

## Evaluating crowding in individual train cars using a dynamic transit assignment model

Peftitsi, Soumela; Jenelius, Erik; Cats, Oded

**DOI**

[10.1080/21680566.2021.1939808](https://doi.org/10.1080/21680566.2021.1939808)

**Publication date**

2021

**Document Version**

Final published version

**Published in**

Transportmetrica B

**Citation (APA)**

Peftitsi, S., Jenelius, E., & Cats, O. (2021). Evaluating crowding in individual train cars using a dynamic transit assignment model. *Transportmetrica B*, 9(1), 693-711.  
<https://doi.org/10.1080/21680566.2021.1939808>

**Important note**

To cite this publication, please use the final published version (if applicable).  
Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

**Takedown policy**

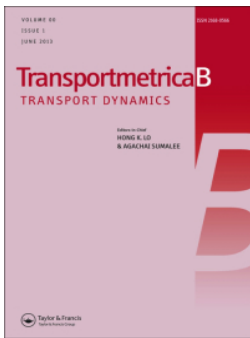
Please contact us and provide details if you believe this document breaches copyrights.  
We will remove access to the work immediately and investigate your claim.

***Green Open Access added to TU Delft Institutional Repository***

***'You share, we take care!' - Taverne project***

**<https://www.openaccess.nl/en/you-share-we-take-care>**

Otherwise as indicated in the copyright section: the publisher is the copyright holder of this work and the author uses the Dutch legislation to make this work public.



## Evaluating crowding in individual train cars using a dynamic transit assignment model

Soumela Pefitsi, Erik Jenelius & Oded Cats

To cite this article: Soumela Pefitsi, Erik Jenelius & Oded Cats (2021) Evaluating crowding in individual train cars using a dynamic transit assignment model, Transportmetrica B: Transport Dynamics, 9:1, 693-711, DOI: [10.1080/21680566.2021.1939808](https://doi.org/10.1080/21680566.2021.1939808)

To link to this article: <https://doi.org/10.1080/21680566.2021.1939808>



Published online: 16 Jun 2021.



Submit your article to this journal [↗](#)



Article views: 30



View related articles [↗](#)



View Crossmark data [↗](#)



# Evaluating crowding in individual train cars using a dynamic transit assignment model

Soumela Pefitsi<sup>a</sup>, Erik Jenelius <sup>a</sup> and Oded Cats <sup>a,b</sup>

<sup>a</sup>Division of Transport Planning, KTH Royal Institute of Technology, Stockholm, Sweden; <sup>b</sup>Department of Transport and Planning, Delft University of Technology, Delft, The Netherlands

## ABSTRACT

As travel demand grows in many cities around the world, overcrowding in public transport systems has become a major issue and has many negative effects for both users and operators. Measures to address on-board congestion span from large-scale strategic investments (e.g. increasing infrastructure capacity), through tactical planning (e.g. stopping pattern) to real-time operational measures (e.g. information provision, gate and escalator control). Thus there is a need to evaluate the impact of these measures prior to their implementation. To this end, this study aims at capturing the effective capacity utilization of the train, by considering passengers' distribution among individual train cars into an agent-based simulation model. The developed model is validated and applied to a case study for the Stockholm metro network. The findings suggest that an increase in peak hour demand leads to a more even passenger distribution among individual train cars, which partially counteracts the increased disutility caused by the higher passenger volumes. Interestingly, the closure of the most popular entrance point at one of the stations leads to lower train crowding unevenness at the downstream stops and consequently reduces passengers' experienced discomfort. We find that the user cost is significantly underestimated when passenger distribution among cars is not accounted for.

## ARTICLE HISTORY

Received 27 June 2020  
Accepted 30 May 2021

## KEYWORDS

Public transport; transit assignment; crowding; agent-based simulation; passenger distribution

## 1. Introduction

Many public transport systems are subject to overcrowding during peak periods. On-board crowding is associated with many negative effects on passengers, such as increased discomfort and stress, delays and denied boarding (Tirachini, Hensher, and Rose 2013) as well as perceived insecurity (Tirachini et al. 2017; Márquez, Alfonso, and Poveda 2019). There is a need to address on-board congestion through infrastructure or operational measures in order to improve system performance and the level of service. This calls for the development of models that are able to adequately capture the impacts of alternative interventions on on-board congestion level and the distribution thereof.

Passenger loads can be highly unevenly distributed along platforms and in individual cars of trains and metros even during peak hours (TRB 2014; Zhang, Jenelius, and Kottenhoff 2017). Kim et al. (2014) investigated the causes behind the uneven passenger distribution between train cars, concluding that passengers' motivation for minimizing the walking distance at the destination is the most decisive factor for choosing a specific train car. Some studies aim to reduce the unevenness of passenger distribution through tactical planning methods, by determining the optimal train stop location along a platform (Sohn 2013), by installing an one-way gate on the middle of the platform to control passenger

car boarding choices (Muñoz et al. 2018), or through real-time crowding information system (Zhang, Jenelius, and Kottenhoff 2017).

Transit assignment models (TAM) are used for describing how passengers are distributed in the public transport network, predicting passengers' travel route decisions and modeling platform and on-board crowding. However, these models usually do not evaluate crowding distribution among individual cars of a train. These models implicitly assume that train cars are evenly crowded, hence yielding an unrealistic vehicle capacity utilization and result with an underestimation of user cost. Since strategies to reduce crowding in transit vehicles require considerable infrastructure or operational investments, it is important to develop tools that can capture the vehicle capacity utilization in a more realistic manner and evaluate the effectiveness of such strategies in advance to smooth crowding between train cars.

The objective of this study, motivated by the aforementioned shortcoming of the state-of-the-art TAM, is to propose a quantitative methodology for analyzing capacity utilization of individual train cars. The main contributions of the paper are:

- The development of a quantitative approach for describing passengers' car-specific boarding choices and evaluating crowding in individual train cars.
- Incorporating the passenger path choice modeling in a dynamic and stochastic TAM which captures train car choices.
- The developed model accounts for day-to-day learning, where passengers' decisions are made in a repetitive way, taking the impact of car-specific crowding into account. This allows to embed the 'local' train car loadings into the 'global' passenger route choice.
- The validity of the model is investigated by comparing the simulated output against an empirical data set.
- The effect of demand and infrastructure changes on crowding distribution among individual train cars is investigated using simulation scenarios.

In the proposed model, transit stops (i.e. rail platforms) are divided into sections, each corresponding to a specific train car. Each car is modeled as a separate unit with a corresponding seating and total capacity. Using an agent-based simulation model, the distribution of passengers among individual cars is dynamically and stochastically modeled. The rapid rise of information and communication technologies enables the collection of massive and continuous mobility data that facilitate calibration and validation of car-specific transit assignment models.

The remainder of the paper is structured as follows. In Section 2, we review the relevant literature. The dynamic model for evaluating the distribution of crowding in a multi-car vehicle is presented in Section 3. Next, we present in Section 4 the application of the proposed model to a case study in Stockholm, followed by the presentation of results in Section 5. Section 6 concludes with a discussion of the key findings, reflecting on the limitations of the study and providing an outline of directions for further research.

## 2. Literature review

Public transport users make travel decisions considering a variety of factors, namely travel time, the number of transfers, comfort level, station physical layout characteristics and topological factors (Raveau et al. 2014). To avoid on-board crowding, passengers make trade-offs, i.e. choose an alternative travel path, board a less crowded train car, wait for the next train service or adapt their departure time to ensure a seat (Pownall, Prior, and Segal 2008; Kim et al. 2015). The skewness of passengers distribution along the station platform and eventually among individual cars of the train is closely related to the physical layout of the station and the location of entrance and exit points (Szplett and Wirasinghe 1984; Krstanoski 2014; Liu, Li, and Wang 2016). Data-driven approaches for studying the behavioral aspects of passengers' boarding choices find that passengers choose a specific train car

to board aiming to minimize the walking distance and on-board discomfort (Kim et al. 2014; Pefitsi, Jenelius, and Cats 2020).

