Cracking and induced steel stresses of reinforced and prestressed piles during driving

Dr.-Ing. Nils F. Zorn
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SUMMARY

The problem of steel stresses during driving of reinforced and prestressed piles in case of concrete failure is analysed in this report using a momentum trap model that includes amplitude and shape of the reflected compressive wave. Special reference is made to the different performance of reinforced and prestressed concrete piles with respect to concrete and steel failure - cracking and yielding. The significant phenomena, bond behavior, prestressing level, prestressing rate and ratio of stress wave length and pile length, are clearly distinguished for assumed linear elastic material behavior which may serve as a basis for further numerical nonlinear analysis.
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<td>A</td>
<td>area</td>
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<tr>
<td>α</td>
<td>coefficient</td>
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<tr>
<td>B</td>
<td>constant</td>
</tr>
<tr>
<td>c</td>
<td>wave velocity $\sqrt{\frac{E}{\rho}}$ index: concrete</td>
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<tr>
<td>δ</td>
<td>damping coefficient</td>
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<tr>
<td>Δ</td>
<td>displacement</td>
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<tr>
<td>E</td>
<td>modulus of elasticity</td>
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<tr>
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<tr>
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<td>I</td>
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<td>K</td>
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<td>ratio</td>
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<td>length</td>
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<td>m</td>
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<td>μ</td>
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<tr>
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<tr>
<td>r</td>
<td>index: reflected</td>
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<tr>
<td>ρ</td>
<td>mass density</td>
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<tr>
<td>σ</td>
<td>stress</td>
</tr>
<tr>
<td>s</td>
<td>index: steel</td>
</tr>
<tr>
<td>sp</td>
<td>index: spall</td>
</tr>
<tr>
<td>T</td>
<td>duration, period</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>u</td>
<td>displacement in x direction index: tensile</td>
</tr>
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<td>index: ultimate</td>
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V
velocity
w
index: wave
\(\omega\)
circular frequency \(\frac{2\pi}{T}\)
x
coordinate
y
index: yield
Z
mechanical impedance \(\frac{EA}{C}\)
1 INTRODUCTION

Reinforced and prestressed prefabricated concrete piles are designed to withstand the expected service loads. These may be only static compressive as in cases of house foundations but may be tensile as in case of tunnel foundations in ground water uplift, and may include alternating loading as in case of crane foundations and offshore constructions. So the requirements for the pile behavior in service results from very different conditions but can be defined rather precisely for design purposes. The design process includes additional load cases as transport and pile lifting into the driving apparatus but little reference is made in codes to stresses during driving.

In general a design philosophy similar to the design for imposed stresses is adopted [3], which relates a minimum reinforcement to a ultimate strength of the concrete cross-section. Only two cases in [3] include the maximum tensile wave force as a parameter, and little reference is made to the shape of a reflected compressive wave on the magnitude of the induced steel stresses in case of concrete tensile failure, i.e. cracking. An analytical approach to evaluate the stresses in the reinforcing steel is presented, that allows to analyse the system until a third crack occurs, is presented in this report. It shows the significant phenomena and may act as a basis for more elaborate numerical analysis that then should also take nonlinear material behavior and the interaction between soil and pile into account.
2 WAVE PROPAGATION IN PILES

2.1 Basic assumptions

It is assumed that the stress wave propagation in piles can be modelled as a one dimensional problem, thus neglecting radial inertia effects that lead to dispersion of the wave shape. This assumption is valid for cases where the ratio of pile diameter (or side length) and wave length is small. In the case of pile driving this ratio is in the order of $0.4:40=0.01$ which is small enough.

A second assumption necessary to apply the following analysis is that the convective term in wave propagation analysis, the derivation of the velocity with respect to the direction of propagation, can be neglected. This means that a resulting translatory velocity is small compared to the wave induced particle velocity and can also be accepted.

Furthermore the uncracked pile is modelled as homogeneous material, neglecting that the wave speed in the reinforcement is higher than in concrete and the resulting bond forces.

These assumptions allow to model the stress wave propagation using the one dimensional linear wave equation:

\[
\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}
\]  

(1)

The solution of this partial differential equation of second order is:

\[
u = f(x-ct) + F(x+ct)
\]

(2)

in which $f$ and $F$ are independent arbitrary shape functions of waves that travel forward resp. backward in $x$ direction with the wave speed $c$.

Generally the functions may be interpreted as initial stress wave $f$ and reflected component $F$ that are superposed in $x$ and $t$ domain.

This leads to the next assumption: there is no interaction at the point of stress wave generation between initial and reflected stress wave. This means the wave length $L_w$ is always shorter than twice the pile length $2L$.

The effect of an interaction between a reflected stress wave and the stress wave generation can be considered [2], however it makes things more complicated and is not necessary to study steel stresses at cracks.
Dealing with a linear and constant (changes will be discussed later) system the effects of a propagating wave, i.e. the induced particle velocity and the force, can be evaluated and superposed. The force acting in a cross-section due to wave propagation can be determined as follows

\[ F = A \cdot E \cdot \varepsilon = A \cdot E \cdot \frac{\partial u}{\partial x} + A \cdot E \cdot \frac{\partial u}{\partial t} = Z \cdot V_p \]  

(3)

It is further assumed an initial stress wave will propagate without being influenced by skin friction forces until it reaches the pile end which has the boundary condition 'no force can be transferred'. These two last assumptions are very conservative with respect to the later calculated steel stresses, and may be changed later. For the effect of soil reactions at the pile end see ref.[1].

2.2 Stress wave analysis

Based on these assumptions a rather straightforward stress wave propagation analysis can be performed. Knowing the shape of the induced stress wave, for example a finite step with exponential decay resulting from the impact of a rigid mass \( m \) with velocity \( v_0 \) (fig.1)

\[ F_1 = V_0 \cdot Z \cdot e^{-\frac{Z}{m} t} = F_0 \cdot e^{-at} \]  

(4)

the stress and particle velocity can be evaluated in the pile as a function of time. After \( t_1 = L/C \) the stress wave reaches the free end of the pile and is reflected as a wave of same shape and amplitude however with opposite sign to fulfill the boundary condition 'no resulting force'. Now for a duration \( T = \frac{W}{Z^2} \) the resulting forces or stresses in the pile are the superposition of a compressive and tensile wave of same shape, and amplitude but opposite sign and propagation direction. Since the particle velocity in a tensile wave is in the opposite direction of the wave propagation, the particle velocity at the pile end doubles (\( F_r = -F_i \))

\[ V_p = V_i + V_r = \frac{F_i}{Z} - \frac{F_r}{Z} = 2F_i \]  

(5)

