Resistivity and $1/f$ noise in nonmetallic phase-separated manganites

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A simple model is proposed to calculate the resistivity, magnetoresistance, and noise spectrum in nonmetallic phase-separated manganites containing small metallic droplets (magnetic polarons). The system is taken to be far from the percolation transition into a metallic state. It is assumed that the charge transfer occurs due to electron tunneling from one droplet to another through the insulating medium. As a result of this tunneling, the droplets acquire or lose extra electrons forming metastable two-electron and empty states. In the framework of this model, explicit expressions for dc conductivity and noise power of the system are derived. It is shown that the noise spectrum has $1/f$ form in the low-frequency range.

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I. INTRODUCTION

Recent experimental^{1–6} and theoretical^{7–12} papers provide strong evidence for the existence of the phase separated state in the perovskite manganites with the colossal magnetoresistance. Both experiment and theory demonstrate that under certain conditions the material in the phase-separated state consists of small ferromagnetic metallic droplets, or magnetic polarons, embedded into the insulating antiferromagnetic (AFM) matrix. The droplet in the ground state contains one charge carrier (electron or hole) confined in a potential well of ferromagnetically ordered local spins. In Refs. 11 and 12 the model allowing the estimation of the droplet radius and droplet concentration was proposed. According to this model, the radius of the droplet a is defined from the minimization of the energy $E \sim t(\pi d/a)^2 + JS^2(4\pi/3)(a/d)^3$, where the first term is related to the kinetic energy of electron delocalization in the bubble with the radius a , and the second term corresponds to the loss in the energy of the Heisenberg AFM exchange due to ferromagnetic ordering of local spins S inside the bubble. The minimization of the energy with respect to the droplet radius yields the following estimate for a in three-dimensional (3D) case: $(a/d) \sim (t/JS^2)^{1/5}$, where J is the AFM Heisenberg exchange, t is the bandwidth, and d is the intersite distance. The number of charge carriers is proportional to the electron (hole) doping x . The volume concentration of metallic droplets increases with x and with the decrease of the temperature since the droplet radius a decreases with temperature growth. As a result, at some critical concentration of holes $x_c \sim (d/a)^3$ the droplets start to overlap and the percolation metal-insulator transition occurs in the system.⁸ In Ref. 6 it was observed that heating and cooling of the perovskite manganites in the phase-separated state is accompanied by strong hysteresis in the magnetization and the resistance. In addition, the giant $1/f$ noise was measured in these experiments. The noise power is very high even far from the percolation threshold and drastically increases in its vicinity. The noise

spectrum is close to $1/f$ form in the 1–1000 Hz frequency range.

In this paper, we calculate the conductivity, magnetoresistance, and noise spectral power of the system in the phase-separated state. The calculations are based on the results of Refs. 11 and 12 and on a simple model for the tunneling conductivity of the material accounting for electron jumps from one polaron to another. A concentration range not too close to the percolation transition is considered. We use the terms (*magnetic*) *polarons* and *droplets* interchangeably throughout the paper.

II. CONDUCTIVITY

Let us consider an insulating antiferromagnetic sample of volume V_s in electric field $\mathbf{E}$. The total number of magnetic polarons in the volume is N , and thus their spatial density is $n = N/V_s$. As mentioned before, the number of polarons is assumed to be equal to the number of charge carriers introduced by doping. Neglecting the conductivity of the insulating phase, we assume that charge carriers are only located within the droplets. The charge transfer can thus occur either due to the motion of the droplets or due to electron tunneling. The former mechanism is less effective: Indeed, the motion of a droplet is accompanied by a considerable rearrangement of the local magnetic structure, which results in the large effective mass of magnetic polarons. In addition, the droplets are expected to be easily pinned by crystal lattice defects. Thus, it is realistic to assume that the charge transport is essentially due to electron transitions between the droplets.

