

Coping with the uncertainties in the climate change adaptation of river dikes using risk-aversion economic optimization

L. Wang¹, P.H.A.J.M Van Gelder¹, J.K.Vrijling¹, R. Ranasinghe^{1,2}, S. Maskey²

1 Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands

2 UNESCO-IHE, Delft, the Netherlands

Abstract: To guarantee a safe flood defence in a changing environment, the adaptation to climate change needs to be considered in the design of river dikes. However, the large uncertainty in the projections of future climate leads to varied estimations of future flood probability. How to cope with the uncertainties in future flood probability under climate change is an inevitable question in the adaptation. In this paper, the uncertainty introduced by climate projections was integrated into the 'expected predictive flood probability', and the risk-aversion attitude was introduced in the adaptation of river dikes. In detail, the uncertain effect of climate change on flood probability was represented by the uncertainty in the parameters of the probabilistic model. This parameter uncertainty was estimated based on the outputs from the GCMs participated in IPCC AR4. The parameter uncertainty estimated from different GCMs under selected scenarios was integrated into the expected predictive probability of flooding, which was used in the risk-aversion economic optimization. Different optimal results were obtained based on varied values of the risk-aversion index. The case of Bengbu Dike in China was studied as an example using the proposed approach. The results show that the uncertain effect of climate change causes an increase of optimal dike height but a decrease of the optimal safety level. The proposed approach enables decision makers to cope with the uncertain effects of climate change by adjusting their risk-aversion attitude.

Key words: Climate change, uncertainty, risk-aversion optimisation

1. INTRODUCTION

There has been a growing concern on the adaptation to the impact of climate change due to the recent overwhelming consensus on global warming. Since the occurrence of floods is highly susceptible to climatic condition, much attention should be put on the adaptation to the uncertain effect of climate change in the field of flood prevention.

Since van Danzig (1956) originally applied the method of economic optimization in the decision problem for flood prevention, risk-based design has become a useful tool in the design of flood defences in the Netherlands. The environmental condition is usually assumed as unchanged when this method is used. However, climate change can influence the probability of future floods and hence flood risk; furthermore, the projections of future climate are very uncertain. In other words, climate change introduces uncertainty to the cost of flood risk by having an uncertain effect on flood probability. In the usual application of risk-based design, the uncertainty of costs (the sum of investment and risk) is not considered because only the expectation of total costs is optimized (i.e., minimized). In order to integrate the uncertainty of costs, the usual method can be updated by minimizing a risk-aversion cost function (Vrijling et al. 1995): i.e., the sum of expectation of total cost and the product of the risk aversion index and the standard deviation of the total cost. This risk-aversion cost function was initially proposed as a simple measure of social risk based on loss of life; it can also be used in economic decision making (Jonkman et al. 2003). However, only a few applications of this risk-aversion function consider the economic optimization for the design of dikes (Kuijper and Kallen 2012; Slijkhuis et al. 1997; van Gelder and Vrijling 1998; Vrijling and van Gelder 1999).

On the other hand, there has been an abundance of research on how climate change will influence extreme floods. The projections from Global Climate Models (GCMs) have been used in

many papers in this field, and the results all show that climate change will introduce significant uncertainty. However, most of these papers focus on analysing the uncertainty in hydrological responses, and the approach of including this uncertainty in the design of flood defences has not been well developed.

In order to fill this gap, this paper attempts to present the uncertain effect of climate change on flood probability and to include this uncertainty in the risk-based design of river dikes. The result will also show how much the optimal dike height is influenced by this kind of uncertainty. To achieve this, the outputs from GCMs and a hydrological model were used to provide the projections of future river stages. A method was proposed to project the probabilistic models of the river stage in the future. The uncertain effect of climate change was represented by the uncertainty of the parameters of probabilistic model. The risk-aversion cost function was minimized to obtain the optimal dike height. Following this introduction, the approach of this research is introduced in Section 2, and the results are presented in Section 3. Discussion and conclusions are provided in the final section.

2. METHODS

The classic risk-based design and the method of risk-aversion optimization are firstly introduced in this section. Then, the section shows how to include the uncertain effect of climate change in the risk-aversion optimization. The rest of the section briefly introduces the method of projecting the probabilistic model of river stage influenced by climate change.

