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Options for the Implementations of Data Assimilation for Geotechnics

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Abstract. Data assimilation methods have been implemented on a slope stability problem, and the performance of different constitutive models and data assimilation schemes has been investigated. In the first part, a data assimilation scheme called the ensemble Kalman filter (EnKF) is implemented using a finite element model (FEM) and its performance with different constitutive models (the Mohr-Coulomb (MC) and Hardening Soil (HS) material models) is investigated to study their effect on the parameter and the factor of safety (FoS) estimation. Measurements of horizontal displacement are assimilated. The results from a synthetic example show that the HS model can generally be used to get reliable results for parameter and FoS estimation. However, using the MC model does not always output reliable parameter and FoS estimation. In the second part, the performance of different data assimilation schemes, *i.e.*, the EnKF and ensemble smoother with multiple data assimilation (ESMDA), is studied with the preferred constitutive material model (the HS model). The results of a synthetic case show that the EnKF results in a narrower distribution for the FoS than the ESMDA method, while the latter results in FoS estimation which is closer to the 'truth'.

Keywords: Slope stability · Data assimilation · Constitutive models · Ensemble Kalman filter · Ensemble smoother

1 Introduction

Soil slopes are common geotechnical structures in many infrastructural applications such as embankments, transport facilities, flood protection and open-pit mining. The stability of these slopes is of great concern due to the high consequences of their failure. There are various assessment methods for slope stability, e.g. the limit equilibrium method, the finite element method, empirical methods and probabilistic methods.

Nowadays, FEM is a popular method to assess slope stability. FEM has several features which make it popular, for example the inclusion of advanced constitutive models, the ability to model coupled processes, the modelling of complex geometries etc. Despite these features of FEM, the slope behaviour obtained from FEM analysis often differs from what is observed in reality. This can be due to many reasons, e.g. numerical approximation (both in FEM and/or in constitutive models), complex geometry, complex

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initial and boundary conditions, poorly known parameters or the combination of these. These limitations can be mitigated by using data assimilation.

Data assimilation is an approach in which measurements are assimilated into a numerical model to estimate a system's state, parameters, or both by considering the uncertainties in the model and measurements.

An increasing number of geotechnical structures are equipped with measurement devices, for example measuring surface displacements, pore water pressure, strain, etc. Such measurements can be used to improve the estimate of uncertain model parameters.

Various types of constitutive models can be used in an FEM slope stability model. These constitutive models vary from simpler models, such as the MC model, to advanced soil models such as the HS model. In addition, various ensemble-based data assimilation schemes exist, such as the EnKF and ESMDA. In this study, we investigate (i) what sort of constitutive model should be used in a data assimilation scheme for FoS estimation (this summarises work presented in Mohsan et al. (2021), and (ii) what sort of data assimilation techniques should be used in slope stability (geotechnical) problems.

2 Methodology

The overall methodology consists of a forward model and a data assimilation scheme. The forward model is used to simulate the problem physics, and data assimilation is used to assimilate the measurements into the forward model to estimate the model parameters. In this study, measurements of horizontal displacements are assimilated into an FEM model to estimate the strength and stiffness parameters of the constitutive model. The estimated parameters are then used to estimate the system state and the FoS.

2.1 Forward Model

In this study, the forward model is an FEM model of slope stability that considers the hydro-mechanical coupled system of equations as implemented in Plaxis (Brinkgreve et al. 2016). The change of hydraulic boundaries in the slope system cause slope deformations, which are computed by performing a hydro-mechanical analysis. Stability analysis follows this hydro-mechanical analysis to compute the FoS, using the strength reduction method. The mechanical constitutive behaviour is modelled by either of the two following models:

Mohr-Coulomb (MC) Model: The MC model is a linear elastic perfectly plastic model which is widely used to model the material in geotechnical analyses. The linear elastic part of the MC model is based on Hooke's law and is modelled by the stiffness parameters. The onset of the plasticity in this model is based on the failure criterion, and hence the perfect plasticity is considered. This model does not consider several soil features, such as stress-dependent stiffness, nonlinear elasticity, pre-failure plasticity, etc. A total of five parameters are required for the MC model in geotechnical analyses: Young's modulus (E), Poisson's ratio (ν), cohesion (c), friction angle (ϕ) and dilatancy angle (ψ). The representation of this model in a deviatoric stress vs. strain plot is shown in Fig. 1a.