A variety of models exist which describe the assignment of passengers to a transit network, predict travelers' decisions and model crowding at station and inside transit vehicles. The growing literature on TAM which are broadly classified into frequency-based and schedule-based approaches for modeling transit path choice, is reviewed by Fu, Liu, and Hess (2012). Frequency-based models represent transit services at the line level. Spiess and Florian (1989) proposed the first frequency-based assignment framework based on passengers' optimal strategies for minimizing the generalized travel time. Frequency-based approaches for congested networks were presented by Lam et al. (1999), accounting for the total passenger travel cost and by Cepeda, Cominetti, and Florian (2006), accounting for service capacity. Schmöcker, Bell, and Kurauchi (2008) introduced a dynamic frequency-based transit assignment model, considering passengers that fail to board due to insufficient remaining capacity.

Schedule-based assignment models represent individual train trips that their availability is dictated by timetables. A schedule-based model, presented by Nuzzolo, Russo, and Crisalli (2001), accounts for on-board congestion by using a passenger discomfort factor. Vehicle capacity constraints were introduced by Nguyen, Pallottino, and Malucelli (2001), Papola et al. (2009), Khani, Hickman, and Noh (2015), Cats, West, and Eliasson (2016) and Ranjbari, Hickman, and Chiu (2020), to model boarding passengers considering the residual capacity of the vehicle and the possibility of denied boardings. Poon, Wong, and Tong (2004) used a schedule-based traffic assignment model for congested transit networks with capacity constraints, to predict the queuing time per passenger, assuming that passengers are queuing according to the first-in-first-out (FIFO) rule. Another model that takes into account transit schedules and vehicle capacity to assign passengers to paths and model the impact of priority rules was proposed by Hamdouch and Lawphongpanich (2008). On-board passengers have priority and waiting passengers are assumed to board the vehicle in an FIFO or at a random manner. Seat capacity constraints have been considered to model the effect of on-board discomfort on the sitting and standing passengers (Sumalee, Tana, and Lam 2009; Leurent 2010).

Agent-based simulation models mimic individual passengers' behavior and choices and allow thus to model dynamic congestion effects. Wahba and Shalaby (2005) were among the first to propose a framework based on the assumption that individual passengers adjust their travel behavior based on their experience. Zhang, Han, and Li (2008) developed an agent-based simulation model to capture passenger boarding and alighting movements at stops. Hänseler et al. (2020) presented a model for describing the interaction between passenger movements on platform and inside the train. Rexfelt et al. (2014) have focused on modeling the behavior of individual passengers at stops and on-board buses, assessing the effect of vehicle layout on boarding and alighting passenger movements. A dynamic and stochastic TAM, which captures congestion and crowding effects (denied boarding, on-board crowding and service irregularity), was proposed by Cats, West, and Eliasson (2016). Cats and Hartl (2016) compared the ability of schedule-based and agent-based TAM to model on-board congestion, finding that the latter is more sensitive to variations in demand. Although tools for modeling and predicting passenger flows in public transport networks are widely used, there is limited knowledge on how to model crowding distribution among individual cars of a train. This motivates the need for developing a dynamic model to capture passengers' boarding choices of individual train cars and evaluate the effects of crowding unevenness.

### 3. Methodology

Modeling the distribution of passengers on-board multi-car trains involves changes to both supply and demand representation and processes. From the supply side, transit vehicles and station platforms representation requires the identification of car units and platform sections, respectively, to enable modeling the distribution of passengers inside the vehicles. From the demand side, path choice modeling in an existing public transit simulation model requires extension to capture passengers' car

boarding choices. The developed model takes the effect of car-specific crowding through a day-to-day learning process into account.

### 3.1. Simulation modeling approach

A dynamic agent-based public transport operations simulation model, BusMezzo, is used in this study for modeling the congestion effects on-board transit vehicles, considering vehicle capacity constraints (Cats 2013). Crowding is evaluated at the vehicle level and the distribution of passengers over vehicle units (e.g. train cars) is not captured. The model simulates individual passenger path decisions and the movements of individual transit vehicles. The movement of transit vehicles between stops is modeled within a mesoscopic simulation model including traffic dynamics and public transport operation (Toledo et al. 2010). Different public transport modes, i.e. metro, commuter rail, bus and tram, are modeled using different vehicle types with distinct capacity characteristics and dwell time functions. A set of trips is assigned to each vehicle type and hence, BusMezzo models also the propagation of delays caused by trip chaining.

The transit network in BusMezzo includes a set of transit stops  $S$  and a set of transit lines  $L$ . Each transit stop  $s \in S$  represents a rail platform which may be served by more than one transit line. In this study, we extend model functionality to allow for modeling passenger distribution over trains. To enable modeling passenger distribution over trains, each transit stop  $s$  is divided into  $K_s$  sections. Each transit line  $l$ , defined by an Origin–Destination (OD) pair and a sequence of stops  $S_l$ , is served by a set of trips denoted by  $J_l$ . Each train serving trip  $j \in J_l$  consists of  $l_j$  cars. It is assumed that all transit lines serving a given transit stop have the same number of car units per train (i.e.  $l_j = K_s$  for all  $s \in S_j$  and  $j \in J_l$ ) and hence, each platform section corresponds to a car unit. Similarly, transit stops that are served by the same transit line have the same number of platform sections.

### 3.2. Passenger path choice modeling

Passengers are generated stochastically according to Poisson processes based on OD matrices. Each origin and destination is specified as a pair of platform sections located at certain stations. Throughout the simulation, each passenger makes a sequence of path decisions, namely boarding, alighting and walking decisions, that combined yield the realization of a path. The path decisions are described with random utility discrete choice models. Each alternative is associated with a utility, evaluated based on the passenger's preferences and expectations, which are shaped by prior knowledge, gained experience and available provided information (Cats and Gkioulou 2017).

In BusMezzo, each transit path alternative  $a$ , which connects an origin location  $o$  to a destination  $d$  and is included in path set  $A^{od}$ , is defined as a combination of stops, lines and walking links (Cats, West, and Eliasson 2016). To capture crowding in individual cars of a train, the path alternative is further defined in this study as an ordered combination of transit stops associated with a platform section, transit lines associated with a car unit and a set of walking links between stops as well as platform sections.

Each feasible path alternative  $a$  is associated with a utility; the deterministic part of the utility for passenger  $y$  of a feasible path  $a$  is:

$$v_{y,a} = \beta_y^{\text{inv}} t_{y,a}^{\text{inv}} + \beta_y^{\text{wait}} t_{y,a}^{\text{wait}} + \beta_y^{\text{walk}} t_{y,a}^{\text{walk}} + \beta_y^{\text{transfer}} N_{y,a}^{\text{transfer}} \quad \forall y \in Y, a \in A^{od} \quad (1)$$

where  $t_{y,a}^{\text{inv}}$  is the expected total perceived in-vehicle time,  $t_{y,a}^{\text{wait}}$  is time-dependent waiting time that depends on passenger arrival process and service frequency,  $t_{y,a}^{\text{walk}}$  is the expected total walking time that includes access and egress walking time as well as on-platform walking time between platform sections,  $N_{y,a}^{\text{transfer}}$  is the number of transfers included in the path alternative and  $\beta$ 's are the corresponding utility function coefficients. The expected total perceived in-vehicle time  $t_{y,a}^{\text{inv}}$  depends on passenger's accumulated experienced in-vehicle time that reflects the on-board crowding conditions.

For each journey leg, the perceived in-vehicle travel time is computed as the nominal in-vehicle time weighted by the respective on-board crowding factor. The value of the latter is defined as a non-linear function of the car load factor, given as the ratio of on-board car passenger load to the car seated capacity, and varies between sitting and standing passengers (Wardman and Whelan 2011).