2.1 Risk-aversion economic optimization in the design of dikes

The economic optimization of dike heightening was originally proposed by van Danzig (1956). The total cost consists of flood risk and the investment of construction. Flood risk is calculated as the product of annual flood probability and potential damage, and the cost of investment is determined by the initial costs and the variable cost. The total cost can be written as

$$C = I_0 + I_1 \cdot X + p \cdot S \cdot disc(r) \quad (1)$$

where I_0 is the initial investment, I_1 is the variable investment, X is the height which the dike is raised by, p is annual flood probability, S is damage (i.e., the expected economic loss), and $disc(r)$ is the discounting function with the discount rate r .

In this function, I_0 , I_1 , S and r are determined as constant based on the social information. p , the occurrence probability of a flood per year, is calculated by an exponential function given the dike height. Thus, the total cost can be seen as a function of increased dike height X . The optimal dike height is determined by the value of X that minimizes the total cost. No uncertainty of the component costs is considered.

Vrijling (1995) proposed a linear function as a simple risk-averse measure of the societal risk. It was originally applied based on the loss of life. Slijkhuis and van Gelder (1997) used this risk-aversion function in the economic optimization of design of sea dikes and write the function as

$$Min[\mu(C) + k \cdot \sigma(C)] \quad (2)$$

Where $\mu(C)$ is the expectation of total cost, $\sigma(C)$ is standard deviation of total cost, and k is the index of risk aversion. The selection of index of risk aversion reflects the risk-averse attitude of the decision maker. The larger the index is, the more risk-averse the decision making is.

2.2 Including the uncertainty of probability in the economic optimization

In the common economic optimization, it is assumed that inundation occurs when water level exceeds the height of the dike. Thus, flood probability is assumed to be equal to the exceedance probability of water level, and it can be modelled by GEV probabilistic distribution

$$p = 1 - e^{-\left[1 + c\left(\frac{H-A}{B}\right)\right]^{-\frac{1}{c}}} \quad (3)$$

where, p can be considered an exponential function of shift parameter A , scale parameter B and shape parameter C at a given river stage H .

The occurrence of a flood per year can be treated as a Bernoulli trial Y with occurrence probability p . In a stationary environment, p is considered constant for a certain dike height. However, changes in climate introduce uncertainty to the occurrence probability of floods, because how climate will change is highly uncertain and the changing climate has significant effect on floods. That is to say, a certain flood probability cannot be provided given a specific dike height. Expressed mathematically, in a changing climate, the occurrence of a flood per year is still a Bernoulli trial Y with occurrence probability P , but P is an uncertain value rather than a constant. The uncertainty of P reflects the uncertain effect of climate change.

Assuming P is a random variable with the expectation $\mu(P)$ and the standard deviation $\sigma(P)$, the expectation and variance of the Bernoulli variable Y can be given by

$$E(Y) = E(E(Y | P)) = \mu(P) \quad (4)$$

$$D(Y) = D_p[E(Y | P)] + E_p[D(Y | P)] = \mu(P) \cdot [1 - \mu(P)] \quad (5)$$

The above formulas show that the expectation and variance of Y are only determined by the expectation of P . Based on the above formulas, the expectation and variance of total cost can be given by

$$\mu(C) = I_0 + I_1 \cdot X + \frac{\mu(P) \cdot S}{r} \quad (6)$$

$$\sigma^2(C) = \frac{\mu(P) \cdot [1 - \mu(P)] \cdot S^2}{(1+r)^2 - 1} \quad (7)$$

where $\mu(C)$ is the mean of total cost, $\sigma(C)$ is the standard deviation of total cost, S is loss, and r is discount rate. The cost is discounted on a very long time horizon.

