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Yang, Yuguang; van der Veen, Cor; Hordijk, Dick; de Boer, A.

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Investigation of v_{min} based on experimental research

Yuguang Yang, Cor van der Veen, Dick Hordijk

Delft University of Technology, the Netherlands

Ane de Boer

Ministry of Infrastructure and the Environment, the Netherlands

ABSTRACT

This paper presents an experimental research program aiming at assessing the current code provisions of v_{min}. A large test series was carried out, by varying the reinforcement ratio and loading conditions, the minimum average shear stress that can cause shear failure of a specimen with different depth, concrete strength is determined. The three test series cover two types of concrete mixtures and a moderate structure depth range. In addition, the Eurocode and ModelCode 2010 provisions on v_{min} are discussed. They are compared with the test results. Recommendations on the present code expression are given.

Keywords: shear capacity, boundary of failure modes, v_{min}, without shear reinforcement

1. Introduction

The Eurocode v_{min} expression defines the shear capacity of reinforced concrete members without shear reinforcement and with relatively lower longitudinal reinforcements. It has been widely applied in the evaluation of the concrete slabs in buildings or bridges. In the recent years, the residual capacity of the existing reinforced concrete bridges is of concern around many European countries. Many of the bridges designed according to the old design code could not fulfill the requirements of the Eurocode. In the case of the concrete slab bridges, preliminary analysis according to the Dutch ministry of Infrastructure and the Environment (Rijkswaterstaat) showed that a large amount of the slab bridges in the Dutch highway system do not have sufficient shear capacity according to the v_{min} expression in Eurocode. These bridges have to be evaluated with more advanced approaches, which are more costly. Thus an accurate expression of v_{min} turns out to be economically important for the existing structures.

In this paper, a review of the expression regarding v_{min} in the Eurocode (Eurocode 2, 2004) and the Model Code 2010(fib, 2012) is given. Base on the theoretical study, an experimental program has been developed at Delft University of Technology. The target of the research program is to determine the actual v_{min} through experiments. The research program and the resultant v_{min} in the program are presented. The values of v_{min} under different configurations are compared with the code prediction. Recommendations on the present code expression are given.

2. Code provisions of v_{min}

2.1 Eurocode

In the Eurocode shear provision, the shear capacity of reinforced concrete members without shear reinforcement is evaluated by $V_{Rd,C}$:

$$V_{Rd,\mathrm{C}} = C_{Rd,c}(k_h \sqrt[3]{\rho_l f_{ck}} + \sigma_{cp}) b_w d \tag{1}$$

Where, k_h is the size effect factor, $k_h = 1 - \sqrt{200/d} \leq 2.0$; $C_{Rd,c}$ is a regression factor, $C_{Rd,c} = 0.18/\gamma_c$.

For the structures with limited amount of longitudinal reinforcement, their shear capacity should not be lower than $v_{min} b_w d$

$$v_{\min} = 0.035 k_h^{3/2} f_{ck}^{1/2} \quad (2)$$

The term $v_{\min}$ is introduced into Eurocode as the lower bound the shear capacity when the longitudinal reinforcement ratio ρ_l becomes very low. It is defined as the minimum average shear stress level at which the yielding of the longitudinal reinforcement and the opening a flexural shear crack occurs simultaneously. Assuming a simply supported member loaded by a single point load *P* located at *a* from the support. Here the term *a* is the center to center distance between the load and the support. It is slightly different from the a_v Eurocode expression, which is the face to face distance between the loading plate and the support. Consequently the expression: $a = M/V$ becomes possible. The maximum support reaction force *V* is defined by the lower one of the following two.

$$V \leq \frac{M_y}{a} = \frac{f_{yk} \rho_l A_c z}{a} \quad (3)$$

$$V \leq V_{Rd,\mathrm{C}} = \beta C_{Rd} k_h \sqrt[3]{\rho_l f_{ck}} b_w d \quad (4)$$

Where, the value β is the reduction factor for the point load that has a face to face distance $a_v < 2.0d$, in that case, $\beta = 2d/a_v$. For the convenience of comparison, the factor is shifted to the resistance side, and it is approximated by $\beta = 2d/a$.