A magnetic polaron in the ground state contains one electron. As a result of the tunneling process, droplets with more than one electron are created, and some droplets become empty (the lifetime of such excitations is discussed in the end of this section). If the energy of an empty droplet $E(0)$ is taken to be zero, then the energy of a droplet with one electron can be estimated as $E(1) \sim t(d/a)^2$. This is essentially the kinetic energy of an electron localized in the sphere

of radius a . In the same way, the energy of a two-electron magnetic polaron $E(2) \sim 2E(1) + U$, with U the interaction energy of the two electrons. In all these estimates, we have disregarded the surface energy, which is expected to be small.¹¹ Thus, $E(2) + E(0) > 2E(1)$, and the creation of two-electron droplets is associated with the energy barrier of the order of $A \equiv E(2) - 2E(1) \sim U$. It is clear that the interaction energy U of two electrons in one droplet is determined mainly by the Coulomb repulsion of these electrons; hence $A \sim e^2/\epsilon a$, where ϵ is the static dielectric constant, which in real manganites can be rather large ($\epsilon \sim 20$). We assume below that the mean distance between the droplets is $n^{-1/3} \gg a$ (the droplets do not overlap). Then, A is larger than the average Coulomb energy $e^2 n^{1/3}/\epsilon$. Since the characteristic value of the droplet radius a is of the order of $10 \text{ \AA}$,^{11,12} we have $A/k_B \sim 1000 \text{ K}$ and $A > k_B T$ in the case under study. In the following, we assume that the temperature is low, $A \gg k_B T$, and we do not consider a possibility of the formation of the droplets with three or more electrons. Even in the case when these excitations are stable, it can be shown that far from the percolation threshold the strong Coulomb interaction suppresses their contribution to the conductivity [giving rise only to the next-order terms with respect to $\exp(-A/k_B T)$].

Let us denote the numbers of single-electron, two-electron, and empty droplets as N_1 , N_2 , and N_3 , respectively. According to our model, $N_2 = N_3$, $N_1 + 2N_2 = N$, and N is constant. Before turning to conductivity, we evaluate the thermal averages of N_1 and N_2 . To this end, we note that the number P_N^m of possible states with m two-electron droplets and m empty droplets equals $C_N^m C_{N-m}^m$, with C_N^m being the binomial coefficients. Since the created pairs of droplets are independent, we write the partition function in the form

$$Z = \sum_{m=0}^{N/2} P_N^m \exp(-m\beta), \quad \beta = A/k_B T. \quad (1)$$

Though the sum can be evaluated exactly and expressed in terms of the Legendre polynomials for arbitrary N , it is more convenient to use the Stirling formula for the factorials and the condition that the sample is macroscopic, $N \gg 1$. Approximating the sum by an integral,

$$Z = \int_0^{N/2} dm \exp \left[-m\beta - N \ln \left(1 - \frac{2m}{N} \right) + 2m \ln \left(\frac{N}{m} - 2 \right) \right],$$

calculating it in the saddle-point approximation, and subsequently evaluating in the same way the statistical average of N_2 ,

$$\bar{N}_2 = Z^{-1} \sum_{m=0}^{N/2} m P_N^m \exp(-m\beta) = -\frac{\partial}{\partial \beta} \ln Z, \quad (2)$$

we easily obtain

$$\bar{N}_2 = N \exp(-A/2k_B T),$$

$$\bar{N}_1 = N - 2\bar{N}_2 = N[1 - 2 \exp(-A/2k_B T)]. \quad (3)$$

FIG. 1. Elementary tunneling processes.

Now we calculate the conductivity. Within the framework of the proposed model the electron tunneling occurs via one of the four following processes illustrated in Fig. 1.

(i) In the initial state we have two droplets in the ground state, and after tunneling in the final state we have an empty droplet and a droplet with two electrons.

(ii) An empty droplet and a two-electron droplet in the initial state transform into two droplets in the ground state (two droplets with one electron).

(iii) A two-electron droplet and a single-electron droplet exchange their positions by transferring an electron from one droplet to the other.

(iv) An empty droplet and a single-electron droplet exchange their positions by transferring an electron from one droplet to the other.