Apart from the uncertainty introduced by climate change, the uncertainty of the costs of investment and damage can also be considered. This can be achieved by assuming the related parameters in Formula (1) as normal distributed variables (Slijkhuis et al. 1997), and the expectation and standard deviation of total cost can be calculated as

$$\mu(C) = \mu(I_0) + \mu(I_1) \cdot X + \frac{\mu(P) \cdot \mu(S)}{r} \quad (8)$$

$$\sigma^2(C) = \sigma^2(I_0) + \sigma^2(I_1) \cdot X^2 + \frac{\mu(P) \cdot [\sigma^2(S) + (1 - \mu(P)) \cdot \mu^2(S)]}{(1+r)^2 - 1} \quad (9)$$

where $\mu(I_0)$ is the mean of initial investment, $\sigma(I_0)$ is the standard deviation of initial investment, $\mu(I)$ is the mean of variable investment, $\sigma(I)$ is the standard deviation of variable investment, $\mu(S)$ is the mean of loss, $\sigma(S)$ is the standard deviation of loss,

Assuming the parameters A , B and C in GEV probabilistic model (Formula 3) are independent normally distributed variables, the expectation of P can be calculated by

$$\begin{aligned} \mu(P) &= \int_{-\infty}^{\infty} \int_0^{\infty} \int_{C_{limit}}^{\infty} (1 - e^{-\left[1 + C \left(\frac{H-A}{B}\right)\right]^{\frac{1}{c}}}) f(C) f(B) f(A) dC dB dA, & H > A & \quad C_{limit} = \frac{B}{A-H} \\ \mu(P) &= \int_{-\infty}^{\infty} \int_0^{C_{limit}} \int_{-\infty}^{\infty} (1 - e^{-\left[1 + C \left(\frac{H-A}{B}\right)\right]^{\frac{1}{c}}}) f(C) f(B) f(A) dC dB dA, & H < A & \quad C_{limit} = \frac{B}{A-H} \\ \mu(P) &= \int_{-\infty}^{\infty} \int_0^{\infty} \int_{-\infty}^{\infty} (1 - e^{-\left[1 + C \left(\frac{H-A}{B}\right)\right]^{\frac{1}{c}}}) f(C) f(B) f(A) dC dB dA, & H = A & \end{aligned} \quad (10)$$

where $f(A)$, $f(B)$ and $f(C)$ are the density functions of A , B and C .

The calculation of $\mu(P)$ is similar with that of van Gelder's method (2000) which copes with the statistical uncertainty caused by the shortage of information. In his method, the statistical uncertainty in the parameters of probabilistic model was integrated out to determine the 'predictive' distribution in a stationary environment. By doing this, it is avoided that 'using point estimators for uncertain parameters underestimates the variance in the random variable'.

In contrast to van Gelder's method, the proposed approach aims at integrating the uncertain effect of climate change. Since historical records cannot represent the status of future climate, the uncertainty of parameters should be estimated based on the projections of future climate rather than from historical observation. The projections from GCMs will be used to provide the parameter uncertainty, and this will be described in Section 2.3.

2.3 Quantification of the uncertainty introduced by climate change

To explore the changes in climate, the period of 2070-2099 (2080s) is set as future target period. During this period, climate is described by different possible scenarios, and different Global Climate Models (GCMs) were selected to provide climate projections (precipitation and temperature) for each scenario. These scenarios are IPCC SRES scenarios A1B, A2 and B1 that represent the mid, high and low GHG (greenhouse gas) emissions, respectively. To examine the simulative ability of GCMs, the period of 1961-1990 is set as baseline period that represents the present climate, and each GCM provides retrospection of climate during the baseline period. To transfer the monthly climate series to monthly runoff series, Xin'anjiang monthly hydrological model was used.

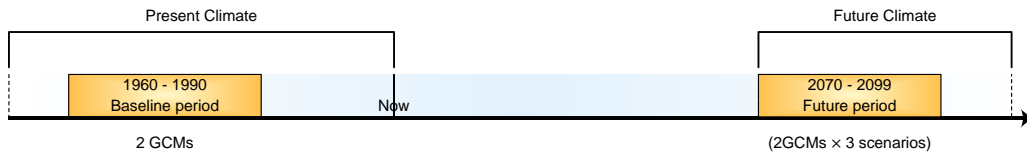


Figure 1 Timeline of GCMs' simulation

The parameter sets A , B and C from the projection of one GCM for one future scenario was obtained by the below steps (Wang et al. 2012):

Step 1 The monthly runoff ($m^3/month$) series was calculated from the hydrological model driven by outputs from one GCM for both the baseline period and the future period, respectively.

