Stellingen behorende bij het proefschrift

Aerospace-Plane Flight Dynamics
Analysis of Guidance and Control Concepts

E. Mooij


2) Het volgen van een sterk gedempt referentie model bij toepassing van MRAC kan tot een te grote regelinspanning van het (onbekende) systeem leiden.

3) De korte ontwikkelingstijd van sub-orbitale voertuigen, zoals de X-33, betekent niet dat herbruikbare lanceersystemen snel zullen volgen. De ontwikkeling van deze systemen zal namelijk worden gedomineerd door de bijzonder complexe door-ontwikkeling van de voortstuwingssystemen.

4) Computermodellen van voortgestuwde ruimtevliegtuigen met geleidings- en besturingsystemen zijn dermate complex, dat voor het begrijpen van de multi-parameter input-output relatie, methoden volgens het principe 'ontwerpen met experimenten' onontbeerlijk zijn. Om de rekentijd te beperken is het gebruik van optimale methoden, zoals de Taguchi Methode, aan te bevelen.

5) Ten einde een interview met voetbalspelers vlot en informatief te laten verlopen verdient het de voorkeur zowel de interviewer als de geïnterviewde een cursus mondeling communiceren te laten volgen.

6) De kans op overlijden door een hartinfarct is vele malen groter dan betrokken te raken bij een vliegtuigongeluk. Echter, de kans op een hartinfarct neemt tijdens een vliegtuigongeluk schrikbarend toe.

7) Het afroepen van patiënten op de Keel-, Neus- en Oorheelkunde (KNO) afdeling in het ziekenhuis dient op zijn minst visueel ondersteund te worden.

8) Het licht dat je aan het einde van de tunnel ziet komt meestal van de trein die op je afkomt.

9) Als het doen van wetenschappelijk onderzoek alleen bepaald zou worden door economische en maatschappelijke relevantie zou de mensheid nog steeds in de vorige eeuw leven.

10) Het feit dat er op introductie van nieuwe modellen in de wetenschap vaak emotioneel afwijzend wordt gereageerd, geeft aan dat er een sterke discipline nodig is om wetenschappelijk te blijven denken.

11) De populariteit van klassieke muziek als kunstvorm is te danken aan het feit dat het gehoorzintuig vrijwel volledig kan worden gemanipuleerd.
Propositions, part of the thesis

Aerospace-Plane Flight Dynamics
Analysis of Guidance and Control Concepts

E. Mooij

1) The principle of Model Reference Adaptive Control (MRAC) is based on simplicity and elegance. Unfortunately, the large number of design parameters and the lack of a concrete design philosophy makes application of this control concept as yet a complex and time consuming procedure.

2) The tracking of a well-damped reference model as applied in MRAC can lead to a too large control effort of the (unknown) plant.

3) The short development time of sub-orbital vehicles, like the X-33, does not mean that reusable launch systems are soon to follow. The development of these systems will be dominated by the very complex evolution of the propulsion systems.

4) Computer models of propelled space planes with a guidance and control system are so complex that to understand the multi-parameter input-output relation methods based on design of experiments are mandatory. To limit the CPU time, use of optimal methods, for instance, the Taguchi Method is recommended.

5) A mandatory course in oral communication for both the interviewer and the interviewee would do much towards conducting meaningful interviews with soccer players.

6) The chance of dying a heart attack is much larger than the chance of getting involved in a plane crash. However, the chance of a heart attack increases dramatically during a plane crash.

7) One would expect that calling for waiting patients in the Throat, Nose and Ear ward in the hospital would be visually supported.

8) The light that welcomes you at the end of the tunnel comes from the train that is speeding towards you.

9) If scientific research would be determined only by issues of economic and social relevance, mankind would still be living in the previous century.

10) The fact that in science one often rejects new models emotionally, indicates that a strict discipline is required for a purely scientific approach.

11) The popularity of classical music as a form of art can be ascribed to the fact that the ear can almost be completely manipulated.
Aerospace-Plane Flight Dynamics

Analysis of guidance and control concepts

ERWIN MOOIJ

Delft University of Technology

June 1998
Cover: The best-known aircraft designed by Leonardo da Vinci are certainly his man-powered ornithopters. The cover illustration shows his Type A, prone ornithopter. The pilot lies on a board and is secured by hoops, beneath a framework supporting the fulcrum or hinges of the system of beating wing-spars. With alternate action of the legs the wings are lowered and raised. The design originated between 1486 and 1490. (Text based on C.H. Gibbs-Smith's Leonardo da Vinci's Aeronautics, a Science Museum Booklet, 1967.)

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Aerospace-Plane Flight Dynamics

Analysis of guidance and control concepts

Proefschrift

Ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof.ir. K.F. Wakker,
in het openbaar te verdedigen ten overstaan van een commissie,
door het College voor Promoties aangewezen,

op donderdag 4 juni 1998 te 13:30 uur
door Erwin MOOIJ
ingenieur luchtvaart en ruimtevaart
geboren te Koog aan de Zaan
Dit proefschrift is goedgekeurd door de promotoren:

Prof.ir. K.F. Wakker
Prof.dr.ir. P.G. Bakker

Samenstelling promotiecommissie:

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<tr>
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<th>voorzitter</th>
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<tr>
<td>Prof.ir. K.F. Wakker</td>
<td>Technische Universiteit Delft, promotor</td>
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<td>Prof.dr.ir. P.G. Bakker</td>
<td>Technische Universiteit Delft, promotor</td>
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<td>ESA/ESTEC, Noordwijk</td>
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<td>Technische Universiteit Delft</td>
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<td>University of Maryland at College Park</td>
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<td>Dr.-Ing. U.M. Schöttle</td>
<td>Universität Stuttgart</td>
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<tr>
<td>Prof.dr.ir. P.Th.L.M. van Woer kom</td>
<td>Technische Universiteit Delft</td>
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'Praw!' zo sprak hij verslagen, 'als mens weet ik dat dit alles waar gebeurd is. Maar als wetenschapsman moet ik vaststellen, dat het bestaan ener droon niet is bewezen. En der kukel? Ach, we zullen nimmer weten, wat der kukel is.' Professor Priwytzkofsky in Het Kukel (Marten Toonder)
Horizontal take-off single- and two-stage vehicles using air-breathing propulsion, also known as aerospace planes, seem to be promising concepts for more economical delivery of payloads to orbit. The effective use of air-breathing propulsion engines leads to ascent trajectories at lower altitudes as compared to conventional launchers, resulting in higher dynamic pressures, higher skin-surface temperatures and higher acceleration loads. Apart from innovating fields such as heat-resistant materials and structures, the development of guidance logic and attitude controllers poses a challenging problem, in particular due to the small performance margin and the strongly varying flight environment.

The main question or problem definition of this study has been formulated as follows:

How and to what extent do design uncertainties and environmental disturbances influence the mission performance of space planes, and in what way can a guidance and control system contribute to mission success?

To compute the trajectory and related performance of space planes in sufficient detail, the need of a flight-simulation tool was identified, the development of which can be seen as one of the major goals within the current study. This tool, the Simulation Tool for Ascent and Re-entry Trajectories (START), can be used to do simulations in 3 or 6 degrees of freedom, thereby treating the vehicle as a mass point or a non-elastic body with variable mass properties due to the use of a propulsion system.

One of the key elements that had to be developed for the use in START is a guidance and control (G&C) system. Starting point is that its performance does not have to be optimised, since a system with a reasonable performance is already sufficient to do the research. The applied guidance systems are partly based on existing models and partly designed while keep-
ing the complexity as low as possible. As attitude control system, two concepts have been studied. The first concept is based on linear state feedback with gain scheduling, thereby using optimal control theory to compute the gains (a so-called Linear Quadratic Regulator, LQR). The second concept is that of direct Model Reference Adaptive Control (MRAC), and has been selected to assess its potential as control system to be used in relation with space planes. This method aims at equating the output parameters of the actual vehicle with those of a simplified reference model, where the gains are continuously adapted to the output error.

To analyse the output from the simulations statistical techniques are used, i.e., analysis of variance (ANOVA) and regression analysis. Applying these techniques with a certain level of confidence is only reliable when sufficient data are available. In this study, alternatives to the commonly applied Monte-Carlo Method are investigated to assess the potential use in the study of flight mechanics in general, and the analysis of G&C systems in particular. Two methods that have been identified are the Taguchi Method (TM) and the related Response Surface Methodology (RSM), and in particular Central Composite Design (CCD).

START has become a tool, with which the powered and unpowered flight of non-elastic vehicles moving in a planetary atmosphere with wind can be studied, thereby using a G&C system or not. Extensive capabilities are implemented to model the geometry and parameters of the vehicle, to define the environment and to describe the mission. Moreover, facilities for output processing have been provided for.

To prove that the algorithms have been correctly implemented and that the simulated output is a good approximation of the reality, START is verified, validated and evaluated. To this extent results are compared with the output of a validated software (RATT/ESTEC), similar results found in literature, and judged on their physical correctness. The outcome of this exercise is that START may be used to study the guided and controlled ascent and descent of (un)powered space planes.

Two major missions are studied, i.e., the unpowered entry and descent of the winged entry vehicle HORUS using the LQR, and the powered ascent of the Winged Cone Configuration (WCC) SSTO space plane, applying MRAC. Most of the simulations have been executed according to the TM. Experience gained in this study indicated that initially one can get insight in the linear and higher-order effects and possible interactions by varying only a few input parameters. The TM and CCD can be used early in the design to get a feeling for parameter sensitivities and interactions. Later in the design, once the majority of the parameters are frozen a verification analysis can be carried out by applying the Monte-Carlo Method.

In case of simulating the controlled flight of an aircraft or spacecraft, it is possible that a three-level analysis leads to instabilities while a two-level analysis gives only stable results. This is caused by the non-linear nature of the system. With statistical analysis it will not be possible to trace control instabilities, unless they actually occur. Moreover, one such 'bad' data point, has a significant influence on the results and should therefore be eliminated before ANOVA is applied or a response surface is computed. But, in case there are more of these 'bad' data
points, the results can be used to discover a trend, and in this study it lead to a guidance-system improvement for HORUS.

Beside using the TM as a sensitivity-analysis tool, it has also been used as a design method. First, a robust design technique, i.e., a dual-loop with respect to design parameters and perturbations, was used to optimise the performance of the guidance system of HORUS under the influence of disturbances without eliminating these disturbances. The optimum settings of the guidance parameters were defined, and it was confirmed that the original design was indeed the optimum one. Second, the ascent trajectory of the WCC has been improved by dividing the trajectory into a number of flight segments defined by a flight-path angle profile. The resulting trajectory parameters were then varied. A large payload-mass variation was found, which indicates that a proper selection of the trajectory is very important to ensure mission success. It may be obvious that the found optimum is only valid within the accuracy of the defined trajectory segmentation and guidance modelling. Therefore, the TM cannot replace a numerical trajectory optimisation method, since the analysis is based on constant-parameter segments that do not have to hold for the optimum trajectory. Moreover, knowledge on the trajectory is required to set up the segmentation. However, by doing a Taguchi analysis for a given trajectory segmentation one can increase the insight in the influence of the trajectory parameters on a selected response.

The LQR has a reasonable performance although the current design has a problem to handle non-linearities in the aerodynamics of the control surfaces. Especially at large deflections this leads to control errors. In case there are consecutive large deflections this may lead to diverging oscillations and a complete loss of control. MRAC seems to be a promising concept when applied to hypersonic space planes. Although it is too early to say that the current configuration of the controller is robust with respect to design uncertainties and environmental perturbations, there are still many improvements possible that will enhance the performance.

Concluding, mission success of space planes is to a large extent dependent on the performance of the guidance and control system. A poorly designed system can directly lead to the loss of the vehicle. An adequate performance for a nominal mission is in itself not sufficient to conclude that we have a good system: extensive analysis for off-nominal conditions is essential. Moreover, to avoid cross couplings between the guidance system and the attitude controller it is mandatory to address the guidance-system performance before extensive analysis of the attitude controller is started. Obviously, this does not have to be true when the performance of the attitude controller is the study goal and one is positive to separate the effects induced by the guidance system from the ones originating from the attitude controller.
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<td>$a$</td>
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<tr>
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<td>Nm</td>
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\( T_z \) yaw-thruster moment \( \text{Nm} \)
\( u \) control vector 
\( U \) column-orthogonal SVD matrix 
\( v \) velocity component \( \text{m/s} \)
\( V \) orthogonal SVD matrix 
\( V \) modulus of velocity vector \( \text{m/s} \)
\( \mathbf{V} \) velocity vector \( \text{m/s} \)
\( \sigma \) variance 
\( \bar{V}_i \) mean inlet velocity \( \text{m/s} \)
\( \bar{V}_e \) mean exhaust velocity \( \text{m/s} \)
\( W \) diagonal SVD matrix 
\( x, y, z \) unit vectors 
\( x, y, z \) cartesian position components \( \text{m} \)
\( x \) state vector 
\( X, Y, Z \) aerodynamic forces in body frame \( \text{N} \)
\( \mathbf{X} \) regression matrix 
\( X, Y, Z \) axes 
\( y \) output vector 
\( y \) response 
\( y \) cross range \( \text{m} \)
\( \bar{y} \) mean response value 
\( Y \) response vector 
\( z_p \) augmented plant output vector

**Greek**

\( \alpha \) angle of attack \( \text{rad} \)
\( \alpha \) axial distance in CCD 
\( \beta \) angle of sideslip \( \text{rad} \)
\( \beta \) regression coefficient 
\( \gamma \) flight-path angle \( \text{rad} \)
\( \gamma_r \) controlling parameter for skipping flight \( \text{rad} \)
\( \delta \) geocentric latitude \( \text{rad} \)
\( \delta^* \) geographic latitude \( \text{rad} \)
\( \delta_a \) aileron deflection angle \( \text{rad} \)
\( \delta_b \) body-flap deflection angle \( \text{rad} \)
\( \delta_e \) elevator/elevon deflection angle \( \text{rad} \)
\( \delta_r \) rudder deflection angle \( \text{rad} \)
\( \delta_T \) throttle setting
\( \Delta \) factor variation
\( \Delta_\text{..} \) perturbation
\( \Delta_E \) energy dead band
\( \Delta_h \) altitude dead band
\( \varepsilon \) perturbation parameter
\( \varepsilon_T \) elevation of thrust force
\( \zeta \) damping ratio
\( \theta \) pitch angle
\( \mu \) gravitation parameter
\( \rho \) (atmospheric) density
\( \rho_f \) density of fluid
\( \sigma \) bank angle
\( \sigma \) standard deviation
\( \tau \) geocentric longitude
\( \tau \) time constant
\( \phi_T \) fuel equivalence ratio
\( \chi \) heading
\( \chi_e \) heading error
\( \psi \) yaw angle
\( \psi_T \) azimuth of thrust force
\( \omega_{cb} \) rotational rate of the central body
\( \omega \) rotation vector
\( \omega \) eigenfrequency

Indices

0 initial condition
1, 2, 3 flap number
A airspeed-based
a aileron
C Coriolis
c commanded
c compensator
cb central body
circ circularisation
cm centre of mass
d derivative
E, e error
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Notations

\( x, y, z \)
along \( X, Y, \) and \( Z \)-axis

\( y \)
cross range

\( \gamma \)
flight-path angle

\( \delta \)
littitudinal direction

\( \tau \)
longitudinal direction

\( \chi \)
heading

Reference frames

For a detailed description of the reference frames that are used in this thesis, the reader is referred to Appendix A. Below, a list of the reference frames is given.

\( AA \)
aerodynamic frame, airspeed-based

\( AG \)
aerodynamic frame, groundspeed-based

\( B \)
boby frame

\( I \)
inertial planetocentric frame

\( R \)
rotating planetocentric frame

\( TA \)
trajectory frame, airspeed-based

\( TG \)
trajectory frame, groundspeed-based

\( V \)
vertical frame

\( W \)
wind frame

Glossary

ACS
Attitude Control System

ANOVA
ANalysis Of VAriance

ARD
Ariane Re-entry Demonstrator

ASPR
Almost Strictly Positive Real

CCD
Central Composite Design

CFD
Computational Fluid Dynamics

CIRA
COSPAR International Reference Atmosphere

c.o.m.
centre of mass

c.o.p.
centre of pressure

c.o.t.
centre of thrust

CPU
Central Processing Unit

d.o.f.
degree of freedom
<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<td>European Space Agency</td>
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<td>ESTEC</td>
<td>European Space research and TEchnology Centre</td>
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<tr>
<td>FCS</td>
<td>Flight Control System</td>
</tr>
<tr>
<td>GNC</td>
<td>Guidance, Navigation and Control</td>
</tr>
<tr>
<td>HAC</td>
<td>Heading Alignment Cylinder</td>
</tr>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
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<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
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<td>MIMO</td>
<td>Multiple Input, Multiple Output</td>
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<td>MRAC</td>
<td>Model Reference Adaptive Control</td>
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<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
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<td>NASP</td>
<td>National AeroSpace Plane</td>
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<tr>
<td>OREX</td>
<td>Orbital Re-entry EXperiment</td>
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<tr>
<td>PID</td>
<td>Proportional, Integral and Derivative</td>
</tr>
<tr>
<td>PR</td>
<td>Positive Real</td>
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<tr>
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<td>Reaction Control System</td>
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<td>Response Surface Methodology</td>
</tr>
<tr>
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<td>Single Input, Single Output</td>
</tr>
<tr>
<td>SPR</td>
<td>Strictly Positive Real</td>
</tr>
<tr>
<td>SSTO</td>
<td>Single-Stage-To-Orbit</td>
</tr>
<tr>
<td>START</td>
<td>Simulation Tool for Ascent and Re-entry Trajectories</td>
</tr>
<tr>
<td>TAEM</td>
<td>Terminal Area Energy Management</td>
</tr>
<tr>
<td>TMC</td>
<td>Thrust Magnitude Control</td>
</tr>
<tr>
<td>TPS</td>
<td>Thermal Protection System</td>
</tr>
<tr>
<td>TSTO</td>
<td>Two-Stage-To-Orbit</td>
</tr>
<tr>
<td>TVC</td>
<td>Thrust-Vector Control</td>
</tr>
<tr>
<td>UI</td>
<td>User Interface</td>
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Chapter 1

Introduction

I have made this letter longer than usual, only because I have not had the time to make it shorter. *Blaise Pascal*

This thesis deals with a new type of launchers that are known under the name space planes. Due to the alternative mission profile and the strongly varying flight environment as compared with conventional aircraft and conventional launch vehicles, a major field of interest is flight mechanics and the simulation thereof in the design phase of such a vehicle, which is the main topic of the current study. Moreover, it is the intention to develop a generic model which allows for a fast analysis of flight mechanics of space vehicles.

The scope of the thesis is presented in Section 1.1 and gives a survey of the topics discussed in more detail in the remainder of this chapter. Issues relevant for the flight mechanics of space planes are discussed in Section 1.2, whereas the analysis of flight mechanics is addressed in Section 1.3. Section 1.4, finally, presents an outline of the thesis.

1.1. Scope

Nowadays, the launch of a payload into Earth orbit is very expensive and highly inflexible. Every launch of a (conventional) expendable launcher like Ariane 4, Proton, Delta and Titan/Centaur, represents a considerable waste of materials, and manufacturing and processing capabilities. Moreover, the launch is restricted to take place only at a limited number of specially designed launch sites; weather conditions at these sites have often caused intolerable delays for time-critical launches.

It is believed that to significantly reduce the costs of space transportation, an advanced space launcher or even a variety of advanced space launchers has to be developed, e.g., see Branscome and Reese (1990), Darwin *et al.* (1991), and Feustel-Büechli *et al.* (1991). Two developments in this field can be identified. On one hand, the design and manufacturing of
conventional launchers can be further improved and optimised, with attention to modular design (e.g., the family of Ariane launchers), high reliability, ease of manufacturing and safety, reusability, and reduced dry mass (Zandbergen and Mooij, 1994). On the other hand, one can think of fully reusable vehicles, which take off and land horizontally like aircraft, operate from airports and have a short mission turn-around time. The latter vehicles are commonly called space planes and form the main issue of this thesis.

From the mid-eighties onwards, many different concepts of reusable launch systems have been studied throughout the world. Berry and Grallert (1995) give an overview of ESA’s Winged Launcher Configuration studies (1988-1994) which focuses on a variety of concepts and vehicles. The basic differences are the number of stages (one versus two, giving the so-called Single-Stage-To-Orbit (SSTO) and Two-Stage-To-Orbit (TSTO) vehicles), method of take-off and landing (vertical versus horizontal), and the type of propulsion system (rocket versus air-breathing engines). An example of an SSTO space plane is the American X-30, see Fig. 1.1.

Fig. 1.1 - The X-30 (courtesy of NASA).

A typical mission of a TSTO space plane consists of the following phases for the first stage (Fig. 1.2): i) horizontal take-off, ii) climb, iii) cruise flight, iv) manoeuvres, v) acceleration, vi) pull-up, vii) separation from second stage, viii) post-separation descent, ix) return cruise-flight, x) deceleration, xi) horizontal landing. For the second stage we may add: i) post-separation ascent, ii) orbit adjustment (e.g., circularization), iii) orbital operations, iv) deorbit-burn manoeuvre, v) re-entry and descent flight, and vi) horizontal landing. Similar phases can be discerned for the
mission of an SSTO space plane. Since the analysis of a complete mission of a space plane is a very broad field with many specific problems, in this thesis only the ascent and descent phases of an SSTO space plane including cruise flight and manoeuvres will be studied. So, the phases of take-off and landing, exoatmospheric flight and the separation manoeuvre are not discussed explicitly.

Fig. 1.2 · Sänger ascent trajectory with Horus stage separation (based on Koelle and Kuczera, 1990).

Space planes are basically unstable vehicles, which need a Guidance, Navigation and Control (GNC) system to ensure mission success\(^1\). Because of the large velocity and altitude range, the stability and control properties of the vehicle change significantly during the mission. A further complication is the integrated propulsion/airframe concept, which gives rise to serious dynamic interactions between the propulsion system, the aerodynamic forces and moments\(^2\) on the vehicle and consequently the GNC system. Moreover, since space planes are usually slender bodies aeroelasticity plays an important role in attitude and propulsion control. Since the space-plane propulsion system will probably have only a small performance margin left, it is necessary to fly along a well-defined trajectory that is extensively analysed with respect to the space-plane’s flight mechanics, especially in relation with design uncertainties and environmental disturbances.

\(^1\) In Section 1.2, the flight mechanics of space planes is discussed in more detail. For this reason, we will not give any references in this section but refer the reader to those given in the next section.

\(^2\) In this thesis the term moment is used as a shorter form of the term moment of force, that has been introduced to distinguish from moment of inertia.
The main question or problem definition of this thesis work can be formulated as follows:

How and to what extent do design uncertainties and environmental disturbances influence the mission performance of space planes, and in what way can a GNC system contribute to mission success?

To find an answer to this question it is required to analyse the ascent and descent trajectory of space-plane-like vehicles, with means to establish and possibly explain the influence of a system parameter on the trajectory and/or the performance of the vehicle. In doing so at least a simulation tool is required, with which the trajectory and related performance can be computed with sufficient accuracy. This tool should include a mathematical representation of (components) of the vehicle with enough detail such that the effects of the parameters of interest can be investigated. Components are, for instance, a set of equations describing the flight mechanics, a GNC system, a propulsion system, etc.

Statistics can be of assistance in analysing the results of simulation, but only when there are sufficient data to apply the statistical techniques with a certain level of confidence. Key word here is sufficient. Since generating data by means of simulation can be time consuming, it is preferable to minimise the number of simulations. A popular method in the field of simulation is the Monte-Carlo method, that uses a random-number generator to vary the system parameters. However, this method requires a relatively large number of simulations which makes this method less practical in a feasibility design phase when many design changes ask for more than one batch of simulations. Therefore, we want to investigate alternatives to the Monte-Carlo method to assess the potential use in the study of flight mechanics. Two methods that have been identified are the Taguchi Method and the related Response Surface Methodology. In Section 1.3, these methods are introduced.

1.2. Flight mechanics of space planes

Flight mechanics describes the motion of a flying vehicle in a general sense. In the different fields of flight transportation the starting point is always the mission objective: go from A to B without endangering either cargo, passengers or environment, and, if possible, in the most efficient way. When the Space Shuttle Orbiter returns from space with the objective to land safely at Edwards Air Force Base, it enters the atmosphere at a high angle of attack to minimise the heat load (Havey, 1982). Once the peak heat load has been passed, the Orbiter has to fly at a much smaller angle of attack to obtain the required cross- and down-range by increasing the lift-to-drag ratio. Furthermore, by banking the lift force the Orbiter can regulate its altitude. The induced lateral motion can be controlled by executing so-called bank reversals such that the vehicle remains to be targeted at its landing site. To control the vehicle's attitude and to
steer it towards its target, a GNC system is required: an uncontrolled Orbiter will exhibit a large variation in the angle of attack and would burn in the atmosphere.

The operation of the GNC system is complicated by the severe aerothermodynamic environment. Maus et al. (1984) describe that at high Mach numbers, the values of the Space Shuttle's pressure coefficients decrease, resulting in a more nose-up pitch moment and a decrease in axial and normal force. Furthermore, the centre of pressure (c.o.p.) shifts forward, especially at lower angles of attack, which means a slower response of the vehicle to control inputs. A similar shift in the c.o.p. is caused by real-gas effects (the inert degrees of freedom of a gas, i.e., vibration, dissociation and ionization are activated, causing serious alterations in the thermal properties of air) at Mach numbers in excess of 10. These real-gas effects increase the pressure coefficients, especially on the forebody. The resulting (destabilizing) nose-up moment - more significant with higher angles of attack - may cause a longitudinal stability problem. In addition, because of the higher pressures the aerodynamic loading on the body flap can be about 20% higher (Maus et al., 1984).

These Mach-number and real-gas effects are characteristic of the hypersonic flight regime and will therefore also play an important role during the powered ascent flight of space planes. But the picture is even more complicated than in the case of unpowered re-entry vehicles. Whitehead (1989) discusses the NASP aerodynamics in combination with the propulsion system. For an acceptable hypersonic propulsive efficiency, the space plane must fly at high dynamic pressures (for SSTO vehicles this dynamic pressure could be as high as 95 kPa), which means a shallow flight at lower altitudes. However, this requirement cannot be treated independently from the high aerothermodynamic heating associated with a high dynamic pressure, because around Mach 12 temperature constraints will force the mission profile to deviate from a flight with maximum dynamic pressure.

Fig. 1.3 - Airframe/engine interaction and resulting forces and moments (based on McRuer, 1991).

Furthermore, the propulsion efficiency demands that a large part of the vehicle contributes to the propulsion system (Fig. 1.3), i.e., the forebody serving as a compressive surface forward of the inlet and the portion of the vehicle aft of the combustor acting as a tailored expansion
nozzle (Whitehead, 1989). A substantial portion of the vehicle's thrust is developed there, which makes the interaction of these propulsion components with the aerodynamic controls a major design issue. An example of the relative pitch moments generated by the airframe and propulsion system is shown in Fig. 1.4. It is easy to understand that the propulsion system will have a major effect on the space-plane stability, especially if we realise that, for instance, the freestream area covered by the inlet of a SCRAM-jet at cruise conditions increases with the Mach number (Johnston et al., 1987). This means that at high Mach numbers the aerodynamic configuration is primarily determined by the propulsion system.

![Diagram showing longitudinal moments of propulsion system and airframe](image)

Fig. 1.4 - Longitudinal moments of propulsion system and airframe (based on Heitmeir et al., 1992).

The net propulsive thrust is the relatively small difference between large ram drag and gross thrust. If the propulsion system fails, the execution of an abort return flight is very difficult, because termination of the propulsive force will automatically change the forces and moments on the vehicle drastically. This poses a serious problem that has to be investigated, both analytically, numerically and experimentally. The exhaust flow does not only increase the lift significantly, but also the lift-to-drag ratio and the cruise performance. Basically, the lift-to-drag ratio is low, because of the large fuselage volume that is required for the low-density hydrogen fuel that has to be taken along (Johnston et al., 1987). The propulsive lift, however, gives also rise to a nose-down moment which has to be compensated for to guarantee moment equilibrium. This is possible with the aerodynamic-control surfaces, for instance, but deflecting these will give rise to additional drag, the so-called trim drag, which has a negative influence on the performance. Of course, aerodynamic trim should not be studied as a stand-alone problem, because pitch or roll manoeuvres may require additional deflections. An extra problem might be that at higher Mach numbers the aerodynamic control effectiveness is significantly reduced.

Cribbs (1990) showed with a very simple analysis, that because of the many required technological breakthroughs, the performance of an SSTO concept with air-breathing propulsion is
at best marginal. Outcome of the analysis was that a weight margin of 30% is required for mission success at a 90%-level of confidence. A conclusion which can be drawn from this analysis is that the uncertainties in the performance parameters should be minimised as far as possible, and that any SSTO design should be robust, meaning that built-in margins have to assure that the flight vehicle will achieve its design goal. Moreover, the guidance design should be robust enough to minimise deviations from a flight along fuel-minimal trajectories.

For a proper performance of the propulsion system an accurate angle-of-attack control is required, because of the large variations in thrust force and moment that can result from minor angle-of-attack perturbations. Therefore, the angle of attack cannot be used freely as a guidance variable, but can only vary within certain limits about some value holding for the nominal operating condition of the propulsion system. Raney and Lallman (1992) interpret these angle-of-attack limits as a limit on the commanded load factor, thus restricting the hypersonic manoeuvring capabilities of air-breathing space planes. To counteract the influence of wind, gusts and turbulence, a more advanced performance-sensitive or adaptive control system may have to be considered (McIver and Morrell, 1990). Also the guidance along an optimal, minimum fuel trajectory in the presence of changing atmospheric conditions will be a difficult but unavoidable task for the on-board GNC system.

Powell et al. (1991) discuss the ascent performance of an air-breathing, horizontal take-off SSTO launch vehicle equipped with a simple guidance and control system. The outcome of their research was that the total drag losses are a significant fraction of the total required velocity increment. In that respect the drag losses due to aerodynamic trim by means of the elevons contributed a great deal to the total drag and thus trimming by alternative means of control (c.o.m. management, Thrust-Vector Control (TVC), reaction control) should be considered. In addition, they studied the effect of atmospheric density variations and monthly varying winds. The trimmed deflection angles of the elevons showed variations of more than 5°, emphasising the need for a carefully selected trim strategy.

The interactions of airframe, propulsion system and controls may define many regions where the vehicle cannot be flown (McRuer, 1991). The attainable performance envelope of space planes can therefore not be defined solely by point-mass performance or considerations related to the operating point of the propulsion system. As an example, Fig. 1.5 shows a possible excluded region in flight corridors and operational areas, bounded by, for instance, thermal and structural limits. This is a region with low angles of attack in the RAM/SCRAM area that may be excluded because of bow and other shock impingements on the vertical control surfaces, in combination with directional divergencies and reduced directional control effectiveness and control power. Other impacts may be due to manoeuvring needs (e.g., a 2-g turn) which can cause a further reduction of the admissible flight corridor.

Hypersonic vehicles are primarily designed to maximize performance, with considerations of stability and control playing at best a secondary role. Accordingly, there is usually a large number of airframe-engine dynamic deficiencies left to be corrected by control means, after a trimmed state for the point-mass performance has been established (McRuer, 1991). However,
inattention to the overall control issues may lead to an optimistic estimate of the performance, which can have a severe influence on design changes at a later stage. Control limitations on older aircraft have sometimes limited the available flight envelope (Schwanz and Cerra, 1984).

![Graph showing flight performance envelope](image)

Fig. 1.5 - The attainable performance envelope of an air-breathing space plane (based on McRuer, 1991).

Schmidt et al. (1991) and Schmidt (1993) go even further. They state that the problems encountered during the control design for hypersonic vehicles may ultimately limit the feasibility of some configurations. Therefore, the flight-dynamic modelling and analysis must be performed early in the design cycle, such that critical dynamics and control issues associated with a candidate configuration may be exposed. For this reason, such dynamics and control analyses cannot be postponed until detailed numerical models are available, because configuration changes may already be too expensive at that stage. For instance, they found a strong dynamic relation over a wide frequency range between the vehicle attitude and the propulsion system. This relation is not one way, however, but it is a strong two-directional coupling between airframe and engine, e.g., a dynamic thrust response to aerodynamic pitch control and an airframe pitch-rate response to changes in fuel-flow rate and the diffuser area. One of their conclusions was that there may be a necessity for some form of high-bandwidth, active control of the inlet/diffuser. Unfortunately, manoeuvring may affect each engine module differently, resulting in possible additional yaw and roll moments. Integrated, active control of each inlet separately should then be considered.
During the control-system design, an additional source of uncertainty arises from the limited accuracy of mathematical dynamic models used to describe the vehicle. Hattis and Malchow (1992) note that some unusual characteristics of the air-breathing propulsion models were shown to have potentially large effects on the vehicle dynamics (high Mach-number control history and flight profile), and identify an extreme sensitivity of the desired hypersonic vehicle reference trajectories to details of aerodynamic and propulsion performance models. This, coupled with the limited availability of empirical data above Mach 8 in aerodynamics, propulsion, aeroelasticity, heating and their combined effects on the vehicle’s mission performance, dictate the need for a robust yet performance-oriented control system (Gregory et al., 1992). The small performance margins give rise to a need for optimizing the control system, resulting in extensive needs for a software testing and evaluating capability.

The dynamic characteristics of lifting vehicles with air-breathing engines in hypersonic flight show substantial differences when compared to relationships for conventional aircraft (Sachs, 1993), because of the wide and different range of operating conditions and mass distributions. Responses of the airspeed to steps in elevon deflection and throttle setting, for instance, are basically determined by so-called aperiodic height-mode characteristics, with little contribution to the long-term period oscillation (phugoid), as is the case in the conventional speed regimes. These unique characteristics require specific control systems to guarantee adequate overall dynamic properties for these type of vehicles. A rate-command/attitude-hold control system that is known for its positive effect on phugoid stability and damping characteristics in subsonic flight, has a destabilising effect on the height mode in hypersonic flight, which already shows an instability in the unaugmented case.

Another important aspect while comparing with conventional aircraft is aeroelasticity. A space plane has an elongated fuselage that produces low-frequency elastic modes that may cause perturbations in the combustor inlet conditions due to oscillation of the forebody compression surface. Raney et al. (1995) studied the effect of this aeroelastic-propulsive interaction on the flight dynamics of a NASP-like configuration in hypersonic flight, whereby the angle-of-attack sensitivity of the propulsion-system performance was taken into account. They found considerable variations in longitudinal force and moment components produced by turbulence-induced aeroelastic geometry perturbations. Furthermore, the variation in the propulsion characteristics exhibited a profound impact on rigid-body pitch mode (short-period dynamics). The magnitude of this (uncertain) variation shows the need for robust control to manage the complex dynamic interactions, especially since precise control of aeroheating, drag and propulsion is required to achieve adequate mission performance. Because of the high aerodynamic heating the vehicle’s structure will de-stiffen, so that the significant coupling between rigid-body and elastic modes results in lower flutter speeds and more distinct aeroelastic response characteristics. The accurate determination of elastic mode shapes and natural frequencies will be a critical requirement for control-system design.

Similarly to the Space Shuttle, a space plane will return unpowered from space and at high angles of attack so as to reduce the entry heating. A question that arises is, whether the space
plane with its ascent-stability characteristics can still be trimmed, since much larger deflections are now required. In addition, because of the high temperatures the inlets are closed to protect the interior of the propulsion system. This changes the aerodynamic configuration significantly: apart from the increased inlet drag, the absence of the exhaust flow will also have its impact. These changing aerodynamics should of course be included in the analysis of the trim problem. At high angles of attack, the c.o.p. moves radically aft giving a positive stability. However, during the (shallow) ascent, the angle of attack is basically small which means a negative stability. Active computer control is therefore required. Last but not least, the rudder will be shielded and ineffective, similar to the situation of the Space Shuttle. Therefore, a Reaction Control System (RCS) is required to control the vehicle, and the rudder will only be activated later during the descent.

At least one conclusion is obvious: some of the ascent stability characteristics are influenced by the descent flight requirements. Especially the stability margin should get attention, since high static margins at lower speeds result in large deflection angles and a decreased manoeuvrability. On the other hand, the alternative would be an unstable vehicle at hypersonic speeds. Many of the aerodynamic (and propulsion) uncertainties can only be determined in flight, as it is not possible to accurately predict them or fully model them in ground tests. Because of these uncertainties, the control system of the test vehicle must be robust even though the performance range will only be gradually expanded.

Summarising this section, we have concluded that space planes are basically unstable vehicles. GNC is an important issue in the design of space planes, but how and to what extent GNC can ensure mission success has to be analysed by means of an extensive study of the flight mechanics. How this can be achieved will be discussed in the next section.

1.3. Study of flight mechanics

1.3.1. Flight mechanics and design

In general, a vehicle flying through the atmosphere may be considered an elastic, mass-varying body. In the previous section, we considered a space plane, which is an aircraft-like, (partially air-breathing) propelled space launcher that incorporates a high level of integration of the airframe, the propulsion system and the control system. Many aspects of hypersonic flight in relation with guidance, and propulsion and attitude control came forward that necessitate a detailed analysis of flight mechanics at an early stage of the design process.

However, within the framework of this thesis it is just not possible to study all aspects of flight mechanics that were mentioned before. For instance, the inclusion of elastic-body dynamics and modelling the aero-propulsive-elastic interaction that influences the longitudinal flight dynamics and/or excite the elastic modes requires a concerted aerothermodynamic, structural and propulsion analysis effort, which is considered to be out of the scope of the current
study. Also the lack of experimental data to verify (and validate) the models and corresponding output makes it very difficult to study the effect of elastic-body deformations. Moreover, to understand the above mentioned effects one needs to have a thorough understanding of the rigid-body dynamics, so therefore we will concentrate on non-elastic vehicles in this thesis and leave the inclusion of aeroelasticity and aero-propulsive-elastic interaction for future research. Note that Waszak and Schmidt (1988), Waszak et al. (1990), Schmidt (1993), Raney et al. (1995) and Billimoria and Schmidt (1995) do discuss the importance of aeroelastic effects and emphasise the strong interactions between aerodynamics and propulsion systems and the need for an integrated guidance and control system. The reader is referred to those references for more details.

As common practice, the three-dimensional motion of the space plane under the influence of external forces can be split into two parts, namely the motion of the c.o.m. of the vehicle and the motion of the vehicle around its c.o.m. For a non-elastic body, we can distinguish six components of motion, also called degrees of freedom (d.o.f.). These consist of three translations (the variation with time of the position coordinates of the c.o.m.) along the axes of an arbitrary but orthonormal reference frame, and three rotations of the vehicle (the variation with time of the attitude angles) around these axes.

Consequently, studies of space-plane motion may be separated into two parts. On one hand, we can discern the study of the translation of the c.o.m. only, i.e., the space plane is considered to be a mass point. The corresponding motion is the so-called 3-d.o.f. motion. On the other hand, the combined translations and rotations of the space plane can be examined, requiring a so-called 6-d.o.f. analysis. Both 3-d.o.f. and 6-d.o.f. analyses are used in special areas of interest. In the first place, a 3-d.o.f. analysis is an integral part of the conceptual-design process. The trajectory is considered to be a subsystem within this design process. In this case, the task is to generate a (near) optimal trajectory for given design and operations requirements to get an overview of the performance of the vehicle and to serve as input for the design (and performance) of thermal protection systems, the construction, the propulsion system, the attitude control system and others.

In the second place, this kind of analysis can give insight in the guidance capabilities of a given design. Is it possible, for instance, that for a specified payload (e.g., 7000 kg) and mission (e.g., a Low Earth Orbit (LEO) target orbit), the space plane can still fulfil its mission despite uncertainties in the design parameters (i.e., a variation in mass, geometry, aerodynamics, and/or propulsion properties)? What will be the influence of a variation in mission constraints (e.g., a maximum acceleration, maximum dynamic pressure, or maximum heat load) for a given design? The (manned) space plane should always be able to abort its mission in case of

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3 A vehicle is called rigid when next to being non-elastic also its mass remains constant. Since the space planes in this thesis include a propulsion system which implies a variable mass, they will be referred to as non-elastic vehicles.
failures. Can that be achieved? And last but not least, should either TVC, aerodynamic control, or a combination of both be used for guidance?

6-d.o.f. analysis is used when knowledge about the attitude and inertia of the vehicle is of importance. The results of the 3-d.o.f. simulations are only meaningful if the assumptions that are made can be proven to be correct. For instance, when in 3-d.o.f. studies guidance of a space plane is investigated, it is assumed that the control system can generate the required moments to change the attitude of the vehicle towards the on-coming flow. Moreover, it is often assumed that these attitude changes take place instantaneously. Clearly, the steerability and controllability for (at least) the nominal mission and payload should be verified. This includes the verification of the size and location of the control surfaces, the location of attitude-control thrusters, and the range of thrust-vector angles. Furthermore, when aerodynamic control is used, the induced trim drag should be taken into account when the effectiveness of the control system or the performance of the space plane is studied.

Another relevant study topic is the robustness of a given control-system design. What is, for example, the influence of wind on the angles of attack and sideslip? How will the mass properties change due to fuel consumption and the (alternative) location of fuel tanks? Sensors, which provide information about the angle of attack, for instance, show a certain time delay. How will this delay influence the controllability of the space plane? What will happen when a control surface fails and cannot be moved any more? Can, in that case, the control function be taken over by another system, either backup or alternative? In addition, uncertainties in the design (with respect to mass properties, aerodynamics and propulsion) might create corresponding problems for the control system. As can be concluded from the above (incomplete) list of study topics, the analysis of flight mechanics is an indispensable part of the design process.

1.3.2. Simulation and analysis

When the flight of an ascent or descent vehicle is studied, its accelerations due to external forces and moments are described by a number of ordinary non-linear differential equations. Many authors have given simplifying analytical solutions of these equations (e.g., see Chapman (1959), Loh (1968, 1969), Busemann et al. (1976), Vinh (1981) and Regan (1984)). Although the use of these analytical solutions contributes significantly to the understanding of flight-mechanics related phenomena, for an accurate flight analysis, however, one should not use these approximate solutions, but work with the complete set of differential equations. These equations can only be solved numerically, so a computer program is required, which includes many aspects of the space plane, the environment and the flight mechanics.

Nguyen et al. (1990) give an overview of the key simulation programs and facilities that were used for the Space Shuttle Orbiter descent-flight verification by simulation. First, they mention linear stability-analysis programs, mass-point stability tools that are used to assess
rigid-body as well as flexible stability margins. Second, the non-real-time simulation programs that are very useful for parametric and sensitivity studies as well as anomaly resolution are reported. These programs can include a variety of details, such as emulations of flight software, sensors, non-linear effectors, Earth motion, and gravitational and atmospheric environment. Third, they list the Man-In-Loop (MIL) engineering simulators, to ensure man-machine-mission compatibility and to assess flying and handling qualities. A fourth category is formed by MIL-verification simulators which are used for flight-verification of all mission phases. These simulators use a combination of actual flight hardware, digital computers to simulate the vehicle dynamics and the environment, and analogue computers to model aerosurface actuators, hinge moments and turbulence.

As we indicated in Section 1.2, a space plane will be faced with many uncertainties, resulting from the hypersonic environment, aeroelasticity and the propulsion/airframe interaction. One way to overcome these uncertainties is to develop a robust flight-control system, that will guarantee a safe flight. But the question that arises then, is: how robust should this system be? A too robust system might negatively influence the flying qualities and manoeuvrability, and if the system is not robust enough, we can still end up with a severe problem. It is therefore important that already during the design process as many uncertainties as possible are studied to see how the guidance and control system will deal with them. Once the GNC system has been developed for the nominal mission, common practice is to simulate a number of test cases with different error sources included, and with all dynamic, vehicle and environment characteristics modeled as accurately as possible. This kind of simulations is usually done with non-real-time simulation programs that fall in the second category mentioned by Nguyen et al. (1990).

The execution of these simulations and the analysis of the results, usually combined under the term sensitivity analysis, can be done in a number of ways. Stone and Powell (1976) discuss a sensitivity analysis on the entry guidance and control system of the Space Shuttle Orbiter. Their study objectives were: i) to identify the Shuttle guidance and control system's tolerance to variations in the stability and control aerodynamic parameters, ii) to determine the key parameters or indicators of system performance and resulting vehicle behaviour as these parameters are varied, and iii) to identify system modifications that will increase system tolerance to off-nominal aerodynamics. Their approach was to first investigate each of the stability and control aerodynamic parameters to identify any flow-field interdependencies which would affect how the parameters should be varied relative to another. Next, separate analyses for both the longitudinal and lateral motion were carried out. For example, for the longitudinal analysis the related aerodynamic parameters were first varied independently to establish the system's sensitivity for each parameter. Then parameters were combined and each combination was varied until critical combinations were found that would lead to a violation of one of the criteria, in one form or the other. As a result, they could identify boundary values for each of the aerodynamic parameters and recommend system modifications that would increase system tolerance to off-nominal aerodynamics.
The above example shows that a detailed analysis can lead to many simulations to be executed. Moreover, it is obvious that the more error sources included, the higher the confidence that can be attached to the results. However, more error sources in principle also translate into more possible combinations. If we want to study each error combination with one simulation (a so-called factorial design), then the total number of simulations will rise drastically. For this reason, different sensitivity-analysis methods have been developed, of which the Monte-Carlo analysis is undoubtedly the most familiar one. With this simulation technique, the parameters that can be subjected to errors are defined with a mean value and a standard deviation (related to, for instance, a normal or uniform distribution) and a sufficiently large number of simulations is executed while using a pseudo random-number generator to define the errors for each simulation. After the simulations have been executed, statistical analysis of the results will give insight in the performance of the system under study. Note that a random generator is initialised with a seed number, and different seed numbers give a different sequence of (pseudo) random numbers. Only increasing the number of simulations will minimise the difference between two batches of simulations.

One of the problems of the Monte-Carlo method is related to the total number of simulations (or run size). How many simulations are required to have confidence in the calculated mean and standard deviation of the output response? Basically, the Monte-Carlo method is applied as a robust method, i.e., the run size is chosen as sufficiently large. This number is usually based on experience from previous analysis cases. A more deterministic approach is to increase the run size with a fixed number until the mean and standard deviation of the output do not change significantly, as compared with a pre-defined tolerance level. However, if the number of error sources changes, does that mean that we also have to adjust the run size? It is likely to be true for many additional error sources, but what is the minimum number that can be added without having an effect on the required run size? To be on the safe side one usually selects a run size that will definitely be large enough. As a consequence, it is quite well possible that too many simulations will be executed resulting in a higher computer load and a larger number database with output parameters\(^4\). Another way to approach this problem is to do two Monte-Carlo analyses, where the second one takes twice the number of runs. Then, by doing a so-called Wilcoxon test (Press et al., 1989), the equality or inequality of the resulting two output distributions can be estimated.

A possible alternative method, initially developed for design and production-process optimization, is the Taguchi Method (Taguchi, 1988 and Phadke, 1989). This method uses a pre-defined scheme of cleverly selected combinations to define parameter-setting combinations. The use of such a scheme enables a rapid search over the experimental region (i.e., the \(n\)-dimensional parameter space defined by all possible values of the parameters), because of the

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\(^4\) The run size is not a serious problem when the CPU time per simulation is short. Unfortunately this is usually not the case with controlled-flight simulations. In addition, post-processing of the results may lead to a problem in case a least-squares problem has to be solved which means that large matrices have to be processed.
limited number of simulations that has to be executed. Stanley et al. (1992) applied the Taguchi design-method to optimize the propulsion system for SSTO vehicles. Instead of 2187 design cycles for a full factorial design of 7 design parameters with three values (also called levels) each, only 50 runs were required. These runs included interactions between several design parameters. Mistree et al. (1993) made a comparison between the Monte-Carlo and Taguchi Method applied to trajectory simulation and concluded that the Taguchi Method can be successfully used as an alternative to Monte-Carlo analysis.

After conducting a sensitivity analysis, it is possible to compute a response surface, i.e., a polynomial function in one or more dimensions that describes the relation between the selected output parameter (the response) and the applied variation on the input parameters (the factors). By minimising the functional representation of the response surface, it is possible to find the optimal value of the response and the corresponding setting of the factors, of course only in the experimental region. Here, we can think of a maximum acceleration, or a maximum allowable heat flux. If we want to make sure that for a given set of uncertainties the maximum response will not be higher than a certain predefined value, we can find the maximum value in the experimental region as we just described. The methodology to set up experiments and derive response surfaces is known as Response Surface Methodology (RSM). Khuri and Cornell (1987) give an extensive treatment of RSM, thereby focusing on first and second-order response surfaces of multiple factors.

One particular method that is worth mentioning is the Central Composite Design (CCD), a design method to derive second-order response surfaces. Stanley (1993) incorporated Taguchi’s orthogonal arrays in part of the CCD and applied it to the configuration selection and design of a rocket-powered single-stage vehicle. For five three-level factors, they needed a total of 27 experiments, as opposed to 81 required by the stand-alone Taguchi method. Therefore it is desirable to know, whether this method can also be successfully applied to trajectory simulation and the development and testing of GNC systems.

The intention in this thesis work is to develop a generic 6-d.o.f. model which allows for a fast analysis of flight mechanics of space vehicles. This methodology will be translated into a computer tool with which the guidance and control of a hypersonic space plane during ascent and descent can be studied, and with which the results of the simulations can be processed. To facilitate the use of the program and to make it suitable to study arbitrary vehicle geometries and missions, it will include an extensive user interface. The development of the flight-simulation tool will be a step-by-step process, wherein evaluations of the models are included. The simulation tool should consist of some key building blocks that have been schematically depicted in Fig. 1.6. Depending on the availability of simulation packages, more or less modules have to be added and/or adapted to allow for the kind of mission analysis to be conducted. A serious problem in obtaining flight-simulation packages that have been developed by industry is that most of the time they are company confidential, and even if they are available it is usually not possible to get the source code, which is necessary to adapt or extend the software.
The only simulation tool that already included some of the modules listed in Fig. 1.6 and which was available at the beginning of this study at Delft University of Technology, Faculty of Aerospace Engineering, was the Simulation Tool for Atmospheric Re-entry Trajectories, START, which has seen a gradual development over the past few years (Mooij, 1991a, 1992, 1993a and 1993b). The layout of the program is such, that it can relatively easy be extended with the models required for the guided and controlled flight of a propelled, winged vehicle (Mooij, 1994a). For this reason, it was decided to select START as the basis for the simulation tool to be developed and used for the analysis of ascent and descent of space planes. The original version of START had the capability of simulating the open-loop behaviour of re-entry vehicles in 6 d.o.f. (Mooij, 1991a). With open loop it is meant that no guidance and control capabilities are included. Furthermore, the vehicles were assumed to have a constant mass and a simple
2-dimensional parachute model could be used for studying some aspects of parachute descent. The mission environment was not restricted to the Earth alone, but included the Moon, Mars and Titan. An extensive user interface made it a flexible tool to simulate different re-entry vehicles and missions.

For a mission analysis of a space plane, several extensions to START were required. The major ones are related to guidance and control, propulsion systems and variable mass properties. To do a sensitivity analysis in an efficient manner it was furthermore required that the simulation definition and execution, and post processing of the results are incorporated in the software. The implementation in START should be done in a step-by-step manner, according to a tool-development plan as discussed by Mooij et al. (1993) and Mooij (1994a). Each of the steps should be verified and, if possible, compared with results published in open literature. Validation of the results, i.e., comparison with actual flight data is unfortunately not possible because of unavailability of such data.

As was stated before, one of the key elements that was missing in START is a guidance and control system. Such a system is required to study guided and controlled flight of atmospheric space vehicles. This means, that although we want to control the vehicle, it does not have to be the best possible controller which guarantees mission success under all circumstances. In fact, it is no problem if the sensitivity analysis will show the limitations of the controller. To put it in other words, not the performance of the controller is the study objective, but the analysis technique and what the impact of the G&C system is on the flight mechanics.

For this reason we will look at available G&C systems, and if there are no suitable ones they will be developed where we will keep the complexity as low as possible. As attitude control system, two concepts will be studied. The first concept is based on linear state feedback with gain scheduling, a popular concept that has been applied many times and which is relatively easy to design and implement. This concept will be used to get experience with control systems. The second concept is somewhat more complex but is said to yield a robust controller, i.e., the concept of Model Reference Adaptive Control. This concept has been selected because the results obtained in various industrial projects seem to be promising and it has not been applied to space planes before.

1.4. Layout of thesis

The main question or problem definition of this study has been formulated as follows:

How and to what extent do design uncertainties and environmental disturbances influence the mission performance of space planes, and in what way can a GNC system contribute to mission success?

This problem definition has been translated into three study goals, i.e.,
1) develop a 6-d.o.f. flight-simulation tool, which can be used to compute the ascent and
descent trajectories of space planes,
2) assess the potential use in the study of flight mechanics in general, and the analysis of
G&C systems in particular, of the Taguchi Method and the related Response Surface
Methodology as an alternative to the commonly applied Monte-Carlo Method, and
3) study the flight mechanics of space planes in ascent and descent missions and identify
possible shortcomings in the GNC-system design.

In Fig. 1.7, the key elements of the study have been identified. Several limitations apply to each
of the elements, some of them which have been excluded from the current research earlier in
this chapter, and they will be listed below.

Fig. 1.7 - Key elements of the thesis study.
GNC system

- no control-surface delays
- no individual modelling of RCS thrusters
- infinite accuracy of control-surface deflections and RCS moments
- ideal navigation system
- separate propulsion and attitude controller
- simple propulsion controller

Vehicle

- no aeroelasticity
- HTO/HL SSTO space plane
- use of available models from literature
- simple propulsion model
- air-breathing propulsion
- no aero-propulsive-elastic interactions
- no fuel-tank modelling and sloshing

Mission

- no terminal area
- no take-off and landing
- no orbital operations
- no stage separation

Analysis and design

- no Monte-Carlo analysis

The current study incorporates in principle three fields of study, i.e., flight mechanics, guidance and control, and design and analysis. Since not every reader may be an expert in each of these fields, it is possible that in some cases attention is paid to apparent trivial issues. The theoretical background for the three fields of study is found in Chapters 2, 3 and 4, as summarised below.

- Chapter 2 introduces the mathematical core of the flight-simulation model, i.e., the equations which describe the motion of a mass-varying non-elastic body under the influence of aerodynamic, gravitational and propulsive forces and moments in a planetary atmosphere with wind. This chapter is divided into three parts. The first part gives the equa-
tions of translational motion with respect to a planet-fixed (and rotating) frame. A discussion on the induced forces due to variable mass properties is included. The need for guidance and the execution of related commands results in the definition of so-called control forces. The second part presents the Euler equations of rotational motion, including a discussion on the induced moments due to variable mass properties. We will comment on how changing the attitude by applying control moments can shape the control forces. The third and last part defines the parameters of the wind vector, and how inclusion of wind will change the equations of translational and rotational motion.

- In Chapter 3 the mathematical foundation of the guidance and control concepts that are used in the thesis study are given. The re-entry guidance will be based on a combination of energy and altitude control for the vertical motion, and applying bank reversals (i.e., changing the sign of the bank angle) for the lateral motion. For the ascent phase, two guidance systems will be introduced. One deals with guidance in a vertical plane, and is based on flight-path angle control while satisfying flight-path constraints (e.g., a maximum heat flux). The other concerns hypersonic manoeuvres. The last part of the chapter presents two attitude-control concepts. The first one is based on optimal-control theory, the so-called Linear Quadratic Regulator (LQR), and will be applied to an unpowered, winged re-entry vehicle in Chapter 7. The second one is based on Model Reference Adaptive Control (MRAC) and will be applied to an SSTO space plane in Chapter 8.

- Chapter 4 gives the theoretical background of the Taguchi Method and basic definitions of response surfaces are given. The Central Composite Design (CCD) method as one particular methodology for setting up second-order response surfaces is discussed, in combination with the method of least squares to actually compute the response surface. Furthermore, the statistical analysis of the results is briefly mentioned. This analysis is based on the ANalysis Of Variance (ANOVA), which is a common technique to derive statements about the results in a statistical manner. The chapter concludes by introducing a robust design methodology as a means to make a system or process as insensitive to disturbances as possible, without eliminating the source of the disturbances.

The developed flight-simulation tool START is discussed in Chapters 5 and 6.

- The developed flight-simulation and analysis model as presented in Chapters 2, 3 and 4 is coded and incorporated in START. The resulting flight-simulation software tool is discussed in Chapter 5. This chapter gives an overview of the layout of the flight-simulation software and its capabilities.
\textbullet{} In Chapter 6, START is applied to a number of selected test cases to validate, verify and evaluate the software step by step. Successively, the free-fall, 6-d.o.f. re-entry of the Apollo capsule, the entry and parachute descent of the Huygens probe in the atmosphere of Titan and the aerodynamic controllability of a moderate lift-to-drag re-entry test vehicle will be reviewed. The aspects that are addressed are the following:

\hspace{1em} \textbullet{} uncontrolled, spinning entry in 3- and 6-d.o.f.,

\hspace{1em} \textbullet{} offset in the location of the c.o.m.,

\hspace{1em} \textbullet{} steady-state wind and horizontal wind gusts,

\hspace{1em} \textbullet{} trimmed entry and descent in 3- and 6-d.o.f.,

\hspace{1em} \textbullet{} open-loop guidance,

\hspace{1em} \textbullet{} aerodynamic attitude control based on LQR, and

\hspace{1em} \textbullet{} limited exploration of Taguchi's orthogonal arrays.

The mission of an SSTO space plane is analysed in Chapters 7 and 8. The analysis of the ascent and descent trajectory of a space plane is discussed separately. Since no aerodynamic data are available that cover the angle-of-attack range required for both the powered ascent and unpowered descent mission, two reference vehicles are used. The analysis will start with the entry and descent, because this is easier in the sense of vehicle modelling and GNC.

\textbullet{} In Chapter 7, the guided and controlled descent of HORUS, an unpowered, winged re-entry vehicle similar to the Space Shuttle Orbiter is presented. The discussion includes the design of a Linear Quadratic Regulator and a detailed application of the developed sensitivity-analysis technique. Aspects that will come forward are:

\hspace{1em} \textbullet{} winged vehicle, with 5 independent control surfaces,

\hspace{1em} \textbullet{} high-angle of attack entry,

\hspace{1em} \textbullet{} closed-loop guidance,

\hspace{1em} \textbullet{} combination of reaction and aerodynamic control based on LQR,

\hspace{1em} \textbullet{} trim with body flap and elevons,

\hspace{1em} \textbullet{} extensive application of Taguchi method, and

\hspace{1em} \textbullet{} robust design of the guidance system.

\textbullet{} In Chapter 8, the powered guided and controlled flight of the space plane will be discussed. This space plane is the Winged Cone Configuration, a reference concept developed by NASA. Two mission aspects are studied, i.e., the direct ascent to orbit, which is basically a flight in a vertical plane bounded by constraints on the dynamic pressure, axial load and heat flux, and hypersonic manoeuvres such as attitude transitions and turns. In either case, an attitude controller based on Model Reference Adaptive Control is used to stabilise the vehicle. Aspects that are addressed, are:

\hspace{1em} \textbullet{} complete definition of the propulsion system,

\hspace{1em} \textbullet{} trim with canards, elevons, and/or the thrust vector,
• propulsion control based on tracking the trajectory constraints,
• reference-trajectory design using Taguchi’s orthogonal arrays, and
• aerodynamic attitude control based on MRAC.

• Chapter 9 concludes this thesis by summarising the major results of the study and giving recommendations for future research.
Chapter 2

Flight Mechanics

Everything in space obeys the laws of physics. If you know these laws and obey them, space will treat you kindly. And don't tell me that man doesn't belong out there. Man belongs wherever he wants to go; and he'll do plenty well when he gets there. Wernher von Braun

In Chapter 1, the need of a flight-simulation model to study the flight mechanics of space planes was identified. This chapter will introduce the mathematical core of this flight-simulation model, i.e., the equations that describe the motion of a mass-varying non-elastic body, under the influence of aerodynamic, gravitational and propulsive forces and moments in a planetary atmosphere with wind.

The organization of the equations depends on the choice of the state variables. Position and velocity can be defined by, for instance, cartesian or spherical components. For the attitude of the body, on the other hand, Euler angles, directional cosines or quaternions can be used. In this study, we will use spherical coordinates for the position, whereas the velocity is defined by its modulus and two direction angles. For the attitude of the vehicle we use aerodynamic angles, a special kind of Euler angles. The reason for this choice is that they are directly interpretable, and can also serve as a basis for the design of the guidance and control system.

The external forces and moments can be expressed in several reference frames. Propulsion forces, for example, are usually expressed in a body-fixed frame, whereas the aerodynamic forces are given with respect to a so-called aerodynamic frame. Moreover, although defining the equations of motion with respect to an inertial frame avoids the use of apparent forces it is more convenient to express them to a rotating frame that is fixed to, for instance, the Earth. In conclusion, to express the various relevant scalar and vector expressions several reference frames are required.

Of course, there are many other publications available that give the equations of motion, e.g., Cornelisse et al. (1979), Etkin (1972), Hughes (1986), Regan (1984) and Vinh (1981). However, some of them do not give the equations expressed in the related state variables, whilst others take simplifying assumptions into account or derive the equations for special cases. Moreover, how to include wind in relation to the selected state variables is not commonly described.
For this reason, a detailed treatment of the equations of motion is given in this chapter, to begin with the general formulation in an inertial frame of reference (Section 2.1). Section 2.2 will introduce all definitions that are required for an unambiguous derivation of the equations of motion. The selected state variables are introduced, as well as environment-related parameters. In Appendix A the related reference frames to express the various relevant scalars and vectors in the most efficient way are defined. Section 2.3 gives an overview of the external forces and moments, which can act on a vehicle moving in a planetary atmosphere. These forces and moments can be of aerodynamic, gravitational and propulsive origin. In Section 2.4, the equations of translational motion will be presented. In a similar manner, the equations of rotational motion are discussed in Section 2.5. Whereas in the previous two sections the equations of motion for an atmosphere without wind are discussed, Section 2.6 will present how to include wind in the equations of motion.

2.1. General formulation of the equations of motion

With the relatively low speeds encountered by present-day spacecraft (that is, when compared to the speed of light), we can use classical or Newtonian mechanics to describe the motion of a vehicle flying in a planetary atmosphere. This kind of mechanics is based on the three Laws of Motion of Newton and Galileo’s principle of relativity.

The motion of a non-elastic body can be divided into the motion of the c.o.m. and the one around this c.o.m. The equations describing the first part are called the equations of translational motion, giving information about position and velocity in three directions (a position coordinate is also called a degree of freedom). The second part of the motion is described by the equations of rotational motion, resulting in information on the angular rates around three axes and the corresponding attitude of the body (similarly, an ‘attitude coordinate’ is a degree of freedom). The complete set of equations incorporates therefore 6 degrees of freedom (6 d.o.f.), i.e., three translations and three rotations.

![Fig. 2.1 - The motion of a body with respect to inertial space.](image-url)
Let us assume, that an arbitrary vehicle with variable mass $m$ is moving about a celestial body and is subjected to a number of external forces. The total sum of the external forces is denoted by $F_I$. The position vector of the c.o.m. of the vehicle is given by $r_{cm}$ relative to an arbitrary inertial frame, whereas its velocity with respect to this frame is given by $V_I$. Furthermore, the vehicle is rotating with an angular velocity $\omega$ with respect to the inertial frame. The situation is depicted in Fig. 2.1. Starting with Newton's second law of motion, the translational motion of this mass-varying body under the influence of an external force can be described by (Cornelisse et al., 1979):

$$F_I + F_C + F_{rel} = \ddot{F}_I = m\frac{d^2r_{cm}}{dt^2}$$  \hspace{1cm} (2.1.1)

with

$F_I$ \hspace{1cm} total of the external forces expressed in components of the inertial frame (N)

$\dot{F}_I$ \hspace{1cm} pseudo external-force vector in inertial frame (N)

$F_C = -2\omega \times \int_m \frac{\delta r}{\delta t} dm$ \hspace{1cm} Coriolis force, due to time variations in mass distribution (N)

$F_{rel} = -\int_m \frac{d^2r}{dt^2} dm$ \hspace{1cm} relative force, due to time variations in mass distribution (N)

$\frac{d^2r_{cm}}{dt^2}$ \hspace{1cm} acceleration of the c.o.m. with respect to the inertial frame (m/s$^2$)

$\omega = (p, q, r)^T$ \hspace{1cm} the rotation vector of the body frame with respect to the inertial frame, expressed in components along the body axes

$r$ \hspace{1cm} the location of a mass element with respect to the c.o.m. of the vehicle (m)

The notation $\frac{\delta .}{\delta t}$ denotes a derivative of a vector relative to the local (body) frame, as compared with $\frac{d .}{dt}$, which denotes a derivative relative to the inertial frame.

The general form of the equation of rotational motion is expressed as (Cornelisse et al., 1979):

$$M_{cm} + M_C + M_{rel} = \ddot{M}_{cm} = \int_m \hat{r} \times \left( \frac{d\omega}{dt} \times \hat{r} \right) dm + \int_m \hat{r} \times (\omega \times (\omega \times \hat{r})) dm$$  \hspace{1cm} (2.1.2)

with

$M_{cm}$ \hspace{1cm} total external moment about the vehicle's c.o.m. (Nm)

$\dot{M}_{cm}$ \hspace{1cm} pseudo external moment about c.o.m. (Nm)
\[ M_C = -2 \int \frac{\mathbf{F} \times \left( \mathbf{\omega} \times \frac{\delta \mathbf{F}}{\delta t} \right)}{m} \, dm \]

Coriolis moment due to time variations in mass distribution (Nm)

\[ M_{rel} = -\int \frac{\delta^2 \mathbf{F}}{\delta t^2} \, dm \]

relative moment due to time variations in mass distribution (Nm)

\[ \int \frac{m \left( \frac{d\mathbf{\omega}}{dt} \times \mathbf{F} \right)}{m} \, dm \]

apparent moment due to the angular acceleration of the vehicle with respect to the inertial frame (Nm)

\[ \int \frac{m \left[ \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{F}) \right]}{m} \, dm \]

apparent moment due to the angular velocity of the vehicle with respect to the inertial frame (Nm)

Eqs. (2.1.1) and (2.1.2) describe the motion of a body under the influence of external forces and moments. The corresponding change in position and attitude can be described by:

\[ \frac{dr}{dt} = \mathbf{V} \]

(2.1.3)

\[ \frac{d\mathbf{\Phi}}{dt} = f(\mathbf{\Phi}) \mathbf{\omega} \]

(2.1.4)

In Eq. (2.1.4), we have used the vector \( \mathbf{\Phi} \) to denote the attitude of the body in the inertial space.

We note that the dynamic equations of motion for an elastic mass-varying body have the same form as those for a rigid body by introducing a pseudo external force and moment vector, that includes the Coriolis and relative force and moment vector. An important principle is involved here, the Principle of Solidification. This principle states (Comelisse et al., 1979) that in general, equations of translational and rotational motion of an arbitrary variable mass system at time \( t \) can be written as the translational and rotational equations for a rigid body with mass \( M \) equal to the mass of the system at time \( t \), while in addition to the true external forces and moments, two apparent forces and moments are applied: the Coriolis and relative forces and moments, respectively.

2.2. Definitions

In this section, state variables for position, velocity, attitude and angular rates, and some environmental parameters will be presented. In Appendix A the reader can find the definition of the reference frames that are used in this study, as well as the relations between these reference frames in the form of transformation matrices.
2.2.1. State variables

The position and velocity of the vehicle are expressed with respect to the rotating planetocentric reference frame with the origin in the c.o.m. of the central body (the so-called $R$-frame), see also Fig. 2.2. The position is given by the distance $R$, longitude $\tau$ and latitude $\delta$, whereas the velocity is expressed by its modulus, the groundspeed $V_G$, and two direction angles, i.e., flight-path angle $\gamma_G$ and heading $\chi_G$. The longitude is measured positively to the east ($-180^\circ \leq \tau < 180^\circ$). The latitude is measured along the appropriate meridian starting at the equator, positive in north direction ($0^\circ \leq \delta \leq 90^\circ$) and negative to the south. $R$, finally, is the distance from the c.o.m. of the central body to the c.o.m. of the vehicle. The relative velocity $V_G$ (i.e., the modulus of the velocity vector $\mathbf{V}$) is expressed with respect to the $R$-frame. $\gamma_G$ is the angle between $\mathbf{V}$ and the local horizontal plane; it ranges from $-90^\circ$ to $+90^\circ$ and is negative when $\mathbf{V}$ is oriented below the local horizon. $\chi_G$ defines the direction of the projection of $\mathbf{V}$ in the local horizontal plane with respect to the local north and ranges from 0° to 360°. When $\chi_G = +90^\circ$, the vehicle is moving parallel to the equator to the east.

![Diagram](image)

Fig. 2.2 - Definition of the six spherical flight parameters, the position ($R, \tau, \delta$) and velocity ($V_G, \gamma_G, \chi_G$). Here, both $\tau$, $\delta$, $\gamma$ and $\chi$ are positive. The indicated frame is the rotating planetocentric frame (index $R$), with its origin in the c.o.m. of the central body and the $Z_R$-axis aligned with the central body's rotation vector.

The attitude of a vehicle, or, in mathematical terms, the orientation of the body-fixed reference frame with respect to the trajectory reference frame, is expressed by the so-called aerodynamic angles, i.e., the angle of attack $\alpha_G$ ($-180^\circ \leq \alpha_G < 180^\circ$, for a 'nose-up' attitude $\alpha_G > 0^\circ$), the angle of sideslip $\beta_G$ ($-180^\circ \leq \beta_G \leq 180^\circ$, $\beta_G$ is positive for a 'nose-left' attitude) and the bank angle $\sigma_G$ ($-180^\circ \leq \sigma_G < 180^\circ$, $\sigma_G$ is positive when banking to the right), see also Fig. 2.3. The above mentioned angles define the attitude of the vehicle with respect to the groundspeed when they are used in the equations of motion. Otherwise, they can also define the orientation of the body with respect to the airspeed.
Fig. 2.3 - Definition of the aerodynamic attitude angles $\alpha_G$, $\beta_G$ and $\sigma_G$, and the angular rates $p$, $q$ and $r$. Here, all states are positive.

The angular rate of the body is here defined as the rotation of the body frame with respect to the inertial frame, expressed in components along the body axes. These components are called roll rate $p$, pitch rate $q$ and yaw rate $r$ (see again Fig. 2.3).

2.2.2. Flight environment

The environment for the flight simulations consists of a celestial body, i.e., a planet or its moon. Parameters that are of importance deal with the shape, size, mass distribution and rotational rate of this so-called central body. The possible presence of an atmosphere implies that there can be wind. In this sub-section the relevant parameters related to the environment are defined.

