Abstract
Simulators play a crucial role in EOR reservoir modeling. One of the big problems with the simulation of gas injection in SAG foam processes, is that foam collapse near the injection well is poorly represented in the simulator well model. This paper focuses on the question whether a simulator using the Peaceman equation for a homogeneous reservoir is accurate enough to simulate the increase in injectivity as foam near an injection well collapses at decreasing water saturation $S_w$. Four scenarios are examined; they have different foam-strength parameters and different wellbore grid-block radii. As a comparison to the simulation model, fractional-flow theory [Buckley and Leverett, 1941, Rossen et al., 2011] is used to represent the same scenarios. The differences, which represent errors in the simulator case with the Peaceman equation, are discussed. This comparison shows that the Peaceman equation gives very inaccurate injectivity for foam SAG processes.

Introduction
Enhanced Oil Recovery (EOR) processes which use gas injection can be very efficient in recovering almost all oil in place where gas sweeps. The only big disadvantage of gas injection is that it has a very low sweep efficiency [Lake, 1989]. Recent studies have pointed out that when foam is made from the injected gas, the sweep efficiency of the mixture increases enormously. The reason is that foam drastically reduces gas mobility [Rossen, 1996]. The most efficient way to sweep the reservoir is by alternating large slugs of surfactant and water (SAG injection) and fix the injection rate at the highest possible level [Shan and Rossen, 2004]. This injection rate maintains injection pressure just under fracturing pressure of the formation.

Foam is a very complex matter. Often it can be contradictory in its properties, but it can be described in a unified model [Alvarez, 2001]. Identified in the model are two distinct regimes; a high-quality (dry) and a low-quality (wet) regime [Osterloh and Jante, 1992]. They each have a particular mechanism which dominates its behavior. Dry foam depends on capillary pressure and coalescence, which gives a relatively high mobility for foam. Wet foam is controlled mainly by bubble trapping and mobilization. The transition between the regimes is sensitive to the porous medium and the ability of the surfactant to stabilize the foam.

Injectivity is an important issue in foam EOR. Simply injecting a very-low-mobility fluid can force a reduction in injection rate to avoid fracturing the injection well. Moreover, injection rate is crucial to the ability of foam to overcome gravity override of injected gas [Rossen et al., 2010]. The high injectivity of a SAG process is a major advantage for this injection method in overcoming gravity override [Shan and Rossen, 2004].

A couple of issues arise in the modeling of injectivity in SAG foam processes. First is the reaction of foam to changing water saturation close to the well. Foam dries out and collapses very suddenly as water saturation falls below a certain value [Rossen and Zhou, 1995]. This means that the mobility of the foam increases near the injection well and therefore this process increases injectivity. Second, foam is a non-Newtonian fluid. Its shear-thinning properties can reduce the pressure gradient near the well, which increases injectivity. This is because in the near-well region the shear-thinning fluid has a low effective viscosity, which is larger further away from the well.

The Peaceman equation used in reservoir simulators to describe injectivity misses both these issues: it assumes a uniform water saturation in the grid block and Newtonian mobility. Sharma et al. (2011) describe how to adjust the parameters of this equation on an ad hoc basis to account for non-Newtonian mobility in the near-wellbore region. Lake (1989) gives an equation for injectivity of non-Newtonian power-law fluids for use in modeling polymer injection, but this equation is not commonly implemented in simulations. In reality, both effects occur simultaneously: saturation varies with position and time near the well, and mobility at each saturation is a non-Newtonian function of superficial velocity at each position. This thesis is part of a bigger research effort; therefore not all of these issues are addressed here. Rossen et al. (2011) describe a method to solve for changing saturation near a well for non-Newtonian fluids, but do not solve for injectivity. Moreover, to simplify their analysis of foam injection, they exclude the huge effect of foam dry-out near the well on foam mobility. Latooij (2012) uses these results to determine the effects of non-Newtonian fluid properties on injectivity. In this thesis, the focus is on the effect of changing water saturation, and especially on foam dry-out and collapse near the well, and, for simplicity, non-Newtonian effects are ignored. This work continues a series of studies of injectivity of foam in SAG injection [Shan and Rossen, 2004; Kloet et al., 2009; Stolwijk and Rossen, 2009; Rossen et al., 2010]