Step 2 The curve of exceedance probability of annual maximum monthly runoff was determined for both periods based on the above time series (see Figure 2: the red solid line for the baseline period and the red dashed line for the future period).

Step 3 The ratio between the curve of the future period and that of the baseline period was calculated at each percentile. By doing this, the error introduced by climate model and hydrological model is assumed to be eliminated.

Step 4 Then future probability curve of runoff was projected by scaling the present-status curve (the black line in Figure 2) by the ratio at each percentile.

$$R_{Future,p} = R_{present,p} \cdot ratio_p \quad (12)$$

$$ratio_p = \frac{R_{future-GCMs,p}}{R_{baseline-GCMs,p}} \quad (13)$$

Where, $R_{Future,p}$ is the future annual maximum monthly runoff at the percentile of p ; $R_{present,p}$ is the historical annual maximum monthly runoff at percentile of p under present climate; $ratio_p$ is the relative change between baseline and future period at percentile p ; $R_{baseline-GCMs,p}$ is the annual maximum monthly runoff projected by the GCMs under present climate, $R_{future-GCMs,p}$ is the annual maximum monthly runoff predicted by the GCMs under future climate.

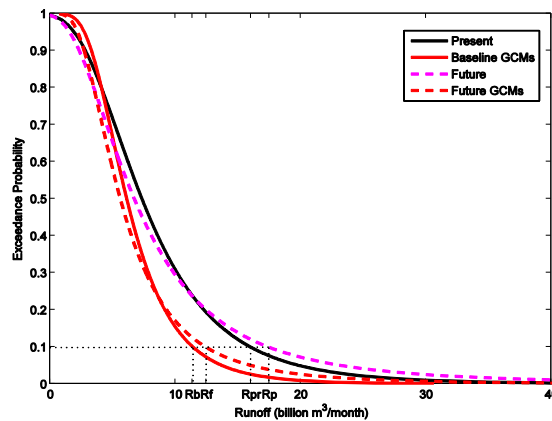


Figure 2 Projection of future probability-runoff curve

Step 5 Then the projected probability curve of runoff was converted to the river stage ~ probability relationship through the derivation process as shown in Figure 2. The relationships in Figure 3 were derived based on observations.

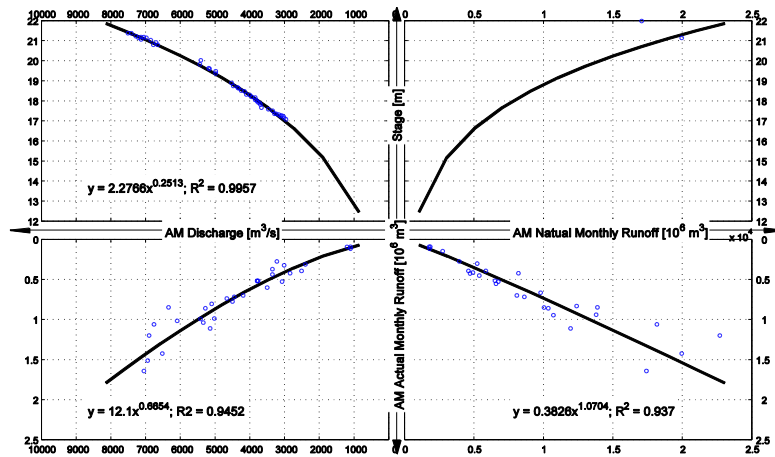


Figure 3 Conversion from monthly runoff to river stage

(From bottom right to top right in the counter clockwise direction)

Step 6 The parameter set of A, B and C was obtained by fitting the river stage ~ probability relationship to the GEV cumulative distribution function.

Repeating above six steps, the parameter sets from the projections of all GCMs for all scenarios can be obtained. Then, the distribution of each parameter can be derived and will be used to determine the expected predictive probability $\mu(P)$ (Formula 10). In the rest of the paper, $\mu(P)$ will be denoted as the expected predictive probability P^* .