In the linear regime, all these processes contribute to the current density j independently, $j = j_1 + j_2 + j_3 + j_4$. The contributions of the first two processes read

$$j_{1,2} = en_{1,2} \left\langle \sum_i v_{1,2}^i \right\rangle, \quad (4)$$

where $n_{1,2} = N_{1,2}/V_s$ are the densities of the single- and two-electron droplets, and $\langle \dots \rangle$ stands for statistical and time averages. The appearance of the factors $n_{1,2}$ reflects the fact that the electron tunnels *from* a single-electron droplet [process (i)] or two-electron (ii) droplet. The summation in Eq. (4) is performed over all magnetic polarons the electron can tunnel *to*—one-electron droplets for process (i) and empty droplets for process (ii). Finally, the components of the average electron velocity $\langle v_{1,2}^i \rangle$ along the direction of the electric field are obviously found as¹³

$$\left\langle \sum_i v_{1,2}^i \right\rangle = \left\langle \sum_i \frac{r^i \cos \theta^i}{\tau_{1,2}(r^i, \theta^i)} \right\rangle, \quad (5)$$

where r^i and θ^i are the electron tunneling length (the distance between the droplets) and the angle between the electric field and the direction of motion, respectively, and $\tau_{1,2}(r^i, \theta^i)$ are characteristic times associated with the tunneling processes. The relation between $\tau_1(r, \theta)$ and $\tau_2(r, \theta)$ can be found from the following considerations. Near equilibrium, the number of two-electron droplets, excited per unit

time, equals the number of the decaying two-electron droplets. We thus have the detailed balance relation,

$$\frac{\bar{N}_1^2}{\tau_1(r, \theta)} = \frac{\bar{N}_2^2}{\tau_2(r, \theta)}, \quad (6)$$

where we have taken into account that the probability of the formation of a two-electron droplet is proportional to the total number N_1 of the single-electron states multiplied by the number of available hopping destinations, which also equals N_1 . Similarly, the probability of decay of a two-electron droplet is proportional to $N_2 N_3 = N_2^2$. Equation (6) implies $\tau_2(r, \theta) = \tau_1(r, \theta) \exp(-A/k_B T)$. We write then the conventional expression for the tunneling times¹³ in the following form,

$$\tau_{1,2}(r, \theta) = \omega_0^{-1} \exp\left(\frac{r}{l} \pm \frac{A}{2k_B T} - \frac{eEr \cos \theta}{k_B T}\right), \quad (7)$$

where l and ω_0 are the characteristic tunneling length and magnon frequency, and we have taken into account the contribution of the external electric field to the tunneling probability.

To perform the averaging, we assume that the centers of the magnetic polarons are randomly positioned in space and the average distance $n^{-1/3}$ between them is much larger than the droplet radius a . Both assumptions seem to be perfectly justified far below the percolation threshold. Then the averaged sum in Eq. (4) is essentially the space average of v^i , multiplied by the number of droplets available for hopping [N_1 for the process (i) and N_2 for process (ii)]. Expanding in $eEl/k_B T \ll 1$, we obtain

$$\left\langle \sum_i v_{1,2}^i \right\rangle = \frac{eE\omega_0}{k_B T} N_{1,2} e^{\mp A/2k_B T} \langle r^2 (\cos^2 \theta) e^{-r/l} \rangle_V, \quad (8)$$

$$\langle \dots \rangle_V = V_S^{-1} \int \dots d^3 r.$$

In Eq. (8), the electric field is outside the averaging. Rigorously speaking, this means that the characteristic hopping length l is larger than the interdroplet distance $n^{-1/3}$ and our approach is valid only when the droplet concentration is not too small. Substituting Eq. (8) into Eq. (4) and performing the integration, we find

$$j_{1,2} = \frac{32\pi e^2 E \omega_0 l^5 n_{1,2}^2}{k_B T} \exp(\mp A/2k_B T). \quad (9)$$

In processes (iii) and (iv) the free energy of the system is not changed after the tunneling, and we write the characteristic times as

$$\tau_{3,4}(r, \theta) = \omega_0^{-1} \exp(r/l - eEr \cos \theta / k_B T). \quad (10)$$

The contribution of these two processes to the current is calculated similarly to that of (i) and (ii). For process (iii) the number of magnetic polarons from which the electron may tunnel is N_2 , whereas the number of accepting droplets is