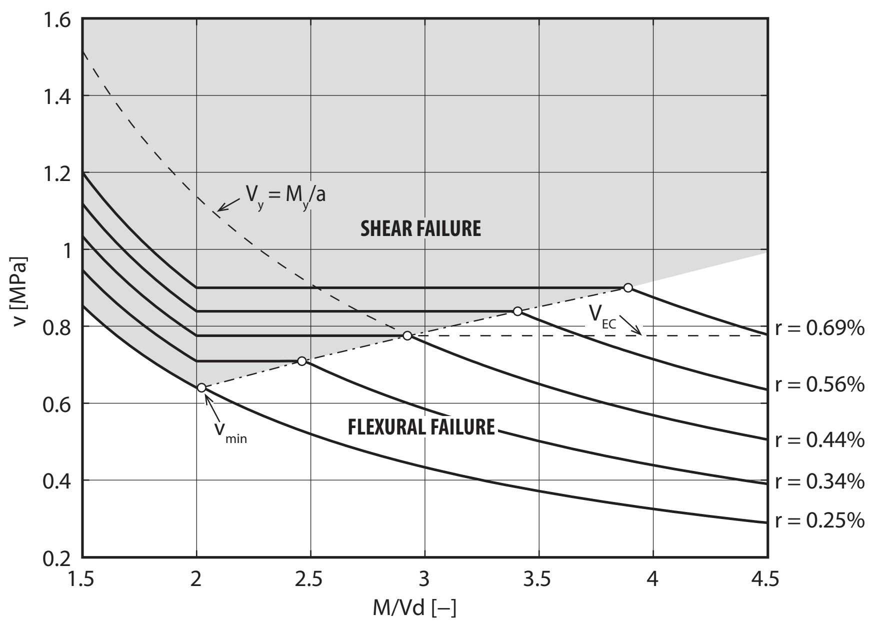

Fig. 1. Determination of $v_{\min}$ according to Eurocode.

The capacity of a beam with given reinforcement ratio is defined by the lower value from Eq.(3) and Eq.(4), see Fig. 1. The intersection of the two curves is the boundary between the two failure modes, the shear stress at that point is defined as v_{bd}. The joint of Eq.(3) and Eq.(4) yields the expression of v_{bd}:

$$v_{bd} = 10.54 \left(C_{Rd,c} k_h \right)^{3/2} \left(\frac{2}{\beta} \frac{f_{ck}}{f_{yk}} \right)^{1/2} \quad (5)$$

The value of v_{bd} decreases with the reduction of the reinforcement ratio ρ_l of the member. That is due to the fact that the value of $V_{Rd,c}$ is independent to the bending moment distribution according to Eurocode. Consequently, it is Eq.(3) defines the value of *a* and ρ_l when v_{bd} is determined. When the loading point is very close to the support, the shear capacity increases because that the compressive strut is less affected by the inclined crack. That defines the minimum value of v_{bd}, which is $v_{\min}$ according to Eurocode. During the determination of Eq(2) from Eq(5), several simplifications have been introduced, which includes that the yielding strength of the rebar is f_{yk} = 500 MPa, arch action starts when a/d = 2.5, following (Kani, 1964). The choices of these two values are supposed to be conservative, which should lead to a conservative $v_{\min}$ expression in general.

Here, the choice of 5% fractal characteristic yielding strength f_{yk} = 500 MPa can be discussed. Since a lower yielding strength in Eq(5) actually results in a higher v_{bd}. For more conservative estimation, the 95% fractal characteristic strength should be used. On the other hand, when reinforcement with lower yielding strength is used in the structure, the actual yielding strength should be taken into account. For that reason, the recommendation on assessment of existing bridges (RBK) given by the Dutch ministry of infrastructure and environment (Rijkswaterstaat, 2013) suggested a modified formula based in Eq(5):

$$v_{\min,RBK} = 0.83 k_h^{3/2} (f_c / f_y)^{1/2} \tag{6}$$

The expression takes the actual yielding strength of the reinforcement into account, which increases the expected $v_{\min}$ of many existing concrete bridges reinforced by lower yielding strength reinforcement.

2.2 Model Code 2010

In Model Code 2010 (fib, 2012), a very different approach regarding the shear capacity expression is proposed. The shear capacity of concrete members without shear reinforcement is generally expressed by

$$V_{Rd,c} = k_v f_{ck}^{1/2} z b_w \tag{7}$$

In which the value of k_v can be estimated with two levels of approximation, depending on the requirement of accuracy. Higher level of approximation also requires more efforts in calculation. At level II approximation, k_v is

$$k_v = \frac{0.4}{1+1500\varepsilon_x} \frac{1300}{1000 + k_{dg} z} \tag{8}$$

Where ε_x is the strain at the mid depth of the cross section. k_{dg} is the factor related to aggregate size. At level I approximation, the value of k_v is simplified as

