Space Charge Effects
Near Inhomogeneous Cathodes

David Nijkerk
Space charge effects
near inhomogeneous cathodes

TR 3887

P ro e fs c h r i ft

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof. dr. ir. J. T. Fokkema,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen op donderdag 13 juni 2002 om 13:30 uur

door

Michiel David NIJKERK

doctorandus in de natuurkunde
egenomen te Amsterdam.
The work described in this thesis has been made possible with financial support of LG.Philips Displays, Eindhoven

The cover shows a Monte Carlo simulation of electrons near a pointed cathode. The erratic meandering trajectories are the result of Coulomb interactions.

Back: based on the work of R.D.E. Oxenaar

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## Contents

1 Introduction ................................. 1
   1.1 The cathode-ray tube .................. 1
   1.2 Simulation and design of electron optical systems .... 1
   1.3 Simulations of CRTs ................... 4
   1.4 Outline of this thesis ................. 5

2 Approximations in present thermionic emission models .... 7
   2.1 Theory of electron emission .......... 7
      2.1.1 Emission function ................ 7
      2.1.2 Virtual emitter .................. 8
      2.1.3 Thermionic emission from metals ... 9
   2.2 From emission to space charge ......... 11
      2.2.1 Self-induced influences on the electron beam ... 11
      2.2.2 Space charge lenses ............... 13
      2.2.3 Numerical representation of space charge ... 14
      2.2.4 Self-consistent solution .......... 16
      2.2.5 Particle-particle interactions ..... 16
   2.3 Space charge limited emission ......... 16
      2.3.1 The Child model .................. 17
      2.3.2 Variation on the Child model ....... 18
      2.3.3 The Langmuir model ............... 19
      2.3.4 Exact space charge models in nonplanar diode geometries ... 22
   2.4 Space charge models for practical geometries .... 23
      2.4.1 Virtual emitter based on a diode array .... 23
      2.4.2 Emission properties of the diode array ... 23
   2.5 Particle optical simulations with space charge .... 26
      2.5.1 Simulation of CRT's with SCELOP ...... 26
      2.5.2 Other software based on space charge models ... 28
      2.5.3 Concluding remarks ............... 29
   2.6 Summary and outlook ................. 29

3 Influence of emission properties on spot size .......... 31
   3.1 Optics of a CRT gun .................. 31
      3.1.1 Configuration of the CRT .......... 31
      3.1.2 Contributions to the spot size ..... 32
   3.2 Variations in the virtual emission function .... 35
   3.3 Classifying the parameter space ....... 36
      3.3.1 Overview of spot size measures ... 36
      3.3.2 Quantification of the start conditions .... 38
      3.3.3 Quantification of the spot size change ... 39
      3.3.4 Calculation of spot distributions ... 39
   3.4 Simulation results ................... 41
      3.4.1 RS gun .......................... 41
      3.4.2 CMT gun .......................... 42
      3.4.3 TVT gun .......................... 42
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4.4 Spot shape change</td>
<td>42</td>
</tr>
<tr>
<td>3.4.5 Consistent spot shape and potential distribution</td>
<td>46</td>
</tr>
<tr>
<td>3.4.6 Validation of the computation method</td>
<td>47</td>
</tr>
<tr>
<td>3.5 Conclusions</td>
<td>48</td>
</tr>
<tr>
<td>4 Emission function of a virtual cathode</td>
<td>51</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>51</td>
</tr>
<tr>
<td>4.2 Self-consistent solution in the presence of a virtual cathode</td>
<td>52</td>
</tr>
<tr>
<td>4.3 Virtual emission function in a triode</td>
<td>53</td>
</tr>
<tr>
<td>4.4 Influence of the velocity correction</td>
<td>56</td>
</tr>
<tr>
<td>4.5 Conclusions</td>
<td>56</td>
</tr>
<tr>
<td>5 Monte Carlo simulation of the emitter region</td>
<td>59</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>59</td>
</tr>
<tr>
<td>5.2 Model of the emitter region</td>
<td>60</td>
</tr>
<tr>
<td>5.2.1 CRT emission parameters</td>
<td>60</td>
</tr>
<tr>
<td>5.2.2 Boxed electrons in a charge continuum</td>
<td>60</td>
</tr>
<tr>
<td>5.2.3 Choosing the box size</td>
<td>61</td>
</tr>
<tr>
<td>5.2.4 Mean-field approximation</td>
<td>62</td>
</tr>
<tr>
<td>5.3 Emission from a metallic surface</td>
<td>62</td>
</tr>
<tr>
<td>5.3.1 Image potential</td>
<td>62</td>
</tr>
<tr>
<td>5.3.2 Work function</td>
<td>63</td>
</tr>
<tr>
<td>5.3.3 Saturation current</td>
<td>64</td>
</tr>
<tr>
<td>5.4 Mathematical model</td>
<td>66</td>
</tr>
<tr>
<td>5.4.1 The electron box</td>
<td>67</td>
</tr>
<tr>
<td>5.4.2 Representation of the cathode</td>
<td>67</td>
</tr>
<tr>
<td>5.4.3 The cathode field</td>
<td>67</td>
</tr>
<tr>
<td>5.4.4 Cyclic boundary conditions</td>
<td>68</td>
</tr>
<tr>
<td>5.5 Numerical implementation</td>
<td>69</td>
</tr>
<tr>
<td>5.5.1 Emission algorithm</td>
<td>69</td>
</tr>
<tr>
<td>5.5.2 Mean-field calculator</td>
<td>70</td>
</tr>
<tr>
<td>5.5.3 Cathode point charges</td>
<td>71</td>
</tr>
<tr>
<td>5.5.4 Integrator</td>
<td>72</td>
</tr>
<tr>
<td>5.5.5 Particle manager</td>
<td>73</td>
</tr>
<tr>
<td>5.6 Field approximations</td>
<td>73</td>
</tr>
<tr>
<td>5.6.1 Induced cathode charge</td>
<td>74</td>
</tr>
<tr>
<td>5.6.2 Smooth space charge</td>
<td>74</td>
</tr>
<tr>
<td>5.6.3 Mean-field mirror charge</td>
<td>74</td>
</tr>
<tr>
<td>5.6.4 Particle bunches</td>
<td>75</td>
</tr>
<tr>
<td>5.7 Program organization</td>
<td>75</td>
</tr>
<tr>
<td>5.8 Consistency checks</td>
<td>75</td>
</tr>
<tr>
<td>5.9 Conclusions</td>
<td>82</td>
</tr>
<tr>
<td>6 Influence of statistical Coulomb interactions</td>
<td>85</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>85</td>
</tr>
<tr>
<td>6.2 Coulomb interactions in a planar diode</td>
<td>86</td>
</tr>
<tr>
<td>6.3 Coulomb interactions in a CRT beam</td>
<td>88</td>
</tr>
<tr>
<td>6.3.1 Interactions in the beam forming region</td>
<td>90</td>
</tr>
<tr>
<td>6.3.2 Interactions in the CRT crossover</td>
<td>91</td>
</tr>
<tr>
<td>6.4 Relaxation of beam temperature</td>
<td>94</td>
</tr>
<tr>
<td>6.5 Conclusions</td>
<td>97</td>
</tr>
<tr>
<td>7 Influence of surface roughness</td>
<td>99</td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td>99</td>
</tr>
<tr>
<td>7.2 Laplace field of bumpy cathode</td>
<td>100</td>
</tr>
<tr>
<td>7.3 Emission properties of bumpy cathode</td>
<td>100</td>
</tr>
<tr>
<td>7.4 Conclusions</td>
<td>106</td>
</tr>
</tbody>
</table>
8 Influence of work function inhomogeneity
  8.1 Introduction .................................................. 109
  8.2 Laplace field of patchy cathode ............................. 111
  8.3 Emission properties of patchy cathode ....................... 112
     8.3.1 Current density profile .................................... 112
     8.3.2 Current-voltage characteristic ........................... 117
  8.4 Conclusions .................................................. 120

9 Shot noise reduction in space charge limited emission ....... 123
  9.1 Introduction .................................................. 123
  9.2 Simulation model .............................................. 124
  9.3 Shot noise reduction .......................................... 125
  9.4 Spatial range of shot noise suppression ....................... 127
  9.5 Discussion ................................................... 128
  9.6 Conclusions .................................................. 129

References ....................................................... 131

Summary .......................................................... 137

Samenvatting ..................................................... 141
Chapter 1

Introduction

When Johann Hittorf discovered cathode rays in 1869 [1] he could not have foreseen that within a 100 years, a ‘cathode-ray tube’ could be found in every household that could afford one. With this discovery the first step towards the field of physics concerned with television tubes, the most familiar of cathode-ray tube applications, was made.

With the realization that the trajectories of electrons, the constituents of cathode rays, can be influenced with electric and magnetic fields, the foundation of electron optics was established, as the action of fields on electrons could be described in similar terms as the action of glass lenses on light rays.

The vast variety of modern day electron optical appliances include instruments for material analysis like electron microscopes, energy analyzers and mass spectrometers, instruments for welding, lithography machines, particle accelerators, microwave tubes, television cameras, image intensifiers, computer monitor tubes and television tubes. Today, nearly 750 million television tubes exist in the world. Over two hundred million computer monitor and television tubes are produced each year.

1.1 The cathode-ray tube

The cathode-ray tube (CRT) is generally considered to have been invented by William Crookes in 1879 [2]. It consists of an evacuated glass bottle, with an electron gun emitting cathode rays at the neck of the bottle, and a screen, coated with phosphor on the inside, at the base. Any point on the screen that is struck by the cathode rays becomes luminous. Cathode rays can be deflected by means of deflection coils, and a picture is formed by modulating the luminous intensity of the electron beam as the beam sweeps across the entire screen at a rate of about 50 times per second (see Fig. 1.1).

At the time of writing, the CRT is after more than 100 years, still the dominant display device for video applications. However, the CRT has a great disadvantage in its bulky size. Recent developments of thin and flat alternatives like liquid-crystal displays and plasma tubes are rapidly filling in a void in the market in that respect. Nevertheless, the CRT is favorable to the current alternatives with respect to brightness, viewing angle, picture quality and cost.

The electron gun is the heart of a CRT. In the world of CRTs the ‘electron gun’ includes both extraction and focussing optics, in other words, it indicates the entire optical system that determines the on-axis spot properties. For the development of new electron guns computer simulations have become an indispensable design tool. Although computer simulations for most electron optical applications have been well-established over the last years, the accuracy of CRT computations still leaves something to be desired, hampered as they are by specific electron optical properties of the electron guns (operating in so-called ‘space charge limited’ mode) and the accuracy required by the designers.

To maintain their appeal as display devices, CRTs must keep up with the trend towards thinner, flatter and wider displays. This imposes increasingly stringent demands on the electron gun which in turn dictates that CRT computer simulations must be performed faster and with greater accuracy.

1.2 Simulation and design of electron optical systems

The design process of electron guns with computer simulations can be described with the flow chart depicted in Fig. 1.2. Starting from an initial design, which may be an ‘educated guess’, or a prior
Figure 1.1: A cathode-ray tube is a glass tube in which electrons are accelerated and formed into a beam by an electron gun at the neck of the tube and projected onto a phosphor coated screen at the base. The picture in a cathode-ray tube is formed by sweeping the electron beam over the screen using deflection coils and modulating the current.
1.2. Simulation and design of electron optical systems

![Flow chart for the design of an electron gun with the aid of computer simulations. If the computer simulations are known to be accurate, the part enclosed in the dashed box is superfluous.](image)

The design that needs to be improved upon, the desired specifications of the electron gun are imposed by the requirements of the complete optical system.

With the aid of computer simulations, electron optical properties are determined by tracking the electron beam with appropriate conditions from a source plane to a target plane through the intermediate optical components that are formed by the electromagnetic fields. Thus, two quantities are required: the initial conditions with which electrons are emitted from the electron source, and the electromagnetic fields through which the emitted electrons traverse.

The explicit implementation of the box in Fig. 1.2 ‘Calculate properties of configuration’, relating to electron beam tracking and the manner in which the optical components are represented, depends on the desired level of accuracy. Electron beam tracking here is a generic designation for several distinct calculation mechanisms. In some simulations, it may be sufficient to keep track of a few particles that are characteristic of the beam and model the optical components by transfer matrices. This is typically the case in the preliminary design phase of an optical system, when a simple spot size measure is sufficient. On the other hand, if an accurate spot description is required, as in the design process of CRT guns, the beam needs to be described with a large number of particles that must be tracked accurately through the electromagnetic fields by numerical ray-tracing to acquire sufficient details in the spot shape.

Ideally, computer simulations are sufficiently reliable, in the sense that the calculated properties of a design are in good agreement with its intended properties. In this case, the flow part in the dashed box in Fig. 1.2 can be dispensed with and the designer can work according to a ‘first time right’ principle, where a design finalized using nothing but simulations can be built, fitted into a CRT and meet specifications without further modifications.

If, for whichever reason, computer simulations are not 100% accurate, the flow part in the dashed box may become necessary to complement the design process. A design loop consisting solely of computer simulations serves to approach the intended design, up till the point where construction and measurements of a prototype are needed to provide the decisive answer whether the design meets the requirements.

In the next chapter, some comparisons between measurements and simulations of CRTs will be reviewed, showing that the discrepancy between the two still necessitates the presence of the dashed box. In practice, in the course of the design process of an electron gun, the dashed box is in fact entered more
than once. Depending on the innovation and the capability of the simulation tool, as much as eight times is not exceptional for a new design. Even using a state-of-the-art simulation tool, prototype construction is needed at least two times before mass production of an actual design.

Constructing and measuring prototypes calls for running a pilot factory to manufacture prototypes and running an experimental setup to perform measurements. Inaccuracies in the computer simulations can thus be responsible for considerable extra development time and increased cost. Also, they lead to the unfortunate necessity of including the word ‘expected’ in the first choice-diamond: to make use of calculation results the designer has to have some experience in interpreting the results with regard to their validity and possible systematic errors.

Every passage through the dashed box costs hundreds of thousands of euros, and more importantly, it increases the time to market by several months. Obviously, there is significant gain in ridding the design process of the dashed box altogether by ensuring that computer simulations are sufficiently accurate and compare well with measurements.

1.3 Simulations of CRTs

Since simulations of a variety of electron optical systems can be performed successfully, it is important to indicate the specifics of CRT optics that make those simulations in particular difficult to perform accurately. For future reference, the computations involved in electron optical simulations will be reviewed very briefly and the box ‘Calculate properties of configuration’ will be explicated somewhat.

The motion of electrons in optical systems is governed by the Lorentz force, \( \mathbf{F} = -e/m(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \). The electromagnetic fields \( \mathbf{E} \) and \( \mathbf{B} \) can be determined from Maxwell's field equations. For simplicity disregarding magnetic fields, the determination of electron optical properties consists of calculating the electric field and electron trajectories. The shapes of the electrodes and the applied electrode voltages determine the boundary conditions and consequently the electric field, in case no beam is present in the system. The electric field in the absence of an electron beam can be calculated by solving the Laplace equation

\[
\nabla^2 V = 0.
\]

The initial conditions of the emitted electrons depend on the physical properties of the emitter. The electron trajectories are then obtained by numerical integration of the equations of motion,

\[
\dot{r} = e/m \nabla V.
\]

The presence of charge in the system introduces a source term \(-\rho/\varepsilon_0\) in the field equation in which case it is known as Poisson's equation.

Solving the field equation and integrating electron trajectories are straightforward numerical exercises that present no fundamental uncertainties except for their intrinsic numerical inaccuracies. To obtain the potential distribution one can resort to the well-established finite-difference, finite-element or charge-density methods. The difficulty lies in the fact that the source term of the Poisson equation is given by the shape of an electron beam, which thus affects the field, and the field in turn governs the shape of the electron beam. For many electron optical systems, this mutual dependence can be ignored as the source term introduces only a negligible contribution to the potential distribution equation. In the computation of the electric field in a CRT gun however, the influence of the electron beam on the field is large, giving an essential coupling between field equation and equations of motion.

A further source of inaccuracies may be the initial conditions of the electron trajectories. The initial conditions determine source properties like size, opening angle and energy spread, which in turn determine the performance of the optical system. The shape and physical properties of the emitter are often not known accurately. Most computer simulations of electron optical systems rely on approximations of the emitter properties. However, if the desired accuracy of a computation is high, the approximations that are perfectly adequate for some cases may fail in others. The inherent optical difficulties of CRT computations necessitate an examination of the description of the physics in CRTs to determine whether the approximations lead to unacceptable computational errors.
1.4 Outline of this thesis

The work in this thesis can be divided into two parts. The first part, Chapters 2 and 3, deals with emission models applicable to thermionic emission and their influence on the optical calculations. In the second part, Chapters 5 through 8, the assumptions that underly these models are examined.

In Chapter 2 models of thermionic emission are reviewed. As their application to electron optical systems is discussed on the basis of CRT guns, the physical assumptions that form the basis of electron optical calculations will be encountered. Simulations of CRTs of the type that can be found in television tubes and computer monitor tubes are discussed in more detail in Chapter 3, and the influence of variations in emission models on the optical properties are charted. Along the way it will be shown how emission properties are commonly approximated using only a few parameters.

As the validity of some of the physical assumptions is doubtful, a computation model has been developed to be able to perform calculations without those assumptions. This model and its numerical implementation are discussed in Chapter 5. The influences of the assumptions on emission is addressed in the last three chapters, which consequently provide an answer to the question whether optical calculations suffer insurmountably from the use of emission models.
Chapter 2

Approximations in present thermionic emission models

In this chapter we will start with a discussion on thermionic electron emission. The transition from thermally limited to space charge limited emission will be discussed and some well-known exact space charge models will be reviewed. The application of exact space charge models in computer simulations of the cathode region will be discussed. The application to CRT guns will be discussed in more detail on the basis of a CRT gun design computer program. Lastly all implicit and explicit approximations common to many computer programs that take space charge into account are indicated in order to reveal possible sources of inaccuracies.

2.1 Theory of electron emission

2.1.1 Emission function

An electron source can formally be described in terms of an emission function \( j_s(r, v) \) that prescribes the initial energy and angular distribution of the emitted electrons over a source surface \( s \).

The six-dimensional space of configuration coordinates \( r \) and velocity coordinates \( v \) is called phase space. To obtain an explicit form of the emission function it is convenient to introduce the particle density per unit of velocity space, or phase space density, denoted \( \varrho^*(r, v) \).

The emission function provides the number of emitted electrons \( d^6N \), with a velocity in velocity interval \( v + d^3v \), from an infinitesimal surface element \( d^2s \) of the source plane \( s \) during an infinitesimal time interval \( dt \)

\[
e d^6N = j_s(r, v) d^2s \ d^3v \ dt. \tag{2.1}
\]

The emission function is related to the phase space density through the continuity equation. This relation can be written

\[
j_s(r, v) d^3v = c \varrho^*(r, v) v \cdot \mathbf{s}(r) d^3v, \tag{2.2}
\]

in which \( \mathbf{s} \) is the local orientation of the source plane. This can be recognized by determining \( d^6N \) as follows:

\[
d^6N = \int_{d^3V} \varrho^*(r, v) d^3r d^3v. \tag{2.3}
\]

\( d^3V \) denotes the volume from which the emitted particles have originated,

\[
d^3V = d^2s \cdot v dt. \tag{2.4}
\]

see Fig. 2.1. Taking the limit \( dt \to 0 \) and \( |d^2s| \to 0 \) yields

\[
d^6N = d^2s \cdot v dt \varrho^*(r, v) \ d^3v. \tag{2.5}
\]

Comparison with Eq. (2.1) leads to Eq. (2.2).

This general form for the emission function allows for a complicated dependency on phase space coordinates \( r \) and \( v \). In practice, the six-dimensional phase space can often be decoupled in several lower dimensional ones, simplifying the emission function considerably.
Chapter 2. Approximations in present thermionic emission models

Figure 2.1: Geometrical visualization for the relation between phase space density and emission function. Electrons emitted during time interval $dt$ can only come from a volume with size indicated in the figure.

One practical example occurs when describing emission from a planar surface, where the momentum distribution of electrons is assumed to be isotropic and does not depend on the configuration coordinate. The phase space density can then be decomposed into a space component and an isotropic velocity component,

$$\phi^*(r, v) = \phi(r)n(v).$$  \hspace{1cm} (2.6)

Defining the $z$-axis to be along the surface normal, Eq. (2.2) simplifies to

$$j(r, v)d^3v = e\nu_s\phi(r)n(v)d^3v. \hspace{1cm} (2.7)$$

In this case, the direction of $\hat{s}$ is fixed and the index $s$ can be suppressed. An explicit expression for $\phi^*$ will be used in Section 2.1.3 to obtain the total emitted current from an ideal metal. But even without an explicit expression the emission function can be simplified using the isotropy of $n(v)$. Using spherical coordinates for $v$ with the $z$-axis normal to $d^3s$ Eq. (2.2) becomes

$$j(r, v)d^3v = e\phi^*(r, v)v^2\cos\theta dv d\Omega. \hspace{1cm} (2.8)$$

The emission function can be separated into an angular part and a velocity magnitude part

$$j(r, v)d^3v = j(r, v)dv f(\Omega)d\Omega, \hspace{1cm} (2.9a)$$

with

$$f(\Omega) = \frac{1}{\pi} \cos\theta, \hspace{1cm} (2.9b)$$

$$j(r, v) = \pi ev^3\phi^*(r, v), \hspace{1cm} (2.9c)$$

in which the factors $\pi$ are introduced for future normalization purposes. Equation (2.9) shows that the current in a certain direction is proportional to the cosine of the emission angle relative to the surface normal. This is Lambert's cosine law [3]. It holds locally if the phase space is locally isotropic and therefore its validity is not restricted to planar surfaces. Equation (2.9b) is called cosine distribution.

However, for the most general emission function one cannot make use of internal symmetries in the phase space density. Later in this thesis we will encounter a situation where the emitter surface is nonplanar and the velocity distribution contains no simple symmetries. The determination of a useable emission function for various physical situations is a fundamental issue addressed in the first part of this thesis.

2.1.2 Virtual emitter

The emission function governs the initial conditions of the emitted electrons at the emitter source plane. The choice of the source plane is open as long as the initial conditions are consistent with the six-dimensional phase space density according to Eq. (2.2). If the source plane coincides with the emitter plane, the initial conditions are given directly by the phase space density of the electrons at the emitter surface.
2.1. Theory of electron emission

![Diagram](image)

**Figure 2.2:** Virtual emitter concept: An electron source emits electrons that are subjected to electromagnetic fields. Whether the electron beam originates from real source, or virtual emitter 1, or virtual emitter 2 is of no concern to the electron optical system, as long as the emission function of the virtual emitter is consistent with the phase space density at the virtual emitter plane (either of the dotted lines).

It is sometimes advantageous to choose a different source plane, in which case it is referred to as virtual emitter or virtual cathode (see Fig. 2.2). For instance, if the shape of the emitter is complicated or not known exactly, one can substitute a virtual emitter, whose properties are known from rigorous calculations or measurements. This also applies if the electron beam properties are altered significantly by mechanisms that are difficult to describe exactly, like statistical Coulomb interactions.

Emission functions relating to a virtual emitter will be referred to as virtual emission functions.

### 2.1.3 Thermionic emission from metals

In this section, we will derive an explicit form for the phase space density and use it to obtain the angular and energy distribution of electrons emitted from a metallic surface.

Electrons in a metal behave approximately as a gas of free electrons. As such, this electron gas follows Fermi-Dirac statistics, meaning the distribution over the energy levels in the metal is given by the Fermi-Dirac function

\[
f(E, T) = \frac{1}{1 + \exp[(E - E_F)/k_BT]}.
\]

The electron gas is confined to the metal by a potential barrier called the work function, \( \Phi \). The work function is defined as the amount of energy necessary to remove an electron from the Fermi level in the metal to vacuum at infinity. At \( T = 0 \text{ K} \), no electron in the metal has sufficient energy \( E_F + \Phi \) to escape from the metal, since \( f(E, 0) = 0 \) for \( E > E_F \).

If the metal is heated, energy states above the Fermi level will be occupied. A number of electrons have sufficient energy to overcome the work function and be emitted from the metal. To derive the current density of emitted electrons we need to determine the number of occupied energy states in the metal. For an isotropic momentum distribution, the number of allowed states per unit volume corresponding to an element \( d^3p \) in momentum space is equal to [4, 5]

\[
2dp_x dp_y dp_z / h^3.
\]

Multiplying with the Fermi-Dirac function gives the number of electrons per volume occupying states with momentum in momentum element \( p + d^3p \)

\[
\frac{2}{h^3} \frac{d^3p}{1 + \exp[(E - E_F)/k_BT]}.
\]

Transforming to velocity space an explicit expression for the phase space density is obtained

\[
\rho^*(r, v) d^3v = \frac{2m^3}{h^3} \frac{d^3v}{1 + \exp[(E - E_F)/k_BT]}.
\]

To determine the emission current we need the velocity distribution of electrons that have overcome the work function. Electrons at the metal-vacuum interface that have overcome the work function with
remaining kinetic energy \( \frac{1}{2} m v^2 \) have total energy \( E = E_F + \Phi + \frac{1}{2} m v^2 \). Choosing the z-axis normal to the emission surface, the emission function at the metal-vacuum interface follows from Eq. (2.7)

\[
j(r, v) = \frac{2m^3e}{\hbar^3} \frac{v_z}{1 + \exp[(\Phi + \frac{1}{2} m v^2)/k_B T]}. \tag{2.14}
\]

We can explicitly separate the emission function into its angular and its velocity magnitude part as outlined in Eq. (2.9) to facilitate obtaining the angular and energy current density distribution. Equation (2.9b) is universal for an isotropic emitter and Eq. (2.9c) becomes for this specific case

\[
j(v) = \frac{2πm^3e}{\hbar^3} \frac{v^3}{1 + \exp[(\Phi + \frac{1}{2} m v^2)/k_B T]}. \tag{2.15}
\]

Note that the argument \( r \) has been dropped since the spatial electron density is constant over the source plane. The current density in a given direction is obtained by integrating the emission function over all velocity magnitudes

\[
j(\Omega) = \int_{v=0}^{\infty} f(\Omega) j(v) \, dv = f(\Omega) \int_{v=0}^{\infty} \frac{2m^3e}{\hbar^3} \frac{v^3}{1 + \exp[(\Phi + \frac{1}{2} m v^2)/k_B T]} \, dv. \tag{2.16}
\]

To evaluate the integral we need to introduce the following approximation. Thermionic emission occurs for temperatures of about 1000 K and the work function for most materials is higher than 1 eV, which means that \( \Phi/k_B T \approx 10 \). The exponential in Eq. (2.16) becomes \( \approx 10^9 \) and we can neglect the 1 in the denominator. In other words, the Fermi-Dirac distribution is replaced with the Boltzmann distribution. Equation (2.16) then evaluates to the angular current density distribution

\[
j(\Omega) = \frac{4me^2\hbar^2 T^2}{\hbar^3} e^{-\Phi/k_B T} \cos \theta, \tag{2.17}
\]

The velocity magnitude current density distribution is obtained by integrating the emission function over all solid angles in the positive \( z \) half space. With the imposed normalization of \( f(\Omega) \) this precisely yields \( j(v) \). Replacing the Fermi-Dirac distribution with the Boltzmann distribution as before and rewriting the result as a function of energy, \( E = \frac{1}{2} m v^2 \) yields the energy current density distribution

\[
j(E) = \frac{4πme}{\hbar^3} E \exp[-(\Phi + E)/k_B T], \tag{2.18}
\]

also known as the spectral radiance. Performing the remaining integration of either of the two previous equations yields the total current density

\[
j = AT^2e^{-\Phi/k_B T}, \tag{2.19}
\]

with the so-called universal thermionic constant

\[
A = 4πme^2\hbar^3 / h^3 = 120 \text{ A/cm}^2\text{K}^2. \tag{2.20}
\]

Equation (2.19) is known as the Richardson-Dushman equation. The quantity \( j \) is called the saturation current density and will be denoted \( j_{\text{sat}} \). The constant \( A \) is also known as Richardson constant.

The derivation of the Richardson-Dushman equation as performed here makes no assumptions on the shape of the emitter surface. If one assumes that the curvature of the emitter surface can be ignored it becomes advantageous to separate the emission function into an axial and a transversal component rather than an angular and a magnitude component. In a similar fashion to Eq. (2.9), using the Boltzmann tail approximation, Eq. (2.14) can be written

\[
j(v) = j_z(v_z)n(v_z)n(v_y), \tag{2.21a}
\]

in which

\[
n(v_{x,y}) = \sqrt{\frac{m}{2πk_B T}} e^{\frac{1}{2}mv_y^2/k_B T}. \tag{2.21b}
\]

and \( j_z(v_z) \) is the one-dimensional current density distribution

\[
j_z(v_z) = \frac{4πm^2ek_B T}{h^3} v_z \exp\left[-(\Phi + \frac{1}{2} mv_z^2)/k_B T\right]. \tag{2.21c}
\]
The one-dimensional current density distribution is a helpful quantity in approximations of emission functions in the neighbourhood of a planar emitter. For later notational purposes it is useful to write \( j_z(v_z) \) in the following manner

\[
j_z(v_z) = v_z \bar{n}(v_z), \tag{2.22a}
\]

with

\[
\bar{n}(v_z) = e^{-\frac{1}{2} \frac{m v_z^2}{k_B T}}, \tag{2.22b}
\]

\[
\bar{\rho}_e = \frac{4 \pi m^2 e k_B T}{h^3} e^{-\frac{\Phi}{k_B T}} = j_{\text{sat}} \frac{m}{k_B T}. \tag{2.22c}
\]

The factors in Eqs. (2.22) are written with bars to indicate that they are not normalized as one would expect. The spatial electron density at the emitter surface is not \( \bar{\rho}_e \) but \( \bar{\rho}_e \sqrt{\pi k_B T/2m} \).

The Richardson-Dushman equation is a specific simplification of the general emission function Eq. (2.2) in which the source plane coincides with the emitter surface and integrations over all possible velocities have been performed.

The thermionic constant \( A \) can be determined experimentally by measuring \( j_{\text{sat}} \) over a range of temperatures. For most materials, the thermionic constant is significantly lower than the theoretical value (2.20). Traditionally this discrepancy has been ascribed to nonuniform emission and the difficulties in measuring the true saturation current, resulting from space charge or Schottky effect [6, 7].

Also, in the derivation of the Richardson-Dushman equation the free electron model was assumed without any band structure considerations. Due to the wave nature of electrons, there is a probability that an electron with sufficient \( z \)-momentum is reflected at the metal-vacuum interface. Usually the wave reflection is taken into account by including a reflection probability \( r(E) \) in a transmission coefficient factor \( (1 - r) \) in Eq. (2.21c). To still be able to evaluate the integration over \( v_z \) the transmission coefficient is replaced by its mean value \( (1 - \langle r \rangle) \). One can then absorb the mean reflection coefficient in the thermionic constant to account for a lower value for \( A \), consistent with most observations for metals. Experiments and wave calculations show that the mean reflection coefficient is close to zero but can deviate considerably in some cases [4, 5, 8].

Furthermore, the application of Eq. (2.19) as the functional relationship between saturation current and temperature is hampered by the fact that in reality both the work function and the thermionic constant depend on the temperature. A more rigorous approach taking into account the shape of the potential step and other band structure effects shows that the power dependency of the saturation current on temperature is not always quadratic [9, 10]. Since the exponential temperature dependency is dominant it is difficult to observe the temperature power factor. If we limit the application of the Richardson-Dushman equation to a relatively small temperature range the precise power dependency is not an important issue and the variation in work function and thermionic constant can be ignored.

Table 2.1 lists some measured values for \( \Phi \) and \( A \). It clearly shows the characteristic of \( A \) as a material property rather than a universal constant.

As band structure effects influence the total current density, uncertainty arises whether the current density distribution is affected. One must realize that the characteristic energy width of the emitted electrons is in the order of \( k_B T \), which is much smaller than the total band width from which the electrons originate. Features of the band structure are most likely not noticeable on a scale with width \( k_B T \).

### 2.2 From emission to space charge

In this section the electromagnetic influences of the beam, the space charge field, will be discussed. It is advantageous to separate space charge influences into a global influence, on the length scale of the entire beam, and a local influence, on the scale where the discreteness of the constituents of the beam becomes important. We will describe a number of procedures to include space charge in calculations. A measure for the relative importance of space charge will be introduced.

#### 2.2.1 Self-induced influences on the electron beam

The emitted current forms an electron beam progressing through the optical system. The electron beam is subjected to electromagnetic forces of the following origins: the average beam charge, the self-generated
magnetic field, the discrete charges in the beam and the system of electron-optical elements. These four contributions will briefly be discussed in turn.

Since the beam consists of moving charged particles, it feels an averaged electric field due to its own charge. This electric field is commonly referred to as space charge field or self-field. The space charge field will cause spreading of the beam since the outer electrons experience Coulomb repulsion due to the inner part of the beam. This effect is called space charge effect. Depending on the beam profile it can also cause deformation of the beam profile.

On the other hand, moving electrons generate a magnetic field that counteracts the beam spreading, since outer electrons experience an attracting Lorentz force towards the center of the beam. The magnetic attraction is in fact a relativistic correction to the electric repulsion with correction factor $1 - v^2/c^2$ [17]. For electrons with initial thermal velocity the relativistic effect is completely negligible. For a beam energy of 1 kV the correction factor is 99.6%. The relativistic effects can be safely ignored if we remain at lower beam energies.

In addition to the average component discussed above, the electric field due to the beam charge has a statistical part that manifests itself in effects that must be described separately from beam spreading. These effects are called statistical Coulomb interactions. They are caused by the discrete nature of electrons and the consequent local charge density fluctuations, in contrast to the beam spreading, which is caused by the global average of the entire beam. Statistical Coulomb effects cause a stochastic shift in the electron positions and velocities, resulting in decreased resolution and energy broadening.

The system of electron-optical elements consists partly of shaped electrodes that produce electrostatic fields by applying boundary potentials and serve to manipulate the beam. The fields of the electrodes themselves are influenced by the electron beam because the space charge induces a surface charge distribution on the electrodes. This rearrangement of conductor surface charge distribution serves to maintain the boundary potentials and is usually automatically included in subsequent calculations. In principle, if the space charge field fluctuates rapidly, for instance in a bunched electron beam in a particle accelerator, the surface charge falls behind with respect to the beam shape, causing wakefields [18] that affect the trailing charge distribution. This is an important effect for relativistic beams in which the space charge field is significantly reduced by the Lorentz force, but can be neglected for the type of low energy beams that are being studied here.

Summarizing, for the studies presented here, the magnetic contraction and the wakefields can be ignored, whereas the statistical Coulomb effects and the self-field may have influence on the emission functions. To what extent statistical effects, the self-field, or both, have to be taken into account depends on the beam current, shape and energy. A measure for the space charge influence will be given below.
2.2.2 Space charge lenses

If the electric field due to the space charge is small with respect to the Laplace field, the space charge field may be considered as a perturbation on the Laplace field.

The effect of a weak space charge field can be taken into account by introducing a negative lens effect based on the beam shape, as the beam traverses along the optical axis. In the particular case of a cylindrical electron beam with a uniform current density distribution the lens is ideal, with no aberrations. Just like an ordinary lens, this ideal space charge lens causes defocusing and magnification. For a nonuniform current density profile, the space charge lens introduces aberrations in addition to defocusing and magnification.

To obtain a more quantitative notion of space charge lenses we take as an example a homocentric rotationally symmetric beam. A beam is homocentric if the particles cross the axis at the same axial position. The lens effect can be described quantitatively by radially expanding the beam profile [19, Chapter 11],

\[
\rho(r, z) = \begin{cases} 
\frac{e \lambda}{\pi r_0(z)^2} \left[ a(z) + b(z) \left( \frac{r}{r_0(z)} \right)^2 + c(z) \left( \frac{r}{r_0(z)} \right)^4 + \cdots \right] & \text{for } r \leq r_1(z) \\
0 & \text{for } r > r_1(z),
\end{cases}
\]  

(2.23)

in which \( \lambda \) is the linear particle density defined as

\[
\lambda = \frac{I}{ev_{\parallel}^2} = I \sqrt{\frac{m}{2e^2 V}},
\]  

(2.24)

with \( v_{\parallel} \) the axial particle velocity, \( I \) and \( V \) the beam current and potential, respectively, implying that the transversal velocity can be neglected with respect to the axial velocity. We write \( r_0(z) \) for a characteristic measure of the beam width at position \( z \) and \( r_1(z) \) for the outer beam radius at \( z \), which may be infinity. The expansion coefficients \( a(z), b(z), c(z), \ldots \) may still depend on \( z \). From Gauss’ law we have

\[
\int_{\partial V} \mathbf{F} \cdot d\mathbf{s} = \frac{e}{\varepsilon_0} \int_{V} \rho \, d^3x.
\]  

(2.25)

To obtain the lens action of the self-field we apply Gauss’ law on a cylinder with length \( \Delta z \) and radius \( r \) centered around the beam axis and we ignore the flux in the axial direction. Substituting the expansion for \( \rho \) in Eq. (2.25) and performing the integrations we obtain

\[
F_r(r, z) = \frac{e^2 \lambda}{2\pi \varepsilon_0} \left[ a(z) \frac{r}{r_0(z)^2} + b(z) \frac{r^3}{2r_0(z)^4} + c(z) \frac{r^5}{3r_0(z)^6} + \cdots \right].
\]  

(2.26)

The first term between brackets in Eq. (2.26) is proportional to the radial distance \( r \). This term can be identified with the first order lens effect: the deflection in the lens is proportional to the distance to the axis. Similarly, the second term can be related to a third order lens effect like spherical aberration, and so on.

We use Eq. (2.26) to obtain the trajectory equation. Restricting ourselves to a beam in drift (i.e. equipotential) space, the relation between the equation of motion (which is an expression for \( d^2r/dt^2 \)) and the trajectory equation (an expression for \( d^2r/dz^2 \)) becomes trivial. Introducing the **perveance**

\[
P = \frac{1}{4\pi \varepsilon_0} \sqrt{\frac{m}{2eV^{3/2}}},
\]  

(2.27)

and transforming to normalized coordinates and coefficients by scaling

\[
R = r/w, \quad Z = z\sqrt{P/w^2},
\]  

(2.28)

(2.29)

with \( w = r_0(0) \) the characteristic beam width at \( z = 0 \), we obtain a normalized trajectory equation

\[
\frac{d^2R}{dZ^2} = A(Z) \frac{R}{R_0(Z)^2} + B(Z) \frac{R^3}{2R_0(Z)^4} + C(Z) \frac{R^5}{3R_0(Z)^6} + \cdots.
\]  

(2.30)
Chapter 2. Approximations in present thermionic emission models

![Image](image.jpg)

**Figure 2.3:** Normalized shape for a uniform beam that expands under influence of its own charge. The R axis is in units of beam width at Z = 0. The Z axis is in units of beam width divided by the square root of the perveance.

In some texts, $I/V^{3/2}$ is referred to as the perveance. The definition (2.27) is conveniently dimensionless. The numerical prefactor relating the two perveances is

$$\frac{1}{4\pi\varepsilon_0} \sqrt{\frac{m}{2e}} = 15154 \text{ V}^{3/2}/\text{A}.$$ 

Equation (2.30) prescribes the beam spreading under the influence of space charge. The normalization of the trajectory equation shows the merit of the perveance as a measure for the degree in which space charge is important. Since the axial coordinate scales with $\sqrt{P}$ increasing the perveance shortens the length along the axis over which variations in the beam shape take place.