N_1 . In the same way, for process (iv) these numbers are N_1 and $N_3 = N_2$, respectively. Consequently, the factors $n_{1,2}^2$ in Eq. (9) are replaced by $n_1 n_2$,

$$j_{3,4} = \frac{32\pi e^2 E \omega_0 l^5 n_1 n_2}{k_B T}. \quad (11)$$

From Eqs. (9) and (11) we now obtain the dc conductivity $\sigma = j/E$,

$$\sigma = \frac{32\pi e^2 \omega_0 l^5}{k_B T} [2n_1 n_2 + n_1^2 e^{-A/2k_B T} + n_2^2 e^{A/2k_B T}]. \quad (12)$$

In this section we are only interested in the average conductivity; fluctuations lead to the appearance of noise and are considered in Sec. IV. Using Eq. (3), we find that all four processes illustrated in Fig. 1 give identical contributions to the conductivity; for $A \gg k_B T$ the average conductivity (for which we retain the notation σ) reads

$$\sigma = \frac{128\pi e^2 n^2 \omega_0 l^5}{k_B T} \exp(-A/2k_B T). \quad (13)$$

We see that the conductivity increases with temperature as $\sigma(T) \propto T^{-1} \exp(-A/2k_B T)$, which is typical for tunneling systems (see, e.g., Ref. 13).

At this point, let us discuss the applicability range of our model. The essence of our picture is the existence of different types of droplets. Only single-electron droplets are stable. Obviously, an empty droplet decays during the time of the order of $1/\omega_0$. On the other hand, following the above discussion, the empty droplet should acquire an electron from neighboring one-electron or two-electron droplets during the characteristic time τ_0 , which can be easily calculated based on the following considerations. The probability P per unit time for an empty droplet to acquire one electron can be written as

$$P = 4\pi\omega_0 \int_0^\infty e^{-r/l} [n_1 + n_2 \exp(A/k_B T)] r^2 dr, \quad (14)$$

where the terms with n_1 and n_2 correspond to the electron transfer from single- and two-electron droplets, respectively. Performing integration in Eq. (14) and using Eq. (3), we find

$$\tau_0 = \frac{\exp(-A/2k_B T)}{8\pi\omega_0 l^3 n}. \quad (15)$$

Just the same estimate can be obtained for the characteristic time of electrons leaving two-electron droplets. For our picture with empty and two-electron droplets to be valid, the following condition must be met: $\tau_0 \ll \omega_0^{-1}$. Thus, our approach is valid at sufficiently low temperatures, $k_B T \ll A$, and for a not too small droplet density n .

The applicability of our approach also implies that $l > a, n^{-1/3}$. It is of interest to consider also the case of $l \sim a$ and/or low droplet concentrations. In this situation, in usual hopping systems, the conductivity strongly depends on the geometry of current paths.¹⁴ This causes an exponential dependence of conductivity on the carrier concentration. How-

ever, our system turns out to be more complicated than those commonly invoked for hopping conductivity. It involves different types of hopping centers giving rise to an unusual geometry of current paths. Therefore, the conventional approaches used for hopping cannot be applied straightforwardly to the analysis of our model at low droplet concentration or at $l \sim a$. Despite these complications, we believe that the expression for the conductivity in the case $l \lesssim n^{-1/3}$ includes the percolation-related factor $\exp(-\beta/n^{1/3}a)$, with β of the order one,¹⁴ though currently we have no rigorous proof of this statement. The results below for magnetoresistance and noise are insensitive to this factor, and therefore we expect them to be valid in a general case.

III. MAGNETORESISTANCE

As we already discussed, below the percolation threshold when the volume fraction of droplets $n < n_c$, a typical value of A/k_B is mainly determined by Coulomb interactions between two electrons inside the droplet $A \sim e^2/\epsilon a$ and has a typical value of 1000 K. Now we can use this estimate to analyze the magnetoresistance in nonmetallic phase-separated manganites. To do that, we use the expression for the radius of the magnetic polaron, obtained in the Introduction, $a \propto d(t/JS^2)^{1/5}$. Recall once more that here $J \sim 100$ K is an AFM Heisenberg exchange between the local spins $S = 3/2$. It is natural to conclude that in the magnetic field H the Heisenberg exchange integral J decreases according to the formula $J(H)S^2 = J(0)S^2 - g\mu_B HS$, where μ_B and g are the Bohr magneton and the gyromagnetic ratio, respectively. Consequently, the value of A is decreasing linearly in the experimentally accessible range of magnetic fields, and for the excitation energy we obtain

$$A(H) = A(0)[1 - bH], \quad b = \frac{1}{5} \frac{g\mu_B}{J(0)S}. \quad (16)$$

It follows now from Eq. (13) that the magnetoresistance is negative and for temperatures $T < A/k_B$ reads

$$\begin{aligned} |MR| &\equiv \frac{\rho(0) - \rho(H)}{\rho(H)} \\ &= \exp\left[\frac{A(0) - A(H)}{2k_B T}\right] - 1 = \exp\left(\frac{bHA}{2k_B T}\right) - 1. \end{aligned} \quad (17)$$