$$k_v = \frac{180}{1000 + 1.25z} \tag{9}$$

The simplification is done by assuming $\varepsilon_x = f_{yk}/2E_s$, and the maximum aggregate size about 10 mm. The value of f_{yk} is set to be 500 MPa in Model Code. Similar to the Eurocode simplification, the choice of f_{yk} = 500 MPa can be discussed. Since ε_x is the strain at the mid depth of the cross section and a linear strain distribution is accepted in Model Code. In the level I approximation, it actually assumes that the strain of the longitudinal reinforcement reaches the yielding strain. Thus, Eq(9) is in fact the equivalent of v_{bd} as in Eurocode. Based on Eq(8), the shear capacity of a structural member is related to the average strain of reinforcing bar. Therefore the shear capacity of the member is dependent to the bending moment, but on the other hand, the value of v_{bd} becomes independent to the reinforcement ratio or the loading conditions. Thus, v_{bd} is equivalent to $v_{\min}$. In this paper, v_{bd} is used to express as a general term of the shear stress at the boundary of bending and shear failure. The expression of which can be derived by combining Eq(7) and (9):

$$v_{\min,mc} = v_{bd,mc} = \frac{162}{1000 + 1.125d} f_{ck}^{1/2} \tag{10}$$

2.3 Discussion

Comparing Eq(10) with Eq(2) or the more general expression Eq(5) in EC, the influence of concrete strength is quite comparable. However, the size effect relationship and the effect of shear slenderness ratio *a/d* are very different. As explained above, one of the origins of the differences is related to how the two approaches deal with the shear capacity. Eurocode considers the shear capacity as a cross sectional property, thus it is content over the whole span, and is independent to the loading condition. While, in Model Code, the shear capacity of a section is related to the average steel strain ε_x, and thus related to the bending moment of the cross section. As a result, the shear capacity is dependent on the loading condition. Besides, the size effect relationship of Eurocode is a regression expression, and the one in Model Code is related to the dimension of the critical crack (the terms ε_x and z). The differences of the two approaches become pronounced when the value of v_{bd} is checked regarding specimens with the same concrete quality and height but difference reinforcement ratio. By definition, the average steel strain is constant at $\varepsilon_s = f_{ym}/E_s$, when the boundary between bending and shear failure is reached. Thus, by checking the differences of v_{bd} for beams with different ρ_l, the difference between the two codes can be verified.

3. Experimental program

3.1 Introduction

Since the study of the two codes shows different results regarding the boundary shear stress v_{bd}. Experimental verification turns out to be essential, to accurately assess the value of v_{bd}. Thus, an experimental program aiming at experimentally obtaining the value of v_{bd} and eventually $v_{\min}$ has been carried out at Delft University of Technology. In total 61 tests were carried out on 16 reinforced concrete beam specimens without shear reinforcement in the research program. A detailed description of the test program and the test results is given in (Yang, 2015). All the specimens are 5000 mm long and 300 mm wide prismatic beams. Most of the specimens have the same longitudinal reinforcement in both tensile and compressive zone. The concrete cover is 25 mm. Very limited stirrups are placed in the center and the ends of the specimens to guarantee the anchorage and support the reinforcement cage. No shear reinforcement is present in the tested span.

Three test series are planned in the research program. Within each test series, several specimens with different reinforcement configurations are casted. The differences between the specimens are listed in Table 1. Test series A is the reference series, it was designed to validate the assumption on the relationship between v_{bd} and reinforcement ratio ρ_l. Test series B is included to investigate the effect of beam depth. And series C is defined to assess the effect of concrete strength.

Table 1. Differences between the test specimens, HC: High strength concrete; LC: low strength concrete.

Specimen	***h*** [mm]	***d* [mm]**	***Concrete***	***Rebar***	ρ_l	***number***
A12	300	265	HC	3Ø20	1.16%	3
A90	300	265	HC	1Ø12+2Ø20	0.90%	2
A75	300	267	HC	3Ø16	0.74%	2
A60	300	267	HC	1Ø10+2Ø16	0.58%	2
B70	500	465	HC	3Ø20	0.67%	2
B50	500	465	HC	1Ø16+2Ø20	0.58%	2
C90	300	265	LC	1Ø12+2Ø20	0.90%	1
C75	300	267	LC	3Ø16	0.74%	1
C60	300	269	LC	3Ø12	0.42%	1

Within each test series, the reinforcement ratio of the specimens varies. For each reinforcement ratio, several tests with different shear slenderness ratio were executed. The names of the tests are defined to indicate the basic variables of the tests. An example is given in Fig. 2, where the meaning of each part of the name is given as well.