In order to obtain passenger loads in individual train cars, we need to incorporate the train car choice into the dynamic individual path choice making. In the following, we detail how the path choice model accounts for train car choice and related travel attributes. Any decision in the simulation model involves a passenger making a decision regarding the following element, i.e. stop associated with platform section, line associated with car unit and walking link, considering all the expected future travel attributes of the path alternatives that are associated with the specific element. When passengers reach the end of a path element, they choose the next path element that maximizes their expected utility. The utility that passenger  $y$  associates with a path element  $c$  ( $c \in C$ ), denoted by  $u_{y,c}$ , is given as composite utility of all path alternatives  $A^{cd}$ .

$$u_{y,c} = \ln \sum_{a \in A^{cd}} e^{v_{y,a}} \quad (2)$$

The probability that a passenger  $y$  will choose the next path element  $c$  is then

$$P_{y,c} = \frac{e^{u_{y,c}}}{\sum_{c \in C} e^{u_{y,c}}} \quad (3)$$

The multinomial logit (MNL) model assumes the independence of irrelevant alternatives (IIA) property, while transit route choice might violate this property. The choice-set generation model applied in BusMezzo partially mitigates this shortcoming by merging overlapping paths – for example corridors with common lines and lines with common transfer locations – which exercise high correlations into single hyperpaths (Cats et al. 2011; Cats and West 2020). Furthermore, passengers do not choose between paths or hyperpaths in the choice process in BusMezzo but rather choose between travel actions (e.g. boarding vs staying), further offering a remedy for unaccounted correlations stemming from the IIA property of the MNL model. The choice structure partially mitigates this problem, as the most correlated alternatives (i.e. same visited stops) will be in the same branch of the tree.

Passenger path choices involve three types of decisions as described in the following.

*Walking decision:* The passenger path choice process starts with the walking decision. Passenger  $y \in Y$  decides whether to stay at the origin location or to walk to platform section  $k$  of a nearby transit stop  $s$ . Each time a passenger alights from a transit vehicle, a new origin location is set and another walking decision needs to be made. The walking utility is based on the walking distance to a given platform section of the first stop that the passenger wants to walk to and on the expected downstream travel attributes, including in-vehicle, walking and waiting times as well as the number of transfers for all path alternatives between this section and traveler's final destination. The total walking distance of the downstream walking links includes the on-platform walking distance at the destination stop. This allows capturing travelers' choice to minimize walking time at the downstream stop while waiting, depending on the location of the desired exit. In the current model implementation, section-to-section walking distances are computed based on the shortest section-to-section walking path available between any pair of sections, within the same station or belonging to different stations.

*Boarding decision:* Each time a train  $j$  arrives at transit stop  $s$ , passenger  $y$  makes a boarding decision; board the train or stay on the platform. In the boarding decision process, the utility associated with boarding is compared to the utility associated with staying and waiting for other train. Expected in-vehicle, walking and waiting times as well as the number of transfers are involved in the boarding utility function. The number of passengers boarding car  $i$  of train run  $j$  at stop  $s$ , denoted by  $q_{ijs}^{\text{board}}$ , is given by the number of passengers that make a positive boarding decision if the car has not reached its total capacity; otherwise, the number of boarding passengers is equal to the remaining capacity of the car. If the car, that is adjacent to the waiting platform section  $s_k$ , has reached its total capacity, the



passenger boards the next closest car if this has residual capacity, otherwise the passenger stays on the same platform section waiting for subsequent vehicles. If the passenger is denied from boarding one of the middle cars of the train, they randomly choose one of the two adjacent cars to board if both offer residual capacity.

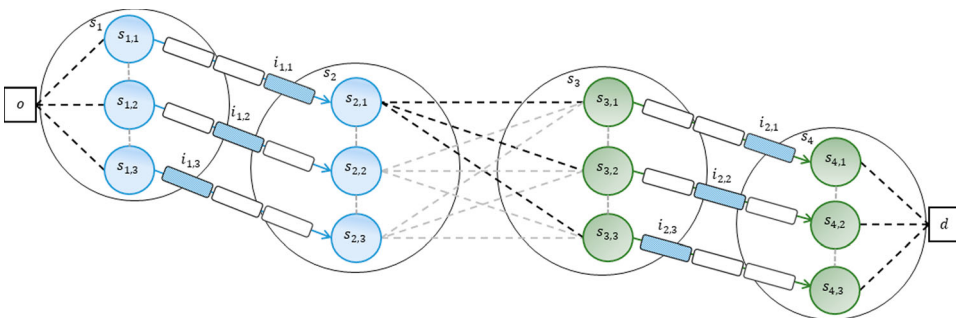
*Alighting decision:* Upon boarding train  $j$ , passenger  $y$  decides at which downstream transit stop to alight. The platform section  $k$ , that the passenger will alight at, is already determined by the car  $i$  that the passenger has boarded, under the assumption that passengers do not move between cars (this may also not be possible for some vehicle configurations). The number of passengers alighting from car  $i$  at stop platform section  $s_k$  equals to the total number of passengers that make a positive alighting decision.

The number of passengers on-board car  $i$  of train run  $j$  when the train departs from stop  $s$ , denoted by  $q_{ijs}^{onboard}$  is a function of alighting and boarding flows to and from car  $i$ .

Figure 1 illustrates the path alternative definition for passengers that start their trip at origin location  $o$  and aim to reach destination location  $d$ . For illustration, it is assumed that the transit stops are served by 3-car trains and hence, the platforms are divided into three sections. The passenger, starting at  $o$ , has three alternative connection choices that can be accessed by walking; the first, second and third platform sections of transit stop  $s_1$ , denoted by  $s_{1,1}$ ,  $s_{1,2}$  and  $s_{1,3}$ , respectively. The stop is served by the transit line  $l_1$ , while each platform section is served by the corresponding train car unit of the line, denoted by  $i_{1,1}$ ,  $i_{1,2}$  and  $i_{1,3}$  for the first, second and third cars, respectively. A transit user that decides to make a walking connection to the first section of the stop  $s_{1,1}$  will board the first car unit  $i_{1,1}$ , if they make a boarding decision, considering car capacity constraints, and will alight at the first platform section of the transfer transit stop  $s_{2,1}$ , which is then set as a new origin transit location. From the alighting platform section, the passenger makes a new walking decision to a platform section of the same or different stop. For the origin–destination pair illustrated in Figure 1, nine path alternatives are available.

**3.2.1. Perceived in-vehicle travel time**

The developed model accounts for day-to-day dynamics, where an iterative within-day network loading, with constant daily passenger demand, is performed to mimic passengers’ adaptive travel behavior in the real world (Cats and West 2020). The dynamic and stochastic interaction between transit vehicles and passenger demand is simulated in the within-day loop, yielding passenger car assignment which is used as an input to the day-to-day simulation loop. This allows passengers to store information about on-board crowding in individual train cars based on the experience gained on a day-to-day basis and to alter their expectations and travel strategy accordingly. Car-specific on-board crowding level affects the perception of in-vehicle travel time in individual train cars and the expected total in-vehicle time  $t_{y,d}^{inv}$  which is part of Equation (1). Passenger’s expectation about the perception of in-vehicle travel time in a train car on the current day is based on the expected and



**Figure 1.** Illustration of path alternative definition (Dashed lines – walking links).

experienced in-vehicle travel time on the previous day. The nominal in-vehicle travel time is weighted using an on-board crowding multiplier. According to a meta-study by Wardman and Whelan (2011), the crowding multipliers depend on the ratio of car occupancy to the car seated capacity, and whether the passenger is sitting or standing. In BusMezzo passengers are allocated to seats assuming a First-In-First-Out (FIFO) rule, where seats of each car are filled sequentially and standing passengers exist when the car seated capacity has been reached (Cats, West, and Eliasson 2016). Seated passengers are assigned with crowding multipliers between 0.95 and 1.71 when the ratio of car occupancy to the car seated capacity increases from 50% to 200%. The multipliers for standing passengers range from 1.78 to 2.69 and are only considered when all seats are occupied. The simulation terminates when the day-over-day change of perceived in-vehicle time is considered negligible.