For the purpose of flight simulation we may approximate the shape of the Earth as an ellipsoid of revolution (Regan, 1984), with its minor axis along the Earth’s polar axis. We assume that this approximation is also valid for other planets. The parameter defining this ellipsoid is called the ellipticity $e$ and is defined as

$$
e = \frac{R_e - R_p}{R_e} = 1 - \frac{R_p}{R_e}$$  \hspace{1cm} (2.2.1)

with

$R_p = \text{mean radius at the pole (m)}$

$R_e = \text{mean radius at the equator (m)}$
For the Earth, $R_p$ is about 21 km smaller than $R_e$. We can use the ellipticity to derive an expression for the radius at an arbitrary point along the surface, $R_s$. Regan (1984) finds the following expression:

\[
R_s = R_e \left[ 1 - \frac{\theta}{2} (1 - \cos 2\delta^*) + \frac{5}{16} \theta^2 (1 - \cos 4\delta^*) + \ldots \right] \tag{2.2.2}
\]

Here, $\delta^*$ is the geographic latitude which is for the ellipsoid not the same as the geocentric latitude $\delta$. However, since the difference is very small it is justified to approximate $\delta^*$ by $\delta$. In addition, for first-order analyses we may approximate Eq. (2.2.2) by

\[
R_s = R_e \left[ 1 - \frac{\theta}{2} (1 - \cos 2\delta) \right] = R_e (1 - \varepsilon \sin^2 \delta) \tag{2.2.3}
\]

The expression for $R_s$ can be used to define the height above the planetary surface. As has been discussed in Section 2.2.2, the position of the vehicle is given by $(R, \tau, \delta)$. The height $h$ can then be derived from

\[
h = R - R_s = R - R_e (1 - \varepsilon \sin^2 \delta) \tag{2.2.4}
\]

For the rotation of the central body, we assume that it is rotating with a constant angular velocity $\omega_R$ directed along the $Z$-axis of the rotating planetocentric frame\(^5\), so

\[
\omega_R = (0, 0, \omega_{cb})^T \tag{2.2.5}
\]

For central bodies with an atmosphere, we assume the possibility of the occurrence of wind. The wind vector is defined with respect to the $R$-frame, and can either be defined by a modulus and two direction angles, i.e., $V_W \gamma_W$ and $\chi_W$ (see Fig. 2.4), or in cartesian components expressed in the vertical frame, depending in which format these (input) data are available. Between the two sets there is a simple relationship, namely

\[
\begin{align*}
\nu_x &= V_W \cos \gamma_W \cos \chi_W \\
\nu_y &= V_W \cos \gamma_W \sin \chi_W \\
\nu_z &= -V_W \sin \gamma_W
\end{align*} \tag{2.2.6}
\]

and vice versa

\(^5\) The rotation vector of the Earth is known to vary in time, both in magnitude and direction. However, this variation is so slow in comparison with the time scale of the flight simulation that it may be neglected.
Fig. 2.4 - The relation between the wind frame (index $W$) and the vertical frame (index $V$).

\[ V_W = \sqrt{v_x^2 + v_y^2 + v_z^2} \]

\[ \gamma_W = -\arcsin \left( \frac{v_z}{V_W} \right), \text{ with } \gamma_W \in [-90^\circ, +90^\circ] \]  \hspace{1cm} (2.2.7)

\[ \chi_W = \arctan \left( \frac{\sin \chi_W}{\cos \chi_W} \right), \text{ with } \sin \chi_W = \frac{v_y}{\sqrt{v_x^2 + v_y^2}} \text{ and } \cos \chi_W = \frac{v_x}{\sqrt{v_x^2 + v_y^2}} \]

2.3. External forces and moments

So far, external forces and moments that are of aerodynamic, gravitational and propulsive origin were mentioned. Before we start discussing them in detail in the next three sub-sections, three remarks have to be made.

First, the propulsion forces and moments are partly internal, i.e., they are due to a variation in mass distribution, and therefore a component of them is included in the Coriolis and relative force and moment introduced in Section 2.1.

Second, it is not trivial in case of an air-breathing space plane which part of the vehicle contributes to the aerodynamic forces and moments and which part to the propulsion forces and moments. This problem, that is also known as bookkeeping the forces and moments, is illustrated in Fig. 2.5. In this figure, the choice of the control surfaces is shown for a particular space plane. The impulse and pressure forces acting on ABCDEFGHIJA define the propulsion forces and corresponding propulsion moments after summation of the individual forces (and moments) acting on
SECTION 2.3  EXTERNAL FORCES AND MOMENTS

AB, BC, etc. It is clear that, for instance, the aerodynamic drag on surface FG is included in the propulsion force and will in this example therefore not be part of the aerodynamic force. It should be noted that for the space plane in Chapter 8, the database with aerodynamic and propulsion forces is predefined, where the problem of bookkeeping has already been addressed. For other space-plane models, careful definition of the control surfaces is required. For more information, see, for instance, the papers by Chavez and Schmidt (1992, 1993 and 1994).

Fig. 2.5 - Control surfaces to determine the forces and moments due to the propulsion system (Kremer, 1991).

Third, there may be other sources of external forces and moments as well, e.g., of magnetic origin. We assume that these additional forces and moments can be neglected compared to the other three. Also the perturbing gravitational attraction of other bodies than the central body will be neglected.

In the remainder of this section, the aerodynamic (Section 2.3.1), gravitational (Section 2.3.2) and propulsion (Section 2.3.3) forces and moments are discussed.

2.3.1. Aerodynamic origin

In addition to the atmospheric density, the aerodynamic forces and moments depend on the velocity of the vehicle with respect to the atmosphere. The forces are usually given by

\[
F_{AA} = \begin{pmatrix}
-D \\ -S \\ -L
\end{pmatrix} = \begin{pmatrix}
-C_D q_{dyn} S_{ref} \\ -C_S q_{dyn} S_{ref} \\ -C_L q_{dyn} S_{ref}
\end{pmatrix}
\] (2.3.1)

designated as drag \(D\), side force \(S\) and lift \(L\). These forces are defined with respect to the (airspeed-based) aerodynamic frame (index AA), and have a positive value along \(-X_{AA}\), \(-Y_{AA}\) and \(-Z_{AA}\), see also Fig. 2.6. The variables in Eq. (2.3.1) have the following meaning:
Fig. 2.6 - Definition of the aerodynamic forces in the aerodynamic frame (for the sake of convenience, the origin of the aerodynamic frame is considered to be coincident with the origin of the body frame). The vehicle has a relative velocity $V_A$ with respect to the atmosphere. The direction of the relative flow velocity is defined by the angle of attack $\alpha_A$ and the angle of sideslip $\beta_A$, both positive in the convention used here.

$$q_{\text{dyn}} = \frac{1}{2} \rho V_A^2$$

- $\rho$ = atmospheric density (kg/m$^3$)
- $V_A$ = airspeed (m/s)
- $S_{\text{ref}}$ = aerodynamic reference area (m$^2$)

The aerodynamic coefficients, $C_D$, $C_S$ and $C_L$, the value of which can be obtained by wind-tunnel measurements or from in-flight measurements and some parameter-identification process, are usually functions of the Mach number $M$ and the aerodynamic angles $\alpha_A$ and $\beta_A$. $M$ is defined as

$$M = \frac{V_A}{a}$$

(2.3.2)

where $a$ is the local speed of sound.

As we will see in Section 2.4, where the equations of translational motion are discussed, the aerodynamic forces must be expressed in the $AG$ frame. If there is no wind, $V_A$, $\alpha_A$, etc., are equal to their groundspeed equivalents $V_G$, $\alpha_G$, etc. This implies that

$$F_{A,AG} = F_{A,AA}$$

(2.3.3)

$F_{A,AG}$ can directly be used for the spherical equations of translational motion. How to treat $F_{A,AA}$ in case there is wind, will be discussed in Section 2.6.
The aerodynamic moments are expressed in the body frame and are defined as

\[
M_{A,B}^M = \begin{pmatrix} L \\ M \\ N \end{pmatrix} = \begin{pmatrix} C_l q_{dyn} S_{ref} b_{ref} \\ C_m q_{dyn} S_{ref} c_{ref} \\ C_n q_{dyn} S_{ref} b_{ref} \end{pmatrix}
\]

(2.3.4)

where \( L \) is the rolling moment, \( M \) is the pitching moment and \( N \) is the yawing moment. In the equation above, \( b_{ref} \) and \( c_{ref} \) are the aerodynamic reference lengths for roll/yaw and pitch, respectively (in relation to aircraft, \( b_{ref} \) is usually the wing span and \( c_{ref} \) the mean aerodynamic wing chord; in case of, for instance, an axisymmetric re-entry capsule as reference length for all three moments usually the maximum diameter \( d \) of the capsule is used).

From wind-tunnel measurements one usually gets the forces and moments with respect to a so-called aerodynamic reference point (e.g., the suspension point of the model in the wind tunnel). In case the location of this point does not coincide with the location of the c.o.m. the contribution of the forces to the moment about the c.o.m. has to be computed separately. This contribution can be obtained as follows. Suppose that the forces are defined with respect to a reference frame with the origin in the aerodynamic reference point and the axes collinear with the corresponding axes of the \( B \)-frame. The location of the c.o.m. in this reference frame is given by

\[
r_{cm} = (x_{cm}, y_{cm}, z_{cm})^T
\]

(2.3.5)

The moment vector due to the aerodynamic forces results from the following vector multiplication (Fig. 2.7):

\[
M_{A,B}^F = r_{cm} \times F_{A,B}
\]

(2.3.6)

yielding a total aerodynamic moment vector of

\[
M_{A,B} = M_{A,B}^M + M_{A,B}^F
\]

(2.3.7)

The relation between \( F_{A,B} \) and \( F_{A,AA} \) is given by

\[
F_{A,B} = C_{B,AA} F_{A,AA}, \quad \text{with} \quad C_{B,AA} = C_2(\alpha_A) C_3(-\beta_A)
\]

(2.3.8)

For the definition of the transformation matrix \( C_{B,AA} \), the reader is referred to Appendix A.
Fig. 2.7 - Moments due to the aerodynamic forces arise, when the c.o.m. and the aerodynamic reference point are not coincident: \((L,M,N)_F^T = (x_{cm},y_{cm},z_{cm})^T \times (X,Y,Z)^T\)

2.3.2. Gravitational origin

The gravitational force acting on the vehicle is equal to

\[ F_{G,R} = mg \]  \hspace{1cm} (2.3.9)

with

\[ m = \text{mass of vehicle (kg)} \]
\[ g = \text{gravitational acceleration vector (m/s}^2) \]

The gravitational force is expressed with respect to the \(R\)-frame. To find component expressions for \(g\), we approximate the central body by an ellipsoid with its minor axis along the polar axis and with a rotationally symmetric mass distribution. For the spherical equations of translational motion, \(g\) is defined as\(^6\)

\[ g = (g_r,0,g_b)^T \]  \hspace{1cm} (2.3.10)

\(^6\) The (spherical) components of the gravity vector are given in a vehicle based frame; the \(X\)-axis is positive pointing downwards, perpendicular to the tangential plane to the ellipsoid, i.e., the local horizontal plane. The \(Z\)-axis lies in this plane and is positive in northern direction, whereas the \(Y\)-axis completes the right-handed system (and is positive pointing eastward).
where, because of the assumptions we made concerning the rotational symmetry of the central body, there is no component of the gravity vector out of the meridian plane. $g_r$ and $g_\delta$ are given by (based on Regan, 1984):

$$g_r = \frac{\mu}{R^2} \left[ 1 - \frac{3}{2} J_2 \left( \frac{R_e}{R} \right)^2 (3 \sin^2 \delta - 1) \right] \quad (2.3.11a)$$

$$g_\delta = -3 J_2 \frac{\mu}{R^2} \left( \frac{R_e}{R} \right)^2 \sin \delta \cos \delta \quad (2.3.11b)$$

where

$\mu$ = gravitational parameter (m$^3$/s$^2$) = $G m_{cb}$

$G$ = universal gravity constant = $6.668 \cdot 10^{-11}$ m$^3$/kg s$^2$

$m_{cb}$ = mass of central body (kg)

$R$ = distance to the centre of the central body (m)

$R_e$ = mean equatorial radius of the central body (m)

$J_2$ = numerical coefficient related to the flattening of the central body

$\delta$ = geocentric latitude (rad)

Because the gravitational force varies with $R$ and $\delta$ there will be a gravitational force gradient over a space vehicle, which, in general, will produce a gravitational moment. However, this gradient is very small so that the moments of gravitational origin can be neglected compared with the aerodynamic and thrust moments acting on a space plane (Duhamel, 1989).

2.3.3. Propulsive origin

In Eq. (2.1.1), two apparent forces due to the varying mass of the vehicle - in this study due to the use of a propulsion system using mass expulsion to propel the vehicle - were mentioned, i.e., the Coriolis force $F_C$ and the relative force $F_{rel}$. Expressions for $F_C$ and $F_{rel}$ are dependent on whether the propulsion system is a rocket or air-breathing system. Here, we apply air-breathing propulsion systems\(^7\) of which a schematic view is given in Fig. 2.8. The propulsion system has an inlet area $A_{in}$ and an exhaust area $A_e$. The inlet mass flow $\dot{m}_{in}$ (with density $p_{in}$) is thought to enter the inlet with a velocity $\bar{V}_{in}$ (expressed in, for instance, the $B$-frame) at the so-called *centre of inlet mass flow*, which is located at a distance $r_{in}$ from the c.o.m. of the space plane. Similarly, the exhaust

---

\(^7\) For deriving the Coriolis and relative force for reaction-control thrusters, we can do a similar derivation with the difference that there will only be an exhaust mass flow. The inlet mass flow should be set to zero, in this case.
mass flow \( \dot{m}_e \) (\( \rho_e \) and \( \mathbf{V}_e \), also with respect to the B-frame) is leaving the propulsion system at the centre of exhaust mass flow, located at \( r_e \) from the c.o.m. Inside the combustion chamber, \( \dot{m}_{in} \) (the oxidizer) will react with a fuel mass flow \( \dot{m}_f \), thus forming \( \dot{m}_e \). Note that \( \dot{m}_{in} \) is positive, whereas \( \dot{m}_e \) is negative.

![Diagram of propulsion system of a space plane](image)

Fig. 2.8 - Schematic model of the propulsion system of a space plane.

In Mooij (1994b), expressions for \( F_C \) and \( F_{rel} \) are derived, following a similar line of thought of Cornelisse et al. (1979) who derived expressions for rocket engines without inlet. The results are

\[
F_C = -2 \omega \times (\dot{m}_{in} \mathbf{r}_{in} + \dot{m}_e \mathbf{r}_e) \tag{2.3.12}
\]

\[
F_{rel} = -(\dot{m}_{in} \mathbf{V}_{in} + \dot{m}_e \mathbf{V}_e) \tag{2.3.13}
\]

The latter force is known as the impulse thrust. In addition, the thrust force also includes a pressure term that is truly external, i.e.,

\[
- \int_{A_i} (p_i - p_a) \, n \, dA_i - \int_{A_e} (p_e - p_a) \, n \, dA_e \tag{2.3.14}
\]

with

- \( A_i \) = inlet area (m²)
- \( \rho_i \) = pressure in the flow at \( A_i \) (N/m²)
- \( A_e \) = exit area (m²)
- \( \rho_e \) = pressure in the flow at \( A_e \) (N/m²)
- \( p_a \) = atmospheric pressure (N/m²)
- \( n \) = outward unit normal on either \( A_i \) or \( A_e \) (-)

When we are talking about the thrust force, we assume that this force includes both the impulse thrust and the pressure thrust.
For rockets, the centre of mass flow is most of the time located on the $X_B$-axis, so that the Coriolis force arises from rotations about the $Y_B$ and $Z_B$-axis, which are usually small. As a result, even if the spin rate (rotation about the $X_B$-axis) is relatively high, $F_C$ is small in particular compared with $F_{rel}$ so that it can be neglected (Wittenberg, 1995). In the case of space planes, it is likely that the centres of mass flow are not located on the $X_B$-axis (although they will be located in the $X_BZ_B$-plane). However, the roll rate during manoeuvres is usually small, so also in this case we can neglect $F_C$ (see Chapter 8 for numerical values of the apparent forces and moments due to variable mass properties). For reaction-control thrusters the mass flow is so small, that the contribution to $F_C$ is negligible.

For practical purposes, we treat the thrust force as follows. A spacecraft may have a number of thrusters and/or engines, in order to influence both the motion of the c.o.m. and the one around the c.o.m. Each of the thrust forces is defined by its magnitude and two direction angles. Parameters, considered to be known, are (see also Fig. 2.9):

- $r_{T,i}$, the location of thruster (or engine) $i$ with respect to the c.o.m.
- $T_i$, the magnitude of the thrust force, generated by thruster $i$
- $\xi_{T,i}$ and $\psi_{T,i}$, the direction of thrust force, generated by thruster $i$, with respect to the body frame

![Diagram of thrust force](image)

**Fig. 2.9** - The $i^{th}$ thrust force, expressed with respect to a local body frame (with the origin coincident with the thruster and the axes collinear with those of the vehicle body frame), is defined by its magnitude $T_i$ and two direction angles, $\xi_{T,i}$ and $\psi_{T,i}$. The thruster is located at $r_{T,i}$ from the c.o.m.

Summation of all thrust forces results in a total thrust force that acts in the so-called centre of thrust (located at $r_T$ in the body frame). The total thrust is defined by a total $T$, and an effective direction in the form of $\xi_T$ and $\psi_T$.

When a propulsion force is not acting through the c.o.m. of the vehicle, a moment due to this force is introduced. To compute the propulsion moments, we need to know the cartesian components of the propulsion force in the body frame $F_{T,B}$. It can easily be verified, see Fig. 2.9, that these components for thruster $i$ are:
\[ F_{Ti,j} = T_i \cos \psi_{Ti,j} \cos \varepsilon_{Ti,j} \]
\[ F_{Ty,i} = T_i \sin \psi_{Ti,j} \cos \varepsilon_{Ti,j} \]
\[ F_{Tz,j} = -T_i \sin \varepsilon_{Ti,j} \]

The corresponding moment is given by the vector product:

\[ M_{T,B}^i = r_T^i \times F_{T,B}^i \]  \hspace{1cm} (2.3.16)

The total thrust moment simply follows from the summation over all \( n \) thrusters,

\[ M_{T,B} = \sum_{i=1}^{n} M_{T,B}^i \]  \hspace{1cm} (2.3.17)

When we study the general formulation of the equations of rotational motion, Eq. (2.1.3), we find both a Coriolis (\( M_C \)) and a relative moment (\( M_{\text{rel}} \)) due to the variable mass properties, i.e., the propulsion system. Expressions for \( M_C \) and \( M_{\text{rel}} \) can be found in Cornelisse et al. (1979) and Mooij (1994b)

\[ M_C = -\frac{\delta I}{\delta t} \cdot \omega - \dot{m}_{in} r_{in} \times (\omega \times r_{in}) - \dot{m}_e r_e \times (\omega \times r_e) \]  \hspace{1cm} (2.3.18)

\[ M_{\text{rel}} = -\dot{m}_{in} r_{in} \times \vec{V}_{in} - \dot{m}_e r_e \times \vec{V}_e \]  \hspace{1cm} (2.3.19)

\( M_C \) will usually be of the same order of magnitude as the aerodynamic moment and the thrust misalignment moment (Cornelisse et al., 1979), so it should be included in the equations of motion. The last term of the Coriolis moment is a damping moment due to the exhaust jet, and is therefore called the jet damping moment. The first term is smaller than the last one, but cannot be neglected since it can decrease the damping by some 30%. The middle term, finally, is a moment due to the inlet flow, and has a decreasing effect on the damping moment. The reader is referred to Chapter 8 for numerical values of the individual terms of \( M_C \). \( M_{\text{rel}} \) is the moment due to the inlet flow and the impulse thrust. As was mentioned before, this moment is considered to be external and will include any moment due to the misalignment of the pressure thrust vector.

2.4. Translational motion

In Section 2.1, the general vector formulation of the equations of translational motion was given. These equations are the result of the application of Newton's second law to a vehicle moving with respect to inertial space. However, when we want to study the motion of a space plane in the
Earth's atmosphere, it is much more convenient to express the equations with respect to the rotating planetocentric frame (index \( R \)). So let us consider the situation as indicated in Fig. 2.10. A vehicle with (variable) mass \( m \) is moving with a velocity \( \mathbf{V}_R \) with respect to the \( R \)-frame at a distance \( r_{cm} \) from the centre of the central body. The vehicle is subjected to an external force \( \mathbf{F}_R \) and has a rotation \( \omega \) (expressed in body-frame components) with respect to the inertial planetocentric frame. The \( R \)-frame is fixed to the central body (with the origin in its c.o.m.) and rotates with an angular velocity \( \omega_R = (0, 0, \omega_{cb})^T \).

![Diagram](image)

Fig. 2.10 - Definition of a vehicle moving about a celestial body. The inertial planetocentric frame is indicated with index \( I \). The rotating planetocentric frame, with index \( R \), is rigidly attached to the central body and rotates with an angular velocity \( \omega_R \), which is identical to the rotational rate of the central body.

Newton's second law, which describes the translational motion of a point mass with respect to an inertial frame, can be adjusted to be valid for a mass-varying system with respect to the rotating \( R \)-frame, yielding:

\[
\mathbf{F}_R = m \frac{d^2 r_{cm}}{dt^2} + 2m \omega_R \times \mathbf{V}_R + m \omega_R \times (\omega_R \times r_{cm})
\]  
(2.4.1)

with
\[ F_R \] summation of all external forces acting on the vehicle, expressed in the \( R \)-frame (N)

\[ \frac{d^2 r_{cm}}{dt^2} \] acceleration of the vehicle in the rotating frame \( (m/s^2) \)

\[ V_R = \frac{dr_{cm}}{dt} \] velocity of the vehicle in the rotating frame \( (m/s) \)

\[ 2 \omega_R \times V_R \] apparent Coriolis acceleration due to the rotation of the frame \( (m/s^2) \)

\[ \omega_R \times (\omega_R \times r_{cm}) \] apparent transport acceleration of the vehicle due to angular rate of the rotating frame \( (m/s^2) \)

\[ \omega_R \] the rotational rate of the \( R \)-frame, which is equal to the rotational rate of the central body \( (rad/s) \)

Eq. (2.4.1) resolves the velocity vector in the \( R \)-frame. To obtain the position of the vehicle with respect to the same frame, we can simply write

\[ \frac{dr_{cm}}{dt} = V_R \] \hspace{1cm} (2.4.2)

the so-called kinematic equation.

In Mooij (1991a) and Mooij (1994b), detailed discussions on the derivation of the differential equations of translational motion based on spherical components for the position and the velocity can be found. In short, an intermediate reference frame is defined in which all related components, i.e., \( \frac{d^2 r_{cm}}{dt^2}, \frac{dr_{cm}}{dt}, F_R, \omega_R, r_{cm} \) and \( V_R \) are expressed. Two systems of three relations each are then obtained, which can be solved to yield the dynamic and kinematic equations. The dynamic equations are given by (with 's', 'c' and 't' used for sine, cosine and tangent, respectively):

\[ \dot{V}_G = \frac{F_V}{m} + \omega_{eb}^2 R c_\delta (s\gamma_G c_\delta - c\gamma_G s_\delta c_x G) \] \hspace{1cm} (2.4.3a)

\[ V_G \dot{\gamma}_G = \frac{F}{m} + 2 \omega_{eb} V_G c_\delta s_x G + \frac{V_G^2}{R} c_\gamma G + \omega_{eb}^2 R c_\delta c_\gamma G + s\gamma_G s_\delta c_x G \] \hspace{1cm} (2.4.4a)

\[ V_G c_\gamma G \dot{c}_G = \frac{F}{m} + 2 \omega_{eb} V_G (s_\delta c_\gamma G - c_\delta s_\gamma G c_x G) + \frac{V_G^2}{R} c^2_\gamma G \delta_\delta s_x G + \omega_{eb}^2 R c_\delta s_\delta s_x G \] \hspace{1cm} (2.4.5a)

with

\[ F_V = -D + T c_\alpha c_\beta c_\psi T c_\epsilon T + T s_\beta s_\psi T c_\epsilon T - T s_\alpha c_\beta c_\delta s_\epsilon T - mg s_\gamma G - mg_\delta c_\gamma G c_x G \] \hspace{1cm} (2.4.3b)
\[ F_\gamma = -(S + Tc_\alpha G \beta G c \psi_T c e_T - Tc_\beta G c \psi_T c e_T) \cdot \sigma_G + (L + Tc_\alpha G \beta G c \psi_T c e_T - Tc_\beta G c \psi_T c e_T) \cdot \sigma_G - mg \cdot \psi_G + mg \cdot s \psi_G c \chi_G \]  

(2.4.4b)

\[ F_\chi = -(S + Tc_\alpha G \beta G c \psi_T c e_T - Tc_\beta G c \psi_T c e_T) \cdot \sigma_G + (L + Tc_\alpha G \beta G c \psi_T c e_T - Tc_\beta G c \psi_T c e_T) \cdot \sigma_G + mg \cdot \chi_G \]  

(2.4.5b)

The corresponding kinematic equations are given by:

\[ \dot{R} = V_G \sin \gamma_G \]  

(2.4.6)

\[ \dot{\chi} = \frac{V_G \sin \chi_G \cos \gamma_G}{R \cos \delta} \]  

(2.4.7)

\[ \dot{\delta} = \frac{V_G \cos \chi_G \cos \gamma_G}{R} \]  

(2.4.8)

Note that in Eq. (2.4.5a) and (2.4.7) there are singularities for \( \gamma_G = \pm 90^\circ \) (vertical flight) and \( \delta = \pm 90^\circ \) (north or south pole of the central body). Since these flight conditions do not occur in the missions studied in this thesis, they are not further considered.

So far we have treated the issue of vehicle motion by assuming some external force and studying the resulting path of the vehicle. If, in some way, we could change the magnitude and direction of the external force, this would mean that we can actively influence the path of the vehicle, or in other words, that we can guide the vehicle towards a target or along a pre-defined trajectory. This notion forms the basis for guidance systems. The two forces that can serve as a means to guide the vehicle are the aerodynamic and propulsion force. We will call these forces control forces.

As we stated in Section 2.3, the aerodynamic force varies with the angles of attack and sideslip, and the Mach number. By changing these variables we can influence the magnitude of the aerodynamic force. Since the angle of sideslip is usually considered to be a disturbance since it gives rise to a cross-range deviation from the nominal path, and it is difficult to vary the Mach number on a short time scale, the angle of attack is the variable with which to vary the magnitude of the aerodynamic force. Furthermore, when we inspect the external force equations, Eqs. (2.4.4b) and (2.4.5b), we find that by banking the vehicle we can split the lift force into a vertical and lateral part. (Note that the side force is in principle depending on the angle of sideslip and is therefore nominally zero.) The aerodynamic controls are thus the angle of attack and bank angle. How to change these angles will be made clear in Section 2.5.

The magnitude of the propulsion force can be varied by throttling the engines or by changing the fuel/oxidizer ratio (i.e., the equivalence ratio), also known as Thrust Magnitude Control (TMC). Furthermore, the direction of the propulsion force depends on the thrust elevation and azimuth angles. These angles can be varied by changing the direction of the exhaust flow (e.g., gimbaling the nozzles of rocket engines), an operation that is usually known as Thrust Vector Control (TVC).
So, the propulsion controls are the throttle setting or equivalence ratio, and the two propulsion angles.

### 2.5. Rotational motion

Expanding the dynamic equations of rotational motion and expressing the components along the body axes, the so-called Euler equations are obtained, which give information about the angular accelerations. The full set of (non-linear) equations as derived in, for instance, Cornelisse et al. (1979) holds

\[
\dot{\omega} = I^{-1}(\tilde{M}_{cm} - \omega \times I \omega)
\]  

(2.5.1)

with

\[
\tilde{M}_{cm} = (M_x, M_y, M_z)^T = \text{sum of external, Coriolis and relative moments about the c.o.m., expressed in components along the body axes.}
\]

\[
l = \begin{bmatrix}
l_{xx} & -l_{xy} & -l_{xz} \\
-l_{xy} & l_{yy} & -l_{yz} \\
-l_{xz} & -l_{yz} & l_{zz}
\end{bmatrix}
\]

\(l\) = inertia tensor of the vehicle, referenced to the body frame.

\(\omega = (p, q, r)^T\) = the rotation vector of the body frame with respect to the inertial frame, expressed in components along the body axes.

Duke et al. (1988) have derived an expression for the inverse inertia tensor, so that \(p, q\) and \(r\) can be explicitly stated as a function of \(p, q\) and \(r\) and the inertia elements. The corresponding equations are given in Appendix B. These equations can be simplified when one or more planes of mass symmetry are present. In that case, two or all three products of inertia are zero.

The kinematic attitude equations are based on a particular set of Euler angles, the so-called groundspeed-based aerodynamic angles \(\alpha_G, \beta_G, \text{ and } \sigma_G\). The following set of equations can be derived (Mooij, 1991a):

\[
\dot{\alpha}_G c \beta_G = -p c \alpha_G s \beta_G - q s \alpha_G c \beta_G + s \sigma_G \left[ \dot{\gamma}_G c \gamma_G - \delta s \chi_G s \gamma_G + (t + \omega_{ab}) (c \delta c \chi_G s \gamma_G - s \delta c \gamma_G) \right] +
\]

\[
- c \sigma_G \left[ \dot{\gamma}_G - \delta c \chi_G - (t + \omega_{ab}) c \delta s \chi_G \right]
\]

(2.5.2)

\[
\dot{\beta}_G = \rho s \alpha_G - r c \alpha_G + s \sigma_G \left[ \dot{\gamma}_G - \delta c \chi_G - (t + \omega_{ab}) c \delta s \chi_G \right] +
\]

\[
+ c \sigma_G \left[ \dot{\gamma}_G c \gamma_G - \delta s \chi_G s \gamma_G + (t + \omega_{ab}) (c \delta c \chi_G s \gamma_G - s \delta c \gamma_G) \right]
\]

(2.5.3)

\[
\dot{\sigma}_G = -p c \alpha_G c \beta_G - q s \beta_G - r s \alpha_G c \beta_G + \dot{\alpha}_G s \beta_G - \dot{\gamma}_G s \gamma_G - \delta s \chi_G c \gamma_G + (t + \omega_{ab}) (c \delta c \chi_G c \gamma_G + s \delta s \gamma_G)
\]

(2.5.4)
In these equations, $\dot{\gamma}$, $\dot{\gamma}$, $\dot{\gamma}$, $\dot{\gamma}$, $\dot{\gamma}$, and $\dot{\gamma}$ are given by Eqs. (2.4.4), (2.4.5), (2.4.7) and (2.4.8) in the previous section. Note that in Eq. (2.5.2) there is a singularity for $\beta = \pm 90^\circ$; however, this flight condition will not be encountered in our studies.

In the previous section we introduced the term control forces, being the aerodynamic and propulsion force. As aerodynamic controls to influence the control forces, we mentioned the angle of attack and bank angle. These angles can be changed by rotating the vehicle with respect to the on-coming flow, thus by exerting moments on the vehicle. Also these control moments can be of aerodynamic or propulsive origin. A vehicle can be equipped with aerodynamic control surfaces or reaction-control thrusters. By either deflecting these surfaces or activating the thrusters, forces are generated that give rise to moments around the c.o.m. in case they do not act through the c.o.m. Furthermore, by gimballing the engines or mechanical exhaust-jet deflection also a propulsion moment may arise. It should be noted that moments are only required in the presence of guidance signals. Otherwise, we want to have moment equilibrium, or in other words, we want the vehicle to maintain a stable attitude. The concept of guidance and control will be discussed in much greater detail in the next chapter.

### 2.6. Wind equations

Usually, the motion of a vehicle passing through a planetary atmosphere is expressed with respect to a planetary fixed frame. The velocity of the vehicle is ground-related and solving the equations of motion directly gives us the trajectory with respect to the planet. Wind, however, that can be regarded as a disturbance in the nominal environment, can change the trajectory considerably. The aerodynamic forces are based on the velocity with respect to the atmosphere, and if the wind velocity and direction differ notably from the ground velocity, the forces change significantly. As a result, the motion with respect to the planetary surface changes. When a vehicle is moving through a region with severe (vertical) wind gusts, it can seriously alter the local flow conditions, albeit only for a short while. However, when there is a continuous downburst, the effect is lasting longer and can be more serious.

In the presence of wind, the velocity of a vehicle with respect to the planetary surface (or ground) can be expressed by the following vector equation:

$$ V_{vehicle/ground} = V_{vehicle/air} + V_{air/ground} \quad (2.6.1) $$

or, in words, the velocity of the vehicle with respect to the ground is the vectorial summation of the velocity of the vehicle with respect to the air and the velocity of the air with respect to the ground. The latter term is also called the wind velocity. Since our equations of motion and kinematic

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8 In principle there is also a concept called c.o.m. management, i.e., changing the location of the c.o.m. by actively displacing units of mass in the vehicle. However, we will not consider this type of control.
attitude variables are groundspeed based, we have to derive some additional equations. The aero-
dynamic forces, for instance, are airspeed based, so we have to resolve the airspeed vector from
Eq. (2.6.1). Then the forces have to be transformed to the groundspeed aerodynamic frame. Given
the wind vector and the groundspeed vector, both in spherical components, the components of the
airspeed vector are given by (Mooij, 1994b), see also Fig. 2.11:

\[ V_A = \sqrt{V_G^2 + V_W^2 - 2V_GV_W\cos \gamma_G \cos \gamma_W \cos(\chi_W - \chi_G) + \sin \gamma_G \sin \gamma_W} \]  
\[ \gamma_A = \arcsin \left( \frac{\sin \gamma_G V_G - \sin \gamma_W V_W}{V_A} \right) \text{ with } \gamma_A \in [-90^\circ, +90^\circ] \]  
\[ \chi_A = \arctan \left( \frac{\sin \chi_A}{\cos \chi_A} \right) \]  

with

\[ \sin \chi_A = \frac{\sin \chi_G \cos \gamma_G V_G - \sin \chi_W \cos \gamma_W V_W}{\cos \gamma_A V_A} \]  
\[ \cos \chi_A = \frac{\cos \chi_G \cos \gamma_G V_G - \cos \chi_W \cos \gamma_W V_W}{\cos \gamma_A V_A} \]

Fig. 2.11 - The relation between the groundspeed- and airspeed-based trajectory frame (index TG and TA,
respectively) and the vertical frame (index V).

To compute the airspeed-based aerodynamic angles \( \alpha_A, \beta_A \) and \( \sigma_A \), we need their ground-
speed counterparts \( \alpha_G, \beta_G \) and \( \sigma_G \), which follow from the kinematic equations of rotational motion
Eqs. (2.5.2) through (2.5.4). Using the velocity parameters \( \gamma_G \) and \( \chi_G \) given by Eqs. (2.4.4) and
(2.4.5), we can compute $\mathbf{C}_{B,V}$ the transformation matrix from the vertical to the body frame. From Appendix A, we find that this matrix can be expressed as

$$
\mathbf{C}_{B,V} = \mathbf{C}_2(\alpha_G) \mathbf{C}_3(\beta_G) \mathbf{C}_1(\gamma_G) \mathbf{C}_2(\sigma_G) \mathbf{C}_3(\chi_G)
$$

(2.6.5)

Another way to express $\mathbf{C}_{B,V}$ is to decompose it into

$$
\mathbf{C}_{B,V} = \mathbf{C}_{B,TA} \mathbf{C}_{TA,V}
$$

(2.6.6)

$\mathbf{C}_{B,TA}$ can be written as (Appendix A):

$$
\mathbf{C}_{B,TA} = \mathbf{C}_2(\alpha_A) \mathbf{C}_3(\beta_A) \mathbf{C}_1(\sigma_A)
$$

(2.6.7)

which includes the three variables we want to calculate. To isolate them, we postmultiply $\mathbf{C}_{B,V}$ by $\mathbf{C}_{V,TA}$, i.e.,

$$
\mathbf{C}_{B,V} \mathbf{C}_{V,TA} = \mathbf{C}_{B,TA} \mathbf{C}_{TA,V} \mathbf{C}_{V,TA} = \mathbf{C}_{B,TA}
$$

(2.6.8)

Computing $\mathbf{C}_{B,V}$ with Eq. (2.6.5), only $\mathbf{C}_{V,TA}$ remains to be calculated, which is defined by

$$
\mathbf{C}_{V,TA} = \mathbf{C}_3(-\gamma_A) \mathbf{C}_2(-\chi_A)
$$

(2.6.9)

with $\gamma_A$ and $\chi_A$ given by Eqs. (2.6.3-4). $\mathbf{C}_{B,TA}$ fully written out gives us

$$
\mathbf{C}_{B,TA} = 
\begin{bmatrix}
\alpha_A \mathbf{c}_A \beta_A & -\alpha_A \mathbf{c}_A \beta_A \mathbf{c}_A \sigma_A - \mathbf{s}_A \beta_A \mathbf{s}_A \sigma_A & \alpha_A \mathbf{s}_A \beta_A \mathbf{s}_A \sigma_A - \mathbf{s}_A \beta_A \mathbf{s}_A \sigma_A \\
\mathbf{s}_A \beta_A & \mathbf{c}_A \mathbf{s}_A & -\mathbf{c}_A \mathbf{s}_A \\
\mathbf{s}_A \alpha_A \beta_A & \alpha_A \mathbf{s}_A \sigma_A - \mathbf{s}_A \beta_A \mathbf{c}_A \sigma_A & \alpha_A \mathbf{c}_A \sigma_A - \mathbf{s}_A \beta_A \mathbf{s}_A \sigma_A
\end{bmatrix}
$$

(2.6.10)

From this matrix, we can derive

$$
\alpha_A = \arctan \left( \frac{\mathbf{C}_{B,TA}(3,1)}{\mathbf{C}_{B,TA}(1,1)} \right), \quad \text{with} \quad \alpha_A \in \left[ -180^\circ, +180^\circ \right]
$$

(2.6.11)

$$
\beta_A = \arcsin(\mathbf{C}_{B,TA}(2,1)), \quad \text{with} \quad \beta_A \in \left[ -90^\circ, +90^\circ \right]
$$

(2.6.12)

$$
\sigma_A = \arctan \left( -\frac{\mathbf{C}_{B,TA}(2,3)}{\mathbf{C}_{B,TA}(2,2)} \right), \quad \text{with} \quad \sigma_A \in \left[ -180^\circ, +180^\circ \right]
$$

(2.6.13)
So if we take for the elements of $C_{B,TA}$ the corresponding elements given by Eq. (2.6.8), we can compute $\alpha_A$, $\beta_A$ and $\sigma_A$. In Fig. 2.12, the relation between airspeed- and groundspeed-based aerodynamic angles is depicted.

![Diagram](image)

Fig. 2.12 - The relation between the body frame (index $B$) and the airspeed- and groundspeed-based aerodynamic frames (index $AA$ and $AG$, respectively). Here, all angles are plotted in the corresponding positive directions.

To compute the aerodynamic forces and moments the aerodynamic coefficients are required, which are usually a function of $M$ (based on $V_A$), and $\alpha_A$ and $\beta_A$ (apart from any possible damping terms, that also depend on the angular rate of the vehicle), and $q_{dyn}$ (based on $V_A$), as discussed in Section 2.3.1. In other words, the resulting aerodynamic force vector is defined with respect to the airspeed-based aerodynamic frame (index $AA$). In the equations of translational motion, however, the aerodynamic-force components are expressed with respect to the $AG$ frame. To use the airspeed-based aerodynamic force vector, it has to be transformed to the $AG$-frame. If we define the aerodynamic force vector in the $AA$-frame to be $F_{A,AA}$, then we have to solve the following equation:

$$F_{A,AG} = C_{AG,AA}F_{A,AA}$$  \hspace{1cm} (2.6.14)

in which $C_{AG,AA}$ is defined by:

$$C_{AG,AA} = C_{AG,TG}C_{TG,AA}, \quad \text{with} \quad C_{AG,TG} = C_1(-\sigma_G)$$  \hspace{1cm} (2.6.15)

Substituting $F_{A,AG} = [D, S, L]_G$ into Eqs. (2.4.3-5), gives us a new state vector for position and velocity.
2.7. Summary

- To study the flight mechanics of a vehicle in a planetary atmosphere with wind, under the influence of aerodynamic, gravitational and propulsive forces and moments, a set of ordinary differential equations is required that forms the mathematical core of the flight-simulation model. The general formulation of these equations in an inertial reference frame can directly be derived from Newton's laws. Due to a variation in mass distribution - here the use of a propulsion system - Coriolis and relative forces and moments are introduced. By including these two components in the external force and moment vector (Solidification Principle), the equations take the same form as those for a rigid body.

- The position and velocity of the vehicle are expressed in spherical components with respect to the rotating planetocentric reference frame with the origin in the c.o.m. of the central body. Due to the transformation of the equations of motion from an inertial to a rotating frame of reference, an extra Coriolis and relative force are introduced. The attitude of the vehicle is expressed by the so-called aerodynamic angles, whereas the angular rate of the vehicle is defined as the rotation of a body-fixed frame with respect to the inertial frame, expressed in components along the body axes.

- The external forces and moments that are acting on the vehicle are of aerodynamic, gravitational and propulsive origin. The relative force due to a variation in mass is also known as the impulse thrust and and is combined with the pressure thrust in the external force. The Coriolis force due to the use of the propulsion system is neglected. The relative moment due to a variation in mass is the moment due to the impulse thrust and is included in the external moment; the internal Coriolis moment is also known as jet damping and is too large to be ignored.

- The two forces that can serve as a means to guide the vehicle are the aerodynamic and propulsion force. These forces are called control forces, and can be changed by issuing guidance commands, such as a commanded angle of attack, bank angle and throttle setting. To influence the aerodynamic angles, the vehicle must be rotated with respect to the oncoming flow, which requires the exertion of so-called control moments on the vehicle. Also these moments are of aerodynamic or propulsive origin.

- The velocity components that appear in the equations of translational motion are defined with respect to the central body, whereas the aerodynamic forces and moments are, amongst others, dependent on velocity components with respect to the atmosphere. In case there is wind, these two sets of velocity components are not identical. Given the wind vector, transformation formulas can be used to compute the airspeed-based aerodynamic force from the groundspeed-based force.
Chapter 3

Guidance and Control of Space Planes

Adapt or die. Unknown

The process of trajectory design and analysis is centred around a nominal (and often optimal) trajectory which usually satisfies trajectory constraints (e.g., a maximum allowable dynamic pressure or thermal load) and final conditions (e.g., a target orbit for a space plane or the landing place in case of a re-entry mission). Once this nominal trajectory has been defined it must be verified that the vehicle can actually fly this trajectory, or, in other words, whether the vehicle can execute the required manoeuvres without violating any constraints. Moreover, it must be guaranteed that the vehicle can still fulfil its mission when it encounters (unforeseen) disturbances which make it deviate from its nominal path. To ensure mission success, the vehicle is equipped with a so-called Guidance, Navigation and Control (GNC) system (Fig. 3.1).

The task of the guidance system is to generate steering commands, e.g., a commanded attitude, thereby taking a reference state, trajectory constraints and/or a final state into account. For this task, the system needs input from the outside world, e.g., the current actual state. These data have to be provided by the navigation system, using sensor information and pre-defined theoretical models. The control system has to take care that the steering commands are carried out, such that, for example, the actual attitude approaches the commanded attitude with a certain tolerance in a finite time and that this attitude is stable (trim stability)\(^9\). To achieve this, the control system may include aerodynamic control surfaces, thrusters, etc. Due

\(^9\) The guidance loop is usually called the outer loop, whereas the control loop is known as the inner loop.
to the complex nature of each of the elements of GNC, we have to limit ourselves within the framework of this study. Therefore, a perfect navigation system is assumed that provides all the relevant data that are required for the execution of guidance and control (G&C).

The operation of the controllers will be influenced by errors from many sources, e.g., resulting from measurement inaccuracies or model mispredictions, although in this study we assume that these errors are non-existent. Of course, it remains to be verified what is the influence of this assumption on the performance of the G&C system.

![Diagram of GNC system](image)

**Fig. 3.1 - Schematic overview of a GNC system.**

In the previous chapter, a mathematical model describing the flight mechanics of a non-elastic vehicle moving in a planetary atmosphere possibly with the aid of a propulsion system. It was concluded that to have the capability to influence the motion the possibility must exist to change the aerodynamic and propulsion force (the control forces). Whenever an attitude change is required to alter the control forces, control moments are necessary to achieve this. In Section 3.1, the relation between the control forces and moments on one hand, and effectuators that
are available to the G&C system on the other hand, will be introduced in more detail.

As was stated in Chapter 1, the G&C system is an essential module that was still missing in the available flight-simulation software. To study the influence of G&C on the flight mechanics of space planes - and in parallel also to test the correct implementation of G&C in the simulation tool - a mathematical model of the G&C system related to the mission of space planes is necessary. Two important parts of the mission are the re-entry and ascent flight, which are studied in Chapters 7 and 8, respectively. Due to the different nature of these parts, two guidance models will be described, i.e., in Section 3.2 and Appendix C one for unpowered re-entry and in Section 3.3 one for powered ascent. In the latter section, attention will also be paid to hypersonic manoeuvres and propulsion control, since they form part of the ascent mission studied in Chapter 8. Last but not least, in Section 3.4 two concepts for attitude-control are presented, i.e., a Linear Quadratic Regulator and Model Reference Adaptive Control.

3.1. Guidance parameters and control effectors

The underlying notion of the guided and controlled flight of a space plane is the notion that the control forces can be manipulated such that their magnitude and direction result in a required acceleration or deceleration. To use the aerodynamic and propulsion force for guidance and control purposes, it is necessary to study their origin in more detail.

The aerodynamic force, with as main components lift and drag, is basically a function of the aerodynamic shape of the vehicle, its velocity and attitude with respect to the incoming flow, and atmospheric properties such as air pressure and temperature. So to change lift and drag we can, for instance, vary the aerodynamic shape by deflecting control surfaces, although the effect is not so large unless the control surfaces are large. An efficient and fast way to control the magnitude (and at a slower time scale also the direction) of the aerodynamic force is by changing the vehicle's attitude and by that to make use of the aerodynamic dependency on the angle of attack. In addition, the vehicle can be rolled around its velocity vector to change the orientation of the lift component, a concept that is known as banking.

The magnitude of the thrust vector produced by an air-breathing propulsion system depends on a number of parameters, i.e., directly on the throttle setting or the fuel-equivalence ratio, and indirectly on the attitude of the inlet with respect to the incoming flow, the flight velocity and altitude. As a matter of fact, the operation of the propulsion system is also largely dependent on the aerodynamic shape of the vehicle, but in this study a given shape is assumed. The direction of the thrust vector can be controlled by changing the direction of the exhaust flow.

So far, we have established a number of guidance variables: the angle of attack \((\alpha_c)\) and bank angle \((\sigma_c)\) to control the aerodynamic force, and the throttle setting \((\delta_{T,c})\), equivalence ratio \((\phi_{T,c})\), thrust elevation \((e_{T,c})\) and azimuth \((\psi_{T,c})\) to control the propulsion force. Note that the subscript \(c\) stands for commanded (by the guidance system). Due to the different nature of the guidance commands, usually two separate (but interfaced) controllers are used to execute
them: an attitude-control system and a propulsion-control system. The operating principle of the former is simple: generate a moment such that an attitude change is initiated and guarantee that this attitude is stable once the commanded attitude has been reached. The moments can be acquired with several means, e.g.:

- Aerodynamic-control surfaces: depending on the vehicle, these surfaces may consist of canards, rudder(s), elevators, ailerons, elevons (combined elevator and aileron function) and body flap(s). A deflection of these surfaces results in a change of the aerodynamic shape. The induced forces are usually small, but since the moment arms can be large, also the moments can be significant. Of course, the control surfaces are only efficient when the dynamic pressure is sufficiently high.

Fig. 3.2 - Entry control modes for the Space Shuttle (based on Cooke, 1982).

- Reaction-control jets; in case the control surfaces are not efficient to guarantee a fast response, cold- or hot-gas thrusters (also called reaction-control system) can be used to generate moments around the three body axes. Furthermore, to minimise the entry heating, space planes will enter the atmosphere at high angles of attack. In case of a configuration like the Space Shuttle, the vertical tail including the rudder is shielded from the incoming flow by the body, and is therefore less effective as a moment producer. For that reason, the yaw jets are used throughout a large portion of the descent (see also Fig. 3.2).

- Moving mass points; by displacing a mass inside the vehicle, relative to the c.o.m., a gravitational moment is induced. It should be noted that when the vehicle has a large inertia and roll damping, either large displacements or a large mass is required to generate any moments of significance. For the roll-control system of a small re-entry vehicle, this method was studied by Petsopoulos et al. (1996) and the results were promising. Since space planes are probably too large for this method to be effective, it
is not considered any further.

- Active c.o.m. management; related to the previous control mode is the active control of the location of the c.o.m. by pumping fuel from one tank to another. In that way not only the stability margin can be influenced but also the trim angle of attack, i.e., that angle of attack for which the resulting pitch moment is zero. Obviously, this control mode can only be applied when sufficient (but also not too much) fuel is on-board the space plane. In this study fuel tanks are not modelled, which means that this mode of control is discarded. Note that therefore also fuel sloshing is not taken into consideration.

- Thrust-Vector Control (TVC); note that although this mode of control is directly related to the propulsion system - and should therefore be discussed in relation to the propulsion-control system - it can also be part of the attitude-control system. Whenever the thrust vector is not operating through the c.o.m., a (passive) propulsion moment results. This moment has to be compensated for by the attitude control system. However, it is also possible to actively control the direction of the thrust vector and thus create a moment that can be used to change the attitude of the space plane in a controlled manner. By varying the geometry of the exhaust the flow can be deflected. An advantage of TVC is that the induced drag is smaller as compared with the trim drag resulting from deflecting control surfaces. In addition, in case a turn manoeuvre needs to be executed, the use of TVC can increase the performance, in the sense that major reductions in turning times can be achieved (Schneider and Watt, 1989). Note that it is not clear how TVC will influence the performance of the propulsion system.

The propulsion-control system takes care of executing the propulsion-related commands from the guidance system to regulate the magnitude and direction of the thrust vector. The thrust magnitude can be controlled by either adjusting the throttle setting or by varying the fuel equivalence ratio, whereas the thrust direction is defined by two thrust-vector angles. In the present study, we will not consider a detailed design of the propulsion-control system. Therefore, the propulsion commands are directly input to the vehicle model, although rate and amplitude limiters are included.

### 3.2. Re-entry guidance

This section gives a brief overview of the re-entry guidance system. A summary of the mathematical model can be found in Appendix C.

The reference vehicle that is used for the re-entry phase (Chapter 7), the HORUS-2B, is an unpowered, winged, high lift-to-drag vehicle similar to the Space Shuttle Orbiter. In principle, two guidance systems are available from literature that can be applied: the Space Shuttle Orbiter guidance system discussed by Harpold and Graves (1979), and a Shuttle-based guidance system for HORUS presented by MBB (1988). For the following reasons it has been
decided to use the latter system: i) it is tailor-made for HORUS and all numerical guidance and reference parameters are at hand, ii) the mathematical model of the guidance system is well-documented, and iii) using this guidance system enables a comparison with the results obtained by MBB (1988).

The mission of HORUS starts in orbit with a de-orbit manoeuvre and a successive coasting arc. The actual re-entry starts at the atmospheric entry interface at an altitude of 120 km, followed by a hypersonic descent targeted towards a landing site. Some 80 km from the landing site guidance control is transferred to the Terminal Area Energy Management (TAEM) guidance system. Here, we concentrate on the hypersonic entry and descent.

The global task of the hypersonic guidance system is to steer the vehicle along a nominal trajectory and to adjust the steering profile of angle of attack and bank angle to satisfy the flight constraints which mainly concern thermo-mechanical loads, and to arrive at a suitably defined TAEM interface with the relevant flight parameters, i.e., velocity, altitude and heading, in the prescribed range. To achieve these objectives, the guidance logic is divided into a horizontal and a vertical part. It is the task of the horizontal entry guidance to bring the vehicle in the vicinity of the landing site with a small heading error, so that the TAEM guidance can take over and steer the vehicle along a right- or left-turn to bring the vehicle in line with the runway and towards a safe landing. The energy state with which the vehicle reaches the TAEM interface, i.e., velocity and altitude, is controlled by the vertical guidance.

To restrict the heading error, a so-called heading-error dead-band is defined (Fig. 3.3), which provides the maximum allowable error. Exceeding this error initiates a manoeuvre known as bank reversal, during which the commanded bank angle changes from an initial value $\sigma_0$ to the final value $-\sigma_0$ with a constant theoretical bank rate $\dot{\sigma}_0$; $\sigma_0$ is the last value generated by the vertical guidance.

![Fig. 3.3 - Heading-error dead-band. Partly based on flight experience gained with the Space Shuttle, dead-band values between 10° and 30° seem to give a reasonable compromise.](image)

All quantities that are important for the vertical guidance are dependent on velocity and altitude. This is true for the final conditions and since the atmospheric density is dependent on
the altitude, also for the encountered loads. By simultaneously controlling $V$ and $h$ by adjusting $\alpha$ and $\sigma$, MBB states that the resulting control histories are not very smooth, meaning quite a heavy load for the flight control system. Therefore, separate energy- and altitude-control loops have been introduced. The total energy is controlled with the angle of attack such that the final value at the TAEM interface will be met, having no direct effect on the constraints during the flight. The internal sharing of potential and kinetic energy, on the other hand, will affect the constraints through the altitude-velocity relation. Altitude control is done by variation of the vertical lift component by means of the (absolute) bank angle.

Briefly, the operating principles of the two guidance laws is as follows. The actual total energy of the vehicle is compared with a distance-to-target dependent reference value that is stored in the on-board flight-computer. In case of an energy excess or deficiency, more or less energy should be dissipated. Use is made of the relation between dissipated energy and drag on one hand, and drag and the angle of attack on the other. The result is a commanded variation of the angle of attack which is added to the nominal value to give $\alpha_c$.

The altitude-control law takes the difference between actual altitude and flight-path angle and their corresponding total-energy based reference values as input and computes the variation of vertical lift that is required to follow the reference trajectory. Combined with the nominal value of the vertical lift and the actual magnitude of the lift, the absolute commanded bank angle can be computed. Together with the sign of $\sigma$ that follows from the horizontal guidance, $\sigma_c$ is fully defined. Note that the reference trajectory can be shifted in altitude in case of a severe energy deficiency, which means that in principle an infinite number of trajectories in the three-dimensional space are possible. Since also the final energy state is a target value, this guidance system is based on a combination of tracking a reference trajectory and steering towards a final state.

What remains to be described is the input to the guidance system, i.e., the reference trajectory and the guidance-law parameters, which are stored in the on-board flight computer. The reference trajectory consists of the following parameters:

- $t$ time spent since atmosphere entry (start of reference trajectory)
- $d$ distance to targeting point along planetary surface
- $E_{\text{tot}}$ total energy per unit mass
- $h$ height
- $\gamma$ flight-path angle
- $\int_0^t \frac{\partial E_{\text{diss}}}{\partial \alpha} \, dt$ total dissipated energy per unit mass and radian angle of attack at time $t$
- $(C_L \cos \sigma)_{\text{nom}}$ nominal vertical lift coefficient
- $\alpha$ angle of attack

The parameters that are used in the guidance laws (see Appendix C) are listed in Table 3.1.
In addition, the on-board database consists of a distance-to-target based heading-error deadband (see also Section 7.3.1), limit values on the commanded angle of attack (function of Mach number) and bank angle, and the theoretical bank rate (function of dynamic pressure).

<table>
<thead>
<tr>
<th>Nr</th>
<th>guidance parameter</th>
<th>comment</th>
<th>nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K_E$</td>
<td>energy gain amplification factor</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>$c_{E,1}$</td>
<td>energy control parameter</td>
<td>$1.75 \times 10^5$ kg rad/J</td>
</tr>
<tr>
<td>3</td>
<td>$c_{E,2}$</td>
<td>energy control parameter</td>
<td>0 kg rad/J</td>
</tr>
<tr>
<td>4</td>
<td>$c_{h,1}$</td>
<td>altitude control parameter related to eigenfrequency</td>
<td>0.03</td>
</tr>
<tr>
<td>5</td>
<td>$c_{h,2}$</td>
<td>altitude control parameter related to damping</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>$E_{r,1}$</td>
<td>region parameter</td>
<td>1000</td>
</tr>
<tr>
<td>7</td>
<td>$E_{r,2}$</td>
<td>region parameter</td>
<td>2000</td>
</tr>
<tr>
<td>8</td>
<td>$\gamma_r$</td>
<td>controlling parameter for skipping flight</td>
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</tr>
<tr>
<td>9</td>
<td>$\Delta E$</td>
<td>energy dead band</td>
<td>0.0005 J/kg</td>
</tr>
<tr>
<td>10</td>
<td>$\Delta h$</td>
<td>altitude dead band</td>
<td>0.001 m</td>
</tr>
</tbody>
</table>

Table 3.1 - Guidance parameters.

3.3. Ascent guidance

In this study we will restrict ourselves to two aspects of powered space-plane guidance, i.e., the direct ascent to orbit, which, for reasons of simplicity, will be limited to vertical-plane motion, and hypersonic manoeuvres, which include cruise and turning flights. By combining the two it is possible to fully cover an ascent mission of a space plane, including a take off from a launch site that is located north or south of the maximum orbital latitude. Note that for missions with launch latitudes above inclination value of the target orbit, turning flights are usually required to change the heading to one due east that guarantees orbit insertion at the target inclination.

This section presents the mathematical model of the ascent guidance system. First, in Sub-section 3.3.1 a guidance model for the vertical-plane ascent is introduced. Then, guidance related to hypersonic manoeuvres is discussed in Sub-section 3.3.2, whereas in Sub-section 3.3.3 a throttle-control law is derived that is used to satisfy the trajectory constraints.

3.3.1. Vertical guidance

As for SSTO launchers, the ascent is the most strenuous flight phase; in fact, the design and sizing is largely determined by the ascent mission goals (Cribbs, 1990). Because of the very small performance margin, the guidance should be based on fuel-optimal trajectories to ensure
mission success (Lovell and Schmidt, 1995, and Schmidt and Lovell, 1997). Also frequent on-board reoptimisation and high-gain perturbation guidance that rapidly responds to perturbations and therefore closely tracks the optimal trajectory will likely be required (Wagner et al., 1990). For TSTO vehicles, on the other hand, the requirements are less demanding since the burden of achieving mission success is divided over two vehicles. Last but not least, since it is unlikely that SSTO and TSTO vehicles will go all the way to orbit on the very first mission, also flight test-type missions should be considered. Therefore, guidance must also be provided for more conventional flight modes, e.g., lower-performance climbs and descents, altitude holds, and achieving and maintaining course to area navigation targets.

New developments in guidance algorithms for space planes basically start from the idea that the algorithms should result in optimal (or near optimal) trajectories. For instance, Calise et al. (1988) and Corban et al. (1991) apply the singular-perturbation technique to achieve a rapid near-optimal trajectory generation and guidance scheme for space planes (zero-order solution). Van Buren and Mease (1992) extended this model to a first-order solution and included feedback control. Drawback of these methods is that they lean heavily on trajectory optimisation that is considered to be beyond the scope of this study.

A relatively simple alternative can be found in a so-called inverse-dynamics approach, which can be described as follows. Given a differential equation of motion that relates the time derivative of a state variable to some external force, this time derivative can be prescribed such that the external forces (and therefore also the controlling variables) can be determined, instead of the usual way of computing the time derivative for a given value of the external force (hence the term inverse). The inverse-dynamics approach has been applied by Lu (1995) an proved to be quite effective in solving the difficult problem of trajectory optimisation, and by Hess et al. (1991) who developed a generalised technique for inverse simulation applied to aircraft manoeuvres.

In this thesis, it will be studied whether this method can be successfully applied to vertical-plane guidance. The approach is to consider a reference trajectory given as \( \gamma \) as a function of \( h \). An inverse-dynamics modelling will then be used to obtain a commanded angle of attack. Furthermore, flying at the maximum values of the flight-path constraints maximum dynamic pressure, axial load and heat flux (so-called constraint tracking) will be considered since that results in a sub-optimal trajectory (Van Buren and Mease, 1992). The flight-path constraints will be controlled by adjusting the thrust, which is discussed in more detail in Section 3.3.3.

Starting point for describing the guidance model is the notion that a reference trajectory is available, i.e., \( \gamma \) as a function of, for instance, \( h, V \) or \( M \). This \( \gamma \)-profile is to be followed as closely as possible, thereby responding to deviations by controlling \( \alpha \). This so-called gamma-alpha steering, is based on the inverse-dynamics approach, i.e., a prescribed \( \dot{\gamma} \) yields \( \alpha_c \) by solving the corresponding differential equation for \( \gamma \), Eq. (2.4.4):

\[
V_G \dot{\gamma}_G = \frac{F_y}{m} + 2 \omega_{cb} V_G \delta \kappa \chi_G + \frac{V_G^2}{R} c \gamma_G + \omega_{cb}^2 R c \delta (c \delta c \gamma_G + s \gamma_G s \delta \chi_G)
\]

(3.3.1a)
where

\[ F_\gamma = -(S + T \alpha G \beta G \psi T \epsilon T + T c_b G \psi T \epsilon T - T s \alpha G \beta G \psi T \epsilon T) \sigma_G + \]  \( + (L + T s \alpha G \psi T \epsilon T + T c_b G \psi T \epsilon T) \sigma_G - m g \tau G + m g s \lambda G \sigma_G \)  \( (3.3.1b) \)

For the symmetric flight in a vertical plane, \( \sigma, \beta, \psi_T, \epsilon \) and \( S \) are equal to zero. In addition, for guidance purposes the term with \( \omega_{cb}^2 \) can be treated as a perturbation and can therefore be neglected. The centrifugal and Coriolis acceleration, on the other hand, are too large in hypersonic flight to be ignored. The resulting equation is:

\[ V_\gamma = \frac{L}{m} + \frac{T}{m} \sin(\alpha + \epsilon_T) - g \cos \gamma - 2 \omega_{cb} V \cos \delta \sin \chi + \frac{V^2}{R} \cos \gamma \]  \( (3.3.2) \)

Note that we have omitted the subscript \( G \) (groundspeed), since for guidance-law development winds are not considered. For the small angles of attack encountered during the major part of hypersonic flight, a linear lift curve can be assumed, i.e., \( L = (C_{L_0} + C_{L_\alpha}) q_{\text{dyn}} S_{\text{ref}} \), see, for instance, Shaughnessy et al. (1990). If also the thrust elevation is small then \( \sin(\alpha + \epsilon_T) \) can be approximated by \( \alpha + \epsilon_T \). After rearranging terms, a functional relationship between \( \alpha \) and \( \dot{\gamma} \) is obtained from Eq. (3.3.2):

\[ \alpha = K_{\dot{\gamma}1} \dot{\gamma} + K_{\dot{\gamma}2} + K_{\dot{\gamma}3} \]  \( (3.3.3) \)

with

\[ K_{\dot{\gamma}1} = \frac{m V}{T + C_{L_0} q_{\text{dyn}} S_{\text{ref}}} \]  \( K_{\dot{\gamma}2} = -\frac{T \epsilon_T - m g \cos \gamma + C_{L_0} q_{\text{dyn}} S_{\text{ref}}}{T + C_{L_\alpha} q_{\text{dyn}} S_{\text{ref}}} \]  \( K_{\dot{\gamma}3} = \frac{2 \omega_{cb} \cos \delta \sin \chi + V \cos \gamma}{T + C_{L_\alpha} q_{\text{dyn}} S_{\text{ref}}} \)

\( K_{\dot{\gamma}1} \) is the actual \( \dot{\gamma} \)-to-\( \alpha \) gain, whereas \( K_{\dot{\gamma}2} \) can be seen as the trim value of \( \alpha \) at which the lift and aerodynamic forces balance the component of the gravity normal to the velocity vector and thus at which \( \dot{\gamma} \) is zero. The \( K_{\dot{\gamma}3} \) term represents the influence of the Coriolis and centrifugal acceleration.

What is left to be done now is to compute a commanded \( \dot{\gamma} \) which drives the actual \( \gamma \) to its reference value, \( \gamma_c \). This is done in a general manner by applying a combination of proportional, integral and derivative control laws, the so-called PID family of control laws (D'Souza, 1988).
Consider a time-varying error $e_c(t)$, i.e., the difference between $\gamma_c$ and $\gamma$. In proportional control, the output of the controller is simply related to its input by a proportional constant, $u(t) = K_{ip} e_c(t)$. Hence, a large current error signal will result in a large corrective action. The integral control law is represented by $u(t) = K_{ii} \int_0^t e_c(t) dt$, with its output proportional to the accumulation of the past error. This can be very effective when the error has the same sign for most of the time. The derivative control law, finally, relates its output to the time derivative of the error signal, i.e., $u(t) = K_{id} \frac{de_c}{dt}$. A large slope of the error will give a large corrective action, implying that this control law anticipates on a large future error. With slowly varying errors, this control law is not very effective. In summary, the flight-path angle steering loop is given by:

$$\dot{\gamma}_c = K_{ip} e_i + K_{ii} \int_0^t e_i dt + K_{id} \frac{de_i}{dt}, \quad \text{with } e_i = \gamma_c - \gamma \quad (3.3.4)$$

To determine the feedback gains, $K_{ip}$, $K_{ii}$, and $K_{id}$, the guidance law is Laplace transformed to the $s$-domain. Suppose that the transfer function of the vehicle is given by $G_p(s)$, then we can write (see also Fig. 3.5):

$$\gamma(s) = \frac{K_{1\gamma} G_p(s) (K_{ip} + \frac{K_{ii}}{s} + K_{id}s)}{1 + K_{1\gamma} G_p(s) (K_{ip} + \frac{K_{ii}}{s} + K_{id}s)} \gamma_{\alpha}(s) + \frac{(K_{2\gamma} + K_{3\gamma}) G_p(s)}{1 + K_{1\gamma} G_p(s) (K_{ip} + \frac{K_{ii}}{s} + K_{id}s)} \quad (3.3.5)$$

To analyse this transfer function and to determine the PID gains, it is necessary that $G_p(s)$ is known. This can fairly easily be done by linearising the differential equations of vertical motion (i.e., $V$, $\alpha$ and $R$), neglecting higher-order terms and bringing the linearised equations in state-space form. Then, the transfer function can be obtained for the equation relating the input variable $\alpha$ to the output variable $\gamma$. However, following a similar line of thought as Carmona-Castillo and González Sánchez-Cantalejo (1993), who determine the feedback gains based on the open-loop transfer function of the gamma-error to gamma-rate block, Eq. (3.3.4) becomes:

$$\dot{\gamma}_c(s) = s \gamma_c(s) = (K_{ip} + \frac{K_{ii}}{s} + s K_{id}) (\gamma_c(s) - \gamma(s)) \quad (3.3.6)$$

so:

$$\frac{H_{\gamma_c(s)}}{\gamma(s)} = \frac{\gamma_c(s)}{\gamma(s)} = \frac{s^2 K_{id} + s K_{ip} + K_{ii}}{s^2 (K_{id}-1) + s K_{ip} + K_{ii}} = \frac{s^2 + s \frac{K_{ip}}{K_{id}-1} + s \frac{K_{ip}}{K_{id}-1}}{s^2 + s \frac{K_{ip}}{K_{id}-1} + s \frac{K_{ip}}{K_{id}-1}} \quad (3.3.7)$$
Equating the denominator with a standard second-order characteristic equation, i.e.,

$$s^2 + 2 \zeta \omega_s s + \omega_s^2 = s^2 + s \frac{K_{yp}}{K_{yp} - 1} + \frac{K_{yi}}{K_{yi} - 1}$$  \hspace{1cm} (3.3.8)

gives us two equations with three unknowns. To solve for $K_{yp}$, $K_{yi}$ and $K_{yp}$, the damping and frequency of the open-loop harmonic response, $\zeta$ and $\omega_s$, as well as one of the three gains has to be specified. Using a PI regulator, Carmona-Castillo and González Sánchez-Cantalejo (1993) found that for $\zeta = 0.7$ and $\omega_s = 1.35$ rad/s, the system was indeed functioning properly. We will therefore start with similar values, adjust the gains when the response is not satisfying, and leave a full closed-loop analysis for future research (see Chapter 8 for the application of this guidance system).

3.3.2. Hypersonic manoeuvres

Raney and Lallman (1992) combine both altitude and cross-range commands such that a unified approach can be used for all kinds of powered hypersonic manoeuvres, e.g., altitude and cross-range transitions, and turn flights. In particular, they discuss an interface between the inner-loop controls that provide stability augmentation and the outer-loop controls which track guidance commands and reject disturbances. This interface or resolver translates vertical and lateral acceleration commands into a lift-vector command that is specified by a normal-load factor and bank-angle combination. This concept will be adapted for a number of manoeuvres, i.e., altitude hold (cruise flight) and altitude and cross-range transitions. The command generator that has to produce the corresponding steering commands will be discussed later.
A dual-loop PID architecture that regulates altitude and cross-range is used for closely following the guidance commands (see Fig. 3.6). In the discussion, we assume that the inner-loop dynamics are sufficiently fast so that they can be ignored in the integrated system design. Note that this is a valid assumption, since trajectory parameters are slow to respond in hypersonic flight. By taking a transfer function of unity for the inner-loop dynamics, the closed-loop transfer function for the altitude loop can be written as (Fig. 3.6):

\[
\frac{h}{h_c} = \frac{K_{hp}K_{hp}(s+K_{hi})}{s^3 + K_{hd}s^2 + K_{hd}K_{hp}s + K_{hd}K_{hp}K_{hi}} \quad (3.3.9)
\]

The third-order characteristic equation of the above transfer function can be expressed as the product of a first- and second-order term, i.e.,

\[
(s^2+2\zeta_h\omega_h s + \omega_h^2)(s+\tau_h) = s^3 + (\tau_h + 2\zeta_h\omega_h)s^2 + (2\zeta_h\omega_h\tau_h + \omega_h^2)s + \omega_h^2\tau_h \quad (3.3.10)
\]

where \(\zeta_h\) and \(\omega_h\) are the damping and frequency of the harmonic response, and \(\tau_h\) is the real root associated with the aperiodic portion of the response. By equating the corresponding terms of the denominator of Eq. (3.3.9) and the characteristic equation Eq. (3.3.10), the PID gains can be solved in terms of these parameters:

\[
K_{hp} = \frac{\omega_h^2 + 2\zeta_h\omega_h\tau_h}{\tau_h + 2\zeta_h\omega_h} \quad \text{and} \quad K_{hp} = \frac{\omega_h\tau_h}{\omega_h^2 + 2\zeta_h\tau_h} \quad \text{and} \quad K_{hd} = \tau_h + 2\zeta_h\omega_h \quad (3.3.11)
\]

Obviously, with a similar loop for the cross-range, the cross-range gains are determined by analogous equations, only with the subscript \(h\) replaced by \(y\). The PID-control laws produce a
vertical and lateral commanded acceleration, \( \dot{h}_c \) and \( \dot{y}_c \), that are input to the resolver.