2.3 Wave momentum

An important factor in the following analysis will be the momentum of a
wave, the product of mass and velocity, and the conservation of this momentum. The latter requires that the momentum of a rigid mass, \( m \cdot v_0 \), that is used to generate a stress wave, is conserved in the wave

\[
I_0 = m \cdot v_0 = \int_0^T F(t) dt = v_0 Z \int_0^T e^{-\frac{z}{m} t} dt
\]  

(6)

The momentum in part of a pile due to wave propagation can be evaluated as follows:

\[
I = m \cdot v = \int_0^L \mu \frac{F}{Z} dx
\]

(7)

substituting for \( dx = c \cdot dt \)

for a pile with constant mass density \( \mu \) leads to

\[
I = \frac{c \cdot \mu}{Z} \int_0^T F(t) dt
\]

(8)

Since the factor \( \frac{c \cdot \mu}{Z} \) is equal to unity this corresponds to summing up the momentum passing through a cross section for a specified duration. Once the wave propagation is interrupted i.e. through a crack, the properties of the system may significantly change and not allow the application of linear one dimensional wave theory anymore. A property not influenced by this effect is the momentum, it either results in a velocity of the spall or in the case of restrained movement will produce a force. In contrast to energy, momentum cannot be dissipated. The resulting system can be analysed in the ms range, a time step rather large for wave propagation analysis.

Fig.1 Initial wave
3 BEHAVIOR OF CONCRETE PILES

3.1 Concrete tensile failure

As stated before tensile wave stresses have been found to reach 10N/mm²[3] during driving. These measurements of course include the effects of skin friction and soil reaction at the pile foot leading to lower stress levels than for the conditions assumed in this study. This stress however is already higher than a value that could be considered the tensile strength of concrete.

In contrast to static loading conditions, however, the sole exceedence of a stress level is not sufficient to define a failure criterion since the process of failure takes time. It consists of the following basic stages [5]:

i) Rapid nucleation of microfractures at a large number of locations in the material
ii) growth of the fracture nuclei in a rather symmetric manner
iii) coalescence of adjacent microfractures
iv) spallation or fragmentation by formation of one or more continuous fracture surfaces through the material

So during the first propagation of a tensile wave it is possible that not all four steps may be completed and a crack will not form. However following stress waves, with resulting tensile peaks at the same location, may find significantly decreased tensile strength, since part of the failure process has already been completed. This was also found in experimental analysis [4], where the initial tensile stress decreased to 40-60% after 1000 test cycles in repeated impact tensile testing.

Due to the lack of information about dynamic crack criteria of concrete (energy, amplitude and or duration of the wave), the sole exceedence of a stress level is adopted as failure criterion in the following, and used for steady and differentiable wave shapes.

Independent of this failure history, the failure path of concrete depends strongly on the strain rate [6]. In static tensile loading the fracture surface has time to form along a weakest link connection, i.e. the regions of low bond strength between aggregate particles and cementpaste. It will not be a surface equal to the plane cross sectional area but larger.
Under impact strain rates the fracture surface minimizes the area and thus fractures aggregate particles leading to a higher tensile strength [9]. To take advantage of this effect, that may compensate the above mentioned strength decrease under repeated loading, a mean strain rate should be used, since it is not constant during wave propagation. Given the force as function of time, the strain rate can be easily evaluated, as for the case of rigid mass impact:

\[ F = V_0 Z e^\frac{-z}{m} t = F_0 e^{-at} \]  

\[ \varepsilon = \frac{1}{EA} \cdot \frac{dF}{dt} = -\frac{z^2 V_0}{m A E} e^\frac{-z}{m} t = \frac{A V_0 \rho}{m} e^\frac{-z}{m} t \]  

For a pile 0.4 x 0.4[m²] with \( \rho = 2400[kg/m^3] \) impacted by a mass \( m=3000[kg] \) with \( V_0=2[m/s] \), ignoring the finite step from zero to \( F_0 \) at the beginning, that results in an infinite strain rate, the maximum strain rate is \( \varepsilon_{max}=0.26[1/s] \). Failure however will occur in reality sometime during a finite rise time to \( F_0 \) or shortly after, which requires more information on the true wave shape. Assuming a rise time of \( 2 \times 10^{-3}[s] \), the strain rate during the rise is \( \varepsilon=0.23[1/s] \). Since strain rates should only be referred to as an order of magnitude a value in the range of \( 10^{-1}-10^0 \) should be used evaluating the tensile strength. Ref.[4] presents test results for this range under single and repeated loading.

In case of prestressed piles the failure stress level or failure concrete strength for resulting tensile stresses will simply be increased by the prestressing stress level. A tensile stress wave of this level results in unstressed concrete if the bond interaction is neglected.

Dealing with concrete failure during wave propagation it seems necessary to define a characteristic time, that then also determines the mechanical model. If as in this study wave effects during fracture are ignored it seems sufficient to choose \( 10^{-3}[s] \) as characteristic time. This will not allow to model bond during wave propagation since the wave will propagate \( \approx 4m \) in this time unit and it also implicitly ignores the effects of very high but extremely short (range up to \( 100.10^{-6}[s] \)) tensile spikes that may
result during fracture or from reflection at a very stiff soil.

3.2 Concrete pile failure

A compressive driving wave that propagates in a pile will be reflected as a tensile wave upon reaching a free end, and the resulting tensile stresses may exceed the failure criterion. Three different failure histories must be distinguished: the primary wave induced crack, secondary wave included crack and bond induced cracks. The first can occur after reflection of the pile end, which in case of soil action will not be total [ref.1]. This effect will possibly lead to elaborate procedures to determine when or where the failure will occur and is only mentioned here, since a free surface is assumed. In contrast to the primary crack and for the case that it occurs before the compressive wave has totally crossed the failure plane the rest of the wave will in every case be totally reflected at this new failure surface and then may cause secondary failures. A bond induced crack may occur parallel and independently when the bond induced tensile forces in the pile exceed the concrete failure criterion. The later case will be treated separately in order to show the different effects clearly. A spall produced by a primary and secondary crack will contain a portion of the wave momentum and thus have an initial velocity. A bond induced spall however will not have an initial velocity, unless it occurs while the wave is still propagating in the spall.

3.2.1 Primary wave induced crack

The primary wave induced crack can be analysed most easily using a rectangular wave that is reflected from the free end surface of the pile. After

\[ t_1 = \frac{L_w}{2} \]

the resulting pile stress will be tensile and if the failure criterion is exceeded under the assumptions stated before a crack will immediately occur. The spall will have a momentum, \( I_{sp} \), equal to the total wave momentum and thus a velocity at \( t=0 \).