Hardening Soil (HS) Model: The HS model is a nonlinear elasto-plastic model. The model considers the nonlinear elasticity and irreversible plasticity (see Fig. 1b). The stress path due to the primary loading is highly nonlinear and modelled by a parameter E_{50}, the stress-dependent stiffness due to the primary loading:

$$E_{50} = E_{50}^{ref}\left(\frac{c\cos(\phi) - \sigma_3\sin(\phi)}{c\cos(\phi) - p_{ref}\sin(\phi)}\right)^m, \tag{1}$$

where E_{50}^{ref} is the reference stiffness modulus corresponding to the reference stress p_{ref} and m is the stress dependency parameter. The nonlinear elastic part is modelled by the stress-dependent unloading reloading stiffness E_{ur} as:

$$E_{ur} = E_{ur}^{ref}\left(\frac{c\cos(\phi) - \sigma_3\sin(\phi)}{c\cos(\phi) - p_{ref}\sin(\phi)}\right)^m. \tag{2}$$

This model is a double hardening plasticity model, *i.e.*, it models both shear hardening and cap hardening. As soon as the material starts yielding, the material's shear strength is mobilised, and the load generates plastic strains. The material fails once the failure criterion is reached after shear strength is fully mobilised.

The input parameters of the HS model are the strength parameters: cohesion (c), friction angle (ϕ) and dilatancy angle (ψ). The stiffness parameters are the secant stiffness at reference pressure in a standard drained triaxial test (E_{50}^{ref}), the tangent stiffness at reference pressure for primary odometer loading (E_{oed}^{ref}), the loading-unloading stiffness at reference pressure (E_{ur}^{ref}), the Poisson's ratio for unloading-reloading (ν_{ur}) and a parameter m which controls the stress-level dependency for stiffness parameters.

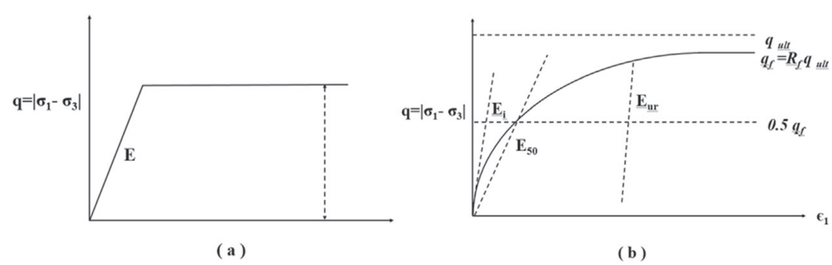

Fig. 1. Representation of constitutive models (a) MC model (b) HS model

2.2 Data Assimilation Scheme

In a data assimilation scheme, the measurements from the field are assimilated into a numerical model to estimate the parameters, the state, or both the parameters and the state of a system given the uncertainty in the model and measurements. In an ensemble-based data assimilation method, multiple realisations of a forward model are considered to approximate the model error covariance. There is a variety of ensemble-based methods currently available, e.g. the EnKF, ensemble smoother (ES) and ESMDA.

Ensemble Kalman Filter (EnKF) (Evensen 1994): the EnKF is an ensemble based Kalman filter formulation that sequentially assimilates the measurements (Fig. 2a). In this case, multiple ensembles of the forward model are run from the start to a certain time when measurements are available. These measurements are then assimilated into the numerical model to update parameter estimates. With these updated parameters, the numerical model is run from the start and continues until the next time when measurements are available. The parameter update is performed every time new measurements are available.

Ensemble Smoother (ES): the ES (Van Leeuwen and Evensen 1996) is an alternative data assimilation method that does not assimilate the data sequentially. Instead, it assimilates all the measurements at all the time steps in a single assimilation step and provides a global update of the system's parameters and/or state. The ES provides a single potentially large Gauss-Newton correction that may not provide satisfactory results.

Ensemble Smoother with Multiple Data Assimilation (ESMDA): ESMDA (Emerick and Reynolds 2012) is an adjusted form of the ES in which the same measurements are assimilated multiple times (Fig. 2b) with an inflated measurement error covariance matrix to ensure that the ensemble update is based on multiple linear smaller corrections rather than a single linear correction like in the ES. In this study, we compare the performance of the EnKF and ESMDA.