The commanded accelerations will be converted to an aerodynamic lift- and propulsion-vector command, defined by a normal load factor, \( n_{z,c} \) and bank angle, \( \sigma \) (note that normal means in this case normal to the flight path). \( n_z \) can be decomposed into a vertical and a lateral component, \( n_v \) and \( n_l \) which holds for small \( \gamma \). In case \( n_l \) is zero and \( n_v \) is just large enough to maintain a constant altitude, we speak of a straight (i.e., no turn) and level flight. At subsonic speeds, this corresponds with a vertical load factor of about 1 g. However, at hypersonic speeds an effect known as centrifugal relief starts playing an important role: this apparent acceleration, with a magnitude of \( \frac{V^2}{R} \), decreases the load factor required for straight and level flight (note that the centrifugal relief approaches 1 at orbital velocities). So the desired load-factor commands can be written as:

\[
n_{v,c} = \frac{\dot{h}_c}{g} + 1 - \frac{V^2}{Rg} \tag{3.3.12}
\]

\[
n_{l,c} = \frac{\dot{y}_c}{g} \tag{3.3.13}
\]

The normal load factor and bank angle are easily obtained from \( n_{v,c} \) and \( n_{l,c} \) by

\[
n_{z,c} = \sqrt{n_{v,c}^2 + n_{l,c}^2} \tag{3.3.14}
\]

\[
\sigma_c = \arctan \left( \frac{\sin \sigma_c}{\cos \sigma_c} \right), \text{ with } \sin \sigma_c = \frac{n_{l,c}}{n_{z,c}} \text{ and } \cos \sigma_c = \frac{n_{v,c}}{n_{z,c}} \tag{3.3.15}
\]

So, if the vehicle comes to the angle of attack required to achieve \( n_{z,c} \) and banks to \( \sigma_c \), then the desired accelerations \( \dot{h}_c \) and \( \dot{y}_c \) are produced. To convert \( n_{z,c} \) to a commanded angle of attack \( \alpha_c \) (and possibly a commanded thrust elevation, \( \varepsilon_{T,c} \)), we must inspect the external-force components of Eq. (3.2.9) at the beginning of this section. For banking flight, only \( n_{v,c} \) is of importance to define \( \alpha_c \) and \( \varepsilon_{T,c} \):

\[
n_{v,c} = \frac{L}{mgV} + \frac{T}{mgV} \sin(\alpha_c + \varepsilon_{T,c}) \tag{3.3.16}
\]

Again, we will assume a linear lift curve and a small angle of attack and thrust elevation. Therefore we may write
\[ n_{v,c} = \frac{(C_{L_0} + C_{L_a} \alpha_c) q_{dyn} S_{ref}}{mgV} + \frac{T}{mgV(\alpha_c + \epsilon_{T,c})} \]

(3.3.17)

The ratio of \( \alpha_c \) and \( \epsilon_{T,c} \) should come from the guidance logic; in case \( \epsilon_{T,c} \) is specified, solving Eq. (3.3.17) yields for \( \alpha_c \):

\[ \alpha_c = \frac{n_{v,c} mg V - C_{L_0} q_{dyn} S_{ref} - T \epsilon_{T,c}}{C_{L_a} q_{dyn} S_{ref} + T} \]

(3.3.18)

Fig. 3.7 - The resolver axis system and the implication of load-factor limits (based on Raney and Lallman, 1992).

The discussion on the guidance logic of the resolver could end here, if it were not for the consideration of flight-condition constraints. The sensitivity of the propulsion system to large variations in \( \alpha \) forces upper and lower limits on this parameter, which result in normal-load factor limits for a given dynamic pressure. The use of load-factor limits define a region of allowable \((n_{v,c}, n_{L,c})\) combinations that ensure that the computed \( \alpha_c \) never violates the constraints. In Fig. 3.7, this is represented by regions bounded by concentric circles. Simply limiting the load-factor command by a radial mapping of all points outside the allowable region onto a bounding circle results in an undesirable effect in the sense that a pure cross-range command would also
result in a descending flight. To maintain altitude, the points outside the allowable region will be mapped horizontally, thereby giving priority to the altitude regulation. This reduces the activity required to compensate for the effect of density variation with altitude on dynamic pressure. In case of a violation of a minimum load factor, a strategy is invoked whereby the vehicle is banked to attain the commanded descent rate. The induced transient cross-range error has to be compensated for after leaving the 'forbidden region'. This practice is referred to as a 'bank-to-dive' strategy. For more details on the implementation of flight-condition limitations the reader is referred to the report by Raney and Lallman (1992).

The last part of the discussion on hypersonic manoeuvres is dedicated to generating the altitude and cross-range commands, a topic not covered by Raney and Lallman. Altitude and cross-range transitions can simply be input as a step function. To avoid excessive commands and large overshoots when a vehicle, which simply cannot manoeuvre that fast, is subjected to large step commands, a command-shaping pre-filter is included in the guidance loop. This compensator is a simple first-order lag filter, with a transfer function equal to

\[ G_c(s) = \frac{K_c}{\tau_c s + 1} \]  (3.3.19)

with \( K_c \) and \( \tau_c \) the gain and time constant of the filter, respectively. Numerical values for the filter parameters are dependent on the response characteristics. Without going into great detail, we will assume a unity gain and a time constant in the order of 10 s (so \( \tau_c = 5/\pi \text{ rad/s} \)).

A heading change, either in the horizontal plane or in combination with altitude commands (ascending and descending turns), is basically a (series of) cross-range step command(s). An additional point of concern may be that not only a heading change is the goal but also a final latitude at which the heading change should be complete, e.g., in case of a turn due east ending at the latitude corresponding with the inclination of the target orbit. This is in principle a two-point boundary optimisation problem, which will, due to the nature of the current research, not be considered here. In fact, we will limit ourselves to cross-range transitions.

### 3.3.3. Throttle control

To ensure mission success, it is important that the so-called flight-path constraints maximum dynamic pressure, maximum axial acceleration and maximum heat flux are satisfied. These constraints are, amongst others, a function of the velocity and since the velocity is directly dependent on the magnitude of the thrust, the throttle setting can be used to regulate the velocity. Successively, a feedback control law to regulate each of the three mentioned constraints will be developed.

The throttle-control law for regulating \( q_{\text{dyn}} \) is derived from the expression of \( q_{\text{dyn}} \), which is given by
\[ q_{\text{dyn}} = \rho V \dot{V} + \frac{V^2}{2} \dot{\rho} \] (3.3.20)

Since the trajectory of the space plane is shallow \((\gamma = 0)\), flown at \(\alpha\) close to zero, for guidance purposes \(\dot{V}\) can be approximated from Eq. (2.4.3):

\[ \dot{V} = \frac{T - D}{m} \] (3.3.21)

Furthermore, the atmospheric density is assumed to be exponential so that the density variation with time can be written as

\[ \frac{d \rho}{d t} = \frac{d \rho}{d h} \frac{d h}{d t} = -\frac{\rho}{H_s} \dot{h} \] (3.3.22)

where \(H_s\) is the density scale height \((H_s = 7,050 \text{ m} \text{ for the Earth's atmosphere})\). Substitution of Eqs. (3.3.21) and (3.3.22) into (3.3.20) yields

\[ q_{\text{dyn}} = \frac{\rho V}{m} T - \frac{\rho V D}{m} - \frac{\rho V^2}{2 H_s} \dot{h} \] (3.3.23)

For a flight along the dynamic-pressure constraint, the above rate is zero yielding the commanded thrust, \(T_c\), to maintain equilibrium. However, to compensate for variations in \(q_{\text{dyn}}\) compensation terms must be added to \(T_c\). Considering a PI regulator, \(T_c\) can be written as

\[ T_c = D + \frac{m V}{2 H_s} \dot{h} + K_{qp} e_{q_{\text{dyn}}} + K_{qi} \int_0^t e_{q_{\text{dyn}}} \, dt \text{ with } e_{q_{\text{dyn}}} = q_{\text{dyn}} - q_{\text{dyn, max}} \] (3.3.24)

Substituting the above equation in Eq. (3.3.23) gives an expression for the rate of change of the error in \(q_{\text{dyn}}\), i.e.,

\[ \dot{e}_{q_{\text{dyn}}} = \frac{\rho V}{m} \left( K_{qp} e_{q_{\text{dyn}}} + K_{qi} \int_0^t e_{q_{\text{dyn}}} \, dt \right) \] (3.3.25)

To determine the feedback gains, the above equation is Laplace transformed to the \(s\)-domain:

\[ \left( s^2 - \frac{\rho V}{m} K_{qp} s - \frac{\rho V}{m} K_{qi} \right) e_{q_{\text{dyn}}}(s) = 0 \] (3.3.26)

which yields the following second-order characteristic equation:

\[ s^2 + 2 \zeta_q \omega_q + \omega_q^2 = 0 \] (3.3.27)
Combining the two equations yields for the proportional and integral gain:

\[ K_{qp} = -\frac{2m}{p} \zeta_q \omega_q \quad \text{and} \quad K_{qi} = -\frac{m}{p} \omega_q^2 \]  

(3.3.28)

By specifying the damping coefficient, \( \zeta_q \), and natural frequency, \( \omega_q \), the (dynamics-depending) gains can easily be determined.

In a similar manner, a throttle-control law is derived for tracking a maximum heat flux, approximated by Chapman’s equation (Chapman, 1959):

\[ \dot{Q} = \frac{c_{Q,1}}{\sqrt{\frac{\rho}{\rho_0} \left( \frac{V}{V_c} \right)^{c_{Q,2}}}} \]  

(3.3.29)

where \( c_{Q,1} \) and \( c_{Q,2} \) are constants (\( c_{Q,1} = 1.06584 \times 10^8 \) W/m\(^{3/2}\) and \( c_{Q,2} = 3 \)). \( r_N \) is a characteristic radius, \( \rho_0 \) is the Earth’s atmospheric density at sea level (\( \rho_0 = 1.225 \) kg/m\(^3\)), and \( V_c \) is the local circular velocity with respect to the rotating frame. Taking the time derivative of Eq. (3.3.29) results in

\[ \ddot{Q} = \dot{Q} \left( \frac{c_{Q,2} \dot{V}}{V} - \frac{\dot{h}}{2H_s} \right) = \dot{Q} \left( \frac{c_{Q,2} T - D}{mV} - \frac{\dot{h}}{2H_s} \right) \]  

(3.3.30)

\( T_c \) is written as the sum of an equilibrium value, derived from Eq. (3.3.30), and the PI compensation terms:

\[ T_c = D + \frac{mV}{2c_{Q,2}H_s} \dot{h} + K_{Qp} e_Q + K_{Qi} \int_0^t e_Q \, dt \quad \text{with} \quad e_Q = \dot{Q} - \dot{Q}_{\text{max}} \]  

(3.3.31)

Substituting \( T_c \) in Eq. (3.3.30) yields an expression for the rate of change of the error in heat flux. After Laplace transforming the resulting equation, and equating it with a second-order characteristic equation similar to Eq. (3.3.27), we obtain for the proportional and integral gains:

\[ K_{Qp} = -\frac{2mV}{c_{Q,2}Q} \zeta_Q \omega_Q \quad \text{and} \quad K_{Qi} = -\frac{mV}{c_{Q,2}Q} \omega_Q^2 \]  

(3.3.32)

Finally, given the axial acceleration and its time derivative by

\[ n_a = \frac{\dot{V}}{g_0} \quad \dot{n_a} = \frac{\ddot{V}}{g_0} = \frac{T - D}{mg_0} - \frac{T - D}{m^2g_0} \dot{m} \]  

(3.3.33)

with \( g_0 \) the acceleration due to gravity at the equatorial sea level (\( g_0 = 9.798 \) m/s\(^2\) for the Earth), \( T_c \) is written as
\[ T_c = D + \frac{m}{\dot{m}}(\ddot{T} - \dot{D}) + K_{np} \sigma_{\eta_a} + K_{nl} \int_{0}^{t} \sigma_{\eta_a} \, dt \quad \text{with} \quad \sigma_{\eta_a} = n_a - n_{a_{\text{max}}} \]  

(3.3.34)

For \( K_{np} \) and \( K_{nl} \) it follows:

\[ K_{np} = \frac{2m^2g_0}{\dot{m}} \xi_n \omega_n \quad \text{and} \quad K_{nl} = \frac{m^2g_0}{\dot{m}} \omega_n^2 \]  

(3.3.35)

### 3.4. Attitude-control concepts

In this section, the mathematical foundation of two attitude-control concepts is presented, without focusing on a particular vehicle or mission (the actual application of the theory will be discussed in Chapters 7 and 8). To this end, Sub-section 3.4.2 describes a control concept based on linear state feedback and Sub-section 3.4.3 presents a concept based on Model Reference Adaptive Control.

#### 3.4.1. Introduction

Feedback control systems have found widespread use in, amongst others, aeronautical engineering (Bryson, 1985). Simple forms of feedback are (a combination of) P, I and D feedback of the output of the system that has to be regulated. Classical control theory of linear systems was based on frequency response and root-locus techniques, see, for instance the books by Kuo (1987) and D'Souza (1988). A set of general performance requirements, that were not optimal in a mathematical sense but rather aimed at a reasonable performance, were commonly used.

Initially, the older concepts were not easy to apply to multi-variable systems. State feedback systems, however, are particularly suitable for systems with Multiple Inputs and Multiple Outputs, so-called MIMO systems. The parameters that define the control-system performance, the gains, can be obtained by pole placement or, alternatively, based on mathematically defined optimisation criteria. The Linear Quadratic cost criterion is well known in this respect, resulting in the so-called Linear Quadratic Regulator (LQR), see, for example, the books by Bryson and Ho (1975), Lewis (1986) and Gopal (1989).

An advantage of LQR is that it is a systematic method for designing MIMO systems. Furthermore, the implementation of the control laws in flight-simulation software is fairly simple, and the computational load for on-line simulation is low. The problems dealing with pole assignment linked with MIMO systems have been replaced by an optimisation problem, and pole selection is now changed to the selection of the optimisation parameters (weighting matrices). However, when not all the states of the controlled system can be measured, then most of the
attractive properties of the LQR methodology are lost. In that case an estimator is introduced to estimate the unavailable states, see Shahian and Hassul (1993), which then results in a Linear Quadratic Gaussian (LQG) controller. However, the LQR seems to be a very appealing concept for our purposes, i.e., designing- and implementation-wise, so we will select this method and thereby assuming an ideal navigation system.

The problem of self adjusting the parameters of a controller in order to stabilise the dynamic characteristics of a feedback control system when drift variations in the plant parameters occur, was the origin of Model Reference Adaptive Control or MRAC (Landau, 1974). With this technique, a reference model serves as the basis to generate the steering commands for the (unknown) plant. The parameters of the controller are adjusted in such a way that the difference between the model output and the plant output are minimised. The performance of the controller is in this way less sensitive to environmental changes, modelling errors and non-linearities within the system. A drawback might be, however, that a large control effort can be required to make the plant follow the model (Messer et al., 1994).

A recent work on direct adaptive control algorithms, and especially a simplified form of MRAC, is given by Kaufman et al. (1994). In their introductory chapter they state that model reference adaptive methods might be classified in three groups, originating from three different approaches:

- full-state access method, assuming that the complete state is measurable,
- input-output method, incorporating adaptive observers in the controller to overcome the problem of partial access to the state, and
- simple adaptive control based on output feedback, requiring neither full state feedback nor adaptive observers.

It may be obvious that the third method is the most appealing, since it puts the fewest restrictions on the controller. Three important properties of this concept are:

- It is applicable to non-minimum phase systems, which means that the plant's open-loop transfer function may have poles and zeros in the right-hand plane.
- The order (i.e., number of states) of the physical system may be much larger than the order of the reference model, which means that the actual plant could be a highly non-linear and elastic space plane, while the reference model could be a series of Linear Time Invariant systems. It should be noted, however, that the number of model outputs must be equal to the number of plant outputs.
- It considers plants with multiple inputs and outputs, so-called MIMO systems.

Since the implementation of the simplified MRAC algorithm is supposed to be relatively easy, this method will also be selected for application.
3.4.2. Linear Quadratic Regulator

To apply classic control theory to design a linear state feedback controller, it is necessary that the system of equations of motion is Linear Time Invariant (LTI). Since the general equations of motion are time varying and highly non-linear, some assumptions need to be made. We consider our nominal trajectory to be an equilibrium path that the vehicle follows under ideal conditions. Looking only at small deviations from the nominal trajectory, the equations can be linearised. Furthermore, by discretising the trajectory into a number of sufficiently small time intervals, in each of the intervals the linearised state of the vehicle can be assumed to be constant. As a result, the overall system has been divided into a sequence of LTI systems.

For a time point $k$, the LTI system can be written in state-space form as

$$\Delta \mathbf{x}_p = A_p(t_k) \Delta \mathbf{x}_p + B_p(t_k) \Delta \mathbf{u}_p$$  \hspace{1cm} (3.4.1)

In this equation, $\Delta \mathbf{x}_p$ is the $n_p \times 1$ state vector, $\Delta \mathbf{u}_p$ is the $m \times 1$ input or control vector, and $A_p$ and $B_p$ are the system and control matrix of appropriate dimensions, respectively. For each time point $t_k$, $A_p$ and $B_p$ are constant, and can be obtained from the $n_p$ equations of motion $f_i$ by

$$A_p = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{n_p}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{n_p}}{\partial x_1} & \cdots & \frac{\partial f_{n_p}}{\partial x_{n_p}} \end{bmatrix} \quad \text{and} \quad B_p = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{n_p}}{\partial u_1} & \cdots & \frac{\partial f_{n_p}}{\partial u_m} \end{bmatrix}$$  \hspace{1cm} (3.4.2)

For each $t_k$, control laws that take commands from the guidance system as input must be selected. To apply optimal control theory, the attitude-control laws are also linearised and possible small terms are neglected, such that they can be approximated by

$$\Delta \mathbf{u}_p = -K(t_k) \Delta \mathbf{x}_p$$  \hspace{1cm} (3.4.3)

where the feedback gains, $K$, are constants for time point $t_k$. Substitution of Eq. (3.4.3) into Eq. (3.4.1) yields an expression of the closed-loop system:

$$\Delta \dot{x}_p = [A_p(t_k) - B_p(t_k) K(t_k)] \Delta x_p$$  \hspace{1cm} (3.4.4)

From the characteristic equation the eigenvalues $\lambda$ and corresponding eigenmotion can be obtained:
\[ \text{det} | A_p(t_k) - B_p(t_k) K(t_k) - \lambda I | = 0 \quad (3.4.5) \]

As becomes obvious when studying the above equation, the eigenvalues of the closed-loop system can be changed by varying the feedback matrix \( K \). Whether the system will be controllable, however, is not only depending on the values of these gains, but also whether the open-loop system in itself is at least controllable, a condition depending on the coefficient matrices \( A_p \) and \( B_p \) (Kuo, 1987). The gains are obtained with an indirect method, i.e., Quadratic Optimal Control, in which a mathematically defined cost criterion is minimised on an indefinite control time interval (Gopah, 1989):

\[ J = \lim_{t_f \to \infty} \int_{t_i}^{t_f} (\Delta x^T Q(t_k) \Delta x + \Delta u^T R(t_k) \Delta u) \, dt \quad (3.4.6) \]

where the term \( \Delta x^T Q \Delta x \) represents the control deviation and the term \( \Delta u^T R \Delta u \) the control effort. \( Q \) is a real positive semi-definite matrix, whereas \( R \) is a real symmetric positive definite matrix, so any \( x, u \neq 0 \) cannot give a negative contribution to \( J \). By varying \( Q \) and \( R \) more importance can be attached to the control deviation, resulting in a faster response, or the control effort, giving smaller control signals. By varying each of the elements of \( Q \) and \( R \) the corresponding elements of \( x \) and \( u \) can be addressed. Brandt and Van den Broek (1984) state that defining \( Q \) and \( R \) is usually done in an iterative manner, and that a good first choice is given by 'Bryson's Rule':

\[ Q = \text{diag} \left\{ \frac{1}{\Delta x_{1, \text{max}}^2}, \frac{1}{\Delta x_{2, \text{max}}^2}, \ldots, \frac{1}{\Delta x_{n, \text{max}}^2} \right\} \quad \text{and} \quad R = \text{diag} \left\{ \frac{1}{\Delta u_{1, \text{max}}^2}, \frac{1}{\Delta u_{2, \text{max}}^2}, \ldots, \frac{1}{\Delta u_{n, \text{max}}^2} \right\} \quad (3.4.7) \]

with \( \Delta x_{\text{max}} \) and \( \Delta u_{\text{max}} \) the maximum allowable amplitude of the \( i \)-th and \( j \)-th element of the state and control vector, respectively. In case of a controllable system, \( K(t_k) \) is given by

\[ K(t_k) = R^{-1}(t_k) B_p^T(t_k) P(t_k) \quad (3.4.8) \]

The constant, symmetric matrix \( P(t_k) \) follows from solving the Algebraic Riccati Equation,

\[ A_p^T P + P A_p - P B_p R^{-1} B_p^T P - Q = 0 \quad (3.4.9) \]

Response tests must prove that these weighting matrices have been correctly selected. If not, the values of the diagonal elements should be adjusted. Frangos and Yavin (1992) propose
a synthesis procedure that automatically varies the weighting matrices and compute the gains in an iterative manner, based on minimisation of J. Luo and Lan (1995), finally, describe a systematic method to determine the weighting matrices, so as to produce specified closed-loop eigenvalues. Implementation of either algorithm, however, is considered to be beyond the scope of the current study.

3.4.3. Model Reference Adaptive Control

In this section, we will give a brief overview of the concept of direct MRAC. This overview is based on the book by Kaufman et al. (1994). The reader is referred to this book for detailed derivations and proofs of applied theorems. Here, we will just summarise the mathematical modelling of the basic concept. Before we come to the mathematical modelling, a brief problem description is given.

The space plane considered in this thesis is only a simple representation of a realistic space plane. Basically, long slender bodies suffer from aeroelastic effects that can influence the performance significantly. Although the effect of aeroelasticity is not covered in this study, the motion of the remaining non-elastic, mass-varying body is described by a set of complex, non-linear differential equations. For the development of a control system, it is common practise to linearise the non-linear model. However, when this controller is applied to the original system, some kind of performance feedback should be included to account for the non-linearities. In the subsequent discussion, the following linearised model of the space plane will be used:

\[ \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \]  \hspace{1cm} (3.4.10a)

\[ y_m(t) = C_m x_m(t) \]  \hspace{1cm} (3.4.10b)

where \( x_m(t) \) is the \( n_m \times 1 \) model state vector, \( u_m(t) \) is the \( m \times 1 \) model input or command, \( y_m(t) \) is the \( q \times 1 \) model output vector, and \( A_m, B_m \) and \( C_m \) are the state, control and output matrix of appropriate dimensions. The non-linear space plane will be defined as a set of non-linear ordinary differential equations, i.e.,

\[ \dot{x}_p = f(t, x_p(t), u_p(t)) \]  \hspace{1cm} (3.4.11a)

\[ y_p = g(x_p(t), u_p(t)) \]  \hspace{1cm} (3.4.11b)

where \( x_p(t) \) is the \( n_p \times 1 \) plant state vector, \( u_p(t) \) is the \( m \times 1 \) plant input or command vector, \( y_p(t) \) is the \( q \times 1 \) model output vector, and \( f \) and \( g \) are \( n_p \times 1 \) and \( q \times 1 \) vector functions. Note that the number of model inputs and outputs equals the corresponding number of plant inputs and outputs.

MRAC is based on matching the response of the system that is to be controlled (the plant) to that of a reference model (the model). In its simplest form, an MRAC system has been
depicted in Fig. 3.8. From this figure it can be seen that the model input $u_m$ and model state $x_m$ are required to form part of the input signal $u_p$ to the plant. Moreover, the so-called output error $e_y$ serves as a feedback quantity to form the third element that composes $u_p$ (note that the adaptive system uses all values that can be measured). The three gains, i.e., $K_x$, $K_u$ and $K_y$ are adaptive. Starting with the concept of perfect model following, Kaufman et al. (1994) derive an algorithm to compute the adaptive gains. The ideal model trajectories are considered to be reference target trajectories that the plant attempts to reach. The resulting adaptive controller can maintain small (bounded) tracking errors in non-ideal environments and remains stable.

![Diagram](image_url)

**Fig. 3.8 - Model reference control system (after Kaufman et al., 1994).**

The adaptive algorithm to compute the input $u_p$ to a LTI plant is summarised as follows. Let us define a vector $r(t)$ that combines the three measurable signals, and a matrix $K_r(t)$ that combines the adaptive gains:

$$r(t) = [e_y(t), x_m(t), u_m]^T \quad \text{and} \quad K_r(t) = [K_o(t), K_x(t), K_u(t)]$$

Using the above definitions, $u_p$ can be written as

$$u_p(t) = K_r(t) r(t)$$  \hspace{1cm} (3.4.12)
To compute the adaptive gains, $K_r$ is defined to be the sum of an integral and proportional component, i.e.,

$$K_r(t) = K_I(t) + K_P(t)$$  \hspace{1cm} (3.4.13)

with

$$K_I(t) = e_y r^T(t) T$$  \hspace{1cm} (3.4.14)

$$K_P(t) = e_y(t) r^T(t) \bar{T}$$  \hspace{1cm} (3.4.15)

In Eq. (3.4.14)-(3.4.15), the weighting matrices $T$ and $\bar{T}$ are positive definite symmetric and positive semi-definite symmetric, respectively.

Before we continue, it is important to note that the adaptive control algorithm cannot be applied to just any system. To guarantee that all states and gains in the adaptive system are bounded and the output error is asymptotically stable, it is necessary that the plant is *almost strictly positive real* (ASPR)\(^\text{10}\). Unfortunately, it is not trivial whether a system is ASPR or not. To apply the adaptive algorithm to a much wider class of systems, various modifications have been developed. One modification is to augment the plant with a so-called feedforward compensator, such that the class of ASPR systems is increased.

A second extension to the adaptive algorithm can be to apply not only feedforward around the plant but also around the reference model. In general, the model following error will be bounded but not zero in steady state, which can be alleviated by the model feedforward. We will not include model-feedforward dynamics to keep the design somewhat simpler. In addition, the dynamics of the space plane are continuously changing, which might make the pursuit of a zero steady-state error an unnecessary luxury.

So far, we considered an ideal environment. To cope with disturbances in an environment

\(^{10}\) Suppose that the plant to be controlled is linear time-invariant and given in state-space form by

$$\dot{x}_p = A_p x_p + B_p u_p$$

$$y_p = C_p x_p$$

Considering a constant output feedback controller, i.e., $u_p = -K_y y_p$ and setting up a Lyapunov equation for the closed-loop system, one can claim that the adaptive system is stable if there exist two positive definite symmetric matrices $P$ and $Q$ such that the fictitious closed-loop system using the unknown gain $K_y$ satisfies simultaneously the following conditions:

$$P[A_p - B_p K_y C_p] + [A_p - B_p K_y C_p]^T P = -Q < 0$$

$$PB_p = C_p^T$$

If these relations are satisfied for a positive semidefinite matrix $Q$, the system is called positive real (PR), whereas for a positive definite matrix $Q$, the system is denoted strictly positive real (SPR). Because it is not the original plant, but some stabilised closed-loop plant that is required to be SPR, and only a constant feedback gain separates the plant from being SPR, the plant is denoted as *almost strictly positive real* (ASPR).
that lead to a persistent non-zero error and therefore to a continuous change in the integral gain $K_i$, we will apply a robust design to adjust the integral gain to prevent it from reaching very high values. The integral term of Eq. (3.4.14) is adjusted in the following manner:

$$K_i = e_y(t) r^T T - \sigma K_i(t)$$  \hspace{1cm} (3.4.19)

Without the $\sigma$-term, $K_i(t)$ is a perfect integrator and may steadily increase (and even diverge) whenever perfect output following is not possible. Including the $\sigma$-term, $K_i(t)$ is obtained from a first-order filtering of $e_y(t) r^T T$ and therefore cannot diverge, unless the output error diverges.

The discussion on the adaptive algorithm has been restricted to linear, time-invariant plants. The space plane to be controlled, however, is non linear and time varying. Kaufman et al. (1994) give additional theorems for time-varying systems and non-linear systems, using the similar concept of almost passivity instead of almost positivity. It is considered to be far beyond the scope of this thesis to discuss this concept here. Although we are aware of the important 'white spot' in the algorithm, i.e., not being able to prove that the space plane is almost strictly passive, we will not pursue this proof. In line with an example of the adaptive, longitudinal control for a relaxed static stability aircraft discussed by Kaufman et al., we will develop the adaptive system ad hoc and validate it by computer simulation.

3.5. Summary

- In the flight-simulation model that has to be developed, a Guidance and Control system is an essential module. The task of the guidance system is to generate steering commands to influence the translational motion of the vehicle by applying control forces, thereby taking a reference state, trajectory constraints and/or a final state into account. The control system has to take care that the steering commands are carried out by applying control moments. The control forces and moments are of aerodynamic and propulsive origin.

- The re-entry guidance, based on tracking of a reference trajectory, is split into a horizontal and vertical part. The horizontal guidance must steer the vehicle towards a target point, by assuring that the heading error stays within a predefined dead-band. Exceeding this dead-band results in the execution of a bank reversal. The vertical guidance consists of two loops: energy control and altitude control. The total energy is controlled such that only the final value at the targeting point will be met, having no direct effect on the constraints during the flight. The internal sharing of potential and kinetic energy, on the other hand, will affect the constraints through the altitude-velocity relation.

- The ascent guidance for vertical plane motion is based on inverse dynamics, i.e., given a
flight-path angle rate, inverting the corresponding differential equation will give the solution in terms of external forces and consequently the guidance command (commanded angle of attack). The commanded flight-path angle rate is obtained from a PID guidance law for the error in flight-path angle, i.e., the difference between the actual and the reference value.

- A suboptimal ascent to orbit can be achieved by tracking the trajectory constraints, i.e., the maximum dynamic pressure, the maximum heat flux and the maximum axial acceleration. By applying a PI guidance law to the error in the constraints, the throttle setting of the propulsion system is adjusted.

- The guidance system for hypersonic manoeuvres is based on a combination of altitude and cross-range control. The corresponding commanded accelerations follow from two separate PID guidance laws. Conversion of the commanded accelerations to a normal load and bank angle are executed by the so-called resolver, thereby taking the centrifugal relief into account.

- Two attitude-control concepts are the Linear Quadratic Regulator and Model Reference Adaptive Control. The LQR is based on a linearisation of the reference trajectory and belongs to the class of perturbation control. The linearised state is fed back and converted to attitude-control commands by multiplying them with a gain value. These gains are scheduled to take the changing flight environment into account, and are computed by applying optimal control theory and a quadratic cost criterion based on a weighting of the state deviation and control effort. The basis for MRAC is that a system of which only the outputs are known has to track a reference model that can be a simple approximation of the actual system. By combining the output error, the model state and model control parameters into a reference signal, proportional and integral gains can be computed that are adapted to the value of the reference signal. To apply MRAC it is necessary that the controlled non-linear system is almost strictly passive.
Chapter 4

Experimental Design

Under the strictest conditions concerning pressure, temperature, volume and other parameters, the organism will just do that it happens to feel like doing. Jean de la Rivière

In the design of complex systems such as the G&C system of a space plane, a large amount of engineering effort is consumed in conducting experiments to obtain sufficient information that is required for making design decisions. These experiments can be done in a laboratory set up, where measurements on a prototype must give insight in the actual system, or they can be executed on a computer, with which simulation of a mathematical model representing the actual system will give the required knowledge. Often, only combinations of the two will give sufficient information to make the right decisions. In this thesis, we refer to both prototype experiments and computer simulations to assist in the design process when we speak of experimental design.

Efficiency in the experimental design process is a key to keeping development and manufacturing costs low, and having high-quality products. In Chapter 1 the Taguchi Method was mentioned, possibly in combination with Response Surface Methodology (RSM), as a potential methodology for achieving efficiency in experimental design, which is, in this thesis, the analysis of flight mechanics of space planes in relation with G&C-system design. To understand the methodology and what to do with the results, it is important that we give a complete description of the method, its background and a mathematical foundation for data processing. This chapter has six more sections.

In Section 4.1, the terminology related to experimental design is explained. Section 4.2 discusses Taguchi's orthogonal arrays. Several matrices for 2- and 3-level factors will be introduced, as well as an algorithm to derive this kind of orthogonal arrays in an automated fashion. Section 4.3 presents the RSM; basic definitions of response surfaces are given, and the Central Composite Design as one particular methodology to set up second-order response surfaces is discussed. In Section 4.4, a brief overview of the analysis of the results is given, whereas a more detailed discussion is included in Appendix E. A special form of experimental design is
robust design, as discussed in Section 4.5, which is a means to make a system or process as insensitive to disturbances as possible without eliminating the source of the disturbances. Finally, Section 4.6 summarises the important points of this chapter.

### 4.1. Terminology related to experimental design

In discussing experimental design, it is important that the used terms are completely clear and will not give rise to confusion. For this reason, some definitions of terms, that are used throughout this and successive chapters, are given. These terms are commonly used in the field of experimental design and can be found in, for instance, Taguchi (1988), Phadke (1989), Khuri and Cornell (1987), and Montgomery (1984).

- **Factors** are processing conditions or input variables. When the values or settings of the factors can be controlled by the experimenter, these factors are also called control factors (e.g., gains of a guidance system or switching times of the propulsion system). In case they cannot be influenced we speak of noise factors (e.g., dispersions in atmospheric density, initial conditions or the location of the c.o.m.). Note that at one time factors can be control factors (e.g., aerodynamic coefficients during the aerodynamic design of a vehicle) and at another time noise factors (uncertainties in the aerodynamic coefficients for a given vehicle design).
- **The factor level** is a setting of the input variable, that can either be randomly or deterministically defined. In the latter case, one can think of a (predefined) minimum, nominal and maximum value of the factor (e.g., the nominal entry flight-path angle is γ₀ = -1.29°, and it is of interest to know the influence of γₘᵢₙ = -1.49° and γₘₐₓ = -1.09°).
- The use of **coded variables** facilitates the construction of experimental designs. Coding removes the units of measurements of the factors and normalises their range. A convenient formula for defining the coded variable, \( X_p \) is (Khuri and Cornell, 1987):

\[
X_j = \frac{2x_j - (x_{IL} + x_{IH})}{x_{IH} - x_{IL}} \quad (4.1.1)
\]

where \( x_{IL} \) and \( x_{IH} \) are the low and high levels of \( x_p \) respectively. Two of the more obvious advantages of using coded variables are: i) computational ease and increased accuracy in estimating the model coefficients, and ii) enhanced interpretability of the coefficient estimates in the model. The coded variables range from -1 (low level, e.g., \( γ_{min} \)) to 1 (high level, \( γ_{max} \)); the nominal value (\( γ₀ \)) corresponds with 0.
- **The experimental region** is the range over which all factors can be varied, e.g., for a single factor this would be between its minimum and maximum value.
- **The response variable** is the measured quantity whose value is assumed to be affected by changing the levels of the factors, e.g., the maximum heat load or maximum accelerations.
during the re-entry of a spacecraft.

- The overall mean response is the mean value of the response due to all factor variations, and is obtained from the sum of response values divided by the number of experiments.
- The effect of a factor level is the response deviation it causes from the overall mean response.
- An interaction between two (input) variables or factors is said to exist, when a variation in the first factor results in a different variation of the response for each level of the second factor. Examples of various degrees of interaction can be found in Fig. 4.1. Note that the concept of interaction between two factors can be generalised to apply to interaction among three or more factors.

![Diagram](image)

**Fig. 4.1 - Examples of interaction between factor A (levels A₁, A₂ and A₃) and B (levels B₁, B₂ and B₃), based on Phadke (1989).**

- An \(n^{th}\)-order response surface is an \(n^{th}\)-order fit of a response \(y\) as a function of \(k\) factors \(x_i\) and their interactions. Example: a first-order response surface in 3 factors without interactions is written as
  \[
  y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 
  \]  
  \hspace{1cm} (4.1.2)

In case of interactions, the first-order model may be written as
  \[
  y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 
  \]  
  \hspace{1cm} (4.1.3)

- One possible way to estimate the unknown \(\beta_i\) and \(\beta_{ij}\) is the method of least squares, that aims at minimizing the sum of squares of the difference between each of the observed responses and the corresponding response given by the response surface. Anticipating on the discussion in Section 4.3, the least-squares estimator of \(\beta\), i.e., \(b\), is given by
  \[
  b = (X^T X)^{-1} X^T y 
  \]  
  \hspace{1cm} (4.1.4)

where \(X\) is the matrix with the levels of the factors, including interactions.
- The variance of the prediction is a direct measure of the likely error associated with the estimate at the point \(x_0\) produced by the response surface.
- A matrix experiment consists of a set of experiments where the levels of the various factors
under study, change from one experiment to the other according to a pre-defined matrix.

- An orthogonal design is a design for which the variances of the coefficient estimates $b$ are minimised. This gives the optimal design for the selected order of the response surface on a per observation basis, i.e., for the number of observations (or experiments) that are included in the design. A first-order orthogonal design is one for which $X^TX$ is a diagonal matrix.

- In a rotatable design the variance of the prediction at $x_0$ is the same for all points that are at the same distance from the centre of the design.

- Orthogonal arrays form a special family of factor-level matrices that are used for matrix experiments resulting in an orthogonal design. Matrix orthogonality, in this context, should be considered in the combinatorial sense, namely: for any pair of columns, all combinations of factor levels occur an equal number of times, the so-called balancing property (Phadke, 1989).

<table>
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<tr>
<th>Exp.</th>
<th>A</th>
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</table>

Table 4.1 - Orthogonal array $L_8$ with 7 factors (A-G) on two levels.

Taguchi (1988) has derived many orthogonal arrays, most of them based on 2- or 3-level factors (4- or 5-level arrays are sometimes used as well). As an example, anticipating the discussion in Section 4.2, the so-called $L_8$ array is given in Table 4.1 (note that the index '8' indicates the number of rows, or, similarly, the number of experiments). Seven 2-level factors (A through G), with levels -1 and 1 are varied over eight experiments. For columns 1 and 2, the 4 possible combinations of factor levels, i.e., (-1,-1), (-1,1), (1,-1) and (1,1), occur in experiments (1,2), (3,4), (5,6) and (7,8), respectively.

### 4.2. The Taguchi Method

To determine the influence of factors on a particular response, the factors can be varied randomly or deterministically. The latter has the advantage that the values of the factors are known and can be reproduced. The factor levels are then chosen as the minimum or maximum value, in other words, they define the boundaries of the experimental region. When the factors
do not interact, we can vary one factor at a time, do a simulation and continue with the next one. Selecting the minimum or maximum factor level depends on the worst-case influence on the response. The total number of simulations equals the number of factors under investigation (if a priori it is not known whether the minimum or maximum factor level results in the worst-case influence, two simulations per factor are required). The influence of each of the factors can be superimposed, and we can determine the combined effect on the response.

But the assumption that the factors do not interact does not hold all the time, and furthermore it might be difficult to simultaneously study the effect of the factors on multiple responses, since conflicting factor levels might result. An alternative is the so-called factorial arrangement of the factors. In this kind of design, the factors are varied simultaneously according to a pre-defined scheme and for each of the combinations of factor levels a simulation (or experiment) is executed (the so-called full factorial design). Two factors, one with two levels and the other one with three, would require six simulations to cover all the combinations. For factorial experiments, rules have been developed to determine the main factor effects and the effect of the interactions (Keppel, 1973). For three factors $A$, $B$ and $C$ this not only means the interactions $A \times B$ and $A \times C$, but also the interaction $A \times B \times C$.

However, a serious drawback of the full factorial design is that the number of experiments increases rapidly with the number of factors. Whereas two three-level factors require 9 experiments, with three factors the number increases to 27, and so on. Thirteen three-level factors would require $3^{13}$ (1,594,323) experiments! Apart from practical implications with prototype experiments, one can imagine that in case of computer simulations this will be a significant CPU load: even if one run takes a mere 1 second of CPU time, completing the simulations would take more than 18 days.

Fortunately, there is an alternative for the factor combinations of a factorial design, resulting in a so-called fractional factorial design. The related method, the Taguchi Method, found its origin in the field of design optimisation and uses orthogonal arrays to define the combinations of factor levels (Taguchi, 1988). In addition, the influence of interactions can be studied by treating the related interactions as separate factors. It must be noted that when one wants to study all possible interactions, the total number of simulations will equal the number required for a full factorial design. However, if one is not interested in all interactions and/or if it is known that some factors do not exhibit any interactions, one may achieve a large simulation reduction.

Taguchi’s use of orthogonal arrays enables a rapid search through many design options to find the design with the best signal-to-noise ratio, i.e., the design that performs consistently on target and is relatively insensitive to factors that are difficult to control (Taguchi, 1988). The main benefits of using orthogonal arrays are (Mistree et al., 1993):

- Conclusions derived from the experiments are valid over the entire experimental region spanned by the control factors and their settings with a minimum of effort.
- There is a large saving of experimental effort and therefore, a reduction of computational time for simulations.
• Analysis of the results can be done easily, e.g., the effect of individual factors on a response can be established in a simple manner.
• Orthogonal arrays and their experiments are designed deterministically, not randomly, which means that results are reproducible.

Stanley et al. (1992) apply the Taguchi design method to propulsion system optimisation for SSTO vehicles. They perform a series of parametric trade studies - varying lift-off thrust-to-weight ratio, engine mode transition Mach number, mixture ratios, area ratios, and chamber pressure values - to optimise both a dual mixture ratio engine and a single mixture ratio engine of similar design and technology level. Seven parameters were studied at three levels, which normally would have taken $3^7$ (2187) trajectory and vehicle sizing runs. The methodology developed by Stanley et al. proved to be very flexible and required only 50 runs. These runs included the interactions between several parameters. The sensitive factors could be identified and the optimum factor levels were determined that reduced the vehicle dry weight.

Taguchi (1988) gives an example of (sub)optimising an electrical design by varying 13 parameters at 3 levels, a low, medium and high value. Instead of $3^{13}$ simulations to do a full factorial design, he arrived at a total of 36 (!) combinations, which give virtually the same information. Marée et al. (1993) have applied the same combinations of parameters to the optimal-trajectory problem of a 360-ton space plane, to determine the influence of the trajectory parameters on the payload mass. The outcome of this study was, that the payload mass varied from -1.9 (!) ton to 5.8 ton, clearly indicating the importance of flying a well-defined trajectory.

To create orthogonal arrays for three-level factors (or, equivalently, two-level factors), an algorithm has been developed by Mistree et al. (1993), see also Appendix D. The basic information is given by so-called Latin Squares that present the smallest orthogonal array for a given number of factor levels. When there is a complete orthogonal system of $n-1$ Latin Squares of dimension $n \times n$ (with $n$ the number of factor levels), denoted by $L_1$, $L_2$, ..., $L_{n-1}$, we can construct an orthogonal array with $nf$ rows ($r = 2, 3, ...$) and $\frac{n^r - 1}{n-1}$ columns. As an example, we refer again to Table 4.1 that shows the two-level $L_9$ orthogonal array ($n = 2$ and $r = 3$ results in 8 rows and 7 columns).

In principle, the Taguchi Method does not take any interactions between factors into account. In case there is physical evidence of interactions it is possible to define these interactions as independent factors. They will occupy pre-defined columns in the orthogonal arrays, one column in case of an interaction between two 2-level factors, and two columns for a three-level one. As Stanley et al. (1992) concluded, the Taguchi Method can successfully determine the existence of interactions between pairs of variables and quantify the effects. This can be particularly useful when non-linearities make it very difficult to initially estimate which pairs exhibit strong interactions. Their method consisted of dividing the total number of trajectory and vehicle-sizing runs into two parts. First, the two-level $L_{32}$ orthogonal array was used to examine the interaction effects between the seven related parameters, and to locate the strong interaction.
Second, this strong interaction was included in the final three-level $L_{18}$ optimisation study of all seven parameters.

In assigning factors to columns, the corresponding interaction columns should then be kept empty in case that interaction is to be studied as well. The columns that are related to the interactions are given by the so-called interaction tables, which can be found in most books on orthogonal arrays, see, for instance, Phadke (1989). An example of the interaction table for the $L_8$ array is given in Table 4.2. Suppose factor $A$ and $B$ are assigned to the left-most columns (1 and 2) of the $L_8$ array, then from this table it follows that the interaction $A \times B$ is represented by column 3. However, two factors $C$ and $D$ assigned to columns 5 and 6 result in an interaction also given by column 3. This means that one has to be careful in assigning the factors, as will be discussed in more detail.

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Table 4.2 - Interaction table for the $L_8$ orthogonal array.

The selection of the proper orthogonal array is not trivial, especially if many factors are included and there are potential interactions. In that case the column assignment can be complicated and it is possible that not all of the factors and interactions can be studied in one go. Basically, this is due to an effect called confounding. When two factors $A$ and $B$ are varied at the same time, the interaction $A \times B$ can be studied by not assigning any factor to the appropriate column as specified by the related interaction table. However, when a factor $C$ is assigned to that column, the corresponding factor variation will be influenced or confounded by the interaction $A \times B$. In case there are many more factors, in general more than one interaction will have its effect on a particular column of the array, and it is not possible to tell in mathematical sense which column interaction is responsible for the confounding. In that case, physical insight and experience of the designer combined with extra experiments must give more information about the interactions.

When for a two-level array the factors are assigned to the columns such that all the two-level interactions are free of confounding with other two-level interactions and main effects, this results in a so-called Resolution-V array, see also Fowlkes and Creveling (1995). Basically, this
is required when no information about the interactions is available. As an example, the $L_8$ array is Resolution V when a maximum of three factors are assigned to columns 1, 2 and 4. In case two-level interactions are confounded with two-level interactions but not with main effects, the array is referred to as Resolution IV (for the $L_8$ a fourth factor is assigned to column 7), whereas the case where two-level interactions confound with main effects, the array is referred to as Resolution III (a fifth factor is assigned to, for instance, column 3). It is easy to understand that for many factors that are possibly dependent, a Resolution-V array will be very difficult to establish. Note that orthogonality of a matrix experiment is not lost by keeping one or more columns of an array empty (Phadke, 1989).

### 4.3. Response Surface Methodology

#### 4.3.1. Introduction

After conducting a matrix experiment, it is possible to compute a response surface, i.e., a polynomial function in one or more dimensions that describes the relation between a response and the applied factors. This response surface can include pure linear (e.g., $x_1$), quadratic (e.g., $x_1^2$), and higher-order terms ($x_1^n$), as well as interactions (e.g., $x_1x_2$). Response Surface Methodology (RSM) is a set of techniques that encompasses (Khuri and Cornell, 1987): i) setting up a series of experiments (design) that will yield adequate and reliable measurements of the response of interest, ii) determining a mathematical model that best fits the data collected from the design chosen under i), by conducting appropriate tests of hypotheses concerning the model’s parameters, and iii) determining the optimal settings of the experimental factors that produce the maximum (or minimum) value of the response.

When this methodology is applied to a trajectory sensitivity analysis, it can be used to find sub-optimal performance parameters or maximum values of constraints. We can then find those factor settings that will give the best (or worst!) result in the experimental region. In case of a constraint violation, a variance analysis can lead to the major contribution of the related factors and/or interactions. The outcome might enable us to drive the design, or rather to minimise the error in those factors, such that the constraint will not be violated any more. This can relieve us from the urge to minimise all the errors, also the ones that do not or hardly contribute to the constraint violation. Note that only continuous factors can be included in the response surface.

After measuring a selected response, it can be expressed as the sum of the true value $\eta$ and some error $\epsilon$. When the true response depends on the levels of $k$ quantitative factors $x_1$, $x_2$, ..., $x_k$, then

$$\eta = \phi(x_1, x_2, ..., x_k) + \epsilon \quad (4.3.1)$$
The function $\phi$ is called the *true-response function* and is assumed to be a continuous function of the $x_i$. When a response function is continuous and smooth, it may be represented locally to any degree of approximation with a Taylor Series expansion about some arbitrary point $(a_1, a_2, ..., a_k)$, to result in a function that can be applied over the experimental region. In the case of only one factor $x_1$ we can write for $\eta = \phi(x_1)$ about the arbitrary point $a_1$:

$$
\eta = \phi(a_1) + (x_1 - a_1) \phi'(a_1) + \frac{1}{2} (x_1 - a_1)^2 \phi''(a_1) + \ldots \tag{4.3.2}
$$
or

$$
\eta = \phi(x_1) = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2 + \ldots \tag{4.3.3}
$$

where the coefficients $\beta_0$, $\beta_1$ and $\beta_{11}$, also called the *regression coefficients*, are parameters which depend on $a_1$ and the derivatives of $\phi(x_1)$ at $a_1$.

A first-order model can be defined by taking only terms up to degree 1, and we get for a response function depending on $k$ factors:

$$
\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k = \beta_0 + \sum_{i=1}^{k} \beta_i x_i \tag{4.3.4}
$$

The second-order model is defined by

$$
\eta = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij} x_i x_j \tag{4.3.5}
$$

which represents what we refer to as a *second-order response surface*. In case of $k$ factors this surface can be called a *hypersurface*. In each case, the model is *linear* in the regression coefficients.

The order of the model can be driven by the goodness-of-fit of the experimental data. For instance, in case all linear terms of a first-order model have been included and the fit is not satisfactory, then the model can be extended with interaction and quadratic terms, thus forming a second-order model. Of course, the number of experiments should be sufficiently large to allow for the computation of the regression coefficients.

### 4.3.2. Least-squares modelling

Suppose we do $N$ simulations to compute the response $y$. We need some mathematical tool to estimate the unknown coefficients $\beta_0$, $\beta_1$, etc. The first-order model can be expressed in matrix notation

$$
Y = X\beta \tag{4.3.6}
$$
\[ Y = (y_1, y_2, \ldots, y_N)^T, \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{N1} & x_{N2} & \cdots & x_{Nk} \end{bmatrix}, \quad \beta = (\beta_0, \beta_1, \ldots, \beta_k)^T \]

The method of least squares enables us to find the estimates \( b_0, b_1, \) etc., of the unknowns \( \beta_0, \beta_1, \) etc., by minimising the following quadratic criterion:

\[
\sum_{i=1}^{N} (y_i - \beta_0 - \beta_1 x_{i1} - \beta_2 x_{i2} - \cdots - \beta_k x_{ik})^2
\]

(4.3.7)

Expanding the criterion, we can find \( b_0, b_1, \) etc., from the \( k+1 \) so-called normal equations. In matrix notation they are written as

\[
X^T X b = X^T Y
\]

(4.3.8)

which has as a solution

\[
b = (X^T X)^{-1} X^T Y
\]

(4.3.9)

For a first-order orthogonal design, \( X^T X \) is a diagonal matrix, so Eq. (4.3.9) is easy to solve. However, when orthogonality is lost, e.g., when an interaction is modelled in the response surface while it was not defined in the matrix experiment, or when the responses are the result of a batch of Monte-Carlo simulations the inverse matrix \( (X^T X)^{-1} \) can be singular, which means that Eq. (4.3.9) cannot be solved. The diagonal elements of the inverse \( (X^T X)^{-1} \), which are called variance inflation factors, can have very high values in the presence of high correlations. So if we can reduce these variance inflation factors in case the matrix is singular, then the coefficient estimators \( b_y \) will be stabilised, i.e., we can find the (approximation) of the inverse \( (X^T X)^{-1} \). One suggestion to do this, as found in literature (Brook and Arnold, 1985), is to apply a method known as ridge regression. However, one of the disadvantages of this method is that there is no mathematically defined algorithm for determining the value of the ridge estimator \( k \). We will therefore proceed in a different manner.

Returning to the original equation defining the response surface, Eq. (4.3.6), Singular Value Decomposition (SVD) can be applied, which is a common method for solving most linear least squares problems (Press et al., 1989). In short, with SVD one can write any \( m \times n \) matrix \( A \) whose number of rows \( m \) is larger than or equal to its number of columns \( n \), as the product of an \( m \times n \) column-orthogonal matrix \( U \), an \( n \times n \) diagonal matrix \( W \) with positive or zero elements, and the transpose of an \( n \times n \) orthogonal matrix \( V \) (Stoer and Bulirsch, 1980), or
\[ A = U \cdot W \cdot V^T \]  
\[ (4.3.10) \]

So, applying SVD to Eq. (4.3.6) yields:

\[ Y = Xb = U \text{ diag}(w_j) \cdot V^T b \]  
\[ b = V \text{ diag}(1/w_j) \cdot U^T Y \]  
\[ (4.3.11) \]

Press et al. (1989) give an extensive discussion on the least-squares method and the application of SVD. One of the major problems in least-squares problems is, that these kind of problems are both overdetermined, i.e., the number of response values is greater than the number of coefficients) and underdetermined, i.e., there exist ambiguous combinations of the coefficients. In the case of an overdetermined system, SVD produces a solution that is the best approximation in the least-squares sense, and in case of an underdetermined system, SVD gives a solution whose values \( b_j \) are smallest in the least-squares sense, or in other words: *when some combination of basic functions is irrelevant to the fit, that combination will be driven down to a small, innocuous, value, rather than pushed up to delicately cancelling infinities.* The proof of these statements is beyond the scope of this general discussion. Again, we refer to the books of Stoer and Bulirsch (1980) and Press et al. (1989) for the mathematical background.

The same method can be applied to higher-order response surfaces by adjusting the matrix \( X \), i.e., treating products (e.g., \( x_1 x_2 \)) and higher-order terms (e.g., \( x_1^2 \)) as individual parameters, having values that correspond with the value of its composing variables. For a second-order response surface, to be estimated from \( N \) responses, \( X \) will have the form

\[
X = \begin{bmatrix}
1 & x_{11} & x_{11}x_{12} & \ldots & x_{1k-1}x_{1k} & x_{11}^2 & \ldots & x_{1k}^2 \\
1 & x_{21} & x_{21}x_{22} & \ldots & x_{2k-1}x_{2k} & x_{21}^2 & \ldots & x_{2k}^2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
1 & x_{N1} & x_{N1}x_{N2} & \ldots & x_{Nk-1}x_{Nk} & x_{N1}^2 & \ldots & x_{Nk}^2
\end{bmatrix}
\]

\[ (4.3.12) \]

### 4.3.3. Central Composite Design

Any second-order design must involve at least three levels of each input variable, as the model described by Eq. (4.3.6) contains pure quadratic terms. A possible second-order design in \( k \) factors is the \( 3^k \) factorial design. In this design, the number of experiments can be very large, especially when the number of input variables under study is large. As one of the alternatives, Box and Wilson (1951) introduced the (empirical) class of Central Composite Designs (CCDs):

- a complete (or fraction of a) Resolution-V \( 2^k \) factorial design, where the factor levels are
coded to the usual -1, +1 values (the \textit{factorial portion} of the design),

- $n_0$ centre points ($n_0 \geq 1$), and
- two axial points on the axis of each design variable at a distance $\alpha$ from the design centre (the \textit{axial portion} of the design).

The maximum number of design points is thus $N = 2^k + 2k + n_0$. Values of $n_0$ and $\alpha$ are chosen appropriately; Khuri and Cornell (1987) state that the CCD can be made orthogonal by choosing $\alpha$ to be equal to

$$\alpha = \left[ \frac{1}{\frac{1}{2} \frac{(FN)^2 - F}{2}} \right]^{\frac{1}{2}}$$

(4.3.13)

where $F$ is the number of points in the factorial portion of the design. Montgomery (1984) mentions that a CCD is made rotatable, the preferred class of second-order response-surface designs, by choosing

$$\alpha = \frac{1}{F^4}$$

(4.3.14)

Last but not least, by equating Eqs. (4.3.13) and (4.3.14) the proper value for $n_0$ in case of a design that is both orthogonal and rotatable is given by the integer closest to:

$$n_0 = 4\sqrt{F + 4} - 2k$$

(4.3.15)

To maximise the efficiency of the CCD, Box and Draper (1971) indicated that the factorial and axial points should be positioned on the boundaries of cuboidal region bounded by the coded $X_i = \pm 1$, i.e., the extremes of the experimental region. This can be applied, for instance, when from a physical point of view, there is no larger variation possible then $X_i = \pm 1$.

If Taguchi’s orthogonal arrays can be applied for the fractional portion, then we can still obtain the same information as with a full factorial design while doing far less experiments. It is known that Stanley \textit{et al.} (1993) have indeed combined the RSM with Taguchi’s orthogonal arrays. They applied this combined method to a rocket-powered single-stage vehicle configuration selection and design. For five three-level factors, a total of 27 experiments was required, as opposed to 81 required by Taguchi’s method and $3^5$ (243) required by a full factorial design. Because of the simulation reduction this CCD became a powerful screening method to quickly come to a configuration selection before a more detailed design was made.

As an example, consider a design with three 3-level factors. A full factorial design would require $3^3$ simulations, see Fig. 4.2a. CCD requires a $2^k$ factorial design, with the factor levels coded to $\pm 1$, see also Fig. 4.2b. We choose $n_0 = 1$ centre point, and $\alpha$ is taken to be $\sqrt{3}$ for a rotatable design (to make the design both orthogonal and rotatable, there are $n_0 = 9$ centre
points). The resulting points are shown in Fig. 4.2c. Last, but not least, we can maximise the efficiency of the CCD design by placing the axial points on the boundaries of the experimental region ($\alpha = 1$), Fig. 4.2d.

![Diagram](image)

Fig. 4.2 - (A) A $3^k$ factorial design in $k = 3$ factors, (B) a $2^k$ factorial design in $k = 3$ factors, (C) a rotatable CCD design in $k = 3$ factors, and (D) a non-rotatable but efficient CCD design in $k = 3$ factors. Note that open circles indicate design points on the factor axes, and solid circles design points off the axes.

### 4.4. Analysis of results

Once the simulations have been executed, the output has to be analysed. Since we will not do a full factorial design, but use Taguchi's orthogonal arrays to do a fractional factorial design, we will only have a sample of the full experiment. Hence, the analysis of this sample has to include an estimate of the statistics and an analysis of the confidence of the results. Primarily we want to answer three questions:

- Which factors contribute to the results and how much?
- What is the optimum condition with respect to quality?
- What will be the expected result at the optimum condition?
To perform this task, ANalysis Of VAriance (ANOVA) using orthogonal arrays, as is discussed by Taguchi (1988), will be used. With respect to the optimum conditions, answers can be obtained through analysis of the least-squares surface (ANOLSQ). Since both ANOVA and ANOLSQ comprise standard techniques, the reader is referred to Appendix E for more details. In this section, a summary is given.

A statistical description of a number of observations of a response can be given by the mean response \( \bar{y} \) and its standard deviation \( \sigma \). The sum of the squared deviation from this mean is represented by the total sum of squares \( S_T \). When the factors are varied according to an orthogonal array, it is easy to compute the factor sum of squares \( S_i \), i.e., the sum of the squared deviation from the mean due to the variation of a single factor. This factor sum of squares can be used to assess the response sensitivity to a factor variation. The error sum of squares \( S_E \) is simply the difference between \( S_T \) and the sum of \( S_i \) \( (i = 1, \ldots, k) \). A degree of freedom \( f \) in a statistical sense is related with each piece of independent information that is estimated from the data. A three-level factor contributes to two degrees of freedom and a two-level factor one. The variance \( V \) of a response is the sum of squares divided by the corresponding degrees of freedom, e.g., the factor variance \( V_i \) is equal to \( S_i \) divided by \( f_i \). The percent contribution \( P \) is the percent contribution of \( S_i \) or \( S_E \) to \( S_T \).

To get an impression of how well the least-squares fit matches the responses three sums of squares can be defined, i.e., the total sum of squares, \( S_T \), the sum of squares due to regression, \( S_R \), and the sum of squares unaccounted for by the response surface, \( S_E \). The coefficient of determination \( R^2 \) is a measure of the proportion of total variation of the values \( y_i \) about the mean \( \bar{y} \) explained by the fitted model. When \( R^2 \) is adjusted by using the degrees of freedom corresponding to \( S_E \) and \( S_T \), then the adjusted \( R^2 \) statistic is obtained, a measure of the drop in the magnitude of the estimate of the error variance achieved by fitting a model other than \( y = \beta_0 + \varepsilon \) relative to the estimate of the error variance that would be obtained by fitting the model \( y = \beta_0 + \varepsilon \).

Assuming that the response surface is an adequate representation of the variation of the responses, then finding the maximum (or minimum) response will be of importance when that extremum will be found at a different factor combination than was included in the original analysis. To solve the problem of finding the maximum of a function of \( k \) independent variables that are bounded to the experimental region, a truncated Newton method (Nash, 1984) is used. Note that this minimisation method finds a local rather than a global maximum. To be certain that we have found the global maximum it is therefore required to repeat this process a number of times, each time with different starting values of the stationary point \( \mathbf{X}_0 \). Finally, a verification experiment is required to check the validity of the computed extremum.

4.5. Robust design

So far, we have discussed methods for analysing a pre-defined response, concentrating on the
sensitive factors, possible interactions, and the maximum (or minimum) value of the response in the experimental region. It has been assumed that the factors are model uncertainties and that the responses are performance parameters or design constraints. The Taguchi Method and CCD served as design methods to determine an optimal (or sub optimal) design.

Robust design is an engineering methodology for improving productivity during research and development so that high-quality products can be produced quickly and at low cost (Phadke, 1989). This methodology raises two points, namely how to reduce economically the variation of a product's function in the customer's environment, and how to ensure that decisions found to be optimum during laboratory experiments will prove to be so in manufacturing and in customer environments.

![Block diagram of a product/process](image)

Fig. 4.3 - Block diagram of a product/process (based on Phadke, 1989).

The underlying notion of robust design is that any parameter that influences a response can be classified into three classes (Fig. 4.3): i) *signal factors* (or set points), parameters that are set by the user to express the intended value for the response of the product, ii) *noise factors*, that cannot be controlled by the designer, either external (environment and load factors), manufacturing non-uniformity or wear-out and process drift, and iii) *control factors*, that can be specified freely by the designer. Basic principle of robust design is to improve the quality of a product by minimising the effect of the cause of variation without eliminating the causes (Phadke, 1989). A practical example is the following. The product is a guidance and control system that we want to perform well, not only under nominal conditions but also in the presence of vehicular and environmental uncertainties. A guidance system includes a number of design parameters, such as gains, damping factors, filters, set points, etc. With robust design we want to find the optimal combination of these design parameters so that the guidance and control system will perform well under all circumstances, i.e., taking all uncertainties into account.
The two major tools that are used in robust design are orthogonal arrays to study many parameters simultaneously, and the signal-to-noise ratio, which is a measure for quality. The signal-to-noise ratio $\eta$ is expressed in decibels (dB) and is defined as:

$$\eta = 10 \log_{10} \left( \frac{\bar{y}^2}{\sigma^2} \right)$$ (4.5.1)

where $\bar{y}$ is the mean value of the response, which can be seen as the desirable value, and $\sigma$ is the standard deviation that represents the effect of the noise factors. Maximising the signal-to-noise ratio therefore corresponds to minimising the sensitivity to noise factors.

How do we set up an appropriate matrix experiment while using orthogonal arrays? Suppose we define one particular combination of control factors (design parameters), and a range of noise factors. These noise factors can now be assigned to the columns of the proper orthogonal array, and the matrix experiment can be conducted. The outcome of this exercise is that we can compute the signal-to-noise ratio for this particular design of the system. In a second matrix experiment, we set the control factors to another combination, keep the ranges of noise factors the same, and conduct the matrix experiment resulting in another signal-to-noise ratio. We continue doing so, until we have covered a sufficient number of control-factor combinations. The question that comes up now is, of course, what is sufficient?

Fig. 4.4 - Schematic representation of robust design.

It is not difficult to imagine that the control-factor combinations will also be based on an appropriate orthogonal array. As a result, we end up with a double Taguchi Method, an outer loop that varies the control factors and an inner loop that represents the noise factors (Fig. 4.4).
After conducting all experiments the outcome consists of a number of signal-to-noise ratios, equal to the number of outer-loop combinations. Processing these ratios can be done in several ways. The easiest manner is just to select the maximum value, representing the best combination of control factors that is least sensitive to the noise factors. A bit more complex is to make a least-squares fit of the signal-to-noise ratios and to optimise this model. In that way we find that control-factor combination inside the experimental region that may give a better yield, provided that the model has an adequate fit and we have found the global maximum (opposed to a local maximum). Analysis of a least-squares model has been discussed in Section 4.4 and Appendix E.

4.6. Summary

- In the design of complex systems a large effort is consumed in conducting experiments to obtain sufficient information needed to make design decisions. Efficiency in this so-called experimental design process is a key to keeping development and manufacturing costs low, and having high-quality products. The use of Taguchi's orthogonal arrays, possibly in combination with Response Surface Methodology, is a potential method for achieving this efficiency.

- When conducting experiments according to orthogonal arrays, factors are assigned to the columns whereas the rows of the array represent the individual experiments. Only in case all factors are independent they can be assigned to arbitrary columns. Factor dependency can lead to confounding, i.e., the inability to distinguish between factor and interaction effects. Interactions can be taken into account by using interaction tables in assigning factors to the columns.

- In Response Surface Methodology, a response surface that includes linear, quadratic, and higher order terms as well as interactions can be computed with the method of least squares. Singular Value Decomposition is a robust way of computing the regression coefficients.

- Key steps in analysing data from experiments are:
  1) perform ANOVA to evaluate the relative importance of the factors and the error variance,
  2) for the response surface, determine the optimum level for each factor that maximises or minimises a selected response, and
  3) compare the results of a verification experiment with the prediction.
Robust design is an engineering methodology for making a system or process as insensitive to disturbances as possible without eliminating the source of the disturbances. The two major tools used in robust design are the signal-to-noise ratio, which measures quality, and orthogonal arrays which allow the study of many design parameters under the influence of many disturbances simultaneously by means of an inner and outer loop.
Chapter 5

A Versatile Tool for the Study of Flight Mechanics

A small number of zeros is the cause of most problems.
Jean de la Rivière

Computer simulation of systems can be done at several levels of complexity and for different purposes. For launch and re-entry systems, one can perform mass-point simulations in three degrees of freedom (3 d.o.f.) to get an impression of the performance of the vehicle at pre-defined attitudes, or to assess the performance and robustness of the guidance system. At a higher level of complexity, one can do rigid-body simulations in 6 d.o.f., thus taking the inertia of the vehicle and its rotational motion into account, to verify design assumptions for an attitude-control system. An even higher level of complexity can take elastic-body modes into account to study aeroelastic effects.

In this study we have restricted ourselves to 3- and 6-d.o.f. simulations. Because there was no other flight-simulation software, with which different types of vehicles and (ascent and re-entry) missions can be studied in a flexible way, available to us it was decided to use the Simulation Tool for Ascent and Re-entry Trajectories (START) as a basis. The development of the START began in 1991, as a tool to study the open-loop behaviour of re-entry vehicles by means of 6-d.o.f. simulations (Mooij, 1991a). With open loop it is implied that no guidance and control capabilities were included. Since then, the tool has seen a gradual development that finally has led to the current Version 3.0, with which guided and controlled re-entry and ascent missions can be simulated, as has been discussed in theory in Chapter 2 and 3. Furthermore, an extensive post-processing facility has been added to the tool to aid the user. This facility was discussed in detail in Chapter 4.
In this chapter, a brief overview of the software architecture is given (Section 5.1). Basically, the software can be divided into three main elements, i.e., pre-processing facilities (Section 5.2), simulation core (Section 5.3) and post-processing facilities. The discussion will be global of nature; the mathematical algorithms that are used by the simulation core, e.g., integration and interpolation methods, are commonly applied and will therefore not be discussed. It should be noted that all simulations in this thesis have been executed using the fifth-order variable step-size Runge-Kutta-Fehlberg method.

Note that this overview is not meant to serve as a user's manual. For that we refer to a previous publication that describes Version 2.1 of the software (Mooij, 1993b). A user's manual that fully covers Version 3.0 is planned but currently not available.

5.1. Software architecture

START is an interactive tool that can be used to simulate a variety of ascent and entry missions for arbitrarily defined vehicles. Three major blocks can be identified, as has been schematically depicted in Fig. 5.1: facilities for pre-processing the simulation-input data, the simulation core and the facilities for post-processing of the simulation output. Both pre- and post-processing facilities are embedded in a user interface, which is based on textual menus that are linked in a hierarchical structure. Each of the aspects of vehicle, environment, mission, simulation and output processing can in this way be accessed via different menus (Section 5.2). The input data can be stored in and retrieved from databases that are stored on disk, in either binary or ASCII format. A facility to store and retrieve subsets of the input data, e.g., a propulsion system, an aerodynamic configuration or a wind model has also been incorporated for flexible and quick changing of input configurations.

The input data are used by the simulation core to do the actual simulation (Section 5.3). This can either be a single run or a predefined batch of runs, the results of which can be used for a sensitivity analysis. During a simulation run, some predefined output variables can be monitored, facilitating preliminary judgement. The output data are written to two types of binary files. The first type deals with selected output variables as a function of time, e.g., the vehicle state, forces, moments, thermodynamic parameters and guidance commands (trajectory parameters). The second type is related to limit or final values of selected variables, e.g., maximum deceleration, maximum heat load and final altitude (performance parameters). In case of a sensitivity analysis, each of the trajectories can be written to a separate file of the first type, whereas the limit and final values will be combined in one file of the second type.

These output files serve as input to the post-processing facilities. First, the binary trajectory files can be converted to ASCII-column format, that can be read by most graphical plotting programs. It should be noted that START itself does not have plotting capabilities. Second, the sensitivity-analysis data can be used to do an analysis of variance or regression analysis (which will be discussed in more detail in Section 5.4). The output of the analysis is an ASCII report
file, that can serve as input to a word processor.

Remark

During the development of START an effort was made to keep the software architecture modular. This makes it relatively easy to extend START with other atmosphere models, integration methods, guidance systems, attitude controllers, etc.

![Diagram of START architecture]

Fig. 5.1 - General architecture of START.

5.2. Pre-processing facilities

The input data can be divided into four blocks, as has been schematically indicated in Fig. 5.2. The first block is related to the vehicle, which is defined by specifying its mass properties, reference geometry, aerodynamic configuration, parachute systems, externally induced forces and moments, propulsion system, control surfaces and thermodynamic properties. Specifying the mass properties of the vehicle can be done in two ways. First, a vehicle can be described
as a number of mass elements, each with its own mass, c.o.m. and inertia tensor. This way of specifying the vehicle enables a user to 'build' a vehicle on basis of fundamental geometrical shapes with readily available inertia tensors. The global inertia tensor will be computed during the simulation. Second, since the first way of defining does not allow for variable mass properties, the complete vehicle can be entered as one configuration. Variable mass properties are taken into account by defining the location of the c.o.m. and the elements of the inertia tensor as a tabulated function of the total mass. The reference geometry of the vehicle consists of key dimensions, such as the wing span and wing area, that are required to compute the aerodynamic forces and moments, Reynolds number and thermodynamic properties.