On the first simulated day, in the absence of past experience, passengers choose a platform section and thereby a train car based solely on the walking distance to the section and on the expected utilities of the path alternatives available between this section and the desired exit at the destination, i.e. passengers expect equally utilized cars of the next arriving train. Performing an iterative network loading, car boarding choice is also affected by passenger's expectations about car crowding. In this case, perceived in-vehicle travel times, weighted with the car-specific crowding factor based on passenger's gained accumulated experience, are included in the utility associated with a path alternative. Consequently, passengers expect different crowding levels on-board individual train cars.

### 3.3. Performance evaluation

The impact of alternative scenarios on the performance of the system is evaluated by considering the average on-board crowding unevenness across the vehicle, the average boarding passengers unevenness and the average generalized travel cost per passenger.

#### 3.3.1. Passenger distribution unevenness

Having a single metric for measuring crowding unevenness facilitates comparisons between different passenger distributions. The distribution of passenger load among the cars  $i \in I$  of a train run  $j$  upon departure from stop  $s$  is systematically measured using the Gini coefficient  $G_{js}$ .

$$G_{js} = \frac{1}{2|I| \sum_{i=1}^I q_{ijs}^{\text{onboard}}} \sum_{i=1}^I \sum_{i'=1}^I |q_{ijs}^{\text{onboard}} - q_{i'js}^{\text{onboard}}| \quad (4)$$

where  $q_{ijs}^{\text{onboard}}$  and  $q_{i'js}^{\text{onboard}}$  denote the passenger occupancy of car  $i$  and car  $i'$ , respectively, of a train run  $j$  upon departure from stop  $s$ . This train crowding unevenness metric measures how much the passenger load distribution deviates from the totally even distribution, i.e. when all train cars are equally utilized. The metric takes the value 0 in case of perfect evenness in the train, i.e. passengers are equally distributed over all train cars – and the value 1 in case of perfect unevenness, i.e. passengers are filling cars in succession.

On-board train crowding distribution is based on passengers' boarding behavior at the stop and hence it is essential to evaluate the performance of the system by considering the distribution of boarding passengers. Similarly, the distribution of the boarding passengers among individual train cars  $i \in I$  of train  $j$  at stop  $s$  is given by

$$G_{js}^{\text{board}} = \frac{1}{2|I| \sum_{i=1}^I q_{ijs}^{\text{board}}} \sum_{i=1}^I \sum_{i'=1}^I |q_{ijs}^{\text{board}} - q_{i'js}^{\text{board}}| \quad (5)$$

where  $q_{ijs}^{\text{board}}$  and  $q_{i'js}^{\text{board}}$  denote the number of passengers boarding car  $i$  and car  $i'$ , respectively, of a train run  $j$  upon departure from stop  $s$ . The metric takes the value 0 when boarding passengers are perfectly evenly distributed among train cars and the value 1 when all passengers waiting on a platform board the same car.

### 3.3.2. Generalized travel cost

The generalized travel cost per passenger is defined as the weighted sum of all travel path attributes, i.e. in-vehicle, walking and waiting times as well as the number of transfers.

The disutility of in-vehicle time reflects on-board passenger discomfort. The discomfort experienced by a passenger is given as the summation of the nominal in-vehicle travel time of each trip segment weighted with the corresponding user-specific parameter for in-vehicle time  $\beta^{inv}$ , which reflects the value of uncrowded in-vehicle time, and the crowding multiplier for the same trip segment. The crowding multiplier value depends on the car occupancy and whether this passenger has a seat or not (Wardman and Whelan 2011).

Walking and waiting times are weighted with the corresponding user-specific parameters for walking  $\beta^{walk}$  and waiting time  $\beta^{wait}$ , respectively. Walking time valuation is assumed to be the same for walking to/from a transit stop and on-platform walking. Each transfer is penalized with the corresponding user-specific parameter for transfers  $\beta^{transfer}$ . Based on the time valuations reported in the literature, walking and waiting times are valued as twice the value of in-vehicle time in uncrowded conditions, while the transfer penalty is valued five times the in-vehicle time and they are set to:  $\beta^{inv} = -1$ ,  $\beta^{walk} = \beta^{wait} = 2 \cdot \beta^{inv} = -2$ ,  $\beta^{transfer} = 5 \cdot \beta^{inv} = -5$  (Wardman 2004).

### 3.4. Data and computational requirements

Advances in information and communication technologies in public transport enable the generation of big data sources, such as cell phone data, social media data or smart-card data, that can be utilized for modeling travel behavior and predicting passengers' movements as well as investigating travel patterns (Chen et al. 2016).

For model application, the platform section-level OD information is required to represent the passenger demand for each pair of platform sections of a given OD pair. If this information is not readily available, three types of data are required to represent the demand.

Average station-to-station travel demand data for each OD pair describe the average number of trips between a given origin and destination. Such data may be obtained through cell phone data (Alexander et al. 2015; Toole et al. 2015; Bachir et al. 2019) or automated fare collection (AFC) data. AFC data play an increasingly important role in estimating travel demand in public transport systems (Munizaga and Palma 2012; Alsgar et al. 2016).

Pedestrian incoming and outgoing flows at each access point of the station are useful to describe the passenger movements at the entrance level and can be used to estimate the probability that a passenger initiates or ends the trip at a certain section of the platform. This information may be obtained through passenger counts, AFC data that has been utilized in Ingvardson et al. (2018) to model passengers' arrival patterns and in Peftitsi, Jenelius, and Cats (2020) to analyze how passengers make metro car boarding choices or cell phone data that has been used in Aguilera et al. (2014) to measure passenger flows in a public transit system.

The physical infrastructure characteristics of the network, including the dimensions of the platform and location of entrance and exit points, are also required to define the stop characteristics and walking distances within stations as well as between stops.

For model validation, passenger load data for each car unit, describing the crowding level on-board individual cars, are required. Car load data may be collected from car weight measurements or obtained through sensors installed at the car doors.

Since the transit simulation model BusMezzo is stochastic, each simulated scenario needs to be evaluated based on a number of simulation replications. The number of replications required  $N(m)$ , given  $m$  initial runs, as it is given in Cats et al. (2010), is determined by

$$N(m) = \left( \frac{\sigma(m)t_{m-1,1-\frac{\alpha}{2}}}{\mu(m)\varepsilon} \right)^2 \quad (6)$$

where  $\sigma(m)$  is the standard deviation of the average generalized travel cost per passenger of  $m$  simulation runs,  $t_{m-1,1-\frac{\alpha}{2}}$  is the critical value of the  $t$ -test for  $m-1$  degrees of freedom and level of

significance  $\alpha$ ,  $\mu(m)$  is the mean generalized travel cost per passenger of  $m$  simulation runs and  $\varepsilon$  is the allowed error. Given significance level and allowed error of 5%, 10 simulation runs were found sufficient, yielding a maximum error of 5%.

## 4. Application

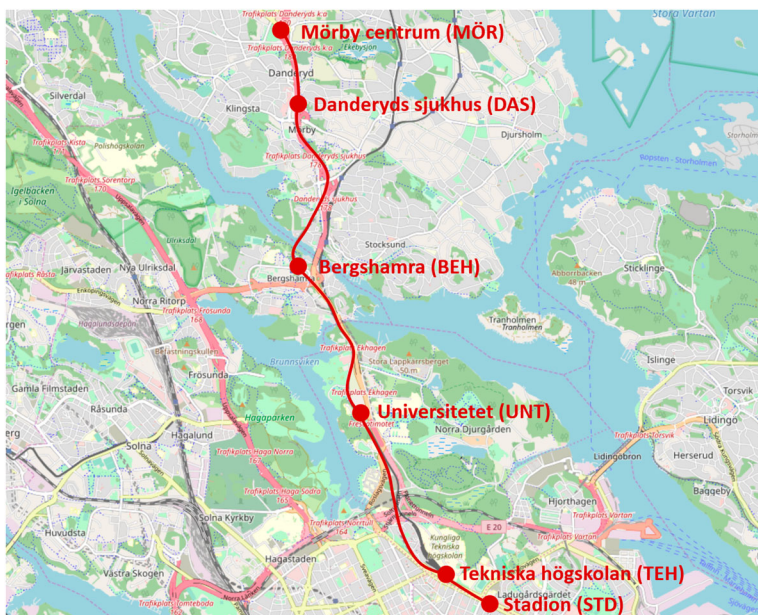
### 4.1. Case study description

The proposed modeling framework is applied to a case study for the metro network in Stockholm. The Stockholm metro system is used by more than 1 million passengers per workday. Although passenger loads are close to capacity during peak hours, passengers are often unevenly distributed among train cars and 20% of the seats remain unoccupied during the morning peak hour (SL 2017). The model is applied to the southbound segment of metro line 14 between Mörby centrum (MÖR) and Stadion (STD), which operates with a planned headway of 5 minutes during the morning peak period (06:00–09:00 am). The segment exhibits high average on-board passenger load. The studied area is shown in Figure 2.