If the spall is short compared to the pile or the pile can be considered fixed in the soil the system can be modelled as a single degree of freedom (SDOF) system shown in fig. 2.
fig. 2 SDOF system

The equation of motion for this system is

\[ m\ddot{x} + Kx = 0 \]  \hspace{1cm} (11)

and with \( \omega^2 = \frac{K}{m} \) it has the general solution

\[ x = A\sin\omega t + B\cos\omega t \]  \hspace{1cm} (12)

where A and B have to be calculated from the initial conditions

\[ x(0) = U \text{ generally}=0 \text{ for pile problems and} \]
\[ \dot{x}(0) = V = \frac{I_{sp}}{m} \text{ first crack} \]

leading to

\[ A = \frac{V}{\omega} = \sqrt{\frac{m}{K}} \]  \hspace{1cm} (13)
\[ B = U \]  \hspace{1cm} (14)

This solution will lead to rather large displacements since the damping due to frictional forces acting on the pile skin will in reality reduce this displacement. In addition the undamped solution would lead to a collision between pile and spall after half of the oscillation period.

A realistic assumption seems to be critical damping of the oscillation, which means a spall displacement takes place and is reduced to zero. The solution then becomes

\[ x(t) = e^{-\delta t} (At + B) \]  \hspace{1cm} (15)
with \( \delta = \delta_{\text{crit}} = \sqrt{\frac{K}{m}} \) and \( A \) and \( B \) to be determined from the initial conditions

\[
x(0) = U = B \\
x'(0) = V_0 = A - \delta B
\]

The time of the maximum displacement can be evaluated from (for \( U = 0 \))

\[
x(t) = A e^{-\delta t} (1 - t\delta) = 0
\]

\[
t(\dot{x} = 0) = \frac{1}{\delta}
\]

which leads to

\[
x_{\text{max}} = e^{-1} \cdot \frac{A}{\delta}
\]

For the case of a prestressed pile the system to be considered is shown in fig. 3

\[
\begin{align*}
m\ddot{x} + Kx &= -F_p \\
\end{align*}
\]

leading to

\[
\begin{align*}
x(t) &= \frac{V_0}{m} \sin \omega t + \frac{F_p}{K} \cos \omega t - \frac{F_p}{K}
\end{align*}
\]

It can be seen that for the same initial velocity a prestressed pile will have a significantly lower maximum displacement, however again this solution would lead to a collision of pile and spall, so for this case the
critically damped system is also analysed. The solution being generally the same as in the homogeneous case, but only valid until contract \((x=0)\) is reached again

\[
x(t) = e^{-\delta t}(B + At) - \frac{F}{K} \tag{19}
\]

with

\[
x(0) = U = B - \frac{F}{K} \quad \Rightarrow \quad B = U + \frac{F}{K}
\]

\[
x(0) = V_o = A - \delta B \quad \Rightarrow \quad A = V_o + \delta B
\]

The maximum displacement occurs at the same time \((t=0)\) as in the homogeneous case, but has a lower value.

\[
x_{\text{max}} = e^{-1} \left( A \frac{F}{\delta K} + \frac{F}{K} \right) - \frac{F}{K} \tag{20}
\]

The qualitative effect of prestressing and critical damping are shown in fig. 4.

![Fig. 4 SDOF displacements qualitative comparison.](image)

The SDOF system is very convenient to analyze spall behavior, but it should always be checked if the system may still be considered a SDOF. The boundary conditions may change due to a second crack resulting from the reflection of a still oncoming compressive wave at the new free surface, or due to a second crack resulting from bond induced tensile forces in the
pile. The result then is that the new system must be analysed as a two degree of freedom system (2DOF) which makes numerical results a bit more complicated and does not allow the introduction of critical damping that easily.

3.2.2 Secondary cracks

Both secondary cracks will result in the same mechanical system, the difference between bond and wave induced cracks will only be found in the initial condition of the secondary spall. A wave induced spall has an initial velocity - a bond induced does not. In both cases the initial conditions of the primary spall must be evaluated with the SDOF system - they resemble the final SDOF conditions at the time the second crack occurs. The first secondary crack will produce a 2DOF as shown in fig. 5, that still can be treated analytically. For further secondary spalls numerical solutions are advised, however the phenomena can be presented most clearly in an analytical 2DOF solution.

![2DOF system for secondary cracks](image)

The equations of motion for masses \( m_1 \) and \( m_2 \) are:

\[
\begin{align*}
\dot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) &= 0 \\
\dot{x}_2 + \omega_1^2 x_2 + \omega_2^2 (x_1 - x_2) &= 0
\end{align*}
\]
\( \dot{x}_2 + \omega_2^2 (x_2 - x_1) = 0 \)  

(24)

\( x_1 = R_1 e^{i(\omega t - \phi_1)} = R_1 (\cos(\omega t - \phi_1) + i \sin(\omega t - \phi_1)) \)  

(25)

\( x_2 = R_2 e^{i(\omega t - \phi_2)} = R_2 (\cos(\omega t - \phi_2) + i \sin(\omega t - \phi_2)) \)  

(26)

and inserted into the equations which leads to

\[
R_1 e^{i(\omega t - \phi_1)} (\omega_1^2 + \mu \omega_2^2 - \omega^2) + R_2 e^{i(\omega t - \phi_2)} (- \omega_2^2) = 0
\]

(27)

\[
R_2 e^{i(\omega t - \phi_1)} (- \omega_2^2) + R_2 e^{i(\omega t - \phi_2)} (- \omega_2^2 + \omega_2^2) = 0
\]

(28)

This can be interpreted as a linear set of equations for the two unknown quantities \( R_1 e^{i(\omega t - \phi_1)} \) and \( R_2 e^{i(\omega t - \phi_2)} \). A nontrivial solution is only obtained, if the determinant of the coefficients is equal to zero.

\[
\begin{vmatrix}
(\omega_1^2 + \mu \omega_2^2 - \omega^2) & - \omega_2^2 \\
- \omega_2^2 & \omega_2^2 - \omega^2
\end{vmatrix} = 0
\]

(29)

which results in an equation for the eigenfrequencies \( \omega_{I,II}^2 \) of the coupled dynamic system

\[
\omega^4 - \omega^2 (\omega_1^2 + (1 + u) \omega_2^2) + \omega_1^2 \omega_2^2 = 0
\]

(30)

and has the following solutions

\[
\omega_{I,II}^2 = \frac{1}{2} (\omega_1^2 + (1 + u) \omega_2^2) \pm \sqrt{\frac{1}{4} (\omega_1^2 + (1 + u) \omega_2^2)^2 - \omega_1^2 \omega_2^2}
\]

(31)
for the eigenfrequencies. Knowing the eigenfrequencies of the system, the
general solution of the coupled system of differential equations can be
formulated as follows:

\[ x_1 = A_1 \cos(\omega_1 t - \varphi_1) + B_1 \cos(\omega_{II} t - \varphi_2) \] (32)

\[ x_2 = A_2 \cos(\omega_1 t - \varphi_1) + B_2 \cos(\omega_{II} t - \varphi_2) \] (33)