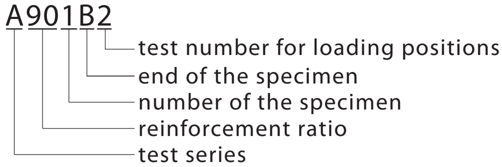

Fig. 2. Definition of test numbers

3.2 Materials

Two types of concrete mixture have been employed in the research. Since the effect of concrete strength to the shear capacity is limited, to investigate the influence of the concrete strength, a large difference of concrete strength was applied between test series A/B and C. Compressive tests and splitting tensile tests were carried out on 150 mm cubes. A summary of the concrete strengths of both mixtures is given in Table 2. Other than the splitting tensile strength of the high strength concrete, the COV of the tests is limited. The equivalent cylinder strengths of the two mixtures are $f_{cm,1}$ = 62.9 MPa, $f_{cm,2}$ = 22.3 MPa respectively, based on the conversion factor of 0.82 according to the recommendation on assessment of existing bridges (RBK) given by the ministry of infrastructure and environment (Rijkswaterstaat, 2013). The densities of the two concrete mixtures are: 2337.8 kg/m^3 (low strength concrete) and 2429.6 kN/m^3 (high strength concrete).

The normal ribbed bar with f_{yk} = 500 MPa is employed in the test. The yield strength from the back analysis of M_y in the tests shows that the mean yielding strength f_{ym} is about 550 MPa.

Table 2. Concrete strengths of both concrete mixtures

Concrete type	***High strength***		***Low strength***	
	cube strength	***COV***	***cube strength***	***COV***
Compressive strength	76.67 MPa	4.34%	27.2 MPa	3.49%
Tensile strength	5.9 MPa	16.87%	2.9 MPa	4.19%

3.3 Test Program

The target of the experimental research is to search the boundary shear stress v_{bd}, under which the flexural shear failure occurs on the specimen while the strain of the longitudinal reinforcement approaches or reaches the yielding strain $\varepsilon_y = f_{ym}/E_s$. Within the test program, all the specimens were loaded by three point bending. The configurations of the tests are indicated in Fig. 3. In order to find the critical point, a special testing program was introduced. It is summarized in Fig. 3 as well. The test series started from placing the point load at a location relatively further away from the support so that flexural failure is obtained, which is defined by the yielding of the longitudinal reinforcement. The specimen was unloaded afterwards, and the point load was moved to a loading position closer to the support. The same loading procedure was repeated until flexural shear failure was obtained. The distance between two loading points was usually constant. After the shear failure was reached, the integrity of the shear span of the specimen was affected. Additional tests have to be carried out on specimens with different supporting conditions. The additional tests were executed with the point load located between the last flexural failure and the first shear failure, so that a refined critical position is obtained. The test procedure assumes that when the supporting condition does not change, and there is no clear flexural shear crack, the crack pattern of the previous test does not affect the shear capacity of the next test. Nevertheless, when it is possible an additional test with the same boundary conditions was carried out on uncracked span, so that the effect of the existing cracks can be evaluated.

Considering that the length of each specimen is 8 m while the actual testing span is 5 m, it was possible to have at most 2.5 m cantilever during the test. The uncracked cantilever is placed at the far end to the loading point. That guarantees that after the critical shear span is damaged by the shear failure, the cantilever at the far end can still be tested without cracks. In addition to that, for beams with 300 mm height, relatively smaller shear span is needed to reach the same *a/d*. For these specimens, an additional cantilever of 1.5 m was firstly reserved at the close end to the loading point. After the first shear failure, the damaged span was strengthened, and the support was moved for 1 m towards the close end. In Fig. 3, a flowchart is presented to summarize the test program.