The passenger distribution on-board the trains is highly skewed towards the front car during the morning peak hour (Figure 3). On average, 41% of the on-board passengers occupy the front train car, while the rear car is occupied by only 25% of the passengers.

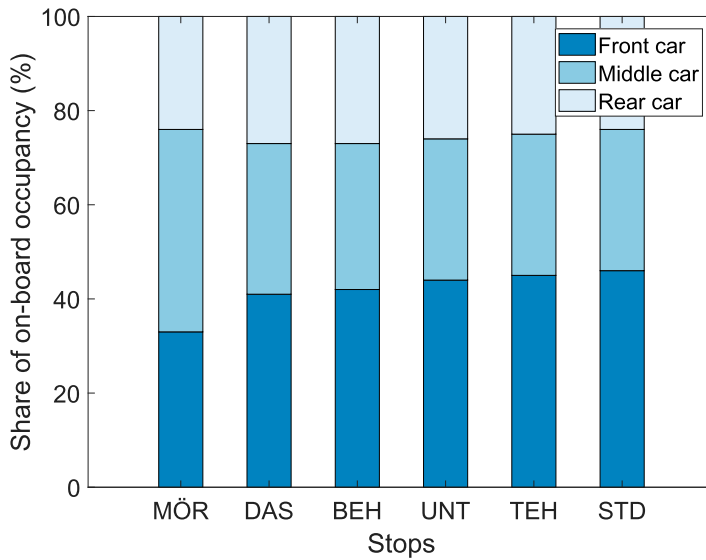
### 4.2. Network representation

The transit network representation in BusMezzo includes both directions of the metro line 14 in Stockholm that serve 38 stops i.e. rail platforms. The stops are served by 72 vehicle trips over the 3-hour morning peak period. Each train is composed of three cars and hence, each transit stop is divided into three platform sections. According to the train manufacturer, the design capacity of a car unit is 126 seated passengers and 288 standees. The transit network is represented in BusMezzo with detailed train scheduling, the connectivity of stops by walking as well as the shortest section-to-section walking



**Figure 2.** Map of the studied segment of Stockholm metro network.

Source: OpenStreetMap



**Figure 3.** Average on-board passenger distribution based on historical data (morning rush hour (06:00-09:00am) in October 2016).

path between these stops, required to compute the section-to-section distance of the walking links. A six-stop-segment of the metro line 14 has been selected as a case study.

### 4.3. Demand representation

Passenger demand is simulated for the morning peak hour. The OD travel demand data for the morning peak period at the station-to-station level are taken from the transit assignment model Visum, based on the official planning zonal OD matrix. For each OD pair, a platform section demand matrix indicating the probability that a passenger starts and ends the trip at a certain platform section at the origin and destination stop, respectively, was produced based on the total incoming and outgoing passenger flows at each entrance point of the station, which are obtained through passenger counts in the morning peak hour.

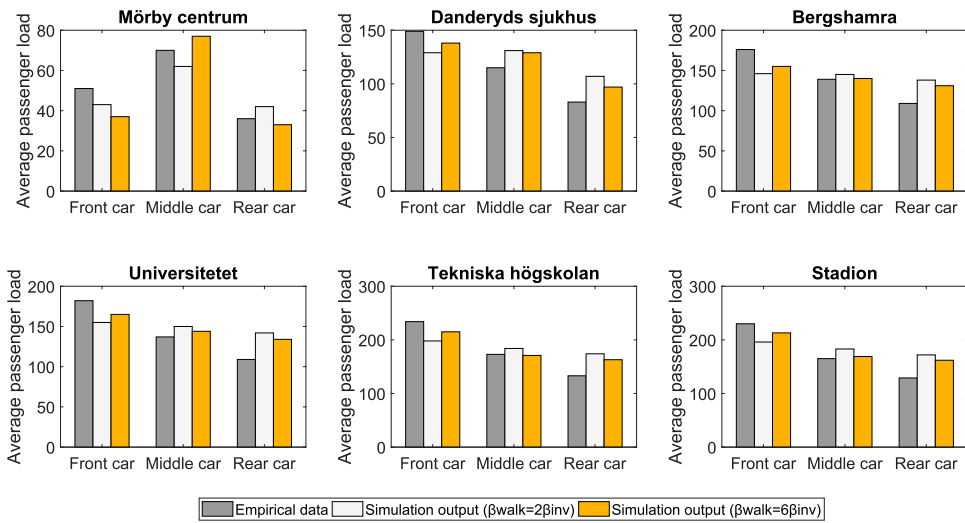
### 4.4. Scenarios design

We simulate the following three scenarios in BusMezzo:

- (1) *Base scenario*, where the case study is simulated with the current average morning peak hour demand.
- (2) *Increased demand scenario*, where the case study is simulated with the current average morning peak hour demand increased by 50%.
- (3) *Intervention scenario*, where an infrastructure change is considered at one of the metro stations, namely Danderyds sjukhus (DAS), which is a station with two access points located at the south and north ends. The south, which is the most popular access point, is considered as temporarily unavailable in this scenario. Elevator and escalator maintenance is one of the factors that might require the temporary closure of a station entrance point.

## 5. Results

We first study the performance of the model in the studied metro line segment to assess the validity of the model, and then investigate the effect of demand and infrastructure changes on crowding unevenness using simulation scenarios.



**Figure 4.** Average car passenger load of the simulation output for different walking time valuations and empirical data.

### 5.1. Model validation

The simulation outputs are tested against an empirical data set of car load data to investigate the validity of the simulation model (Figure 4). Empirical car load data estimated through the car weight measurements, based on an average of 78 kg per passenger including luggage, are available at train departure from each stop on the studied line segment for the morning peak period in October 2016. The outputs of a simulation hour are tested for the highest peak hour of the morning rush period (07:30–08:30 am). The relation between in-vehicle and walking time valuation is calibrated to investigate the sensitivity of the model and assess the goodness of fit. We find that the model, which accounts for day-to-day learning and takes the crowding effect into account in the decision making process, best reproduces the empirical car passenger loads when passengers value walking time higher.