For low magnetic fields and not very small temperatures the absolute value of the magnetoresistance (MR) is small, $|MR| = bHA/2k_B T \ll 1$. In higher fields (but still $bH \ll 1$) the absolute value of magnetoresistance eventually exceeds 1 and behaves in exponential fashion, $|MR| = \exp(bHA/2k_B T)$. Note that for temperatures $T \lesssim A/k_B$ and for typical gyromagnetic ratios $g \sim 10$ the magnetoresistance in our region of doping becomes larger than 1 by absolute value only in relatively high magnetic fields $H \sim 10$ T.

Though our simple model, which only takes into account magnetic field dependence of the radius of a droplet, cannot explain various types of behavior that the magnetoresistance shows in manganites, it still captures some common features of MR at temperatures above the resistivity peak: Its de-

crease with temperature, power-law magnetic field dependence in low fields, and exponential behavior in higher fields (for review, see Ref. 15). To describe the actual experimental situation in more detail, we need to take into account other important physical mechanisms, in particular, spin-dependent tunneling.

IV. 1/f NOISE POWER

Recently, Podzorov *et al.*⁶ reported the observation of giant 1/f noise in perovskite manganites in the phase-separated regime. Generally, systems with distributed hopping lengths are standard objects that exhibit 1/f noise (for review, see Refs. 16 and 17). The purpose of this section is to study low-frequency noise within the framework of the model used to calculate the conductivity in Sec. II and show that it has, indeed, 1/f form.

Starting from the Ohm's law $U = IL/\sigma S$ (where L and S are the sample length and the cross-section, respectively) and assuming that the measuring circuit is stabilized ($I = \text{const}$), we can present the voltage noise at the frequency ω , $\langle \delta U^2 \rangle_\omega$, in the following way:

$$\langle \delta U^2 \rangle_\omega = U_{dc}^2 \frac{\langle \delta \sigma^2 \rangle_\omega}{\sigma^2}, \quad (18)$$

where U_{dc} is the time-averaged voltage and $\langle \delta \sigma^2 \rangle_\omega$ is the noise spectrum of the fluctuations of the conductivity.

If we disregard possible fluctuations of temperature in the system, the only source of the fluctuations in our model is those of the occupation numbers n_1 and n_2 . Using the conservation law $n_1 + 2n_2 = n$, we find from Eq. (12)

$$\delta \sigma = \sigma \frac{\delta n_2}{n_2} [1 - 2 \exp(-A/2k_B T)]. \quad (19)$$

We thus need to find the fluctuation spectrum $\langle \delta n_2^2 \rangle_\omega$. Following the general prescription,¹⁸ we recollect that the two-electron droplets decay via process (ii), and the relaxation equation has the form

$$\delta \dot{n}_2 = -\frac{\delta n_2}{\tau(r)}, \quad \tau(r) = \omega_0^{-1} \exp(r/l - A/2k_B T), \quad (20)$$

where we have neglected the effect of the electric field. The fluctuation spectrum then reads¹⁸

$$\langle \delta n_2^2 \rangle_\omega = \langle \delta n_2^2 \rangle_T \left\langle \sum_i \frac{2\tau(r^i)}{1 + \omega^2 \tau^2(r^i)} \right\rangle, \quad (21)$$

where $\langle \delta n_2^2 \rangle_T$ is the thermal average of the variation of n_2 , and the summation is performed over the ‘‘empty droplet–two-electron droplet’’ pairs, with r_i being the distance between the sites in a pair. Since all such pairs contribute to the noise, the average in Eq. (21) is essentially a spatial integral, with the main contribution coming from short distances,

$$\langle \delta n_2^2 \rangle_\omega = 8\pi \bar{n}_2 \langle \delta n_2^2 \rangle_T \int_0^\infty \frac{\tau(r)}{1 + \omega^2 \tau^2(r)} r^2 dr. \quad (22)$$

Note that Eq. (22) is valid for an arbitrary relation between l and a , not necessarily for $a \ll l$.

We are interested below in the frequency range

$$\tilde{\omega}_0 \exp(-L_s/l) \ll \omega \ll \tilde{\omega}_0, \quad \tilde{\omega}_0 \equiv \omega_0 \exp(A/2k_B T), \quad (23)$$

where L_s is the smallest of the sample sizes. In this case, with the logarithmic accuracy we obtain for $A \gg k_B T$,

$$\langle \delta U^2 \rangle_\omega = U_{dc}^2 \frac{\langle \delta n_2^2 \rangle_T}{\bar{n}_2} \frac{4\pi^2 l^3}{\omega} \ln^2 \left(\frac{\tilde{\omega}_0}{\omega} \right). \quad (24)$$

Thus, in the wide range of sufficiently low frequencies, Eq. (23), the noise power spectrum for our system has almost a $1/f$ form.

The variation $\langle \delta n_2^2 \rangle_T = V_s^{-2} (\bar{N}_2^2 - \bar{N}_2)$ is easily found in the same way as Eq. (3),

$$\langle \delta n_2^2 \rangle_T = \frac{\bar{n}_2}{2V_s}. \quad (25)$$

Combining this with Eq. (24), we write the final expression for the spectral density of noise for $A \gg k_B T$ in the form

$$\langle \delta U^2 \rangle_\omega = U_{dc}^2 \frac{2\pi^2 l^3}{V_s \omega} \ln^2 \left(\frac{\omega_0 e^{A/2k_B T}}{\omega} \right). \quad (26)$$

V. DISCUSSION

For the further discussion, it is convenient to rewrite Eq. (26) in the form

$$\alpha = \frac{\langle \delta U^2 \rangle_\omega V_s \omega}{U_{dc}^2} = 2\pi^2 l^3 \ln^2 \left(\frac{\tilde{\omega}_0}{\omega} \right). \quad (27)$$

It is remarkable that the noise spectrum in our model has a $1/f$ form up to very low frequencies. This is due to fluctuations in occupation numbers of droplets, associated with the creation and annihilation of extra electron-hole pairs. This mechanism of $1/f$ noise is specific for our model and is not present in standard hopping conduction.

Let us estimate the numerical value of the parameter α , which is the standard measure of the strength of $1/f$ noise. This parameter is proportional to the third power of l . Simple estimates (analogous to that presented in the Introduction for a) reveal that, in general, l is of the order or higher than a . Assuming again that the excitation energy is of the order of the Coulomb energy $A \sim e^2/a\epsilon$, taking ω_0 to be of the order of the Fermi energy inside droplets (which means $\hbar\omega_0 \sim 300$ K for $n < n_c$), and estimating the tunneling length l as being $l \gtrsim 2a \sim 20$ Å, we arrive at the conclusion that the parameter α is of the order $\alpha \approx 10^{-17} - 10^{-16}$ cm³ for $T < A/k_B$ and $\omega \sim 1$ Hz–1 MHz. This value of α is by several orders of magnitude higher than that in the usual semiconducting materials (see Ref. 16). Such a large magnitude of the noise can be attributed to the relatively low height of the potential barrier A and to the relatively large tunneling length l . Formally, it is also related to the large value of the logarithm squared in Eq. (27).

According to Eqs. (26) and (27), the noise power and the noise parameter α are independent of the volume fraction occupied by the droplets. This result is valid in the intermediate range of n , when the droplet density is not too high and not too low. First, we assumed that the droplets are isolated point objects and that the tunneling between the two droplets is not affected by a third polaron. This is only valid provided the droplet density is far from the percolation threshold, $n \ll n_c$. On the other hand, the droplet density must not be too low since the conditions $N, N_1, N_2 \gg 1$ are assumed to be met. Moreover, we neglected the possibilities of the disappearance of a droplet without an electron, the formation of a new droplet due to the electron tunneling, and the decay of two-electron droplets. Thus, the characteristic times of these processes should be longer than the characteristic tunneling time, and the average tunneling distance cannot be too high [see Eq. (15) and the discussion below it].