With SAG injection, a Buckley–Leverett shock front forms at the leading edge of the gas bank. This front is followed by a two-phase spreading wave that extends back to the well, in which foam dries out and collapses. All in all, two regions are present; a spreading wave with two-phase flow, and ahead of it brine-only flow. In this study foam dries out near the well because of water displacement and flow. Evaporation of water into the gas is another mechanism of dry-out near the well, as examined in another study of gas injectivity without foam [Pickup, 2012].
The focus of this thesis is the near-wellbore area, so assumed is that surfactant concentration is uniform and constant in the water phase as a result of earlier surfactant injection. Injectivity during gas injection is modeled in a SAG process in two ways. At first the region of interest is seen as a single grid block, as in reservoir simulators [Computer Modeling Group, 2010]. The injection pressure is calculated from the Peaceman equation, which assumes a cylindrical geometry for a rectangular shaped grid block.

The second part of this thesis uses the Method of Characteristics (MoC) to examine saturation and mobility near the well and overall injectivity in the same region. This analytical theory is used to check the accuracy of the Peaceman equation. Both models are described in the next section.

Theory

For this thesis the following assumption are made for both cases:

1. All phases are incompressible, as is the reservoir, and components are not soluble in each other
2. The reservoir has isotropic and uniform permeability: \( k_x = k_y \)
3. The surfactant concentration \( C_s \) in the water phase is uniform and constant in the region of interest
4. There are only two phases flowing, though a third immobile phase may be present. This immobile phase is oil and \( S_o \) is uniform and constant. For simplicity, assume here is that \( S_o = 0 \)
5. The well radius is \( r_w \). Well skin factor is zero
6. The reservoir is of uniform height \( H \); the vertical well penetrates the whole interval
7. There are no chemical or biological reactions
8. The effect of gravity is negligible
9. Fluids are injected with a constant total volumetric rate \( Q \)
10. Foam properties immediately take their steady-state values corresponding to the given water saturation

For the first case, in which the region of interest is treated as one grid block, the following assumptions are added to the assumptions listed above:

a) Uniform saturation in the grid block at any time
b) For the injection-pressure calculations, the reservoir is represented as a cylinder, which is homogeneous and extends from inner radius \( r_w \), where the fluids are injected, to an open boundary at \( r_e = W/2 \), where \( W \) is the width of the (square) grid block.

\( P_w - P_{re} = \frac{Q}{2\pi H k \lambda_e} \cdot \ln \left( \frac{r_e}{r_w} \right) \)

where \( \lambda_e \) is determined by the water saturation in the grid block at a given time

For the second case where the Method of Characteristics is used, the first ten assumptions are made; added to those, these following assumption also apply;

a) The 1D cylindrical reservoir extends from inner radius \( r_w \), where the fluids are injected, to open outer boundary \( r_e \). The open boundary is far larger than the inner radius: \( r_e >> r_w \).

b) Dispersive processes, including fingering, capillary diffusion and dispersion are negligible

The properties of the reservoir are as follows. The uniform height of the reservoir is \( H=10 \) m, \( Q=0.02 \) \( m^3/\) is the fixed volumetric injection rate, porosity is \( \phi=0.2 \) and permeability \( k=158 \) \( mD \) [as in Ma et al., 2012]. The wellbore radius is \( r_w = 0.1 \) m. When presented in dimensionless form below, only the value of wellbore radius affects the results.

The foam model used here is that in the Computer Modeling Group simulator STARS and is described in Appendix A. In that model, gas mobility is dependent on a foam mobility reduction factor, \( f_m \), and the effect the drying-out water saturation has on gas mobility. This dry-out effect depends on the critical water saturation at which foam dries out, \( fmdry \), and another parameter, \( epdry \), which determines how abruptly foam collapses for water saturation in the vicinity of \( fmdry \). As water saturation at any location in the reservoir decreases to \( fmdry \), the foam weakens more or less abruptly (depending on \( epdry \)). The greater the \( epdry \) value is, the more abruptly foam decay occurs. Two scenarios are developed, with values of \( epdry=10^2 \) and \( 10^3 \). For \( epdry=10^2 \) there is a very gradual dry-out effect; foam at irreducible water saturation has mobility about one third that of water. For \( epdry=10^3 \) the dry-out effect is greater, as shown below.
**Case 1:**
In this first case the studied area is described as one grid block of a certain fixed size surrounding the well. Two different sizes for the grid block were selected, as pressure rise is sensitive to grid resolution. One scenario has a 5m radius and the second a 50m radius.

