EFFECTS OF NUMBER OF EVENTS AND RELAY POINT DENSITY ON ACCURACY OF THREE-DIMENSIONAL AE-TOMOGRAPHY

Y. Kobayashi 1, T. Shiotani 2 and K. Oda 3

1 Department of Civil Engineering, College of Science and Technology, Nihon University, Tokyo, Japan / Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands. Email: kobayashi.yoshikazu@nihon-u.ac.jp
2 Graduate School of Engineering / Graduate School of Business, Kyoto University, Kyoto, Japan.
3 Department of Civil Engineering, College of Science and Technology, Nihon University, Tokyo, Japan.

ABSTRACT

This paper introduces results of numerical investigations on accuracy of elastic wave velocity distribution in Three-dimensional AE-Tomography. A series of numerical analyses is conducted by changing number of events and density of relay points for the investigation. AE-Tomography is an identification problem and its number of observation equations is in proportion to the number of events, and further, the density of the relay points is immediately correlated with the resolution of the source location that is significant for calculation of travel time from the source location to receivers. The investigation is carried out on a model of tetrahedron that is meshed by using tetrahedral cells, and 8 receivers are settled at apexes and middle of the edges of surfaces of the model. The results of the investigation demonstrate that the accuracy of the source location and wave velocity distribution is strongly correlated with the number of events. It was also demonstrated that the density of the relay points affects to the estimated elastic wave velocity distribution, and better estimation of the elastic wave velocity distribution is confirmed in case of dense installation of relay points. However, these tendencies are confirmed qualitatively and suggest that more investigation is required to figure out the effects of them to the estimated wave velocity distribution.

KEYWORDS

AE-Tomography, Elastic wave tomography, system identification, relay points, source location, ray-trace.

INTRODUCTION

Elastic wave tomography has been applied for soundness evaluation of concrete structures because of its capability that enables to figure out elastic wave velocity distribution. Generally, the correlation between the elastic wave velocity and deterioration of the material is well known, and the extent of the deterioration can be figure out from the elastic wave velocity distribution. It is note that source information such as source location and stimulated time are required for the elastic wave velocity tomography and it raises the cost of investigation. On the other hand, as NDT by the elastic wave, AE Testing has also been adopted for the soundness evaluation. This technique assesses the deterioration of the concrete from the distribution of AE events since it is a natural that AE events are concentrates in the vicinity of the defects such as cracks. However, the elastic wave velocity distribution must be assumed or given for computing the source locations in this technique. Thus, since each technique requires the information that is obtained by another technique, these techniques form circular references each other. AE-Tomography was proposed by Schubert (2006) to unite the advantages of the both techniques without the circular references each other. AE-Tomography figures out the source location of AE events and elastic wave velocity distribution simultaneously, although the estimation of the source location and elastic wave velocity distribution has been conventionally conducted separately as AE-Testing and Elastic wave velocity tomography. This algorithm of AE-Tomography assumes straight ray-path on cross section of interest to simplify the computation. However, it would cause error of estimated elastic wave velocity distribution since the ray-path is not straight in actual cases because the ray-path cannot be defined as a straight line on heterogeneous elastic wave velocity field as well known, and the heterogeneity generally exists in the concrete structures. Hence, algorithms were proposed by the authors for AE-Tomography to take the heterogeneity of the wave velocity distribution and reflection and diffraction of the elastic wave into consideration in two- and three-dimensional manner (e.g. Kobayashi and Shiotani, 2012). Although it was demonstrated that these algorithms
estimate the source locations of AE events and elastic wave velocity distribution adequately, the effects of observation and model condition for the result of AE-Tomography have not been discussed yet. Therefore in this paper, a series of numerical analyses is carried out to investigate the characteristic of the effect of observation and model condition on the accuracy of estimated elastic wave velocity tomogram.

THREE-DIMENSIONAL AE-TOMOGRAPHY

Ray-Trace technique

Conventionally, straight ray-path has been assumed to simplify the computational procedure in the source location and elastic wave velocity tomography. However, it is well known that the ray-path is not straight on heterogeneous wave velocity distribution, and it would affect results of them since the heterogeneity of the wave velocity distribution is always observed in actual cases such as severely deteriorated concrete structures. The authors have proposed a ray-trace technique (Kobayashi, 2011) that considers the heterogeneity of the elastic wave velocity. This method carries out the ray-trace on finite element mesh, and relay points are introduced to reduce its mesh dependency of the ray-trace as illustrated in Figure 1. It is assumed that elastic wave velocity is constant in individual cells and ray-path is defined as a polyline. This polyline is formed by connecting segments that are defined as a straight line that connects arbitrary combination of two nodal or relay points. Then, travel time is computed in all of ray-paths from a source to a receiver, and the ray-path that gives minimum travel time is adopted as the ray-path from the source to the receiver. Finally, the travel time on the ray-path is obtained as follows.

\[ t_{ij} = T_i - T_{ij} \] (1)

This algorithm is used for both of estimation of the source location and wave velocity distribution. This algorithm has been extended for three-dimensional problem, and hexahedral and tetrahedral cells are available in present.