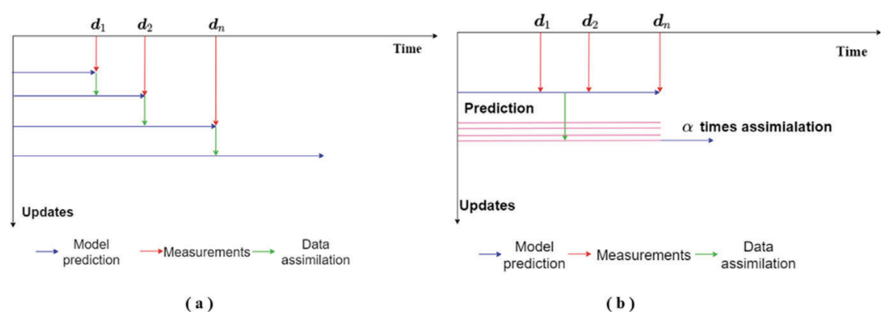

Fig. 2. Illustration of the data assimilation schemes (a) EnKF-modified from Evensen (2009) (b) ESMDA

3 Experimental Plan

This study assimilates synthetic measurements at the crest and slope of a simplified slope geometry into a numerical model of this slope to update the strength and stiffness parameters. The synthetic measurements are in the form of horizontal nodal displacements and are produced from a single simulation using 'true' model parameters. These synthetic measurements mimic the observed reality. The data assimilation methods aim to find the 'truth' within the assumed accuracy of measurements. The experimental plan is divided according to the research questions:

i. **What sort of constitutive model should be used in a data assimilation scheme?**

In the first data assimilation experiments, the EnKF is implemented with an FEM slope stability model. This FEM model considers different material models (the MC and the HS models). The data assimilation estimates parameters and the resulting FoS estimation gives insight into which constitutive models are preferable. We prefer results where the posterior distribution of the FoS incorporates the true FoS (accurate) and is narrow so the estimate is precise. These results imply that the evidence contains sufficient information to improve the prior estimation. In addition, we prefer a distribution of results consistent with the prior estimate and Bayes rule, ensuring that the data assimilation is performing well.

ii. **What sort of data assimilation technique should be used in the slope stability model (geotechnical problems)?**

In the second data assimilation experiments, the preferred constitutive model from the first part results is used in the FEM model. Then, two different data assimilation schemes are tested with this model, *i.e.,* the EnKF and ESMDA. The presented results are the parameter- and FoS estimation. Two synthetic examples are considered to perform these experiments. The geometrical properties for these cases are mentioned in Table 1 and as shown in Fig. 3 with water level fluctuation on the side indicated with the letters CE on the top right. The measurements for case **i** and **ii** are considered on the crest (i) and on both the crest and the slope (ii) respectively.

Table 1. Geometrical properties of slope in meters, water level fluctuation and measurement location

Case	h1	h2	w1	w2	w3	Dw	Measurement location
(i)	16	4	24	24	72	Water level 1	Crest
(ii)	7	2	8	5	18	Water level 2	Crest and slope

4 Results and Discussions

i. **What sort of constitutive model should be used in a data assimilation scheme?**

The EnKF has been implemented with the MC and the HS constitutive models to estimate the parameters (stiffness and strength) and the FoS. The (effective) friction angle (ϕ') estimation and FoS estimation are summarised in this section for brevity, for more details see Mohsan et al. (2021).

In Fig. 4a, the true, initial (prior) distribution and estimated distribution of ϕ' are presented. The prior estimation of ϕ' is distant from the true parameter and has a wide spread. This means that our initial guess of parameters is poor and far from reality.

It can be seen from Fig. 4a that with the HS model, the data assimilation moves the mean of ϕ' towards the true value and narrows the spread of the ϕ' distribution. On the other hand, the MC model does not show any improvements with data assimilation,

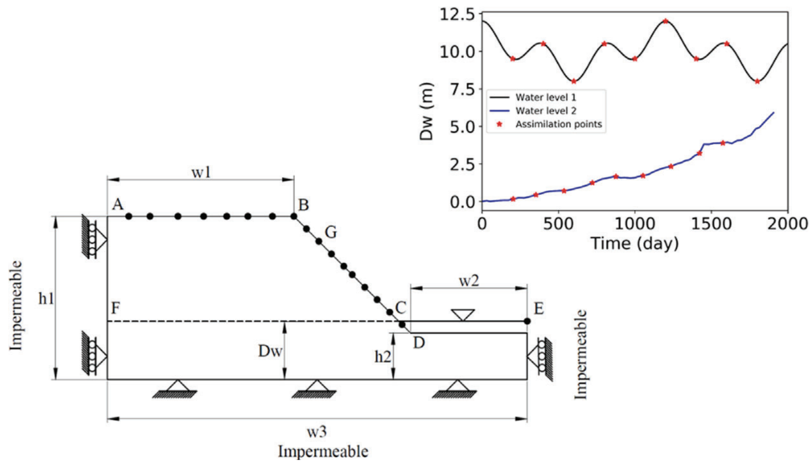

Fig. 3. Slope geometry with the geometrical properties mentioned in Table 1. The measurements points are indicated with black circles and the water-level fluctuation on the side indicated with the letters CE are presented in the inset on the top right. The red stars are the measurements considered in the data assimilation.

i.e., the estimated parameter distribution mean does not move towards the true ϕ' value. This implies that when assimilating into the HS model, the information contained in the measurements can effectively improve the estimation, and not when using the MC model. The FoS (in Fig. 4b) is shown for the same analysis and shows the same trend as the ϕ' estimation. This is because the FoS is computed with the strength reduction method, and ϕ' has a major effect on its computation. The results suggest that if the MC model is used in data assimilation, it does not give a reliable estimation of FoS. This can be explained by the sharp switch between elastic and plastic behaviour in the MC model, which does not occur in the HS model (Fig. 1).