All parameters used are constant and chosen from previous studies [Lake 1989; Rossen, 1996; Latooij, 2012; Boeije 2012], if they aren’t specifically discussed. The goal of this thesis is to calculate the pressure drop in the reservoir as a function of dimensionless time. This will be described step by step, starting from a reservoir fully saturated with water. At the starting point, gas is injected and water saturation in the grid block decreases over time.

**Material Balance and Injectivity**
A material balance on the grid block determines how water saturation changes with time. Inflow minus outflow determines the change in water saturation in the grid block:

\[
Q(f_{wo} - f_w)dt = -H\pi \left(\frac{r_e}{r_w}\right)^2 \phi \, dS_w \tag{1}
\]

where the injected fractional flow of water, \(f_{wo} = 0\) and the slight volume of the wellbore in the grid block volume is neglected. Rearranging this turns the equation into

\[
\frac{dS_w}{dt} = \frac{Q}{H\pi \left(\frac{r_e}{r_w}\right)^2 \phi} \times [1 - f(S_w)] \tag{2}
\]

The pressure drop is determined by the Peaceman equation.

\[
P_w - P_{re} = \frac{Q}{2\pi Hk\lambda_{rt}} \times \ln \left(\frac{r_e}{r_w}\right) \tag{3}
\]

The results can be expressed in dimensionless units as follows:

\[
t_D = \frac{Qt}{\pi \left(\frac{r_e}{r_w}\right)^2 H\phi} \tag{4}
\]

\[
P_D = \frac{P_w - P_{re}}{P_w - P_{re} \left[\lambda_{rt} = \frac{1}{0.001}\right]} = \frac{1000}{\lambda_{rt}(S_w)} \tag{5}
\]

The rise in injection pressure is made dimensionless by dividing by that for injecting water (viscosity 0.001 Pa s) at the same injection rate with relative permeability 1. Thus the dimensionless injectivity is the same as the effective viscosity of the foam in cp. In dimensionless units, the material balance (Eq. 1) can be computed using increments in either time or saturation as follows:

\[
(0 - f_{w,i})(t_{D,i} - t_{D,i-1}) = S_{w,i} - S_{w,i-1} \tag{6}
\]

In this thesis, the small increments in water saturation \(S_w\) were preferred and so the corresponding dimensionless time is calculated. The increments in water saturation were smaller in the vicinity of connate water saturation:

\[
S_{w,i} = S_{w,i-1} - 10^4 \quad \text{if } S_{wc} - 0.02 < S_{wm,i-1} < S_{wc} + 0.02 \tag{7}
\]

\[
S_{w,i} = S_{w,i-1} - 10^3 \quad \text{if } S_{w,i-1} > S_{wc} + 0.02 \tag{7}
\]

These small increments led to a maximum jump of the total relative mobility of 1.4% compared to its previous value and average absolute increment of 0.2% compared to its previous value. The only unknown in the numerical dimensionless material balance is the fractional flow, \(f_w\), which is given by

\[
f_w = \left(1 + \frac{k_{rg}(S_w)}{\mu_g} \times \frac{\mu_w}{k_{rw}(S_w)}\right)^{-1} \tag{8}
\]
These relative permeabilities and mobilities are also calculated for each water saturation as described in Appendix A.

If the calculated $f_w$ values are plotted in a graph with their corresponding water saturation increments, a $S_w - f_w$-graph is constructed. This graph describes how the $f_w(S_w)$ curve changes as a function of water saturation. It runs from the initial condition, I, at point (1,1) in that graph, to the injection condition, J, where $f_w=0$. At the point of tangency an imaginary line is drawn to the initial condition. The point of tangency is the point at which a tangent to the $f_w(S_w)$ curve extends to point I, at (1,1). This point of tangency is the saturation just behind the foam collapse shock. At first $f_w$ is almost constant at decreasing water saturation, until it reaches this shock. Hence the $f_w$ value plummets to the point of tangency in very few increments of the water saturation. The size of the grid block doesn’t matter for the simulation of this graph. The value of the foam parameter, $epdry$, does alter the path of the graph as can be seen in graph 1. The greater this parameter, the more abrupt the fall of the $f_w(S_w)$ curve. In Figures 1 and 2, the tangent line is plotted for the curve with $epdry=10^4$.