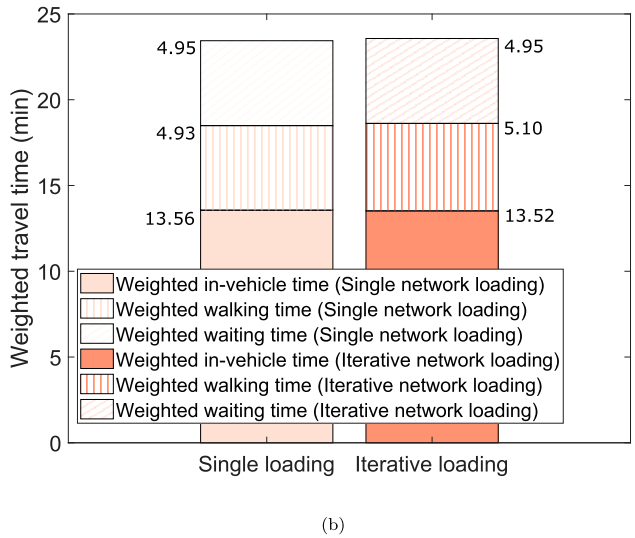
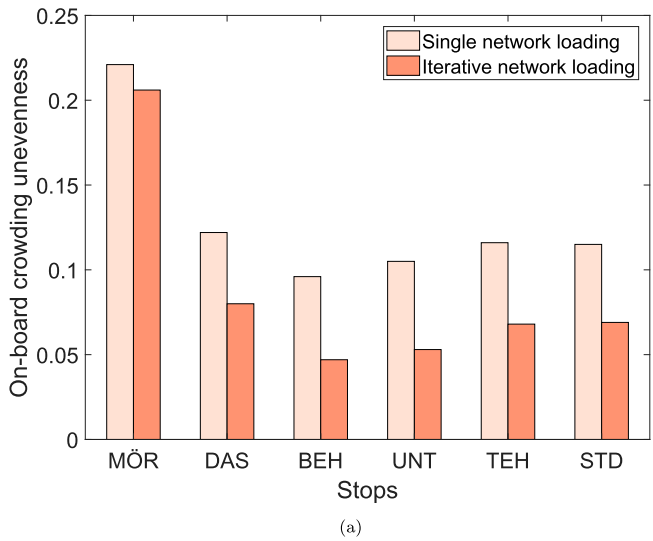
The impact of modeling car-specific perceived in-vehicle travel times on train crowding unevenness, based on the Gini coefficient, is illustrated in Figure 5(a). Lower Gini coefficient values, indicating more even average passenger distribution inside the train, are observed when car-specific perceived in-vehicle travel times are taken into account in the iterative network loading process. In particular, the average on-board crowding unevenness decreases by 4 percentage points. A  $t$ -test at 5% significance level shows that this is a statistically significant difference between single and iterative network loading. The average perceived in-vehicle time per passenger is 0.3% shorter when car-specific perceived in-vehicle travel times are accounted for in passengers' decisions; the decrease in passengers' experienced discomfort is statistically significant (Figure 5 b).

Experienced passengers are expected to alter their travel behavior aiming to minimize car-specific discomfort when train passenger volumes are higher. In increased demand conditions, experienced passengers walk more, making trade-offs between on-platform walking and on-board crowding level (Figures 6 a,b). The average walking time significantly increases by 22%, leading to passengers experiencing 3% lower on-board discomfort due to the more even passenger distribution. In particular, we find a statistically significant decrease by 7 percentage points in crowding unevenness at 5% significance level.

### 5.2. Model application

#### 5.2.1. Crowding unevenness

Figure 7(a) shows the average unevenness of boarding passengers, for the three simulated scenarios: the *Base scenario*, the *Increased demand scenario* and the *Intervention scenario* (Section 4.4). In all

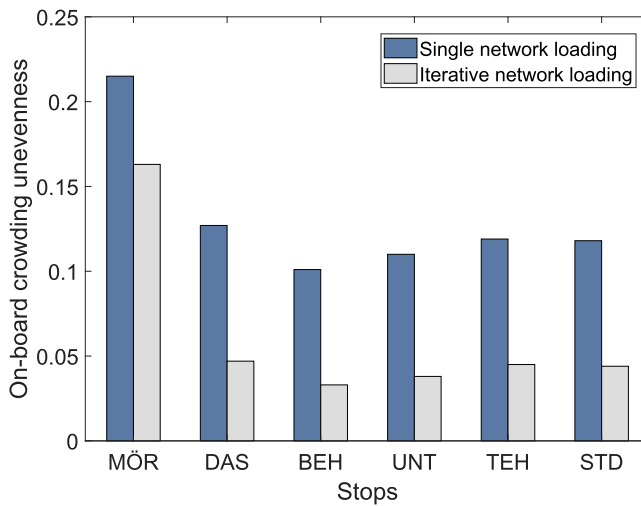


**Figure 5.** (a) Average on-board train crowding unevenness, given by the Gini coefficient; (b) Average generalized travel time components per passenger; (with and without day-to-day dynamics).

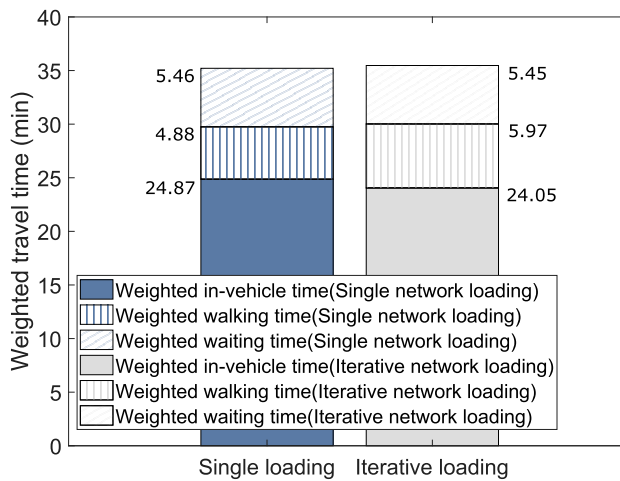
simulated scenarios, day-to-day learning is used to incorporate car-specific perceived in-vehicle travel times in passengers’ train car choices.

On average, boarding passenger unevenness drops by 5 percentage points under increased demand conditions. Considering the large passenger volumes in some of the cars of the arriving train, experienced boarding passengers are skewed towards the next closest cars, leading to a more even passenger distribution. This stems from the increasing discomfort associated with increased crowding as implied by the established values of in-vehicle time multipliers in the literature. The intervention scenario has a significant impact on the distribution of boarding passengers at DAS, leading to higher unevenness, since passengers’ preference has switched to the car located close to the single access point. Moreover, passengers’ car choice at Universitetet (UNT), a station with a single access point at the south part, is significantly affected by the intervention at the upstream





(a)



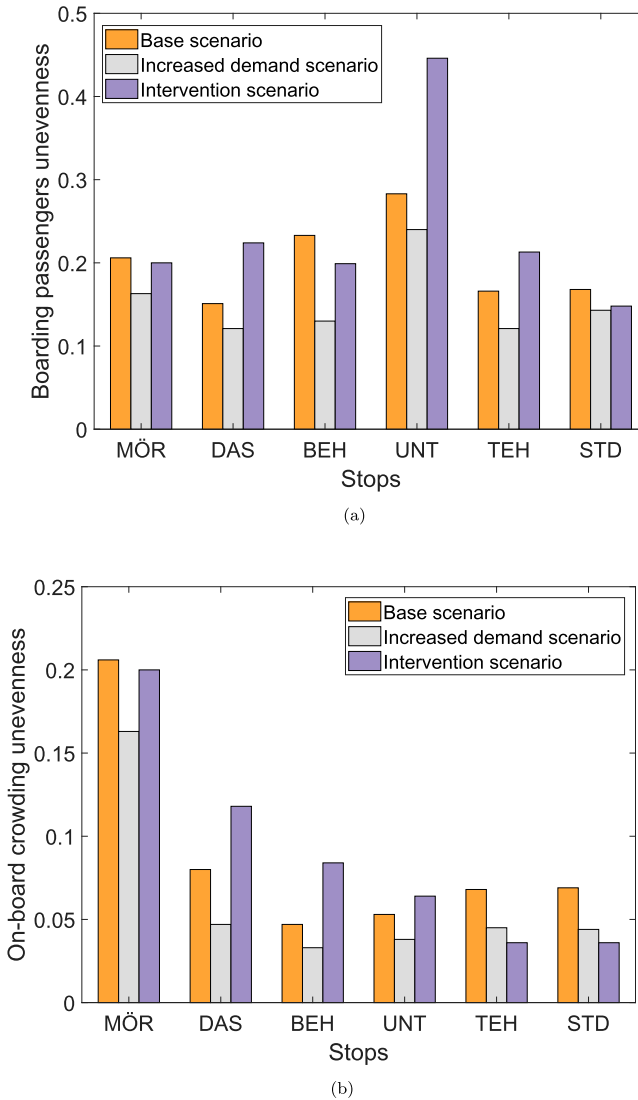
(b)

**Figure 6.** (a) Average on-board train crowding unevenness, given by the Gini coefficient; (b) Average generalized travel time components per passenger; (with and without day-to-day dynamics) in increased demand conditions (150%).

station. Experienced passengers at UNT, who have a preference for the front car located close to the access point, expect different on-board crowding conditions, i.e. less crowded front car, due to passengers’ switched car preference at DAS, which explains the highly skewed distribution of boarding passengers.