3 A CASE STUDY

A case study was carried out over the Old Dike Ring area in Bengbu, China. Old dike ring was built to protect the most urbanized part of Bengbu city on the south bank of Huai River. Old dike ring is a “U”-shape dike with the length of 12.6 km, and it protects the area of 49 km². The mean crest height of the dike is 25.5m, which was designed to with stand the historical flood in 1954 with return period of about 40 years.

3.1 Uncertainty in flood probability under climate change

The projections of future climate from two GCMs, CSIRO-Mk3.0 (CSMK3) and GFD-CM2.0 (GFCM20), were selected based on their performances of simulating the historical climate (Wang et. al. 2011). Both of the GCMs provided the projections from 2070 to 2099 under three GHG emission scenarios.

Since no record of historical river stage was available, through step 5 and 6 in Section 2.3, the parameters of the present-status probabilistic model of river stage were derived based on the historical records of runoff from 1956 to 2000. According to the derived probabilistic model, the return period of river stage in 1954 is 60 years, and the return period of river stage at present dike height is about 1600 years.

Through the six steps in Section 2.3, the parameters of GEV probabilistic distribution of future river stage were obtained, respectively (see Table 1). The spread of the probability curves is significant (see Figure 4). The shape parameters from the GCM-projections have much larger variation (with variance coefficient of 185%) than the scale and shift parameters. GCM-projections predict higher flood probability than the present status except the projection of GFCM20 under low emission scenario (B1). Projections of high emission scenario (A2) present higher probability than the other scenarios. CSMK3 generally projects higher flood probability than GFCM20.

Table 1 Parameter sets of present status and from GCMs' predictions

GCMs-scenarios	Parameters of GEV distribution		
	Shape parameter (C)	Scale parameter (B)	Shift parameter (A)
Present Status	-0.0169	0.9705	18.7568
CSMK3-A1B	0.0124	1.3305	17.475
GFCM20-A1B	0.0564	1.1302	17.301
CSMK3-A2	0.0878	1.7044	16.3553
GFCM20-A2	0.0613	1.4385	18.5171
CSMK3-B1	0.0483	1.4745	16.9612
GFCM20-B1	-0.0767	0.7247	18.9283
Predictive_Mean	0.0316	1.30	17.59
Predictive_S.D	0.0584	0.339	0.97
Variance Coefficient	185%	26.1%	5.5%

Here the projections from both of the models under three scenarios were taken as equal opportunities of future condition. By assuming the parameters obey normal distribution, the

expected predictive flood probability in the future, P^* , was calculated using Formula 10 (blue solid curve in Figure 4). The predictive flood probability, P^* , is considered as the probability that including the uncertainty of the projections of future climate. It will be used in the following analysis of risk-aversion optimisation.

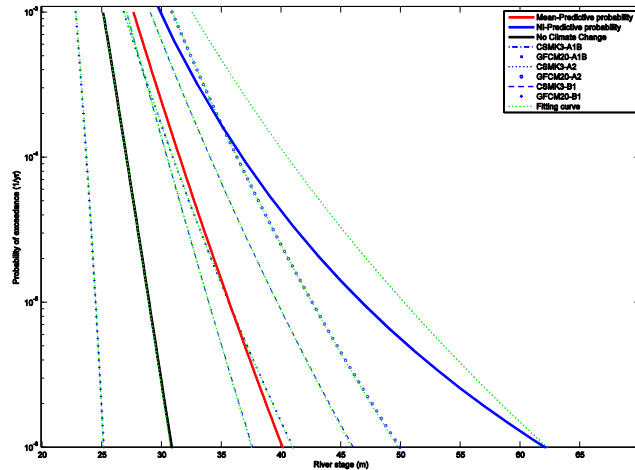


Figure 4 Exceedance probability of river stage under present climate and predictive climate.

3.2 The effect of climate change on risk-aversion optimization

To prevent floods in a changing climate, the assumed measure was to build a flood wall on the top of the dike in the year 2013. It was highly hypothesized that climate will change as soon as the flood wall is built.