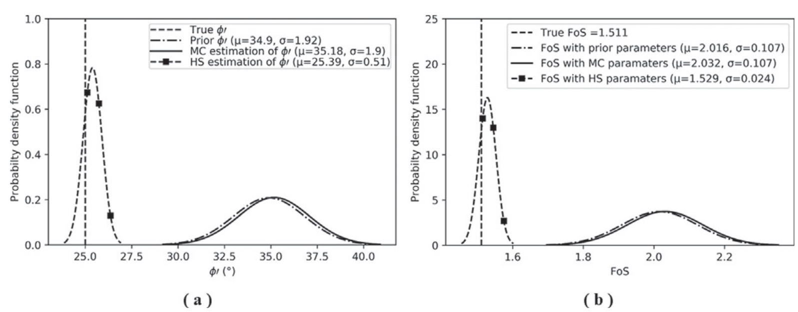

Fig. 4. Data assimilation results from the different constitutive models (a). Parameter estimation: True, prior distribution and estimated (effective) frictional angle (ϕ') distribution with MC and HS model (b). Factor of safety estimation (FoS) with true, prior and estimated parameters with MC and HS

ii. **What sort of data assimilation technique should be used in the slope stability model (geotechnical problems)?**

Two different types of data assimilation techniques are tested using the HS constitutive model, *i.e.*, the EnKF and ESMDA. In the ESDMA model, the same measurements are assimilated four times (this is termed ESMDA(x4d)). Furthermore, again only the ϕ' estimation and FoS estimation is presented in this section for simplicity.

It can be seen in Fig. 5a that the initial guess (prior estimate) is again a poor estimate, *i.e.*, far from the true value. With both data assimilation methodologies, the ϕ' mean approaches the true value. On the other hand, a narrower spread in parameter distribution is seen to be obtained by the EnKF. Figure 5b shows the resulting FoS estimate. It can be seen that both data assimilation schemes estimate parameters and FoS reasonably well when using the HS model. ESMDA gives a good mean (approximately equal to the true value of FoS), and the EnKF gives a narrower distribution, both encompassing the true value. This narrower distribution of EnKF can be due to the sequential parameter update and could be a form of over-fitting, *i.e.*, over-relying on the last data set. On the other hand, ESMDA can represent the non-linearity of the model more easily resulting in a more accurate mean.

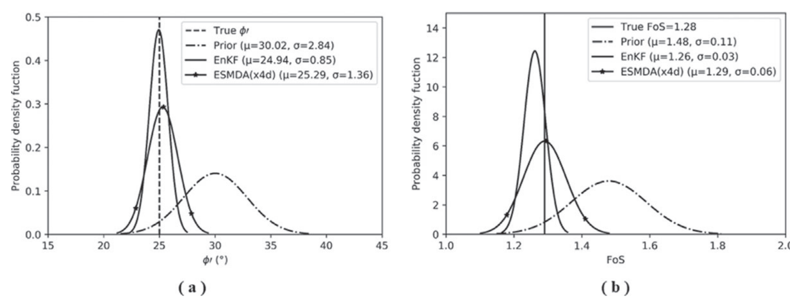

Fig. 5. Data assimilation results from different data assimilation methods (a). Parameter estimation: True, prior distribution and estimated (effective) frictional angle (ϕ') distribution with EnKF and ESMDA(x4d) (b). Factor of safety (FoS) estimation with true, prior and estimated parameters EnKF and ESMDA(x4d).

5 Conclusions

In this study, data assimilation is implemented in a synthetic slope-stability analysis. The suitability of different constitutive models and data assimilation schemes are studied. The results suggest that the HS models give more reliable FoS estimation than more conceptual models when using a data assimilation scheme, due to the more realistic soil features which require a correlation between the model parameters. The EnKF and ESMDA(x4d) data assimilation methods both give reliable FoS estimation with the HS constitutive model. ESMDA(x4d) gives a better mean estimation, whereas the EnKF gives a narrower spread. It can be concluded that data assimilation can improve the reliability of FoS assessment, but that attention should be paid to the constitutive model, especially when the prior parameters are highly uncertain.

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