![Graph 1: Fractional flow as a function of water saturation.](image1)

The following graph is a zoomed in version of the graph above. The scenario where $epdry=10^2$ is left out, so the focus lies on the scenario where $epdry=10^4$ and its point of the tangency, where the foam shock takes place.

![Graph 2: $S_w - f_w$-graph for $epdry=10^4$ zoomed in at the point of tangency.](image2)

Point of tangency is at a water saturation of $12.6 \times 10^{-2}$ and at fractional flow value of around $6.6 \times 10^{-3}$. In Case 2 the foam bank consists of saturations below this point of tangency, moving with velocity equal to $df_w/dS_w$ for the given saturation.
**Case 2**

In the second part of this thesis, the same parameters as in Case 1 are used. The difference is that the Method of Characteristics is used to determine $S_w$ in the region of interest, rather than simulating the region of interest as a single grid block as has been done in Case 1. In Case 2, for given dimensionless time, the dimensionless position is calculated for different water saturations. The same water saturation increments as in Case 1 are used, this time applied to consecutive characteristics. The relative permeability functions, fluid properties, and foam model are also the same as in Case 1. First the Method of Characteristics will be discussed briefly.

**Method of Characteristics**

The Method of Characteristics, or fractional-flow theory, is a model used to analyze foam and other EOR processes. It was developed over the years, after Buckley and Leverett developed a theory for a waterflood [Buckley and Leverett, 1941]; many others elaborated and applied it to specific EOR processes [Pope, 1980; Lake 1989; Zhou & Rossen, 1995; Rossen, 1996]. Here the model is restricted for two-phase-flow, i.e. of gas and water. A third phase may be present, such as oil, but it is assumed to be immobile and at residual saturation. The assumptions of the theory are given above. Even with all those assumptions results provide valuable insights to foam EOR and have proved to be accurate in analyzing a field test in the Snorre Field [Martinsen and Vassenden, 1999].

Key to this method is the fractional flow curve, $f_w(S_w)$ for the given process. These figures are found in Figures 6 and 7 below. More information about how this graph is constructed from experimental data, can be found in Kibodeaux and Rossen (1997).

The results can be plotted in a dimensionless time-distance diagram (see Figures 7 and 8), where the progress of waves through the porous medium can be seen. The dimensionless time, $t_D$, is defined as the volume of gas injected since the start of displacement divided by the pore volume of the formation. Dimensionless position, $x_D$, is defined as:

$$x_D = \frac{r^2 - r_w^2}{t_D}$$

[10]

The slope $dx_D/dt_D$ of these characteristic lines match the slopes of the fractional flow curve at the given saturations. Local mobilities along these lines plotted in Figures 7 and 8, are those at the $f_w, S_w$ values, with the given slope tangent to the $f_w(S_w)$-curve [Kibodeaux and Rossen, 1997]. For further explanation on the dimensionless time-dimensionless distance plots, please look at Appendix B.
Fractional flow theory applied to Case 2

In the Method of Characteristics solution, the mobility ahead of the shock is uniform at the value at the initial condition. Water saturation and mobility in the region of interest are examined at several dimensionless times. The water saturation behind the shock extends from connate water saturation (at the well) to the saturation at the point of tangency defining the shock.

