of \( C_d \) on the square of the wave steepness, in the range investigated. The steepness has been multiplied by the term \( \coth \frac{2\pi d}{L} \), where \( d \) is the undisturbed water depth. This is a semi-empirical corrective term which takes into account the difference in shape between waves of the same steepness but with different length-depth ratios.

SETUP MEASUREMENTS

In figure 10 is shown a sample series of wave runs in which the shear stress is computed both from the momentum analysis and from the setup measurements according to equation (2). It is apparent that the shear stresses deduced from the setup are too high in the low wind speed range,
Figure 9. Drag coefficients vs. wave steepness.

Figure 10. Shear stresses from setup and momentum analysis.
since a linear extrapolation of these data would appear to yield a measurable shear stress with zero wind speed. Further investigation revealed that, because of the mass transport associated with the wave motion, a significant setup could be measured for the waves, even with no wind. These measured setups are shown in figure 11. Although there is difficulty in obtaining reliable measurements when the setup is small, the shape of the curves (setup proportional to square of wave height) suggests that there is truly a substantial mass-transport induced setup, and that the measurements are probably not recording an extraneous phenomenon which can be misconstrued as setup.

When measured setups from figure 11 are subtracted from the measured setup for each wave run, and difference results are converted to surface shear stress, the results appear as in figure 12. Allowing for scatter, the points fall on a single curve, which is only very slightly below the shear stress curves obtained from setups for the no-wave runs. A likely conclusion is that none of the additional surface stress (attributable to the presence of waves) is reflected in an increase in setup.

WAVE DAMPING MEASUREMENTS

A sample of the results of wave height measurements is shown in figure 13. Wave damping with no wind, in a laboratory channel, is normally due primarily to laminar boundary layer friction; consequently the damping equation is of exponential form. It is realized that, in the presence of wind forces, the wave height-distance curves need no longer exhibit viscous damping characteristics. Nevertheless, over short distance, it is felt that the net damping can be approximated by a viscous behavior curve (i.e., by a straight line on a semi-logarithmic plot).

Considering two control stations along the wave channel, the net mean rate of energy dissipation within the control section must equal the gradient of energy flux between the stations. In equation form,

\[
\frac{dE}{dx} \frac{dt}{dt} = - \frac{dE_y}{dt} + \frac{dE_w}{dt}.
\]

The term \( \frac{dE}{dx} \frac{dt}{dt} \) represents the wave energy flux, which can be expressed as

\[
\frac{dE}{dt} = \frac{1}{8} \rho g H^2 \frac{n C}{n}
\]

where \( C \) is the wave celerity (or phase velocity) and \( n \) is the ratio of group to phase velocity. The term \( \frac{dE_y}{dt} \) is the rate of viscous damping of the waves, which is in this case due primarily to the wall and bottom boundary layers, and which can be evaluated from the no-wind damping information, adjusted for average wave height. The term \( \frac{dE_w}{dt} \)
**Figure 11.** Measured setup for waves with no wind.

**Figure 12.** Summary of shear stresses from adjusted setups.
Figure 13. Sample wave-height curves.

Figure 14. $\Delta T_s$ from momentum analysis and wave-height curves.
represents the rate at which the wind adds energy to a unit area of the wavy surface. Treating this surface as a series of solid undulations moving with a velocity, \( C \), the power absorbed is given by \( \Delta r_C \).

Assuming, as have others, that the energy so absorbed is added to that of the wave, we write

\[
\frac{dE_W}{dt} = \Delta r_C C. \tag{10}
\]

The additional surface stress, \( \Delta r_C \), was evaluated for the wave runs from equations (8), (9), and (10) and compared with the results obtained directly from the momentum analysis. Sample comparisons are shown in figure 1. In all cases it is seen that agreement is acceptable only when the wind velocity is low. Part of this behavior can be ascribed to two effects. First, when wind-produced roughnesses appear on the water surface at higher wind speeds, some portion of the mechanical wave energy may be transferred to these ripples by an interaction process described by Longuet-Higgins (1963). Such an occurrence would result in an increase in the damping term of equation (8) over the viscous value. The second, and smaller, effect is a consequence of the generation of a wind driven surface current. Because of nonlinear interactions between currents and waves, as described by Longuet-Higgins and Stewart (1960), it is not entirely correct to write an energy balance equation for the wave train alone. The error in neglecting this effect results in the \( dE/dt \) term in equation (8) being too small, by an amount which depends upon the ratio of the energy transport velocity of the current to the group velocity of the waves. Without detailed data on the wind-induced currents, the former velocity cannot be evaluated. Thus it can be concluded, at this juncture, that only when the wind velocity is small can the entire additional surface stress due to waves be considered to be effective in increasing the wave height.

CONCLUSIONS

It can be concluded that there exists an additional wind shear stress on a water surface due to the presence of waves—a shear stress which, at least for the range of variables investigated here, depends upon the square of the wave steepness. This additional stress can be used directly to compute a decrease in wave damping only at low wind speeds. The total wind setup in the channel appears to be the sum of the setup which would exist without waves and the setup due to the mass transport of the wave train.

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