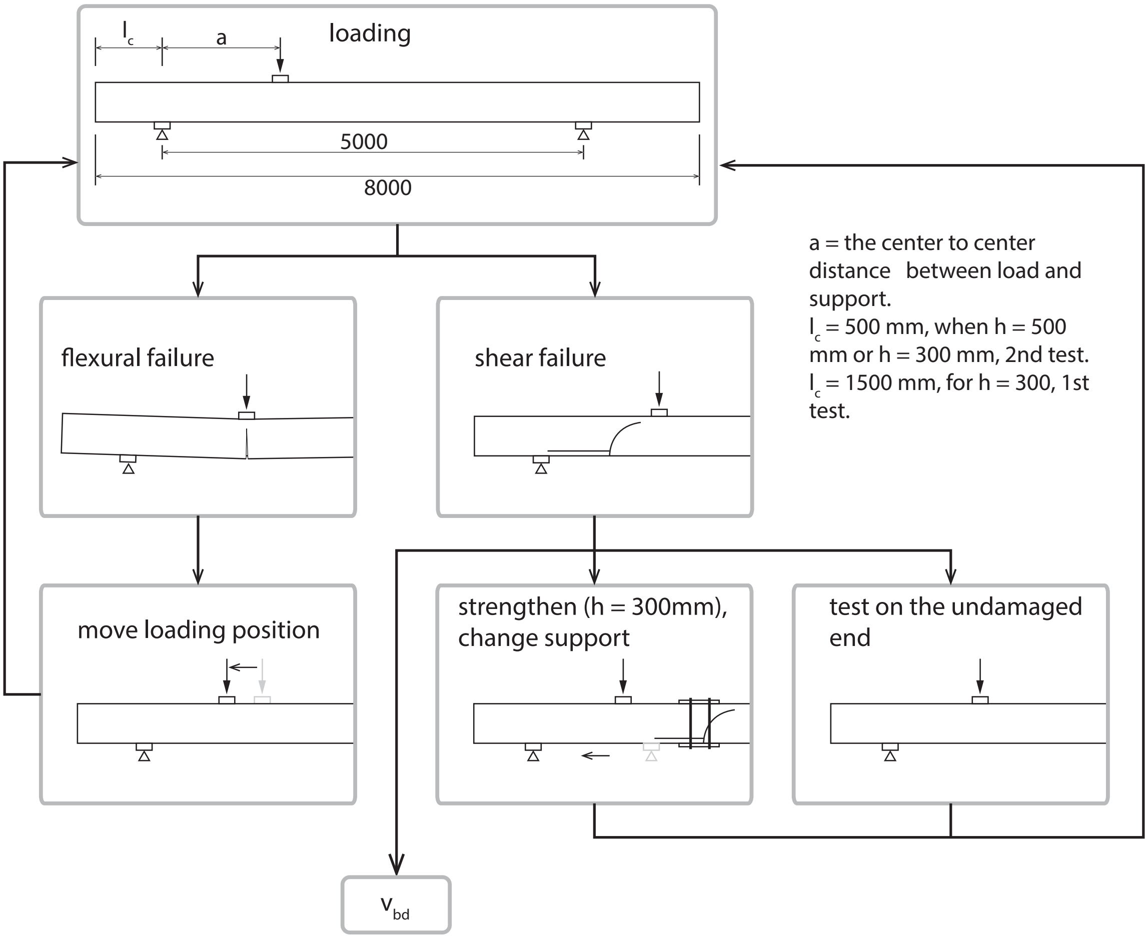

Fig. 3. Flow chat of the test program for a given beam configuration.

4. Test results and discussions

4.1 Typical failure modes

From the total 61 tests, three typical failure modes can be distinguished. They are flexural failure, (flexural) shear failure and mixed failure. The definition of the three failure modes are given as follows:

The so called flexural failure is defined by the yielding of the longitudinal reinforcement in this test program. At the defined failure, a teeth shape yielding plateau is observed in the load – deflection relationship. Additional deflection will not result in other failure modes. Although the ultimate capacity of such specimen is higher, the tests usually stopped after an additional deflection of 10 mm so that the integrity of the specimen is kept for further testing.

The shear failure here refers to the flexural shear failure. As suggested by (Yang, 2014), in such failure mode, the specimen loses its capacity at the moment when unstable secondary cracks develop along the tensile reinforcement and the compressive zone. When such failure occurs, no additional test was carried out in the same span. The position of the critical shear crack was measured at both the middle of the beam height x_{crm} and at the level of the longitudinal reinforcement x_{crb}. x_{crm} is used to calculate the shear force resulted by the self-weight: $V_u = P_u(l - a)/l + q_g(l/2 - x_{crm})$. The shear forces are converted to equivalent average cross sectional shear stress $\tau_u = V_u/bd$.

The mixed mode is, on the other hand a failure mode between flexural failure and shear failure. With this failure mode, the yielding of the tensile reinforcement usually occurred first, meanwhile, the secondary cracks initiated at the tips of one of the flexural cracks further away from the loading point. Further increase of the deflection or load level resulted in the unstable development of the secondary crack branches (shear crack), which lead to the loss of the global capacity. Such failure mode typically occurs when the position of the point load is between that of a flexural failure test and a shear failure test. It can be considered as representative of the boundary between the two failure modes. In the test program, this failure mode is considered as shear failure as well. However, the shear stress v_u measured from tests with this failure mode is considered as v_{bd} directly.