As an example we examine the trajectory of an electron at the edge of a uniform beam, $R = R_0$. For a uniform beam, $A(Z) = 1$ and the other coefficients vanish. Equation (2.30) can be integrated with initial condition $R(0) = 0$ to give the beam shape as plotted in Fig. 2.3. It is important to note that a cylindrical beam with uniform density that is somewhere parallel to the axis always follows this shape, regardless of the beam width or perveance. The latter two only influence the scale.

We can apply the preceding discussion to the beam parameters in the drift space of a CRT. For a typical TVT beam, $V = 30$ kV, $I = 1$ mA, resulting in a perveance of $P = 2.92 \cdot 10^{-6}$. The beam radius is $w \approx 1$ mm. The length of the drift space is 0.5 m which corresponds to $Z = 0.85$ in units of $w/P^{1/2}$. If we assume that the beam is uniform and enters the drift space precisely parallel, we can determine from the curve in Fig. 2.3 that the beam width has increased by 34%. For a realistic nonuniform beam the beam spreading is larger than for a uniform beam [20], therefore we cannot attach a quantitative meaning to this result. We can nevertheless expect Eq. (2.30) for uniform beams to provide a qualitative indication of the influence of space charge and it does show that space charge in a CRT has pronounced influence.

In summary, the strength of a space charge lens thus depends on the beam width, perveance and the length of the beam segment. If the influence of the space charge field is too substantial to be considered a perturbation, it may not be advantageous to use a space charge lens description. Below, some methods for a full description of space charge are discussed.

### 2.2.3 Numerical representation of space charge

The most well-established fashion to obtain a space charge distribution resulting from an electron beam requires the electron beam to be modelled as a set of trajectories of 'super particles' of properly chosen charge. The initial conditions and the charge of the particles must be consistent with the emission function. In this manner, the electron beam is divided into small filaments of charge.

In a mesh-type calculation, a super particle thus deposits charge in a mesh cell determined by the dwell time inside the cell, which in turn is determined by the size of the mesh cell and the particle velocity
2.2. From emission to space charge

![Diagram of current flow with points A and B]

**Figure 2.4:** Numerical representations of charge: the square on the left represents a single mesh cell. The curved arrow represents a filament carrying current $I$. The dwell time inside the cell $\Delta t = t_B - t_A$, represented by the solid part of the curve, gives the charge $Q = I \Delta t$ deposited in the cell. The figure on the right represents the subdivision of a filament into small charge line segments.

![Diagram of beamlet method]

**Figure 2.5:** Beamlet method: the electron beam is represented by a collection of charge tubes, one of which is shown. A typical idealized intensity profile of the cross-section of the tube is a Gaussian, shown here.

(see Fig. 2.4, left). This method is commonly known as the particle-in-cell method or counting method. In a charge density method-type calculation, often a local mesh is defined around the trajectory, onto which the particle-in-cell (PIC) method is applied [21, 22]. Alternatively, a trajectory can be represented by a sequence of charge line segments [23]. The PIC cells or line segments then serve as charge sources, additionally to the electrodes.

To maintain the smoothness of the beam a large number of filaments is required [20, 24]. The smoothness can be enhanced by assigning the PIC charge not to a single cell, but rather smearing it out over several neighbouring cells with a weighting factor [25].

A more sophisticated version describes the complete electron beam in terms of (possibly overlapping) beamlets with a certain fixed intensity distribution [26]. By tracking both the center and the width of each beamlet the full electron beam can be modelled as a sum of charge density distributions. The widths of the beamlets are identified with the characteristic width of the intensity distribution. The advantage of beamlets over filaments is the intrinsic smoothness. A smooth space charge field can be obtained with a smaller number of beamlets than would be required using filaments [24].
2.2.4 Self-consistent solution

The charge density distribution is a representation of the electron beam. The evolution of the electron beam in the optical system depends on the potential distribution, and the potential distribution is affected by the electron beam. It is this mutual dependency that complicates the problem of finding the potential distribution. The solution to this problem is called the self-consistent solution; the potential distribution and the space charge density are consistent with respect to the equations of motion governing the shape of the electron beam.

Once the space charge density has been acquired with e.g. the filament or beamlet method, the space charge field can be obtained by solving the Poisson equation. The total resulting potential distribution, including self-field, can be used to determine a new beam shape, thus corrected for the initially calculated self-field. The corrected beam shape in turn implies a modified self-field. Iterating the space charge density determination and the electron beam evolution until convergence one obtains a self-consistent solution of space charge density and potential distribution [27]. This procedure is outlined in Fig. 2.6, where computational subtleties to enhance numerical stability are left out.

![Flow chart for algorithm to obtain a self-consistent solution. In practice, the space charge density is updated with under-relaxation, that is, the change in space charge density is damped with a correction factor.](image)

In Section 2.3 we will review a few simple cases in which an analytical solution can be pursued rather than the iterative approach.

2.2.5 Particle-particle interactions

The most precise manner to incorporate the self-field of the beam is by modelling the complete beam in terms of individual electrons that exert a mutual Coulomb force. The large number of electrons that are present in the system (in a CRT typically millions) makes this a very elaborate method. A lot of performance can be gained by a complex grouping algorithm introduced in particle optics by Munro [28] that combines the Coulomb force due to a large number of electrons at large distances to the force of a single super particle at a weighted distance.

In contrast with this method, since the discrete character of the electrons is not taken into account, the methods of including the beam charge discussed in the previous sections are intrinsically incapable of modelling statistical Coulomb interactions. Statistical interactions are the topic of Chapter 6.

2.3 Space charge limited emission

Let us now picture an emitter with an anode to apply an electric field in the negative axial direction and to collect the current. The emitter is homogeneous in the sense that it is perfectly flat and that the saturation current is constant across the entire surface.

If a low electric field is applied, the current flowing to the anode is lower than the saturation current. This is a consequence of the emitted charge that creates a repelling space charge cloud in front of the cathode, forcing some electrons back. This situation is an important example in which space charge has a pronounced influence on the electron beam.

When the electric field is increased, the influence of the space charge field will decrease, leading to an increased current. If the electric field is so strong that the space charge cloud becomes too rare to have a repelling influence on the emitted electrons, the current that will flow in the system is equal to the saturation current. Increasing the field even more does not lead to an increased current. In this case, the current is said to be saturated.

Footnote 1: For the moment, the Schottky effect will be neglected.
2.3. Space charge limited emission

![Graph of the emission current as a function of emitter temperature. The emitter is a dispenser cathode with a Tungsten-Iridium matrix. At low temperature, the emitter is temperature limited. At high temperatures, the emitter is space charge limited and the temperature has little influence on the emission [29]](image)

Figure 2.7: Graph of the emission current as a function of emitter temperature. The emitter is a dispenser cathode with a Tungsten-Iridium matrix. At low temperature, the emitter is temperature limited. At high temperatures, the emitter is space charge limited and the temperature has little influence on the emission [29].

A different way to look at the two current regimes is by keeping the field constant and adjusting the saturation current by changing the emitter temperature. Starting in the saturated current regime, the current depends on the temperature via Eq. (2.19) and is thus also referred to as temperature limited current. When the temperature is increased, the emitted current increases and the space charge cloud becomes more dense. At a certain temperature, the increase in saturation current is counteracted by the increase in space charge cloud density. Increasing the temperature hardly influences the current. The emission current is limited due to the influence of space charge and referred to as space charge limited current. In Fig. 2.7 this behaviour is plotted.

We will briefly discuss several models known from literature to calculate the space charge limited current.

### 2.3.1 The Child model

In the Child model [30] a system of two infinite parallel plates representing the cathode and anode is considered. Cathode and anode are called a diode. The potential difference between cathode and anode is $V_d$. The distance between cathode and anode is $d$. Below, $V_d$ and $d$ are referred to as the diode parameters. In literature, the term ‘diode parameter’ is usually reserved for $d$. The electron beam is infinitely wide which makes the model one-dimensional, i.e. independent of the transversal coordinates.

The space charge in the diode is determined by solving the Poisson equation in a planar geometry

$$\frac{d^2V}{dz^2} = -\frac{\rho}{\varepsilon_0}, \quad (2.31)$$

with the assumption that the initial velocity of the emitted electrons can be neglected. The validity of the Child model is restricted to the regime of unsaturated current, i.e. no assumptions are made on the saturation current. Using

$$j = \rho(z)v(z) \quad \text{continuity equation,} \quad (2.32)$$

$$eV(z) = \frac{1}{2}mv(z)^2 \quad \text{energy conservation,} \quad (2.33)$$

the Poisson equation becomes

$$\frac{d^2V}{dz^2} = -\frac{1}{\varepsilon_0} j \sqrt{\frac{m}{2eV(z)}}, \quad (2.34)$$
with boundary conditions

\[ V(0) = V'(0) = 0. \]  

(2.35)

The initial conditions are associated with the assumptions of zero velocity at the cathode and space charge limited emission: \( j \) adjusts itself such that the electric field at the cathode vanishes. Zero electric field at the cathode is precisely the condition that electrons with zero initial velocity can still penetrate the space charge barrier. Henceforth we will adapt the convention that the current due to electrons is positive, irrespective of the flow direction. Equation (2.34) can be solved to yield

\[ V(z) = \frac{9}{4} \frac{j}{\varepsilon_0} \left( \frac{m}{2e} \right)^{2/3} z^{4/3}. \]  

(2.36)

Imposing the anode boundary condition \( V(d) = V_d \) and solving for \( j \) yields Child’s law for the space charge limited emission current density,

\[ j = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{V_d^{3/2}}{d^2}. \]  

(2.37)

also known as Child-Langmuir’s law.

According to Child’s model, the charge density at the cathode is infinite. This pathological behaviour is caused by the assumption of zero initial velocity.

### 2.3.2 Variation on the Child model

In reality, electrons have a nonzero initial velocity, which affects the potential distribution at the cathode. The space charge limit is no longer obtained with vanishing electric field at the cathode since electrons can overcome a potential barrier if they have sufficient initial energy. In the space charge limit the potential decreases at the cathode, forms a minimum \( V = V_m \) at \( z = z_m \) between cathode and anode, and increases to \( V_d \) at the anode. This description also incorporates Child’s model, in which the potential minimum coincides with the cathode.

A simple variation on the Child model assumes nonzero, monovalent initial velocity of the electrons [31–33]. In this case, all electrons come to a stop at the potential minimum. A fraction \( \alpha \) proceeds towards the anode, the remaining fraction \( 1 - \alpha \) is reflected back towards the cathode. The initial velocity determines the depth of the minimum, \( -eV_m = \frac{1}{2}m v_m^2 \). The applied anode voltage determines the location of the minimum and the space charge limited current.

The relation between anode voltage and space charge limited current can easily be found by separating the diode into a region \( \odot \) between cathode and potential minimum and a region \( \oplus \) between minimum and anode. The derivation leading to Child’s law can be followed if we alter Eq. (2.33) to account for the fact that some electrons do not contribute to the current, but do contribute to the charge density. For the region between minimum and anode Eqs. (2.34) and (2.35) hold if we make the following replacements:

\[ V \rightarrow V - V_m \]  

(2.38)

\[ j \rightarrow \alpha j_{\text{sat}} \]  

(2.39)

\[ z \rightarrow z - z_m \]  

(2.40)

For the region between cathode and potential minimum the charge density is made up of electrons going towards the minimum and electrons going towards the cathode. Thus, the replacement

\[ j \rightarrow (1 - \alpha)j_{\text{sat}} + j_{\text{sat}} \]  

(2.41)

must be made. The two resulting Eqs. (2.37\(\odot\)) and (2.37\(\oplus\)) for regions \( \odot \) and \( \oplus \) can be rearranged to give the relation between the fraction of the saturation current and the diode parameters

\[ \frac{2}{\alpha} - 1 = (d/z_m - 1)^2 (1 - V_d/V_m)^{3/2}. \]  

(2.42)

This expression provides an implicit interdependence between the diode parameters \( V_d \) and \( d \) and the emitter parameters \( j_{\text{sat}} \) and \( \eta \), so that one can be obtained from the other three.

The infinite charge density pathology that plagues the Child model is still present, since at some point, the electron velocity is zero.
2.3.3 The Langmuir model

The space charge limited current between two infinite parallel plates taking into account the velocity distribution of the emitted electrons was originally derived by Langmuir [34]. He assumed that the initial distribution of emitted electrons was Maxwellian, in agreement with the results of Section 2.1.3. This model is also known as Langmuir-Fry’s model. In view of the planar symmetry, the emission is governed by Eq. (2.21) where the transversal components can be ignored. The space charge limited current in a Langmuir diode with potential minimum $V_m$ is

$$j = j_{sat} \exp(eV_m/k_BT),$$  

(2.13)

which can be found by direct integration of Eq. (2.22) over all velocities $v > \sqrt{-2eV_m/m}$ or by realizing that in the derivation of the Richardson-Dushman equation (2.19) no assumptions were made on the shape of the potential barrier. To include an additional potential barrier on top of the work function we can therefore replace the work function $\Phi \rightarrow \Phi - eV_m$ in Eq. (2.19).

The determination of $V_m$ and $z_m$ is complicated by the fact that the spatial charge density $\rho$ can no longer be expressed as a simple equation of current density and beam potential. Rather, it must be obtained by integrating the phase space density over all possible velocities.

The phase space density at a point $z$ with potential $V_z$ with respect to the cathode can be derived from the continuity equation as follows: the contribution to $\rho$ due to electrons that were emitted at the cathode with a velocity in interval $v + dv$ is

$$d\rho(z) = \frac{j(v)}{u} dv,$$  

(2.44)

where $u$ is the velocity of the electrons when they have reached $z$.

$$|u| = \sqrt{v^2 + 2eV_z},$$  

(2.45)

which follows from energy conservation. For convenience we introduce

$$v_m = \sqrt{2e(V_z - V_m)/m}.$$  

(2.46)

The explicit form for $d\rho$ in terms of $u$ can be found by substituting Eqs. (2.22) and (2.45) in (2.44),

$$d\rho(z) = \frac{j(u)}{u} \left( \frac{du}{dv} \right) = \bar{\rho}_e e^{V_z/k_BT} e^{-\frac{1}{2}mu^2/k_BT} du.$$  

(2.47)

To calculate $\rho$ the regions at both sides of the potential minimum must be considered separately. In the region between cathode and minimum we encounter electrons moving towards the anode with velocity $u \geq 0$ and electrons that were reflected by the potential minimum with velocity satisfying $-v_m \leq u < 0$. The total spatial density is therefore

$$\rho_-(z) = \int_{u = -v_m}^{\infty} d\rho = \bar{\rho}_e e^{V_z/k_BT} \int_{u = -v_m}^{\infty} e^{-\frac{1}{2}mu^2/k_BT} du,$$  

(2.48a)

where the index $-$ denotes the region between cathode and minimum (see Fig. 2.8). In the region between minimum and anode, only electrons moving from minimum towards anode are present with $u \geq v_m$, resulting in a spatial density

$$\rho_+(z) = \int_{u = v_m}^{\infty} d\rho = \bar{\rho}_e e^{V_z/k_BT} \int_{u = v_m}^{\infty} e^{-\frac{1}{2}mu^2/k_BT} du,$$  

(2.48b)

with index $+$ indicating the region beyond the minimum. Transforming to integration variable

$$t = u \sqrt{\frac{m}{2k_BT}},$$

and introducing Langmuir’s dimensionless potential

$$\eta = \frac{c}{k_BT} (V_z - V_m),$$  

(2.49)
Equations (2.48) become

\[
\begin{align*}
\rho_-(z) &= \rho_e \sqrt{\frac{2k_B T}{m}} e^{V_e / k_B T} \int_{t=-\eta}^{\infty} e^{-t^2} \, dt, \\
\rho_+(z) &= \rho_e \sqrt{\frac{2k_B T}{m}} e^{V_e / k_B T} \int_{t=\eta}^{\infty} e^{-t^2} \, dt.
\end{align*}
\tag{2.50a, b}
\]

Using the error function, \(\frac{1}{2} \pi^{1/2} \text{erf } x = \int_0^x \exp(-t^2) \, dt\), these equations can be rewritten and combined to

\[
\rho_\pm(z) = \rho_e \sqrt{\frac{2k_B T}{m}} e^{V_e / k_B T} \left[ 1 \mp \text{erf}(\sqrt{\eta}) \right].
\tag{2.51}
\]

Using Langmuir’s dimensionless distance parameter

\[
\xi = (z - z_m) \sqrt{\frac{j e}{\varepsilon_0 k_B T \left( \frac{2\pi m}{k_B T} \right)^{1/2}}},
\tag{2.52}
\]

the Poisson equation (2.31) becomes

\[
\frac{d^2 \eta_{\pm}}{d\xi^2} = \frac{\eta}{2} \left[ 1 \mp \text{erf}(\sqrt{\eta}) \right],
\tag{2.53}
\]

where we made use of Eqs. (2.22c) and (2.43). The – sign of \(\mp\) applies to the region between cathode and minimum, the + applies to the region between minimum and anode. The initial conditions for both equations are

\[
\eta(0) = \left. \frac{d\eta}{d\xi} \right|_{\xi=0} = 0.
\tag{2.54}
\]

The numerical solution tabulated by Langmuir has been extended with tables and approximants by others [35–37]. Similarly to Eq. (2.42) Langmuir’s \(\eta\) function with the normalized potential and distance provides an implicit interdependence between the diode parameters \(V_d\) and \(d\) and the emitter parameters \(T\) and \(j_{sat}\), so that one can be obtained from the other three. The \(\eta\) function is plotted in Fig. 2.9.

In Fig. 2.10, left, the space charge limited current for a diode with anode distance 100 \(\mu\)m, a temperature \(T = 1160\) K and a saturation current of \(10^5\) A/m\(^2\) are plotted as calculated with the Child model, the Child model with modification for initial velocity, and the Langmuir model.
2.3. Space charge limited emission

Figure 2.9: Langmuir's dimensionless potential $\eta$ and space charge density $\eta''$

Figure 2.10: Left: current flowing in a space charge limited (scl) diode according to the three space charge models. The saturation current is $j_{sat} = 10 \text{ A/cm}^2$, the anode distance is 100 $\mu$m. Right: Position of the space charge minimum according to the Langmuir and the modified Child model.
Figure 2.11: Smooth transition from space charge limited to temperature limited current as a result of nonuniform emission. Solid curves: I-V characteristics of ideal Langmuir diodes with uniform saturation current 8, 9, 10, 11, 12 A/cm². Dashed curve: I-V characteristic of an ensemble of Langmuir diodes with \( j_{\text{sat}} \) = 8–12 A/cm². Note that the solid curves overlap in the space charge limited regime, which shows that an increase in saturation current does not increase the space charge limited current.

It can be seen that the modification to the Child model does not bring a significant improvement. Contrary to the Child model, it predicts a potential minimum in front of the cathode, and is capable of calculating the space charge limited current for a weak negative anode voltage. It is clear from the comparison with Langmuir’s model in Fig. 2.10, right, that these values for current and position of the minimum are of no quantitative use. For increasing anode voltage, the Child predicted minimum moves away from the cathode very slightly, whereas in the Langmuir model it moves towards the cathode.

Since the initial velocity distribution is continuous, the Langmuir model does not suffer the charge density singularities of the previous models.

Figure 2.10 shows a sharp transition from the space charge limited regime to the saturated regime when increasing the electric field, as discussed in Section 2.3. In reality, the transition is smooth. The smoothness can partly be attributed to nonuniform emission, resulting in a superposition of current curves with different transition points. This behaviour is shown in Fig. 2.11. In fact, a measure for the emission nonuniformity may be deduced from the smoothness of the transition [38–40].

A second, more fundamental cause for the smooth transition, giving an explanation for its occurrence also in the case of uniform emission, is the image potential of an electron in front of a conductor, which is not taken into account in the Langmuir model. The image potential is responsible for the Schottky effect that describes enhanced emission in the presence of a strong accelerating field. A number of attempts to incorporate image forces in the space charge potential distribution are known [41–44], predicting transitions with varying degree of smoothness.

2.3.4 Exact space charge models in nonplanar diode geometries

Langmuir and Blodgett [45] derived an expression for the space charge limited current in a diode consisting of concentric cylinders or spheres. Both models neglect the initial velocity of the electrons at the cathode, like the Child model.

Expressions for concentric cylinder and sphere diodes that do consider a Maxwellian initial distribution have been derived by Porter et al. [46]. Space charge limited current in the vicinity of sharp edges or corners can be described by solutions of the one-dimensional systems obtained by scaling the
equations [47].

Self-consistent space charge fields in other geometries can only be approximated with the aid of exact space charge models. This will be discussed in the following section.

2.4 Space charge models for practical geometries

The above described exact models share a common trait: they are all valid in basically one-dimensional geometries. To calculate the space charge limited current in a realistic geometry using an exact space charge model, some necessary approximations must be made. Alternatively, one can resort to the iterative approach discussed in Section 2.2.4. The disadvantage of using the latter is the fact that the large number of particles that are reflected back towards the cathode cause a significant calculation overhead in determining the space charge limited beam shape. It is possible to improve the effectiveness of the iterative calculation in the space charge limit considerably by an efficient super particle scheme [48]. It may however still be preferable to utilize exact space charge models. The real cathode can then be replaced by a virtual emitter, discussed in Section 2.1.2, situated at the potential minimum, so that no particles emitted from the virtual emitter are reflected back towards the real cathode. The term 'virtual cathode' is usually reserved for this particular type of virtual emitter.

The feature that no particles are reflected back towards the real cathode is in fact the great advantage of using space charge models in simulations of space charge limited emission. The virtual emitter is sometimes placed a small distance past the potential minimum [49], or on a certain equipotential [50,51] rather than on the potential minimum.

2.4.1 Virtual emitter based on a diode array

Since a realistic geometry is not one-dimensional, no exact space charge model is available to model the emitter region. The emitter must be represented by a set of adjoining small cathodes, which locally may be considered to be part of a one-dimensional geometry on which an exact model may be applied.

With an understandable preference for the models that do not neglect the Maxwellian velocity distribution of the emitted electrons, either of the discussed diode models may be selected, to suit the local emitter geometry best with respect to surface curvature. For a weakly curved cathode, the space charge field near the emitter can be modelled as a collection of sectors of cylindrical or spherical diodes [52,53]. Here, we restrict ourselves to a situation in which the emitter is planar. The emitter region is modelled as an array of local planar Langmuir diodes that are each bounded in the transversal as well as the axial direction.

The cathode end of the diode is attributed to the emitter surface, but the anode end is undetermined. Each diode needs to be equipped with its own virtual anode, with anode potential $V_a$, that behaves similarly to the anode in the diode model, i.e. it must be a flat equipotential plane.

To preserve the local planarity, the distance between the emitter and the virtual anode must be small with respect to the transversal size of the local Langmuir diode. It can not be too small however, otherwise the amount of charge that is considered in the space charge model is underestimated, leading to an overestimation of the current. This can easily be understood qualitatively by the fact that a smaller space charge cloud is able to reflect fewer particles.

The virtual emitter is formed by the union of the potential minimums of the local diodes. Local application of the Langmuir model supplies the space charge cloud between real emitter and virtual emitter.

2.4.2 Emission properties of the diode array

The use of a virtual emitter is sensible only when the emission properties of the virtual emitter are known. The first step in establishing the virtual emission function of a diode array is to determine the virtual emission function of a single planar diode. To this end we first ascertain that a potential barrier in a one-dimensional geometry does not affect the current density distribution (2.21c). We can determine from Eq. (2.47) that at a potential barrier with depth $V_m$ the current due to electrons in velocity interval $u + du$ is

\[ j(u)du = u \frac{d p}{d u} = u \bar{p}_e e^{V_m/k_B T} e^{-1/2 u^2/k_B T} du, \]  
\[ (2.55) \]
Figure 2.12: Potential distribution in a triode. The lower edge of the picture coincides with the optical axis. The extraction field due to the extractor (not shown) penetrates through the aperture. The penetration can be tuned by changing the potential of the aperture electrode. In the situation shown here space charge is neglected; the emitting area is bounded by the equipotential line hitting the cathode.

which is similar to (2.22) if we replace

$$\rho_e \rightarrow \rho_e e^{V_m/k_B T}.$$  \hspace{1cm} (2.56)

The current density distribution thus maintains its Maxwell-Boltzmann character. In a planar diode, the transversal velocity remains unaffected since there are no transversal forces. Thus, the emission function (2.21) remains invariant, except for a correction factor $\exp(eV_m/k_B T)$. The emission properties of the virtual emitter in a single planar diode are identical to the emission properties of the real emitter, given by Eq. (2.21).

Consider now a triode consisting of a planar cathode, an extractor and an aperture in between that limits the extracting field at the cathode. The aperture serves to bound the emitting area of the cathode (Fig. 2.12). The cathode region is modelled as an array of diodes described by the Langmuir model.

Moving from the optical axis towards larger transversal distances, the electric field at the cathode in the absence of space charge decreases. Consequently the local emission decreases implying that the Langmuir minimum is deeper and lies further away from the cathode. Note that although "the Langmuir minimum" is here referred to as singular, it is not a single valued potential and the Langmuir minimum plane is not an equipotential plane. In Fig. 2.13 a schematic diagram of the diode array is sketched.

The Langmuir model relates the axial potential distribution in a diode to the space charge density. Each of the diodes has its own axial potential distribution. The array of axial potential distributions represents the global potential distribution and space charge density in front of the cathode under the assumption that there is no coupling between neighbouring diodes. Since electrons feel a force directed towards the optical axis in the region between cathode and minimum this assumption is not strictly accurate.

Due to this force, the transversal velocity distribution at the minimum is subjected to a shift that depends on the spatial coordinate. Therefore the phase space density at the minimum can not be decoupled in a spatial and a velocity component. Although the virtual emission function of a single Langmuir diode is well-defined, the emission function of the complete virtual emitter is not.

One method to address this problem is to introduce an estimated transversal velocity shift at the virtual emitter [20]. The emission function then has the form

$$j(r, v) = j_z(v_z) \exp(eV_m/k_B T) n(v_x - \Delta v_x) n(v_y - \Delta v_y),$$  \hspace{1cm} (2.57)

with $n$ and $j_z$ given in Eqs. (2.21b) and (2.21c). The velocity shifts $\Delta v_x, y$ depend on $r$ to impose coupling. $r$ is constrained to the virtual emitter plane.

Alternatively, one can use an emission plane situated at the real source, where no coupling between transversal velocity and spatial coordinate exists, and to introduce an axial shift at the real source such, that only particles with sufficient energy to overcome the potential minimum are emitted [49]. In that
2.4. Space charge models for practical geometries

Figure 2.13: Schematic of the cathode region modelled as an array of Langmuir diodes. The transversal diode boundaries are indicated by the horizontal dotted lines. The Langmuir minimums are indicated with the vertical solid lines. The distance between cathode and minimum varies typically from 2 µm near the optical axis (dash-dotted line) to 20 µm near the edge of the emitting area. The virtual cathode (dashed curve) is formed by interpolation through the midpoints in the diodes of the Langmuir minimums. The virtual anodes have increased shading towards the optical axis to indicate the increase of \( V_A \). The space charge limited current \( j_{\text{sc}} \) increases accordingly. In numerical computations, the transversal dimensions are given by the dimensions of the finite-element or finite-difference mesh, typically 5-15 µm.

In general, the emission function has the form

\[
j(r, \mathbf{v}) = j_z(v_z + \Delta v_z) \exp(eV_m/k_B T)n(v_z)n(v_y),
\]

with \( \Delta v_z \) depending on \( r \). \( r \) is constrained to the real emitter plane.

The iterative approach to obtain a self-consistent result described in Fig. 2.6 can be applied with this virtual emitter if care is taken that the properties of the virtual emitter change per iteration, since the virtual emitter must be determined from the potential distribution. The local virtual anode potentials change under the influence of the surrounding space charge, whereas a real anode potential would be fixed by an electrode. The iterative approach must be extended with one step:

- determine virtual emission function
- and space charge cloud

between solving the Poisson equation and tracking the beam (we use the term space charge cloud here specifically for the space charge between real emitter and virtual emitter).

But contrary to the situation with a real cathode, as a result from the approximations in the virtual emission function, if the iterative procedure has converged there is still uncertainty as to whether the self-consistent result is the correct solution. The electron beam may very well be consistent with the virtual emission function and the space charge field, but if the virtual emission function is not correct, neither is the space charge field.

The validity of the virtual emission function in realistic simulations will be addressed in the next chapter, where we will consider the issue of general self-consistency in more detail.
2.5 Particle optical simulations with space charge

Computer programs that are able to calculate a self-consistent solution for an arbitrary particle optical system mostly follow the iteration scheme discussed in the previous section with minor differences. Several programs are discussed below. The focus lies mainly on programs suitable for CRT simulations. For a description of the configuration of a CRT the reader may skip ahead to Section 3.1.

2.5.1 Simulation of CRTs with SCELOP

The computer program SCELOP [54] used by Philips Display Components for the design of CRT electron guns will be discussed in detail, since this program has been used for the calculations in Chapter 3.

SCELOP is largely based on the work of van den Broek [29]. It is equipped with a finite-difference Poisson solver. Local Langmuir diodes are used to calculate the position of the virtual emitter. The virtual emitter coincides with the Langmuir minimum plane. To improve the convergence of the iteration, local Child diodes are used as an initialization for the Langmuir calculations.

The emission properties of the virtual emitter are based on the emission properties of planar Langmuir diodes. The current density at the virtual emitter surface is given by the depth of the potential minimum via Eq. (2.43). The coupling between position and velocity distribution is enforced by estimating the transversal acceleration an electron would experience in the space charge field between cathode and virtual cathode. The estimate is based on the transversal electric field at the minimum and the depth and position of the minimum. The expression for $\Delta v_{x,y}$ in Eq. (2.57) is [20]

$$\Delta v_{x,y} = -\frac{2}{3} \frac{e}{m} E_{x,y}(z_{\text{cat}} - z_{\text{em}}) \sqrt{\frac{2m}{e(V_{\text{cat}} - V_{\text{em}})}},$$

(2.59)

in which $E_{x,y}$ is the transversal electric field at the Langmuir minimum and $z_{\text{cat}}$ and $V_{\text{cat}}$ are cathode position and potential.

The axial velocity distribution is simplified from a Maxwellian to a monovalent distribution.

The space charge field past the virtual cathode is determined by assigning charge density to the corners of the grid cell encompassing the trajectory, on the basis of its current and velocity. This method bears most resemblance to the beamlet method, with a beamlet that envelops exactly one grid cell and a uniform intensity distribution for each beamlet.

For a smooth space charge field thousands of trajectories are necessary. A fast fitting mechanism is used to calculate a large set of trajectories on the basis of a smaller set that has been calculated by numerical trajectory integration.

When the perveance drops below a certain value due to the acceleration in the spot forming region of the CRT, the space charge field can be considered a perturbation on the Laplace field, as explained in Section 2.2.2. The iterative search for a self-consistent solution is then abandoned in favor of a fast 'on-the-fly' space charge lens correction to the beam shape. The lens action is determined by calculating the space charge potential distribution in a local mesh moving along with the beam.

SCELOP is especially suited for simulating common CRT electron guns. CRT electron guns normally have a triode part consisting of a planar cathode, a Wehnelt aperture biased around -20 V to -160 V and an extractor at 500 to 1000 V, as schematically depicted in Fig. 3.1. In Chapter 3 the optics of CRT guns will be discussed in more detail. Electrodes in CRTs are referred to as grids, Wehnelt and extractor are therefore known as G1 and G2, respectively. The distance between the cathode and the first grid is denoted $s_0$. The cathode-Wehnelt bias is effected by keeping the Wehnelt on a fixed potential and varying the cathode voltage. This is called cathode drive, as opposed to grid drive, in which case the cathode is maintained at a constant potential and the Wehnelt potential is varied.

The user input includes geometry information, meshing for the finite difference method, field calculation parameters, fit parameters, cathode temperature, saturation current density and Langmuir model parameters.

The following characteristics are often used to describe electron gun performance:

cathode drive current vs. cathode voltage

fixed focus characteristic spot size vs. current for given focus voltage

focus characteristic spot size vs. optimum focus voltage for given current
projected beam characteristic spot size of the projected beam vs. current (main lens switched off)

The spot size is an objective measure classifying the spot distribution at the screen. The definition will be given in Chapter 3. With the cathode drive an additional quantity is associated: the cut off voltage, which is the cathode-Wehnelt bias at which no current is emitted.

Comparison between measurements and simulations

The reliability of SCELOP has not been tested systematically. Through the years, several simulation quirks have been encountered, as well as major and minor discrepancies with measurements. In general, simulations of rotationally symmetric guns are more accurate than simulations of non-symmetric guns. Below some reported comparisons are summarized. It should be noted that some major improvements to SCELOP have been introduced since, that have increased the accuracy of computations considerably [54].

Super triode in 21” CMT As a means to test rotationally symmetric computer simulations, two triode geometries that had been modelled with SCELOP were fitted into a 21” computer monitor tube (CMT). The triodes differed with respect to electrode spacings, most notably cathode-Wehnelt (60 µm for triode 1, 40 µm for triode 2). Measured and computed focus characteristics agree reasonably well, apart from an offset in focus voltage of up to 50 V.

Calculations of the spot size as a function of beam current for triode 2 differ up to 10%. For triode 1 the difference is 5% for high currents and up to 20% for low currents. It should be noted that two measurements per triode were performed for two guns with nominally identical triodes, giving significantly different measurement results for triode 2 but not for triode 1. Triode 1 calculations were systematically too low, whereas triode 2 calculations were in between the two measurements[55].

Distributed Main Lens in HD1 gun The DML HD1 gun is used in a television tube (TVT) and consists of a RS triode, a dynamic stigmator grid and a RS main lens. The calculated cathode drive yielded a systematic overestimation of the current. In subsequent simulations, the cathode-Wehnelt distance was adjusted to yield the extractor voltage and cut off voltage corresponding with the measurements, leading to an s01 of 85 µm instead of the nominal value of 70 µm. Subsequent simulations yielded an accurately corresponding cathode drive characteristic. Unfortunately, it is not confirmed whether the actual value for s01 matches the nominal (design) value.

The calculated fixed focus spot size agrees within 10% with measurements. For high currents the calculated spot sizes are too large, for low currents too small.

The qualitative effect of the stigmator could well be predicted with simulations[56].

P1 gun The P1 TVT gun contains an astigmatic Wehnelt, a two-part extractor and a so-called polygon main lens. By adapting s01 in the simulations the cathode drive characteristic could accurately be reproduced. At intermediate currents the calculated projected beam spot size (i.e. main lens switched off by setting the anode voltage to the same value as the focus bus) was 30% larger than the measured value. At high currents the calculated spot size was 30% larger than the measured value.

Modifying the geometry (e.g. the radius of the Wehnelt aperture and the distance between Wehnelt grid and extractor grid) in the same fashion as s01, within the prescribed assembly tolerance, did not lead to a significant improvement in simulation results[57].

15” CMT MkIIa The 15” MkIIa gun has a round Wehnelt, a two-part astigmatic extractor with extra prefocussing grid and a polygon main lens. Three slightly differing triodes were built and simulated. Cathode drive characteristics computations corresponded well with measurements.

In the optimum prefocussing characteristics differences of up to 20% were found. More disturbing was the fact that the trend of the spot size versus focus voltage in simulations and measurements was reversed. The calculated projected beam spot sizes are in fair agreement, within 6% of the measured values[58].

P³ gun The p³ TVT gun contains an astigmatic Wehnelt, astigmatic extractor and polygon main lens. The systematic comparison between p³ simulations and measurements has been carried out more extensively than previous gun studies: horizontal and vertical spot sizes in both center and corner of the screen have been examined, unlike the comparisons discussed earlier, in which mostly only horizontal
center spot sizes were examined. It should be noted that deflected spot size calculations are facilitated by means of matrix methods rather than by trajectory calculations in a magnetic deflection field.

If the triode geometry was adapted to obtain the best correspondence in horizontal spot size at the screen center and corner, the deviation for the entire current range could be kept below 10%. The vertical spot size with this triode geometry then turned out to be up to 30% too large for the center spot size and 30% too small for the corner spot size. The cut off voltage was 15 V too low. The focus voltage was 100 V too high.

If the extractor voltage was tuned to yield the proper cut off voltage, $s_0$ had to be reduced from 80 $\mu$m to 70 $\mu$m to obtain the best correspondence with the horizontal spot size measurements. If $s_0$ was maintained at 80 $\mu$m, the extractor voltage had to be set at 1028 V compared to 940 V in the measurements.

Repeating the analysis for different anode voltage yielded similar results: although the deviation in horizontal spot size could be kept around 10% the vertical spot size simulation remained inadequate.

For all vertical spot sizes the computed center spot size was too large and the corner spot size was too small. This distortion could be reduced by changing the dimensions of a hole in the main lens. However, even a change far outside the normal range of assembly tolerances could not reproduce the measurement results with the desired accuracy [59].

Influence of input parameters

All user input and model parameters not explicitly relating to the geometry and electrode voltages are called input parameters. Dependence on input parameters like finite-difference mesh width and field calculation parameters is a common characteristic for all electron optical calculations. The influence of SCELOP specific input parameters has been observed in numerous simulations. Some interesting results are the following [60]:

- Increasing the number of fit trajectories yielded closer agreement with measurements, but not always. In some instances, increasing the number of fit trajectories gave worse results.

- Increasing the cathode temperature could sometimes lead to a lower current.

- Modifying the Langmuir model parameters affected the current.

2.5.2 Other software based on space charge models

Below we discuss briefly some simulation programs that make use of virtual emitters.

BEFORE is a CRT design program developed by Hitachi Ltd. [24, 49]. The potential distribution near the cathode is solved using the finite difference method. The space charge density near the cathode is determined from the Langmuir model. The virtual emitter is situated at the cathode plane with an emission function with axial component corrected for the potential barrier. The local charge density at the virtual emitter follows from the local Langmuir minimum. The space charge field past the Langmuir diode is determined with the beamlet method. Similarly to SCELOP, the axial velocity distribution is simplified to a monovalent distribution.

SCALA by Vector Fields Software [50] is a finite-element based Poisson solver. The space charge limited emission functionality is implemented with local Langmuir diodes. The virtual cathode is defined as the plane in front of the cathode with voltage equal to the real emitter. The space charge field past the virtual cathode is determined with a particle-in-cell scheme.

EGUN by SLAC is finite-difference based, with local Langmuir diodes for the emission function. The virtual emitter is defined as the equipotential plane in front of the cathode with the same potential as the cathode. The space charge field is determined from particle-in-cell.

CPO by RB consultants Ltd [22] is a charge-density based electrostatic field solver. The virtual emitter is identified with the anode of local Langmuir diodes. The diode spacing is identical for each diode. The space charge past the virtual emitter is determined with a particle-in-cell scheme on a local mesh, or with the previously discussed charge line segment method [23].
2.6. Summary and outlook

Most commercially available software packages (e.g., CPO, SCALA) supply virtual emitters as an additional functionality to space charge calculators with real emitters. If the emission is space charge limited, not using a virtual emitter has some disadvantages as discussed above. On the other hand, dispensing with the virtual emitter also eliminates any doubts to its validity. Some simulations of space charge limited emission without a virtual emitter are discussed in Refs. [48, 61–63].

2.5.3 Concluding remarks

This list is by no means complete but it serves to indicate that, apart from the variety of physical models and mathematical algorithms, the basic framework for space charge calculations is very similar for all computer programs and the problems that have been encountered with SCELOP are representative for all space charge programs. To recapitulate, simulations of space charge limited emission model the cathode region as an array of local Langmuir diodes. The local emission density is determined from the depth of the Langmuir minimum. The initial energy of the electrons, given by a virtual emission function, is corrected for the depth of the minimum. The converged solution is obtained with the iterative scheme Fig. 2.6 with the extension introduced in Section 2.4.2.

2.6 Summary and outlook

A self-consistent solution including space charge can in general be obtained using an iterative approach that consists of calculating the potential distribution, determining the emission and resulting electron beam and recalculating the potential distribution (Fig. 2.6).

If the emission is space charge limited, it is preferable to use the emission function of a virtual cathode formed in an array of local planar diodes, since most current from the real cathode is reflected back towards the cathode, taking up computer time without contributing to the current.

The following approximations, all but the second of which are implicitly present in the Langmuir model, are used in this virtual cathode:

1. the emitted electrons follow Maxwell-Boltzmann statistics.
2. the emitter region is represented by an array of independent planar diodes with possibly an estimated coupling imposed.
3. the space charge field behaves as a smooth jelly. Electrons feel an averaged space charge force and are not subjected to statistical Coulomb interactions.
4. the emitter surface is smooth. No surface irregularities or lattice structure are taken into account.
5. the emitter structure is homogeneous. The cathode emission function is uniform. Saturation current density, work function and temperature are uniform.

The first assumption was made plausible in Section 2.1.3 and has also been experimentally been verified [64, 65]. Electrons in the emitter in reality follow Fermi-Dirac statistics. However, electrons that are actually emitted have a high energy with respect to the Fermi level so that their statistics can be approximated with Boltzmann statistics. The error in the distribution function is below 0.01%. Furthermore, the effects of band structure are generally small [66] as the shape of the energy bands is not noticeable on the scale of the energy width of the emitted electrons.

Using the proposition that an emitter can be replaced by a virtual emitter, one can presuppose that the shape of the virtual emission plane is correctly given by current space charge models and that nevertheless the velocity component of the emission function is erroneous. This provokes the following question: how does the spot distribution respond to a perturbation of the emission function? Effects of changes in the emission distribution on the optics will be researched using SCELOP in the next chapter.

One can obviously define a virtual emitter plane provided the emission function is consistent with the phase space density at the virtual emitter plane. An analysis of the correctness of present emission functions can be achieved by using a model that abandons the remaining assumptions. Such a model is then expected to reproduce the space charge barrier and obtain the five-dimensional phase space distribution at the barrier. The development of the model and the simulation results will be discussed in the subsequent chapters.
Chapter 3

Influence of emission properties on spot size

In the previous chapter, we discussed the assumptions that are used in most calculations of space charge limited beams. We wish to study the validity of these assumptions in a CRT, and to examine the extent to which they have an influence on the optics of the system.

In this chapter we will address the question whether the spot distribution at the target plane is sensitive to inaccuracies in the virtual emission function. In Section 3.1 the optics of a CRT will be discussed on the basis of a simple rotationally symmetric CRT gun. Following that, the crucial point of uncertainty in using a virtual emitter is demonstrated in Section 3.2. In Section 3.3 we will formulate the systematization of the subsequent analysis. In order to get a handle on the variation of a general spot distribution and a general emission function, we will seek to characterize both in terms of a small number of parameters. Also, the mathematical procedure to calculate spot distributions with SCELOP is discussed. In Section 3.4 the variations on the spot distribution due to modifications of the emission function are examined for three guns, a rotationally symmetric CMT gun, a quadrant symmetric CMT gun and a quadrant symmetric TVT gun. The influence of the emission function on the spot distribution depends strongly on the gun settings. The results of the analysis will be discussed in Section 3.5.

3.1 Optics of a CRT gun

3.1.1 Configuration of the CRT

The schematic set-up of a CRT gun is shown in Fig. 3.1. This is a so-called bipot gun. It consists of a cathode, a current limiting aperture, an extractor, a focus bus and an anode bus. For historical reasons, current limiting aperture and extractor are referred to as G1 and G2 (grid 1 and grid 2) respectively. Cathode, G1 and G2 are collectively known as the triode. The electrons emitted from the cathode are focussed to a crossover by a lens formed in the triode, referred to as triode lens. The crossover is imaged to a spot at the viewing screen by a prefocus lens, formed by the extractor and the focus bus, and the so-called main lens, formed by the focus bus and the anode. The region between triode lens and prefocus lens is sometimes referred to as the beam forming region, the region past the prefocus lens is called the spot forming region. The region between anode and screen is drift space, the screen being at anode potential.

Relating to common electron optical nomenclature, G1 is a Wehnelt electrode, and the triode lens is equivalent to a cathode lens. The name ‘bipot’ stems from the main lens type: formed by two electrodes it is a bipotential lens.

This is the basic arrangement for most CRT guns. Other guns may have complex prefocus and main lenses consisting of more electrodes, but other than that the layout is similar. Figure 3.1 shows typical values for the electrode voltages. Anode, G1 and G2 voltage remain constant during operation of a gun. G1 is always at ground potential. The anode, focus and cathode voltages are denoted $V_a$, $V_{foe}$, and $V_{cat}$ respectively.

The current is regulated by biasing the cathode potential, in the range from 20 V for high currents, to 100 V for low currents. An intermediate current is 3 mA for TVT guns. For CMT guns the current is
Figure 3.1: Schematic drawing of a bipot gun. In the interest of clarity, the scale is severely compromised. The diameters of the G1 and G2 aperture are ≈ 0.35 mm, the diameter of the focus bus aperture is ≈ 1.5 mm. The diameter of both focus bus and anode bus is ≈ 6 mm. The dotted box adjacent to the cathode indicates the location of the virtual emitter.

about a factor of ten lower since the spot size must be smaller and the shorter viewing distance allows for a lower spot intensity. The G2 voltage is tuned to obtain cut-off at a particular cathode voltage, often 160 V.

With the focus bus voltage, the prefocus lens and main lens settings can be varied to obtain the smallest possible image of the crossover at the screen. The optimum focus bus voltage (focus voltage at which minimum spot size occurs) thus is largely determined by the position of the crossover and distance between main lens and screen. More sophisticated guns have a segmented focus bus to enable dynamic focussing as the spot is scanned over the screen to correct for the varying image distance.

3.1.2 Contributions to the spot size

A CRT gun for display applications is designed to produce a small, high intensity spot at the screen. The current density at the screen is limited by a number of factors, namely the size of the crossover, the spherical aberration of the lenses and the space charge effect. The size of the crossover is determined by the thermal spread of the emitted electrons (resulting in a so-called thermal spot size), space charge effect in the beam forming region and the aberrations of the cathode lens. Generally, the main contributions to the spot at the screen are the thermal spread, the spherical aberration of the main lens and the space charge effect in the drift space between main lens and screen. Chromatic aberration can be neglected [67, 68].

**Thermal spot size** The origin of the thermal spot size is the fact that electrons from the cathode are not all emitted perpendicularly to the cathode surface. The optical equivalent of cathode and cathode lens system is shown in Fig. 3.2, which illustrates the characteristic of the crossover as the image of the angular distribution at the cathode. The contribution of the thermal spread to the spot size at the screen can be approximated by considering the image of the crossover without aberrations of the subsequent optical elements. The size of the imaged crossover can then be obtained from the conservation of reduced brightness in the system. Replacing the current density distribution at the cathode by an effective average current density \( j \), the reduced brightness can be written [69]

\[
B_r = \frac{j}{\pi k_B T/e},
\]

with \( T \) the cathode temperature. The introduction of \( j \) is necessary to account for the gradient of the space charge limited current density over the emitting area. The reduced brightness at the screen is given
3.1. Optics of a CRT gun

![Figure 3.2: Optical equivalent of the cathode and cathode lens. The initial angular distribution of the electrons is imaged to a finite sized spot.](image)

by

\[ B_r = \frac{I}{\pi \alpha_s^2 \pi r_s^2 V_s}, \]  

(3.2)

in which \( I \) is the current, \( \alpha_s \), \( r_s \), and \( V_s \) are the beam half opening angle, spot radius and beam potential at the screen, respectively. Ignoring the effects of aberrations and space charge to the crossover size, \( \alpha_s \) can be approximated as the beam radius in the main lens \( r_{ml} \) divided by the distance between main lens and screen \( l \). Solving for \( r_s \) yields

\[ r_s = 2 \sqrt{\frac{I k_B T}{\pi e V_{sl} r_{ml}}} l, \]

(3.3)

showing that the contribution to the spot size at the screen due to the thermal motion of the electrons is inversely proportional to the beam radius in the main lens.

**Spherical aberration** A perfect lens deflects rays such, that the difference in gradient is linearly proportional to the beam radius in the lens. Spherical aberration is a lens defect that causes an additional deflection proportional to the third power of the beam radius in the lens. An axial point is not imaged to a point by an imperfect lens, but to a so-called aberration disk with radius

\[ r_{sa} = M C_s \alpha_s^3, \]

(3.4)

with \( M \) the magnification of the lens system and \( \alpha_s \) the half opening angle at the object side, defining the coefficient of spherical aberration \( C_s \). For optical systems dominated by spherical aberration, the minimum spot size radius is \( \frac{1}{2} r_{sa} \), obtained a short distance in front of the image plane.

The half opening angle at the object side is related to the half opening angle at the image side via the angular magnification, \( \alpha_i/\alpha_o = M_o \). In a CRT, \( \alpha_i = \alpha_s \) and Eq. (3.4) can be written

\[ r_{sa} = M^4 C_s \alpha_s^3 (V_s/V_{foc})^{3/2}, \]

(3.5)

where the relation \( M_o M = \sqrt{V_{foc}}/V_s \) was used. Approximating \( \alpha_s \) with \( r_{ml}/l \) it can be seen that a smaller beam radius in the main lens results in a smaller spherical aberration spot contribution.

**Space charge effect** The influence of space charge has been discussed in the previous chapter, where the effect was estimated by assuming that the beam leaves the main lens parallel.

To obtain an estimate for the space charge spot contribution in closed form, it is assumed that the beam radius increase due to space charge is small compared with the beam radius. Equation (2.30) can simply be integrated over the drift space length to yield the beam radius increase, which can then be identified with the spot contribution due to space charge

\[ r_{sc} = \frac{1}{2} \frac{l^2 P}{r_{ml}}, \]

(3.6)
with \( l \) the distance between main lens and screen as before.

In Section 2.2.2 it was shown that the space charge of a uniform beam acts as an aberration-free negative lens. This may lead one to conclude that space charge leads to defocus, but that Eq. (3.6) can be compensated by refocussing. In contrast with reality, computer simulations are equipped with a knob that turns off space charge defocussing simply by not including the self-field in the computations in the spot forming region. Indeed, Fig. 3.3 shows that space charge defocus can partly be compensated by increasing the strength of the main lens. Nevertheless, the minimum spot size is increased. The derivation leading to Eq. (3.6) assumes that electrons are prevented from crossing the axis, a situation that occurs when the charge distribution near the minimum cross section can be considered to be smoothed out, and the unperturbed crossover size (i.e. in the absence of space charge) is small [70]. In a CRT, these conditions are fulfilled, so that rays converging past the main lens cannot cross over due to the strong lateral space charge forces that ensue. A point is imaged to a disk, thus contributing to spot size, even if the imaging system is aberration free.

Equation (3.6) underestimates the beam radius increase. In reality, the effect of space charge will be greater, since the beam does not remain parallel but converges past the main lens. The decrease in beam width results in an increased space charge effect. Also, nonuniformity of the current density profile gives a larger beam spreading. Schwartz [71] derived the so-called universal beam curve from Eq. (2.30) to calculate the minimal possible spot size at a screen at given distance, for given beam diameter and pervaence. For CRTs, the minimal spot size is obtained by focussing the beam such, that its minimum cross section occurs a short distance in front of the screen rather than precisely at the screen. The universal beam curve is based on an idealized uniform radial beam profile. The spot sizes given by the universal beam curve cannot be obtained in practice and the curve is of very limited value for predicting spot sizes in actual CRTs. Nevertheless, the result that the minimum cross section occurs a short distance in front of the screen is valid for real CRTs.

All approximations notwithstanding, the scaling introduced in Eq. (2.30) demonstrates that the contribution of the space charge effect is inversely proportional to the beam radius in the main lens, which is made more explicit in Eq. (3.6).

**Total spot size** Several theories on how to add the different contributions to obtain a qualitative notion of the total spot size are known from literature [68, 72, 74]. Let us first determine the relative
importance of the contributions on the basis of typical CRT parameters. Values for the beam current, screen voltage, beam length and beam radius in the main lens are mentioned in the previous section, \( I = 1 \text{ mA}, V_x = 30 \text{ kV}, l = 0.5 \text{ m} \) and \( r_{ml} = 1 \text{ mm} \). Additionally, \( V_{loc} \) and \( M \) are needed, typical values for which are \( V_{loc} = 0.28 V_x \) and \( M = -9 \) respectively. Finally, the average current density at the cathode and the cathode temperature are needed, for which the values \( \bar{J} = 3 \text{ A/cm}^2 \) and \( T = 1000 \text{ K} \) are used respectively.

For an indication of the influence of spherical aberration \( C_s \) can be calculated accurately with particle optical computations. For a simple order-of-magnitude calculation a book of tables \cite{75} will suffice. A two cylinder main lens with lens diameter 8 mm and gap 0.8 mm, dimensions that are comparable to the rotationally symmetric CRT gun that will be discussed later in this chapter, has a coefficient of spherical aberration \( C_s = 1.3 \text{ m} \). Substitution in Eq. (3.5) yields \( r_{sa} = 0.45 \text{ mm} \). The space charge effect contributes \( r_{sc} = 0.37 \text{ mm} \) which follows from Eq. (3.6). The thermal spot size follows from Eq. (3.3), \( r_t = 0.35 \text{ mm} \).

Obtaining an exact total spot size by combining \( r_s, r_{sa} \) and \( r_{sc} \) is a hopeless exercise in view of all approximations that underlie the separate contributions. Since the contributions are of similar magnitude, accurate calculations of total spot size can only be performed using sophisticated computer simulations that take all effects into account as precisely as possible.

In considering the three aforementioned contributions separately, \( r_{ml} \) emerged as an important parameter of electron gun design. The spherical aberration increases with \( r_{ml} \), both thermal spot size and space charge effect on the other hand decrease with \( r_{ml} \). Regardless of the inaccuracies of whichever simple spot addition model, it is obvious that there exists an optimal \( r_{ml} \) that minimizes the spot size at the screen. Optimizing the beam radius in the main lens is an important function of the prefocus lens. The spot at the screen will always be a compromise between the thermal contribution and the space charge effect on the one hand, and the spherical aberration of the main lens on the other hand. As the specific current density or beam shape determines the effect of space charge and aberrations, the optimal lens settings for given screen voltage and distance depend not only on the position of the crossover, but also on its size and on any effect that will cause a change in beam shape, like the emission properties of the virtual cathode. This argues the importance of an accurate virtual emission function when determining the optimum lens settings with computer simulations.

### 3.2 Variations in the virtual emission function

We discuss in more detail the use of the simple one dimensional planar Langmuir model to describe more complex geometries operating in space charge limited mode, employed by SCELOP. In this computational method the real emission plane is replaced with a virtual emitter plane that is positioned overlaying the curved potential minimum plane. The curvature of the potential minimum plane is dependent on the virtual emission function since the latter determines the space charge field in front of the emitter.

Even if the intended shape of the emission plane, i.e. the curved potential minimum, is not reproduced exactly, the self-consistent solution obtained with such a virtual emission model can nevertheless be correct. Replacing a real emitter with a virtual emitter is always a valid possibility. The particular choice of shape and position of the virtual emitter plane is open, provided that the emission function at the virtual emitter plane is consistent with the phase space of the electron beam.

A small change of virtual emission function could significantly influence subsequent calculations of the spot distribution and if this is indeed the case, it is imperative that the virtual emission function is correct, i.e. consistent with the phase space.

This realization provokes two questions: firstly, do small changes in the virtual emission function influence the spot distribution significantly? Secondly, is the virtual emission function based on the onedimensional Langmuir model correct? The first question is addressed in the remainder of this chapter. In the case that a spot distribution turns out to be insensitive to changes in the emission function, the precise form of the emission function is not important for the spot size calculation. We will investigate the sensitivity of the spot calculation by introducing variations about the emission function that is used customarily in SCELOP. The effect of practical errors in the emission function can then be determined via interpolation of the variations.

The usefulness of such an analysis depends on whether the customary emission function is at least approximately correct. If the emission function is totally inappropriate, leading to a potential distribution with considerably differing optics, the analysis of the subsequent spot calculations is pointless for the
optical system under consideration. In Chapter 4 it will be shown that the virtual emission function is remarkably consistent with the phase space. The variations that are introduced intentionally for the analysis of the sensitivity to emission properties are much larger than a possible inaccuracy in the emission function. The change in spot distribution due to a modified emission function is thus representative for a change that would appear as a result of small misrepresentations of the emission properties.

3.3 Classifying the parameter space

The examination of the dependency of spot distributions on emission properties comprises a massive amount of degrees of freedom. The general spot distribution is given by a two-dimensional function and the general emission properties are given by a five-dimensional function. To reduce the vast parameter space of a classification of the spot shape in terms of the emission properties, one may impose some constraints on the emission properties and some restrictions on the spot shape measure.

The freedom in the five-dimensional function that prescribes the emission properties can be reduced by assuming that any effect that occurs in the region between the cathode and the virtual cathode does not induce a major change from the physical cathode emission properties. The distributions will shift or change in width, but other than that the shape will remain the same.

To reduce the parameter space of the spot distribution, one can use the spot size to characterize a spot shape partly in terms of a small set of numbers. To attach a quantitative meaning to the spot size value of a spot distribution it is important to observe the concept of various spot size measures.

The spot size measures and the characterization of the emission functions will be discussed in turn below.

3.3.1 Overview of spot size measures

To define the different concepts of spot size, we introduce a number of auxiliary functions. The most general representation of a spot distribution is the current density function, \( J(x, y) \), providing the current density distribution at a given plane, parametrized with variables \( x \) and \( y \). We will assume that the plane of interest is at all times a flat screen perpendicular to the optical axis and \( x \) and \( y \) are Cartesian coordinates in a system with the \( z \) axis along the optical axis. More specific to CRTs, the \( x \) and \( y \) directions are horizontal and vertical with respect to the viewing screen, respectively.

From the spatial current density the line spread functions are defined as follows:

\[
LSF_x(x) = \int_{-\infty}^{\infty} J(x, y) \, dy
\]

\[
LSF_y(y) = \int_{-\infty}^{\infty} J(x, y) \, dx.
\]

Furthermore, we will make use of the accessory edge integral

\[
I_x(x) = \int_{x'=-\infty}^{x} LSF_x(x') \, dx'
\]

\[
I_y(y) = \int_{y'=-\infty}^{y} LSF_y(y') \, dy'.
\]

With the term 'spot size' one commonly refers to a characteristic width of the spot distribution based on the functions \( J \), \( LSF_{x,y} \) or \( I_{x,y} \). Spot distributions are thus described in terms of a \( x \) and a \( y \) spot size. For rotationally symmetric spot distributions, the \( x \) and \( y \) variants are identical and the index is left out.

Usually, as a measure for spot size, the \( LSF \)-5 width is taken, i.e. the full width of the line spread function at 5% of the maximum height. During the design process of a CRT gun in addition the \( LSF \)-30 is used as an important measure. Due to the presence of numerical noise, it is difficult to obtain a definite value for the spot size, which can be seen from Fig. 3.4. The noise gives an inaccuracy in the \( LSF \)-5 width, since both the maximum and the position of the 5% levels are susceptible to noise.

The noise originates from the nature of Monte Carlo simulations. For different simulations, both position and height of the maximum will differ. But even within one simulation, the position and value will depend on the manner of discretization of the data (the amount of bins in which the data is sampled).
3.3. Classifying the parameter space

Figure 3.4: Gaussian distributed data (1000 samples) plotted together with the distribution function.

Figure 3.5: Same data as Fig. 3.4 but now the integral is plotted.
Objective, a preferable classification of the intensity distribution is based on the edge integral. Experimentally, the edge integral can be obtained by measuring the current when a spot is scanned over a knife-edge. The spot size can be defined as the full width of the interval containing a fraction 1-2f of the total current, positioned as shown in Fig. 3.5. The number f is referred to as the knife edge value. Integrating the line spread function provides a natural smoothing of the Monte Carlo noise and it reduces the influence of discretization. The advantageous influence of integration is clearly visible when comparing Figs. 3.4 and 3.5.

To confirm that the knife edge spot measure gives better results quantitatively, both spot size definitions were applied on a Gaussian test spot. As spot size measures, the LSF-5% and the knife edge-0.718% were taken. The exact spot size for these measures is $2\sqrt{\ln 0.05} \approx 3.46$ as obtained from the analytical distribution function. The discretization was varied by distributing the samples over 20, 30, 40, ..., 140 bins. The spot size was determined several thousand times, each for a spot containing 1000 samples. The average and the standard deviation of the resulting spot sizes are plotted in Fig. 3.6. The discretization is quantified as the number of bins over which the samples are distributed. The knife edge spot measure is generally closer to the exact result of 3.46 and the variance in the results is smaller. The discretization has less influence on the knife edge spot measure than on the linear spread spot measure.

For this consideration the knife edge value was chosen such, that the resulting spot size is equal to the corresponding LSF spot size. This can only be done if the shape of the distribution is known in advance. Therefore in general one can not compare LSF spot widths to knife edge spot widths, only knife edge spot widths among each other.

The adopted use of the LSF-5 spot size measure in the CRT industry is based on the way a spot at the screen is perceived by the human eye [73]. In the following section, we are interested solely in an objective measure for the change in spot distribution. The knife edge spot measure, being less sensitive to numerical artifacts, is especially advantageous to use for calculations based on a small number of samples.

### 3.3.2 Quantification of the start conditions

The adapted Maxwell-Boltzmann distribution as discussed in Section 2.5.1 is consistent with the Maxwell-Boltzmann distribution at the cathode, which will be explicitly shown in Chapter 4, and it therefore serves as a starting point for the perturbed emission function. The spot size changes pertaining to variations in the axial velocity distribution and the transversal velocity distribution will be investigated. In the
variations, the velocity distributions maintain their Gaussian character, only the widths are modified. The x and y component of the transversal velocity are not considered separately.

The mathematical independence of the two velocity distributions can be employed to provide three different modes of introducing changes:

Changing the axial velocity distribution  This can be related to a work function inhomogeneity. Sharply localized work function variations will lead to an increased axial velocity distribution. Within the presumed virtual emitter model, the transversal velocity distribution will not change since the work function is decoupled from the transversal direction.

Changing the transversal velocity distribution  This can be related to a surface graininess. The angular distribution of thermionic emission can be described by a cosine distribution. If the surface is rough, the emission distribution will be a convolution of the cosine distribution with the distribution of surface normals. At the real source plane, the transversal velocity width can increase at the cost of the axial velocity. This will not lead to a decreased axial velocity width at the virtual emitter since the virtual emitter acts as a filter for low axial velocity electrons.

Changing the total velocity distribution  Changing the transversal and axial velocity distributions by the same amount represents a temperature variation.

3.3.3 Quantification of the spot size change

We wish to quantify the change in spot size when the start condition distribution width pertaining to one of the three modes is increased. The spot size change can formally be described by a functional $d_{\text{spot}}(j)$ of the emission distribution function $j$. The choice of the spot size measure on which $d_{\text{spot}}$ is based is in principle still open.

If we constrain ourselves to the three independent distribution variations introduced above, the emission function can be characterized by a single parameter and the functional can be replaced by a function of the distribution width $w_i$, in which $i$ denotes one of the three modes of the distribution change: axial velocity, transversal velocity, or both. The functional is normalized so that $w = 1$ signifies the unperturbed distribution. A value of e.g. $w = 1.1$ indicates that the velocity distribution is 10% broader than the unperturbed velocity distribution.

To condense the parameter space further, we look for a single parameter that assumes a high value when the spot size is sensitive to changes in the start conditions, and low otherwise. High sensitivity to the start conditions appears as a steep rise in the function $d_{\text{spot}}(w_i)$.

The sensitivity of the spot size to changes in the start conditions can thus be described by a dependency parameter $\Delta$ defined as the overall slope of the function $d_{\text{spot}}$ divided by the unperturbed spot size $d_{\text{spot}}(w_i = 1)$. The slope is determined in the region around the unperturbed width. For example, a value of $\Delta = 0.1$ means that an increase in the distribution width of 10% leads to an increase in spot size by 1%.

Summarizing, the parameter space is reduced by characterizing both emission function and spot distribution in terms of a single scalar representing the width. In this manner, the dependency of the spot size on the emission function can be expressed in a single number representing the rate of change of the spot distribution as a functional of the emission function. This reduction of the parameter space is depicted in Fig. 3.7.

3.3.4 Calculation of spot distributions

Spot size calculations in SCELOP are separated in a field calculation part and a spot calculation part. In normal operation, the field calculations and the spot calculations make use of an identical emission function. This guarantees the consistency between field and spot distribution with respect to the emission function.

SCELOP makes use of a mapping mechanism to determine both the space charge field and the spot distribution on the basis of a relatively small number of trajectories [76]. This enables fast field calculations and a spot distribution analysis that allows for different types of spot measures. In the subsequent analysis, unless otherwise stated, the spot size measure that will be used is the knife edge 1%.
The mapping algorithm is applied with a virtual emission function as discussed in Section 2.5.1. Regardless of the particular perturbed emission function that will be used for the calculation of the change in spot distribution, the calculation of the space charge field arises from an unperturbed distribution. This approach has the advantage that the spot calculations for various emission functions can be performed without a recalculation of the potential distribution. The disadvantage is, that the spot distribution obtained in this manner is not consistent with the potential distribution, in contrast with the normal use of SCELOP.

This is nevertheless the method we will employ to examine the influence of the virtual emission function. As a check whether the lack of consistency has any effect on the spot size dependency, the mapping algorithm can be altered so that the spot distribution and the space charge field remain mutually consistent when changing the emission function. The space charge field may be influenced by artifacts in the mapping mechanism but if it turns out that the effect on the spot distribution is much larger than the effect on the space charge lens action this can be disregarded. This will be verified later in this chapter.

To calculate \( d_{\text{spot}} \) as a function of \( w_l \), several thousands rays with the appropriate current and start conditions corresponding to the particular emission distribution are tracked from the virtual cathode to the screen for each value of \( w_l \).

The attributes of a single ray are set as follows: the start position is sampled uniformly from the emitting area. The velocity is sampled from the distributions with width \( w_l \), with a velocity perturbation according to the position in the emitting area. The total current carried by the ray is determined from the potential minimum at the position of emission with Eq. (2.43).

The spot at the screen comprises a set of weighted two-dimensional data, in which the current of the ray serves as a weight factor.
3.4 Simulation results

Three types of electron gun have been investigated: a rotationally symmetric (RS) CMT gun, a quadrant symmetric (3D) CMT gun and a 3D TVT gun, in Chapter 2 referred to as 21" CMT, 15" CMT MkIIa and pi3 respectively.

We use the procedure outlined in the previous section to attach a single number $\Delta$ to each optical configuration, $\Delta$ representing the sensitivity to emission function variations. The optical configuration of an electron gun depends on its geometry and its electrode potentials. For a given gun, the optical system must therefore be characterized for a range of cathode and focus bus voltages.

The analysis will be performed for three cathode voltages, representative for the current range of each gun, and a range of focus bus voltages around the optimum focus.

To calculate $\Delta$, the knife edge-1% spot size is calculated for a set of four values of $w$: 0.6, 1, 1.5, 2, where $w = 1$ indicates that the distribution function is unperturbed. All spot size calculations are repeated a number of times for different random seed. This provides a measure for the error in the spot size data and reduces the numerical noise. $\Delta$ is set to the slope of the linear least squares fit to the four $d_{\text{spot}}(w_i), w_i$ pairs. In Fig. 3.7 $\Delta$ is represented by tan $\phi$.

The spot distributions are calculated using 300 fit trajectories and mapping 20000 trajectories for smoothening of the space charge distribution. These values hold for the quadrant symmetric guns; for rotationally symmetric guns respectively 100 and 1000 trajectories are used.

3.4.1 RS gun

The RS gun in question is the bipot gun, discussed in Section 3.1. The calculated focus characteristics are shown in Fig. 3.8. The focus characteristics shown in Fig. 3.3 were obtained with this gun as well.

In Fig. 3.9 $\Delta$ as a function of focus voltage is depicted for the three modes of distribution changes.

We notice that the spot size is influenced most by an increase in total velocity width. An increase in solely axial velocity hardly has any influence. We see that the spot size increase is mainly due to the transversal velocity. Furthermore, $\Delta$ is very sensitive to the focus voltage. Near the optimum focus, $\Delta$ is sharply peaked.

In Fig. 3.10 focus loops for start conditions with different energy width are shown together with the resulting $\Delta$ for cathode voltages $V_{\text{cat}}=25$ V, 50 V, 80 V. For all three cathode voltages the spot increase due to an axial velocity increase is very small and only the spot increase due to a transversal velocity
Figure 3.9: The influence of start conditions for three modes versus focus voltage for $V_{cat}=80$ V

width increase shown.

For all three currents, $\Delta$ is peaked at the optimum focus voltage.

3.4.2 CMT gun

The 15° CMT MKIa gun is a quadrant symmetric gun with a circular current limiting aperture but no other rotational symmetry in either triode or main lens. In Fig. 3.11 the effect of variations in the emission function on the focus characteristics and spot sizes are shown for three current settings, $V_{cat} = 30$ V corresponding to $I = 3.32$ mA, $V_{cat} = 90$ V ($I = 0.376$ mA) and $V_{cat} = 110$ V ($I = 0.087$ mA).

For the $x$ spot size, similar behaviour is observed as in the RS case. The spot size change is prominently visible when the transversal velocity is changed, whereas an axial velocity change hardly affects the spot size. For this reason, only the $\Delta$ corresponding to a transversal velocity width increase is shown. $\Delta$ is peaked near the optimum focus voltage and the peak value is higher for lower currents.

For the $y$ spot size, strongly deviating behaviour is visible. For high current, optimum focus in the $y$-direction occurs for lower voltage than optimum focus in the $x$-direction. For lower currents, and consequently less space charge effect in the spot forming region, the optimum focus voltage increases, in agreement with the fact that the defocus due to space charge decreases. The fact that this is especially obvious for the $y$ spot size can be explained by the particular prefocus lens in this gun, which shapes the electron beam such that the cross section in the main lens is flattened in the $y$-direction, resulting in a more pronounced space charge lens action in this direction.

3.4.3 TVT gun

Figure 3.12 shows $\Delta$ and focus characteristics for the quadrant symmetric TVT gun at three cathode settings $V_{cat} = 20$ V corresponding to $I = 5.755$ mA, $V_{cat} = 50$ V ($I = 2.807$ mA) and $V_{cat} = 100$ V ($I = 0.525$ mA). The features that are present also in the plots of the CMT gun reappear here, in that the $x$ slope peaks in optimum $x$ focus and that the $y$ optimum focus increases for decreasing current. In comparison with the results for the CMT gun, the main difference for the TVT gun is the fact that the $\Delta$ peak value is much lower, which means that the sensitivity to the start conditions is less.

3.4.4 Spot shape change

The graphs in Figs. 3.10–3.12 show that the knife edge-1% spot size changes with when the transversal velocity component in the emission function is modified, and that the change itself, quantified with
Figure 3.10: Focus loops for RS gun for various $w$, for transversal velocity distribution and the resulting $\Delta$ for $V_{\text{col}} = 25$ V, 80 V and 100 V (from top)
Figure 3.11: x and y spot size focus loops for 3d CMT gun for various $w_i$ for transversal velocity distribution and the resulting $\Delta$ for $V_{cat}=50$ V, 90 V and 110 V (from top)
Figure 3.12: $x$ and $y$ spot size focus loops for 3d TVT gun various $w$, for transversal velocity distribution and the resulting $\Delta$ for $V_{\text{ext}}=20 \text{ V}, 50 \text{ V}$ and $100 \text{ V}$ (from top)
parameter $\Delta$, varies with focus voltage. In Fig. 3.13 similar plots of $\Delta$ are shown, but here with different spot size measures, namely the \textit{LSF-5} and \textit{LSF-30} measure together with the knife edge-1%. From the observation that the curves have different shape one can conclude that the spot at the screen changes shape as well as size when the emission function is modified.

Furthermore, the maximum of $\Delta$ is higher for the \textit{LSF} measures. This is also true for low currents: while in Fig. 3.11 $\Delta_{x}$ based on the knife edge peaks at 0.3, the maximum $\Delta$ based on the \textit{LSF-5} and \textit{LSF-30} is 0.8 and 0.95, respectively. For the TVT gun the values are somewhat lower, but still higher than the values based on the knife edge plotted in Fig. 3.12.

### 3.4.5 Consistent spot shape and potential distribution

The spot shape calculations that have been performed thus far are not consistent with the potential distribution, in the sense that the potential distribution is obtained using the customary, unperturbed emission function throughout the analysis, even as the spot shape is obtained with an intentionally deformed emission function. It can be expected that tracing electrons in a consistent potential distribution, obtained with corresponding emission function for each spot shape calculation, gives different spot shapes, but it is not obvious to what extent.

In Section 2.2.2 the effect of space charge on the shape of the beam was characterized optically as a weak negative lens effect. For discussion purposes the same characterization can be made here, although the space charge lens is not weak in the region near the virtual cathode.

With varying emission function a spot size change is observed, which undeniably implies a beam shape change. This beam shape change must result in a change in optical effect of the space charge field, which is not taken into account in the triode section because of the specific, nonconsistent calculation method that is used. If a change in the emission function results for instance in a narrower beam, the space charge lens will become stronger. However, the effect on the optics is very small.

The influence of the altered space charge lens can be examined by changing the mapping mechanism to incorporate a changed emission function. Rather than repeating the complete analysis of the three guns for a consistent potential distribution, we can test whether the spot distribution changes significantly if the space charge lens is changed.

Previously, the potential distribution (and thus space charge lens) was based on an unperturbed emission function and the spot distribution was based on a perturbed emission function, now the roles will be reversed: the potential distribution will be calculated for an emission function with a broader
total energy distribution while the spot distribution will be determined from the unperturbed emission function. This is again a nonconsistent approach in order to keep the number of changing parameters limited.

In Fig. 3.14 the LSF curves of the quadrant symmetric TVT gun are shown for matching potential distribution, and a potential distribution calculated with a perturbed emission function (transversal velocity distribution width $w = 0.6$). Calculation of the curve with $w = 0.6$ has been performed twice with a different random seed to show the noise that is inherently present. The differences in spot shape due to the changes in the potential distribution are small, which makes a complete analysis of the dependency parameter $\Delta$ with consistent spot shape simulations unnecessary. The nonconsistent simulations are sufficiently accurate for the analysis of $\Delta$.

### 3.4.6 Validation of the computation method

The parameter $\Delta$ is subject to two sources of error: the numerical error in the spot size data and the goodness of the linear fit used to determine the sensitivity to the start conditions, the slope $\tan \phi$ in Fig. 3.7. The error in the spot size is determined from repeated spot size calculations with different random seed. The error in the slope is determined directly from the least squares fit. It turns out that the resulting total error in $\Delta$ is usually below 0.05.

With regard to the quality of the linear fit we also note the following. All calculations have been performed using a arguably small set of distributions $w = \{0.6, 1, 1.5, 2\}$. This gives a global impression of the dependency in the range around 1 with distribution widths changes in the order of 50% but it tends to conceal the behaviour for smaller changes in $w$. This may give an error if the resulting spot size $d_{\text{spot}}(w)$ shows a behaviour that is significantly nonlinear with $w$. As a falsification, some calculations of $\Delta$ were repeated for a set $w = \{0.8, 0.9, 1, 1.2\}$, yielding similar results. In Fig. 3.15 two graphs of $\Delta$ are shown for varying sets of $w$.

A further point of attention is the mapping algorithm. One has to be careful that the mapping algorithm does not absorb the variations introduced when varying the emission properties. To ensure that this is not the case, the spot size can also be calculated without mapping algorithm on the basis of a small number of trajectories. Results obtained in this manner, that is much more time-consuming, are very similar.
3.5 Conclusions

As discussed in Section 3.1, the spot of an electron gun is formed by imaging the crossover with two lenses onto the screen. The spot size is influenced by

- thermal spot size,
- aberrations and space charge effect in the triode,
- aberrations of the main lens,
- space charge effect in the drift region, between main lens and screen,
- defocus

In this chapter the influence of a change in velocity distribution at the virtual emitter on the spot size at the screen was analyzed. The analysis has been performed rather thoroughly for an increase in transversal velocity distribution width. The influence of the axial velocity distribution is much smaller, which is a confirmation of the statement made earlier that the influence of chromatic aberration on the spot size can be neglected. This is a consequence of the fact that the initial axial velocity can be ignored with respect to the gain in axial velocity everywhere in the gun except very close to the virtual emitter. The axial velocity of a particle is determined almost completely by the potential distribution rather than the initial velocity. For the transversal velocity this is not the case since the space charge potential distribution is steeper in the axial direction than in the transversal direction, following the assumption that the emitter region can be modelled with locally planar diodes.

The overall behaviour that is visible in the simulations is, that $\Delta$ is high in focus and low out of focus. This can be explained by the thermal spread contribution to the spot size.

For an ideal cathode lens, the crossover size is directly related to the transversal velocity width at the virtual emitter. Increasing the transversal velocity distribution thus leads to an increased crossover size and consequently an increased spot size. Based on the thermal spot size, $\Delta$ must everywhere be positive and smaller than unity and drop to zero when moving far out of focus. Figure 3.11, bottom left, is a good example of this feature of $\Delta$. The focus characteristics for different velocity widths are shifted vertically.

Two distinct types of deviation from this behaviour are visible. In some graphs, no peak in $\Delta$ at optimum focus is present. This can be understood by regarding the shift of the focus characteristics towards higher or lower focus potential. Due to spherical aberration and space charge effect in the
beam forming region the minimum cross section moves with respect to the conjugate plane of the screen. Figure 3.12, bottom right, is a good example of this behaviour. When changing \( w \), the focus characteristics shift not only vertically, along the spot size axis, but also horizontally, along the focus voltage axis. An increase in \( w \) coincides with a decrease in optimum \( V_{\text{opt}} \). When decreasing \( V_{\text{opt}} \) the main lens becomes stronger and consequently the conjugate plane of the screen moves towards the screen. We can thus conclude that increasing \( w \) shifts the minimum crossover towards the screen.

Furthermore, in some cases \( \Delta \) becomes negative: an increase in velocity width leading to a decrease in spot size. This unexpected behaviour can be explained by considering the space charge effect in the spot forming region. The beam radius in the main lens increases for increasing \( w \), and consequently the space charge lens decreases in strength. This is consistent with the observation that the negative \( \Delta \) appears for high currents only. For both the CMT gun and the TVT gun the space charge effect is not isotropic since the beam shape in the main lens is elliptical. The increase of the beam radius in the main lens is the result of the spherical aberration in the triode that results in a wider beam due to the increased angular distribution.