These solutions have too many unknown coefficients for the order of the
differential equations, so there are further dependencies that results
from the original equations. They are

\[ \frac{A_2}{A_1} = x_1 = \frac{\omega_2^2}{\omega_2^2 - \omega_1^2} \] (34)

\[ \frac{B_2}{B_1} = x_2 = \frac{\omega_2^2}{\omega_2^2 - \omega_{II}^2} \] (35)

This leaves four coefficients, \( A_1, B_1, \omega_1 \) and \( \varphi_2 \) to be determined by
boundary conditions \( x_1(0), x_1'(0), x_2(0) \) and \( x_2'(0) \), for the case analysed.
This results in a set of four equations as follows

\[ A_2 \cos \varphi_1 + B_1 \cos \varphi_2 = x_1(0) = U_1 \] (36)

\[ A_2 \omega_1 \sin \varphi_1 + B_1 \omega_{II} \sin \varphi_2 = x_1'(0) = V_1 \] (37)

\[ x_1 A_1 \cos \varphi_1 + x_2 B_1 \cos \varphi_2 = x_2(0) = U_2 \] (38)

\[ x_1 \omega_1 A_2 \sin \varphi_1 + x_2 \omega_{II} B_1 \sin \varphi_2 = x_2'(0) = V_2 \] (39)

Which results in the following relations for the coefficients:

\[ \tan \varphi_2 = \frac{V_2 - X_1 V_1}{\omega_{II} (U_2 - X_1 U_1)} \] (40)

\[ B_1 = \frac{U_2 - X_1 U_1}{(X_2 - X_1 \cos \varphi_2)} \] (41)
\[ \tan \varphi_1 = \frac{V_2 - X_2 \varpi_1 B_1 \sin \varphi_2}{X_1 \varpi_1 (U_1 - B_1 \cos \varphi_2)} \] (42)

\[ A_1 = \frac{U_1 - B_1 \cos \varphi_2}{\cos \varphi_1} \] (43)

which allow to analyse the time dependent behavior of the free vibration system.

This analytic solution now can be easily applied to analyse a system with two spalls, the second either induced by bond or by the reflection of the wave rest at the free surface of the first crack. It should be noted that the stress in the steel is proportional to the relative displacement \((x_2 - x_1)\) as soon as the second spall exists and becomes a function of the momentum trapped in the spalls and of the eigenfrequencies of the system that depend on the ratio of the spall masses. Fig. 6 gives a qualitative impression of how the displacements develop with time, and also indicates that the system is only valid for a positive value of \((x_2 - x_1)\). Once the two masses are in contact \((x_2 - x_1 = 0)\) the property of the spring \(K_2\) must be changed from steel to concrete compression properties.

![Displacement History](image)

Fig. 6 Qualitative 2 DOF displacement history.

In cases of bond included secondary spalls and very good bond between steel and concrete the second spall will only have a small mass and there-
fore little influence. However there then is a strong probability of a third and further spalls.

For the case of a prestressed pile the system that has to be considered to analyse the behavior once a second spall occurs is shown in fig. 7

![Prestressed 2DOF system](image)

**Fig. 7** Prestressed 2DOF system.

The prestressing force does not fundamentally change the system however it makes terms for a particular solution necessary in addition to the homogeneous solution. It is hereby important to note that the direct positive effects on the displacement will not occur for the second and following spalls, since the prestressing force is in equilibrium - it just passes through. A positive effect, the existence of a particular solution also for the second spall only results from the coupling of the two equations of motion. Since the inhomogeneity is constant with time, two constants $C_1$ and $C_2$ are used for particular solutions and added to the general solutions eq. (32, 33). Inserting the new solutions into the system of differential equations eq. (23,24) leads to

$$\{\omega_1^2 + \mu \omega_2^2\} C_1 - \mu \omega_2^2 C_2 = 0 \tag{44}$$

$$- \omega_2^2 C_1 - \omega_2^2 C_2 = -\frac{F_p}{m_2} \tag{45}$$

resulting in

$$C_1 = -\frac{F_p}{K_1} \left(1 + \frac{K_2}{K_1}\right)$$

$$C_2 = -\frac{F_p}{K_2} \left(1 + \frac{K_1}{K_2}\right)$$
The general solutions then for the case of $K_1 = K_2 = K$ is

$$x_1 = A_1 \cos(\omega_1 t - \phi_1) + B_1 \cos(\omega_2 t - \phi_2) - \frac{F_p}{K}$$  \hspace{1cm} (46)$$

$$x_2 = X_1 A_1 \cos(\omega_1 t - \phi_1) + X_2 B_1 \cos(\omega_2 t - \phi_2) - \frac{2F_p}{K}$$  \hspace{1cm} (47)$$

If the initial conditions $u_1$ and $u_2$ are modified to take the prestressing into account as follows

$$\ddot{u}_1 = u_1 + \frac{F_p}{K}$$  \hspace{1cm} (48)$$

$$\ddot{u}_2 = u_2 + \frac{2F_p}{K}$$  \hspace{1cm} (49)$$

the relations to evaluate the coefficients for the homogenous case can be used as before, eq. (40-43).

At this point it must be stated, that the introduction of damping into this system causes analytical problems. Since they can only be overcome under assumptions concerning the size of the damping and the mass ratio it is left to numerical solutions to introduce this effect.

3.3 Bond behavior steel-concrete

A point that has not been stressed so far is the assumption of a spring with constant stiffness to model the reaction of the reinforcing steel. Any other assumption makes analytical solutions intractable, however the reality is certainly not a constant stiffness. The spring stiffness is evaluated as

$$K = \frac{EA}{L_i}$$  \hspace{1cm} (50)$$

where $L_i$ is the free length for extension and this for the case of reinforcing steel in concrete is a function of bond and the steel force. The way the system is loaded resembles closely a pull out test with long embedment length, but as result a relation between the steel force and the free steel extension length is needed, not a relation between bond stress and relative displacement at the free surface. This can be evaluated from bond-displacement relations, performing an analytical analysis, sufficient
insight can be gained from straightforward qualitative investigations.

The free length of the reinforcement at a crack is initially zero, however depending on the shape an initial length in the order of 1-2 rib distances may be assumed. Note that the free length extends into both, the pile and the spall. This initial free length then extends according to the induced force and the bond relationship between reinforcement and concrete. In static loading cases a transmission length can be evaluated indicating at which distance from a specified surface (i.e. the crack) the relative displacement of reinforcement and concrete is zero. This is characterised by zero bond stress $\tau$ (fig. 8).

Since the transmission length will change with time a detailed analysis would require to take this into account and evaluate a free length $L_1$ at every time step. A simplification of this process is to use a mean value of 50% of the maximum transmission length on both sides independent of time and force. This will underestimate the stiffness in the beginning and overestimate it in the end of the displacement history. The evaluated maximum displacement will therefore be a good approximation, and can be evaluated analytically.