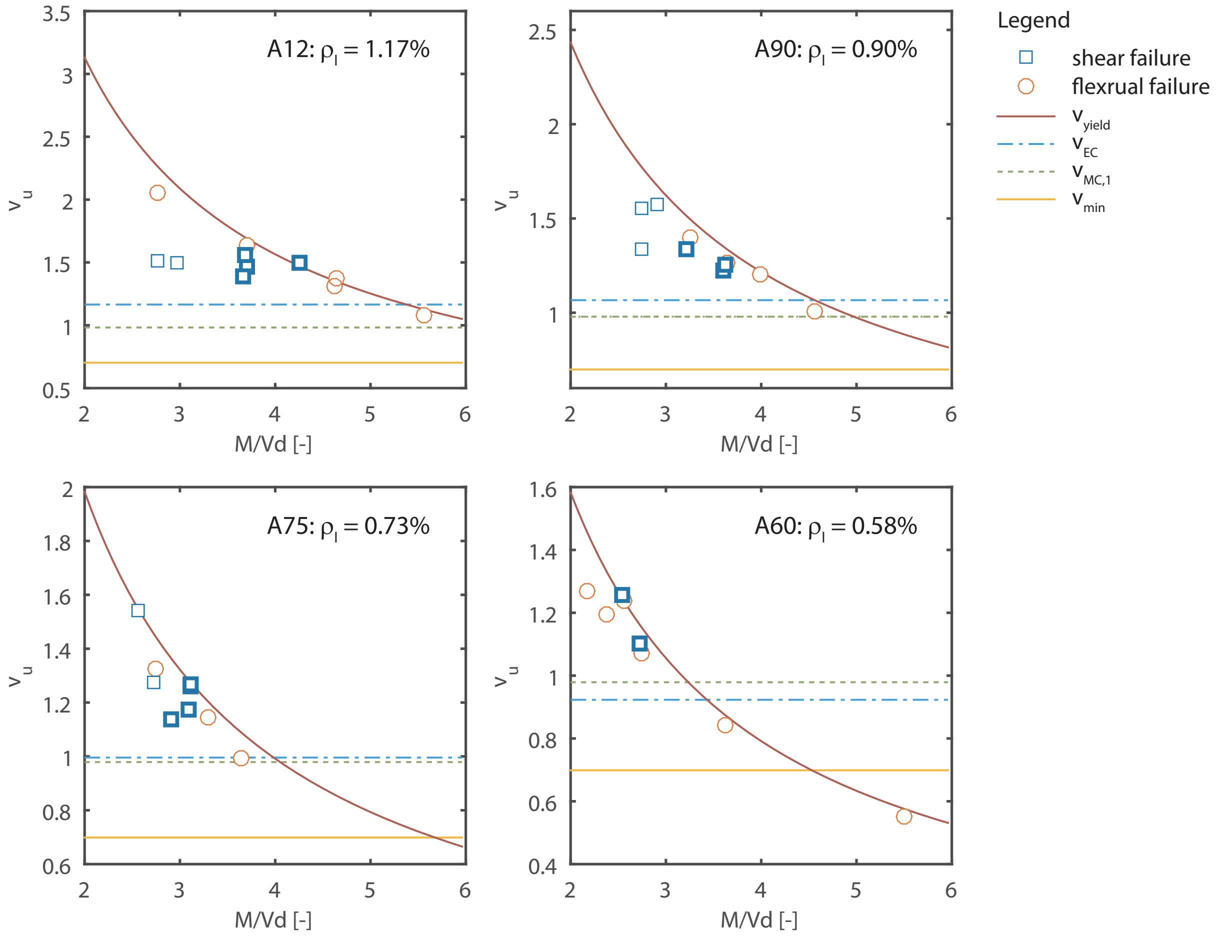

Fig. 4. Determination of v_{bd} based on the results of test series A.