To calculate the pressure, Darcy’s law is expressed in terms of the pressure gradient:

\[ \frac{\partial p}{\partial r} = \frac{u}{k \lambda_{rt}} = \frac{Q}{2\pi r H k \lambda_{rt}} \]  \[ 11 \]

Numerical integration of Eq. 10 is as follows. For any given time, the time-distance diagram (in tabular form) gives saturation as a function of dimensionless position. For each dimensionless position \( j \), the dimensional radius is

\[ r_j = \sqrt{x_D j * r_e^2 + r_W^2} \]  \[ 12 \]

From this radius, the dimensionless pressure difference between radii for consecutive values of \( S_w, r_{j-1} \) and \( r_j \), can be estimated as follows:

\[ P_j - P_{j-1} = \frac{Q}{2\pi k h} \frac{1}{2} \left( \frac{1}{\lambda_{rt,j-1}} + \frac{1}{\lambda_{rt,j}} \right) \ln \left( \frac{r_{j-1}}{r_j} \right) \]  \[ 13 \]

By adding all the pressure differences for all the different water saturations behind the shock, the total injection pressure rise for that time is constructed:

\[ (P_W - P_e) = \sum_{j=1}^{N} (P_j - P_{j-1}) \quad \text{with} \quad P_1 = P_W \quad r_1 = r_w \quad P_N = P_e \quad r_N = r_e \]  \[ 14 \]

As in Case 1, a dimensionless pressure is used to see its rise for single-phase water injection, which is given by the Peaceman equation. These pressure values are now ready to compare with Case 1.
Results
A distinction is made between the simulator case (Case 1) and the Method of Characteristics case (Case 2). Hence, their results will be analyzed apart in different sections. In the section “Discussion of Results”, later on, comparisons between the cases will be made.

Case 1
In the simulator case, the total relative mobility is a function of water saturation. At 100% water saturation, the mobility starts at the mobility value of water, but when gas is injected, the total relative mobility decreases. It decreases until the foam bank collapses and total relative mobility is a minimum for $S_w$ near $f_{mdry}$ [Cheng et al., 2000]. After this collapse, the total relative mobility rises quickly as it now approaches the mobility of gas, which is released from the foam. The point of the collapse is different for the two foam parameters chosen in this thesis. Total relative mobility on this plot is inversely related to effective viscosity of foam; $\lambda_{rt} = 10$ corresponds to an effective viscosity of 100 cp. For single-phase gas flow, $\lambda_{rt} = 50,000$.

![Figure 3: Plot of total relative mobility $\lambda_{rt}$ vs $S_w$ for the foam model with the two values of $epdry$.](image)

The numerical integration used to analyze this case is based on small increments in the water saturation over dimensionless time. The step size is reduced in the region of the inflection points of the fractional-flow curves. This makes the simulation more accurate, but it also costs a lot more time.

As shown in Figure 4, when $epdry=10^2$, the simulation approaches near connate water saturation a lot quicker than when $epdry=10^4$. After 6 dimensionless time steps, the scenario where $epdry=10^2$, is close to the connate water saturation, but will reach this after around $10^3$ dimensionless time steps. In contrary, when $epdry=10^4$ the simulation reaches near connate water saturation after about $10^6$ dimensionless time steps.

A variation in radius of the grid block doesn’t affect results as presented in the dimensionless variables.

![Figure 4: Decrease of the water saturation with dimensionless time for the two values of $epdry$. The results are independent of grid-block size.](image)
Figure 5 describes the course of the dimensionless injection pressure over dimensionless time. The dimensionless pressure builds up to the same dimensionless pressure for all scenarios, but decreases abruptly at the dimensionless time just after the foam shock. With the scenario where \( epdry=10^4 \), the foam collapse is more intense and therefore a larger drop in the dimensionless injection pressure is noticed than the \( epdry \) value of \( 10^2 \) does. Just as in the previous figures, the radius of the grid block doesn’t change the results of the graph as expressed in dimensionless variables.

![Figure 5: Pressure according to dimensionless time. Only the different epdry value scenarios are plotted and no distinction in grid block radius is made, as this variation doesn’t alter the course of the graph.](image)
**Case 2**

In Case 2, the fractional-flow curves for the foam model for the two values of $epdry$ also differ, as can be seen in Figures 6a and 6b. The greater the value of $epdry$, the more abrupt the shift is in the fractional-flow curve near $fmdry$. This abrupt shift describes the dry-out of the foam bank. After foam dries out, its maximum strength is immensely reduced. In Figure 6a a total overview of the decreasing fractional flow according to falling water saturation is given. Figure 6b zooms in on the dry-out region, here the difference between the two foam parameters can be clearly seen.