Fig. 5.2 - Overview of input data.

A major part of the vehicle data consists of the aerodynamic database. The aerodynamic forces and moments consist of the drag, side and lift forces $D$, $S$, and $L$ (expressed in the airspeed-based aerodynamic frame), and the roll, pitch and yaw moments $L$, $M$ and $N$ (defined in the body frame), respectively. Each of the force and moment coefficients are written as a Taylor series including first and second derivatives:

$$CF_{tot} = CF_0 + \frac{\partial CF}{\partial X_1}X_1 + \frac{\partial CF}{\partial X_2}X_2 + ... + \frac{\partial^2 CF}{\partial X_1 \partial X_2}X_1X_2 + ... + \frac{\partial^2 CF}{\partial X_1^2} + \frac{\partial^2 CF}{\partial X_2^2} + ... + O(h^3)$$
The variables $X_i$ are called derivation variables (e.g., $\alpha$, $\beta$, $p$, $q$, $r$). In addition, each of the coefficient components (including the derivatives) can be tabulated as a function of 0, 1 or 2 table variables (e.g., $\alpha$, $\beta$, $q_{\text{dynt}}$, $M$). By storing the data in a separate file, the aerodynamic data can be updated after each configuration change, by simply reading in an aerodynamic database file.

In addition, a 3D parachute model has been implemented. Both the parachute and the vehicle are considered to be rigid bodies, connected by a rigid bar. This rigid-bar connection prevents the two bodies from rotating with respect to each other. However, along this hypothetical bar the two bodies can roll, because a (non-ideal) swivel has been implemented in the connection between parachute and vehicle. Non-ideal here means, that the swivel causes a friction moment that counteracts the spinning motion. The air in and under the parachute is taken into account as added mass. The shape of the canopy is defined to be a hemisphere. The model may also include reefing (i.e., step-wise opening) of the parachute. Up to three parachute systems can be defined, e.g., to model pilot, drogue and main parachutes.

To create additional external moments, a simplified spin-vane model has been implemented. Each vane is considered to be a rectangular flat plate. The lift and drag contribution for a single vane is computed, based on flat-plate theory. These forces result in a roll moment that drives the vehicle. The contribution of all vanes is taken into account by multiplying the computed roll moment by the number of vanes. This implies, that the flow direction for each vane is assumed to be identical, so no transverse winds can be studied.

The definition of the propulsion system is divided into two parts. The first part deals with the main propulsion system. For each of the engines (maximum 15), $T$ and $I_{\text{sp}}$ can be entered as a tabular function of at most 3 independent variables ($M$, $q_{\text{dynt}}$, $h$ or $\phi_\tau$). Furthermore, the location of the thrust centre and the initial thrust-vector angles can be specified. The second part is related to the reaction-control thrusters, that are part of the attitude-control system. A total of 18 thrusters can be defined at most, for each of which the location, constant $T$ and $I_{\text{sp}}$, and the direction of $T$ can be specified. Note that the use of these thrusters will only affect the total mass of the vehicle; the influence on the inertia tensor is neglected.

In case of winged vehicles or lifting bodies, control surfaces that are to be used by the attitude-control system for trimming purposes or corrective control can be defined. Four sets of (1 or 2) control surfaces are implemented, designated as canard, elevon, rudder and body flap. Per set, a number of aerodynamic force and moment increments can be specified. These increments can be a tabulated function of at most three independent variables ($\alpha$, $\beta$, $M$ or the deflection angle $\delta$). The current implementation also allows for a first-order derivative, e.g., $\Delta C_{\rho}$, to be defined. The last vehicle-definition entry is related to thermodynamics. In this version of START, only the simple Chapman heat-flux equation has been implemented, for which the constants can be varied.

The second block of input data deals with the environment. To start with, a central body can be selected. This can either be the Earth, the Moon, Mars or Titan. Characteristic values, such as the equatorial radius, the ellipticity and the rotational rate can be edited. Depending on the central body is the choice of the gravity model (central field plus harmonics up to $J_4$) and that
of the atmospheric model. For the Earth, four models are available: an exponential atmosphere, the US Standard Atmosphere 1962 (US62), US76 and a simple tabulated model based on US76 (up to 90 km) and CIRA86 (above 90 km), evaluated for Kourou. The Martian atmosphere is also tabulated and based on Viking-1 data (Seiff and Kirk, 1977). For the Titan atmosphere, the tabulated minimum, nominal and maximum Lellouch-Hunten model of October 1987 are available (Lellouch and Hunten, 1987).

A major part of the environmental data consists of the wind database. Presently, the wind model may exist of a steady-state wind and horizontal wind gusts. The steady-state wind can be defined as having two components, either a zonal and a meridional component, or a modulus and direction component. The components can be entered in tabular form, being a function of two independent variables at the most (i.e., the atmospheric pressure $p$, $h$ or $\delta$). It is also possible to define a zonal component only, being a (simple) function of the planetary rotation. In addition, the so-called Flasar wind model (Lunine et al., 1991) has been implemented for Titan. This is an engineering model, defining a zonal steady-state wind as a function of altitude (atmospheric pressure) and latitude. A total of 10 horizontal wind gusts can be specified. Main parameters are the initial altitude, the thickness, the maximum velocity and the direction of each gust. Two shapes have been predefined.

The mission block enables the user to define the mission of the vehicle. To this end, data correspond with the initial state of the vehicle, the trajectory definition, mission events, and targeting data. Furthermore, an attitude controller can be selected. The initial state of the vehicle relates to position and velocity (mass-point parameters), and attitude and angular rates (finite-body parameters). For position and velocity, both spherical, cartesian and orbital parameters can be specified. To define a trajectory, it can be divided into a number of segments. Per segment, a guidance law can be selected from more than 10 pre-programmed laws and key guidance parameters can be edited. Moreover, operational constraints that apply to the trajectory as a whole (e.g., $q_{\text{dyn, max}}$) can be specified. At this point we should also mention the targeting data. Some guidance laws guide the vehicle towards a landing site, in which case the location of the landing site must be defined.

Mission events constitute parachute operations (i.e., deployment, reefing and release), configuration changes and propulsion operations. Configuration changes are in this sense restricted to discretely changing mass properties, reference geometry and aerodynamic shape. In case the vehicle is defined as a number of mass elements, a configuration change means deleting one of the elements (corresponding with, for example, the jettison of a heat shield). For more complex mass-varying systems, when the total configuration has been defined as one set of data, a configuration change just means loading another mass database. In that way, e.g., the (discrete) staging of two winged vehicles can be simulated. Propulsion operations deal with defining the on/off conditions of the engines of the main propulsion system.

For attitude control, four options are provided for, i.e., no control, ideal control, a Linear Quadratic Regulator with gain scheduling or Model Reference Adaptive Control. For the latter two, a choice between reaction, aerodynamic, thrust-vector and hybrid (a combination of two
or three modus operandi) control can be made. Related parameters of the attitude controller, e.g., the gains as a tabular function of $q_{dyn}$, can be entered or changed. For each of the attitude-control elements, the operational range (on/off conditions) can be specified.

The fourth block of the input data, simulation, contains data for executing the actual simulation, i.e., the choice of the integrator, selection of output variables, stop criteria for a simulation run, simulation parameters and the simulation/analysis method. Eight integration methods have been implemented, i.e., the fourth-order fixed-step Runge-Kutta, the fifth-, sixth- and seventh-order variable-step Runge-Kutta-Fehlberg, the variable-order variable-step Adams-Bashforth, Adams-Moulton, and the second-, third- and fourth-order, fixed-step Adams-Bashforth. For the variable step-size integration methods, the tolerances of the state variables must be defined. Output variables can be selected from a list of more than 100 entries. They will be written to the binary plot file at a user-defined filing rate. Definition of the simulation parameters has to do with the choice of attitude state variables (aerodynamic angles or quaternions) and simulation mode (3 d.o.f., trimmed 3 d.o.f. and (trimmed) 6 d.o.f.). Finally, the simulation/analysis method enables the user to do a sensitivity analysis in a predefined way. Simulation methods that are included are regular simulation and error functions (both one run), and Monte Carlo analysis, factorial design, Taguchi method and Central Composite Design (all four multiple runs). Input facilities to specify uncertainties, e.g., according to a normal or exponential distribution, have been provided for.

5.3. Simulation core

Once a simulation run or batch of runs has been defined, the simulation can be initiated from the user interface or as a batch job. A simulation run consists of three main elements, as has been schematically depicted in Fig. 5.3: event scheduling, guidance, navigation and control, and state-vector propagation. These three elements are in principle controlled by a simulation-batch controller. Whenever within a run an event occurs that results in termination of the run (e.g., a stop criterion or an interpolation error), control is returned to the batch controller that either decides to continue with the next run or to finish the complete batch. Furthermore, the batch controller controls the integration process, including step-size control. All parameters that are computed during the simulation run are in principle submitted to the output controller, that decides which ones are selected to be written to the binary plot file or the screen (simulation monitoring). Furthermore, it also adjusts the integration step size (within the given tolerances, of course) to reach history points, thereby taking the user-defined filing rate as input. In this way, an equidistant output grid is obtained.

The event scheduler controls the mission events per simulation run, see also Fig. 5.4. Four main events are continuously checked, i.e., the transition from one flight segment to another, propulsion operations (on/off criteria of the engines), configuration changes (mass properties and aerodynamic shape), and parachute operations (deployment initiation, reeling and para-
chute release). All these events can be triggered by several event criteria, i.e., \( t, h, V_G, M, q_{dyn}, \gamma_G \) and trajectory-segment number. In practice this means that whenever the predefined criterion value is passed (either from above or below), an event flag is set. If so, the event is executed, the event flag is reset and the simulation run continues. The use of the segment number as criterion, enables us to operate an engine for one or more segments (i.e., guidance laws) in succession, thus defining a turbo, ram-jet or scram-jet phase.

![Diagram](image)

Fig. 5.3 - Main simulation loop.

![Diagram](image)

Fig. 5.4 - Event scheduler.
Furthermore, the event scheduler also decides whether a stop criterion for the run has been reached. If so, control of the simulation is returned to the batch controller that can either initiate the next run, or return control to the user interface when the last run has been executed.

The next element in the main simulation loop is the guidance, navigation and control system. The guidance system generates the steering commands that drive the attitude and propulsion controller, thereby taking measurements and estimates from the navigation system as input. For this reason, the navigation module is the first element in the loop, as has been indicated in Fig. 5.5. Parameters that are required by the guidance system and attitude controller are in principle related to the state of the vehicle (position, velocity, attitude and angular rates), forces and moments, the environment and some derived parameters, such as the total energy of the vehicle or the vertical component of the lift force). The navigation system takes the actual values as input, and in order to simulate sensor noise or offsets, the actual values can be adjusted so that the guidance system and attitude controller can only use approximations of the real values.

![Fig. 5.5 - Guidance, Navigation and Control subsystem.](image-url)
The guidance system must ensure that the vehicle follows a trajectory that is dictated by predefined guidance laws, thereby taking operational constraints into account. Basically, this results in a commanded attitude and main engine setting. The output of the guidance system serves as input to two controllers, i.e., the propulsion and attitude controller. However, the dynamics of a propulsion controller are not modelled in this study, so variations in $\delta_T$, $\epsilon_T$ and $\psi_T$ take place instantaneously or at a predefined constant rate. The commanded attitude must be realised by the attitude controller by deflecting control surfaces or operating the reaction control thrusters. Furthermore, the realised attitude should be stable so a trim algorithm will compute additional commands. Deflection angles and reaction-thruster forces are output of the
attitude controller and become available to the main simulation loop, and in particular to the last element: state-vector propagation.

The element state-vector propagation, schematically depicted in Fig. 5.6, is a numerical solution to simulate a physical process on a computer. For a particular point in time, the state derivatives are computed and according to a predefined scheme, a new state vector is computed. Key element in the state-vector propagation is the computation of external forces and moments, which can be of gravitational, aerodynamic and propulsive origin. These are a function of the actual state of the vehicle as well as the environment. The computation of the aerodynamic forces and moments may include contributions of control surfaces.

In this thesis, we consider three types of guidance and control (or lack of it), i.e., none, ideal or realistic. In case there is no control, the vehicle is left to itself. Whatever attitude it has, defines the current aerodynamic forces and moments. In general, there are no propulsion forces in this mode, which can be applied in both 3 and 6 d.o.f. In case of ideal control (3 d.o.f. only), the actual state of the vehicle is equated to the corresponding commands generated by the guidance system. Forces and moments are then computed based on this new actual state. In a realistic attitude-control system additional forces and moments due to the actual settings of control surfaces and reaction-control thrusters are taken into account. Due to the vehicle inertia the new attitude is not reached immediately but needs a finite amount of time, resulting in deviations that have to be compensated for by the G&C system in the next time step.

5.4. Post-processing facilities

Post-processing of the results can in principle be divided into four parts, as can be seen in Fig. 5.7. Each of the elements can be accessed through the user interface. Output files resulting from different simulation sessions can serve as input, so the user is not restricted to the latest executed simulation or batch of simulations. In principle, three of the four elements only apply to a combination of multiple runs (as is the case with a sensitivity analysis). The fourth element, i.e., to convert a binary plot file (with trajectory data) to a file in ASCII-column format applies to the output of a single run. The resulting ASCII file can be used as input to practically any plotting or spreadsheet program, to visualise the results.

5.5. Summary

- The Simulation Tool for Ascent and Re-entry Trajectories (START) is an interactive tool that can be used to simulate a variety of ascent and entry missions for arbitrarily defined vehicles. A vehicle can have varying mass properties due to the use of a propulsion system. Any simulation is restricted to either 3 or 6 d.o.f. A choice can be made from a number of guidance and control algorithms.
A Versatile Tool for the Study of Flight Mechanics  Chapter 5

Fig. 5.7 - Post-processing of results.

- All input data to the simulation can be edited by means of a user interface. The mass properties, aerodynamic coefficients of both the vehicle and control surfaces, propulsion system and wind can be represented by extensive data bases.

- It is possible to do a single simulation run or a predefined batch runs, the results of which can be used for a sensitivity analysis. Simulation methods that are included are regular simulation and error functions (both one run), and Monte-Carlo analysis, factorial design, Taguchi method and Central Composite Design (all four multiple runs). Input facilities to specify uncertainties, e.g., according to a normal or exponential distribution, have been provided for.

- To integrate the differential equations of motion, the fifth-order, variable-step Runge-Kutta-Fehlberg integration method has been selected, because of the combination of robustness, accuracy and computational speed. Data extraction from tabular inputs are done by linear interpolation, or in case of the atmosphere models, cubic-spline interpolation.

- Post-processing of the results include the creation of ASCII plot files for single runs, and ANOVA and response-surface modelling for multiple runs.
Chapter 6

Verification, Validation and Evaluation

It was very successful, but it fell on the wrong planet. 
Wernher von Braun (Referring to the first V2 rocket to hit London during World War II)

Once a software package has been developed, it has to be proven that the algorithms have been implemented correctly and that the simulated output is a good approximation of reality. When each of the algorithms and modules of the software are tested separately, and the input/output relation of some simple test cases are compared with precomputed expected values, these parts of the software can be verified to be correctly implemented for these type of problems, and enhance the confidence when more complicated problems are studied. Of course, this does not shed light on how the output compares with real-life problems.

To that end, the best way to test a flight-simulation package and the corresponding input models would be to simulate the mission of an existing space vehicle, and to compare the output with actual flight data. In that way, the software package and input models can be validated. To further increase the confidence in the software, a number of different missions and vehicles can be simulated. If evaluation of the results does not leave major white spots, it may be concluded that the software can also be applied to advanced vehicles and missions that are not commonly published in literature.

In principle, the software that has been developed in this thesis is to study the flight mechanics of space planes. However, before the developed software is applied to the complex case of the unpowered entry and descent of a winged vehicle (Chapter 7) and the powered ascent of an SSTO space plane (Chapter 8), a number of simpler vehicles and missions is studied with this software. This is done to evaluate flight-mechanics related issues encountered
in some typical missions of space vehicles, and, whenever possible, to verify and validate the computer code by comparing our results with results obtained by others. Note that a validation of the applied input databases, which is, of course, an essential step in simulating real-life problems, is not pursued here. The goal of evaluating the test cases is to try and find an explanation for the output as it is, such that it increases the confidence in applying the input models and the software to study these kind of vehicles and missions.

To this extent, the chapter has been divided into three sections. Section 6.1 begins with the verification and validation of (part of) the software. In Sections 6.2 and 6.3, a summary of two mission analyses is given: the entry and parachute descent of a scientific probe in the atmosphere of Saturn’s moon Titan, and the aerodynamic controllability of a blunted-cone, moderate lift-to-drag re-entry vehicle. A third mission analysis, i.e., the influence of the deorbit-burn manoeuvre on the footprint of a low lift-to-drag rescue vehicle is not presented here, but is published elsewhere (Mooij, 1993). Section 6.4, finally, gives a summary of the findings.

6.1. Verification and validation

All of the numerical algorithms implemented in START have been thoroughly tested on some fundamental problems, for instance: inverting a matrix twice should give the original matrix, multiplication of the three decomposed matrices obtained from SVD should give the original matrix, adjoining points from the linear-interpolation algorithm should lie on a straight line, and so on. The integration methods were applied to some differential equations of which the analytic solution was known, and showed to give correct output because of identical curves. However, it is beyond the scope of this thesis to give a full report on this verification. It suffices to conclude that all of the algorithms are properly implemented.

Because no flight data were available to validate START, an alternative approach is used, namely to compare the output with that of other flight-simulation packages as published in literature, of course while using the same input database. If the output matches, it does not mean that the software is validated but that we can have confidence in the results since independent research led to the same results. This other tool, called Re-entry and Atmospheric Transfer Trajectories flight-dynamics software or RATT (Leon, 1989), is used at ESTEC to do mission analyses and has already been (partially) validated by the industrial consortium, which developed it (GMV, Dornier and Matra Espace). This validation was based on some flight phases of the Apollo capsule returning from the Moon, where part of the aerodynamic database was based on wind-tunnel measurements.

As validation case for part of the software, the same Apollo mission was simulated. Once entered in the Earth atmosphere, the vehicle is left to itself (free-fall re-entry), so no control is exerted. This test case was considered to be representative for validation, because of its large speed and altitude range. Details on the validation can be found in Mooij (1991a). The results for translational motion (i.e., position, velocity and external forces versus time) showed a close
similarity between the two programs, especially when the nature of the physical problem itself is taken into account, e.g., sensitivity for (small) differences in initial conditions. This sensitivity translates itself into a shift of the integral curves, which was indeed observed. An additional verification of the translational motion was done by simulating an exoatmospherical Keplerian orbit. Both circular and elliptical orbits appeared to be repetitive, with groundtracks matching with the theory. Also the total energy remained constant throughout the orbit, because of the absence of aerodynamic drag.

The results for rotational motion (i.e., angular rate, attitude and external moments) showed some differences in the oscillation pattern between START and RATT. For each related state variable a phase shift was observed, in some cases resulting in a different oscillation pattern. The difference may partly be explained by the numerical sensitivity for the tolerances on the state variables, which were not the same for the two programs, because RATT uses cartesian position and velocity components and quaternions for the attitude. Furthermore, $\alpha$ and $\beta$ are computed from velocity components, whereas $\sigma$ is computed in an indirect manner from the orientation of the aerodynamic force. Differences in the attitude angles will have their impact on the rotational motion. In any way, integrating oscillatory motions appeared to be sensitive to small phase shifts. Regarding the evolution of the average value of the variables, however, the results were similar. A further verification of the equations of rotational motion consisted of the implementation of kinematic equations based on quaternions (see also Mooij, 1994b) instead of $\alpha$, $\beta$ and $\sigma$. For both cases the results were identical, which increases the confidence of a proper implementation.

It should be mentioned once more, that to have flight data of a free-fall re-entry would greatly improve the possibility to validate the program and input models with such a test case, as mentioned above. Also the RATT software cannot have been validated for the oscillations we encountered during the validation process, since part of the Apollo flight was controlled to prevent these kinds of oscillations. Taking the uncertainties mentioned above into account, and looking at the results for this (numerically difficult) test case, it can be concluded that START may be used for re-entry simulations.

### 6.2. Entry and parachute descent of a scientific probe

The mission analysis of the scientific probe Huygens as discussed below, has embedded three aspects related to flight mechanics in general and the implemented models in particular, i.e.,

- uncontrolled, spinning entry in 3- and 6-d.o.f.,
- offset in the location of the c.o.m., and
- steady-state wind and horizontal wind gusts.
Apart from evaluating these aspects, some of the obtained results will also be compared with similar results obtained by industry as a part of the validation of START. The mission analysis of Huygens has been extensively discussed in two reports (Mooij, 1991b and 1992). Here, some of the findings of this study are summarised, separating the results of the entry phase from those of the parachute descent phase.

6.2.1. Mission overview

Cassini/Huygens is a planetary-exploration mission to Saturn and Titan (ESA/NASA, 1988). It consists of a Saturn orbiter (Cassini), to be provided by NASA and a Titan-atmosphere probe, called Huygens, which is the responsibility of ESA. The Cassini/Huygens mission is designed to explore the Saturnian system and all its elements, namely the planet Saturn and its atmosphere, its rings, its magnetosphere, and a large number of its moons, namely planet-sized Titan and the icy satellites. At October 15, 1997, Cassini/Huygens was successfully launched, to arrive in the Saturnian system in 2004 and then to be inserted into an initial highly eccentric Saturn-centred orbit with a period of 100 days. During this first orbit, the Huygens probe mission starts. It is targeted for Titan atmospheric entry, spun up to 10 RPM and separated from the orbiter. The orbiter itself will perform a deflection manoeuvre to establish and maintain the probe relay link during the descent of Huygens through the Titan atmosphere.

Fig. 6.1 - Artist impression of the Huygens probe entering the atmosphere of Titan (courtesy of ESA).
The entry phase of the Huygens probe (Fig. 6.1 and Table 6.1) begins at the reference altitude of 1250 km, where the flight-path angle will be within a permissible range of -90° to -60°, resulting in a steep entry that keeps the probe within line of sight with the orbiter for the duration of the probe mission. The entry velocity relative to the surface will have a value between 5.72 and 6.47 km/s. Because of low atmospheric density, drag is low at these high altitudes. Gravitational forces will accelerate Huygens until drag is high enough to initiate deceleration at an altitude of 600 km (note that the Titan atmosphere is much denser than the one of Earth, which explains this high altitude). During the entry, the descent module which contains the experiments, is protected by a heat shield/aerodynamic decelerator and aft cover.

| Mass descent module          | 177.6 kg |
| Mass aft cover               | 15.3 kg  |
| Mass aerodynamic decelerator | 93.8 kg  |
| Entry mass                   | 286.7 kg |
| Diameter aerodynamic decelerator | 3.0 m   |
| Reference area               | 7.069 m² |
| Diameter descent module      | 1.393 m  |
| Reference area               | 1.524 m² |
| x-location c.o.m. of entry configuration | -0.405 m (from nose) |
| x-location c.o.m. of descent module | -0.394 m (from nose) |
| Main parachute: mass (incl. lines) | 4.935 kg |
| Diameter                     | 9.91 m   |
| Stabilising drogue: mass (incl. lines) | 0.314 kg |
| Diameter                     | 2.09 m   |
| Minimum stabilising drogue: mass (incl. lines) | 0.164 kg |
| Diameter                     | 1.36 m   |

Table 6.1 - Main characteristics of Huygens.

When an axial deceleration level of 8.25 m/s² is encountered, the Descent Control Sub-System (DCSS) is initiated. This event marks the end of the entry phase (for γ₀ = -60° and V₀ = 5.8 km/s, the duration is about 4½ minute). By then, the altitude will be about 180 km. First event since DCSS initiation will be the deployment of the pilot chute in the supersonic regime (at a Mach number of M ≈ 1.5) and release of the aft cover as soon as the pilot chute has been fully inflated. The aft cover pulls out the main parachute that decelerates the probe through Mach 1. When the probe has stabilised in the subsonic regime (30 seconds after pilot-chute deployment), the aerodynamic decelerator is jettisoned. After a further time, the main parachute is released, which in its turn deploys a stabilising drogue chute.

For stabilising-drogue deployment, there are two design criteria. The first one assumes the release of the main parachute after 10 minutes at an altitude of about 142 km and subsequent drogue-chute deployment, and for the second one, the main chute is retained for 2550 s until
a lower altitude (about 90 km), where a minimum-sized stabilising drogue, required to maintain probe stability, is deployed. For each of the scenarios, the descent phase may not last too long (orbiter data-relay link constraint). The maximum descent duration, starting from parachute deployment, will be 3 hours. Ground contact occurs with a velocity of about 7 m/s. Three minutes later, the mission of Huygens has officially ended.

6.2.2. Entry phase

For the entry phase, an analysis was conducted to determine the sensitivity of Huygens’ trajectory, i.e., flight time, altitude of parachute deployment, maximum deceleration and maximum dynamic pressure, to some key parameters. It appeared that for steeper entries, \( g_{\text{load}} \) and \( q_{\text{dyn, max}} \) are higher, and the deployment altitude lower. The same behaviour was observed when \( C_D \) was decreased, \( m \) increased or \( \rho \) decreased. At \( M > 1.5 \), the probe appeared to be stable with respect to rotational motion: after a deviation from its initial equilibrium attitude, the probe will return to this attitude. It appeared that for a non-spinning probe the oscillations in rotational motion were less severe than in case of a spinning probe. This is mainly due to the coupling effect of \( \rho \) with \( q \) and \( r \). It should be noted, however, that close to \( M = 1.2 \), the probe becomes very unstable which makes parachute deployment at a higher Mach number an important design driver.

Moreover, two worst-case missions were defined by ESA to cope with uncertainties in the knowledge of the atmosphere, entry parameters and some design values, to identify the initial flight domain of the DCSS, i.e., the minimum dimensioning case (shortest descent time) with

- minimum atmosphere (see Lellouch and Hunten, 1987)
- maximum \( \gamma_E \), i.e. \( \gamma_E = -66.1^\circ \) (nominal \( \gamma_E = -64.0^\circ \))
- minimum \( C_D \ (\Delta C_D = -10\%) \) and maximum \( m \ (\Delta m = +5\%) \)

and the maximum dimensioning case (longest descent time) with

- maximum atmosphere
- minimum \( \gamma_E \), i.e. \( \gamma_E = -61.6^\circ \)
- maximum \( C_D \ (\Delta C_D = +10\%) \) and minimum \( m \ (\Delta m = -5\%) \)

Martin-Baker (1992), the contractor for the DCSS, did similar simulations, so we used their results for comparison. Starting with the nominal dimensioning case, Table 6.2 shows a close similarity of the results. The small differences are due to a simplified model of Mach-number computation used by industry. However, comparing the minimum and maximum dimensioning cases, shows larger differences which cannot readily be explained by a difference in Mach-number computation. It appeared that the minimum and maximum dimensioning cases were not
equally defined. Martin-Baker used the atmosphere-definition of Aerospatiale (1991), where the \( \rho_{\text{min}} \) was combined with \( T_{\text{max}} \) to form the minimum atmosphere, and \( \rho_{\text{max}} \) with \( T_{\text{min}} \) to define the maximum atmosphere. Here, \( \rho_{\text{min}} \) and \( T_{\text{min}} \) are used for the minimum atmosphere, and \( \rho_{\text{max}} \) and \( T_{\text{max}} \) for the maximum atmosphere (which is the original definition of Lellouch-Hunten, 1987). The results of Table 6.2 indicate that mixing minimum and maximum profiles will lead to large differences.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum dimensioning case</th>
<th>Nominal dimensioning case</th>
<th>Maximum dimensioning case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Martin-Baker</td>
<td>Mooij</td>
<td>Martin-Baker</td>
</tr>
<tr>
<td>( t ) (s)</td>
<td>not available</td>
<td>256.7</td>
<td>not available</td>
</tr>
<tr>
<td>( h ) (km)</td>
<td>155.0</td>
<td>154.0</td>
<td>176.1</td>
</tr>
<tr>
<td>( V ) (m/s)</td>
<td>378.5</td>
<td>358.5</td>
<td>401.5</td>
</tr>
<tr>
<td>( M ) (-)</td>
<td>1.34</td>
<td>1.43</td>
<td>1.51</td>
</tr>
<tr>
<td>( q ) (N/m²)</td>
<td>276.5</td>
<td>256.2</td>
<td>226.4</td>
</tr>
<tr>
<td>( \gamma ) (°)</td>
<td>-59.2</td>
<td>-59.2</td>
<td>-56.6</td>
</tr>
</tbody>
</table>

Table 6.2 - Comparison with Martin-Baker results at parachute deployment.

The last topic that will be discussed for the entry phase is the effect of an offset in the location of the c.o.m. of Huygens. The design specifications give the location of the c.o.m. with some tolerance values. It is nominally located on the \( X \)-axis (the spin axis) and it would be of interest to see the effect of especially an offset in \( Y \)- and \( Z \)-direction on the attitude motion. In that case, Huygens is not an axisymmetrical body any more, which will introduce off-diagonal terms in the inertia tensor (products of inertia). Simulations showed that an offset in \( X \)-direction only, does not have a large influence on the rotational motion. Only a few minor differences in the oscillatory motion were observed. However, the attitude motion changes radically if a \( Y \)- or a \( Z \)-offset is introduced (a positive shift of 10 mm in each direction was simulated). In these cases, the aerodynamic force components give rise to moments that can only be counteracted if the angle of attack and sideslip have non-zero equilibrium values and thus introduce compensating moments. Also the equilibrium values for \( q \) and \( r \) will differ from zero. Note that the above description only applies to the flight phase where atmospheric density is sufficiently high to introduce aerodynamic forces and moments.

When \( \rho \) is that low, that \( q_{\text{dyn}} \) and therefore the aerodynamic forces and moments are virtually zero, resulting in so-called moment-free motion, the following reasoning holds. Due to an offset of the c.o.m., the principal axes of inertia will be rotated with respect to the \( B \)-frame. In
the nominal case (no offset), the principal $X$-axis (the spin axis) was aligned with the $X_B$-axis. Due to the rotated orientation of the principal axes, and therefore also the actual spin axis, there will be non-zero $q$- and $r$-components, also when there are no moments. For moment-free motion, the angular momentum and rotational rate, $B$ and $\omega$, are aligned with the principal $X$-axis. Decomposing $\omega$ into components along the body axes, gives a non-zero $p$, $q$ and $r$. For a $z$-offset only, the resulting so-called transverse rotational rate $\omega_t$, defined as

$$\omega_t^2 = \omega_y^2 + \omega_z^2$$

will describe a circular motion in the $Y_B-Z_B$-plane (Hughes, 1986). Because the $X$-axes of the body frame and the principal frame are not collinear, $\omega$ will describe a (constant) conical motion about the $X_B$-axis. We can observe this, for instance, in Fig. 6.2 showing the time history of the pitch rate, which resembles the yaw-rate history for similar offsets in both $y$- and $z$-direction. Note that the first 90 s of the trajectory are dominated by the moment-free motion.

![Pitch rate versus flight time](image-url)

**Fig. 6.2** - The pitch rate versus flight time for a combined c.o.m. offset (dashed line), 10 mm in all three directions, compared with the no-offset case (solid line).

When $q_{dyn}$ increases, aerodynamic forces and moments are introduced, resulting in a change of $B$, both in size and orientation. In case of no offsets, $\omega$ starts moving towards $B$, in the meantime performing a conical motion about $B$ (Hughes, 1986). This rotation is called nutation. The damping moments drive $B$ towards the spin-axis again, with $\omega$ following, resulting in decreasing values for $q$ and $r$. In case of offsets, on the other hand, there is a combined conical motion of $\omega$. When $B$ and $\omega$ are aligned again, $q$ and $r$ are stabilised at the equilibrium values, as mentioned before. The angle of attack and angle of sideslip will be stabilised at a value differing from 0°, as can readily be seen for $\beta$ in Fig. 6.3. The oscillation in the first flight
phase (no aerodynamic forces and moments) are again due to the gyroscopical effects, as we saw with \( \omega \). Due to continuously changing aerodynamic properties - the aerodynamic coefficients are dependent on \( M \) - the stabilised values are not constant, but keep on decreasing.

![Graph](image)

**Fig. 6.3** - The angle of sideslip versus flight time for a combined c.o.m. offset (dashed line), 10 mm in all three directions, compared with the no-offset case (solid line).

### 6.2.3. Descent phase

For the descent phase a number of study objectives was defined (Mooij, 1992):

i) to determine the flight conditions at heat-shield separation to see whether interference with the descent module is possible,

ii) to establish the total descent time (which may not exceed 3 hours),

iii) to obtain the spin-rate profile for the two stabilising-drogue scenarios: at higher altitudes (i.e., above 10 km) it should be between 1 and 15 RPM, whereas below 10 km, it should be between 2 and 4 RPM, and

iv) to get insight in the influence of a steady-state wind and horizontal wind gusts on the drift over the surface and the stability of the parachute-payload system.

Of these four, we will not discuss the first objective. The simulations have been executed applying a (simple) 6 d.o.f. parachute model, whereas the descent module was equipped with so-called spin vanes. A detailed description of the related models can be found in Mooij (1992); a brief description is given in Section 5.2.

In Table 6.3, the descent times are compared with results obtained by Martin-Baker (1992). The general trend is that the Martin-Baker descent times are smaller than our results. At least
three sources which introduced the difference can be identified. First, the deployment of the stabilizing drogue took place 600 s (2550 s) after heat-shield jettisoning, instead after initiation of the DCSS. As a result, the altitude at which the main chute was released was higher in our case, and the corresponding velocity lower. Both parameters increase the total descent time. Second, Martin-Baker simulated the wake behind the descent module, which was neglected in this study. This means that the performance of their parachute was worse than the one used here, and this reduced drag area resulted in a higher velocity and thus shorter descent time. Third, alternate definitions of the minimum and maximum atmosphere also contribute to the differences. Unfortunately, it was not possible to check the influence of the mentioned aspects. Note that the descent time remains well below the 3-hour maximum.

<table>
<thead>
<tr>
<th>Drogue-deployment scenario</th>
<th>Minimum atmosphere</th>
<th>Nominal atmosphere</th>
<th>Maximum atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Martin-Baker</td>
<td>Mooij</td>
<td>Martin-Baker</td>
<td>Mooij</td>
</tr>
<tr>
<td>$t = 600$ s</td>
<td>7014 s</td>
<td>7932 s</td>
<td>8144 s</td>
</tr>
<tr>
<td>$t = 2550$ s</td>
<td>7285 s</td>
<td>8080 s</td>
<td>8982 s</td>
</tr>
</tbody>
</table>

Table 6.3 - Comparison of descent times found by Martin-Baker.

![Graph](image)

Fig. 6.4 - The spin-rate profile for the minimum (dashed), nominal (solid) and maximum (dash-dot) atmosphere. The stabilising drogue deploys after $t = 2550$ s.

Fig. 6.4 shows the spin-rate profiles for the 2550-s scenarios of stabilising drogue deployment and the three atmosphere models, with the nominal spin-vane configuration of 24 vanes. Starting at an initial value of 5 RPM, $\rho$ first decreases. This is due to the friction moment...
of the swivel. Because the heat shield has not yet been jettisoned, there is no spin-inducing moment due to the spin vanes. Once the heat shield has been separated, $p$ increases rapidly. It should be noted, that for the descent module the aerodynamic roll damping is not zero, although the effect on the spin rate is small. It can be seen, that the peak values are too high for each of the curves. The conclusion is, that the nominal spin-vane configuration will not meet the mission requirements. The end values of $p$, on the other hand, do meet the specifications.

Compared with the 600-s scenario, the 2550-s scenario results in a lower $p$: because of the longer descent under the main parachute, $q_{\text{dyn}}$ will be lower, hereby decreasing the driving moment. During the main-chute descent the driving moment is not even sufficient to increase $p$. Once the stabilizing drogue has been deployed, $V$ increases and thus also $q_{\text{dyn}}$, which results in a higher spin-vane-induced moment. However, compared with the 600-s scenario, $q_{\text{dyn}}$ will not reach the same value, so $p_{\text{max}}$ is smaller. Since $q_{\text{dyn, max}}$ occurs at a lower altitude, also $p_{\text{max}}$ rate is found at a lower altitude. The end values of $p$ are higher for the 2550-s scenario, because the descent time left from peak value down to impact is smaller, so the integrated effect of the damping is smaller.

The wind climate on Titan is severe, as became obvious from, amongst others, Voyager data. Lunine et al. (1991) developed an engineering model for a steady-state zonal wind that became known as the Flasar model. This model results in $V_{\text{wind}} = 180 \text{ m/s}$ for $h = 180 \text{ km}$, and a wind velocity at the surface of $2 \text{ m/s}$. It has been found that this steady state wind does not have a noticeable effect on the descent time. However, for an equatorial entry large drift values as compared to the no-wind case were found:

- $\Delta \tau = 9.97^\circ$ (or $\Delta R = 448.1 \text{ km}$) for the 600-s scenario
- $\Delta \tau = 12.92^\circ$ (or $\Delta R = 580.7 \text{ km}$) for the 2550-s scenario

Fig. 6.5 - The 6t horizontal wind-gust model.
Note that when the Cassini orbiter is moving in the direction of the drift, the duration of data contact will increase.

According to the system requirements, the parachute system should be stable enough to withstand any possible (horizontal) wind gusts in the atmosphere of Titan. The maximum pendulum oscillation should therefore be less than $\pm 10^\circ$. This criterion may be exceeded for periods up to 10 s (no data could be found on the maximum allowable peak values). The maximum pitch rate must be below 6 $^\circ$/s (no data were available for the yaw rate). To study the stability of the parachute system, the influence of three high-level gusts at $h_{hg} = 160, 140$ and 120 km, and four low-level gusts at $h_{hg} = 90, 60, 30$ and 2 km for each of the two descent scenarios was simulated. The gusts have the so-called 6t-shape (Fig. 6.5), with a maximum velocity of 25 m/s (6 m/s) for the high(low)-level gust and a thickness $t_{hg}$ of 250 m (60 m).

Fig. 6.6 - The angle-of-attack profile for a low-level gust at $h = 2$ km. The 600-s deployment scenario for the stabilising drogue was used.

While studying the pitch-rate profile it became obvious that there was an overshoot of the maximum value of 6 $^\circ$/s, but for each of the occurrences the duration of the overshoot was less than 5 s. The conclusion drawn from the results was that the parachute system is sufficiently stable. The major oscillations occur when the gust is active; once the wind velocity is back to zero again, the oscillations are damped in a very short time (as an example, in Fig. 6.6 the angle-of-attack profile for a low-level gust at $h = 2$ km has been depicted). The peak values of the total angle of attack have been compared with similar results obtained by Martin-Baker. However, only results due to a gust of 6 m/s were available, so the comparison for the high-level gust could only be qualitative. For $h = 160$ km, Martin-Baker found a peak value of 2$^\circ$ ($V_w = 6$ m/s). Our value of just over 8$^\circ$ ($V_w = 25$ m/s) seems to be consistent with this. Martin-Baker found $\Delta \alpha_{tot} = 2.5^\circ$ for $h = 100$ km ($V_w = 6$ m/s), whereas we obtained $\Delta \alpha_{tot} = 3^\circ$ for $h = 90$ km ($V_w = 6$ m/s), again a good similarity. A difference was found for $h = 2$ km: 22$^\circ$ (Martin-Baker)
versus 16°. However, the 6-d.o.f. parachute model showed some limitations, which should first be removed before this difference can be explained.

6.3. Aerodynamic controllability of a re-entry test vehicle

In the next step of the evaluation process, the controllability of a moderate lift-to-drag re-entry test vehicle is studied. The complete study, as published by Mooij et al. (1995), focuses on a simple vehicle design process, the development of a G&C system, and flight simulation of the vehicle in 3- and 6-d.o.f. With respect to the use of START, four new aspects are evaluated, i.e.,

- trimmed entry and descent in 3- and 6-d.o.f.,
- open-loop guidance,
- aerodynamic attitude control,
- limited exploration of Taguchi’s orthogonal arrays.

Consideration of each of these aspects is included in the global discussion.

6.3.1. Introduction

A major problem in hypersonic aerodynamics concerns the limited capabilities of ground-based test facilities to simulate hypersonic flow. The technology to design and develop a wind tunnel that can fully reproduce hypersonic flow as encountered during an atmospheric re-entry, is not available now or in the near future. Furthermore, the hypersonic aerothermodynamic database gained in flight tests is rather limited and not always available in the open literature. Thus the aerodynamic design of a hypersonic vehicle heavily depends on Computational Fluid Dynamics (CFD) with computer codes that are not sufficiently validated by ground based facilities. As a result the design of hypersonic vehicles must apply a new methodology that is based on a clever combination of CFD, wind-tunnel tests and flight tests (Kraft and Chapman, 1993).

Apart from testing the aerothermodynamic design methodology and expanding the hypersonic database, flight tests can be used for the qualification of Thermal Protection Systems (TPS) and TPS materials, and for gaining experience in re-entry guidance and control. For these goals, (semi-)ballistic re-entry vehicles have already been designed, like Japan’s Orbital Re-entry EXperiment, OREX (Asada et al., 1994), and ESA’s Atmospheric Re-entry Demonstrator, ARD (Cazaux et al., 1995). A disadvantage of such a test vehicle, however, is the limited manoeuvrability, which makes it difficult to predict the landing site, and the small flexibility to cope with flight-path constraints like the heat flux or total load. These difficulties can only be overcome by actively controlling the aerodynamic forces acting on the re-entry vehicle.
The feasibility study presented by Mooij et al. (1995) concerns the aerodynamic controllability of such a small re-entry test vehicle, designated Hyperion. The vehicle has a triangular cross section with rounded corners, a spherical nose and three control flaps mounted at the base (Fig. 6.7). The three flaps can be independently deflected to allow for roll, pitch and yaw control, and are treated as flat plates. Hyperion will fly in a bank-to-turn mode where the bottom flap is used for trim-angle control and both upper flaps are used for yaw and roll control. The geometry of the vehicle, defined by the semi-cone angle $\theta$, the base radius $R_B$ and the nose radius $R_N$, has been determined such that it is capable of re-entering the atmosphere without violating a heat-flux constraint ($\dot{Q}_{\text{max}} \leq 1600 \text{ kW/m}^2$) and with the highest possible $L/D$-ratio (= 0.9) to maximise the cross- and down-range capability. The resulting geometry is depicted in Fig. 6.7. The mass of the vehicle has been estimated at 547 kg (which results in a ballistic parameter of 1818 kg/m$^2$). The aerodynamic properties of the vehicle have been analysed with an analytical model, developed by Wells et al. (1962). Basically, it means that the vehicle is a blunted cone in Newtonian flow. The dynamical damping is assumed to be zero.

![Fig. 6.7 - Artist impression of Hyperion, including final geometry.](image-url)
As a reference trajectory for the design of the G&C system a minimum heat-load trajectory is assumed, because the weight of a TPS tends to decrease with decreasing heat load. This means a flight along the path constraints $\dot{Q}_{\text{max}}$ and $g$-$\text{load}_{\text{max}}$, because along these path constraints the drag, and therefore the deceleration, is maximised resulting in the shortest entry time (Havey, 1982). To minimise the maximum occurring $\dot{Q}$ the vehicle should fly with maximum lift (Loh, 1969), which means a high angle-of-attack entry, similarly to the entry of the Space Shuttle. So, at entry ($h = 120$ km, $V = 7,782.5$ m/s, $\gamma = -2.87^\circ$) $\alpha$ is selected to be $45^\circ$. When the highest $\dot{Q}$ will be reached, a flight at maximum $L/D$ will be started ($\alpha = 20^\circ$) to maximise the cross- and down-range capability (Vinh, 1981). The bank angle is modulated, first to fly along $\dot{Q}_{\text{max}}$, and later on to fly along the $g$-$\text{load}$ constraint. The $\alpha$-control to follow the path constraints has been derived in an analytical form, based on simplified equations of motion and with the use of an exponential atmospheric model. The solution is based on the fact that for both path constraints the term $\rho V^n$ is constant$^{11}$. It should be noted that the guidance is basically open loop. This implies that deviations from the nominal trajectory are not compensated for. Furthermore, violation of the path constraints will only be prevented to the extent of the (analytical) modelling and related assumptions.

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$^{11}$ Chapman (1959) derived an expression for the heat flux that shows a dependency of $\dot{Q}$ with the square root of $\rho$ and the third power of $V$, so $\dot{Q}^2 \sim \rho V^3$ hence $n = 6$. The $g$-$\text{load}$ is expressed as the aerodynamic force divided by the vehicle mass and normalised to the gravitational acceleration at sea level. Since the aerodynamic characteristics of Hyperion are almost constant down to $M = 5$, the $g$-$\text{load}$ is only depending on $q_{\text{dyn}}$ hence $n = 2$. Note that along the $g$-$\text{load}$ constraint $q_{\text{dyn}}$ is constant for that reason.

---

Fig. 6.8 - The reference trajectory in relation to the path constraints.
Flying the nominal trajectory resulted in $Q_{\text{max}} = 2000$ kW/m$^2$ instead of the design value of 1600 kW/m$^2$. One of the main reasons for this large overshoot is that for the vehicle design, an equilibrium glide was assumed since this enabled an analytical optimisation formulation. However, the actual reference trajectory is somewhat steeper resulting in a higher $Q$. However, since this study is considered to be the first (iteration) step in the design process, with still sufficient margin to adjust the design, no further attention is paid to this aspect. Therefore, the high value of $Q_{\text{max}}$ is taken as path constraint for now. In Fig. 6.8, the resulting reference trajectory has been plotted (note that the final Mach number is $M = 3$).

For attitude control three independent aerodynamic control surfaces are available. Since all three flaps generate pitch moments, in principal all three can be used for pitch stability. Flaps #2 and #3 (see Fig. 6.7), however, have to be used for roll/yaw control and should therefore not be used for trim, because the remaining control effectiveness might not be large enough. Moreover, flap #1 can generate larger pitch moments which results in smaller deflection angles. The trimmed deflection of flap #1 is computed taking the commanded angle of attack $\alpha_c$ as input and employing an analytical model to compute the pitch moment for both the vehicle and the flap. A Newton-Rhaphson iteration procedure is used to compute the deflection angle. An accuracy of 0.3° and using the previously computed deflection angle as starting value for the new iteration process, gives a quick convergence. The trimmed deflection of flap #1 will simply be added to the deflection resulting from the controller described below.

To respond to disturbances, an LQR, as described in Section 3.3.1, is employed. A detailed discussion on the design of a similar attitude controller is included Appendix E; here, only a brief description of the current design is given. We assume pitch control with flap #1, and combined roll/yaw control with flaps #2 and #3. The induced pitch moment by the latter two is considered to be a perturbation, which will be compensated for by flap #1, after a deviation in $\alpha$ has occurred. After dividing the reference trajectory in a series of LTI systems, for each of these time points the control laws are chosen to be of the form as depicted in Fig. 6.9. The commanded $\alpha$ and $\sigma$ are output from the guidance system, whereas $\beta_c = 0^\circ$. The commanded angular rate is such that $\dot{\alpha} = \dot{\beta} = \dot{\sigma} = 0$ °/s.

Assuming constant weighting matrices throughout the trajectory, with maximum state deviations of $5^\circ$ for the attitude and infinite for the angular rates, and maximum deflection angles of $40^\circ$, the gains are computed by solving the Algebraic Riccati Equation. A representative number of computed gains has been implemented in the simulation model and scheduled, i.e., interpolated, as a function of $q_{\text{dym}}$. Anticipating on the controllability study, the gains are reduced by 30%, because of too strong oscillations in the deflection angles due to non-linear effects. The control system is activated when $q_{\text{dym}} > 100$ N/m$^2$.

Before doing the controllability study, a suitable location of the c.o.m. has to be found. Shifting the location of the c.o.m. in longitudinal direction, i.e., $X$, has its impact on $\alpha_{\text{trim}}$, whereas a shift in Z-direction will increase or decrease the maximum available roll moments of flaps #2 and #3. At all times the c.o.m. should be in the plane of symmetry to avoid a non-zero
Final values are \( x_{cm} = -1.1 \) m (from the nose) and \( z_{cm} = 0.1 \) m. This location of the c.o.m. requires only a small deflection of flap #1 (= -3°) to have trim stability at \( \alpha = 20^\circ \).

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**Fig. 6.9** - Schematic overview of the attitude controller.

### 6.3.2. Nominal mission

To begin with the nominal re-entry mission (see Fig. 6.10 for the nominal \( \alpha \) and \( \sigma \)), the control error in \( \alpha \), i.e., the difference between \( \alpha_c \) and \( \alpha \), shows two major deviations from the ideal, zero-deviation control (Fig. 6.11). In the first place, there is a sinusoidal oscillation up to \( t \approx 90 \) s. This is due to the absence of aerodynamic control, since the flaps are only activated when \( q_{dy} > 100 \text{ N/m}^2 \) to avoid large-amplitude oscillations in flap deflection because of low effectiveness (note that the oscillation in \( \alpha \) will diverge if after 90 s the aerodynamic control is not activated). However, in case \( \delta_1 \) is fixed at 0°, a large-amplitude oscillation in \( \alpha \) will occur (\( \Delta \alpha = \pm 20^\circ \)). Therefore, by giving flap #1 a fixed non-zero deflection (here, \( \delta_1 = -27^\circ \)), this oscillation can be strongly reduced. The remaining control error of between -1.5° and 2° is due to the
fact that the fixed setting of flap #1 is only an average value. Once the control system is activated, the control error rapidly decreases to zero. In the second place, at $t = 210$ s, $\alpha$ is reduced by the guidance system from $45^\circ$ to $20^\circ$, with a commanded rate of change of $10$ $^\circ$/s. Due to the inertia of the vehicle, the actual attitude cannot follow the commanded one that quickly, although the error is only $0.5^\circ$ which is well within the design margin of $5^\circ$.

Due to the strong coupling between the roll and yaw motion, and therefore $\beta$ and $\sigma$, the corresponding control errors are discussed simultaneously (see Figs. 6.12 and 6.13). Both initial attitude angles are $0^\circ$. At $t = 210$ s, the guidance system generates a large $\sigma_c$ of almost $100^\circ$, with a commanded rate of change of $10$ $^\circ$/s. Apparently, this theoretical rate is a bit too high for the applied attitude control system, since due to the inertia of the vehicle an error of about $9^\circ$ is introduced. The control action in roll induces a disturbance in $\beta$ of about $\pm 1.5^\circ$, that is, however, reduced to zero in a short time.

At $t = 830$ s, a small peak in both $\beta$ and $\sigma$ shows. At this moment, the guidance logic starts the transition of $Q$-control to $g$-load control. Since this transition is a simple weighted averaging of the two control signals, it results in a small discontinuity in the bank-angle rate (and therefore also $\beta$). At the end of the flight there is a steep increase of $\sigma$ up to $140^\circ$. Again, the inertia of the vehicle causes a control error. Moreover, due to the coupling of roll and yaw, a non-zero $\beta$ will be induced whenever there is severe roll control. This angle is always small, however, and in case of a slowly varying $\sigma$ it is very close to zero.

The discussion of the time histories of the flap deflections, plotted in Fig. 6.14, will of course closely follow the discussion of the control angles. The first $90$ s of flight, the three flaps are not actively used and have constant deflections. For reasons of trim stability, flap #1 is set to $-27^\circ$. Due to the control error in $\alpha$, there is a brief deflection of flap #1 (with an amplitude of $\pm 1.5^\circ$ around the trim value). After that, $\delta_1$ remains stable at $-28^\circ$. When $\alpha$ is reduced from $45^\circ$ down to $20^\circ$, also $\delta_1$ gradually reduces towards the new trim deflection of $-2^\circ$. Throughout the flight, $\delta_1$ remains almost constant. Only towards the end of the flight there is a need to compensate some deviations, because of the increasing $\sigma$ and the rapidly increasing $q_{dyr}$. However, the deflection remains small: only $\pm 1^\circ$ round the trim value.

Because of the symmetric design, the deflection of flap #2 and #3 will be symmetric as well, at least on the face of it, although there is a sign difference. Once $\sigma$ starts increasing at $t = 210$ s, flap #2 and #3 are deflected at $\pm 10^\circ$. After acquiring the commanded attitude, the deflection angles become zero, indicating a stable roll rate. At $t = 830$ s, the peak deflection of $5^\circ$ is due to the transition of heat-flux control to $g$-load control. The remaining peak is due to the increasing $\sigma$ at the end of the flight. Overall, it has been verified that the vehicle is very well capable of flying the nominal trajectory, especially when we consider the very crude guidance and control system.
Fig. 6.10 - The nominal angle of attack and bank angle.

Fig. 6.11 - The control error in the angle of attack as a function of flight time.

Fig. 6.12 - The angle of sideslip as a function of flight time.

Fig. 6.13 - The control error in the bank angle as a function of flight time.
Fig. 6.14 - The time history of the flap deflection angles. Flap #3 has an identical, but negated, history as flap #2.

6.3.3. Sensitivity analysis

To identify the sensitive parameters and to find the maximum allowable errors, first a screening analysis is conducted. To allow for a rapid exploration of the parameter space, as discussed in detail in Chapter 4, the parameters are varied over two levels according to Taguchi's $L_8$ array (8 simulations with a maximum of 7 two-level factors). With deviations in the initial attitude and angular rates of $10^\circ$ and $10^\circ$/s, respectively, and no control while $q_{dy} < 100$ N/m$^2$ (the first 90 s of flight), the vehicle ended up in a tumbling state, that took long to stabilise after control was initiated (about 100 s). An offset in the location of the c.o.m. appeared to be very critical. The c.o.m. could not be moved towards the base, because its position was very close to the centre of pressure (c.o.p.). In addition, a shift forward of 1 cm was as far as we could go.

The vehicle was even more sensitive to a shift in $Y$-direction. For a shift larger than ±0.5 cm, the induced $\beta$ demanded such deflections of flaps #2 and 3, that the resulting roll moment made the vehicle end up in an uncontrollable spin$^{12}$. The controllability of the vehicle was less sensitive to a shift in $Z$-direction, but also in this case the c.o.m. could not be moved more than 1 cm up (too small roll moment) or down (geometrical constraint). An error of 10% in the aerodynamic moment coefficients resulted in some minor oscillations and somewhat larger deflection angles, but no serious problems were encountered. Larger perturbations could not be handled by the control system, because the flaps were deflected at their maximum.

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$^{12}$ This extreme sensitivity to a small shift in the location of the c.o.m. indicates that the current shape of Hyperion is by no means an optimal one and should be improved before any serious effort is put into the design of an attitude controller and the successive controllability study. At the end of this section, the drawbacks of the current design will be discussed in more detail.
Fig. 6.15 - The minimum and maximum heat-flux curves, as a function of flight time.

A second analysis to address possible quadratic effects combined the error sources mentioned above, with the exception of a shift of the c.o.m. in $X$-direction. We took only half the value of the errors in initial conditions, and a shift of 0.5 cm in both $Y$- and $Z$-direction. We simulated over three levels, thus requiring the $L_{27}$ array for the 11 involved parameters. In Fig. 6.15, the curves of the minimum (longest flight, $t_f \approx 1070$ s) and maximum (shortest flight, $t_f = 680$ s) $\dot{Q}$ have been plotted, showing a difference of about 600 kW/m$^2$, which is sure to result in a loss of the vehicle unless the thermal protection system is overdimensioned. Due to the errors, the attitude of the vehicle varied such that different trajectories were flown. In case of the shortest flight, the path was steeper and the vehicle dived deeper into the atmosphere before following a shallower path, thereby increasing $\dot{Q}$. It must be noted that because of the much shorter flight time, the total heat load is smaller than for the longest flight.

With ANOVA the contribution of each of the error sources to the variance of $\dot{Q}_{\text{max}}$ could be derived (see Fig. 6.16). The major contributors are the $r_0$ (27%) and $q_0$ (14%), $\beta_0$ (13%), and the moment coefficients (between 7% and 9%). About 12% was not accounted for, which indicates that there may be substantial (linear) interactions between the parameters. Quadratic effects contributed significantly to the total variation. It should be noted that the slow control of the initial attitude and angular rates after control initiation is likely to lead to the application of a Reaction Control System for the first flight phase, as to keep $\dot{Q}_{\text{max}}$ within bounds. At first sight it might be surprising that the initial $r$ has a larger influence on $\dot{Q}_{\text{max}}$ than the initial $q$. One should bear in mind, however, that $\dot{Q}$ is determined by the orientation of the aerodynamic force vector. A rotation about the yaw axis results in compensating deflections of flap #2 and #3, which also induces pitch moments. Apparently, the yaw-rate error cannot be controlled too well,
resulting in quite large pitch-rate errors and, consequently, a large $\dot{Q}$, larger than would be the case of an isolated pitch-rate error.

![Graph showing factor contributions for the maximum heat flux](image)

**Fig. 6.16 - Factor variations for the maximum heat flux.** Note that $a_c$, $b_0$ and $s_0$ stand for $\alpha_c$, $\beta_0$ and $\sigma_0$

The error in the initial conditions resulted in very strong oscillations in both $\alpha$ and $\sigma$, which damp out very slowly after control activation. This is mainly due to the fact that the control system cannot cope with them because the flap deflections have reached their limiting values. After $t = 210$ s, when $\alpha_c$ has decreased to $20^\circ$, the control error becomes almost zero. For $\sigma$ there is a similar pattern. Only after $t = 210$ s, the vehicle becomes stable in roll and yaw. More control effort is required, however, because of the fact that the shift of the c.o.m. has more impact on roll/yaw control. For the first part of the flight, the flaps are severely oscillating between $\pm 40^\circ$. Once stable, the minimum and maximum deflection angles of flap #2 and 3 are between $-10^\circ$ and $15^\circ$, while strongly oscillating.

As it seems, the control system of the re-entry vehicle does not allow for large deviations from the nominal configuration. On one hand this is caused by the relatively simple design of the controller, and on the other hand the vehicle configuration as it is now cannot cope with them. One of the reasons for the poor response to disturbances is the location of the c.o.p. that is very close to the c.o.m.. The distance between the c.o.p. and the c.o.m. could be enlarged by giving all flaps a nominal deflection of some degrees outward. The strong coupling of roll and yaw motion can be addressed by changing the Z-position of the c.o.m. and/or by adjusting the aerodynamic shape of the vehicle.

In fact, the roll-yaw coupling is a direct consequence of the small and even negative restoring moment about the Z-axis ($C_{n_z}$), as a follow-up study indicated (Wentink, 1996, and Mooij et al., 1998). To minimise the roll-yaw coupling it was found that Hyperion's cross section


should not be an equilateral triangle, but much flatter, with a base angle of only 30\(^\circ\) (the base angle is the angle between flap #1 and flap #2 or #3, when they are not deflected). In addition, it appeared that the effectiveness of the flaps is not large enough, resulting in large controller gains and therefore a large sensitivity to parameter variations. Also the effectiveness in yaw is much larger than the one in roll, which gives large deflections (and thus large induced yaw rates) for small deviations in roll. In case flaps #2 and #3 are asymmetrically split, such that the larger parts are used for yaw control alone, and the smaller ones for roll control. Then, larger deviations in yaw might not result in an uncontrollable spin (roll) of the vehicle. The stability around the Z\(_G\) axis can be improved by either adding a vertical tail of some sort, or to make the vehicle’s bottom side rounder (such that it resembles more the shape of the lifting bodies of the sixties). Still, the results of this feasibility study encourages further research of this new configuration, including a detailed analysis of the aerodynamic properties and different control concepts.

Note that during the initial two-level analyses we found parameter combinations that resulted in an unstable spin. For the combined analysis with other, although similar, parameter combinations, this spin did not show. This indicates that the results of simulations while varying more parameters at a time, should be considered carefully. Statistical analysis will usually give the parameter contributions to some defined response, but control instabilities cannot be traced by such analysis. A verification by means of a Monte-Carlo analysis could in this case be applied, after the vehicle and controller design have reached a more mature level.

6.4. Summary

- To prove that the algorithms have been correctly implemented and that the simulated output is a good approximation of the reality, START is in parts verified, validated and evaluated. The validation part restricted to compare the results of an open-loop, 6 d.o.f. re-entry with the results obtained with the RATT software package. The position and velocity curves showed a close similarity, whereas a phase shift was observed in the time history of angular rates and attitude angles, resulting in different oscillation patterns. An additional comparison with a quaternion implementation to define the attitude led to the conclusion that START can be used for 6-d.o.f. open-loop re-entry simulations.

- The first of two mission analyses, i.e., the entry and parachute descent of the scientific probe Huygens, included the evaluation of three flight-mechanics related issues, i.e., 1) uncontrolled, spinning entry in 3- and 6-d.o.f., 2) offset in the location of the c.o.m., and 3) steady-state wind and horizontal wind gusts. The results allow for the conclusion that the above issues can indeed be studied with START. Moreover, part of the results was compared with results obtained by industry
(Martin-Baker). The results correspond to an acceptable degree and for the differences a plausible explanation could be given, although this remains to be verified.

- The second mission analysis dealt with the aerodynamic controllability of a moderate lift-to-drag re-entry test vehicle. Three issues related to flight-mechanics and one to experimental design were covered in this study:
  1) trimmed entry and descent in 3- and 6-d.o.f.,
  2) open-loop guidance,
  3) aerodynamic attitude control, and
  4) limited exploration of Taguchi's orthogonal arrays.
Each of the above listed topics can efficiently be studied with START.

- General conclusion of this chapter: START may be used to study the guided and controlled ascent and descent of powered/unpowered space planes.
Chapter 7

Guidance and Control for Space-Plane Re-entry

One kept on thinking that this piece of machinery had 400,000 parts, all constructed by the lowest bidder.

David Scott, astronaut, about Apollo 15.

A typical mission of a space plane consists of a powered ascent, cruise flight, manoeuvres to change the heading to one due east, orbital operations, re-entry into the Earth's atmosphere and finally the descent towards the Earth's surface. This chapter will deal with the re-entry and descent part of the mission. A re-entry mission of a space plane is characterised by a high angle-of-attack entry to minimise the heating rate followed by a lower angle-of-attack descent to improve the cross-range and down-range capabilities. The major part of this flight will be unpowered. Analysis of such a mission is a logical next step in using the software, after the evaluation test case discussed in Section 6.4. There, open-loop guidance and aerodynamic control of a re-entry test vehicle was studied. This mission analysis will go further, in the sense that closed-loop guidance, horizontal-plane manoeuvres, and hybrid attitude control will be included. Hybrid in this context means a combination of reaction control and aerodynamic control.

The goal of this chapter is threefold. In the first place, it serves as a basis to verify a correct implementation of a complete closed-loop guidance and control system. To this end, results are compared with results obtained by MBB\textsuperscript{13} (1988), the company that developed the applied

\textsuperscript{13} MBB discusses two attitude controllers, of which the first is a simplified PD controller that \textit{directly} computes the required control moments for given attitude deviations, instead of computing control-surface deflections which will \textit{result} in control moments. This is an optimistic design, since non-linearities in the aerodynamics of the control surfaces as well as asymmetric deflections are not taken into account. The second attitude controller is based on linear state feedback with gain scheduling, where the gains are computed by pole
reference vehicle HORUS-2B. In the second place, the use of Taguchi's orthogonal arrays for mission analysis is further extended and evaluated. In the third place, the mission itself is analysed to get an understanding of the related flight mechanics and the performance of the guidance and control system, also because this topic is not covered by MBB (1988).

In addition to elements that were introduced in the previous chapter, the following aspects are included in the re-entry study:

- winged vehicle, with 5 independent control surfaces and reaction-control thrusters,
- closed-loop guidance,
- lateral manoeuvres,
- combination of reaction and aerodynamic control based on LQR,
- trim with body flap and elevons (elevator function),
- extensive application of Taguchi Method, and
- performance optimisation by robust design of the guidance system.

The lay-out of this chapter is as follows. In Section 7.1, the reference vehicle HORUS-2B will be briefly described. The nominal re-entry mission is presented in Section 7.2. In Section 7.3, a sensitivity analysis using Taguchi's orthogonal arrays is conducted, and the results are discussed. In Section 7.4 the orthogonal arrays are applied to a robust (re)design of the guidance system and Section 7.5 states the main points of this chapter.

7.1. The HORUS-2B reference vehicle

7.1.1. Mass, geometry and aerodynamics

Initially, HORUS-2B was designed as a fully reusable second stage to the Ariane-5 launcher. Basically, the vehicle was unpowered although it had been equipped with a deorbitation engine and attitude-control thrusters. Later on, a rocket engine was added to the design and this adapted version became the manned, second stage of Sänger, the German TSTO reference concept. In this thesis, we use the original design being that of a winged, unpowered re-entry vehicle (see Fig. 7.1). In Table 7.1, the main characteristics of HORUS are listed.

The complete database of HORUS is described by MBB (1988), where the aerodynamic coefficients are given in graphical form; the corresponding numerical values are presented by Mooij (1995). We will briefly summarise the key elements of the vehicle. The gross mass of the vehicle at re-entry is \( m = 26,029 \) kg. The c.o.m. is located in the vehicle's symmetry plane at 13 m from the nose (Fig. 7.1). This configuration has the following principal moments of inertia:

placement. This controller resembles the Linear Quadratic Regulator that is used in this thesis, but unfortunately it was not used by MBB. For this reason, only the 3-d.o.f. results can be compared.
Table 7.1 - Main characteristics of HORUS-2B.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total vehicle length</td>
<td>25 m</td>
</tr>
<tr>
<td>Maximum fuselage width</td>
<td>5.4 m</td>
</tr>
<tr>
<td>Maximum fuselage height</td>
<td>4.5 m</td>
</tr>
<tr>
<td>Wing span</td>
<td>13.0 m</td>
</tr>
<tr>
<td>Wing chord</td>
<td>23.0 m</td>
</tr>
<tr>
<td>Wing area</td>
<td>110 m²</td>
</tr>
<tr>
<td>Maximum payload mass</td>
<td>7,000 kg</td>
</tr>
<tr>
<td>Re-entry mass</td>
<td>26,029 kg</td>
</tr>
</tbody>
</table>

The inclination of the principal X-axis with respect to the X-axis of the body frame (negative for 'nose up') is \(-1.7°\).

The HORUS-2B has a number of control surfaces, which have schematically been depicted in Fig. 7.2: two rudders, two elevons and one body flap. The sign definitions for the deflection angles are as follows:

- Left and right rudder, deflection angles $\delta_{r,l}$ and $\delta_{r,r}$: positive outboard
- Left and right elevon, deflection angle $\delta_{e,l}$ and $\delta_{e,r}$: positive down
- Body flap, deflection angle $\delta_{p}$: positive down

\[ \bar{I}_{xx} = 119,000 \text{ kg m}^2 \quad \bar{I}_{yy} = 769,000 \text{ kg m}^2 \quad \bar{I}_{zz} = 806,000 \text{ kg m}^2 \]
The rudders are outward movable only, which means that for yaw control only one rudder is active at a time. Deflecting both rudders at the same time would result in the so-called speed-brake function, but is not considered in this study\textsuperscript{14}. The elevons combine both the elevator and aileron function. The deflections commanded by the attitude controller should be combined to give the corresponding left and right elevon deflection.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{diagram.png}
\caption{Definition of the control surfaces (based on MBB, 1988).}
\end{figure}

The aerodynamic coefficients are given by tabulated functions of $\alpha$ and $M$, and include sideslip derivatives; in addition, the increments due to a control-surface deflection are also dependent on the corresponding deflection angle. Note that the current aerodynamic model of HORUS-2B does not comprise any damping coefficients, i.e., $C_{l_p} = C_{m_q} = C_{n_r} = 0$.

\subsection{7.1.2. The guidance and control system}

For the high angle-of-attack entry and subsequent descent through the hypersonic, supersonic and transonic flight regime towards a selected landing site, a closed-loop guidance and control system is required to guarantee mission success. The re-entry guidance system has been briefly described in Section 3.2, and in more detail in Appendix C (Section C.1). In Sections 3.4.2 and 3.4.3, the underlying theory for two attitude-control concepts has been presented: LQR and MRAC. Of these two, the former is the simpler method to implement, so for a first assessment of attitude control this concept is further detailed for the re-entry mission. In Appendix C, Section C.2, the design of the LQR for attitude control of HORUS has been

\textsuperscript{14} Speed brakes can be used in the approach and landing phase to increase the drag and therefore decrease the velocity. Since the entry guidance of HORUS is based on energy control by regulating the drag through the angle-of-attack dependency, it might be worthwhile to study the use the speedbrakes in this phase as well.
described. Below, the design logic is summarised.

In the current design, the two tasks of the attitude controller, i.e., corrective control towards a commanded attitude and guaranteeing that this attitude is stable (the so-called trim stability) are separated. Moreover, only trim stability in pitch will be considered, because the related moments are largest. Assumed is that roll and yaw stability is guaranteed by the corrective control. The results later in this chapter justify the assumption. The trim stability is primarily taken care of by the body flap of HORUS, assisted by the elevons whenever required (especially at lower Mach numbers). Because the body flap is only activated when $q_{dny} > 100 \text{ N/m}^2$, in the first part of the flight there is no active trim control. As was done for Hyperion (Section 6.4), the body flap is set in a fixed position, which gives a pseudo equilibrium for the complete first phase.

The design of the linear-state feedback attitude controller is centred around a reference trajectory (for details, see Section 7.2.1). Starting point for the design is a local stability model, that was also used to study the open-loop motion of HORUS. The results indicated that the symmetric motion can be decoupled from the asymmetric motion, such that the longitudinal and lateral corrective controller can be designed separately. Each controller makes use of both reaction-control jets and aerodynamic-control surfaces. As was mentioned in Chapter 1, instantaneous operation of the control effectuators with an infinite accuracy is assumed. Since RCS thrusters can in principle only be switched on and off, this means that some kind of pulse-width modulation should be applied. However, this is not studied here.

![Schematic lay-out of the HORUS-2B attitude controller.](image-url)
Computation of the feedback gains is based on optimal control theory (see Section 3.3.1). The adaptation of the control system to the rapidly changing flight state is achieved by redesigning a new controller at regular intervals. As a practical consequence this means that the gains are computed off-line for a number of points in the trajectory. These gains are then stored in on-board reference tables as a function of \( q_{\text{dyn}} \). In-flight computation of the actual gains is based on linear interpolation for the local value of \( q_{\text{dyn}} \). This scheme is also known as gain scheduling. A schematic overview of the attitude controller for HORUS is depicted in Fig. 7.3.