4.2 Determination of v_{bd} and v_{min}

As the major target in this experimental research, the value of v_{bd} is determined from the test results. As an example, the determination process of v_{bd} in the test series A is illustrated in Fig. 4. Where, the maximum shear stress v_u of specimens with four different ρl are plotted against the *M/Vd* of the span. The circles represent flexural failure and the squares represent shear failure. As expected, the transition between the two failure modes is strongly related to *M/Vd* (or shear slenderness *a/d*) of the span. The v_u of the shear failures with the largest *M/Vd* are considered as v_{bd}. Although the definition of v_{bd} is the minimum shear stress that can cause shear failure for beams with given ρ_l, it is apparently not appropriate to only take the minimum shear stress as the experimental determined v_{bd} or $v_{\min}$. In order to take into account the scatter of the tests, all the v_u from shear failure tests that is close to the boundary *M/Vd* are all considered as v_{bd}. Fig. 4 shows that with the reduction of ρ_l, the boundary *M/Vd* reduces. When ρ_l reduced to 0.58%, it was rather difficult to generate shear failure anymore because of the low *M/Vd*. In all the tests with that reinforcement ratio, yielding of longitudinal rebars were observed. The only two shear failures were obtained by further loading of the specimen after the yielding of the longitudinal rebars. In order to generate sufficient shear force in the critical span in these tests, the loading point was located at a relatively small distance *a* from the support. The consequence is that more shear force was carried by uncracked compressive zone. That effect increased the shear capacity.

For the tests with flexural failure, at failure, the yielding moment M_y determined from is translated by average shear stress v_{yield} in Fig. 4, with the expression $v_{yield} = M_y/(M/V)/b/d$. The comparison between test results and the theoretical v_{yield} shows that the initial assumption of f_{ym} = 550 MPa turns out to be a reasonable estimation.

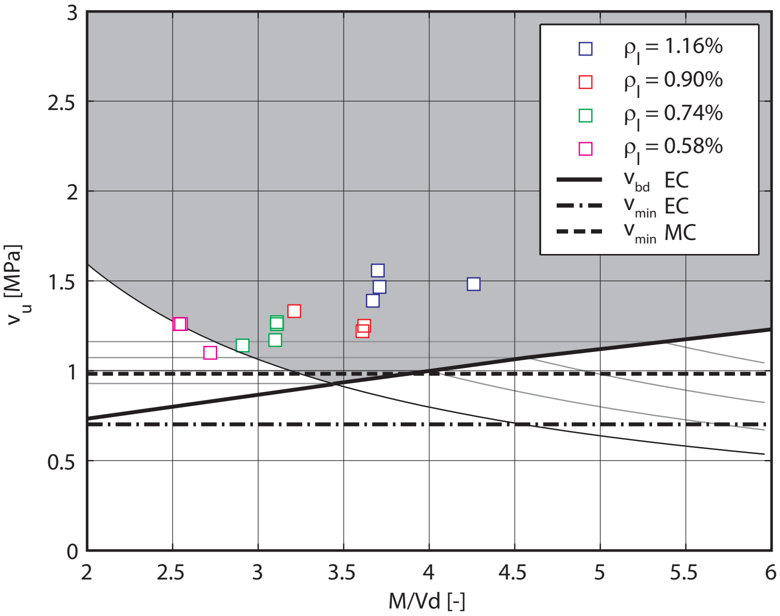

Fig. 5. Determined $v_{\min}$ through test series A in comparison with code previsions.

In Fig. 5, v_{bd} derived from Fig. 4 are plot together, which gives the relationship between v_{bd} and *M/Vd*. As comparison, the value of $v_{\min}$ from Eq (2), the full expression of v_{bd} based on Eurocode according to Eq (5) and the expression based on MC2010 level I approximation given by Eq. (10) are plotted in the same graph. The mean material properties are employed in the formulas. In the case of Eq (5), the value of $C_{Rd,c}$ is chosen to be 0.15 as discussed in (Yang et al., 2011). Similar procedure yields the values of v_{bd} in test series B and C. They are plotted in Fig. 6.

4.3 Discussions

Fig. 5 and Fig. 6 clearly show that in all test series, the present code provisions give conservative prediction. As one of the point of discussion, weather the value of v_{bd} is a constant or it varied with ρ_l and *M/Vd* is of interest. The comparison of the three test series shows very different conclusions. In test series A and C, it is clear that the determined v_{bd} reduces with the reduction of ρ_l. The inclination compares well with Eurocode. However, the test series B shows very different results. The v_{bd} from specimens with higher depth seems to increase with the reduction of ρ_l. Before more experimental proofs are available, it is hard to have solid conclusion at the moment. Nevertheless, since most of the measured v_{bd} are determined from specimens with very low reinforcement ratio whose shear capacity is already very close to the theoretical v_{min}, the average value of the v_{bd} given in Fig. 5 and Fig. 6 can be considered as v_{min} from tests. The resultant v_{min} of the three test series are summarized in Table 3.