Summarizing, the sensitivity of the spot size on the start conditions at the virtual emitter is in general, but not always, high in focus and low out of focus. The behaviour of \( \Delta \) is influenced by the space charge effect in the spot forming region, and the extent of this influence is strongly dependent on the type of gun. \( \Delta \) reaches values of up to 0.4 for the knife edge-1% spot measure. For more common spot measures \( \Delta \) can reach values as high as 0.95, based on the LSF-30 spot measure. The observation that different spot size measures have a different sensitivity reflects the fact that the spot also changes shape even though the emission function does not. If we want spot shape calculations to be accurate to within say 10\%, the transversal velocity distribution at the virtual emitter must then be accurate to within 10\%.

Since the emission properties of the virtual cathode do notably influence the spot shape it is important to know the emission function of the virtual cathode quite accurately. Whether the customary emission function is correct depends on the validity of the local application of Langmuir's model, and on the approximations that the space charge field is smooth, statistical Coulomb interactions play no role, and the cathode surface is flat and homogeneous. These remaining assumptions will be discussed in the next chapters.
Chapter 4

Emission function of a virtual cathode

Computer simulations including space charge limited emission in a self-consistent manner often make use of analytical space charge models like Child’s or Langmuir’s, applied in a small region in front of the emitter. These models are valid in a one-dimensional diode geometry with infinite parallel cathode and anode. To overcome this restriction, the emitter region is split into small parallel diodes on which the restricted models are assumed to be completely valid. In reality, space charge influences the potential distribution in adjacent diodes whereas in the parallel diode model, the diodes are uncorrelated and charge flow parallel to the emitter are not taken into account. The distinction results in a different potential distribution near the emitter and a difference in velocity distribution of the space charge limited current. In this chapter it will be shown that these differences are minute. For a planar emitter in a nonplanar geometry, the representation using sets of planar Langmuir diodes describes the emitter region adequately.

4.1 Introduction

For the determination of the properties of an electron optical imaging system, a characterization of the emission properties of the source is essential to determine the spot distribution. When the emission from the source is limited by space charge, the emission properties not only determine the spot distribution directly, but also indirectly, via the influence on the optical properties of the region in front of the emitter.

A solution of potential distribution in which the space charge of the electron beam is consistent with the equations of motion in that potential distribution is called a self-consistent solution. In practice, in self-consistent calculations with emission that is known to be space charge limited, one almost always makes use of models for space charge limited emission to describe the emitter region. These models predict how the emission properties of the source affect the field near the emitter. The advantage of the space charge models is the fact that reflection towards the cathode by the space charge cloud in front of the emitter can be accounted for by absorbing it in a virtual cathode.

The space charge models that are commonly used are valid in one-dimensional geometries. Their application to other geometries is justified by the assumption that locally, the geometry is planar and the region immediately in front of the emitter can be described in terms of an array of adjacent local planar diodes. The emission function of the virtual cathode can then be estimated from the combination of the individual virtual cathodes. Examples of programs that utilize an array of local planar diodes in this fashion are the CRT simulation programs SCELOP [54], BEFORE [49] and the commercially available SCALA [50].

The assumption of planarity is fair when the potential distribution in the absence of space charge is planar, that is, when the equipotential lines close to the emitter run along the emitter surface. In a triode geometry consisting of emitter, symmetric current limiting aperture and extractor, this condition is met only on the optical axis. At the edge of the emitting area the unperturbed equipotential line is perpendicular to the emitter surface (if the emitter is a conductor), and the local planarity is violated to the extreme. Furthermore, the individual diodes, that are considered to be independent, are in fact
coupled since current can flow from one diode into another. This lack of transversal invariance is enhanced by the nonplanar potential distribution.

In this chapter, we want to show whether the nonplanarity of the potential distribution does interfere with the validity of local planar diodes and whether the assumption of local planarity in a triode geometry gives accurate results. As it turns out, the virtual emission function is consistent with the the emission properties of the real cathode and the potential distribution in front of the cathode.

### 4.2 Self-consistent solution in the presence of a virtual cathode

In order to identify where the self-consistent solution with real emission function departs from the solution with virtual emission function, let us explicate the self-consistent calculation scheme using a virtual emission function introduced in Section 2.4.2.

In a triode, the emission properties of the virtual emitter are determined by the fields of a large planar cathode, an aperture and an extractor. The region depicted in Fig. 2.13 corresponds to the dotted box in Fig. 3.1. It is useful to make the distinction between the region between cathode and virtual emitter, region $\ominus$ and the region past the virtual emitter, region $\oplus$.

![Diagram](image)

**Figure 4.1:** Iteration loop scheme for a self-consistent calculation using a virtual emission function.

The iterative scheme to obtain the space charge field in the presence of a virtual emitter is depicted in Fig. 4.1. Ignoring all inherent difficulties of an iteration scheme, like oscillatory behavior or multiplicity of solutions, when convergence is obtained we have acquired a self-consistent solution in the sense that

- the Poisson equation is satisfied, by virtue of box 1,
- the virtual emission function is consistent with the local diode approximation and transversal velocity perturbation, by virtue of box 2,
- the electron beam shape past the virtual emitter is governed by the solution to the equations of motion in the space charge field in region $\oplus$, by virtue of box 3.

There is no consistency aspect related to box 4 since its only purpose is to make the tracked beamlets or superparticles suitable for input in box 1. It introduces no additional information but translates a physical concept into a numerical representation.

Within the scope of smooth space charge and homogeneous emission from the real source, the only approximation in the self-consistent solution is the virtual emitter. We cannot get around this if we wish to utilize a virtual emitter based on one of the exact space charge models discussed in Chapter 2. The approximations in the virtual emitter are a cause of uncertainty in the solution. The calculated field is self-consistent, but not necessarily correct.

We regard the field as correct when the potential distribution in the entire system is consistent with the real emission function and the equations of motion. To verify whether this is the case, we need to examine whether the real emission properties are mapped onto the virtual emission properties by the equations of motion in region $\ominus$. This can be understood as follows.

Let us consider the implications of a converged solution obtained without virtual emission function, that is, emission from the real source, according to the procedure of Section 2.2.4. Such a solution is usually not available because of the computational inefficiency, as discussed in Section 2.4, but for explanatory purposes we will theorize about it nevertheless. If we track an electron beam with emission properties given by the real emission function Eq. (2.21) (spatially uniform, Maxwellian cosine distribution), from the real source, and determine the space charge, the newly obtained $\rho$ must be identical to the previous $\rho$ in view of the convergence criteria. Turning this argument around, if we have a given
space charge field, possibly obtained by clever guessing, in which \(\rho\) does not change from one iteration to the next, we have by accident stumbled upon the true self-consistent solution.

Now instead of clever guessing, we use the self-consistent potential distribution obtained with virtual emission function. If we track a beam from the real source to the screen, and \(\rho\) does not change, we have a true self-consistent solution. It is important to note that we do not have to track the beam all the way to the screen. We need only track the beam up to the virtual emitter plane and compare the phase space of the super particles to the approximate virtual emission function. If the two are consistent, the demand ‘track beam up to screen and verify that the change in \(\rho\) is zero’ is implicitly satisfied since the field in region \(\oplus\) is self-consistent.

The self-consistency aspect inferred from box 3 can then be extended to read ‘the complete electron beam is governed by the solution to the equations of motion in the complete space charge field’, box 2 becomes irrelevant, and we recover the solution of scheme Fig. 2.6.

In other words, the verification that the virtual emission function is recovered from the real emission function in conjunction with the potential distribution in region \(\ominus\) shows not only that the virtual emission function is correct, but also that the complete potential distribution is correct.

### 4.3 Virtual emission function in a triode

We follow the argument of the previous section to examine the validity of the approximate virtual emitter. SCELOP was used to calculate the self-consistent potential distribution of a rotationally symmetric gun. The virtual emission function has the general form of Eq. (2.57) with transversal velocity perturbation as discussed in Section 2.5.1.

The exact triode dimensions are given in Fig. 4.2. For the calculation of the potential distribution, the cathode temperature was set to 1350 K with a saturation current density of \(j_{\text{sat}} = 10 \text{ A/cm}^2\). The electrode voltages are chosen such that the space charge limited current is about 1 mA.

Using a virtual emission function composed of local Langmuir diodes as calculated with SCELOP, in this situation the position and depth of the Langmuir minimum on the optical axis are 1.005 \(\mu\)m and \(-0.0436 \text{ V}\) respectively. The current density at the center of the cathode is 6.873 \(\text{A/cm}^2\) and the total space charge limited current is 1.103 mA. The emitting area of the cathode, defined as the radial position at which the space charge limited current has dropped to 0.04% of the saturation current, has a radius 0.108 mm.

The virtual emission consistency check is achieved by tracing particles from the cathode with spatially uniform, Maxwellian cosine initial velocity distribution and collecting them at the Langmuir minimum plane to examine the phase space. The particles do not generate an electric field. They do not deposit charge in region \(\ominus\) and they do not interact with one another.

To obtain sufficiently smooth statistics, the number of particles traced from the cathode must be very
large, especially considering that most current will be reflected by the potential barrier. Two million trajectories with random initial conditions were started at a circular area of the cathode with a radius of \( r_e = 0.125 \) mm, a moderate overfill with respect to the calculated emitting area. At an emission rate of \( j_{\text{inc}} \pi r_e^2 = 4.9 \) mA this number of electrons represents an emission duration of 65 ps.

In Fig. 4.3 the number of electrons at the potential minimum plane is plotted together with the current density that would be expected on basis of the depth of the local Langmuir minimum. The number of trajectories that reach the virtual cathode is 452689, corresponding to a space charge limited current of 1.111 mA. The radial electron distribution at the virtual cathode is in perfect agreement with the local Langmuir minimum. Since the number of particles close to the axis is small, a spike at \( r = 0 \) due to numerical noise is visible.

The gradual change in transversal velocity distribution along the Langmuir minimum is shown in Fig. 4.4. The eleven curves, corresponding to eleven radial intervals, are normalized with respect to the maximum of the distribution function. The coupling of the diodes is clearly visible in the shift of the means of the velocity distributions. The shift in transversal velocity is most prominent far from the axis. It is everywhere much smaller than the width of the transversal velocity distribution. The velocity shift based on Eq. (2.59) is plotted in the same figure. It follows the means of the velocity distributions well up to 90 \( \mu m \) from the axis. At larger radial distances, the velocity shift appears to be overestimated. It is not likely that this lack of consistency is very serious since the current density far from the axis is low. In the following section we will go into this subject.

The virtual emission function as assumed using uncoupled Langmuir diodes is well consistent with the emission function from the real cathode. This is not yet a rigorous examination since the space charge field past the virtual emitter is calculated using other approximations as discussed in Section 2.5.1, like a fitting mechanism, non-self-consistent approach at high potential and uniform beamlet method for space charge calculation.

To make plausible that the approximations in the beam shape past the virtual emitter do not introduce a significant error in the region between cathode and virtual emitter, the simulation was repeated in a space charge field calculated with a slightly different virtual emission function. The transversal velocity perturbation in the virtual emission function was removed and the analysis was repeated for the resulting self-consistent potential distribution. The resulting velocity distributions are superimposed on the previous results in Fig. 4.5. No significant differences arise from the adapted virtual emission function.

Recalling the observation of Chapter 2 that rotationally symmetric simulations are more accurate than quadrant symmetric simulations, it is worthwhile to perform a similar analysis for a quadrant symmetric TVT gun. This gun has a G1 with a rectangular aperture as well as a nonrotationally symmetric focus bus and main lens. The emitting area of the cathode assumes an elliptical shape. Ten million particles
Figure 4.4: Phase space coupling at the virtual emitter. The solid lines are the transversal velocity distributions at the Langmuir minimum in 11 annular intervals with inner radii $r = 0 \, \mu m, 10 \, \mu m, \ldots, 100 \, \mu m$ and radial width $\Delta r = 10 \, \mu m$. The dashed line is the average of the transversal velocity in the 11 intervals. The dotted line is the approximated velocity shift based on the potential distribution, Eq. (2.59). The vertical axes are scaled such, that the apexes of the distributions follow the $v_{\text{ave}}$ curve.

Figure 4.5: Comparison between velocity shifts in potential distributions calculated with ($\Delta v = v_\gamma(r)$) and without ($\Delta v = 0$) velocity correction. The start conditions of the trajectories at the real source are identical, only the potential distribution differ.
were traced from the cathode and collected in a narrow band across the emitting area at the Langmuir minimum. In Fig. 4.6 the current density in this band is plotted together with the average \( v_x \) and the estimated velocity shift in the \( x \)-direction. At the edge of the emitting area, the velocity shift is overestimated by Eq. (2.59), just as in the rotationally symmetric case. Since the amount of current from the edge of the emitting area is low, this will have only little influence on the space charge field.

### 4.4 Influence of the velocity correction

Both for the rotationally symmetric and quadrant symmetric triode it was found that the velocity correction, which is used to incorporate the shape of the potential distribution between cathode and Langmuir minimum, deviates from the simulated velocity shift, especially close to the edge of the emitting area. One may expect two influences from this deviation. Firstly, spot distribution calculations, based on emission properties that do not agree completely with the virtual cathode, are incorrect. Secondly, the self-consistent potential distribution is inaccurate to some extent, since we inferred the correctness of the potential distribution from the correctness of the virtual emission function.

Both issues have in a sense been addressed in the previous chapter. The first effect can be attributed to the influence of a change of velocity width on spot distribution. The second was part of the check that the absence of consistency between field calculations and spot calculations do not invalidate the computation method used.

Nevertheless, it is worthwhile to perform an explicit check. It is rather straightforward to calculate the field and spot distribution with or without velocity correction and compare the line spread functions. Figure 4.7 shows the results for a calculation of the quadrant symmetric TVT gun in optimum \( x \) focus. The line spread functions are nearly identical. This is by no means exclusive for the TVT gun in focus; other currents, focus voltages or guns show the same characteristic. The velocity correction has little influence and the overestimation of the velocity shift at the edge of the emitting area does little harm.

### 4.5 Conclusions

We have analysed the virtual emitter based on an array of local planar diodes onto which the Langmuir model is applied, as used in a triode in a CRT. The coupling of the diodes has only a minor influence on
4.5. Conclusions

Figure 4.7: Line spread functions for quadrant symmetric TVT gun for intermediate current in optimum x focus, with and without velocity correction. The correction has been applied to both field and spot calculations (solid curves), field only (dashed curves), spot only (dotted curves).

the potential distribution. The emission function at the Langmuir minimum can be well approximated by joining the individual virtual cathodes and imposing a transversal velocity correction. The correction used in SCELOP, Eq. (2.59), overestimates the velocity shift near the edge of the emitting area, but this has no noticeable influence on the spot distribution.

A CRT triode can very well be represented with a local planar diode array, in spite of the fact that the potential distribution at the edge of the emitting area is far from planar. This justifies the advantageous use of a virtual cathode in a practical, space charge limited, optical system.
Chapter 5

Monte Carlo simulation of the emitter region

In this chapter, a description is given of the numerical tool SWARM (Statistical interactions With Axial Rounded Mean field) that has been developed to perform the analyses of various physical effects occurring in the emitter region. In the first part of the chapter, the mathematical models that form the basis of the simulation set-up are discussed and the numerical implementation is described. In the latter part some simulation results are discussed for verification of the models. Several test cases like Langmuir’s space charge model for planar diodes and beam spreading under the influence of space charge are reproduced.

5.1 Introduction

Most simulation tools for electron optical computations are based on a number of assumptions that some aspects of the real physical situation near the electron emitter can be approximated or even ignored completely. As we have seen in Chapter 3, the emitter properties notably determine the characteristics of the optical system. The combined approximations may thus lead to inaccuracies in the computation results. Approximations can be such that the emitter is represented with as few as three parameters: source size, opening angle of the emission cone and univalent kinetic energy of the emitted electrons, a common approximation when modelling complete optical systems. On the other hand, some simulation models take into account the real physical properties of the emitter region: the potential distribution near the emitter as imposed by the geometry of the emitter and the extraction electrodes, the spatial and velocity distribution of emitted electrons and the extent to which the emitted electrons are affected by the potential distribution near the emitter.

We are especially concerned with the latter kind of simulation models, that take the physics of the emitter region into account. In Chapter 2 assumptions that are often used in computations of electron guns were listed. Electrons are generally considered to be emitted from a planar surface with uniform emission properties and after emission, the electron cloud behaves as a smooth jelly. On the basis of these assumptions the Langmuir model predicts the space charge field in front of the emitter. In Chapter 4 it was shown that the assumption of planarity in the Langmuir model does not lead to inconsistencies, when applied locally in a nonplanar geometry. Nevertheless simulations making use of the Langmuir model are not free of errors, as reviewed in Chapter 2, which makes the applicability of the Langmuir model debatable.

In space charge limited mode, local emission properties are determined by the depths and positions of the local minimums of the potential distribution. The minimum lies typically several microns from the cathode. Consequently, this length provides a characteristic length scale of potential calculations in this region. We demand that a calculation model describe the space charge field adequately with sufficient detail at least down to this characteristic length scale. Mass-produced oxide cathodes present in CRT guns have a surface roughness that is not negligible to the characteristic length scale. The influence of surface roughness can thus not a priori be overlooked. By the same logic, it can be reasoned that work function inhomogeneities in the same order of magnitude as the depth of the potential minimum may not be overlooked. Thirdly, the inter electron distance in the region in front of the emitter is on the same order of magnitude as the characteristic length scale. On this scale therefore, the number of discrete
constituents of the space charge field is low and the assumption of a smooth space charge field is locally incorrect. This third point is closely correlated with statistical Coulomb interactions.

The assumptions in the Langmuir model that are questionable are surface smoothness and work function uniformity of the emitter, and smoothness of the space charge cloud. The first two correspond to the common approximation that the emission surface can be represented by a flat equipotential plane with homogeneous emission properties. The third corresponds to the approximation that a cloud of discrete electrons can be represented by a smooth scalar field. To determine the extent in which these approximations are valid one would prefer to abandon the simplifications underlying each assumption and examine the influence on the computation results when including those facets of the emitter that are often disregarded.

Thus, a simulation model is needed that can describe an emitter region in which the emitter surface is not necessarily homogeneous, and the space charge cloud is not represented by a smooth jelly. To this end, a tool has been developed in which the space charge cloud is described in terms of its individual constituents, where electron-electron interactions and emitter surface structure inhomogeneity are incorporated. In the following sections, we will discuss the computer model that has been developed, and review the numerical implementation of the physical models. Simulation results will be presented in the following chapters.

5.2 Model of the emitter region

In this section the basic layout of the simulation tool is discussed. When developing a simulation tool to evaluate the behaviour of an electron cloud near a thermionic emitter, it is necessary to make specific design choices. The simulation model is specifically intended for a CRT and should therefore be capable of describing an emitter with cathode properties that are characteristic for CRT emitters.

No assumptions on the gun itself are required since the simulation tool is not intended to perform simulations of a complete electron gun. The main purpose is to obtain a description of the region of interest in front of the emitter.

5.2.1 CRT emission parameters

CRT emitters operate in the space charge limited regime with peak current densities in the range of 1–10 A/cm² and an emitting area of 20–200 µm radius. Using the Langmuir model, we can calculate a number of relevant quantities, like the expected particle density and the characteristic length scale of the potential distribution. The expression for the charge density near the cathode of a planar diode is

$$\rho(z \to 0) = j_{\text{sat}} \sqrt{\frac{2m}{k_BT}} \left[ 1 + \text{erf}(\sqrt{-eV_m/k_BT}) \right],$$

(5.1)

following from Eq. (2.51). Favouring round numbers, a saturation current density $j_{\text{sat}} = 10$ A/cm² and a temperature of $T = 1160$ K (0.1 eV/kB) can be substituted, leading to an electron density at the cathode of $\rho_{\text{sat}}/e \approx 5$–10 µm⁻³. In space charge limited operation the distance from cathode to minimum is $z_{\text{min}} \approx 1$ µm. The applied electric field is in the range of 5–50·10⁴ V/m.

5.2.2 Boxed electrons in a charge continuum

The most simplified model that should be within the capabilities of the simulation tool describes a planar emitter in a perpendicular extracting electric field. When we study only the behaviour of electrons in the vicinity of the emitter, no anode or extracting electrode needs to be defined. The electric field can be introduced either as a negative surface charge at the emitter that 'pushes' the electrons outward, or as an external vector field of unknown origin. The latter can be thought of as caused by a 'ghost' electrode, i.e. an electrode with unspecified shape and position, that is not susceptible to the electron beam. An anode placed at infinity exerting a finite electric field is an example of such a ghost electrode.

To study the effects of discrete space charge, the electron beam should ideally be described completely in terms of a cloud of individual electrons. A CRT electron beam typically consists of several million electrons. It is neither practical nor necessary to keep track of all electrons as we are mainly interested in the electron cloud behaviour between cathode and virtual cathode.
5.2. Model of the emitter region

![Diagram of emitter plane and infinite toroid](image)

**Figure 5.1:** Schematic of the particle box with discrete electrons (crosses) encompassed by homogeneous charge toroid (gradient fill). The charge density of the toroid follows the same distribution as the charge in the electron box: as the electron density in the box decreases towards the right, so does the charge density of the toroid.

We will initially look at a small box-shaped region at the center of a symmetric beam. In this region electrons will be represented by moving point charges. The charge outside the simulation box is represented by a smooth space charge distribution. The advantage of the region surrounding the axis is the fact that at the beam center there will be no net effect of global space charge. If any deviations with respect to the initial distributions are observed, they must be due to the surface structure and the discreteness of the space charge.

The complete system of boxed electrons and charge continuum can be described as a cylinder enclosing a rectangular block. The block represents the region containing the discrete charge and the remainder of the cylinder represents the continuous charge. In the axial direction both cylinder and block are bounded by the emitter plane and a target plane. If only the discrete effects in the paraxial beam region are investigated, the edge of the beam, given by the cylinder radius, can be placed at infinity. Then effectively a planar diode geometry is obtained. This representation is depicted in Fig. 5.1. A finite cylinder radius can be used to model a uniform cylindrical beam, like a rotationally symmetric Pierce gun [77]. The cylinder excluding the box will be referred to as a toroid.

5.2.3 Choosing the box size

In order to investigate the effects of surface structure and the discreteness of space charge on the virtual cathode, the electron box should be chosen at least large enough to include the Langmuir minimum. The distance between physical cathode and Langmuir minimum is several microns, and the typical size of cathode surface grains is also several microns. A cubic box with side 10 μm is large enough to provide an accurate description of the space charge cloud in front of the emitter. The total number of electrons in this box according to the Langmuir model is several thousands for the range of saturation currents and electric fields in the operating regimes of CRTs. Simulating this number of electrons is within the capabilities of a workstation or personal computer.
For optical systems other than CRTs, with a smaller beam, it may be possible to describe the complete electron beam by moving point charges. In that case, the simulation box encompasses the electron beam, and the surrounding smooth space charge distribution is absent.

5.2.4 Mean-field approximation

The assumption that the discrete charge distribution in the toroid can be replaced by a smooth mean charge density is the essence of the mean-field approximation. In a planar geometry with homogeneous emitter, the charge distribution inside the toroid follows the same distribution function as the charge distribution inside the box for symmetry reasons. The shape of the mean field distribution function can be determined from the positions of the electrons inside the box, for instance by dividing the box into axial slices and then counting the number of electrons in each slice. The mean field in each corresponding toroid slice is then given by the mean charge density in the box slice.

The application of the mean-field approach as discussed here discounts a subtle detail. Due to the discrete nature of electrons stochastic fluctuations are present on the mean charge density in the box slice. The relative effect of these fluctuations decreases with increasing number of particles. Simply mapping the distribution inside the box onto the toroid, which represents a greater volume, may thus result in an overestimation of the fluctuations of the surrounding charge.

In the present model, the fluctuations cannot be reduced by sampling a greater volume since the size of the box is fixed in the simulation. If the box size influences the results we need to devise a scheme to mimic infinite box size for obtaining the mean field without modifying the actual box size. One way to obtain the reduced fluctuations associated with a greater volume is by averaging repeated simulations. Since we are mainly interested in the stationary situation, it is justified to sample the mean field repeatedly within a single simulation rather than to average over multiple simulations. Therefore, if it proves necessary, we may choose to reduce the fluctuations by averaging over a time interval within a single simulation, effectively replacing the space average by a time average.

5.3 Emission from a metallic surface

In Chapter 2 the theory of electron emission was reviewed for continuous electron current. The saturation current was derived as the fraction of electrons in the metal that can overcome the work function and be emitted into vacuum. The work function was assumed to be a discontinuous barrier. The discreteness of the electrons was disregarded. When electrons are to be regarded as individual particles, one has to take into account the interaction with the charge distribution at the emitter surface, which gives rise to a continuous work function barrier. A refinement to the previously discussed theory is necessary, emerging as a consequence of the description of an electron cloud in terms of individual particles.

5.3.1 Image potential

A charge distribution in vacuum near a conductor induces a charge distribution at the conductor surface in such a way that the total electric field at the surface is perpendicular to the surface. In calculations with a static electron cloud, for instance in an iterative space charge solver, the induced conductor charge is implicitly taken into account by imposing the correct boundary conditions on the solution of the Poisson equation.

In the present simulations of a dynamic electron cloud, the Poisson equation is not solved explicitly. If the electron cloud is far away from the electrodes, for instance in calculations of statistical Coulomb interactions in electron beam columns, the influence of the electron cloud on the electrodes can be neglected. For simulation of the electron cloud close to the emitter this is clearly not the case. An explicit form of the induced conductor charge is needed to take the boundary conditions into account.

When the conductor surface is an infinite plane, a simple representation is obtained by replacing the conductor and its surface charge distribution with the mirror image of the charge cloud in the conductor surface and negating the sign of the charge. Outside the emitter surface, this so-called mirror charge gives the exact same field as the induced surface charge density.

We will make the unconventional distinction between the induced surface charge, the image charge and the mirror charge. The induced surface charge is the actual charge distribution on the conductor surface, top left in Fig. 5.2. The image charge is any charge distribution on one side of the conductor
plane that causes a field on the other side that is identical to the field that is produced by the induced surface charge, e.g. the right two pictures in Fig. 5.2. The mirror charge is that particular representation of image charge that is obtained by mirroring and negating the charge (Fig. 5.2, top right).

In general, the charge distribution is referred to as image charge and the resulting potential distribution is the image potential.

### 5.3.2 Work function

Considering the case of an individual emitted electron near a planar metallic emitter, the induced charge distribution exerts an attracting force on the electron and thus is a constituent to the work function. The image potential contribution to the work function is obtained by integrating the electric field due to the image point charge

\[ V_{\text{im}}(z) = \frac{e}{16\pi\varepsilon_0 z}. \]

Effectively, the step shaped work function is replaced by a continuous one. This form of the image potential allows for the description of the Schottky effect [78]. According to the Schottky effect, the emission in an accelerating field is enhanced due to the fact that the surface barrier is lowered by the image potential. Incorporating image potential in our simulation tool has the appealing side effect that the dynamics of the electron cloud including Schottky enhanced emission may be studied.

For an electron approaching the conducting surface the mirror charge potential diverges. The classical image point charge model is not suited to describe electron emission since electrons cannot escape the diverging potential.

Therefore, if one wants to maintain a simple classical model for the emitting surface, a cut-off in the potential singularity must be introduced. A well-known cut-off method consists of moving the mirror plane from the emitting surface inwards to \( z_{\text{mirror}} = -z_0 \),

\[ V_{\text{im}}(z) = \frac{e}{16\pi\varepsilon_0 (z + z_0)}. \]  

Now the singularity is situated inside the emitter material but this region is not part of the simulation and therefore presents no problem. Another method is replacing the singularity by a linear segment [78],

\[ V_{\text{im}}(z) = \begin{cases} 
\frac{e}{16\pi\varepsilon_0} \left( \frac{2}{z_0} - \frac{z}{z_0^2} \right) & \text{if } z < z_0 \\
\frac{e}{16\pi\varepsilon_0} \frac{1}{z} & \text{if } z \geq z_0.
\end{cases} \]  

Both of these methods provide the facility to represent the work function solely in terms of the image potential. An arbitrary known value for the work function prescribes the distance the mirror plane needs to be shifted in Eq. (5.3) or the length of the linear potential piece in Eq. (5.4). For instance, to describe a work function \( \Phi = 4 \) eV in terms of the image potential we can substitute \( z_0 = 0.9 \) Å in Eq. (5.3) or \( z_0 = 1.8 \) Å in Eq. (5.4).

The applicability of this classical picture at the boundary of an emitter is questionable. Quantum mechanically the external electron is not described by a localized point particle but by a spatially extended wave function that overlaps with the wave functions of the electrons inside the material. Although the two aforementioned potential cut-offs do represent a spatially extended surface charge, the wave function interaction is not present.

However, we do not seek to describe accurately the behavior of electrons at the precise moment of emission. We do wish to include a model for the interaction between electrons in vacuum and their image charges. For this interaction we expect a classical model to suffice.

Both image potentials Eqs. (5.3) and (5.4) give the axial electric field of the image charge and provide the interaction between an electron and its own image charge. To account for the nonaxial electric field, to describe interactions between electrons and neighboring image charges, we have to relate the image potential to an image charge distribution.

In general, there is no unique image charge distribution that corresponds to a given axial image potential. For Eq. (5.4) no simple charge distribution presents itself. For Eq. (5.3) an obvious choice for the image charge distribution is a point charge at \( z_{\text{im}} = -z_0 - z \). However, we have to be careful not to attribute a fundamental meaning to the shifted image plane, other than that of an effective mirror point.
for an external electron with respect to its own image charge. It would be helpful to have an indication of the areal extension of the induced surface charge independently from Eq. (5.3).

Therefore, we present a third simple method to resolve the image charge singularity in a classical picture. From a mathematical point of view, the singularity is caused by the fact that the image charge is a point. Replacing the image charge by a disk will resolve the singularity. We can attribute a physical meaning to this disk. An electron that is emitted from the metal leaves a hole in the charge distribution at the metal surface. Since the surface charge is made up of discrete electrons rather than a charge jelly, at an infinitesimal distance from the surface, the vacancy resembles a homogeneous disk exerting an imaging field

\[ E_{im} = \frac{e}{2\pi R^2 \varepsilon_0} \left( \frac{z}{\sqrt{4z^2 + R^2}} - 1 \right). \]  

(5.5)

The effective size of this disk is on the order of the inter electron distance of electrons in the surface layer. An ‘electron surface density’ can be obtained from the lattice spacing and the free electron density. As the electron moves away from the emitter, the surface density appears smoother and the image charge will asymptotically resemble a mirror point charge. The image potential of an image disk with radius \( R \) is

\[ V_{im}(z) = \frac{e}{2\pi R^2 \varepsilon_0} \left( \frac{1}{2} \sqrt{R^2 + 4z^2} - z \right). \]  

(5.6)

which reduces to the classical point image charge for \( R \to 0 \). This simple model relates the lattice spacing to the work function. A work function of \( \Phi = 4 \) eV corresponds to \( R = 3.6 \) Å. This value for \( R \) is indeed a realistic lattice spacing.

Equation (5.6) provides us with an intuitive model for the image potential to describe the interaction between emitted electrons and their induced surface charges. It should be pointed out that the differences between the image point, the shifted image point and the image disk become irrelevant at distances \( z > 2 \) nm.

5.3.3 Saturation current

Using a continuous work function rather than a step function at the emitter surface introduces a difficulty in the interpretation of the saturation current. In Section 2.1.3, the saturation current was shown to be the current density of electrons that can overcome the work function. For a step work function this simply is the current density of electrons emitted at the material-vacuum interface into vacuum.

For a continuous work function the material-vacuum interface must be considered as a finite region rather than an infinitesimal plane. There is a lack of specificity as to where the emission surface is in reference to the subsequent field calculations. If one includes electrons that escape the emitter lattice due to their thermal energy, but are reflected by their image charge barrier even in the absence of any other field one can more or less freely choose an emission surface within the interface region. This can in fact be considered as an application of the virtual emitter concept. The effective current from this emission surface then has to be increased with respect to the zero-field thermionic saturation current to compensate for the residual work function barrier (see Fig. 5.3). If the image potential at the mathematical emission surface is \( V \), the effective emission current, following from Eq. (2.19), is \( j_{sat} = \exp(eV/k_B T)j_{sat} \). The zero-field thermionic saturation current is here referred to as the macroscopic saturation current. The effective current will not contribute to the macroscopic saturation current, but it will partly contribute to the Schottky enhanced emission current when a strong extracting field is applied. If one would want to analyse the different image charge models, the emission plane must be closer to the mirror plane otherwise the different image potentials can not be resolved. However, if we move onto the mirror plane the effective emission current becomes several orders of magnitude larger than the saturation current. Attributing for instance 1 eV of the work function solely to the image potential, the effective current increases five orders of magnitude. Resolving the singularity at the mirror plane has not prevented this.

For the moment we will make the ad hoc decision to design the tool for macroscopic saturation current, thus several thousands of electrons rather than millions. This means that an auxiliary cut-off distance is needed that specifies the position of the emission surface with respect to the mirror plane. Consequently, the emission plane can not be too close to the mirror plane and the image charge models cannot be distinguished.

In choosing the cut-off distance, we have to keep in mind the object of incorporating mirror image charge. In first instance, describing electrons near a conductor as point charges rather than a smooth
Figure 5.2: Image charge and surface charge. Clockwise, from top left: surface charge induced by a negative point charge; image charge, represented by a mirrored negated point charge, Eq. (5.3); image charge, represented by a mirror disk charge, Eq. (5.6); image potential near the surface represented with a linear potential segment, Eq. (5.4).
Figure 5.3: Energy distribution of electrons in the emitter and image potential. The energy axis is not labeled since only the exponential tail is relevant. At $z = 2$ nm the image potential with respect to infinity is $-0.18$ V. The macroscopic saturation current is composed of particles in the shaded area, with energy level higher than the image potential at infinity. At the plane $z = 2$ nm there is an additional contribution of particles in the dash-filled area to the effective emission current, which will eventually be reflected by their image charge barrier. At $T = 1160$ K, $j_{\text{eff}}$ is about six times higher than the macroscopic saturation current.

charge jelly, necessitates an image charge that is accurate on a small length scale. Furthermore, a mirror image charge enables implementation of Schottky enhanced emission.

A lower limit to the length scale on which the image potential needs to be accurate is provided by the de Broglie wavelength $\lambda_B = h/mv$. There is no point in describing electrons as classical point particles on a scale smaller than $\lambda_B$.

A value of $\lambda_B = 4$ nm corresponds to an electron kinetic energy of 0.1 eV, which is on the same order as the average thermal energy at 1160 K. At $z = 4$ nm the residual work function barrier is 0.09 V, meaning that at 1160 K the corrected saturation current is a factor of 2.5 higher than the macroscopic saturation current.

A much smaller cut-off distance will bring the number of simulated electrons outside the computable range, and will also bring the length scale out of the range that can be described using classical mechanics. We therefore set the cut-off around $z = 4$ nm. The cut-off value sets an upper limit to the obtainable emission enhancement due to the Schottky effect. No matter how high the applied field, the current enhancement in a simulation can never be more than a factor of 2.5 because the number of electrons is simply limited by the effective saturation current.

5.4 Mathematical model

In the previous section certain aspects of the space charge field in the vicinity of an emitter were discussed. In the following sections we will outline how those aspects are incorporated in the simulation model. The model is designed to be capable of describing thermionic emission, including the Schottky effect. No secondary electron emission or backscattering is present in the source model. The source model can handle surface roughness, work function variations, temperature variations and temperature limited current. The potential distribution incorporates global space charge, statistical interactions and image charges.
5.4. Mathematical model

![Diagram](image)

**Figure 5.4:** Examples of surfaces that cannot be defined by means of a function $z(x,y)$. Both functions are multivalued at some $(x,y)$.

5.4.1 The electron box

In Section 5.2.2 we introduced the electron box, acting as a container for a part of the electron cloud that is representative for the complete electron beam. The numerical model is set up to monitor the electron cloud composition in the source region, which is different from most other simulations of Coulomb interactions in particle optical systems [19, 28, 79] in which the evolution of a bunch of particles traversing along the beam path is monitored. In the present model, electrons have to be injected into the system continuously through one plane and, to obtain the statistics, collected if they cross a certain end plane. If electrons are reflected into the cathode, for instance by a space charge barrier, they are simply removed from the system. No backscattering has been taken into account. It is practical to use the designations ‘field’ particle and ‘test’ particle from Jansen [19]. A field particle is a particle that exerts a force, a test particle is a particle that feels a force. In the present simulation, all particles are both field and test particles. The distinction is used for descriptive purposes only.

The optical axis is aligned with the $z$ axis. The positive $z$ direction is the direction from cathode to anode.

5.4.2 Representation of the cathode

One side of the particle box performs the role of cathode surface. Particles are continuously being emitted according to the local emission properties of the cathode. To describe a cathode surface, a set of functions representing the four quantities work function, shape of the cathode surface, saturation current and temperature must be supplied. The emission properties, specifically the rate of emission and the emission distribution, are given implicitly by these four quantities.

The surface shape is represented by a functional dependency of the surface protrusion in the $z$ direction. The saturation current, temperature and work function are trivially represented by their respective values. The crystal structure of the cathode will not be considered, therefore the surface quantities can be parameterized as a set of four smooth functions of $x$ and $y$.

The functional representation imposes the restriction that the outward directed cathode surface normals must always have a positive $z$ component. This makes it impossible to model for instance a grain on top of the cathode surface, or a crater, shaped as a partly embedded cavity, shown in Fig. 5.4.

The temperature and the saturation current only affect the emission conditions and can be considered separately from the electric field, whereas the perpendicular protrusion and the work function are interconnected with the field at the cathode: the work function affects the surface potential, while the surface potential and the shape of the surface define the field in front of the cathode.

5.4.3 The cathode field

The cathode is modelled as a charge distribution consisting of point charges and infinitely thin planar charges, combined with the four surface function parameterizations. The charge distribution must be positioned inside the emitter, behind the emission surface in order not to interfere with the electron cloud.
Figure 5.5: A uniform charge sheet (thick grey line) gives planar equipotential lines. B. Adding a point charge (solid dot) creates a disturbance in the equipotential line. C. An arbitrarily shaped ‘disturbance’ can be effected by choosing the potential distribution of the distribution of point charges to satisfy the desired shape.

The implementation of the cathode in terms of a charge distribution combined with surface functions can best be explained taking the example of an arbitrarily shaped cathode surface with uniform work function inside a uniform extracting field. In this situation the potential distribution in front of the cathode is bounded by the cathode surface, which is an equipotential plane, and by a boundary potential defined by the applied extracting field. These fully specified boundary conditions determine the potential distribution through the Poisson equation. If no charge is present inside the system, this potential distribution is unique, regardless of the origin of the boundary conditions. Choosing a charge distribution such, that the resulting potential distribution has an equipotential plane coinciding with the cathode surface, the electric field near the cathode surface must by the principle of uniqueness properly be represented by the charge distribution.

A nonuniform work function can then be represented by choosing the charge distribution such, that the resulting potential at the cathode surface plane is not an equipotential but rather corresponds to the desired work function distribution over the surface.

A cathode model that imposes the correct potential distribution in front of the cathode automatically includes effects like field enhancement near a sharp corner and patch fields due to work function inhomogeneities.

The method is clarified in Fig. 5.5. The simplest case, a planar cathode with uniform work function, can be modelled with a uniform planar charge sheet. Deviations on the surface planarity and the work function uniformity can be effected by adding point charges. This simple combination of a planar charge sheet and a collection of point charges is well suited for simulation of a small region near the emitter surface. Even if the complete emitter is to be modelled, as opposed to only the small part of the emitter enclosed by the particle box, this type of charge distribution works adequately, if the shape of the emitter is not too exotic.