Evaluating the transmission length the strain rate effects on the bond behavior must be considered [ref.7]. Fig. 9 shows how different the strain rate influences the bond behavior of ribbed (deformed) and smooth reinforcing steel.
Fig. 9 Bond displacement relations for steel pull-out tests.

So when evaluating the bond behavior again a mean strainrate should be considered.
4 ANALYTICAL CONCLUSIONS

Since it is generally accepted that resulting tensile stresses occur and lead to cracks during driving of reinforced and prestressed concrete piles, it is of interest to analyze which parameters significantly influence the resulting steel stresses at the cracks. Based on the approach to determine these stresses some general conclusions can be made, others have to be reviewed carefully, since they may be valid only for a short period of time and then after a change of the system, i.e. a new crack or contact between two spalls, have to be reviewed under new aspects. In the following case examples, that can will cover a few possible combinations, the conclusions will be checked.

The spall behavior is always analyzed in terms of relative displacement $\Delta x$ of the crack surfaces, and referring to the reinforcement as a spring yield

$$ F = K \cdot \Delta x = \frac{E_s A}{L_i} $$

(51)

So a large free extension length $L_i$ resulting from low bond stresses $\tau$ will lead to low steel stresses. However for the case of a bond induced second crack this will not significantly reduce the relative displacement of the two spalls compared with the maximum displacement of only one spall since the second spall will have a large mass that has to be accelerated from zero velocity.

The influence of damping and prestressing is shown in fig. 4 for a spall of identical mass and initial velocity. This case however only will occur for a rectangular wave. A rectangular wave will only produce one crack and thus the mass and initial velocity of the spall depend only on the wave length and amplitude. Generally these parameters will also depend on the failure criterion and therefore be different for prestressed and reinforced piles. Assuming a wave of the shape shown in fig. 1 that can produce one wave crack in both cases (fig. 10) is one possibility to compare reinforced (2) and prestressed (1) piles.
Fig. 10 Spall creation for reinforced and prestressed pile.

Due to the increased failure stress in case of the prestressed pile the spall will have a larger mass and contain more of the wave momentum. For the reinforced concrete pile the maximum displacement results from eq.(13)

\[ \Delta x_{\text{max}} = \frac{V_0}{\omega} = \frac{I_{\text{sp}}}{\sqrt{K m_{\text{sp}}}} \]

In the prestressed case the maximum results from eq.(18)

\[ \Delta x_{\text{max}} = \frac{I_{\text{sp}}}{\sqrt{K m_{\text{sp}}}} \cdot \sin \omega t_{\text{max}} + \frac{F_p}{c} \cos \omega t_{\text{max}} - \frac{F_p}{c} \]

with

\[ t_{\text{max}} = \frac{1}{\omega} \arctan \left( \frac{I_{\text{sp}}}{F_p} \omega \right) \]

Noting that both equations refer to different masses, \( m_{\text{sp}} \), and therefore different frequencies \( \omega \) not much can be compared. Generally the displacement of the prestressed pile will be lower, however in terms of total stress the prestressing must be considered, so evaluating the the safety against exceeding the yield stress of the reinforcement requires case studies. Some more insight can be gained if critical damping and identical spring constants are assumed; The ratio between the max displacement of a prestressed (1) and a reinforced (2) pile then becomes (eq. 16, 20)
\[
\begin{align*}
    n &= \frac{e^{-1} \left( \frac{I_{\text{sp}}(1)}{m_{\text{sp}}(1)} \right)}{\sqrt{\frac{m_{\text{sp}}(1)}{K} + \frac{2F_p}{K} + \frac{F_p}{K}}} \\
    n &= \frac{e^{-1} \left( \frac{I_{\text{sp}}(2)}{m_{\text{sp}}(2)} \right)}{\sqrt{\frac{m_{\text{sp}}(2)}{K}}} 
\end{align*}
\]

which inserting \( I = \int F dt \) and \( m = c \cdot u \cdot T \) can be rewritten as follows

\[
\begin{align*}
    n &= \frac{e^{-1} \left\{ \frac{\int_{0}^{T_1} F dt}{\sqrt{c \cdot u \cdot T_1 \cdot K}} - \left( e^{-2} \right) \frac{F_p}{K} \right\}}{e^{-1} \left\{ \frac{\int_{0}^{T_2} F dt}{\sqrt{c \cdot u \cdot T_2 \cdot K}} \right\}} 
\end{align*}
\]

This shows that the ratio depends on the momentum-integral and the square root of the duration till failure, and on the prestressing force. Neglecting the second term that will surely decrease the ratio yields

\[
\begin{align*}
    n^* &= \frac{\int_{0}^{T_1} F dt}{\sqrt{\int_{0}^{T_2} F dt}} \\
    T_1 &\geq T_2 
\end{align*}
\]

showing this more clearly. Inserting \( F = F_0 \cdot e^{-\alpha t} \)
leads to

\[
\begin{align*}
    n^* &= \left( \frac{1 - e^{-\alpha T_1}}{1 - e^{-\alpha T_2}} \right) \sqrt{\frac{T_2}{T_1}} 
\end{align*}
\]

This ratio is smaller than 1.0 for times of interest because the square root function decays faster than the first factor, that converges to 1.0 with increasing \( T \).

It must be noted at this point, that the lower boundary of the integrals eq.(54) is only zero if the failure occurs at the wave front. In other cases, i.e. when part of the wave momentum passes through the later crack-plane, this duration defines the lower boundary.
Taking the second term eq.(53) into account again it can be stated that the displacement of prestressed spalls will be lower than for only reinforced spalls. However this does not make reference of the safety against exceeding the yield stress-depending on the degree of prestressing there may be significantly less allowable displacement left compared with unstrained reinforcement. So in addition to the fact that prestressing increases the spall momentum due to the higher failure criterion, it reduces the spall displacement because there is an active force acting from the moment of cracking. But the degree of prestressing must be carefully observed to be able to take advantage of the prestressing during driving. This holds also for multiple cracking, even though it seems that the prestressing force will not act on the second and following spalls, its action is passed by the coupling of the equations.

As shown in this section, even neglecting soil actions does not allow to make conclusions that are generally valid, since too many nonlinearities influence the spall behavior in amplitude and frequency behavior. The latter becomes very important for the case of multiple spalling since the steel stress is governed by relative displacement of two spalls. Therefore in the following example calculations are presented for different wave shapes, reinforced and prestressed piles and steel stress levels evaluated.
5 EXAMPLE CALCULATIONS

5.1 Pile properties

The pile used in all examples is assumed to have the same dimensions and will differ only in steel properties. Since the wave propagation in the reinforcement is ignored the uncracked properties are all the same. The pile chosen for this analysis has the following dynamic properties:

\[
\begin{align*}
A_c &= 0.4 \cdot 0.4 = [0.16 \text{m}^2] \\
E_c &= 45.10^9 [\text{N/m}^2] \\
\varphi_c &= 2400 [\text{kg/m}^3] \quad (\mu = 384 [\text{kg/m}]) \\
c &= 4330 [\text{m/s}] \\
Z &= 1.67 \cdot 10^6 [\text{Ns/m}] \\
\sigma_{t,u} &= 4 [\text{N/mm}^2] \quad (F_{t,u} = 640 [\text{kN}])
\end{align*}
\]

The reinforcement, or in case of a prestressed pile the prestressing force and the degree of prestressing are chosen for each example, together with assumptions on free extension length and bond behavior.