The comparison of tested v_{min} and code previsions in Fig. 5, Fig. 6 and Table 3 shows that, the expression of v_{min} given by Eq (2) seems to underestimate v_{min}. Although the intention of Eq (2) is to give a lower bound of the shear capacity, it is not economy when the estimated v_{min} from Eq (2) is significantly lower than the measured on even when the mean values of the material propertied are used in the expression. The difference is as high as a factor of 2.0 in the case of test series C. The adjusted expression Eq(6) by Rijkswaterstaat improves the accuracy, nevertheless, the differences are still significant.

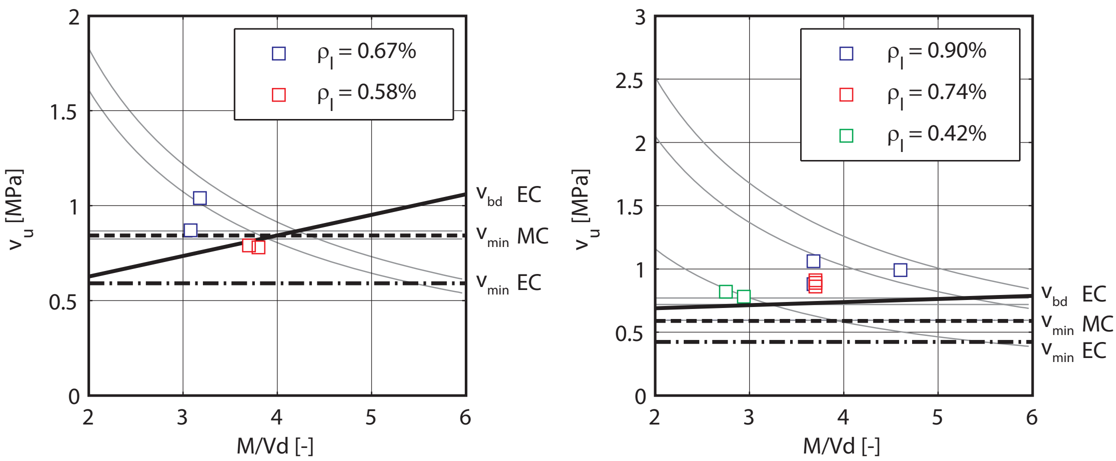

Fig. 6. v_{bd} determined from test series B and C in comparison with code previsions (Eurocode, and Model Code).

Table 3. Summary of the values of v_{min} and model calculations, unit: MPa.

Test series	$v_{min,exp}$	$v_{min,EC}$	*ratio*	$v_{min,RBK}$	*ratio*	v_{MC}	*ratio*
A	1.21	0.70	1.73	0.74	1.64	0.98	1.23
B	0.78	0.59	1.32	0.62	1.26	0.84	0.93
C	0.87	0.42	2.07	0.44	1.98	0.59	1.47

On the other hand, the difference between the MC2010 level I approximation and test results turns out to be much smaller. It implies that the model code approach seems to be more realistic. Similarly, the approach of combining $v_{Rd,c}$ and v_{yield} gives rather close estimation to v_{min}. However, the difficulty of employing Eq (5) directly is that it requires a proper estimation of minimum *M/Vd* at the boundary. The MC2010 approach seems to be a reasonable alternative of the original v_{min} expression in Eurocode.

In addition, regarding the basic parameters involved in the determination of v_{min}, it turns out that both models are able to evaluate the effect of concrete strength accurately. However, regarding the size effect, both codes do not represent the test results properly. Comparing series B and A or C, the ratios between test results and code prediction are clearly dropped with the increase of specimen depth. Though additional test proof is still necessary, if this observation is extended towards even deeper specimens, the risk of structures with very large depth could become rather high.

5. Conclusions

This paper presents an experimental research program aiming at assessing the current code provisions of v_{min}. A large test series was carried out, by varying the reinforcement ratio and loading conditions, the minimum average shear stress that can cause shear failure of a specimen with different depth, concrete strength is determined. The three test series cover two types of concrete mixtures and a moderate structure depth range.

In addition, two different approaches regarding the shear capacity are discussed. They are based on the Eurocode and ModelCode 2010. They are compared with the test results. One may conclude that without further adjustment, the Eurocode v_{min} expression is not economic for design practice. It significantly underestimates the value of v_{min} especially for structures with smaller depths. The adjusted expression by Rijkswaterstaat and the Model Code 2010 level I approximation improves the accuracy, nevertheless, the differences are still significant. Besides, both expressions turn out to underestimates the size effect when the specimen size is increased. Although not in the presented test series, further increase of the specimen size might cause overestimation of the structural shear capacity.

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