If charge is present in the electron box, image charges have to be introduced to conserve the boundary conditions and the uniqueness of the potential distribution. An analytical surface charge model is feasible, as long as the surface is flat, yet possibly inhomogeneous with respect to temperature, work function or saturation current. Analytical expressions for the action of the surface charge were given in Section 5.3.

In contrast, since it is not possible to describe the induced surface charge of a point charge near an arbitrary surface by an analytical image charge, a shaped cathode surface can be described only at the cost of an exact image charge model. If a shaped cathode surface is to be modelled, we have to accept an unprecise induced surface charge. Section 5.6.1 discusses how to implement the induced surface charge on a shaped cathode surface.

5.4.4 Cyclic boundary conditions

When the tool is used to examine the influence of discrete space charge in a planar diode, no focussing lens field is taken into account. The transversal motion of the particles is influenced by stochastic interactions
5.5 Numerical implementation

The implementation of the model can be clarified by discussing the separate parts of the simulation tool: the emission algorithm that continuously inserts particles into the particle box with the proper spatial and velocity coordinates, representing the emission current; a field calculator that computes the mean field potential by application of the mean-field approximation to the electron distribution in the particle box; a field calculator to compute the potential distribution imposed by the cathode; the integrator for the equations of motion; and a "particle manager" that keeps track of the statistics of the electron cloud.

5.5.1 Emission algorithm

Electron emission from a thermionic emitter is a statistical waiting time process. The number of emitted electrons during a time interval follows a Poisson distribution with expectation value given by the saturation current and the length of the time interval.

The probability that an electron is emitted at a certain position of the cathode surface depends on the local emission properties. The initial velocity of the particles is given by the local temperature and the local orientation of the emitter surface: the velocity follows a cosine distribution as derived in Section 2.1.3 with a mean direction that is everywhere perpendicular to the surface.

The emission algorithm governs the number of emitted electrons and their initial positions and velocities at the time of emission. First, the number of electrons $k$ that is emitted during a time step $\Delta t$ is
sampled from the Poisson distribution,
\[ f_k(k) = \frac{\mu^k e^{-\mu}}{k!}, \]  
(5.7)
with mean \( \mu = I_{\text{sat}} \Delta t \) and \( I_{\text{sat}} \) the total emitted current. Then, the initial positions in the source plane are sampled from a distribution with a probability density defined by the emission properties,
\[ f_{x,y}(x,y) = C \frac{j_{\text{sat}}(x,y)}{n_s(x,y)} \exp \left( -\frac{e\Phi(x,y)}{k_B T(x,y)} \right), \]  
(5.8)
with \( C \) a normalization constant. The z-component of the surface normal, \( n_z(x,y) \), appears as a correction factor to account for the surface orientation: surface elements that appear to have equal size when projected onto the \( xy \)-plane are actually larger and consequently emit more current.

The velocity is sampled from the cosine distribution by sampling the individual components from a distribution with distribution functions given in Eqs. (2.21)
\[ f_{v_x}(u) = n(u) \]
\[ f_{v_y}(u) = n(u) \]
\[ f_{v_z}(u) = C' j_z(u) \]  
(5.9)
(\( C' \) a normalization constant) and rotated such, that the mean lies along the surface normal,
\[ v \longrightarrow R_n v, \]  
(5.10)
with \( R_n \) a rotation matrix that depends on the normal vector \( \hat{n} \) defined such that \( R_n \hat{e}_z = \hat{n} \).

If the emitter plane is flat, a mirror image point charge model can be used. The use of a mirror image charge model introduces some freedom as to where the numerical plane of emission must be located. The numerical plane of emission can not coincide with the physical emitter plane in order to keep the number of electrons in the system within reasonable bounds. As we have seen in Section 5.3.3, a cut-off distance, the distance between the physical emitter and the numerical emitter surface is required, and the macroscopic saturation current must be corrected for the residual work function.

To ensure that the electrons arrive at the emission plane at random times, they are drifted from the emission plane back into the emitter. This prevents the unphysical situation that all electrons are emitted at the same point in time. The time instant of emission is sampled uniformly in the time interval.

Rather than using the thermionic emission type that generates cosine distributed initial directions and Maxwellian initial energies, it is also possible to supply individual start conditions, for instance when a different emission mechanism needs to be examined. The emission algorithm then reads the start conditions cyclically from a file. The output of a previous computation can be used as an emitter file.

### 5.5.2 Mean-field calculator

The mean field is approximated using slices of toroid of uniform charge density. Due to the periodic boundary conditions, only the axial field component of the field on axis caused by the toroid slices is relevant. The axial field \( dE_z \) of an infinitesimally thin toroid slice with charge density \( \rho \), inner halfwidth \( h \) and outer radius \( R \), as shown in Fig. 5.7, can be obtained by direct integration in cylindrical coordinates
\[ dE_z = \frac{1}{4\pi \varepsilon_0} \int_{\phi=0}^{2\pi} \int_{r=h}^{R} \frac{z\rho}{(z^2 + r^2)^{3/2}} r \, dr \, d\phi \, dz \]
\[ = \frac{2}{\pi \varepsilon_0} \left( z \arctan \frac{z}{\sqrt{2h^2 + z^2}} + \frac{1}{2} \arctanh \frac{2h\sqrt{2h^2 + z^2}}{3h^2 + z^2} - \frac{\pi}{4} \sqrt{z^2 + R^2} \right). \]  
(5.11)

The charge density per slice is determined from the spatial distributions of the electrons in the electron box by dividing the electron box into \( z \)-intervals and counting the number of electrons per \( z \)-interval. Each \( z \)-interval corresponds to one toroid slice.

Due to the discreteness of the number of electrons in a slice, the mean field is subject to fluctuations. As discussed in Section 5.2.4 the mean field may suffer from the fact that it is determined from a relatively small number of electrons, thereby increasing the fluctuations. The fluctuations can be reduced by time-averaging. To implement a mean-field approximation for the charge distribution in the toroid, we make use of a moving time window.
The time window is implemented by keeping track of the number of electrons per slice at the most recent few hundreds of time steps. say, 200 steps. The charge density is determined from the average of 200 time steps. At the starting stage of a simulation, during the first 200 time steps, no earlier time steps are available, and the averaging is performed over all available time steps.

The short term fluctuations caused by the Poisson stochastic nature of the emission are thus reduced. However, replacing a space average by a time interval average can introduce a long term fluctuation due to the contrived inerts of the mean field. As a result of Poisson stochastics it is inevitable that, even in the stationary state, the number of particles in the box is sometimes different than specified by the stationary solution. Assuming the number of particles is at some point too high, if no time-averaging is performed there is an immediate response to this excess of electrons since the space charge barrier increases, acting to reduce the number of electrons. When time-averaging is applied the increase of the space charge barrier lags behind and the number of particles can increase even further. By the time the space charge barrier kicks in its repelling influence has grown too strong, causing a decrease in the number of particles below the stationary solution, which causes a subsequent delayed underestimation of the space charge barrier.

For diode settings corresponding to CRT conditions, these long term oscillations are not visible. They can however occur when the contribution of the toroid field is especially dominant. This situation occurs when the transversal dimensions are small and the number of electrons is nonetheless large, which is the case when simulating extremely high currents. To suppress the oscillations for these cases, one can reduce the length of the time interval, or by using a weighted time average in which the relative contribution of the present charge to the mean field distribution is increased. We will experiment with the latter alternative at the end of this chapter.

The axial electric field or potential distribution caused by the toroid is stored in a table, and the actual mean-field external force on each electron is calculated by cubic interpolation.

### 5.5.3 Cathode point charges

The cathode is defined by four functions: work function, saturation current, temperature and surface shape. The cathode surface potential is anchored at a number of points. The potential at these points is given by the work function subtracted from a fixed potential value. This fixed value may be arbitrary since electrons only feel a potential difference.

The cathode charge distribution is created by adding charges at well-chosen positions behind the cathode surface. When the number of charges is equal to the number of anchored surface points, a solution for the charge distribution can be found, since the number of degrees of freedom is equal to the number of boundary conditions. This approach is similar to the common charge density scheme [80].
with a few differences. In the charge density method (CDM), the charges are (often uniformly) charged surface segments that also define the electrode shape. The boundary conditions are usually equipotential type conditions. In our cathode model the boundary conditions need not be equipotentials, and since the charges are point charges, it is not permitted to position them exactly at the boundary, as a point charge at the emitter boundary creates a potential singularity.

When the boundary conditions depend linearly on the charges, obtaining the charges for given boundary conditions is a straightforward linear algebra computation which can be solved using standard techniques [81].

In practice, defining a number of potential boundary conditions and solving the charges, does not automatically lead to a useable cathode surface. Taking again the example of uniform work function, and consequently a surface equipotential plane, it can easily be understood that the resulting equipotential plane may be invalidated as cathode surface. It is after all conceivable that an equipotential line, intended as emission surface, curves back towards the cathode charges, as pictured in Fig. 5.5C, to such an extend that a cavity is formed, or that it becomes disjunct, if the point charge positions are chosen arbitrarily. A suitable charge distribution needs to be found for a large part by trial and error. The trials are more successful when the point charges are placed at a small distance behind the surface on the order of the distance between the charges.

If the net charge of the cathode is negative, it also provides an extracting field, or rather, a propelling field.

5.5.4 Integrator

The integrator is the ray tracing routine that calculates the trajectories of the particles from emission to the point in time where they leave the particle box and are discarded. It solves 3N coupled differential equations numerically for a system containing N electrons.

The total force exerted on a particle is the sum of all Coulomb interactions from the particles in the particle box, an external Laplace field, and the mean space charge field,

$$ F_i = \frac{e^2}{4\pi\varepsilon_0} \sum_{j \neq i}^{N_{\text{tot}}} \frac{r_{ij}}{r_{ij}^3} + F_{i}^{\text{ext}} + F_{i}^{\text{mf}}, $$

(5.12)

where $r_{ij} \equiv r_i - r_j$ the relative position vector of particles $i$ and $j$, $F_{i}^{\text{mf}}$ denotes the forces due to the mean space charge field and $F_{i}^{\text{ext}}$ denotes all other forces. The latter two terms together are referred to as the global fields.

The mean-field force is discussed below. The external forces are due to the cathode charges and the 'ghost' electrode, mentioned in Section 5.2.2. Mathematically, the ghost electrode can be implemented as a constant electric field for a planar geometry, or as an axial potential distribution that defines the paraxial electric field for a nonplanar geometry.

The main part of the mutual interaction force integrator is based on the ray trace routine from MONTEC [19, Chapter 13]. The time-derivative of the Coulomb forces can be expressed in the relative position and velocity vectors of the particles, enabling the use of a third order integration in the position vectors and a second order integration in the velocity vectors. The time-derivative of the external field is zero, the time-derivative of the mean field is ignored as it vanishes in the stationary state. Using Eq. (5.12) and differentiation with respect to time then gives the following equations

$$ r_i^{(2)} = \frac{d^2r_i}{dt^2} = \frac{e^2}{4\pi\varepsilon_0 m} \sum_{j \neq i}^{N_{\text{tot}}} \left( \frac{r_{ij}}{r_{ij}^3} + \frac{F_{i}^{\text{ext}}}{m} + \frac{F_{i}^{\text{mf}}}{m} \right), $$

(5.13a)

$$ r_i^{(3)} = \frac{d^3r_i}{dt^3} = \frac{e^2}{4\pi\varepsilon_0 m} \sum_{j \neq i}^{N_{\text{tot}}} \left( \frac{v_{ij}}{r_{ij}^3} - \frac{(r_{ij} \cdot v_{ij}) v_{ij}}{r_{ij}^3} \right), $$

(5.13b)

with $v_{ij} \equiv v_i - v_j$ the relative velocity vector of particles $i$ and $j$. These equations are used to obtain the new positions $r'$ and velocities $v'$ advancing a time step $\Delta t$:

$$ r_i' \approx r_i + v_i \Delta t + \frac{1}{2} r_i^{(2)} \Delta t^2 + \frac{1}{6} r_i^{(3)} \Delta t^3 + \frac{1}{2} \left( F_{i}^{\text{ext}} + F_{i}^{\text{mf}} \right) \Delta t^2/m, $$

(5.14a)

$$ v_i' \approx v_i + r_i^{(2)} \Delta t + \frac{1}{2} r_i^{(3)} \Delta t^2 + \left( F_{i}^{\text{ext}} + F_{i}^{\text{mf}} \right) \Delta t/m. $$

(5.14b)
There is an abundance of alternative integration algorithms for the numerical integration of the equations of motion. The method used here is straightforward to implement and since the expression for the time derivative of the Coulomb forces is simple, the number of function evaluations needed per time step is low in comparison with other methods, that use numerical evaluations of midpoints to obtain equal order.

The integration routine maintains a measure of integration error by keeping track of the average contribution to the position update as a result of the acceleration, \( \langle \| r^{(2)} \| \Delta t^2 / 2 \rangle \) and the time derivative of the acceleration, \( \langle \| r^{(3)} \| \Delta t^3 / 6 \rangle \). The average is taken over the ensemble of particles and the norm is defined as \( \| r \| = (x^2 + y^2 + z^2)^{1/2} / 3 \). This form of integration error is motivated by the recommendations for the time step adaptation of the MONTEC program [19, Ch. 13], where the time step is adapted to \( \Delta t_{\text{new}} \) such that \( \langle \| r^{(2)} \| \Delta t_{\text{new}}^2 / 2 \rangle \) and \( \langle \| r^{(3)} \| \Delta t_{\text{new}}^3 / 6 \rangle \) are smaller than a preset value. After a simulation run has finished one can determine whether this value has been exceeded.

In ignoring the time derivative of the global fields in the stationary solution a subtle issue was disregarded. In Eqs. (5.14) the Coulomb interaction contributes to a higher order than the global fields. Neglecting one term of third order while taking into account another seems insensible. One should realize the object of the trajectory integration. We are interested in the effects that statistical Coulomb interactions have on the optics of the global fields, rather than in exact trajectories of all electrons. It is advantageous to determine the differences in the electron distribution as a result of statistical collisions to high order, but it is not necessary to maintain equal order for the global fields. Higher order terms due to the global fields do influence the trajectories, but not the occurrence of collisions.

### 5.5.5 Particle manager

The particle manager routine is responsible for keeping track of the position of each electron in the box. To perform statistical analyses on the current density in the box, it is possible to define a number of diagnostic planes. These planes can be of arbitrary shape, provided they can be defined in terms of a function \( z(x,y) \), like the cathode surface.

The electron box is divided into a number of zones, equal to the number of diagnostic planes plus two. Zone 1 is the region before the emitter surface. This zone is necessary to hold electrons prior to emission, when they are drifted back into the cathode by the emission algorithm. Electrons in zone 1 experience no electric force and do not contribute to the electric field. Zone 2 is the region between cathode and the first diagnostic plane, zone 3 is the region between the first and the second diagnostic plane, and so on.

The particle manager detects when a particle crosses a zone boundary in positive and negative \( z \)-direction and updates the zone number for each individual particle. This ensures that if a single particle crosses a diagnostic plane several times, for instance when it is reflected by a space charge barrier, each crossing will be detected.

The phase space coordinates of an electron that crosses a diagnostic plane is stored, together with the acceleration it underwent at the time it crossed the plane. Storing the acceleration gives an indication of the space charge field due to the discrete electrons and provides an additional check on a possible integration error.

The electron distribution can be sampled by storing the complete electron cloud at regular time intervals. If the time interval between to storage operations is small, the composition of the electron cloud has not changed much and no improved statistics are obtained. To prevent correlation between two sampled clouds, the particle manager keeps count of the number of particles that have left the electron box, either through the anode plane or through the emitter plane. The complete electron cloud is stored only then, after all electrons in the previous store have been 'refreshed'. Also, the trajectories of individual electrons can be stored to examine the occurrence of different collision events.

### 5.6 Field approximations

The computation model to simulate the region in front of a nonuniform emitter contains several distinct sources of electric fields. These field sources can individually be 'switched on or off' as applicable to the particular simulation that is being performed. There is a certain redundancy as some simulations can be performed using either of two field sources. For instance, a uniform extraction field can be implemented as a static external field or as a propelling uniform cathode surface charge. Likewise, the induced surface charge can be represented using mirror electrons or cathode charge distribution as explained below.
Chapter 5. Monte Carlo simulation of the emitter region

When describing identical physical situations, the redundancy in field sources provides an advantageous mechanism to isolate certain physical aspects. Taking the example of the two representations of the induced surface charge, one can determine the particular influence of discreteness of the image charge, which is appropriately modelled in a mirror electron representation, but not in a cathode charge representation. An inventory of the approximations using different field sources is presented in this section.

5.6.1 Induced cathode charge

At the end of Section 5.4.3 it was mentioned that the induced surface charge in a shaped cathode surface can only be modelled with an approximated image charge, because no simple analytical mirror charge exists to represent the image charge distribution. The cathode model described above enables accounting for the induced cathode charge due to the space charge field. At regular time intervals, the induced cathode potential is measured at several sampling points. When the sampling points are identical to the points on which the vacuum surface potential was defined, the induced surface charge can be taken into account by adapting the cathode charge distribution. In this manner the surface boundary potential can be kept constant, which is the object of an image charge model.

There are a number of approximations involved in this approach. The induced charge may ‘lag behind’ with respect to the space charge field, since the cathode charge usually is not re-induced at every time step. Also, the constancy of the surface potential is limited by the number of sampling points and cathode charges.

When simulating a flat cathode surface, where an exact image potential method is available, this method may sometimes also be favored because it allows for speedier computations.

5.6.2 Smooth space charge

To allow for comparison of computations with and without statistical space charge effects, two methods that do not incorporate discrete space charge have been devised.

The simplest method uses the mean-field approximation that is already present for the surrounding charge and apply it to the volume of the electron box as well as the mean-field toroid. No time averaging is performed for the mean field inside the box, because there is no need to reduce the stochastical fluctuations. The mean field does not contain a transversal structure. This method is therefore not well suited when the cathode properties are not uniform.

As an alternative, the total space charge potential distribution in the electron box can be calculated on a cubic mesh. The space charge forces are then calculated by cubic interpolation. Cathode inhomogeneities do show up on the space charge mesh which makes this method suited to simulate a nonuniform cathode. The number of mesh nodes has to be sufficiently large to adequately represent emission inhomogeneities.

This method is particularly useful in the stationary state, when reevaluation of the cubic mesh is not required, as calculating the mesh is computationally intensive due to the large number of mesh nodes involved.

5.6.3 Mean-field mirror charge

We have described two methods that are capable of modelling the induced surface charge: representing image charge distribution by mirror point charges (as discussed in Section 5.3.1); and representing the image charge by the charges that were used to model the cathode field (Section 5.6.1). When using the mean-field approximation on the charge inside the electron box (the first method in Section 5.6.2) a third method becomes evident, as the image charge can be taken into account by mirroring the mean-field charge slices.

Mirror slices are especially suited to represent the image charge when performing computations with smooth space charge inside the electron box. In fact, the image potential is exact in that case. For discrete space charge inside the box the mirrored slices are an image charge approximation. As the mean-field mirror charge is one-dimensional, this method is not suited when the cathode is not uniform.
5.6.4 Particle bunches

In some simulations, discrete space charge effects can be properly determined by following a finite bunch of electrons from the emitter to the end plane. The electrons in the bunch experience mutual interactions and no charge is present outside the bunch. Consequently the space charge field is approximated by neglecting the charge outside the bunch. This is the conventional manner of performing Monte Carlo Coulomb interaction calculations in complete particle optical columns.

This type of calculation can also easily be performed using the simulation tool by allowing particles to be emitted only once, rather than continuously in each time step. In that case, the number of particles in the bunch is set by the user, and the length of the first time step is adapted to make the number of emitted particles consistent with the current.

5.7 Program organization

The complete simulation tool SWARM (Statistical interactions With Axial Rounded Mean field) is organized in three separate programs.

`makecathode` is an interactive program to create the point charge distribution with accessory surface files that define a nonuniform cathode. The user inputs the boundary conditions that specify the surface potential at various points on the cathode surface. The program determines the weights of the point charges that satisfy the boundary conditions and calculates the equipotential plane that serves as emitter surface in the simulations. Surface data can be output suitable for plotting programs to check on the shape of the cathode, the position of the point charges or boundary points, or the potential distribution due to the cathode charges.

The boundary conditions that can be input prescribe potentials at a point, so-called Dirichlet conditions. The boundary condition points can be grouped such that the average of a group of points, rather than all points individually, satisfies a Dirichlet condition. This reduces the complexity of the charge density matrix, and it gives a more stable solution of charges. Additionally, the potential at a plane due to a pair of infinite charge sheets can be set. The freedom in the cathode definition provides the facility to define a fixed anode potential at the end plane. The anode is susceptible to space charge and is not a ghost electrode as introduced in Section 5.2.2.

`cloud` contains the simulation routine that performs the actual particle emission and ray tracing, and generates output files for additional data processing. The layout of `cloud` is shown in Fig. 5.8. The driving routine is `evolve`, which calls all routines that handle emission, field calculations and integration. `evolve` runs until a specified time, a specified number of integration steps, or a specified amount of output has been generated. The latter is implemented as a criterion whether sufficient statistics have been obtained. No convergence criteria are used.

`field` is an interactive program for post-simulation data processing. It reads files created by `cloud` in order to examine the evolution of the composition of the electron cloud. The potential distribution in the electron box can be plotted including mean field and cathode surface charge at the time instants where those data have been stored by `cloud`. Also, the emission function created by `makecathode` be checked by generating particles with the proper distributions for position and velocity. Space charge distribution, electron trajectories and emission statistics, and phase space distribution at user-defined planes can be analyzed with simple statistical measures.

5.8 Consistency checks

Without the intention to examine the physical effects for which the simulation tool has been developed we can perform some consistency checks on simple, uniform diodes. Simulations of nonuniform, nonplanar, or nondiode geometries are discussed in the following chapters.

Convergence to stationary state

As an example a simple planar diode without cathode inhomogeneities will be examined. The electron box is cubic with sides 10 μm. The saturation current is \( j_{\text{sat}} = 10 \, \text{A/cm}^2 \) and the cathode temperature is \( T = 1160 \, \text{K} \). If a weak electric field is applied, the emission will be space charge limited and there will both be a positive flow of particles through the virtual anode plane and a negative flow back in the
Figure 5.8: Schematic of the cloud program set-up. The first column contains the emission algorithm and the particle manager, the second column marks the calculation of the mean field, the image charge and the mutual interactions, the third column illustrates the trajectory integration, the fourth column describes the halt criterion.
Figure 5.9: The number of particles and the transmitted and reflected current for a low field (left) and a high field (right) in an electron box with sides 10 µm. The low field is chosen such, that the current is severely limited by space charge. The high field is chosen such, that the current is almost temperature limited.

emitter plane. Figure 5.9 shows the charge and currents in the particle box, starting with an empty box. At \( t = 0 \) the number of electrons gradually increases. At \( t = 0.02 \) ns the first electron reaches the end of the particle box. To build up the space charge barrier takes a little longer. Because of this, the initial current is slightly higher than the space charge limited current and then falls off to the stationary value.

When the current is temperature limited, it takes a little while longer for the space charge cloud to become stationary since the driving influence of the space charge field is smaller compared with the extraction field.

**Influence of the approximations in mean field**

The computations presented in Fig. 5.9 were performed using the moving time window to suppress Poisson statistics, and applying the weighted time average — in which the most recent time step is weighted more strongly — to prevent the possible occurrence of long term oscillations due to the delayed response of the space charge cloud. In fact, the weighted time average is not necessary since the statistics of the electron cloud are simply 'too good' to observe the long term oscillations.

To make the oscillations visible, the width of the particle box must be decreased and the current must be increased. As an extreme example the width of the particle box is decreased to 0.2 µm and the current is increased to \( 10^4 \) A/cm², which is three orders of magnitude higher than a typical thermionic emitter used in a CRT. In Fig. 5.10, left, the influence of the weighting in the time average is shown. The weighting is set with a mixing parameter \( f \) and the averaging is performed using

\[
\rho_{\text{mean field}} = (1 - f) \rho_t + f \langle \rho \rangle_T,
\]

in which \( \rho_t \) is the charge distribution in the electron box at the present time step and \( \langle \rho \rangle_T \) represents the average charge distribution over the moving time window. It should be stressed that the oscillations visible in Fig. 5.10 are caused by the moving time window and unrelated to a physical effect.
Figure 5.10: Left: graph with the number of particles in the electron box for three settings for the mean field determination. In case 1 a time window of 200 time steps (200Δt) is used and the mean field is not mixed with the current time step. In case 2, the time window is increased to 500Δt and the mean field mixing parameter $f = 0.8$. In case 3, $f = 0$. Increasing the time window increases the period of the oscillation. Decreasing $f$ decreases the amplitude. Case 2 and 3 are shifted vertically by 100 and 200 respectively in the interest of clarity. Right: time window with length of one time step up till 0.09 ns, 500 time steps afterwards.
Figure 5.11: Emission rate spectra for three simulations with box length 10 μm and box widths 1 μm, 2 μm, 10 μm. The saturation current is set at \( j_{\text{sat}} = 100 \, \text{A/cm}^2 \) and the extraction field is 1 V/μm leading to a space charge limited current of about 50% of the saturation current. Each spectrum is obtained by Fourier transformation of the emission rate and smoothed over by averaging over repeated simulations and filtering with a sliding mean filter after removing the DC term.

The oscillations were explained in terms of a deviation with respect to the equilibrium state of the amount of charge in the electron box, in particular, the deviation as a result of the Poisson stochastical nature of emission. A more obvious reason for nonequilibrium is the initial absence of charge and thus potential barrier. As the left graph in Fig. 5.10 shows that the amount of charge in the electron box initially rises to higher than the stationary state and then starts oscillating, one may be inclined to attribute the oscillations solely to the initial charge build-up that occurs when there is no charge in the system. If this were the case, oscillations would not appear if the space charge field is stationary to start with. Fig. 5.10, right, shows the charge in the system in which oscillations are initially suppressed by setting \( f = 0 \) in order to obtain a stationary solution. At \( t = 0.03 \) ns the time window is 'switched on' (\( f = 1 \)) and increasing oscillations appear. Thus, Poisson emission statistics are indeed a sufficient source of nonequilibrium to cause unphysical oscillations when the surrounding mean field is determined using an averaging time window.

The time window was intended to dampen short term fluctuations but it introduces unphysical long term fluctuations. The reason for introducing the time window was the concern that the amount of charge in a narrow electron box would be too low, thus overestimating the Poisson fluctuations in the mean field. Whether this is a valid concern can be determined by looking at the emission rate spectrum of the space charge limited current. If fluctuations in the mean field affect the current they must show up in the emission spectrum. If nevertheless the emission spectrum does not depend on the electron box width, the statistics of the electron box are sufficient to appropriately represent the mean field.

Three simulations were performed with fixed electron box length and differing widths. The emission rate spectra were determined by Fourier transformation of the emission per unit time. An example of the latter was encountered earlier in Fig. 5.9, the solid line in the left graph. One has to take care to dismiss the initial time interval, at which the current is not yet stationary. The results, plotted in Fig. 5.11, show that the choice of electron box width has no effect on the fluctuations in current. The leading edge of the spectrum reflects the settling time of the space charge barrier. Note that here is referred not to the settling time at the start of a simulation, when the space charge barrier is completely absent, but to the settling time throughout the stationary state, when the space charge barrier continuously adjusts for nonequilibrium as a result of Poisson emission statistics.
Figure 5.12: I-V characteristics of planar diode with diode spacing 10 μm. The four curves represent the characteristics of the Langmuir model, the Monte Carlo model including mirror charge, the Monte Carlo model including image charge in a cathode point charge distribution and the 'ghost electrode' model. The Langmuir curve gives a sharp transition between space charge limited and temperature limited current. The image charge curve shows a slightly more gradual transition. The mirror charge curve shows enhanced emission due to the Schottky effect. It lacks the presence of a sharp transition which makes it difficult to define the saturation current for a realistic emitter.

Consistency with planar Langmuir diode

In Fig. 5.12 the consistency of the Monte Carlo program with the Langmuir model is shown for a planar cathode without surface structure. The four curves are I-V characteristics. The initial part of each curve shows a transmitted current lower than the saturation current (624 electrons per ns for an emitting square with sides 1 μm) as a result of the space charge barrier. Two different surface charge methods are shown: 'mirror' which uses a mirror point image charge representation of the induced surface charge (as discussed in Section 5.3.1) and 'image' which uses the cathode model (Section 5.6.1) that can be adapted to compensate the induced surface charge. In these two curves the anode voltage is varied by imposing different Dirichlet boundary conditions. The mirror curve has been obtained with a mirror plane at z = 4 nm, and the saturation current was corrected to compensate for the image charge barrier. The 'ghost' curve shows the I-V characteristic in the case that all electrons are subjected to a constant external acceleration (Section 5.2.2). Here the anode voltage is varied by changing the acceleration and defining it as the potential difference between cathode and anode due to the resulting stationary space charge barrier.

The Monte Carlo curves follow the Langmuir model nicely in the space charge limited regime. Near the knee of the curve some deviations are observed. The Langmuir model contains no coupling with the image charge in the cathode. When the Langmuir minimum lies far from the cathode, in the deep space charge limited regime, one expects no influence from the image potential. But when the minimum is close to the cathode on a scale where the image forces are large the transmitted current is lower than one would expect from the Langmuir model.

When the current is temperature limited and an accelerating field is present at the cathode surface, the superposition of the image potential and the external field will cause a lowering of the effective work function, thereby enhancing the emission according to the Schottky effect.
Beam spreading in a uniform beam

The spreading of a beam under influence of the negative space charge lens was discussed in Section 2.2.2. A uniform cylindrical beam in drift space is shaped according to a universal curve shown in Fig. 2.3. If the particle box is equipped with a homogeneous circular virtual emitter at nonzero potential with no external fields we obtain a situation that can be described with the universal curve.

In Fig. 5.13 an explicit example is plotted. The particle box in this particular case has dimensions length 20 µm and infinite width, thus no cyclic transversal boundary conditions are present. The beam energy is 2 V. The initial beam radius is 1.5 µm and the beam converges to a virtual crossover at \( z = 10 \mu m \). For practical situations with higher beam energy or a larger initial beam radius the thermal transversal velocity can be neglected with respect to the axial velocity but at these microscopic circumstances the electrons have to be stripped of their thermal transversal velocity in order to satisfy the assumption of homoeentricity used in the derivation of the universal curve.

The simulated beam radius is obtained by combining electron clouds from repeated computations, determining the radius containing 50% of the particles in small axial slices, and multiplying with \( \sqrt{2} \) to obtain the outer beam radius. The first order space charge lens effect is reproduced correctly in the simulation.

Pierce gun

A Pierce electron gun is a particular electrode configuration designed to constrain beam spreading due to space charge. Beam spreading occurs since the space charge forces acting on electrons at the edge of the beam are unbalanced. In a Pierce gun the electrodes are shaped such, that the space force at the edge of the beam is counteracted by the Laplace field of the electrode configuration. If one considers the potential distribution in a planar diode with a circular emitting area, the field lines will be curved due to the lateral finiteness of the electron beam, resulting in an outward radial force. If the emitting electrode is funnel-shaped, the field lines can be made planar so that an electron at the edge of the beam feels the same force as an electron at the center of the beam. Consequently, the electron beam is constrained to a cylinder that has the emitting area as base.

The particular angle of the funnel originates from the demand that the potential distribution along the cylinder edge is identical both with and without space charge. When the space charge distribution is taken to follow Child’s model, it can be shown that the equipotential encircling the emitting area must be at a 67.5° angle, the so-called Pierce angle [77].
Figure 5.14: Circular cathode surrounded by a collar at the same potential. The angle between collar and cathode is the Pierce angle, $67.5^\circ$. The stars indicate the positions of the point charges, the surface is the resulting equipotential surface. The z-axis is inverted for clarity.

An emitting equipotential plane shaped according to these rules can be simulated using the point charges cathode representation, as shown in Fig. 5.14. The anode in this example is placed at $z = 10 \mu m$. Two simulations are performed, one in which the emitter is planar, and one in which the emitter is shaped like a Pierce-funnel. The emitting area is identical in both situations. In the Child model, it is assumed that electrons are emitted with zero initial velocity. Therefore, the emitter temperature is set to almost zero in the simulations. The radius of the emitting area is $4 \mu m$. The shape of the space charge field and the electron beam are plotted in Fig. 5.15.

The specific shape of the emitting electrode straightens the equipotential lines and counteracts beam spreading. In fact, the beam spreading is slightly overcompensated and the beam radius decreases by about 4% over $10 \mu m$. The beam convergence is due to the fact that for a perfect Pierce geometry, the anode must also be nonplanar, a subtlety that is not included in the model shown in Fig. 5.14. Also, the funnel should ideally extend to infinity, which it does not.

5.9 Conclusions

In this chapter a simulation tool was discussed that can calculate the dynamics of an electron cloud near a thermionic emitter. It was shown that discreteness of the electron cloud leads to the necessity of describing the work function partly in terms of image potential. The simulation tool takes into account statistical Coulomb interactions, image charge and space charge effect due to the cumulative action of charge at large distances. It is shown that when a weak electric field is applied to a uniform planar emitter, the emitted current follows Langmuir’s theory of space charge limited emission. The specific influence of statistical interactions and surface inhomogeneities will be discussed in the following chapters.
Figure 5.15: Equipotential lines and beam radius in a Pierce funnel (left) and in a planar diode (right) in the plane \( y = 0 \). The potential distribution is obtained by averaging over a large number of electron clouds in order to smoothen the equipotential lines.
Chapter 6

Influence of statistical Coulomb interactions

Emission models that form the basis of self-consistent field computations make use of the approximation that emitted electrons form a smooth space charge jelly. In reality, electrons are discrete particles that are subject to statistical Coulomb interactions. The simulation tool described in the previous chapter is used to evaluate the influence of discrete space charge effects on self-consistent calculations of CRT optics. We find that interactions in the space charge cloud affect the electron trajectories such that the velocity distribution is Maxwellian, regardless of the current density. Coulomb interactions in a CRT crossover have a negligible effect with regard to increase of energy spread and angular distribution width.

6.1 Introduction

In the previous analyses, an electron beam was considered to be a smooth jelly of charge without any internal structure. In Chapter 2 the distinction between the average, smooth space charge and the discrete space charge was made. Smooth space charge was discussed in terms of optical self-influence of the beam. The optical effects were explained on the basis of a cylindrically symmetric beam, but also in the absence of geometric symmetries, the space charge effect of an electron beam can be described in terms of first order optical parameters and aberrations of the imaging system.

A possible source of inaccuracies in virtual emitter models is the approximation that the discreteness of space charge may be neglected. The effect of this approximation is two-fold. Firstly, the statistical Coulomb interactions are ignored. However, it is known that the discrete character of the charged particles has an influence on the phase-space distribution. Interacting electrons arrive at the target plane with a transversal displacement with respect to noninteracting electrons. This displacement is stochastic and cannot be related to an optical effect. The displacement is called trajectory displacement [82]. A further effect of statistical interactions is the increase in axial velocity width [83]. Generally, Coulomb interactions can lead to a decreased resolution of the imaging system.

Secondly, fluctuations in the spatial current distribution that are inherently present due to the discreteness of the emitted charge can not be taken into account. The effect of Poisson fluctuations in the current is known as shot noise. These fluctuations apply equally well to the amount of charge in the system, and thus the optics. In the space charge limited beam in a CRT, the virtual emitter properties are determined by features in the space charge field with a length scale of a few microns. In a cube with 1 μm size the number of electrons is as low as ten, so statistical fluctuations may well play a role.

In Chapter 5 a computer program was discussed that is capable of calculating the local stationary electron current from a thermionic emitter in space charge limited mode, including electron-electron interactions and cathode surface inhomogeneities. This simulation tool can be used to examine the influence of discrete space charge effects in a CRT. In Section 6.2 the effects of interactions in the region surrounding the virtual cathode will be examined. The influence in the remainder of the beam will be discussed in Section 6.3. Finally, a general consideration of the increase of axial velocity width known from plasma physics is presented in Section 6.4.
6.2 Coulomb interactions in a planar diode

The basis of the virtual emission function is the Langmuir model for an infinite planar cathode in space charge limited operation. The Langmuir model predicts a potential minimum in front of and parallel to the cathode. The potential minimum performs the role of virtual emitter, its depth determines the emitted space charge limited current with the relationship given in Eq. (2.43)

$$j_{\text{Langmuir}} = j_{\text{sat}} \exp(eV_m/k_BT),$$

(6.1)

showing that the emission is uniform over the entire virtual emitter, provided that the saturation current, $j_{\text{sat}}$, is homogeneous over the entire emitter surface.

In reality the potential minimum is distorted from a perfect plane due to the discreteness of the space charge cloud. The distorted potential distribution affects the emission properties of the virtual emitter since the emitted particles experience different potential minimums. Also, the statistical interactions in the region between the cathode and the virtual emitter may change the electron velocity distribution.

We have simulated a planar cathode with $j_{\text{sat}} = 10$ A/cm$^2$ with temperature $T = 1160$ K (0.1 eV/k_B). The simulation has been performed for a cubic electron box with edge 10 μm, which is large enough to encompass the characteristic features of the expected potential distribution. A cubic region of this size in front of an emitter contains about one thousand electrons.

At the start of the simulation, the electron box is empty. Electrons are being emitted into the box continuously, at a rate corresponding to the saturation current. Apart from the mutual Coulomb interactions, the electrons experience an extracting electric field of $F = -10^5$ V/m. This particular value for the extracting field is chosen such that in the stationary state, current flows both through the end plane in the positive $z$-direction and through the cathode plane in the negative $z$-direction, in other words, the current is space charge limited. After about 0.05 ns a state of equilibrium is reached, that is, there is no systematic change in the total number of particles in the box. The current flowing through the end plane, determined by counting the number of electrons passing the end plane during a short time interval after equilibrium, is 2.74 A/cm$^2$. The potential difference between emitter and end plane is 0.32 V.

This result is in agreement with Langmuir’s theory if the emission of a 10 μm diode with anode voltage 0.32 V is calculated. These settings yield a space charge minimum $V_m = -0.133$ V at $z_m = 2.35$ μm, resulting in a space charge limited current of 2.65 A/cm$^2$. The difference with respect to the Monte Carlo current can be explained in terms of numerical noise. Note that the applied electric field $F = 10^5$ V/m corresponds to an anode voltage of 1 V, only in the absence of space charge. The reduction to 0.32 V is due to the space charge distribution in the diode.

In Fig. 6.1 equipotentials of the stationary electron cloud after 0.4 ns are plotted. Instead of a well-defined equipotential line corresponding to the minimum, parallel to the cathode, the minimum plane is a disjunct surface with multivalued potential, indicated with the dotted lines.

The velocity distributions at the plane $z = 10$ μm are plotted in Fig. 6.2. The dotted lines correspond to the Maxwellian velocity distributions one would expect in a continuous space charge field without statistical interactions.

The transversal velocity distribution of the electrons is indistinguishable from a Maxwellian distribution, except for the presence of Monte Carlo noise. In contrast, the axial velocity distribution of the electrons clearly shows a leading edge instead of a discontinuous step.