5.2 Reinforced concrete piles

5.2.1 Primary wave induced crack

As first example a stress wave resulting from rigid mass impact (fig. 1) that will produce concrete failure once is analysed.

\[
F(t) = -V.Z.\frac{Z}{m} t = - F_0 e^{-\alpha t} = -800.10^2 e^{-556.6 t} [\text{N}]
\]

Counting from beginn of reflection, failure occurs when the resulting force of reflected tensile and oncoming compressive wave is equal to the ultimate tensile resistance of the pile concrete

\[
t = \frac{1}{2\alpha} \cdot \ln \frac{160}{800} = 1.4 \cdot 10^{-3} [\text{s}]
\]

This value of \( t \) allows to evaluate the spall momentum and mass, the initial velocity of the spall

\[
I_{sp} = \int_0^{2t} F(t) dt = -F_0 \int_0^{2t} e^{-\alpha t} dt = -\frac{F_0}{-\alpha} \cdot e^{-t} \bigg|_0^{2t} = -1147.9 [\text{Ns}]
\]
The negative sign results from the fact that the force of a compressive wave is defined negative even though the particle velocity is in positive x direction, it can be ignored.

\[ m_{sp} = \mu c t = 384.4330 \cdot 1.44 \cdot 10^{-3} = 2403.6 \text{ [Kg]} \]

These values result in an initial spall velocity of

\[ V_0 = \frac{I_{sp}}{m_{sp}} = 0.47 \text{ [m/s]} \]

At this point it becomes necessary to define the reinforcement properties and the free length. The reinforcement is chosen according to the philosophy that it reaches its yield stress for the ultimate concrete tensile load \( f_{sy} = 420 \text{ N/mm}^2 \)

\[ A_s = \frac{F_{tu}}{\sigma_y} = \frac{640 \cdot 10^3}{420} = 1523.8 \text{ [mm}^2\text{]} \]

\[ \frac{A_s}{A_c} = 0.95\% \]

The free steel length \( l_i \) is assumed to be 0.4m, 0.2m in the spall and in the pile. The spring stiffness then is

\[ K = \frac{E_s A_s}{l_i} = \frac{0.21 \cdot 10^6}{0.4} 1523.8 = 799.9 \cdot 10^6 \text{ [N/m]} \]

For the undamped case this will result in a maximum displacement eq.(12) of

\[ \Delta_{max} = V_0 \sqrt{\frac{m}{K}} = 0.83 \cdot 10^{-3} \text{ [m]} \]

and a steel stress of

\[ \Delta_{max} = \frac{E \cdot \Delta_{max}}{l_i} = \frac{0.21 \cdot 10^6 \cdot 0.83 \cdot 10^{-3}}{0.4} = 434.5 \text{ [N/mm}^2\text{]} \]

This value is already above the yield stress, it however ignores the damping by skinfriction forces. Introducing critical damping for this case leads to a reduction of the crack-opening and steel stress as follows
eq. (16)

\[ \Delta_{\text{max}} = e^{-1} \left( \frac{V_0}{\sigma_{\text{crit}}} \right) = e^{-1} \left( 0.47 \sqrt[7]{\frac{2403.6}{799.9 \times 10^{-6}}} \right) = 0.29 \times 10^{-3} \text{ [m]} \]

\[ \sigma_{\text{max}} = 157.3 \text{ [N/mm}^2\text{]} \]

This is only 37.5% of the yield stress and a significant reduction compared with the undamped case. For the assumed spring properties a bond induced crack will occur when exceeding a crack width of

\[ \Delta = \frac{F_{\text{t},u}}{K} = 0.8 \times 10^{-3} \text{ [m]} \]

This is the case for the undamped system, and will be analysed later.

5.2.2 Secondary wave induced crack

Since the maximum compressive stress in the above example is only 5N/mm², an idea to increase driving performance may be to use a higher force. This will be analysed for two following examples, one just increasing \( F_0 \) of the above used wave, the other changing \( F_0 \) and adding a trapez shape wave which may result from different driving apparatus. Both cases will lead to secondary spalls.

\[ F(t) = -1500 \times 10^3 e^{-556.6t} \text{ [N]} \]

The first spall will occur analog to 5.2.1 after

\[ t_1 = \frac{-1}{2.556.6} \ln \frac{860}{1500} = 0.5 \times 10^{-3} \text{ [s]} \]

Now once this has happened the rest of the compressive wave will be reflected at the crack-surface. This wave has a maximum amplitude of \( 860 \times 10^3 \text{N} \), and a shape that is part of the initial wave. A second crack will result from the reflected tensile stress wave.

\[ t_2 = \frac{-1}{2.556.6} \ln \frac{220}{860} = 1.2 \times 10^{-3} \text{ [s]} \]

thus determining that the first spall only can be modelled as a SDOF for the duration \( t_2 \). The initial properties of the first spall are
Since damping is not included in the 2DOF it is also neglected when evaluating the conditions of the first spall when the second spall occurs. Due to the high initial velocity the free extension length of the reinforcement is increased to $l_1 = 2 \times 0.3 = 0.6 \text{ m}$. The initial conditions of the first spall for the 2DOF system then result to

$$x_1(t_2) = V_0 \sqrt{\frac{m}{K}} \sin \sqrt{\frac{K}{m}} \cdot t_2 = 1.42 \cdot 10^{-3} \text{ [m]}$$

$$\dot{x}_1(t_2) = V_0 \cos \sqrt{\frac{K}{m}} \cdot t_2 = 0.79 \text{ [m/s]}$$

The displacement of the spall is rather large, which is due to the eigenfrequency of this SDOF.

$$\omega_{SDOF} = \sqrt{\frac{K}{m}} = 801.8 \text{ [s}^{-1}]$$

It has a fundamental period of $T = 7.8 \cdot 10^{-3} \text{ [s]}$ and the maximum value is reached after $T/4 = 1.9 \cdot 10^{-3} \text{ [s]}$ a value rather close to $t_1$.