![Initial condition I](image1)

![Injection condition J](image2)

**Figure 6a:** Fractional flow with decreasing water saturation of the two scenarios with different foam parameters.

![Injection condition J](image3)

**Figure 6b:** Zoomed in on the dry-out section of the different foam parameters.
Figures 7 and 8 show the $x_D$-$t_D$ graph for the different $epdry$ values. The slope of a characteristic is the slope of fractional flow with respect to water saturation for its given value of water saturation. The initial condition I is on the y-axis at infinite slope $\frac{df_w}{dS_w}$ and the injection condition is found at slope 0 of $\frac{df_w}{dS_w}$, which is on the x-axis. In the graph below, the total mobility is labeled on each characteristic, which corresponds to a fixed water saturation. As water saturation drops, the corresponding total relative mobilities rise. The scenario where $epdry=10^2$ corresponds to a larger mobility at the leading edge of the foam bank and a greater increase of the mobility back to the injection well.

The region in front of the first characteristic is the region before the dry-out zone, hence the mobility of this area is the mobility of water with no gas present: $1000 \text{ m}^2/\rho_w \cdot s$.

The value of $epdry$ affects these results significantly. The relative mobility of the foam, for $epdry=10^2$, rises only 2.18 times from the shock front to connate water saturation (at the well), while for $epdry=10^4$ the mobility increases by 18.2 times its original value at the shock at connate water saturation.
Figure 9 compares dimensionless injection pressure of the four scenarios of Case 2. For Case 2 the dimensionless injection pressure does depend on radius of the region of interest. The larger the radius of the grid blocks, the larger the relative decrease in dimensionless injection pressure. The dimensionless pressure build-up is not so severe as Case 1. Nevertheless, the drop of the dimensionless pressure for $epdry=10^4$ is greater than for $epdry=10^2$. The dimensionless injectivity pressure hardly decreases with time for the scenarios where $epdry=10^2$, because mobility hardly increases as foam dries out in the region of interest (see Figure 7).

The value of $epdry=10^4$ gives a relatively large decrease of the dimensionless pressure between dimensionless times $t_D=1$ and 5. The dimensionless pressure decreases to 36.6% of its largest value for $epdry=10^4$ and a grid block radius of 50m, from 1.83 at dimensionless time 1 to 0.67 at dimensionless time 5. With the 5m radius, the dimensionless pressure falls from 1.28 to 0.53, which means only 41.3% is left of the original value of the dimensionless injectivity pressure. With $epdry=10^2$ the dimensionless pressure decreases from 27.4 at dimensionless time 1 to 20.7 at dimensionless time step 5. Hence 75.7% of the value of the dimensionless injectivity pressure is left at dimensionless time step 5 with a 50m grid block radius. In the 5m grid block radius, the dimensionless injectivity pressure drops from 33.7 to 28.4 on the same trajectory, which means that at dimensionless time step 5 the dimensionless pressure decreased to 84.2% of the value at dimensionless time 1.

![Figure 9. Pressure according to dimensionless time, Case 2](image-url)
Discussion of results
In Figure 10, a comparison is made between the two cases described in this thesis. Figure 10a shows the different cases of the dimensionless injectivity pressure with the foam parameter $epdry=10^2$ and Figure 10b shows those different cases for the scenario where $epdry=10^4$. The figures show that the dimensionless injection pressure results differ a lot between the simulator case and Method of Characteristic case. The dimensionless pressure in the simulator case at dimensionless time step 1, its highest point, is 9.5 times larger than the Method of Characteristics for $epdry=10^2$ and even 174.3 times larger than $epdry=10^4$ on the same trajectory.

The values for $epdry$ would have been even more interesting if an extra, ever larger value for $epdry$ than $10^4$ would have been chosen. In that way the simulator case and the Method of Characteristics case would approach each other even closer than in these graphs. However, the Peaceman equation hasn’t got the dry-out effect of foam incorporated well enough to fully approach the more realistic Method of Characteristics. The $epdry$ value of $10^2$ is too small and hardly lets the foam collapse and is therefore unrealistic. While when $epdry=10^4$, the dimensionless pressure build up is too severe to approach the Method of Characteristics case in few dimensionless time steps.