Figure 7(b) presents the average crowding unevenness on-board trains upon departure from each stop which is the result of the distribution of boarding passengers at the same stop, since walking between train cars is not possible in the Stockholm’s metro system. We find that under increased demand conditions the average train crowding unevenness decreases by 3 percentage point at the studied stops, explained by passengers’ car boarding choices shown in figure 7(a). On average, 12% of the train seats are available in the base scenario, showing a highly uneven distribution of passengers even when the total load exceeds train seated capacity. Experienced passengers adapt their car choices based on their crowding expectations and hence the seated capacity utilization increases by 5 percentage points in the increased demand scenario. Crowding unevenness is large on-board trains





**Figure 7.** Average (a) boarding passengers unevenness; (b) on-board train crowding unevenness; for base, increased demand and intervention scenarios.

departing from DAS due to the increased passengers' preference for the rear train car caused by the closure of the southern station entrance point, but it gradually decreases at the downstream stops where boarding passenger distribution is on average skewed towards the front car (Figure 7 b). The infrastructure intervention has the most significant impact at the two most crowded stops, Tekniska högskolan (TEH) and Stadion (STD), where train crowding unevenness halved with a drop of approximately 3.3 percentage points, leading to a much more even passenger load distribution (Figure 8). The impact of the infrastructure intervention arguably depends on the entrance point which is disrupted as well as the unevenness of crowding at the downstream stops.

### 5.2.2. Generalized travel cost

The average generalized time components per passenger are used to evaluate the effects of the simulated scenarios on user cost (Figure 9).

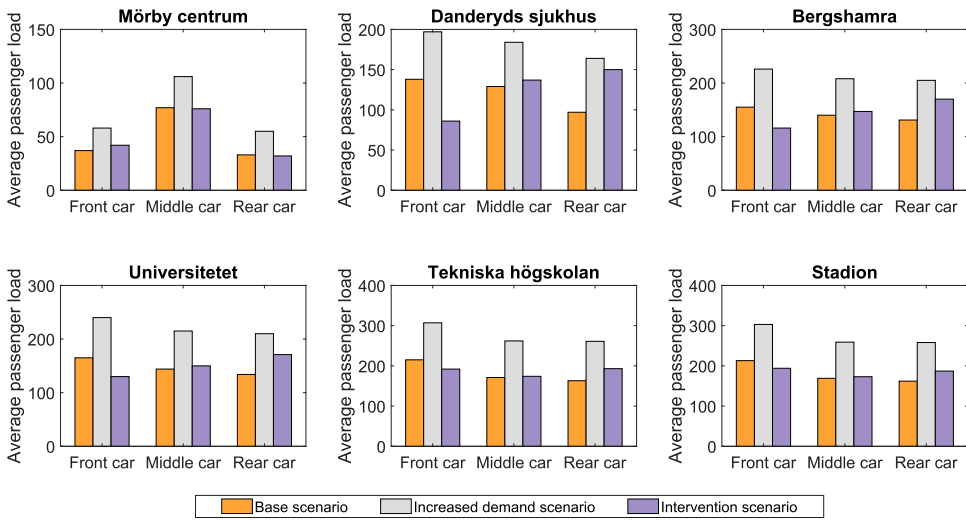


Figure 8. Average car passenger load for base, increased demand and intervention scenarios.

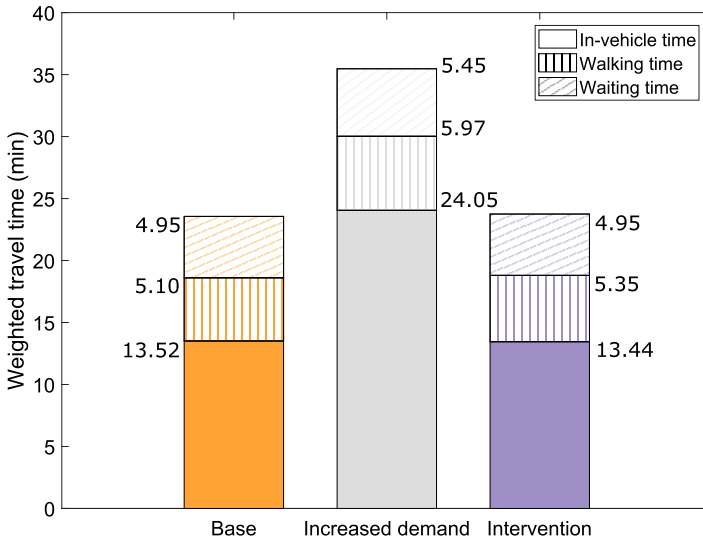


Figure 9. Average generalized travel time components per passenger for base, increased demand and intervention scenarios.

In the increased demand scenario, the average perceived in-vehicle time per passenger increases by 78%. This finding suggests that the more even passenger distribution among individual train cars, partially counteracts the increased disutility caused by higher passenger volumes. In increased demand conditions, experienced passengers walk more to attain lower on-board discomfort. The highly skewed demand distribution towards the rear train car at DAS, due to the closure of the most popular entrance point, cancels out the crowding unevenness at the downstream stops. As expected, passengers experience lower crowding discomfort at the downstream stops and consequently the average perceived in-vehicle time per passenger decreases by 0.6% when compared to the base scenario. The effectiveness of the infrastructure intervention with respect to passenger’s experienced discomfort is statistically significant at the 5% significance level.

If individual train cars were not modeled, the average generalized travel time per passenger would be 20% shorter for the Base scenario, indicating that the user cost is significantly underestimated, when passenger distribution among cars is not accounted for.

## 6. Discussion and conclusion

We propose a modeling framework that evaluates and forecasts crowding distribution on-board trains. Each train car is modeled as a separate unit with its own capacity constraints, which enables capturing on-board crowding unevenness. The path choice modeling implemented in a transit assignment simulation model has been extended to allow for the inclusion of platform section and car unit in the walking and boarding decision making process, respectively.

An application of the proposed modeling framework to a six-stop segment of the southbound direction of metro line 14 in Stockholm, where on-board crowding is on average highly skewed towards the front train car, is conducted. The developed model accounts for day-to-day dynamics, where platform section and eventually car choices are affected by passengers' expectations about car-specific on-board crowding based on the experience they gain during the course of successive network loadings. The validity of the model and its sensitivity to time valuations as well as day-to-day learning has been examined. We conclude that the model can better reproduce the average empirical car passenger load data when crowding effect is taken into account in the iterative network loading model and walking time induces a higher disutility than typically assumed in transit route choice models. This finding suggests that passengers may dislike walking along the platform more than walking from/to the station. Walking disutility may be affected by the conditions in which walking is spent and hence, it may increase with crowding on platforms and around the access points of the rail stations. There is, however, limited empirical knowledge concerning the valuation of within-station walking times under different circumstances (see suggested values and discussion in Hänseler et al. 2020). A stated-preference study by Douglas (2006) shows that the within-station walking time is valued more than 6 times in-vehicle time under very high crowding conditions along the platform, reflecting the greater effort involved.

In our case study, increased demand level reduces crowding unevenness on-board trains upon departure from the studied stops. This finding suggests that experienced passengers choose a specific car, making trade-offs between walking and waiting to minimize on-board discomfort under high demand conditions, in line with results reported by Pownall, Prior, and Segal (2008) and Kim et al. (2015). Although passengers are more evenly distributed among individual train cars when travel demand increases, the perception of in-vehicle travel time increases, due to more severe on-board car crowding conditions. The closure of a popular station entrance point due to, for example, maintenance work required, leads to uneven distribution of boarding passengers at the specific stop, which is skewed towards the single access point, but it cancels out the crowding unevenness on-board trains when departing from the downstream stops. We find that for the same demand level, passengers' experienced discomfort is lower when crowding is more evenly distributed across the train. However, the effectiveness of such infrastructure intervention is critically affected by the disrupted access point and the distribution of boarding passengers at the downstream stops. Alternative crowding management strategies, such as controlling passenger flow to station platforms through different gates (Xu et al. 2016) could be evaluated.