The second spall has no initial displacement, but a velocity due to the wave momentum

$$I_{sp2} = - \frac{1500 \cdot 10^3}{556.6} \cdot e^{-556.6 \cdot t} \left|^{2t_1}_{2t_1} = -835.9 \text{ [Ns]} \right.$$ 

$$m_{sp2} = 384.4330 \cdot 1.2 \cdot 10^{-3} = 1995.2 \text{ [Kg]}$$

$$V_{02} = 0.42 \text{ [m/s]}$$
Neglecting the portion of the wave that is reflected at the second crack, the conditions for the analysis of the 2DOF are defined. Note that in the 2DOF masses are counted from the fixed end and not according to their formation sequence. So inserting into eq. (40-43) the initial conditions are

\[
\begin{align*}
    u_1 &= 0 \\
    v_1 &= 0.42 \\
    u_2 &= 1.42 \cdot 10^{-3} \\
    v_2 &= 0.79
\end{align*}
\]

The further constants are

\[
\begin{align*}
    \omega_1^2 &= 267.30 \cdot 10^3 \quad (T_1 = 12.1 \cdot 10^{-3} [s]) \\
    \omega_2^2 &= 642.87 \cdot 10^3 \quad (T_2 = 7.8 \cdot 10^{-3} [s])
\end{align*}
\]

The fundamental frequencies result from eq. (31)

\[
\begin{align*}
    \omega_1^1 &= 1006.8 \cdot 10^3 \text{ rad} \\
    \omega_1^2 &= 170.6 \cdot 10^3 \text{ rad}
\end{align*}
\]

and the amplitude ratios from eq. (34,36)

\[
\begin{align*}
    x_1 &= -0.468 \\
    x_2 &= 1.361
\end{align*}
\]

Using these values the coefficients of the total 2DOF solution can be obtained eq. (40-43)

\[
\begin{align*}
    \tan \varphi_2 &= 1.682 \quad \varphi_2 = 1.0345 \text{ rad} \\
    B_1 &= 1.518 \cdot 10^{-3} \\
    \tan \varphi_1 &= 0.153 \quad \varphi_1 = 0.1519 \text{ rad} \\
    A_2 &= -0.7848 \cdot 10^{-3}
\end{align*}
\]
Inserting these constants into the general 2DOF solution eq.(32,33) the displacements can be calculated, as shown in fig. 11, including a scale for the steel stress $\Delta(10^{-3}m)$ $\sigma[N/mm^2]$.

![Diagram](image)

Fig. 11 Displacement time history for secondary wave spall example.

For the assumed steel properties the reinforcement exceeds the yield stress before the second crack occurs in the undamped case. Ignoring this since the skin friction will significantly reduce the displacement some qualitative conclusions can be made: the steel stress at the first crack increases after generation of the second spall, the rate of further increase depends on the dynamic properties of the second spall, and the steel stress at the second crack exceeds the stress at the first crack before contact between both spall is obtained. The effect of compressive contact between the spalls requires to change the properties of the connecting spring for further calculation. This is not performed for this example since the qualitative important effects have happened before.

If the wave shape is changed by adding a trapez-shape function this will influence the spall masses a little but primarily increase the trapped momentum and increase the initial velocity.

Results can be easily obtained following the presented procedure.
5.2.3 Secondary bond induced crack

As indicated for the undamped SDOF in 5.2.1 a bond induced crack will occur when the steel stress reaches the yielding value for the chosen design philosophy. This bond induced second crack changes the system from a SDOF to a 2DOF. The failure criteria is reached for

\[
\Delta_{cr} = 0.8 \cdot 10^{-3} = V_0 \cdot \sqrt{\frac{m}{K}} \cdot \sin \omega t
\]

\[
t = \frac{1}{\omega} \arcsin \left( \frac{\Delta_{cr}}{V_0} \sqrt{\frac{K}{m}} \right) = 2.4 \cdot 10^{-3} \text{ [s]}
\]

The rest of the wave has travelled 10m until the bond forces are high enough to produce a second crack and contains so little momentum that the wave rest is neglected. For reasons of comparison two cases will be analysed, the first assuming very good bond \( \tau = 6.0 \text{N/mm}^2 \) and a second assuming fair bond \( \tau = 2.0 \text{N/mm}^2 \). In both cases the spring stiffness is unchanged and constant.

The reinforcement chosen are 6 bars of diameter 18mm that have a circumference of 5.65 cm each, i.e. a bond force of 2034, resp. 678N can be transferred per mm. This results in transfer lengths of 314,6mm and 943,9mm, determining the properties of the 2DOF system.

- \( \tau = 6.0 \text{N/mm}^2 \)

\[
L_{sp1} = 0.31 + \frac{1}{2} L_1 = 0.31 + 0.2 = 0.51 \text{ [m]}
\]

\[
m_{sp1} = 384 \cdot 0.51 = 195.8 \text{ Kg}
\]

Initial conditions:

\[
 u_1 = 0 \\
 V_1 = 0 \\
 u_2 = 0.8 \cdot 10^{-3} \text{ [m]} \\
 V_2 = 0.08 \text{ [m/s]}
\]

- \( \tau = 2.0 \text{N/mm}^2 \)

\[
L_{sp1} = 0.94 + 0.2 = 1.14 \text{ [m]}
\]

\[
m_{sp1} = 437.7 \text{ Kg}
\]
initial conditions as above.
The resulting displacement time history is shown in fig. 12 for both cases.

Fig. 12 Desplacement time history for bond induced spall examples.

Due to the properties of the first spall the bond induced spall occurs when $x_2$ is almost at the maximum value and more or less stays while $x_1$ grows until contact of the two spalls is gained. For this rather special case (velocity of first spall almost zero) the influence of the bond properties on the frequency of the 2DOF can be clearly seen. For the case of very good bond, the force in the steel at the second crack at $t=3.5$ ms is equal to the failure criterion and will cause a second bond induced spall, thus result in a 3DOF system. This corresponds very well to an observation quoted in [8] under 'problems and failures': "Puffs of "dust" are seen about one third of the length from the top and horizontal cracks appear at half meter intervals."

If the steel force by large exceeds the concrete failure criterion bond induced cracks will occur until equilibrium with inertia forces of the spalls is reached.

5.3 Prestressed concrete piles

5.3.1 Rate of prestressing, failure criterion
As already stated in 3.1 the failure criterion of concrete will be increased by the prestressing. This in some cases will totally prevent cracks, in others lead to different spall properties, compared with reinforced
concrete piles, generally increasing the momentum, which may lead to large displacements. For the latter cases the initial strain of the reinforcement reduces the allowable steel stress for safety against exceeding the yield stress. So when analysing prestressed piles the prestressing forces that influences the concrete failure criterion and the prestressing rate, that influences the total pile failure criterion-exceedence of the yield stress must be considered. For the following examples a constant prestressing level for the concrete of $\delta_{cp} = 5 \text{ N/mm}^2$ is assumed and the rate of prestressing in the steel varies, as well as the steel yielding stress. Due to the lower bond properties of prestressing steel the free extension length $l_1$ is increased compared with reinforcing steel.