The following values were used for the variables in Formula 8 and 9:

Initial investment: $I_0 = 10$ million RMB

Variable investment: $I_x = 11$ million RMB

Damage: $S = 14.8$ billion RMB

Standard deviation of damage: $\sigma(S) = 30\% * S$

Discount rate: $r = 8\%$

To compare the effects of different uncertainties on the optimal result, three situations were analyzed:

- 1) Only uncertainty of damage is included (assumed as 30% of damage)
- 2) Only uncertainty of flood probability caused by climate change is included
- 3) Both uncertainties of damage and climate change are included

The results are shown in Figure 5 and 6:

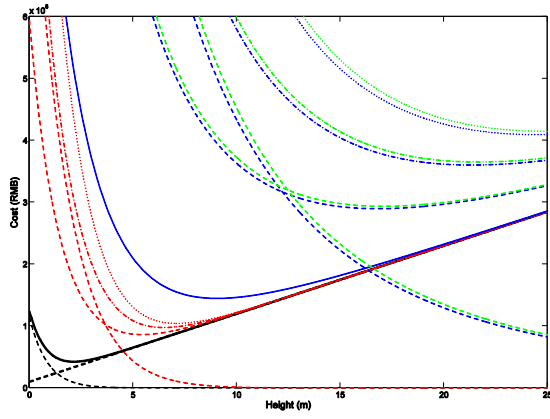


Figure 5 Costs V.S. Height for three situations with different risk-aversion indexes

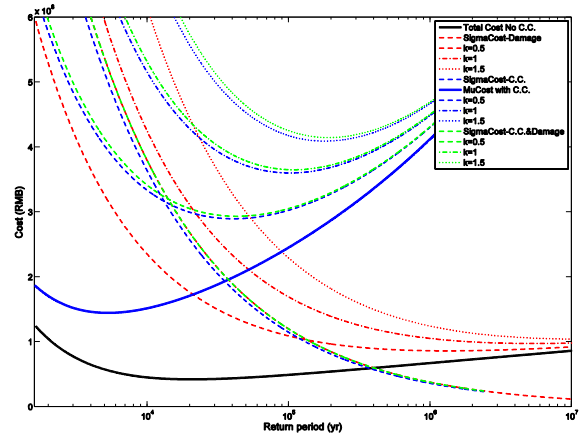


Figure 6 Costs V.S. Return period for three situations with different risk-aversion indexes

Generally speaking, the more uncertainties are included and the larger the risk-aversion index is, the more investment and the higher the dike is. The effect of the uncertainty in climate projections is much larger than that of uncertainty in damage, which can be seen in Figure 5, the curves of Situation 3 (both uncertainties are included) are close to the curves of Situation 2 (uncertainty caused by climate change is included), but deviate significantly from Situation 1 (uncertainty in damage is included). In terms of safety level, the effect of climate change reduces the optimal return period (blue solid curve in Figure 6), which is due to the increase of flood probability (see in Figure 4), while the effect of including the uncertainty in damage leads to the increased optimal safety level (red curves in Figure 6).

The detail optimal results of three situations are summarized in Table 2, in which the columns correspond to the risk-aversion indexes k from 0 to 1.5. Situation 1 with k equals to 0 represents the optimal results under present climate without uncertainty in damage. It can be seen that when risk-aversion index increases, its influence decreases. For instance, for S1, the optimal height increases by 3.3m when k increases from 0 to 0.5, but only by 0.62m when k increases from 1 to 1.5. There are similar phenomena in terms of optimal total cost and safety level. The effect of climate change leads to wild results on optimal height and total cost: increase by 325% and 240%, respectively. Meanwhile, the optimal safety level is reduced significantly from one in 20620 to one in 5263. However, if the decision maker is more risk avertable ($k=0.5$), the risk-aversion optimal safety level is higher than the no risk-aversion situation under present climate.

Table 2 The optimal results of three situations

Optimal height (m)				
k	0	0.5	1	1.5
S1: U in Damage	2.13	5.42	6.48	7.1
S2: U in C.C.	9.06	16.83	21.35	24.55
S3: D & C.C.	9.06	17.07	21.67	24.91
Optimal total cost (million RMB)				
k	0	0.5	1	1.5
S1: U in Damage	42.4	86.3	97.7	104.3
S2: U in C.C.	144.8	289.8	360.4	409.4
S3: D & C.C.	144.8	293.7	365.3	415.0
Optimal safety level (1/yr)				
k	0	0.5	1	1.5
S1: U in Damage	20620	1.3E+06	5.3E+06	1.2E+07
S2: U in C.C.	5263	40000	100000	166667
S3: D & C.C.	5263	43478	111111	200000

3.3 The influence of the pattern of climate change

The underlying assumption of the above results is climate will change right after the construction of the flood wall, which is not realistic. To analyze the influence of timing of climate change, a pattern of gradually climate change and a sudden change in 2070 were also considered.