In order to exaggerate the effects of statistical interactions, the current was increased thousandfold to a value uncharacteristic for CRTs of $j_{\text{sat}} = 10^4$ A/cm$^2$. The applied electric field was increased to $F = 10^7$ V/m and the box width was decreased to 0.5 μm in order to obtain a sufficiently high space charge limited current and to limit the number of particles. The results are shown in Fig. 6.3. The transversal velocity distribution is similar, whereas the statistical axial velocity distribution now deviates significantly from the Maxwellian distribution.

The Maxwellian axial velocity width is much smaller than the transversal velocity width due to the axial acceleration, in contrast to the previous situation (Fig. 6.2) with low saturation current and low electric field. We will return to this phenomenon in Section 6.4 where the increase of the axial velocity width will be addressed in a different manner.

One may be inclined to conclude that statistical Coulomb interactions do not change the transversal velocity component even in situations with current densities as high as $10^4$ A/cm$^2$. However, this is counterintuitive, considering the fact that Coulomb interactions have in particular effect at low beam energy.
Figure 6.1: Stationary space charge potential distribution at the plane \( y = 0 \) (solid equipotential lines), plotted together with the solution according to the Langmuir model (dashed line). The discontinuous dotted line shows the positions of the local potential minima. The deviation from the planar potential distribution due to the discreteness of the electrons is most clearly visible at the potential minimum.

Figure 6.2: Velocity distributions at \( z = 10 \ \mu m \) (solid lines). The dotted lines correspond to the Maxwellian distribution in a continuous space charge cloud.
In Fig. 6.4, left, electron trajectories near the virtual emitter are plotted, projected in the plane perpendicular to the extraction field. On the right, the same trajectories are plotted without the influence of statistical interactions. This graph was obtained with a current density of $10^4$ A/cm$^2$. The region plotted is a square with sides 0.4 μm.

The influence of Coulomb interactions on the trajectories is clearly visible. Due to the planar symmetry, trajectories without interactions form straight lines in the figure, since the curvature that results from the space charge barrier is not visible in this plane. Electrons experiencing interactions on the other hand, are pushed from a straight line by their neighbours.

The notion that Coulomb interactions in the virtual emitter region influence the trajectories of the electrons without affecting the velocity distribution seems contradictory. This notion can be falsified in the following manner. We changed the velocity distribution of emitted electrons at the emission plane artificially from a Maxwellian to a top hat shaped distribution and saw the Maxwellian velocity distribution reappear when performing the simulation with a current density of $10^4$ A/cm$^2$. Apparently, Coulomb interactions do play a role, but their effect is precisely to maintain the Maxwell distribution. This is in agreement with the fact that random interactions bring about a relaxation towards the most probable distribution, which is the Maxwell distribution.

The calculation with a top hat initial velocity distribution was repeated for different current densities. The plots in Fig. 6.5 show, that at low current densities, the velocity distribution is far from Maxwellian. The fact that at a current density of 10 A/cm$^2$, which is more realistic for a CRT, the velocity distribution has remained top hat-like, indicates that at realistic current densities the space charge cloud is too rare to be susceptible to statistical interactions.

Therefore, if a Maxwellian emission distribution at the cathode is assumed, the transversal velocity distribution of the emitted electrons at the virtual emitter of a planar cathode is effectively not influenced by Coulomb interactions in front of the emitter, no matter how high the saturation current density.

### 6.3 Coulomb interactions in a CRT beam

As we have determined in the previous section, Coulomb interactions do not influence the velocity distribution near the emitter. The particle density in a CRT is highest near the emitter, but this does not necessarily imply that Coulomb interactions can also be neglected elsewhere in the beam. Interactions near the emitter effectively conserve the Maxwellian distribution. But if the distribution is not
6.3. Coulomb interactions in a CRT beam

Figure 6.4: Trajectories in front of an emitter with $j_{\text{sat}} = 10^4$ A/cm$^2$ in the $xy$-plane with (left) and without (right) interactions. All deviations from a straight line are due to Coulomb interactions.

Figure 6.5: Transversal velocity distribution at $z = 10$ μm of electrons emitted with a tophat-shaped distribution. When the current density is high, the tophat distribution relaxes towards a Maxwellian. For low currents, the particle cloud becomes too rare for interactions to be noticeable.
Maxwellian to start with, interactions can have a noticeable effect, since there is a tendency to transform that distribution to a Maxwellian distribution.

The space charge cloud near the emitter was found to be too rare to have a noticeable influence over a length of 10 \( \mu \text{m} \). As the influence of Coulomb interactions depends not only on the particle density but also on the interaction time, interactions in the rest of the gun may still be important. The research of Coulomb interactions in particle beams is well established [19, 28, 79, 84] but Monte Carlo simulations of Coulomb interactions near the emitter are somewhat more complicated for reasons that will be outlined below.

The continuous stream of electrons can often be replaced by a single pulse of current, forming a comparatively small sample of electrons traversing in a bunch through the optical column. The interactions in the total electron beam are thus approximated by neglecting all electrons outside of the bunch. Travelling along the optical axis, the length of the electron bunch will increase due to the unbalanced space charge forces, resulting in an unphysical axial space charge effect. Furthermore, the initial energy distribution of the emitted causes an additional spatial dispersion. The dispersion is physical, but the increase in sample length causes an unphysical decrease in Coulomb forces and a consequent underestimation of statistical interactions.

The unwanted effect of the energy distribution can be remedied by increasing the number of electrons in the sample, thereby increasing the length of the electron bunch. In general, the length of the current pulse should be chosen such that the spatial dispersion due to the energy spread is negligible with respect to the length of the sample. The lower the starting beam energy, the larger the number of particles required to keep the spatial dispersion small relative to the sample length.

A correction for the unbalanced space charge forces can be implemented as long as the axial space charge effect on both sides of the bunch in the complete beam is similar. For a space charge limited electron beam near the emitter, where the space charge forces are different at the edges, this is decidedly untrue. Furthermore, space charge limited emission requires some build-up of charge, which can not be provided by a bunch of electrons that is limited in size.

For these two reasons the method of simulating Coulomb interactions with a short pulse of current works for particle beams after some acceleration, past the extraction electrode, but is not well-suited near the source. Therefore, simulations of interactions are computations in which one generally makes use of a virtual emitter. Space charge effects, space charge limitation and Coulomb interactions must then be accounted for in the virtual emission function.

### 6.3.1 Interactions in the beam forming region

To calculate all space charge effects including Coulomb interactions in the beam forming region starting from the real emitter, one would ideally calculate all mutual interactions in a region that includes the crossover, for instance with the computation model discussed in Chapter 5 by enlarging the electron box to envelope the complete beam forming region, taking into account a cathode lens field and determining the stationary current.

The difficulty with this approach is the amount of particles that are present in the beam forming region. In the first millimeter from the cathode, in the rotationally symmetric CRT gun discussed in Chapter 3 the number of electrons is about \( 10^6 \).

Evaluating all mutual interactions in a stationary space charge cloud consisting of this number of electrons is computationally demanding. It is possible with more advanced methods that take exact mutual interactions into account only in a small region around each particle. In one of these methods, recently introduced in simulations of interactions in electron beam columns [28], the space charge field due to particles at large distances is taken into account by clustering them into a single superparticle. An alternate method, used in simulations of charged particle plasmas, represents the space charge cloud with a smooth charge distribution on a mesh and applies a local correction for the charge that has already been taken into account via mutual interactions [85].

Here, we will discuss how the concept of the virtual emitter can be exploited to use our computation model to determine the influence of Coulomb interactions in the beam forming region, without having to resort to simulations of millions of electrons. The computation is broken down in a two stage simulation. First, the — possibly space charge limited — virtual emission function is determined by calculating the stationary current in a small diode. Particles collected at the virtual anode then make up the distribution of the virtual emitter in the second stage of the simulation.
6.3. Coulomb interactions in a CRT beam

The merit of this two-step approach is that the number of electrons is contained, since in the first stage the simulated region is small, and in the second stage the spatial dispersion due to the energy distribution is small because of the higher beam energy. Also, in the first stage one may exploit the fact that Coulomb interactions can be ignored, which allows the use of additional approximations like grouping electrons together into super particles. In the second stage, where the influence of interactions needs to be determined, the current pulse approach can be used.

This method is not without its flaws. Most notably, even in a small diode the number of particles becomes very large when a realistic emitting area of a CRT is to be simulated. Thus, for a CRT only a situation close to cut-off can be calculated where the current is very low, otherwise the advantage of calculating a small region in the first step is lost. Furthermore, the virtual emitter is calculated while completely discounting the shape of the space charge field past the virtual anode. Any curvature of the virtual emitter can therefore not be regarded as being accurate.

On the other hand, for different emitter geometries where the current is lower and the extraction field is higher the number of particles in a diode may be within computable range. When the effect of space charge past the virtual anode on the particles in the diode can be ignored the two-step approach is valid.

6.3.2 Interactions in the CRT crossover

As explained above, the electron beam in a CRT contains too much charge for both a simulation of all mutual interactions in the complete beam forming region and a simulation performed separately in the two regions before and after a virtual emitter.

Abandoning the notion of calculating the exact optical properties of a CRT including statistical interactions we can instead examine the differences in phase space with and without interactions to obtain a quantitative measure of their influence. Note that including interactions does not automatically imply including the space charge effect, since the optical effect of the self-field is different in the absence of charge on both ends of a particle bunch.

For this analysis we accept the calculated space charge effect to be erroneous. Statistical effects will show up regardless. Their influence depends mostly on the beam potential, beam diameter and half-opening angle in the crossover, rather than on the specific spatial current density distribution. As we have seen in Chapter 2 a nonuniform spatial current density introduces aberrations that will be ignored in this analysis. Thus, we dispense with the necessity for an accurate virtual emission function that prescribes the correct emission current density distribution, as long as the diameter of the emitting area and the average current density are similar to the situation including space charge.

The computer program allows the inclusion of an axial Laplace potential distribution to provide a paraxial focussing field. The axial Laplace potential distribution of the rotationally symmetric gun discussed in Chapter 3 has been calculated using SCELOP.

For the first step a virtual emission function is determined by starting electrons at the emitter plane and collecting them after they have travelled 2 μm in the paraxial field. At z = 2 μm the beam potential is about 10 V, which is high enough to limit the spatial dispersion that occurs due to initial energy spread.

For the second step 1000 electrons distributed according to the emission function at z = 2 μm are traced through the paraxial field and collected at five equidistant axial positions from z = 0.2 mm to 1.0 mm by defining diagnostic planes perpendicular to the optical axis at those positions. The rate of emission is chosen such, that the electron bunch corresponds to a current of 1 mA.

The unbalanced space charge forces are corrected for by removing the correlation between the axial coordinate and the axial Coulomb force. This procedure is identical to the one used in MONTEC [19].

Figure 6.6 shows some trajectories plotted together with the electron distribution at five different times. The electron positions are plotted at the time the foremost electron crosses a diagnostic plane.

To make the influence of interactions visible, electrons are traced two times, once including and once excluding Coulomb interactions.

Boersch effect

The interactions in the crossover give rise to a broadening in the energy distribution of the electrons. In Fig. 6.7 the energy distribution in the first plane past the crossover is plotted for the simulations including and excluding Coulomb interactions. A significant energy broadening is visible, 230%.

This energy broadening in the beam forming region in a CRT is the result of a broadening of the axial velocity distribution. It has no noticeable influence on the spot size at the screen since the spot
Figure 6.6: Electron trajectories and positions projected onto the xz-plane. The positions plotted are five 'snapshots' of the electron bunch as the electrons travel along the optical axis. In the simulation, the bunch contains 1000 electrons; here, only 400 positions and 20 trajectories are plotted.

Figure 6.7: Energy distribution at the plane \( z = 0.6 \) mm for electrons including and excluding Coulomb interactions. The energy widths (FW50) are indicated in the figure. The axial potential at \( z = 0.6 \) mm is 822.6 V.
is mainly determined by space charge effect, spherical aberration in the main lens and the size of the
crossover, as mentioned in Chapter 3. The contribution of chromatic aberration to the spot size does
contribute significantly only at energy widths at least an order of magnitude larger than the few tenths
of volts resulting from Boersch effect in the crossover. Furthermore, as a result of the global space charge
field the relative energy broadening due to Coulomb interactions is in practice smaller than 230%; the
variations in the space charge field across the beam are several tens of volts which is not taken into
account in Fig. 6.7 since only a small beam slice is modelled rather than the entire beam; moreover
the energy variation across the virtual emitter due to the varying depth of the space charge barrier is
also several tenths of electron volts. The latter two effects are adequately modelled in a self-consistent
calculation without the necessity to include discrete space charge.

Trajectory displacement

Coulomb interactions in the crossover affect the transversal velocity distribution of the electrons as well as
the axial velocity distribution. These statistical angular deflections give rise to a lateral displacement at
the screen plane, referred to as trajectory displacement. As the lateral displacement is purely stochastic,
it can not be compensated by refocussing and it leads to a larger spot.

In Fig. 6.8, left, a scatter plot of the angular distribution of the electrons in the absence of Coulomb
interactions at the plane $z = 1$ mm is plotted. When interactions are present, the directions of the
individual electrons are slightly different. A small change in direction could over a large distance amount
to a significant lateral displacement.

On the right, a scatter plot of the angular displacement of the electrons as a result of interactions
is presented. The Coulomb forces lead to an increase in opening angle of 0.5% due to the global and
stochastic space charge combined.

The distribution of the angular deflection by itself does not allow any conclusions to be drawn relating
to the influence of trajectory displacement as the global space charge effect also affects the angular
distribution, yet the latter is not conventionally associated with trajectory displacement. One is often
interested in the isolated contribution of stochastic space charge on the spot size. After all, the relative
importance of this contribution determines whether it is necessary to consider the electron beam as a
cloud of particles rather than as a jelly of charge.

Separating the stochastic space charge from the global space charge is not possible from these simu-
lation results. One can however apply a correction to the lateral interaction force during the course of
a simulation to remove the global space charge lens effect. This is similar to the correction to the axial
force that is used to prevent unbalanced space charge forces at the ends of the particle bunch, discussed at the beginning of this section. The first order lens effect of the space charge lens manifests itself as a linear correlation between a particle's radial position and the radial component of the Coulomb force. This correlation can be removed for the ensemble of particles at each integration step which will then cancel out the first order global space charge effect. The aberrations of the space charge lens still remain, but since the spatial current distribution started out uniform at the virtual emitter higher order effects are expected to be negligible.

In Fig. 6.9 the beam diameter (defined as the FW50 of the spatial distribution, the diameter containing 50% of the current) drifted back from \(z = 1\) mm is plotted. Three curves are shown, corresponding to the cases when no interaction is present, interactions are present but no global space charge lens correction is applied, and interactions are present with space charge lens correction. The former two curves clearly show the effect of the space charge lens. The virtual crossover is smaller and lies more to the right when space charge forces are present as a result of the negative lens action of the global space charge. The curve without interactions and the curve with lens correction are not significantly different, indicating that trajectory displacement in a CRT crossover is a negligible effect.

### 6.4 Relaxation of beam temperature

The concept of relaxation of the velocity distribution as mentioned in Section 6.2 is also interesting in the discussion of the axial velocity distribution. Under the influence of an external axial electric field, the axial velocity distribution of the electron cloud is contracted, illustrated in the graphs of the Maxwellian axial velocity distribution in Figs. 6.2 and 6.3. This is a consequence of the quadratic dependency of the kinetic energy on the particle velocity. Examine the difference in kinetic energy between two particles with axial velocity \(v_z\) and \(v_z + \Delta v_z\),

\[
\Delta E = \frac{1}{2} m (v_z + \Delta v_z)^2 - \frac{1}{2} m v_z^2 \approx m v_z \Delta v_z,
\]

when \(\Delta v_z \ll v_z\). The acceleration of the two particles travelling a short distance in an electric field leaves their energy difference invariant. The ensuing increase in \(v_z\) must be accompanied by a decrease in \(\Delta v_z\).

Random interactions serve to redistribute the two transversal velocity components individually to an equilibrium distribution. When including the axial component in the consideration of the random
exchange of momentum it is useful to regard the electron cloud in the frame of reference moving along with the beam. There is no preferential direction in this frame and the equilibrium velocity distribution is isotropic.

It would be interesting to see how the velocity components redistribute as the beam progresses along the optical axis. To facilitate a more quantitative consideration of relaxation towards equilibrium it is useful to introduce the beam temperature. The velocity distribution of a particle beam can be specified in terms of an longitudinal and a transverse beam temperature.

\[
T_{\parallel} = \frac{m}{k_B} \left( \langle v_{\parallel}^2 \rangle - \langle v_{\parallel} \rangle^2 \right),
\]

\[
T_{\perp} = \frac{m}{2k_B} \left( \langle v_{\perp}^2 \rangle - \langle v_{\perp} \rangle^2 \right),
\]

respectively. Here, \(v_{\parallel}^2 = v_{\parallel x}^2 + v_{\parallel y}^2\). Relaxation of the velocity distributions, as a result of interactions, can now be described via temperature relaxation of the longitudinal and transverse temperature. The longitudinal temperature, like the generic axial velocity spread discussed above, reduces with acceleration.

Regarding the simple case of relaxation in a planar diode, one realizes that it can be considered as a particular beam that is confined by fields that have no effect on the relaxation. Consequently, as far as beam temperature relaxation is concerned, no accelerating or focussing axial field needs to be considered. The temperature relaxation in this case is linked to the velocity distribution in a volume of particles in a beam at a certain energy, moving along the optical axis.

As an approximation the relaxation can be determined by inserting a bunch of particles in a very long particle box at certain energy and observing its velocity distribution as it is allowed to progress freely along the optical axis. Of course, what will happen then is that the particle bunch will increase in length due to the finite energy width, as a result of which the particle density will decrease. Fast particles move to the front of the bunch while slow particles move to the back, thus no sufficient collisions can take place to cause relaxation.

However error-prone, the beam temperature relaxation for a bunch of particles was simulated in this manner, the result plotted in Fig. 6.10. The bunch consisted of 2000 particles with beam energy 80 V forming a sample of initial length 37 \(\mu\)m in an electron box with width 1.6 \(\mu\)m. The particular choice of beam energy and sample length is chosen to correspond to the space charge limited current plotted in Fig. 6.3, followed by drift space. In view of the excessively high saturation current this is not a realistic physical situation occurring in a CRT but as we will see below, beam temperature relaxation is scaleable and the results can be adjusted for application to a CRT.

Before discussing the outcome of the simulation it is informative to analyze the temperature relaxation adopting a theory originally applied to plasma physics. The relaxation between the longitudinal and transverse temperature can be described using the equation [86]

\[
\frac{dT_{\perp}}{dt} = - \frac{1}{2} \frac{dT_{\parallel}}{dt} = - \frac{T_{\perp} - T_{\parallel}}{\tau},
\]

where the factor \(\frac{1}{2}\) is due to the fact that \(T_{\parallel}\) changes twice as fast as \(T_{\perp}\). Equation (6.5) defines the relaxation time \(\tau\). It is a function of the particle charge and mass, as well as \(T_{\parallel}, T_{\perp}\) and \(T_{\parallel}\) [87] with

\[
\frac{1}{\tau} = \frac{8\pi^{1/2}ne^4}{15(4\pi\varepsilon_0)^2m^{1/2}(k_BT_{\text{eff}})^{3/2}} \ln \Lambda,
\]

where \(n\) is the particle density, \(\ln \Lambda\) is the Coulomb logarithm

\[
\ln \Lambda = \ln \frac{12\pi(\varepsilon_0k_BT)^{3/2}}{e^3n^{1/2}}
\]

and the effective temperature \(T_{\text{eff}}\) is defined through

\[
\frac{1}{(T_{\text{eff}})^{3/2}} = \frac{15}{4} \int_{-1}^{1} \frac{x^2(1-x^2)dx}{[(1-x^2)T_{\parallel} + x^2 T_{\perp}]^{3/2}}.
\]

The temperature relaxation can be determined by solving Eq. (6.5) numerically. It is plotted in Fig. 6.10 together with the temperature relaxation as determined from Monte Carlo simulations. It is to be
expected that the two types of curves are not completely similar since the Monte Carlo curves suffer from the problem with the spatial dispersion, as well as the unbalanced space charge force.

Initially, the transverse beam temperature is equal to the cathode temperature and the longitudinal beam temperature is negligible due to the beam acceleration. Advancing along the optical axis, Coulomb collisions cause relaxation of the beam temperatures until equilibrium is reached. According to the plasma theory, at $z = 2$ mm, the temperatures differ by 0.5%.

The time scale of relaxation is reasonably well reproduced in the simulations. The transverse temperature stays behind somewhat, since the transverse dimensions of the electron bunch are much smaller than the axial dimensions. One would prefer the dimensions to be of equal order but this would result in extremely long computation time. Convergence to equilibrium does not occur in the simulation. This is mainly due to the unbalanced axial space charge forces that are present in a finite length bunch.

The fact that the plasma theory is not exactly reproduced by the simulation is not a cause for concern considering the applicability to a practical beam. The problem calculated here, an extremely high current beam running unhampered at 80 eV for several millimeters, is not particularly realistic. An electron bunch in the spot forming region of a CRT has to run for 14 km to come to the same level of equilibrium as the fictitious beam achieves at 2 mm. For practical beams only a very small interval at the left of Fig. 6.10 is relevant, where the differences between simulation and plasma theory are small.

Using the concept of beam temperature we can reconsider the axial velocity broadening in the space charge limited diode discussed in Section 6.2. Now the particles do not propagate in drift space but experience acceleration traveling from cathode towards anode. Furthermore, the initial longitudinal beam temperature can not be neglected. It can be determined by direct application of Eq. (6.3) to the longitudinal velocity distribution introduced in Chapter 2, Eq. (2.21c) to yield $T_{\|,i} = 2(1 - \pi/4)T_{\text{cat}}$, or $498$ K at $T_{\text{cat}} = 1160$ K.

Acceleration causes the longitudinal beam temperature to reduce inversely proportional to the beam energy. It also influences the particle density by virtue of the continuity equation. Thus, when integrating Eq. (6.5) the particle density appearing in Eqs. (6.6) and (6.7) is changed accordingly, and a correction based on the increase of particle velocity is applied to the longitudinal beam temperature at successive integration steps.

Thus one finds a longitudinal beam temperature at $z = 10$ $\mu$m of $T_{\|} = 100$ K for $j_{\text{cat}} = 10$ A/cm$^2$ and $F = -10^5$ V/m, and $T_{\|}= 35$ K for $j_{\text{cat}} = 10^4$ A/cm$^2$ and $F = -10^5$ V/m, corresponding to axial velocity width $3.9 \cdot 10^4$ m/s and $2.3 \cdot 10^4$ m/s, respectively. The calculated beam temperatures depend somewhat
on the particular potential distribution in the diode. In both cases, a potential distribution according to Child's model has been assumed.

In the former case, interactions hardly have any influence as one could see from Fig. 6.2. The axial velocity spread can solely be attributed to the initial beam temperature contracted due to the acceleration as the kinetic energy of the electrons is increased by a factor of five from the thermal kinetic energy in the space charge field. In the latter case, the observed axial velocity spread is consistent with the calculated longitudinal beam temperature. Thus, the increase of axial velocity spread can be understood in terms of the relaxation of beam temperature.

6.5 Conclusions

We have used the simulation tool discussed in Chapter 5 to take into account all effects relating to discrete space charge in front of a thermionic emitter operating in the space charge regime. Disturbances in the potential distribution due to the discreteness of the charge do not show up in the transversal velocity distribution of the electrons. Statistical Coulomb interactions do not interfere with the Maxwellian transversal velocity distribution.

The dynamics of the space charge potential distribution gives rise to a broadened axial velocity distribution. In Chapter 3 we saw that the axial velocity distribution of the virtual emitter does not have a large influence on the spot distribution in a CRT. Furthermore, the axial broadening due to the space charge cloud dynamics is very small for CRT conditions.

For extremely high current densities there may be a visible effect of the broadened axial velocity distribution. Although these current densities can in practice occur for thermionic emitters [88], they are applicable to completely different situations than CRT conditions.

Statistical interactions in the crossover lead to increased energy width even for ordinary CRT emission conditions. The energy broadening is a few tenths of volts for a typical CRT current. Since the chromatic aberration in a CRT gun is of minor importance relative to the spherical aberration, space charge effect and the crossover size, statistical interactions in the crossover do not influence the spot distribution at the screen.

This allows us to conclude that possible errors in the spot distribution simulations of CRTs, obtained with a virtual emitter, can not be due to the discreteness of the space charge cloud near the emitter, nor to the interactions in the crossover.
Chapter 7

Influence of surface roughness

Simulation models of space charge limited currents with thermionic emitters often rely on simple one-dimensional analytical models. The geometric limitations of these models are circumvented by applying them locally over the emitter surface. The validity of the analytical models is compromised by the presence of surface roughness. In this chapter we will calculate the self-consistent potential distribution and emission current of a bumpy surface. It will be shown that, although surface roughness influences the emission function in the temperature limit, a space charge barrier causes smoothening in the current density so that a rough emitter, operated in space charge limited mode has an emission function that is very similar to a perfect planar emission function.

7.1 Introduction

Display CRTs are equipped with mostly two types of cathode; oxide cathodes, consisting of a barium-strontium compound sprayed on top of a nickel cylinder, or impregnated cathodes (I-cathodes) that consist of a porous tungsten structure in which barium oxide, aluminium oxide or calcium oxide is dispersed. The layer on top of the nickel or tungsten acts to reduce the work function, enhancing the emission.

The emitter surface is not perfectly flat due to crystal growth effects, or the structure of the work function reducing agent. This bumpiness causes inhomogeneities in the emission. It is well-known that cathode inhomogeneities do not show up in the cathode image further along the optical axis if the cathode is operated in the space charge limited regime. However, the calculations in Chapter 3 have shown that small changes in the emission properties of the virtual emitter do show up in the spot of a CRT. Consequently, not only the spatial distribution of the virtual emitter, but also its velocity distribution is important.

The features of the surface give rise to differences in electric field and the mean direction of the velocity of the emitted electrons. Although the direction of the electrons farther along the optical axis is usually determined almost completely by the focusing field, surface roughness may be a notable source of electron energy spread even in space charge limited mode [89]. The space charge cloud in front of the emitter may be distorted in comparison with the space charge cloud in front of a flat surface. Certainly, the differences in electric field result in differences in space charge cloud density. A virtual cathode defined by the space charge minimum is thus likely to be deformed by the surface roughness of the real cathode.

The initial velocity spread of electrons at the cathode affects the beam characteristics in the rest of the system. The resulting variation of space charge lens effect in a CRT beam as discussed in Chapter 3 is but one example of the influence of initial velocity spread. Discrepancies between measured beam velocity spread and predictions based on trajectory calculations bring an interest in understanding sources of additional velocity spread as may occur for instance in magnetron injection electron guns [90,91].

Increased spread of the initial velocity is likely to be caused by roughness of the emitter surface. As such it is interesting to examine the influence of surface roughness on the velocity distribution of electrons near the source. Calculating the current and electric field self-consistently is a laborious task. The effect of small bumps at an emitter surface on the electron beam quality has earlier been estimated with a non-self-consistent approach revealing an increase in velocity spread, both in the temperature limited and, to lesser extent, in the space charge limited regime, thus showing that a space charge barrier reduces the influence of surface roughness on velocity spread [92]. An estimate based on self-consistent solutions

99
in the neighbourhood of edges and conical points, representing surface roughness as a series of cones, has been discussed in Ref. [47], corroborating the increase in velocity spread. Both analyses neglect the initial electron velocity distribution.

In this chapter, the influence of surface roughness on the shape and emission properties of the virtual cathode is examined by calculating the emission current from a surface with smooth bumps in a self-consistent manner. The simulation tool discussed in Chapter 5 is used to determine the stationary space charge limited current in a small box taking into account cathode surface image charges, all mutual interactions and the initial electron velocity distribution. No Schottky enhanced emission is taken into account.

In practice, the roughness of the surface is often formed by work function reducing grains, therefore the roughness, saturation current and work function are interrelated. For the present discussion, the effects of saturation current, work function and surface roughness will be considered separately. Saturation current and work function as well as temperature are taken to be constant over the surface.

Initially, the electrostatic field near a surface irregularity is analyzed without space charge, with the aid of the cathode model present in the simulation tool. Then, the space charge limited current is calculated by injecting individual electrons from the curved surface into the simulation box and determining the stationary charge distribution.

### 7.2 Laplace field of bumpy cathode

A real cathode surface consists of irregularly shaped sharp grains, with a grain size of typically 1 μm. To model a rough cathode surface we use the SWARM simulation model discussed in Chapter 5, specifically with cathode plane of the electron box deformed by a single smoothly shaped protrusion. Employing the simulation tool with periodic boundary conditions, the features of the cathode plane are repeated periodically. The rough cathode is thus modelled as a flat surface with periodic identical bumps. The cathode face of the electron box is the unit cell of surface roughness.

The protrusion of the electron box cathode plane is implemented as an equipotential plane of a well-chosen cathode charge distribution. Thus, the charge distribution consists of an infinite planar charge sheet with properly distributed point charges to mould a specific equipotential plane along the planar sheet into the desired bumpy shape. The emission is assumed to be uniform over the surface. The image charge in the cathode surface is taken into account by adjusting the cathode point charges such, that the potential over the cathode surface plane is constant, regardless of the space charge distribution in the electron box, as discussed in Section 5.6.1.

In Fig. 7.1, a region of 4 by 4 μm is plotted. About 80 cathode point charges are distributed in regular grids in the xy-plane with z-position a few tenths of a micron below the cathode surface. The charge distribution is chosen such, that an equipotential plane exists that is shaped as a plane with smooth bump 1 μm high. This equipotential plane serves as emitter surface.

The emitter is, on a global scale, planar with local geometric disturbances. The extraction field, caused by a fixed boundary potential at 10 μm from the cathode surface, is set at $5 \times 10^4$ V/m. The geometric disturbances give rise to inhomogeneities in the electric field at the cathode surface, which in turn locally influence the space charge limited current density.

In Fig. 7.2, the graph of the axial field at the emitter surface shows that the field enhancement at the apex of the protrusion is about a factor of five. The field decrease near the base of the protrusion is about a factor of 0.5. The distortion of the field can be seen in a different representation in Fig. 7.3.

### 7.3 Emission properties of bumpy cathode

In Section 2.4.2 we discussed the concept of the local planar diode array that is often used for self-consistent space charge limited computations. This concept was verified in Chapter 4 specifically for a perfectly flat and homogeneous emitter surface. The diode array concept is applied under the assumption that the surface is flat, or at any rate that surface roughness has little influence on the electric field. Figures 7.2 and 7.3 show that the electric field of a bumpy cathode in the absence of space charge is rather different from the uniform field in a planar geometry, both in direction and magnitude. Yet a planar geometry with a uniform field forms the basis of the planar diode array. However, this does not a priori invalidate the diode array model in the presence of space charge for a rough cathode surface. It is
Figure 7.1: Cathode surface protrusion defined as an equipotential plane of an array of point charges. The point charges are placed on sticks to show the z-displacement.

Figure 7.2: Axial field enhancement and decrease with respect to the electric field at large distance ($F = 10^5$ V/m) due to the local surface curvature.
necessary to know if, and to what extent, the space charge field adjusts itself such, that it appears locally planar. In other words, we must find out whether the virtual emission properties of a bumpy cathode differ much from the emission properties of a planar emitter.

To examine the emission properties the self-consistent space charge field is computed. The field is calculated by gradually filling an initially empty box with electrons with the appropriate initial velocity, i.e. Maxwellian and cosine with respect to the local surface orientation. In the box all electrons are susceptible to mutual interactions and the field exerted by the cathode surface charge. A mean-field approach is used to take charge outside the simulation box into account. The saturation current of the emitter is set at $J_{sat} = 20 \text{ A/cm}^2$ and the temperature is $T = 1160 \text{ K}$. When an extraction field of $5 \cdot 10^4 \text{ V/m}$ is applied, the cathode is space charge limited. The stationary state is obtained after about 0.04 ns. The space charge limited current is $3.8 \text{ A/cm}^2$, determined by counting the electrons crossing the anode plane in a time interval of a few tenths of a nanosecond.

In Fig. 7.4 the stationary space charge potential distribution in the plane $y = 0$ is shown. It is obtained by averaging over a large number of discrete electron clouds in order to accentuate its smooth features. Close to the emitter surface, the equipotential lines follow the shape of the protrusion, just like in the case when no space charge is present. The curved potential minimum does not follow the curvature of the real surface exactly since the field enhancement at the apex of the protrusion pulls the space charge minimum closer to the surface. This is consistent with the Langmuir model predicting that the potential minimum moves towards the cathode with increasing anode voltage. The distance between potential minimum and emitter surface varies from 2.0 \mu m at the edge of the electron box to 1.4 \mu m near the apex of the protrusion. The potential depth is around $-0.17 \text{ V}$. For a planar surface, the Langmuir model predicts a potential minimum with depth $-0.16 \text{ V}$ at a distance 2 \mu m from the cathode and a space charge limited current 3.9 A/cm$^2$, which is within error limits of the Monte Carlo result.

Some curvature in the potential minimum plane remains visible in Fig. 7.4 but it is greatly reduced by the space charge distribution. Considering the equipotential plane $V = -0.17 \text{ V}$ it can be seen that at a distance less than 2 \mu m from the apex the surface roughness is almost completely shielded by the space charge.

The surface curvature may nevertheless cause a deviating current density profile at the virtual emitter. At the apex one may expect increased emission due to the field enhancement. Also, the mean direction of the emitted electrons surrounding the apex is governed by the slope of the protrusion.

The spatial current density profile can be obtained by tracing electrons in the self-consistent potential distribution. No mutual interactions need to be taken into account to determine the current profile as the space charge cloud is too rare for interactions to have effect, following the results from Chapter 6.
Figure 7.4: Space charge potential distribution in the $y = 0$ plane. Equipotential lines from $-0.17$ are plotted, 0.02 V apart. The dents and spikes originate from the discreteness of the electron cloud. The arrow indicates a contour line of $-0.17$ V.

The spatial density profile of the current crossing the anode plane at $z = 10 \mu m$ is uniform. However, one should bear in mind that spatial nonuniformity, if present at all, is blurred out by the transversal motion of the electrons. It is thus not sufficient to examine the spatial density solely at the end plane, but rather one ought to determine if the spatial density remains uniform when drifting back towards the cathode, not unlike trying to focus onto a virtual source.

Below we will discuss how nonuniformity of the drifted current density can be detected in a systematic fashion. Let us first look at the resulting current density profile. In order to make the influence of space charge discernible the computation is performed both with and without space charge. The latter can be thought of as the extreme situation that the saturation current is infinitesimal. In Fig. 7.5 the profiles are plotted. At the anode plane the current density is uniform, apart from Poisson statistical noise. Drifting back, one can indeed find a plane in which a nonuniformity is visible, as a result of the bump at the cathode surface, if space charge is neglected. If on the other hand space charge is taken into account, the current density is smoothened as a result of the reflecting space charge barrier. The dotted surface on the right appears just as ‘straight’ as in the left figure, in spite of the field enhancement at the apex. Note that the number of electrons in both graphs corresponds to 0.1 $\mu s$ of space charge limited current.

Analysis of the velocity distribution shows similar smoothening action of the space charge barrier. The velocity takes on a cosine distribution at the real emitter surface. As the surface normal has a distribution of its own, an increased angular width at the real emitter surface ensues. The 1 $\mu m$ bump of the previous simulation is too small to show significant velocity broadening. Therefore the velocity distribution near a 2 $\mu m$ bump was analysed. At the emitter surface, the surface roughness gives rise to a 13% increase in transversal velocity width with respect to the Maxwellian distribution. The nonplanar field near the bump exerts a converging action so that the transversal velocity width surplus reduces to 9% at the anode plane even when no space charge is taken into account.

Any smoothening action of space charge can be understood as follows. Electrons with a large transversal velocity have a deficit in longitudinal velocity and will, as a result of the space charge barrier, have a greater chance of being reflected back towards the cathode. Figure 7.6 shows the width reduction due to the space charge effect. The space charge field straightens the velocity distribution such, that the spread in surface normals is no longer evident and the current appears as if it were emitted from a planar surface. Thus, not only the spatial density but also the velocity distribution is homogenized by the space charge field.
Figure 7.5: Spatial current density profile (dotted surface) of cathode with protrusion. On the left, the density at the end plane is plotted. On the right, the density at a plane drifted back towards the cathode is plotted. The plane is chosen so as to obtain maximum contrast in the density. The solid surface plane represents the situation ignoring the effect of space charge.

Figure 7.6: Transversal velocity distribution of electrons emitted from a bumpy surface, bump height 2 µm, with and without space charge, and compared with a planar surface. The transversal velocity widths ($v_{T,50\%}$) are $1.55 \cdot 10^5$ m/s, $1.69 \cdot 10^5$ m/s and $1.56 \cdot 10^5$ m/s, respectively.
7.3. Emission properties of bumpy cathode

As Fig. 7.5 shows it is quite difficult to determine a measure of residual nonuniformity in the spatial current density. Any feature in the current density is blurred by the transversal motion of the electrons, and on top of that, the space charge barrier smoothes the irregularities in phase space such that — even if irregularities remain they are not clearly observable above the Poisson statistical noise. There is a way to make a feature in the current density stand out more, without having to resort to the computation of an extreme number of particles, which would reduce the Poisson noise. One can make use of advance knowledge on the specific shape of the current density. If an irregularity remains, it will assume a shape like the bumpy surface itself.

Integrating the spatial current density over the cathode surface unit cell, using the two-dimensional function describing the bumpy surface as a weighting function, one can distinguish two situations. If the current density is uniform, the integral will yield the average of the surface function. If there is a high concentration of particles in the center of the unit cell, the integral will be larger. Equivalently, if there are relatively few particles in the center, the integral gives a smaller value.

The aforementioned procedure can be described as follows: indicating the surface function with \( f(x, y) \) and the current density function with \( j(x, y) \), the integral quantifying the amount of nonuniformity is written

\[
b = \int_U f(x, y) j(x, y) \, dx \, dy.
\]

in which \( U \) indicates the cathode surface roughness unit cell. For a particular emission function the integral needs to be evaluated at a range of drift distances. In the present specific case that the emission function is represented with a discrete electron cloud in six-dimensional space it is plausible to evaluate the integral by Monte Carlo integration. Writing the positions and slopes of \( N_{\text{sam}} \) particles in the end plane in the traditional manner one obtains a function \( b(\Delta z) \) quantifying the nonuniformity in a drift interval

\[
b(\Delta z) = \sum_{i=1}^{N_{\text{sam}}} f(\ldots) \delta_{(\Delta z)},
\]

in which \( \ldots \) indicates a possible translation to shift the drifted particle into the unit cell, accounting for the periodicity of the simulation. The function \( f(x, y) \), taken here to be the representation of the surface shape, can in fact be any correlator function that is expected to resemble the spatial nonuniformity.

The \( b \)-function can now be used to examine the smoothening influence of the space charge barrier with varying degree of space charge. It is thought that the regulating action of the space charge field is a feature of space charge limited emission, occurring particularly when a potential barrier is present, not simply when space charge has a significant influence on the field. Let us examine a diode, as usual with anode spacing 10 \( \mu \)m and anode voltage \( V = 1 \) V. Langmuir's model predicts that a planar system is temperature limited if the saturation current is lower than 4.2 A/cm\(^2\). Thus, if the saturation current is slightly lower than this value, space charge is a considerable influence on the field but still not so large that a potential barrier is present.