5.3.2 Primary wave induced crack

A wave induced crack will occur when the resulting tensile wave force exceeds a value of

$$F_{t,u} = A_c \cdot (\sigma_{t,u} + \sigma_{cp}) = 1440 \text{ kN}$$

This value is larger than the maximum value of the wave used for the reinforced concrete pile (5.2.1) which means that due to prestressing no failure will occur. Now using the wave that produces two wave induced crack in reinforced concrete piles (5.2.1) with a maximum value of 1500kN one crack will occur and a spall produced. The spall will occur after

$$t_1 = -\frac{1}{2.556.6} \cdot \ln \frac{60}{1500} = 2.9 \cdot 10^{-3} \text{ [s]}$$

The momentum trapped in the spall results to

$$I_{sp} = \frac{1500 \cdot 10^3}{556.6} \cdot e^{-556.6 \cdot t} \left|_{2t_1}^0 \right. = 2155.9 \text{ [Ns]}$$

the mass is

$$m_{sp} = 384.4330 \cdot 2.9 \cdot 10^{-3} = 4821.9 \text{ [Kg]}$$

and the initial velocity is

$$V_o = 0.45 \text{ [m/s]}$$
So far just assuming a level of prestressing was sufficient to evaluate the properties, at this point however it becomes necessary to define two properties of the prestressing steel, the steel cross section and the prestressing rate or the quality, i.e. the yield stress. For this example two rates of prestressing will be analysed for one type of prestressing steel with a yield stress of 1600N/mm², i) \( \sigma_{sp} = 0.5f_{sy} \) and ii) \( \sigma_{sp} = 0.75f_{sy} \).

i) \[
F_p = A_c \cdot \sigma_{cp} = 800 \text{ [kN]}
\]
\[
A_s = \frac{F_p}{0.5f_{sy}} = 1000 \text{ [mm}^2]\]

Since bond of prestressing strands is low than for deformed reinforcing bars the free steel length 1 is assumed to be 0.8m
\[
K = \frac{E_s A_s}{l_1} = 262.5 \times 10^6 \text{ [N/m]}
\]

For the undamped case this will result in a maximum displacement eq.(12) of
\[
\Delta_{max} = 1.92 \times 10^{-3} \text{ [m]}
\]

and an induced additional steel stress of
\[
\sigma_{\Delta_{max}} = 506.3 \text{ [N/mm}^2]\]

leaving the total stress in the steel
\[
\sigma_{total} = \sigma_{\Delta_{max}} + \sigma_{sp} = 1306.2 \text{ [N/mm}^2]\]

below the yield stress of the prestressing steel.

ii) \[
A_s = \frac{F_p}{0.75f_{sy}} = 666.6 \text{ [mm}^2]\]

for reasons of comparability 1 is kept constant and the maximum displacement results to
\[
\Delta_{max} = 2.36 \times 10^{-3} \text{ [m]}
\]
and are induced additional steel stress of

\[ \sigma_{\Delta_{\text{max}}} = 620,1 \, \text{[N/mm}^2\text{]} \]

resulting in a total steel stress of

\[ \sigma_{\text{total}} = 1820 \, \text{[N/mm}^2\text{]} \]

which is above the yield strength and would lead to failure of the pile due to fracture of concrete and prestressing steel.

The cases of secondary wave and bond induced cracks develop analoge to the reinforced concrete pile examples, they only require the consideration of the particular solutions (eq.46,47). The results will develop similarly and again show the influence of prestressing on the spall momentum and of the prestressing rate on the total steel stress.
6 CONCLUSIONS

The presented analysis shows that a critical length for piles exists, depending on the duration of the induced stress wave. Piles shorter than half of the wave length will always show interaction between initial and reflected stress wave and therefore are less likely to crack since damping effects become significant after few reflections. Assuming a wave duration of \( \approx 10 \text{ ms} \) and a wavespeed of 4300 m/s the critical length is 21.5 m. Prestressed piles show this effect significantly since due to the higher failure criterion they can withstand higher resulting tensile stresses, that may result for the case of interaction.

In general it can be stated that prestressed piles provide a higher safety against cracking, however for the case that one crack occurs, multiple bond induced cracks may follow as shown in the example and mentioned in [8] and, if the rate of prestressing is high the safety against yielding of the reinforcement is very small. So if uncracked piles are the aim, prestressing is necessary, however to ensure that the steel stays elastic in the case of cracking it is necessary to use a low (\( \approx 50\% \)) rate of prestressing. This value may vary if more information on the initial stress wave and the soil is available.

To gain further insight and include skinfriction effects a numerical analysis is necessary, however the significant effects, and important parameters are shown in this analytical approach.
REFERENCES

1 Zorn, N.F., Stress wave propagation in reinforced concrete piles during driving. Report 5-83-21, Stevin Laboratory, Delft University of Technology 1983.


3 STUVO-rapport nr. 65, Berekeningsmethoden Voorgespannen Betonnen Heipalen, sept. 1982.


Stevin-reports published by the division of concrete structures:


SR - 2 Froon, M. "Hoogwaardig beton" (1972). out of print. (5-72-1)

SR - 3 Walraven, J.C. "De meewerkende breedte van voorgespannen T-balken" (1973). out of print. (5-73-1)


SR - 6 Bruggeling, A.S.G. "De constructieve beïnvloeding van de tijdsafhankelijke doorbuiging van betonbalken" (1974). (5-74-2)


SR - 9 Uijl, J.A. den, Bednár, J. "Onderzoek naar het verankeringsgedrag van gebundelde staven" (1974). (5-74-5)


(5-77-21)
SR -20 Corrosie van wapening in beton; de kwestie "Monoliet" (1977).
(5-77-2)
SR -21 Corrosie van wapening in beton; Proefresultaten (1977).
(5-78-2)
(5-77-6)
SR -23 Uijl, J.A. den. "Krachtsoverdracht tussen beton en voor-
spanstreng" (1978).
(5-78-6)
(5-78-9)
SR -25 Stekelenburg, P.J. van, Walraven, J.C., Mathews, M.S.
"Development of a semicylindrical shaped roof in ferro-
(5-78-11)
(5-78-12)
(5-79-1) out of print.
SR -28 Pat, M.G.M., Fontijn, H., Reinhardt, H.W., Stroeven, P.
"Erosie van beton" (1979).
(5-79-30)
(5-79-3)
(5-79-10)
SR -31 Gremmen, C. "Beton met grof grind als toeslagmateriaal".
(5-79-5)
(5-78-10)
(5-80-2)
"Experiments on concrete under single and repeated uniaxial impact tensile loading" (1980).
(5-80-3)
SR -35 Vos, E., Reinhardt, H.W. "Bond resistance of deformed bars, plain bars and strands under impact loading" (1980).
(5-80-6)