The developed model can be used for decision support by public transport authorities and operators at the planning stage of possible infrastructure or operational changes to evaluate their impact on crowding on-board trains. Crowding effects are evaluated by taking into account that passengers are not evenly loaded among individual cars. Emerging data sources, such as pedestrian counts and pedestrian density measurements available from cameras and sensors, can be utilized in the future to further calibrate and validate the developed car-specific transit assignment model.

The simulation model implementation is currently limited to situations where each transit stop (i.e. rail platform) is served by transit lines with a fixed number of car units per train. However, we can handle situations where trains that serve different transit lines do not have the same number of cars as long as they serve different platforms even at the same rail station. Future developments may consider parallel virtual queues for different lines and line combinations with different vehicle compositions or an integration with a pedestrian simulation model with a continuous-space representation. There is

lack of behavioral knowledge on the trade-off between walking and waiting when those are not exact substitutes, i.e. whether passengers choose to walk while waiting on the platform at the origin station as opposed to walking on the platform at the destination station. Future research on the disutility of walking when it is substituting waiting will thus allow for refining model specification. Future research should also include the modeling of passenger movements along the platform in order to evaluate on-platform crowding unevenness and its interaction with on-board crowding unevenness. Another direction for future research is to evaluate the impact of novel solutions, such as real-time crowding information systems on passengers' travel behavior and car boarding choices. Such a passenger information system has been tested in a pilot study, finding that real-time crowding information provision has a statistically significant impact on the car choice (Zhang, Jenelius, and Kottenhoff 2017). From a simulation perspective, Drabicki et al. (2020) formulated a path choice model accounting for passengers' access to crowding information at the vehicle level that is consistent with the agent-based modeling approach adopted in this study. This model was applied to Krakow public transport network, showing that real-time crowding information has the potential to reduce passengers' perceived travel disutility by 3%. However, passengers' access to car-specific crowding information is expected to result in a more efficient vehicle capacity utilization and hence, larger improvements of passengers' travel experience. Thus modeling the effects of car-specific real-time crowding information would be an interesting research direction.

## Acknowledgments

The authors would like to thank MTR Nordic for providing the metro train car load data.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

## Funding

This work was funded by Stockholm County Council (SLL), Swedish National Transport Administration (Trafikverket) and TRENOP Strategic Q2Research Area.

## ORCID

Erik Jenelius  <http://orcid.org/0000-0002-4106-3126>

Oded Cats  <http://orcid.org/0000-0002-4506-0459>

## References

- Aguilera, V., S. Allio, V. Benezech, F. Combes, and C. Milon. 2014. "Using Cell Phone Data to Measure Quality of Service and Passenger Flows of Paris Transit System." *Transportation Research Part C: Emerging Technologies* 43 (2): 198–211.
- Alexander, L., S. Jiang, M. Murga, and M. C. González. 2015. "Origin-destination Trips by Purpose and Time of Day Inferred From Mobile Phone Data." *Transportation Research Part C: Emerging Technologies* 58: 240–250.
- Alsger, A., B. Assemi, M. Mesbah, and L. Ferreira. 2016. "Validating and Improving Public Transport Origin-Destination Estimation Algorithm Using Smart Card Fare Data." *Transportation Research Part C: Emerging Technologies* 68: 490–506.
- Bachir, D., G. Khodabandelou, V. Gauthier, M. El Yacoubi, and J. Puchinger. 2019. "Inferring Dynamic Origin-Destination Flows by Transport Mode Using Mobile Phone Data." *Transportation Research Part C: Emerging Technologies* 101: 254–275.
- Cats, O. 2013. "Multi-Agent Transit Operations and Assignment Model." *Procedia Computer Science* 19: 809–814.
- Cats, O., W. Burghout, T. Toledo, and H. N. Koutsopoulos. 2010. "Mesoscopic Modeling of Bus Public Transportation." *Transportation Research Record: Journal of the Transportation Research Board* 2188 (1): 9–18.
- Cats, O., and Z. Gkioulou. 2017. "Modelling the Impacts of Public Transport Reliability and Travel Information on Passengers-Waiting Time Uncertainty." *EURO Journal on Transportation and Logistics* 6 (3): 247–270.
- Cats, O., and M. Hartl. 2016. "Modelling Public Transport On-board Congestion: Comparing Schedule-based and Agent-based Assignment Approaches and Their Implications." *Journal of Advanced Transportation* 50: 1209–1224.
- Cats, O., H. N. Koutsopoulos, W. Burghout, and T. Toledo. 2011. "Effect of Real-Time Transit Information on Dynamic Path Choice of Passengers." *Transportation Research Record: Journal of the Transportation Research Board* 2217: 46–54.

- Cats, O., and J. West. 2020. "Learning and Adaptation in Dynamic Transit Assignment Models for Congested Networks." *Transportation Research Record: Journal of the Transportation Research Board* 2674 (1): 113–124.
- Cats, O., J. West, and J. Eliasson. 2016. "A Dynamic Stochastic Model for Evaluating Congestion and Crowding Effects in Transit Systems." *Transportation Research Part B: Methodological* 89: 43–57.
- Cepeda, M., R. Cominetti, and M. Florian. 2006. "A Frequency-based Assignment Model for Congested Transit Networks with Strict Capacity Constraints: Characterization and Computation of Equilibria." *Transportation Research Part B: Methodological* 40: 437–459.
- Chen, C., J. Ma, Y. Susilo, Y. Liu, and M. Wang. 2016. "The Promises of Big Data and Small Data for Travel Behavior (AKA Human Mobility) Analysis." *Transportation Research Part C: Emerging Technologies* 68: 285–299.
- Douglas, N. 2006. "Value and Demand Effects of Rail Service Attributes." *Report to RailCorp*, Sydney, Australia.
- Drabicki, A., R. Kucharski, O. Cats, and A. Szarata. 2020. "Modelling the Effects of Real-Time Crowding Information in Urban Public Transport Systems." *Transportmetrica A: Transport Science* 17 (4): 675–713.
- Fu, Q., R. Liu, and S. Hess. 2012. "A Review on Transit Assignment Modelling Approaches to Congested Networks: A New Perspective." *Procedia - Social and Behavioral Sciences* 54: 1145–1155.
- Hamdouch, Y., and S. Lawphongpanich. 2008. "Schedule-based Transit Assignment Model with Travel Strategies and Capacity Constraints." *Transportation Research Part B: Methodological* 42: 663–684.
- Hänseler, F. S., J. P. A. van den Heuvel, O. Cats, W. Daamen, and S. P. Hoogendoorn. 2020. "A Passenger-Pedestrian Model to Assess Platform and Train Usage From Automated Data." *Transportation Research Part A: Policy and Practice* 132: 948–968.
- Ingvardson, J. B., O. A. Nielsen, S. Raveau, and B. F. Nielsen. 2018. "Passenger Arrival and Waiting Time Distributions Dependent on Train Service Frequency and Station Characteristics: A Smart Card Data Analysis." *Transportation Research Part C: Emerging Technologies*. 90: 292–306.
- Khani, A., M. Hickman, and H. Noh. 2015. "Trip-Based Path Algorithms Using the Transit Network Hierarchy." *Networks and Spatial Economics* 15: 635–653.
- Kim, K. M., S. P. Hong, S. J. Ko, and D. Kim. 2015. "Does Crowding Affect the Path Choice of Metro Passengers?" *Transportation Research Part A: Policy and Practice* 77: 292–304.
- Kim, H., S. Kwon, S. K. Wu, and K. Sohn. 2014. "Why Do Passengers Choose a Specific Car of a Metro Train During the Morning Peak Hours?" *Transportation Research Part A: Policy and Practice* 61: 249–258.
- