STRUCTURAL DEPENDENCE OF ROTATION CAPACITY
OF PLASTIC HINGES IN RC BEAMS AND SLABS

AGNIESZKA JOANNA BIGAJ
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OF PLASTIC HINGES IN RC BEAMS AND SLABS

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Dit proefschrift is goedgekeurd door de promotor:

Prof. dr. ir. J.C. Walraven

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Subject headings: reinforced concrete / plastic hinge / size dependence / reinforcing steel / bond

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DEDICATED TO MY MOTHER AND FATHER
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Agnieszka Bigaj
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INTRODUCTION

1 INTRODUCTION

1.1 Research background

As stated in the CEB-FIP Model Code 1990 (MC90), the approach to structural modelling of reinforced concrete (RC) should be based on physical models of the internal forces and external reactions of the structures. The models should represent continuous systems of internal forces in equilibrium under design conditions and should, at least approximately, consider compatibility of deformations. These requirements, however, are far from being fulfilled simply. RC must be looked at as a compound material with a complicated behaviour, which is a result of the properties of its constitutive materials, concrete and steel, and their composite action. Hence, the non-linearity of RC depends mainly on bond between reinforcement and concrete, interaction of concrete between cracks, shear transfer behaviour between cracks and, last but not least, non-linear characteristics of reinforcing steel and concrete itself. Numerous experimental investigations proved that with increasing load RC structures show a pronounced non-linear behaviour. Regardless that, until now mostly linear elastic theory has been used to analyse members behaviour. There are, however, other static analysis methods allowed by the codes, which give a better representation of the real member behaviour (linear analysis with moment redistribution, plastic analysis, non-linear analysis).

The mechanical basis for the analysis of a static system is the theory of continuum mechanics, predominantly with equilibrium conditions, compatibility equations and constitutive laws of concrete, steel and bond. The description of the complex material behaviour of RC controls the numerical approximation of the real physical structure's behaviour. If linear elastic relations are applied to describe the behaviour of steel and concrete in a cracked state, one speaks about the linear analysis method. It is a relatively simple method of analysis, which is however a very poor tool to describe the behaviour of RC structures in their ultimate limit state, since it neglects the non-linearity in RC structures behaviour. To take advantage of this phenomenon, besides the non-linear analysis method, which implies that non-linearity arising from a realistic consideration of the structural behaviour is taken into account in the response relationship (non-linear constitutive laws for materials, non-linear force deformation relationships for cross-sections due to material properties, cracking and second order effects, etc.), for statically indeterminate structures the elasto-plastic analysis method has been introduced and is nowadays commonly used in design. In this thesis particular attention is given to the linear analysis method followed by limited moment redistribution applied to the analysis of linear members (beams and one-way slabs). In fact notion could be made
that, rather than stipulating that intentional redistribution may be carried out, limited moment redistribution should be carried out in order to correct the linear analysis for the consequences of redistribution due to the non-linear behaviour of RC. There are several benefits of utilising a moment redistribution, the following of which can be given as examples:

- savings of reinforcing steel, as there is no need to design for the full moments of the moment envelope obtained for different load arrangements
- less reinforcement placed in the negative moment zones, thus a reduced magnitude of the internal compression force (in particular beneficial for narrow webs of T-sections)
- reduction of congestion of bars over supports of continuous beams or slabs and therefore improving the conditions for attaining a good concrete quality in these critical areas
- more freedom for the designer in arranging the reinforcement

Besides the plastic analysis method, based on the assumption of indefinite plasticity of the member, or the elastic analysis with moment redistribution, which obviously needs a certain ductility of the plastic hinges, some other design procedures can be justified and some engineering problems which can be solved only when sufficient deformation capacity can be assured. The plastic deformation capacity of a RC member is needed for:

- elastic analysis (linear analysis without moment redistribution) which demands a certain rotation capacity in the plastic hinges, because the distribution of moments differs from the assumed distribution for elastic behaviour due to concrete cracking and the subsequent reduction in member stiffness
- equilibrium methods which are valid only if compatibility of displacement can be guaranteed (e.g. truss models, strut and tie models) - in these models a particular demand for ductility regards the shear reinforcement
- resistance against imposed deformations, which demands plastic adaptability of the structure (e.g. temperature, shrinkage, creep)
- ability to withstand unforeseen local impact and accidental loading without collapse (robustness)
- energy dissipation under cyclic (e.g. seismic) loading
- fire resistance
- deformability of the rebars needed for bending of the reinforcement

Provided the sufficient understanding of structural and material influences on both the available and the required deformation capacity of RC structures is available, the limits
for the application of these simplified design and detailing provisions can be determined. Hence, within a determined set of assumptions the practitioner is able to design the structure such as to take advantage of the freedom provided by the chosen procedure, e.g. in the arrangement of the reinforcement.

It is absolutely necessary to realise that any of the different design strategies has a well defined range of validity, that follows from the necessity of ensuring equilibrium and compatibility of deformations at all possible loading conditions, and depends on the deformation capacity of the structure. It is therefore not surprising that the reduced deformation capacity of reinforced concrete, attributed to the considerable changes of the properties of structural materials in recent years, has become the subject of an intensive discussion (Eligehausen et al. 1988, CEB TG 2.2 1993, 1998). Although a lot has been done to improve the properties of structural materials, some less desired developments have to been observed as well. Increasing the quality of reinforcing steel, (e.g. obtaining a higher steel strength and extending the elasticity range, bettering weldability) or industrialising steel production, went along with reducing the elongation of the steel at failure and lowering its strain hardening ratio. The combination of these two negative effects resulted in lower ductility of the RC members (e.g. reduced length of the plastic hinge and limited extent of plastification in the structure) (Van der Vlugt 1992). Furthermore, the application of a new high-strength type of concrete (HSC), which shows a more brittle fracture behaviour than the ordinary normal-strength concrete (NSC), may additionally attribute to a loss of member ductility (Markeset 1993a, Markeset 1996, Fabroccino and Pecce 1997). The last, but not the least, aspect is the dependence of the rotation capacity on the member size. Investigations have proven that the ductility of plain concrete members and concrete members with unreinforced areas (i.e. no shear- or punching reinforcement) decreases with an increase of the member size. In the field of reinforced concrete this field of research is still partially standing out.

The presently used codes of practice do not limit their application field to some selected range of member dimensions and allow the application of similar design procedures for a wide range of construction materials. A reconsideration of the existing models is therefore more than justified if one aims at a safe design and is aware of the changes of the properties of the structural materials and new developments in the design field. Ductility of RC members is the basic requirement of various design approaches, which seem to lose their validity (and certainly - their safety) when no sufficiently ductile response of the structure is to be expected. Hence, the adequacy of conventional methods for the calculation of the deformation capacity of RC members is questioned since it shows the lack of sufficient understanding of the phenomenon of plastic hinging and of the structural (i.e. member size) dependence of the rotation capacity of plastic hinges.
1.2 Scope and contents

This investigation is intended to achieve a better understanding of non-linear behaviour of reinforced concrete members on the basis of experimental work and extensive parameter studies, using a rational model developed to analyse the phenomenon of plastic hinges in reinforced concrete members. In this thesis only the analysis of flexural beams and slabs under monotonic loads is considered. It is assumed that all but bending failure modes are excluded due to sufficient member resistance against shear, torsion etc. In particular slender members without confining reinforcement are studied.

Chapter 2 deals with the physical and theoretical background of the phenomenon of plastic hinging in RC structures. Special emphasis is laid on the question of structural dependence of deformation capacity of plastic hinges in RC members. In particular member size dependence and influence of properties of construction materials are addressed. The increase of ductility with decreasing member size is interpreted from the viewpoint of fracture mechanics of concrete. The influence of the ductility characteristics of reinforcing steel is discussed in relation to bond of yielded reinforcement and to reinforcement detailing. An introductory test series on simply supported slender beams loaded in three-point bending is reported and the influence of member size, reinforcement ratio and concrete type on deformation capacity of plastic hinges is discussed, based on the results of those tests. A size effect on the rotation capacity of plastic hinges is determined from experimental results. The localisation process in the hinge region is analysed, in particular in the compression zone of the members.

In Chapter 3 the essential components of the model for calculating the rotation capacity of plastic hinges in RC members are discussed. Depending on the magnitude of the shear force in the critical region of the RC member, a distinction is made between two significantly different types of plastic hinges: the flexural crack hinge and the shear crack hinge. Considering the scope of this work, a calculation model is proposed for the first type of plastic hinge. A distinction is made between two distinct failure modes: ductile failure with dispersed cracking and non-ductile failure with a single major crack, which give, respectively, upper and lower bound values of the available rotation capacity of the plastic hinge. The behaviour of the plastic hinge is analysed taking into account the strain localisation in the damage zones in the hinge region. The concrete is modelled by virtue of a fracture mechanics approach: the Fictitious Crack Model (FCM) and the Compressive Damage Zone Model (CDZ) are adopted to describe the behaviour of concrete in tension and compression, respectively. A new, fracture mechanics based, bond model for ribbed bars in concrete is introduced.
In Chapter 4 the new bond model is presented. The importance of taking into account the influence of steel yielding when modelling the bond of reinforcement is shown, in particular in relation to the development of rotation in a plastic hinge. The incapability of the CEB-FIP Model Code 1990 to capture this behaviour is discussed. An analytical bond model capable of predicting the bond behaviour of reinforcing bars in concrete in a wide range of steel strains (in particular in the post-yield regime) is established. The way of taking into account the influence of concrete fracture properties, the bar contraction (significant after steel yielding), the degree of confinement and the corresponding mode of bond failure is described. The experimental investigation of bond is reported in the sequel. Test data on the effect of steel yielding and bar contraction on the bond behaviour of ribbed bars are provided. The test results are used to examine the bond model recommended by CEB-FIP Model Code 1990 and to verify the new bond model. Finally, the new bond model is validated against a broad range of other experimental data for various types of concrete and steel.

Chapter 5 is devoted to the general analysis of cracking in reinforced concrete. The development of a stabilised crack pattern, crack opening and the tension stiffening effect are analysed for the case of direct tension and bending. A definition of the effective concrete tension area in specimens where the reinforcement is not uniformly distributed (e.g. beams or slabs) is proposed, that takes into account the influence of the member size. A comparison with experimental results is made, the role of the properties of the construction materials is examined and the potentials of the proposed bond model are demonstrated.

Chapter 6 reports the results of the verification analysis for the calculation model for rotation capacity. The numerical procedure applied for calculating the plastic rotation of the flexural crack hinge is described. The ability of the model to predict the deformation capacity of reinforced concrete in an accurate way is shown for a broad scope of variables. The predictions of the member size dependence and of the influence of construction materials characteristics are verified against results of three point bending tests for different reinforcement ratios, concrete types and steel ductility classes.

Chapter 7 presents the results of a parameter study performed to determine the influence of the member size and of the construction materials characteristics on the ductility of RC members. The importance of the size effect in practical design situations is evaluated and the need for alteration of existing design rules in the light of a possible member size dependence of the rotation capacity of plastic hinges is investigated. The significance of detailing of the reinforcement is shown and a method is proposed to include the reinforcement detailing in analysing the member sensitivity to size dependence. The
influence of the reinforcing steel properties on the crack widths and tensile cord deformations for yielding steel and on the available rotation capacity of plastic hinges is examined, while varying the main steel ductility characteristics in a wide range. A proposal is made for an improvement of the definition of the equivalent steel ductility parameter for the case of hot rolled and cold worked steel.

Chapter 8 responds to the need, expressed by the practitioners, for a clear explanation of the relation between the steel ductility properties, demand and supply of the ductility of the RC members and allowable methods of structural analysis. It is aimed at determining the precise limits related to steel quality and structural performance, that need to be introduced into codes of practice to guarantee safe design. With respect to structural design procedures main attention is given to the linear analysis method followed by limited moment redistribution applied to analyse linear members (continuous beams and slabs). An attempt is made to arrive at the conditions for analysis of this type of statically indeterminate systems, that directly follow from the discussion of plastic hinge behaviour presented here. In this perspective the limitations of the presently used design standards (CEB-FIP Model Code 1990, Eurocode No.2 and VBC 1995) are verified and revised rules for allowable degree of moment redistribution are proposed.

Chapter 9 summarises the conclusions from this research and recommends some related topics that need to be investigated in the coming years. With respect to the latter, particular attention is given to new developments in the field of structural materials and the possibility to optimise member design with respect to ductility requirements.
2 PLASTIC HINGING IN RC MEMBERS

2.1 Problem statement

In spite of all attention recently paid to the problem of deformation capacity of reinforced concrete members, not much progress has been made with regard to other structural influences except for the dependence on the ductility of reinforcing steel. However, it has been suggested by some researchers (Hillerborg 1988, Hillerborg 1989) that rotation capacity of plastic hinges is member size dependent, approximately inversely proportional to the beam height. Some experimental work has been done in the past, where such a tendency has been noticed (Mattock 1965, Corley 1966, Cederwall and Sobko 1990, Bosco and Debernardi 1992, Bosco et al. 1992). However, so far this phenomenon remains not explained nor understood.

One of the plausible explanations of the increase of ductility with decreasing member size originates from studies on failure localisation and size dependence of the constitutive relationship of concrete in compression. It has been confirmed in uniaxial compression tests on concrete prisms that the ductility of the specimen increases when its height decreases i.e. the compressive failure tends to be less brittle with decreasing size of the specimen (Van Mier 1984, Van Mier and Vonk 1991, Vonk 1992). It can be seen from the test results presented in Fig. 2.1, that the post-peak behaviour becomes more ductile when the size of the specimen decreases. However, the differences in stress versus post-peak displacement curves for specimens of different sizes are almost negligible. Such a phenomenon can be explained by the approach based on strain localization, similar to the one used to explain softening in tension. Just like cracking in tension, the deformation in compression in the post-peak regime is localised to certain zones, whereas the material outside these zones unloads. The strain measured in the post-peak regime therefore depends on the measuring length and the stress-strain relation obtained becomes size dependent.

The principle of failure localisation in compression and the consequent size dependence of the stress-strain behaviour of compressed concrete may be adopted for the analyses of members loaded in flexure as well. Following the concept of Hillerborg (1989), recently further developed by Markeset (1993a), the size-related increase of ductility can be associated with the size dependent value of the ultimate strain of the concrete. This value, approximately inversely proportional to the beam height, includes the effect of the fracture processing within a damage zone of limited extension. The complete stress-strain curve of concrete in compression, determined on the basis of this approach, has thus a size dependent softening branch and is not a pure material but rather a structural
characteristic. This is an essential issue with regard to the influence of concrete quality and fracture characteristics on the member ductility, pondering that according to Hillerborg (1989) the beam size is likely to be at least as important as the material characteristics for the shape of the stress-strain curve of concrete in the compression zone of the RC beam. This may have a significant implication for the analysis of size-related differences in performance of NSC and HSC members.

![Graph 1](image1)

**Figure 2.1** Influence of the specimen height on the stress-strain curves for concrete under uniaxial compression (after Van Mier 1984).

Discussing the fracture process in the compression zone of the flexural member it must however be remembered that compressive failure of concrete is a three-dimensional process, highly sensitive to the boundary restraints. Both the degree of confinement provided by the structure and the crack formation at the tensile side of the beam influence the compressive strain development and the location of the final compressive damage zone. Accordingly, the strain localisation in the tensile zone of the member has to be considered as well when evaluating the phenomenon of plastic hinging in RC members.

This leads directly to the problem of deformability of reinforcing units embedded in concrete, or - more specific - to the correlated problems of deformation of stressed reinforcement and bond between steel and surrounding concrete. While the characteristics of the reinforcing steel can easily be established, a lot of uncertainties remain concerning the bond behaviour in a hinge region, especially at advanced loading stages. Up till now bond models have mostly been based on pull-out tests with short embedment length (Eligehausen et al. 1983). Since for normal concrete grades and
normal types of reinforcing steel yielding of a bar will not be reached in such tests, the experimental bond stress versus slip relationships will reflect softening of the concrete surrounding the bar, but not the effects associated with 'softening' of the bar itself. Deformations in the plastic hinge are, however, associated with the steel strains within the plastic range. To be capable of modelling these deformations, a bond model must express the bond properties in a wide range of steel strains, including the post-yield regime.

It has been experimentally observed (Shima et al. 1987a, Engström 1992) that the local bond stress-slip relationship is considerably effected by yielding of the reinforcing steel. It was found that the yield front penetration and the plastic deformations were strongly underestimated when calculations were based on the schematic bond stress-slip relationship provided in the CEB-FIP Model Code 1990. Modified bond stress-slip relationships, with a reduced bond strength for yielding steel, were proposed (Shima et al. 1987b, Cederwall and Engström 1993), but the mechanism of the steel-concrete interaction in the post-yield regime remains not fully explained. Additionally, since in most of the tests the bar diameter has not been varied systematically, experimental bond stress - slip relationships generally cannot represent the effect of the bar size. Yet, internal cone-shape cracking, which is likely to cause a significant part of the local slip, is related to the bar diameter. Furthermore, the implications of the use of HSC are of great concern given the empirical nature of most available bond models and considering that most of the experimental evidence comes from tests carried out on bars embedded in concrete of low and medium strengths. It can be expected that the bond properties of HSC, with its different fracture behaviour, are not reliably described on this basis. Adopting bond characteristics of NSC for HSC may lead to major errors or at least to an essential loss of accuracy in the analyses based on such ad hoc adjusted models.

In order to generate the basis of a consistent model for the calculation of the rotation capacity of plastic hinges in RC members a systematic experimental investigation has been carried out. In this study the mechanism of plastic hinging has been analysed taking into account the strain localisation in the damage zones in the hinge region, both in compression and in tension. This complex approach should permit a fundamental description of various aspects of structural dependence of plastic hinge behaviour. In this way both the member size dependence and the influence of construction materials on the rotation capacity of plastic hinges can be studied and explained. An indispensable step formed the development of a bond model capable of predicting the bond behaviour of reinforcing bars in concrete of various fracture properties in a wide range of steel strains (in particular in the post-yield regime). Such a general bond model serves both as an independent tool in studying cracking behaviour in reinforced concrete and as an aid in
analysing the deformation capacity of reinforced concrete members up to the ultimate loading stage.

2.2 Introductory experimental study

2.2.1 Objectives and scope of the test series

A first test series has been conducted in order to determine the size effect on the rotation capacity of plastic hinges and to investigate the localisation process in the hinge region, in particular in the compression zone of the member. It has been intended as a principal orientation step into the development of an improved model, which is dealt with in the following chapters of this thesis. This study has been limited to the case of bending without axial load, utilising simply supported slender beams loaded at mid-span. The scope of the test series covered the investigation of three major parameters: member size, reinforcement ratio and concrete type.

<table>
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<tr>
<th>Specimen</th>
<th>$d$ [mm]</th>
<th>$h$ [mm]</th>
<th>$b$ [mm]</th>
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<td>100</td>
<td>18000</td>
<td>1$d_s$ 16mm</td>
<td>201</td>
<td>1.117</td>
<td>2100</td>
<td>2000</td>
</tr>
<tr>
<td>B.1.3.4</td>
<td>490</td>
<td>450</td>
<td>250</td>
<td>112500</td>
<td>4$d_s$ 20mm</td>
<td>1256</td>
<td>1.116</td>
<td>5500</td>
<td>5200</td>
</tr>
<tr>
<td>B.1.3.16</td>
<td>490</td>
<td>450</td>
<td>250</td>
<td>112500</td>
<td>4$d_s$ 20mm</td>
<td>1256</td>
<td>1.116</td>
<td>5500</td>
<td>5000</td>
</tr>
</tbody>
</table>

Table 2.1 Geometry of test specimens

where: $d$ - total height of the beam, $A_c$ - concrete cross-section, $l_o$ - total length of the beam, $h$ - effective height of the beam, $A_s$ - steel cross-section, $l$ - span of the beam, $b$ - width of the beam, $\rho_s$ - reinforcement ratio, $d_s$ - bar diameter

Specimens of three different sizes were tested, each of them having the same ratio of width, effective height and span. The dimensions of the medium and the large specimens were determined by multiplying those of the smallest one with a factor 2 and 5 respectively. The same factors were applied to the dimensions of the loading plates so that for the whole test series the ratio of the loading plate length to its width was kept constant. Table 2.1 gives the characteristics of all tested beams, Fig. 2.2 shows a comparison of specimen dimensions, for set of beams with equal reinforcement ratio.
Figure 2.2 Geometry of test specimens (series with reinforcement ratio 0.28%)

The specimens were provided with longitudinal bars at the tensile side of the beam only. Two reinforcement ratios were chosen, namely 0.28% and 1.12%. All specimens were reinforced with ribbed bars. Use was made of hot rolled reinforcing steel FeB 500 HWL for 8, 10, 16 and 20 mm diameter bars (the average steel properties varied per bar diameter: the yield stress from $f_y = 550$ to 573 MPa, the tensile strength from $f_t = 641$ to 661 MPa, the ratio of steel strain hardening from $f_t / f_y = 1.13$ to 1.18 and the elongation of the steel at maximum load from $\varepsilon_u = 9.17$ to 9.36%). Cold worked steel FeB 500 HKN was used for 4 mm diameter bars ($f_y = 590$ MPa, $f_t = 678$ MPa, $f_t / f_y = 1.15$, $\varepsilon_u = 3.6%$). In general no stirrup reinforcement was used. The mechanical characteristics of the reinforcing steel were determined for all types of bars in an extensive series of standard tests. The actual values obtained are used in further analysis. No bond tests have been performed at this stage.

| Table 2.2 Mechanical characteristics of the reinforcing steel |
|------------------|------------------|------------------|------------------|------------------|------------------|
| **Steel type**   | $d_i$ [mm]       | $f_y$ [MPa]      | $f_t$ [MPa]      | $f_t/f_y$        | $\varepsilon_u$ [%] | $f_R$ |
| FeB 500 HKN      | 4                | 590              | 678              | 1.15             | 3.60             | 0.070 |
|                  | 8                | 562              | 641              | 1.14             | 9.17             | 0.079 |
|                  | 10               | 568              | 641              | 1.13             | 9.36             | 0.069 |
|                  | 16               | 573              | 661              | 1.15             | 9.31             | 0.073 |
|                  | 20               | 550              | 650              | 1.18             | 9.27             | 0.072 |

where: $d_i$ - bar diameter
$f_y$ - yield strength
$f_t$ - ultimate strength
$\varepsilon_u$ - elongation at maximum stress
$f_R$ - projected rib area
Two different NSC mixes were used (the maximum aggregate size $d_{a,max}$ varied from 16 mm for MIX 1 to 4 mm for MIX 2; the average concrete cube compressive strength after 28 days $f_{cc}$ reached respectively 38.3 and 33.4 MPa). Specification of the materials characteristics is given in Table 2.2 and 2.3. Out of four values presented in Table 2.3, $f_{cc}$ and $f_{cts}$ correspond to the 28 days strength determined on standard cubes stored in the fog room, while $f_{cem}^*$ and $f_{cts}^*$ refer to the time when the beams were tested and was determined on specimens stored under the same conditions as the beams. The values given are the average from three tests.

<table>
<thead>
<tr>
<th>Mix type</th>
<th>Test specimen</th>
<th>$f_{cc} / f_{cc}^*$ [MPa]</th>
<th>$f_{cts} / f_{cts}^*$ [MPa]</th>
<th>$E_c$ [MPa]</th>
<th>Age at testing [days]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIX 1</td>
<td>B.0.2.16</td>
<td>39.18 / 40.57</td>
<td>2.66 / 3.16</td>
<td>34690</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>B.1.2.16</td>
<td>38.94 / 39.76</td>
<td>2.72 / 2.91</td>
<td>35867</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>B.0.3.16</td>
<td>34.77 / 37.25</td>
<td>2.48 / 2.77</td>
<td>32070</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>B.1.3.16</td>
<td>31.58 / 35.43</td>
<td>2.61 / 2.73</td>
<td>32060</td>
<td>29</td>
</tr>
<tr>
<td>MIX 2</td>
<td>B.0.1.4</td>
<td>32.82 / 31.71</td>
<td>2.47 / 2.47</td>
<td>31137</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>B.1.1.4</td>
<td>32.60 / 33.12</td>
<td>2.51 / 2.51</td>
<td>32393</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>B.0.2.4</td>
<td>32.87 / 34.40</td>
<td>2.43 / 2.37</td>
<td>31823</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>B.1.2.4</td>
<td>33.60 / 35.27</td>
<td>2.52 / 2.33</td>
<td>31320</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>B.0.3.4</td>
<td>32.45 / 33.52</td>
<td>2.30 / 2.31</td>
<td>31407</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>B.1.3.4</td>
<td>31.52 / 32.26</td>
<td>2.20 / 2.26</td>
<td>32440</td>
<td>29</td>
</tr>
</tbody>
</table>

In total ten beams were tested, each one with a unique combination of test variables. The specimens were assigned an identifying code indicating the type of element, according to Fig.2.3. An overview of the whole test series is given in Table 2.4 (the member size is given as $b \times h \times l$, where $b$, $h$, $l$ are respectively the width, the effective height and the span of the member).

![B.a.b.c](image)

**Figure 2.3** Member identification codes
Table 2.4 Specification of test parameters and specimen codes

<table>
<thead>
<tr>
<th>Type of concrete</th>
<th>Type of concrete mix</th>
<th>Member size [mm]</th>
<th>Reinforcement ratio [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSC</td>
<td>MIX 1</td>
<td>50×90×1000</td>
<td>B.0.2.16</td>
</tr>
<tr>
<td></td>
<td>d_{a max}=16mm</td>
<td>100×180×2000</td>
<td>B.0.3.16</td>
</tr>
<tr>
<td></td>
<td>MIX 2</td>
<td>250×450×5000</td>
<td>B.1.2.16</td>
</tr>
<tr>
<td></td>
<td>d_{a max}=4mm</td>
<td></td>
<td>B.0.2.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B.1.1.4</td>
</tr>
</tbody>
</table>

2.2.2 Test assembly

Due to the big differences in beam dimensions and carrying capacity, two different testing frames have been used. The tests on beams B.0.3 (B.1.3) were performed in a 100-ton testing frame (see Fig. 2.4). The load was applied with a hand-operated hydraulic jack which could be controlled by either load or deflection. The beams were simply supported on both sides utilising pendulum bars. A felt layer was used to give an even surface between tested specimen and supports. For specimens B.0.1 (B.1.1) and B.0.2 (B.1.2) a 20-ton testing frame was used and a hand-operated hydraulic jack permitted both load- and deformation-controlled testing. The beams were simply supported at both sides on rollers, without any constraint. In all cases the load was transmitted to the beam through a stiff steel loading beam and steel loading plates. A 5 mm thick plaster layer provided an even surface between the loading plate and the test specimen. All tests were carried out in a load-controlled way until the specimen reached its maximum capacity and in a displacement-controlled way afterwards. The load was applied to the beams in steps: about ten loading steps up to the point of yielding of the reinforcement, and a number of deformation steps after the yield stress of the reinforcing steel had been exceeded. The size of the deformation step was adapted during the test.

After each load increment, when deflections and cracking had stabilised, a set of load, displacement and deformation readings was taken automatically, followed by the measurements with a demountable displacement transducer (if applicable) and by observation of the crack pattern development. The steel strains were determined on the basis of the deformations registered at both side faces of the beams at the level of the reinforcing bars. For this purpose extensometers (clip gauges) of a nominal length 50, 100 and 150 mm were basically used. For these extensometers the maximum displacement that can be measured equals 2 mm, both in tension and in compression, and the accuracy reaches 0.01 mm. To eliminate erroneous readings such as might occur when the measuring points were not properly fixed at the concrete surface, displacement transducers (LVDT’s) were additionally used to execute control steel elongation...
measurements, wherever it was possible. LVDT’s reading up to 20 mm, with an accuracy of 0.001 mm, were positioned in the zone where the hinge was expected. In order to provide information on the deformation development beyond the limit of the clip gauges, additional measurements were taken on beams B.0.3 (B.1.3) with a demountable displacement transducer. A hand-held measuring device with a build-in opto-electric displacement transducer, characterised by 0.001 mm accuracy at 5 mm range, was used here. Reliable data were achieved by repeating each recording and the measuring error was estimated to 0.03 mm. The actual number and location of measuring instruments depended on the member size.

![Diagram](image)

*Figure 2.4 100-ton testing frame used in tests B.0.3 (B.1.3)*

Close attention has been paid to the observation of the concrete deformations in the compression zone of the beam in the hinge region. Longitudinal deformations were measured using extensometers (clip gauges) with a nominal length of 50 and 100 mm, allowing a measurement of the displacement to a maximum of 2 mm both in tension and compression, with an accuracy of ± 0.01 mm. The displacement was measured between measuring points glued to the concrete surface. To register deformations in the transverse (lateral) direction LVDT’s (Sangamo type AG5, 0.001 mm accuracy at 10 mm range). These measurements were taken from the reference frame fixed to the stiff steel beam through which the load was applied.
In Fig. 2.5 an example of the final arrangement of the measuring devices in the compression zone of the beam is shown. Although an effort has been made to stay as close as possible to the same pattern, keeping the distances proportional to the member size, large differences in sizes of the specimens required a slightly different arrangement of the measuring devices than desired. The final number and location of the measuring instruments arranged in the compression zone depended on the member size, as indicated in Appendix 1. Longitudinal concrete deformations were also measured on the top surface of the beam. For more details reference is made to Bigaj (1992).

Figure 2.5 Arrangement of the measuring devices in the hinge region of the test specimen B.0.2.4

2.2.3 Discussion of test results

As far as the load-deflection response is concerned, in most cases the transition from the uncracked state to the cracked state was hard to distinguish. Contrary to that, the point of yielding of the reinforcement could be identified very well.

As it was expected, all but one (specimen B.0.1 reinforced with cold-worked steel FeB 500 HKN) members showed a distinct ductile behaviour. A long, nearly horizontal plastic branch characterizes all load-deflection diagrams, however a distinction has to be made between the high reinforcement percentage, where the peak load was reached at an early stage, and the low reinforcement percentage, where the point of peak load was noticed rather late (Fig. 2.6).
The final crack patterns are shown in Fig. 2.7 and 2.8. A typical flexural crack pattern was observed, except for specimen B.1.3.16, where shear failure took place before any yielding of the reinforcement had occurred. In all other cases an extensive degree of steel yielding was registered. There had been a major loss of bond over a considerable length of the reinforcing bars in most cases. Actually, only in the case of specimen B.0.1.4, reinforced with low ductility cold rolled steel, a very little sign of any bond failure was found. Comparing the crack width development a strong concentration of deformations at the midspan crack was found. Moreover, after the peak load had been reached, the deformation in the main midspan crack increased while some other crack widths decreased. A comparison of the measured crack width values with these estimated on the basis of the CEB-FIP Model Code 1990 formulation showed a strong discrepancy, in particular in the range beyond steel yielding.

With respect to the strain development in the compression zone of the beams, the formation of localisation areas was studied using the measurements of concrete deformation in the hinge region (see Fig. 2.9). It appeared that Bernoulli’s principle of plain sections remaining plain loses its validity at a certain stage of loading. This finding corresponds with earlier observations and analyses of Mattock (1965) and Corley (1966) who investigated the size effect on beams that failed due to concrete crushing. In Table 2.5 the maximum measured concrete compressive strains are compared, obtained in the own test series and in those of Mattock and Corley, for specimens of different sizes with for each test series similar geometry and amount of stirrup reinforcement (in tests of Mattock and
Corley equalled the effective height of the beam, the results from the own test series are presented as the actually obtained maximum values (gauge length given) and as the values calculated over the length equal to one and two times the effective height of the members.

Figure 2.7 Final crack patterns for specimens with reinforcement ratio of 0.27%

Figure 2.8 Final crack patterns for specimens with reinforcement ratio of 1.12%
Concrete strain (measurements at upper gauge line) [mm]

*location of measuring points
- concrete strains plotted over the member height at mid-span
- concrete strains plotted over the member height at sections selected in the hinge region
- location of reinforcement

Figure 2.9 Strain localization in the compression zone of specimen B.0.2.16 obtained from the measured elongation at the onset of steel yielding (left), maximum load (middle) and post peak load at the 0.98 of the maximum load (right)

The favourable effect of confining reinforcement in the compression zone on the increase of the ultimate concrete strain is evident (in the own test series no stirrup reinforcement was used). Moreover, the test results by Mattock and Corley show a trend toward a larger concrete strain at failure load with smaller member size for geometrically similar specimens. In analysing the results obtained in the own investigation, a distinction should be made between members which failed due to concrete crushing (as in the tests of Mattock and Corley) and that which failed due to exceeding the steel strain capacity. In this first case no direct link is found between the member size and the extreme concrete strain values, although it should be noted that the strain values are far in excess of the conventionally assumed value of the ultimate concrete compressive strain of 0.0035 and amounted even up to around 0.01. As far as the average strain values are concerned (both calculated over one and two times the effective member height) clearly larger strains are found for the smallest specimens, however the tendency of increased strain with decreasing specimen size is not clearly confirmed for the medium and large size specimens.

In this respect it must be stressed, that the values specified as the concrete compression strain are assessed here as the member surface shortening per unit gauge length (displacement measured between the measuring points at the outer face of the member divided by the gauge length). Besides the clearly visible effect of the measuring length
(compare average over \( l_m = h \) and \( l_m = 2h \)), also the position of the measuring device with respect to the failure localisation zone will strongly influence the measurements and must be considered by interpretation. Consequently, the localisation pictures should only be interpreted as indicative average localisation fields. However, although the computed values are not true strains in the generally accepted sense of the word, they do have a real meaning in terms of the calculation of curvature and rotation, when used in conjunction with the measured elongation of the tensile cord of the member at the same cross-section.

<table>
<thead>
<tr>
<th>Test</th>
<th>Failure type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2, A5, C2, C5, E2, F2, K6, K8</td>
<td>concrete crushing</td>
</tr>
<tr>
<td>B1, B3, D1, D3, G1, G3</td>
<td></td>
</tr>
<tr>
<td>M1, M3, M6, M8</td>
<td></td>
</tr>
<tr>
<td>N1, N3</td>
<td></td>
</tr>
</tbody>
</table>

<p>| Tests reported by ( \text{Mattock (1965) and Corley (1966)} ) |
|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>Effective height ( h ) [mm]</th>
<th>Smallest ( \varepsilon_c^{\text{max}} ) ( (l_{\text{gauge}} = h) )</th>
<th>Average ( \varepsilon_c^{\text{max}} ) ( (l_{\text{gauge}} = h) )</th>
<th>Largest ( \varepsilon_c^{\text{max}} ) ( (l_{\text{gauge}} = h) )</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.007</td>
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</tr>
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<td>508</td>
<td>0.011</td>
<td>0.015</td>
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<td>0.008</td>
<td>0.010</td>
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</table>

<p>| Tests reported by ( \text{Bigaj and Wairaven (1993)} ) |
|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>Effective height ( h ) [mm]</th>
<th>Average ( \varepsilon_c^{\text{max}} ) ( (l_m = 2h) )</th>
<th>Average ( \varepsilon_c^{\text{max}} ) ( (l_m = h) )</th>
<th>Largest ( \varepsilon_c^{\text{max}} ) ( (l_m = l_{\text{gauge}}) )</th>
<th>Test</th>
<th>( l_{\text{gauge}} ) [mm]</th>
<th>Failure type</th>
</tr>
</thead>
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<td>0.00425</td>
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<tr>
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<td>0.00546</td>
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</tr>
<tr>
<td>450</td>
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<tr>
<td>0.00054</td>
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<td>70mm</td>
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</tbody>
</table>

In the case of members that fail due to steel rupture a huge variation in maximum measured strain values is found. This is not surprising considering that in this case the equilibrium at the maximum load level depends on the actual steel properties and that the ductility of the rebars used in this test series differed (with the extreme case of specimen B.0.1.4, where low ductility steel was used).
The relation between rotation capacity and effective height of the beam has been analysed as well. Here the rotation capacity has been defined as the difference between the total rotation of a member at peak load and its total rotation at the onset of yielding of the reinforcement neglecting the post-peak deformation capacity, which is conform the definition given in CEB-FIP Model Code 1990. It has been calculated by integration of the curvature along the longitudinal axis of the member. The curvature was obtained directly at each location from measurements of the strain in the compression zone and at the position of the reinforcing bars.

Figure 2.10  Curvature distribution along the beam axis for specimens manufactured in concrete MIX 2 with reinforcement ratio 0.27% (left) and 1.12% (right)
For comparison of the curvature distributions see Fig. 2.10, where the curvature values computed on the basis of the strains measured in the compression zone (upper measurements) and at the position of the reinforcing bars are presented for some selected tests. Data are shown for two characteristic load stages, namely at the onset of the yielding of reinforcement and at the moment the member reached its maximum load carrying capacity. For detailed values of measured strains and calculated curvatures and rotations see Appendix 1.

There appeared to be a clear relation between the member size and its rotation capacity. As expected, members with smaller dimensions showed a much more ductile response to the load than members with the same characteristics but larger dimensions. This relation has been observed for both reinforcement ratios investigated. The experimental curves for NSC showed for a reinforcement ratio of 0.28% a twofold increase of the rotation capacity as the effective height of the beam decreased from 450 to 180mm, while for a reinforcement ratio of 1.12% a fourfold increase of the rotation capacity was observed with the effective height of the beam decreasing from 450 to 90mm, see Fig. 2.11. The general trend of increasing values of rotation capacity with decreasing beam height is thus verified both for members which fail due to exceeding deformation capacity of the steel bars and for members in which crushing of concrete prevails after yielding of steel.

![Diagram](image)

**Fig. 2.11  Plastic rotation at peak load versus effective height of the member**

The low value of the rotation capacity obtained for specimen B.0.1.4 proves that the use of cold worked steel can reduce the member ductility significantly, compared to cases where hot rolled (or heat treated) steel is used. A variation of the concrete mix
composition by increasing the maximum aggregate size in the range investigated does not seem to have a significant influence on the available plastic rotation of the members. For a detailed discussion of the test results reference is made to Bigaj and Walraven (1993) and Bigaj (1994).

2.2.4 Conclusions

The most significant observation from this test series is that the rotation capacity of plastic hinges is very sensitive to the member size. This effect is observed for both reinforcement ratios investigated, hence, it takes place for both types of flexural failure: due to steel rupture and due to concrete crushing. The explanation for this phenomenon is likely to be found in strain localisation in the hinge region. The size dependence of the concrete in compression arising from the member size independent dimensions of the localisation zone may be a possible answer to this question. In this respect the extremely high measured values of the concrete compressive strain provide important information for the validation of models applied to describe the concrete behaviour in the compression zone of a flexural member. The observations of the crack width development drew attention to the steel-to-concrete bonding in the range of high steel stresses. Pondering that the development of plastic hinges is directly associated with yielding of reinforcement, the obvious incapability of the CEB-FIP Model Code 1990 formulation to take the possible effect of steel yielding into account while describing the bond stress versus slip relation calls for a more detailed study of this phenomenon. Furthermore, while speaking about strain localisation, both in compression and in tension (including bond behaviour), the effect of concrete brittleness on the available plastic hinge rotation and on its size dependence asks for a better understanding. Finally, speaking about the construction materials, not only concrete but also steel mechanical characteristics play an important role in limiting the deformation capacity of plastic hinges, especially in cases where a limited steel deformation capacity controls member failure. Evidence available from this and from other research (Clarke 1990, Bühler and Eibl 1991, Calvi et al. 1993, Beeby 1994) show that high ductility steel is usually associated with the development of a long hinge region with a number of major cracks, whereas low ductility causes very localized yielding with much smaller tensile cord elongation. The need for a clear explanation of the relation between the steel ductility properties and the member ductility, expressed by the practitioners, underlines the importance of further studies in this field. In this investigation it is intended to achieve a better understanding of these problems on the basis of experimental work and extensive parameter studies, using a rational model developed to analyse the phenomenon of plastic hinging in reinforced concrete members.
3 MODELLING OF PLASTIC HINGES

3.1 General scope

In analysing the mechanism of plastic hinging in reinforced concrete and, in particular, studying the size dependence of the rotation capacity of plastic hinges, it is essential to consider strain localisations in the damage zones in the hinge region, both in compression and in tension. In this way the material and the structural influences on the available rotation capacity can be thoroughly studied. In particular a bond model capable of predicting the bond behaviour of reinforcing bars in concrete in a wide range of steel strains, suitable for the analysis of the deformation capacity of reinforced concrete members up to the ultimate loading stage is needed.

In the course of a systematic experimental and analytical study (Bigaj 1992, Bigaj and Walraven 1993) a basis for the formulation of a consistent model of plastic hinges in RC members has been provided and a calculation model for the rotation capacity can be formulated. The basic components of this model (i.e. the material models and the bond model) need to be chosen in such a way that the effect of strain localisation can be reliably incorporated. Preliminary analyses shall prove the ability of the model to predict the deformation capacity of reinforced concrete in an accurate way. In a subsequent analytical parameter study the influence of member size and construction materials characteristics on the ductility of RC members shall be determined. It is believed that the phenomenon of size dependence of rotation capacity can reliably be studied and finally fully explained by virtue of this approach.

3.2 Modelling approach

From a number of experimental and theoretical studies (Dilger 1966, Bachmann 1967, 1970, Sveinsson and Dilger 1991, Sigrist and Marti 1994, Sigrist 1995) it can be concluded that, depending on the magnitude of the shear force in the critical region of the RC member, two significantly different types of plastic hinges can develop. So-called flexural crack hinges (FC-hinges) occur in the member zone in which the bending moment is predominant, while shear crack hinges (SC-hinges) develop in the member zone where in addition to a bending moment a considerable shear force is acting. Contrary to the FC-hinges, where plastic deformations concentrate in a single or a very few cracks, so that their rotation capacity remains relatively low, SC-hinges exhibit a significantly increased rotation capacity due to the inclined flexural-shear cracks, provided that the member possesses a sufficient shear capacity. This improvement of the behaviour of the hinge is achieved by the shift of the tensile force as a result of the
inclined cracks, enlarging the length of the plastic hinge. Basing the analysis on a FC-type of hinge, lower limit values for the ultimate rotation are obtained. Keeping in mind that the main application of this model will be the analysis of slender (slab) members, where FC-hinges usually prevail, the model for the FC-hinge has been adopted.

![Figure 3.1 Modelling approach - member discretisation](image)

The reinforced concrete hinge is then approached as follows. A RC member with a crack spacing \( s_r \) is discretised as shown in Fig. 3.1. It is assumed that each of the discrete flexural crack elements (FC-element) is subjected to a bending moment only. In order to calculate the deformations of the FC-element the following fundamental relationships are needed: the stress-strain relationship of the reinforcing bars, the stress-strain relationship of the concrete representative for the material behaviour including the localisation effects (both in compression and in tension), and the bond stress-slip relationship appropriate for the bond behaviour in the hinge region. With the well defined elementary models listed above, a sectional analysis may be performed. In this analysis the validity of Bernoulli's principle of plain sections remaining plain is assumed. The stress and the strain distribution within each FC-element can be derived in an iterative calculation procedure. The elementary rotations of the FC-elements \( \Theta_{el} \) are then obtained integrating the calculated strains in the reinforcement \( \varepsilon_s \) and in the upper fibre of the member compression zone \( \varepsilon_c \), over each element length \( s_r \), according to:

\[
\Theta_{el} = \frac{\int_0^{s_r} \varepsilon_s \cdot dx + \int_0^{s_r} \varepsilon_c \cdot dx}{h} \tag{3.1}
\]

where \( h \) is the effective height of the member.
The summation of the elementary $\Theta_{el}$ rotations gives the total rotation in the hinge $\Theta_{tot}$.

In this step each FC-element, in which plastic steel deformations occur at the ultimate loading state, must be taken into account:

$$\Theta_{tot} = \sum_{i=1}^{n} \Theta_{el}^i$$  \hfill (3.2)

where $i$ is the FC-element number, $i = 0...n$.

Accordingly, the rotation capacity of the hinge $\Theta^{(p)}$ may be computed as:

$$\Theta^{(p)} = \Theta^{(u)} - \Theta^{(v)}$$  \hfill (3.3)

where $\Theta^{(u)}$ and $\Theta^{(v)}$ are the total rotations in the hinge at ultimate load and at the onset of yielding of the reinforcement, respectively.

Important with respect to member discretisation is to realise that the method of member subdivision into discrete FC-elements and thus the approach used to describe the cracking behaviour of the reinforced member is crucial. However, with respect to crack development, not only the crack spacing but also the crack location will be of influence on the attained member deformation. Depending on the position of the cracks in the presence of a moment gradient, the length of the bars where plastic deformations localise may significantly differ. As a consequence the resulting deformation of the member will also be different. This has been pointed out by other researchers as well. Beeby (1994) made even a distinction between two distinct failure modes: so-called ductile failure with dispersed cracking and non-ductile failure with a single major crack. Consequently in cases where member failure is due to steel rupture, there may not be a steady increase in member ductility as the ductility of the reinforcement increases, but a significant scatter may be observed due to the typography of the developed crack pattern. Similar conclusions resulted from the investigation of Clarke (1990), who came with the postulate that since the ductility depends on the number of cracks developed in the hinge region, which has to be an integral number, there tend to be step changes in the total plastic rotation achieved.

As it has been observed in the experiments carried out within the scope of this research project, there is nearly an equal possibility of developing a symmetrical crack pattern with and without crack at the mid-span (see Fig. 3.2 and 3.3, where for the same specimen size, reinforcement ratio and bar type two different crack patterns evolved). Although the variation of material strength along the member is obviously responsible for this effect, it may be modelled only with a very refined statistical analysis. For the
case of simplicity only two cases will be analysed within the scope of this work, namely
a member with a crack at a mid span providing a lower bound value for the deformation
capacity (non-ductile failure), and a member with a symmetrical crack pattern with no
crack at mid-span yielding an upper bound value for the available deformation capacity
(ductile failure).

Figure 3.2 Plastic hinge with crack at mid-span (test B.0.2.4)

Figure 3.3 Plastic hinge with no crack at mid-span (test B.0.2.16)

3.3 Modelling of concrete behaviour

In order to attain the required accuracy of the model for rotation capacity it is necessary
to simulate the material behaviour of concrete and reinforcing bars, and the bond
between these two materials in an appropriate manner. In this modelling approach it is aimed at taking into account the localisation effects in reinforced concrete. To fulfil this primary requirement, the concrete is modelled by virtue of a fracture mechanics approach. Such a routine offers an additional possibility to study explicitly the influence of concrete fracture toughness on the member behaviour. The constitutive relationships used in the elementary model for tension and compression are discussed below.

The essential issue in studying size effects in structural concrete is the failure localisation. By means of fracture mechanics this localisation phenomenon can be described and included in the analysis. Therefore in this study the mechanical behaviour of concrete is characterised using a fracture mechanics approach, both in tension and in compression. For this purpose the Fictitious Crack Model (FCM) and the Compressive Damage Zone Model (CDZ) are adopted, as described in the following sections.

3.3.1 Concrete in tension

(1) General considerations

To characterise the fracture of concrete in tension the Fictitious Crack Model (FCM) is used (Hillerborg 1983). This general tension softening model is based on the following fundamental principles. Up to the peak load the deformations of the concrete under a tensile load are considered to be approximately homogeneous along the whole length of the stressed specimen. When the peak stress is reached a localised damage zone (also called a fracture process zone) starts to develop. In the following post-peak regime further deformation is confined to the localised damage zone. With increasing imposed deformation this zone undergoes progressive softening, which manifests in a gradual decrease of stiffness and load carrying capacity, while the remaining parts of the body exhibit continuing unloading (see Fig. 3.4).

Contrary to the pre-peak range, at this stage the stress-deformation properties of the material cannot be described by means of one single general stress-strain curve, since the deformations along the specimen length cannot be assumed to be uniformly distributed any more. The composition of a complete stress-strain relationship of FCM is yet possible by means of two diagrams. The stress-strain diagram describes the deformation properties prior to the peak load and includes the unloading branch, to describe the response of the material outside the damage zone. The stress-displacement diagram represents the development of the additional deformation $w$ inside the damage zone beyond the peak load (stress-crack opening softening branch). With the defined shapes of the $\sigma_r e_{ct}$ and $\sigma_r w$ curves the material properties of concrete in tension are
fully defined by the tensile strength $f_{ct}$, the modulus of elasticity $E_c$ and the fracture energy $G_F$. With these two diagrams a resulting complete stress-strain curve can be obtained, which is size dependent and should be interpreted not as a pure material feature but as a specific structural characteristic.

![Diagram of FCM on a prism in uniaxial tension and schematisation for a calculation model](image)

**Figure 3.4 Illustration of the Fictitious Crack Model (FCM) on a prism in uniaxial tension and schematisation for a calculation model**

The total energy dissipated in completely fracturing a specimen is given as the summation of the energy absorbed inside and outside the damage zone. If the FCM with bulk dissipation is applied the total tensile fracture energy per unit volume $W'$ follows from equation 3.4:

$$W' = \frac{G_{FA}}{L} = \omega_{DU} + G_F \frac{1}{L} \quad \text{(3.4)}$$

where
- $G_{FA}$ is the apparent fracture energy per unit area
- $\omega_{DU}$ is the energy per unit volume absorbed in the bulk
- $G_F$ is the fracture energy per unit area absorbed in the fracture zone during the complete fracture process
- $L$ is the specimen length

For FCM with no bulk dissipation the energy per unit volume absorbed in the bulk $\omega_{DU}$ becomes zero and the total energy supply needed to fracture the specimen equals:
\[ W' = G_F \frac{1}{L} \]  \hfill (3.5)

In the following analyses the FCM with no bulk dissipation is applied, further discussed in *Elías and Planas* (1989). The average tensile strain \( \varepsilon_{ctm} \) on a certain length \( L \) containing one localisation zone becomes then:

\[ \varepsilon_{ctm} = \varepsilon_{ct} + \frac{w}{L} \]  \hfill (3.6)

where \( \varepsilon_{ct} \) is the uniform strain

\( w \) is the deformation localised in the fracture zone, interpreted as a fictitious crack opening, in which \( w_o \) is the localised deformation in the loading direction at failure.

With the defined shapes of the \( \sigma_r \varepsilon_{ct} \) and \( \sigma_r w \) curves the material properties of concrete in tension are fully defined by the tensile strength \( f_{ctp} \), the modulus of elasticity \( E_c \) and the fracture energy \( G_F \). It is worthwhile to combine these values into a characteristic length \( l_{ch} \), used as a measure for the brittleness, which is defined as:

\[ l_{ch} = \frac{E_c \cdot G_F}{(f_{ct})^2} \]  \hfill (3.7)

The characteristic length \( l_{ch} \) is a pure material property with no direct physical correspondence. It can be used to describe the toughness of the material. It is also worth noticing that the length of the fracture zone is likely to be proportional to \( l_{ch} \) and that the length of the fracture zone at crack growth can be estimated to be about 0.3-0.5 \( l_{ch} \) (Hillerborg 1983).

(2) Application for calculations

In the numerical simulations the above mentioned FCM is used with the following simplifications. Linear elastic response in tension is assumed prior to peak stress (see Fig. 3.5). It is modelled as a straight line with a slope equal to the modulus of elasticity \( E_c \). For normal density gravel concrete (both for NSC and HSC) the modulus of elasticity is calculated as a function of compressive strength of concrete \( f_c \):

\[ E_c = 9000 \ (f_c)^{0.33} \]  \hfill (3.8)

A bi-linear softening relation, as proposed in *Roelfstra and Wittmann* (1986), is chosen to model the softening behaviour in tension. If this shape of the softening branch is chosen, the fictitious crack width \( w \) can be expressed as:
\[ w = \frac{w_o}{a_i} \left( \frac{\sigma_i}{f_{ct}} - b_i \right) \]  

(3.9)

where \(a_i, b_i\) \((i = 1, 2)\) are the model constants for two branches of the softening curve. With respect to \(\sigma_i\) this yields:

\[ \frac{\sigma_i}{f_{ct}} = a_i \frac{w}{w_o} + b_i \]  

(3.10)

Expressing the constants \(a\) and \(b\) in the relative coordinates \(\alpha\) and \(\beta\) of the intersection point yields for the two branches of the softening curve with respect to \(w\):

\[ w = -w_o \frac{\alpha}{1 - \beta} \left( \frac{\sigma_i}{f_{ct}} - 1 \right) \]  

(3.11)

and

\[ w = -w_o \frac{1 - \alpha}{\beta} \left( \frac{\sigma_i}{f_{ct}} - \frac{\beta}{1 - \alpha} \right) \]  

(3.12)

and with respect to \(\sigma_i\):

\[ \frac{\sigma_i}{f_{ct}} = -\frac{1 - \beta}{\alpha} \frac{w}{w_o} + 1 \]  

(3.13)

and

\[ \frac{\sigma_i}{f_{ct}} = -\frac{\beta}{1 - \alpha} \frac{w}{w_o} + \frac{\beta}{1 - \alpha} \]  

(3.14)

The fracture energy \(G_F\) and the characteristic length \(l_{ch}\) can be expressed in the same coordinates as:

\[ G_F = 0.5 \left( \alpha + \beta \right) f_{ct} w_o \]  

(3.15)

\[ l_{ch} = 0.5 \left( \alpha + \beta \right) \frac{E_c w_o}{f_{ct}} \]  

(3.16)

With the strain value at peak stress \(\varepsilon_{cr}\) defined as:

\[ \varepsilon_{cr} = \frac{f_{ct}}{E_c} \]  

(3.17)

the characteristic length \(l_{ch}\) becomes:

\[ l_{ch} = 0.5 \left( \alpha + \beta \right) w_o \varepsilon_{cr} \]  

(3.18)

For normal density concrete (both for normal strength and high strength concrete) the
localised deformation at failure $w_o$ is assumed to be equal to 0.20 mm, the value of the
model constant $\alpha$ is set to 0.14, while the value of $\beta$ is linked to the strength of the
concrete and is approximated by:

$$
\begin{align*}
\text{for } f_c < 30 \text{ MPa} & \quad \beta = 0.25 \\
\text{for } f_c \geq 30 \text{ MPa} & \quad \beta = 0.25 - 0.0015 \cdot (f_c - 30)
\end{align*}
$$

These values were derived from experimental results reported by Roelfstra and Wittmann
(1986) for normal strength concrete and by König and Remmel (1992) for high strength
concrete.

![Diagram](image)

*Figure 3.5 FCM schematization for application - composition of the $\sigma$-$\varepsilon_{cim}$ curve used
in calculation model for concrete in tension*

In order to obtain a complete $\sigma_t$-$\varepsilon_{cim}$ relationship, which could be used in section
analyses, a discretisation procedure has to be considered and an assumption has to be
made concerning the development of damage regions in the tensile zone of the bent
member. In this modelling approach (FCM) it is justified to assume that the distance
between the successive damage zones $L^i$ equals the crack spacing $s_r$. This results, for the
two branches of the post-peak curve, in the following relationships for the average
tensile strain:

$$
\varepsilon_{cim} = \frac{\sigma_t}{E_c} + \frac{w_o}{s_r} \frac{\alpha}{1 - \beta} \left( 1 - \frac{\sigma_t}{f_{ct}} \right)
$$

(3.19)
and

\[ \varepsilon_{ctm} = \frac{\sigma_t}{E_c} + \frac{w_o}{s_r} \frac{1 - \alpha}{\beta} \left( \frac{\beta}{1 - \alpha} - \frac{\sigma_r}{f_{ct}} \right) \]  \hspace{1cm} (3.20)

and with respect to \( \sigma_r \):

\[ \sigma_r = \frac{\varepsilon_{ctm} - \frac{w_o}{s_r} \alpha}{f_{ct} - \frac{w_o}{s_r} \frac{\alpha}{1 - \beta}} \]  \hspace{1cm} (3.21)

and

\[ \sigma_t = \frac{\varepsilon_{ctm} - \frac{w_o}{s_r}}{f_{ct} - \frac{w_o}{s_r} \frac{1 - \alpha}{\beta}} \]  \hspace{1cm} (3.22)

3.3.2 Concrete in compression

(1) General considerations

To model the failure of concrete under compression the Compressive Damage Zone Model (CDZ) is applied (Markeset 1993a). This model describes the response of concrete in compression using the principal ideas illustrated in Fig. 3.6. Similarly to the tensile case, prior to the peak load the longitudinal strain in the concrete is regarded as uniformly distributed along the whole length of the specimen. After the peak load, longitudinal tensile cracks and lateral deformations occur within a limited part of the specimen (damage zone). Along with the axial splitting in the damage zone, inclined micro-cracks coalesce to form a localised shear-band and the adjoining parts tend to slide relatively to each other.

The composition of the complete stress-strain relationship of the CDZ model proceeds as follows: the behaviour of the material prior to the peak load is described by the conventional ascending branch and the deformations are assumed to be uniformly distributed; beyond the peak load a distinction is made between the response of the material inside and outside the damage zone: outside the damage zone an unloading branch is assumed, starting from the peak stress; inside the damage zone the tensile fracture process beyond the peak is assumed to be uniformly distributed and is described by the softening branch of the stress-strain relationship. A sliding failure, confined to the
inclined shear-band, is described by a stress-displacement relationship. This means that, similarly to the approach used in modelling concrete in tension, the descending branch of the complete stress-strain relation may not be considered as a material property but rather as a structural characteristic.

Figure 3.6 Illustration of the Compression Damage Zone Model (CDZ) on a prism in uniaxial compression and schematisation for a calculation model

The total energy absorbed in the structure is then given as the summation of the energy absorbed in the longitudinal tensile cracks and the energy absorbed in the inclined shear-band, inside and outside the damage zone. The compressive fracture energy per unit volume $W^c$ may be given as:

$$W^c = W^{in} + W^s \frac{L^d}{L} + G^l \frac{1}{L}$$  \hspace{1cm} (3.23)

where $W^{in}$ is the energy per unit volume absorbed in the longitudinal cracks up to peak stress

$W^s$ is the energy per unit volume absorbed in the longitudinal cracks after peak stress

$G^l$ is the energy per unit area perpendicular to the $\sigma_c$ consumed in the shear-band

$L^d$ is the length of the damage zone

$L$ is the specimen length

The average compressive strain $\varepsilon_{cm}$ for a specimen with length $L > L^d$ is given as:
\[ \varepsilon_{cm} = \varepsilon_c + \varepsilon_d \frac{L^d}{L} + \frac{w}{L} \]  

(3.24)

where \( \varepsilon_c \) is the uniform strain  
\( \varepsilon_d \) is the additional compressive strain after peak stress caused by the axial splitting process in the damage zone (\( \varepsilon_{du} \) indicates the strain at failure)  
\( w \) is the additional deformation in the fracture zone caused by the sliding failure, interpreted as the vertical component of the sliding deformation along the inclined shear-band, in which \( w_c \) is the localised deformation in the loading direction at failure

The primary material parameters of the model are the values of energy absorption that characterize the fracture processes distinguished inside and outside the damage zone, i.e. the areas under the three curves given in Fig. 3.6: \( W^{in} \), \( W^s \) and \( G^l \). Introducing some secondary material parameters, the characteristic energy absorption can be defined in the following way:

\[ W^{in} = \frac{G_F}{r \left( 1 + k \right)} \]  

(3.25)

\[ W^s = \frac{k \cdot G_F}{r \left( 1 + k \right)} \]  

(3.26)

\[ G^l = \xi \cdot f_c \cdot w_c \]  

(3.27)

where \( k \) is the proportionality factor, that describes the relation between the energy dissipated in the longitudinal tensile cracks up to the peak stress and after peak stress; it accounts for the fact that at the peak stress the longitudinal cracks are only partially opened and only a part of the available fracture energy is consumed; the value is defined as:

\[ k = \frac{W^s}{W^{in}} \]  

(3.28)

\( r \) is the parameter proportional to the average distance between successive longitudinal cracks, most likely decreasing with decreasing maximum aggregate size; the value is defined as:

\[ r = \frac{G_F}{W^{in} + W^s} \]  

(3.29)

\( f_c \) is the compressive strength
\( \zeta \) is the shape factor; if a straight line relation is assumed, the shape factor equals 0.5 and the localised deformation at failure \( w_c \) is being estimated between 0.4 and 0.7 mm in the case of normal density concrete (both for NSC and HSC).

Considering the above mentioned relations, the compressive fracture energy per volume \( W^c \) can be expressed as a function of secondary material parameters of the CDZ model in the following way:

\[
W^c = \frac{G_F}{r \left( 1 + k \right)} \left( 1 + k \frac{L^d}{L} \right) + \zeta \cdot f_c \cdot \frac{w_c}{L} \tag{3.30}
\]

In modelling cases where a strain gradient is present, i.e. the bending case, it has to be considered that the difference in deformation between different fibres influences both the strength and the ductility. As soon as the localization starts to develop, the less stressed fibres partially take over the forces. The redistribution of internal stresses consequently leads to a decrease of stress in the mostly stressed fibre over a distance \( L^d \) in the stress direction, i.e. over the length of the damage zone. An additional assumption regarding the development and the interaction of several damage zones is needed to formulate the complete \( \sigma-\varepsilon_{cm} \) relationship in the case of bending. In this analysis it is assumed that the damage zones are close together and that the distance between the successive damage zones \( L^l \) is equal to the damage zone length \( L^d \), which yields:

\[
\varepsilon_{cm} = \varepsilon_c + \varepsilon_d + \frac{w}{L^l} \tag{3.31}
\]

\[
W^c = \frac{G_F}{r} + \zeta \cdot f_c \cdot \frac{w_c}{L^l} \tag{3.32}
\]

Furthermore, it is likely that the length of the damage zone \( L^d \) is proportional to the depth of the damage zone \( d^l \). With the proportionality factor \( k^l \) it gives:

\[
L^d = L^l = k^l \cdot d^l \tag{3.33}
\]

In the case of bending the value of parameter \( k^l \) is assumed to be equal to 5. The depth of the damage zone \( d^l \) in a bent member can be derived from the following relation:

\[
d^l = \frac{\left( \varepsilon_{cm} - \varepsilon_o \right)}{\varepsilon_{cm}} \cdot x \tag{3.34}
\]

where \( \varepsilon_{cm} \) is the compressive strain of the most stressed fibre
\( \varepsilon_o \) is the compressive strain at peak stress
$x$ is the depth of the compression zone

Based on the assumption of a linear strain distribution in the cross-section of a flexural member, the depth of the damage zone $d'$ can be calculated as:

$$d' = \frac{(\varepsilon_{cm} - \varepsilon_o)}{(\varepsilon_{cm} - \varepsilon_s)} \cdot h$$

(3.35)

where $\varepsilon_s$ is the tensile strain in the longitudinal reinforcement

$h$ is the effective height of the member.

It must be remembered that in cases where a strain gradient is present (e.g. in the compression zone of a bent member) no unique $\sigma_c$-$\varepsilon_{cm}$ relationship exists, since the boundary conditions are not the same for all the fibres of the joint. Nevertheless, considering the minor influence of the variation of boundary conditions on the results in terms of section analysis, it is justified to assume the same $\sigma_c$-$\varepsilon_{cm}$ relationship for the whole compression zone. Consequently, one $\sigma_c$-$\varepsilon_{cm}$ diagram with a descending branch may be used. Such a curve is size dependent, or more precisely strain gradient dependent, which makes it to an appropriate instrument in studying the size effects on the overall member behaviour.

(2) Application for calculations

The simplified stress-strain and stress-distribution curves as well as the resulting complete stress-strain characteristics used in the numerical analysis are shown in Fig. 3.7. The ascending branch in compression is represented by a bi-linear curve. The slope of the elastic unloading curve is assumed to be equal to the modulus of elasticity $E_c$. The softening curves, both $\sigma_c$-$\varepsilon_d$ and $\sigma_c$-$w$ are approximated with straight lines. The average compressive strain in the post-peak region is expressed as:

$$\varepsilon_{cm} = \varepsilon_o - \left( \frac{f_c - \sigma_c}{E_c} \right) + \varepsilon_{du} \left( 1 - \frac{\sigma_c}{f_c} \right) + \frac{w_c}{k^l \cdot d^l} \left( 1 - \frac{\sigma_c}{f_c} \right)$$

(3.36)

with respect to $\sigma_c$ this can be rewritten as:

$$\frac{\sigma_i}{f_c} = 1 + \frac{\varepsilon_m - \varepsilon_o}{f_c} - \frac{\varepsilon_{du}}{E_c} - \frac{w_c}{k^l \cdot d^l}$$

(3.37)
The value of compressive strain at peak stress $\varepsilon_o$ is being estimated to:

$$\varepsilon_o = \frac{f_c}{E_c} + \gamma \frac{G_F}{f_c}$$  \hfill (3.38)

where $\gamma$ is the parameter representing the non-linearity of the stress-strain curve in the pre-peak region, with a value approximately equal to 0.25 mm$^{-1}$ for normal density concrete with max. aggregate size 16 mm and 0.50 mm$^{-1}$ for normal density concrete with max. aggregate size 4 mm (both for NSC and HSC)

Figure 3.7 CDZ model schematisation for application - composition of the $\sigma$-$\varepsilon_m$ curve used in calculation model for concrete in compression

The strain softening parameter $\varepsilon_{du}$ is estimated as follows:

$$\varepsilon_{du} = \frac{2}{1 + k} \frac{G_F}{f_c} \frac{r}{r}$$  \hfill (3.39)

where $k$ is the proportionality factor discussed above, with a value of approximately 3 for normal density concrete (both for NSC and HSC)

$r$ is the material parameter discussed above, with a value of approximately 1.25 mm for normal density concrete with max. aggregate size 16 mm and of 0.625 mm for normal density concrete with max. aggregate size 4 mm (both for NSC and HSC)
The localised deformation at failure $w_e$ is being chosen equal to 0.5 mm for normal density concrete (both for NSC and HSC). The proportionality factor $k^l$ in the case of bending equals to 5. For a more detailed discussion of the parameters choice see Markeset (1993a), where the CDZ model verification is reviewed for the case of uniaxial and eccentric compressed concrete prisms and unconfined concrete in the compressive zone of a beam in bending.

3.4 Modelling of steel and bond behaviour

3.4.1 General considerations

The deformations in a reinforced cracked section are strongly affected by the deformation characteristics of the reinforcing bars itself as well as by its bond behaviour. It is indispensable to provide a proper description of these both elements in order to secure a satisfactory representation of the member behaviour. As far as the properties of the reinforcement are concerned, there is enough information available on variation of steel properties for different steel ductility classes (see for instance Rußwurm and Martin 1993). This independent part of the model for rotation capacity can thus reliably be defined - using the actual steel properties if available, or incorporating the results of a statistical analysis of the steel material characteristics. However, as far as the bond model for reinforcing units is concerned, an essential lack of information has been found, especially with regard to the bond behaviour of yielding steel in concretes of different toughness.

With this concern an additional study was performed, aiming at developing a general bond model for ribbed bars, which accounts for the effect of steel-yielding on bond in normal strength and high strength concrete (Den Uijl and Bigaj 1996). The new bond model takes into account the concrete quality, the bar contraction (significant after steel yielding), the degree of confinement and the corresponding mode of bond failure. In Chapter 4 an outline of the systematic study on bond of ribbed bars in concrete is given, the experimental investigation is briefly presented and a new generalized bond model is discussed in detail.

(1) Steel model for calculation

For the purpose of application in the calculation model the stress-strain relation of the reinforcing steel is simplified by a polygon. A polygon with 6 points permits a sufficiently accurate representation of the characteristic shape of this curve, which is significantly different for various types of steel. It also allows to vary the shape of this
diagram in such a way that the influence of specific material characteristics can separately be studied. As shown in Fig. 3.8, major differences appear in the stress-strain relation of a hot rolled or a heat treated type of steel (left) and that of cold worked bars (right). The elastic part of the ascending branch is defined by the modulus of elasticity $E_y$, which can be taken equal to 200000 MPa if no actual value from standard material tests is provided. A significant yield plateau followed by a distinct hardening part and an often nearly horizontal second part of the pre-peak branch characterises the post-yield part of the typical diagram of hot rolled or heat treated steel. In this case the shape of the stress-strain diagram is defined by a number of parameters: yield stress $f_y$, tensile strength $f_p$, ratio between these two values $f_t / f_y$, steel strain value at the onset of strain hardening $e_{sh}$ and at the peak stress $e_u$. On the contrary, for cold worked steel types no specific yield point can be defined and a gradual transition from the pre-yield to the post-yield range is found. In this case the characteristic parameters are the yield stress $f_y$ (conventionally defined as the stress value at 0.02% plastic strain), tensile strength $f_p$, the ratio $f_t / f_y$, and the value of the steel strain at the peak stress $e_u$.

![Figure 3.8](image)

*Figure 3.8 Typical steel stress-strain relationships for heat treated and hot rolled bars (left) and cold worked bars (right); actual diagrams from tensile tests (top) are compared with the 6-point polygon model (bottom)*

While the steel strains are assumed to be uniformly distributed along the whole equally stressed length of the bar until the peak stress is reached, in the post peak range strain localisation takes place. Finally it leads to brittle steel failure at the point of total exhaustion of the strain capacity in the failure localisation zone. Pondering the
uncertainties in defining this part of the steel stress-strain curve and considering that steel necking under constant load leads to brittle failure, only the ascending part is taken into account.

In this context it is important to remind that steel properties should not be judged in absolute terms, but according to the structural requirements for reinforced concrete members. Each of the characteristic material parameters plays an important role in defining the structural performance of the bar with regard to ductility and deformation capacity. The comparison of different steel types within one steel ductility class and ductility classes with each other is only possible if a new parameter is introduced, that combines all the above listed characteristics. In an attempt to facilitate such a comparison Cosenza et al. 1992, 1993, Beeby 1992, Ortega 1993 and Cosenza and Manfredi 1996 proposed the equivalent steel concept, where a single ductility parameter is introduced to describe the overall steel quality. The basic idea behind this concept is that various steel types, characterised by a different set of conventional characteristics provide the same deformation capacity of a structure, which fails due to the steel rupture, if the overall ductility parameter value is the same. Although different approaches are followed to arrive at the final definition, basically very similar formulations are given. To facilitate the comparison of different steel types performed in the course of the parameter studies (Chapter 7), in this work the formulation according to Cosenza and Manfredi 1996 is adopted, and the overall steel ductility parameter $p$ is defined as follows:

\begin{equation}
    p = \varepsilon_u^{0.75} \cdot \left( \frac{f_t}{f_y} - 1 \right)^{0.9}
\end{equation}

- for cold worked steel

\begin{equation}
    p = ((\varepsilon_u - \varepsilon_{sh}) + 4 \cdot \varepsilon_{sh})^{0.75} \cdot \left( \frac{f_t}{f_y} - 1 \right)^{0.9}
\end{equation}

- for hot rolled or heat treated steel

Finally, an important remark regarding the modelling of reinforcing steel is made with respect to the influence of lateral strains in the bar. While bar contraction before steel yielding is very limited, softening of steel goes along with largely increased lateral strains. This effect is of importance in analysing bar to concrete bonding, as discussed in detail in Chapter 4. In the analyses a constant value of the Poisson ratio $\nu = 0.3$ is assumed for the whole range of steel stresses.
4 BOND OF RIBBED BARS IN CONCRETE

4.1 Problem definition

4.1.1 Introduction

The length of a plastic hinge, which is of importance for the rotational capacity of reinforced concrete members, depends among other things on the force transfer from reinforcement to concrete between subsequent bending cracks (Bigaj and Walraven 1996). For a refined analysis of this phenomenon as well as for the analysis of other phenomena, in which bond plays a dominant role (cracking and crack width development, tension stiffening, bar anchorage etc.) a bond stress versus slip relation is required that accurately takes into account the response of the concrete surrounding the bar to the bar displacement. To that end two important components have to be known: the first one is the magnitude of the force induced by the displaced bar on the concrete, the second one is the capacity of the concrete to resist this force. In analysing the first component, attention is focused on the mechanism of force transfer from a ribbed bar to the surrounding concrete, its connection with the bond failure mode and the state of stresses in the pulled bar. To evaluate this second component, the concrete confining capacity is estimated.

4.1.2 General description of bond behaviour

When bond between a ribbed bar and concrete is activated three consecutive stages can be observed. First, the initial contact between steel and concrete is maintained by adhesion and interlocking of the cementitious matrix and the steel surface. In this stage elastic bond behaviour is assumed, which is connected to small bond stress values. In the second stage, which starts when the initial bond is broken, bond is mainly governed by bearing of the ribs of the bar against the concrete. The concentrated bearing forces in front of the ribs cause the formation of cone-shaped cracks starting at the crest of the ribs. The resulting corbels between the ribs transfer the bearing forces into the surrounding concrete. In this stage the displacement of the bar with respect to the concrete (slip) consists of bending of the corbels and crushing of the concrete in front of the ribs, see Fig. 4.1 (Goto 1970). The bearing forces, that are inclined with respect to the bar axis, can be decomposed into the directions parallel and perpendicular to the bar axis. The parallel component equals the bond force, whereas the radial component induces circumferential tensile stresses in the surrounding concrete, which may result in radial cracks.
Now two failure modes have to be considered. If the radial cracks propagate through the entire cover bond splitting failure is decisive. In that case the maximum bond stress follows from the maximum radial stress delivered by the surrounding concrete. Further crack propagation results in a decrease of the radial compressive stress. At reaching the outer surface - which marks the beginning of the third stage of the bond splitting failure mode - this stress strongly reduces resulting in a sudden drop of the bond stress. Yet, the load bearing mechanism remains the same as in the previous stages.

*Figure 4.1 Deformations around the bar for splitting bond failure, after Goto (1970)*

When the confinement is sufficient to prevent splitting of the concrete cover bond failure is caused by pull-out of the bar. In that case a new sliding plane originates around the bar, shearing off the concrete corbels and the force transfer mechanism changes from rib bearing into friction, see Fig. 4.2. The shear resistance of the corbels can be considered as a criterion for this transition, which in this case of pull-out bond failure mode marks the beginning of the third stage. Due to the lower roughness of the new sliding plane compared to that of the ribbed bar, the occurrence of this surface goes along with a considerable reduction of the radial compressive stress and, hence, with a reduction of

*Figure 4.2 Deformations around the bar for pull-out bond failure*
the bond stress. Under continued loading the sliding surface is smoothened, due to wear and compaction, and the attendant volume reduction will result in a release of the radial strain and in further reduction of the bond stress.

In general it is assumed that the Poisson effect has a negligible influence on the bond resistance of ribbed bars. As long as rib bearing is the force transferring mechanism (splitting bond failure) this statement can be justified, considering that the transverse deformation resulting from the local change of steel stress is small compared to the rib height. However, when the force transfer mechanism changes from rib bearing to friction (pull-out failure) the local transverse deformation can not be disregarded considering the roughness of the sliding plane. In this case the Poisson effect may considerably influence the development of the radial compressive stress and, hence, the bond stress. For an increasing steel stress this will result in a release of the radial strain and, thus, in a reduction of the bond stress, which may become pronounced when the steel starts to yield.

Parallel to the analytical study of the bond phenomenon an experimental investigation has been performed, in particular meant to provide information on the effect of steel yielding and bar contraction on the bond behaviour of ribbed bars. The tests are described in the sequel. The data obtained in this test series were used to tune and verify the new bond model.

4.2 Experimental investigation

4.2.1 Object and scope of test series

A systematic experimental investigation of the bond behaviour of rebars in concrete was conducted. Since in practical cases two significantly different bond failure mechanisms take place, namely bar pull-out and concrete cover splitting, the test series were designed in such a way that both types of bond failure could be studied. Two test series were performed: the first one on bars with large embedment lengths, cast into massive specimens, and the second one on beam-type elements with a usual concrete cover (see Fig. 4.3 and 4.5 for test set-up). The scope of the test series was limited to the three prime variables: confining conditions (confined and unconfined concrete), concrete type (NSC and HSC) and bar diameter. The local and the overall bond behaviour for both types of bond failure was reconstructed, based on the local steel strain measurements. The test results served the validation of the currently used design bond model, recommended by the CEB-FIP Model Code 1990. It also provided sufficient data for a thorough verification of a new analytical concrete confinement based bond model.
4.2.2 Test assembly

To investigate the influence of the confining conditions on the bond behaviour two types of test specimens were chosen. The bond performance of deformed bars embedded in confined concrete (no occurrence of splitting cracks permitted) was studied using a massive concrete cylinder with a diameter of 500 mm. The length of the test specimens depended on the bar diameter $d_s$ and equalled $60d_s$, with an embedded length of $50d_s$ and a bond-free length at the loaded end of $10d_s$. Table 4.1 and Fig. 4.3 give the details of the test specimen geometry for both bar diameters used (testing devices schematically represented in Fig. 4.3).

<table>
<thead>
<tr>
<th>Specimen codes</th>
<th>Concrete cylinders dimensions</th>
<th>Reinforcing bar dimensions</th>
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<tr>
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<td>$d_c$ [mm]</td>
<td>$h_c$ [mm]</td>
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<tr>
<td>P.16.16.1</td>
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<td>P.20.HS.2</td>
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</table>

where: $d_c$ - diameter of concrete cylinder
$h_c$ - total height of concrete cylinder
$h_{db}$ - bond-free part of concrete cylinder
$l_b$ - embedded length of reinforcing bar

One bar was concentrically cast in the concrete cylinder. The concrete was cast in a vertical direction parallel to the bar (the pull-out force was acting in the opposite direction). A steel cone, removed after demoulding, provided a duct of so-called bond-free length at the loaded end. To ensure that yielding of the bar would take place at the edge of the specimen (at the beginning of the embedded part of the bar) a LENTON taper threaded splicing coupler was used for the connection of the embedded reinforcing bar with the loading device. It transmitted the pull-out force from the loading device to the bar practically up to the 100% of the maximum bar capacity ($P_{max}$) (the average value of $P_{max}$, as determined in standard bar tensile tests, was reached in all tests performed).

In order to follow the steel strain development along the whole embedded length of the bar, specially prepared and instrumented steel bars were used. For this purpose, as shown in Fig. 4.4, the longitudinal ribs of the ordinary deformed reinforcing bars were removed.
and the grooves were made on both sides all along the bar. In the space created in this way strain gauges, used to measure the steel strain development, and electrical wires were placed. To assure a proper insulation of the strain gauges the grooves were filled with epoxy coating before casting. Post-yield strain gauges YFLA-2 (gauge length 2 mm, gauge width 1.8 mm), capable of measuring strains up to 10% were used (in the experiments the maximum measured strain limit was reduced to 5% due to the limited wires elongation).

**Figure 4.3** Specimen geometry and test set-up for pull-out bond tests

The location of the strain gauges on the bar is indicated in Fig. 4.3 (7×2 gauges at the interval of 2.5d_s, 3×2 gauges at the interval of 5d_s and 2×2 gauges at the interval of 10d_s).

**Figure 4.4** Test bar geometry
In order to study the process of concrete cover splitting and bond behaviour of bars in unconfined concrete for large steel deformations, modified beam tests were performed (see Fig. 4.5 for test set-up). All beams had a rectangular cross-section: 100×212 mm and 160×220 mm for the bar $d_e = 16$ and 20 mm respectively, so that the reinforcement ratio remained constant and the bottom concrete cover on the bar equalled $1.5d_e$ in all cases. The beams were reinforced with a single reinforcing bar; no transverse reinforcement was used. The reinforcing bar was prepared in the same way as the bars used in the pull-out tests and a series of 2×14 strain gauges was applied in the middle part of the specimen at equal distances of 40 mm. There was a threaded LENTON coupling, connecting the rebar at mid-span, and a knife-edge hinge in the upper part of the beams. The span of a coupled beam between the supports amounted to 2000 mm. The casting position (perpendicular to the bar, with the bar located at the bottom of the specimen) corresponded to what is usual in practice.

![Diagram of specimen geometry and test set-up for bending bond tests](image)

*Figure 4.5* Specimen geometry and test set-up for bending bond tests

The other two prime test parameters - concrete strength in connection with mix composition and reinforcing bar diameter were chosen in the following way. Normal strength and high strength concrete with mechanical characteristics as given in Table 4.2 were used. The concrete strength was determined in two sets of standard tests (cubes 150×150×150 mm and prisms 400×100×100 mm), performed on specimens cured in a fog room after 28 days (set 1) and on specimens cured under similar conditions as the
test specimens and tested at the same age (set 2). The cube compressive strength $f_{cc}^*$ and the tensile splitting strength $f_{cts}^*$ given in Table 4.2 are the average characteristics obtained in the first set of standard tests, while $f_{cc}, f_{cts}$ and $E_c$ (Young’s modulus of concrete) are the results of the second set of standard tests. The compressive and the tensile strengths of the concrete, in conformity with the CEB-FIP Model Code definitions ($f_c^*$ being the uniaxial compressive strength of cylinders, 150 mm in diameter and 300 mm in height and $f_{ct}^*$ being the axial tensile strength, determined in accordance with RILEM CPC 7) are computed as follows:

for NSC \[ f_{ct} = 0.85 f_{cts} \] and for HSC \[ f_{ct} = 0.70 f_{cts} \]

\[ f_c = 0.80 f_{cc} \]

\[ f_c = 0.88 f_{cc} \]

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<th>Concrete type</th>
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<th>$f_{cc}^<em>/f_{ct}^</em>$ [MPa]</th>
<th>$f_{cc}/f_{cts}$ [MPa]</th>
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<th>$f_{ct}$ [MPa]</th>
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<tr>
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<td>96.88</td>
<td>4.12</td>
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Only hot rolled reinforcing steel FeB 500 HWL was used in the experiments and two bar diameters $d_r$ were chosen, namely 16 and 20 mm. The mechanical characteristics of the steel were determined in tensile tests (measuring length of $10d_r$) both for the original and the prepared bars. The actual values obtained for the prepared bars, which are used
in the further analysis, are specified in Table 4.3. Each value is the average of three tests results. In total eight pull-out tests and four (double) bond bending tests were performed. All specimens were assigned an identifying code written as X.a.b.c, which denotes the current combination of test parameters according to Fig. 4.6. A complete overview of the test parameters and the specimen codes is given in Table 4.4.

| Table 4.3 Mechanical characteristics of reinforcing steel (prepared bars) |
|---|---|---|---|---|---|---|
| $d_s$ [mm] | at yielding $f_y$ [MPa] | $e_y$ [%] | at strain hardening $f_{sh}$ [MPa] | $e_{sh}$ [%] | at maximum load $f_y$ [MPa] | $e_u$ [%] | $f_t / f_y$ | $f_R$ |
| 16 | 539.67 | 0.42 | 542.54 | 2.84 | 624.35 | 11.50 | 1.16 | 0.077 |
| 20 | 526.24 | 0.35 | 531.22 | 2.10 | 612.87 | 11.20 | 1.17 | 0.075 |

| Table 4.4 Specification of test parameters and specimens codes |
|---|---|---|
| Type of concrete | Confining conditions | Bar diameter $d_s$ [mm] |
| | confined concrete | unconfined concrete |
| NSC $f_c = 28.45$ MPa | P.16.16.1 | M.16.16.L | 16 |
| | P.16.16.2 | M.16.16.R |
| | P.20.16.1 | M.20.16.L |
| | P.20.16.2 | M.20.16.R |
| HSC $f_c = 96.9$ MPa | P.16.HS.1 | M.16.HS.L | 16 |
| | P.16.HS.2 | M.16.HS.R |
| | P.20.HS.1 | M.20.HS.L |
| | P.20.HS.2 | M.20.HS.R |

Figure 4.6 Member identification codes
4.2.3 Discussion of test results

In this paragraph emphasis is first put on evaluation of the pull-out test results. During all tests, at the subsequent loading levels, steel strains were measured at fixed locations along the embedded part of the bar. Besides, relative displacements of the loaded end and of the free end of the bar were measured. They were used in order to check the correctness of the bar slip acquisition. An accurate determination of steel strains is a crucial point in these experiments, since the steel strain values were used to determine local slip and local bond stress distribution curves, and consequently to obtain bond stress - slip relationships.

![Graphs showing strain distribution](image)

\textbf{Figure 4.7} Steel strain distribution along the bar at two characteristic load stages and the unique steel strain distribution curve derived by parallel translation - test \textit{P.16.16.1}

From the comparison of the steel strain distributions measured at different load stages it was noticed that by a parallel translation of the measured sets of data (steel strain versus location) for each test one unique steel strain distribution curve can be obtained, which fits all data points. This proves that for this specific case (pull-out tests with long embedment length) there is a unique bond stress - slip relationship, which is independent of the location at the bar. The unique (experimental) strain distribution curves, obtained in the way described above, are smooth in the elastic range of steel strains (Fig. 4.7).
The curves show a notable change in the slope at the location where yielding of the bar takes place. The shape of the steel strain distribution curves is about similar to that of the steel stress-strain relation. Yet, the proportions between the elastic and the plastic part of the stress-strain diagram of steel are different from this between the parts of the bar transfer length which hold elastic and that which hold plastic strains. This suggests different (lower) values of the bond stress in the post-yield range of the steel strains than in the elastic range. Both for NSC and HSC a similar form of the unique steel strain distribution curves is observed, however the proportion between the portions of the bar which hold elastic strains and that which hold plastic strains at comparable load stages (e.g. at the maximum load $P_{\text{max}}$) are different. Also differences in the transfer length of the bar due to the change of the concrete type are significant: in specimens where HSC is used the transfer length of the bar is much shorter than in specimens made of NSC. There is no major influence of the bar diameter $d_s$ on the shape of the unique strain distribution curves, if the length scale (location along the bar) is normalised with respect to $d_s$.

![Graphs showing bond stress and slip distribution along the bar](image)

**Figure 4.8 Test results for test P.16.16.1 - slip distribution curve, bond stress distribution curve and experimental bond stress versus slip relationship**

The slip and bond stress distributions along the bar are derived from the unique steel strain distribution curve. In this analysis the slip is defined as the relative displacement from a fixed point in the concrete. The slip at each location along the bar is therefore
computed by integration of steel strains from the free-end of the bar to the point considered. The values obtained in this way are verified by comparison with the measured loaded end displacements, which are corrected for the slip in the threaded coupling. To arrive at the unique steel stress distribution curves the experimentally determined accurate stress - strain relationship of the steel bars is used. Using those distribution curves the bond stress distribution along the bar is calculated in an iterative smoothening procedure. Local bond stress values are calculated, averaged over the distance between two subsequent data points on the unique steel strain distribution curve. An example of slip and bond stress distribution curves is given in Fig. 4.8.

The slope of the slip distribution curve changes at the location where the bar yields and the slip substantially increases in the range where steel strains are beyond the yield strain. In this range also the shape of the bond stress distribution curve in this range is significantly different from that which holds in the elastic steel strain range. The bond stress in the post-yield range of the steel strain is smaller than that in the elastic range, where the bond stress gradually increases with the steel strain. A decrease of bond stress is observed starting from the location where the bar reaches its yield point. In the part of the bar with plastic strains just beyond yield level a sudden drop in the bond stress takes place, which is followed by a part with slower, yet still noticeably decreasing bond stress. A similar character of slip and bond stress distribution curves is observed in all tests.

Slip and bond stress distribution curves can be converted to a unique bond stress - slip relationship, which is independent from the location along the bar (see Fig. 4.9 and 4.10). For low slip values (in the elastic range of steel strains) the bond stress increases with increasing slip. The slip value, at which a sudden change in the form of the bond stress versus slip relation takes place, is associated with yielding of the steel bar. Following from this point the descending branch of the bond stress - slip diagram shows a substantial reduction of the bond stress with increasing slip (a sudden drop of bond stress value for slip values close to the point of yielding and a much slower, almost monotonic decrease of the bond stress for larger slip values). The same tendencies are noted for all combinations of parameters tested, however there are some major differences between bond stress - slip relationships obtained for NSC and HSC. In the stage associated with steel strains below the yield stress a larger stiffness of bond is observed for HSC than for NSC. Also the extreme bond stress values are much higher for HSC, however when the bond strength is normalised with respect to $f_c^{1/2}$ about 25% lower values for HSC are obtained. The degree of bond stress decrease caused by yielding of the steel differs for NSC and HSC, and a more sudden drop of the bond stress in the case of HSC is noticed. In the following stage the value of the residual
bond stress remains higher for HSC, yet the slope of the descending branch differs. A reduction of the bond stress with increasing slip values, in the range corresponding to advanced steel strains in the post-yield range, is stronger for HSC than for NSC. All those significant differences can be explained only when, besides the differences in the strength of NSC and HSC, also the differences in their fracture properties are taken into account.

![Graph showing bond stress vs slip for NSC and HSC](image)

**Figure 4.9** Comparison of experimental bond stress versus slip relations for NSC

**Figure 4.10** Comparison of experimental bond stress versus slip relations for HSC

The influence of the bar diameter on the bond stress - slip relation is not pronounced for the investigated range of variables. Nevertheless, it is likely to be fully revealed if the slip $\delta$ is normalised with respect to the bar diameter $d_s$ and if the nondimensional slip $\Delta = \delta / d_s$ is used to present the bond stress - slip relationships (Morita et al. 1994).
In the beam bond tests, the loads and deflections, as well as the local steel strains and the overall deformations of the tensile cord were measured. To register the overall deformations a series of displacement transducers (LVDT's) was used. Readings were taken on the targets attached to the surface of the concrete at both side faces of the beams along the lines corresponding to the position of the bars. Furthermore, the crack pattern development was observed and the width of the splitting cracks was registered. Some characteristic crack patterns and a typical set of steel strain/deformation measurements are given in Fig. 4.11 and 4.12. The data obtained in this test series were used to verify the applicability of the new bond model to the case of a splitting type of bond failure in unconfined concrete. For more details concerning the experimental investigation see Bigaj (1995a, 1995b, 1997).

![Front Face](image1)
![Front Face](image2)
![Rear Face](image3)
![Rear Face](image4)
![Bottom Face](image5)
![Bottom Face](image6)

Figure 4.11  Crack pattern - test M.16.16.R (left) and M.16.HS.R (right)

![Graph](image7)
![Graph](image8)

Figure 4.12  Local steel strains and overall deformations of tensile cord at characteristic load levels - test M.16.16.R (left) and M.16.HS.R (right)
4.2.4 Conclusions

The bond stress - slip relationships experimentally obtained for confined concrete are compared with the corresponding recommendations of the CEB-FIP Model Code 1990 (MC90). For the purpose of this comparison the bond stress is normalised with respect to $f_{e}^{1/2}$. Fig. 4.13 and 4.14 present test results in comparison with the MC90 bond stress - slip relationship recommended for good confinement and good bond conditions for NSC and HSC. Since the application of the MC90 formulation of the bond stress - slip relationship will lead to different results for different steel bar diameters and geometry, the average of the relations obtained for 16 and 20 mm diameter bars is used for this comparison. In order to account for the concrete strength variation, the mean relationship as well as the upper and the lower bounds, obtained using the CEB-FIP Model Code formulations for $f_{cma}$, $f_{ck0.05}$, $f_{ck0.95}$ are shown.

The shape of the ascending branch of the MC90 diagram is close to the experimental results and for low slip values in the elastic range of the steel strains, the general tendency of increasing bond stress with increasing slip is confirmed, both for NSC and HSC. However, the MC90 underestimates the bond stiffness in this range in both cases. It should be noticed that this deviation is much larger for HSC than for NSC. These differences can be explained from the different confinement and bond conditions provided in the test series described here and the experiments, which gave the basis for the MC90 formulation, and obviously also by the fact that the Code was mainly based on experiments performed using NSC. It is quite significant, that the MC90 does not reflect the experimentally observed change in the bond stress - slip relation due to the yielding of steel. Furthermore, the descending branch and the residual bond stress value, given in the MC90 to represent the reduction of bond resistance, fall far above the experimentally determined values. The overestimation of bond stress in the post-yield range is found for both NSC and HSC.

Remarkably good agreement is found between the test results for NSC and the modified bond stress - slip relationship proposed by Engström (1992). This model, which assumes both a lower residual strength and a steeper slope of the falling branch than postulated in the MC90, takes into account the influence of yielding of the steel on the bond resistance. The application of Engström's model to HSC does not give a good agreement with the experimental results, which proves that the extrapolation of empirical relations based on NSC to HSC may lead to the incorrect results. Hence, it is indispensable to account for the concrete fracture characteristics, when modelling bond behaviour.
Figure 4.13 Comparison of test results and CEB-FIP Model Code 1990 formulation of bond stress - slip relation for NSC (modified relation according to Engström (1992) shown)

Figure 4.14 Comparison of test results and CEB-FIP Model Code 1990 formulation of bond stress - slip relation for HSC

In conclusion it is stated that the application of the CEB-FIP Model Code 1990 bond stress - slip relationship to the case of confined concrete leads to a considerable overestimation of bond stress when the yield stress of the bar is exceeded. Moreover, in the elastic range of steel strains the MC90 relationship shows a clearly larger disparity for HSC than for NSC. This discrepancy found for the case of confined concrete will obviously also hold true for unconfined concrete pondering the empirical character of this code formulation and the fact that it has been tuned on experiments carried out on concrete with medium strength and fracture properties different from that of HSC.
Furthermore, with respect to the estimation of the degree of confinement, the bond stress-slip relation in the *CEB-FIP Model Code 1990* has not been defined as a continuous function of the degree of confinement (quantified using the value of the concrete cover on the bar), but it has only been given for two limit values, viz. for unconfined concrete $c/d_s = 1$ (requiring a minimum amount of transverse reinforcement) and for confined concrete $c/d_s = 5$. Such an arbitrary restriction has consequences for the accuracy of the bond strength estimation in cases qualified as unconfined concrete, ponder the significant influence of concrete cover thickness for the appearance and propagation of splitting cracks, and thus on the resulting bond capacity. It is therefore necessary to quantify the confining capacity of the concrete surrounding the bar and to include this quantity in the description of the bond behaviour in order to develop a bond model that will be fully capable of representing the influence of confinement on bond strength and force transfer by bond.

4.3 Modelling of bond behaviour

4.3.1 Concrete confinement model

(1) Formulation of confinement model

It has been recognised in earlier research that the confining capacity of concrete surrounding a bar not only affects the magnitude of the ultimate bond stress but also the bond stress-slip relation (*Morita and Fuji 1982*, *Robins and Standish 1982*, *Eligehausen et al. 1983*, 1989, *Gambarova et al. 1989*, *Malvar 1992*, *Cairns and Jones 1995*). This has been acknowledged in the *CEB-FIP Model Code 1990* as well, where both characteristics have been defined as a function of the bond failure modes: splitting ("unconfined concrete") and pull-out ("confined concrete") and the bond conditions ("good" and "all other cases"). Hence, the bond stress-slip relation is not a continuous function of the degree of confinement, but it has been given for the upper and lower limits only.

The importance of the concrete confining capacity for the bond resistance of a ribbed bar directly follows from the analysis of the force transfer between the bar and the surrounding concrete. The radial components of the concentrated forces, that radiate from the bar into the concrete, are equilibrated by circumferential tensile forces in the concrete and by any additional confinement, such as transverse reinforcement and external forces. This confining action determines the mode of bond failure and specific bond characteristics as well. In the bond model given hereafter the concrete confining capacity is used as a starting point to describe the bond behaviour of a ribbed bar.
embedded in concrete. Here only the confinement delivered by the surrounding concrete is considered. This confining capacity is analytically estimated taking into account the softening behaviour of concrete loaded in tension. It is expressed by the relation between the radial stress and the radial displacement of the bar to concrete interface. For the transition from this relation to a bond stress-slip relation the ribbed bar is conceived as a smooth conical bar transferring the bond stress through dry friction to the concrete. The "bar-to-concrete interface" is further denoted as "interface".

Figure 4.15  Partially cracked thick-walled-cylinder

To describe the resistance of the concrete cover against splitting due to bond the model of a thick-walled cylinder is used, in which the radial component of the bond action makes equilibrium with the circumferential tensile stresses across the cylinder wall. As long as these hoop stresses remain below the tensile strength a linear elastic stress state is present, for which analytical expressions are available. When the circumferential stress reaches the tensile strength one or more radial cracks start to grow and the response of the concrete to the internal pressure becomes non-linear. For this stage Tepfers (1973) considered a lower and an upper bound solution. The lower bound was given by assuming that in the cracked part of the cylinder the circumferential tensile stresses are equal to zero and that the uncracked part behaves linear elastically. The upper bound followed from the assumption of a purely plastic material behaviour, which yields a uniformly distributed circumferential tensile stress equal to the tensile strength. A more refined approximation for the non-linear state was developed by Van der Veen (1990), extending Tepfers' (1973) cracked-elastic model by taking into account the softening behaviour of concrete. The splitting resistance found with this approach falls between Tepfers' (1973) lower and upper bounds and shows good resemblance with experimental results. The aforesaid models, however, only deal with the concrete stresses around a bar, which suffices to establish a criterion for splitting failure. In the present approach
the bond stress is considered as a function of the concrete response to the radial displacement of the interface. Therefore, the concrete deformations involved have to be considered as well.

Uncracked stage (stage I)

The response of a thick-walled cylinder to an internal pressure can be subdivided into three stages: the uncracked, partly cracked and entirely cracked stage, respectively. In the uncracked stage a linear elastic behaviour of the cylinder can be assumed, for which the stresses and deformations have been given by Timoshenko (1976):

$$\sigma_{r,r} = \frac{r_i^2 \sigma_{r,r_i}}{r_e^2 - r_i^2} \left(1 - \frac{r_e^2}{r^2}\right)$$  \hspace{1cm} (4.1)

$$\sigma_{t,r} = \frac{r_i^2 \sigma_{r,r_i}}{r_e^2 - r_i^2} \left(1 + \frac{r_e^2}{r^2}\right)$$  \hspace{1cm} (4.2)

$$u_{r,r_i} = \frac{r_i \sigma_{r,r_i}}{E_c} \left(\frac{r_e^2 + r_i^2}{r_e^2 - r_i^2} + \nu_c\right)$$  \hspace{1cm} (4.3)

with \( r_i = r_s = \frac{d_s}{2} \) and \( r_e = c + r_s = c + \frac{d_s}{2} \)

where

- \( r \) is the radius
- \( r_e \) is the radius of cylinder outer face
- \( r_i \) is the radius of cylinder inner face
- \( r_s \) is the bar radius
- \( \sigma_{r,r} \) is the radial stress at radius \( r \)
- \( \sigma_{r,r_i} \) is the radial stress at radius \( r_s \)
- \( \sigma_{t,r} \) is the tangential stress at radius \( r \)
- \( u_{r,r_i} \) is the radial displacement of cylinder inner face
- \( \nu_c \) is the Poisson constant of concrete
- \( c \) is the clear concrete cover
- \( c_1 \) is the clear concrete cover plus bar radius

Partly cracked stage (stage II)

To find the \( \sigma_r - \varepsilon_r \)-relationship at the interface in the second stage the cylinder is
subdivided into a cracked part and an uncracked part (Fig. 4.15). At the crack front \( r = r_{cr} \) the circumferential stress is - per definition - equal to the tensile strength. With \( r_{cr} \) denoting the radius of the crack front and taking \( r_1 = r_{cr} \) and \( r_2 = c_1 \), substitution of \( \sigma_{t,r} = f_{ct} \) and \( r = r_{cr} \) into equation 4.1 yields for the radial stress at the crack front \( \sigma_{r,r_{cr}} \):

\[
\sigma_{r,r_{cr}} = f_{ct} \frac{c_1^2 - r_{cr}^2}{c_1^2 + r_{cr}^2} = f_{ct} C_1 \tag{4.4}
\]

where \( C_1 = \frac{c_1^2 - r_{cr}^2}{c_1^2 + r_{cr}^2} \)

Hence, at the interface the contribution of the uncracked cylinder (corresponding terms indicated by the superscript \( LE \) - linear elastic) to the radial stress \( \sigma_{r,s}^{LE} \) equals:

\[
\sigma_{r,s}^{LE} = \frac{r_{cr}}{r_s} \sigma_{r,r_{cr}} = \frac{r_{cr}}{r_s} f_{ct} C_1 \tag{4.5}
\]

Taking \( \frac{f_{ct}}{E_c} = \varepsilon_{cr} \), substitution of equation 4.4 into equation 4.3 gives for the radial displacement of the cylinder inner face at the crack front \( u_{r,r_{cr}} \):

\[
u_{r,r_{cr}} = \frac{r_{cr}}{E_c} \varepsilon_{cr} \left( 1 + \frac{c_1^2 - r_{cr}^2}{c_1^2 + r_{cr}^2} \right) = r_{cr} \varepsilon_{cr} (1 + \nu_c C_1) \tag{4.6}
\]

where \( \varepsilon_{cr} \) is the concrete strain for \( \sigma_i = f_{ct} \)

Thus, the contribution of the uncracked cylinder to the radial strain at the interface \( \varepsilon_{r,s}^{LE} \):

\[
\varepsilon_{r,s}^{LE} = \frac{r_{cr}}{r_s} \varepsilon_{cr} (1 + \nu_c C_1) \tag{4.7}
\]

To estimate the contribution of the cracked cylinder to the radial stress and strain at the interface the softening behaviour of concrete loaded in tension is taken into account by applying Hillerborg’s (1983) fictitious crack model (FCM). A bi-linear softening relation as proposed by Roelfstra and Wittmann (1986) is chosen (see Chapter 3). By rewriting equation 3.10 the fictitious crack width \( w_i \), at which the tensile stress \( \sigma_i \) is transferred, can be expressed as:

\[
w_i = \frac{w_0}{a_i} \left( \frac{\sigma_i}{f_{ct}} - b_i \right) \tag{4.8}
\]

where \( a_i, b_i (i = 1,2) \) and \( w_0 \) are defined as in Chapter 3
The total elongation $\Delta_{t,r}$ of a circular fibre with radius $r$, consists of the total width of the fictitious cracks and the elastic deformation of the concrete between those cracks. Neglecting the influence of the radial stress on the tangential deformation, the fibre elongation can be written as:

$$\Delta_{t,r} = 2\pi r e_{t,r} + n \frac{w_0}{a_i} \left[ \frac{\sigma_{t,r}}{f_{ct}} - b_i \right]$$

(4.9)

where $e_{t,r}$ is the tangential strain in the circumferential fibre with a radius $r$

$n$ is the number of fictitious radial cracks

![Figure 4.16](image-url)  
*Rigid body movement of segments in the cracked part of a cylinder (left) and deformed segments due to the tangential tensile stresses transmitted by cracks (right) after Van der Veen (1990)*

To establish the distribution of the circumferential tensile stresses across the wall of the cracked cylinder, *Van der Veen (1990)* first assumed the concrete segments between the cracks to be unstressed and considered the rigid body movement of these segments shown in Fig. 4.16, which resulted in a constant crack width across the wall. These cracks are partly closed, however, by circumferential tensile stresses as long as the width of the individual cracks is smaller than $w_0$. At the crack front, where the circumferential stress equals the tensile strength, the cracks are entirely closed and the total fibre elongation is only caused by elastic deformation. Neglecting the Poisson effect this deformation equals to $2\pi r_c e_{cr}$, thus:

$$2\pi r_c e_{cr} = 2\pi r e_{t,r} + n \frac{w_0}{a_i} \left[ \frac{\sigma_{t,r}}{f_{ct}} - b_i \right]$$

(4.10)

which becomes after rewriting:
\[
\frac{\sigma_{cr}}{f_{ct}} = a_i \frac{2\pi e_{crr}}{nw_0} \left( r_{cr} - \frac{e_{crr}}{e_{cr}} \right) + b_i = a_i C_2 \left( r_{cr} - \frac{e_{crr}}{e_{cr}} \right) + b_i
\]  
\( (4.11) \)

where \( C_2 = \frac{2\pi e_{cr}}{nw_0} \)

The radial stresses which make equilibrium with the circumferential tensile stresses in the cracked part of the cylinder (corresponding terms indicated by the superscript \( NL \) - non linear) follow from:

\[
\sigma_{cr}^{NL} = \frac{1}{r} \int_{r}^{r_{cr}} \sigma_{cr} \, dr
\]  
\( (4.12) \)

In the following considerations the case is addressed where the concrete in the thick-walled cylinder remains in the stage described by the first branch of the softening curve defined in Chapter 3, i.e. \( \sigma_{cr} \geq \sigma'_{ct} \). This limitation is justified pondering that both for normal and high strength concrete this condition is met if the cylinder wall thickness is bellow 150 mm, that is to say for any bar cover thickness in the practical cases.

To simplify the integration operation, the ratio \( e_{cr}/e_{cr} \) in equation (4.11) is put equal to 1, which involves a certain overestimation of the circumferential strain in the cracked cylinder. On the other hand, this strain was underestimated by neglecting Poisson's effect at the crack front. Hence, both effects partly neutralise each other.

Substitution of equation (4.11) into equation (4.12) and integration of the latter yields:

\[
\frac{\sigma_{cr}}{f_{ct}} = a_i C_2 r \left( \frac{r_{cr}}{r} - 1 \right)^2 + b_i \left( \frac{r_{cr}}{r} - 1 \right)
\]  
\( (4.13) \)

Substitution of \( r = r_s \) into equation (4.13) gives the contribution of the cracked cylinder to the radial stress at the interface:

\[
\frac{\sigma_{cr}}{f_{ct}} = a_i C_2 r_s \left( \frac{r_{cr}}{r_s} - 1 \right)^2 + b_i \left( \frac{r_{cr}}{r_s} - 1 \right)
\]  
\( (4.14) \)

Now the confining stress of the entire cylinder in the partly cracked stage (stage II) \( \sigma_{rr}^{II} \) is given by the sum of equation 4.5 and equation 4.14:

\[
\frac{\sigma_{rr}}{f_{ct}} = \frac{\sigma_{rr}^{LE}}{f_{ct}} + \frac{\sigma_{rr}^{NL}}{f_{ct}}
\]  
\( (4.15) \)
The radial deformation of the cracked cylinder is evaluated assuming Poisson’s effect to play a minor role compared to the influence of the fictitious cracks. Therefore the change of the wall thickness of the cracked cylinder can be written as:

\[
\Delta r_{cr}^{NL} = \int_{r_s}^{r_{cr}} \sigma_{rr}^L \frac{dr}{E_c} = \varepsilon_{cr} \int_{r_s}^{r_{cr}} \frac{\sigma_{rr}^L}{f_{ct}} \frac{dr}{r} = \varepsilon_{cr} \int_{r_s}^{r_{cr}} \frac{\sigma_{rr}^{LE}}{f_{ct}} \frac{dr}{r} + \varepsilon_{cr} \int_{r_s}^{r_{cr}} \frac{\sigma_{rr}}{f_{ct}} \frac{dr}{r} \tag{4.16}
\]

with \( \frac{f_{ct}}{E_c} = \varepsilon_{cr} \)

From equation 4.4 the uncracked cylinder’s contribution to the radial stress in the cracked cylinder is known:

\[
\frac{\sigma_{rr}^{LE}}{f_{ct}} = \frac{\sigma_{rr}}{r} \frac{r_{cr}}{f_{ct}} = \frac{r_{cr}}{r} C_1 \tag{4.17}
\]

and the contribution of the circumferential tensile stresses in the cracked cylinder to the radial stress is given by equation 4.13. Substitution of equation 4.4 and equation 4.13 into equation 4.16 after rewriting yields:

\[
\Delta r_{cr}^{NL} = \Delta r_{cr,1} + \Delta r_{cr,2} \tag{4.18}
\]

with:

\[
\Delta r_{cr,1} = \varepsilon_{cr} C_1 \int_{r_s}^{r_{cr}} \frac{r_{cr}}{r} \frac{dr}{r} = \varepsilon_{cr} C_1 r_{cr} \ln \left( \frac{r_{cr}}{r_s} \right) \tag{4.19}
\]

and:

\[
\Delta r_{cr,2} = \varepsilon_{cr} \int_{r_s}^{r_{cr}} \frac{a_1 C_2 r_{cr}}{2} \left( \frac{r_{cr}}{r} - 1 \right)^2 \frac{dr}{r} + \varepsilon_{cr} \int_{r_s}^{r_{cr}} b_1 \left( \frac{r_{cr}}{r} - 1 \right) \frac{dr}{r} = \tag{4.20}
\]

\[
= \varepsilon_{cr} a_1 C_2 \left[ \frac{2 r_{cr}^2 \ln \left( \frac{r_{cr}}{r_s} \right) - 4 r_{cr} (r_{cr} - r_s) + (r_{cr} - r_s)^2}{4} \right] + \varepsilon_{cr} b_1 \left[ r_{cr} \ln \left( \frac{r_{cr}}{r_s} \right) - (r_{cr} - r_s) \right]
\]

Now, the radial strain at the interface due to the change of the wall thickness of the cracked cylinder follows from:

\[
\varepsilon_{r,r_s}^{NL} = \frac{\Delta r_{cr}^{NL}}{r_s} = \frac{\Delta r_{cr,1}}{r_s} + \frac{\Delta r_{cr,2}}{r_s} \tag{4.21}
\]

The total radial strain at the interface \( \varepsilon_{r,r_s}^H \) is the sum of the strain due to the
deformation of the uncracked part of the cylinder and the aforementioned change in wall thickness of the cracked part of the cylinder, as given by the equations 4.7 and 4.21, respectively:

$$
\varepsilon_{r,r_s}^H = \varepsilon_{r,r_s}^{LE} + \varepsilon_{r,r_s}^{NL}
$$

(4.22)

**Entirely cracked stage (stage III)**

Up to now the radial stress-strain behaviour that occurs in the first two stages, during which radial cracks are initiated and driven through the entire wall of the cylinder, has been derived. In the third stage these cracks become wider and the confining action of the concrete diminishes as a consequence of the softening behaviour. The derivation of the radial stress-strain relation for this stage is analogous to that for the cracked part of the cylinder in the second stage. First, the circumferential tensile stress is estimated assuming the total elongation of the fibres across the cylinder wall to be constant ($\Delta_{t,r} = \Delta_{tot}$) and the tangential strain $\varepsilon_{t,r}$ to be equal to the fracture strain $\varepsilon_{cr}$. Substituting these values into equation 4.9 and rewriting yields:

$$
\frac{\sigma_{t,r}}{f_{ct}} = a \left[ \frac{\Delta_{tot}}{nw_0} - \frac{2\pi E_{cr} r}{nw_0} \right] + b = a\left(C_3 + C_2 r\right) + b
$$

(4.23)

where $C_3 = \frac{\Delta_{tot}}{nw_0}$

To simplify the solution procedure, it is assumed that concrete in the cracked cylinder is entirely in the stage that can be described either by the first or by the second branch of the softening curve defined in Chapter 3. This assumption leads to slight underestimation of $\sigma_{r,r}$ in the case where the stress in the concrete cylinder wall is to be described by both branches of the softening curve. However, it results only in negligible inaccuracy in the estimation of $\sigma_{r,r_s} - \varepsilon_{r,r_s}$ response in the neighbourhood of the inflexion point on its descending branch (Fig.4.17).

Hence, the radial stress is found after substitution of equation 4.23 into equation 4.12 and integration between $r$ and $c_1$:

$$
\frac{\sigma_{r,r}}{f_{ct}} = (a_i C_3 + b_i)\left(\frac{c_1}{r} - 1\right) - \frac{a_i C_2 r}{2}\left(\frac{c_1}{r}\right)^2 - 1
$$

(4.24)

Substituting $r = r_s$ into equation 4.24 yields the radial stress acting at the interface:
\[
\frac{\sigma_{r,s}^{III}}{f_{ct}} = (a_i C_3 + b_i) \left( \frac{c_1}{r_s} - 1 \right) - \frac{a_i C_2 r_s}{2} \left( \frac{c_1}{r_s} \right)^2 - 1
\]  

(4.25)

The radial strain at the interface \( \varepsilon_{r,s}^{III} \) is derived from the radial displacement, which consists of two parts: the rigid body movement (RBM) equal to the radial displacement of the outer fibre and the change of the wall thickness (\( \Delta c \)):

\[
\varepsilon_{r,s}^{III} = \varepsilon_{r,s}^{RBM} + \varepsilon_{r,s}^{\Delta c}
\]

(4.26)

The strain due to the rigid body movement can be written as:

\[
\varepsilon_{r,s}^{RBM} = \frac{\Delta r_s}{r_s} = \frac{\Delta_{tot}}{2\pi r_s} = C_3 \frac{nw_0}{2\pi r_s}
\]

(4.27)

The strain connected with the change of the wall thickness is obtained by integrating equation 4.24 over the cylinder wall and dividing it by the elastic modulus and the bar radius:

\[
\varepsilon_{r,s}^{\Delta c} = \frac{\Delta c}{r_s} = \frac{1}{r_s} \int \frac{\sigma_{r,s}}{E_c} dr = \frac{\varepsilon_{cr}}{r_s} \left( a_i C_3 + b_i \right) \int_{r_s}^{c_1} \left( \frac{c_1}{r} - 1 \right) dr - \frac{a_i C_2}{2} \int_{r_s}^{c_1} \left( \frac{c_1}{r} - r \right) dr
\]

\[
= \varepsilon_{cr} \left( a_i C_3 + b_i \right) \left[ \frac{c_1 \ln\left( \frac{c_1}{r_s} \right) - \frac{c_1}{r_s}}{r_s} + 1 \right] - \frac{a_i C_2 \varepsilon_{cr} r_s^2}{4} \left[ 2 \left( \frac{c_1}{r_s} \right)^2 \ln\left( \frac{c_1}{r_s} \right) - \left( \frac{c_1}{r_s} \right)^2 + 1 \right]
\]

(4.28)

Discussion

An estimate of the confining capacity found with the thick-walled-cylinder model given in Section 4.3.1 (1) is shown in Fig. 4.17. The clamping action of the concrete is expressed by the radial compressive stress as a function of the radial strain at the interface, the latter being the radial displacement of the interface divided by the bar radius. The three stages mentioned in Chapter 3 are indicated. During stage I the cylinder remains uncracked and a linear elastic material behaviour is assumed. The transition into stage II is marked by the initiation of radial (macro)cracks starting at the interface. In this stage the cylinder is partly cracked and for the cracked part the softening of concrete in tension is taken into account. The penetration depth of the radial cracks is the control parameter in this stage. The maximum radial stress is reached after penetration of about 70% of the cylinder wall. Further crack penetration results in a
decrease of the radial stress, which is accelerated as soon as the entire cylinder wall is cracked. Then stage III begins, which is controlled by the radial crack width at the outer perimeter of the cylinder.

![Graph showing stress-strain relationship](image)

**Figure 4.17** Confining capacity estimated with the thick-walled-cylinder model (stage I: uncracked, stage II: partially cracked, stage III: entirely cracked)

(2) **Experimental verification of the confinement model**

**Reference experiments**

A series of tests have been performed to verify and tune the confinement model. A conical steel bar (1:500, 12, 16 and 20 mm diameter) with a smoothly grinded surface was embedded in a 25 mm thick concrete disc (80, 100, 120 and 140 mm diameter, 4 to 6 MPa splitting tensile strength). The bar was pulled through the disc measuring the pull-through force as a function of the free bar end displacement, thus obtaining a relationship between the average bond stress and the radial displacement of the interface. Under the assumption of a constant coefficient of friction this relationship is proportional to the radial stress versus radial strain relation found with the thick-walled-cylinder model. Some of the results are presented here, for more details see Den Uijl (1992a).

**Simulations**

In general 4 to 5 radial cracks started at the interface. In the beginning the penetration depth of all cracks was about the same, but at the end one of these cracks developed faster, clearly showing a greater depth and a greater width. In the model, however, the cracks are supposed to open uniformly. To count for this discrepancy a smaller number of cracks is assumed in the model than observed in practice. As can be seen in Fig. 4.18,
the choice of this parameter has a strong influence on the descending branch of the confining capacity curve. Good simulations, bearing in mind that the experiments showed considerable scatter, have been obtained by fixing the number of cracks to three, see Fig. 4.19.

![Graph showing influence of assumed number of radial cracks on confinement capacity estimated with thick-walled-cylinder model](image)

**Figure 4.18** Influence of assumed number of radial cracks on confinement capacity estimated with thick-walled-cylinder model

![Graphs showing comparison of measured bond stress versus radial strain relation (left) and calculated radial stress versus radial strain relation (right)](image)

**Figure 4.19** Comparison of measured bond stress versus radial strain relation (left) and calculated radial stress versus radial strain relation (right)

Note that the thick-walled-cylinder model is used to simulate the concrete clamping action on a reinforcing element. This requires a choice of the wall thickness that takes into account the real configuration with respect to cover and spacing. For this purpose an approach is presented in Section 4.4.1. Furthermore, bond conditions vary as a function of the bar position with respect to the bottom of the mould due to sedimentation of the fresh concrete. This aspect can be simulated by extending the thick-walled-
cylinder model described here with a thin boundary layer with a lower modulus of elasticity, see Den Uyl 1994.

4.3.2 Bond model formulation

In the formulation of the bond model the following steps are taken (see Fig. 4.20). To evade the difficulty of describing the highly non-linear local stresses and strains around the pulled bar, induced by rib bearing, a boundary layer is assumed in which these stresses and strains are smoothened. The wedging effect that occurs when the bar is displaced with respect to the concrete, is incorporated by conceiving the interface between the boundary layer and the surrounding concrete as conical. The radial stress, acting perpendicularly to the interface, is the response of the surrounding concrete to the radial displacement of the interface. By assuming dry friction at the interface, the bond stress is directly proportional to the radial stress. For practical reasons the thickness of the boundary layer is neglected. The response of the concrete to the radial displacement of the interface is calculated by assuming a thick-walled cylinder under internal pressure.

\[ d_{sl} = d_s \cdot V_s \cdot E_s \cdot r_s \]

*Figure 4.20  Bond model formulation - modelling steps*

The relationship between the radial displacement and the radial compressive stress is given by the confinement model discussed in Section 4.1.1. In this confinement model the radial displacement of the interface is normalised to the bar radius, which results in a radial strain at the interface. From the radial compressive stress \( \sigma_r \) the bond stress \( \tau_b \) is found by:

\[ \tau_b = \sigma_r \cdot \cot(\phi) \quad (4.29) \]

where \( \cot(\phi) \) is the coefficient of friction
The relationship between slip and radial displacement is given by the cone angle, thus representing the wedging action of the ribbed bar when displaced with respect to the surrounding concrete. This wedging action becomes active after the transition from adhesion to rib bearing, so with the beginning of the second stage. In the model the first stage, which comprises of adhesion and interlocking, is neglected.

**Figure 4.21  Relationship between slip \( \delta \) and radial stress \( \sigma_r \) for splitting bond failure**

In the third stage the cone angle depends on the failure mechanism. With bond splitting, rib bearing continues to be the force transferring mechanism and in that case the cone angle remains the same. On the contrary, the development of the cylindrical sliding plane around the bar in the case of pull-out failure is connected with a much smaller roughness, which involves a reduction of the cone angle. Moreover, as the pull-out failure develops, the progressive smoothening of the sliding plane will cause a further
reduction of the cone angle. It is assumed that this process is a function of the slip, and that the rate with which it occurs decreases as the slip increases. This holds also for the radial displacement of the interface, which reduces as the smoothening of the sliding plane continues.

These considerations result in different relationships between slip and radial strain for splitting failure and pull-out failure as shown in Fig. 4.21 and 4.22, respectively. Furthermore, it can be seen that with increasing slip the entire confinement capacity curve is followed in the case of splitting failure, whereas in the case of pull-out failure only a part of the ascending branch of this curve is followed, first upward and then downward. The subscript \(s\) in Fig 4.22 denotes the longitudinal steel strain and the influence of the Poisson's effect on the radial strain. In fact, this influence is proportional to the change of the local steel strain. Assuming the initial value of the local steel strain to be zero, the change of the local steel strain corresponds to the local steel strain itself.

For the mathematical formulation of the bond model a clear distinction is made between both bond failure modes: splitting and pull-out. For the splitting mode the most important influence is the wedging effect connected with the rib bearing mechanism. It has been argued that the influence of the Poisson's effect is small and therefore it can be neglected. Wear and compaction do not play a role for this failure mode. Hence, for the splitting mode the radial displacement of the interface is expressed by:

\[
\varepsilon_{r_s} = \delta \tan(\phi) \tag{4.30}
\]

where \(\varepsilon_{r_s}\) is the radial strain at the interface, i.e. radial displacement divided by radius,

\[r_s\] is the bar radius,

\[\delta\] is the slip,

\[\phi\] is the cone angle between cone surface and bar axis

For the pull-out mode of bond failure it has been reasoned that after the occurrence of a cylindrical sliding plane the cone angle diminishes as the slip increases, that the Poisson effect can not be disregarded and that wear and compaction of the sliding plane shall be taken into account. Thus in this case the radial displacement of the interface is influenced by a number of factors that can be connected with the slip and the local steel strain:

\[
\varepsilon_{r_s} = \delta \tan(\phi(\delta)) - \alpha_p \varepsilon_s v_s r_s - F(\delta) \tag{4.31}
\]

where \(\phi(\delta)\) is the slip dependent cone angle
\( \alpha_p \) is a coefficient representing the effectiveness of the release of the radial strain at the interface due to the bar contraction

\( \varepsilon_s \) is the longitudinal steel strain in the bar

\( \nu_s \) is the Poisson's coefficient of steel

\( F(\delta) \) is a slip dependent function representing the radial strain release due to the wear and compaction of the sliding plane

In absence of the information needed to quantify precisely all the above mentioned influences an expression is used that represents the combined influences of wedging, Poisson's effect and wear of the sliding plane:

\[
\varepsilon_{r,s} = F_1(\delta, \varepsilon_s)
\]  \hspace{1cm} (4.32)

where \( F_1(\delta, \varepsilon_s) \) is a function of the local slip and steel stress.

![Graph](image)

**Figure 4.23** Radial strain - slip relationship for pull-out bond failure

A graph described by this function for a constant steel strain \( \varepsilon_s \) is displayed in Fig. 4.23. It consists of three sections marked by the points a, b, c and d. The subscript s in the coordinates of these points refers to the steel strain. Section A and B are cubic parabolas, section C is an exponential function. In point b the tangent is horizontal, whereas c is a point of inflection. The coordinates of points b, c and d are derived from the position of the boundary curve shown with the dashed line, applying a reduction function, which implicitly takes into account the influences of the Poisson's effect and of the sliding plane wear on the radial deformation. The coordinates of the boundary curve are discussed later. First, the expressions for the coordinates of the points a, b, c and d are given.
The horizontal shift of point a is connected with the free space that occurs in front of a rib when only the Poisson’s effect is taken into account and no slip, see Fig. 4.24:

\[ \delta_{0,s} = \frac{L_{ele} v_s \varepsilon_s r_s}{r_s \tan(\gamma_R)} \]  

(4.33)

where \( L_{ele} \) is length of finite element  
\( \gamma_R \) is rib face angle

The horizontal coordinate of point b equals:

\[ \delta_{1,s} = \delta_{0,s} + \delta_{1,0} \]  

(4.34)

The vertical coordinate of this point follows from:

\[ \varepsilon_{r,l,s} = (\varepsilon_{r,l,0} - \varepsilon_{r,3}) e^{-K(\varepsilon_{r,l})} + \varepsilon_{r,3} \]  

(4.35)

where \( K(\varepsilon_{r,l}) \) is a constant with which the reduction rate of \( \varepsilon_{r,l} \) is controlled.

The coordinates of point c are arbitrarily chosen as follows:

\[ \delta_{2,s} = \frac{\delta_{3,s}}{2} \]  

(4.36)

\[ \varepsilon_{r2,s} = \frac{\varepsilon_{r,l,s} + \varepsilon_{r,3,s}}{2} \]  

(4.37)

The coordinates of point d are derived in a similar way as the vertical coordinate of point b:
\[ \delta_{3,e} = (\delta_{3,max} - \delta_{3,min}) e^{-K(\delta_3)\varepsilon_{s} + \delta_{3,min}} \tag{4.38} \]

where \( K(\delta_3) \) is a constant with which the reduction rate of \( \delta_3 \) is controlled.

\[ \varepsilon_{r3,s} = (\varepsilon_{r3,max} - \varepsilon_{r3,min}) e^{-K(\varepsilon_r)\varepsilon_{r} + \varepsilon_{r3,min}} \tag{4.39} \]

where \( K(\varepsilon_r) \) is a constant with which the reduction rate of \( \varepsilon_r \) is controlled.

The boundary curve is derived on the basis of a number of assumptions and considerations, which will be discussed hereafter. The criterion for the transition from the splitting failure mode into the pull-out failure mode is a critical bond stress \( \tau_{b1} \), which is assumed to be proportional to the tensile strength of the concrete. This bond stress is related to a radial stress \( \sigma_{r1} \) according to equation 4.29. The radial strain \( \varepsilon_{r1}(=\varepsilon_{r1,0}) \) that corresponds to this radial stress is directly found from the confinement model, see Section 4.3.1. For the splitting failure mode the slip \( \delta_1 \) connected to this radial strain is found with equation 4.30. Considering that the transition from one failure mode into the other is a gradual process, for pull-out failure this slip value is multiplied by a factor 2:

\[ \delta_{1,0} = 2 \delta_1 = \frac{2\varepsilon_{r1,0}}{\tan(\varphi)} \tag{4.40} \]

The value of \( \delta_{3,max} \) is put equal to the length of the concrete corbels between two subsequent ribs \( L_{key} \), which value is proportional to the bar diameter, in general. The value of \( \delta_{3,min} \) is arbitrarily put to 2.1 times \( \delta_{1,0} \). The limit values of the radial strain, \( \varepsilon_{3,max} \) and \( \varepsilon_{3,min} \), are found from the assumed residual bond stress levels \( \tau_{b3,max} \) and \( \tau_{b3,min} \) after the slip induced strain release and the combined slip and steel stress induced strain release, respectively (conversion from bond stress to radial stress with equation 4.29, and from radial stress to radial strain with the confinement model (Section 4.3.1)).

**Parameter choice**

To calibrate the bond model and to set the values of model parameters, a number of calculations has been performed for various confinement and load arrangement conditions, as well as for concrete and steel with different material characteristics. Based on this comparative analyses the parameter values have been fixed, as listed in the table below.

Besides the parameters specified here, also material characteristics of reinforcing steel and concrete, rebar geometry and concrete cover thickness are the input data of this bond
model. In this way it becomes possible to perform reliable simulations for different combinations of material properties and member geometry.

<table>
<thead>
<tr>
<th>notation</th>
<th>parameter name</th>
<th>value / expression</th>
<th>remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cot(\phi) )</td>
<td>coefficient of friction</td>
<td>1</td>
<td>constant</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>cone angle [deg]</td>
<td>0.1( f_c )</td>
<td>concrete strength dependent</td>
</tr>
<tr>
<td>( \gamma_R )</td>
<td>rib face angle</td>
<td>60°</td>
<td>bar geometry dependent</td>
</tr>
<tr>
<td>( K(e_{f1}) )</td>
<td>constant for reduction of ( e_{f1} )</td>
<td>30</td>
<td>constant</td>
</tr>
<tr>
<td>( K(e_{f3}) )</td>
<td>constant for reduction of ( e_{f3} )</td>
<td>8.5</td>
<td>constant</td>
</tr>
<tr>
<td>( K(\delta_3) )</td>
<td>constant for reduction of ( \delta_3 )</td>
<td>100</td>
<td>constant</td>
</tr>
<tr>
<td>( \tau_{b1} )</td>
<td>critical bond stress</td>
<td>5( f_{cy} )</td>
<td>concrete strength dependent</td>
</tr>
<tr>
<td>( \tau_{b3,\text{max}} )</td>
<td>max. residual bond stress</td>
<td>2.5( f_{cy} )</td>
<td>concrete strength dependent</td>
</tr>
<tr>
<td>( \tau_{b3,\text{min}} )</td>
<td>min. residual bond stress</td>
<td>0</td>
<td>constant</td>
</tr>
<tr>
<td>( \delta_{3,\text{max}} )</td>
<td>characteristic slip value</td>
<td>( L_{key} = 0.33 \times d_s )</td>
<td>bar geometry dependent</td>
</tr>
</tbody>
</table>

### 4.4 Bond model application in member analysis

#### 4.4.1 Effective concrete cover

The present bond model for ribbed bars is based on the confinement delivered by the surrounding concrete. The bond force is limited by the maximum radial component that can be equilibrated by the radial compressive stresses resulting from the concrete confinement. After the occurrence of radial cracks a redistribution of the internal stresses may take place due to local changes in stiffness. The internal stress distribution around a bar will strongly depend on the geometry of the member cross section and the position of the reinforcing bars.

It has been observed in experiments with equal bottom cover that both the crack pattern around a bar and the ultimate resistance against splitting are influenced by other cross-sectional dimensions (Morita et al. 1994, Den Uijl 1992). This observation supports the supposition that the confinement delivered by the concrete not only depends on the smallest concrete cover, but a greater part of the concrete cross-section may contribute to it.

This effect is taken into account by considering a fictitious concrete cover, which is related to the dimensions of the concrete cross section and the lay-out of the reinforcing elements. This effective concrete cover is used to simulate the confinement conditions of the considered bar by means of the thick-walled cylinder model, as already described.
The following expression has been chosen:

\[
    c_{\text{eff}} = \frac{1}{m} \sum_{i=1}^{m} \left[ c_i \chi(c_i) + c_{\text{eff,max}}(1 - \chi(c_i)) \right] \tag{4.41}
\]

where \( c_{\text{eff}} \) is the effective concrete cover

\( c_{\text{eff,max}} \) is the maximum effective concrete cover to be taken into account

\( m \) is the number (3 or 4) of equally spaced directions to be taken into account depending on section geometry

\( c_i \) is the cover thickness in any of \( m \) directions

\( \chi(c_i) \) is an indicator function, defined as follows:

\( \chi(c_i) = 1 \) if \( c_i \leq c_{\text{eff,max}} \)
\( \chi(c_i) = 0 \) if \( c_i > c_{\text{eff,max}} \)

The maximum effective concrete cover is defined as the length of the most probable splitting crack:

\[
    c_{\text{eff,max}} = \frac{c_{i,\text{min}} + r_s}{\cos(\alpha_s)} - r_s \tag{4.42}
\]

where \( c_{i,\text{min}} \) is the smallest concrete cover to be taken into account

\( \alpha_s \) is the angle between the critical splitting plane and normal to the closest concrete surface (\( \alpha_s = 45-60^\circ \))

![Diagram](image)

Figure 4.25  Effective concrete cover for different member geometries

If more than one bar is located in a section a fictitious cover of \( 0.75s_i \) is assumed, \( s_i \) being the clear bar spacing in the direction considered. This assumption takes into
account that the concrete in between the bars is more effective in resisting the circumferential tensile stresses than the concrete in the cover, for the stress distribution is more uniform in the former case. In Fig. 4.25 examples of effective concrete cover, determined for members with different cross-sectional dimensions, are given.

### 4.4.2 Load introduction zone

The local bond stress-slip response is affected by the state of stress in the surrounding concrete. Since this state of stress significantly differs with different loading and boundary conditions, it is not surprising that differences in bond stress-slip relations along the transfer length of an embedded bar can occur. In the bond model described in Section 4.3 the local bond stress is conceived as being based upon the response of a concrete disc surrounding the bar at that particular position. This is a simplification of reality, for the rib bearing forces are spread into the concrete under a certain angle. In fact concrete cones shall be considered that equilibrate the radial forces that are introduced by bond (Fig. 4.26). In the approach taken here it is interpreted as activating the concrete in adjacent concrete discs perpendicular to the bar. This "linkage effect" shall be taken into account since it may result in significant differences in bond strength and stiffness, depending on the degree of restraint near the end of a concrete member or near a crack, and on the direction of bar movement with respect to the entrance plane. This has experimentally been shown by Cowell et al. (1982) in the case of pull-out and push-in tests.