There is no difference in dimensionless injectivity between different sizes of grid radii for the Peaceman equation case. In the Method of Characteristics a small difference does occur in the dimensionless injectivity pressure, because the well is relatively smaller in the larger grid block radius than it is for the smaller grid block radius.

![Figure 10a: Comparison of the path of the dimensionless injection pressure of both cases with the foam parameter epdry set at 10^2.](image1)

![Figure 10b: Comparison of the path of the dimensionless injection pressure of both cases with the foam parameter epdry set at 10^4.](image2)
Conclusions
This research examines the injectivity of foam EOR. Two cases were compared: Case 1 uses the Peaceman equation to estimate injectivity in a grid block; Case 2, with the Method of Characteristics, is more reliable because it accounts for variations of saturation within a grid block. Hence, the thesis looks at the question, “Does simulation give an “accurate” solution of the foam model using the Peaceman equation?”

Four scenarios were used to answer this question, involving different grid sizes and different values of the foam parameter $epdry$. What can be seen is that the size of the grid block doesn’t matter in Case 1, but it does contribute to differences in Case 2. This is because the radius of the well is relatively larger when a smaller grid block radius is chosen.

The differences in the value of $epdry$, contribute to the big differences between the different scenarios. For $epdry=10^2$, the mobility of the foam bank is almost uniform and constant and never gets close to the mobility of gas, which should be approached near the well if the foam bank would collapse at connate water saturation. The scenario where $epdry=10^4$ does a better job of representing dry-out in foam, but also doesn’t fulfill its job totally. An even larger value of $epdry$ should be studied to see if it is more realistic. However, the Peaceman equation is too inaccurate to simulate the dry-out effect of SAG foam well enough.

One of the big problems with the simulation of gas injection in SAG processes is that the foam collapse is not represented well in the model. The Peaceman equation, and the assumption of uniform saturation in the grid block of the injection well, lead to a great underestimation of the true amount of gas which should be injected. In this thesis, at the peak in dimensionless injection pressure, the mobility in the grid block is almost a 100 times lower than anywhere within the foam bank as represented in the Method of Characteristics.

In the foam model used, even for $epdry=10^4$, gas mobility is reduced by a factor of at least 10 even at connate water saturation $S_{wc}$. If instead foam collapsed completely at $S_{wc}$, and gas mobility at the well would have been 10 times greater, the contrast between true behavior and that simulated using the Peaceman equation would be even greater.
Nomenclature

\begin{align*}
H & = \text{reservoir height [m]} \quad (1, 2, 3, 4, 11, 13) \\
k & = \text{reservoir permeability [Da]} \quad (3, 8, 9, 11, 13, 16, 18) \\
e_{\text{dry}} & = \text{STARS foam parameter [-]} \quad (19) \\
\phi & = \text{reservoir porosity [-]} \quad (1, 2, 4) \\
f_{\text{m mobility}} & = \text{reference mobility reduction factor [-]} \quad (19) \\
FM & = \text{foam mobility factor [-]} \quad (18, 19) \\
S & = \text{saturation [-]} \quad (1, 2, 6, 7, 8, 9, 15, 16, 18, 20) \\
f & = \text{fractional flow} \quad (1, 2, 6, 8, 9, 15, 19, 20) \\
Q & = \text{injection rate [m}^3/\text{s}] \quad (1, 2, 3, 4, 11, 13) \\
t & = \text{time [s]} \quad (1, 2, 4, 6, 15) \\
\lambda & = \text{mobility [m}^2/\text{Pa}*\text{s}] \quad (3, 5, 9, 11, 13) \\
r & = \text{radius [m]} \quad (1, 2, 3, 4, 11, 12, 13) \\
P & = \text{pressure [Pa]} \quad (3, 5, 11, 13, 14) \\
\mu & = \text{viscosity [Pa}*\text{s}] \quad (8, 9) \\
x & = \text{position [m]} \quad (12, 15) \\
\end{align*}

Subscripts

\begin{align*}
i & = \text{account for time} \\
j & = \text{accounts for space} \\
g & = \text{gas} \\
r & = \text{relative} \\
w & = \text{water} \\
c & = \text{connate} \\
e & = \text{end} \\
o & = \text{start} \\
t & = \text{total} \\
D & = \text{dimensionless} \\
\end{align*}
References


• Shu, J., “Comparison of various techniques for computing well index”, Report PE, Standford University, 2005.
Appendix A: Fluid and Foam model

In the absence of foam, the relative permeabilities of gas and water are as follows:

\[
k_{rg}(S_w) = \left(\frac{1-S_w-S_{gr}}{1-S_{wc}-S_{gr}}\right)^{2.2868}\]

\[
k_{rw}(S_w) = 0.7888 \left(\frac{S_w-S_{wc}}{1-S_{wc}-S_{gr}}\right)^{2.6844}\]

With foam, the water relative permeability and viscosity are not altered, but gas mobility is greatly reduced (Rossen, 1996). Here the foam model in STARS\textsuperscript{TM} is used (Cheng et al. 2000; Computer Modeling group, 2006):

\[
k_{rg} = FM \ast k_{rg}(S_w)\]

\[
FM = \frac{1}{1+(fmmob+F_w)}\]

For simplicity here, other terms in the STARS model are ignored, such as the effects of oil, changing surfactant concentration, capillary number and other effects.

This reference foam mobility reduction factor, \(fmmob\), is set to 3.4\ast10\textsuperscript{4}; this value corresponds to a model fit of Boeije (2012) to sandpack foam data. Function \(F_w\) in Eq. 19 accounts for the effect of water saturation (dry-out) on gas mobility. As noted, assumed is that surfactant concentration and oil saturation are uniform in the region of interest, i.e. near the injection well. Since assumed is that oil saturation is uniform in this region, accounting for the effect of oil on foam is equivalent to adjusting the value of \(fmmob\).

The function \(F_w\) gives an abrupt or gradual transition in gas mobility as water saturations enter in the vicinity of \(fmdry\), according to the value of \(epdry\):

\[
F_w = 0.5 + \frac{\arctan(epdry*(S_w-fmdry))}{\pi}\]

In these calculations \(fmdry\) was set at 0.13. The larger the value of \(epdry\), the more abruptly foam weakens in the vicinity of \(fmdry\). Here two scenarios are examined, \(epdry=10^2\) and \(10^4\). Figure 11 shows how \(epdry\) affects gas relative permeability for \(S_w\) near \(fmdry\). A significant weakness of the STARS model is that foam does not collapse even at Swc. Figure 11 shows that gas relative permeability is reduced by foam by about one and three orders of magnitude even at Swc for these two values of \(epdry\). This has a significant effect on injectivity, as shown in the text.

Figure 11: Relative permeabilities of water and of gas without foam, and for the two foam models used here.
Appendix B: Numerical Method for the Method of Characteristics (Case 2)

This method is simulated at 9 discrete dimensionless times, namely: 0.01, 0.1, 0.3, 0.5, 0.75, 1, 2, 3 and 5. This is done for all 4 scenarios: the combination of two different radii per grid block and two different epdry values, e.g. the 5m radius with epdry of $10^2$. The dimensionless position for each characteristic is calculated by multiplying the fixed dimensionless time by the slope of water fractional flow with respect to water saturation. When dimensionless time exceeds its maximum value of 1, the value is set to 1; this ensures that characteristics that have left the region of interest have no effect on the calculation of injection pressure:

\[
x_D = \frac{\partial f_w}{\partial s_w} \cdot t_D \quad \text{for} \quad x_D < 1 \quad \text{.................................................................}[15a]
\]

\[
x_D = 1 \quad \text{for} \quad \left(\frac{\partial f_w}{\partial s_w} \cdot t_D\right) > 1 \quad \text{.................................................................} [15b]
\]

With this equation the position of each characteristic can be monitored at each dimensionless time step. See Figures 7 and 8. One would expect that at injection condition J, i.e. at connate water saturation $S_{wc}$, foam would have totally collapsed and the total relative mobility would be equal to gas mobility. For the given foam model and values of epdry used here the foam hasn’t (completely) collapsed, even at $S_{wc}$. With epdry=$10^4$ (see Figure 8), the total mobility at the well ($x_D = 0$) is approaching the mobility of gas, which is at around $5 \times 10^4$ m$^2$/Pa s, but it still doesn’t reach that value.