Pattern 1: Sudden change after heightening (2013) Discounting time horizon: 2014~very long

Pattern 2: Climate gradually changes until 2070 and stays stationary after 2070. Discounting time horizon is from 2014 to far future (2200). The flood probability during 2014~2069 is assumed to increase exponentially.

Pattern 3: Climate stays stationary until the sudden change in 2070. Discounting time horizon: 2014~ very long

Optimal results of the three climate patterns without risk-aversion attitude are summarized in Table 3. If the sudden change of climate occurs right after the construction of heightening, more investment is required to reach the optimal safety standard. For the pattern of gradually change of climate, the required investment of the heightening is much lower than Pattern 1; the optimal safety level is lowered as well. In Pattern 3 where climate will not change until 2070, the optimal dike heightening is further decreased and is close to the results of the no climate change status. However, the optimal safety level of Pattern 3 is dramatically lower than that of present, because the flood probability increases because of the change of climate.

Table3 Optimal results of different climate patterns (no risk-aversion)

Pattern	Added Height (m)	Costs (million RMB)	Return period
No climate change	2.13	42.4	20620
Pattern1: Sudden change in 2013	9.06	144.8	5263
Pattern2: Gradually change until 2070	3.37	62.6	662
Pattern3: Sudden change in 2070	2.34	48.5	422

The optimal dike heightening considering risk-aversion attitude is shown in Table 4. The more risk-aversion attitude is held, the higher the dike should be. The influence of risk-aversion index decreases when its value increases.

Table 4 Optimal added height with different risk-aversion indexes (m)

Pattern	Risk- aversion index: k			
	0	0.5	1	1.5
No climate change	2.13	5.42	6.48	7.1
Pattern1: Sudden change in 2013	9.06	16.83	21.35	24.55
Pattern2: Gradually change until 2070	3.37	6.3	7.48	8.23
Pattern3: Sudden change in 2070	2.34	5.4	6.38	6.93

Figure 7 shows the flood probability over time with risk-aversion levels of 0, 0.5 and 1 for gradually climate change pattern. It can be seen that if the risk-aversion index is taken as 0.5, the optimal safety level can be higher than the present safety level, even when climate change occurs.

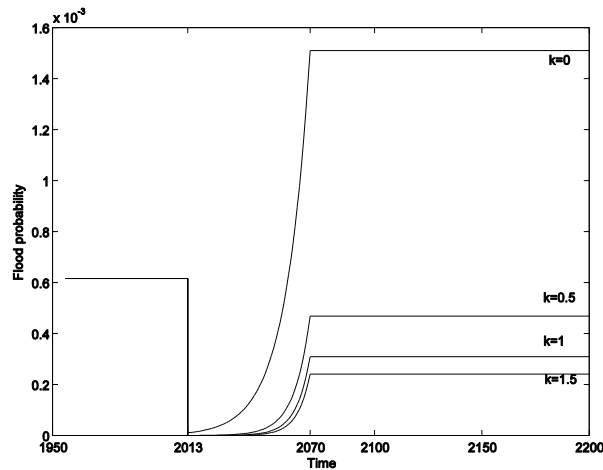


Figure 7 Flood probabilities over time with heightening the dike under Climate Pattern 2

4 CONCLUSION

This paper attempts to answer two questions: How can the uncertain effect of climate change be included in the risk-based design, and how much will it influence the result? To quantify the uncertainty of flood probability caused by climate change, a method was proposed based on the information from the projections of future climate. The underlying assumption was that the probabilistic distribution of future river stage will change, and climate models are able to project the source of this change through the projections of climate. Furthermore, the error introduced by hydrological model was expected to be eliminated (or reduced) by this method. Then, a method of risk-aversion economic optimization was used to include the uncertainty in risk caused by climate change. This method provides a way of taking account of uncertainty in the economic optimization associated by holding different levels of risk-aversion attitude. A case study of Old Dike Ring in Bengbu, China, was carried out by using the proposed approach.