**Longitudinal controller**

As control variables, the symmetric elevon deflection angle \( \delta_e \) (elevator function) and the thruster torque about the pitch axis, \( M_{T,y} \), are available. The elevators are activated at \( q_{\text{dyn}} > 100 \text{ N/m}^2 \), whereas the pitch jets will start working at the entry interface (in principle \( q_{\text{dyn}} = 0 \text{ N/m}^2 \) and will continue to do so until \( q_{\text{dyn}} > 1000 \text{ N/m}^2 \), an operation scheme that is based on that of the Space Shuttle (see also Fig. 3.1). The state-feedback laws for longitudinal control are selected to be simple proportional laws, as has been schematically depicted in Fig. 7.4.

![Fig. 7.4 - The longitudinal controller.](image)

These control laws are a linear combination of the state variables, which is the right form for the LQR:

\[
\Delta u = -K \Delta x
\]  

(7.2.1)

The gains can be computed by solving the Riccati Equation, Eq. (3.3.9), as discussed in Section 3.3.1. The selected weighting matrices \( Q \) and \( R \) are chosen to be

\[
Q = \text{diag}\left\{ \frac{1}{\Delta q_{\text{max}}^2}, \frac{1}{\Delta \alpha_{\text{max}}^2} \right\} \quad R = \text{diag}\left\{ \frac{1}{\Delta \delta_{e_{\text{max}}}}, \frac{1}{\Delta M_{T,y_{\text{max}}}} \right\}
\]  

(7.2.2)
where
\[ \Delta q_{\text{max}} = \infty \quad \Delta \alpha_{\text{max}} = 2^\circ \]
\[ \Delta \delta_{\text{e}_{\text{max}}} = 40^\circ \quad \Delta M_{T, y_{\text{max}}} = 10,400 \text{ Nm} \]

The value of \( \Delta \alpha_{\text{max}} \) has been selected in accordance with the small margin that is left, since the nominal \( \alpha \) is close to its maximum. Note that the selection of the weighting matrices is in principle an iterative procedure that is not pursued here.

**Lateral controller**

As control variables for the lateral controller, the aileron (asymmetric-elevon) deflection angle \( \delta_{\text{a}} \), the rudder deflection angle \( \delta_{r} \), and the thruster moments about the roll and yaw axis, \( M_{T,x} \) and \( M_{T,z} \), are available. Since the rudder is outward movable only, \( \delta_{r} \) is defined to be equal to \( \delta_{r,c} \) when \( \delta_{r,c} \) is positive, and equal to \( -\delta_{r,c} \) when \( \delta_{r,c} \) is positive. The ailerons are activated at \( q_{\text{dyn}} > 100 \text{ N/m}^2 \) and the rudder at \( q_{\text{dyn}} > 150 \text{ N/m}^2 \). The roll and yaw jets will start working at the entry interface and will continue to do so until \( q_{\text{dyn}} > 150 \text{ N/m}^2 \) (roll jets) and \( M < 1 \) (yaw jets).

The logic of the lateral controller is schematically shown in Fig. 7.5. It should be noted that the aerodynamic- and reaction-control part are identical, but for the gains. After linearising, also these control laws can be written in the form of Eq. (7.2.1). Solving the gains for the equilibrium points is done according to Eqs. (3.3.8-9), with

\[
Q = \text{diag} \left[ \frac{1}{\Delta p_{\text{max}}^2}, \frac{1}{\Delta r_{\text{max}}^2}, \frac{1}{\Delta \dot{\beta}_{\text{max}}^2}, \frac{1}{\Delta \sigma_{\text{max}}^2} \right]
\]

\[
R = \text{diag} \left[ \frac{1}{\Delta \delta_{\text{a}}_{\text{max}}^2}, \frac{1}{\Delta \delta_{r,c}^2}, \frac{1}{\Delta M_{T,x_{\text{max}}}^2}, \frac{1}{\Delta M_{T,z_{\text{max}}}^2} \right]
\]

(7.2.3)

where
\[ \Delta p_{\text{max}} = \infty \quad \Delta \dot{\beta}_{\text{max}} = 2^\circ \]
\[ \Delta r_{\text{max}} = \infty \quad \Delta \sigma_{\text{max}} = 5^\circ \]
\[ \Delta \delta_{\text{a}}_{\text{max}} = 40^\circ \quad \Delta M_{T,x_{\text{max}}} = 1,600 \text{ Nm} \]
\[ \Delta \delta_{r,c} = 40^\circ \quad \Delta M_{T,z_{\text{max}}} = 7,600 \text{ Nm} \]
A sensitivity analysis based on tabulated error functions for the atmospheric density, position measurements, and the lift and drag coefficient, as well as dispersions in the initial conditions (Mooij, 1998b), resulted in an improvement of the lateral controller. By including a phase-lead filter in the loop for $\beta$. The transfer function $G_c(s)$ of the phase-lead filter is:

$$G_c(s) = \frac{K_c \tau_1 s + 1}{\tau_2 s + 1}, \text{ with } K_c = 10, \quad \tau_{c,1} = 1 \text{ and } \tau_{c,2} = 10$$

Remark

To minimise strong oscillations in the controls, the gains of the control laws are adjusted as to lower the response time. Although application of the Riccati Equation leads to optimal gains, they can sometimes be too large for the non-linear vehicle and environment. Instead of varying the weighting matrices to alter the gains, they are decreased directly. Throughout the sensitivity analysis we will use 80%-values of the gains.
7.2. Nominal trajectory

The operational principle of the guidance system is based on tracking a reference trajectory and computing a commanded attitude that must guarantee this. In addition, the reference trajectory is required for analysis of the open-loop behaviour of the vehicle and the subsequent design of the attitude control system. In Section 7.2.1 the set-up of this 3-d.o.f. trajectory will be discussed. Since a reference trajectory was not readily available from MBB, in Section 7.2.2, it is verified whether the vehicle can actually fly the reference trajectory in 6 d.o.f., which means a first test for the attitude controller applied to the non-linear vehicle and flight environment.

7.2.1. Three-degrees-of-freedom reference trajectory

The guidance system takes a reference trajectory as input, as we discussed in Section 3.2.1. The related parameters are

\[ t, d, E_{tor}, h, \gamma, \int_0^t \frac{\partial E_{diss}}{\partial \alpha} \, dt \text{ and } \langle C_L \cos \alpha \rangle_{\text{nom}} \]

From the MBB documentation a nominal control history of \( \alpha \) and the absolute value of \( \sigma \) as a function of time is available. When this control history is put into the simulation model it appears that HORUS does not reach the TAEM interface, a fact that has been contributed to differences in the applied aerodynamic model, because the coefficients were measured from graphs.

By adjusting \( \sigma \) to compensate for these differences (trial and error), the control history of Table 7.2 was obtained, such that the Terminal Area is reached at about 25 km altitude. With this control history as guidance output and applying ideal control, it is possible to generate a realistic re-entry trajectory. Since only \( |\sigma| \) is available, the sign of \( \sigma \) is determined by flying inside a heading-error corridor (Table 7.3).

The initial conditions for the simulation are:

\[ V_0 = 7435.5 \, \text{m/s} \]
\[ \gamma_0 = -1.43^\circ \]
\[ h_0 = 122 \, \text{km} \]
\[ \chi_0 = 70.75^\circ \]
\[ \delta_0 = -22.3^\circ \]
\[ \tau_0 = -106.7^\circ \]

The vehicle is assumed to be heading towards a target point in Kourou (French Guyana), with \( \tau_{rw} = -53.0^\circ \) and \( \delta_{rw} = 5.0^\circ \). The heading of the runway is \( \chi_{rw} = 180^\circ \) (landing direction south). The target point lies somewhere on a cylinder, with the runway at its centre. The radius of this Heading Alignment Cylinder (HAC) is about 83 km (0.75°), so when the distance-to-target is 0.75° the simulation will stop. In Fig. 7.6 the groundtrack for the reference trajectory has been plotted.
<table>
<thead>
<tr>
<th>t (s)</th>
<th>$\alpha_{\text{ref}}$ (°)</th>
<th>$\sigma_{\text{ref}}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40.0</td>
<td>0.0</td>
</tr>
<tr>
<td>264</td>
<td>40.0</td>
<td>0.0</td>
</tr>
<tr>
<td>290</td>
<td>40.0</td>
<td>79.6</td>
</tr>
<tr>
<td>554</td>
<td>40.0</td>
<td>56.0</td>
</tr>
<tr>
<td>686</td>
<td>40.0</td>
<td>59.8</td>
</tr>
<tr>
<td>924</td>
<td>40.0</td>
<td>59.8</td>
</tr>
<tr>
<td>1319</td>
<td>11.5</td>
<td>54.0</td>
</tr>
</tbody>
</table>

Table 7.2 - Nominal control history.

<table>
<thead>
<tr>
<th>coarse heading-error dead band</th>
<th>low-distance dead band</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$d$ (°)</td>
<td>$\chi_{e,\text{db}}$ (°)</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------</td>
</tr>
<tr>
<td>0.0</td>
<td>23.0</td>
</tr>
<tr>
<td>10.0</td>
<td>23.0</td>
</tr>
<tr>
<td>30.0</td>
<td>15.0</td>
</tr>
<tr>
<td>60.0</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Table 7.3 - Heading-error dead bands as a function of the distance to target.

Fig. 7.6 - The groundtrack from the entry interface to the Terminal Area near Kourou for the reference mission.
7.2.2. Six-degrees-of-freedom controlled trajectory

The previously discussed reference trajectory is taken as input to the guidance controller. With the same initial and final conditions, a 6-degrees-of-freedom simulation including attitude control is executed. We found that the sample frequency of the (digital) attitude controller is an important parameter. A frequency of 10 Hz resulted in discrete jumps in the commanded deflection angles and reaction-control moments, inducing oscillations that required additional control. Doubling the frequency, i.e., execution of the attitude-control laws every 0.05 s, gave sufficiently smooth signals without any oscillations due to the discretisation. A sample rate of 10 Hz for executing the guidance logic is used.

Comparison with MBB results can only be done in a qualitative manner, since their plots were not very detailed. Moreover, they did not use a linear state-feedback controller, so only some 3-d.o.f. results can be compared. Overall, the corresponding curves match well which indicates a correct implementation of the guidance model (see also Mooij, 1998b). Details of the MBB results can be found in MBB (1988). Since we are more interested in the full picture, i.e., including attitude control, we will concentrate on the 6-d.o.f. results.

Position and velocity

In Fig. 7.7 the height as a function of flight time has been plotted, both for the controlled and the nominal trajectory. As we can see, the two curves match well, apart from two visible differences at \( t = 750 \) s and \( t = 1,100 \) s. The explanation for these two differences can be found in the time history of the bank angle (Fig. 7.11). As we already stated in the previous sub-section, the nominal bank reversals are executed in zero time (ideal control). Of course, in a realistic simulation the inertia of the vehicle will result in a finite time required for the reversals. The duration is, amongst others, depending on the control effectiveness (for aerodynamic control, this is basically determined by the dynamic pressure). The duration of a bank reversal can be in the order of 20 seconds. During that time, the bank angle has a smaller value than it should have, implying a larger vertical lift coefficient \( C_L \cos \alpha \). A larger vertical lift means that the descent rate decreases, or in other words, the vehicle flies at higher altitudes than it should. The height difference, however, is corrected by the guidance system, since after the deviation the vehicle comes back to the nominal height.

In principle, a similar reasoning applies to the deviation from the nominal ground track. Due to the vehicle inertia the maximum heading error will be exceeded before it starts decreasing again (see also Fig. 7.14). Far away from the target, the effect is small, but the closer the vehicle is to the target, the more rapidly it will deviate from its nominal ground track.

Sofar, we have briefly discussed the influence of additional degrees of freedom on the mass-point motion of HORUS. The additional dynamics resulted in deviations from the nominal trajectory that had to be compensated for by the guidance system. In Fig. 7.8, the difference between the actual (vehicle inertia taken into account) and reference (no vehicle inertia or ideal
control assumed) flight-path angle has been plotted. As can be seen, during each of the bank reversals there is an increase in $\gamma$. This error is proportional to the inverse of the bank rate, see also Appendix C (Section C.1.4), so the faster the vehicle can bank, the smaller the error will be. It is obvious that an error in $\gamma$ will have its influence on the altitude control, and therefore the corrective $\sigma$-control. Furthermore, the energy dissipation will differ from the nominal dissipation, which might lead to corrective $\alpha$-control. In conclusion, especially during bank reversals significant attitude errors may occur.

**Attitude**

Fig. 7.9 shows the first of the three attitude angles which are controlled by the attitude control system, i.e., $\alpha$. In this figure we see more of a difference between the controlled value and the nominal one. During the first 200 s, $\alpha$ is slightly diverging, caused by the fact that the trim law is not active. This effect can be better observed in Fig. 7.10 (where the difference between $\alpha_c$ and $\alpha$ is plotted). As discussed earlier, trim is primarily guaranteed by deflecting the body flap which is only activated when $q_{\text{dyn}} < 100$ N/m². The moment the body flap is activated ($t = 194$ s), $\alpha$ is stabilised at the nominal value. The diverging $\alpha$ is only partly compensated for by the pitch jets, but apparently due to the design assumptions and simplifications, this offset is not properly controlled. On the other hand, it should be noted that the gains are computed allowing a 2° overshoot of $\alpha$, so the offset is within range. A revision of the trim law (trim with pitch jets, when the body flap is not active) or a gain computation for the first 200 s with a smaller allowable overshoot is not required, considering the small divergence.

Returning to the time history of $\alpha$, up to $t = 480$ s the controlled $\alpha$ is almost equal to the nominal one. Then, noticeable differences occur, which can be related to either the guidance system or the attitude controller. Fig. 7.10, which shows $\alpha_c - \alpha$, can be consulted in finding the answer, since this figure indicates how well the attitude controller performs. As we see, the differences are small at all times, from which it can be concluded that the 'problem' is not related to the attitude controller but a result of guiding the vehicle towards the target along the reference trajectory.

In the same figure, four peaks in the right half of the graph need more explanation. During bank reversals, the guidance system keeps the commanded variation in $\alpha$ at a constant value, which means that during the reversal no correction other than a change in the reference value takes place. It was discussed above, that $\sigma$ is smaller than it should be during a reversal, resulting in a larger vertical lift. Any resulting error can only be compensated for after the reversal has been completed. This shows as an abrupt change in the commanded angle of attack at $t = 730$ s, $t = 1,080$ s, $t = 1,200$ s and $t = 1,250$ s.

In Fig. 7.12, the difference between $\sigma_c$ and $\sigma$ has been plotted, again indicating how well the attitude controller performs. The first difference between actual and nominal $\sigma$ is found just before $t = 100$ s, which is due to a relatively sudden activation of the guidance system. Since during the first 100 s, the actual $\alpha$ is not identical to the nominal value, a small altitude differ-
ence will be built up. Altitude control is activated after some 80 s to avoid excessive commands to the attitude controller in this low-dynamic pressure regime. Once altitude control is active, it is faced with a discrete altitude error resulting in a sudden change in $\sigma_c$. However, only a small peak value of $2^\circ$ is observed, well within the gain overshoot-value of $5^\circ$. The limitations of the attitude control system in case of sudden changes that are relatively large is shown at $t = 270$ s, where $\sigma_c$ is linearly increased at a rather large rate. An adjustment of the guidance scheme seems to be required; on the other hand, neither the ailerons nor the roll jets have been used at their maximum capacity, so there is still a margin in the attitude controller.

The four remaining peaks are all related to the bank reversals. It suffices to say that the change in $\sigma_c$, as generated by the guidance system based on a predicted bank rate, is too large for the attitude control system to achieve. Moreover, the reversals induce a significant altitude error, and each time a rather large corrective $\alpha$ is required to compensate for this (Fig. 7.11).

The last attitude angle to be discussed is $\beta$ (Fig. 7.13). While designing the control system, it became clear that there is a coupling between $\beta$ and $\sigma$, the so-called roll-yaw coupling (Mooij, 1997). Therefore, an induced $\beta$ can be expected in case there is a sudden change in $\sigma$. Indeed, a number of peaks is observed during the bank reversals, although they damp out relatively quickly. At $t = 1,000$ s, however, $\beta$ diverges slowly, without coming back to zero again. It is assumed that this diversion is due to the simplifications made during the control-system design, e.g., neglecting terms due the rotation of both the Earth and the local horizontal plane (Mooij, 1997). This indicates that the translational motion has at least some influence on the rotational motion, which the LQR in its current design cannot compensate for completely.

Controls

Figs. 7.15 through 7.19 show the deflections of the control surfaces and the moments due to the yaw jets. Most activity takes place during the bank reversals, or when a sudden change in commanded attitude arises. The ailerons (Fig. 7.15) show peak values when bank reversals are initiated; smaller deflections are required when $q_{dyn}$ increases towards the end of the flight. The peaks in the elevator deflection (Fig. 7.16) are also due to the bank reversals, although only indirectly. During the bank reversal the commanded variation of $\alpha$ is kept constant, which means no energy control. To compensate for induced energy errors a sudden change in $\alpha_c$ (and hence elevator deflection) right after completing the reversal is the result.

Note that the elevator and aileron deflection should be combined to get the left and right elevon deflection, and since the related deflections are due to corrective control only, also the trimmed deflection of the elevator should be included. The trimmed deflection of the body flap has been depicted in Fig. 7.17. After a constant value of $\delta_{b,trim} = 15^\circ$, active body-flap trim is started at $q_{dyn} = 100$ N/m$^2$. Mach effects result in a slowly decreasing deflection. When $\alpha$ starts

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15 The moments due to the roll and pitch jets have been omitted, because of a marginal activity. The interested reader is referred to Mooij (1998b) for a complete discussion on the control variables.
decreasing (at \( M = 12 \)), \( \delta_{b,\text{trim}} \) increases because of a larger nose-up moment (the maximum pitch moment in that Mach range lies around \( \alpha = 25^\circ \)). Below \( M = 3 \), the pitch moment changes to nose down, resulting in negative trim deflections. Towards the end of the flight, there is a strong nose-down moment that results in a maximum upward deflection of the body flap (\( \delta_{b,\text{trim}} = -20^\circ \)). As a result, the trimmed elevator deflection is about \(-10^\circ\) in the very last phase of the mission which gives an elevon deflection of about \(-20^\circ\). It remains to be studied in more detail, whether the margins in manoeuvrability capability are sufficient.

The rudder (Fig. 7.18) is twice deflected at its maximum value to start a bank reversal and to compensate for the induced \( \beta \). Unfortunately, the two rudders are only outward deflectable, which means that only half the available control moment can be used. It is therefore advised, since not even uncertainties in the vehicle design have been included, that both rudders should be used for yaw control or that the effective rudder area is increased. Increasing equilibrium deflection angles are found after \( t = 900 \) s, to compensate for the slowly diverging \( \beta \). Large control signals are also found for the yaw jets (Fig. 7.19), similarly to those of the rudder, when a strong variation of \( \sigma \) is commanded. The total fuel mass that is used by the RCS is 10 kg.

To conclude with, some performance parameters that are used for comparison in the discussion of the sensitivity analysis in the next section, are discussed. In Fig. 7.20, the heat flux at the nose (\( R_N = 0.8 \) m) is plotted. The maximum value is \( \dot{Q} = 530 \) kW/m\(^2\). The influence of attitude dynamics can be seen, although the differences are quite small. Origin of the differences is that in 6-d.o.f. the vehicle banks slower and has therefore a larger vertical lift component. This tends to decrease \( \dot{Q} \). In addition, after completing the bank reversal, a larger \( \sigma \) is required to compensate for the induced altitude error resulting in a slightly higher \( \dot{Q} \). Fig. 7.21 shows the total aerodynamic load, the \( g \)-load (normalised to the gravitational acceleration on the Earth's surface, \( g_0 = 9.79 \) m/s\(^2\)). A maximum value of 1.9 is encountered. Differences between the reference and controlled trajectory (of more than 5%) can basically be traced back to deviations from the nominal \( \alpha \), and the bank reversals. Also, in controlled flight the induced \( \beta \) results in small lateral loads, that are higher when the dynamic pressure increases.

The above results give enough confidence in a good implementation of the G&C system. We are now ready to study the sensitivity of HORUS to deviations from the nominal vehicle and environment. This sensitivity analysis will be discussed in the next section.
Section 7.2  Nominal Trajectory

Fig. 7.8 - The flight-path angle error as a function of time.

Fig. 7.9 - Height as a function of time for the controlled (solid) and the reference trajectory (dashed).

Fig. 7.10 - The difference between the commanded and the actual angle of attack, as a function of time.
7.3. Sensitivity analysis

7.3.1. Introduction

The ability of the G&C system to compensate the errors induced by deviations from nominal vehicle and environmental parameters is a measure for the performance of this system. A better performance means a more robust G&C system, and this can increase mission success. Some questions that can serve as the basis for a first analysis are:

- Can the guidance system track the reference trajectory that well that the flight-path constraints are not violated?
- Does the guidance system show deficiencies?
- Are the design parameters of the guidance system well chosen, or can by optimising these parameters the performance be improved?
- Are there dynamic interactions between the guidance system and the attitude controller?
- Is the attitude controller capable of handling non-linearities?

Of the above questions, the third one will be answered in Section 7.4. Here, the other questions are addressed, thereby using the Taguchi Method and Response Surface Methodology. An additional study aspect will be the use of alternative column assignments while using orthogonal arrays to define the simulations.

In Mooij (1998b), results from a simulation using tabulated error functions for the atmospheric density, position measurements, and the lift and drag coefficient, as well as dispersions in the initial conditions, were used to address the robustness and for verification by qualitatively comparing some of the 3-d.o.f. results of MBB (1988), which showed a fair agreement. The results indicated that, although the guidance system had no problems at all for properly guiding the vehicle, the deviations were such that at $t = 930$ s, the vehicle could not be controlled any longer and started diverging oscillations around all three axes. These oscillations could be traced back by a combined $\alpha$ and $\beta$ corrective control. The former requires a symmetric deflection of the elevons, whereas the latter needs an asymmetric deflection. As a result, the absolute deflection of the left and right elevon, that are a simple summation of elevator and aileron deflection, differed substantially. Since the aerodynamic moments due to elevon deflection are quite non linear for larger deflection angles, the attitude controller assumed roll and pitch moments that deviated substantially from the actual moments, besides the fact that the longitudinal is not decoupled from the lateral motion any more. The induced differences resulted in diverging, oscillating elevon deflections and an uncontrollable vehicle. (Note that the aerodynamics are such that for some flight conditions the commands for $\alpha$-control and $\beta$-control are conflicting.)

Although the above simulation led to a design change of the LQR, i.e., the inclusion of a
lead filter in the loop for $\beta$ (see Section 7.1.2), it was not possible to properly analyse the results and to draw conclusions of the sensitivity of the G&C system to a particular error. For that reason a multiple-run sensitivity analysis using orthogonal arrays (see the theory of Chapter 4) is conducted. In principle we want to assess both linear and quadratic factor effects, as well as (linear) interactions between the factors. Since no prior knowledge about the existence of interactions is available, a Response Surface Method such as Central Composite Design would be the best method to use. In this way, all factor interactions can be taken into account. However, for the relatively large number of factors (and thus also many interactions) considered in this analysis a Central Composite Design is considered to be beyond the current computer facilities. Therefore, the relative importance of the factors and possible interactions will be assessed, before a response-surface method is to be applied.

In preparing the sensitivity analysis, we define a total of 23 factors, as specified in Table 7.4. The factor levels are chosen in line with Mistree et al. (1993), i.e.,

$$\mu-\Delta, \mu, \mu+\Delta$$

since these are levels that are often chosen in orthogonal array experiments (Taguchi, 1987, and Phadke, 1989). It should be noted that these numerical values, partly based on MBB (1988), are given these particular numerical values and combined to identify the potential of the Taguchi Method and at the same time to see whether the G&C system can cope with (a combination of) these dispersions. It is stressed that we do not imply that all dispersions will occur at the same time. Moreover, the dispersions may be too pessimistic.

<table>
<thead>
<tr>
<th>Error source</th>
<th>Description</th>
<th>Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-B</td>
<td>$\tau_0$, $\delta_0$</td>
<td>$\pm 3^\circ$</td>
</tr>
<tr>
<td>C</td>
<td>$V_0$</td>
<td>$\pm 20 \text{ m/s}$</td>
</tr>
<tr>
<td>D</td>
<td>$\gamma_0$</td>
<td>$\pm 0.2^\circ$</td>
</tr>
<tr>
<td>E</td>
<td>$\chi_0$</td>
<td>$\pm 5^\circ$</td>
</tr>
<tr>
<td>F-H</td>
<td>$\alpha_0$, $\beta_0$, $\sigma_0$</td>
<td>$\pm 5^\circ$</td>
</tr>
<tr>
<td>I-K</td>
<td>$p_0$, $q_0$, $r_0$</td>
<td>$\pm 5 \text{ %/s}$</td>
</tr>
<tr>
<td>L-Q</td>
<td>$C_D$, $C_S$, $C_L$, $C_p$, $C_m$, $C_n$</td>
<td>$\pm 10%$</td>
</tr>
<tr>
<td>R</td>
<td>$m$</td>
<td>$\pm 80 \text{ kg}$</td>
</tr>
<tr>
<td>S-U</td>
<td>$X_{cm}$, $Y_{cm}$, $Z_{cm}$</td>
<td>$\pm 0.1 \text{ m}$</td>
</tr>
<tr>
<td>V</td>
<td>$p$</td>
<td>$\pm 10%$</td>
</tr>
<tr>
<td>W</td>
<td>$T$</td>
<td>$\pm 10%$</td>
</tr>
</tbody>
</table>

Table 7.4 - Selected factor levels (index '0' means initial condition).
To study the influence of column assignment two batches of simulations are executed, i.e., batch #1 where the 23 factors are quite arbitrarily assigned to the columns of an $L_{32}$ array\(^{16}\) and batch #2 where the factors are assigned to the left-most columns. In principle, it is assumed that the factors are independent from each other such that the main effects (factor variance) can be determined from each column. However, since it is unlikely that the factors are truly independent these main effects will be confounded with interactions. Taguchi (1988) states that main effects are usually much stronger than interactions, so the results can still be used to identify the most important factors. The empty columns are used for error analysis, and, by computing the individual column variances possible groups of interactions can be identified. As an example, one of these groups will be studied in more detail. Finally, a CCD of a subset of factors will be executed to assess quadratic factor effects in a selected response.

7.3.2. Linear effects

The first series of 32 simulations with 'arbitrary' column assignment resulted in 5 crashes of the vehicle, i.e., a final altitude $h_f$ of zero before the TAEM interface was reached. Wondering about the cause(s) of these crashes, it is possible that either the guidance system failed, or the attitude controller, or the combination of the two. Another possibility could be that the selected factor variations were just too large for the vehicle to handle, e.g., the total down- and/or cross-range capability of the vehicle is not large enough. ANOVA of the results can assist in analysing the cause of the crashes.

'Crash' analysis

To begin with $h_f$ five crashes mean five zero altitudes whereas the average $h_f$ should be around 24 km, so the result is a large standard deviation. The variation in final altitude appears to be dominated by three main factor effects, i.e., a variation in the initial heading $\chi_0$, the drag coefficient $C_D$ and the roll-moment coefficient $C_I$. One could conclude that due to $\Delta C_D$ the energy dissipation might be too large to maintain sufficient velocity to reach the TAEM interface, due to $\Delta \chi_0$ the guidance system might generate too large commands and due to $\Delta C_I$ the vehicle cannot bank fast enough, which means a problem for the attitude controller. Unfortunately, this reasoning only confirms that the possibilities that were listed before may be correct. However, after studying the control effort it turned out that the elevator and rudder deflections have a large standard deviation. This indicates oscillating deflections, which is confirmed by inspecting the individual trajectories. Oscillating control surfaces usually mean a loss of control of the vehicle such that guidance commands cannot be properly executed any more, so failing of the attitude

\(^{16}\) The column assignment for the 23 factors in the $L_{32}$ array is from left to right: $C, D, E, A, B, I, J, K, F, G, H, R$, empty, $S, T, U, L$, empty, $M, N$, empty, $O, P, Q, V$, empty, $W$, empty, empty, empty, empty.
controller seems to be the main problem.

To verify whether the three mentioned factors are the only cause for the oscillations, the 'crash-simulations' are repeated with smaller dispersions. However, even though the variation in the two aerodynamic coefficients was reduced to 5%, the oscillatory deflections of the control surfaces remain a problem even for a zero $\Delta x_0$, although in two cases the vehicle did not crash any more. Due to the combinations of the dispersions, the energy-control logic of the guidance system commands a large variation in $\alpha$, which is limited to $\pm1^\circ$ in the first part of the trajectory. However, the moment this limit is relaxed the combination of a large $\delta \alpha_c$ and a bank reversal results in non-linear aerodynamic effects of the elevons and consequently oscillatory behaviour.

So how do $\Delta x_0$, $\Delta C_D$ and $\Delta C_l$ fit into this picture? Due to the initial heading error, the times for bank reversals are shifted and one of the reversals is executed at the moment the limit on $\delta \alpha_c$ is relaxed. The deficiency in $C_l$ is directly related to the bank reversals, so any aileron deflection will not have the pre-computed effect. Either the deflections are too large, resulting in a too high bank rate, or too small, which results in a large bank-angle error and thus a larger commanded aileron deflection with stronger non-linear aerodynamic effects. A variation in $C_D$ will have its effect on the energy control, because the operating principle of this part of the guidance is based on more or less dissipation of energy through the $\alpha$-drag relation.

Since the problem of a too large $\delta \alpha_c$ continues to give rise to attitude-controller problems, it was decided to adjust the limit on $\delta \alpha_c$. This limit is defined as a polynomial grid as a function of Mach number with an absolute value of $1^\circ$ from $M = 30$ down to $M = 15$. At $M = 15$, the limit linearly increases to $10^\circ$ until $M = 13$, and then further to $20^\circ$ until $M = 0$. Since the control problem occurs around $M = 15$ it is obvious that we have to reduce the function values for lower Mach numbers. The selected (experimental) values are: $\delta \alpha_c = 5^\circ$ ($M = 13$) and $\delta \alpha_c = 10^\circ$ ($M = 0$). Redoing the analysis shows that this particular problem of energy control has indeed been solved, but only to make place for other energy-control related problems. The fact that this did not show before may be related to the fact that there may be some significant factor interactions that give rise to compensating effects in one case, but become dominant in case a certain effect is suppressed.

At this moment there are still three 'crash trajectories' combined with control-surface oscillations. The combination of a drag deficiency (large $\alpha_c$) and a too large lift (large variation in $\alpha$ to reduce the vertical lift component) result again in large deflections. Moreover, in one of the trajectories the total energy of the vehicle appeared to be too low, but since the elevon was deflected at its limit value nothing could be done to reduce $\alpha$ as to decrease energy dissipation. Last but not least, in one of the trajectories it was required to deflect the elevon to $-20^\circ$ to assist the body flap in trimming the vehicle. Of course, this leaves only a small margin for attitude control before reaching the non-linear region of aerodynamics.

The origin for the above mentioned problems can be a combination of many things. However, we already indicated that the dispersion in $\tau_0$ and $\delta_0$ also seemed to have a large effect on $h_\tau$. One can imagine that if the entry point is far away from the nominal entry point a larger distance has to be covered to reach the TAEM interface. As a result the integrated dissipated
energy will be larger and unless $\alpha$ can be reduced this will result in a crash of the vehicle. Reducing $\Delta \tau_0$ and $\Delta \delta_0$ to $1.5^\circ$ and doing another analysis finally gives 32 two trajectories that do not result in a crash. In case the dispersions are fixed input values, the conclusion can only be that mission success cannot be guaranteed for HORUS with the current configuration of the guidance system and especially the attitude controller. On the other hand, by revealing the sensitivities it is possible to focus a redesign of the vehicle and/or G&C system.

**Final two-level analysis with arbitrary column assignment**

We will conclude this section by presenting the other results of this final analysis, and by comparing some of the results with the batch-#2 analysis. The final column assignment for the 22 remaining factors in the $L_{32}$ array is from left to right: $C, D, \text{empty, A, B, I, J, K, F, G, H, R,}$\empty, $S, T, U, L, \text{empty, M, N, empty, O, P, Q, V, empty, W, empty, empty, empty, empty}$. Note that factor $E (\Delta \chi_0)$ has been omitted.

The results will be presented in two categories, as listed above: guidance and attitude-control related responses. Since the main factor effects on $h_T$ have already been established for the original dispersions, and after reducing the dispersions $h_T$ does not vary so much any more it will not be further discussed. The same applies to $V_T (\bar{V}_T = 618.3 \text{ m/s, } \sigma_v = 10.7 \text{ m/s})$.

The final flight-path angle, on the other hand, shows a larger variation ($\bar{\gamma}_T = -11.16^\circ, \sigma_\gamma = 1.77^\circ$), but that is because close to the TAEM interface the final bank reversal is given. Due to the dispersions, the guidance system has more or less problems with $\gamma_{err}$ hence the large standard deviation. So because execution of the final bank reversal is the reason for the large $\sigma_\gamma$ we will suffice by stating the major effects: $\Delta Z_{cm}$ (69.4%) and $\Delta \tau_0$ (9.9%).

The maximum $g$-load - a measure for the maximum total load acting on vehicle and astronauts - has a mean of $\bar{y}_{g:\text{load}} = 1.94$ and a standard deviation of $\sigma_{g:\text{load}} = 0.26$. The total variation is influenced by many factors, as can readily be seen in Fig. 7.22. The largest variation is caused by $\Delta C_L$ (26.1%) and $\Delta C_D$ (18.4%), which was to be expected considering the definition of $g$-load. The third important factor is $\Delta X_{cm}$ (10.0%), of which the influence is indirect. A shift in the location of the c.o.m. in $X$-direction introduces an attitude-control problem, as will be discussed in more detail later. For now, it is sufficient to know that the influence of $\Delta X_{cm}$ on the variation of the maximum control errors $(\alpha_{c-\alpha})_{max}$ and $(\beta_{c-\beta})_{max}$ is 7.4% and 51.4%, respectively. This means that due to deviations in $\alpha$ and $\beta$ the magnitude of the aerodynamic force and thus the $g$-load is influenced.

The remaining guidance-related responses are thermodynamic parameters, i.e., $\dot{Q}_{max}$ and $Q$. $\dot{Q}_{max}$ has a mean of 536.5 kW/m² and a standard deviation of 27.8 kW/m², which shows that despite the fact that $\dot{Q}$ is not actively involved as a trajectory constraint, the guidance system is quite well capable of limiting $\dot{Q}_{max}$ by simply trying to follow the reference trajectory. The standard deviation in $Q$ is less than 1% of the mean and is not further discussed.
The control-related responses are divided into three groups, i.e., attitude deviations, aerodynamic-control parameters, and reaction-control parameters. For the design of the LQR, maximum attitude errors of $\Delta \alpha = 2^\circ$, $\Delta \beta = 2^\circ$, and $\Delta \sigma = 5^\circ$ were assumed to compute the controller gains, after which they were reduced to 80% to relax the transient response and to lower the chance that a sudden control action induces unwanted oscillations. This means that the maximum attitude error can be somewhat larger. Consequently, when there is an overshoot of a few degrees of the maximum attitude errors this does not necessarily mean that the attitude controller is poorly performing.

The maximum control error in $\alpha$, i.e., $(\alpha_c-\alpha)_{max}$ has a mean value of $\bar{\gamma}_{\alpha_c-\alpha} = 18.2^\circ$, whereas the standard deviation is $\sigma_{\alpha_c-\alpha} = 10.7^\circ$. This implies that in general the maximum error in $\alpha$ is too large for the designed LQR, or, in other words, the current attitude controller cannot deal with deviations from the nominal parameters that well. In addition, the large value of $\sigma_{\alpha_c-\alpha}$ indicates that there are consistent larger errors in $\alpha$. We do not know whether these large errors are single-occasion or multiple-occasion events, since the response only gives the maximum control error. To check that, either the individual trajectories should be inspected or the integrated control error should be studied, but unfortunately the latter is not available right now.

After inspecting the main factor effects, it becomes clear that especially the dispersions in the initial $\alpha$ and $\sigma$ are responsible, i.e., $\Delta \alpha_0$ (44.2%) and $\Delta \sigma_0$ (10.6%). The fact that the most important factors are attitude related may imply that the RCS is not capable of reducing these errors to zero\(^{17}\). Dispersions in the aerodynamic moment coefficients have a negligible

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\(^{17}\) It is noted that the RCS thrusters are not individually modelled. This means that the actual RCS moments are not computed using the principle of force times distance, but are directly outputted from the control laws. As
influence on the control errors, which confirms that it is indeed a problem related to the initial conditions.

In Fig. 7.23, $\alpha$, $\beta$ and $\sigma$ have been plotted as a function of flight time (run #25, which exhibits the maximum $(\alpha_c-\alpha)_{\text{max}}$). After 60 s, $\alpha$ has reached its nominal value, which is not that bad considering the deviations in the other two attitude angles. Concluding, despite the large control error the RCS does not have a poor performance, although it could be improved. Continuing with $(\beta_c-\beta)_{\text{max}}$ ($\bar{\beta}_c-\bar{\beta} \pm \sigma_{\beta_c-\beta} = 21.6^\circ \pm 11.6^\circ$) and $(\sigma_c-\sigma)_{\text{max}}$ ($\bar{\sigma}_{\sigma_c-\sigma} \pm \sigma_{\sigma_c-\sigma} = 69.0^\circ \pm 26.7^\circ$), we see in Fig. 7.23 an equally fast response as for $\alpha$. Also in this case the RCS is performing well. Due to the initial flight-path angle error ($\gamma_0 = -1.33^\circ$ for run #25), the guidance system commands a maximum $\sigma$ ($|\sigma| = 87^\circ$) to correct this. However, due to the initial roll rate, there is a significant overshoot, but still $\sigma$ stabilises at $\sigma = -87^\circ$. The error in $\beta$ is partly due to the error in $\beta_0$, $\rho_0$ and $r_0$, and partly due to the large $\sigma$ through the roll-yaw coupling. $\beta$ stabilises at a value close to zero because of the offset in the location of c.o.m.. Again, only by inspecting the individual trajectories conclusions about the total control deviation can be made.

In principle, large deviations are only expected during the bank reversals and possibly at the moment that the energy-control logic commands a large $\alpha$.

As an example, in Fig. 7.24 the control errors for run #25 are plotted (note that $\alpha_{\text{err}}$ has been omitted since the major error only occurred during the first 60 s; during the rest of the flight the attitude controller managed $\alpha$ very well). The larger errors occur only during the bank reversals; the error in $\beta$ is close to zero, but the error in $\sigma$ has a persistent non-zero value. In fact, this value is exceeding the design value of $5^\circ$ although not that much. So, considering the

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As a result, an offset in the location of the c.o.m. does not have an influence on the RCS moments. Future studies should include more sophisticated thruster models, which enables a more elaborate study of the RCS.
design specifications, the LQR is performing well for this particular simulation (apart from during the bank reversals).

![Graph showing control error angles as a function of time for run #25.](image)

Fig. 7.24 - The control error in the angle of sideslip (solid) and bank angle (dashed) as a function of time for run #25.

To conclude the discussion on the maximum control errors, the main factor effects are given for $(\beta_c-\beta)_{\text{max}}$ and $(\sigma_c-\sigma)_{\text{max}}$. The former is mainly dominated by $\Delta X_{cm}$ (51.4%), $\Delta Y_0$ (15.7%), and $\Delta \alpha_0$ (12.8%). The (indirect) effect of $\Delta Y_0$ is clearly understood, namely because of the large commanded bank angle and corresponding overshoot, a large angle of sideslip is induced. A shift of the c.o.m. in X-direction results in a change of $I_{yy}$ and $I_{zz}$, and also of $I_{xy}$ and $I_{xz}$ in combination with shifts in Y- and Z-direction. The Euler equations of rotational motion are dynamically coupled through these inertia parameters, which is at least one of the reasons for a larger induced angle of sideslip. This coupling is in itself not so strong, unless the rotational rates about the three axes are relatively large. Due to the errors in initial conditions this can indeed be true. However, further analysis of the dynamic coupling is required to draw final conclusions. Finally, it is interesting to note that the three main effects are in principle caused by factors that are related to symmetrical motion. The main effect for $(\sigma_c-\sigma)_{\text{max}}$ is caused by $\Delta Y_0$ (62.5%) which has already been discussed.

To assess the activity of the control surfaces, we have many responses at our disposal, i.e., the maximum, cumulated and mean deflection, and standard deviation related to the control-surface deflection. To avoid a lengthy discussion, we will focus on the rudder. Details on the elevons and the body flap can be found in Mooij (1998b).

It is obvious that in all simulations the rudder is at some times deflected at its maximum, because of the zero standard deviation. In principle this happens during the bank reversals. It should be considered whether dual-rudder operations are required to avoid yaw-control problems during these reversals. The cumulated deflection, i.e., the integrated product of time and absolute deviation from zero, can give an indication of which factor is the main cause for
attitude correction. The rudder usage is dominated by $\Delta C_L$ and $\Delta C_R$ which is directly related to the induced $\beta$ due to deviations in $\alpha$. In addition, $\Delta C_D$ has also a large impact which can be an indirect effect of the energy control. Due to this dispersion, control errors in $\alpha$ are introduced that can be compensated for, but only at the expense of larger deflections of the elevator. In combination with the aileron commands, there will be a significant asymmetry between the two elevons which will introduce a significant yaw moment. This moment must then be compensated for by the rudder. Note that the large standard deviation indicates an intensive use of the rudders for some dispersion combinations, again implying that the rudder effectiveness may have to be increased.

<table>
<thead>
<tr>
<th>response</th>
<th>mean</th>
<th>standard deviation</th>
<th>factor #1</th>
<th>factor #2</th>
<th>factor #3</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{r,\text{max}}$</td>
<td>40.0°</td>
<td>3.403°/s</td>
<td>$\Delta C_L$ (44.9%)</td>
<td>$\Delta C_D$ (26.4%)</td>
<td>$\Delta C_I$ (13.0%)</td>
<td>2.0%</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>40.0°</td>
<td>3.403°/s</td>
<td>$\Delta Y_{cm}$ (92.8%)</td>
<td>-</td>
<td>-</td>
<td>0.7%</td>
</tr>
<tr>
<td>$\Sigma_r$, $\Sigma_r$</td>
<td>8.0°</td>
<td>5.9°</td>
<td>$\Delta Y_{cm}$ (95.7%)</td>
<td>-</td>
<td>-</td>
<td>1.2%</td>
</tr>
<tr>
<td>$\Sigma_r$, $\Sigma_r$</td>
<td>8.2°</td>
<td>4.6°</td>
<td>$\Delta Y_{cm}$ (90.4%)</td>
<td>-</td>
<td>-</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\Sigma_r$, $\Sigma_r$</td>
<td>5.8°</td>
<td>4.5°</td>
<td>$\Delta Y_{cm}$ (93.0%)</td>
<td>-</td>
<td>-</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

Table 7.5 - Results for rudder-related responses.

The mean rudder deflection angle represents a trim value and should nominally be zero (corrective function). However, the mean is non-zero with a large standard deviation, which can almost completely be traced back to the Y-offset in the c.o.m.: $\Delta Y_{cm}$ introduces a persistent non-zero $\beta$, that is compensated for by aileron and rudder. The rudder has a large trim value, because yaw control is the primary task of the rudder. Note that interactions are hardly influencing the trim value. The standard deviation related to the rudder deflection should in principle be zero or close to zero, which indicates (almost) constant mean deflections, maybe exhibiting a single peak. The larger the standard deviation, the more the control surfaces are oscillating which might lead to attitude-control problems in real-life flight situations. Since we more or less 'removed' any severe oscillations from the results, because they resulted in a complete loss of control and consequently a loss of the vehicle, we expect not so large values for the standard deviation. The rudder, however, has quite a large standard deviation (≈ 6°), and on top of that a significant $\Sigma$ which suggests persistent larger values and therefore stronger oscillations. Evidently the smaller oscillations in the elevons induce an angle of sideslip that are hard to control by the rudder, because of the offset in the c.o.m. ($\Delta Y_{cm}$). Also before we have seen the strong coupling between roll and yaw motion, but this motion is amplified by controlling the induced motion. This should be a focus for further study, especially since yaw control is already relatively weak. The results show further that a deviation from the nominal location of
the c.o.m. introduces a point of attention for the control system.

Due to the weak rudder control, the yaw thrusters are used significantly, and more than once at their maximum level. This indicates that during operation of these thrusters the maximum control is not sufficient to compensate the attitude errors, although no instabilities are introduced. Still, it is a point of concern in combination with the rudder-control problems. The cumulated effort of the RCS is shown in the fuel consumption. During the nominal trajectory \( m_{\text{fuel, tot}} \) is 10 kg. Due to the dispersions, the average consumption increases to 138.7 kg (\( \sigma_{m_{\text{fuel, tot}}} = 19.8 \) kg), most of it consumed by the yaw thrusters, since they operate throughout the trajectory. Most fuel is consumed to correct an induced \( \beta \) due to strong bank control. It is interesting to note that \( \Delta Y_{\text{crn}} \), which gives a persistent non-zero \( \beta \) does not have much of an influence on the variation of \( m_{\text{fuel, yaw}} \); the effect of compensating for \( \beta \) is shown directly in the mean fuel consumption and is in fact the largest contributor.

Left-most column assignment

Before we try to identify the interactions and quadratic effects of a selected response in a three-level analysis, we will briefly describe the results of the second batch\(^{18}\). Using the same factor dispersions, a peculiar result was obtained from this analysis. One of the trajectories, namely, resulted in a crash of the vehicle. The fact that this happened is in itself not so strange and can be explained as follows. By using different columns for assigning the factors, different (but similar) factor combinations are input to the simulations. If one combination will lead to a crash, this means that factor influences cannot be studied independently but only in combinations, and in principle all combinations should be investigated. Moreover, when also the levels and possible interactions have an influence, then only a Monte-Carlo type of analysis will suffice. But, since many simulations will be involved it is advised to do this only later once the system is understood better and a verification analysis is required.

Another aspect that is more critical is the following. By inspecting the results from the ANOVA it appears that the factor variations do not resemble the variations of the batch-#1 analysis. Each of the responses that is directly affected by a crash, e.g., the distance to target, the final position and velocity, or indirectly, e.g., the maximum decelerations and thermodynamic parameters that are large because of the control problem that caused the crash, will show the effect. In many of the responses, each of the column variations appeared to be more or less the same (about 3% for the 31 columns). Furthermore, for some of the responses with one or two strong main effects, the factor variation was considerably reduced while other, small variations were increased. This means that one 'bad' data point, i.e., the crash, has a large impact on the 'good' data points, i.e., normal descents, and the factor contribution to the total variation cannot be relied upon.

---

\(^{18}\) The column assignment of the \( L_{3b} \) array is, from left to right: \( C, D, A, B, I, J, K, F, G, H, R, S, T, U, L, M, N, O, P, Q, V, W \), empty, empty, empty, empty, empty, empty, empty, empty.
An important conclusion can be drawn from the above results. Taguchi (1988) and Phadke (1989) state that a poor result is also a result and should be included in the analysis. This is basically true if these results can be used to discover a trend, as done in the 'crash' analysis. However, when there is one physically poor result then it cannot be used to identify main effects; on the contrary, because it is an isolated case it will have effect on all the results. One can compare this with a least-squares approximation when the number of data is only slightly larger than the number of model coefficients. It is easy to understand that in case one of the data is deviating significantly, it will pull the surface away from the other data points resulting in a worse approximation.

7.3.3. Interactions and quadratic effects

To address the issue of potential interactions and quadratic effects, we will do some additional simulations but then only applied to one response, i.e., the g-load, to serve as an example for the analysis process. For an analysis of more than one response at the same time, the column assignment can become very complicated. In that case it is wise to begin with only a few factors such that without confounding all interactions can be studied, a result that holds for all responses. Here, with a two-level analysis we try to identify the interactions and repeat this with a response-surface method. The latter will also give information about the quadratic effects.

The main factor effects on the g-load are given by $\Delta C_L$ (26.1%), $\Delta C_D$ (18.4%) and $\Delta X_{cm}$ (10.0%). The error variation is 14.3% and comes mainly from columns #26 (6.7%) and #31 (2.7%). The magnitude of these two column variations serves as a warning to treat smaller main effects such as $\Delta \alpha_0$ (5.2%) with caution, because they are smaller than the larger interaction variation. Of course the other main effects can be confounded with interactions as well, but since these variations are rather large they can be called main effects without absolute reference to the individual variations. Column #26 represents the following possible interactions, as taken from the interaction table for $L_{32}$ (Phadke, 1989): $\Delta V_0 \times \Delta T$, $\Delta \gamma_0 \times \Delta C_m$, $\Delta r_0 \times \Delta C_L$, $\Delta \alpha_0 \times \Delta Z_{cm}$, $\Delta \beta_0 \times \Delta C_D$, $\Delta m_0 \times \Delta C_i$ and $\Delta X_{cm} \times \Delta C_L$.

So far we listed 6 factors that give main effects, and an additional 10 factors that are included in the 7 interactions. For a first analysis an $L_{32}$ is selected. This array has 31 columns which can be used for as many factors and interactions. To avoid confounding only 6 factors can be assigned to these columns, although some interactions may be discarded such that more columns are available to assign factors. It is assumed that beside the listed ones no other interactions are present, at least not for the selected response. Note that when the identified main effects and interactions are indeed the only ones, it is possible to use the results of the $L_{32}$ analysis as basis for a CCD. By adding the related axial points and a centre point, the quadratic effects can be addressed. This would reduce the number of simulations if in comparison with a full 3-level analysis, because in the latter case the simulation results of $L_{32}$ cannot be included.
The factors and interactions are assigned to the columns in the following order: $\Delta C_L$, $\Delta C_D$, $\Delta X_{cm}$, $\Delta C_L \times \Delta X_{cm}$, $\Delta V_0$, $\Delta r_0$, $\Delta C_L \times \Delta r_0$, $\Delta \beta_0$, $\Delta T$, $\Delta C_D \times \Delta \beta_0$, $\Delta \gamma_0$, $\Delta V_0 \times \Delta T$, $\Delta \alpha_0$, $\Delta m$, $\Delta \sigma_0$, $\Delta C_m$, $\Delta Z_{cm}$, $\Delta \rho_0$, $\Delta C_p$, $\Delta \gamma_0$ (empty, empty, empty, empty, empty, empty, empty, $\Delta \gamma_0 \times \Delta C_m$, $\Delta \alpha_0 \times \Delta Z_{cm}$), $\Delta m \times \Delta C_r$ (empty, empty, empty, empty). The simulations give $\bar{y}_{g-load} = 1.91$ and $\sigma_{g-load} = 0.17$, which is close to the original 1.94 and 0.26. The differences can easily be explained by the omission of some of the factors, thereby also ignoring other interactions. Looking at the factor variations, some differences are found. In decreasing order of magnitude the variations read, with the original value listed as second entry: $\Delta C_L$ (45.4%, 26.1%), $\Delta C_D$ (18.6%, 18.4%) and $\Delta X_{cm}$ (10.2%, 10.0%). It appears that especially $\Delta C_L$ has changed, which indicates that the corresponding column must have been (or is) confounded. It can therefore be concluded that it is difficult to address the issue of interactions in this manner, because these very interactions obfuscate the results in the first place. This makes it difficult to plan the next step in the analysis. Note that with the current analysis the error variation is 7.3%, with the main contribution of column #31 (4.0%).

An alternative to the above approach is to assign only a limited number of factors to the columns such that confounding is avoided. To illustrate this, a second analysis is based on five factors ($\Delta \gamma_0$, $\Delta X_{cm}$, $\Delta C_D$, $\Delta C_L$ and $\Delta T$) assigned to the columns of the Resolution V array $L_{16}$. The contribution to the total variation in $g$-load ($\bar{y}_{g-load} \pm \sigma_{g-load} = 1.89 \pm 0.14$) are as follows: $\Delta C_L$ (67.4%), $\Delta C_D$ (13.5%), $\Delta X_{cm}$ (7.8%), $\Delta \gamma_0$ (1.3%) and $\Delta T$ (0.5%). The three largest contributors are still the same, but the percentages have changed again, also because fewer factors are studied. Since these values are free from confounding, we can at least be certain of them. The 10 interactions contribute 9.6% to the total variation, divided as follows: $\Delta \gamma_0 \times \Delta X_{cm}$ (1.1%), $\Delta \gamma_0 \times \Delta C_D$ (1.2%), $\Delta \gamma_0 \times \Delta C_L$ (1.3%), $\Delta \gamma_0 \times \Delta T$ (0.9%), $\Delta X_{cm} \times \Delta C_D$ (0.6%), $\Delta X_{cm} \times \Delta C_L$ (1.0%), $\Delta X_{cm} \times \Delta T$ (0.9%), $\Delta C_D \times \Delta C_L$ (0.5%), $\Delta C_D \times \Delta T$ (1.0%) and $\Delta C_L \times \Delta T$ (1.1%). So although the individual interactions in case of the $g$-load are not very strong, the combined effect cannot be ignored. Note that in case of $h_{err,max}$ ($\bar{y}_{h_{err,max}} \pm \sigma_{h_{err}} = 2.240 \pm 0.225$ m), the two main effects are caused by $\Delta \gamma_0$ (81.0%) and $\Delta C_L$ (9.0%), whereas the only significant interaction $\Delta \gamma_0 \times \Delta C_L$ contributes 9.5%. So in this case it is a single interaction that cannot be ignored.

To address the quadratic effects of $\Delta \gamma_0$, $\Delta X_{cm}$, $\Delta C_D$, $\Delta C_L$ and $\Delta T$ on the $g$-load, two approaches are possible. In the first place, the results of the $L_{16}$ analysis can be used as a basis for a CCD. The computed response surfaces will give an idea about the quadratic effects by inspecting the coefficients of the quadratic terms. In this case, 11 additional simulations are required. In the second place we can do a three-level analysis. If we want to avoid confounding, however, an $L_{81}$ array is required for the 5 factors. On the other hand, by ignoring the smaller interactions the smaller $L_{27}$ array suffices.

Here, we will stick to the first approach. The coefficients of the response surface are given in Table 7.6, thereby reminding that subscripts '1' .. '5' are related to $\Delta \gamma_0$, $\Delta X_{cm}$, $\Delta C_D$, $\Delta C_L$ and $\Delta T$. After inspecting the coefficients, comparing the relative magnitudes and taking the variation contributions into consideration, it can be concluded that quadratic effects are present in the total variation of the maximum $g$-load. The relative influence ranges from 10 to 100% of the
linear terms. Note that $b_4$, representing $\Delta C_L$, has also in this case the largest impact on the variation of $g\text{-load}_{\text{max}}$. Its value is about twice as large as $b_3$ ($\Delta C_D$) in absolute sense, as compared with five times the value that we found before with the two-level analysis. This means that conclusions drawn from results obtained from a linear analysis should be regarded carefully, if the existence (or, equivalently, the absence) of quadratic effects has not been addressed. Similarly, the current results including quadratic effects should also be treated carefully until higher-order effects have been studied.

<table>
<thead>
<tr>
<th></th>
<th>linear</th>
<th>interaction</th>
<th>quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>1.913</td>
<td>$b_{12}$</td>
<td>$b_{11}$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-0.011</td>
<td>$b_{13}$</td>
<td>$b_{22}$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.033</td>
<td>$b_{14}$</td>
<td>$b_{33}$</td>
</tr>
<tr>
<td>$b_3$</td>
<td>-0.064</td>
<td>$b_{15}$</td>
<td>$b_{44}$</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0.119</td>
<td>$b_{23}$</td>
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</tr>
<tr>
<td>$b_5$</td>
<td>-0.006</td>
<td>$b_{24}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.6 - Response-surface coefficients for $g\text{-load}_{\text{max}}$.

In the above discussion we have only briefly addressed the effect of interactions and quadratic terms, and then only applied to a single response. A full analysis of all the responses, including higher-order effects would not serve the purpose of the current study and is therefore not pursued. This exercise merely addressed the analysis techniques rather than the physical aspects of the related effects. We do not imply that these physical aspects can be neglected, but only that they remain to be studied.

7.4. Robust design of a guidance system

7.4.1. Introduction

The guidance system of HORUS is supposed to steer the vehicle along a pre-defined nominal trajectory, whenever deviations from that nominal trajectory occur due to uncertainties in vehicle parameters, initial conditions or atmospheric conditions. The guidance system does not directly keep track of trajectory constraints, such as a maximum heat flux, a maximum $g\text{-load}$ or a maximum integrated heat load. It would therefore be interesting to have a guidance system that
will minimise the deviations from the maximum allowable constraints in case disturbances occur that cannot be controlled or predicted. In other words, the guidance system should be robust with respect to the disturbances.

To illustrate the benefits of robust-design techniques two sets of simulations will be executed. The first set contains a sensitivity analysis with respect to the guidance input parameters applied to the nominal mission. By varying the guidance parameters it must become clear which combination of parameters results in the minimum values of the trajectory constraints. The second set contains a double sensitivity analysis, i.e., the first one will be based on varying the guidance input parameters (control factors), in a similar manner as for the first set of simulations, and the second one will be based on varying some vehicle and mission dependent parameters (noise factors). As discussed in Section 4.5, the two sensitivity analyses are to be considered as an outer loop and an inner loop: for each of the outer-loop control-factor combinations, an inner-loop sensitivity analysis is conducted resulting in a signal-to-noise ratio. Maximising the signal-to-noise ratios must then lead to the most robust design of the guidance system with respect to the considered disturbances.

The outer-loop design will be based on a Central Composite Design that must enable us to compute a second-order response surface, i.e., a relation between the trajectory constraints and linear, interaction and quadratic terms of the guidance parameters. An additional result of this exercise will then also be whether interactions between and quadratic effects of the guidance parameters play an important role in the design of this particular guidance system. The related guidance parameters including their nominal values are given in Table 3.1. Not all of these parameters will be taken into consideration, because of the relatively large number of simulations that is involved in a CCD. For that matter, the main intention is to show the benefits of robust design which can also be achieved with a limited number of parameters.

Part of the CCD consists of a two-level factorial design that should give enough data to study the main effects of the control factors and interactions between them. Since we do not know whether some interactions can be excluded from the response surface, it is necessary to take all interactions into account. Stanley et al. (1993) used an $L_{16}$ array for the inclusion of five design parameters and their interactions. Applying this array results in a Resolution-V array, as we discussed before.

Suppose we want to include all of the 10 guidance parameters of Table 3.1 in the CCD. Ten factors have 45 interactions, so one would expect that for these 55 degrees of freedom the $L_{64}$ could be used, but to avoid confounding a much larger array will have to be used. Usually, there is only a limited number of interactions that can be assigned to columns to guarantee a Resolution-V array. This makes the simultaneous study of a larger number of design parameters a complicated and time consuming job, because of the column assignment and the relatively large number of experiments.

Resolution-V arrays are $L_{16}$ for five factors and $L_{32}$ for six. However, Fowlkes and Creveling (1995) indicate that by allowing some two-level interactions a near Resolution-V experiment can be obtained for 7 factors assigned to $L_{32}$, where the confounded columns are
3, 5 and 6. Since the total number of simulations in this CCD is still acceptable, i.e., \( N_{\text{min}} = 47 \), we will select seven out of the ten available guidance parameters and try to establish a second-order response surface. The column assignment for factors and interactions in the factorial part \((L_{32})\) is, from left to right: \(A, B, A \times B\) and \(C \times D, C, A \times C\) and \(B \times D, A \times D\) and \(B \times C, D, E, A \times E, B \times E\), empty, \(C \times E, F \times G,\) empty, \(D \times E, F, A \times F, B \times F,\) empty, \(C \times F, E \times G,\) empty, \(D \times F, E \times F, C \times G, D \times G,\) empty, \(A \times G, G,\) empty, \(B \times G.\) From this column assignment it is also clear that by removing factor \(D,\) the array can be made \(\text{Resolution V}.\) The guidance parameter dispersions can be found in Table 7.7. Note that the nominal value of \(c_{E,2}\) has been changed from 0 to 2.5 to avoid that this parameter becomes negative at its minimum level.

<table>
<thead>
<tr>
<th>factor</th>
<th>guidance parameter</th>
<th>dispersion</th>
<th>% of nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(K_E)</td>
<td>4.0</td>
<td>40.0</td>
</tr>
<tr>
<td>(B)</td>
<td>(c_{E,1})</td>
<td>1.75 \times 10^{-6}</td>
<td>10.0</td>
</tr>
<tr>
<td>(C)</td>
<td>(c_{E,2})</td>
<td>1.0</td>
<td>40.0</td>
</tr>
<tr>
<td>(D)</td>
<td>(c_{n,1})</td>
<td>0.005</td>
<td>16.7</td>
</tr>
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<td>(E)</td>
<td>(c_{n,2})</td>
<td>0.05</td>
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</tr>
<tr>
<td>(F)</td>
<td>(\gamma_r)</td>
<td>1.0</td>
<td>10.0</td>
</tr>
<tr>
<td>(G)</td>
<td>(\Delta_n)</td>
<td>10^{-4}</td>
<td>10.0</td>
</tr>
</tbody>
</table>

Table 7.7 - Guidance-parameter dispersions.

For the inner-loop array containing the noise factors it is not important to determine the interactions between those factors, since it is sufficient that the fractional factorial design covers the experimental region (or, in this context, noise region) as well as possible. As noise factors, dispersions in the initial conditions \(V_0 (\pm 20 \text{ m/s}), \gamma_0 (\pm 0.1^\circ), \chi_0 (\pm 2^\circ), \tau_0 (\pm 1.5^\circ)\) and \(\delta_0 (\pm 1.5^\circ),\) mass \(m (\pm 80 \text{ kg}),\) aerodynamic coefficients \(C_D (\pm 5\%)\) and \(C_L (\pm 5\%),\) and atmospheric properties \(\rho (\pm 5\%)\) and \(T (\pm 5\%)\) are considered. These noise factors are assigned to the first 10 columns of an \(L_{16}\) array. In the discussion of the responses, we will focus on \(g_{load_{\text{max}}}\) to show the benefits of this robust design method. Some remarks will be made on \(\dot{Q}_{\text{max}}\) and \(Q,\) which are used to identify the variation in the trajectory constraints, since a large variation may result in severe damage of the vehicle. In addition, \(h_{\text{err}_{\text{max}}}\) and \(\gamma_{\text{err}_{\text{max}}}\) are included to get an impression of how closely the guidance system can steer the vehicle towards the reference trajectory, although integrated errors would have been better. For more details on other selected responses, the reader is referred to Mooij (1998b).

7.4.2. Single-loop design

The first design, i.e., a single-loop CCD for the guidance parameters in a nominal mission, is
a rotatable design. This means that 1 centre point is included, whereas the axial points are located at \( \alpha = 2.3 \). Notice that this results in maximum dispersions that are 2.3 times higher than the ones indicated in Table 7.7, which is considered to be a satisfactory size of the 7-dimensional design space. The computed second-order response surfaces (Mooij, 1998b) indicate that for the given variations in guidance parameters the guidance system is robust when it concerns the trajectory constraints. The computed minimum and maximum value of \( g:\text{load}_{\text{max}} \) are 1.87 and 1.90, respectively (for \( \hat{Q}_{\text{max}} \) and \( Q \) the two extremes are even closer. The quality of the response surface is given by the coefficient of determination \( R^2 \), which is 99.99%, 99.28% and 96.60% for the three constraints.

When it comes to the performance of the guidance system with respect to following the reference trajectory, \( h_{\text{err, max}} \) \( (R^2 = 99.97%) \) and \( \gamma_{\text{err, max}} \) \( (R^2 = 67.61%) \) show more dependence on the guidance parameters. For both responses, there are significant interaction and quadratic terms and in case of \( \gamma_{\text{err, max}} \) higher-order effects are likely to exist because of the low value for \( R_2 \). Since the trajectory constraints will not be influenced, the optimum settings of guidance parameters can be calculated that minimise the maximum errors. This results in a minimum \( h_{\text{err, max}} \) of 346.4 m (normalised values of \( K_E = -2.30 \), \( c_{E,1} = 0.80 \), \( c_{E,2} = -1.72 \), \( c_{h,1} = 2.30 \), \( c_{h,2} = 2.30 \), \( \gamma_r = 1.09 \) and \( \Delta_n = 0.05 \)). The maximum \( h_{\text{err, max}} \) would be 2,146.3 m. For what is worth it, the minimum \( \gamma_{\text{err, max}} \) would be 1.55° (normalised values of \( K_E = -2.30 \), \( c_{E,1} = 2.30 \), \( c_{E,2} = -2.30 \), \( c_{h,1} = -2.30 \), \( c_{h,2} = 2.30 \), \( \gamma_r = -0.01 \) and \( \Delta_n = -2.30 \)), and its maximum 2.15°.

7.4.3. Dual-loop design

The second design, i.e., a double-loop analysis with a rotatable CCD for the control factors and a two-level \( L_{16} \) for the noise factors, is executed for the same guidance-parameter settings as in the first design. For each of the 47 designs, i.e., combination of control factors, 16 simulations for the noise factors are executed \( (N = 47 \times 16 = 752) \). Fig. 7.25 shows \( \bar{y}_{\text{h},f} \) and \( \sigma_{\text{h},f} \) for each of the designs. It appears that in many cases the standard deviation is rather large. This can only indicate that out of the 16 simulations per design, one or more lead to a crash of the vehicle. Inspecting the individual final altitudes confirms this. So here we already see the first advantage of the robust-design methodology, since variation of the guidance parameters in the nominal mission did not lead to any crashes. Note that in case of a sensitivity analysis for the nominal guidance system, i.e., design #47 or the centre point of the CCD, the standard deviation is small, indicating no crash trajectories. However, for this design the standard deviation is not smallest in absolute sense. In many other designs, the standard deviation is practically zero, which means that the final altitude is insensitive to the noise factors. This fact can be used, if one of the design goals of the guidance system is to guarantee a specific final altitude.
Fig. 7.25 - Mean and standard deviation of $h_f$ for the 47 designs.

Fig. 7.26 - Mean and standard deviation of $g_{load_{max}}$ for the 47 designs.

Fig. 7.26 shows the maximum $g$-load. The differences between the designs is even more striking. Most crash trajectories have large mean values and standard deviations, some of them being very large. This basically indicates that for a substantial number of trajectories within that design (i.e., #5, #6, #29, #30, #38), $g_{load_{max}}$ is far too high resulting from an incapable guidance system, whilst for the others the $g$-load is within limits. Design #47 exhibits a rather large standard deviation, so that also for the nominal guidance parameters in some trajectories the maximum allowable $g$-load will probably be exceeded. In this sense, the 'best' design is #37,
which has the smallest mean value and a standard deviation of practically zero. This means that for that combination of guidance parameters the vehicle becomes insensitive to $g\text{-load}_{\text{max}}$

Note that design #37 represents the nominal settings for the guidance parameters, apart from factor $C$ (i.e., $c_{E,2}$) that has the value of the negative axial point. And this value is the original value as defined by MBB, since for the sake of the analysis we shifted the nominal value. So with this exercise we have confirmed that the MBB design is the most robust with respect to the maximum $g\text{-load}$. As for the other trajectory constraints $Q_{\text{max}}$ and $Q$ despite the crash trajectories, they are not so much influenced. Therefore, we will not further discuss them.

A measure for the robustness of the design is the signal-to-noise (S/N) ratio. In Fig. 7.27, the Smaller-The-Better (STB) and Nominal-The-Better (NTB) S/N have been plotted for the maximum $g\text{-load}$. In a way, a large STB S/N is related to a small mean value, whereas a large NTB S/N ratio is related to a small standard deviation. Improvement of the design is shown in increasing the S/N ratio. In case of design #37, both the STB and the NTB S/N ratio are the largest which gives the best overall performance. However, it is also possible to compute the second-order response surface for these 47 S/N ratios and fine tune the parameter settings. Maximising the resulting surface should yield the optimal guidance parameters with respect to robustness.

![Figure 7.27 - S/N ratios of $g\text{-load}_{\text{max}}$ for the 47 designs.](image)

In doing so, an STB S/N-ratio response surface is obtained in which both linear, interaction and quadratic terms are represented. With $R^2 = 93.42\%$, the response surface seems to be an adequate approximation. When the surface is maximised, the following normalised guidance-parameter settings are found: $K_E = -2.300$, $c_{E,1} = -0.002$, $c_{E,2} = -2.300$, $c_{h,1} = -2.300$, $c_{h,2} = 2.300$, $\gamma_r = 0.000$ and $\Delta_h = -0.015$ for a maximum STB S/N ratio of 2.9 dB, as compared with STB S/N = -5.7 for design #37. However, a verification analysis with these guidance parameters
and the same noise factors as before yields a STB S/N ratio of -5.7 dB. In trying to explain this, we found that the optimisation algorithm is quite sensitive for the initial guess of the location of the extreme for the type of functions that is examined here. Application of other optimisation algorithms should be studied and compared with the current one. Furthermore, the coefficient of determination is an indication of the quality of the approximation, but can apparently not be used in an absolute sense. This means that other terms may have to be included in the model. One conclusion is obvious: a verification run/design is always required to check numerical predictions!

Note that while optimising the response surfaces for the STB S/N ratios for $g_{\text{load}}_{\text{max}}$ and, for instance, $\gamma_{\text{err}}$, conflicting values for the guidance parameters can be found. This can of course always happen when multiple responses are to be optimised. When both (or more) responses should be optimised simultaneously, the single-objective optimisation algorithm that was used here should be replaced by a multiple-objective optimisation algorithm. Application of such an algorithm is considered to be beyond the scope of the current study.

7.5. Summary

- Flight simulation of HORUS-2B, an unpowered, winged re-entry vehicle, includes closed-loop guidance with horizontal and vertical manoeuvres, combination of reaction and aerodynamic attitude control by means of a Linear Quadratic Regulator, and the use of a body flap combined with elevons for trim. For executing the simulations the Taguchi Method is fully used, whereas ANOVA and RSM is employed to analyse the data. By means of a robust-design method the guidance-system performance is studied in combination with disturbances.

- The 3-d.o.f. reference trajectory is computed with a specified angle-of-attack and bank-angle profile. The operating principle of the guidance system is such that by tracking this reference the trajectory constraints will be automatically met. Confining the heading error to a pre-defined dead band forces the vehicle to execute bank reversals that will guide the vehicle to the landing area. A verification simulation in 6 d.o.f. proves that the G&C system does not have problems in reaching the landing area. Moreover, the results show that the G&C system has been correctly implemented.

- The major findings from the 6-d.o.f. results are the following. Because the bank reversals are executed in a finite time, $\gamma$ errors up to $2^\circ$ are induced that result in altitude errors. The guidance system is robust enough to correct these. The commanded bank rate is too large to be initially handled by the attitude controller, which results in a large $\sigma$-error (up to $25^\circ$), although this error can be reduced to zero in a short time. A roll-yaw coupling is present that shows as an induced $\beta$ of up to $2.5^\circ$ during the bank reversals. This $\beta$ can be cor-
rected although both the yaw jets and the rudder are used at their maximum capacity.