Source location and wave velocity distribution

The authors proposed two-dimensional algorithm for AE-Tomography (Kobayashi and Shiotani, 2012). In this algorithm, the source locations and occurrence times of AE events are computed by using arrival time at receivers and given elastic wave velocity distribution on first stage, and then theoretical travel times are calculated by carrying out ray-trace from the source location to the receivers. Finally, the elastic wave velocity distribution is updated by minimizing difference between the observed arrival times and theoretical arrival times in all events. This procedure is iteratively conducted until adequate criteria are achieved. In this section, the algorithm is briefly introduced.

The source location technique is a significant part of the algorithm of AE-Tomography. Conventionally, the source location has been estimated by assuming homogeneous elastic wave velocity distribution and straight ray-path from the source to the receiver on a cross section of interest. Although these assumptions would be useful in term of reduction of computational cost, it would be a cause of inaccurate result because the ray-path is not straight in actual case especially if the wave velocity shows strong heterogeneity as well as the ray-trace. Thus, the ray-trace algorithm that was introduced in previous section has been adopted for the algorithm of AE-Tomography. In this algorithm, the ray-trace is carried out from a receiver on a given elastic wave velocity distribution on a primary stage. This procedure gives travel time \( T_i \) from the receiver to all of the nodal and relay points as illustrated in Figure 1. In which, \( i \) is a number of nodal or relay point at the receiver and \( j \) is a
number that is assigned to a nodal or relay point of destination. Then, the possible occurrence time of the AE event \( t_{ij} \) can be estimated as follows at the nodal or relay point of destination \( j \).

\[
t_{ij} = T_i - T_{ij}
\]

(2)

in which, \( T_i \) is the arrival time at receiver \( i \). This procedure is conducted from all of the receivers, then each nodal or relay point has plural possible occurrence time of AE events. The number of the possible occurrence time is identical to the number of receivers, and these times must be the same if the ray-path and elastic wave velocity are exactly represented. However, it is impossible in almost case because the resolution of the representations depends on the mesh. Therefore, variance the possible occurrence time at the nodal or relay point \( j \) is computed at all of the nodal and relay point at first, then a nodal or relay point that gives minimum variance of the possible occurrence time is chosen as the source location since the possible occurrence times would be close each other near the source location and the variance would be small in consequence. And furthermore, the occurrence time is estimated as average of the possible occurrence time as follows.

\[
t_j = \frac{\sum_{i=1}^{N} t_{ij}}{N}
\]

(3)

in which, \( N \) is number of receivers.

Because the source location and occurrence time of the AE events, the estimation of the elastic wave velocity can be estimated by conventional technique of elastic wave velocity tomography. In this stage, the theoretical arrival time at receivers can be obtained by using the estimated occurrence time and the travel time. Since the travel time from all of the receivers to the source location are already obtained as \( T_{ij} \), the theoretical arrival time \( T'_i \) is given as

\[
T'_i = t_j + T_{ij}
\]

(4)

Finally, the elastic wave velocity distribution is updated by using Simultaneous Iterative Reconstruction Technique (SIRT). SIRT is simple and robust method, and updates the elastic wave velocity distribution as follows.

\[
\Delta S_k = \sum_{i=1}^{N} \frac{(T_i - T'_i)l_{ki}}{L_i} \left/ \sum_{i=1}^{N} l_{ki} \right.
\]

(4)

in which, \( \Delta S_k \) is variation of slowness in cell \( k \), \( l_{ki} \) is a length of the segment of ray-path \( i \) in cell \( k \) and \( L_i \) is a total length of the ray-path \( i \). Consequently, the slowness of cell \( k \) is updated by the variation \( \Delta S_k \) as follows.

\[
S'_k = S_k + \Delta S_k
\]

(5)

where \( S'_k \) is a updated slowness of cell \( k \). This procedure is iteratively carried out until the adequate criteria are achieved. It is noteworthy that the source location is updated in every iterative procedure as well as the elastic wave velocity distribution.

**NUMERICAL VERIFICATION**

A series of numerical analyses is carried out to figure out the characteristic of the algorithm for three-dimensional AE-Tomography on its accuracy in a tetrahedral model that is illustrated in Figure 2. For the AE-Tomography, the number of events and resolution of the source location are significant parameter on its accuracy under the same model because AE-Tomography is an identification problem and the number of events and resolution of the source location correlates the number of observation equations and accuracy of the observations, respectively. Thus, the number of events and the resolution of the source location are adopted as parameters for the investigation. It is note that the resolution of the source location can be controlled by changing the number of the relay points. The conditions of cases on the investigation are shown in Table 1. For the arrival time at the receivers, firstly virtual source locations are randomly generated in the model as illustrated in Figure 2 as black spheres, the arrival times are obtained by carrying out the ray-trace from the virtual source location. If the number of events is less than the number of generated sources, adequate number of events is chosen from the generated source. 8 receivers are installed as illustrated in Figure 2 as green sphere. Homogeneous wave velocity distribution is assumed in the analyses as the initial wave velocity distribution for AE-Tomography, and it is set to 4000 m/s.

**RESULTS AND DISCUSSIONS**

Figures 3 to 6 show the estimated wave velocity distribution in each case. In these figures, it is shown that the computed elastic wave velocity distribution in the cases of that number of events is 100 are closer to the target.
elastic wave velocity distribution than the cases of that number of events is 12. It is natural that the better elastic wave velocity distribution is obtained in cases of large number of events because number of observation equations is larger if the number of events is larger.

![Figure 2 Configuration of target model](image)

**Table 1 Parameters for Cases**

<table>
<thead>
<tr>
<th>Case</th>
<th>Num. of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>3</td>
</tr>
<tr>
<td>Case 2</td>
<td>3</td>
</tr>
<tr>
<td>Case 3</td>
<td>5</td>
</tr>
<tr>
<td>Case 4</td>
<td>5</td>
</tr>
</tbody>
</table>

The number of variables that is identified in these analyses is 71 because number of cells is 71 and it is assumed that the wave velocity is constant in individual cell in this model. Since the number of receiver is 8 in all cases, the number of observation equation is 96 if the number of events is 12. Thus, the number of observation equations to the number of variables ratio is 1.35. This ratio is very low in application of the elastic wave tomography, and it would be the reason of the estimated elastic wave velocity distribution. Even in cases 1 and 3, the slow velocity area is not detected at lower left area of the figures although it should be identified as well as the target elastic wave velocity distribution. On the other hand, the estimated elastic wave velocity It is caused because insufficient number of events. Generally, the low wave velocity area is detected in case 2 and 4 although the velocity is higher than the target wave velocity because the number of observation functions is
1200 and the ratio is 16.90. However, although this ratio is extremely higher than the case 1 and 3, the estimated wave velocity and target wave velocity are not quantitatively consisted with each other. This may be possible that the resolution of the estimated wave velocity is also depend on the configuration of the receiver installation. Thus, the optimized receiver configuration would be required to study. Furthermore, it would be necessary to consider about the possibility of the shadow zone problem during the identification procedure. On the effects of the density of the relay points, the weak correlation is found in the cases of $N = 3$ and $N = 5$. In contrast of case 1 and case 3, while the low velocity area is found in case 1, low velocity area was more averagely identified in case 3. In the case of case 2 and case 4, the same tendency is observed. The reason of that the low velocity area is locally estimated in case 1 and 2 would be from the distribution of the events since the events are randomly generated in the model and consequently the distribution is biased. Thus, the effect of $N$ should be more deeply considered by carrying out more cases of analyses with various $N$ and changing the distribution of the events to figure out the cause of this result.

![Wave velocity distribution in Case 3](image1)

![Wave velocity distribution in Case 4](image2)

Figure 5 Wave velocity distribution in Case 3

Figure 6 Wave velocity distribution in Case 4

**CONCLUSIONS**

This paper presented the results of numerical investigation on three-dimensional AE-Tomography. Consequently the following conclusions were drawn.

1. The quality of the estimated elastic wave velocity distribution is affected by the number of events that is directly correlates with the number of observation equations.
2. The slow velocity area was correctly detected in case that the number of events is sufficient.
3. The estimated elastic velocity distribution was not quantitatively consistent with the target elastic wave velocity distribution even in the case of sufficient number of events. The cause of this result should be more deeply considered.
4. The effect $N$ was not clearly figure out by the series of analyses. This should be studied more deeply by using more wide range of $N$.

**ACKNOWLEDGMENTS**

The authors gratefully acknowledge the financial support provided by Tobishima Corporation.

**REFERENCES**