![Concrete cone equilibrating radial forces introduced by bond](image)

![Concrete disc representing activated concrete cone](image)

**Figure 4.26** Schematic representation of the approach to the "linkage effect" in case of pull-out for free-edge (left) and for push-in (right)

Pull-out loading with relative displacement into the direction of the free edge will result
in an earlier occurrence of radial cracks due to lack of restraint. In those cases often a concrete cone is pulled out. Both the occurrence of the radial cracks and the cone pull-out are connected with a considerable reduction of the bond stress and this effect is often taken into account by assuming a bond free length of $2d_i$; e.g. König and Fehling (1988). In the present model a more refined approach is used, however.

![Graph showing load introduction angle-dependent effective concrete cover along the embedded bar for $45^\circ \leq \alpha_c < 90^\circ$. Left in the case of $c_{eff} < \kappa d_s \tan(\alpha_c)$ and right in the case of $c_{eff} \geq \kappa d_s \tan(\alpha_c)$]

Figure 4.27 Load introduction angle-dependent effective concrete cover along the embedded bar for $45^\circ \leq \alpha_c < 90^\circ$: left in the case of $c_{eff} < \kappa d_s \tan(\alpha_c)$ and right in the case of $c_{eff} \geq \kappa d_s \tan(\alpha_c)$

To account for the influences related to the introduction of bond forces near the entrance plane it is assumed that the activated concrete area is proportional to the distance from the entrance plane. This is in agreement with the approach of Schlaich and Schäfer (1981) to determine the extension of disturbed regions in concrete. It is assumed that the force is spread out in a conical volume with a cone angle $\alpha_c$ (angle between surface and axis of the cone), which varies in the case of pull-out from $45^\circ$ for a free edge to $90^\circ$ for high boundary restraint. For push-in $\alpha_c$ is taken as $90^\circ$ irrespective of the boundary conditions. Hence, for $\alpha_c = 90^\circ$ the activated concrete area remains constant and its cover equals $c_{eff}$ whereas in the case of $\alpha_c < 90^\circ$ the activated concrete cover depth increases from zero near the entrance plane to a maximum value when $c_{eff}$ is reached. The extension of the load introduction zone is limited to $\kappa d_s$, where $\kappa = 3$ is arbitrarily chosen.

With this assumption the effective concrete cover at any point of the embedded rebar $c_{eff}(x)$, i.e. the wall thickness used in the thick-walled cylinder model, may be determined, depending on the actual member geometry, distance of the point considered from the edge $x$, boundary conditions and direction of the relative displacement of the bar in concrete, according Fig. 4.27 and equation 4.43:
\[ c_{\text{eff}}(x) = c_{\text{eff}} \quad \text{if} \quad \alpha_c = 90^\circ \]
\[ c_{\text{eff}}(x) = x \tan(\alpha_c) \quad \text{if} \quad 45^\circ \leq \alpha_c < 90^\circ \quad \text{and} \quad c_{\text{eff}} < \kappa d_s \tan(\alpha_c) \quad (4.43) \]
\[ c_{\text{eff}}(x) = x \tan(\alpha_c) + \frac{c_{\text{eff}} - \kappa d_s \tan(\alpha_c)}{\kappa^2 d_s^2} \quad \text{if} \quad 45^\circ \leq \alpha_c < 90^\circ \quad \text{and} \quad c_{\text{eff}} \geq \kappa d_s \tan(\alpha_c) \]

4.4.3 Additional confinement

Among the parameters affecting bond efficiency the confinement plays a major role. When analysing the behaviour of a structure it shall be considered whether, besides the clamping action of the concrete surrounding the bar, also additional confinement is available. This can be the active confinement resulting from loads transverse to the bar, i.e. resulting from a support or from the column force in a beam-column joint, or passive confinement generated by transverse reinforcement. In practice often both active and passive confinement will be present.

The active confinement is more efficient than the passive one, since its effect does not depend on the mobilised bond stress. Passive confinement, on the contrary, depends on concrete dilatancy corresponding to the radial stress at the concrete-to-steel interface. Furthermore, the influence of additional confinement depends on the bond failure mode. *Maeda et al.* (1995) observed an increase of bond strength proportional to the confining stress delivered by the transverse reinforcement in the case of bond splitting failure. As far as the pull-out failure is concerned it is usually assumed that once the transition from rib bearing to friction has taken place the bond strength can not be increased by transverse reinforcement (*Eligehausen et al.* 1983). On the contrary, active confinement will contribute to the bond strength in both failure modes (*Eligehausen et al.* 1983; *Malvar* 1992). It is remarked that tensile stresses perpendicular to the bar may result in a negative contribution to the confinement (*Nagatomo* 1992).

Although the present confinement model given in Section 4.3.1 only deals with the clamping action of the concrete surrounding the bar it seems feasible to include the passive and active confinement as well.

4.4.4 Bond stress versus slip relation

In applying the actual bond stress versus slip relation in member-analysis the following steps are taken. In the thick-walled cylinder concept the wall thickness is a weighted average of the concrete cover around the bar, taking into account the real member
geometry (see Section 4.4.1). The response of the thick-walled cylinder is found with the smeared crack approach and the FCM is incorporated (see Section 4.3.1). The influence of the transverse deformation of the pulled bar and of the smoothening of the sliding surface on the release of the radial strain in the surrounding concrete are included in the estimation of the radial pressure at the inner hole (see Section 4.3.2). In this way the degree of the confinement delivered by the surrounding concrete (and thus the bond strength) is related to the state of stress in the bar itself and to the relative displacement of the interface.

**Figure 4.28** Bond stress versus slip relations for pull-out bond failure for $d_s = 20$ mm for normal strength concrete with $f_{cc} = 35$ MPa (left) and high strength concrete with $f_{cc} = 107$ MPa (right) (simulations for effective cover thickness of $c_{eff} = 100$ mm)

**Figure 4.29** Bond stress versus slip relations for bond splitting for $d_s = 20$ mm for normal strength concrete with $f_{cc} = 35$ MPa (left) and high strength concrete with $f_{cc} = 107$ MPa (right) (simulations independent of the steel strain)
The concrete confining capacity has a decisive influence on the ultimate bond resistance and on the mode of bond failure. Depending on the degree of confinement from the surrounding concrete, and thus on the actual member geometry, two modes of bond failure can occur: bond splitting and pull-out. Bond splitting prevails when the confinement is not sufficient to prevent the radial cracks from penetrating the whole concrete cover. On the contrary, with enough confinement pull-out failure takes place. In that case the concrete teeth in front of the ribs are sheared off resulting in a cylindrical sliding plane through the crowns of the ribs. At the moment of shearing of the concrete teeth radial cracks have not crossed the entire cover and their further growth (i.e. further cover penetration) is terminated.

The ability of the model to distinguish between these two models is of great importance for the analysis of member behaviour, pondering the profound differences of the bond resistance in both cases. Fig. 4.28 shows the bond stress as a function of the slip and the steel strain for pull-out failure. Since along the bond activated part of the bar both parameters may vary, depending on the stress-strain characteristics of steel and on the boundary conditions, no general and unique bond stress versus slip relation can be proposed in this case. Note that the influence of the steel strain and, hence, of the steel properties is small as long as no yielding takes place.

Splitting failure usually occurs before the steel yields and in general the bond strength in this case is considered to be hardly influenced by steel strain. The occurrence of bond splitting is strongly related to the effective cover, as can be seen in Fig. 4.29. Owing to a fracture mechanics based description of the concrete fracture, not only the bar-size effect is explicitly included, but also the differences in response of concretes with various toughness can accurately be represented by the new bond model (in Fig. 4.28 and 4.29 examples are given for bar diameter $d_s = 20$ mm for normal strength and high strength concrete with $f_{ce}$ of 35 and 107 MPa, left and right of the figures, respectively).

Considering the application of the proposed bond model in the modelling of plastic hinges in RC members, its ability to predict the actual bond stress-slip relation in a wide range of steel deformations, and in particular taking account of the yielding of the steel is of major importance. Also the fact that the actual member geometry is involved in determining the degree of confinement and the mode of bond failure is an important improvement comparing to other presently used bond models (e.g. MC90, Eligehausen et al. 1983, Eligehausen et al. 1989). With respect to the boundary conditions, in the member simulations the effect of load introduction and the possible so-called cone pull-out are accounted for (see Section 4.4.2). Considering that the presented confinement model deals with the clamping action of the concrete surrounding the bar and does not
include the passive and active confinement effect only members without confining reinforcement are analysed in the verifications and parameter studies discussed hereafter (see Section 4.4.3).

4.5 Bond model verification

The concrete confinement based bond model enables the prediction of the actual bond stress - slip relationship for a particular loading path (the steel stress-strain characteristics dependence included) and for particular geometrical characteristics of the specimen (the confinement and boundary conditions dependence involved). The validity of this model has been verified by comparing model predictions with results of tests, in which well defined confinement conditions had been created. By this means also the potentials of the model, further discussed in Bigaj et al. (1996), have been shown.

Reference experiments and simulations

In order to demonstrate the ability of the model to simulate the bond resistance in a wide range of steel deformations the calculated results are compared with those obtained in pull-out tests with long embedment length, described in Section 4.2 and in Bigaj (1995).

Results of this comparison are shown in Fig. 4.30 and 4.31. An equally good agreement is found for test series with NSC and HSC - both concerning the bond stress and slip development along the transfer length and the bond stress - slip relationships. In diagrams showing the bond stress versus slip relationship besides the experimentally obtained lines and the final simulation results also the bond stress - slip curves valid for a number of constant steel strain values are shown. It is of great importance to realise that the bond stress - steel strain - slip relationships (as in Fig. 4.28 and 4.29) are unique for a given set of material properties and well defined specimen geometry, whereas the bond stress - slip relationship is not unique but dependent on the actual loading and boundary conditions.

To prove the ability of the bond model to account for these effects other load arrangements and boundary conditions were also studied - besides the beam bending tests (Bigaj 1996) referred to in Section 4.2, also experiments have been analysed, where conditions as between two cracks (short embedment length, zero slip values with non-zero steel stress value) have been created in tests on short prismatic specimens (Manfredi and Pecce 1996). In these tests concrete prisms with a cross-section of 150x150 mm² and length of 600 mm were used. One steel bar $d_s = 14$ mm was cast either at the centre or 30 mm from one of the specimen faces. Steel sheets traversed the full cross-section at
Figure 4.30  Comparison of test results and simulations for long embedment length -
d_s = 16 mm, normal strength concrete f_{cc} = 27.6 MPa (Bigaj 1995)

Figure 4.31  Comparison of test results and simulations for long embedment length -
d_s = 16 mm, high strength concrete f_{cc} = 94.5 MPa (Bigaj 1995)
distances of 100 or 150 mm to pre-form cracks at fixed locations. Normal strength concrete ($f_c = 31$ MPa) and hot rolled steel ($f_y = 540$ MPa, $f_u = 625$ MPa, $\epsilon_u = 13.7\%$) were used. In the tensile tests the bond behaviour in the post-yield range of steel deformations had been studied. The slip and steel strain distribution along the bar after failure were reported. The test results are used to validate the developed bond model for the case of a short embedment length combined with large steel strains in the post yield range (in this case failure is defined as to occur by steel rupture) and a good agreement between test results and simulations is found both in terms of steel strain and slip values, see Fig. 4.32. Here the results are given for a crack spacing of 100 mm and a concentric bar position.

![Graph showing strain and slip distribution](image)

**Figure 4.32** Comparison of test results and simulations for short embedment length (Manfredi and Pecce, 1996)

The potentials of the model to simulate the effect of reinforcing steel characteristics on the bond stress - slip relationships has also been proven by comparison with test results (Shima et al. 1987a), where this effect had explicitly been studied in pull-out tests with long embedment lengths. It the test set-up similar to that used in the pull-out tests with long embedment length described in Section 4.2 the effect of steel stress - strain relationship on the bond behaviour in the post yield range of steel strains was studied. Three kinds of hot rolled reinforcing steel, denoted as SD30, SD50 and SD70, were used. Major differences concern the yield strength $f_y$ (350, 610 and 820 MPa, respectively), the strain from which strain hardening starts $\epsilon_{sh}$ (0.0165, 0.0140 and 0.0060, respectively) and the stiffness in the strain hardening range (that followed from the strength at a steel strain of 0.04, which amounted to 450, 730 and 900 MPa, respectively). Fig. 4.33 shows that the model predictions agree well with the test results. The significant influence of the reinforcing steel deformation on the bond strength development evidently appears from the comparison of relationships obtained for
different kinds of steel. The obvious distinction is the slip at which the bond stress drops down. The range of slip which holds the highest bond stress is wider for steel with higher strength. In the post-yield range a strong reduction of the bond strength is found and almost constant bond stress values are observed with increasing slip. The ability of the bond model to reproduce these effects is a basic requirement when striving at a proper analysis of the influence of the construction materials characteristics on a phenomenon like plastic hinging in reinforced concrete.

![Graph showing bond stress vs. slip for different steel types](image)

**Figure 4.33** Effect of reinforcing steel characteristics on bond stress versus slip relation - comparison of test results (left column, after Shima et al. 1987) and simulations (right column)

Keeping in mind that one of the aims of this investigation is getting a better understanding of member size effects on structural performance of reinforced concrete members, attention is also given to the effect of bar diameter variation on bond behaviour. The model discussed is fully capable of representing this effect, which is implicitly included, owing to the fracture mechanics based description of the propagation process of the splitting crack. There is not so much experimental evidence available concerning this phenomenon, since most of the tests have been performed with only one bar size at the time, or the bar size variation has been very limited. Some more extensive research (Morita et al. 1994) confirms, however, that a bar size effect on bond behaviour
indeed exists. The reported tendencies are also captured in the simulations with the concrete confinement based bond model, see Fig. 4.34. A change in the rate of bond strength decrease with an increase of the bar diameter is found for a changing concrete cover thickness, as it was observed in the experiments. Note that also the insensitivity of the ultimate bond stress to the bar size in cases where a pull-out type of bond failure takes place is confirmed.

\[
\frac{\tau_{bu}}{f_{ct}}
\]

<table>
<thead>
<tr>
<th>Cover:</th>
<th>Failure mode:</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥ 3.0 (d_s)</td>
<td>pull-out</td>
</tr>
<tr>
<td>2.5 (d_s)</td>
<td>splitting</td>
</tr>
<tr>
<td>2.0 (d_s)</td>
<td>splitting</td>
</tr>
<tr>
<td>1.5 (d_s)</td>
<td>splitting</td>
</tr>
<tr>
<td>1.0 (d_s)</td>
<td>splitting</td>
</tr>
</tbody>
</table>

Figure 4.34  Simulation of bar size effect on ultimate bond strength

In all the simulations to which is referred here, the above-mentioned assumptions concerning the load introduction zone and the effective concrete cover have been adopted. Consequently, in the case of pull-out tests with long embedment length and a 10 bar diameter bond free length at the loaded end of the bar, the load introduction angle \(\beta\) value was taken equal to 90°. On the contrary, in the simulation of bond between two cracks in beams and short prismatic specimens a \(\beta\) value of 45° was assumed. In all cases the actual mechanical characteristics of steel and concrete were used as input data. In this respect it must be stressed that as soon as the steel reaches in the post-yield range both the simulations and the actual behaviour are very sensitive to the constitutive model of steel. A small variation of the steel stress induces a large variation of the steel strain, and strongly influences the conditions of equilibrium and compatibility. Therefore it is of great importance to use reinforcing steel characteristics as close to reality as possible when simulating phenomena where steel yielding may occur, if one aims at a reliable analysis.
5 CRACKING BEHAVIOUR OF RC MEMBERS

5.1 Crack development and crack spacing

Contrary to various semi-empirical relations, which estimate the crack spacing roughly on the basis of the bar diameter, concrete cover and effective reinforcement ratios (CEB-FIP Model Code 1990, Eurocode 1990, see also Braam 1990), in this calculation model the crack spacing follows from the actual member geometry, bond characteristics and the material strength.

Important steps in the simulation are the definition of the transfer length, which is the distance required to develop the cracking force in the concrete and the definition of the average crack spacing. The transfer length follows on the one hand from the bond stress versus slip relation that results from the bond model, described in Chapter 4, and on the other hand from the tensile strength and effective concrete area taken into account. For the estimation of the bond strength it is considered that the cracking load is developed over a certain length, along which weaker sections alternate with stronger ones. This is best simulated by taking into account the average tensile strength. However, the characteristic lower bound value of the tensile strength is used to define the load at which the member starts to crack, since this may occur in the weakest section. The origin of cracks will continue until the distance between two subsequent cracks is too small to generate new cracks, which concludes the so-called primary crack stage. With the load which marks the end of the primary crack stage corresponding to a concrete stress $\sigma_{cr}$, the steel stress $\sigma_{s2}$ at the crack that settles the conditions of a stabilised primary crack pattern is calculated according to equation 5.1, where $\alpha_e = E_s / E_c$ is the ratio between the moduli of elasticity of steel and concrete, and $\rho_{s,ef} = A_s / A_{c,ef}$ is the ratio between the cross-sectional area of steel and concrete effective in tension (member size dependence included):

$$\sigma_{s2} = \sigma_{cr} \cdot \frac{1 + \alpha_e \rho_{s,ef}}{\rho_{s,ef}}$$  (5.1)

When aiming at determination of the smallest average crack spacing, the lower bound value of the transfer length has to be used for the simulation of the origin of the subsequent cracks in members subjected to bending. In this case the characteristic lower bound value of the flexural tensile strength $f_{ct,fl,k.05}$ is assumed to be decisive and it is used as a cracking criterion in flexural members (for reinforced concrete tensile members the characteristic lower bound value of the uniaxial tensile strength $f_{ck,05}$ has to be used):
\[ \sigma_b(l_t) \leq f_{ct,fl} k_{0.5} \]
\[ \text{or} \]
\[ \sigma_b(l_t) \leq f_{ck,0.5} \]

where \( \sigma_b(l_t) \) is the concrete stress along the transfer length induced by the pulled bar due to bond.

The flexural tensile strength is estimated according to the \textit{CEB-FIP Model Code 1990}:

\[ f_{ct,fl} = f_{ctm} \cdot \left[ 0.66 \left( \frac{h}{h_o} \right)^{-0.7} + 1 \right] \]  \hfill (5.3)

where \( h_o = 100 \text{ mm}, h > 50 \text{ mm} \)

For the development of a stabilised primary crack pattern no reduction of the bond stress near the crack faces is applied. On the contrary, in the analyses at a higher steel stress such a reduction connected to the pull-out of a concrete cone is applied.

After the crack pattern has stabilised the maximum crack spacing amounts to two times the transfer length and the minimum spacing equals one transfer length. In a member of infinite length the number of maximum and minimum spacings will be the same, yielding an average crack spacing of 1.5 times the transfer length. However it is necessary to account for the limited member length and, for the members in bending, for the effect of moment gradient. Hence, the higher probability of the occurrence of small spacings must be considered. Following the statistical analysis of \textit{Maier (1983)} a relation between the crack location at the initiation of the primary cracks and the average crack spacing may be derived. If the initial crack distance \( s_{r,l} \) falls between 2 and 3 times the transfer length, only one more crack can be developed in between. In this case the average crack spacing \( s_{r,a} \) equals \( 1.25 l_t \). However, if the initial crack distance \( s_{r,l} \) falls between 3 and 4 times the transfer length, more than one crack can be developed there. Than, the average crack spacing \( s_{r,B} \) must be calculated accounting for the probability of the occurrence of smaller and larger spacings according to:

\[ s_{r,B} = \frac{(k_s + 2)}{2} \cdot l_t \cdot \left( \frac{2}{k_s} - 1 \right) + \frac{(k_s + 2)}{3} \cdot l_t \cdot \left( 2 - \frac{2}{k_s} \right) = \frac{(k_s + 2)^2}{6k_s} \cdot l_t \]  \hfill (5.4)

where the ratio \( k_s \), that follows from \( l = (k_s + 2)l_2 \), varies between 1 and 2 (see also Fig. 5.1). After integration within these limits the average crack spacing \( s_{r,B} \) follows from:
\[ s_{r_0} = \frac{1}{6} \cdot \frac{2}{1} \left( k_s^2 + 4 + \frac{4}{k_s} \right) \, dk_s = 1.38 \cdot l_t \]  
(5.5)

The average crack spacing \( s_r \) is therefore taken as:

\[ s_r = \frac{1.25 + 1.38 \cdot l_t}{2} = 1.3 \cdot l_t \]  
(5.6)

\[ \text{Figure 5.1 Schematic interpretation of the probable crack location in case of initial crack distance } s_{r_1} \text{ between 3 and 4 times the transfer length } l_t \]

In the simulation of the member behaviour it is conceded that secondary cracking may take place when at a higher loading level an increased slip is associated with a sufficient growth of the bond stress resulting in a strong reduction of the transfer length. In such cases the average secondary crack spacing is taken equal to half the mean primary crack spacing.

### 5.2 Effective tension concrete area

In specimens where the reinforcement is not uniformly distributed (e.g. beams or slabs) the crack pattern is not similar along the whole specimen circumference. For the estimation of the crack spacing in such cases a specimen with non-uniformly distributed reinforcement is treated as a tensile member by defining an effective tension concrete area \( A_{c,ef} \) around the main reinforcement. This approach basically follows the method deduced by Leonhard (1976) or Schießl and Wölfl (1986). An important extension is, however, that the chosen formulation explicitly includes the member size in defining the
effective concrete tension area in an implicit and consistent way.

The cracking force acting on the effective concrete tension area follows from:

\[ N_{cr} = A_{c,ef} \cdot \sigma_{cr} \cdot (1 + \alpha_e \cdot \rho_{s,ef}) \]  \hspace{1cm} \text{(5.7)}

where \( \sigma_{cr} = f_{cm} \) is taken as the concrete stress value, that agrees with the formulation in the CEB-FIP Model Code 1990 for the stabilised primary crack pattern condition and complies with equation 5.1.

From the equilibrium conditions it follows that:

\[ N_{cr} = \frac{M_{cr}}{z} \] \hspace{1cm} \text{(5.8)}

where \( z \) is the inner lever arm.

Then, the cracking moment is equal to:

\[ M_{cr} = \sigma_{cr,fl} \cdot W_{cs,2} \] \hspace{1cm} \text{(5.9)}

where the flexural tensile stress \( \sigma_{cr,fl} \) is taken equal to the flexural tensile strength \( f_{ct,fl} \) and the resistance moment of the uncracked cross-section \( W_{cr,2} \) is defined as:

\[ W_{cs,2} = \frac{I_{cs}}{\frac{1}{2} d - e_a} \] \hspace{1cm} \text{(5.10)}

where \( d \) is the total height of the cross-section

\( e_a \) is the distance between the gravity points of the plain concrete section and of the reinforced section

\( I_{cs} \) is the moment of inertia of the uncracked reinforced cross-section

For rectangular sections it holds:

\[ I_{cs} = I_c + A_c \cdot e_a^2 + \alpha_e \cdot A_s \cdot \left( \frac{d}{2} - c_1 \right) \] \hspace{1cm} \text{(5.11)}

\[ e_a = \left( h - \frac{d}{2} \right) \cdot \frac{\alpha_e \cdot \rho_s}{1 + \alpha_e \cdot \rho_s} \] \hspace{1cm} \text{(5.12)}

\[ I_c = \frac{b \cdot d^3}{12} \] \hspace{1cm} \text{(5.13)}

where \( h \) is the effective height of the cross-section

\( b \) is the width of the cross-section

\( c_1 = d - h \) is the concrete cover
\[ \rho_s \] is the reinforcement ratio

Substitution of these expressions into equation 5.10 yields:

\[ W_{cs,2} = \frac{bd \cdot (d^2 + 4 \alpha_e \rho_s d^2 - 12 \alpha_e \rho_s d c_1 + 12 \alpha_e \rho_s c_1^2)}{12 \alpha_e \rho_s c_1 + 6d} \] \hspace{1cm} (5.14)

Assuming that the cross-section remains in the linear-elastic stage, the inner lever arm \( z \) follows from:

\[ z = h - \frac{1}{3} h \left[ -\alpha_e \rho_s + \sqrt{(\alpha_e \rho_s)^2 + 2 \alpha_e \rho_s} \right] \] \hspace{1cm} (5.15)

From this, the effective tension concrete area is computed as:

\[ A_{c, ef} = \frac{\sigma_{cr, flt} \cdot bd \cdot (d^2 + 4 \alpha_e \rho_s d^2 - 12 \alpha_e \rho_s d h + 12 \alpha_e \rho_s h^2)}{2 \sigma_{cr} \cdot h \cdot (2 \alpha_e \rho_s d - 2 \alpha_e \rho_s h + d) \cdot \left[ 3 + \alpha_e \rho_s - \sqrt{(\alpha_e \rho_s)^2 + 2 \alpha_e \rho_s} \right]} - \alpha_e \rho_s b h \] \hspace{1cm} (5.16)

In order to account correctly in further calculations for the member size dependence it is important to define the concrete flexural tensile strength using a formulation that explicitly considers this effect. For this reason the previous formulation according to CEB-FIP Model Code 1990 (equation 5.3) is chosen.

### 5.3 Approach verification

#### 5.3.1 Tension stiffening in tensile members

To show the applicability of the discussed approach in analysing the formation of cracks in reinforced members under tensile load and, in particular, its accuracy in estimating the crack width, crack spacing and tension stiffening effect, the load versus elongation relationships of concentrically reinforced concrete prisms were simulated. Experiments from a comparative study into the bond properties of ribbed bars in lightweight aggregate and gravel concrete were used as a reference (Stroband 1991).

**Reference experiments**

Within the framework of a comparative study into the structural behaviour of lightweight aggregate concrete, the bond behaviour of ribbed bars and the tension stiffening was
investigated in lightweight and gravel concrete. Here the comparison between experiments and calculation results is restricted to the tests with gravel concrete. The experiments comprised pull-out tests on 12, 16 and 20 mm diameter bars concentrically embedded over three bar diameters in 200 mm cubes and displacement controlled tensile tests on reinforced concrete prisms (100×100×1000 mm). The concentrically embedded reinforcement consisted of one 12, 16 or 20 mm diameter bar or four 10 mm diameter bars, which resulted in a reinforcement ratio of 1.13%, 2.01% and 3.14%, respectively. The relative rib area of the 10, 12, 16 and 20 mm diameter bars amounted to 0.048, 0.065, 0.080 and 0.053, respectively. The mix designs aimed at 30 and 60 MPa cube strength after 14 and 28 days, respectively.

In the reinforced prism tensile tests the overall deformations were measured by means of LVDT's on two opposite faces with 935 mm measuring length. Local deformations, from which crack initiation and crack width were deduced, were measured with 110 mm long clip gauges attached to the other two faces. Besides, at some load levels crack widths were measured by means of a magnifying glass with a gradation scale.

**Simulations**

In the simulations the characteristic lower bound value of the tensile strength is used to define the load at which the prism starts to crack, since this occurs in the weakest section. For the simulation of the origin of the following cracks two approaches have been chosen. In the first one the characteristic lower bound value of the tensile strength \( f_{ck,0.5} \) is assumed to be decisive, which will result in the smallest average crack spacing. The probability that the end of the transfer zone coincides with the weakest cross-section is very small, however. Therefore, cracking has also been simulated using the average tensile strength \( f_{ctm} \), which will yield an upper bound value of the average crack spacing.

The simulation has been effectuated by calculating the elongation at a series of steel stresses. This is illustrated in Fig. 5.2. Point A=a corresponds to the origin of the first crack, which follows from the characteristic lower bound value of the tensile strength, the elastic moduli of steel and concrete and the reinforcement ratio. Point B (b) marks the end of the primary crack stage, which is assumed to correspond with the characteristic upper bound value of the tensile strength. If no further cracking would occur, line BD (bd) was followed. However, when the increased slip at a higher load level is connected with sufficient growth of bond stresses, the transfer length becomes shorter and new cracks may develop. In the simulations it was not tried to estimate the exact load at which these secondary cracks arise, but at fixed load levels it was verified whether they had occurred. For example, at point C (c) in Fig. 5.2 no secondary
cracking had taken place, but at point E (e) it had happened. As a consequence, point C (c) lays on line CD (cd), but point E (e) is shifted to the right, thus indicating the decrease of the concrete contribution in carrying the load. The two different crack stress levels assumed result in different load levels at which secondary cracks may have occurred; compare C-E and c-e in Fig. 5.2. The calculated lines for both crack stress levels (ABCEF and abcef) indicate the boundaries between which the simulated load-elongation curve would be situated if more load levels and a continuously increasing crack stress level had been taken into account. Therefore, the area between both curves is shaded in the following presentations.

Figure 5.2 *Steps in simulation of load elongation behaviour of reinforced prism loaded in tension*

In the simulations the cube compressive strength is used as a reference basis, from which the tensile strength and elastic modulus were calculated according to the *CEB-FIP Model Code 1990* expressions, see Table 5.1. Although these values and the measured ones differ, preference has been given to the calculated ones because of unexpected results in the measured values, e.g. with respect to the ratio between splitting tensile strength and cube strength and the ratio between splitting tensile strength and uniaxial tensile strength. The measured load-elongation curves in the reinforced prism tests compare well with the calculated ones, as can be concluded from Fig. 5.3. Differences with respect to the onset of cracking and the course of the curves during the primary crack stage can be explained from the simulation procedure discussed above. In Table 5.2 the average crack spacing and crack width calculated with the two crack stress values considered are listed as well as the values found in the experiments at 300 MPa bare steel stress. The calculated results can be compared with the measured ones if the stabilised cracking stage had been achieved.
Table 5.1 Concrete properties in bond simulations

<table>
<thead>
<tr>
<th>property</th>
<th>B30</th>
<th>B60</th>
<th>B100</th>
</tr>
</thead>
<tbody>
<tr>
<td>cube strength</td>
<td>23</td>
<td>62</td>
<td>100</td>
</tr>
<tr>
<td>average tensile strength</td>
<td>2.19</td>
<td>3.75</td>
<td>4.73</td>
</tr>
<tr>
<td>5% char. tensile strength</td>
<td>1.76</td>
<td>3.01</td>
<td>3.79</td>
</tr>
<tr>
<td>elastic modulus</td>
<td>25616</td>
<td>34492</td>
<td>39811</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>fracture energy</td>
<td>85.5</td>
<td>135</td>
<td>146.6</td>
</tr>
</tbody>
</table>

1) corresponds to average value in experiments
2) tests: splitting tensile strength 2.04 MPa, uniaxial tensile strength 1.97 MPa
3) tests: splitting tensile strength 3.56 MPa, uniaxial tensile strength 2.86 MPa

Figure 5.3 Steel stress of naked bar versus total elongation for cube strength of 23 MPa (left) and 62 MPa (right)

Figure 5.4 Tension stiffening as a function of concrete strength and reinforcement ratio $\rho_s = 1.13\%$ (d_s = 12 mm) (left) and $\rho_s = 2.01\%$ (d_s = 16 mm) (right)
<table>
<thead>
<tr>
<th>prism specifications</th>
<th>steel stress level [MPa]</th>
<th>simulation for $\sigma_{cr}-f_{ctm}$</th>
<th>simulation for $\sigma_{cr}-f_{ck05}$</th>
<th>test results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s_r$ [mm]</td>
<td>$w$ [mm]</td>
<td>$s_r$ [mm]</td>
<td>$w$ [mm]</td>
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<tr>
<td>B30</td>
<td>200</td>
<td>93.5</td>
<td>0.087</td>
<td>134.4</td>
</tr>
<tr>
<td>$f_{ce} = 23$ MPa 4 d$_2$ 10</td>
<td>300</td>
<td>93.5</td>
<td>0.131</td>
<td>67.2</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>93.5</td>
<td>0.176</td>
<td>67.2</td>
</tr>
<tr>
<td>B30</td>
<td>200</td>
<td>186.1</td>
<td>0.166</td>
<td>271.8</td>
</tr>
<tr>
<td>$f_{ce} = 23$ MPa 1 d$_2$ 20</td>
<td>300</td>
<td>186.1</td>
<td>0.258</td>
<td>135.9</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>186.1</td>
<td>0.346</td>
<td>135.9</td>
</tr>
<tr>
<td>B30</td>
<td>200</td>
<td>325.0</td>
<td>0.224</td>
<td>231.3</td>
</tr>
<tr>
<td>$f_{ce} = 23$ MPa 1 d$_2$ 16</td>
<td>300</td>
<td>162.5</td>
<td>0.226</td>
<td>231.3</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>162.5</td>
<td>0.303</td>
<td>115.6</td>
</tr>
<tr>
<td>B30</td>
<td>253</td>
<td>256.3</td>
<td>0.226</td>
<td>186.7</td>
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<tr>
<td>$f_{ce} = 23$ MPa 1 d$_2$ 12</td>
<td>300</td>
<td>256.3</td>
<td>0.273</td>
<td>186.7</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>256.3</td>
<td>0.375</td>
<td>186.7</td>
</tr>
<tr>
<td>B60</td>
<td>200</td>
<td>205.6</td>
<td>0.155</td>
<td>150.0</td>
</tr>
<tr>
<td>$f_{ce} = 62$ MPa 1 d$_2$ 20</td>
<td>300</td>
<td>205.6</td>
<td>0.239</td>
<td>150.0</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>102.8</td>
<td>0.196</td>
<td>150.0</td>
</tr>
<tr>
<td>B60</td>
<td>250</td>
<td>181.3</td>
<td>0.173</td>
<td>131.6</td>
</tr>
<tr>
<td>$f_{ce} = 62$ MPa 1 d$_2$ 16</td>
<td>300</td>
<td>181.3</td>
<td>0.211</td>
<td>131.6</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>181.3</td>
<td>0.189</td>
<td>131.6</td>
</tr>
<tr>
<td>B60</td>
<td>283$^1$</td>
<td>153.1</td>
<td>0.250</td>
<td>112.0</td>
</tr>
<tr>
<td>$f_{ce} = 62$ MPa 1 d$_2$ 12</td>
<td>300$^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>423</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSC</td>
<td>209</td>
<td>153.0</td>
<td>0.127</td>
<td>111.1</td>
</tr>
<tr>
<td>$f_{ce} = 100$ MPa 1 d$_2$ 20</td>
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<td>0.187</td>
<td>111.1</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>153.0</td>
<td>0.262</td>
<td>111.1</td>
</tr>
<tr>
<td>HSC</td>
<td>310</td>
<td>135.2</td>
<td>0.171</td>
<td>98.2</td>
</tr>
<tr>
<td>$f_{ce} = 100$ MPa 1 d$_2$ 16</td>
<td>400</td>
<td>135.2</td>
<td>0.225</td>
<td>98.2</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>135.2</td>
<td>0.288</td>
<td>98.2</td>
</tr>
<tr>
<td>HSC</td>
<td>354$^1$</td>
<td>131.8</td>
<td>0.256</td>
<td>92.4</td>
</tr>
<tr>
<td>$f_{ce} = 100$ MPa 1 d$_2$ 12</td>
<td>500$^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>529</td>
<td>120.3</td>
<td>0.260</td>
<td>89.1</td>
</tr>
</tbody>
</table>

1) origin of first crack  2) primary crack pattern not yet stabilised

The simulations for the characteristic lower bound value of the tensile strength as a cracking criterion agree well with three out of six tests that complied with this condition. In the other cases crack spacing and crack width fall between the upper and the lower bound or are slightly overestimated, which can possibly be explained by differences between the tensile strength in the experiments and the values used in the simulations, see Table 5.1.

By distributing the steel cross-sectional area over more bars smaller crack spacings and
crack widths are found. This was simulated very well as shown by comparison of cases B30-4d_j10 and B30-1d_j20.

Both experiments and simulations show a tendency of decreasing crack spacing and crack width with increasing concrete strength. This tendency is clearly continued in predictions for 100 MPa cube strength. On the contrary, tension stiffening after stabilised cracking is hardly influenced by the concrete strength according to these predictions, as shown in Fig. 5.4.

5.3.2 Cracking in flexural members

The simulations described below show, that the approach presented can successfully be applied when analysing stiffness and crack width in reinforced concrete in bending.

Reference experiments

For the verification of the above-mentioned approach for cracking in flexural members test results from three point bending tests conducted on members with different sizes were used (detailed description in Chapter 2), as well as the measurements from the beam bond tests carried out within the framework of the bond investigation where also the concrete strength was varied (see Chapter 4 for details on test results).

Simulations

In the simulation only the average minimum crack spacing was calculated, using the lower bound flexural tensile strength $f_{ct,flk,05}$ as the cracking criterion. This approach is further used in the analysis of the deformation capacity of members, pondering the good agreement with the average values registered in the experiments. Moreover, it must be remembered that the casting position of the beams (with bars located at the bottom) favourably influences the bond strength, reducing the transfer length and thus also the crack spacing.

Simulations have been executed in two steps: at the end of the formation of the primary crack pattern and at the onset of yielding of the reinforcement. In the first case, conform the CEB-FIP Model Code 1990, the cracking force acting on the effective concrete tension area follows from $\sigma_{cr} = f_{cin}$ and the corresponding value of steel stress $\sigma_s$ at the crack is calculated according to the equation 5.1, using the effective tensile concrete area approach, described above. In the second case the steel stress value follows directly from the given reinforcing steel characteristics. In this case, considering the possibility
of pull-out of a concrete cone at the free crack edge when the secondary crack pattern develops at a higher steel stress, a reduction of bond strength is assumed to simulate this effect, see the discussion in Section 4.4.2.

![Graph](image)

Figure 5.5 Crack spacing - simulations and test results for flexural members (member size variation studied)

![Graph](image)

Figure 5.6 Crack spacing - simulations and test results for flexural members (concrete strength and bar diameter variation studied)

Simulation results are compared with the measured values, as shown in the Fig. 5.5 and 5.6. It is very important to conclude that for members with different sizes the correlation of measured and calculated values is equally good. This proves that the way of incorporating the member size influence is appropriate and sufficiently accurate. This is of significant importance considering the possible error propagation and the consequences that the estimation of crack spacing (and corresponding crack width) has for the computation of total member deformation capacity. Concerning the effect of
concrete strength and bar diameter (both investigated in the second series of simulations) the phenomena noticed in the experiments are also well captured by the model: the effect of the concrete strength on the cracking load and on the corresponding variation in primary and secondary crack pattern development level is confirmed; the effect of bar diameter reduction on the crack spacing (and crack width) decrease is also evident. All these influences are of significant importance for the development of the structural members deformation, and therefore this step-by-step verification procedure has been applied in this study.
6 Verification of Calculation Model for Rotation Capacity

6.1 Calculation procedure

The numerical model used for the calculation of the rotation capacity is based on the assumptions discussed in the Chapters 3, 4 and 5. To predict the deformation capacity of a member with given dimensions, reinforcement layout and material characteristics, firstly the crack spacing $s_r$ must to be determined. For this the procedure described in Chapter 5 is followed. Starting from the member geometry the effective concrete tension area $A_{c, eff}$ is calculated (see Section 5.2), the effective concrete cover on the bar $c_{eff}$ is estimated (see Section 4.4.1) and the load introduction angle $\beta$ is considered (see Section 4.4.2). Secondly, the distribution of bending moments $M$ along the member axis is calculated at the loading level considered, taking into account the width of the loading plate. Considering the range of slenderness ratios analysed, only flexural cracking is accounted for. Therefore no shift of the moment line due to shear is taken into account. Following the member discretisation principle, described in Chapter 3, the member is subdivided into cracked elements (FC-elements) of length $s_r$. As discussed in Section 3.2, two types of feasible crack patterns are considered, with and without crack at mid-span. In the following they are referred to as the lower and upper bound crack patterns.

Sectional forces at each crack (eg. at the edge of each FC-element) are determined by sectional analysis, assuming the validity of Bernoulli's principle of plane sections remaining plane. For this analysis the material laws are applied as described in Section 3.3 for concrete and in Section 3.4 for steel. Since the constitutive law of concrete in compression takes into account failure localization in the damage zone an iterative procedure is required. After assuming the compressive strain of the mostly stressed fibre $\varepsilon_m$, the depth of the compression zone $x$ as well as the depth $d^l$ and the length $L^l$ of the localization zone (eg. the shape of the stress versus strain relation for concrete in compression) can be determined. The value of $\varepsilon_m$ is corrected until the conditions with regard to equilibrium of forces and deformation compatibility are satisfied. Due to the limitations of the chosen method of member discretisation the limit for the length of the localization zones is such, that the two neighbouring zones do not overlap, hence that the condition $L^{l(n)} + L^{l(n+1)} < 2s_r$ is fulfilled. It is noted that in the analysed range of member sizes, reinforcement ratios and material characteristics this was always the case.

Knowing now the stresses and strains at every crack, the distribution of strains within each FC-element can be computed, and hence the rotations of all elements can be obtained by an integration procedure. With respect to the tensile chord, its elongation is obtained solving the differential equations of bond using the bond model described in
Chapter 5. As schematically shown in Fig. 6.1, a tensile element with an effective concrete area $A_{c,ef}$ and length $s_r$ subdivided into $n = 50$ segments, is analysed. The deformations of the concrete are accounted for according to the description given in Chapter 5, so that the tension stiffening effect is implicitly included. The total elongation of the tensile chord of the FC-element considered is a product of the integration of the elongation of all $n = 50$ segments.

**Figure 6.1** Schematization of tensile chord for calculation of steel strain distribution within the FC-element and the calculated steel strain distribution between two cracks

**Figure 6.2** Concrete stress distribution at three sections considered along the FC-element and schematized procedure for calculation of total upper fibre deformation

Strain localization in compression is accounted for when calculating the total deformation of the mostly stressed compressive fibre along the FC-element. Therefore,
as schematically shown in Fig. 6.2, a constant strain value \( \varepsilon_m \) is assumed over the length corresponding to the computed extension of the localization zone \( L' \), for the case the damage localization in compression is found to occur. Additionally to the concrete strains at both edges of the FC-element also the value at the mid-length is computed using as a starting point the steel stress value at this location, computed in the previous calculation step (i.e. estimation of the elongation of tensile chord). A linear concrete strain distribution is assumed in the range outside the damage zone. Integration along the whole FC-element length of the strains approximated in this way gives the total deformation in the upper fibre.

Subsequently to the calculation of the deformation of the tensile cord and of the upper fibre, the elementary rotations of all FC-elements and the total rotation of the hinge are obtained according to the definitions given in Section 3.2 (equation 3.1 and 3.2, respectively). Calculations performed at different loading levels provide the corresponding plastic hinge rotations. In the simulations discussed in the sequel only three loading levels are considered: the loads at the onset of steel yielding at the critical section, at the maximum load carrying capacity and at the ultimate state corresponding to the concrete compressive failure, in the cases where this failure mode prevails. The rotation capacity is then defined (according to equation 3.3) as the difference between the total rotation at the maximum load (or ultimate load, if specifically indicated) and that at the onset of yielding of the reinforcement. This step completes the calculation.

The adequacy of the model is evaluated in a step-by-step verification procedure. Firstly, the elementary models included in the calculation model for rotation capacity (such as the CDZM and the bond model) are separately verified using the results of the initial tests developed especially to study these phenomena, as already reported in Chapter 2 and 3, and in relevant literature (Bigaj 1995, Bigaj et al 1996, Markeset 1993a). Secondly, the proper link between the elementary models is verified and its ability to model a selected aspect of the structural member behaviour is checked (cracking and tension stiffening modelling verification in Chapter 5). Finally, the prediction of the deformation capacity for a broad scope of variables is examined.

6.2 Member size dependence

In this paragraph emphasis is put on the prediction of member size dependence using the calculation model for rotation capacity. In this model a number of size-effect related factors has been included in order to be able to describe the influence of the member size and geometry on its deformation capacity. To this aim, the basic components of the model (i.e. the material models and the bond model) were chosen in such a way that the
effect of strain localization is reliably incorporated. The latter is responsible for the size effect observed on the macro (structure) level. To check to what extent the goal has been accomplished the calculation model developed is verified against the results of the specially designed test series, where the deformation capacity of beams of different dimensions was studied in three point bending.

Reference experiments and simulations

The member size, the reinforcement ratio and the concrete mix composition were varied in a test series described in detail in Chapter 2. Three sizes of geometrically similar members were used (the effective member height $h$ was equal to 90, 180 and 450 mm, respectively), two reinforcement ratios were applied ($\rho_s$ equal to 0.28% and 1.12%) and two maximum aggregate sizes were taken in two concrete mixes with similar strength ($d_{a,\text{max}}$ equal to 4 and 16 mm). For the purpose of this verification only tests carried out with hot rolled reinforcing steel FeB 500 HWL are considered. Simulations are performed according to the model description given in Chapter 3. To describe the cracking behaviour the procedure, verified in Chapter 5, is followed.

Table 6.1 summarises the values of the concrete characteristics used in these simulations of all reference tests. Similar to the results referred to in Chapter 5, also in this case the cube compressive strength $f_{cc}$ was used as a basic value, from which the tensile strength $f_{ct}$, the elastic modulus $E_c$ and the fracture energy $G_f$ were calculated according to the CEB-FIP Model Code 1990. The behaviour of reinforcing steel is modelled as described in Section 3.4. In Table 6.2 the points defining the constitutive stress versus strain relationship are given for all types of reinforcing bars used.

<table>
<thead>
<tr>
<th>Table 6.1 Concrete properties in test simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member code</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>B.1.1.4</td>
</tr>
<tr>
<td>B.1.2.4</td>
</tr>
<tr>
<td>B.1.2.16</td>
</tr>
<tr>
<td>B.1.3.4</td>
</tr>
<tr>
<td>B.0.2.4</td>
</tr>
<tr>
<td>B.0.2.16</td>
</tr>
<tr>
<td>B.0.3.4</td>
</tr>
<tr>
<td>B.0.3.16</td>
</tr>
</tbody>
</table>
Table 6.2 Steel properties in test simulations

<table>
<thead>
<tr>
<th>$d_s$ [mm]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [%]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [%]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [%]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [%]</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [%]</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>530</td>
<td>0.27</td>
<td>560</td>
<td>2.00</td>
<td>580</td>
<td>3.00</td>
<td>625</td>
<td>6.00</td>
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<td>9.00</td>
<td>0.0423</td>
</tr>
<tr>
<td>10</td>
<td>565</td>
<td>0.28</td>
<td>575</td>
<td>2.00</td>
<td>600</td>
<td>3.00</td>
<td>631</td>
<td>6.00</td>
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<td>9.00</td>
<td>0.0344</td>
</tr>
<tr>
<td>16</td>
<td>570</td>
<td>0.29</td>
<td>580</td>
<td>2.00</td>
<td>610</td>
<td>3.00</td>
<td>651</td>
<td>6.00</td>
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<td>9.00</td>
<td>0.0410</td>
</tr>
<tr>
<td>20</td>
<td>550</td>
<td>0.28</td>
<td>560</td>
<td>1.50</td>
<td>605</td>
<td>3.00</td>
<td>640</td>
<td>6.00</td>
<td>660</td>
<td>9.00</td>
<td>0.0472</td>
</tr>
</tbody>
</table>

In Table 6.3 the measured and calculated values of the rotation at the onset of reinforcing steel yielding $\Theta^{(i)}$ and at the maximum loading level $\Theta^{(u)}$, as well as the rotation capacity values $\Theta^{(p)} = \Theta^{(u)} - \Theta^{(i)}$, are listed. The two measured values given for each test refer to the measurements taken at the front and at the rear face of the member. The upper and the lower simulation bounds are given, corresponding to the two feasible crack patterns - with and without a crack at mid span ($LB$ - denotes the lower bound and $UB$ - the upper bound result).

Table 6.3 Summary of test and simulation results

<table>
<thead>
<tr>
<th>Member code</th>
<th>$\Theta^{(i)}$ [rad]</th>
<th>$\Theta^{(u)}$ [rad]</th>
<th>$\Theta^{(p)}$ [rad]</th>
<th>$\Theta^{(i)}$ [rad]</th>
<th>$\Theta^{(u)}$ [rad]</th>
<th>$\Theta^{(p)}$ [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1.1.4</td>
<td>0.023044</td>
<td>0.060225</td>
<td>0.037181</td>
<td>0.021846</td>
<td>0.023428</td>
<td>0.043718</td>
</tr>
<tr>
<td></td>
<td>0.024179</td>
<td>0.062520</td>
<td>0.038341</td>
<td>0.021872</td>
<td>0.023428</td>
<td>0.043718</td>
</tr>
<tr>
<td>B.1.2.4</td>
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<td>0.032679</td>
<td>0.031560</td>
<td>0.018718</td>
<td>0.018832</td>
<td>0.032218</td>
</tr>
<tr>
<td></td>
<td>0.018498</td>
<td>0.031187</td>
<td>0.012688</td>
<td>0.018718</td>
<td>0.018832</td>
<td>0.032218</td>
</tr>
<tr>
<td>B.1.2.16</td>
<td>0.018729</td>
<td>0.031064</td>
<td>0.012335</td>
<td>0.020574</td>
<td>0.020668</td>
<td>0.030010</td>
</tr>
<tr>
<td></td>
<td>0.019676</td>
<td>0.032096</td>
<td>0.012420</td>
<td>0.020574</td>
<td>0.020668</td>
<td>0.030010</td>
</tr>
<tr>
<td>B.1.3.4</td>
<td>0.006201</td>
<td>0.016486</td>
<td>0.010285</td>
<td>0.007056</td>
<td>0.008258</td>
<td>0.012994</td>
</tr>
<tr>
<td></td>
<td>0.007832</td>
<td>0.015675</td>
<td>0.007843</td>
<td>0.007056</td>
<td>0.008258</td>
<td>0.012994</td>
</tr>
<tr>
<td>B.0.2.4</td>
<td>0.013629</td>
<td>0.076000</td>
<td>0.063291</td>
<td>0.012182</td>
<td>0.013726</td>
<td>0.045234</td>
</tr>
<tr>
<td></td>
<td>0.012743</td>
<td>0.076440</td>
<td>0.063697</td>
<td>0.012182</td>
<td>0.013726</td>
<td>0.045234</td>
</tr>
<tr>
<td>B.0.2.16</td>
<td>0.012928</td>
<td>0.086194</td>
<td>0.073206</td>
<td>0.011550</td>
<td>0.013032</td>
<td>0.044700</td>
</tr>
<tr>
<td></td>
<td>0.011178</td>
<td>0.087839</td>
<td>0.076121</td>
<td>0.011550</td>
<td>0.013032</td>
<td>0.044700</td>
</tr>
<tr>
<td>B.0.3.4</td>
<td>0.004192</td>
<td>0.032880</td>
<td>0.028688</td>
<td>0.004044</td>
<td>0.005066</td>
<td>0.025238</td>
</tr>
<tr>
<td></td>
<td>0.004192</td>
<td>0.032880</td>
<td>0.028688</td>
<td>0.004044</td>
<td>0.005066</td>
<td>0.025238</td>
</tr>
<tr>
<td>B.0.3.16</td>
<td>0.010023</td>
<td>0.047068</td>
<td>0.037045</td>
<td>0.012996</td>
<td>0.012626</td>
<td>0.031986</td>
</tr>
<tr>
<td></td>
<td>0.010184</td>
<td>0.046767</td>
<td>0.036583</td>
<td>0.012996</td>
<td>0.012626</td>
<td>0.031986</td>
</tr>
</tbody>
</table>

In the simulations it is taken into account that during the tests measurements used to determine the rotations have only been taken along the central part of the beams.
Therefore the calculated rotation values reported here refer also to the central part of the beams with a length actually involved in the measurements.

\[ \text{rotation capacity [rad]} \]

\[ \text{max. rotation [rad]} \]

\[ \begin{align*}
\text{test results} \\
\text{simulations} \\
\ell_{efg} = 4 \text{ mm} \\
\ell_{efg} = 16 \text{ mm} \\
UB - \text{upper bound solution} \\
LB - \text{lower bound solution}
\end{align*} \]

\[ \begin{align*}
\text{test results} \\
\text{simulations} \\
\ell_{efg} = 4 \text{ mm} \\
\ell_{efg} = 16 \text{ mm} \\
UB - \text{upper bound solution} \\
LB - \text{lower bound solution}
\end{align*} \]

Figure 6.3 Rotation capacity and rotation at max. load versus the effective height of the member - simulations and test results for reinforcement ratio 1.12%

\[ \begin{align*}
\text{test results} \\
\text{simulations} \\
\ell_{efg} = 4 \text{ mm} \\
\ell_{efg} = 16 \text{ mm} \\
UB - \text{upper bound solution} \\
LB - \text{lower bound solution}
\end{align*} \]

\[ \begin{align*}
\text{test results} \\
\text{simulations} \\
\ell_{efg} = 4 \text{ mm} \\
\ell_{efg} = 16 \text{ mm} \\
UB - \text{upper bound solution} \\
LB - \text{lower bound solution}
\end{align*} \]

Figure 6.4 Rotation capacity and rotation at max. load versus the effective height of the member - simulations and test results for reinforcement ratio 0.27%

In Fig. 6.3 and 6.4 the simulation results for the tests performed are compared with the test results for both applied reinforcement ratios. In these diagrams also the lower and the upper simulation bounds are indicated. These bounds are shown as well in Fig. 6.5, where the comparison for the whole test series illustrates the correlation of measured and calculated values as far as the rotation at yielding and at the maximum load is concerned, and in Fig. 6.6, where the comparison of the measured and calculated values of the rotation capacity is given (the solid and open marks represent the crack pattern dependent simulation bounds, between which the actual value is to be expected and the
solid marks indicate the simulation corresponding to the crack pattern actually obtained in the test).

Figure 6.5  Comparison of measured and calculated values of rotation at yielding and at max. load for two reinforcement ratios: 1.12% (left) and 0.27% (right)

Figure 6.6  Comparison of measured and calculated values of rotation capacity for two reinforcement ratios: 1.12% (left) and 0.27% (right)

In general, a very good correlation between the measured and the calculated values is found. The results of the simulations at the maximum loading level for the lower reinforcement ratio tend slightly to fall below the measured values - which is likely to be the result of assuming in the simulations the lower bound ultimate steel strain value of 9%, in order to account for significant scatter with respect to this value. In this respect it should be stressed that for the interpretation of the measured values of the rotation capacity the accuracy of the estimation of the maximum loading level is significant. Considering the shape of the load - deflection diagram, which is almost
horizontal at the loading level close to the maximum, a possible inaccuracy in data acquisition may take place. This is specially noticeable in the cases of members with a low reinforcement ratio where steel rupture is critical, ponder the very low hardening ratio in the range of highest steel strain. Nevertheless, since when analysing the experiments reported here it has been aimed at limiting measuring errors as much as possible, it is believed that the tendencies found in the tests are the representation of real structural effects. In the following some of these effects are further elaborated using the outcome of the numerical simulations.

The results of the simulations clearly show that there is a very strong effect of the crack pattern type (with or without crack at mid-span) on the available rotation capacity. Especially in cases with low reinforcement ratio this effect becomes very pronounced, for the combination of parameters used in this simulation. For the members analysed here the difference in the available rotation capacity may amount in some cases to more than 50%, which is of the similar magnitude as the variation attributed to the member size influence in the range investigated. This should be kept in mind, considering that in practice the location of the first crack, and thus the type of crack pattern, depends on the stochastic distribution of the material strength or is enforced by the presence of transverse reinforcing bars. In both cases this is hard to predict (contrary to laboratory tests, in practice the location of transverse bars is never known) and it is therefore not conservative to assume anything but the lower bound simulation values when evaluating the available rotation capacity of flexural members in a general case.

In the simulations the effect of the aggregate size on the deformation capacity of concrete in compression is included and modelled with the CDZM, as explained in Section 3.3.2. It finds its confirmation in the test results, where slightly increased rotations were observed in the members with the smaller maximum aggregate size. Both test results and simulations suggest that this effect is not of a meaningful magnitude in practical cases. However, it is important to realize that similar to the concrete properties in tension also the response of concrete in compression is sensitive to the concrete mix composition, and that this will be reflected in the overall structural performance.

It is believed that with this verification the validity of the model has been confirmed so that it can be used for further parameter studies. In this way the member size dependence of the rotation capacity of plastic hinges can be analysed in detail for an extended range of structural and material variables. This includes also the possibility to study the effect of the properties of structural materials on the available deformation capacity of plastic hinges.
6.3 Influence of material properties

One of the primary goals of this research is the formulation of requirements for the structural materials, that follow from the demand of structural ductility. The significance of the properties of reinforcing steel has clearly been acknowledged. Various design codes introduce a classification of the steel based on its ductility characteristics. However, not an individual material properties but the total structural performance should be of importance in this respect. Numerous tests have been carried out to study the correlation between the deformation capacity of the structure and the reinforcing steel ductility, and a clear difference in the response of members reinforced with cold worked and hot treated steel types has been found. Yet, it must be remembered that ponder the variety of steel characteristics, it is not possible to arrive at a reliable solution only by evaluating test results. A full understanding of the mechanism of plastic hinging, bond and cracking behaviour is required in order to be able to cope with this problem. Before a parameter study of this problem can be undertaken, it is necessary to support it with a model validation against the results of experimental investigation.

Reference experiments

The test series reported in Chapter 2 and simulated with good agreement, as discussed above, were carried out with a very ductile type of steel. For the validation of the calculation model for rotation capacity in a wider range of steel ductility classes tests reported by Bühler and Eibl (1991) are analyzed. The main intension of these experiments was to study the influence of different steel characteristics on the rotation capacity of reinforced concrete slabs, in particular when low ductility cold worked steel wired meshes are used. The tests were therefore conducted on six single span slabs applied to three point bending. The geometry, structural materials and type of loading were thus representative for a one-way continuous slab arrangement typically used in house construction. Test specimens, that represent the situation between the two points of inflection at both sides of the support, were 800 mm wide, 180 mm thick and had a span of 2000 mm (total length was equal to 2200 mm). The bottom concrete cover was equal to 20 mm so that the effective depth of the cross-section was 160 mm. The reinforcement ratio was chosen such, that it corresponded to the commonly used types of meshes, with both longitudinal bars \( d_y = 8 \text{ mm} \) and transverse bars \( d_y = 6 \text{ mm} \) spaced at 150 mm. This resulted in a reinforcement ratio \( \rho_s = 0.24\% \).

Two types of ribbed and welded wire meshes, two types of ribbed unwelded bars and smooth welded wire mesh were used. The steel ductility characteristics were varied in the tests and, hence, numerous samples were taken from the steel bars and mesh fabric.
to determine the main steel properties in series of independent standard tests performed in three laboratories. Table 6.4 gives an overview of the steel characteristics as determined in the standard test series for all steel types used (test specimens codes given). Considering the scatter in the reported values in some extreme cases both the average for the whole set of standard tests and for a selected single series are given. In the following Section the effect of the scatter of the steel properties on the structural response of the members is discussed, using this results in the numerical simulations.

<table>
<thead>
<tr>
<th>Specimen code</th>
<th>Steel type</th>
<th>$f_y$ [MPa]</th>
<th>$f_t$ [MPa]</th>
<th>$f_t/f_y$</th>
<th>$\epsilon_u$ [%]</th>
<th>$f_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPL1 (average)</td>
<td>ribbed welded mesh</td>
<td>703</td>
<td>732</td>
<td>1.04</td>
<td>2.00</td>
<td>0.052</td>
</tr>
<tr>
<td>RPL1 (series Braunschweig)</td>
<td>ribbed welded mesh</td>
<td>672</td>
<td>716</td>
<td>1.07</td>
<td>2.75</td>
<td>0.052</td>
</tr>
<tr>
<td>RPL2 (average)</td>
<td>ribbed unwelded bars</td>
<td>502</td>
<td>594</td>
<td>1.18</td>
<td>10.4</td>
<td>0.075</td>
</tr>
<tr>
<td>RPL2 (series 8)</td>
<td>ribbed unwelded bars</td>
<td>521</td>
<td>607</td>
<td>1.17</td>
<td>10.4</td>
<td>0.075</td>
</tr>
<tr>
<td>RPL3 (average)</td>
<td>smooth welded mesh</td>
<td>644</td>
<td>692</td>
<td>1.07</td>
<td>2.30</td>
<td>0.000</td>
</tr>
<tr>
<td>RPL4 (average)</td>
<td>ribbed welded mesh</td>
<td>590</td>
<td>629</td>
<td>1.07</td>
<td>4.40</td>
<td>0.056</td>
</tr>
<tr>
<td>RPL4 (series 2)</td>
<td>ribbed welded mesh</td>
<td>585</td>
<td>626</td>
<td>1.07</td>
<td>4.50</td>
<td>0.056</td>
</tr>
<tr>
<td>RPL5 (average)</td>
<td>ribbed welded mesh</td>
<td>590</td>
<td>629</td>
<td>1.07</td>
<td>4.40</td>
<td>0.056</td>
</tr>
<tr>
<td>RPL5 (series 6)</td>
<td>ribbed welded mesh</td>
<td>590</td>
<td>636</td>
<td>1.08</td>
<td>5.20</td>
<td>0.056</td>
</tr>
<tr>
<td>RPL6 (average)</td>
<td>ribbed welded mesh</td>
<td>532</td>
<td>612</td>
<td>1.15</td>
<td>4.30</td>
<td>0.066</td>
</tr>
</tbody>
</table>

where:

- $d_r$ - bar diameter
- $\epsilon_u$ - elongation at maximum stress
- $f_y$ - yield strength
- $f_t$ - ultimate strength
- $f_R$ - projected rib factor

The concrete mix was designed to satisfy the requirements of a B25 concrete (the average cube compressive strength obtained for the whole test series was $f_{cc} = 33$ MPa). The slabs were cast with the reinforcement placed near the top face in order to simulate the conditions of the part of the slab located over the support (top bar effect). It was attempted to obtain a lower bound for the rotation capacity by localizing the crack at a position in the welded mesh fabric where local defects are likely to occur as a result of the welding process. Therefore a sheet-metal strip was used to fix the first crack in the middle section of the slab at the location of a transverse bar.

The test specimens were instrumented with a number of strain gauges and displacement transducers (LVDT) to obtain all important values. In order to localize the observed rotation, strains on the top and on the bottom face of the slab were measured in two rows, using electrical resistance strain gauges, and with two rows of LVDT's
respectively. The measuring length, both for the strain gauges and the LVDT’s was reduced from 90 mm to 60 mm in the mid-zone of the member where the hinge was expected. The curvature was obtained directly from the local strain and elongation measurements and the rotation was calculated by integrating the calculated curvature along the member axis.

Simulations

A procedure identical to that described in Section 6.1 was used to simulate the above-mentioned tests. Considering the negligible scatter on the concrete strength for the whole test series the same value of the cube compressive strength $f_{cc} = 33$ MPa was taken. Using this as a basic value, a tensile strength of $f_{ct} = 2.71$ MPa, an elastic modulus of $E_c = 28547$ MPa and a fracture energy of $G_f = 105.7$ N/m were obtained. Contrary to the concrete, a significant scatter characterized the properties of the reinforcing steel in some cases. This was accounted for in the simulations by repeating the calculations, taking as the input both the average steel properties as obtained in the series of standard tests and the characteristics that follow from a single selected test of standard measurements. It was intended to show in this way the effect of inherent variation of material properties on the member response. In Table 6.5 the points defining the constitutive steel law are given, as used in the simulations of all tested members (in the steel identification the letters a and s denote the average characteristics from all standard tests and from the selected series, respectively). Considering the fact that the calculation model is suitable only for analysing members with ribbed reinforcement, test RPL 3 was not included in this verification.

<table>
<thead>
<tr>
<th>Steel</th>
<th>$\sigma_1 = f_c$ [MPa]</th>
<th>$\varepsilon_1 = \varepsilon_y$ [%]</th>
<th>$\sigma_2$ [MPa]</th>
<th>$\varepsilon_2$ [%]</th>
<th>$\sigma_3$ [MPa]</th>
<th>$\varepsilon_3$ [%]</th>
<th>$\sigma_4$ [MPa]</th>
<th>$\varepsilon_4$ [%]</th>
<th>$\sigma_5 = f_t$ [MPa]</th>
<th>$\varepsilon_5 = \varepsilon_u$ [%]</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
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<td>RPL1a</td>
<td>703</td>
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<td>715</td>
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<td>725</td>
<td>1.00</td>
<td>730</td>
<td>1.50</td>
<td>732</td>
<td>2.00</td>
<td>0.0030</td>
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<td>RPL1s</td>
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<td>690</td>
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<td>1.00</td>
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<td>716</td>
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</tr>
<tr>
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<td>530</td>
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<td>560</td>
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<td>580</td>
<td>7.50</td>
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</tr>
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<td>570</td>
<td>2.00</td>
<td>595</td>
<td>4.50</td>
<td>602</td>
<td>7.00</td>
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<td>10.40</td>
<td>0.0362</td>
</tr>
<tr>
<td>RPL4a</td>
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<td>605</td>
<td>0.40</td>
<td>615</td>
<td>1.00</td>
<td>625</td>
<td>2.50</td>
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<td>4.40</td>
<td>0.0083</td>
</tr>
<tr>
<td>RPL4s</td>
<td>585</td>
<td>0.29</td>
<td>605</td>
<td>0.40</td>
<td>615</td>
<td>1.00</td>
<td>620</td>
<td>2.50</td>
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<td>4.50</td>
<td>0.0089</td>
</tr>
<tr>
<td>RPL5a</td>
<td>590</td>
<td>0.30</td>
<td>605</td>
<td>0.40</td>
<td>615</td>
<td>1.00</td>
<td>625</td>
<td>2.50</td>
<td>629</td>
<td>4.40</td>
<td>0.0083</td>
</tr>
<tr>
<td>RPL5s</td>
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</table>

The results of the simulations are compared with the reported experimental values. Rotations at characteristic load levels (at the onset of steel yielding $\Theta^{(\psi)}$ and at maximum load $\Theta^{(\psi)}$) and the rotation capacity $\Theta^{(\psi)}$ are given in Table 6.6, both measured and calculated for the average steel characteristics $a$ and for the selected standard tests series $s$. Considering the imposed crack pattern only the lower bound simulation results are listed. In interpreting the simulation results it must be remembered that in a number of cases the simulated members were reinforced with welded wire mesh (RPL1, RPL4, RPL5). The bond model used here does not fully account for this type of reinforcement, since it considers only bonding of longitudinal bars. However, while for ordinary straight reinforcing bars the anchorage is achieved by pure bond action along the embedded bar length, for other types of reinforcement bond action of longitudinal bars can be combined with mechanical action of other anchorage elements (i.e. hooks, loops, transverse reinforcement). Welded transverse bars are very effective in this respect, as it was proven in tests where the anchorage length was investigated as a function of method of anchorage (Martin and Schießl 1970, Schießl 1982). The increase of the bond capacity was found to be dependent on the diameter ratio and spacing of the transverse bars. Considering that in all tests by Bühler and Eibl (1991) that are discussed here the same type of meshes has been used, the magnitude of the anchorage improvement due to the welded transverse bars action is likely to be the same.

With respect to the modelling of bond in these simulations, one more remark must be made. The bond of the longitudinal bars is directly related to the bar’s geometry. In the developed bond model the variation in bar geometry, e.g. rib form and rib factor, is not accounted for, though it has been established that the bar deformations pattern influences bond characteristics to some extent. Numerous experiments carried out in this field prove that an increase of the related rib area $f_R$ causes the improvement of bond strength.
combined with an increase of bond stiffness in the initial stage and a stronger splitting action (Martin 1973, Martin and Noakowsk 1981). The proposed bond model has been tuned in accordance to the test results obtained using bars with medium relative rib area ($f_R \approx 0.07$), which is more than in the case of the welded mesh used in the tests by Bühler and Eibl (1991). Therefore, a decrease of bond strength and a lower bond stiffness would be expected for the longitudinal bars in these cases. Yet, it is believed that in combination with the influence of the transverse bars the total bond capacity is in acceptable agreement with the applied estimate.

**Figure 6.7 Comparison of measured and calculated values of rotation at yielding and at maximum load**

In Fig. 6.7 the measured and the calculated values of the rotation at yielding and at maximum load are shown, the latter obtained in calculations where the average steel
properties were used as input data. In Fig. 6.8, in which the values for the rotation capacity are compared, the variation found in the simulations due to the change in the steel characteristics is indicated. It must be remembered that still not the single extreme standard test results, but the average values over the selected test series served the determination of the steel properties. Although the differences are not of an excessive magnitude, they clearly show the sensitivity of the structural response to changes in the specific material characteristics. The agreement is satisfactory, thus showing the capability of the model to take account of the steel characteristics. The tendencies found in the simulations are confirmed by the experimental evidence. It is believed that in this manner the capability of the calculation model for rotation capacity to represent the effect of steel characteristics on the deformation capacity of reinforced concrete members is fully proven.

Both test results and simulations show that the steel properties are directly related to the deformation capacity of the members. A decrease in ductility of the steel bars becomes manifest in a reduction of the rotation capacity at maximum load. However, a comparison of the ratio of the rotation capacity reduction with the ratio of the reduction of the ultimate steel strain shows that it is misleading to focus on isolated steel characteristics: a decrease of the ultimate steel strain with a factor of more than two does not result in a rotation capacity reduction of the same ratio (compare RPL1 - RPL4, 5, 6 - RPL2). It confirms that steel properties must not be judged in isolated terms, but according to the deformation capacity of the reinforced members, which represents an overall assessment of their structural performance. To quantify this property a combination parameter is required, for instance as approached in Section 3.4.

In Fig. 6.9 the relation between the equivalent steel parameter $p$, given by equation 3.40, and the calculated rotation capacity at maximum load is plotted for all cases simulated. There is a clear proportionality between the value of the equivalent steel parameter $p$ and the computed rotations. This tendency evidently proves that not the absolute difference in the values of the individual steel characteristics, but its combination is crucial for the attained deformation capacity of a member. A small variation of a single property may in some cases mean a substantial change for the value of the overall ductility parameter, and thus for the performance delivered. Test results confirm this finding. The magnitude of the variation of the rotation capacity found in the tests can clearly be coupled with the overall ductility measure $p$, that characterizes reinforcing steel. Fig. 6.10 shows the relation between the measured rotation capacity and the equivalent steel parameter $p$ for the whole test series. Here the whole range of $p$ values, found in the standard tests is indicated, with the two values for which simulations were conducted being marked.
A huge scatter in the steel ductility indicator is obviously one of the explanations for the scatter in the test results (compare e.g. RPL4 and RPL5). Furthermore, it is clear why even a minor adjustment of the steel characteristics used for the simulations significantly influences the outcomes. In the experiments, however, it is by no means possible to avoid this natural variation of reinforcing steel properties. Therefore parameter studies are likely to be the best way to determine the relation between the reinforcing steel characteristics and the ductility of reinforced concrete members. Such an approach allows to focus on one isolated question only and in this way the scatter due to simultaneous variation of more parameters, inherent to any experimental investigation,
can be avoided.
7 PARAMETER STUDIES

7.1 Member size dependence

The initial tests and the analytical study allow to identify the main parameters which influence the phenomena studied. Once the behavioural laws regarding the main parameters had been formulated by means of suitable models, they were implemented into a specifically-conceived calculation model. In order to validate this model, results were used obtained in laboratory tests, in which conditions assumed in the model were recreated and selected influences were studied. By numerical modelling and parameter studies the importance of various factors can now be evaluated and further insight into the structural behaviour can be gained. Finally, the analysis may be extended to different structural conditions and design problems.

One of the design problems that needs a further clarification is the member size dependence of the rotation capacity of plastic hinges. For a long time, the size effect has been explained statistically as a consequence of the randomness of material strength, particularly by the fact that in a larger structure it is more likely to encounter a material point of smaller strength (e.g. Mihashi and Zaitsev 1981, Mihashi 1983). Later, however it has been demonstrated that the structural size effect is a consequence of cracking. Hence, it has been postulated that the size effect can properly be explained by energy release due to fracture growth, producing damage localisation instabilities and that the randomness of material fracture properties plays only a negligible role in setting the size effect (e.g. Bazant 1984, Bazant and Xi 1991). However, contrary to the single mode-I crack growth in plain concrete, the most reinforced concrete structures exhibit complex cracking and instead of one single crack a system of cracks must be considered. Their existence and growth strongly depend on the fracture properties of the construction materials, as well as on the structure geometry and the reinforcement area and detailing.

Consequently, interpreting the size effect in terms of system toughness is a necessary step that must be taken when reinforced concrete members are analysed. In reinforced concrete a brittle and a ductile material, concrete and steel, are inherently combined and both deliver their contribution to the structural performance. If the toughness of a reinforced structure is expressed by the quotient of the energy needed to fracture the structure and the stored elastic energy (Elfgren 1989), the fracture characteristics of both materials, the structural dimensions and the stiffness, related to bond characteristics and reinforcement ratio, will prove to be influential. Such a large number of independent influential parameters causes a difficulty in formulating a simple member size dependence rule for reinforced concrete members. The purpose of this parameter study
is thus only to evaluate the importance of the size effect in practical design situations, and in particular to determine whether justification exists for alteration of existing design rules in the light of a possible member size dependence of the rotation capacity of plastic hinges. It should be emphasised that only single reinforced slender members without shear and confining reinforcement are analysed. This study is also limited to normal strength concrete, considering that due to a lack of experimental evidence the calculation model for rotation capacity could not be verified for concretes of high strengths.

7.1.1 Parameters choice

It is acknowledged that a number of parameters will be important for the magnitude of member size dependence of rotation capacity. In this study, however, some limitations apply. The concrete strength is not included as a parameter in the analysis and only cases where normal strength concrete with a compressive strength $f_{cc} = 35$ MPa is used are examined. For this type of concrete the following properties apply: tensile strength $f_{ct} = 2.80$ MPa, modulus of elasticity $E_c = 29055$ MPa and fracture energy $G_f = 109.2$ N/m. The values of the CDM parameters, involved in the modelling of concrete under compression, are taken equal to these of a concrete mix with a maximum aggregate size $d_{a, max} = 16$ mm (see Section 3.3.2).

Reinforcing steel characteristics are likely to have a minor influence on the magnitude of the size effect. Therefore less attention is given to this parameter, and most of the simulations are performed with one type of hot rolled steel, called A in the following. To give an indication of the range of variation that can be introduced when varying individual steel characteristics, four more steel types are included in the limited series of simulation. Steel types A1 and A2 are identified by the same value of the overall ductility parameter $p$ as steel A, but have quite different characteristics in terms of stress and strain relation. In particular the ratio between yield stress and tensile strength $f_y / f_{ct}$ amounts to 1.08, 1.06 and 1.12 whereas the ultimate steel strain $\varepsilon_u$ reaches 5.0%, 7.0% and 3.0% for steel types A, A1 and A2, respectively. To show the effect that yield strength variation can have on the rotation capacity and the magnitude of member size dependence two more steel types are used in the simulations, namely steel A3 and A4. These steel types are characterized by the same value of the parameter $p$, have the same hardening ratio and ultimate strain as steel A, however the yield stress ranges from 500 MPa for steel A3 to 600 MPa for steel A4. The data points defining the constitutive relations used in the simulation for all these steel types are given in Table 6.1. The value of the parameter $p$ is determined according to equation 3.41.
The major parameters in this analysis are related to the geometry of the members. The effect of a proportional increase of the member dimensions (effective height $h$, width $b$, span $l$) while keeping the reinforcement ratio $\rho_s$ constant is studied. The simulations are performed for one slenderness ratio $l/h = 12$, for five $h$ values (varying from 100 to 1200 mm), and for three reinforcement ratios $\rho_s$ (0.25%, 0.50% and 1.00%). With respect to the latter, the difficulty in scaling the members, as encountered in practice with regard to the detailing of the reinforcement, is minded. Numerous possibilities to achieve the same reinforcement ratio with different number of bars - and thus different bonded area - ask for a separate analysis of the effect of reinforcement layout on the deformation capacity of the member. It is well known that in general increasing the number of bars while decreasing the single bar diameter causes a more dense crack spacing and smaller crack widths at the serviceability limit state. It is however important to know to what extend the ultimate deformation capacity will be influenced and what impact it will have on magnitude of the member size dependence. Different reinforcement layouts were studied in combination with different member sizes in order to find the answer to this last question. The following has been chosen:

I. constant bar diameter $d_s = 16$ mm, bar spacing adjusted to the actual member size and to the required reinforcement ratio

II. constant bar spacing $s_s = 100$ mm, bar diameter adjusted to the actual member size and to the required reinforcement ratio

III. constant bar spacing $s_s = 50$ mm, bar diameter adjusted to the actual member size and to the required reinforcement ratio

IV. constant bar diameter $d_s = 8$ mm, bar spacing adjusted to the actual member size and to the required reinforcement ratio

In order to limit the number of geometry-dependent variables, the bottom cover on the bars was in all cases kept equal to 2 $d_s$. 

---

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7.1.2 Simulation results

In this Section the basic findings from the numerical simulations are presented. In Fig. 7.1 - 7.3 the effect of member size on the rotation at maximum load and on the rotation capacity is illustrated. The calculated rotation values are plotted as a function of the effective height of the member. In all cases both the lower and the upper bound values are shown, that follow from assuming different critical crack patterns in the simulations (discussed in Chapter 3). The actual layout of the reinforcement is indicated. For the purpose of general comparison essentially only the layouts I, II and III are considered.

Figure 7.1 Rotation at the onset of yielding of the reinforcement and at maximum load (top and centre) and rotation capacity (bottom) as a function of effective member height - simulations for $\rho_s = 0.25\%$ and reinforcement layouts I-IV.
The simulations prove the empirical observation that the member size influences the deformation capacity. Both rotation at maximum load and rotation capacity decrease with increasing member size, while the rotation at the onset of yielding of reinforcement is almost size independent. The related to effective member height decrease of the rotation at maximum load and of the rotation capacity is fast for small members and tends to be slower as the dimensions of the members increase, which agrees with experimental evidence. The reinforcement ratio has an obvious effect on reached rotation values. For the combination of parameters studied in these simulations, failure is governed by concrete crushing for $\rho_s = 0.50\%$ and $1.00\%$, and by steel rupture for $\rho_s = 0.25\%$.

*Figure 7.2* Rotation at the onset of yielding of the reinforcement and at maximum load (top and centre) and rotation capacity (bottom) as a function of effective member height - simulations for $\rho_s = 0.50\%$ and reinforcement layouts I-III.
Here attention is payed to the important differences found in the magnitude of the size effect in different cases. There is no meaningful distinction between the intensity of the size effect observed for the reinforcement ratios 0.25% and 0.50%, although the effect is slightly less pronounced for the lower of the two ratios. This slight difference it is not very surprising, ponder the fact that for a lower reinforcement ratio, in the case of steel rupture, the concrete in compression will be less exhausted, so that its contribution to the member size dependence will be effectively reduced. This corresponds well with the theories that explain member size dependence of the rotation capacity through size dependence of the concrete response in compression.

Figure 7.3 Rotation at the onset of yielding of the reinforcement and at maximum load (top and centre) and rotation capacity (bottom) as a function of effective member height - simulations for $\rho_s = 1.00\%$ and reinforcement layouts I-III
The comparison of the two cases where this concrete compressive failure is critical is therefore very interesting. It is found that the size effect is clearly stronger for 0.50% reinforcement and is much less pronounced, yet still visible for a reinforcement ratio of 1.00%. This is an important conclusion, since it points at the effect that the deformation of the tensile cord of the member can have on the size dependence of the overall member response. While in the case of $\rho_s = 1.00\%$ concrete failure takes place at small steel elongations, this is not any more the case for $\rho_s = 0.50\%$, where almost all deformation capacity of the steel is consumed at the failure stage.

Because of requirements of compatibility and equilibrium, the steel elongations are governing for the depth of the compression zone and the corresponding concrete deformations. Considering the effect of strain localization in concrete in compression, it can be understood that a decrease of the depth of the compression zone must result in an active increase of the member size dependence. In this sense, the deformations of the tensile cord enhance the effect of strain localization in the concrete compression zone on the overall size dependence of the plastic hinge behaviour. It should be remembered that general tensile cord deformations are, on their own, sensitive to a number of size and geometry dependent factors. There is bar size and effective concrete cover thickness dependence involved in setting the bond between steel and concrete. Furthermore, there is a bar contraction effect on the bond strength reduction in cases where the pull-out bond failure mode prevails. Finally - a crack spacing, related to bond strength, bonded area and tensile cord reinforcement ratio, effects the overall tensile cord deformations.

The importance of the bond dependent size effect can be evaluated comparing the results of the simulations obtained for different reinforcement layouts. In the case of the lowest reinforcement ratio 0.25%, the influence of the reinforcement detailing is found to be very pronounced. A specific choice of the reinforcement layout can nearly eliminate the size dependence of the rotation capacity, if understood as a relation to the effective member depth. Here it must be stressed that this does not mean, that the size effects are eliminated from the plastic hinge performance, but they rather are superimposed in such a way that the overall member response is finally affected by a mixed type of size effect. This can be clearly seen when in Fig. 7.1 the relations corresponding to the layouts I and IV or II and III are compared. In the first case the reduction of bar diameter and doubling of bar number results in a decrease of the rotation capacity and a reduction of the degree of size dependence. A similar effect is observed in the second case, where again the bar number is doubled and the bar diameter reduced, but with a different initial set of values. Therefore, the major conclusion is that when speaking about member size dependence of reinforced concrete members not only the overall member dimensions must be regarded but the reinforcement configuration as well.
Figure 7.4 Degree of size dependence determined for a reference member size $h = 200$ mm as a function of the effective member height (upper and lower bounds shown)

Figure 7.5 Degree of size dependence determined for a reference member size $h = 400$ mm as a function of the effective member height (upper and lower bounds shown)

From a design point of view it is important to know to what extent member size dependence may manifest in usual engineering practice. Here an attempt is made to answer this question using a simple statistical evaluation of all simulations results obtained. For this purpose a degree of size dependence is defined, which is the ratio between the actual member rotation capacity and a reference value, defined below. In design no account is taken of the influence that the reinforcement detailing can have on the available member rotation. Of all geometry-related factors only the reinforcement
ratio is accounted for. Therefore here, for each reinforcement ratio the representative reference value is determined, being a mean value of the average rotation capacity predicted for all members with the reference size, despite the reinforcement layout used. Then, relating the determined degree of size dependence to the member effective height information is obtained about the possible spread of size sensitivity when changing the reinforcement layout within the investigated limits.

Now the following consideration is made: members are classified as hazardously sensitive to size effect if the size dependence degree is lower than 1. In this way it is considered that attaining a higher rotation capacity than predicted is a desired situation, conservative in the sense of design safety. In Fig. 7.4 and 7.5 it is shown how the degree of size dependence varies when the size of the reference member varies. In both figures the values for the upper and the lower bound solutions are used to determine the feasible range of variation of the degree of size dependence in relation to the developed crack pattern. Two reference members are used, namely these with effective depth $h = 200$ and $400$ mm.

It is essential to compare the extreme values of the degree of size dependence obtained in both cases. When taking the reference member size $h = 200$ mm, the minimum values of the degree of size dependence reach 0.54 (for the lower bound solution) and 0.58 (for the upper bound solution), while the maximum values range from 1.66 (for the lower bound solution) and 3.32 (for the upper bound solution). For the reference member size $h = 400$ mm the range that holds a size dependence degree lower than 1 is smaller and the minimum values of the degree of size dependence are 0.78 (for the lower bound solution) and 0.84 (for the upper bound solution). On the contrary, the range where the values are higher than 1 is larger and maximum size dependence degrees as high as 2.31 (for the lower bound solution) and 4.54 (for the upper bound solution) are found. The practical conclusion from this evaluation is that, when the estimate of available rotation capacity is based on the evaluation of the behaviour of a single reference member, an overestimation of the actual member rotation is possible. It can amount up to about 20% if $h = 400$ mm is used or up to almost 50% if $h = 200$ mm is taken as the reference value. Here it must be stressed that this conclusion follows from the evaluation of a limited number of cases, and does not cover all the extremes e.g. with respect to the reinforcement layout. In any case, however, it is clear that taking a larger member size as a reference for further parameter studies is more conservative, c.q. a more safe approach. Therefore fixing the reference member size at $h = 400$ mm seems a justified choice, considering the type of members studied (slabs with no shear reinforcement and slender beams with light confinement) and the dimension limits following from the range of their application.
It has been already stressed that not only the overall member dimensions but also the way of arranging the reinforcement strongly influences the deformation capacity of the member. Therefore, in the next step it is attempted to show a way to include the reinforcement detailing in analysing the member sensitivity to size dependence. To that end a number of geometry characteristics is examined that are likely to be directly influential. Here the proportion between the reinforcement ratio of the tensile cord and the reinforcement ratio of the member is considered, meant to involve the effect of non-uniformly distributed reinforcement on the cracking behaviour of members with different sizes, reinforcement ratios and layouts. Furthermore the effect of the concrete confinement on the bond strength is involved by incorporating the ratio between the effective concrete cover and the bar diameter. With these components the geometry dependent parameter \( g \) is defined as follows:

\[
g = \frac{\rho_{s,ef}}{\rho_s} \frac{c_{eff}}{d_s}
\]  

(7.1)

where \( \rho_{s,ef} = A_s / A_{c,ef} \) is the ratio between the cross-sectional area of steel and concrete effective in tension (member size dependence included, \( A_{c,ef} \) determined according to equation 5.16)

- \( \rho_s \) is the reinforcement ratio referred to the member cross-section
- \( c_{eff} \) is the effective concrete cover, determined according to equation 4.41
- \( d_s \) is the bar diameter

Although very simplistic, this method allows to compare members with different dimensions and detailing in order to judge the influence that the choice of reinforcement layout can have on their size dependence sensitivity. In Fig. 7.6 and 7.7 the relation is shown between the proposed parameter \( g \) and the degree of size dependence, determined as in the preceding evaluation for the same set of simulation results. Here only the average of the values obtained using the upper and the lower bound solutions are plotted. Again two reference member sizes are used, namely \( h = 200 \) and \( 400 \) mm. Although no general size effect law which could cover all practical situations can be proposed, it is clear that in both cases with decreasing values of the parameter \( g \), the degree of size dependence increases, i.e. the actual rotation capacity raises with respect to that of the reference member. The bigger the reference member size the larger the range of \( g \) values that satisfied the safety criteria of size dependence degree > 1. Hence, also from this comparison the choice of larger member size to reduce the range of members potentially vulnerable to size effect proves justified.
More attention deserves the effect of reinforcement layout on the size dependence sensitivity of members with similar dimensions and reinforcement ratio. This is better visible in Fig. 7.8, where for three reinforcement ratios analysed here the range of rotation sensitivity possibly attained is plotted as a function of the parameter $g$, with the actual member effective height indicated (similar as in the Fig. 7.4 and 7.5 the lower and the upper bound solution values are used to determine the critical degree of size
dependence). Although member dimensions are likely to be a dominant factor governing the size dependence of the rotation capacity, it is obviously not the only one important parameter. The detailing of reinforcement, represented here by the parameter $g$ can increase the degree of member size dependence, e.g. reduce its negative consequences for the design.

![Diagram](image)

**Figure 7.8** Degree of size dependence determined for the reference member size $h = 400$ mm as a function of the geometry dependent parameter $g$ for reinforcement ratios $1.00\%$ (top left), $0.50\%$ (top right) and $0.25\%$ (bottom) (upper and lower bound solutions shown).

This leads to the conclusion that an optimisation of reinforcement layout from the point of view of the potential member size dependence must be possible. For each of the reinforcement ratios studied a range for the parameter $g$ can be determined that results in acceptable (moderate) values for the degree of size dependence. For the range of variables studied here it is justified to conclude that only if the $g$ values exceed $\sim 4$ or fall below $\sim 2$ a strong member size dependence may manifest. Once again it is remarked that, due to the size dependent behaviour of concrete in the compression zone and the degree of its exhaustion, connected to the reinforcement ratio, the intensity of the size
dependence related to this phenomenon may strongly differ. In practice one should however be aware of the fact that apart from the unavoidable size effect on the reinforced members deformation capacity luckily a lot depends on the designer's choice.

![Image](image.png)

**Figure 7.9** Rotation capacity as a function of effective height of the member - simulations for reinforcement ratio 0.25% for steel types A, A1 and A2

![Image](image.png)

**Figure 7.10** Rotation capacity as a function of effective height of the member - simulations for reinforcement ratio 0.25% for steel types A, A3 and A4

In the last step the effect of steel characteristics on the member size dependence is shortly evaluated. Within this parameter study a limited series of simulations is conducted varying the reinforcing steel characteristics. As described in the preceding Section, a large difference in individual characteristics typify the steel types A, A1 and A2, while the overall ductility parameter $p$ is kept constant. The results of the simulations performed for these steel types are presented in Fig. 7.9, where the rotation capacity (both upper and lower bound solutions) is shown as a function of the effective
member height. The rotation capacity predicted varies considerably for these three steel types. The degree of member size dependence obtained is not very much influenced, however. Its magnitude seems to be related to the absolute value of the rotation capacity and slightly increases with increasing rotation capacity. The explanation for this tendency in the case of members with low reinforcement ratio (0.25%) that fail due to steel rupture is likely to be found both in the concrete compressive strain mobilised at the ultimate limit state that follows from the equilibrium conditions, and in the differences that arise in the deformation of the tensile cord due to the huge differences in the ultimate steel strain for these three types of steel.

The results of simulations where steel types A3 and A4 are used are very important from a practical point of view, considering that in reality a considerable scatter is observed with respect to the yield stress values, and usually much higher values than guaranteed are reported. In Fig. 7.10 the relations between the rotation capacity and the member size predicted for these steel types are compared with the estimated ones, obtained for steel A (simulation for reinforcement ratio 0.25%). Differences in yield strength by as much as 100 MPa are likely to have no meaningful influence on members rotation capacity, although the rotation at the onset of yielding and at the maximum load increase with increasing yield stress value, as expected. Also the magnitude of size dependence is rather nonsensitive to this parameter.

The relative influences of reinforcing steel properties on crack width, tensile cord deformation and member deformation capacity are discussed in more detail in Section 7.2. With respect to the size dependence of the rotation capacity it is believed that if the member reference size is kept relatively high no additional risk is introduced by the change of steel properties with respect to this phenomenon that would essentially influence the safety of the estimation.

7.2 Influence of material properties

7.2.1 Crack width development

Structural materials properties on their own are not a topic for structural engineers but rather for material scientists. However, as soon as they are placed in the perspective of structural performance they become a fundamental civil engineering issue. In particular reinforcement properties are frequently brought to the attention in connection with the deformation capacity of reinforced members. Some of the aspects of the structural behaviour, where the influence of reinforcing steel properties manifests, are discussed here.
At first, attention is given to the development of the crack width in relation to certain steel characteristics in the post yield range of the steel strains. Two examples of the numerical simulation are given where for a constant crack spacing of 150 mm the crack width development and the steel strain distribution between the cracks are studied, for different reinforcing steel characteristics. The calculations are performed for a concrete strength $f_{cc} = 35$ MPa. Significantly different steel types are used, namely steel 1 with a distinct yield plateau ($\varepsilon_{sh} = 2.75\%$) and a high strain hardening ratio $f_y / f_y = 1.15$, and an almost elasto-plastic steel 2 type, with a hardening ratio of 1.03 (details in Table 6.2). To focus the analysis on the effect of steel properties only, all other influences, such as cone pull-out or splitting cracking are excluded by taking a large concrete cover ($c_{eff} = 5d_s = 80$ mm) and assuming a high restraint at both edges of the simulated cracked element ($\beta = 90^\circ$). In Fig. 7.11 the computed results are presented in terms of steel strain versus location diagrams, while in Fig. 7.12 the crack width versus the steel strain at the crack is plotted.

<table>
<thead>
<tr>
<th>Steel code</th>
<th>$\sigma_1 = f_y$ [MPa]</th>
<th>$\varepsilon_1 = \varepsilon_y$ [%]</th>
<th>$\sigma_2$ [MPa]</th>
<th>$\varepsilon_2$ [%]</th>
<th>$\sigma_3$ [MPa]</th>
<th>$\varepsilon_3$ [%]</th>
<th>$\sigma_4 = f_y$ [MPa]</th>
<th>$\varepsilon_4 = \varepsilon_u$ [%]</th>
<th>$p$</th>
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</thead>
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<td>553</td>
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<td>8.00</td>
<td>630</td>
<td>12.50</td>
<td>0.0542</td>
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<td>steel 2</td>
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<td>0.28</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>566</td>
<td>12.50</td>
<td>0.0087</td>
</tr>
</tbody>
</table>

The combination of both charts is very useful, since it directly points at the reasons for the dissimilarities found. As long as the steel strains remain low no significant difference is observed both with respect to steel strain distribution and crack width development. As soon as the steel strains exceed the strain hardening criterion for steel 1 important differences are observed in the steel strain distribution curves predicted for both steel types. In the case of steel 1 due to the increased load carrying capacity in the steel hardening range an accelerated shift of the yield front towards mid-length takes place. This results in an increase of the length of the highly stressed bar that contributes to the total slip, compared to the case of steel 2, where no accelerated yield penetration is observed. With increasing steel strain at the crack also in the case of the yield front for steel 2 a gradual propagation along the bar takes place, and yet, even for the extremely high steel strains at the crack, the mid part of the bar remains in the elastic stage. This is not the case for steel 1, where already at the half of the assumed steel strain capacity the whole bar is yielding.

In terms of crack widths, differences in crack width versus steel strain relations occur directly after the steel hardening range has been reached in the case of steel 1. The
differences arise fast as long as steel 1 remains in the strain hardening range and slightly less intense as soon as the top plateau of the stress-strain diagram of steel 1 is reached. Although at the very advanced stages the crack widths computed in both cases tend to approach one another, still at a level $\varepsilon = 10\%$ the relation between the attained crack openings is about $1.5 : 1$ (steel 1 : steel 2). However, this ratio can be as high as $3 : 1$ at a strain level of $\varepsilon = 5\%$. The latter is a meaningful observation, ponder that in practice reinforcing steel with the characteristics of the steel 2 type hardly ever permit higher strains to be developed.

![Graphs showing steel strain distribution and crack width](image)

**Figure 7.11** Steel strain distribution along the embedded bar between two cracks for crack spacing 150 mm, simulated for steel 1 (left) and steel 2 (right)

![Graphs showing crack opening versus steel strain](image)

**Figure 7.12** Crack opening versus steel strain at the crack, simulated for crack spacing 150 mm for steel 1 (left) and steel 2 (right)

Although very simplistic, this comparison indicates which consequences for the deformation capacity of the tensile cord can arise when steel with a low hardening ratio
is used as reinforcement. The numbers given above strengthen, what already has been said, that not only the ultimate strain values but also other characteristics need to be considered when analysing the change of deformation capacity of reinforced structures due to the change in the reinforcing steel behaviour. This topic is the theme of the following Section.

7.2.2 Available rotation capacity

It has been discussed in the preceding Chapters that a number of parameters is involved in limiting the ductility of plastic hinges in reinforced concrete members, both of the material and the geometrical type. The influence of all these factors can be independently studied. In this Section the correlation between the properties of the reinforcing steel and the attained rotation capacity is analysed. Some tendencies can be deduced from the reviewed series of simulations, that could help to clarify the relative importance of the variables depending on the steel quality. Yet still, it is necessary to value these effects in terms of available rotation capacity. There is commonly a fairly strong correlation between steel characteristics, so that the relative influences of each are not easy to perceive in an experimental way. This can best be achieved on the basis of a systematic parameter study.

(1) Parameters choice

A primary question, that needs to be answered when designing a parameter study, concerns the final goal of such an analysis. Here it is aimed at determining the precise limits related to steel quality and structural performance, that need to be introduced into codes of practice to guarantee safe design. This intention has direct consequences for the choice of initial parameters. Following the preceding analysis, the effect of member dimensions and reinforcement detailing is acknowledged. Since in this analysis member dimensions are not going to be varied, a relatively large member size (effective height \( h = 400 \text{ mm} \)) is chosen as the basis. In this way a possible reduction of the rotation capacity attributed to size dependence and a stronger sensitivity of smaller members to reinforcement detailing can be accounted for, which is in agreement with the final conclusions formulated in Section 7.1. Here the reinforcement layout defined as II in Section 7.1 is implemented (constant bar spacing \( s_y = 100 \text{ mm} \), bar diameter adjusted to the actual member size and to the required reinforcement ratio). The ratio between span and effective member height \( l / h = 12 \) is applied, though the member slenderness effect on the rotation capacity is studied in a separate series of simulations, reported in Chapter 8. The analysis is performed for reinforcement ratios ranging from a minimum value of 0.15% to a maximum of 1.75%, which guarantees that for all combinations of
materials considered the reinforcement yields before failure occurs.

The main variables in this series of simulations are the reinforcing steel characteristics. As basic steel types the ones defined in the *CEB-FIP Model Code 1990* are chosen, denoted in the following as A, S, and B. Out of the three, the first two are of the hot rolled steel type with a distinct yield plateau and hardening branch, whereas the last one is a cold worked steel with no apparent yield limit and low hardening ratio. For the hot rolled steel the following characteristics are varied (range of variation indicated in the parenthesis): hardening ratio $f'_y / f_y$ (1.05 ÷ 1.40), strain at the onset of steel hardening $\varepsilon_{sh}$ (0.30% ÷ 3.00%) and ultimate strain $\varepsilon_u$ (3.00% ÷ 7.00%).

<table>
<thead>
<tr>
<th>$f'_y / f_y$</th>
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<th>1.055</th>
<th>1.073</th>
<th>1.08</th>
<th>1.087</th>
<th>1.095</th>
<th>1.15</th>
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<th>1.40</th>
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</table>

In Table 7.3 an overview of the actual combinations of steel characteristics used in the simulations is given and the steel codes are indicated. Here also the values for the equivalent steel parameter $p$ are listed, calculated according to equation 3.41. For the steel types used in these simulations $p$ varies from 0.0120 for A5 to 0.0837 for A12. A specification of data points defining the constitutive relations of all hot rolled steel types analysed is given in Table 7.5.
For cold worked steel a similar procedure is followed. Here the characteristic steel parameters are kept within the following range: the hardening ratio $f_y / f_y$ varies from 1.03 to 1.08) and the ultimate strain $\varepsilon_u$ from 2.00% to 5.00%. This results in a variation of the equivalent steel parameter $p$ for the steel types used in the simulations from 0.0034 for B4 to 0.0123 for B1 (values calculated according to equation 3.40).

<table>
<thead>
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<th>$\varepsilon_u$ [%]</th>
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<th>1.05</th>
<th>1.054</th>
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</table>

$\varepsilon_{sh} = \varepsilon_y$

$\varepsilon_y = 550$ MPa

An overview of all combinations of cold worked steel characteristics considered is given in Table 7.4 and the details of the constitutive relations employed follow from Table 7.6. Considering the minor effect of the yield stress level on the rotation capacity, as explicitly shown in the preceding Section, where the results obtained for steel A3, A and A4 were compared, in this series of simulations a constant yield stress value of $f_y = 550$ MPa is chosen, which is likely to be a realistic estimate of the value actually attained for steel with a guaranteed yield stress of 500 MPa.

The broad scope of this parameter study allows to analyse a number of isolated influences. Very critical with respect to the discussion on steel classification is determining the influence of the hardening ratio and the ultimate steel strain on the member ductility. Both in the case of hot rolled and cold worked steel, these influences can be determined based on the evaluation of the results obtained for the steels A5, A6, A, A7 and A8 (ultimate strain effect for hot rolled steel type), A9, A, A10, A11 and A12 (hardening ratio effect for hot rolled steel), B6, B, B7 and B8 (ultimate strain effect for cold worked steel), B3, B2, B1 and B4, B, B6 (hardening ratio effect for cold worked steel). Last but not least, the influence of the length of the yield plateau can be found on the basis of the simulation results for the steel types A13, A14, A and A15.
In the following a number of representative results of the simulations is presented and discussed.

<table>
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<tr>
<th>Steel code</th>
<th>$\sigma_{t}^{e}$ [MPa]</th>
<th>$\epsilon_{t}^{e}$ [%]</th>
<th>$\epsilon_{s}^{e}$ [%]</th>
<th>$\sigma_{t}^{e}$ [MPa]</th>
<th>$\epsilon_{t}^{e}$ [%]</th>
<th>$\sigma_{s}^{e}$ [MPa]</th>
<th>$\epsilon_{s}^{e}$ [%]</th>
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<table>
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<th>$\epsilon_{t}^{e}$ [%]</th>
<th>$\epsilon_{s}^{e}$ [%]</th>
<th>$\sigma_{t}^{e}$ [MPa]</th>
<th>$\epsilon_{t}^{e}$ [%]</th>
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<td>B8</td>
<td>550</td>
<td>0.28</td>
<td>550.1</td>
<td>0.30</td>
<td>566</td>
<td>0.90</td>
<td>574</td>
<td>1.70</td>
<td>584</td>
<td>4.00</td>
</tr>
</tbody>
</table>
(2) Simulation results

A systematic variation of the steel characteristics allows not only to study the influences of specific steel characteristics on the available member deformation capacity, but also to draw conclusions concerning the critical combination of steel properties with respect to structural application requirements. In this way questions important both for design and manufacturing practice can be answered. Moreover, the clearly promising concept of equivalent steel can be further verified and commented.

The rotation capacity predicted with the calculation model for all types of reinforcing steel considered is shown in Fig. 7.13 - 7.34 as a function of the mechanical reinforcement ratio of the section. Since in the cases studied no compressive reinforcement is applied the mechanical reinforcement ratio of the section is defined as follows: \( \omega_s = \rho_s \cdot \frac{f_y}{f_c} \) where \( \rho_s \) is the tensile reinforcement ratio, \( f_y \) is the actual steel yield strength and \( f_c \) is the actual concrete compressive strength. Note that in the general case the effective mechanical reinforcement ratio of the reinforced section \( \omega_{eff} \) should rather be used, that incorporates the effect of both tensile and compressive reinforcement. E.g. in Pommerening (1996) the following definition has been proposed: \( \omega_{eff} = \omega_{su} - \omega_s' \), where \( \omega_{su} = \omega_s \cdot \frac{f_y}{f_c} \) is the factored mechanical tensile reinforcement ratio and \( \omega_s' = \rho_s' \cdot \frac{f_y}{f_c} \) is the mechanical compressive reinforcement ratio.

The curves given in Fig. 7.13 - 7.34 are fitted lines based on the values calculated for the following percentages of reinforcement: 0.15%, 0.25%, 0.38%, 0.50%, 0.63%, 0.75%, 0.88%, 1.01%, 1.50% and 1.75%. In the majority of cases members with an effective height \( h = 400 \text{ mm} \) are used in the simulations. Only for steel A additional simulations are performed for a smaller member with similar slenderness ratio \( (h = 200 \text{ mm}, l / h = 12) \). The conclusions from the parameter studies on the member size dependence are fully confirmed by this set of simulations, where a substantial increase of the rotation capacity is found with a reduction of member dimensions. Furthermore, in the case of steel types A, S and B results are reported that show the reserve in rotation capacity with respect to the value reached at the maximum load level, that arises due to the softening of concrete under compression. In the cases where concrete failure is governing (i.e. for higher reinforcement ratios) a substantial capacity of the plastic hinge to rotate after the maximum load carrying capacity has been reached is predicted. Since the descending branch of the steel stress-strain relationship is not modelled and steel failure is defined when reaching the ultimate stress value, no reserve capacity in cases where steel rupture prevails can be possibly found in these simulations. There is a direct relation between member size and the values of the reserve rotation capacity, which is found to increase with decreasing member effective height.
Figure 7.13  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A (h = 400 mm)

Figure 7.14  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A (h = 200 mm)

Figure 7.15  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A5 (h = 400 mm)
Figure 7.16 Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A6 (h = 400 mm)

Figure 7.17 Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A7 (h = 400 mm)

Figure 7.18 Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A8 (h = 400 mm)
Figure 7.19  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A9 (h = 400 mm)

Figure 7.20  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A10 (h = 400 mm)

Figure 7.21  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A11 (h = 400 mm)
Figure 7.22  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A12 (h = 400 mm)

Figure 7.23  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A13 (h = 400 mm)

Figure 7.24  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A14 (h = 400 mm)
Figure 7.25  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel A15 (h = 400 mm)

Figure 7.26  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel S (h = 400 mm)

Figure 7.27  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B (h = 400 mm)
Figure 7.28  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B2 (h = 400 mm)

Figure 7.29  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B3 (h = 400 mm)

Figure 7.30  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B4 (h = 400 mm)
Figure 7.31  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B5 (h = 400 mm)

Figure 7.32  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B6 (h = 400 mm)

Figure 7.33  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B7 (h = 400 mm)
Figure 7.34  Rotation capacity as a function of mechanical reinforcement ratio - lower bound (left) and upper bound solution (right) for steel B8 (h = 400 mm)

In Fig. 7.13-7.34 lower and upper bound solution results are given, that correspond to the crack pattern with and without crack at mid-span, respectively as explained in the preceding Chapters. The crack pattern is found to be a very significant influence in setting the available rotation capacity. In all cases a huge supplementary rotation capacity is foreseen if a favourable crack pattern is regarded.

The primary aim of this parameter studies, i.e. the determination of steel ductility requirements for safety, has consequences for the further evaluation of the simulation results. Out of the two - the lower and the upper - bound rotation values, the earlier is considered to give a safe estimate of the available member ductility and thus to be conservative in the sense of design specifications. The same caution applies to a defining the failure criteria. In the further analysis the maximum load level is chosen as a limit defining a safe boundary, despite of the significant reserve capacity expected in cases where concrete compressive failure prevails. Hence, this post-peak rotation capacity is used as a safety margin, in particular to accommodate imposed deformations, which agrees with the approach taken in CEB-FIP Model Code 1990.

In the following the influences of the specific steel properties are discussed. Ponder the foregoing comment, in the Fig. 7.35 - 7.40 only the lower bound rotation capacity at the maximum load estimates are shown. At first the effect of the ultimate steel strain variation for the hot rolled steel type is studied, based on the comparison of results obtained for the steel types A5, A6, A, A7 and A8, see Fig. 7.35. The same post yield curve is followed, which means that the initial ratio of strain hardening at the onset of the hardening range is equal in all cases and the ultimate strain is terminated at 2%, 3%, 4%, 5%, 6% and 7%, respectively. Hence, for this set of steel types in fact not only the
ultimate steel strain capacity varies, but also the hardening ratio increases slightly as the 
$\varepsilon_u$ value increases. As expected, the increase of the ultimate strain has a direct effect 
on the shift of the point of transition from concrete to steel failure: as $\varepsilon_u$ increases, the 
transition point shifts towards lower reinforcement ratios. In the range where concrete 
in compression is critical no difference is found in the available rotation capacity, as 
long as the corresponding steel strain at failure can be developed for the steel types 
compared. Contrary to this, in the range where steel rupture is the governing failure 
mode, an increase of $\varepsilon_u$ is also accompanied by a strong proportional increase of the 
rotation capacity values. For a reinforcement ratio $\rho_s = 0.15\%$ (failure due to the steel 
rupture) for the steel types considered, the rotation capacity appears to increase in the 
following proportion: $1 : 1.5 : 1.7 : 2 : 2.4 \ (A5 : A6 : A : A7 : A8)$. A comparison of 
the values of the equivalent steel parameter $p$ for the same steel types results in the 
following sequence: $1 : 1.4 : 1.6 : 1.9 : 2.1$, which is slightly slower raising than the 
rotation values. This leads to the conclusion that the definition of $p$ tends to slightly 
underestimate the effect that an increased ultimate strain capacity combined with an 
increased hardening ratio has on the member ductility.

In the next step the influence of a variation in hardening ratio for hot rolled steel is 
analysed, comparing the rotation capacity found for $A9, A, A10, A11$ and $A12$, see Fig. 
7.36. All these steel types are characterised by the same length of the yield plateau and 
equal ultimate strain value. The prime difference is the hardening ratio in the strain 
hardening range, which changes in the order: $1.05, 1.08, 1.15, 1.25$ and $1.40$, 
respectively. A very lean shift of the transition point from the concrete to the steel 
failure towards lower reinforcement ratios is found as the hardening ratio increases. For 
the extreme case of the hardening ratio of $1.40$ concrete failure is governing for all 
reinforcement ratios analysed. The major difference, however is a very strong increase 
of the rotation capacity as soon as steel strains higher than the strain at the onset of 
strain hardening are activated at failure. This means that already in the range that 
corresponds with concrete compressive failure large differences in rotation capacity arise. 
In the range of reinforcement ratios where steel rupture causes member failure a fairly 
steady correspondence of the rotation capacity values is found. For a reinforcement ratio 
of $\rho_s = 0.15\%$ (failure due to the steel rupture for all steel types but $A12$) the rotation 
capacity increases in the following proportion, as the hardening ratio rises: $1 : 1.4 : 2.5 
: 4.2 \ (A9 : A : A10 : A11 : A12)$. The corresponding values of the equivalent steel 
parameter $p$ show the following sequence: $1 : 1.5 : 2.7 : 4.3$, which is a faster increase 
than that found for the rotation capacity values. Hence, it is believed that the effect of 
the hardening ratio for hot rolled steel is slightly overestimated by the Equation 3.41.

In Fig. 7.36 the calculated rotation capacity for steel $S$ is also shown. The higher
hardening ratio than for steel A explains the differences between the two steel types found as soon as reinforcement ratios are reached, where higher steel strains are mobilised. Compared to steel type A10 a larger ultimate strain capacity favourably influences the member ductility in the range where steel rupture prevails. This is in agreement with the conclusions from the evaluation discussed above.

Figure 7.35 Influence of ultimate steel strain on rotation capacity developed by hot rolled steel - lower bound solution as a function of mechanical reinforcement ratio for A5, A6, A, A7 and A8 (left) and corresponding stress - strain diagrams of the steel (right)

Figure 7.36 Influence of strain hardening ratio on rotation capacity developed by hot rolled steel - lower bound solution as a function of mechanical reinforcement ratio for A9, A, A10, A11 and A12 (left) and corresponding stress - strain diagrams of the steel (right)
The last aspect studied with respect to the hot rolled steel characteristics is the effect of extension of the yield plateau. The analyses are based on the results obtained for steel A13, A14, A and A15, presented in Fig. 7.37. A strain at the onset of strain hardening equal to 0.3%, 1%, 2% and 3% is assumed in these simulations. A variation of this parameter seems to have no meaningful effect on the shift of the failure type transition point. It has, however, a strong influence on the available rotation. Already at high reinforcement ratios, where concrete failure is governing, differences are found. There, due to early hardening in the case of steel with a short yield plateau, a relatively larger part of the reinforcing bars are in the yield range resulting in additional deformation capacity, compared to the cases where due to a low hardening ratio all deformation localises in the crack. As the reinforcement ratios decrease this effect diminishes and finally disappears as soon as sufficiently high steel strains are attained in the critical section at the failure stage. In the range where steel failure prevails, the yield plateau expansion is proportional to the rotation capacity reached. In the case of a reinforcement ratio \( \rho_s = 0.15\% \) the following increase rate is found for the steel types considered: \( 1 : 1.1 : 1.2 : 1.3 \) (A13 : A14 : A : A15). A comparison of the parameter \( p \) values results in a rather different relation: \( 1 : 1.3 : 1.6 : 1.9 \). The favourable effect of the yield plateau length on the attained member ductility is thus strongly overestimated, although the tendency is represented well.

![Graphs showing the influence of extension of yield plateau on rotation capacity developed by hot rolled steel - lower bound solution as a function of mechanical reinforcement ratio for A13, A14, A and A15 (left) and corresponding stress - strain diagrams of the steel (right).](image)

Figure 7.37  Influence of extension of yield plateau on rotation capacity developed by hot rolled steel - lower bound solution as a function of mechanical reinforcement ratio for A13, A14, A and A15 (left) and corresponding stress - strain diagrams of the steel (right).

Before an attempt is be made to modify the definition of the parameter \( p \), given in Chapter 3, the results obtained for the cold-worked steels are analysed. In this case mainly two parameters are of importance: the ultimate strain and the hardening ratio.
Figure 7.38  *Influence of ultimate steel strain on rotation capacity developed by cold worked steel - lower bound solution as a function of mechanical reinforcement ratio for B6, B, B7 and B8 (left) and corresponding stress - strain diagrams of the steel (right)*

Figure 7.39  *Influence of hardening ratio on rotation capacity developed by cold worked steel - lower bound solution as a function of mechanical reinforcement ratio for B4, B and B5 (left) and corresponding stress - strain diagrams of steel (right)*

With respect to the first one, a systematic evaluation can be performed using the results of the simulations for B6, B, B7 and B8 (Fig. 7.38). Here, similar to the first of the sets for hot rolled steel, the same stress - strain curve is followed with the strain capacity being limited to 2%, 2.5%, 3% and 4%, respectively. Corresponding to the ultimate strain, an increase of the hardening ratio results. In general the conclusions from this comparison are similar to that deduced in case of hot rolled steel. Of most interest is that
also here a significant increase of the rotation capacity is found in the range where steel failure is governing. The rate of increase for members with $\rho_s = 0.15\%$ appears to be: $1 : 1.1 : 1.3 : 1.6$. When this is compared with the ratio between the corresponding parameter $p$ values ($1 : 1.2 : 1.4 : 1.9$) a large difference is found. Thus, in the case of cold worked steel, the influence of an increase of the ultimate strain combined with an increase of the hardening ratio seems to be overestimated.

More data for judging the adequacy of the parameter $p$ follow from the analysis of two sets of steel types (B4, B and B5 in Fig. 7.39 and B3, B2 and B1 in Fig. 7.40) where for two values of the ultimate strain (2.5% and 5%, respectively) the effect of a variation of the hardening ratio is studied. In both cases an increase of the hardening ratio results in a substantial increase of the rotation capacity. An enlargement of the ultimate strain values causes a shift of the failure type transition point towards lower reinforcement ratios, as expected. Most interesting, however, is the comparison of the rotation increase ratio and its relation to the values of the parameter $p$. In series B4, B, B5 for $\rho_s = 0.15\%$ the rotation capacity increases with $1 : 1.2 : 1.5$ respectively, while the parameter $p$ varies with $1 : 1.6 : 2.4$. For the series B3, B2, B1 this rate is, respectively, $1 : 1.3 : 1.6$ for the rotation capacity and $1 : 1.6 : 2.4$ for the parameter $p$. Hence, in both cases a less substantial increase of the rotation capacity is found than expected on the basis of the parameter $p$.

![Figure 7.40 Influence of hardening ratio on rotation capacity developed by cold worked steel - lower bound solution as a function of mechanical reinforcement ratio for B3, B2 and B1 (left) and corresponding stress-strain diagrams of steel (right)](image)

In this respect the manner of involving the effect of the hardening ratio and the ultimate strain in the overall steel ductility parameter $p$ is a very interesting issue. The definition
according to equation 3.40 includes both effects in separate terms. It seems, however, that there is a correlation between these two parameters, considering the observation that a stronger increase of the rotation capacity is found in cases where a higher ultimate strain was assumed. This appears more directly, when cases with equal hardening ratio are compared with one another. The following ratios of the rotation capacity are found: for B3 : B4 - 1 : 1.5, for B : B2 - 1 : 1.6, for B5 : B1 - 1 : 1.7. Clearly with the same variation of ultimate strain, the rotation capacity increases faster if the hardening ratio increases, while not such a tendency is foreseen in the p definition (in these cases a constant proportion of 1 : 1.5 is supposed). It seems however not to be very practical to try to involve this possible dependence in the definition of the equivalent steel parameter.

It is clear that the equivalent steel concept in its present version is generally correct but there is still considerable room for adjustments and corrections. On the basis of the previous discussion, here a proposal is made for some improvements in the weight factors involved in the formulations of the parameter p for the case of hot rolled and cold worked steel. As discussed above, the effect of the length of the yield plateau has been found to be overestimated for the hot rolled steel type. In the case of cold worked steel, however, a sufficiently accurate estimate is found. It seems therefore sufficient to correct the definition of p only for the effect of $\varepsilon_{sh}$. With respect to the hardening ratio, an overestimation applies as well, for both steel types. Concerning this, the following formulations are proposed:

- for cold worked steel

$$ p = \varepsilon_u^{0.75} \cdot \left( \frac{f_t}{f_y} - 1 \right)^{0.8} \quad (7.2) $$

- for hot rolled or heat treated steel

$$ p = \left( (\varepsilon_u - \varepsilon_{sh}) + 3 \cdot \varepsilon_{sh} \right)^{0.75} \cdot \left( \frac{f_t}{f_y} - 1 \right)^{0.8} \quad (7.3) $$

To show the reliability of the new formulation and to demonstrate the achieved improvement, the values of the overall steel ductility characteristic p, computed according to the old and new formulations (Equation 3.40 - 3.41, and 7.2 - 7.3, respectively) are compared with the results of simulations for all steel types obtained for a reinforcement ratio $\rho_c = 0.15\%$. Since it is expected that the parameter p is proportional to the plastic rotation in cases where steel failure is prevailing, a plot of the
predicted available rotation values versus $p$ should show a straight line through the origin. This is confirmed in Fig. 7.41 and 7.42. For both the old and the new formulation a very good agreement is found, with standard errors of 0.0317 and 0.0306, respectively. The improvement concerns thus not a better average correlation acquired with the modified formulation. More important is, that the new formulation of the parameter $p$ better captures the tendency of increasing rotation capacity with increasing overall steel ductility. A number of such cases is indicated in the figures with the empty dots. This occurrence suggests that specific steel characteristics are better incorporated in the new definition of the overall steel ductility characteristic.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7_41.png}
\caption{Rotation capacity versus parameter $p$ (old formulation) for reinforcement ratio 0.15\%}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7_42.png}
\caption{Rotation capacity versus parameter $p$ (new formulation) for reinforcement ratio 0.15\%}
\end{figure}
A similar conclusion can be drawn from the reevaluation of the response of the members reinforced with steel A, A1 and A2 (see Fig. 7.9). Although originally meant to represent steel types with the same overall ductility performance, despite equal values of the parameter $p = 0.0197$, calculated in accordance with Equation 3.41, differences in rotation capacity appeared: it was found highest for A1 and lowest for A2. Yet, this can be explained by the similar trend found in the new values of the parameter $p$, calculated according to Equation 7.2 ($0.0225 : 0.0218 : 0.0209$ for A1 : A : A2, respectively). In Appendix 2 the overall ductility parameter $p$ calculated according to the new formulation is given for all steel types referred to in this Thesis.

From this parameter study it is concluded that steel properties are undoubtedly one of the key issues related to the phenomenon of rotation capacity. The equivalent steel concept offers a very practical tool in quantifying the importance of specific reinforcing steel characteristics. With the introduced improvement the global ductility parameter $p$ represents well the relative importance of specific steel characteristics on attained structural response. Although, considering the complexity of the problem, it seems to be too early to develop a standard formula that would directly provide the allowable rotation capacity in a general case, the proposed formulations allow to evaluate the influence of steel properties in a consistent way. A general quantitative assessment of the steel properties provided by the equivalent steel ductility parameter $p$ is certainly a measure that allows for a proper comparison of different steel types from the perspective of their structural performance. By virtue of this potential it is a very practical criterion to be implicated in the material standards and codes of practice.

Finally, it must be remembered that the ability of the structure to develop in critical sections plastic hinges with required deformation capacity is exploited in some structural analysis techniques and engineering methods. It seems worthwhile to revise related parts of the design standards, considering the results of the parameter studies performed here, both with respect to the member geometrical characteristics (size and detailing), and the structural materials currently used. In the following Chapter an attempt is made to arrive at the conditions for analysis of statically indeterminate systems, that will directly follow from the discussion on the plastic hinging mechanism presented here. In this perspective the limitations of the presently used design standards may be verified and revised rules can be proposed.
8 Final remarks on structural analysis

8.1 Structural analysis according to CEB-FIP Model Code 1990

Apart from non-linear analysis methods, which take account of a realistic description of the physical non-linear behaviour of structural materials and of second order effects by virtue of suitable incremental and/or iterative numerical methods, other more simplified approaches to the structural analysis are allowed in the CEB-FIP Model Code 1990. Among these, the methods of linear analysis method has the longest tradition in design practice. Based on the assumption of linear-elastic behaviour it provides realistic results only under the condition that actions are low and members are uncracked. In the ultimate limit states, however, this cannot always satisfy the conditions of compatibility in view of the invalidity of the assumptions relating to the corresponding deformations. In this respect an important development in the calculation of structures was the introduction of plastic analysis. A basic principle of this calculation method is that a structure just then collapses, when such a number of plastic hinges has been formed, that the structure is not stable any more. A condition for the creation of such a collapse-mechanism is that the plastic hinges can deform enough, i.e. can sufficiently rotate. In other words, the hinges which have arisen first must have such a rotation capacity that the structure will not collapse, but will deform so much that also the last hinges, necessary for the collapse-mechanism can occur (see Fig. 8.1).

![Figure 8.1 Mechanism with sufficient (left) and insufficient (right) rotation capacity](image)

In order to account for the redistribution of forces in a structure, resulting from the formation of a collapse-mechanism, corrections were introduced into the linear analysis method, leading to the introduction of the so-called linear analyses followed by limited redistribution. In this method an assumption is made for the amount of redistribution of
bending moments with respect to the distribution according to an elastic calculation. That amount is in fact proportional to the overload that a structure can sustain, while the moment capacity of the sections remains constant. The degree of moment redistribution \( \delta \) is defined as the ratio between the moment over the support after redistribution and its value calculated according to the theory of elasticity for a given load. The allowable degree of moment redistribution is mainly controlled by the member ductility and therefore its estimation involves the evaluation of the available and required rotation capacity of plastic hinges. In the following sections attention will be given to this last topic, evaluating the results of parameter studies performed within the scope of this project and comparing them with some other proposals, in particular with the CEB-FIP Model Code 1990 guideline.

### 8.1.1 Linear analysis followed by limited redistribution

In general the structural analysis is qualified as linear with redistribution, if the actions derived from a linear analysis are redistributed over the structure. It is therefore allowed to reduce the moments in the section subjected to the highest action effect with the reduction coefficient \( \delta \), provided that in the other sections the moments are increased to maintain equilibrium. Suitable ductility conditions, linked to the corresponding required rotation capacity, should be satisfied. Various models give different criteria in this respect, however the type of reinforcing steel (ductility class) and the reinforcement ratio (relative compression zone depth \( x/h \)) are commonly agreed to be the main influential parameters (see Fig. 8.2).

The CEB-FIP Model Code 1990 distinguishes between three reinforcing steel ductility classes, depending on the characteristic value of the ratio between the steel tensile strength \( f_t \) and yield stress \( f_y \) and the characteristic value of steel strain at maximum load \( \varepsilon_{uk} \):

- **steel type S:** \( (f_t / f_y)_k \geq 1.15 \) and \( \varepsilon_{uk} \geq 6\% \)
- **steel type A:** \( (f_t / f_y)_k \geq 1.08 \) and \( \varepsilon_{uk} \geq 5\% \)
- **steel type B:** \( (f_t / f_y)_k \geq 1.05 \) and \( \varepsilon_{uk} \geq 2.5\% \)

The requirements for the reduction coefficient \( \delta \), to be used for multiplying the moments in the sections subjected to the highest moments, are specified as follows:

- for steel type S and A:
  - for concrete grades C12 to C35: \( \delta \geq 0.44 + 1.25 \frac{x}{h} \) \( (8.1) \)
  - for concrete grades C40 to C60: \( \delta \geq 0.56 + 1.25 \frac{x}{h} \) \( (8.2) \)
and

- for continuous beams: \(0.75 \leq \delta \leq 1.00\)
- for non-sway frames: \(0.75 \leq \delta \leq 1.00\)
- for sway frames: \(0.90 \leq \delta \leq 1.00\)

- for steel type B:
  - for concrete grades C12 to C60: \(\delta \geq 0.75 + 1.25 \times/h\) (8.3)
  and \(0.90 \leq \delta \leq 1.00\)

In this context it is interesting to recall some models that served as a basis for the formulation of the rules currently included in the CEB-FIP Model Code 1990. Resulting from the work of Langer (1987), in Eligehausen and Langer (1987) a recommendation for the reinforcement ductility classes was given, followed by a suggestion for the allowable degree of redistribution. In this study a distinction was made between two steel ductility types:

- **Type 1:** \(f_t/f_y \geq 1.10 + 1.15\) and \(\varepsilon_g \geq 5\%\)
- **Type 2:** \(1.05 \leq f_t/f_y \leq 1.10\) and \(2.5\% \leq \varepsilon_g \leq 5\%\)

where \(\varepsilon_g = 2\varepsilon_{10} - \varepsilon_5\) is the unit ultimate steel strain, determined from the \(\varepsilon_{10}\) and \(\varepsilon_5\), being the strain at steel rupture measured over a length of 5 and 10 times the bar diameter, respectively.

The reduction coefficient \(\delta\) was then limited as follows:

- for steel **Type 1:** \(\delta \geq 0.44 + 1.25 \times/h\) (8.4)
  and \(0.70 \leq \delta \leq 1.00\)
- for steel **Type 2:** \(\delta \geq 0.44 + 1.25 \times/h\) (8.5)
  and \(0.85 \leq \delta \leq 1.00\)

A more recent proposal presented by Eligehausen and Fabritius (1993) recommended for the steel ductility classes defined as in the CEB-FIP Model Code 1990 the following allowable degrees of moment redistribution:

- for steel type S: \(\delta \geq 0.5 + \times/h\) (8.6)
  and \(0.70 \leq \delta \leq 1.00\)
- for steel type A: \(\delta \geq 0.6 + \times/h\) (8.7)
  and \(0.80 \leq \delta \leq 1.00\)
- for steel type B: \(\delta \geq 0.7 + \times/h\) (8.8)
  and \(0.90 \leq \delta \leq 1.00\)
Figure 8.2 Comparison of various proposals for allowable degree of moment redistribution

To demonstrate the way in which the presently used codes of practice limit the use of linear design with moment redistribution the Dutch code for design of concrete structures VBC 1995 and the European standard Eurocode No.2 (1989) are referred. VBC 1995 allows linear design with limited redistribution and recognised different steel ductility types. However, contrary to the steel ductility classification given above, VBC 1995 limits only the characteristic ultimate steel strain values $\epsilon_{uk}$, leaving the ratio $f_y$ over $f_t$ unspecified:

- FeB 500 HWL, HK: $\epsilon_{uk} \geq 3.25\%$
- FeB 500 HKN: $\epsilon_{uk} \leq 2.75\%$
- FeB 500 HKN (fabric): $\epsilon_{uk} \leq 2.75\%$

With respect to the allowable moment redistribution no special requirements are stated and a redistribution up to a maximum of 20% ($\delta = 0.8$) is allowed, providing that the equilibrium is maintained and the redistributed moments are used to determine reactions and shear forces along the member. This is in basic disagreement with the CEB-FIP
Model Code 1990 and other models discussed here, which all relate the degree of redistribution to the steel ductility and strongly reduce its value for higher values of \(x/h\).

The European standard for the design of concrete structures Eurocode No.2 (1989) classifies the steel into two ductility classes:

- **H** (High-Ductility Steel): \[ \frac{f_t}{f_y} \geq 1.08 \quad \text{and} \quad \varepsilon_{uk} \geq 5\% \]
- **N** (Normal-Ductility Steel): \[ \frac{f_t}{f_y} \geq 1.05 \quad \text{and} \quad \varepsilon_{uk} \geq 2.5\% \]

The simplified redistribution expression follows the main line of the CEB-FIP Model Code 1990:

- for concrete grades not higher than C35/45: \[ \delta \geq 0.44 + 1.25 \frac{x}{h} \quad (8.9) \]
- for concrete grades higher than C35: \[ \delta \geq 0.56 + 1.25 \frac{x}{h} \quad (8.10) \]

and

- for steel **H**: \[ 0.70 \leq \delta \leq 1.00 \]
- for steel **N**: \[ 0.85 \leq \delta \leq 1.00 \]

All the above equations are valid for members (beams and slabs) with straight axes in the horizontal plane. In curved members flexural yielding may produce a sudden increase of torsion, which can lead to a brittle failure before the redistribution of flexural moments is fully exploited. A generally applied condition for a structure subjected to various loading cases is that only one redistribution can be assumed.

### 8.2 Rotation capacity and degree of moment redistribution

#### 8.2.1 Behaviour of statically indeterminate RC members

The allowable degree of moment redistribution results from the evaluation of required and available rotation capacity of the plastic hinges necessary to develop a collapse-mechanism in a statically indeterminate RC member. For this purpose it is necessary to consider non-linear effects that take place with increasing load. A proper understanding of the flexural stiffness development is required.

As illustrated in Fig. 8.3, with increasing load the member at first remains uncracked (member with rectangular cross-section and statical system as shown in Fig. 7.4). Up to the cracking moment the stiffness along the member length is almost constant and the moments are distributed according to the theory of elasticity. After the cracking moment has been reached in one of the critical sections \(M_{crl}\) the reduction of bending stiffness
associated with cracking results in a redistribution of moments. The lower flexural stiffness after cracking causes also a less rapid increase of the moment in this section with a further increase of the load. To maintain the equilibrium, the moment in the other (stiffer) section grows faster. With further increasing load, however, also in this section cracking takes place ($M_{crII}$). Subsequently the distribution of the moments depends entirely on the then existing stiffness ratio along the member. In general, after cracking the stiffness will be approximately proportional to the corresponding reinforcement ratio. The difference in stiffness will determine which of the critical sections first attains the yielding moment ($M_{pl}$). In this section no further increase of the moment can occur with further increasing load (for sake of simplicity the influence of steel hardening is neglected in this description). A plastic hinge develops and starts to rotate. To develop a collapse-mechanism, in the other critical section a plastic hinge must form as well with further increase of the load ($M_{II}$).

![Development of moments in critical sections as a function of applied load (schematization)](image)

**Figure 8.3** Development of moments in critical sections as a function of applied load (schematization)

uniformly distributed load $q$

![Deflection curve and compatibility of deformations in a continuous member under uniformly distributed load (schematization)](image)

**Figure 8.4** Deflection curve and compatibility of deformations in a continuous member under uniformly distributed load (schematization)
For reasons of compatibility in the assumed static system (see Fig. 8.4) the rotation in the span must be equal to the rotation over the support. Therefore, if the stiffness along the member is known at any stage of the loading, the corresponding span and support rotations may be computed. From the criterion for the development of a collapse-mechanism and the compatibility condition, the required plastic rotation of the hinge that develops first can be calculated.

In design, the assumed redistribution ratio has a direct influence on the bending moments values considered and, thus, on the necessary amount of reinforcement and the resulting member stiffness. It is important to realise, however, that even if the member is designed according to the theory of elasticity ($\delta = 1.0$), plastic rotation must occur in the hinge over the support to allow the formation of the desired collapse-mechanism. A schematic representation of the moment-rotation relationship for two different degrees of redistribution (and, hence, two different stiffness ratios) is presented in Fig. 8.5 (stiffness in the cracked state over the support equal to about twice the value in the span for $\delta = 1.0$ and about 50% for $\delta = 0.5$). Also in this Figure the effect of steel hardening is not neglected.

![Figure 8.5](image)

**Figure 8.5** Development of the moment in the span and over the support as a function of the rotation depending on the degree of moment redistribution (after Macchi 1976)

In the following paragraph a procedure is introduced to calculate the value of the required rotation capacity of a plastic hinge as a function of the slenderness ratio of the member and the design value of the degree of moment redistribution (which is thus implicitly a function of the reinforcement ratio and the members stiffness). This is then used to evaluate the allowable degree of moment redistribution that results from the structural dependent available deformation capacity of the member.
8.2.2 Required rotation capacity

A simple procedure to determine the relation between the assumed degree of redistribution $\delta$ and the required rotation capacity in a plastic hinge, necessary to form a collapse mechanism in a statically indeterminate system under a uniformly distributed load $q$ at the required load-carrying capacity, as shown in Fig. 8.6, is followed.

When the moment redistribution has been fully accomplished, for the support and span moment holds respectively $M_A = \gamma_M M_p$ and $M_B = M_p$, where $\gamma_M$ is the ratio between moment resistance at the support and in the span, dependent on the assumed degree of redistribution $\delta$ (e.g. $\delta = 1.0 \gamma_M = 2.0$, $\delta = 0.75 \gamma_M = 1.0$).

\[
M_A = \gamma_M M_p \\
R_A = \frac{1}{2}ql \\
M_B = M_p
\]

$A, D$ - supports  \\
$B$ - mid-span  \\
$C$ - inflection point

Figure 8.6 The ideal mid-span schematization for calculation of required rotation

The relation between the bending moments at the fixed support and at the midspan corresponds with the assumed redistribution degree and is given with the equation 8.11:

\[
\gamma_M M_p + M_p = \frac{1}{8} q l^2
\]  

(8.11)

where $q$ is the uniformly distributed load  
Compatibility of deformations requires:

\[
\Theta_A = \Theta_B
\]  

(8.12)

where $\Theta_A$ is the required rotation at the support  
$\Theta_B$ is the corresponding rotation over the span

If the member span is denoted by $l$, the ratio between the length of the support zone and that of the span zone follows from the location of the moment inflection point (linear-
elastic analysis with moment redistribution) and is given by the parameter $\alpha_9$. The ratio between the flexural stiffness $(EI)_g$ of the support zone and of the span zone is defined by the parameter $\beta_9$. Considering the equilibrium of forces and compatibility of the deformations the following expressions can be obtained (see Fig. 8.7):

$$
\Theta' = -\frac{M' \cdot l \left(\frac{1}{2} - \alpha_9\right)}{(EI)_g} + \frac{D' \cdot l^2 \left(\frac{1}{2} - \alpha_9\right)^2}{(EI)_g} - \frac{q \cdot l^3 \left(\frac{1}{2} - \alpha_9\right)^3}{(EI)_g}
$$

(8.13)

with

$$
D' = \frac{1}{2} q \cdot l - q \cdot \alpha_9 \cdot l = q \cdot l \left(\frac{1}{2} - \alpha_9\right)
$$

(8.14)

$$
M' = \gamma_M \cdot M_p - \frac{1}{2} q \cdot \alpha_9 \cdot l^2 + \frac{1}{2} q \cdot \alpha_9^2 \cdot l^2 = \gamma_M \cdot M_p + \frac{1}{2} q \cdot l^2 \cdot \alpha_9 \left(\alpha_9 - 1\right)
$$

(8.15)

and

$$
\frac{\Theta_A}{2} = \Theta' - \frac{\gamma_M \cdot M_p \cdot \alpha_9 \cdot l}{\beta_9 \cdot (EI)_g} + \frac{\frac{1}{2} q \cdot l \left(\alpha_9 \cdot l\right)^2}{2 \beta_9 \cdot (EI)_g} - \frac{q \left(\alpha_9 \cdot l\right)^3}{6 \beta_9 \cdot (EI)_g}
$$

(8.16)

where $\Theta'$, $M'$, $D'$ are, respectively, the rotation, moment and shear force at the predefined point of transition from the span to the support zone.

$M_a = \gamma_M \cdot M_p \left(\frac{q \cdot l}{\beta_9 \cdot (EI)_g}\right) \quad R_a = \frac{1}{2} q l$

$M_c = \frac{1}{2} q l \left(\alpha_9 \cdot l\right)$

$M_s = M_p$

$\Theta_A = \frac{1}{2} \Theta' - \frac{\gamma_M \cdot M_p \cdot \alpha_9 \cdot l}{\beta_9 \cdot (EI)_g} + \frac{\frac{1}{2} q \cdot l \left(\alpha_9 \cdot l\right)^2}{2 \beta_9 \cdot (EI)_g} - \frac{q \left(\alpha_9 \cdot l\right)^3}{6 \beta_9 \cdot (EI)_g}$

$\Theta_A = \frac{1}{3} \left(\frac{M_p \cdot l}{(EI)_g} \beta_9 \left(2 - \gamma_M\right) + 6 \gamma_M \cdot \alpha_9 \left(\beta_9 - 1\right) + \left(\gamma_M + 1 - \beta_9 - \beta_9 \cdot \gamma_M\right) \cdot (12 - 8 \alpha_9) \alpha_9^2\right)$

(8.17)

Yet, an estimate of the required rotation in the plastic hinge above the support may be
obtained with equation 8.17 when the moment resistance in the span $M_p$ and the flexural stiffness along the member $(EI)g$ are determined. Here the following approach is chosen. $M_p$ is calculated in a conventional way using the actual values of material properties and a parabolic-rectangular stress-strain diagram for the concrete with $\varepsilon_{cu} = 3.5\%$ and taking yielding of the steel in the span as the critical condition. To estimate the flexural stiffness of the reinforced member, the results of extensive studies performed by Monnier (1970) and by De Bruijn (1985) are used. The bending stiffness in the cracked stage is an average of both cracked cross-sections and uncracked reinforced concrete between the cracks. In the course of those studies and the subsequent numerical analysis the major structural effects on the bending stiffness have been examined. It has been concluded that the grade of the steel is found not to have a noticeable effect upon the stiffness in the cracked stage. Also the concrete strength plays a marginal role, if varied in the range examined (15 to 50 MPa). This holds also for the amount of compressive reinforcement. As main influencing factors the percentage of tensile reinforcement, the magnitude of the bending moment and the loading history are identified. De Bruijn proposed a fictitious modulus of elasticity $E_f$ which is a tool in calculating the overall member deformations on the basis of a critical section geometry, represented by the moment of inertia of the concrete section $I_c$. $E_f$ is defined as:

$$ E_f = \frac{(EI)g}{I_c} \quad (8.18) $$

In the situation considered in this study for (concrete grade around C35 and steel grade 500) and if the creep effect is disregarded, De Bruijn recommends the following relationship, derived by a linear regression of values obtained for a wide range of the reinforcement ratios $\rho_s$:

$$ E_f = 4800 + 760 \cdot 10^3 \cdot \rho_s \quad (8.19) $$

With this function the ratio of the average bending stiffness at the support and in the span zone $\beta_g$, defined above, can be determined depending on the design degree of moment redistribution and corresponding percentage of reinforcement. Finally the required rotation capacity of the plastic hinge above the support can be calculated.

Although rather simple, this procedure gives results that in general agree well with the more extensive studies on the required rotation capacity presented by Cosenza et al. (1991) and Eligehausen and Fabritius (1993). Influential parameters such as the degree of redistribution, the slenderness ratio of the member and the reinforcement percentage are involved and the tendencies are similar to that predicted by these two sources referred. In Fig. 8.8 a comparison is made between the required plastic rotation capacity according to the latter one and the results obtained using the procedure described above.
Here the rotation is given as a function of the relative depth of the compression zone in the support section $\xi = x/h$, calculated using a parabolic-rectangular stress-strain diagram for the concrete with $\varepsilon_{cu} = 3.5\%$ and a bilinear stress-strain relationship for steel with $\varepsilon_u = 1.00\%$. The effect of the moment redistribution on the slenderness of the statically determinate beam cut-out, representing the support region, is considered (the modelling steps taken to define a support region beam cut-out and to determine the relation between the redistribution degree, continuous member slenderness and cut-out beam extension are discussed in the following Section). Assuming a constant slenderness of the beam cut-out equal to $l/h = 6$, the slenderness of the statically indeterminate system increases with decreasing degree of moment redistribution from $l^*/h = 14.3$ for $\delta = 1.00$ to $l^*/h = 26.7$ for $\delta = 0.60$. Satisfactory good agreement is found between the two required rotation estimates. Nevertheless, it is admitted that in general the calculation of the required rotation capacity is highly sensitive to the assessment of the member stiffness. This should be kept in mind when coming to final conclusions.

\[\text{Figure 8.8 Required rotation capacity related to a degree of redistribution } \delta \text{ as a function of the effective depth of the compression zone } \xi = x/h \text{ - estimate according to Elighausen and Fabritius (1993) (left) and the approach used here (right) in case of a constant slenderness } l/h = 6 \text{ of beam cut-out representing the support region} \]

### 8.2.3 Degree of moment redistribution

If the evaluation of the required rotation is completed on a theoretical basis, the allowable degree of redistribution following from the structural safety considerations can be deduced, ponder that the allowable rotations are provided on the basis of analytical and experimental studies. To permit a safe design with a given moment redistribution percentage the plastic rotation at the critical hinge, required to obtain structural
compatibility, must be lower than the allowable one. In this respect also the problem of the redistribution due to cracking requires careful analysis, because a linear design completely neglects the plastic rotation that also in this case is required. It is of fundamental importance to establish the limitations to the depth of the compression zone (or directly to the reinforcement ratio, in cases of members with no compressive reinforcement) to prevent an uncontrollable loss of structural safety in the absence of a compatibility check in the case of linear analysis without redistribution.

(1) Parameter study

In Chapter 7 an extensive parameter study of the available deformation capacity of plastic hinge was presented. In order to use this information in analysing statically indeterminate systems some modelling steps must be taken, as already mentioned. Firstly, in the statically indeterminate structure a statically determinate beam that represents the region over the support is cut out. The beam cut-out length is thus equal to the distance between two adjacent points of zero moments (Fig. 8.9).

Figure 8.9 Statically determinate beam cut-out of the statically indeterminate system to represent the support region

Assuming a moment distribution according to the theory of elasticity ($\delta = 1$) the distance between the inflection points will be 0.42 of the continuous member span. This means that for a statically indeterminate system with slenderness a $l^* / h = 20$ the cut-out beam slenderness will amount to $l / h = 8.4$, while for instance for $l^* / h = 30 l / h$ it will be equal
to 12.6. An evaluation of the moment distribution along the member for different degrees of moment redistribution leads to the conclusion that with increasing δ the slenderness l/h of the cut-out statically determinate beam will increase (see Table 7.1).

| Table 7.1 Slenderness l/h of cut-out beam as a function of redistribution degree |
|------------------------------------|---|---|---|---|---|
|slenderness of indeterminate system| δ = 1.0 | δ = 0.9 | δ = 0.8 | δ = 0.7 | δ = 0.6 |
|l'/h = 20| 8.4 | 7.3 | 6.3 | 5.4 | 4.5 |
|l'/h = 30| 12.6 | 11.0 | 9.5 | 8.0 | 6.7 |

To account for this effect a relation between the available rotation capacity of the plastic hinge and the member slenderness is proposed, for the case of the static system considered. This relation follows directly from the results of the simulations, where the member slenderness ratio was used as a variable (see Fig. 8.10). A series of calculation similar to those described in Chapter 7 was carried out for member effective height h = 400 mm, using as input the concrete and steel A characteristics as given in Section 7.1.1. For three reinforcement ratios the member slenderness was varied from l/h = 12 to 6 (ρx equalled to 0.25%, 0.50% and 1.00%, with reinforcement layout II according to the classification given in Section 7.1.1). With a relation derived by linear regression the rotation capacity Θ^(p) of the member with specified slenderness l/h can be determined on the basis of the Θ^(p)_{l/h=12} value determined for the beam with l/h = 12, according to:

\[
Θ^{(p)} = Θ^{(p)}_{l/h=12} \cdot \left( \frac{l/h}{12} \right)^{K_Θ}
\]

(8.20)

where K_Θ = 0.85 is an empirically determined constant.

![Graph showing effect of member slenderness on available rotation capacity for three reinforcement ratios: 0.25%, 0.50% and 1% (simulation results)]
The procedure for determining the allowable degree of moment redistribution follows directly from the discussion presented before. The condition that the available rotation capacity should at least be equal to that required by the given degree of redistribution must be fulfilled in order to meet the compatibility criteria. Fig. 8.11 illustrates the procedure to determine the allowable degree of redistribution in case of steel A, for the lower bound rotation values (crack pattern with crack at mid span) and for the effective member height \( h = 400 \text{ mm} \) (statically indeterminate system slenderness ratio \( l^*/h = 20 \)). The limitation of both member size and feasible crack pattern is intended to give conservative values for the allowable degree of redistribution, following from the discussion on the structural effect on the rotation capacity presented in Chapter 7. The available and required rotation capacities are presented as a function of the actual value of mechanical reinforcement ratio of the section, which - in the absence of compressive reinforcement - is defined as \( \omega_s = \rho_s \cdot f_y/f_c \), where \( \rho_s \) is the tensile reinforcement ratio, \( f_y \) is the actual steel yield strength and \( f_c \) is the actual concrete compressive strength.

Considering the effect that the degree of moment redistribution has on the slenderness of the beam cut-out used in the modelling of the statically indeterminate system, the available rotation capacity is determined on the basis of the values shown in Fig. 7.13 (left), evaluated using equation 8.20. Intersection points of the corresponding curves mark the succeeding points of the resulting \( \delta - \omega_s \) curve.

![Figure 8.11](image)

*Figure 8.11 Allowable degree of redistribution for steel A - characteristic lower bound for \( h = 400 \text{ mm} \), \( l^*/h = 20 \)*

It is once more stressed that a safe estimate is aimed for and that the given degrees of redistribution are computed assuming the mean material strength and corresponding to this redistribution of the bending moment, and should therefore not be interpreted as design values. No margin is accounted for steel overstrength (i.e. the yield strength of
the reinforcement used being higher than the nominal value assumed in design) or overreinforcement (i.e. sections of a structure containing a greater amount of reinforcement than required from the design), that most probably will occur in practice. Using the full range of simulations presented in Section 7.2, the allowable moment redistribution can be studied in relation to a number of parameters. Starting from geometrical effects, a decrease of the member size (for the same slenderness ratio) results in an increase of the available rotation capacity. Consequently, a substantial increase of the allowable degree of moment redistribution results, see Fig. 8.12.

![Graph showing allowable degree of redistribution](image1)

**Figure 8.12** Allowable degree of redistribution for steel A - characteristic lower bound for $h = 200 \text{ mm}$, $l^*/h = 20$

![Graph showing available degree of redistribution](image2)

**Figure 8.13** Available degree of redistribution for steel A - characteristic upper bound (no crack at mid-span) for $h = 400 \text{ mm}$, $l^*/h = 20$

Almost the same effect has the crack pattern: if the upper bound crack pattern is assumed in the calculations for $h = 400 \text{ mm}$ again an increase in the redistribution
capacity is concluded, see Fig. 8.13. Interesting is also the result of assuming different failure criteria of the plastic hinge, in particular for higher reinforcement ratios. As already discussed, for the reinforcement ratios that correspond with concrete compressive failure, a considerable rotation can be developed after the maximum moment capacity has been reached. If one allows to consider concrete crushing as a limit condition, an increased capacity for moment redistribution follows, as can be seen when comparing Fig. 8.11 and 8.14.

![Figure 8.14 Available degree of redistribution for steel A - characteristic upper bound (concrete crushing as limit condition) for h = 400 mm, t*/h = 20](image1)

![Figure 8.15 Available degree of redistribution for steel A - characteristic lower bound for h = 400 mm, t*/h = 30](image2)

Last but not least the effect of member slenderness is evaluated, simulating the allowable degree of redistribution for a statically indeterminate system with a slenderness ratio of t*/h = 30 (see Fig. 8.15). A comparison of Fig. 8.11 and 8.15 leads to the conclusion...
that within this range the slenderness ratio of the system $l^*/h$ does not play a very dominant role with respect to the allowable degree of moment redistribution. This is likely to be due to the fact that both the available and the required rotation capacity tend to be dependent, in a very much similar way, on the ratio $l^*/h$.

Considering that in this Chapter a safe, in the design sense, solution is aimed at, in the following figures only the characteristic lower bound for the allowable redistribution capacity is evaluated for different reinforcing steel characteristics ($h = 400$ mm and $l^*/h = 20$, lower bound crack pattern with crack at mid-span, failure condition defined by the maximum load carrying capacity). Similar effects as discussed for steel A, are again associated with a change in member size, feasible crack pattern or limit criterion, which is however not further estimated. Continuing the discussion, initiated in Chapter 7, on the influence of specific reinforcing steel properties on member behaviour, the characteristic lower bound for the allowable degrees of moment redistribution are first compared for all hot rolled steel types described in Section 7.2.2. The following influences are separately studied: the ultimate steel strain (the same relationship between stress and strain in post-yield range) (see Fig. 8.16), the hardening ratio (the same length of yield plateau and equal ultimate strain value) (see Fig. 8.17), the yield plateau extension (equal strain hardening ratio and ultimate strain value) (see Fig. 8.18).

The results of this analysis can be best interpreted in combination with the study discussed in Chapter 7. As far as the influence of the ultimate strain is concerned (see Fig. 8.16 and 7.35 for comparison), with the increase of its value a continuous increase in the allowable degree of redistribution is observed in the range of low reinforcement ratios (steel failure governing). In particular, an increase of the ultimate strain in the lower range of values investigated ($\varepsilon_u = 3.0$ to $5.0\%$) causes a substantial improvement of the moment redistribution capacities. In the upper range ($\varepsilon_u = 5.0$ to $7.0\%$) still a positive influence of the increase of the ultimate strain on the allowable degree of redistribution is observed, although with less fast advancement. Nevertheless, in the analysed range where the steel ductility limits the plastic hinge deformations, the allowable degree of redistribution is ranges from $\delta = 0.92$ to $0.80$ for steel A5 ($\varepsilon_u = 3.0\%$), from 0.78 to 0.68 for steel A ($\varepsilon_u = 5.0\%$) and from 0.72 to 0.63 for steel A8 ($\varepsilon_u = 7.0\%$).

As soon as the steel strain activated at the ultimate stage falls in the strain hardening range (also in cases where concrete crushing governs the failure of the plastic hinge) an improvement of the moment redistribution capacity is caused by an increase of the strain hardening ratio (see Fig. 8.17 and 7.36 for comparison). A considerable increase of the allowable degree of redistribution is found already for a small (realistic) raise of the hardening ratio values: while for steel A9 ($f_y^*/f_y = 1.05$) an allowable redistribution
degree within the range where steel rupture prevails ranges from $\delta = 0.84$ to 0.80, for steel A ($f_t/f_y = 1.08$) it ranges from 0.78 to 0.68 and for A10 ($f_t/f_y = 1.15$) it falls below 0.66. For high hardening ratios (A11 with $f_t/f_y = 1.25$ and A12 with $f_t/f_y = 1.40$) the allowable degree of redistribution is smaller than $\delta = 0.60$ already in the range of a concrete crushing type of plastic hinge failure.

Here it also appears clearly that by virtue of the favourable effect of the ultimate strain the degree of redistribution becomes stronger as the hardening ratio increases. This follows from a comparison of A and A7 ($f_t/f_y = 1.08$) with A10 and S ($f_t/f_y = 1.15$), where in both cases the ultimate strain value varies from 5.0% to 6.0%, yet with a much more favourable effect for lower reinforcement ratios in the latter case.

As a step into the evaluation of the influence of cold worked steel properties, the effect of the length of the yield plateau is separately studied (see Fig. 8.18 and 7.37 for comparison). Although a variation of this parameter influences the values of the available rotation capacity both for the low and high reinforcement ratios, in the latter case the influence on the allowable degree of redistribution is rather moderate. For the lower reinforcement ratios the extension of the yield plateau plays some role, and a clearly higher redistribution capacity is found for the hot rolled steel type A15 than for the cold worked steel A13 = B1, regardless the equal values of the ultimate strain and hardening ratio. In this case both values are rather high, considering the realistic properties of the cold worked steel. The influence of these characteristics in a wide range is suitably described by the sets of steel properties variation used to obtain the curves in Fig. 8.19 - 8.21.

In Fig. 8.19 the allowable degree of redistribution is given as a function of the ultimate steel strain. For the steel types B6, B, B7 and B8 the same post-yield curve of the steel stress - strain diagram is followed with an ultimate strain value ranging from 2.0% up to 4%. The conclusions that follow from this analysis are similar to those drawn for the hot rolled steel type: an increase of the ultimate steel strain value improves the redistribution capacity in cases where steel rupture governs the deformation capacity of the plastic hinge. For this type of failure mode the range of the allowable degree of redistribution varies from $\delta = 0.94$ to 0.85 for steel B6 ($\epsilon_u = 2.0\%$) till 0.92 to 0.77 for steel B8 ($\epsilon_u = 4.0\%$) (for steel B ($\epsilon_u = 2.50\%$) in the range considered $\delta$ ranges from 0.92 to 0.94 to 0.83). In order to judge about the combined effect of an increase of both the ultimate strain and the hardening ratio an other set of simulations must be used. Before that, however, a separate analysis of the hardening ratio effect is performed for two levels of the ultimate steel strain: in Fig. 8.20 for steel types B4, B and B5 ($\epsilon_u = 2.50\%$) and in Fig. 8.21 for steel types B3, B2 and B1 ($\epsilon_u = 5.0\%$).
Figure 8.16 Allowable degree of redistribution - characteristic lower bound as influenced by the ultimate steel strain (steel A5, A6, A, A7, and A8)

Figure 8.17 Allowable degree of redistribution - characteristic lower bound as influenced by the strain hardening ratio (steel A9, A, A10, A11 and A12)

Figure 8.18 Allowable degree of redistribution - characteristic lower bound as influenced by the extension of yield plateau (steel A13, A14, A and A15)
Figure 8.19 Allowable degree of redistribution - characteristic lower bound as influenced by the ultimate steel strain (steel B6, B, B7 and B)

Figure 8.20 Allowable degree of redistribution - characteristic lower bound as influenced by the strain hardening ratio (steel B4, B and B5)

Figure 8.21 Allowable degree of redistribution - characteristic lower bound as influenced by the extension of yield plateau (steel B3, B2 and B1)
Once again the positive effect of an increasing hardening ratio on the allowable degree of redistribution is confirmed. Both for a lower and a higher value of the ultimate strain, an increase of the $\delta$ value follows from the increase of the ratio $f_t/f_y$. The comparison of both simulation sets is very interesting, because it confirms that the ultimate strain increase is more effective with respect to the allowable degree of redistribution if the hardening ratio also increases.

It should also be noticed that in the range of higher reinforcement ratios the value of the initial hardening ratio is the important parameter. This also explains why within a certain range of reinforcement ratios a better redistribution capability is obtained with the cold worked steel type than with the hot rolled one which is usually characterised by a higher "global" strain hardening ratio $f_t/f_y$. A comparison between different steel types is, however, more a subject for the next paragraph, where an attempt is made to arrive at provisions for the design standards. There also the basic steel types A, S and B are compared with one another, and judged on the basis of the structural performance that they can guarantee.

(2) Comparison with other models

The rotation capacity of a critical section, and thus also the moment redistribution capacity, depends on the depth of the compression zone at the ultimate limit state $x$ and, hence, on the mechanical reinforcement ratio of the section. Proven that the reinforcement in the tension and in the compression zone are properly incorporated, the mechanical reinforcement ratio of the reinforced section can be equally effective in capturing the same effects on the rotation capacity as the commonly used relative compression zone depth $\xi = x / h$. Moreover, contrary to $\xi$, the mechanical reinforcement ratio is independent of the sectional analysis method. Note that the value of $\xi$ strongly depends on the assumptions taken with respect to the basic constitutive laws of concrete and steel, that are not always evident and straightforward. For example, while for reinforcing steel the CEB-FIP Model Code 1990 permits to replace the actual diagram for a particular steel with an idealised bilinear elasto-plastic stress - strain curve, for concrete under compression two alternative simplifications of the stress - strain relationship are allowed: a parabola-rectangle stress-strain diagram and a uniform stress block. It is obvious that the calculated position of the neutral axis at failure differs when different approaches are taken.

Yet, to make it possible to compare the results of this study with proposals derived from other models and that included in some currently used design codes, the relative compression zone depth $\xi = x / h$ is used as a parameter. The values of $\xi$ are calculated
in the conventional way, assuming for steel a bilinear elasto-plastic stress - strain relationship with a strain limit of \( \varepsilon_u = 1.0\% \) and for concrete under compression a parabola-rectangle stress - strain diagram with a strain limit according to the CEB-FIP Model Code 1990 (for \( f_{ck} \leq 50 \text{ MPa} \) \( \varepsilon_{cu} = 3.5\% \)). The design values for concrete and steel strength are used and, following the provisions of the CEB-FIP Model Code 1990, they are generally defined as \( f_d = f_k / \gamma_m \), where \( f_k \) is the characteristic strength value and \( \gamma_m \) is the partial safety factor. Here \( \gamma_c = 1.5 \) is applied for the concrete under compression and \( \gamma_s = 1.15 \) for the steel. The same approach has been taken when formulating the relations for allowable degree of moment redistribution that are given in the CEB-FIP Model Code 1990 and in Eurocode No.2, as well as that proposed by Eligehausen and Langer (1987) and Eligehausen and Fabritius (1993). The design value of the mechanical reinforcement ratio \( \omega_{sd} \) for the reinforced section with no compressive reinforcement is calculated as \( \omega_{sd} = \rho_s \cdot f_{yd} / f_{cd} \), where \( \rho_s \) is the tensile reinforcement ratio, \( f_{yd} \) is the design value of the yield strength of the steel and \( f_{cd} \) is the design value of the concrete compressive strength (compare definition in Section 7.2.2 (2)).

In the preceding discussion on moment redistribution, a realistic description of the member behaviour was aimed at. Consequently the given values for \( \delta \) were related to the actually redistributed forces. These values represent a realistic lower boundary solution, in the sense discussed above. A major question which has yet to be seriously addressed is how design rules should be derived from either an experimental or a theoretical description of real member behaviour. For this question there is no straightforward answer. The design values for the degree of moment redistribution must be related to the desired safety level. An evaluation of this issue requires a complicated statistical analysis of probability of the failure being acceptably low from a safety point of view. Some studies of this problem have been initiated: e.g. Beeby (1991) considered the influence of the frequency distribution of rotation capacities on the frequency distributions of the resistance of statically indeterminate structures. Such a complex analysis is, however, beyond the scope of this project. Here a rather simplistic approach is taken, where on the one hand it is acknowledged that design values of the allowable degree of redistribution \( \delta_d \) may differ from the actual ones, due to the stochastic nature of material strength and bending resistance, and on the other hand the lack of sufficient information to quantify this dependence is admitted. When the global influence of these stochastic effects is confined in the coefficient \( K_\delta \), equation 8.21 follows:

\[
\delta = \delta_d \cdot K_\delta
\]  

(8.21)

In the lack of sufficient information to quantify coefficient \( K_\delta \), it is believed that assuming a value of \( K_\delta = 1 \) will lead to a reliable approximation. This agrees with the conclusion of Beeby (1991), who suggests that it is the mean value of available rotation
capacity that has to be taken as the design rotation capacity in order to obtain the same safety level (equal probability of failure) for an statically indeterminate structure and for a determinate structure designed to support the same load.

Using this concept as a basis for safety considerations, an appropriate relationships can be established between the allowable degree of redistribution $\delta_d$ and the design value of the mechanical reinforcement ratio $\omega_{sd}$ or the corresponding relative depth of the compression zone $\xi = x/h$. The relations between $\omega_{sd}$ and $\delta = \delta_d \cdot K_\delta$ are firstly obtained for the steel types defined in the former as A, S and B (in Fig. 8.22 both the exact and the schematised relations are given). Secondly, the simplified relations between $\xi = x/h$ and $\delta = \delta_d \cdot K_\delta (K_\delta = 1)$ derived on this basis are compared with the CEB-FIP Model Code 1990 provisions (Fig. 8.23) and with the other models and standards described in Section 8.1.1 (model of Eligehausen and Langer (1987) in Fig. 8.24, model of Eligehausen and Fabritius (1991) in Fig. 8.25, Eurocode No.2 in Fig. 8.26 and VBC 1995 in Fig. 8.27). Major conclusions from this comparison are presented separately for each type of steel. Considering that the simulations were performed for concrete with compressive strength $f_{cc} = 35$ MPa, corresponding relations (equations 8.1 and 8.9 in the case of the CEB-FIP Model Code 1990 and Eurocode No.2, respectively) are used for this evaluation.

The CEB-FIP Model Code 1990 allows in case of low ductility steel B a redistribution of at most $\delta = 0.90$ and for the relative depth of the compression zone $\xi \leq 0.12$ a value of the redistribution degree which is constant and equal to $\delta = 0.90$. For $\xi$ ranging from 0.12 to 0.20 a redistribution is permitted, at the respective low level. This is the most conservative of all models presented: also in the view of this study it seems to be too restrictive. The model of Eligehausen and Fabritius (1993) shows somehow higher redistribution capacities in the case of steel B. Here the range that holds an allowable redistribution degree of $\delta = 0.90$ is extended till $\xi = 0.20$, and the upper limit value of the effective compression zone depth is set at 0.30. The results obtained in the simulations discussed here are in a very good agreement with this approximation. In fact even a higher value of $\xi$ is found to set the upper limit for the application of the linear analysis method. Eligehausen and Langer (1987) do not distinguish between the steel types B and A, yet steel type B fulfils the requirements of their steel Type 2. The allowable degree of redistribution in this case seems to be not conservative, however. In particular the range of $\xi$ that corresponds with a degree of redistribution as high as $\delta = 0.85$ is very wide ($\xi = 0.33$ is also higher than the limit value according to the CEB-FIP Model Code 1990 and Eligehausen and Fabritius (1993)). Eurocode No.2 is very similar in its approach both to the model of Eligehausen and Langer (1987) and to the CEB-FIP Model Code 1990. As in the latter case, a distinction is made with regard to
the concrete strength, categorising them in grades higher and lower than C35. Once again a strong overestimation of the allowable redistribution capacity is concluded, both with respect to the ultimate value ($\delta = 0.85$) and the overall level. Note that the falling branch defined for the higher concrete grades fits much better the simulation results. It is reminded that a sudden reduction of the allowable redistribution degree attributed to the change of concrete quality does not agree with some other studies as well (eg. Cosenza et al., 1991).

As far as steel of higher ductility is concerned, similar conclusions as for lower ductility steel follow. Eurocode No.2 uses High-Ductility-Steel, that has characteristics identical with that of steel A. Comparison of the results of the simulations and provisions of Eurocode No.2 shows in this case a strong overall overestimation of the allowable degree of redistribution. Although the redistribution degree of $\delta = 0.70$ can be possibly reached in the simulations, the range where it can be safely applied seems not to be of practical importance. Eurocode No.2 proposes in the analysed case (i.e. lower concrete grades) for High-Ductility-Steel (comparable with steel A) exactly the same relation as Eligehausen and Langer (1987) propose for steel Type 1 (comparable with steel S). Obviously, due to the considerably larger deformation capacity of steel S, for lower reinforcement ratios the simulation results for steel type S compare better with this curves than those obtained for steel A. While according to the simulations for steel S a limit value of $\delta = 0.70$ is reached for $\xi \leq 0.15$, for steel Type 1 Eligehausen and Langer (1987) predict $\xi \leq 0.20$. However, in the range where concrete crushing governs the member failure (larger values of $\xi$, in the absence of compressive or confining reinforcement) model of Eligehausen and Langer (1987) largely overestimates the redistribution capacity of steel S. This aspect is frequently recalled and it seems that there is a lot of uncertainty considering this range of reinforcement ratios.

In a general sense, there seems to be basic disagreement concerning the available rotation capacity in the range of higher reinforcement ratios and its relation to steel ductility. While the proposal presented here as well as the model of Eligehausen and Langer (1987) and Eurocode No.2 arrive at a steel ductility independent relation for the allowable redistribution in the range of the higher reinforcement ratios, the CEB-FIP Model Code 1990 and Eligehausen and Fabritius (1991) make there a very strong distinction between different steel ductility classes. This distinction is very much unlikely to have a physical explanation, considering the shape of the stress - strain diagrams of the different steel types and the fact that as long as very low steel strains are mobilised at the failure stage, no profound effect of the ultimate steel deformation capacity $\varepsilon_u$ and the "global" strain hardening ratio $f_t/f_y$ can be found.
Figure 8.22  Allowable degree of moment redistribution as a function of the mechanical reinforcement ratio (exact and schematised relations at top and bottom left, respectively) and corresponding stress-strain diagrams of steel (right).

Figure 8.23  Comparison of proposed allowable redistribution degree and CEB-FIP Model Code 1990
Figure 8.24 Comparison of proposed allowable redistribution degree and model according to Eligehausen and Langer (1987)

\[ \delta = \delta_d K_\delta \quad (K_\delta = 1) \]

Figure 8.25 Comparison of proposed allowable redistribution degree and model according to Eligehausen and Fabritius (1991)

\[ \delta = \delta_d K_\delta \quad (K_\delta = 1) \]

Figure 8.26 Comparison of proposed allowable redistribution degree and model according to Eurocode No.2

\[ \delta = \delta_d K_\delta \quad (K_\delta = 1) \]
There are however some other effects that may influence the available rotation capacity in this range, and consequently manifest in terms of allowable degree of redistribution. This has been discussed in Section 8.2.3, and here only some examples will be given for better illustration. In Fig. 8.28 the model according to Eligehausen and Fabritius (1991) is compared with the allowable degree of redistribution obtained from simulations for steel type A. Besides the allowable redistribution that follows from the consideration of the characteristic lower bound solution (effective member height \( h = 400 \) mm, crack pattern with crack at mid span, maximum load level defining failure criteria) (schematised curve from Fig. 8.11) also the curve resulting from the simulations for a member of smaller size (effective member height \( h = 200 \) mm, crack pattern with crack at mid span, maximum load level defining failure criteria) is given (schematised curve from Fig. 8.12). A substantial increase of the allowable degree of redistribution results from the change of the member size all over the range analysed (here shown only up to a limit value of \( \delta = 0.80 \)). In this case perfect agreement is found between the relation proposed on the basis of simulations and that given by Eligehausen and Fabritius (1991). It is noted that almost exactly the same magnitude of additional allowable redistribution capacity is found if the crack pattern with no crack at mid span is assumed for a member effective height \( h = 400 \) mm (for comparison see Fig. 8.13). Changing the assumptions concerning the definition of the rotation capacity by assuming concrete crushing as a limit criterion instead of the moment when the maximum load level is reached would also favourably influence the computed value of the allowable degree of redistribution, in particular in the range of large reinforcement ratios (for comparison see Fig. 8.14). It is however not justified to allow for exploiting these margins in a general case. Structural effects on the rotation capacity and on the degree of moment redistribution must be considered, and as long as they are not explicitly included in the
design process, a safe approach must be chosen. Therefore it is concluded that the model of Eligehausen and Fabritius (1991) in the general case overestimates the allowable degree of redistribution in case of steel A for values of $\xi \geq 0.15$. The same holds true for steel S, which is likely to allow for less redistribution than suggested by Eligehausen and Fabritius (1991), as soon as the $\xi$ values exceed 0.15. Once again it is emphasised that all simulations discussed here were conducted with only one concrete strength and for the case of unconfined concrete in the compression zone.

It is worth noting that, although based on the model of Eligehausen and Fabritius (1991), the CEB-FIP Model Code 1990 allows for more redistribution in the case of steel A than the referred model. When limiting the degree of redistribution, the CEB-FIP Model Code 1990 combines for sake of simplicity the steel types A and S, and proposes an expression which in a way middles the relations given for both steel types in the model of Eligehausen and Fabritius (1991), taking over their deficiencies and adding inaccuracy. The CEB-FIP Model Code 1990 introduces a refinement by proposing a concrete strength dependent falling branch of the redistribution curve. Note that the relation proposed for concrete of higher grades fits the simulation results much better than the relation given for lower concrete grades, similar as in the case of Eurocode No.2. However, it is reminded that on the basis of the studies performed here it is difficult to judge in a general sense on the correctness of the manner of involving the concrete strength in the limitation of the degree of redistribution. This is one of the subjects that require further investigation.

![Figure 8.28](image-url)  
**Figure 8.28** Comparison of proposed allowable degree of redistribution and model according to Eligehausen and Fabritius (1991) - member size effect demonstrated for steel A

Finally, the Dutch code for the design of concrete structures VBC 1995 is discussed. Contrary to all other codes and theoretical models, in this case no account is taken of
steel ductility characteristics, reinforcement ratio (i.e. depth of the compression zone at failure) or concrete strength. Although it may be wondered to what extend some additional safety is provided by safety factors incorporated for loads of material strengths, undoubtedly it is not correct to neglect totally the differences in structural performance of different steel types in cases where steel rupture prevails. Furthermore, the problem of the redistribution due to cracking needs to be addressed. Here no specific requirements are provided to guarantee sufficient member ductility in the case of linear design. However, a considerable amount of rotation is required also in the case where members are not designed for redistribution of moments. The absence of a compatibility check regarding this rotation can reduce the structural safety uncontrollably. It is, hence of fundamental importance to establish limitations on the depth of the design neutral axis at failure, in the case of linear analysis without redistribution.

This is acknowledged in the two other design standards discussed here. The CEB-FIP Model Code 1990 states that for continuous beams and slabs sufficient ductility can be assumed to be present, if the related depth of the neutral axis \( \xi = x/h \) in the critical cross-section in the ultimate limit state is in accordance with the following formulations:

- for steel type S and A:
  - for concrete grades C12 to C35: \( \xi = x/h \leq 0.45 \)
  - for concrete grades C40 to C80: \( \xi = x/h \leq 0.35 \)
- for steel type B:
  - for concrete grades C12 to C80: \( \xi = x/h \leq 0.25 \)

Similar limitations are found in Eurocode No.2, which requires that in elements where no redistribution has been carried out the ratio of \( x/h \) at the critical section should not exceed:

- for any steel type:
  - for concrete grades C12/15 to C35/45: \( \xi = x/h \leq 0.45 \)
  - for concrete grades C40/45 and greater: \( \xi = x/h \leq 0.35 \)

Moreover, the standards recognise that special detailing provisions can increase the ductility and the moment redistribution ability. Hence, higher values of \( \xi \) can be admitted under more complex conditions or adoption of higher lateral confinement. Considering the results of the simulations performed within the scope of this project, this seems to be a safe approach. It still needs to be answered what the influence of the concrete strength can be in this respect, in particular with regard to very high concrete grades. Here also the possible member size dependence can come back into the picture,
since for very high concrete grades the size dependent concrete compressive failure in combination with increased brittleness of concrete can possibly strongly reduce the member ductility and lead to unexpected and explosive failure. This question remains open for the time being. Further theoretical and experimental analysis is necessary in order to evaluate the influence of an increased concrete grade on the deformation capacity of plastic hinges and the moment redistribution ability of continuous members.

Already now, however, some improvements can be suggested with respect to the performance of members made of normal strength concrete. Out of all the models considered in this evaluation, the proposal of Eligehausen and Fabrittius (1991) seems to be most realistic with respect to low ductility steel B (i.e. Normal-Ductility-Steel N according to Eurocode No.2 classification). The limitation of the $\xi$ value to 0.30 seems in this case rather conservative and a value as large as $\xi = 0.35$ is likely to be more appropriate. It seems practical to keep the same limit value for all steel types, making it possibly dependent on the concrete grade or member size. The reduction of the $\xi$ value for an increasing concrete grade and member size is at this stage, however, hard to quantify. Therefore a value of $\xi = 0.35$ is proposed in the general case. This is somehow more conservative than the provisions of the CEB-FIP Model Code 1990 and Eurocode No.2 for low concrete grades, and falls closer to the proposal for medium concrete grades, however agrees well with the results of other researches (Bigaj and Mayer 1998).

In general the explanation for the higher values of the allowable degree of redistribution given by both standards can be sought in the empirical access of available rotation capacity, that was used to validate both codes. Consequently, various combined structural effects (member size, concrete brittleness, feasible crack pattern, confining conditions, reinforcement detailing etc.) have not been explicitly evaluated. Moreover, the reinforcing steel used in experiments and classified to a certain ductility class has usually properties that are better than that required as the limit values.

Ponder that a general reliable safety level is aimed at, the above considerations lead to the following proposal for limiting the degree of moment redistribution (steel ductility classes are defined as in CEB-FIP Model Code 1990 (S, A, B) and Eurocode No.2 (H, N)):

- for steel type $S$:
  
  $\delta \geq 0.45 + 1.6 \times h$  
  $0.70 \leq \delta \leq 1.00$  
  
  (8.22)

- and

- for steel type $A$ (or $H$) and $B$ (or $N$):
  
  $\delta \geq 0.65 + x/h$  
  $0.80 \leq \delta \leq 1.00$  
  
  (8.23)

- and for steel type $A$ (or $H$):

- and for steel type $B$ (or $N$):

  $0.90 \leq \delta \leq 1.00$
Final Remarks on Structural Analysis

Figure 8.29 Proposal for allowable degree of moment redistribution according to equation 8.22 and 8.23 (conservative approach)

Figure 8.30 Proposal for allowable degree of moment redistribution according to equation 8.24 and 8.25 (permissive approach)

Being aware of the fact that a large additional capacity to redistribute moments can be activated due to the structural dependent effects discussed above, it is suggested that in some specific cases an increased allowable degree of redistribution should be introduced. When justified (e.g. considering the usually small depths of slabs without confining reinforcement and presuming the presence of the stirrups, confining the hinge region of larger beams) it seems possible to allow:

- for steel type S: \[ \delta \geq 0.45 + 1.25 \frac{x}{h} \]  \hspace{1cm} (8.24)
  and \[ 0.70 \leq \delta \leq 1.00 \]
- for steel type A (or H) and B (or N): \[ \delta \geq 0.65 + 0.8 \frac{x}{h} \]  \hspace{1cm} (8.25)
and for steel type A (or H): \(0.80 \leq \delta \leq 1.00\)
and for steel type B (or N): \(0.90 \leq \delta \leq 1.00\)

* if needed, it seems to be justified to extend these limits and permit:
  for steel type S: \(0.65 \leq \delta \leq 1.00\)
  for steel type A (or H): \(0.75 \leq \delta \leq 1.00\)
  for steel type B (or N): \(0.85 \leq \delta \leq 1.00\)

In this case a limit value of \(\xi\) required for linear design equal to 0.45 is recommended.

Diagrams showing the final proposal for the allowable degree of moment redistribution as a function of the design value of the relative depth of the compression zone at the critical sections are presented in Fig. 8.29 and 8.30. The former corresponds with the more conservative approach according to the equations 8.22 and 8.23, while the latter follows from the equations 8.24 and 8.25 and refers to the cases where it is justified to assume that due to structural effects an additional moment redistribution capacity can be mobilised. It is up to the practitioners view to decide which of the two approaches better corresponds with the conditions of standard design routine. It may be discussed in this respect whether there is a need of introducing extended code provisions in more specific cases, considering for instance member size or degree of confinement. Still it is not quite clear what is the magnitude of the effect of the concrete grade on the allowable degree of redistribution. The question of the deformation capacity of members in relation to the concrete strength and brittleness in general requires more extensive experimental and analytical investigation.
CONCLUSIONS

This investigation was intended to achieve a better understanding of the behaviour of plastic hinges and its structural dependence on the basis of experimental work and extensive parameter studies, using a rational model developed to analyse the phenomenon of plastic hinging in RC members. It was found essential to consider strain localisation in the damage zones in the hinge region when analysing the deformations in the plastic hinge. To that end a fracture mechanics approach to modelling the behaviour of concrete was used, both in compression and in tension. The phenomenon of size dependence as well as construction materials dependence of rotation capacity could reliably be studied by virtue of this approach.

There is a clear relation between the member size and its rotation capacity. While the rotation at the onset of yielding of the reinforcement is almost size independent, both the rotation at maximum load and the rotation capacity increase with decreasing member size in the case of members which fail due to rupture of steel as well as in the case when crushing of concrete prevails after yielding of steel. This structural size dependence is a consequence of cracking. RC structures exhibit complex cracking and a system of cracks must be considered when analysing their behaviour, contrary to the single mode-I cracks in plain concrete. It is found that in the analysed size range, when the fracture properties of concrete are kept constant, members which posses higher rotation capacity and, hence, exhibit more stable crack growth are more sensitive to the change of size, both in the case of steel and concrete failure. On the contrary, members which posses a lower rotation capacity due to the less stable crack growth are less sensitive to a size effect.

When analysing the member size dependence of RC members not only the overall member dimensions must be regarded but a number of other geometry dependent factors as well. Here the consequences of scaling the members, as encountered in practice with regard to the detailing of the reinforcement, are minded. Note that the bar size and effective concrete cover thickness dependence are implied in setting the bond between steel and concrete. Furthermore, the tension stiffening of steel, which is related to the bond strength, bonded area and tensile cord reinforcement ratio, effects the overall member deformations. Finally, the proportion between the reinforcement ratio of the tensile cord and the reinforcement ratio of the member must be considered, in order to account for the effect of non-uniformly distributed reinforcement on the bending stiffness of members with different sizes, amount and arrangement of reinforcement.

In order to have a ductile RC structure the arrangement of the reinforcement should
be related to the member size. A choice of the reinforcement arrangement can nearly eliminate the size dependence of the rotation capacity, if understood as a relation to the effective member depth. This does not mean, that size effects are eliminated from the plastic hinge performance, but that they are rather superimposed. An optimisation of the distribution of reinforcement from the point of view of the potential member size dependence is possible.

Due to a large number of independent parameters that are involved no general size dependence rule for RC members can be formulated. Besides structural dimensions, the occurrence and the growth of the system of cracks in RC member strongly depends on the fracture properties of the construction materials and on the structural stiffness, related to the bond characteristics, the area of reinforcement and its distribution. The importance of the geometrical effect in practical design situations has been evaluated, showing that no justification exists for alteration of existing design rules in the light of a possible member size dependence of the rotation capacity of plastic hinges. Aiming at future forming of the basis for codes of practice, it is emphasised that an overestimation of the actual member rotation is very likely if the available rotation capacity is based on the evaluation of the behaviour of the reference members within a limited size range. It can amount up to about 20% if $h = 400$ mm or up to almost 50% if $h = 200$ mm is taken as the reference value. The bigger the size of the member used as a reference, the larger the choice of adequate reinforcement arrangement and the wider the size range of proportionally scaled members, which safety is not negatively influenced by consequences of the ductility reduction due to the size dependence. In engineering practice one should be aware of the fact that a poor design in combination with an unavoidable size dependence of the deformation capacity of RC members can endanger the safety margin of the codes provisions.

Mechanical characteristics of steel play an important role in limiting the deformation capacity of plastic hinges, especially in cases where a limited steel deformation capacity controls member failure. Numerical parameter studies are likely to be the best way to determine the relation between the reinforcing steel characteristics and the ductility of reinforced concrete members. Such an approach allows to focus on one isolated question only and in this way the scatter due to simultaneous variation of more parameters, inherent to any experimental investigation, can be avoided.

It is commonly granted that the use of cold worked steel can reduce the member ductility significantly, compared to cases where hot rolled (or heat treated) steel is used. Yet, a decrease in ductility of the steel bars manifests in a reduction of the rotation capacity only within a limited range of reinforcement ratios and as long as the steel
strains remain low no significant difference is observed between hot rolled and cold worked steel, both with respect to steel strain distribution and crack width development. There is a very strong increase of the rotation capacity as soon as steel strains higher than the strain at the onset of strain hardening are activated at failure. Therefore, in members that fail due to concrete crushing, a slightly larger rotation capacity may be reached using cold worked steel, characterised by weak yet early strain hardening, than when hot rolled steel with a distinct yield plateau and strong but late strain hardening is used. In the case of steel failure the ultimate strain increase is more effective with respect to the available rotation capacity if the hardening ratio also increases and in this range the use of currently produced hot rolled steel results in general in a higher rotation capacity than the use of cold worked steel.

It is misleading to focus on isolated steel characteristics and steel must not be judged in isolated terms, but according to the deformation capacity of the reinforced members, which represent an overall assessment of their structural performance. To quantify this property a combination parameter is required. The proportionality between the value of the equivalent steel parameter \( p \) and the plastic rotations proves that not the absolute difference in the values of the individual steel characteristics, but its combination is crucial for the attained deformation capacity of a member. A small variation of a single property may in some cases mean a substantial change for the performance delivered. Although, considering the complexity of the problem, it seems to be too early to develop a standard formula that would directly provide the allowable rotation capacity of members that fail due to steel rupture, a general quantitative assessment of the steel properties provided by the equivalent steel ductility parameter \( p \) allows for a proper comparison of different steel types from the perspective of their structural performance. By virtue of this potential it is a very practical criterion to be implicated in the material standards and codes of practice.

The effect of concrete strength and ductility characteristics on strain localisation, both in compression and in tension, and its influence on the available rotation of plastic hinge and on its size dependence is not fully explained. The impact of a more brittle material with a more linear behaviour should be considered in the design, also in respect to the available deformation capacity. Some critical issues should be minded when comparing the deformation capacity of normal strength and of high strength RC members: (1) flexural failure due to crushing of concrete in the compression zone is likely to be more sudden and abrupt in the case of high strength concrete because of its lower toughness, (2) for small slip values the bond of reinforcing bars in high strength concrete is stronger and the bond behaviour is stiffer, but for large slip values and after yielding of steel the values of the bond stress are lower for high strength concrete and
the tension stiffening effect is presumably less in this case, (3) the member size
dependence of the rotation capacity may manifest for a different size range due to the
different crack propagation in high strength concrete members, (4) due to geometrical
and mechanical reasons and in relation to the concrete quality various local failure
phenomena can occur in case of high strength RC members which influence the rotation
capacity and the reserve post-peak rotation (e.g. spalling of the concrete cover). The
development of a behavioural model for evaluating the ductility of high strength
reinforced concrete members is clearly still standing out.

The deformation capacity of a plastic hinge is directly related to the length of the plastic
hinge, which depends among other things on the force transfer from reinforcement to
concrete between subsequent bending cracks. However, not only the crack spacing and
bond of steel to concrete between the cracks but also the typography of the developed
crack pattern is of influence on the rotation capacity of the plastic hinge. There is nearly
an equal possibility of developing a symmetrical crack pattern with and without crack
at the mid-span. Depending on the position of the cracks in the presence of a moment
gradient, the length of the bars where plastic deformations localise significantly differ
and a huge supplementary rotation capacity is foreseen if a favourable crack pattern
is regarded. It should be therefore distinguished between two distinct failure modes:
ductile failure with dispersed cracking and non-ductile failure with a single major crack.
It is indispensable to mind this phenomenon when interpreting test results, since a
member with a crack at a mid span provides a lower bound for the rotation capacity
(non-ductile failure), while a member with a symmetrical crack pattern with no crack
at mid-span yields, respectively, an upper bound (ductile failure). It is not conservative
to consider anything but the lower bound values when evaluating the available rotation
capacity of flexural members in a general case.

Properties of construction materials, both characteristics of reinforcing steel and concrete,
influence the bond behaviour which, in a turn, is one of the most important factors
controlling deformation capacity of a plastic hinge. In order to analyse the deformation
capacity of reinforced concrete members up to the ultimate loading stage a bond model
that takes into account not only the geometry of the bar surface but also the state of
stresses in the bar, the concrete quality, the degree of confinement and the
 corresponding mode of bond failure is required.

The CEB-FIP Model Code 1990 underestimates the bond stiffness for low slip values
in the elastic range of the steel strains in this range both for NSC and HSC in the case
of confined and unconfined concrete. It does not reflect the change in the bond stress
- slip relation due to the yielding of steel and strongly overestimates the bond of yielding
CONCLUSIONS

steel. The stress-strain behaviour of steel is not taken into account and the confinement is taken into account only in global terms. The empirical character of the CEB-FIP Model Code 1990 formulation and the fact that it has been tuned on experiments carried out on concrete with medium strength is the reason why the extrapolation for high strength concrete leads to incorrect predictions. It is indispensable to account for the concrete fracture characteristics, when modelling bond behaviour.

The concrete confining capacity, attributed both to active and passive confinement, has a decisive influence on the ultimate bond resistance and on the mode of bond failure. It is necessary to quantify the confining capacity of the concrete surrounding the bar (e.g. by means of the thick-walled cylinder model) and to include this quantity in the description of the bond behaviour. Bond splitting prevails when the confinement is not sufficient to prevent the radial cracks from penetrating the whole concrete cover. With enough confinement the concrete teeth in front of the ribs are sheared off resulting in a new cylindrical sliding plane and pull-out failure takes place. The confinement delivered by the concrete not only depends on the smallest concrete cover, but a greater part of the concrete cross-section may contribute to it and the actual member geometry must be involved in determining the degree of confinement.

For splitting bond failure and as long as rib bearing is the force transferring mechanism the Poisson effect has a negligible influence on the bond resistance of ribbed bars. In the case of splitting bond failure and in the absence of additional confinement the bond stress-slip relationship is directly related to the clamping action of the concrete surrounding the bar, which is proportional to the depth of effective concrete cover. When for pull-out failure the force transfer mechanism changes from rib bearing to friction the local transverse deformation can not be disregarded and in this case an increasing steel stress results in a release of the radial strain due to the Poisson effect and, thus, in a reduction of the bond stress, which becomes pronounced when the steel starts to yield. In the case of pull-out failure the bond stress is primarily a function of the slip and the steel strain. There is a significant influence of the stress-strain characteristics of reinforcing steel deformation on the bond strength developed after yielding of steel in case of pull-out failure. For given construction materials characteristics and well defined specimen geometry there is an unique bond stress-steel strain-slip relationship, however the bond stress-slip relationship is not unique but dependent on the actual loading and boundary conditions.

The ductility of RC members can be influenced by changing the bond properties of the reinforcing bars. Since the bar deformations pattern influences the bond characteristics, attention should be given to optimisation of the geometry of reinforcing
bars. The proposed bond model has been tuned for bars with a medium relative rib area, however an increase of the related rib area is expected to cause an improvement of the bond strength combined with an increase of the bond stiffness in the initial stage and a stronger splitting action. When improved by introducing as an independent parameter the variation in bar geometry (e.g. rib form or rib factor), this bond model can successfully facilitate the optimisation of bond properties of reinforcing bars both in the elastic and in the plastic regime. Note that bond should ensure a high structural stiffness and small cracks in the serviceability limit state, generate small splitting forces and allow full utilisation of the reinforcement ductility in the ultimate limit state. Both an experimental and a numerical study in this field are encouraged and the development of new types of rib pattern seems to be the right direction for further investigation.

In order to extend the range of application of the proposed bond model it is important to take in the model into account the effect of the additional confinement, both active (i.e. resulting from loads transverse to the bar, e.g. from a support or from the column force in a beam-column joint) and passive (i.e. confinement generated by transverse reinforcement). Also the mechanical action of other anchorage elements (i.e. hooks, loops, transverse reinforcement) should be considered so that not only the ordinary straight reinforcing bars but also other types of reinforcement can be properly modelled.

It was aimed at determining the conditions for the analysis of statically undeterminate systems, in relation to the steel quality and to its structural performance. The allowable degree of moment redistribution, controlled by the member ductility, was estimated using the evaluation of the available and required rotation capacity of plastic hinges. In this perspective the limitations of the presently used design standards were verified and revised rules were proposed. For the ductility classes defined in the CEB-FIP Model Code 1990 the following is concluded with respect to linear design with moment redistribution: the allowable degree of moment redistribution should not be more than $\delta = 0.85$, 0.75 and 0.65 for steel class B, A and S, respectively. Sufficient ductility for linear design without redistribution may be assumed if in the critical cross-section the relative depth of the neutral axis at failure is lower than $\xi = 0.35$. This value is valid for linear members with no or little confining reinforcement. However, it is recognised that the member ductility can be increased significantly by special detailing or other structural effects. Therefore higher values of $\delta$ and $\xi$ can be admitted under more complex conditions or when the concrete is sufficiently confined by stirrup reinforcement. The relation between $\xi$ and the degree of moment redistribution $\delta$ is constituted for each steel class, as described in Chapter 8. The need for a clear explanation of the relation between the construction materials properties and the member ductility, expressed by the practitioners, underlines the importance of further studies in
this field, in particular with respect the effect of the concrete grade on the allowable degree of redistribution.

Proven that the reinforcement in the tension and in the compression zone are properly incorporated, the mechanical reinforcement ratio can be equally effective in capturing the same effects on the rotation capacity as the commonly used relative compression zone depth $\xi$. Moreover, contrary to the mechanical reinforcement ratio the value of $\xi$ strongly depends on the assumptions taken with respect to the basic constitutive laws of concrete and steel, that are not always unique (e.g. in the CEB-FIP Model Code 1990). Therefore it is strongly supported to consider the use of the mechanical reinforcement ratio instead of $\xi$ when constituting the relationships of the available rotation capacity or allowable degree of moment redistribution.

The limitation of both member size and feasible crack pattern was intended to give conservative values for the allowable degree of moment redistribution. For the same reason no steel overshoot or overreinforcement was taken into account. However, there is a significant variation of steel characteristics with respect to the nominal values and greater amount of reinforcement than required from the design can hardly be avoided in practice. **The steel overshoot and overreinforcement increase the safety of structural members with respect to the bending capacity in terms of applied loads and less redistribution of moments compared to the design value is required in this case to reach the design load capacity of continuous beams or slabs.** However, the overshoot or overreinforcement cannot be used to cover the influence of imposed deformations, which directly demand deformability of the structure. **The safety margin with respect to the force transfer to adjacent members, the shear resistance and the bond resistance of staggered reinforcement can be considerably reduced if the effect of steel overshoot and overreinforcement is not taken into account in design.** Note that the corresponding properties of the steel characteristics (overshoot) are currently missing in the material standards.

To increase the knowledge of structural effects on rotation capacity further experimental and analytical research is needed. In particular attention should be given to the influence of concrete grade and brittleness, covering besides high strength also fibre reinforced concrete. Bond of reinforcing steel should further be researched, both in relation to the rib pattern and construction materials properties. When optimising bond it should be minded that it is necessary to consider the specific fracture characteristics of different types of concrete. The influence of detailing of reinforcement, in particular the effect of confinement must be carefully analysed when estimating the deformation capacity of the members. An effort should be made to harmonise material standards and codes of
practice so that the requirements for construction materials fully comply with the demanded structural performance.
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SUMMARY

Structural Dependence of Rotation Capacity of Plastic Hinges in RC Beams and Slabs

by Agnieszka Joanna Bigaj

The theory of continuum mechanics is the currently accepted mechanical basis for the analysis of static systems. The description of the equilibrium conditions, compatibility equations and constitutive laws of concrete, steel and bond control the accuracy of the numerical approximation of the real physical structure's behaviour. Besides the non-linear analysis method, that implicitly takes advantage of the non-linearity of RC behaviour, for statically indeterminate structure a elasto-plastic analysis methods are nowadays commonly used in design. Each of the different design strategies, however, has a well defined range of validity, that follows from the necessity of ensuring the equilibrium and the compatibility of deformations at each of the possible loading conditions and depends on the deformation capacity of the structure. Also a number of other engineering problems can be solved only when a sufficient deformation capacity of the RC members is assured.

The purpose of the research, presented in this Thesis, is to provide a sufficient understanding of the structural and material influences on both the available and the required deformation capacity of RC structures and to formulate on this basis the application limits for elastic analysis with moment redistribution. The presently used design standards do not limit their application field to a certain selected range of member dimensions and allow the application of similar design procedures for a wide range of structural materials. However, the reduced deformation capacity of reinforced concrete, attributed to the considerable changes of the properties of structural materials in recent years (both concrete and steel) can question the validity of the existing models, especially in combination with the member size dependence of the rotation capacity of plastic hinges. The adequacy of conventional methods for calculating the deformation capacity of RC members is doubtful, since it shows a lack of sufficient understanding of the complex structural dependence of the rotation capacity (Chapter 1).

In this Thesis the increase of ductility with decreasing member size is interpreted from the viewpoint of fracture mechanics of concrete. The influence of the ductility characteristics of the reinforcing steel is studied in relation to bond of the reinforcement and the reinforcement detailing. The results of the own test series on simply supported slender beams loaded in three-point bending as well as references from the literature are used to determine the effect of member size, reinforcement ratio and concrete and steel
type on the deformation capacity of plastic hinges (Chapter 2).

In order to formulate the model for calculating the rotation capacity of plastic hinges in linear RC members the behaviour of plastic hinges is analysed taking into account the strain localisation in the damage zones in the hinge region. The concrete is modelled by virtue of a fracture mechanics approach: the Fictitious Crack Model (FCM) and the Compressive Damage Zone (CDZ) model are adopted to describe the behaviour of concrete in tension and compression, respectively (Chapter 3). To analyze properly the deformation capacity of RC up to the ultimate loading stage a bond model is needed that takes into account not only the geometry of the bar surface and the concrete quality but also the state of stresses in the bar, the degree of confinement and the corresponding bond failure mode. The own experimental study of bond is reported and a new, fracture mechanics based, bond model for ribbed bars is introduced (Chapter 4).

The analysis of crack formation and development and of the tension stiffening effect in direct tension and in bending allow to verify the new bond model and examine the fundamental role of the properties of structural materials. The importance of implementing the influence of steel yielding when modelling the bond of the reinforcement is illustrated, in particular in relation to the rotation capacity of plastic hinges (Chapter 5).

The accuracy of the new calculation model for rotation capacity in predicting the member size dependence and the influence of structural materials characteristics on the rotation capacity of plastic hinges is verified against the results of three point bending tests for different reinforcement ratios, concrete types and steel ductility classes (Chapter 6). The significance of the way of detailing the reinforcement is shown and a method is proposed to include the reinforcement detailing in analysing the member sensitivity to size dependence. The importance of the size effect in practical design situations is studied and the need for a modification of the existing design rules in the light of a possible member size dependence of the rotation capacity of plastic hinges is evaluated. The relation between the isolated steel characteristics and the overall assessment of the structural performance of the steel is investigated analysing the rotation capacity of plastic hinges (Chapter 7).

The relation between the steel ductility properties, demand and supply of the ductility of RC members and the allowable degree of moment redistribution for a linear analysis followed by limited moment redistribution in the case of linear members (continuous beams and slabs) is evaluated (Chapter 8). The limitations of the presently used design standards (CEB-FIP Model Code 1990, Eurocode No.2 and VBC 1995) are verified and
revised rules for allowable degree of moment redistribution are proposed. In the conclusions the main observations from the numerical simulations are summarised, in particular with respect to bond of ribbed steel and the structural dependence of the rotation capacity of plastic hinges. Some prospects of further research are given, mainly related to new developments in the field of structural materials and the possibility to optimise member design with respect to ductility requirements (Chapter 9).
SAMENVATTING

Constructieve afhankelijkheid van de rotatiecapaciteit van plastische scharnieren in balken en platen van gewapend beton.

door Agnieszka Joanna Bigaj

De theorie van de continuüm mechanica vormt tegenwoordig de basis voor de analyse van statische systemen. De beschrijving van de evenwichtskondities, compatibiliteitsvergelijkingen en constitutive wetten van beton, staal en aanhechting bepalen de nauwkeurigheid van de numerieke benadering van het werkelijke constructiegedrag. Naast niet-lineaire analysemethoden, die impliciet gebruik maken van de niet-lineariteit van gewapend beton, worden tegenwoordig elastoplastische analysemethoden toegepast voor het ontwerpen van statisch onbepaalde constructieelementen. Elk van deze ontwerpstrategieën heeft echter duidelijk een afgebakend toepassingsgebied. De grenzen van het toepassingsgebied zijn rechtstreeks afhankelijk van de vervormingscapaciteit van de constructie. Verder kunnen een aantal constructieproblemen alleen opgelost worden wanneer de constructieelementen voldoende vervormingscapaciteit bezitten.

Het doel van het onderzoek, zoals gepresenteerd in dit proefschrift, is voldoende begrip te krijgen van de constructieve en materiële invloeden op zowel de beschikbare als de vereiste vervormingscapaciteit van constructies in gewapend beton. Op deze basis worden de toepasbaarheidsgrenzen voor een elastische analyse met momentenverdeling bepaald. De tegenwoordig gebruikte ontwerpnormen beperken hun toepasbaarheid niet tot een beperkt gebied van constructiedeelafmetingen en staan toe dat voor een groot scala aan constructie materialen dezelfde ontwerpmethodes worden gebruikt. Echter de gereduceerde vervormingscapaciteit van gewapend beton, als gevolg van de duidelijke veranderingen van de eigenschappen van de constructiematerialen in de laatste jaren (zowel van beton als van staal), maakt de toepasbaarheid van de huidige modellen twijfelachtig. Dit geldt met name in combinatie met de veronderstelling dat de rotatiecapaciteit van plastische scharnieren afhankelijk is van de grootte van de constructiedelen. De betrouwbaarheid van conventionele berekeningsmethoden van de vervormingscapaciteit van plastische scharnieren staat ter discussie, gezien het gebrek aan voldoende begrip van de complexe relatie tussen de rotatiecapaciteit en de constructieeigenschappen (Hooftstuk 1).

In dit proefschrift is de toename van de ductilitéit bij een afname van de grootte van de constructiedelen, bekeken in het licht van de betonbreukmechanica. De invloed van de ductiliteitskenmerken op het wapeningsstaal is bestudeerd in relatie tot de
aanhechting van wapening en het wapeningsontwerp. De resultaten van de eigen drie-
puntsbuigproeven op slanke balken evenals van literatuuronderzoek, zijn gebruikt om de
invloed van de constructiedeelafmetingen, het wapeningspercentage en het beton en staal
type op de vervormingscapaciteit van plastische scharnieren te bepalen (Hoofdstuk 2).

Om een berekeningsmodel voor de rotatiecapaciteit van plastische scharnieren in balken
en in-één-richting-dragende platen te formuleren, is het gedrag van plastisch scharnier
geanalyseerd. Hierbij werd rekening gehouden met de spanningsconcentraties in de
deformatie zones van het scharnier. Het beton is gemodelleerd vanuit de verworvenheden
van de breukmechanica: het Fictieve Scheur Model (FCM) en het Compressive Damage
Zone Model (CDZ) zijn gebruikt om het gedrag van het beton te beschrijven (Hoofdstuk
3). Om de vervormingscapaciteit van gewapend beton tot aan bezwijken te analyseren,
is een aanhechtmmodel nodig dat niet alleen rekening houdt met de profilering van het
wapeningsstaal en de betonkwaliteit maar ook met het spanningsniveau in de wapening,
de mate van opsluiting en het corresponderende bezwijktype van de aanhechting. Een
eigen experimenteel onderzoek van het aanhechtmechanisme wordt gepresenteerd en een
nieuw aanhechtmmodel voor geribde wapeningsstaal, gebaseerd op de beton-
breukmechanica, wordt voorgesteld (Hoofdstuk 4).

De analyse van scheurvorming en scheurontwikkeling evenals van het tension stiffening
effect in één-assige trek en buiging maken het mogelijk om het nieuwe aanhechtmmodel
te verifiëren en om de fundamentele rol van de eigenschappen van de constructie-
materialen te onderzoeken. Het belang van het impliciet rekening houden met het vloei-
van het staal bij het modelleren van de aanhechting van de wapening is geïllustreerd,
met name in relatie tot de rotatiecapaciteit van het plastische scharnier (Hoofdstuk 5).

De nauwkeurigheid van het nieuwe rekenmodel voor de rotatiecapaciteit bij het
modelleren van de schaal-effecten en van de invloed van constructiematerialen op de
rotatiecapaciteit van plastische scharnieren is geverifieerd met de resultaten van drie-
puntsbuigproeven voor verschillende wapeningspercentages, betontypes en staal-
ductiliteitsklassen (Hoofdstuk 6). Het belang van het wapeningsontwerp is aangetoond
en een methode is voorgesteld waarmee de invloed van de detailering van de wapening
wordt meegenomen in de analyse van de schaalgevoeligheid van de constructiedelen.
Het belang van schaal-effecten in praktische ontwerpsituaties is bestudeerd en de
noodzaak van het aanpassen van de bestaande ontwerpregels in het licht van mogelijke
schaalafhankelijkheid van de rotatiecapaciteit van plastische scharnieren is geëvalueerd.
De relatie tussen de geïsoleerde staaleigenschappen en de globale prestaties van het staal
is onderzocht vanuit het oogpunt van de rotatiecapaciteit van het plastische scharnier
(Hoofdstuk 7).
SAMENVATTING

De relatie tussen de staalductiliteitskarakteristieken, vraag en aanbod van ductiliteit van het gewapend beton en de toelaatbare momentenverdeling voor doorgaande balken en platen, is geëvalueerd (Hoofdstuk 8). De relevante ontwerpregels in de voorschriften zoals CEB-FIP Model Code 1990, Eurocode No.2 en VBC 1995 zijn geverifieerd en aangepaste regels voor de toelaatbare momentenverdeling worden voorgesteld. In de conclusies zijn de belangrijkste waarnemingen van de numerieke simulaties samengevat, in het bijzonder met betrekking tot de aanhechting van geribd staal en de constructieve afhankelijkheid van de rotatiecapaciteit van plastische scharnieren. Enkele aanbevelingen voor verder onderzoek zijn gegeven, voornamelijk in verband met nieuwe ontwikkelingen op het gebied van de constructiematerialen en de mogelijkheden om het ontwerpen van constructieelementen te optimaliseren voor wat betreft de ductiliteitseisen (Hoofdstuk 9).
Figure A1.1  Position and numbering of the measuring devices (general overview for all specimen sizes)
Figure A1.2  Arrangement of the measuring devices in hinge region of specimens B.0.1, B.1.1

Figure A1.3  Arrangement of the measuring devices in hinge region of specimens B.0.2, B.1.2
Numbering of extensometers in compression zone of the hinge

| Number | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | R | S | T | U | V |
| Front face | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| Rear face   | 22| 23| 24| 25| 26| 27| 28| 29| 30| 31| 32| 33| 34| 35| 36| 37| 38| 39| 40| 41| 42 |

Figure A1.4  Arrangement of the measuring devices in hinge region of specimens B.0.3, B.1.3
<table>
<thead>
<tr>
<th>Table zone</th>
<th>Load level $P_{max} = 3.3,\text{kN}$</th>
<th>Load level $P_{max} = 2.5,\text{kN}$</th>
<th>Load level $P_{max} = 3.3,\text{kN}$</th>
<th>Load level $P_{max} = 2.5,\text{kN}$</th>
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</table>

(*) arrangement of the measuring devices according to Fig. A1.1 and Fig. A1.2
<table>
<thead>
<tr>
<th>Shoreline</th>
<th>Distance of Measuring Device (mm)</th>
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<th>Distance of Measuring Device (mm)</th>
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**APPENDICES**

**Profile Face**

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**Profile Face**

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<th>Shoreline</th>
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**APPENDICES**
<table>
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<th>Layer depth (m)</th>
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**Table Notes:**
- The table provides data on depth, porosity, density, water content, CO₂ content, temperature, and pressure at various layer depths.
- The data is presented in a tabular format with columns for depth, porosity, density, water content, CO₂ content, temperature, and pressure.

**Appendix:**
- The appendix contains additional data related to the above table, including:
  - **Porosity Variations:**
  - **Density Annotations:**
  - **Water Content Analysis:**
  - **CO₂ Content Statistics:**
  - **Temperature Measurements:**
  - **Pressure Calculations:**
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**Measurement from Three-Point-Bending Tests**
### Table 1: Measurements from Three-Point Bending Tests

<table>
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<tr>
<th>Load Level</th>
<th>Left Zone</th>
<th>Zone in Middle</th>
<th>Right Zone</th>
<th>Mean Face</th>
<th>Points in ...</th>
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</table>

**Notes:**
- Points in ... indicates the measurement points or units, likely representing the bending or deflection under different load levels.
- The table likely represents a multi-column format, with each column indicating different zones or measurements under varying load conditions.

### Additional Notes
- **Load Level:** Indicates the magnitude of the load applied to the material.
- **Mean Face:** The average face value across the measured points.
- **Left Zone, Zone in Middle, Right Zone:** Represent different sections or locations under test, possibly measuring deflection or deformation at these points.
A2 Global steel ductility parameter $p$

New formulation for the global ductility parameter $p$ is proposed, that better takes into account relative influences of specific steel characteristics on attained members ductility. Here, for all steel types referred to in these Thesis, comparison is given of parameter $p$ values according to the new formulation (equation 6.2 and 6.2) and to the old one (equation 2.40 and 2.41), proposed by Cosenza and Manfredi, 1996.

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<tr>
<th>Steel code</th>
<th>$\varepsilon_{sh}$ [%]</th>
<th>$\varepsilon_u$ [%]</th>
<th>$f'_s / f'_y$</th>
<th>$p$ (old)</th>
<th>$p$ (new)</th>
<th>remarks</th>
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Simulations of steel ductility dependence on rotation capacity and moment redistribution capability (Chapter 6, 7)
NOTATIONS

The explanations of notations in the text in direct conjunction to their appearance have preference to what is treated here.

Upper case letters

\( A_c \) - concrete cross-section
\( A_{c, ef} \) - effective concrete tension area
\( A_s \) - steel cross-section
\( C_1 \) - variable
\( C_2 \) - variable
\( C_3 \) - variable
\( D' \) - shear force at the transition point from the span to the support zone
\( E_c \) - modulus of elasticity of the concrete
\( E_f \) - fictitious modulus of elasticity
\( E_s \) - modulus of elasticity of the steel
\( (EI)_g \) - flexural stiffness
\( F(\delta) \) - function of local slip
\( F_1(\delta,\varepsilon_s) \) - function of the local slip and steel stress
\( G_F \) - concrete fracture energy per unit area
\( G_{FA} \) - apparent fracture energy per unit area
\( G^1 \) - energy per unit area perpendicular to \( \sigma_c \) absorbed in the shear-band
\( I_c \) - moment of inertia of the concrete section
\( I_{cs} \) - moment of inertia of the uncracked reinforced cross-section
\( K_8 \) - ratio between actual and design value of allowable degree of moment redistribution
\( K_{\Theta} \) - coefficient for the influence of slenderness ratio on rotation capacity
\( K(\varepsilon_{r1}) \) - constant with which the reduction rate of \( \varepsilon_{r1} \) is controlled
\( K(\delta_3) \) - constant with which the reduction rate of \( \delta_3 \) is controlled
\( K(\varepsilon_{r3}) \) - constant with which the reduction rate of \( \varepsilon_{r3} \) is controlled
\( L \) - specimen length; also denotes length containing one localisation zone
\( L^d \) - length of the damage zone
\( L_{ele} \) - length of finite element
\( L_{key} \) - length of the concrete corbels between two subsequent ribs
\( L^i \) - distance between the successive damage zones; under assumption that \( L^i = L^d \) also denotes length of the localization zone
\( L^{(n)} \) - length of the localization zone in FC element \( n \)
\( L^{(n+1)} \) - length of the localization zone in FC element \( n+1 \)
$M_A$ - moment resistance in the support
$M_p$ - moment resistance in the span
$M_{cr}$ - cracking moment
$M_{crI}$ - cracking moment in section I
$M_{crII}$ - cracking moment in section II
$M_{pl}$ - yielding moment
$M_{plI}$ - yielding moment in section I
$M_{plII}$ - yielding moment in section II
$M'$ - moment at the point of transition from the span to the support zone
$N_{cr}$ - cracking force acting on the effective concrete tension area
$P_{max}$ - maximum load carrying capacity of the bar
$W^c$ - total compressive fracture energy per unit volume
$W_{cr,2}$ - resistance moment of the uncracked cross-section
$W^{in}$ - energy per unit volume absorbed in pre-peak in longitudinal cracks
$W^s$ - energy per unit volume absorbed in post-peak in longitudinal cracks
$W^t$ - total tensile fracture energy per unit volume

Lower case letters

$a$ - FCM constant
$b$ - width of the cross-section; also denotes FCM constant
$c$ - clear concrete cover on the bar
$c_1$ - clear concrete cover plus bar radius
$c_{eff}$ - effective concrete cover
$c_{eff,max}$ - maximum effective concrete cover
$c_i$ - cover thickness in each of $m$ directions
$c_{i,min}$ - smallest concrete cover to be taken into account
$cot(\phi)$ - coefficient of friction
$d$ - total height of the cross-section
$d_{a,max}$ - maximum aggregate size of concrete
$d_c$ - diameter of cylindrical specimen
$d^t$ - depth of the compression damage zone
$d_s$ - bar diameter
$e_a$ - distance between the gravity points of plain concrete and reinforced concrete section
$f_c$ - cylinder compressive strength of concrete
$f_{cc}$ - cube compressive strength of concrete
$f_{cc}^*$ - cube compressive strength of concrete in test conditions
$f_{cd}$ - design value of cylinder compressive strength of concrete
$f_{ck0.05}$ - characteristic lower bound of cylinder compressive strength of concrete
$F_{ck0.95}$ - characteristic upper bound of cylinder compressive strength of concrete
$F_{cm}$ - mean value of cylinder compressive strength of concrete
$F_{ct}$ - uniaxial tensile strength of concrete
$F_{ct,f}$ - flexural tensile strength of concrete
$F_{ct,0.05}$ - characteristic lower bound value of the flexural tensile strength
$F_{ct,0.05}$ - characteristic lower bound value of the uniaxial tensile strength
$F_{cm}$ - mean value of uniaxial tensile strength of concrete
$F_{ct}$ - splitting tensile strength of concrete
$F_{ct}^*$ - splitting tensile strength of concrete in test conditions
$F_d$ - design strength value
$F_k$ - characteristic strength value
$F_R$ - projected rib area
$F_t$ - tensile strength of steel
$F_t/F_p$ - strain hardening ratio of steel
$(F_t/F_p)_K$ - characteristic value of the hardening ratio of steel
$F_p$ - yield strength of steel
$F_{yd}$ - design value of yield strength of steel
$g$ - parameter dependent on the detailing of reinforcement detailing
$h$ - effective height of the cross-section
$h_c$ - total height of cylindrical specimen
$h_{db}$ - bond-free part of cylindrical specimen
$h_o$ - constant for calculation of flexural tensile strength of concrete
$i$ - ordinal integer number
$k$ - proportionality factor for the energy dissipation in the longitudinal tensile cracks in CDZ
$k_1$ - proportionality factor for the dimensions of CDZ
$k_s$ - coefficient for crack distribution
$\ell_b$ - embedded length of reinforcing bar
$\ell_{ch}$ - characteristic length in FCM
$\ell_{gauge}$ - gauge length
$\ell_m$ - measuring length
$\ell$ - span of the simply supported member
$\ell^*$ - span of the statically indeterminate system (continuous member)
$\ell_o$ - total length of the member
$\ell_f$ - transfer length
$\ell/h$ - slenderness ratio of simply supported member; also denotes slenderness ratio of beam cut-out in continuous member
$\ell^*/h$ - slenderness ratio of continuous member
\( m \) - number of directions for calculation of \( c_{\text{eff}} \)

\( n \) - number of fictitious radial cracks; also denotes ordinal integer number

\( p \) - overall steel ductility parameter

\( q \) - uniformly distributed load

\( r \) - radius; also denotes parameter proportional to the average distance between successive longitudinal cracks in CDZ

\( r_{cr} \) - radius of the crack front

\( r_{o} \) - radius of thick-walled cylinder outer face

\( r_{i} \) - radius of thick-walled cylinder inner face

\( r_{s} \) - bar radius

\( s_{i} \) - clear bar spacing

\( s_{r} \) - average crack spacing; also denotes length of the FC-element

\( s_{r,l} \) - crack spacing at the initiation of the primary cracking

\( s_{r,A} \) - average crack spacing if \( 2l \leq s_{r,l} \leq 3l \)

\( s_{r,B} \) - average crack spacing if \( 3l \leq s_{r,l} \leq 4l \)

\( s_{s} \) - bar spacing (centre-to-centre)

\( u_{r,ri} \) - radial displacement of thick-walled cylinder inner face

\( u_{r,cr} \) - radial displacement of thick-walled cylinder inner face at crack front

\( w \) - crack width; also denotes fictitious crack opening in FCM and vertical component of the sliding deformation along the shear-band in CDZ

\( w_{o} \) - localised deformation in the loading direction at failure in FCM

\( w_{c} \) - localised deformation in the loading direction at failure in CDZ

\( w_{t} \) - fictitious crack width, at which the tensile stress \( \sigma_t \) is transferred

\( x \) - depth of the compression zone

\( x/h \) - distance of the point considered from the edge, also denotes coordinate

\( z \) - inner lever arm

**Greek letters**

\( \alpha \) - FCM constant

\( \alpha_{c} \) - angle between surface and axis of the load introduction cone

\( \alpha_{e} \) - ratio between the moduli of elasticity of steel and concrete

\( \alpha_{p} \) - coefficient for the effectiveness of the radial strain release at the interface due to the bar contraction

\( \alpha_{s} \) - angle between critical splitting plane and normal to closest surface

\( \alpha_{9} \) - ratio between the length of the support zone and of the span zone in linear-elastic analysis with moment redistribution

\( \beta \) - FCM constant
\( \beta_9 \) - ratio between the average bending stiffness of the support zone and of the span zone

\( \gamma \) - coefficient for non-linearity of the pre-peak stress-strain curve in CDZ

\( \gamma_c \) - partial safety factor for the concrete under compression

\( \gamma_M \) - ratio between moment resistance at the support and in the span

\( \gamma_m \) - partial material safety factor

\( \gamma_R \) - rib face angle

\( \gamma_s \) - partial safety factor for the steel

\( \Delta \) - slip normalised with respect to the bar diameter \( d_s \)

\( \Delta_{t,r} \) - total elongation of circumferential fibre with radius \( r \)

\( \Delta_{tot} \) - constant elongation of circumferential fibres across the thick-walled cylinder wall

\( \delta \) - slip; also indicates degree of moment redistribution

\( \delta_{0,s} \) - initial slip due to the bar contraction

\( \delta_1 \) - slip connected to radial strain \( \varepsilon_{r1} \)

\( \delta_{1,0} \) - characteristic slip

\( \delta_3 \) - characteristic slip

\( \delta_{3,\text{max}} \) - upper limit value of characteristic slip \( \delta_3 \)

\( \delta_{3,\text{min}} \) - upper limit value of characteristic slip \( \delta_3 \)

\( \varepsilon_d \) - design values of the allowable degree of moment redistribution

\( \varepsilon_s \) - strain at steel rupture measured over a length of 5 bar diameter

\( \varepsilon_{10} \) - strain at steel rupture measured over a length of 10 bar diameter

\( \varepsilon_{3,\text{max}} \) - upper limit value of radial strain

\( \varepsilon_{3,\text{min}} \) - lower limit value of radial strain

\( \varepsilon_c \) - concrete compressive strain; also denotes uniform compressive strain

\( \varepsilon_{cm} \) - average concrete compressive strain

\( \varepsilon_{cr} \) - strain value at peak stress

\( \varepsilon_{ct} \) - concrete tensile strain; also denotes uniform tensile strain

\( \varepsilon_{ctm} \) - average concrete tensile strain

\( \varepsilon_c^{\text{max}} \) - concrete compressive strain in most stressed fibre at maximum load

\( \varepsilon_d \) - additional post-peak compressive strain due to axial splitting in CDZ

\( \varepsilon_{du} \) - additional compressive strain due to axial splitting at failure in CDZ

\( \varepsilon_g \) - unit ultimate steel strain

\( \varepsilon_m \) - compressive strain of the mostly stressed fibre

\( \varepsilon_o \) - compressive strain at peak stress

\( \varepsilon_r \) - radial strain

\( \varepsilon_{rl} \) - characteristic radial strain

\( \varepsilon_{r3} \) - characteristic radial strain

\( \varepsilon_{r,r_3} \) - radial strain at the interface
\[ \varepsilon_s \] - steel strain
\[ \varepsilon_{sh} \] - steel strain at the onset of strain hardening
\[ \varepsilon_{t,r} \] - tangential strain in the circumferential fibre with a radius \( r \)
\[ \varepsilon_u \] - steel strain at peak stress
\[ \varepsilon_{uk} \] - characteristic value of steel strain at peak stress
\[ \zeta \] - shape factor
\[ \Theta_A \] - support rotation; also denotes required support rotation
\[ \Theta_B \] - rotation over the span
\[ \Theta_{el} \] - rotation of the FC-element
\[ \Theta(p) \] - rotation capacity of the hinge
\[ \Theta(p)_{ref} \] - rotation capacity of a reference member
\[ \Theta(p)_{lh=12} \] - rotation capacity of member with slenderness ratio \( l/h = 12 \)
\[ \Theta(p) / \Theta(p)_{ref} \] - degree of size dependence
\[ \Theta_{tot} \] - total rotation of the hinge
\[ \Theta(u) \] - rotation of the hinge at ultimate load
\[ \Theta(v) \] - rotation of the hinge at the onset of yielding of the reinforcement
\[ \Theta' \] - rotation at the point of transition from the span to the support zone
\[ \kappa \] - coefficient for limitation of the extension of the load introduction zone
\[ \nu_c \] - Poisson constant of concrete
\[ \nu_s \] - Poisson constant of steel
\[ \xi \] - relative depth of the compression zone
\[ \rho_s \] - reinforcement ratio
\[ \rho_{s,ef} \] - ratio between \( A_s \) and \( A_{c,ef} \)
\[ \sigma_b \] - concrete stress induced due to bond
\[ \sigma_{cr} \] - concrete stress for stabilised primary crack pattern
\[ \sigma_{cr,fl} \] - flexural tensile stress
\[ \sigma_r \] - radial compressive stress
\[ \sigma_{r1} \] - characteristic radial stress
\[ \sigma_{r,r} \] - radial stress at radius \( r \)
\[ \sigma_{r,ri} \] - radial stress at radius \( r_s \)
\[ \sigma_{r,cr} \] - radial stress at the crack front
\[ \sigma_{s2} \] - steel stress at the crack for stabilised primary crack pattern
\[ \sigma_t \] - tensile stress
\[ \sigma_{t,r} \] - tangential stress at radius \( r \)
\[ \tau_b \] - bond stress
\[ \tau_{b1} \] - critical bond stress
\[ \tau_{b3,max} \] - residual bond stress after the slip induced strain release
\[ \tau_{b3,min} \] - residual bond stress after the combined slip and steel stress induced strain release
**Notations**

- $\phi$: friction angle
- $\chi(c_i)$: concrete cover dependent indicator function
- $\phi$: cone angle between cone surface and bar axis
- $\omega$: mechanical reinforcement ratio of reinforced section
- $\omega_{DU}$: energy per unit volume absorbed in the bulk
- $\omega_s$: mechanical tensile reinforcement ratio
- $\omega_s$: mechanical compressive reinforcement ratio

**Special sub- or superscripts**

- $LE$: linear elastic; refers to contribution of uncracked part of the thick-walled cylinder
- $NL$: non-linear; refers to contribution of cracked part of the thick-walled cylinder
- $I$: uncracked stage behaviour of the thick-walled cylinder
- $II$: partly cracked stage behaviour of the thick-walled cylinder
- $III$: entirely cracked stage behaviour of the thick-walled cylinder
- RBM: rigid body movement
- $\Delta c$: change of wall thickness

**Abbreviations and acronyms**

- CDZ: Compressive Damage Zone
- FC: flexural crack
- FCM: Fictitious Crack Model
- HSC: high strength concrete
- LVDT: displacement transducer working on the principle of the linear variable differential transformer
- NSC: normal strength concrete
- RC: reinforced concrete
- SC: shear crack
- $LB$: lower bound solution
- $UB$: upper bound solution
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