The 'bumpiness function' \( b(\Delta z) \) was calculated several times, with the same bump geometry and anode voltage but with different values for the saturation current, the results plotted in Fig. 7.7. Unbiased Monte Carlo integration of the surface function \( f(x, y) \) shows that the standard deviation in \( b \) is \( \approx 1 \). For the moment we will consider the \( j_0 \) curve in the figure, corresponding to the situation in which space charge is not taken into account at all. What one can notice is foremost, that nonuniformities, blurred out by the transversal motion of the electrons at \( z = 10 \) \( \mu \)m, do appear to be recovered by drifting. A uniform spatial current density, without any correlation between the electron positions and the surface function, integrates to a \( b \)-value of around 45. The inhomogeneity of the emission function manifests itself as peaks in \( b(\Delta z) \).

Additionally, one can distinguish two peaks in \( b \), corresponding to a situation in which the current density is high in the center, at \( \Delta z = -20 \) \( \mu \)m, and a situation in which the central current density is low, at \( \Delta z = -11 \) \( \mu \)m. Because of the sloped edges of the bump, more current is emitted at the center of the unit cell, causing the upward peak. The downward peak is the result of the fact that at the apex, electrons spread out rapidly in the transversal direction due to the electric field shape.

Considering the other curves, one notes a variation in peak heights. Since the correlator function \( f \) is unchanged in each of the \( b \) curves one can conclude that the inhomogeneity in the emission function is gradually smoothened by space charge up till a saturation current of \( j_{\text{sat}} = 5 \) A/cm\(^2\), after which point hardly any features are left. Thus, even when there is no space charge barrier, \( j_{\text{sat}} < 4.2 \) A/cm\(^2\),
Figure 7.7: Nonuniformity of drifted spatial current density expressed in terms of $b$ function. $j_0$ stand for the situation without space charge. $j_1, \ldots, j_5$ stand for saturation currents $1, \ldots, 5$ A/cm$^2$. The three dashed curves, not labelled as they are very similar, have $j_{sat} = 7, 10, 20$ A/cm$^2$.

smoothening takes place. When the barrier sets in, $j_{sat} > 4.2$ A/cm$^2$, the emission function is practically homogeneous and no additional smoothening is gained with increasing saturation current.

When the bump size is increased, similar smoothening occurs. For a bump of height $2.5$ $\mu$m Fig. 7.8 shows $b(\Delta z)$ for $j_{sat} = 0, 2, 5$ A/cm$^2$ and the spatial current density in a plane drifted back towards the cathode. When the saturation current is increased to higher than $5$ A/cm$^2$ no significant changes in $b(\Delta z)$ occur. Even the remaining features in $b(\Delta z)$ correspond to merely minute nonuniformities in the phase space as can be seen in the right picture.

7.4 Conclusions

In this chapter, we have calculated the stationary current in a small diode with a nonplanar cathode, including image charges in the cathode surface and electron-electron interactions in the space charge cloud. The diode is representative for the virtual emitter that forms in front of a real cathode with surface features with a size in the order of microns. Although the properties of a virtual emitter are determined locally by features in the potential distribution with the same size as the surface roughness, the virtual emission properties of a rough surface is very similar to the virtual emission properties of a perfectly planar surface. We have established the similarity by considering the spatial density and velocity distribution. Any blurring of spatial nonuniformities that occur as a result of the transversal motion of the electrons is eliminated by drifting.

A space charge cloud evens out any inhomogeneities in the surface. Increasing the amount of space charge makes the emission function gradually become more homogeneous. When a space charge barrier is formed the emission function is practically as homogeneous as the emission function of a planar emitter surface. Increasing the amount of space charge does not homogenize the emission function further.

A virtual emitter model based on perfectly planar emitters can be used for thermionically emitting rough surfaces, provided they are operated in space charge limited mode. The virtual emission function discussed in Chapter 3 is then not affected by surface roughness.
Figure 7.8: Smoothening near a bump with height 2.5 μm. Left: bumpiness function for $j_{\text{sat}} = 0.2, 5$ A/cm². Right: spatial current density of $j_{\text{sat}} = 0$ and 5 A/cm² in plane of maximum contrast.
Chapter 8

Influence of work function inhomogeneity

Most practical thermionic emitters have strongly and weakly emitting patches rather than a uniform emission profile due to work function inhomogeneities. When the emission is space charge limited the emission nonuniformity gives rise to an inhomogeneous space charge field, possibly affecting the applicability of Langmuir’s space charge model to determine the self-consistent potential distribution near an emitter. Furthermore the current-voltage characteristic of an emitter is influenced by nonuniformity of the work function as well as Schottky effect. Using a simulation model that takes into account global space charge, mirror charge and discrete Coulomb interactions it is shown that work function patches indeed play an important role in the shape of the current-voltage characteristic. In contrast, as the emission profile nonuniformity is greatly reduced by both patch fields and the inhomogeneity of the space charge field, work function patches do not influence the space charge limited emission profile considerably.

8.1 Introduction

The most common electron source, used in a variety of electron optical systems, is a thermionic electron emitter, which is present in systems like microwave tubes and cathode-ray tubes. To obtain emission from a thermionic emitter it is heated to about 1000 K at which temperature electrons can escape the bulk material. The emission is increased at higher temperatures but this also increases evaporation of the emitter material which decreases the lifetime. The operating temperature is a compromise between the emission and the operating lifetime of the cathode. The theory of electron emission from metals was discussed in Chapter 2, where the emitting metal was assumed to be a degenerate electron gas, perfectly uniform in bulk and surface. Emission properties can be classified in terms of the work function and Richardson constant. As noted in that chapter, the theory and classification is not only applicable to metals, but also to compound emitters (e.g. LaB₆, ZrC).

Pure metals are hardly used as thermionic electron emitters nowadays, since the saturation current is too low at temperatures at which the evaporation rate is acceptable. The electron sources in the aforementioned appliances are mostly alkaline-earth oxide cathodes or dispenser cathodes which have the advantage of a lower work function and consequently a higher saturation current.

The work function is generally defined as the energy needed to transfer an electron at the Fermi level in the solid to infinity, at rest [93]. Conceptually, this energy can be split into two parts: the energy needed to remove the electron from the solid and the energy needed to bring the electron to infinity. The energy of the electron in the solid is composed of two contributions. The first contribution is the result of the surface dipole layer. The surface dipole is caused by the penetration of the negative charge beyond the surface, while the positive charge is constrained to the crystal lattice. The dipole layer causes an electrostatic potential drop which escaping electrons have to overcome. Secondly, the electron has to overcome the energy involved in formation of the crystal, the chemical potential. To transfer the electron to infinity, finally, it has to overcome the image force exerted by the induced polarization of the surface charge.

The reduction of the surface dipole due to the presence of foreign adsorbed atoms is an important mechanism for increasing the emission of dispenser cathodes. Nonuniform emission can thus be under-
stood in terms of nonuniform adsorption layer, either in concentration or atom species. Inhomogeneity of the work function has effect not only on the emission but also on the surface potential of the emitter. At the boundary of regions with different surface potentials, there is a strong electric field in the direction parallel to the emitter surface as a consequence of the potential jump. These local fields are called patch fields. Thus, the electron emission is affected due to the differences in intensity of the emission current as well as the nonplanar shape of the electric field, affecting the electron trajectories after emission.

Apart from the nonuniformity of the emission profile there is an additional consequence of a nonuniform work function. It is common practice to characterize an emitter by the zero-field thermionic saturation current density, $J_{sat}$. In Langmuir's theory it appears as a sharp transition from the space charge limited to the temperature limited current. In practice a sharp transition point is absent, which makes it difficult to characterize different emitters using Langmuir's model with uniform saturation density \cite{39,94}. The gradual transition from space charge limited to temperature limited emission has often been attributed to nonuniform work function and Schottky enhanced emission at accelerating fields. Some previous work can be found in literature that is dedicated to the understanding of the deviation from the sharp transition as predicted by Langmuir's theory. If the work function is nonuniform, the emitting surface can have patches that are space charge limited while at the same time, neighbouring patches are temperature limited. The cumulative effect of space charge and temperature limited patches gives rise to a rounded transition range. Such a description of nonuniform emission provides a mechanism to identify the roundedness of a measured transition curve with the level of work function inhomogeneity \cite{38,39}.

The effects of nonuniform work function on the space charge density and the electron trajectories have been studied using a model in which the emitter surface is described by a periodic square wave work function variation \cite{95}. The space charge density and electron trajectories are then approximated with a non-self-consistent calculation, predicting that the space charge density shields the patch fields close to the emitter and space charge inhomogeneities are reduced as a result of the electron motion parallel to the emitter surface. This description does not give an explanation for the smooth transition of the current from the space charge limited to the temperature limited regime.

Most cathodes used in practical applications have a considerably nonuniform work function. In a common CRT emitter, the size of work function patches is several microns, the difference in work function of neighbouring patches can be several tenths of volts \cite{38,96,97}. As we have seen in Chapter 2 differences in emission will result in differences in the space charge field, especially in the space charge limited regime. Presently we are interested in the validity of the space charge limited virtual emitter model discussed in that chapter. It assumes uniform emission properties over the full emitter surface. The profile of space charge limited emission due to variations in the extraction field is incorporated by considering the emitter region to be describable by small, effectively one-dimensional subregions. In these subregions Langmuir's model is applicable.

Realistic work function patches are on the same order of magnitude as the features of the potential distribution in the virtual emitter model, that is, the size of the patches is on the same order as the distance from cathode surface to Langmuir minimum, and the potential difference of neighbouring patches is on the same order as the depth of the minimum. This may affect the applicability of the one-dimensional diode model with uniform work function that forms the basis of the virtual emitter model.

As a possible improvement of the virtual emitter one may abandon the assumption of uniformity of the work function in the diode model. The virtual emitter then consists of a diode array in which each subcathode has a different work function and saturation current to match the work function distribution of a real cathode. However, this is very impractical since it entails simulation of optical systems assuming that the work function distribution of individual cathodes is known or does not change much from cathode to cathode, and remains stationary throughout operation. Even then, it is not clear whether the virtual emitter obtained in this manner is consistent with the real cathode emission properties. We will get back to this issue at the end of the chapter.

For the consideration of the applicability of the virtual emitter model in computer simulations the previous studies \cite{38,39,95,96} are insufficient since both the average current density and the local effects on the current density and the velocity distribution of the emitted electrons play a role. In this chapter we will determine whether the local planarity of the emission is conserved in the presence of emission inhomogeneities and patch fields, thus examining the applicability of the common virtual emitter model based on an array of planar diodes.

To take into account the effect of a nonuniform work function including differences in emission, patch fields, space charge and Coulomb interactions we use the simulation model discussed in Chapter 5. The cathode surface is assumed to be a flat plane that is not an equipotential plane. The surface potential
distribution prescribes the work function distribution since a low work function corresponds to a high surface potential and vice versa. The local saturation current density as a function of work function is given by the Richardson-Dushman equation, Eq. (2.19)

\[ j = AT^2 e^{-\Phi/kT}. \]

In the following sections, the potential distribution near a nonuniform cathode will be calculated both with and without space charge. The emission characteristics of a diode will be computed for cathodes with varying degree of nonuniformity.

### 8.2 Laplace field of patchy cathode

The basic shape of the potential distribution at the cathode surface that is used in the subsequent simulations is shown in Fig. 8.1. The simulation area is a cube with sides 10 μm. The cathode surface potential distribution is modelled by placing a grid of point charges of specific charge a distance 1 μm from the emitter surface, inside the emitter. The specific shape of the potential distribution is not based on an actual measured cathode. The method of representing a surface potential distribution with point charges is well-suited for smoothly varying potentials but to adequately model a measured patchy cathode the number of required point charges becomes large. Although this is by no means a fundamental computational difficulty it is felt that no heightened understanding of the emission properties will ensue. The particular shape of the potential distribution is chosen to contain local maximums and minimums as are likely to be present in a real cathode.

Figure 8.1 also shows the work function distribution function \( WFD(\Phi) \). \( WFD(\Phi) d\Phi \) gives the fraction of the emitter surface having work function in the range \( \Phi \) to \( \Phi + d\Phi \). The peak appearing in the work function distribution function of real cathodes is not as sharp but the width is comparable, both for oxide cathodes [98] and dispenser cathodes [40, 96]. The width of the plotted WFD, based on the standard deviation, is 0.1 eV. For comparison, Fig. 8.2 shows a measured WFD for a dispenser cathode.

The electric patch fields at the emitter surface due to the work function variation are shown in Fig. 8.3. The average cathode potential is 0.046 V and the anode potential (at \( z = 10 \) μm) is \( V_d = 0.5 \) V therefore the average axial electric field at the cathode is on the order of \(-50 \) V/mm. Due to the work function...
patches the local axial field in fact varies between $-400$ to $200$ V/mm. Furthermore, a significant electric field component parallel to the surface is present.

Figure 8.4 shows force lines at the emitting surface. Emission from areas with high surface potential is reduced due to the axial patch field forces back towards the emitter. These areas correspond to low work function and high emission. Therefore, the patch fields themselves even out the emission nonuniformity.

8.3 Emission properties of patchy cathode

A cathode with moderate work function patches as shown in the previous section has a wildly varying emission profile since the emission depends exponentially on the work function. The emission current from regions with high surface potential is according to the Richardson-Dushman equation more than thirty fold increased with respect to regions with low surface potential.

A cathode surface with periodic emission inhomogeneity of this degree was modelled with the SWARM model that has been discussed in Chapter 5. The unit cell of inhomogeneity is one face of a cube with sides $10$ μm. The cathode surface charge is modelled as a square grid of 81 point charges placed at $1$ μm behind the flat surface. The self-consistent potential distribution and current density characteristics are obtained by emitting individual electrons from this square according to the nonuniform emission profile and tracing them to a plane $10$ μm from the emitter plane, allowing them to build up the space charge cloud. In the cube all mutual interactions are taken into account, as well as the patch fields. The charge outside the cube is taken into account with a mean-field approximation. Electrons crossing a side plane are reinjected at the opposite plane, consistent with the assumed periodicity of the cathode surface structure.

The emitted charge in the electron box causes a polarization in the surface charge distribution such that the potential distribution at the surface is stationary. The potential distribution and space charge density are obtained by averaging over a large number of electron clouds in order to accentuate the deterministic features in the field over the stochastic effects. From Chapter 6 it is known that statistical Coulomb interactions near the virtual emitter do not influence the electron cloud. Therefore current density profiles can be obtained by using the resulting self-consistent potential distribution directly, rather than evaluating all mutual interactions in-the-fly, saving a considerable amount of computation time.

8.3.1 Current density profile

In Fig. 8.5 the space charge field in front of a cathode with nonuniform work function is plotted. The particular WFD is plotted in Fig. 8.1. The total saturation current of the cathode is $5.3$ A/cm$^2$. The
8.3. Emission properties of patchy cathode

Figure 8.3: Electric patch fields due to work function variation parallel to the emitter surface ($E_\perp$, left) and perpendicular to the emitter surface ($E_z$, right). The labels of the contour lines are in units 100 V/mm.

Figure 8.4: Field lines at emitting surface in xz-plane, showing that patches with high emission current (large box arrow) experience patch fields directed back towards the emitter, whereas the emission from patches with low emission current (small box arrow) is enhanced by the patch fields. The arrows point in the direction of the Coulomb force, contrary to common convention.
extracting field is effected by imposing a voltage difference between cathode and anode of 0.5 V. The distance between cathode and anode is 10 μm. Schottky enhanced emission is not taken into account. In the presence of a space charge barrier it does not play a role, and even when no barrier is present, in the present range of voltages, it may only lead to a minor change in local current density and consequently it will hardly influence the emission nonuniformly.

On the left, the space charge potential distribution is plotted. The potential gradient at the surface is slightly negative at the edges of the zz-plane, strongly negative at the position of high surface potential and slightly positive at the center of the cathode. This reflects the fact that the cathode has some patches that are space charge limited to various extent, and some patches that are temperature limited. The space charge density field is shown on the right. The space charge acts as an emission filter, just as in the case of a uniform emitter, and since the filtering action is more effective at areas where the emission is high, the space charge limited emission profile is smoothened.

The levelling action of the patch fields and the space charge field is visualized in Fig. 8.6, showing the local contribution from the patches on the emitter surface to the current. The rightmost graph shows the emission current as given by the Richardson-Dushman equation. This is the emission profile of emitted electrons very close to the surface, regardless of whether they will be pulled back by patch fields or reflected by a space charge barrier. In the middle graph, the high emission peak is reduced because many of the particles emitted from this area will in fact be reflected by the patch field. The middle graph has been calculated ignoring the repelling effect of space charge. In the left graph, also the space charge barrier is taken into account, effecting a further smoothening. Note that these are current density profiles taken at the cathode surface and not the emission profiles that will appear when imaging the cathode. Patch fields and space charge field form an inhomogeneous potential distribution extending some distance from the cathode, causing a spatial intermixing of the current.

In order to determine the specific smoothening of the space charge barrier also a situation without barrier is calculated. Increasing the anode voltage to 3.2 V the emission is everywhere temperature limited except at the regions where the saturation current is highest, at \( x = \pm 2.5 \mu m \) on the y-axis. Space charge density and potential distribution are shown in Fig. 8.7.

Now we want to determine to what extent the virtual emission function based on planar diodes with uniform emission remains valid when the real emitter is nonuniform. The virtual emitter model assumes a spatially uniform current distribution and a Maxwellian angular distribution. If the spatial current distribution near a patchy emitter surface is not uniform, or the angular distribution deviates considerably from a Maxwellian, one can not simply apply the virtual emitter model to a nonuniform emitting surface. Conversely, when the spatial distribution is uniform and the angular distribution Maxwellian, the virtual emitter model is valid.
8.3. Emission properties of patchy cathode

Figure 8.6: Emission profiles at the cathode surface taking into account the saturation current nonuniformity (right), the influence of patch fields (middle) and the influence of the space charge field (left).

Figure 8.7: Potential distribution (left) and charge density distribution (right) with anode voltage $V_a = 3.2$ V; the emission is mostly temperature limited.
In Fig. 8.8 the current density from a patchy cathode surface with work function spread 0.1 eV is shown for anode voltages 0.5 V and 3.2 V at the plane $z = 2 \mu$m. With these anode settings the current is respectively space charge limited and temperature limited. Both the spatial current density and the transversal velocity distribution are plotted. In the space charge limited case, at $z = 2 \mu$m the spatial current density is well smoothed. The velocity distribution is Maxwellian. The potential distribution past $z = 2 \mu$m is practically planar, thus there is no nonplanar aspect noticeable in the remainder of the diode, apart from the slightly inhomogeneous spatial distribution. The influence of the space charge barrier is strikingly visible when comparing with the temperature limited spatial current density.

In the space charge limit, both spatial distribution and velocity distribution closely resemble the distributions in a truly planar diode. Is this sufficient proof to conclude that the emission functions are similar? Only if the decoupling of the spatial and the velocity component of the emission function is valid can one conclude that the emission functions are similar. Here with decoupling it is meant that the emission function can be written as the product of a function solely of position and a function solely of velocity. In fact, one would prefer to examine the full phase space distribution of the nonuniform emitter. After all, even the thermally limited spatial current density will appear homogeneous eventually, a long distance from the cathode, since neighbouring high intensity peaks spread out and start overlapping.

A direct comparison of the phase spaces is not very useful in this case as the separate components are
Figure 8.9: Spatial current density at \( z = 10 \mu m \) (left) and drifted back towards the cathode (right). Top: \( V_{\text{anode}} = 0.5 \) V. Bottom: \( V_{\text{anode}} = 3.2 \) V. At \( z = 10 \mu m \) the high intensity peaks are overlapping considerably, smoothening the current density, but the inhomogenety is recovered by drifting.

quantitatively very similar. Furthermore, the phase space is four-dimensional, making graphic representation complex. To remove the last shred of doubt, the phase space distribution at the plane \( z = 10 \mu m \) is examined indirectly by determining the spatial distribution obtained by drifting back towards the cathode. In this way, one concurrently ensures that the space charge field past \( z = 2 \mu m \) does not introduce disturbances that will affect the planarity of the emission function.

In Fig. 8.9 the real spatial current density is plotted as well as the current density obtained by drifting, keeping in mind the periodicity of the simulation. The length of drift is chosen such, that features in the current density are most prominently visible. Drifting back further, the high intensity peaks will start to spread out and overlap again.

The space charge limited emission profile has very modest bumps in spite of the huge emission nonuniformity at the cathode. Space charge and patch fields decrease the emission nonuniformity by an order of magnitude.

### 8.3.2 Current-voltage characteristic

In addition to the influence on the current density profile, one can expect that emission nonuniformity affects the current versus voltage characteristic of an emitter. The effect of emission nonuniformity on the transition from space charge limited to temperature limited emission can be studied by applying
an increasing anode voltage to a patchy cathode while keeping the diode spacing constant. Thus an IV-characteristic is obtained.

From the Langmuir model it is known that for increasing anode voltage, the space charge limited current will increase until the current is temperature limited. This transition occurs at a sharp point, in the sense that the derivative of current with respect to anode voltage is zero [42].

Cathode nonuniformity is regarded as one of the causes for the absence of a sharp transition as it is predicted by the Langmuir model. This can easily be understood if one considers a set of diodes with varying transition points. The cumulative effect will result in a transition range rather than a point.

Of course, in this simple line of reasoning, one conveniently disregards the coupling of the diodes. In reality, current flows from one diode into another, and consequently a patch with high saturation current contributes to the space charge barrier of neighbouring patches. As we have seen in the previous section, the space charge field tends to smoothen the differences in emission current that occur for different patches. Thus, one may find that the IV-characteristic bears closer resemblance to the IV-characteristic of a uniform emitter than to the IV-characteristic of cumulative uncoupled diodes. Nevertheless the approach of uncoupled diodes is often used to explain work function inhomogeneity in terms of the measure of smoothness of the IV-characteristic [38–40].

Let us examine the effect of coupling by regarding the current characteristic both in the self-consistent manner and in the uncoupled manner. It is now desirable to include the Schottky effect, because it in itself can partly account for the smoothness of the transition between space charge limit and saturation.

First the uncoupled case is regarded, where no self-consistent space charge field needs to be calculated. The work function distribution is taken as before, plotted in Fig. 8.1. The current characteristic of uncoupled diodes is easily determined by adding the Langmuir current of many small diodes with different saturation currents. The currents in the small diodes can directly be obtained from the differential equations of Langmuir, with extension to accelerating field to include the Schottky effect [42]. In Fig. 8.10 four curves are plotted, corresponding to the case of a large single diode with uniform saturation current and to a set of small, independent diodes with saturation currents conforming to the work function distribution of Fig. 8.1, both graphs excluding and including Schottky effect. The solid lines show a smoother transition than the corresponding dashed lines. The lines with crosses lack a horizontal saturation current limit at high anode voltage.

It is obvious that the nonuniformity of the saturation current causes roundedness of the transition in a set of uncoupled diodes, which is augmented as a result of Schottky enhanced emission. The question is, how correct is this simplification of uncoupled diodes? We find the answer in calculating the self-consistent
stationary current for various anode voltages, using the simulation model taking into account space charge, interactions and mirror charges. The resulting IV-characteristic is shown in Fig. 8.11. Two situations are plotted with a different 'level of nonuniformity'. The graph on the left represents the situation in which the work function spread (width of the WFD) is $\sigma(\Phi) = 0.1$ eV, in the right graph $\sigma(\Phi) = 0.05$ eV. The resulting local saturation current density distributions have widths $\sigma(j) = 6.3$ and $\sigma(j) = 2.9$, respectively. The total saturation currents in the two situations are equivalent. Both simulation results are compared with the IV-characteristic as determined from a set of uncoupled Langmuir diodes including Schottky effect, like the solid line with crosses in Fig. 8.10.

A number of specific characteristics are visible in the graphs. First, we note, as before, the absence of a horizontal saturation current limit at high anode voltage because of the Schottky effect of enhanced emission in the presence of an accelerating field. This in itself makes the transition from space charge limited emission to temperature limited emission smoother than predicted by the Langmuir model. Regarding the smoothness of the transition we note a second feature: a larger spread in work function leads to a more gradual transition. This is consistent with the concept that the cathode consists of small patches that pass from one emission regime to the other at different field strengths. The transition field strength depends on the saturation emission and when the difference in saturation emission of the patches is larger, the interval of transition field strengths will also be larger.

Lastly we note that the current characteristic of the self-consistent model including mirror charge and patch fields corresponds reasonably well to the simple model consisting of uncoupled one-dimensional diodes that take space charge and Schottky emission into account. One may not conclude however that the coupling in the self-consistent field can be neglected. In the present simulation the emission inhomogeneity was the result of differences in work function which translate — as predicted by the Richardson-Dushman equation — into differences in saturation current. One can alternatively attribute differences in saturation current to variations in the Richardson constant, a hypothetical situation that bears little resemblance to an actual physical cathode. It will be used here solely as a device to show the impact of coupling of the neighbouring emission patches. The simplification of the self-consistent field using uncoupled diodes with different saturation current applies equally well, regardless of whether the saturation current inhomogeneity is caused by different work functions, or different Richardson constants. However, as we will see, the IV-characteristics are rather different.

Figure 8.12 shows the IV-characteristics of cathodes with identical saturation current inhomogeneity as in Fig. 8.11, but here the inhomogeneity is the result exclusively of differences in Richardson constant. The surface potential is uniform and consequently no patch fields are present. The two IV-curves for the
differen levels of inhomogeneity are very similar in this case. Also, particularly compared with Fig. 8.11, both dashed curves show a sudden transition behaviour. The IV-curves do resemble more closely the curve of a uniform emitter surface, which is precisely what we remarked at the beginning of this section.

The difference in smoothness of the transition is much less pronounced in the case, that emission nonuniformity is effected by varying the Richardson constant rather than by varying the work function. The reason for this is that the presence of axial patch fields augments the effect of the transition variation. Consider how an intensely emitting local patch affects the transition. Not only does the increased saturation current shift the transition point towards higher applied electric field, but also does the patch field responsible for the high emission tends to lower the apparent applied electric field. For weak patches the converse reasoning holds.

Examining the curves of Fig. 8.10, bearing in mind that the non-self-consistency of those particular results is not very important, it can be seen that nonuniformity of the emitter and Schottky effect are both responsible for the smooth transition from space charge limited to temperature limited emission.

8.4 Conclusions

Surface potential inhomogeneities can be modelled by representing the potential distribution in front of the surface by a combination of point charges and a planar uniform charge distribution. By considering the surface potential distribution as being caused by a work function distribution, and using a surface emission profile locally consistent with the work function, the emission from patchy cathodes can be studied.

The emission of a nonuniform cathode can locally be in the space charge limited range or in the temperature limited range. The nonuniformity of the work function and consequent patched emission gives rise to a variation in the influence of the space charge barrier. Intensively emitting patches are strongly space charge limited while neighbouring weakly emitting patches may be temperature limited at the same time.

This compensating effect equalizes the emission profile that exists at the cathode surface due to the differences in saturation current. Especially when the current is severely space charge limited the current density profile appears smooth. A space charge cloud evens out emission inhomogeneities in the emission profile. The levelling action of the space charge cloud is so effective that the Langmuir minimum, that is commonly identified with a virtual emitter, is practically identical to the Langmuir minimum appearing in front of a uniform emitter.

The emission characteristic of a cathode is strongly influenced by surface patches. The effect of work
function inhomogeneities is twofold. Firstly, variations in work function cause differences in saturation emission and a consequent difference in transition point, i.e. the field strength that separates the space charge limited regime from the temperature limited regime in the emission characteristic. Secondly the surface potential distribution causes variations in the extraction field experienced by an electron at the surface in such a manner that intensely emitting patches are subjected to a lower extraction field and vice versa.

The smoothness of the transition from space charge limited to temperature limited emission that is usually observed can be attributed to nonuniform work function. The effect of the work function patches on the transition is more prominent than the effect of the nonuniform emission or the Schottky effect, especially at low extraction fields.

Since the emission profile from a nonuniform cathode in the space charge limit is homogeneous, the angular emission distribution is conserved and the potential distribution is practically planar, virtual emitter models remain valid undiminished.
Chapter 9

Shot noise reduction in space charge limited emission

A simulation model is discussed for the determination of current fluctuations in a planar diode. Two distinct types of fluctuations are discussed: the dynamics of the relaxation to stationary space charge limited current, and shot noise fluctuations. When the current is space charge limited shot noise may be reduced. The time scale of shot noise reduction is related to the settling time of the space charge barrier. The reduction factor is small at low frequencies and approaches 1, corresponding to full shot noise, at high frequencies.

9.1 Introduction

Electron emission is susceptible to current fluctuations known as shot noise. The origin of shot noise is the discreteness of electrons. Electrons are not emitted regularly in time, but according to a Poisson process. This means that, if the average current flowing from the emitter is \( \bar{I} \), during a time interval \( t_0 \) the average number of emitted electrons is \( \mu = (\bar{I}/e)t_0 \), while the actual number of emitted electrons during a certain time interval \( t_0 \) is an integer stochastic variable \( N_{t_0} \) with distribution function

\[
 f(N_{t_0} = k) = \frac{\mu^k e^{-\mu}}{k!}. \tag{9.1}
\]

The mean square current fluctuation can be expressed in terms of the variance of \( N_{t_0} \)

\[
 \langle I^2 \rangle = \bar{I}^2 \left( N_{t_0}^2 - N_{t_0} \right), \tag{9.2}
\]

which becomes in view of the characteristics of the Poisson distribution

\[
 \langle I^2 \rangle = e^2 \mu = e\bar{I}/t_0. \tag{9.3}
\]

In general, current fluctuations in a frequency band \( \Delta \nu = 1/2t_0 \) are written [99]

\[
 \langle I^2 \rangle = 2e\bar{I}\Delta \nu, \tag{9.4}
\]

signifying full shot noise as it occurs in a temperature limited diode.

It has long been known that in the transition from temperature limited to space charge limited emission, the current fluctuations drop, so-called shot noise reduction [100]. The drop in current fluctuations is generally considered to be due to the interaction of the space charge barrier with subsequently emitted electrons [101–103]. This can be made plausible with the following argument. In Chapter 2 it was mentioned that the cathode temperature — and consequently the saturation current — has little influence on the space charge limited current. The reason for the smoothening of saturation current variations can be found in the depth of the space charge barrier. Suppose in a stationary state a potential minimum is present, resulting from a certain amount of emitted current. If for some reason more current is emitted the space charge cloud becomes more dense, thus preventing the excess current to penetrate the space
charge barrier. As the space charge cloud reacts to fluctuations in the saturation current, it can explain the reduction of fluctuations in the current that passes the barrier. The shot noise reduction is indicated by including a factor $\Gamma^2$,

$$\langle I^2 \rangle = 2e\Gamma^2 I \Delta \nu,$$  \hspace{1cm} (9.5)

where $\Gamma^2$ may depend on frequency.

There have been a number of investigations on the reduction of shot noise in the presence of space charge. One approach considers shot noise as a perturbation event with respect to the average current, represented by an excess charge sheet, moving from cathode to minimum. The perturbation charge influences some subsequently emitted charge sheets such that they precisely do not cross the barrier, compensating the excess charge. The compensation charge in turn affects subsequent charges, causing a noise-compensating feedback [104]. In a similar approach a space charge diode is represented by an equivalent circuit consisting of a retarding field diode in series with a second diode, the former corresponding to the space between cathode and potential minimum, the latter to the space past the minimum. A disturbance current in the retarding field diode induces a response in each of the two diodes, as a small increase in the charge allowed to pass the potential minimum must be accompanied by an equal decrease of charge returning back to the cathode. The fluctuation in the second diode, determined by the response current, displacement current and the impedance of the retarding field diode, is lower than the original disturbance, indicating shot noise reduction [105, 106]. Another approach is a Monte-Carlo method [107] in which disks of charge are traversing in a diode subjected to space charge fields, external extraction field and image charges, and collected at the anode plane. Shot noise reduction is observed in the noise power spectrum as a deviation from a white spectrum. The earliest simulation showed a non-monotonic noise power spectrum, including peaks that would appear to correspond to shot noise stimulation, but these features could not be reproduced in later simulations [108, 109]. The behaviour of space charge limited emission in relation to saturation current can quantitatively be determined by Langmuir’s space charge model. Nevertheless, a direct analytical approach to determine the drop in current fluctuations in a space charge limited diode, based on Langmuir’s model, fails to predict shot noise reduction factors occurring in practice, underestimating $\Gamma^2$ by several orders of magnitude [102]. Measurements of shot noise reduction have been performed by inducing a known amount of current fluctuation at the cathode through photoemission and measuring the current fluctuations at the anode [108].

All aforementioned quantitative theoretical investigations of shot noise reduction assume infinite planar geometry in which particles are effectively represented as sheets of charge, which does not include any local influence of the electron cloud on shot noise. The investigations share the common result that the shot noise reduction is effective at low frequencies and disappears at high frequencies. In this chapter Monte Carlo simulations of space charge limited currents will be described, representing the current with discrete electrons. Statistical interactions are included. Shot noise suppression as a result of fluctuations in the space charge barrier will be shown, both the general reduction and the local effect.

### 9.2 Simulation model

Using the simulation model described in Chapter 5, the self-consistent space charge limited current in a diode is determined. The diode consists of a rectangular box in which two opposite planes play the role of cathode and anode. The box, initially empty, is gradually filled with electrons that are emitted from the cathode plane with starting conditions corresponding to the emission properties of a thermionic emitter. In the box, the electrons experience their space charge field and an extracting field perpendicular to the cathode. Subject to these fields, some electrons are repelled back to the cathode and some reach the anode, where they are removed from the system.

The number of electrons reaching the anode in time steps of equal length $t_0$ can be identified with the stochastic variable $N_{t_0}$. By keeping track of $N_{t_0,i}$ for a large number of time steps $i$, the current fluctuations $I(t)$ are obtained.

Following from Eq. (9.5), the reduction factor can be determined with

$$\Gamma^2 = \langle N^2 \rangle / \bar{N},$$  \hspace{1cm} (9.6)

where the index $t_0$ is implied.

Note that the relative root-mean-square of the fluctuations, $\langle N^2 \rangle^{1/2} / \bar{N}$ decreases with increasing cathode plane area. This has nothing to do with the reduction of shot noise as a result a space charge
Figure 9.1: Shot noise reduction factor $\Gamma^2$ with varying degree of space charge limitation. The transmission is the ratio between space charge limited current and saturation current. Measuring time is $\Delta t = 0.1$ ns.

barrier but is merely a consequence of the stochastic nature of the current fluctuations. The reduction factor $\Gamma^2$ nevertheless does depend on the total cathode plane area simply because the distance and height of the space charge barrier will be different when the electron beam is broader. If a planar geometry is to be modelled, the infinitely extending charge outside the box can be represented with a surrounding static mean field. Even then, $\Gamma^2$ is influenced by the particle box width, but for a different reason: distance and height of the barrier remain the same but the impact of the charge fluctuations is greater when the particle box is wider. The quantitative influence of the box width on $\Gamma^2$ is discussed below.

9.3 Shot noise reduction

A diode setup as discussed above is calculated with diode length 10 $\mu$m, diode width 4 $\mu$m and anode voltage 0.5 V. The cathode temperature is 1160 K. No mean field of surrounding charge is taken into account so that we can be certain that the space charge barrier fluctuations are not artificial, that is, affected by the mean field smoothening. Cyclic boundary conditions at the electron box side planes ensure that the electron beam does not diverge. In a way, the confinement to the electron box via cyclic conditions is similar to a realistic confinement with a focusing geometry, except that the focusing action is not taken into account in the electron trajectories. The saturation current of the cathode is varied to obtain $\Gamma^2$ values for different operating conditions. $\Gamma^2$ is determined from the distribution of $N$ with $t_0 = 10^{-10}$ s. The results are plotted in Fig. 9.1. We observe a sudden drop in $\Gamma^2$ when the current is space charge limited. $\Gamma^2$ continues to decrease with increasing saturation current. For infinite saturation current it tends to zero.

The frequency dependency of $\Gamma^2$ may be determined by repeating the calculation with different values for $t_0$ but there is a simpler way of obtaining the frequency dependency of the reduction factor. One can instead determine the spectrum of the current fluctuation $I(t)$ by Fourier analysis [107]. Full shot noise gives a uniform noise power spectrum whereas shot noise reduction is visible as a nonuniform spectrum. In the following situation the diode length is again 10 $\mu$m, the saturation current is $j_{sat} = 10$ A/cm$^2$ and the anode voltage is $V = 0.5$ V, leading to a transmission of 77%. The power spectrum of $I(t)$ is obtained by Fourier-transforming the discrete $N_i$, taking care to remove an initial interval of time steps, that corresponds to a non-stationary state. The power spectrum gives the frequency dependency of $\Gamma^2$, which is plotted in Fig. 9.2.

Here we see a deviation with respect to the literature. In our computation, the low frequency limit of $\Gamma^2$ remains appreciable, whereas in previous Monte Carlo simulations [107, 108] as well as in theoretical
analyses [104, 105] it tends to zero for low frequency. Vanishing shot noise is an effect particular to infinite one-dimensional geometry that is assumed in the previous analyses. Shot noise is more effectively suppressed in a wide electron beam with large diode spacing. When the diode spacing in the simulations described here is increased the low frequency value of $\Gamma^2$ is significantly lower.

In Fig. 9.3 measurements of the frequency dependency of shot noise suppression at transmission 20% are shown together with simulations. The simulations were performed with parameters diode length 185 $\mu$m, anode voltage 10 V, including surrounding mean field, resulting in a transmission of 20%. Measurements and simulation parameters are taken from [108]. The computations in that paper are based on infinite planar particles rather than point charges.

The time scale involved in the shot noise suppression is determined by the time it takes for a change in the emission current to be reflected in a change in the space charge limited current. A rough measure of this space charge barrier settling time may be obtained by estimating the transit time from the cathode to the potential minimum of an average electron that contributes to the space charge limited current. For a distance between cathode and minimum of several microns and an average thermal velocity one obtains a time scale of $> 10$ ps. Thus, one would expect that fluctuations on a time scale much smaller than 10 ps can not be suppressed by feedback with the barrier. In the noise power spectrum this is reflected as the limiting constant value at high frequency.

A low frequency limit of zero is not observed in the simulations even when a large diode is simulated. It is worth noting that zero shot noise would imply that, taking a measuring time interval sufficiently long and a measuring anode area sufficiently small, the current emitted from the space charge barrier consists of perfectly regular pulses with no stochastic noise whatsoever. Perfect regularity is counterintuitive in view of the stochastic nature of the space charge barrier itself.

According to a simple derivation using Langmuir's theory the shot noise reduction factor in a planar diode should be on the order of $\Gamma^2 \approx 10^{-19}$ whereas the observed value is in the order of a few percent. [102]. The theory corroborates the idea of completely vanishing shot noise. However, this misconception is caused by the planarity of the Langmuir model. For the derivation of shot noise reduction it must be assumed that current fluctuations are synchronous over the complete surface. In reality, there may be excess current at one part of the surface while simultaneously there is a deficiency in an adjacent part. Furthermore, in view of the symmetry of the model the flux of compensating current acts solely perpendicular to the cathode surface. In reality compensating current has a parallel flux component as well, which reduces the effectiveness of the compensation.
9.4 Spatial range of shot noise suppression

The conceptual understanding of shot noise suppression is that of a self-regulating space charge barrier: an excess of emitted current heightens the barrier, decreasing the current and conversely, a shortage of current leads to a reduced barrier, increasing the current.