The above speculations imply that the following set of inequalities should be met, $a \ll n^{-1/3} \ll l$, for formula (13) for the conductivity to be valid. In general, the tunneling length should not be much larger than the droplet radius since just the same physical parameters determine these two characteristic distances. So, these inequalities could not be valid for real physical systems, and it is of interest to consider the situation where $a, l \ll n^{-1/3}$, which is beyond the scope of our model. However, some definite conclusions concerning the magnetoresistance and the noise power can be made at present.

First, the factor $\exp(A/2k_B T)$ in the temperature dependence of the conductivity is related to the number of carriers and appears due to the strong Coulomb repulsion of electrons in the droplet. It seems rather obvious that such a factor appears in the formula for the conductivity below the percolation threshold for an arbitrary relation between a and l . On the other hand, in contrast to common hopping systems, a strong $1/f$ noise in our model results from fluctuations of state occupation numbers. Actually, our result for parameter α , Eq. (27), only relies on the fact that $\delta\sigma/\sigma \sim \delta n_2/\bar{n}_2$. As we have mentioned previously, Eq. (22), which determines the spectral density of fluctuations of n_2 , applies for an arbitrary relationship between a and l . It follows then that the value of the parameter α for $1/f$ noise remains approximately the same under the (experimentally relevant) conditions $a \sim l$.

Another important point is that we disregard the direct Coulomb interaction between the droplets in comparison with the energy A . This is justified if the gas of the droplets is diluted, $n^{-1/3} \gg a$. In this respect, we recollect that in standard hopping conduction systems (doped semiconductors) the main mechanism of low-frequency noise is an exchange of electrons between the infinite cluster and nearby finite clusters. In the absence of interactions it leads to the noise power proportional to $\omega^{-\alpha}$, with the exponent α being considerably below 1.²⁰ To explain $1/f$ noise in these systems, models involving Coulomb interactions have been proposed.^{21,22} These sources of low-frequency noise are thus beyond our discussion. We also did not consider sources of noise different from resistance fluctuations. At least two

other types of noise are inevitably present in the system: Nyquist-Johnson (thermal) noise, which is a consequence of the fluctuation-dissipation theorem, and shot noise due to the discrete nature of electron charge (see Refs. 17 and 19 for review). Both these noises are frequency independent (white) at low frequencies. The magnitudes of Nyquist-Johnson, shot, and $1/f$ noises are governed by absolutely different parameters, and we do not attempt to compare them here, noting only that at low frequencies $1/f$ noise must dominate.

In our model, we assumed that the number of droplets N is fixed and strictly equal to the number of extra electrons. In actual systems, N can also fluctuate, and this can be an additional source of noise and of $1/f$ noise, in particular. However, this contribution depends critically on the heights of corresponding energy barriers and can vary for different systems.

As we have already mentioned, the main motivation of our work was the experimental study,⁶ which observed high $1/f$ noise power at high temperatures far from the metal-insulator transition. In the same experiment, the noise dropped to much lower levels at low temperatures in the metallic phase. This behavior of the noise power is consistent with the present model since in the metallic phase the electron tunneling contribution to the total conductivity is negligible. In the vicinity of the percolation transition the noise power increases drastically.⁶ In this paper we do not attempt to describe the system of magnetic polarons close to the percolation threshold. However, we argue that the amplitude of $1/f$ noise is already large in the phase-separated regime even far from the percolation threshold.

VI. CONCLUSIONS

We emphasize that even in our oversimplified model we get a reasonable behavior of resistivity and magnetoresistance for underdoped manganites. Moreover, we have shown that in the framework of our model $1/f$ noise appears in a natural way. The phase separation ensures a large magnitude of the noise power as compared with homogeneous materials.

Of course, a more sophisticated theory should include both the ferromagnetic structure of the droplet and the antiferromagnetic structure of the insulating matrix. This can lead us to the physics resembling that observed in the process of spin-assisted tunneling, which attracts a considerable interest nowadays (see, e.g., Ref. 23). The work in this direction is in progress.

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