- The advantage of applying Taguchi's orthogonal arrays in a sensitivity analysis is that the number of output data is relatively small, which makes it easier to process and understand the outcome. However, there are some points of attention. The column assignment of factors is important in two ways. In the first place, when interactions between columns are present, the column variances are confounded, which in practice means that main factor effects are influenced. In the second place, assigning factors to different columns gives different (although similar) factor combinations that can result in a different system response. Factor interactions can be studied by leaving the appropriate column(s) of the orthogonal array empty. Linear effects and interactions can be studied by using a 2-level array, whereas for quadratic effects a 3-level array is required. One 'bad' data point, e.g., a system failure, has a significant influence on the results. In case there are more of these 'bad' data points, the results can be used to discover a trend, as we did with the crashes of HORUS.

- During the bank reversals a significant altitude and energy error is induced, that is only corrected after finalising the reversal. This results in steps in $\alpha_c$ and $\sigma_c$ and consequently a rather large attitude-control activity. An adjustment in the guidance logic that foresees the altitude error at the beginning of the reversal and adjusts $\alpha_c$ beforehand is one way to deal with this. The energy control logic includes a constraint on the commanded variation of $\alpha$. Relaxing this constraint when the guidance system is faced with a large energy error leads to loss of control of the vehicle. By adjusting the constraint, this particular control problem could be solved, but it remains to be studied in more detail.

- Towards the end of the trajectory, $\beta$ slowly diverges. This seems assumed that this diversion is due to the simplifications made during the control-system design, e.g., neglecting terms due the rotation of both the Earth and the local horizontal plane. This indicates that the translational motion has at least some influence on the rotational motion, which the LQR in its current design cannot compensate for completely.

- The LQR has a reasonable performance although the current design has a problem to handle non-linearities in the aerodynamics of the control surfaces. Especially at large deflections this leads to control errors. In case of consecutive large deflections this may lead to diverging oscillations that results in a complete loss of control. The attitude controller should be made more robust to avoid this problem.

- The robust design of the HORUS guidance system confirmed that the initial design is most robust when it comes to minimising the maximum $g$-load under the influence of disturbances. In case a single-loop CCD is applied to the nominal mission, no vehicle crashes occur, whereas in the dual-loop robust design crashes did occur. It was found that in
optimising the response surface, the applied algorithm was sensitive to the initial guess of the location of the optimum. Moreover, a verification run for the optimal settings of the design variables did not result in the predicted optimum. On one hand it indicates that the response surface may not have been a good prediction of the results, and on the other hand the importance of the verification run has been clearly established.

- Other recommendations that arose from this study are:
  - Dual-rudder operation of the rudders should be considered for improved yaw control.
  - It is advised to begin a sensitivity analysis with only a few factors to get insight in the linear and higher-order effects and possible interactions. CCD is a good method in that case.
  - In case one has no knowledge about possible interactions and many factors have to be varied, then application of Taguchi may not be so practical. Either one should study only a few factors at a time to avoid confounding, or a Monte-Carlo analysis should be used to try and identify interactions and higher-order effects. Monte-Carlo analysis may also be used for verification of the results after an experimental design has converged.
  - To design the aerodynamic shape and/or the control system it is necessary to define the aerodynamic coefficients as individual factors. This will give more insight where the actual influence on a selected response comes from.
Chapter 8

The Powered Flight of an SSTO Space Plane

A man with wings large enough and duly attached might learn to overcome the resistance of the air, and conquering it succeed in subjugating it and raise himself upon it. Leonardo da Vinci

Whereas in the previous chapter the guided and controlled entry and descent of a space plane was discussed, this chapter deals with the powered flight of an SSTO space plane and has the following goals.

First, of course, it serves as a verification of the mission-analysis capability of START applied to guided, controlled and powered flight. Second, by doing a sensitivity analysis with respect to the trajectory parameters that specify the flight-path angle profile, the relative importance of these parameters is identified and their contribution to the variation in the payload mass is computed. Third, a Model Reference Adaptive Control (MRAC) system is developed to assess the potential of this kind of attitude-control system. Fourth, by simulating the powered ascent to orbit and some related hypersonic manoeuvres, the flight mechanics are studied, of course within the modelling accuracy of the applied reference vehicle.

To this extent, the following topics will be discussed. In Section 8.1, the SSTO reference vehicle that is used for this study is introduced. This Winged-Cone Configuration, developed by NASA to do performance and control studies, consists of numerical models of the aerodynamics, the mass properties and the propulsion system. For this vehicle, a reference trajectory is defined in Section 8.2, which is then further refined by sub-optimising the related trajectory parameters by using Taguchi’s orthogonal arrays. Section 8.3 presents the design of an MRAC system, taking the sub-optimal reference trajectory as input to specify the reference model. The developed control system is applied in Sections 8.4 and 8.5. In the former section, controlled
hypersonic manoeuvres, i.e., cruise flight (altitude hold), altitude transition and heading changes are studied to judge the potential of this kind of controller. The latter section deals with the vertical ascent to orbit. In both cases the influence of disturbances on the performance of the attitude controller are addressed.

8.1. The Winged-Cone Configuration reference vehicle

The Winged-Cone Configuration (WCC) reference vehicle, also known as the Langley accelerator, is a generic, horizontal take-off, SSTO configuration that can be used for point-mass as well as batch and real-time 6-d.o.f. simulations (Shaughnessy et al., 1990), see also Fig. 8.1 for an artist impression. Sofar, the WCC has been successfully used in many optimisation and control-system design studies, see, for instance, Hattis et al. (1991), Powell et al. (1991), Van Buren and Mease (1992), Lu (1995) and Gregory et al. (1994). The combined database consists of aerodynamic, propulsion and mass models, that will be briefly described below. For the numerical values, the reader is referred to Shaughnessy et al. (1990).

The WCC has a dry mass of 58,968 kg and is composed of an axisymmetric 5° half-angle conical forebody, a cylindrical engine nacelle section, and a cone frustum nozzle (Powell et al., 1991). The wing has a leading-edge sweep of 78° and is set at 0° incidence and dihedral. The wing is a 4% thick diamond airfoil. Elevons are located at the trailing edge of the wing with their hinge line perpendicular to the fuselage centre line, and positive deflections are with the trailing edge down. The vertical tail is a 4% thick diamond airfoil with a leading-edge sweep angle of 70° and includes a rudder with a hinge line at the 75% chord position measured from the leading edge. Positive rudder deflections are with the trailing edge left. The canards have a 6% thick symmetrical 65A series airfoil, are deployed only at subsonic speeds, and are kept at 0° incidence relative to the fuselage centre line. Positive canard deflections are with the trailing edge down. The take-off mass of the vehicle is 136,079 kg. The main geometric characteristics can be found in Table 8.1 and Fig. 8.2.

The aerodynamic database for the WCC was generated using the Aerodynamic Preliminary Analysis System (APAS), a collection of algorithms to compute aerodynamic coefficients in the subsonic, supersonic and hypersonic velocity range. Also the control effectiveness and dynamic derivatives can be calculated. Data are given as a function of $M$, $\alpha$, $\beta$, $p$, $q$, $r$ and control-surface deflection angles, and are defined with respect to the moment reference centre.

The propulsion model was developed using a two-dimensional forebody, inlet, and nozzle code with a one-dimensional combustor code. The thrust and specific impulse of the all air-breathing engine, $T$ and $I_{sp}$, were determined as functions of $M$, $q_{dyn}$ and $\phi_T$. The effects of $\alpha$, $\beta$, $p$, $q$, $r$ and control-surface deflections on $T$ and $I_{sp}$ were assumed to be negligible for the current configuration of the WCC.
Fig. 8.1 - An artist impression of the Winged Cone Configuration.
Table 8.1 - WCC geometric characteristics.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>fuselage length</td>
<td>61.0 m</td>
</tr>
<tr>
<td>cone half angle</td>
<td>5°</td>
</tr>
<tr>
<td>cylinder radius</td>
<td>3.9 m</td>
</tr>
<tr>
<td>cylinder length</td>
<td>3.9 m</td>
</tr>
<tr>
<td>boat-tail half angle</td>
<td>9°</td>
</tr>
<tr>
<td>boat-tail length</td>
<td>12.2 m</td>
</tr>
<tr>
<td>moment reference centre from nose</td>
<td>37.8 m</td>
</tr>
<tr>
<td>wing reference area</td>
<td>334.7 m²</td>
</tr>
<tr>
<td>aspect ratio</td>
<td>1.0</td>
</tr>
<tr>
<td>span</td>
<td>18.3 m</td>
</tr>
<tr>
<td>mean aerodynamic chord</td>
<td>24.4 m</td>
</tr>
<tr>
<td>elevon area</td>
<td>8.6 m²</td>
</tr>
<tr>
<td>chord</td>
<td>2.2 m</td>
</tr>
<tr>
<td>rudder area</td>
<td>15.0 m²</td>
</tr>
<tr>
<td>span</td>
<td>7.0 m</td>
</tr>
<tr>
<td>chord (relative to tail)</td>
<td>25%</td>
</tr>
<tr>
<td>canard area</td>
<td>14.3 m²</td>
</tr>
<tr>
<td>aspect ratio</td>
<td>5.48</td>
</tr>
<tr>
<td>span</td>
<td>10.2 m</td>
</tr>
</tbody>
</table>

Fig. 8.2 - Geometry of the Winged-Cone Configuration (after Shaughnessy et al., 1990).

From the available documentation it is not clear what the geometry of the propulsion system is like. However, the Langley Accelerator is known to have engine modules all around the cylindrical engine nacelle section. In principle, each of the inlets are equally effective at $\alpha = 0^\circ$, since only in that case there is a symmetrical flow and none of the modules are shielded from the flow
by the fuselage. Although the major part of the flight of the WCC will be at small $\alpha$ (and preferably at $\beta = 0^\circ$), it is not known to what extent the assumption of independency on both $\alpha$ and $\beta$ holds.

Furthermore, to compute the Coriolis force and moment due to the use of the propulsion system we need to know where the mass flow will enter and leave the propulsion system. Considering an axisymmetric propulsion system means that the effective locations of the centre of inlet mass flow and the centre of exhaust mass flow are located on the $X_G$-axis. Assuming a control surface (see Fig. 2.7) that is put along the geometrical boundaries of the propulsion system, this results in values for $r_{in}$ and $r_e$ (as used in, for instance, Eq. 2.3.12) of:

$$r_{in} = (-7.1,0,0)^T \text{m} \quad \text{and} \quad r_e = (-11.0,0,0)^T \text{m}$$

The above locations are defined in a body-fixed frame with the origin in the moment reference centre ($X_G$-axis pointing towards nose).

The vehicle mass model is based on a rigid structure and distributed fuel. No fuel slosh is considered. The mass properties, i.e., the total mass, the $x$-location of the c.o.m., and the moments of inertia vary due to the consumption of fuel. They are modeled as a function of the current total mass. Note that the products of inertia are assumed to be negligible.

### 8.2. Reference-trajectory design using the Taguchi method

#### 8.2.1. Initial trajectory

At the beginning of this study, we did not have an all-up reference trajectory for an SSTO space plane available, nor did we have the opportunity to use optimisation software to generate one. Based on experience we gained in Marée et al. (1994), it is possible, however, to roughly define a trajectory based on available literature and then to further refine the trajectory using Taguchi's orthogonal arrays. This implies that a sensitivity analysis is executed where the key parameters of the trajectory are the factors, and the fuel mass or the payload mass delivered to orbit are the responses. The outcome of the analysis will be a sub-optimal trajectory for the given definition of the trajectory parameters.

A trajectory can be defined by specifying the flight-path angle as a function of altitude. By comparing guidance and optimisation studies by Hattis et al. (1991), Powell et al. (1991), Van Buren and Mease (1992), and Lu (1995), it was found that the ascent trajectories include a take-off segment consisting of a pull-up manoeuvre towards a large $\gamma$ between $25^\circ$ and $30^\circ$ in the subsonic range, and then falls back to values close to zero not to violate the trajectory constraints and for subsequent flight acceleration. Van Buren and Mease (1993) found that tracking the trajectory constraints $a_{dyn}$, axial acceleration, $\dot{n}_a$, and $\dot{O}$ results in a near minimum-fuel trajectory for the applied vehicle (WCC) and propulsion models. Powell et al. (1991) stated
that $q_{\text{dyn}}$ is the most stringent constraint, and Lu (1995) applied both $q_{\text{dyn}}$ and $\dot{Q}$ as trajectory constraints. To be in line with literature we will apply the three mentioned constraints and judge the importance of each of these constraints later. Moreover, we will base our trajectory design on the trajectory discussed by Powell et al. (1991). The design logic is as follows.

In principle, the WCC flies with full throttle to optimise propulsion-system performance. By varying $\phi_T$ (under- and overfueling) less or more thrust can be generated if required. Constraint tracking can in principle be done by varying $\phi_T$, but since we implemented the propulsion model for $\phi_T = 1$, the engines will be throttled directly. A maximum $\delta_T$ of 100% is used, which implies that the maximum thrust could in principle be higher if overfueling is allowed. The $\gamma$-profile is specified as a function of $h$ and includes segments with constant $\gamma_c$ or constant $\dot{\gamma}_c$. The commanded $\gamma$ is input to the gamma-alpha guidance system as discussed in Section 3.2.2, giving a commanded $\alpha$.

By trial and error we come to a definition of the segments such that a circular LEO at 120 km altitude can be reached. Transition from one segment to the other is triggered by reaching, for instance, a predefined $\gamma$, $M$ or $V$. The final pull-up manoeuvre is started at $V = 6,900$ m/s, a value close to values found in literature, and is performed at $\delta_T, \text{max} = 120\%$. After reaching a specified $q_{\text{dyn}}$ the vehicle is commanded to fly at $\alpha = 0^\circ$, which corresponds with the minimum drag configuration, and the engines are switched off. A coast phase will bring the vehicle in orbit. Obviously, it will not have the proper final state since no targeting guidance is used. Powell et al. (1991) guarantee this final state by applying a predictor-corrector guidance system as outer-loop. It determines the proper conditions for starting the final pull up, such that only minor corrections have to be applied by the circularisation engine. Since we do not have a targeting guidance system at our disposal, the final altitude is considered to be the only final state and circularisation of the target orbit is added as a mass penalty. This penalty is approximated by Tsiockovsky’s Equation, using $I_{sp} = 465$ s for the circularisation engine (Powell et al., 1991). After circularising the orbit, $\gamma$ will be zero and $V$ will be the local circle velocity at 120 km.

The trajectory represents a trimmed flight in the equatorial plane, starting with $V = 170$ m/s. Trim is guaranteed by the canards ($M \leq 0.9$) and elevons ($M > 0.9$). TVC is not considered for the reference trajectory, although the influence of TVC on fuel-mass consumption will be studied later. When $q_{\text{dyn}}$ is low, i.e., during the final ascent to orbit, $\alpha = 0^\circ$ and trim can be restricted to corrective control by reaction-control jets. The applied guidance system is the already mentioned gamma-alpha steering, with constraint tracking by the throttle laws of Section 3.2.2. The applied values for the gains to give a good response and an acceptable overshoot are summarised in Table 8.2.

The trajectory constraints are $q_{\text{dyn, max}} = 95,000$ N/m$^2$, $\dot{Q}_{\text{max}} = 8,000$ kWe$^{-2}$ at the leading edge of the wing ($R_h = 0.1$ m), and $n_a, \text{max}$ due to propulsion and aerodynamic forces of 1 g$_0$. These values are commonly applied to SSTO space planes, see the previously mentioned references. The resulting initial reference trajectory consists of the segments with corresponding parameters and fuel-mass consumption, as indicated in Table 8.3.
### Table 8.2 - Parameter values for guidance and throttle control.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{ip}$</td>
<td>2.0</td>
</tr>
<tr>
<td>$K_{i}$</td>
<td>1.8</td>
</tr>
<tr>
<td>$K_{yd}$</td>
<td>0.0</td>
</tr>
<tr>
<td>$\zeta_q$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\omega_q$</td>
<td>0.75 rad/s</td>
</tr>
<tr>
<td>$\zeta_Q$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\omega_Q$</td>
<td>0.075 rad/s</td>
</tr>
<tr>
<td>$\zeta_{n_a}$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\omega_{n_a}$</td>
<td>0.075 rad/s</td>
</tr>
</tbody>
</table>

### Table 8.3 - Initial reference trajectory.

<table>
<thead>
<tr>
<th>Nr</th>
<th>guidance law</th>
<th>parameter</th>
<th>segment boundary</th>
<th>$\Delta t$ (s)</th>
<th>$\Delta m_{fuel}$ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>constant normal load</td>
<td>$n_{z,c} = 1.5$</td>
<td>$\gamma_1 = 25.0^\circ$</td>
<td>12</td>
<td>761</td>
</tr>
<tr>
<td>2</td>
<td>constant flight-path angle rate</td>
<td>$\gamma_{c2} = -0.2^\circ$/s</td>
<td>$\gamma_2 = 5.0^\circ$</td>
<td>100</td>
<td>6,864</td>
</tr>
<tr>
<td>3</td>
<td>constant flight-path angle rate</td>
<td>$\gamma_{c3} = -0.075^\circ$/s</td>
<td>$\gamma_3 = 3.5^\circ$</td>
<td>20</td>
<td>903</td>
</tr>
<tr>
<td>4</td>
<td>constant flight-path angle rate</td>
<td>$\gamma_{c4} = -0.025^\circ$/s</td>
<td>$\gamma_4 = 1.5^\circ$</td>
<td>80</td>
<td>3,210</td>
</tr>
<tr>
<td>5</td>
<td>constant flight-path angle</td>
<td>$\gamma_{c5} = 1.5^\circ$</td>
<td>$M_g = 7.0$</td>
<td>138</td>
<td>5,892</td>
</tr>
<tr>
<td>6</td>
<td>constant flight-path angle rate</td>
<td>$\gamma_{c6} = -0.025^\circ$/s</td>
<td>$\gamma_6 = 0.5^\circ$</td>
<td>40</td>
<td>1,982</td>
</tr>
<tr>
<td>7</td>
<td>constant flight-path angle</td>
<td>$\gamma_{c7} = 0.5^\circ$</td>
<td>$M_p = 10.0$</td>
<td>136</td>
<td>5,955</td>
</tr>
<tr>
<td>8</td>
<td>constant flight-path angle rate</td>
<td>$\gamma_{c8} = -0.01^\circ$/s</td>
<td>$\gamma_8 = 0.25^\circ$</td>
<td>25</td>
<td>1,126</td>
</tr>
<tr>
<td>9</td>
<td>constant flight-path angle</td>
<td>$\gamma_{c9} = 0.25^\circ$</td>
<td>$M_0 = 14.0$</td>
<td>257</td>
<td>11,643</td>
</tr>
<tr>
<td>10</td>
<td>constant flight-path angle rate</td>
<td>$\gamma_{c10} = -0.001^\circ$/s</td>
<td>$\gamma_{10} = 0.12^\circ$</td>
<td>130</td>
<td>6,865</td>
</tr>
<tr>
<td>11</td>
<td>constant flight-path angle</td>
<td>$\gamma_{c11} = 0.12^\circ$</td>
<td>$V_{11} = 7,000$ m/s</td>
<td>931</td>
<td>20,883</td>
</tr>
<tr>
<td>12</td>
<td>pull-up manoeuvre initial</td>
<td>$\alpha_{z_{12}} = 6.0^\circ$</td>
<td>$\alpha_{12} = 0.0^\circ$</td>
<td>83</td>
<td>765</td>
</tr>
<tr>
<td></td>
<td>transition</td>
<td>$q_{dyn,12} = 2,500$ N/m²</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>final</td>
<td>$\alpha_{f_{12}} = 0.0^\circ$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>constant angle of attack</td>
<td>$\alpha_{\gamma} = 0.0^\circ$</td>
<td>$\gamma_{13} = 0.0^\circ$</td>
<td>317</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>circularisation</td>
<td>$h_{circ} = 120$ km</td>
<td>$h_{circ} = 120$ km</td>
<td>0</td>
<td>7,700</td>
</tr>
</tbody>
</table>

Before we come to optimising the initial reference trajectory, some remarks must be made. The final relative velocity is $V_f = 6,878.7$ m/s, a rather large difference with the local circle velocity ($V_c = 7,358.2$ m/s). A properly selected guidance scheme might therefore reduce the total required fuel mass. The circularisation manoeuvre required an inertial $\Delta V$ of 480 m/s.
\(\Delta m_{\text{fuel}} = 7,700 \text{ kg}\) as compared with \(\Delta V = 162 \text{ m/s} \ (\Delta m_{\text{fuel}} = 2,487 \text{ kg})\), mentioned by Powell et al. (1991). The final mass after circularisation is 62,289 kg, which means a maximum payload of 3,412 kg.

Previously, it was stated that the performance of a space plane is marginal at best, and that small deviations from the nominal design could easily lead to a negative payload and thus mission failure. Here, however, we could relatively easily find a successful trajectory with a positive payload, although not as much as the commonly specified target payload of 7,000 kg. The reason may be found in a too optimistic modelling of the SSTO space plane, the WCC, in the sense that the performance of the propulsion system is overrated with respect to the vehicle’s dry mass.

During the simulations it was found that the discrete jumps in flight-path angle commands caused peaks in the commanded angle of attack, especially after the initial pull up to \(\gamma = 25^\circ\) and the successive commanded flight-path angle rate. To avoid these peaks - and during the sensitivity analysis sometimes even instabilities - any \(\dot{\gamma}_c\) that is input to the guidance law will first be converted to a \(\gamma_c\) and subjected to the guidance law after which the resulting rate command is constrained to twice its input value.

8.2.2. Sub-optimal trajectory

As can been seen from Table 8.3, there are 28 parameters that define the trajectory. However, not all of them are suitable to be varied: i) the \(\gamma\)-boundary of a segment preceding a constant-\(\gamma\) segment should in principle be equal to \(\gamma_c\) to prevent additional discontinuities, ii) \(\alpha_f\) of the pull up should be equal to \(\alpha_c\) of segment #13 and zero, because \(\alpha = 0^\circ\) corresponds with the minimum-drag configuration which is already optimal, iii) the boundary altitude of the last segment is the (fixed) target altitude. Omitting these leaves 20 parameters.

Marée et al. (1994) discuss the shaping process of an air-breathing first stage and rocket-powered second stage of a TSTO space plane by varying the important trajectory parameters according to an orthogonal array. For 13 selected trajectory parameters, a three-level \(L_{36}\) array taken from Taguchi (1988) was applied. Note that in principle, an \(L_{27}\) array would have sufficed for 13 factors; however, 9 simulations were added for the sake of error analysis. The outcome of the sensitivity analysis was that the payload-to-orbit varied between -1,890 and +5,820 kg. Analysis of variance of the payload mass, indicated that the contribution of the quadratic terms was only 0.45%. Unfortunately, no effort was made to assess the influence of interactions although the results implied that this influence was small.

In line with the experience with Taguchi’s orthogonal arrays in the previous chapter, we will start our analysis with a two-level array to assess the linear effects of the trajectory parameters. Nineteen parameters require the \(L_{32}\) array, leaving 12 columns for interactions. However, if it can indeed be assumed that there are no interactions, the number of simulations can be reduced if beforehand it can be decided that some parameters are not so important. On the other
hand, we only want to indicate the potential of this way of trajectory shaping. Therefore, it is decided to use the \( L_{16} \) array. This means that 15 out of the 19 parameters will be varied. The four parameters that are discarded are the \( \gamma_c \) of segments \#2, \#3, \#4 and \#10, because the trajectory shape can also be influenced by the corresponding \( \gamma \)-boundaries.

It can be expected that not all of the combinations of trajectory parameters will result in actual trajectories all the way up to orbit. This is solved by incorporating an additional circularisation manouvre. The stop criterion for all simulations will be a zero flight-path angle after the coasting phase. In any case, one burn will be applied to raise or lower the final altitude and another burn for circularisation, as was discussed in the previous sub-section. The parameter variations are selected quite arbitrarily, and are defined as follows:

\[
A: \quad n_{z,c} = 1.5 \pm 0.15 \\
B: \quad \gamma_1 = 25.0^\circ \pm 1.0^\circ \\
C: \quad \gamma_2 = 5.0^\circ \pm 0.25^\circ \\
D: \quad \gamma_3 = 3.5^\circ \pm 0.25^\circ \\
E: \quad \gamma_4 = \gamma_{c,5} = 1.5^\circ \pm 0.15^\circ \\
F: \quad M_5 = 7.0 \pm 0.5 \\
G: \quad \gamma_6 = \gamma_{c,7} = 0.5^\circ \pm 0.05^\circ \\
H: \quad M_f = 10.0 \pm 0.5 \\
I: \quad \dot{\gamma}_{c,8} = -0.01^\circ/s \pm 0.005^\circ/s \\
J: \quad \gamma_8 = \gamma_{c,9} = 0.25^\circ \pm 0.05^\circ \\
K: \quad M_9 = 14.0 \pm 0.5 \\
L: \quad \gamma_{c,11} = 0.12^\circ \pm 0.02^\circ \\
M: \quad V_{11} = 7,000 \text{ m/s} \pm 50 \text{ m/s} \\
N: \quad \alpha_{12} = 6.0^\circ \pm 0.5^\circ \\
O: \quad q_{d,12} = 2,500 \text{ N/m}^2 \pm 1,000 \text{ N/m}^2
\]

Executing the 16 simulations resulted in the consumed fuel masses as presented in Table 8.4. There, also the integrated heat loads are given. The higher this heat load is, the higher the mass of the thermal protection system. After inspecting the results it is obvious that the minimum-fuel trajectory does not correspond with the minimum heat-load trajectory. Depending on the maximum allowable value of the latter, it should be included in any trajectory optimisation process. Note that in this case the minimum heat-load trajectory takes up 1,401 kg more fuel.

Focusing on the used fuel mass a large variation can be seen, that would result in a payload-mass variation between 991 and 5857 kg, if the remaining fuel mass is considered to be the payload. To assess the relative influence of the trajectory parameters on the fuel mass, an ANOVA is performed. In Fig. 8.3, the linear factor contribution to the total variance has been plotted, for both the fuel mass and the total heat load. Since all columns of the orthogonal array were assigned to factors, no column (or, equivalently, degree of freedom) was left to address the error variance. Note that any variation due to interactions is confounded with the factor variance, although the contribution of the interaction will be smaller than the corresponding main effects. As a result of the analysis with the fuel mass as objective, we find that four factors including possible interactions contribute to about 75% of the variation in fuel mass, i.e.,

- the normal load at take off, factor A (12.5%),
- the flight-path angle at the end of the initial pull up, factor B (45.7%),
- the commanded flight-path angle of segment 9, factor J (7.7%), and
• the pull-up velocity, factor $M$ (10.1%).

<table>
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<th>Nr</th>
<th>$m_{fuel}$ kg</th>
<th>$Q$ MJ/m²</th>
<th>Nr</th>
<th>$m_{fuel}$ kg</th>
<th>$Q$ MJ/m²</th>
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<td>16</td>
<td>73,297</td>
<td>10,492</td>
</tr>
</tbody>
</table>

Table 8.4 - Fuel mass and total heat load for sensitivity analysis (extremes are bold printed).

Fig. 8.3 - The contributions of the linear terms of variation per factor, in percents of the total variation, to the fuel mass and total heat load.

The contribution of the other factors is smaller, though not negligible. Only factors $C$, $D$, $I$ and $O$ have an influence smaller than 1%. Note that only the relative influence of the factors is given. Since the factors are not equally varied it cannot be concluded that the trajectory is more sensitive to one factor than to another. To verify that, the fuel mass is plotted as a function of the factors related to their mean value (Fig. 8.4), i.e., the mean value of the fuel mass at the minimum level of a factor and the maximum level. From this figure it is clear that the factor that contributes most to the total variation ($B$) and the factor that has the largest absolute influence ($M$) are not the same. Moreover, it is also obvious that a larger value of
factor A has an increment of the fuel mass as result, while for the other four factors an increase leads to a decrement.

The two ways of graphically presenting the results can be of great help while trying to interpret the results of a sensitivity analysis. In case the factor levels are given, then the factor contribution to the total variation directly gives the major factors, and relative differences between the factors are easily found from the bar graph. On the other hand, in case the factor levels are chosen quite arbitrarily studying the response lines can give a lot of extra insight, especially when the factors are varied over more than two levels.

![Graph showing fuel mass variation](image)

Fig. 8.4 - The influence on the fuel mass of the five factors that contribute most to the total variation. The factor values are expressed with respect to their nominal values.

Yet another way of analysing the results is a response-surface approach. When the first-order response surface for the fuel mass is computed, we find, using normalised factors:

\[
m_{\text{fuel}} = 73,578 + 473 n_{z,c} - 905 \gamma_1 + 26 \gamma_2 - 34 \gamma_3 - 178 \gamma_{c,5} - 311 M_5
- 245 \gamma_{c,7} - 264 M_7 - 87 \gamma_{c,8} - 371 \gamma_{c,9} - 243 M_9 - 223 \gamma_{c,11}
- 425 V_{11} + 231 \alpha_{i,12} + 14 q_{\text{dyn,12}}
\]

(8.2.1)

Unfortunately, it is not possible to compute the confidence interval for the coefficients in the response surface, because of the zero degrees of freedom for the error. Therefore, this surface should only be seen as a means to get an indication of the extremes in the design region, and not as a final model. Moreover, we included all factors in the model, whereas in principle those factors with a small contribution could be omitted. However, maybe also some interaction terms
should be included in the final first-order response surface. Since it is not our intention to actually optimise the trajectory but merely improve the performance of the WCC to an acceptable level, a refinement of the response surface is not pursued.

Comparing the coefficients in the response surface with the factor contribution, we find a similar relative importance. Computing the minimum fuel mass with Eq. (8.2.1) is simple:

\[ x = (-1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1)^T \] and \( m_{\text{fuel,minimum}} = 69,549 \text{ kg} \)

A verification simulation with these factor combinations results in a fuel consumption of 70,101 kg which is close to the computed minimum. The difference may be explained by the ignored interactions and, of course, the finite accuracy of the response surface. Since the difference is small it can be expected that also the influence of individual interactions is small. For this reason, we will not study them in great detail, moreover because in the previous chapter it was shown, that it is difficult to get an idea of individual interactions unless confounding is eliminated.

To assess the influence of the simulation size on the selected responses and to have some columns available for interaction analysis, a second batch of simulations is executed with twice the number of runs. For the 15 factors an \( L_{32} \) array is used with the following column assignment: A, C, empty, H, D, I, empty, E, F, L, L, M, empty, empty, B, G, J, K, N, empty, empty, O, empty, empty, empty, empty, empty, empty, empty, empty, empty, empty. ANOVA of the results shows that the contribution of the empty columns (i.e., column interactions) to the total variation is small, although not negligible (in the order of a few percents).

One of the aspects that came forward by comparing the results of the two analyses is that there appears to be a shift in factor contribution to the total variation. In Table 8.5, the factor effects are listed for both analyses. It can be seen that major factor effects are still major factor effects, but in case of smaller effects there are some shifts in the order of the factor contributions. The conclusion that was also drawn in the previous chapter is that major factor effects can be extracted from the results, but that the absolute importance cannot be fully assessed. This means that one cannot label a certain factor with a \( p\% \) influence on the total variation. In particular this holds for minor factor influences.

Finally, the \( L_{16} \) and \( L_{32} \) analyses resulted in two different means (73,578 and 73,682 kg) and standard variations (1,382 and 1,280 kg) for the selected responses. Evidently the problem of trajectory (sub)optimisation is a non-linear problem that emerges when different (but similar) factor combinations are used. Of importance is of course the question whether one mean is better than the other, or whether the two means differ significantly from one another. One can imagine that a mean gets more accurate the more samples are used to compute this mean. In that sense, the mean obtained by the \( L_{32} \) analysis would be better. An even more global question is: are the two distributions of the fuel masses consistent with each other or are they different? Statistical tests are available that can address these questions in a formal manner, see Appendix E (Section E.1).
Table 8.5 - Factor contribution to total variance, including ranking.

The generally accepted Kolmogorov-Smirnov test can be applied to continuous distributions to check whether they are consistent or different, as to find out whether the two sets of orthogonal factor combinations that are in principle normally distributed result in the same distribution. Carrying out the test gives as a result a probability of 95.7% that the two distributions are consistent. Apparently, only the number of simulations has an influence on the mean and variance of a selected response.

![Graph](image)

Fig. 8.5 - The ascent reference trajectory, including the dynamic-pressure constraint of $q_{\text{dyn, max}} = 95,000 \text{ N/m}^2$ and the heat-flux constraint of $\dot{q}_{\text{max}} = 8,000 \text{ kW/m}^2$. The flight-segment boundaries are represented by diamonds.
The remaining part of this sub-section will present the reference trajectory, but only after two adjustments are made in the final pull-up and circularisation manoeuvre. During the final pull-up the propulsion system is producing 20% extra thrust. This is reduced by 10% to come closer to the original propulsion model. Furthermore, the circularisation manoeuvre is still taking about 7,000 kg of fuel, which is rather large for an actual mission. Therefore, it should be decreased which can be achieved by increasing the final pull-up velocity, since there is a one-to-one relationship between this velocity and the required $\Delta V$. The $\Delta V$ for circularisation mentioned by Powell et al. (1991) is 162 m/s. We aim at a $\Delta V$ of about 250 m/s. In that case, the pull-up velocity should be 7,200 m/s. Furthermore, the initial pull-up angle of attack is set to 3° to avoid any trim problems. For this new trajectory, the consumed fuel mass is 68,114 kg, leaving a payload mass of 9,146 kg. It might be disputed whether such a high pull-up velocity is feasible. Since our model seems to be capable of achieving that velocity, this is not studied any further. The final reference trajectory is plotted in Fig. 8.5.

To begin the discussion on the reference trajectory, one can observe that the trajectory constraints are not violated. Note that the $n_a$-constraint is not plotted in Fig. 8.5; this constraint is only briefly active during the initial flight phase, which would not show in this graph. But, this constraint is an important one as will become clear later. The $q_{dyn}$-constraint seems only to be followed for a short while. It is obvious that during the major part of the trajectory, the vehicle climbs only up to an altitude of about 49 km, whereas $V$ increases from 170 m/s to 7,200 m/s. During the powered pull up an additional 13 km is gained whereas $V$ remains almost constant. A major part of the remaining altitude is covered during the coasting phase.

The trajectory was specified as a $\gamma$-profile. This profile is depicted in Fig. 8.6 at the end of this sub-section, where for both the initial and final reference trajectory $\gamma$ as a function of $t$ is plotted. Apparent is the large $\gamma$ during the initial climb, which peaks in the subsonic flight regime. $\gamma$ rapidly decreases to very small values in order to lock on the trajectory constraints as good as possible. During the subsequent semi cruise flight, the vehicle can accelerate while gaining altitude only slowly, such that after about 1,700 s the pull-up velocity is reached. The pull up shows as an increase in $\gamma$, decreasing again during the (unpowered) coasting phase. At $h = 120$ km, $\gamma$ is almost zero. Circularisation actually takes place at an altitude of $h = 122$ km. In Fig. 8.6 it can also be seen that major differences between the initial and final $\gamma$-profile can be found at the beginning and end of the trajectory.

Fig. 8.7 plots $\alpha_c$ versus $t$. The angle of attack is high during take off ($\alpha = 12^\circ$) to provide sufficient lift. It reaches a local minimum in the transonic region, which is also mentioned by Hattis et al. (1991). Briefly, $\alpha_c$ increases but the general trend is a decreasing angle of attack as a result of an increasing centrifugal relief. The velocity in the final reference trajectory is higher compared with the initial one at corresponding flight times, such that $\alpha$ of the final trajectory is generally lower. Discontinuities can be found whenever a sudden change in $\gamma$ is commanded. At $t = 1,700$ s, $\alpha$ is commanded to be 3° to provide sufficient lift for the final pull up. Note the difference between the initial and final reference trajectory.

In Fig. 8.8, $\delta_T$ is plotted versus flight time. The minimum $\delta_T$ for the final trajectory is about 0.6, whereas for the initial trajectory this was 0.4. The difference is explained by the two $\gamma$-
profiles: the final trajectory is a steeper one such that more of the available propulsion power is used to gain height so the vehicle can accelerate more while not violating the constraints. After 200 s of flight, $\delta_T$ reaches its maximum, which means that apparently the vehicle can fly with full power without violating the $q_{dyn}$-constraint. For the initial trajectory this is not true: $\delta_T$ is smaller than 1 up to $t = 750$ s, with sharp peaks when a smaller $\gamma$ is commanded. This means a decrease in ascent rate so that also the acceleration rate must be decreased.

This reasoning is confirmed by Fig. 8.9, where $q_{dyn}$ is plotted versus $t$. It can clearly be seen that the initial trajectory is locked on the $q_{dyn}$-constraint of 95 kPa for the major part of the first 1,000 s of flight, whereas $q_{dyn}$ of the final trajectory comes close to the constraint value but does not reach it. It was stated before that flying along $q_{dyn,\text{max}}$ results in a sub-optimal trajectory which means that for our final trajectory this is not the case. And still, it is the more fuel-efficient one. Since the many optimisation studies published in literature all state that a flight along the maximum allowable $q_{dyn}$ is optimal, we can only conclude that despite that fact the current trajectory is the best available for the given trajectory segmentation, parameter variations and applied models. It does not imply that this trajectory cannot be further improved, but for that we may need to refine the segmentation and extend the number of examined factors. One conclusion is obvious, and quite straightforward: the Taguchi method cannot replace a numerical trajectory optimisation method. On the other hand, by doing a Taguchi analysis one can increase the insight in the influence of the trajectory parameters on a selected response.

Returning to Figs. 8.8 and 8.9, in the first 200 s of flight $\delta_T$ is not maximal, while still the dynamic pressure is not at its maximum. In that region, the axial acceleration constraint is active, preventing the vehicle from accelerating faster to cope with the increase in altitude. Thus, the vehicle's initial path is too steep for the air-breathing propulsion system. This is easily verified if one compares the corresponding $\gamma$ profiles for the initial and final reference trajectory. It is interesting to ask ourselves what would happen if the throttle control is released from the acceleration constraint. In that case, $q_{dyn,\text{max}}$ is reached but due to the large $\gamma$ and the relatively high velocity, the vehicle reaches higher altitudes too soon and it does not have enough climbing power to maintain the acceleration. Therefore, a strong drop in $q_{dyn}$ is the result, the vehicle starts descending again and finally crashes. This clearly shows the difference between a conventional rocket and an air-breathing space plane. Whereas the former can accelerate fast enough to reach higher altitudes and to minimise the drag, the latter must remain at lower altitudes and accelerate at high $q_{dyn}$ to maximise the performance of the propulsion system. To conclude these figures, at $t = 1,700$ s the pull up shows as $\delta_T = 1.1$ for the final reference trajectory and a sharp decrease of $q_{dyn}$. The rapid increment in $h$ and thus decline in $\rho$ results in this decrease.

In Fig. 8.10, $\dot{Q}$ is plotted as a function of $t$. It is clear that this constraint is an important one since a considerable part of the trajectory is flown along this constraint. It is therefore not sufficient to take only the $q_{dyn}$-constraint into account as was suggested by Powell et al. (1991), but to include $Q_{max}$ as well. The integrated heat load is for both trajectories almost the same, i.e., the higher velocities of the final trajectory is compensated for by the longer flight time of
the initial trajectory.

Fig. 8.11, finally, shows the deflection angles of the elevons to trim the vehicle (at subsonic speeds trim is solely guaranteed by the canards; during take off \( \alpha = 12^\circ \), \( \delta_{e,\text{trim}} = 5^\circ \)). The history shows a minimum near \( M = 2 \) (\( t = 100 \text{ s} \)) and exhibits a change in vehicle trim as a result of combined changes in aerodynamic loading and a shift in the location of the c.o.m. due to fuel consumption, an effect confirmed by Hattis et al. (1991). There is, of course, a strong relation between \( \alpha \) and \( \delta_e \) to compensate for the induced pitch moment. The maximum \( \delta_e \) is found shortly after the local \( \alpha_{\text{max}} \). Considering the fact that \( \delta_{e,\text{max}} = 20^\circ \), trimming the vehicle with elevons alone is a heavy load for these control surfaces which does not leave so much margin for manoeuvring and compensating deviations in \( \alpha_c \). For this reason, it is wise to assess other means of vehicle trim (see Section 8.2.3). Due to the decreasing \( \alpha \), also \( \delta_{e,\text{trim}} \) decreases. The final pull up results in a deflection of the elevons close to their limit value. Since this pull-up manoeuvre is not optimal in any sense, we cannot draw strong conclusions but state that this manoeuvre should be carefully examined since it might create control problems. During the coasting at \( \alpha = 0^\circ \), the pitch moment is zero, and thus also \( \delta_{e,\text{trim}} \).

8.2.3. Thrust-vector control used for trim

The last topic to be discussed in this sub-section is the already mentioned large trim deflection of the elevons. Two reasons are apparent to decrease the required deflection angles, i.e., it leaves a larger margin for manoeuvring and the induced trim drag can be reduced to save fuel mass. The first reason has already been shown to be a valid one; the second one is relatively easy to illustrate, although some remarks must be made.

In Figs. 8.12 and 8.13, the drag and lift have been plotted for the final reference trajectory, including the induced trim drag and lift. Both the trim drag and lift form a substantial part of the vehicle drag and lift, i.e., up to 50% for the drag and 60% for the lift (excluding the pull-up manoeuvre). The current reference trajectory has been defined such that a large part of the external forces is used to increase the altitude of the space plane. This part consists mainly of the lift force, so it is easy to understand that for this steep ascent trajectory the lift will not be sufficient if the contribution of the elevons is removed from the external-force vector. Verifying this notion indeed results in a crash of the vehicle due to insufficient lift. To avoid the problem of finding a new reference trajectory for an untrimmed vehicle, the influence of trim drag is assessed by flying a trimmed trajectory and by putting the trim drag to zero. It follows that 2,929 kg less fuel is required to reach the target orbit, which is almost 30% of the payload mass of 9,100 kg. Conclusions that can be drawn from these observations is that in performance analysis, the influence of trim drag and lift should both be included. Moreover, the trim drag results in a lot of extra fuel mass which makes it worthwhile looking for alternatives.

One alternative that can be studied is the use of TVC. Of course, this will make the propulsion system more complicated (and possibly heavier), but in principle fuel-mass savings are
possible. The thrust is the basis for the acceleration of the vehicle and contributes to the lift force. When the thrust vector is deflected such that the vertical component gives rise to a compensating pitch moment, two side effects are introduced. First, there is larger contribution to the lift force which may result in a smaller $\alpha$ and therefore a decrease of the global drag force. Second, the horizontal thrust component for acceleration becomes smaller, that means that it will take a larger $\delta_T$ (or, equivalently, a longer flight time) to reach the pull-up velocity and consequently a larger fuel consumption. Obviously, the use of TVC should be included in the trajectory optimisation process.

Since the WCC flies with maximum throttle for a large part of the current trajectory, the use of TVC leads to problems in the sense that there is not enough accelerating power left. For that reason we will use the initial trajectory definition (Table 8.3), because there is some performance margin left and we only want to study the relative influence of TVC. To apply thrust-vector control we need to define the so-called centre of thrust (c.o.t.), i.e., the location where the resulting thrust vector is acting on the vehicle. Since there is no information available on the defined propulsion model, the following is assumed. The actual engine section is considered to be the cylindrical part of the WCC, recall Fig. 8.2. The cone frustrum nozzle is also part of the propulsion system, since the exhaust-gas expansion is an indispensable part of the thrust generation. Therefore, the c.o.t. must be located somewhere on this nozzle. For computational purposes it is assumed that this is at one third of from the end of the nozzle, i.e., $19.1 \text{ m}$ from the moment reference centre, and, since the engine section is axisymmetric, effectively at the $X$-axis. Furthermore, a limit of $\pm 25^\circ$ will be put on the thrust-vector angle, although it is not implied that this limit is a feasible one.

The technical implementation of TVC will not be addressed, nor the influence of TVC on propulsion-system performance which could be that significant that TVC is not feasible at all. The practical implementation of the trim rule is that below $M = 0.9$ the canards are used, and above $M = 0.9$ a defined percentage of the pitch moment is compensated for by the elevons, and the remaining part by TVC. Note that if the limit value of the TVC angle is reached, the remaining moment is also compensated for by the elevons. Since full TVC is not possible, we will study values of 50% down to 0% of the pitching moment that is in principle addressed by TVC. In all cases, the WCC reaches the pull-up velocity at about the same altitude and after almost similar flight times. The results of the simulations are listed in Table 8.6.

From the results it is obvious that the use of TVC can save a substantial amount of fuel. By assigning 50% of the trim moment to TVC, the total $m_{\text{fuel}}$ to reach the pull-up velocity is reduced by 1,109 kg. When the trim parameters are studied, i.e., $\delta_{e}$ (Fig. 8.14) and thrust elevation $\varepsilon_T$ (Fig. 8.15), we see the difference between the all-elevon-trim and 50%-elevon-trim trajectories. It is clear that the maximum $\delta_{e}$ reduces from about 12° down to about 9°. Overall, the decrease $\delta_{e}$ is between 2° and 3°. Although the reduction is not that large it is evident that the manoeuvring capability has increased. In the profile of $\delta_{e}$ some sharp peaks are observed. Again, these peaks are the result of the crude reference-trajectory definition. Transition from one $\gamma_c$ to another results in discrete jumps in $\alpha$ (see also Fig. 8.7) and thus $\delta_{e,\text{trim}}$.
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</table>

Table 8.6 - The influence of TVC on the fuel mass.

Regarding $\varepsilon_{T,trim}$ at activation ($M = 0.9$) $\varepsilon_{T,trim} = -7^\circ$. Up to $t = 1,000$ s it decreases to $-10^\circ$, with some discrete transitions that are linked with discrete $\delta_T$ variations (Fig. 8.8; note that this particular study on TVC is based on the initial reference trajectory). Also in this case, the variations are related to transitions from one flight segment to the other. Just after $t = 1,000$ s, the $Q$-constraint is activated which means a rather sudden decrease in $\delta_T$ and thus $T$. It is obvious that larger deflections of the thrust vector are required to compensate for the pitch moment. This can clearly be seen in Fig. 8.15. After reaching its maximum deflection at about $-20^\circ$, it slowly decreases to $-6^\circ$ due to the influence of $M$ and $\alpha$ on the pitch moment.

From the above results it would be easy to conclude that TVC is a good means to decrease the total fuel mass and to increase the manoeuvrability capability with the elevons. As was indicated before, the technical aspects of TVC have not been addressed at all. Doing so might lead to completely different conclusions. Also the influence of the thrust-vector orientation on the flight mechanics (jet damping) will have to be studied. However, this coarse performance study has indicated that the use of TVC should at least be studied in more detail.
Fig. 8.6 - Flight-path angle versus time for the final (solid) and initial (dashed) reference trajectory.

Fig. 8.7 - Angle of attack versus time for the final (solid) and initial (dashed) reference trajectory.

Fig. 8.8 - Throttle setting versus time for the final (solid) and initial (dashed) reference trajectory.

Fig. 8.9 - Dynamic pressure versus time for the final (solid) and initial (dashed) reference trajectory.
Fig. 8.10 - Heat flux versus time for the final (solid) and initial (dashed) reference trajectory.

Fig. 8.11 - Elevon trim deflection versus time for the final (solid) and initial (dashed) reference trajectory.

Fig. 8.12 - Vehicle drag (solid) and elevon-trim drag (dashed) as a function of flight time.

Fig. 8.13 - Vehicle lift (solid) and elevon-trim lift (dashed) as a function of flight time.
8.3. Design of a Model Reference Adaptive Control system

To develop the adaptive controller for the nominal mission of the WCC, it must be said that the final pull up is not too realistic when compared with literature. There was no targeting guidance involved to determine the pull-up conditions so that quite a large angle of attack was required, resulting in a trim problem. The development and application of the MRAC system must be seen as a first assessment of its potential. For that reason we do not want to include the final ascent phase to orbit, since it might lead to control problems that could better be solved by improving the guidance. As a result, only aerodynamic control will be applied since \( q_{\text{dyn}} \) only starts decreasing significantly after the pull-up manoeuvre.

The remainder of this section is divided into three parts. The first sub-section discusses the reference model, whereas the second one gives an overview of the complete MRAC system and its relation with the non-linear space plane. The third sub-section reflects on some of the experiences, that were gained in the process of designing the MRAC system for the hypersonic manoeuvres and the ascent to orbit.
8.3.1. Reference model

Starting point in the development of the adaptive controller is the notion that we want the nonlinear space plane to behave like a stabilised, linear system where the longitudinal and lateral motion are decoupled. So, as a reference model for the adaptive controller the linearised model of the rotational dynamics, that also formed the basis in the development of the LQR (see Appendix C) is selected. At any point in the trajectory the reference model is supposed to be linearised around its trim equilibrium-state. Summarised, the following equations hold:

\[
\Delta \dot{\beta}_m = \frac{1}{l_{xx}} \left( \frac{\partial C_i}{\partial \beta} \Delta \beta_m + \frac{\partial C_i}{\partial \delta_a} \Delta \delta_{a,m} + \frac{\partial C_i}{\partial \delta_r} \Delta \delta_{r,m} \right) q_{dyn} S_{ref} b_{ref}
\]

\[
\Delta q_m = \frac{1}{l_{yy}} \left( \frac{\partial C_m}{\partial \alpha} \Delta \alpha_m + \frac{\partial C_m}{\partial \delta_e} \Delta \delta_{e,m} \right) q_{dyn} S_{ref} c_{ref}
\]

\[
\Delta f_m = \frac{1}{l_{zz}} \left( \frac{\partial C_n}{\partial \beta} \Delta \beta_m + \frac{\partial C_n}{\partial \delta_a} \Delta \delta_{a,m} + \frac{\partial C_n}{\partial \delta_r} \Delta \delta_{r,m} \right) q_{dyn} S_{ref} b_{ref}
\]

\[
\Delta \dot{\alpha}_m = \Delta q_m - \frac{1}{m V_e} \frac{\partial C_L}{\partial \alpha} q_{dyn} S_{ref} \Delta \alpha_m
\]

\[
\Delta \dot{\beta}_m = \sin \alpha_e \Delta p_m - \cos \alpha_e \Delta r - \frac{g_e}{V_e} \cos \gamma_e \cos \sigma_e \Delta \sigma_m
\]

\[
\Delta \dot{\sigma}_m = -\cos \alpha_e \Delta p_m - \sin \alpha_e \Delta r_m + \left( \frac{g_e}{V_e} \cos \gamma_e \cos \sigma_e - \frac{L_e}{m V_e} \right) \Delta \beta_m + \frac{\tan \gamma_e}{m V_e} \cos \sigma_e L_e \Delta \sigma_m
\]

In the above equations, \( \Delta \cdot \) means that the equations are related to small deviations from the equilibrium state. The subscript \( m \) indicates that the variables are related to the reference model, whereas the subscript \( e \) stands for equilibrium value. Thus, in this case \( \alpha_e \) and \( \sigma_e \) are identical to the guidance commands \( \alpha_c \) and \( \sigma_c \).

By defining a state vector \( \Delta x_m \) and a control vector \( \Delta u_m \), i.e.,

\[
\Delta x_m = (\Delta p_m, \Delta q_m, \Delta r_m, \Delta \alpha_m, \Delta \beta_m, \Delta \sigma_m)^T
\]

\[
\Delta u_m = (\Delta \delta_{a,m}, \Delta \delta_{e,m}, \Delta \delta_{r,m})^T
\]

the above equations can be written in the well-known state-space form:

\[
\Delta \dot{x}_m = A_m \Delta x_m + B_m \Delta u_m
\]
The actual model state is the sum of the equilibrium and the perturbation values. As we mentioned before, the equilibrium attitude is assumed to be equal to the commanded attitude issued from the guidance logic. The equilibrium (or commanded) angular rate is such that the derivatives of the aerodynamic angles are zero. By defining an output vector $y_m$ the reference model is completed with the output equation:

$$y_m = x_c + C_m \Delta x_m$$  \hspace{1cm} \text{(8.3.8)}$$

with

$$y_m = (p_c + \Delta p_m, q_c + \Delta q_m, r_c + \Delta r_m, \sigma_c + \Delta \sigma_m, \alpha_c + \Delta \alpha_m, \beta_m)^T$$

To guarantee a stable reference model, a combination of an optimal PI controller for longitudinal motion and an analytical PD-controller for lateral motion, that decouples the roll and yaw motion, is included in the model.

The reference model is integrated in time to enable an update of the model state and control variables. A simple trapezoid rule that is usually applied to propagate a digital PID controller does not give a stable solution for the reference model. Therefore, the fixed-step fourth-order Runge-Kutta method is used, taking the sample time of the attitude controller as step size. Since the equilibrium state of the reference model is equal to the commanded state, the differential equations of the linearised model will yield zero derivatives once that equilibrium state is reached after an (initial) perturbation. In that case, the reference model state is represented by the commanded attitude and angular rates, and the control variables are given by the trimmed deflection angles. This would even make the reference model superfluous.

However, if there is an abrupt change in the commanded attitude, the response of the model to this change and the corresponding perturbation control should be used as much as possible to shape the non-linear vehicle control vector. For this reason, we take the difference in guidance commands of two successive samples and excite the reference model with a step corresponding with this difference. In the consecutive sample the model controller will try to bring back the model to its equilibrium state, thereby generating perturbation-control commands. Note that the initial state of the model will be put equal to that of the non-linear vehicle. In this way, any abrupt reaction of the vehicle will be minimised. In the remainder of this sub-section, the longitudinal and lateral controller will be discussed.

**Longitudinal controller**

For the sake of clarity, in the following discussion the coefficient derivatives will be written as moment derivatives, i.e., the pitch equations are rewritten as:

$$\dot{q}_m = \frac{M_{\alpha}}{I_{yy}} \alpha_m + \frac{M_{\delta e}}{I_{yy}} \delta_{e,m}$$  \hspace{1cm} \text{(8.3.9a)}$$
\[ \alpha_m = q_m - \frac{L_\alpha}{mV_e} \alpha_m \]  

(8.3.9b)

To compute the controller gains by means of solving the Riccati Equation, the elevator and pitch reaction control law must be written in the form

\[ \Delta u_m = -K \Delta x_m \]

In developing the HORUS longitudinal controller a simple P control law was assumed, based on feedback of both \( \alpha \) and \( q \). Such a control law can in principle also be interpreted as a PD control law, since the inclusion of \( q \) provides physical damping. For the longitudinal control of the space plane, we want a more accurate \( \alpha \)-control than in case of the re-entry of HORUS, because the performance of the propulsion system is dependent on, amongst others, \( \alpha \). For this reason, we include an integral component in the control law that should reduce the steady state error in \( \alpha \) to zero.

![Diagram](image)

Fig. 8.17 - Closed-loop reference model for longitudinal motion.

The control law is defined as (see also Fig. 8.17 for a schematic overview of the closed-loop system):

\[ \Delta \delta_{e,m} = K_{e_q} \Delta q_m + K_{e_\alpha} \Delta \alpha_m + K_{e_i} \int_0^t \Delta \alpha_m \, dt \]  

(8.3.10)

To compute the gains, the state equations Eq. (8.3.9) are augmented to compute the integrated angle of attack error, i.e.,

\[ \frac{d}{dt} \int \Delta \alpha_m \, dt = \Delta \alpha_m \]  

(8.3.11)
such that the system and control matrices, $A_m$ and $B_m$ are written as:

$$A_m = \begin{bmatrix} 0 & M_\alpha \\ \frac{1}{T_{yy}} & 1 \\ -\frac{L_\alpha}{mV_e} & 0 \end{bmatrix}, \quad B_m = \begin{bmatrix} \frac{M_{\delta_e}}{T_{yy}} \\ 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (8.3.12)

In completing the problem definition, two weighting matrices related to the state and control variables are defined, thereby assuming only aerodynamic control for now:

$$Q = \text{diag} \left\{ \frac{1}{\Delta \alpha_{\text{max}}^2}, \frac{1}{\Delta \alpha_{\text{max}}^2}, \frac{1}{\int \Delta \alpha_{\text{max}} \, dt} \right\}, \quad R = \begin{bmatrix} 1 \\ \frac{1}{\Delta \delta_{e_{\text{max}}}^2} \end{bmatrix}$$  \hspace{1cm} (8.3.13)

The gain matrix is calculated using Eq. (3.4.8) and (3.4.9). For more details on the application of optimal control theory, the reader is referred to Section 3.4.2 for the theoretical background, and Section 6.3 and Chapter 7 for applications in re-entry missions.

**Lateral controller**

The lateral controller operates the rudder to control the yaw motion and the ailerons to control the roll motion. To begin with the yaw motion, the related equations are given by:

$$\Delta r_m = \frac{N_\beta}{T_{zz}} \Delta \beta_m + \frac{N_\delta_r}{T_{zz}} \Delta \delta_{r,m}$$  \hspace{1cm} (8.3.14)

$$\Delta \beta_m = -\Delta r_m$$  \hspace{1cm} (8.3.15)

In the above equations, the contribution of the ailerons to the yaw moment is neglected and it is assumed that the space plane is flying at small $\alpha$. After Laplace transformation of both the state and the control variables, and substituting the control law

$$\Delta \delta_{r,m}(s) = K_r \Delta r_m(s) + K_\beta \Delta \beta_m(s)$$  \hspace{1cm} (8.3.16)

in Eqs. (8.3.14-15) we obtain

$$\begin{bmatrix} s - \frac{N_\beta}{T_{zz}} K_r - \frac{N_\delta_r}{T_{zz}} K_\beta \\ \frac{N_\beta}{T_{zz}} K_r - \frac{N_\delta_r}{T_{zz}} K_\beta - \frac{N_\delta_r}{T_{zz}} K_r \end{bmatrix} \begin{bmatrix} \Delta r_m(s) \\ \Delta \beta_m(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (8.3.17)
The characteristic equation is obtained by computing the determinant of the above matrix. Moreover, it can be rewritten in the standard form for a second-order system by introducing the yaw damping and radial frequency, $\zeta_{r,m}$ and $\omega_{r,m}$:

$$s^2 - \frac{N_\beta K_r + K_{z_r}}{I_{zz}} s + \frac{N_\beta + N_{\delta_r} K_\beta + K_{z_\delta}}{I_{zz}} = s^2 + 2\zeta_{r,m} \omega_{r,m} s + \omega_{r,m}^2$$

(8.3.18)

So when $\zeta_{r,m}$ and $\omega_{r,m}$ are specified the gains can be computed with:

$$K_r = -2\frac{I_{zz}}{N_{\delta_r}} \zeta_{r,m} \omega_{r,m} \quad \text{and} \quad K_\beta = \frac{I_{zz}}{N_{\delta_r}} \omega_{r,m}^2 - \frac{N_\beta}{N_{\delta_r}}$$

(8.3.19)

The bank controller drives the ailerons to achieve the commanded bank angle issued by the guidance system. The related state equations are as follows:

$$\Delta \dot{\rho}_m = \frac{L_\beta}{I_{xx}} \Delta \beta_m + \frac{L_{\delta_a}}{I_{xx}} \Delta \delta_{a,m} + \frac{L_{\delta_r}}{I_{xx}} \Delta \delta_{r,m}$$

(8.3.20)

$$\Delta \dot{\delta}_m = -\Delta \rho_m + \left( \frac{g_e}{V_e} \cos \gamma_e \cos \sigma_e - \frac{L_e}{m V_e} \right) \Delta \beta_m + \frac{\tan \gamma_e}{m V_e} \cos \sigma_e L_e \Delta \sigma_m$$

(8.3.21)

Focusing on the related terms of the above equations, the aileron-control law is defined to be

$$\Delta \delta_{a,m} = K_{a_{\beta}} \Delta \beta_m + K_{a_{\delta_r}} \Delta \delta_{r,m} + K_{a_{\sigma}} \Delta \sigma_m + K_{a_{\delta}} \Delta \delta_m$$

(8.3.22)

and is substituted in Eq. (8.3.20):

$$\Delta \dot{\rho}_m = \frac{L_\beta + L_{\delta_a} K_{a_{\beta}}}{I_{xx}} \Delta \beta_m + \frac{L_{\delta_a}}{I_{xx}} \Delta \delta_{a,m} + \frac{L_{\delta_r} K_{a_{\delta_r}}}{I_{xx}} \Delta \delta_{r,m} + \frac{L_{\delta_a} K_{a_{\sigma}}}{I_{xx}} \Delta \sigma_m + \frac{L_{\delta_a} K_{a_{\delta}}}{I_{xx}} \Delta \delta_m$$

(8.3.23)

To decouple the roll and yaw motion, from the first two terms of the right-hand side of Eq. (8.3.23) it follows that

$$K_{a_{\beta}} = -\frac{L_\beta}{L_{\delta_a}} \quad \text{and} \quad K_{a_{\delta_r}} = -\frac{L_{\delta_r}}{L_{\delta_a}}$$

(8.3.24)

As a result

$$\Delta \dot{\rho}_m = \frac{L_{\delta_a}}{I_{xx}} K_{a_{\sigma}} \Delta \sigma_m + \frac{L_{\delta_a}}{I_{xx}} K_{a_{\delta}} \Delta \delta_m$$

(8.3.25)
Laplace transforming Eqs. (8.3.21) and (8.3.25), thereby neglecting the relatively small \( \Delta \beta_m \) component that introduces a bank error due to a non-zero slip angle, gives

\[
\begin{bmatrix}
-\frac{L_{\delta_a} K_a}{I_{xx}} - \frac{L_{\delta_e}}{I_{xx}} K_a \sigma_m(s)

1 - \frac{\tan \gamma_e}{m V_e} \cos \sigma_e L_e
\end{bmatrix}
\begin{bmatrix}
\rho_m(s)

\sigma_m(s)
\end{bmatrix} = \begin{bmatrix}
0

0
\end{bmatrix}
\]  
(8.3.26)

The characteristic equation is given by

\[
s^2 + \left( \frac{L_{\delta_a}}{I_{xx}} K_a - \frac{\tan \gamma_e}{m V_e} \cos \sigma_e L_e \right) s + \frac{L_{\delta_a}}{I_{xx}} K_a = s^2 + 2 \zeta_{a,m} \omega_{a,m} s + \omega_{a,m}^2
\]  
(8.3.27)

so we find for the gains, as a function of \( \zeta_{a,m} \) and \( \omega_{a,m} \):

\[
K_a = \frac{I_{xx}}{L_{\delta_a}} \left( \frac{\tan \gamma_e}{m V_e} \cos \sigma_e L_e + 2 \zeta_{a,m} \omega_{a,m} \right) \quad \text{and} \quad K_d = \frac{I_{xx}}{L_{\delta_a}} \omega_{a,m}^2
\]  
(8.3.31)

### 8.3.2. Total-system model

To apply MRAC, the reference model and the non-linear WCC have to be combined into one dynamic simulation model (see also Fig. 8.17). Moreover, the reference-model controller parameters and the parameters that are required to compute the adaptive gains must be specified. To begin with the reference model, given by Eqs. (8.3.7) and (8.3.8), numerical values for the longitudinal controller are chosen to be

\[
\Delta q_{m,\text{max}} = 0.5 \degree/s \quad \Delta \alpha_{m,\text{max}} = 0.2\degree \quad \left[\int \Delta \alpha_m dt\right]_{\text{max}} = 0.1\degree/s \quad \Delta \delta_e_{m,\text{max}} = 20\degree
\]

For the lateral controller the values of Raney and Lallman (1992), who applied a similar controller directly to the non-linear model of the WCC, are used, i.e.,

- \( \zeta_{r,m} = 0.707 \) and \( \omega_{r,m} = 3.0 \) rad/s
- \( \zeta_{r,m} = 0.707 \) and \( \omega_{r,m} = 3.0 \) rad/s

The control and output vectors of the plant or WCC, \( \mathbf{u}_p \) and \( \mathbf{y}_p \), are defined as:
\[ u_p = (\delta_{a,p}, \delta_{e,p}, \delta_{r,p})^T \]

\[ y_p = (p_p, q_p, r_p, \sigma_p, \alpha_p, \beta_p)^T \]

With respect to the output error \( e_y \), some remarks must be made. In the initial design only the aerodynamic angles were part of the model and plant output vector, thus defining \( e_y \) as the difference between the two. It appeared to be very difficult to get a stable controller while using only the basic MRAC system. Improving the stability can be done in a mathematical manner, i.e., by adding feedforward compensators around the plant and the model, or, more generally, by including supplementary dynamics in cascade, parallel, or feedback with the plant.

However, it can also be done in a more or less physical manner, by using more information of the rotational motion of both model and plant. In principle, the angular rate of the non-linear vehicle will be measured by the IMU and thus be available to the control system. The difference (or error) in angular rate as compared with the reference model can serve as a measure of the damping, e.g., a rather large difference implies that the vehicle is changing its attitude too fast as compared with the model, and it would therefore be better if the deflection angles are restrained as to avoid overshoot and possible oscillations.
Since the output error can only have as many elements as the number of controls, each of the attitude errors is blended with the corresponding angular-rate error to form \( e_y \):

\[
e_y = \begin{pmatrix}
    k_{e\alpha}(\alpha - \alpha_m) + k_{e\alpha}\nu
    
    k_{e\alpha}(\alpha - \alpha_m) + k_{e\alpha}(q - q_m)
    
    k_{e\beta}(\beta - \beta_m) + k_{e\beta}(r - r_m)
\end{pmatrix}
\]

The gains \( k_{e\alpha}, k_{e\alpha} \) and \( k_{e\beta} \) can be selected to be unity, whereas the rate gains \( k_{e\alpha} \) and \( k_{e\beta} \) can be found while doing some response tests. Values between 0.05 and 0.1 are found to yield satisfying results.

Let the plant control vector be computed with (see Sec. 3.4.3):

\[
u(t) = K_r(t) r(t)
\]

with \( r(t) = [e_y(t), x_m(t), u_m]^T \) and \( K_r(t) = [K_e(t), K_x(t), K_u(t)] \); note that the dimensions of \( r \) and \( K_r \) are 12 x 1 and 3 x 12, respectively. The total gain matrix \( K_r \) is decomposed into a proportional and an integral part, which are computed according to the left-to-right multiplication:

\[
K_i = e_y(t) r^T T \sigma_i K_i(t)
\]
\[
K_p(t) = e_y(t) r^T(t) T
\]

The weighting matrices \( T \) and \( T \) are 12 x 12 matrices and, if completely defined, represent a total of 288 weighting coefficients. However, it is common practice for a first design to define only the diagonal elements (Kaufman et al., 1994), thereby reducing the number of coefficients to 24. Furthermore, 12 integral-gain coefficients \( \sigma_{ij} \) \((j = 1..12)\), representing each of the elements of the concatenated vector \( r \), can be selected to increase the robustness of the controller in the presence of persistent steady-state errors.

The numerical values of the controller parameters are usually found by trial and error, based on experience of the designer. Careful selection of the parameters and analysing the response will finally lead to an adequate performance of the adaptive controller. Application of a numerical optimisation technique as applied by Messer et al. (1994), or, alternatively, the Taguchi method or CCD, to determine (sub-)optimal values of the controller parameters is considered to be beyond the scope of the current study. Since the parameter values used here are not taken constant, they will be given at the appropriate places where the control system is applied, i.e., in Section 8.4, where some hypersonic manoeuvres are discussed, and Section 8.5, presenting the all-up ascent to orbit.
Remark

In case of combined aerodynamic and reaction control, there are more controls than outputs and the control problem is overdimensioned. Kaufman et al. (1994) suggest that either the number of outputs should be increased, the number of controls decreased, or a linear combination of the controls should be formed such that there is a one-to-one relation between controls and outputs. In the current study, the stabilised reference model determines the model controls, and as such it is suggested that for hybrid control the moment contribution of the corresponding control surfaces and reaction jets are taken equal. In the previous chapter, where an LQR has been applied, for both reaction and aerodynamic control similar control laws were used that both had the same control deviation as input. Thus, in this case the model and plant output vector can be extended by just doubling it. In combination with the model control, the plant control will be regulated such that the two output vectors will match.

8.3.3. Reflections on the use of Model Reference Adaptive Control

In this sub-section, some aspects that came forward while designing the MRAC system will be evaluated. As was mentioned before, initially the output error was based on only the aerodynamic angles $\alpha$, $\beta$ and $\sigma$. Most of the encountered instabilities were found with the application of this output error, pointing at the low damping that the controller could provide in case only the basic MRAC algorithm is applied. Increasing the damping by including the angular-rate error in the output error improved the results considerably, even completely removing some of the instabilities. However, since the gained experience is worth sharing it will briefly be discussed.

In the process of designing the adaptive controller, it was found that discontinuities in the aerodynamic coefficients can cause instabilities when the weighting factors are not large enough. The first discontinuity was found when the elevon deflection changed sign. The left derivative (negative deflection angles) is smaller than the right one (positive deflections). This means that when the elevon trim deflection is negative (but close to zero) and the corrective elevon deflection is such that the total elevon deflection is slightly positive, the larger right derivative causes a too large pitch moment that induces an oscillation in pitch. This oscillation could be controlled by increasing the proportional gains. However, an adjustment of the interpolation scheme that is used to compute the elevon trim deflection angle - currently linear interpolation is applied - such that continuous derivatives are obtained is advisable. One can think of cubic spline interpolation that is also commonly used in trajectory optimisation. A similar problem occurred while reaching the Mach = 6 flight condition, another node in the aerodynamic tables. Also in this case, the left and right derivatives are different. It should be noted that in principle the attitude controller should be that robust that it can easily handle these discretisations, as is, in fact, the case with the blended output error.

The adaptive controller requires relatively large weighting factors, especially for the integral
gains if accurate $\alpha$-control is pursued. Thus, the integral gains become large and when sudden $\alpha_c$ changes are commanded, oscillations can be induced due to the large commands. This can be prevented if the integral gains are reset (or largely reduced) before the $\alpha_c$ command takes place. However, it should be studied how this can be done in a smooth manner, similarly to the $\sigma_c$ compensation. Note that a large value of $\sigma_c$ reduces the integral gain but also decreases the efficiency of integral control.

Because of the large weighting factors, sudden changes in commanded attitude result in large deflections, basically due to the inertia of the vehicle and therefore increasing attitude errors. This can lead to oscillations in, for instance, the angle of attack and thus the flight-path angle. One way to improve the damping of the pitch control, apart from inclusion of the pitch-rate error in the output-error vector, might be to use non-zero entries for the off-diagonal elements of the weighting matrix. It remains to be studied in what way the response of the system can be affected, although Messer et al. (1994) have found that the results can be positively influenced.

For the nominal ascent mission it appeared that the guidance commands could serve as reference model. This is due to the fact that the attitude controller included in the reference model is relatively fast, i.e., it can track the guidance commands almost instantaneously. This means that the reference state is almost equal to the guidance commands. Since the main contribution to the elevator command is the trim angle, the corrective control coming from the reference model is practically zero. Note that for lateral motion, the nominal aileron and rudder deflections are zero, such that the corrective control can have an impact on the definition of the plant control vector, especially when the corrective control is large, e.g., during hypersonic turns.

Unfortunately, also discrete changes in $\alpha_c$ are immediately followed and this results in some oscillatory behaviour of the WCC, as will be discussed later. It is therefore recommended that designing the reference model including attitude controller should not be regarded lightly. The response of the reference model to the guidance commands should be smooth, especially at those flight conditions where the aerodynamic forces and moments are large. It should also be understood that continuous guidance commands without abrupt changes will enhance the performance of the MRAC system while applied in supersonic and hypersonic flight.

The values for the MRAC weighting coefficients are listed in Table 8.7. However, some explanation needs to be given. First, the parameter values are chosen such that an acceptable response is guaranteed, without trying to optimise this response. We also found that for other flight conditions (higher or lower Mach numbers), different parameter values would improve the response. A future extension could be weighting coefficients as a function of Mach number. Second, we only define non-zero diagonal elements for $T$ and $\hat{T}$. Off-diagonal elements are all zero. Third, we distinguish three sets of coefficients for each of the three rotations roll, pitch and yaw. Kaufman et al. (1994) do not discuss this, since they only consider SISO systems. However, we found that if the weighting coefficient of, for instance, $\alpha_{err}$ is large (which is required for an adequate pitch response) and large errors in both $\alpha$ and $\beta$ are occurring simultaneously, then the contribution to the rudder deflection is also large, and in our case it appeared to be too
large. For that reason, a small weighting coefficient for $\alpha_{err}$ would be favourable, but this would give a slow pitch response. We solved this by defining the weighting coefficients for each of the three controls independently. The remaining coefficients that are not listed in each of the columns are defined to be $10^{-6}$ for the proportional gains (to meet with the positive definiteness of $T$) and 0 for the integral gains (to meet with the positive semi-definiteness of $\overline{T}$). Fourth, the integral-gain coefficients are the same for each of the integral gains.

Concluding this sub-section, we will briefly discuss the results of a flight at constant $\alpha$ ($M = 5.6$; for the initial conditions the reader is referred to Section 8.4.1). In one case, we will perturb the initial $\alpha$ with $\Delta \alpha = 1^\circ$, and in a second case the initial bank angle is perturbed with $\Delta \sigma = 5^\circ$. In Fig. 8.19, the angle-of-attack error is plotted and in Fig. 8.19 the corresponding total elevator deflection, i.e., trim value plus corrective control. It is clear that because of a large initial elevator deflection ($\Delta \delta_e = 9^\circ$), a fast response is obtained, though as a result there is an overshoot in $\alpha$ of almost $0.6^\circ$. The angle of attack stabilises at $2.4^\circ$ after about 8 s.

<table>
<thead>
<tr>
<th>roll</th>
<th>parameter</th>
<th>value</th>
<th>pitch</th>
<th>parameter</th>
<th>value</th>
<th>yaw</th>
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Table 8.7 - MRAC parameter values for the $M = 5.6$ flight condition.

A similar bank-angle response is observed when $\sigma$ is perturbed with $\Delta \sigma = 5^\circ$. The initial response is fast (see Fig. 8.20) whereas the overshoot is about $0.5^\circ$. The aileron deflection (Fig. 8.21), though, is smaller than $\Delta \delta_e$ i.e., $\Delta \delta_e = 4.5^\circ$ which is due to a lower inertia about the $X_B$
axis. The augmented roll damping is lower than the augmented pitch damping, which shows as a somewhat stronger oscillatory behaviour, although the amplitude is quite small. A small induced \( \beta \) is observed \( (\Delta \beta = 0.15^\circ) \), which indicates that although the roll-yaw motion is not completely decoupled the coupling is not so strong. The coupling can probably even be further reduced by optimising the controller. The rudder deflection (Fig. 8.21) is only 0.2\(^\circ\).

Concluding, the first results obtained by using the MRAC system are promising, and form the basis for a more detailed evaluation in the next two sections.

8.4. Hypersonic manoeuvres

The first application of MRAC will not be the reference ascent trajectory of Section 8.2, since the changing flight environment from \( M = 0.5 \) to \( M = 20 \) is considered to be too much, unless we gain more experience first. Therefore, we will simulate some simple manoeuvres, i.e., an altitude transition and subsequent altitude hold (cruise flight), and a cross-range transition. The related guidance system has been described in Section 3.2.2. The simulations are executed in three parts. In Section 8.4.1, a nominal altitude transition and subsequent cruise flight with and without a steady-state wind is studied. In Section 8.4.2, a cross-range transition with and without wind is simulated. Finally, in Section 8.4.3, a sensitivity analysis for a combined altitude and cross-range transition is executed.

8.4.1. Altitude transition and subsequent cruise flight

To reach a target orbit with non-zero inclination the final ascent is started at a latitude equal to the orbit inclination. When the latitude of the launch base is not equal to the target latitude, a cruise flight may be necessary before the pull up that marks the beginning of the final ascent. In this subsection, an altitude transition (step command of \( \Delta h_c = 1,000 \) m) and subsequent cruise flight at a maximum dynamic pressure of 95,000 N/m\(^2\) will be studied for \( M = 5.6 \). The initial state of the WCC is partly based on data given by Raney and Lallman (1992) and partly on the 3-d.o.f. reference trajectory of Section 8.2:

\[
\begin{align*}
V_0 &= 1667.0 \text{ m/s} \\
\gamma_0 &= 0.0^\circ \\
m_0 &= 118,968 \text{ kg} \\
p_0 &= 0.0 \text{ °/s} \\
alpha_0 &= 2.8^\circ \\

c_0 &= 90^\circ \\
h_0 &= 21,641 \text{ m} \\
\delta_{\tau_0} &= 0.2 \\
q_0 &= 0.0 \text{ °/s} \\
\beta_0 &= 0.0^\circ \\
\delta_0 &= 0.0^\circ \\
r_0 &= 0.0 \text{ °/s} \\
\sigma_0 &= 0.0^\circ \\
\tau_0 &= 0.0^\circ 
\end{align*}
\]

The values for the MRAC weighting coefficients are listed in Table 8.7. Moreover, limitations are considered on \( \alpha_c \) (\( \alpha_{c,\text{min}} = 2.4^\circ \) and \( \alpha_{c,\text{max}} = 3.2^\circ \)) and \( \delta_T \) (\( \delta_{T,\text{min}} = 0.1 \) and \( \delta_{T,\text{max}} = 0.5 \)). Also
the influence of a steady-state wind will be studied. The wind model is based on the GRAM-95 model (Justus et al., 1995), that, although it has been defined for Kennedy Space Center, is representative for winds at such an altitude. The wind vector is defined as a number of discrete east-west zonal components as a function of altitude: $V_w = 12.2 \text{ m/s} \ (h = 20.0 \text{ km})$, 10.4 m/s (21.0 km), 10.0 m/s (22.0 km), 10.2 m/s (23.0 km) and 10.6 m/s (24.0 km). Linear interpolation is used to obtain intermediate values.

In Fig. 8.22, $h$ is plotted as a function $t$. It can clearly be seen that the altitude transition is smooth and the space plane takes about 150 s to reach a stable cruise-flight condition. The influence of the steady-state wind shows as a delay in the ascending flight. Since the WCC is flying against the wind, $q_{\text{dyn}}$ is increased and $\delta_T$ will be decreased. This results in a lower ground velocity and hence a longer climb.

Fig. 8.23 shows $\alpha_{\text{err}}$ as a function of time. The largest deviations from $\alpha_c$ occur at the beginning of the climb ($t = 0 \text{ s}$) and at the end of the climb ($t = 90 \text{ s}$), when the guidance system commands a smaller $\alpha$ to decrease the climb rate. For the no-wind case, the error is well damped and in general the control error is very small. In case there is wind, there are some larger errors up to $t = 40 \text{ s}$. Initially, it seems difficult to understand this phenomenon until we have a look at $\delta_T$ (Fig. 8.24). It is obvious that $\delta_T$ is oscillating, to account for the head wind and to lower $q_{\text{dyn}}$. However, due to the relative fast response and the limit on $\delta_T$ (10 \%/s) it takes a while before a stable condition is reached. A review of the throttle-control law should prevent these kind of oscillations (note: $q_{\text{dyn}}$ could be kept at 95 kPa, once equilibrium was reached). Finally, the shift in the two throttle curves corresponds with the shift in the two height curves of Fig. 8.22.

The last figure that we discuss here shows $\delta_e$ as a function $t$. Any sharp peak corresponds with a peak value in $\alpha_{\text{err}}$ and aims at reducing this control error. The oscillating deflection in the presence of wind is of course again related to the oscillating $\delta_T$. It should be noted that $\sigma$ and $\beta$ were not affected by this manoeuvre and remained zero throughout the flight.

Concluding, the attitude controller is very well capable of following guidance commands for an altitude transition and remains stable during the subsequent cruise flight. The presence of a steady state wind does not seem to influence the performance of the attitude controller.

### 8.4.2. Cross-range transition

Starting from the same initial conditions as in the previous sub-section, a cross-range transition of 2,000 m is simulated. This manoeuvre consists of a small initial heading change towards the south and a correcting change to bring back the heading to one due east. A positive cross-range transition results in this case in a target latitude south of the equator.

In Fig. 8.26, the cross-range error is plotted as a function of time. The initial error is of course equal to the commanded 2,000 m. We see a smooth decrease and in the case there is no wind a rather large overshoot. It appeared that the PID controller that is applied in the
cross-range guidance is very sensitive to the integral gain. A very small gain value will decrease the overshoot but will also significantly increase the response time. Since the PID gain values are established by trial and error, it is expected that the performance will be improved once the closed-loop system is properly analysed. It is remarkable to see that the presence of a head wind works stabilising, in the sense that the overshoot is smaller and equilibrium is reached must faster, although it takes somewhat longer to cover the 2,000 m. The explanation can be found in the next figure (Fig. 8.27), where the altitude error is depicted. This error is larger in case there is wind. Since attitude control has priority over cross-range control (see also Fig. 3.7), and the commanded load factor is close or even equal to its maximum allowable value, there is only limited or no cross-range control (which has been verified by inspecting $\sigma_c$). This means that once cross-range control is started again, the vehicle has already covered a significant part of the 2,000 m and can suffice with a smaller $\sigma_c$ which results in the smaller overshoot.

Part of this reasoning is confirmed by Fig. 8.28. The bank-angle error becomes zero after the first 10 s of flight, and the next command shows as a peak in $\sigma_{err}$ at $t = 45$ s. The initial $\sigma_c$ is large, i.e., $-22^\circ$, which explains the large control error. However, the attitude controller is stable and quickly reduces the error to zero. Fig. 8.29 shows the induced $\beta$, caused by the roll-yaw coupling. The MRAC system is apparently not capable of uncoupling the roll and yaw motion. Further study on how the weighting coefficients need to be altered to accomplish this is required.

We finish this sub-section by giving the time histories of the elevon and rudder deflections, which should give similar peaks as the corresponding control errors. In Figs. 8.30 and 8.31 the left and right elevon deflections are plotted. For a major part, both deflections are the same and are equal to the trim value. Only during the first 50 s there is some strong activity, mainly due to the aileron function to initiate the roll manoeuvre for tilting the lift force and hence to start the cross-range transition. Fig. 8.32 shows the rudder deflection, which is basically very small ($\pm 1.2^\circ$). Indeed it is possible to deflect the rudder at larger angles to minimise the induced $\beta$.

Concluding, although it seems that the attitude controller is capable of following $\sigma_c$ and to minimise the induced $\beta$ during a cross-range transition, it seems that the performance should be further improved before it can be called robust.

**8.4.3. Sensitivity analysis**

For the sensitivity analysis, a combined altitude and cross-range transition of 1,000 m each will be considered. Initial conditions and MRAC weighting coefficients are the same as before.

Analysis of the results will focus on the performance of the control system. The selected responses are the maximum control errors $\alpha_{err}$, $\beta_{err}$ and $\sigma_{err}$ and the control-surface activity that is given by the mean and standard deviation of the deflection angles. The 6 factors that we include are the following: $\Delta X_{cm} = \Delta Y_{cm} = \Delta Z_{cm} = \pm 0.1$ m and $\Delta C_l = \Delta C_m = \Delta C_n = \pm 5\%$, and they
will be varied according to the 2-level $L_8$ array. The factors are (arbitrarily) assigned to the leftmost columns of the array. It should be noted that we will not try to establish the existence of particular interactions. Only a general conclusion about possible interactions will be made by inspecting the error variance.

Executing the simulations shows that 4 out of the 8 runs end in unstable oscillations (#2, #4, #5 and #8). Run #2 even resulted in a crash of the vehicle. Part of the explanation can be found in the design of the guidance system. Initially, a cross-range command is generated that results in a small heading change. However, the moment the load limits are reached priority is given to altitude control. Since in the presence of the disturbances it is not quite possible to reduce the attitude error to zero, the guidance system does not issue a commanded bank angle differing from zero for a long time. In the mean time the covered cross range increases due to a heading that deviates from the initial heading. The moment the load limit is relaxed, the guidance system is faced with a large cross-range error, and the subsequent $\sigma_c$, in combination with the perturbations is in some cases the cause of unstable oscillations.

Inspecting the factor contributions to the total variance of the responses shows that due to the runs with oscillations the results are so much influenced that in most cases the factor variances are equal. However, this is also true for the error variance which means that all factors are likely to interact during the control problems. Without further simulations (and by eliminating the confounding) it is not directly possible to draw conclusions from the ANOVA.

In trying to understand to what extent the guidance system is at fault and to what extent the attitude controller, as an example we will analyse one of the trajectories that does not exhibit attitude oscillations (run #1). In Figs. 8.33 and 8.34 the attitude and cross-range error are shown. It is clear that the guidance system is not capable of reducing both errors to zero. The attitude error exhibits an oscillatory behaviour that even seems to diverge at the end of the simulation interval. We already mentioned the ever increasing cross-range error due to a fixed heading and load limits that prevent active cross-range guidance.

A question that arises now is: is the attitude controller responsible for the guidance errors? Fig. 8.35 should give the answer. In this figure the control errors in $\alpha$, $\sigma$ and $\beta$ are depicted. The solid line represents $\alpha_{err}$ and is quite close to zero although the error is slowly diverging and reaches a final value of just over 0.15°. This means that the attitude controller is very well capable of following $\alpha_c$. The long-dashed line shows $\sigma_{err}$. Apart from two peak values of about -5° and -2° when the only two major cross-range guidance commands are issued, the error has a constant bias of about -0.2°. Although the attitude controller cannot reduce this error to zero in the presence of the perturbations, the error remains bounded (and small). The third curve (short-dashed) is difficult to distinguish: $\beta$ is practically zero. A maximum (absolute) value of 0.2° was found at $t = 45$ s.

In Fig. 8.36, the corresponding corrective elevator and aileron deflections are shown. It is obvious that due to the perturbations a non-zero equilibrium value for both is required. Added to the trimmed elevon deflection, an asymmetric deflection of the left and right elevon is the result: the difference is about 2° which is the result of the non-zero aileron deflection.
Concluding, at least for the results of this single run, the attitude controller seems to be performing reasonably well. Apparently, the guidance system cannot adequately reduce the altitude and cross-range error and should be carefully reviewed.

8.4.4. Forces and moments due to mass expulsion

In Chapter 2, the equations of motion have been defined for a non-elastic vehicle with variable mass properties. The apparent Coriolis force, $F_C$, and moment, $M_C$, were introduced to account for this variation in mass properties. To assess the relative importance of $F_C$ and $M_C$ numerical values are computed and compared with the aerodynamic lift force and pitch damping moment. The flight case that is considered is the nominal altitude transition at $M = 5.6$.

Since both $F_C$ and $M_C$ are dependent on the rotational rate of the vehicle, or, to be more precise, only on the pitch rate since the propulsion system is axisymmetric, it can be expected that the largest values occur at the initial pull up ($t = 0$ s) and the corrective pull down at the end of the manoeuvre at $t = 120$ s (e.g., see Fig. 8.22). However, the pitch rate is very small ($q = -0.3$ °/s), partly due to the applied gains in the guidance system, the dynamic-pressure constraint and the limit values on $\alpha_C$. Therefore, only small values may be expected. Indeed, this is the case. The Coriolis force has a component in $Z_B$-direction only with a peak value of 15 N and an average of about 3 N. Compared with a lift of about 900,000 N, it can be neglected, as was already stated in Chapter 2.

The pitch damping moment has peak values of 12,000 and 8,000 Nm, respectively, and in between $t = 0$ and $t = 120$ s an average value of 450 Nm. The total Coriolis moment has peaks of 120 and 100 Nm, with an average value of 15 Nm. Also for $M_C$ the conclusion is that it can be neglected and that apparently the jet damping is far less than in the case of conventional rockets. However, this conclusion may not be true for other vehicles, propulsion systems and/or manoeuvres. At higher flight velocities the damping moment decreases. Moreover, when the exhaust is further away from the c.o.m. the jet-damping moment increases. Also, since the manoeuvre is flown at a throttle setting of $\delta_T = 0.3$, with full throttle the mass flows will be triple the value and thus also $M_C$. Finally, a non-axisymmetric propulsion system and asymmetric manoeuvres may also introduce roll and yaw components of $M_C$. But, for the current modelling of WCC and propulsion system $M_C$ can be neglected.
Fig. 8.22 - Height versus time for $\Delta h_c = 1,000$ m without (solid) and with (dashed) wind.

Fig. 8.23 - The angle-of-attack error versus time for $\Delta h_c = 1,000$ m without (solid) and with (dashed) wind.

Fig. 8.24 - Throttle setting versus time for $\Delta h_c = 1,000$ m without (solid) and with (dashed) wind.

Fig. 8.25 - Elevon deflection versus time for $\Delta h_c = 1,000$ m without (solid) and with (dashed) wind.
Fig. 8.26 - Cross-range error versus time for $\Delta Y_c = 2,000$ m without (solid) and with (dashed) wind.

Fig. 8.27 - Altitude error versus time for $\Delta Y_c = 2,000$ m without (solid) and with (dashed) wind.

Fig. 8.28 - Bank-angle error versus time for $\Delta Y_c = 2,000$ m without (solid) and with (dashed) wind.

Fig. 8.29 - Angle of sideslip versus time for $\Delta Y_c = 2,000$ m without (solid) and with (dashed) wind.
Fig. 8.30 - Left-elevon deflection versus time for $\Delta y_c = 2,000$ m without (solid) and with (dashed) wind.

Fig. 8.31 - Right-elevon deflection versus time for $\Delta y_c = 2,000$ m without (solid) and with (dashed) wind.

Fig. 8.32 - Rudder deflection versus time for $\Delta y_c = 2,000$ m without (solid) and with (dashed) wind.

Fig. 8.33 - Altitude error versus time for run #1 of the sensitivity analysis.
Fig. 8.34 - Cross-range error versus time for run #1 of the sensitivity analysis.

Fig. 8.35 - Control errors in $\alpha$ (solid), $\sigma$ (long-dashed) and $\beta$ (small-dashed) for run #1 of the sensitivity analysis.

Fig. 8.36 - Corrective elevator (solid) and aileron (dashed) versus time for run #1 of the sensitivity analysis.
8.5. Vertical-plane ascent to orbit

8.5.1. Introduction

Before we start the discussion on the results obtained with the adaptive controller, it should be mentioned that the trajectory definition in the form of flight segments with discontinuous transitions from one segment to the other caused some problems for the attitude controller. For instance, during transition from segments with a prescribed $\dot{\gamma}$ to one with a constant $\gamma$, oscillations are induced that were relatively difficult to control by the attitude controller, especially at higher Mach numbers. If during the first part of the transition the proportional $\alpha$-error, $\alpha_m$ and $\omega_m$ gains are increased, a stronger oscillation damping is observed. However, when the weighting factors are chosen too large then an opposite effect can be the result, i.e., a too rapid response resulting in oscillatory behaviour that is poorly damped or even diverging.

While designing the adaptive controller, the transition between the segments to come closer to a more realistic reference trajectory and subsequent guidance commands has been adjusted. These adjustments are very simple and should be regarded as a shaping of $\gamma_c$ such that discontinuities are removed. A review of the guidance system is recommended to avoid possible instabilities in the attitude controller that are induced by a poorly designed guidance system. (A possible starting point could be the paper by Schmidt (1997), who developed guidance laws to robustly control energy height and altitude and follow fuel-optimal trajectories.)

In the process of finding a simple solution for this problem without a complete redesign of the guidance logic, it appeared that when the guidance sample frequency is increased a larger proportional gain can be applied that gives a better response, i.e., $\gamma_c$ can be tracked more closely without abrupt changes in $\alpha_c$. Moreover, the following adjustments were made:

- transition from $\dot{\gamma}_{c,1}$ to $\dot{\gamma}_{c,2}$: during the first two seconds in the next segment, a linear weighting of $\dot{\gamma}_c$ is applied, i.e., $\ddot{\gamma}_c = (1 - 0.5 dt_{seg})\dot{\gamma}_{c,1} + 0.5 dt_{seg}\dot{\gamma}_{c,2}$, where $dt_{seg}$ is the time spent in the next flight segment.
- transition from $\dot{\gamma}_c = \text{constant}$ to $\gamma_c = \text{constant}$ (i.e., $\dot{\gamma}_c = 0 \, ^\circ$/s): slowly decreasing $\dot{\gamma}_c$, similarly as above.
- transition from $\gamma_c = \text{constant}$ to $\dot{\gamma}_c = \text{constant}$: slowly increasing $\dot{\gamma}_c$, similarly as above.

To avoid any remaining peaks in $\gamma_c$ a first-order shaping filter is applied. This filter has the following transfer function:

$$G_{ps}(s) = \frac{1}{\tau_{\gamma}s + 1} \quad (8.5.1)$$

The filter parameter $\tau_{\gamma}$ is selected to be $\tau_{\gamma} = 0.1$, corresponding with a time constant of 0.63 s.
Although this time constant seems to be very small, it appeared to be sufficient to remove sharp peaks and hence improve the response. Finally, at the moment of guidance transition we reset the integral gains of the attitude controller. Since the transition will almost certainly results in a discrete change in $\alpha_c$ and therefore also an angle-of-attack error that is less related to the error of the previous segment, resetting the integral gains avoids excessive deflections.

While flying the 3-d.o.f. reference trajectory (Section 8.2), it was assumed that throttle control was instantaneous. This led to jumps $\delta_T$ of tens of percents. These jumps resulted in a dynamic response of the vehicle while flying in 6 d.o.f., and showed in the form of oscillations in $\alpha$. To prevent these oscillations a finite $\delta_T$ is included, which is of course also more realistic to begin with. However, we could not put a value to this rate of change, but for the sake of simulation we will assume $\delta_T = 10\%/s$. The influence of this limit on the reference trajectory is not further studied here.

Concluding the discussion on the changes that were made as compared with Section 8.2, we found that the influence of the rotational dynamics on the mass point performance was substantial, especially in relation with the $n_g$-constraint. This constraint was encountered more than once in the first part of the 6-d.o.f. ascent flight and resulted in notable variations in throttle setting. It appeared that the rotational dynamics were significantly affected by this, giving additional problems for the control system. Since both the guidance system and the propulsion controller are by no means mature, it was decided to exclude the axial-acceleration constraint from the mission definition, but only for the sake of simulation.

As a result, however, the initial part of the trajectory had to be redefined because the space plane would otherwise climb too fast and would not be able to reach orbit. Changes that were made in the trajectory definition of Fig. 8.5 concerned flight segment #1 ($\gamma_1 = 22.5^\circ$), #2 ($\gamma_2 = 5.5^\circ$) and #3 ($\gamma_3 = 3.6^\circ$). Moreover, two extra segments were included in between #1 and #2 ($\dot{\gamma}_c = -0.001 \text{ }^\circ/\text{s during } dt = 1 \text{ s} )$ and #2 and #3 ($\dot{\gamma}_c = -0.125 \text{ }^\circ/\text{s until } \gamma = 4.4^\circ$).

8.5.2. The nominal mission

While simulating the nominal mission, it appeared that the inclusion of rotational dynamics did not alter the symmetric character of the trajectory. The vehicle kept on flying along the equator in eastern direction, whereas $\beta$ and $\sigma$ remained zero. We will therefore discuss only results that are related to the symmetric motion.

In Fig. 8.37, the time history of $\alpha_c-\alpha$ has been plotted. It appears that for the major part of the trajectory, $\alpha_{err}$ is very small (< 0.01°) which clearly indicates that the MRAC system can accurately track the commanded angle of attack\textsuperscript{19}. However, this figure also shows that in

\textsuperscript{19} Because we compensate the integral gain to avoid overflow in the presence of persistent non-zero perturbations, the angle-of-attack error will reach some equilibrium value: the remaining error will increase the
the first 50 s of flight the error is much larger and exhibits some peaks. A detail of this part of the flight has been plotted in Fig. 8.38. During the initial pull-up $\alpha_{err}$ is rather large, i.e., up to 0.2°, due to the fact that the system has to 'settle down'. Once $q$ has obtained the trim value corresponding with $q_c$, $\alpha_{err}$ starts decreasing. However, at $t = 17$ s there is an almost discrete jump in $\alpha_{err}$ that is caused by the change of trim mode at $M = 0.9$, i.e., from canards to elevons. Again the error decreases smoothly until a second discrete jump in $\alpha_{err}$ indicates the switch between the open-loop guidance for the pull-up phase and the closed-loop gamma-alpha steering. Two conclusions are evident: a change of trim mode should be smooth, possibly using an interpolating function of $M$, and one should take particular care of the transition from one guidance law to another.

In Fig. 8.39 $\delta_e$ is plotted as a function of flight time. Considering the rather small $\alpha_{err}$ no surprises are expected from this profile other than two peaks in the first 25 s of flight (trim-mode switching and guidance transition). The irregularities that can be found at, for instance, $t = 100$ s and $t = 300$ s are due to guidance transitions.

In Fig. 8.40 the flight-path angle error is depicted, both for the 6-d.o.f. nominal ascent mission and the corresponding 3-d.o.f. trajectory. It is obvious that $\gamma_{err}$ is much smaller in the 3-d.o.f. case. While simulating a guided ascent in only 3-d.o.f. one might think that the guidance system is performing rather well and can track the reference flight-path angle. However, only the inclusion of attitude dynamics does already give rise to a much worse performance. Although we knew beforehand that the gamma-alpha PID guidance system is quite coarse, the difference is still surprising. Note that an oscillation in $\gamma$ gives rise to an oscillating $\alpha_c$ to compensate for it. If the damping of the augmented system is not sufficient the oscillations will only damp out slowly, or can even be amplified.

Another effect of simulating in 6 d.o.f. instead of 3 d.o.f. is shown in Fig. 8.41, which gives $\delta_T$ as a function of time. As a whole, it seems that in the 6-d.o.f. simulation the throttle setting is higher. Comparing the two end masses, however, gives a different answer: the 6-d.o.f. simulation gives a final mass of 73,942 kg (flight duration 1,578 s) whereas the 3-d.o.f. simulation gives a final mass of 73,154 kg ($t_{final} = 1,640$ s), almost 800 kg lower (which is more than 10% of the 7,000 kg payload). The difference is obviously the result of the longer flight. The explanation should be found in the fact that the 3-d.o.f. trajectory is by no means an optimal trajectory. For ascent missions the throttle setting should be as high as admissible in order to fly along an optimal trajectory. Since in the 6-d.o.f. simulation $\delta_T$ is larger, the resulting trajectory is more optimal and will thus be closer to the optimum minimum-fuel trajectory. Possibly, due to the use of the elevons additional lift is generated, more than the trim lift in the 3-d.o.f. case. In that case the vehicle is flying at a slightly higher altitude that results in a lower drag and thus a larger acceleration per kg fuel.

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integral gain at the same pace as the integral-gain coefficient $\sigma_i$ will decrease it.
8.5.3. Sensitivity analysis

While designing the MRAC system it became already obvious that the guidance system is very coarse and needs a significant improvement before it will give adequate performance during the ascent mission. As a result, the sensitivity analysis that we want to perform can only be limited to a few factors. Moreover, since the guidance system and the mission profile is in principle two-dimensional, i.e., the ascent is restricted to the equatorial plane, we can only study the symmetric motion of the space plane\(^\text{20}\), and establish the performance of the attitude controller. The 7 factors that we include in the analysis are: \(\Delta X_{cm} = \Delta Z_{cm} = \pm 0.1 \text{ m}, \Delta C_D = \Delta C_L = \Delta C_m = \pm 5\%, \text{ and } \Delta \phi = \Delta T = 5\%.\) We do a simultaneous variation of these parameters thereby using Taguchi’s \(L_8\) array. Stop condition for each of the 8 simulations is either \(V_f = 7,000 \text{ m/s} \) or an excess flight time of 2,000 s.

Out of the 8 simulations, 3 ended up in an oscillating trajectory as can readily be seen in Fig. 8.42, where the 8 height-versus-time curves are plotted. Runs #3 and #7 ended in an oscillatory state (rotations around each of the three axes), because the elevon deflection had reached its maximum while the attitude controller commanded even larger deflections to compensate for the perturbations. Despite the fact that we simulated a symmetric flight with only ‘symmetric’ perturbations, the asymmetric motion was influenced. This shows the dynamic coupling between symmetric and asymmetric motion. Run #5, however, does not exhibit oscillations in the attitude angles - also the elevon does not reach its maximum - but in the mass-point motion (in Fig. 8.42 this shows as the smallest of the three oscillation patterns that continues up to \(t = 2,000 \text{ s}\)). Apparently, one of the eigenmotions is excited that cannot be suppressed by the guidance system. We assume that this oscillation is initiated because of a too low dynamic pressure that is not sufficient to maintain a large enough thrust and thus vertical acceleration.

The angle-of-attack error, as plotted in Fig. 8.43 apart from the curves of runs #3 and #7, shows errors between -0.18° and 0.65° (run #5, which is somewhat different in nature compared with the other runs has a decreasing \(\alpha_{err}\) down to -0.4°). Apparently the basic MRAC system is not capable to reduce the persistent \(\alpha_{err}\). Either the weighting coefficients related to the integral gains are not large enough, or the integral coefficient \(\sigma_i\) is too large, preventing the integral gains from growing too large. More experimenting with the weighting matrices (before the controller is, for instance, extended with supplementary dynamics) is required.

In Fig. 8.44, finally, \(\delta_e\) is depicted. It shows clearly the two runs reaching the maximum deflection of 20°. Also the deflection profile of run #5 is evident: although the deflection is large,

\(^{20}\) A shift of the location of the c.o.m. in \(y\)-direction of 0.1 m, resulted in an angle of sideslip of more than 20° in the initial flight phase. The MRAC system managed to reduce this error to about 2° in 100 s. Then, \(\beta\) increased slowly again, resulting in severe oscillations after a few hundreds of seconds. Due to the induced side force and the induced bank angle (roll-yaw coupling that is not completely eliminated by the attitude controller) a large heading change made the space plane fly back to the west, and finally to the east again. A guidance system that regulates the heading is required to compensate for these asymmetric deviations.
it does not reach 20°. In fact, after the mass-point oscillation has started and the space plane starts descending slowly, the elevon deflection slowly decreases due to Mach effects, until it more or less stabilises at about 16.5°.

Concluding, the performance of the attitude controller seems to be reasonable, although the performance can likely be improved when the weighting matrices are optimised. In addition, it is required to repeat the simulations with an alternative controller, e.g., the LQR of Section 3.4.2, such that a performance comparison can be made (see also Mooij, 1998a).

8.6. Summary

- The simulation and analysis of the powered Winged Cone Configuration includes closed-loop guidance based on inverse dynamics for the ascent to orbit and PID closed-loop guidance that is used for altitude and cross-range transitions. Moreover, a Model Reference Adaptive Control system that makes use of aerodynamic control surfaces and a closed-loop throttle control law to track the trajectory constraints (maximum dynamic pressure, axial acceleration and heat flux) complete the guidance and control system. Trim can be guaranteed by using either elevons or TVC.

- The 3-d.o.f. reference trajectory is computed by dividing the trajectory into a number of flight segments and by specifying the flight-path angle profile for each of the segments. The trajectory is sub-optimised with respect to payload mass by treating the segment parameters as design variables and by doing a Taguchi analysis. A payload-mass variation between 991 and 5857 kg was found, which indicates that a proper selection of the trajectory is very important to ensure mission success. The minimum-fuel trajectory does not correspond with the minimum heat-load trajectory, which takes up 1,401 kg more fuel. Therefore, the maximum allowable heat load should be included in any trajectory optimisation process. The relative ease with which trajectories could be found that could reach orbit, seems to indicate that the performance of the propulsion system is overrated with respect to the vehicle's dry mass.

- ANOVA of the results indicated that the influence of interactions is small for this particular design problem of finding a reference trajectory. Redoing the design with double the number of simulations by using the $L_{32}$ instead of the $L_{16}$ array, gave different means (73,578 and 73,682 kg) and standard variations (1,382 and 1,280 kg) of the selected response, the consumed fuel mass. However, conducting a Kolmogorov-Smirnov test proves that the two distributions of the fuel masses are consistent. On the other hand, there also appeared to be a shift in factor contribution to the total variation which means that major factor effects can be extracted from the results, but that the absolute importance cannot be fully assessed.
The Taguchi method cannot replace a numerical trajectory optimisation method. However, by doing a Taguchi analysis for a given trajectory segmentation one can increase the insight in the influence of the trajectory parameters on a selected response.

Using the elevons for trim, results in a relatively large contribution to the drag and lift. Therefore, for an accurate performance analysis the influence of trim drag and lift should both be included. Moreover, the trim drag increases the consumed fuel mass substantially. Using TVC to generate part of the compensating trim moment results in a significant fuel reduction. However, when major part of the flight is flown with full throttle the trajectory has to be redesigned (or re-optimised) to compensate for the loss of accelerating power. The technical feasibility of TVC was not included in this study.

The fundamental design of an MRAC control system is relatively easy. Unfortunately, there are many design parameters that have to be defined by the designer, and unless his experience is large this can be very time consuming. An alternative would be to define performance parameters for the attitude controller, such as integrated control errors and deflection angles, and maximum allowable values of these parameters. An optimisation process can then be carried out to determine the weighting coefficients, but since this number can be quite large it remains a complicated matter. In this study, we have applied the basic MRAC algorithm and defined the weighting coefficients by trial and error. Only the diagonal elements have been used and it remains to be studied how off-diagonal elements can influence the performance. The damping of the system has been improved by including the angular rates in the output errors, i.e., the difference between model and vehicle attitude angles.

Some aspects that came forward during the design process of the MRAC system are:
- Discontinuities in the aerodynamic coefficients can cause instabilities when the weighting factors are not large enough.
- When the integral gains can increase to large values, a strong response can be expected when there are discontinuities in the input signal. This effect can be reduced by including so-called integral coefficients $\sigma_i$, although a large value of $\sigma_i$ decreases the efficiency of integral control.
- Combination of roll, pitch and yaw control in one MRAC control system, and the application of one set of weighting matrices can give rise to conflicting demands when simultaneously large output errors occur. In this study, this is solved by defining three different sets of weighting matrices.
- Applying the MRAC system to nominal hypersonic manoeuvres showed that the controller is very well capable of following the guidance commands for an altitude and cross-range transition, and to remain stable during the subsequent cruise flight. The presence of a
steady state wind does not seem to influence the performance. However, considering lateral control it seems that the performance should be further improved before the controller can be called robust. During the succeeding sensitivity analysis the limitations of the guidance system were revealed, whereas the attitude controller has a reasonable performance for at least a number of simulations.

- While studying the guided and controlled ascent to orbit, it was found that the influence of rotational dynamics on the point-mass performance was substantial. A relatively fast oscillating throttle induces angle-of-attack perturbations that are difficult to control. These throttle fluctuations were the result of frequently meeting with the axial-acceleration constraint, and as such a redesign of the propulsion controller seems to be required. Integration of flight and propulsion controller as proposed by Schmidt (1995 and 1996) could be a possible approach. Finally, the difference between mass-point and finite-body performance is significant, although this seems to be due to the fact that the 6-d.o.f. trajectory is closer to the optimal trajectory than the 3-d.o.f. one.

- Doing a sensitivity analysis for the ascent mission, we found that the performance of the attitude controller seems to be reasonable, although the performance can likely be improved when the weighting matrices are optimised. In addition, it is required to repeat the simulations with an alternative controller, e.g., a LQR, such that an accuracy and performance comparison can be made.

- Major conclusion from this chapter is that, although still a lot of work remains to be done, Model Reference Adaptive Control seems to be a promising concept when applied to hypersonic space planes.
Chapter 9

Conclusions and Recommendations

The main question or problem definition of this study has been formulated as follows:

How and to what extent do design uncertainties and environmental disturbances influence the mission performance of space planes, and in what way can a GNC system contribute to mission success?

This problem definition has been translated into three study goals, i.e.,

1) develop a 6-d.o.f. flight simulation tool, which can be used to compute the ascent and descent trajectories of space planes,
2) assess the potential use in the study of flight mechanics in general, and the analysis of G&C systems in particular, of the Taguchi Method and the related Response Surface Methodology as an alternative to the commonly applied Monte-Carlo Method, and
3) study the flight mechanics of space planes in ascent and descent missions and identify possible shortcomings in the G&C-system design, thereby treating the space plane as a non-elastic body and using relatively simple input models.

In Section 9.1 the major conclusions and findings of the thesis study are stated whereas in Section 9.2 recommendations for further research are given.
9.1. Conclusions

To compute the trajectory and related performance of space planes in sufficient detail, a computer program, START (Simulation Tool for Ascent and Re-entry Trajectories), has been developed. START can be used to do simulations in 3 or 6 d.o.f., thereby treating the vehicle as a mass point or a non-elastic body with variable mass properties due to the use of a propulsion system. To prove that the algorithms have been correctly implemented and that the simulated output is a good approximation of the reality, START has in parts been verified, validated and evaluated. The validation part restricted to compare the START output with that obtained with the RATT software package, which led to the conclusion that START can be used for 6-d.o.f. open-loop re-entry simulations. In addition, the results of two mission evaluations, i.e., the entry and parachute descent of the scientific probe Huygens, and the aerodynamic controllability of a moderate lift-to-drag re-entry test vehicle, indicated that START may be used to study the guided and controlled ascent and descent of powered/unpowered space planes. It should be noted, however, that exclusion of the effects due to aeroelasticity and aeropropulsion-elastic interactions will put the results of the ascent analysis in a particular context.

The advantage of applying Taguchi's orthogonal arrays in a sensitivity analysis is that the number of output data is relatively low, which makes it easier to process and understand the outcome. However, there are some points of attention. The column assignment of factors is important in two ways. In the first place, when interactions between columns are present, the column variances are confounded, which in practice means that main factor effects are influenced. In the second place, assigning factors to different columns gives different (although similar) factor combinations that can result in a different system response.

Factor interactions can be studied by leaving the appropriate column(s) of the orthogonal array empty. In case one has no knowledge about possible interactions and many factors have to be varied, then application of the Taguchi Method may not be so practical. Either one should study only a few factors at a time to avoid confounding, or a Monte-Carlo analysis should be used to identify interactions and higher-order effects. Based on the experience gained in this study, it is advised to begin a sensitivity analysis with only a few factors to get insight in the linear and higher-order effects and possible interactions. CCD is a good method in that case.

One 'bad' data point, e.g., a system failure, has a significant influence on the results and should therefore be eliminated before ANOVA is applied or a response surface is computed. But, in case there are more of these 'bad' data points, the results can be used to discover a trend, as was done with the crashes of HORUS.

Oscillations in flap deflections or reaction-jet moments are a measure for the inability of a control system to control the deviations from a reference value. These oscillations can be identified by inspecting the mean and standard deviation of the related control signal. A high value for the standard deviation as compared with the mean value signifies that there are persistent high-amplitude oscillations. Note that a single peak value cannot be traced by the standard deviation, because it will be averaged over the complete simulation interval.
In case of simulating the controlled flight of an aircraft or spacecraft, it is possible that a three-level analysis leads to instabilities while a two-level analysis gives only stable results. This is due to the fact that in both cases different factor combinations are used, and the vehicle can be particularly sensitive to certain factor combinations, that can be caused by the non-linear nature of the system. A similar effect can be introduced when the factors are assigned to different columns of the same orthogonal array. This indicates that the results of simulations while varying more parameters at a time, should be considered carefully. Statistical analysis will usually give the parameter contributions to some defined response, but control instabilities cannot be traced by such analysis, unless they actually occur.

In general, it can be concluded that the Taguchi Method and CCD can be used early in the design to get a feeling for factor sensitivities and interactions. Later in the design, once the majority of the design parameters are frozen a verification analysis can be carried out by applying the Monte-Carlo Method.

The Taguchi Method and CCD have not only been applied as a sensitivity-analysis technique but also as a design method. First, a robust design technique, i.e., a dual-loop with respect to control factors and noise factors, was used to optimise the performance of the guidance system of HORUS under the influence of disturbances without eliminating these disturbances. In case a single-loop CCD is applied to the nominal mission no vehicle crashes occur, whereas in the dual-loop robust design crashes did occur. It was found that in optimising the response surface, the applied algorithm was sensitive to the initial guess of the location of the optimum. Moreover, a verification run for the optimal settings of the design variables did not result in the predicted optimum. On one hand it indicates that the response surface may not have been a good prediction of the results, and on the other hand the importance of the verification run has been clearly established.

Second, the ascent trajectory of the WCC has been optimised by dividing the trajectory into a number of flight segments defined by some characteristic parameters such as $\gamma_c$ and $\dot{\gamma_c}$, and boundary conditions. A payload-mass variation between 991 and 5857 kg was found, which indicates that a proper selection of the trajectory is very important to ensure mission success. The minimum-fuel trajectory does not correspond with the minimum heat-load trajectory, which takes up 1,401 kg more fuel. Therefore, the maximum allowable heat load should be included in any trajectory optimisation process. The relative ease with which trajectories were found that could reach orbit, seems to indicate that the performance of the propulsion system is overrated with respect to the vehicle's dry mass. Note that the found optimum is only valid within the accuracy of the defined trajectory segmentation and guidance modelling.

ANOVA of the results indicated that the influence of interactions is small for this particular design problem of finding a reference trajectory. Redoing the design with twice the number of simulations by using the $L_{32}$ instead of the $L_{16}$ array, showed a difference in the mean and standard variation of the consumed fuel mass. Conducting a Kolmogorov-Smirnov test proves that the two distributions of the fuel masses are consistent, although there appeared to be a shift in factor contribution to the total variation. This means that major factor effects can be
extracted from the results, but that the absolute importance cannot be fully assessed.

It should be stressed that the Taguchi Method cannot replace a numerical trajectory optimisation method, since the analysis is based on constant-parameter segments that do not have to hold for the optimum trajectory. Moreover, knowledge on the trajectory is required to set up the segmentation. However, by doing a Taguchi analysis for a given trajectory segmentation one can increase the insight in the influence of the trajectory parameters on a selected response.

The major findings from the HORUS study are the following. Because the bank reversals are executed in a finite time, significant errors in $\gamma$ are induced that result in altitude errors. The guidance system is robust enough to correct these. The $\delta_c$ is too large to be initially handled by the attitude controller. The result is a large error in $\sigma$, although it can be reduced to zero in a short time. A roll-yaw coupling is present that shows as an induced $\beta$ of up to 2.5° during the bank reversals. This $\beta$ can be corrected although both the yaw jets and the rudder are used at their maximum capacity.

Towards the end of the trajectory, $\beta$ slowly diverges. This seems assumed that this diversion is due to the simplifications made during the control-system design, e.g., neglecting terms due the rotation of both the Earth and the local horizontal plane. This indicates that the translational motion has at least some influence on the rotational motion, which the LQR in its current design cannot compensate for completely.

The LQR has a reasonable performance although the current design has a problem to handle non-linearities in the aerodynamics of the control surfaces. Especially at large deflections this leads to control errors. In case there are consecutive large deflections this may lead to diverging oscillations and a complete loss of control. The attitude controller should be made more robust to avoid this problem. An additional aspect that came forward is that the longitudinal and lateral motion that were considered to be decoupled appeared to have some coupling due to these non-linearities.

Using the elevons for trim during the ascent flight of the space plane, results in a relative large contribution to the drag and lift. Therefore, for an accurate performance analysis the influence of trim drag and lift should both be included. Moreover, the trim drag increases the consumed fuel mass substantially. Using TVC to generate part of the compensating trim moment results in a significant fuel reduction. However, when major part of the flight is flown with full throttle the trajectory has to be redesigned (or re-optimised) to compensate for the loss of accelerating power.

The influence of rotational dynamics on the point-mass performance was substantial. A relatively fast oscillating throttle induces perturbations in $\alpha$ that are difficult to control. These throttle fluctuations were the result of frequently meeting with the axial-acceleration constraint, and as such a redesign of the propulsion controller seems to be required. In addition, the difference between mass-point and finite-body performance is significant, because the latter are apparently more optimal than the former.

The fundamental design of an Model Reference Adaptive Control system is relatively easy.
Unfortunately, there are many design parameters that have to be defined by the designer, and unless his experience is large this can be very time consuming. When the integral gains can increase to large values, a strong response can be the result due to discontinuities in the input signal. This effect can be reduced by including so-called integral coefficients, although a large value of this coefficient decreases the efficiency of integral control. The combination of roll, pitch and yaw control into one MRAC system, and the application of one set of weighting matrices can give rise to conflicting demands when simultaneously large output errors occur. In the current study this problem has been solved by defining three different sets of weighting matrices.

Applying the MRAC system to nominal hypersonic manoeuvres and the nominal ascent mission showed that the controller is very well capable of following the guidance commands for altitude and cross-range transitions, and to remain stable during a cruise flight. However, considering lateral control it seems that the performance should be further improved before the controller can be called robust. Studying the influence of model uncertainties on the controller performance showed a reasonable performance for at least a number of simulations. Concluding, MRAC seems to be a promising concept when applied to hypersonic space planes.

To come back to the question that was posed at the beginning of this thesis (and restated at the beginning of this chapter), we found that mission success of space planes is to a large extent dependent on the performance of the guidance and control system. Obviously, a poorly designed guidance system can directly lead to the loss of the vehicle. An adequate performance for a nominal mission is in itself not sufficient to conclude that we have a good guidance system. Extensive analysis for off-nominal conditions is therefore required. Moreover, to avoid cross couplings between the guidance system and the attitude controller it is mandatory to address the guidance-system performance before extensive analysis of the attitude controller is started. Of course, this does not have to be true when the performance of the attitude controller is the study goal and one is positive to separate the effects induced by the guidance system from the ones originating from the attitude controller. Note that in this study, the focus was put on the analysis method rather than on the development of a robust G&C system.

### 9.2. Recommendations

A simulation tool such as the one described in this thesis can of course always be extended with more complex models that give a better approximation of the actual system. However, even the current version of START can be used for more detailed analysis of some aspects of ascent and descent flights. And, of course, similar analyses as discussed in this thesis can be applied to alternative guidance and control models. Below, the recommendations for further research are divided into three groups, more or less related to the study goals listed at the beginning of this chapter.
Input models to START

- Inclusion of aeroelasticity, which is believed to have a significant impact on the design of control system for space planes.
- Development of a navigation system, that includes, amongst others, accelerometers, gyroscopes, GPS receivers, and models for parameter estimation.
- Development of more detailed control effectuators, i.e., control surfaces and reaction-control thrusters. For the former, one can think of deflection delays and limited accuracy and for the latter modelling of the individual thrusters, that can be operated in a pulsed manner.
- A more detailed design of the propulsion controller should be included in the simulations because it was found that throttle fluctuations have a significant impact on the performance and can create unwanted oscillations in the attitude dynamics. This may lead to control problems. Preferably, the development of the propulsion controller should be integrated with the design of the attitude controller.
- Since the propulsion model of the WCC is not very detailed, it is wise to redo the analysis with a more detailed model that includes the inlet and exhaust as separate modules. Also the interaction of the propulsion system with the aerodynamics should get full attention. Finally, the effect of performance degradation of a single engine module as well as the influence of a non-symmetrical flow should be studied in relation with guidance and control.
- The ascent guidance system should be reviewed and made more robust. In addition, the vertical guidance should be extended with lateral guidance to compensate for asymmetric deviations caused by symmetric disturbances.

Taguchi Method

- Detailed analysis of typical linear, quadratic and higher-order effects as well as interactions for re-entry and ascent missions to serve as a basis for a knowledge-based system that can aid the engineer in selecting factors, potential interactions and simulation technique, and in interpreting the results.
- To design the aerodynamic shape and/or the control system it is necessary to define the aerodynamic coefficients as individual factors. This will give more insight where the actual influence on a selected response comes from.

Study goals

- Currently, during the bank reversals a significant altitude and energy error is induced, that is only corrected after finalising the reversal. This results in steps in $\alpha_c$ and $\sigma_c$, and consequently a rather large attitude-control activity. An adjustment in the guidance logic that foresees the altitude error at the beginning of the reversal and adjusts $\alpha_c$ beforehand is one way to deal with this. The energy control logic includes a constraint on the commanded
variation of $\alpha$. Relaxing this constraint when the guidance system is faced with a large energy error leads to loss of control of the vehicle. By adjusting the constraint, this particular control problem could be solved, but it remains to be studied in more detail.

- For HORUS, dual-rudder operation of the rudders should be considered for improved yaw control.
- To address the importance of TVC as an alternative trim method, the technical feasibility of TVC should be addressed as well.
- A feasibility study should be conducted to extend the application of MRAC techniques to guidance of re-entry vehicles and/or space planes.
- A detailed analysis of the contribution of each of the weighting factors in the MRAC system is to be carried out. This should lead to a better understanding which of the components of the controller state vector can influence the damping of the augmented system.
- An optimisation procedure should be developed regarding the weighting matrices of the MRAC system to optimise the system performance.
- Possible inclusion of supplementary dynamics in the MRAC system to improve the system performance.
- Once a well-developed MRAC system is available the performance should be compared with an alternative controller, e.g., an LQR, to make statements about the robustness of one design with respect to the other.
- For the WCC, active c.o.m. management can be studied as a means to trim the vehicle and/or to improve the stability characteristics. This would imply modelling of the individual fuel tanks.
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Appendix A

Reference Frames
and
Frame Transformations

In deriving the differential equations that describe the motion of a vehicle in a planetary atmosphere, several reference frames are required, amongst others to allow for the definition of the state variables, external forces and moments and the wind vector. These reference frames will be defined in section A.1. In section A.2, the relation between the reference frames is given in the form of transformation matrices.

A.1 - Reference frames

*Inertial planetocentric reference frame, index I.*

The origin of the inertial reference frame is located at the centre of mass (c.o.m.) of the central body
around which the vehicle is moving\textsuperscript{21}. The $OX_Y$-plane coincides with the equatorial plane of the central body. The $Z_R$-axis is pointing north and the reference meridian which determines the direction of the $X_R$-axis, is defined by the zero-longitude meridian at zero time. The $Y_R$-axis completes the right-handed system. Note that the $Z_R$-axis is coincident with the rotational axis of the central body (the rotation of the central body is assumed to be constant in magnitude and direction).

*Rotating planetocentric reference frame, index $R$.\*

This frame is fixed to the central body and coincides with the inertial planetocentric frame at zero time. The $Z_R$-axis is pointing north, the $X_R$-axis intersects the equator at zero longitude and the $Y_R$-axis completes the right-handed system.

The origin of the following reference frames is located at the c.o.m. of the vehicle. We assume that the vehicle has at least one plane of (geometrical) symmetry (in longitudinal direction).

*Body reference frame, index $B$.\*

The body frame is fixed to the vehicle. The $X_B$-axis lies in the plane of symmetry and is positive in forward direction. The $Z_B$-axis also lies in the plane of symmetry and is positive in downward direction. The $Y_B$-axis completes the right-handed system.

*Vertical reference frame, index $V$.\*

The $Z_V$-axis is pointing towards the c.o.m. of the central body, along the radial component of the gravitational acceleration. The $X_V$-axis lies in a meridian plane, perpendicular to $Z_V$ and points to the northern hemisphere. The $Y_V$-axis is the right-handed supplement. In case the central body is a true sphere, the $X_VY_V$-plane can be referred to as the local horizontal plane. A small error in flight-path angle is introduced, when an elliptical shape is assumed.

*Trajectory reference frame (groundspeed based), index $TG$.\*

The three axes of the $TG$-frame are defined as follows:

\[ X_{TG} \] : along the velocity vector relative to the rotating planetocentric frame.
\[ Z_{TG} \] : in vertical plane, pointing downwards.
\[ Y_{TG} \] : completes right-handed system.

\textsuperscript{21}In principle, this reference frame is pseudo-inertial because of the motion of the central body itself. When the central body is a planet, it orbits around the Sun and when we are dealing with a moon, it describes an orbit around a planet which in itself revolves around the Sun. However, since this rotation of the pseudo-inertial frame has only a marginal effect on the results, we can consider the frame to be inertial. In case of the Earth, it revolves around the Sun with an angular velocity that is about 365 times smaller than the daily planetary angular rate (0.27\%). Varying the daily rotational rate with this amount does not show any different results.
Aerodynamic reference frame (groundspeed based), index AG.

The \( X_{AG} \)-axis is defined to be oriented along the velocity vector of the vehicle relative to the rotating planeto-centric frame (the groundspeed). This definition implies, that the \( X_{AG} \)-axis is collinear with the \( X_{TG} \)-axis. The \( Z_{AG} \)-axis is collinear with the aerodynamic lift force (based on groundspeed variables), but opposite in direction. The \( Y_{AG} \)-axis completes the right-handed system. Nota bene: when the vehicle is not banking, the AG- and TG-frames are coincident.

The above mentioned TG and AG frames conclude the definition of reference frames that are required in case there is no wind. Wind will have an effect on the aerodynamic forces and moments, and for that reason three more reference frames need to be defined.

Trajectory reference frame (airspeed based), index TA.

The axes of the airspeed-based trajectory reference frame are defined as follows:

\( X_{TA} \) : along the velocity vector relative to the atmosphere.

\( Z_{TA} \) : in vertical plane, pointing downwards.

\( Y_{TA} \) : completes right-handed system.

Aerodynamic reference frame (airspeed based), index AA.

The \( X_{AA} \)-axis is defined along the velocity vector of the vehicle relative to the atmosphere. This definition implies, that the \( X_{AA} \)-axis is collinear with the \( X_{TA} \)-axis. The \( Z_{AA} \)-axis is collinear with the aerodynamic lift force (based on airspeed variables), but opposite in direction. The \( Y_{AA} \)-axis completes the right-handed system. Nota bene: when the vehicle is not banking, the AA- and TA-frame are coincident.

Wind reference frame, index W.

The \( X_{W} \)-axis is collinear with the wind-velocity vector, which is defined as a modulus and two direction angles; for a northern wind, the \( X_{W} \)-axis is positive in northern direction. For a wind in the local horizontal plane, the \( Z_{W} \)-axis is positive pointing downward. The \( Y_{W} \)-axis is the right-handed supplement. Note that for non-horizontal winds, the \( W \)-frame can be obtained from the \( V \)-frame by right-handed rotations over the two wind angles.

A.2 - Frame transformations

The transformation from one right-handed cartesian frame to another one can be expressed by means of, for instance, unit axis-rotations, Directional Cosine Matrices and quaternions. Here, we will concentrate on unit axis-rotations (positive rotations according to the right-hand rule). The unit rotation matrices are, for a rotation about an arbitrary angle \( \alpha \):
\[ C_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} \]  
(A.1)

\[ C_2(\alpha) = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \]  
(A.2)

\[ C_3(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  
(A.3)

In the above equations, the subscripts 1, 2 and 3 indicate the X-, Y- and Z-axis of a particular frame.

Any rotation from frame \(i\) to frame \(j\) can always be decomposed into a number of sequential unit axis rotations. This implies that the resulting transformation matrix can be written as a combination of the matrices \(C_1, C_2\) and \(C_3\). Each of the matrices \(C_1, C_2\) and \(C_3\) is orthonormal. Also the product of orthonormal matrices is orthonormal again. The inverse of this kind of matrix is simply its transpose. For the sake of convenience, we will introduce a short-hand notation for the resulting transformation matrices. A matrix written as \(C_{ij}\) means that the matrix defines the transformation from the \(j\)-frame to the \(i\)-frame. Using the same convention, we can write for the inverse transformation matrix \(C_{ji}\):

\[ C_{ji} = C_{ij}^{-1} = C_{ij}^T \]  
(A.4)

In the remainder of this appendix, Table A.1, we will give the transformation matrices between the most commonly used reference frames. For a detailed treatment of these matrices, the reader is referred to Mooij (1994). It should be noted that matrix multiplications are carried out from right to left.

<table>
<thead>
<tr>
<th>Frame (i) to (j)</th>
<th>Notation</th>
<th>Sequence of unit-axis rotations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotating planetocentric to inertial</td>
<td>(C_{i,R})</td>
<td>(C_3(-\omega_{CB}))</td>
</tr>
<tr>
<td>planetocentric frame</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical to rotating planetocentric</td>
<td>(C_{R,V})</td>
<td>(C_3(-\tau) C_2(\pi/2 + \delta))</td>
</tr>
<tr>
<td>frame</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wind to vertical frame</td>
<td>(C_{V,W})</td>
<td>(C_3(-\chi_W) C_2(-\gamma_W))</td>
</tr>
<tr>
<td>(Groundspeed-based) trajectory to</td>
<td>(C_{V,TG})</td>
<td>(C_3(-\chi_G) C_2(-\gamma_G))</td>
</tr>
<tr>
<td>vertical frame</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Airspeed-based) trajectory to</td>
<td>(C_{V,TA})</td>
<td>(C_3(-\chi_A) C_2(-\gamma_A))</td>
</tr>
<tr>
<td>vertical frame</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frame $i$ to $j$</td>
<td>Notation</td>
<td>Sequence of unit-axis rotations</td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
<td>---------------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>(Airspeed-based) aerodynamic to (airspeed-based) trajectory frame</td>
<td>$C_{TA,AA}$</td>
<td>$C_1(\sigma_A)$</td>
</tr>
<tr>
<td>(Airspeed-based) aerodynamic to vertical frame</td>
<td>$C_{V,AA} = C_{V,TA} C_{TA,AA}$</td>
<td>$C_3(-\chi_A) C_2(-\gamma_A) C_1(\sigma_A)$</td>
</tr>
<tr>
<td>(Airspeed-based) aerodynamic to (groundspeed-based) trajectory frame</td>
<td>$C_{TG,AA} = C_{TG,V} C_{V,AA}$</td>
<td>$C_2(\gamma_G) C_3(\chi_G) C_3(-\chi_A)$ $C_2(-\gamma_A) C_1(\sigma_A)$</td>
</tr>
<tr>
<td>Body to (airspeed-based) aerodynamic frame</td>
<td>$C_{AA,B}$</td>
<td>$C_3(\beta_A) C_2(-\alpha_A)$</td>
</tr>
<tr>
<td>Body to (groundspeed-based) aerodynamic frame</td>
<td>$C_{AG,B}$</td>
<td>$C_3(\beta_G) C_2(-\alpha_G)$</td>
</tr>
<tr>
<td>Vertical to inertial planetocentric frame</td>
<td>$C_{I,V} = C_{I,R} C_{R,V}$</td>
<td>$C_3(-\omega_{zG},f) C_3(-\tau) C_2(\pi/2 + \delta)$</td>
</tr>
<tr>
<td>(Airspeed-based) aerodynamic to rotating planetocentric frame</td>
<td>$C_{R,AA} = C_{R,V} C_{V,AA}$</td>
<td>$C_3(-\tau) C_2(\pi/2 + \delta) C_3(-\chi_A)$ $C_2(-\gamma_A) C_1(\sigma_A)$</td>
</tr>
<tr>
<td>Body to rotating planetocentric frame</td>
<td>$C_{R,B} = C_{R,V} C_{V,AG} C_{AG,B}$</td>
<td>$C_3(-\tau) C_2(\pi/2 + \delta) C_3(-\chi_G) C_2(-\gamma_G)$ $C_1(\sigma_G) C_3(\beta_G) C_2(-\alpha_G)$</td>
</tr>
<tr>
<td>(Groundspeed-based) trajectory to wind frame</td>
<td>$C_{W,TG} = C_{W,V} C_{V,TG}$</td>
<td>$C_2(\gamma_W) C_3(\chi_W) C_3(-\chi_G) C_2(-\gamma_G)$</td>
</tr>
</tbody>
</table>

Table A.1 - Basic frame transformations.

Reference

Appendix B

Analytical Expressions of the Euler Equations

The full set of (non-linear) Euler equations is given by the following matrix equation

\[ \dot{\omega} = M_{cm} - \omega \times \omega \]  
\[(B.1)\]

or, with the components written out:

\[ \begin{bmatrix} l_{xx} & -l_{xy} & -l_{xz} \\ -l_{xy} & l_{yy} & -l_{yz} \\ -l_{xz} & -l_{yz} & l_{zz} \end{bmatrix} \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} - \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} \times \begin{bmatrix} l_{xx} & -l_{xy} & -l_{xz} \\ -l_{xy} & l_{yy} & -l_{yz} \\ -l_{xz} & -l_{yz} & l_{zz} \end{bmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \]  
\[(B.2)\]

Expanding the above matrix equation results into three (complex) scalar equations:

\[ M_x = l_{xx}\dot{p} + (l_{zz} - l_{yy})\dot{q}r - l_{xy}(q - pr) - l_{xz}(\dot{r} - pq) - l_{yz}(q^2 - r^2) \]  
\[(B.3a)\]
\[ M_y = l_{yy} q + (l_{xx} - l_{zz}) p r - l_{xy} (p^2 + qr) + l_{yz} (p^2 - r^2) - l_{yz} (q - pr) \]  
(B.3b)

\[ M_z = l_{zz} p + (l_{yy} - l_{xx}) pq - l_{xy} (p^2 - q^2) - l_{xz} (p - qr) - l_{y} (q + pr) \]  
(B.3c)

These equations can be solved for \( \dot{p}, \dot{q}, \) and \( \dot{r} \) (Duke et al., 1988). The end result is stated below.

\[ \dot{p} = P_{pp} p^2 + P_{pq} pq + P_{pr} pr + P_{qq} q^2 + P_{qr} qr + P_{rr} r^2 + P_x M_x + P_y M_y + P_z M_z \]  
(B.4a)

\[ \dot{q} = Q_{pp} p^2 + Q_{pq} pq + Q_{pr} pr + Q_{qq} q^2 + Q_{qr} qr + Q_{rr} r^2 + Q_x M_x + Q_y M_y + Q_z M_z \]  
(B.4b)

\[ \dot{r} = R_{pp} p^2 + R_{pq} pq + R_{pr} pr + R_{qq} q^2 + R_{qr} qr + R_{rr} r^2 + R_x M_x + R_y M_y + R_z M_z \]  
(B.4c)

The definition of the inertial parameters \( P_{pp}, P_{pq}, ..., R_z \) can be found in Table B.1 on the following pages.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{qq}$</td>
<td>$\frac{(l_{yz}l_1 - l_{xy}l_3)}{\det I}$</td>
</tr>
<tr>
<td>$P_{qr}$</td>
<td>$-\frac{[(l_{zz}-l_{yy})l_1 - l_{xy}l_2 + l_{xz}l_3]}{\det I}$</td>
</tr>
<tr>
<td>$P_{rr}$</td>
<td>$-\frac{(l_{yz}l_1 - l_{xz}l_2)}{\det I}$</td>
</tr>
<tr>
<td>$P_x$</td>
<td>$\frac{l_1}{\det I}$</td>
</tr>
<tr>
<td>$P_y$</td>
<td>$\frac{l_2}{\det I}$</td>
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<tr>
<td>$P_z$</td>
<td>$\frac{l_3}{\det I}$</td>
</tr>
<tr>
<td>$Q_{pp}$</td>
<td>$\frac{-(l_{xx}l_2 - l_{xy}l_3)}{\det I}$</td>
</tr>
<tr>
<td>$Q_{pq}$</td>
<td>$\frac{[l_{xz}l_2 - l_{yz}l_4 - (l_{yy}-l_{xx})l_5]}{\det I}$</td>
</tr>
<tr>
<td>$Q_{pr}$</td>
<td>$\frac{-(l_{yx}l_2 + (l_{xx}-l_{zz})l_4 - l_{yz}l_5)}{\det I}$</td>
</tr>
<tr>
<td>$Q_{qq}$</td>
<td>$\frac{(l_{yz}l_2 - l_{xy}l_3)}{\det I}$</td>
</tr>
<tr>
<td>$Q_{qr}$</td>
<td>$\frac{-(l_{zz}-l_{yy})l_2 - l_{xy}l_4 + l_{xz}l_5}{\det I}$</td>
</tr>
<tr>
<td>$Q_{rr}$</td>
<td>$\frac{-(l_{yz}l_2 - l_{xz}l_4)}{\det I}$</td>
</tr>
<tr>
<td>$Q_x$</td>
<td>$\frac{l_2}{\det I}$</td>
</tr>
<tr>
<td>$Q_y$</td>
<td>$\frac{l_4}{\det I}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$Q_z$</td>
<td>$\frac{l_5}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_{pp}$</td>
<td>$\frac{-\left(l_{x2}l_5 - l_{xy}l_6\right)}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_{pq}$</td>
<td>$\frac{\left[l_{xz}l_3 - l_{yz}l_5 - \left(l_{yy} - l_{xx}\right)l_6\right]}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_{pr}$</td>
<td>$\frac{-\left(l_{xy}l_3 + \left(l_{xx}l_{zz}\right)l_5 - l_{yz}l_6\right)}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_{qq}$</td>
<td>$\frac{\left(l_{yz}l_3 - l_{xy}l_6\right)}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_{qr}$</td>
<td>$\frac{-\left(l_{zz} - l_{yy}\right)l_5 - l_{xy}l_5 + l_{x2}l_6\right]}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_{rr}$</td>
<td>$\frac{-\left(l_{yz}l_3 - l_{xz}l_5\right)}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_x$</td>
<td>$\frac{l_3}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_y$</td>
<td>$\frac{l_5}{\text{det } I}$</td>
</tr>
<tr>
<td>$R_z$</td>
<td>$\frac{l_6}{\text{det } I}$</td>
</tr>
</tbody>
</table>

Table B.1 - Definition of inertia parameters.

Reference

Appendix C

The HORUS Guidance and Control System

This appendix gives a summary of the mathematical models underlying the guidance and control system of HORUS. The design of the guidance system for HORUS has been extensively described by MBB (1988) and has been applied and tested by both MBB (1988) and Mooij (1997b). A detailed description of the design of the attitude-control system can be found in Mooij (1997a). In Section C.1 the guidance system will be discussed and in Section C.2 the attitude-control system.

C.1. Guidance system

C.1.1 - Introduction

Re-entry guidance methods have been described extensively in literature. In the fifties and sixties, many studies were conducted within the framework of the American space programme, and, more specific, the Apollo project. Wingrove (1963) gives a survey of atmospheric re-entry guidance concepts that were most commonly used in those days, some of which still form the basis of currently applied concepts.
Some of the ideas in the design of the Space Shuttle guidance (Harpold and Graves, 1979) have been adopted in the guidance scheme applied to HORUS. The major parts, however, are covered by a different design, basically as a result that the nominal trajectories flown by HORUS are not necessarily composed of sections with constant flight parameters as is the case with the trajectories of the Space Shuttle. A fundamental functional separation in the Space-Shuttle guidance is the subdivision of the horizontal and vertical entry guidance. As a result, the vertical flight path is controlled by adjusting the angle of attack and the absolute value of the bank angle, while the sign of the bank angle is provided by the horizontal guidance. This decoupling of the trajectory guidance has been included in the design of the HORUS guidance system.

It is the task of the horizontal entry guidance to bring the vehicle in the vicinity of the landing site with a small heading error, so that the TAEM guidance can take over and steer the vehicle towards a safe landing. Input to the horizontal guidance is the distance from the targeting point and the heading error, i.e., the difference between the actual heading and the heading of the connecting line of the actual position and the targeting point. To restrict the heading error, a so-called heading-error dead-band is defined, which provides the maximum allowable error. Exceeding the dead-band initiates a bank manoeuvre also known as bank reversal.

The upper boundary of the heading-error dead-band is driven by two notions. In the first place, the length difference between the actual and the nominal trajectory should be kept (relatively) small. This tends to decrease the allowable heading error. In the second place, intersection with the TAEM interface must be guaranteed, which imposes restrictions on the dead-band closer to the target point. The lower boundary is dictated by the fact that a narrow dead-band increases the number of bank reversals, which is not very favourable. Partly based on flight experience gained with the Space Shuttle, dead-band values between 10° and 30° seem to give a reasonable compromise.

During the bank reversal, the commanded bank angle changes with a constant theoretical bank rate $\dot{\sigma}_c$ from an initial value $\sigma_0$ to the final value $-\sigma_0$. The initial value $\sigma_0$ is the last value generated by the vertical guidance. The commanded bank rate is defined as an input function of the dynamic pressure, done so because the dynamic pressure defines the banking capabilities due to deflecting the aerodynamic control surfaces. For higher dynamic pressures, a higher bank rate is applied. It should be noted that during the bank reversal, no variations in commanded angle of attack and absolute bank angle are computed by the vertical guidance logic to prevent oscillating command signals. The commanded variations are kept constant and are adjusted again after finishing the bank reversal.

The objectives of the vertical guidance are i) to arrive at the TAEM interface with a prescribed total energy and altitude (or velocity), and ii) to meet the flight-path constraints during the flight. All quantities that are important for the vertical guidance are dependent on velocity and altitude. This is true for the final conditions and since the atmospheric density is dependent on the altitude, also the encountered loads. However, when $V$ and $h$ are regulated simultaneously by adjusting $\alpha$ and $\sigma$ the resulting control histories are not very smooth meaning quite a heavy load for the flight-control system. To avoid this, two guidance loops with different time constants are employed, i.e., an energy- and

---

22 The targeting point of the horizontal guidance is located at a distance $d_{rw}$ from the centre of the runway, in opposite direction to the landing direction.
altitude-control loop. The total energy will be controlled such that only the final value at the TAEM interface will be met, having no direct effect on the constraints during the flight. The internal sharing of potential and kinetic energy, on the other hand, will affect the constraints through the altitude-velocity relation. In the next two sections, the energy loop (Section C.1.2) and altitude loop (Section C.1.3) will be discussed in more detail. In Section C.1.4, finally, a few remarks are made concerning the bank reversals.

C.1.2. Vertical guidance: energy control

The decrease of total energy, i.e., the sum of kinetic and potential energy, can be traced back to the working of the atmosphere on the vehicle, in the form of the drag force. This dissipation of mass-specific energy can be written as

\[ E_{\text{tot}}(t_2) - E_{\text{tot}}(t_1) = \int_{t_1}^{t_2} \frac{\dot{E}_{\text{diss}}}{m} dt = \int_{t_1}^{t_2} \frac{DV}{m} dt \]  

(C.1.1)

Since one of the parameters with which the dissipated energy can be influenced is the angle of attack, the following, linearised relation between the energy variation and the angle-of-attack variation can be written:

\[ \delta E = \delta \alpha \left[ \int_{t}^{t_f} \frac{\partial \dot{E}_{\text{diss}}}{\partial \alpha} dt \right] = \delta \alpha \left[ \int_{0}^{t_f} \frac{\partial \dot{E}_{\text{diss}}}{\partial \alpha} dt - \int_{t}^{t_f} \frac{\partial \dot{E}_{\text{diss}}}{\partial \alpha} dt \right] \]  

(C.1.2)

\( t_f \) is the (final) time that the TAEM interface is reached. The second term on the right-hand side is generated from the reference trajectory and stored in the on-board reference table; the first term on the right-hand side is the last value of this on-board table denoting the integral value at the TAEM interface. The guidance law can be chosen in the form

\[ \delta \alpha_c = -\frac{K_E}{\int_{t}^{t_f} \frac{\partial \dot{E}_{\text{diss}}}{\partial \alpha} dt} \Delta E, \quad \text{with} \ \Delta E = E_{\text{tot,ref}}(d_{\text{nav}}) - E_{\text{tot,nav}} \]  

(C.1.3)

The denominator of the right-hand side of Eq. (C.1.3) is evaluated from the on-board reference tables. \( E_{\text{tot,ref}}(d_{\text{nav}}) \) is the distance-to-target based reference energy whereas \( K_E \) is the energy-gain amplification factor, which usually (moderate energy disturbances, sufficient margins in \( \alpha \)) has a value of 1 or slightly higher for good energy control. Before entering Eq. (C.1.3) a dead-band operation is applied to \( \Delta E \) to avoid oscillating controls. Note that in case \( t \) is approaching \( t_f \) the denominator gets close to zero; then, the fraction is constrained to a user-defined input value.
In principle, we have finalised the discussion on energy control. However, the atmosphere is very thin in the first phase of the re-entry so the control efficiency is very poor. Furthermore, the total specific energy that serves as a reference value for computing \( h_{\text{ref}} \) and \( \gamma_{\text{ref}} \) used by the altitude-control logic described below, decreases very slowly with altitude in this region. This makes the reference very sensitive to small energy changes and thus also energy errors. For this reason, the descent is divided into three phases according to the actual energy dissipation. In the first region, no energy control takes place, so \( \delta a_{L,C} = 0 \). In the third region, that constitutes the major part of the re-entry, we proceed as we have discussed above. The second region is a transition between region \#1 and \#3, in which we use linear interpolation to compute the energy deficiency \( \Delta E \) of Eq. (C.1.3). Typical transition altitudes are about 94 km (region \#1 \( \Rightarrow \) \#2) and 90 km (region \#2 \( \Rightarrow \) \#3).

### C.1.3. Vertical guidance: altitude control

The altitude control has to balance the kinetic and potential energy such that the final condition at the TAEM interface and the constraints during the flight are met. To achieve this, we use a feedback law for the commanded variation of the vertical part of the lift per unit mass (vertical lift acceleration), \( \delta a_{L,C} \) (MBB, 1988):

\[
\delta a_{L,C} = -c_{h,1}^2 \delta h_{\text{eff}} \frac{\gamma_{\text{ref}}}{\gamma} + (\Delta a_{L,C})_{\text{eff}}
\]  

(C.1.4)

with

\[
\delta h_{\text{eff}} = \delta h + 2 \frac{c_{h,2}}{c_{h,1}} \nu \delta \gamma
\]

\[
\delta h = h_{\text{nav}} - h_{\text{ref}}(E_{\text{tot,nav}}) \quad \text{and} \quad \delta \gamma = \gamma_{\text{nav}} - \gamma_{\text{ref}}(E_{\text{tot,nav}})
\]

The value of the altitude-control parameters \( c_{h,1} \) and \( c_{h,2} \) can be found from a linearised analysis of the altitude-controller dynamics. In Eq. (C.1.4), the exponential function is a correction factor (with \( \gamma_r \) being an input parameter) giving a damping effect for skipping flight. The second term on the right-hand side is included to compensate for observed lift errors, using the on-board reference value of the vertical lift component. The effective altitude error \( \delta h_{\text{eff}} \) is subjected to a dead-band operation, as was the case with the energy variation.

In case of a severe energy deficiency it is even more important to fly at higher altitudes than the reference altitude since this will reduce the energy dissipation. Therefore, not the actual specific energy \( E_{\text{tot,nav}} \) is used to compute \( h_{\text{ref}} \) and \( \gamma_{\text{ref}} \) but an adjusted energy, i.e.,

\[
E_{\text{tot,ref}}(d_{\text{nav}}) = E_{\text{tot,nav}}(d_{\text{nav}}) + c_{E,2} [E_{\text{tot,ref}}(d_{\text{nav}}) - E_{\text{tot,nav}}]
\]  

(C.1.5)

\(^{23}\)In case of a negative altitude error (the vehicle is too low), the altitude controller input \( \delta h \) will be automatically reduced to zero if \( \gamma > 0 \). However, if we let the controller respond to the altitude error in this situation, a skipping flight with significant altitude oscillations can be induced. On the other hand, we cannot refrain from altitude control for positive \( \gamma \) close to zero, because correction of the altitude error might take too long.
but only in case $E_{\text{tot,rel}}(d_{\text{nav}}) > E_{\text{tot,nav}}$. This produces the desired effect by raising the reference altitude with increasing energy deficiency; $c_{E,2}$ is one of two energy-control parameters.

After computing the commanded variation of the vertical lift component, we have to convert this to an absolute value of the commanded bank angle. The nominal value of the vertical lift is given by:

$$a_{L,\text{nom}} = \frac{q_{\text{dyn}} S_{\text{ref}}}{m} (C_L \cos \sigma)_{\text{nom}}$$  \hspace{1cm} (C.1.6)

with $(C_L \cos \sigma)_{\text{nom}}$ is the nominal vertical-lift coefficient referenced to the actual energy and computed from the on-board tables. So the resulting commanded lift acceleration is given by

$$a_{L,c} = a_{L,\text{nom}} + \delta a_{L,c} = \frac{q_{\text{dyn}} S_{\text{ref}}}{m} C_L (\alpha + \delta \alpha_c) \cos \sigma_c$$  \hspace{1cm} (C.1.7)

where the lift curve is assumed to be a linear function of the angle of attack. Substituting Eq. (C.1.6) for $a_{L,\text{nom}}$ in Eq. (C.1.7), the absolute commanded bank angle can be obtained:

$$|\sigma_c| = \arccos \left( \frac{1}{C_L (\alpha + \delta \alpha_c)} \left[ (C_L \cos \sigma)_{\text{nom}} + \delta a_{L,c} \frac{m}{q_{\text{dyn}} S_{\text{ref}}} \right] \right)$$  \hspace{1cm} (C.1.8)

### C.1.4. The influence of bank reversals on the flight-path angle

During a bank reversal there will be an increase in flight-path angle. In this section, it is shown that the extent of this increment is proportional to the inverse of the bank rate. In other words, the faster the vehicle can bank the smaller the error in flight-path angle will be. It is obvious that an error in flight-path angle will have its influence on the altitude control, and therefore the corrective bank-angle control. Furthermore, the energy dissipation will differ from the nominal dissipation, which might lead to corrective angle-of-attack control. Fast execution of the bank reversal would therefore be favourable.

The variation of the flight-path angle with time can be approximated by

$$\dot{\gamma} \approx \frac{1}{m} \left( \frac{V}{R} - \frac{g}{V} \right) \cos \gamma + \frac{L}{mV}$$  \hspace{1cm} (C.1.9)

by considering the motion in a vertical plane (no sideslip and no banking) and assuming a non-rotating Earth. The increase of $\gamma$ can be estimated from the variation of the lift force with time during the bank reversal, since it is reasonable to assume that the variation of velocity and altitude are much smaller during the manoeuvre, so:

$$\Delta \gamma = \int_0^{\Delta t} \dot{\gamma} \, dt = \int_0^{\Delta t} \frac{1}{mV} \Delta L(t) \, dt$$  \hspace{1cm} (C.1.10)
Assuming a constant bank rate \( \dot{\sigma} \) the duration of the bank reversal \( \Delta t \) from \( \sigma = \sigma_0 \) to \( \sigma = -\sigma_0 \) becomes

\[
\Delta t = 2 \frac{\sigma_0}{\dot{\sigma}}
\]  
(C.1.11)

The variation of the vertical lift force with time is\(^\text{24}\)

\[
\Delta L(t) = L(t) - L(t+\Delta t) = C_L q_{dyn} S_{ref} [\cos(-\sigma_0 + \dot{\sigma} t) - \cos(-\sigma_0)]
\]  
(C.1.12)

Eq. (C.1.12) substituted into Eq. (C.1.10) yields

\[
\Delta \gamma = \frac{L}{mV} \int_0^\Delta t [\cos(-\sigma_0 + \dot{\sigma} t) - \cos(-\sigma_0)] \, dt = -\frac{L}{mV} \left[ \frac{\sin(-\sigma_0 + \dot{\sigma} t)}{\dot{\sigma}} + \cos(-\sigma_0) t \right]_0^\Delta t
\]  
(C.1.13)

Solving Eq. (C.1.13) and rearranging terms finally gives us

\[
\Delta \gamma = 2 \frac{L}{mV_0} [\sin \sigma_0 - \sigma_0 \cos \sigma_0]
\]  
(C.1.14)

where we have used Eq. (C.1.11). Since the error in \( \gamma \) is inversely proportional to the bank rate, it must therefore be chosen as large as possible to minimise the flight-path angle error, while on the other hand as small as possible to minimise the demands on the attitude controller. In other words, these requirements have to be balanced.

C.2. Attitude controller

C.2.1. Introduction

The design of the attitude controller is centred around a nominal trajectory, comprising the state of the vehicle, the nominal control variables, the external forces and moments, and environmental parameters. The 12 coupled ordinary differential equations that describe the 6-d.o.f. vehicle motion are non-linear and time varying, which makes it impossible to design a linear state-feedback control system with classical control theory. To apply this theory, the equations of motion have to be linearised and to be made time invariant, by dividing the nominal trajectory into a number of time points in between the vehicle state is assumed to be constant. Per time point, a so-called Linear Time Invariant (LTI) system is thus obtained. To address the time-varying character of the re-entry mission, each of the LTI systems will be combined in a series.

\(^{24}\)It should be noted that a bank reversal will have a variation in the angle of attack as result, so the absolute lift force is in that sense not constant. For the sake of simplicity, however, we assume that the magnitude of the lift is constant and that only the orientation changes due to the banking manoeuvre.
The linearised model for the dynamics of the state space is also known as the local stability model, and will be presented in Section C.2.2. This model was used to study the open-loop behaviour of the re-entry vehicle. It appeared that the dynamics of the attitude subsystem are mainly influenced by a short-period longitudinal motion and a lateral oscillation. Details of this study can be found in Mooij (1997a).

For the sake of re-entry control-system design the translational motion can be decoupled from the rotational motion, since the time scale of the latter is at least one order smaller such that position and velocity can be thought to be constant for a particular point of the trajectory (Wingrove, 1963). The resulting model is given by 6 linearised differential equations of motion in the state variables $p$, $q$, $r$, $\alpha$, $\beta$ and $\sigma$. From the study of the open-loop behaviour, it appeared that the symmetric can be decoupled from the asymmetric motion. For this reason, the longitudinal and lateral corrective controller can be designed separately (Sections C.2.3 and C.2.4). Each controller makes use of both reaction-control jets and aerodynamic-control surfaces.

The trim stability in pitch is primarily taken care of by the body flap of HORUS, assisted by the elevons whenever required (especially at lower Mach numbers). Because the body flap is only activated at $q_{\text{dyn}} > 100 \text{ N/m}^2$, in the first part of the flight there is no active trim control. This is addressed by setting the body flap in a fixed position, which gives a pseudo equilibrium for this phase. Two advantages of this approach are less fuel consumption by the reaction-control thruster and possibly a more stable attitude for small off-nominal design conditions.

![Fig. C.1 - Schematic lay-out of the HORUS-2B attitude controller.](image)
Mathematically, the trim algorithm is implemented as follows. First, the pitch-moment of the vehicle, $C_{mb}$, is computed from the on-board tables taking $\alpha_c$ and $M_{nav}$ as input. Note that $\alpha_c$ is taken as input instead of $\alpha_{nav}$ since the vehicle should be stable for $\alpha_c$ whereas $\alpha_{nav}$ might have an error. Using inverse interpolation, the corresponding deflection angle of the body flap, $\delta_{b,\text{trim}}$, is extracted from the three-dimensional aerodynamic tables. When the body flap does not suffice, the remaining moment coefficient is compensated for by the elevons. Also the elevon deflection angle, $\delta_{e,\text{trim}}$, is computed by inverse interpolation.

Computation of the feedback gains is based on optimal control theory (Gopal, 1989); details can be found in the main text (Section 3.3.1). The adaptation of the control system to the rapidly changing flight state is achieved by re-designing a new controller at regular intervals. As a practical consequence this means that the gains are computed off-line for a selected number of points in the trajectory. These gains are then stored in on-board reference tables as a function of the dynamic pressure. In-flight computation of the actual gains is based on linear interpolation for the local value of the dynamic pressure. This scheme is also known as gain scheduling. A schematic overview of the attitude controller for HORUS is depicted in Fig. C.1.

### C.2.2. Local stability model of HORUS-2B

The local stability model is a linearised model giving the flight dynamics of a vehicle. It describes small deviations from an equilibrium state (indicated with the subscript 'e'). In this section the assumptions under which the local stability model has been derived are stated, and the mathematical formulation of this model is given. To start with, the non-linear equations of motion are derived for an unpowered vehicle of constant mass, with a plane of mass symmetry ($X_BY_B$-plane). Aerodynamic control effectuators are a body flap, two elevons and two rudders; furthermore, there are roll, pitch and yaw reaction-control jets. The linearisation is done under the assumptions that:

- the Earth is not rotating,
- the gravity field is spherical,
- the vehicle is rotationally symmetric in mass,
- the asymmetric translational motion has no effect on the attitude kinematics, i.e., the trajectory is directed along the equator,
- pitch stability is guaranteed throughout the flight, and
- the nominal angle of sideslip is zero.

Due to these assumptions, the equations for $\chi$, $\tau$ and $\delta$ are decoupled from the other nine and are hence of no importance for the design of the controller. The resulting equations are linearised about the nominal values $V_e$, $\gamma_e$, $R_e$, $\alpha_e$ and $\sigma_e$. The nominal angular velocities $p_e$, $q_e$ and $r_e$ are derived from the requirement $\dot{\alpha} = \dot{\beta} = \dot{\sigma} = 0 \text{ }^\circ/\text{s}$, i.e.,

$$p_e = c_1 \sin \alpha_e + c_2 \cos \alpha_e$$  \hspace{1cm} (C.2.1)
\[ q_e = \frac{L_e}{m V_e} - \frac{g_e \cos \gamma \cos \sigma_e}{V_e} \]  
(C.2.2)

\[ r_e = -c_1 \cos \alpha_e + c_2 \sin \alpha_e \]  
(C.2.3)

with

\[ c_1 = \frac{g_e \cos \gamma_e \sin \sigma_e}{V_e} \]

\[ c_2 = \frac{L_e}{m V_e} \tan \gamma_e \sin \sigma_e \]

In addition, all second-order terms are neglected, where \( \rho_e, q_e \) and \( r_e \) are also considered to be small quantities. The outcome, nine coupled linear differential equations, can be written in matrix form:

\[
\Delta x = A \Delta x - B \Delta u
\]  
(C.2.4)

with

\[
\Delta x = (\Delta V, \Delta \gamma, \Delta R, \Delta \rho, \Delta q, \Delta r, \Delta \alpha, \Delta \beta, \Delta \sigma)^T
\]

\[
\Delta u = (\Delta \delta_e, \Delta \delta_\alpha, \Delta \delta_r, \Delta M_{T,x}, \Delta M_{T,y}, \Delta M_{T,z})^T
\]

\[
A = \begin{bmatrix}
  a_{VV} & a_{V\gamma} & a_{V R} & a_{V \rho} & a_{V q} & a_{V r} & a_{V \alpha} & a_{V \beta} & a_{V \sigma} \\
  a_{\gamma V} & a_{\gamma \gamma} & a_{\gamma R} & a_{\gamma \rho} & a_{\gamma q} & a_{\gamma r} & a_{\gamma \alpha} & a_{\gamma \beta} & a_{\gamma \sigma} \\
  a_{R V} & a_{R \gamma} & a_{R R} & a_{R \rho} & a_{R q} & a_{R r} & a_{R \alpha} & a_{R \beta} & a_{R \sigma} \\
  a_{\rho V} & a_{\rho \gamma} & a_{\rho R} & a_{\rho \rho} & a_{\rho q} & a_{\rho r} & a_{\rho \alpha} & a_{\rho \beta} & a_{\rho \sigma} \\
  a_{q V} & a_{q \gamma} & a_{q R} & a_{q \rho} & a_{q q} & a_{q r} & a_{q \alpha} & a_{q \beta} & a_{q \sigma} \\
  a_{r V} & a_{r \gamma} & a_{r R} & a_{r \rho} & a_{r q} & a_{r r} & a_{r \alpha} & a_{r \beta} & a_{r \sigma} \\
  a_{\alpha V} & a_{\alpha \gamma} & a_{\alpha R} & a_{\alpha \rho} & a_{\alpha q} & a_{\alpha r} & a_{\alpha \alpha} & a_{\alpha \beta} & a_{\alpha \sigma} \\
  a_{\beta V} & a_{\beta \gamma} & a_{\beta R} & a_{\beta \rho} & a_{\beta q} & a_{\beta r} & a_{\beta \alpha} & a_{\beta \beta} & a_{\beta \sigma} \\
  a_{\sigma V} & a_{\sigma \gamma} & a_{\sigma R} & a_{\sigma \rho} & a_{\sigma q} & a_{\sigma r} & a_{\sigma \alpha} & a_{\sigma \beta} & a_{\sigma \sigma}
\end{bmatrix}
\]

and

\[
B = \begin{bmatrix}
  b_{V e} & b_{V \alpha} & b_{V r} & b_{V x} & b_{V y} & b_{V z} \\
  b_{\gamma e} & b_{\gamma \alpha} & b_{\gamma r} & b_{\gamma x} & b_{\gamma y} & b_{\gamma z} \\
  b_{Re} & b_{R \alpha} & b_{R r} & b_{R x} & b_{R y} & b_{R z} \\
  b_{pe} & b_{p \alpha} & b_{p r} & b_{p x} & b_{p y} & b_{p z} \\
  b_{qe} & b_{q \alpha} & b_{q r} & b_{q x} & b_{q y} & b_{q z} \\
  b_{re} & b_{r \alpha} & b_{r r} & b_{r x} & b_{r y} & b_{r z} \\
  b_{ue} & b_{u \alpha} & b_{u r} & b_{u x} & b_{u y} & b_{u z} \\
  b_{be} & b_{b \alpha} & b_{b r} & b_{b x} & b_{b y} & b_{b z} \\
  b_{se} & b_{s \alpha} & b_{s r} & b_{s x} & b_{s y} & b_{s z}
\end{bmatrix}
\]

The used notations are:

- \( V \) = modulus of relative velocity vector (m/s)
- \( \gamma \) = flight-path angle (rad)
- \( R \) = modulus of position vector (m)
- \( \rho \) = roll rate (rad/s)
- \( q \) = pitch rate (rad/s)
- \( r \) = yaw rate (rad/s)
- \( \alpha \) = angle of attack (rad)
- \( \beta \) = angle of sideslip (rad)
- \( \sigma \) = bank angle (rad)
- \( \delta_e \) = elevator deflection angle (rad)
\( \delta_a \) = aileron deflection angle (rad)
\( \delta_r \) = rudder deflection angle (rad)
\( M_{T,x} \) = roll-thruster moment (Nm)
\( M_{T,y} \) = pitch-thruster moment (Nm)
\( M_{T,z} \) = yaw-thruster moment (Nm)

The elements of matrix \( A \) are given by:

\[
a_{VV} = -\frac{1}{mV_e} \left( M_e \frac{\partial C_D}{\partial M} q_{dyn e} S_{ref} + 2D_e \right) \tag{C.2.5}
\]
\[
a_{Vf} = -g_e \cos \gamma_e \tag{C.2.6}
\]
\[
a_{VR} = 2 \frac{g_e}{R_e} \sin \gamma_e \tag{C.2.7}
\]
\[
a_{V\alpha} = -\frac{1}{m} \frac{\partial C_D}{\partial \alpha} q_{dyn e} S_{ref} \tag{C.2.8}
\]
\[
a_{V\beta} = a_{Vq} = a_{VR} = a_{V\alpha} = a_{V\gamma} = 0 \tag{C.2.9}
\]
\[
a_{\gamma V} = \frac{1}{V_e} \left(-\gamma_e + \frac{2V_e}{R_e} \cos \gamma_e \right) + \frac{\cos \gamma_e}{mV_e} \left( M_e \frac{\partial C_L}{\partial M} q_{dyn e} S_{ref} + 2L_e \right) \tag{C.2.10}
\]
\[
a_{\gamma V} = -\left( \frac{V_e}{R_e} - \frac{g_e}{V_e} \right) \sin \gamma_e \tag{C.2.11}
\]
\[
a_{VR} = \left( \frac{2g_e - V_e}{V_e} \right) \cos \gamma_e \tag{C.2.12}
\]
\[
a_{\alpha V} = \frac{\cos \gamma_e}{mV_e} \frac{\partial C_L}{\partial \alpha} q_{dyn e} S_{ref} \tag{C.2.13}
\]
\[
a_{\beta V} = -\sin \gamma_e \frac{\partial C_S}{mV_e} \frac{\partial q_{dyn e}}{\partial \beta} S_{ref} \tag{C.2.14}
\]
\[
a_{\gamma \alpha} = -\frac{L_e}{mV_e} \sin \gamma_e \tag{C.2.15}
\]
\[
a_{\alpha \gamma} = a_{\alpha q} = a_{\gamma f} = 0 \tag{C.2.16}
\]
\[
a_{RIV} = \sin \gamma_e \tag{C.2.17}
\]
\[
a_{RI} = V_0 \cos \gamma_e \tag{C.2.18}
\]
\[
a_{RR} = a_{Rq} = a_{Rf} = a_{R\alpha} = a_{R\beta} = a_{R\gamma} = 0 \tag{C.2.19}
\]
\[
a_{\beta \beta} = \frac{1}{I_{xx}} \frac{\partial C_I}{\partial \beta} q_{dyn e} b_{ref} \tag{C.2.20}
\]
\[
a_{PV} = a_{p\gamma} = a_{pR} = a_{pp} = a_{pq} = a_{p\alpha} = a_{p\beta} = a_{p\gamma} = 0 \tag{C.2.21}
\]
\[ a_{qV} = \frac{M_e}{I_{yy} V_e} \frac{\partial C_m}{\partial M} q_{dyn_e} S_{ref c_{ref}} \]  
(C.2.22)

\[ a_{q\alpha} = \frac{1}{I_{yy}} \frac{\partial C_m}{\partial \alpha} q_{dyn_e} S_{ref} c_{ref} \]  
(C.2.23)

\[ a_{qI} = a_{qR} = a_{qP} = a_{qQ} = a_{qR} = a_{qR} = a_{qQ} = a_{qQ} = 0 \]  
(C.2.24)

\[ a_{I\beta} = \frac{1}{I_{zz}} \frac{\partial C_n}{\partial \beta} q_{dyn_e} S_{ref} b_{ref} \]  
(C.2.25)

\[ a_{rI} = a_{rR} = a_{rP} = a_{rQ} = a_{rR} = a_{rR} = a_{rQ} = a_{rQ} = 0 \]  
(C.2.26)

\[ a_{\alpha V} = \frac{g_e}{V_e^2} \cos \gamma_e \cos \sigma_e \left[ 1 - \frac{1}{m V_e^2} \left( M_e \frac{\partial C_L}{\partial M} + C_L \right) \right] q_{dyn_e} S_{ref} \]  
(C.2.27)

\[ a_{\alpha I} = \frac{g_e}{V_e} \sin \gamma_e \cos \sigma_e \]  
(C.2.28)

\[ a_{\alpha R} = -\frac{2g_e}{R_e V_e} \cos \gamma_e \cos \sigma_e \]  
(C.2.29)

\[ a_{\alpha Q} = 0 \]  
(C.2.30)

\[ a_{\alpha \alpha} = -\frac{1}{m V_e} \frac{\partial C_L}{\partial \alpha} q_{dyn_e} S_{ref} \]  
(C.2.31)

\[ a_{\alpha \sigma} = \frac{g_e}{V_e} \cos \gamma_e \sin \sigma_e \]  
(C.2.32)

\[ a_{\alpha \phi} = a_{\alpha \tau} = a_{\alpha \phi} = 0 \]  
(C.2.33)

\[ a_{\beta V} = \frac{g_e}{V_e^2} \cos \gamma_e \sin \sigma_e \]  
(C.2.34)

\[ a_{\beta I} = \frac{g_e}{V_e} \sin \gamma_e \sin \sigma_e \]  
(C.2.35)

\[ a_{\beta R} = \frac{2g_e}{R_e V_e} \cos \gamma_e \sin \sigma_e \]  
(C.2.36)

\[ a_{\beta P} = \sin \alpha_e \]  
(C.2.37)

\[ a_{\beta R} = -\cos \alpha_e \]  
(C.2.38)

\[ a_{\beta \beta} = -\frac{1}{m V_e} \frac{\partial C_S}{\partial \beta} q_{dyn_e} S_{ref} \]  
(C.2.39)

\[ a_{\beta \sigma} = \frac{g_e}{V_e} \cos \gamma_e \cos \sigma_e \]  
(C.2.40)

\[ a_{\beta Q} = a_{\beta \alpha} = 0 \]  
(C.2.41)
\[
a_{\alpha V} = \frac{\tan \gamma \sin \sigma \sigma_e}{mV_e^2} \left( M_e \frac{\partial C_L}{\partial M} + C_L \right) q_{\text{dyn} e} S_{\text{ref}} \tag{C.2.42}
\]

\[
a_{\gamma \gamma} = \frac{L_e}{mV_e} \sin \sigma_e \tag{C.2.43}
\]

\[
a_{\alpha p} = -\cos \alpha_e \tag{C.2.44}
\]

\[
a_{\alpha r} = -\sin \alpha_e \tag{C.2.45}
\]

\[
a_{\alpha \alpha} = \frac{\tan \gamma \sin \sigma \sigma_e}{mV_e} \frac{\partial C_L}{\partial \alpha} q_{\text{dyn} e} S_{\text{ref}} \tag{C.2.46}
\]

\[
a_{\alpha \beta} = \frac{\tan \gamma \cos \sigma \sigma_e}{mV_e} \frac{\partial C_S}{\partial \beta} q_{\text{dyn} e} S_{\text{ref}} - \frac{L_e}{mV_e} + g_e \cos \gamma \cos \sigma_e \tag{C.2.47}
\]

\[
a_{\alpha \sigma} = \tan \gamma \cos \sigma E_e \frac{L_e}{mV_e} \tag{C.2.48}
\]

\[
a_{\alpha R} = a_{\alpha q} = 0 \tag{C.2.49}
\]

The elements of matrix \( B \) are:

\[
b_{V_e} = b_{V_a} = b_{V_r} = b_{Vx} = b_{Vy} = b_{Vz} = 0 \tag{C.2.50}
\]

\[
b_{\gamma e} = b_{\gamma a} = b_{\gamma r} = b_{\gamma x} = b_{\gamma y} = b_{\gamma z} = 0 \tag{C.2.51}
\]

\[
b_{R_e} = b_{R_a} = b_{Rr} = b_{Rx} = b_{Ry} = b_{Rz} = 0 \tag{C.2.52}
\]

\[
b_{pa} = \frac{1}{t_{xx}} \frac{\partial C_i}{\partial \alpha} q_{\text{dyn} e} S_{\text{ref}} b_{\text{ref}} \tag{C.2.53}
\]

\[
b_{px} = \frac{1}{t_{xx}} \tag{C.2.54}
\]

\[
b_{pe} = b_{pr} = b_{py} = b_{pz} = 0 \tag{C.2.55}
\]

\[
b_{qe} = \frac{1}{t_{yy}} \frac{\partial C_m}{\partial \alpha} q_{\text{dyn} e} S_{\text{ref}} c_{\text{ref}} \tag{C.2.56}
\]

\[
b_{qy} = \frac{1}{t_{yy}} \tag{C.2.57}
\]

\[
b_{qa} = b_{qr} = b_{qx} = b_{qz} = 0 \tag{C.2.58}
\]

\[
b_{ra} = \frac{1}{t_{zz}} \frac{\partial C_n}{\partial \alpha} q_{\text{dyn} e} S_{\text{ref}} b_{\text{ref}} \tag{C.2.59}
\]

\[
b_{rr} = \frac{1}{t_{zz}} \frac{\partial C_n}{\partial \sigma} q_{\text{dyn} e} S_{\text{ref}} b_{\text{ref}} \tag{C.2.60}
\]

\[
b_{rz} = \frac{1}{t_{zz}} \tag{C.2.61}
\]
\[ b_{re} = b_{\alpha} = b_{ry} = 0 \]  
\[ b_{ra} = b_{\alpha} = b_{ur} = b_{ax} = b_{uy} = b_{az} = 0 \]  
\[ b_{pe} = b_{\alpha} = b_{pr} = b_{px} = b_{py} = b_{pz} = 0 \]  
\[ b_{ae} = b_{\alpha} = b_{ar} = b_{ax} = b_{ay} = b_{az} = 0 \]  

(C.2.62)  
(C.2.63)  
(C.2.64)  
(C.2.65)  

C.2.3. Longitudinal control

Restricting to the attitude subsystem (i.e., \( \Delta V = \Delta \gamma = \Delta r = 0 \)), and neglecting the asymmetric motion by setting \( \Delta \beta = \Delta \alpha = 0^\circ \), the following approximation can be derived from the linearised equations of motion:

\[
\Delta q = \frac{1}{I_{yy}} \left( \frac{\partial C_m}{\partial \alpha} \Delta \alpha + \frac{\partial C_m}{\partial \delta_e} \Delta \delta_e \right) q_{\text{dyn}} \hat{S}_{\text{ref}} c_{\text{ref}} + \frac{\Delta M_{T,Y}}{I_{yy}}  
\]

(C.2.66)

\[
\Delta \alpha = \Delta q - \frac{1}{mV_e} \frac{\partial C_L}{\partial \alpha} q_{\text{dyn}} \hat{S}_{\text{ref}} \Delta \alpha  
\]

(C.2.67)

The static stability mainly depends on \( \frac{\partial C_m}{\partial \alpha} \), which changes sign at the end of the re-entry phase. This results in an unstable short-period oscillation.

As control variables, the symmetric elevon deflection angle \( \delta_e \) (elevator function) and the thrust moment about the pitch axis, \( M_{T,Y} \), are available. The elevators are activated at \( q_{\text{dyn}} > 100 \text{ N/m}^2 \). The pitch jets will start working at the entry interface (in principle \( q_{\text{dyn}} = 0 \text{ N/m}^2 \)) and will continue to do so until \( q_{\text{dyn}} > 1000 \text{ N/m}^2 \), an operation scheme that is based on that of the Space Shuttle (Cook, 1982). The elevator deflection angle is defined as the symmetric combination of the left and right elevon, i.e.,

\[
\delta_e = \frac{\delta_{e,l} + \delta_{e,r}}{2}  
\]

(C.2.68)

The state-feedback laws for longitudinal control are selected to be simple proportional laws, as has been schematically depicted in Fig. C.2:

\[
\frac{\Delta \delta_e}{\delta_{e_{\text{max}}}} = -K_1 \Delta q - K_2 \Delta \alpha  
\]

(C.2.69)

\[
\frac{\Delta M_{T,Y}}{M_{T,Y_{\text{max}}}} = -K_1 \Delta q - K_2 \Delta \alpha  
\]

(C.2.70)

where

\[
\Delta q = q - q_e = q - q_0 \text{ (rad/s)} \]

\[
\Delta \alpha = \alpha - \alpha_e = \alpha - \alpha_c \text{ (rad)}  
\]
\[ q_c = \text{commanded pitch rate (rad/s)} \]
\[ \alpha_c = \text{commanded angle of attack from the guidance system (rad)} \]

These control laws are a linear combination of the state variables, which is the right form for the LQR:

\[ \Delta u = -K \Delta x \]

Writing both the equations of motion Eqs. (C.2.66-67) and the feedback laws Eqs. (C.2.69-70) in matrix form, the gains can be computed by solving the Riccati Equation (Gopal, 1989), as discussed in Chapter 3. The selected weighting matrices \( Q \) and \( R \) are chosen to be

\[ Q = \text{diag} \left( \begin{array}{cc} 1 & 1 \\ \Delta q_{\text{max}}^2 & \Delta \alpha_{\text{max}}^2 \end{array} \right) \quad R = \text{diag} \left( \begin{array}{cc} 1 & 1 \\ \Delta \delta_{\text{max}}^2 & \Delta M_{T,y}^2 \end{array} \right) \]

where

\[ \Delta q_{\text{max}} = \infty \quad \Delta \alpha_{\text{max}} = 2^\circ \]
\[ \Delta \delta_{\text{max}} = 40^\circ \quad \Delta M_{T,y_{\text{max}}} = 10,400 \text{ Nm} \]

The value of \( \Delta \alpha_{\text{max}} \) has been selected in accordance with the small margin that is left, since the nominal \( \alpha \) is close to its maximum. Note that the selection of the weighting matrices is in principle an iterative procedure that is not pursued here.

**C.2.4. Lateral control**

The lateral dynamics consist of a coupled motion of roll and yaw. This motion can be approximated by setting \( \Delta \alpha \) to zero and neglecting smaller terms. The following equations can be derived:
\[
\Delta \dot{\varphi} = \frac{1}{I_{xx}} \left( \frac{\partial C_I}{\partial \varphi} \Delta \beta + \frac{\partial C_I}{\partial \delta_a} \Delta \delta_a \right) q_{dyn_e} S_{ref} b_{ref} + \frac{\Delta M_{T,x}}{I_{xx}} 
\]

\[
\Delta \dot{\theta} = \frac{1}{I_{zz}} \left( \frac{\partial C_n}{\partial \beta} \Delta \beta + \frac{\partial C_n}{\partial \delta_a} \Delta \delta_a + \frac{\partial C_n}{\partial \delta_r} \Delta \delta_r \right) q_{dyn_e} S_{ref} b_{ref} + \frac{\Delta M_{T,z}}{I_{zz}} 
\]

\[
\Delta \dot{\beta} = \sin \alpha_e \Delta \rho - \cos \alpha_e \Delta r - \frac{g_e}{V_e} \cos \gamma_e \cos \sigma_e \Delta \sigma 
\]

\[
\Delta \dot{\sigma} = -\cos \alpha_e \Delta \rho - \sin \alpha_e \Delta r + \left( \frac{g_e}{V_e} \cos \gamma_e \cos \sigma_e - \frac{L_e}{mV_e} \right) \Delta \beta + \frac{\tan \gamma_e \cos \sigma_e}{mV_e} L_e \Delta \sigma 
\]

Because the damping coefficients such as $C_{lp}$ and $C_{n_l}$, are not defined in the current aerodynamic model, the corresponding terms do not appear in the equations and therefore there is no natural damping. The available controls consist of the aileron, rudder, roll jets and yaw jets.

The aileron deflection angle $\delta_a$ is defined as the asymmetric combination of the left and right elevon, i.e.,

\[
\delta_a = \frac{\delta_{e,l} - \delta_{e,r}}{2} 
\]

The rudder deflection angle $\delta_r$ is defined to be equal to $\delta_{r,l}$ when $\delta_{r,l}$ is positive, and equal to $-\delta_{r,r}$ when $\delta_{r,r}$ is positive. The control laws for aileron and rudder are chosen in the form

\[
\frac{\Delta \delta_a}{\delta_{a_{max}}} = -[K_3 \dot{\sigma} + K_4 (\sigma - \sigma_e)] \cos \alpha - \left[ K_5 (\dot{\beta} + \frac{g}{V} \sin \sigma) + K_6 \beta \right] \sin \alpha 
\]

\[
\frac{\Delta \delta_r}{\delta_{r_{max}}} = -[K_7 \dot{\sigma} + K_8 (\sigma - \sigma_e)] \sin \alpha - \left[ K_9 (\dot{\beta} + \frac{g}{V} \sin \sigma) + K_{10} \beta \right] \cos \alpha 
\]

where

\[
\Delta \beta = \beta - \beta_e = \beta - \beta_c = \beta \\
\Delta \sigma = \sigma - \sigma_e = \sigma - \sigma_c \\
\Delta \rho = \rho - \rho_e = \rho - \rho_c \\
\Delta r = r - r_e = r - r_c 
\]

The lateral controller, which is represented by these equations, is schematically shown in Fig. C.3\textsuperscript{25}. It should be noted that the aerodynamic- and reaction-control part are identical, but for the gains.

\textsuperscript{25} In the loop for the angle of sideslip, a lead filter is included. This filter is not considered in the computation of the feedback gains with optimal control theory. The filter has later been added to improve the sideslip response to rudder deflections.
By linearising the control laws, they can be written as

\[
\Delta u = \begin{bmatrix}
\Delta \delta_d \\
\Delta \delta_r \\
\Delta M_{T,x} \\
\Delta M_{T,z}
\end{bmatrix} = - \begin{bmatrix}
\delta_{a_{max}} K_3^* & \delta_{a_{max}} K_4^* & \delta_{a_{max}} K_5^* & \delta_{a_{max}} K_6^* \\
\delta_{r_{max}} K_7^* & \delta_{r_{max}} K_8^* & \delta_{r_{max}} K_9^* & \delta_{r_{max}} K_{10}^* \\
T_{x_{max}} K_3^* & T_{x_{max}} K_4^* & T_{x_{max}} K_5^* & T_{x_{max}} K_6^* \\
T_{z_{max}} K_7^* & T_{z_{max}} K_8^* & T_{z_{max}} K_9^* & T_{z_{max}} K_{10}^*
\end{bmatrix}
\begin{bmatrix}
\Delta p \\
\Delta \tau \\
\Delta \beta \\
\Delta \sigma
\end{bmatrix} = -K^* \Delta x \tag{C.79}
\]

with
Again, the gains for the equilibrium points follow from solving the Riccati Equation. The applied weighting matrices are given by

\[
Q = \text{diag}\left\{ \frac{1}{\Delta \rho_{\text{max}}^2}, \frac{1}{\Delta \tau_{\text{max}}^2}, \frac{1}{\Delta \phi_{\text{max}}^2}, \frac{1}{\Delta \sigma_{\text{max}}^2} \right\}
\]

\[
R = \text{diag}\left\{ \frac{1}{\Delta \delta_{\alpha_{\text{max}}}^2}, \frac{1}{\Delta \delta_{\delta_{\text{max}}}^2}, \frac{1}{\Delta M_{T,x_{\text{max}}}^2}, \frac{1}{\Delta M_{T,z_{\text{max}}}^2} \right\}
\]

(C.2.80)

where

\[
\Delta \rho_{\text{max}} = \infty \quad \Delta \phi_{\text{max}} = 2^\circ
\]

\[
\Delta \tau_{\text{max}} = \infty \quad \Delta \sigma_{\text{max}} = 5^\circ
\]

\[
\Delta \delta_{\alpha_{\text{max}}} = 40^\circ \quad \Delta M_{T,x_{\text{max}}} = 1,600 \text{ Nm}
\]

\[
\Delta \delta_{\delta_{\text{max}}} = 40^\circ \quad \Delta M_{T,z_{\text{max}}} = 7,600 \text{ Nm}
\]

In Moolij (1997b), a sensitivity analysis based on tabulated error functions for the atmospheric density, position measurements, and the lift and drag coefficient, as well as dispersions in the initial conditions, has been conducted. The results indicated that, although the guidance system had no problems at all for properly guiding the vehicle, the deviations were such that at \( t = 930 \) s, the vehicle could not be controlled any longer and started diverging oscillations around all three axes. These oscillations could be traced back by a combined angle-of-attack and angle-of-sideslip corrective control. The former requires a symmetric deflection of the elevons, whereas the latter needs an asymmetric deflection. As a result, the absolute deflection of the left and right elevon, which is a simple summation of elevator and aileron deflection, differed substantially. Since the aerodynamic moments due to elevon deflection are quite non-linear for larger deflection angles, the attitude controller assumed roll and pitch
moments that deviated substantially from the actual moments. The induced differences resulted in diverging, oscillating elevon deflections and an uncontrollable vehicle.

Without going into great detail of trying to solve this problem, the lateral controller was adjusted such that the above mentioned problem did not occur any more. The angle of sideslip is an induced disturbance that can relatively fast be controlled, since, next to the ailerons, also the rudder and yaw jets are used. Therefore, priority is given to $\beta$ control by including a phase-lead filter in the loop for $\beta$, such that derivative information is taken into account. As a result, a strong variation in $\beta$ (large derivatives) results in larger deflections of rudder and aileron (side-slip compensation) and therefore faster side-slip control. The transfer function $G_c(s)$ of the phase-lead filter is:

$$G_c(s) = \frac{K_c \tau_1 s + 1}{\tau_2 s + 1}, \text{ with } K_c = 10, \tau_{c,1} = 1 \text{ and } \tau_{c,2} = 10$$

C.3. References


Mooij, E., "Linear Quadratic Regulator design for an unpowered, winged re-entry vehicle", Report LR-806, Delft University of Technology, Faculty of Aerospace Engineering, 1997a.

Mooij, E., "Guidance and hybrid control of a winged re-entry vehicle", Report LR-xxx, Delft University of Technology, Faculty of Aerospace Engineering, 1997b.

Appendix D

An Algorithm to Derive Orthogonal Arrays

To generate orthogonal arrays for three-level parameters, an algorithm has been developed by Mistree et al. (1993). The basic information is provided by Latin Squares which present the smallest orthogonal entities (Table D.1). When there is a complete orthogonal system of \( (n-1) \) Latin Squares, with \( n \) the number of factor levels, each Latin Square of dimensions \( n \times n \), denoted by \( L_1, L_2, \ldots, L_{n-1} \), it is possible to construct an orthogonal array of size \( n^r \) \( (r = 2, 3, \ldots) \) and number of columns \( \frac{n^r - 1}{n - 1} \). The relation between the number of experiments \( (L) \) and the number of factors \( (x_i) \) is given in Table D.2 for certain values of \( r \) (3-level arrays).

<table>
<thead>
<tr>
<th>Latin Square 1</th>
<th>Latin Square 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1  0  1</td>
<td>-1  1  0</td>
</tr>
<tr>
<td>0  1  -1</td>
<td>0  -1  1</td>
</tr>
<tr>
<td>1  -1  0</td>
<td>1  0  -1</td>
</tr>
</tbody>
</table>

Table D.1 - The two Latin Squares for three levels.
Table D.2 - Relation between the array size $L_j$ and the number of factors $x_k$ for 3-level experiments.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$L_j$</th>
<th>$x_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>13</td>
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<td>243</td>
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<tr>
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</tr>
<tr>
<td>7</td>
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<td>1093</td>
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<tr>
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<td>6561</td>
<td>3280</td>
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</table>

Starting with the two Latin Squares of Table D.1, we can easily derive the $L_9$, $L_{27}$, and $L_{81}$ arrays for 4, 13, and 40 factors, respectively. In Table D.3, the orthogonal array $L_9$ is divided into three blocks from which the higher orthogonal array ($L_{27}$) is created. Each block represents one first column, A, with the same number. The three other columns (B, C and D) contain an equal value within one row in block 1, the values of Latin Square 1 in block 2, and the values of Latin Square 2 in block 3.

<table>
<thead>
<tr>
<th>Exp.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>-1</td>
<td>-1</td>
<td>-1</td>
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<tr>
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<tr>
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<td>1</td>
<td>1</td>
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</table>

Table D.3 - Orthogonal array $L_9$ with 4 factors (A-D) on three levels.

For creating the higher arrays, four basic steps are involved. These will be discussed below. As an example we will create the $L_{27}$ array, see also Table D.4.

1) Starting at column 1 in the new array, we create three new blocks by assigning level 1 to the first third of rows (Block 1), level 2 to the second third of rows (Block 2), and level 3 to the last third (block 3).

2) In block 1, columns 2 to 4 are created by taking the first column of the old array (which has the same length as one new block) and assigning the same level in one row to all three columns.
Columns 5 to 7 are then created by using the second column of the old array. This continues until all columns in block 1 are filled.

3) In block 2, columns 2 to 4 are created by taking the first column of the old array and by assigning a row of Latin Square 1 starting with the old value. This step is repeated until all columns in block 2 are filled.

4) Block 3 is created similarly to block 2, but instead of Latin Square 1 we use Latin Square 2.

<table>
<thead>
<tr>
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<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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</tbody>
</table>

Table D.4 - Orthogonal array $L_{27}$ with 13 factors (A-M) on three levels.

The algorithm which has been described above can also be used to derive the orthogonal arrays for other-than-three level factors. Since usually only two- and three-level factors are used (Taguchi,
1988), we will give the first four 2-level orthogonal arrays as an example, starting with the 2x2 Latin Square $L_2$ (Table D.5). Successively, we get $L_4$ (Table D.6), $L_8$ (Table D.7) and $L_{16}$ (Table D.8). Note that Step 4 is not required for two-level orthogonal arrays.

![Latin Square](image)

Table D.5 - The Latin Square for two levels.

<table>
<thead>
<tr>
<th>Exp.</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
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<td>-1</td>
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</tr>
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</tr>
<tr>
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</tr>
</tbody>
</table>

Table D.6 - Orthogonal array $L_4$ with 3 factors (A-C) on two levels.

<table>
<thead>
<tr>
<th>Exp.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
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Table D.7 - Orthogonal array $L_8$ with 7 factors (A-G) on two levels.
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Table D.8 - Orthogonal array $L_{16}$ with 15 factors (A-O) on two levels.

References


Appendix E

Analysis of Variance and Analysis of Response Surfaces

E.1. Analysis of variance

In the following we define some parameters which are used with ANalysis Of VAriance (ANOVA), see, for instance, Taguchi (1988). The mean (or first moment) \( \bar{y} \) of the values \( y_j (j = 1, 2, \ldots, N) \) is given by

\[
\bar{y} = \frac{1}{N} \sum_{j=1}^{N} y_j
\]  

(E.1.1)

The total variation in a set of data is called the total sum of squares (\( S_T \)):

\[
S_T = \sum_{j=1}^{N} (y_j - \bar{y})^2 = \sum_{j=1}^{N} (y_j^2 - 2y_j\bar{y} + \bar{y}^2) = \sum_{j=1}^{N} y_j^2 - 2\bar{y} \sum_{j=1}^{N} y_j + N\bar{y}^2
\]  

(E.1.2)
By defining the total sum $T$ (or the zeroth moment) as

$$T = \sum_{j=1}^{N} y_j$$  \hspace{1cm} (E.1.3)

we can write

$$S_T = \sum_{j=1}^{N} y_j^2 - \frac{T^2}{N} = \sum_{j=1}^{N} y_j^2 - CF$$  \hspace{1cm} (E.1.4)

where $CF$ is a correction factor in the magnitude of the sum of squares of the mean.

In case experiments are done according to orthogonal arrays, there is an equal number $n_{x_i}$ of experiments at each of the levels for one factor. The sum of squares $S_i$, e.g., for factor $x_i$ on levels $x_{ij}$, is the sum of all level variations and can be calculated as

$$S_i = \frac{1}{n_{x_i}} \left( \sum_{j=1}^{k_i} \left( \sum_{j=1}^{N} y(x_{ij}) \right)^2 \right) - CF$$  \hspace{1cm} (E.1.5)

Note that the second sum means that from all $N$ responses those at level $j$ are added together, after which the sum is squared. The first sum means that this operation is repeated for all $k_j$ levels. The error sum of squares can easily be computed from

$$S_E = S_T - \sum_i S_i$$  \hspace{1cm} (E.1.6)

which is the total sum of squares minus the sum of squares of all factors and interactions.

A degree of freedom $f$ in a statistical sense is related with each piece of independent information that is estimated from the data. A three-level factor contributes to two degrees of freedom because we are interested in two comparisons, for instance level 1 with level 0, and level -1 with level 1. In that case, the comparison of level 0 with level 1 is not independent, because we can get the information from the other two comparisons. The same is true for the degrees of freedom associated with the error estimate. Suppose we have $N$ experiments, then we can compare the result of experiment 1 with the one of experiment 2, 1 with 3, and so on. The total number of degrees of freedom of a result is therefore

$$f_T = N - 1$$  \hspace{1cm} (E.1.7)

The degrees of freedom for factor $x_i$ with $k_i$ levels is

$$f_i = k_i - 1$$  \hspace{1cm} (E.1.8)
so the degrees of freedom for the error can be defined as

\[ f_E = f_T - \sum_i l_i \]  \hspace{1cm} (E.1.9)

This implies that for the orthogonal \( L_9 \) array with 4 three-level factors \( f_E \) is equal to zero, indicating that when the array is used to its full extent there are no degrees of freedom left to estimate the error.

The variance (or second moment) is defined as

\[ \text{Var}(y_1 \ldots y_N) = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \bar{y})^2 \]  \hspace{1cm} (E.1.10)

Using the definitions as stated before, we can also write for the variance \( V_i \) of a factor \( x_i \)

\[ V_i = \frac{S_i}{l_i} \]  \hspace{1cm} (E.1.11)

For the error it follows that

\[ V_E = \frac{S_E}{f_E} \]  \hspace{1cm} (E.1.12)

under the condition that \( f_E > 0 \).

The variance ratio \( F \) is the variance of the factor divided by the error variance. This ratio is used to measure the significance of the investigated factor with respect to the variance of all factors included in the error term. The expected distribution of this statistic, the \( F \)-distribution, can be found in most statistics handbooks and can be used to determine which factor contributes to the sum of squares within the selected confidence level, see also Montgomery (1984), Khuri and Cornell (1987), Taguchi (1988) and Press et al. (1989).

If we want to know to what extent one factor contributes to the variation \( S_T \), we divide the pure sum of squares by the total sum of squares \( S_T \). The percent contribution \( P \) is then defined as

\[ P_i = 100 \frac{S_i}{S_T}, \quad P_E = 100 \frac{S_E}{S_T} \]  \hspace{1cm} (E.1.13)

Comparing distributions

By means of a \( t \)-test it is possible to assess the question of difference in case the two distributions are assumed to have the same variance (Press et al., 1989). An \( F \)-test can in this case be applied to find out whether we actually have two distributions with the same variance. Wilcoxon's test can be used to verify whether the individual responses of distribution \#1 are consistently larger (or smaller) than the individual responses of distribution \#2. Finally, the generally accepted Kolmogorov-Smirnov test
can be applied to continuous distributions to check whether they are consistent or different.

In this appendix, we restrict to the problem of consistent or different distributions, since it is of interest to know whether the two sets of orthogonal factor combinations that are in principle normally distributed result in the same distribution. A practical problem is that the Kolmogorov-Smirnov test is usually more accurate when the number of data points is larger than 20 when compared with a theoretical distribution, also a critical number for the computation of the test of Wilcoxon. Fortunately, the results are usually good while comparing two distributions with $N_1$ and $N_2$ data points when

$$\frac{N_1N_2}{N_1+N_2} \geq 4$$  \hspace{1cm} (E.1.14)

Press et al. (1989) discuss an algorithm that computes the significance level for the null hypothesis that the responses are drawn from the same distribution. The function that enters into the calculation of the significance is written as the following sum:

$$Q_{KS}(\lambda) = 2\sum_{j=1}^{\infty} (-1)^{j-1} e^{-2j^2\lambda^2}$$  \hspace{1cm} (E.1.15)

For two distributions with $N_1$ and $N_2$ datapoints, the function to be evaluated is

$$Q_{KS}\left(\frac{N_1N_2}{N_1+N_2}, D\right)$$  \hspace{1cm} (E.1.16)

where $D$ is the maximum value of the absolute difference between two cumulative distribution functions.

**E.2. Analysis of Response Surfaces**

In this section we concentrate on two aspects of the analysis of response surfaces, i.e., what is the goodness of fit of the response surface (Section E.2.1) and what will be the optimum condition (Section E.2.2). It should be stressed that response surfaces can be analysed in much more detail, but it is far beyond the scope of this thesis to give a complete description of the regression analysis, both linear and non linear. For more information, the reader is referred to the many books on design and regression analysis, e.g., Keppel (1973), Gunst and Mason (1980), Montgomery (1984), Khuri and Cornell (1987), and Taguchi (1988). However, the description below will suffice for the purpose of assessing the potential of RSM applied to trajectory analysis, and guidance and control system testing.

**E.2.1. Goodness of fit**

To get an impression of how well the least-squares fit matches the raw data (the responses), we can
apply several techniques, as can be found in books on regression analysis. We will conclude this subsection with a brief summary, in its full form presented by Khuri and Cornell (1987). First, we will give some definitions. For each of the factor combinations of all $N$ experiments, resulting in $N$ measured responses, we compute the predicted response $\hat{y}_i$, $i = 1, \ldots, N$ with the response-surface model. The difference between the measured and predicted response is the residual $r_i$, i.e., $r_i = y_i - \hat{y}_i$. We define now three sum of squares, the total sum of squares ($S_T$), the sum of squares due to regression ($S_R$) and the sum of squares unaccounted for by the response surface ($S_E$):

$$S_T = \sum_{i=1}^{N} (y_i - \overline{y})^2, \quad S_R = \sum_{i=1}^{N} (\hat{y}_i - \overline{y})^2, \quad S_E = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

with the mean value of the observed responses, $\overline{y}$, given by Eq. (E.1.1). The degrees of freedom associated with $S_T$, $S_R$ and $S_E$ are $N-1$, $p-1$ and $N-p$ ($= (N-1) - (p-1)$), respectively, where $p$ is the number of parameters of the fitted model.

The coefficient of determination, $R^2$, defined by

$$R^2 = \frac{S_R}{S_T} \quad (E.2.1)$$

is a measure of the proportion of total variation of the values $y_i$ about the mean $\overline{y}$ explained by the fitted model. When the coefficient of determination is adjusted by using the degrees of freedom corresponding to $S_E$ and $S_T$, and defining

$$R_A^2 = 1 - \left( \frac{S_E}{N-p} \right) \left( \frac{S_T}{N-1} \right)^{-1} = 1 - (1-R^2) \left( \frac{N-1}{N-p} \right) \quad (E.2.2)$$

then we get the adjusted $R^2$ statistic, a measure of the drop in the magnitude of the estimate of the error variance achieved by fitting a model other than $y = \beta_0 + \varepsilon$ relative to the estimate of the error variance that would be obtained by fitting the model $y = \beta_0 + \varepsilon$.

Statements about the individual regression coefficients $\beta_i$ can be made if we can make some assumptions about the random errors $\varepsilon_i$: i) the errors $\varepsilon_i$ have zero mean and a common variance $\sigma^2$, ii) they are mutually independent in the statistical sense, and iii) they are normally distributed (so that the $t$- and $F$-statistic can be applied to them). These assumptions approximate practice, and is justified since the model also is just an approximation (Gunst and Mason, 1980). With the above assumptions as a starting point, it is possible to derive an expression for the variance-covariance matrix of the vector of coefficient estimates $\theta$, based on the normal equations:

$$\text{Var}(\theta) = (X^T X)^{-1} \sigma^2 \quad (E.2.3)$$

Since we use SVD to solve the least-squares problem, an equivalent expression based on the SVD...
solution can be used:

\[
\text{Var}(\mathbf{b}) = (V \text{ diag}(w_j) \mathbf{U}^T \mathbf{U} \text{ diag}(w_j) V^T)^{-1} \sigma^2 = C \sigma^2
\]  

(E.2.4)

Note that SVD algorithms usually return a column-orthonormal matrix \( \mathbf{U} \). In that case, \( \mathbf{U}^T \mathbf{U} \) equals the identity matrix.

So \( c_{ij} \) is the variance of \( b_p \) whereas the standard error is given by the positive square root of the variance. The \( j^{th} \) element of \( \mathbf{C} \), i.e., \( c_{ij} \sigma^2 \), is the covariance between \( b_i \) and \( b_j \). What is left to derive is an expression for the estimation of \( \sigma^2 \), because usually it is not known. Khuri and Cornell (1987), or equivalently Gunst and Mason (1980), give an expression for the general case of a fitted model with \( p \) parameters and \( N \) observations \((N > p)\). The estimate \( s^2 \) of \( \sigma^2 \) is computed from:

\[
s^2 = \frac{1}{N-p} \sum_{i=1}^{N} r_i = \frac{1}{N-p} (\mathbf{Y} - \mathbf{X} \mathbf{b})^T (\mathbf{Y} - \mathbf{X} \mathbf{b}) = \frac{1}{N-p} S_E
\]  

(E.2.5)

In Eq. (E.2.5) \( r_i \) is the \( i^{th} \) residual and the divisor \( N-p \) is the degrees of freedom of the estimator \( s^2 \). Now we can give expressions for the confidence limits of the individual coefficient estimates \( b_p \) or, in other words, the interval in which \( b_i \) lies for a given level (percentage) of confidence. In doing so, we assume that the variance \( \sigma^2 \) is unknown and estimated by \( s^2 \). Gunst and Mason (1980) give the following equation for the confidence interval of the coefficient estimate \( \beta_i \) of \( \beta_p \) based on Student's \( t \)-statistic:

\[
b_i - t_{1-\alpha/2}(N-p)s^2 \sqrt{C_{ii}} \leq \beta_i \leq b_i + t_{1-\alpha/2}(N-p)s^2 \sqrt{C_{ii}}
\]  

(E.2.6)

with \( \alpha \) the deviation from 100% confidence (\( \alpha = 0.05 \) means a 95%-confidence level). Tables with values for the \( t \)-statistic can be found in most books on statistics. Press et al. (1989) give a relation between Student’s distribution and the incomplete beta function. This incomplete beta function can be numerically approximated, so that we do not have to use the tables for the \( t \)-statistic. For a given confidence level \( \alpha \) and a given degrees of freedom, we can compute \( t \) in an iterative manner.

E.2.2. Optimum condition

Assuming that the response surface is an adequate representation of the variation of the responses, then finding the maximum response will be of importance when that maximum will be found at a different factor combination than was included in the original analysis. For a function of \( k \) independent variables this implies deriving expressions for \( k \) first derivatives, equating them to zero and solving them simultaneously. Putting the second derivatives to zero will give us information about the nature of the so-called stationary point, whether it is a (local) minimum or maximum, or a saddle (minimax) point. A second-order model can be written in matrix form as

\[
y = b_0 + \mathbf{x}^T \mathbf{b} + \mathbf{x}^T \mathbf{B} \mathbf{x}
\]  

(E.2.7)
where

\[
X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{pmatrix}, \quad \text{and } B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ \frac{b_{22}}{2} & \frac{b_{2k}}{2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \frac{b_{kk}}{2} \end{pmatrix}.
\]

Taking the derivatives \( \frac{\partial y}{\partial x_i} \) results in a vector \( \Delta \) of first derivatives:

\[
\Delta = b + 2BX
\]

(E.2.8)

so that we find for the stationary point

\[
X_0 = -\frac{B^{-1}b}{2}
\]

(E.2.9)

assuming that the coefficients \( b \) and \( B \) are known exactly and the fitted model is not bounded in any of its independent variables \( x_r \).

However, since the response surface will never be 100% accurate, we cannot compute the maximum but rather an interval of maximum values. Since we know the confidence interval of the regression coefficients, this interval can be computed. Finding the combination of minimum and maximum values of regression coefficients is not trivial in case of many linear, quadratic and interaction terms. In case of many factors \( x_i \) that are usually also bounded to the limit values of the experimental region, it is not that straightforward to find the location of the stationary point. One alternative to find the range of maximum values is to do a Monte-Carlo analysis on the regression coefficients and computing the maximum for each case. However, in that case we can also apply the Taguchi method.

For a given total of \( k \) factors, \( b_0 \pm \Delta b_0, b_i \pm \Delta b_i \) and \( b_{ij} \pm \Delta b_{ij} (i, j = 1, \ldots, k) \) represent \((k+1)(k+2)/2\) model parameters that need to be assigned to the columns of the orthogonal array. For each parameter combination the stationary point can now be computed using a numerical optimisation method. In our case, we will use a truncated Newton method that minimises (or equivalently, maximises) a function of \( k \) independent bounded variables, as described by Nash (1984). From the results, we can extract the interval of maximum values of the response. Note that this minimisation method finds a local rather than a global maximum. To be certain that we have found the global maximum it is therefore required to repeat this process a number of times, each time with different starting values of \( X_0 \).

E.3. References

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Samenvatting

Horizontaal opstijgende eentraps- en tweetrapsruimtevoertuigen uitgerust met luchtgebruikende motoren, de zogenaamde ruimtevliegtuigen, lijken veelbelovend om betalende lading in een baan om de Aarde te brengen tegen lagere kosten dan hedentendage mogelijk is. Het gebruik van luchtdemende motoren resulteert in opstijgbanen die voor een aanzienlijk deel op lage hoogte liggen in vergelijking met bestaande lansseervoertuigen. Dit geeft hogere waarden voor de dynamische druk, de wandtemperatuur en de mechanische belasting. Naast innovaties op het gebied van hittebestendige materialen en constructies, vereist dit de ontwikkeling van geleidingslogica en standregelsystemen (G&C-systemen), een enorme uitdaging gezien de kleine marge in de prestaties en de sterk variërende vluchtcondities.

De probleemdefinitie van deze studie is als volgt geformuleerd:

**Hoe en in welke mate beïnvloeden ontwerponzekerheden en verstoringen in de vluchtomgeving de prestaties van ruimtevliegtuigen, en hoe kan een G&C-systeem bijdragen aan het succes van de missie?**

Om de banen en prestaties van ruimtevliegtuigen in voldoende detail te berekenen is er behoefte aan een computerprogramma voor vluchtsimulatie; de ontwikkeling daarvan wordt gezien als een van de belangrijkste doelstellingen van dit proefschrift. Dit programma, het *Simulation Tool for Ascent and Re-entry Trajectories* (START), kan worden gebruikt om simulaties in 3 en 6 vrijheidsgraden uit te voeren. Het voertuig wordt daarbij gezien als massapunt of als niet-elastisch lichaam met variabele massa-eigenschappen door het gebruik van een voortstuwings-systeem.

Een van de belangrijkste modules, die ontwikkeld is voor START, is een G&C-systeem. Uitgangspunt is hierbij geweest dat de prestatie van dit systeem niet behoeft te worden geopti-
Samenvatting

maliseerd, omdat een acceptabele prestatie reeds voldoende is om het onderzoek te kunnen doen. De toegepaste geledingssystemen zijn deels gebaseerd op bestaande systemen, en betreffen deels een eigen ontwikkeling. Voor het standregelsysteem zijn twee concepten bestudeerd. Het eerste is gebaseerd op terugkoppeling van de gelineariseerde toestand met variabele versterkingsfactoren, waarbij optimale regeltheorie wordt gebruikt om deze factoren te berekenen; dit concept staat bekend onder de naam Linear Quadratic Regulator (LQR). Het tweede concept is dat van Model Reference Adaptive Control (MRAC), en is geselecteerd om de mogelijkheid als regelaar voor ruimtevliegtuigen te bestuderen. Deze methode probeert de uitvoervariabelen van het voertuig gelijk te maken aan die van een versimpeld referentie model waarbij de versterkingsfactoren continu worden aangepast aan de hand van de optredende fout.

Om de resultaten van de simulaties te analyseren wordt o.a. gebruik gemaakt van statistische methoden, met name van variantie- en regressie-analyse. Deze methoden kunnen alleen met een zekere mate van betrouwbaarheid gebruikt worden als er voldoende data beschikbaar zijn. In deze studie worden twee alternatieven voor de gewoonlijk gebruikte Monte-Carlo Methode bestudeerd, te weten de Taguchi Methode (TM) en de gerelateerde Response Surface Methodology, en met name het Central Composite Design (CCD).

START is een gereedschap geworden, waarmee zowel de voortgestuwde als de niet-voortgestuwde vlucht van starre voertuigen in een planeetatmosfeer met wind bestudeerd kan worden. Een uitgebreide interface stelt de gebruiker in staat om de geometrie en de parameters van het voertuig te modeleren, de vluchtduur te definieren en de missie te omschrijven. Verder zijn er faciliteiten om de simulatie-uitvoer te verwerken.

Om aan te tonen dat de algoritmes correct zijn geïmplementeerd en dat de gesimuleerde uitvoer een goede benadering is van de werkelijkheid, is START geverifieerd, gevalideerd en geëvalueerd. Hiertoe zijn de resultaten vergeleken met de uitvoer van een gevalideerd softwarepakket (RATT/ESTEC), met vergelijkbare resultaten uit de literatuur, en beoordeeld op fysische juistheid. Het resultaat is dat START gebruikt kan worden om de geregelde stijg- en daalvlocht van (niet-)voortgestuwde ruimtevliegtuigen te bestuderen.

Twee missies zijn uitgebreid onderzocht, namelijk de niet-voortgestuwde terugkeerlucht van het gevleugelde ruimtevoertuig HORUS dat de LQR gebruikt, en de voortgestuwde opstijgbaan van het Winged Cone Configuration (WCC) eentrips ruimtevliegtuig, in combinatie met MRAC. De meeste simulaties zijn uitgevoerd volgens de Taguchi Methode. De opgedane ervaring leert dat men in eerste instantie door variatie van slechts een gering aantal parameters inzicht kan verkrijgen in lineaire en hogere-orde effecten, en mogelijke interacties tussen de parameters. De TM en CCD kunnen gebruikt worden in de voortontwerp fase om inzicht te krijgen in de gevoelige parameters en interacties. Verder in het ontwerpproces, als het merendeel van de parameters bevallen is, kan een verificatieanalyse worden uitgevoerd met de Monte-Carlo Methode.

Bij het simuleren van de geregelde vlucht van een ruimtevliegtuig is het mogelijk dat er instabiliteiten optreden als de parameters over drie niveaus worden gevarieerd, terwijl het variëren over twee niveaus alleen maar stabiele resultaten geeft. Dit wordt veroorzaakt door
niet-lineariteiten in het systeem. Dergelijke instabiliteiten kunnen in principe niet door een statistische analyse getraceerd worden, tenzij deze daadwerkelijk optreden. Voorts heeft een enkele instabiliteit een grote invloed op de resultaten en zou verholpen moeten worden voordat bijvoorbeeld variantieanalyse wordt toegepast. Als de instabiliteiten bij meerdere parametercombinaties optreden kunnen de resultaten worden gebruikt om de oorzaak hiervan te vinden; in deze studie leidde dit tot een modelverbetering van het geleidingssysteem van HORUS.

Behalve als methode voor het uitvoeren van een gevoeligheidsanalyse, is de TM ook als ontwerpmethode gebruikt. In de eerste plaats is een robuuste ontwerpmethode, gebaseerd op een dubbele matrix voor zowel de ontwerpparameters als de verstoringen, toegepast om de prestaties van het geleidingssysteem van HORUS te optimaliseren onder invloed van verstoringen, echter zonder deze te elimineren. Hier werden de optimale waarden van de geleidingsparameters bepaald, en er werd vastgesteld dat het oorspronkelijke ontwerp optimaal is. In de tweede plaats is de opstijgbaan van de WCC verbeterd door deze in een aantal segmenten te verdelen, en per segment het profiel van de baanhoek voor te schrijven. De resulterende baanparameters werden daarna gevarieerd. Dit bleek te resulteren in een grote variatie in ladingscapaciteit, hetgeen aangeeft dat een juiste keuze van de te vliegen baan essentieel is voor het succes van de missie. Het moge duidelijk zijn dat het gevonden optimum slechts geldig is binnen de nauwkeurigheid van de toegepaste baansegmentatie en geleidingsmodellen. Dit is een van de redenen dat de TM geen vervanging kan zijn van numerieke baanoptimalisatie, omdat deze analyse is gebaseerd op segmenten met constante parameters wat niet geldig hoeft te zijn voor de optimale baan. Verder is voorkennis van de te vliegen baan vereist voor het opstellen van de segmentatie. Echter, door een Taguchi-analyse los te laten op een gegeven baansegmentatie kan men het inzicht in de gevoeligheid van baanparameters op prestatievariabelen vergroten.

De LQR geeft een redelijke prestatie alhoewel het huidige systeem een probleem met niet-lineariteiten in de aërodynamica van de stuurlvlakken heeft. Vooral bij grotere uitslagen kan dit tot besturingsproblemen leiden. In het geval dat er herhaaldelijk grote uitslagen nodig zijn kan dit zelfs een compleet verlies van besturing geven. MRAC lijkt een veelbelovend concept te zijn bij toepassing op hypersone ruimtevliegtuigen. Desondanks is het nog te vroeg om te concluderen dat de huidige configuratie van de standregelaar robuust is met betrekking tot ontwerp-onzekerheden en verstoringen in de vluchtomgeving: veel veranderingen zijn nog mogelijk om de prestaties te verbeteren.

Concluderend kan gesteld worden, dat het missiesucces van ruimtevliegtuigen voor een groot deel afhankelijk is van de prestaties van het besturingssysteem. Een slecht ontwerp van dit systeem kan direct tot het verlies van het voertuig leiden. Een redelijke prestatie voor de nominale missie alleen is niet voldoende om te concluderen dat het een goed systeem is: een uitgebreide analyse voor niet-nominale omstandigheden is daarom noodzakelijk. Om koppelingen tussen het geleidingssysteem en de standregelaar te voorkomen is het verder noodzakelijk dat de prestaties van het geleidingssysteem eerst onder de loep worden genomen, voordat men met uitgebreide analyses van de standregelaar begint. Dit hoeft in principe niet als de prestaties
van de standregelaar het doel van de studie zijn en men er zeker van is dat de effecten die door het geleidingssysteem worden geïnduceerd, gescheiden kunnen worden van die van het standregelsysteem.
Curriculum Vitae

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1982-1986: College of Advanced Engineering in Haarlem, Bachelor of Science in aeronautical engineering.
1991: Master of Science thesis work at ESTEC, developing a computer tool to simulate atmospheric re-entry trajectories and to study the entry end descent of ESA's Huygens probe in the atmosphere of Titan.
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