It is interesting to examine the transversal range of the shot noise reducing barrier fluctuations. In order to do so, one can not simply determine $\Gamma^2$ for increasing electron beam width because the geometry of the electron beam affects the current density. It is necessary to simulate space charge limited current with varying electron box width such that the space charge field remains identical. The most straightforward way is to calculate an infinite planar space charge field, by encapsulating the electron box with a surrounding space charge field.

Contrary to the simulations in previous chapters, one cannot use the mean-field approach based on the spatial distribution of electrons in the electron box. The surrounding mean field outside the box must be statistically independent of the electron distribution inside the box so that the shot noise reducing action solely originates from the charge inside the box. Thus, a static surrounding space charge field without any fluctuations is used. The relevant space charge potential distribution can be determined from a stationary one-dimensional space charge distribution by cutting out the electron box and calculating the electric field of the remaining space charge. This requires a separate surrounding potential distribution for each setting of the electron box width.

The one-dimensional diode settings are $V_d = 0.5$ V, $T_{\text{cat}} = 1160$ K and $j_{\text{sat}} = 10$ A/cm$^2$, as before. The electron box width is varied from 2 $\mu$m to 60 $\mu$m. We fill up the electron box, surrounded by appropriate space charge field, and collect the electrons at the anode during time intervals of equal length. The shot noise reduction factor $\Gamma^2$ can be determined from Eq. 9.6 for different values of $t_0$. We expect $\Gamma^2$ to be equal to 1 when the electron box width is zero, and to relax to an asymptotic value for increasing electron box width. One can indeed observe this general behaviour in Fig. 9.4. The asymptote and the ‘relaxation width’ can be quantified by using the following model for the dependency of $\Gamma^2$ on the electron box width $w$:

$$\Gamma^2(w) = \Gamma^2_\infty + (1 - \Gamma^2_\infty) e^{-w \Gamma_w}.$$  \hspace{1cm} (9.7)

This model is not intended to imply a profound underlying mechanism of shot noise reduction but serves purely to uncover a certain order in $\Gamma^2(w)$. The least-squares fit of $\Gamma^2(w)$ is shown in the figure as a solid line.

Repeating the simulation for various values of $t_0$ and with a different anode voltage, $V_d = 0$ V, leading to a transmission 14%, a pattern in fit coefficients $\Gamma^2_\infty$ and $\Gamma_w$ is revealed, shown in Fig. 9.5.
The asymptotic value of $\Gamma^2$ for large electron box increases with increasing $t_0$, as it should in view of the corresponding decreasing frequency. The relaxation constant $\Gamma_{w}$ is larger for increased transmission, indicating that the spatial range of shot noise suppression is smaller by a factor of two for low frequencies.

It has been theorized that the relevant length scale of the shot noise suppression is determined by the particle density [110]. However, the sole difference in the two simulations with different transmission is the distance between cathode and potential minimum, 3.7 $\mu$m and 1.9 $\mu$m for low and high transmission, respectively. As the saturation current is kept equal, the average distance between particles in this region in the situations with high and low transmission differs by less than 5%. Thus, the length scale underlying the boundary of the shot noise reduction is the distance between cathode and potential minimum rather than the interparticle distance.

### 9.5 Discussion

For practical application of the phenomenon of shot noise suppression we note the following. In a CRT for display applications, the current is typically 0.1–1 mA and the modulation frequency is 100 MHz. The number of electrons involved in the formation of a picture is several millions, consequently shot noise fluctuations are irrelevant. However, for other applications like electron projection lithography (EPL) or scanning electron microscopy (SEM) shot noise is relevant.

The resolution of an EPL system is partly determined by the electron spot size. For given resolution the blurring influence of Coulomb interactions limits the amount of current that can be allowed in an EPL system [111]. In order to maintain a competitive throughput of several tens of wafers per hour in spite of the limited current, it is necessary to use extremely sensitive resists. Shot noise may prove to be an important restriction on the sensitivity of resist that can practically be used [112]. Present day specifications demand that fluctuations in the illumination dose be less than 10%. In view of shot noise the number of discrete particles necessary to illuminate a pixel must then be at least 900 for a 3$\sigma$ dose variation of 10%. This fundamental lower limit to the number of particles translates to a minimum required current and consequently a lower limit to the resolution of the EPL system. However, with careful electron gun design, it should be possible to construct an electron beam operating in the space charge limit with significantly less shot noise than the ‘fundamental’ limit. Indeed, this consideration has been followed as a motivation to design a space charge limited electron gun for electron projection lithography systems [113].

The same argument applies to formation of SEM images. Noise can degrade contrast in situations where the signals are small, as is the case for high resolution, involving low current [114]. The signal-to-noise ratio can be improved by increasing the measuring time but if one source of noise could be
removed by appropriate electron gun design, scanning times could reduce, decreasing the influence of environmental noise.

Regarding these advantages one should first note that Coulomb interactions will reintroduce shot noise. This may not be such an important issue in SEMs, because a stream of neatly arrayed electrons do not experience statistical interactions. On the other hand, we have seen that there is a limit in the spatial coherence of suppressed shot noise fluctuations. In particle projection lithography hypothetical regularly spaced electrons coming from one pixel will meet electrons coming from another pixel in the crossover. When the two pixels are far apart at the mask, a situation that will inevitably occur in view of the large image field sizes, both streams combined, randomly arrayed with respect to each other, will experience statistical interactions.

A second effect that can reintroduce shot noise is spatial dispersion due to the energy distribution of the electrons. Due to the variations in velocity electrons that leave the space charge cloud at regularly spaced intervals arrive at the sample randomly, if the transit time through the column is so long that the electrons start overtaking one another. At practical beam energies this is not often the case but these conditions might be met for large currents (> 10 nA) and very low voltages (< 1 kV), occurring in the beam forming region close to the emitter. Whether shot noise remains reduced is thus strongly dependent on the beam shape and acceleration.

It is interesting to note that the fact that shot noise is reduced in space charge limited current has the consequence that increasing the temperature of an emitter operating in the space charge limit reduces noise whereas the average current is only slightly influenced.

9.6 Conclusions

We can model space charge limited currents by calculating all mutual interactions in a particle box with cyclic boundary conditions on the side planes. The front and back plane serve as anode and cathode, respectively. Electrons are emitted at the cathode and collected if they reach the anode. Fluctuations in the space charge limited current can be investigated by analyzing the rate of arrival of electrons at the anode.

The rate of arrival at the anode does not necessarily follow Poisson statistics even though the rate of emission does. The fluctuations in the arrival current are reduced as a result of feedback between the space charge barrier and an excess or a deficiency of current. This is the cause of shot noise reduction. The shot noise reduction factor, denoted $I_0^2$ can be determined by calculating the spectrum of the current fluctuations. It lies typically between a few percent and 1, the latter value corresponding to full shot
noise. The reduction of shot noise is effective at low frequencies. The relevant time scale is determined by the settling time of the space charge barrier. On a time scale shorter than the settling time, the space charge barrier cannot react to excess or deficiency of current, resulting in full shot noise at high frequencies. This behaviour, properly reproduced with the simulation model, is in qualitative agreement with previous theories.

Electron guns operating in the space charge limited regime may prove advantageous in SEMs or EPL. If they can be designed such that shot noise is reduced shorter measuring times or shorter illumination times can be allowed while maintaining signal-to-noise or shot noise induced dose variation.
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Summary

Space charge effects near inhomogeneous cathodes

Background

The fact that the motion of electrons is influenced by electromagnetic fields forms the basis of electron optics. Electromagnetic fields can be considered the optical elements of electrons just as glass lenses are optical elements of light rays. Electron lenses can for instance be created by using electrodes of particular configuration such that a specifically shaped electric field originates. The equivalent of the light source in an electron optical system is the electron emitter. Several mechanisms of electron emission are known. For the work in this thesis only thermionic emission is relevant. Thermionic emission occurs as a result of the fact that electrons can use part of their thermal kinetic energy to overcome the work function barrier.

Characterization of electron optical systems comprises calculation of the electromagnetic fields and computation of the electron trajectories by integration of the equations of motion. Thus, accurate electromagnetic fields are required as well as an accurate description of the initial conditions of the electrons. The mathematical entity that represents the electron initial conditions is called the emission function. It prescribes the probability of finding an electron at a certain position in phase space. Thermionic emission of electrons is assumed to be according to a Maxwellian velocity distribution and a cosine angular distribution.

As electrons cause electrostatic fields themselves, the motion of electrons changes the fields that govern the electron motion. Whereas this annoying mutual dependency can often be disregarded, when the electrostatic field due to the electrons can be neglected with respect to the field due to the electrodes, in other cases it causes the necessity of time-consuming and laborious computations. Considering the electron beam as a smoothed-out charge distribution the electrostatic field resulting from the beam causes a deterministic lens effect called global space charge effect. In reality, the beam consists of individual electrons exerting a Coulomb force on each other. The Coulomb forces result in a stochastic effect called statistical interactions. Both global space charge effect and statistical interactions are manifestations of the mutual dependency mentioned above. They are usually considered separate effects.

In computations requiring great accuracy the global space charge effect is most often dealt with by utilizing an iterative procedure of solving the equations of motion to determine the electron beam charge distribution and solving the Poisson equation to compute the electric field which in turn influences the solution to the equations of motion. The converged solution obtained in this manner is called self-consistent. The effect of statistical interactions is dealt with by representing part of the electron beam by a cloud of discrete electrons and summing up their Coulomb interactions as they traverse along the optical axis.

In the vicinity of the electron emitter electrons have relatively low kinetic energy and consequently are easily affected by space charge, even to the point that they are reflected back towards the emitter by a barrier that exists solely as a result of the charge of the electrons. This behaviour in which the effective current, injected into the system, is lower than the current actually emitted from the source is referred to as space charge limited emission. It occurs at low extraction fields and high emission currents. When the extraction field is increased, or the emission current is decreased, the electrostatic field due to the electrons can not cause a potential minimum and the emission current is said to be temperature limited or saturated. Especially in the space charge limit the iterative procedure to obtain a self-consistent solution is cumbersome and prone to errors.

In view of the complexity of the computations it is advantageous to use analytical models that determine the field, including global space charge, in a small region in front of the emitter, like Child’s and

137
Langmuir's. These models assume that the emitter surface is perfectly flat and homogeneous and that statistical interactions near the emitter can be ignored. They are employed in most calculations in which global space charge effect is known to be considerable, like cathode-ray tubes (CRTs). The models are valid in a one-dimensional geometry consisting of two infinite parallel plates and thus quite restricted in geometrical applicability. They are used locally over the emitter surface to bypass this restriction. The collection of parallel local space charge regions over the emitter surface is referred to as a diode array in this thesis. The most accurate model is Langmuir's model that takes the initial velocity distribution of the electrons into account. Langmuir's model predicts a potential minimum in front of the cathode which occurs as a result of the negative field of the electrons combined with the positive field of the extraction electrodes. Local application of Langmuir's model gives the end conditions of the electrons at the space charge barrier so that the barrier, the so-called Langmuir minimum, can be used as a virtual emitter with virtual emission function that prescribes the 'initial' conditions in subsequent calculations starting from the virtual emitter rather than the real emitter.

Simulations of display CRT systems like computer monitor tubes and television tubes is time-consuming, laborious and often not adequate in the sense that the actual electron spot at the screen cannot accurately be predicted using computations. The fact that these simulations sometimes appear to have questionable reliability is a severe time and cost load on the design process of innovative electron gun designs. One conceivable reason for inaccuracy is the local application of the Langmuir model. Another is the possibility that the assumption of a flat and homogeneous emitter surface with ignorable statistical interactions is unjustified.

Aim

The work in this thesis strived to reveal the source of inaccuracies in simulations of display CRT systems. First it was shown whether the local application of Langmuir's model is allowed in configurations that fall outside the geometric scope of the model, and second, whether Langmuir's model for space charge limited emission is adequate in the presence of emitter surface roughnesses and other inhomogeneities, including the effect of statistical Coulomb interactions and image charges in the emitter.

Methods

To show whether certain approximations in a calculation are justified one can leave out the approximation in favour of an exact description, for each approximation in turn. Local applicability of Langmuir's model was put to the test by comparison of the emission properties as formed by the diode array with the end conditions of electrons traced in the computed potential distribution all the way from the cathode surface. The potential calculations including diode arrays model were performed using a CRT electron gun design software package called SCELOP, developed and used by Philips Display Components. It calculates the self-consistent potential distribution by solving the Poisson equation, including a specific beam shape as source term, redetermining the beam shape in the newly obtained Poisson electric field and iterating the process. The potential distribution in the vicinity of the cathode and the virtual emitter are constructed using two space charge models, Child's and Langmuir's.

Furthermore, a new simulation tool was developed that is capable of calculating the self-consistent potential distribution, space charge field and emission current profile from a cathode suffering surface inhomogeneities, including statistical interactions, image charges and work function patches. Statistical interactions are taken into account in a small beam segment, the so-called electron box, by integration of coupled differential equations containing all Coulomb forces of the particles. The electron box is bounded by the cathode and a virtual anode and is subject to cyclic boundary conditions in the other four planes. The statistics of the electron cloud are determined by detecting electrons crossing the virtual anode or any other plane of interest parallel to the emitter surface. Global space charge is taken into account with a mean-field approach to the space charge field outside the electron box. Surface roughness and work function nonuniformity are modelled by considering a plane of specific shape and potential distribution to be the real emitter surface and allowing electrons to be emitted into the electron box according to the appropriate initial conditions. The particular shape and potential of the modelled emitter plane is created by placing a well-chosen collection of point charges with specific charges such, that the resulting potential distribution corresponds to the intended cathode surface shape and surface potential distribution. Image charges are represented by mirroring the individual electrons in the emitter plane, or by adjusting the
strength of the point charges that are responsible in the emitter surface model for the surface shape and potential.

Results

The complete research can roughly be divided into three parts. First, it is necessary to determine whether local application of the one-dimensional Langmuir model for space charge limited emission in a diode geometry is a valid description for the space charge field in the CRT emitter region. The emitter region is described by a set of small parallel diodes with a common cathode. These diodes prescribe the potential distribution near the emitter using Langmuir's space charge model, and concurrently the emission function at the Langmuir minimum is given. The resulting emission function is used to prescribe the initial conditions in the remainder of the electron optical system. The verification of the diode array is given in Chapter 4. It is shown that emission from the virtual emitter according to the virtual emission function and subsequent solution of the Poisson equation will yield the same results as emission from the real emitter according to the Maxwellian cosine distribution.

The second part shows that the correctness of the initial conditions at the virtual emitter is essential for the correctness of the characterization of the electron optical system. Thus, errors in the virtual emission function propagate into errors in the properties of the calculated spot. This analysis, described in Chapter 3, is essentially a validation for the third part of the investigation.

The third subject addressed in the exploration of inaccuracies in CRT spot calculations makes use of a new simulation tool described in Chapter 5. This tool is designed to calculate the behaviour of a cloud of discrete electrons emitted from a rough and nonuniform surface, taking into account mutual interactions, image charges and patch fields. The influences of interactions, surface roughness and work function inhomogeneity are investigated in turn.

The topic of Coulomb interactions in the space charge cloud is addressed in Chapter 6, in which it is investigated whether the electron cloud in a CRT must necessarily be described in terms of a cloud of individual particles. The discreteness of the electron cloud has the consequence that the individual electrons interact with each other and that the space charge potential distribution is not smooth on the scale of the interelectron distance.

Interactions cause a stochastic push in the electron motion resulting in trajectory displacement or energy broadening. The effect of interactions taking place over a long time is to cause a relaxation towards an isotropic equilibrium distribution. Thus, if the electron cloud is close to equilibrium to start with, interactions have little effect. The electron cloud near a thermionic emitter as used in a CRT is rare in the sense that electrons are far apart, so that the motion of electrons is hardly affected by statistical collisions. It is almost solely governed by deflection resulting from the total, averaged charge of all electrons in the system. Furthermore, the velocity distribution in the virtual emitter region is very close to equilibrium, making any interactions ineffective.

The interelectron distance in front of a thermionic emitter is on the same order as the distance from emitter to potential minimum. Thus, on this length scale the potential distribution looks like a jagged mountain edge with local peaks and valleys. Electrons have to overcome the peaks as well as the smooth barrier so that the effective barrier height is determined by the peaks, rather than the average. This heightening of the effective barrier could cause the space charge limited current to be less than predicted in a smooth space charge field. A lower current has not been observed indicating that the peaks are pushed out of the way with the result that they do not interfere with the average transmission probability of the electrons through the space charge barrier.

Consequently the electron cloud behaves as a smooth jelly, subject to its own global space charge field, and a description in terms of discrete electrons is unnecessary, corroborating the validity of the Langmuir model near a CRT emitter.

In Chapter 7 the influence of surface roughness is discussed. The rough emitter surface is represented by a bumpy equipotential plane of a well-chosen point charge distribution. The point charges are placed inside the emitter, so that they do not interfere with the emitted electrons. As the electrons fill up the electron box moving towards the anode plane they induce a charge distribution on the point charges such that the total potential at the bumpy surface plane remains stationary. In this manner the induced surface charge as it occurs at a metallic emitter surface is represented.

The space charge cloud in front of the rough surface acts as a barrier that homogenizes the nonuniformities occurring in the phase space density. For surface bumps with similar dimensions as the distance
from real emitter surface to Langmuir minimum, the phase space density of a bumpy surface is indistinguishable from the phase space density of a planar surface. Space charge homogenization takes place even without a space charge barrier but it is most effective when a barrier is present.

Work function inhomogeneities are studied in Chapter 8. Variations in work function over the emitter surface cause patch fields. Their influence on the emission current is twofold. The resulting variations of saturation current cause a smooth transition from space charge limited emission to temperature limited emission, whereas this transition is expected to be rather sudden in the presence of a homogeneous work function. Furthermore, the differences in saturation current can result in a wildly varying spatial current density profile at the cathode. The current density is smoothened by the patch fields because electrons emitted from highly emitting patches are pulled back by neighbouring patch fields. Additionally, the space charge barrier becomes deeper at highly emitting patches, effectively smoothening the current even further.

A second smoothening effect that occurs in the space charge cloud is reduction of fluctuations resulting from shot noise. Shot noise is caused by the fact that the actual number of electrons emitted in a short time interval varies according to a Poisson distribution. Therefore, the relative $1/e$ fluctuation of the number of electrons emitted from a thermionic emitter is equal to the inverse square root of the average number of electrons, as prescribed by the Poisson distribution. In the presence of a space charge barrier the fluctuations decrease because a stochastic excess of emitted current heightens the barrier, decreasing the current and conversely, a shortage of current leads to a reduced barrier, increasing the current. This effect is described in Chapter 9. The space charge barrier reduces shot noise fluctuations of low frequency. High frequency noise is passed unsuppressed.

Conclusions

The two questionable assumptions used in the simulation of CRTs using computation tools like SCELOP are the extendibility of the one-dimensional Langmuir model to a complex electron gun geometry by local application, and the validity of the Langmuir model near a CRT emitter, concerning the influence of surface inhomogeneities, image charges and statistical interactions.

Both assumptions were tested over the course of this research. By tracing a large number of electrons from the emitter in a space charge field it was verified that the virtual emission function defined by application of the Langmuir model is consistent with the emission function at the real emitter. To test the second assumption a new simulation tool was developed, described in this thesis, that is capable of calculating diverse effects near the emitter, like statistical interactions, surface roughness including image charges and work function patches including patch fields. Describing the space charge distribution in terms of a cloud of discrete electrons using this tool it was revealed that statistical interactions in the space charge cloud near the emitter do not affect the phase space distribution of the electrons. Work function non-uniformities and small surface bumps are smoothened by the filtering action of the space charge barrier so that the emission function near an inhomogeneous surface is very similar to the emission function of a uniform cathode.

The work in this thesis has shown that the reason for inaccuracies in the simulations can not be found in the application of simple space charge models. Indeed, in parallel to this research, the mathematical implementation of the electron optical problem in SCELOP has been advanced, causing a tremendous improvement in the accuracy of the computation results, without any change in the space charge model.
Samenvatting

Ruimteladingseffecten bij inhomogene kathodes

Achtergrond

De beweging van elektronen wordt beïnvloed door elektromagnetische velden. Dit verschijnsel vormt de basis van de elektrooptica. Elektromagnetische velden kunnen worden beschouwd als de optische elementen voor elektronen, naar analogie van glazen lenzen in lichtoptiek. Men vormt elektronelelenzen bijvoorbeeld door met behulp van een stelsel van elektroden met de juiste vorm een specifiek elektrisch veld te maken. De analogie met lichtoptiek voortzettend kunnen we als lichtbron een zogeheten elektronenemitter benoemen. Er zijn verschillende fysische mechanismen voor elektronenemissie bekend. Voor het onderzoek beschreven in dit proefschrift is uitsluitend thermionische emissie van belang. Thermionische emissie vindt plaats wanneer materiaal zodanig wordt verhit dat de kinetische energie van de elektronen in het materiaal voldoende is om de werkwachter barrière te overwinnen.

Om elektron-optische systemen te karakteriseren is het nodig om de elektromagnetische velden uit te rekenen en de de bewegingsvergelijkingen van de elektronen in de velden te integreren. Voor een nauwkeurige karakterisering zijn dus zowel de velden als de begincondities van de elektronen van belang. De begincondities van de elektronen kunnen worden gekarakteriseerd met behulp van een wiskundige afbeelding die emissie functie wordt genoemd. Deze functie geeft de waarschijnlijkheidsdichtheid in de fase ruimte ter plaatse van het emitter oppervlak. In het geval van thermionische emissie gaat men uit van een Maxwellsche snelheidsverdeling en een cosinusvormige richtingsverdeling.

Aangezien elektronen zelf een elektrostatisch veld veroorzaken, verandert door de beweging van de elektronen het veld dat hen doet bewegen. Deze hinderlijke wederzijdse wisselwerking kan vaak worden verwaarloosd: namelijk wanneer het elektrostatische veld veroorzaakt door de elektronen veel kleiner is dan het veld veroorzaakt door de elektronen. In andere situaties echter kan het tijdlopende en bewerkelijke berekeningen met zich mee brengen. Wanneer men de elektronenbundel als een gladde, uitgesmeerde ladingsverdeling beschouwt, veroorzaakt de bundel een deterministisch lens effect dat globaal ruimteladingseffect wordt genoemd. In werkelijkheid bestaat een elektronenbundel uit individuele elektronen die een Coulomb wisselwerking op elkaar uitoefenen. De Coulombkrachten hebben stochastisch gedrag tot gevolg, aangeduid met statistische interacties. Zowel het globale ruimteladingseffect als de statistische interacties zijn uitingen van de wederzijdse wisselwerking die aan het begin van de paragraaf wordt genoemd. Gewoonlijk beschouwt men de twee als afzonderlijke effecten.

Wanneer nauwkeurige berekeningen gewenst zijn en globale ruimtelading niet verwaarloosd kan worden maakt men vaak gebruik van een iteratieve procedure waarbij de bewegingsvergelijking numeriek wordt opgelost om de vorm van de elektronenbundel — daarmee de ladingsverdeling — te bepalen, en vervolgens de Poissonvergelijkingen om het veld te berekenen, wat zijn weerslag heeft op de oplossing van de bewegingsvergelijking. De aldus na convergentie verkregen oplossing wordt zelfconsistent genoemd. Wanneer statistische interacties in de berekening meegenomen dienen te worden doet men dat vaak door een gedeelte van de elektronenbundel te representeren als een wolk van discrete puntladingen waarin alle Coulombkrachten worden opgeteld gedurende hun wandeling in de optische kolom.

Dichtbij de emitter hebben de elektronen een relatief lage kinetische energie en zijn dichtgegevolge erg vatbaar voor ruimteladingseffecten, zo vatbaar zelfs dat ze onder sommige omstandigheden door een ruimteladingsbarrière deels terug worden geduwd richting bron. In dat geval is de effectieve stroom, die in het systeem wordt geïnjecteerd, lager dan de stroom die door de bron wordt geëmitteerd. We noemen dit ruimteladingsbegrensde emissie. Deze situatie doet zich voor bij lage extractievelden en
hoge emissiestromen. Als het extractieveld wordt vergroot, of de emissiestroom verkleind, is het veld van de gecombineerde elektronen vlak voor de emitter te zwak om een barrière te vormen zodat alle elektronen die uit de emitter komen ook daadwerkelijk in het systeem worden geïjecteerd. Men noemt de effectieve stroom dan temperatuurbegrensd of verzaagd. Vooral in geval van ruimteladingsbegrenzing is de iteratieve procedure van hierboven zeer bewerkelijk en gecijtigd tot onvolkomenheden.


Simulaties van CRT’s voor beeldschermen zoals computerbuizen en televisiebuizen is tijdrovend, werkelijk, en vaak niet goed in dit zin dat de elektronenvleug (-spot) op het scherm niet nauwkeurig genoeg kan worden voorspeld met behulp van berekeningen. Het feit dat deze simulaties niet altijd even goed te vertrouwen lijken te zijn heeft grote gevolgen voor doorklooptijd en de kosten van het ontwerp proces van innovatieve elektronenkanonnen. Een mogelijke bron van onnauwkeurigheid is de lokale toepassing van Langmuir’s model. Een andere is de aannames dat de emitter oppervlak homogeen en perfect vlak is en dat statistische interacties kunnen worden verwaarloosd.

Doel
Het hier beschreven onderzoek is uitgevoerd om de onnauwkeurigheid in simulaties van CRT’s voor beeldschermen te leggen. Als eerste is aangetoon of de lokale toepassing van Langmuir’s model een voldoende nauwkeurig resultaat geeft in een elektrodie configuratie die a priori niet geschikt lijkt om met een één-dimensionaal model te beschrijven; vervolgens is aangetoond of het model correcte resultaten kan opleveren wanneer het emitter oppervlak ruw en inhomogeen is, en statistische Coulomb interacties en spiegelladingen in de emitter ook meegenomen dienen te worden.

Methoden
Om in het algemene geval aan te tonen of een benadering in een berekening is toegestaan, kan men de benadering in het model vervangen door een exacte beschrijving. Door de cindcondities van elektronen die zich bewegen van kathode naar Langmuir minimum te vergelijken met de ‘begin’condities zoals voorspeld door de emitter array model werd de gelijkheid van lokale toepassing van Langmuir’s model getoetst. Hierbij werd gebruik gemaakt van een softwarepakket met de naam SCEL, dat door LG Philips Displays ontwikkeld en gebruikt is om CRT’s mee te ontwerpen. Dit pakket berekent de zelfconsistente potentiaalverdeling door de Poisson vergelijkingen op te lossen met de specifieke vorm van de elektronenbundel als bronterm, de bundelvorm in het resulterende veld opnieuw te bepalen en deze procedure iteratief te herhalen, waarbij gebruik wordt gemaakt van het diode array model bij de kathode. Zowel het model van Langmuir als dat van Child wordt gebruikt om tot een diode array te komen.

Tevens is een nieuw simulatiepakket ontwikkeld dat in staat is om zelfconsistent potentiastadium, ruimteladingsverdeling en emissiestroom dichtheidsprofiel van een kathode met oppervlakruweheid en andere inhomogeniteiten te berekenen, waarbij statistische interacties, spiegellading en werkkrommetie van de deeltjes ook in beschouwing worden genomen. Statistische interacties worden meegenomen in een klein bundelsegmentje, de zogenoemde elektronstrip, door integratie van de gekoppelde differentiaalvergelijkingen die alle Coulomb krachten op de deeltjes bevatten. De elektronstrip is begrensd door de kathode en een
Resultaat

Het complete onderzoek kan ruwweg in drie stukken worden onderverdeeld. Als eerste is het nodig om te bepalen of met de diode array een goede beschrijving kan worden verkregen van het ruimteladingsveld bij de bron in een CRT. Het gebied rondom de emitter wordt geregistreerd door een verzameling kleine parallelle diodes met gemeenschappelijk kathodevlak. Deze diodes geven de potentiaalverdeling bij de emitter door gebruik te maken van Langmuir’s model, en leveren zo de emissie functie ter plaatse van het Langmuir minimum. Deze emissie functie wordt gebruikt voor de begincondities in de rest van het optische systeem. In Hoofdstuk 4 staat de verificatie van het diode array model beschreven. Hier wordt aangetoond dat een berekening vanaf een virtuele emitter met bijbehorende virtuele emissie functie dezelfde resultaten geeft als een berekening vanaf de echte emitter volgens een Maxwellse, cosinusvormige verdeling.

Het tweede stuk laat zien dat een correcte beschrijving van de begincondities op de virtuele emitter essentieel is voor de verdere karakterisering van het elektronenoptische systeem. Vooral in de virtuele emissie functie werken dus door in de eigenschappen van de berekende elektronenvlek. Deze analyse, die beschreven is in Hoofdstuk 3, motiveert uiteindelijk het derde deel van het onderzoek.

Voor het derde onderwerp dat onder handen wordt genomen in deze speurtocht naar onwaarschijnlijke vogels in CRT spotgrootte berekeningen wordt gebruik gemaakt van een nieuw simulatieprogramma, beschreven in Hoofdstuk 5. Dit programma is ontworpen om het gedrag te berekenen van een wolk van discrete elektronen die door een ruw en niet-uniform oppervlak worden geïnitieerd, waarbij de invloed van wederzijdse interacties, beeldladingen en werkfunctie gebiedjes worden meegenomen. Achtereenvolgens komen interacties, oppervlakte ruwweg en werkfunctie inhodenititeit aan bod.

Coulomb interacties in de ruimteladingswolk worden bediscussieerd in Hoofdstuk 6. Hierin wordt onderzocht of de elektronenruwweg in een CRT moet worden geregistreerd als een wolk van discrete deeltjes om tot een correct resultaat te komen. Het discrete karakter van de elektronen heeft als gevolg dat individuele elektronen interacties met elkaar hebben en dat de ruimteladingsverdeling niet glad is wanneer men hem beschouwt op de schaal deeltjes vergelijkbaar met de afstand tussen de elektronen.

Interacties veroorzaken een stochastische duw in de beweging van elektronen die aanvankelijk ‘trajectory displacement’ en energie verbreding tot gevolg hebben. Statistische interacties brengen uiteindelijk een relaxatie tot een isotrop evenwichtsverdeling tewech. Interacties hebben dus weinig effect wanneer een elektronenwolk zich al in isotroop evenwicht bevindt. De elektronenwolk bij een thermonische emitter in een CRT is ij gelijk, zodat de beweging van de deeltjes nauwelijks wordt beïnvloed door statistische botsingen. De elektronenbanen worden vrijwel uitsluitend bepaald door de totale, gemiddelde lading van alle elektronen in het systeem tezamen. Daar komt bij dat de snelheidsverdeling in het emitter gebied bijna in evenwicht is, zodat interacties toch al weinig effectief zijn.

De afstand tussen de elektronen in dit gebied is van gelijke grootte als de afstand van kathode tot potentiaal minimum. Op deze lengtenschaal ziet het potentiaalveld er dus uit als een gekromde bergrug, met plaatselijke pieken en dalen. De elektronen moeten over de pieken heen komen dus de effectieve barrière hoogte wordt bepaald door de piekhoogte en niet zozeer door de gemiddelde hoogte van de barrière. Dit zou aanleiding kunnen geven tot een ruimteladingsbegrachts stroom die lager is dan de stroom in een glad, uitgesmeerde veld. In de simulaties is echter geen lagere stroom waargenomen, wat er op wijst dat de pieken in het potentiaalveld van de weg worden geduwd zodanig dat ze geen invloed hebben op de gemiddelde transmissie van elektronen over de ruimteladingsbarrière.

Hieruit kan geconcludeerd worden dat de elektronenwolk zich gedraagt als een gladde, uitgesmeerde
ladiansverdeling. Een beschrijving in termen van discrete elektronen is dus overbodig, en wat dat betreft is het Langmuir model geldig in de buurt van een CRT emitter.

In Hoofdstuk 7 werd de invloed van oppervlakte ruwheid besproken. ALS ruw emitter oppervlak doet het buitlachtige equipotentiaal vlak van een geschikt gekozen puntdeeljes dienst. De puntdeeljes bevinden zich in de emitter zodat de geëmitteerde elektronen geen last van ze hebben. Terwijl de elektronen zich naar de anode bewegen induceren zij een ladiansverdeling op de puntdeeljes zodanig dat de oppervlakte potentiaalverdeling stationair blijft. Op deze manier wordt rekening gehouden met de geïnduceerde oppervlakte lading zoals die bij een metallisch oppervlak voorkomt.

De ruimteladingswolk bij een ruw oppervlak werkt als een filter dat niet-uniformiteiten in de faseruimte effent. Bultjes aan het oppervlak met gelijke grootte als de afstand tussen kathode en Langmuir minimum worden daardoor aan het zicht onttrokken en de faseruimte verdeling van dit ruwe oppervlak is dan niet te onderscheiden van de verdeling bij een glad oppervlak. De effening treedt ook gedeeltelijk op als er wel ruimtelading is maar geen ruimteladingsbarrière is, maar is vooral effectief in aanwezigheid van een barrière.


Een tweede effening effect dat in de ruimteladingswolk plaats vindt is de onderdrukking van fluctuaties die het gevolg zijn van hagelruit ('shot noise'). Hagelruit wordt veroorzaakt door het feit dat het aantal elektronen dat in vaste tijdsintervalletjes wordt geëmitteerd onderhevig is aan Poisson statistiek. De variantie van het aantal elektronen is daardoor omgekeerd evenredig met het gemiddelde aantal elektronen, zoals uit de Poisson verdeling volgt. Echter, in aanwezigheid van een ruimteladingsbarrière is de variantie kleiner, omdat een overdaad aan elektronen, zoals dat stochastisch kan voorkomen, leidt tot een verhoogde barrière en daardoor een verlaagde stroom, en vice versa. Dit effect wordt beschreven in Hoofdstuk 9. De ruimteladingsbarrière reduceert hagelruit fluctuaties in het lage frequentie regime. Hoge frequentie ruis wordt niet onderdrukt.

Conclusies

Bij de simulatie van CRT's met rekenpakketten als SCELOP worden de volgende twee aanname gemaakt: men veronderstelt dat het één-dimensionale Langmuir model kan worden uitgebreid naar een complexe geometrie door gebruik te maken van een diode array, en dat Langmuir's model geldig is bij een CRT emitter, wat betreft de aanwezige oppervlakte inhomogeniteit, beeld lading en statistische interacties. Aan de geldigheid van deze twee aannamen kan worden gewijzigd gezien de waargenomen onaangewezenheden van de simulaties.

Beide aannamen zijn onder de loof genomen gedurende dit onderzoek. Door de beweging van een groot aantal elektronen in het ruimteladingsveld rond de emitter te berekenen kon bevestigd worden dat de virtuele emissie functie die tot stand komt in het diode array model consistent is met de emissie functie van de fysieke emitter. Ten behoeve van de tweede aanname is in dit proefschrift een nieuw simulatieprogramma beschreven dat in staat is om uiteenlopende effecten als statistische interacties, oppervlakte ruwheid met beeldlading en werkfunctie gebiedjes met patch velden te berekenen. Door de ruimteladingsverdeling te beschrijven als een wolk van deeltjes met behulp van dit programma kwam aan het licht dat statistische interacties in de buurt van de emitter de faseruimte verdeling van de elektronen nauwelijs beïnvloeden. Werkfunctie variaties en kleine bultjes op het oppervlak worden geïffent door de filterwerking van de ruimteladingsbarrière zodat de emissie functie bij een inhomogeen oppervlak erg veel lijkt op die van een uniforme kathode.

Het onderzoek beschreven in dit proefschrift heeft laten zien dat de toepassing van simpele ruimteladingsmodellen niet debiet is aan onaangewezenheden in de simulatie. Het is interessant om op te merken dat tegelijkertijd met dit onderzoek enige verbeteringen in de wiskundige implementatie van het elektron-optische probleem in SCELOP zijn toegepast. Hiermee is enorme vooruitgang geboekt in de nauwkeurigheid van de berekeningen, zonder fundamentele veranderingen in het ruimteladingsmodel.
Dankwoord

Dit proefschrift is niet door mijn inspanning alleen tot stand gekomen. Een ieder die van mening is op de een of andere manier te hebben bijgedragen heeft ongetwijfeld gelijk, dus: hartelijk dank, iedereen! Bij enkelen van jullie wil ik iets langer stilstaan.

Pieter, hartelijk dank voor je enthousiasmerende vermogen om telkensmale met wat streken van een vulpotlood tien grote problemen te veranderen in twintig kleintjes. Dit heeft niet alleen altijd leuke, nieuwe inzichten gebracht, maar ook leuke, nieuwe, ingewikkeldere rekenarijen nodig en mogelijk gemaakt.

Rudi, ik wil jou van harte danken voor onze boeiende discussies over vaderlandse geschiedenis, taal, whisky, wiskunde, economie en warenplo zo nu en dan ook wat ruimtelaadingsmodellen. Jouw stimulerende wetenschappelijke creativiteit heeft zeker een belangrijke rol gespeeld. Minder wetenschappelijk maar minstens zo belangrijk was het heen-en-weer zeulen van de SPARC als die weer eens kuren had.

Philips Display Components ben ik dank verschuldigd voor het bekostigen van dit onderzoek. Aan het begin van dit promotietraject heb ik enige tijd doorgebracht Eindhoven. Dankzij de ‘electron optics’ groep van PPD heb ik het reilen en zeilen van SCELOP geleerd en bovendien een aantal interessante excursies mee kunnen maken. Ook met de CRT groep van Natlab heb ik veel stimulerende gesprekken gehad, waar ik met name Nijs van der Vaart voor wil danken. Danny Roozendaal en Leo Stok wil ik nog noemen voor hun hulp op softwaregebied om SCELOP en ELOP aan de praat te houden.

Mijn collega’s uit de vakgroep Declitjesoptica wil ik bedanken voor de prettige werksfeer de afgelopen jaren. Michiel, bedankt voor je begeleiding in het eerste jaar en je ongezouten commentaar op een van de hoofdstukken in het laatste jaar. Eriks, mijn hartelijke dank voor alle hulp op het vlak van de computerondersteuning, en dan vooral de ‘Rescue 911 Mb’ actie toen de harddisk in de soep liep. Elly, zonder jou was het administratieve gedeelte misschien wel een onoverkomelijk obstakel geweest.

Ik wil mijn naaste collega’s op de TPD bedanken voor hun geduld terwijl ik waardevolle tijd spendeerde aan het reviseren van dit proefschrift en andere vreemdsvoorsoortige zaken.

Mijn dierbare vriendjes en vriendinnen, mijn familie, mijn broer en Babette hebben door hun genoeglijke aanwezigheid in kroegen en aan uitgebreide dissen, en door tot vervolgens toe te vragen “wanneer is het feest?” ook hun bijdrage geleverd om dit alles tot een goed einde te brengen. Mijn ouders hebben onmisbaar bijgedragen door niet al te vaak te vragen “wanneer is het feest?” maar natuurlijk voornamelijk door hun aandacht, zorg en steun, gastronomisch of anderszins. Dat jullie dit boek kunnen vasthouden is hopelijk een tastbaar bewijs dat het geen verspilde aandacht is geweest.
Curriculum Vitæ


Gedurende de periode januari 1996 tot januari 2000 was hij werkzaam als assistent in opleiding (AIO) bij de vakgroep Deeltjesoptica van de faculteit Technische Natuurkunde aan de Technische Universiteit Delft. De resultaten van het hier verrichte onderzoek staan beschreven in dit proefschrift.

Vanaf december 2000 is hij werkzaam bij TNO TPD te Delft.