The result shows that the uncertainty introduced by climate change has much more significant influence on the optimal decisions than the uncertainty in damage does. The optimal dike height taking account of climate change is about 7m higher than that in a stationary climate. With the increase in risk-aversion index, this difference increases significantly. However, in the same pattern of climate change, the influence of risk-aversion index decreases when its value increases. In terms of safety level, climate change reduces the optimal return period from about once in 20000 to once in 5000. However, with the risk-aversion level of 0.5, the optimal safety level is as twice as the present situation. Timing of climate change is also an influential factor because of the discounting to the present value. The sooner climate change is expected, the more uncertainty is taking account of.

It is notable that the uncertainty in the future projections of climate represents the agreement of the climate models rather than the true uncertainty in climate change. Thus, the more uncertainty in the projections, the less confidence is preserved in climate models, and the more risk-aversion attitude should be held in the decision making. The proposed approach enables decision makers to make trade-offs between the confidence in the climate projections and the risk-aversion attitude. The approach may be criticized to be subjective and ambiguous in the aspect of risk-aversion attitude (Aven and Flage 2009). However, it is unavoidable and is a usual way to turn it into a quantifiable index when a “subjective” attitude has to be incorporated. The implication of risk-aversion index should be discussed in further work.

REFERENCES

- Aven, T. and R. Flage (2009). Use of decision criteria based on expected values to support decision-making in a production assurance and safety setting. *Reliability Engineering & System Safety* 94(9): 1491-1498.
- Jonkman, S. N., P. H. A. J. M. van Gelder, et al. (2003). An overview of quantitative risk measures for loss of life and economic damage. *Journal of Hazardous Materials* 99(1): 1-30.
- Kuijper, B. and M. J. Kallen (2012). Uncertainty in optimal decisions for dike maintenance. *Structure and Infrastructure Engineering* 8(4): 317-327.
- Slijkhuis, K. A. H., P. H. A. J. M. van Gelder, et al. (1997). Optimal dike height under statistical -, construction - and damage uncertainty. 7th International Conference on Structural Safety and Reliability. Kyoto.
- Van Danzig, D. (1956). Economic Decision Problems for Flood Prevention. *Econometrica*; 24:276-278.
- Van Gelder, P. H. A. J. M. and J. K. Vrijling (1998). Sensitivity analysis of reliability-based optimization in sea dike designs. *Sensitivity Analysis of Model Output*: 313-315.
- Van Gelder, P.H.A.J.M., (1999). *Statistical Methods for the Risk-Based Design of Civil Structures*. Ph.D. Thesis. Delft University of Technology: the Netherlands.
- Vrijling, J. K. and P. H. A. J. M. van Gelder (1999). Uncertainty Analysis Of Water Levels On Lake IJssel In The Netherlands: A Decision-making Analysis. *Proceedings of the 9th Annual Conference Risk Analysis: Facing the New Millennium*. Rotterdam, The Netherlands.
- Vrijling, J. K., W. van Hengel, et al. (1995). A framework for risk evaluation. *Journal of Hazardous Materials* 43(3): 245-261.
- Wang, L., Reggiani, P., Van Gelder, P.H.A.J.M. and Vrijling, J.K.. GCMs simulative ability on climatic and extreme hydrologic conditions over Chinese Huai River Basin. *Proceedings of EGU General Assembly 2011 (April 3-8 2011, Vienna, Austria)* *Geophysical Research Abstracts* Vol. 13, EGU2011-1068, 2011
- Wang, L. · Van Gelder, P.H.A.J.M. · Vrijling, J.K. (2012). A preliminary analysis of climate change effect on long-term risk-based design of flood defense. *Proceedings of EGU General Assembly 2012 (22-27 April, Vienna, Austria)* *Geophysical Research Abstracts* Vol. 14, EGU2012-2079, 2012 EGU General Assembly 2012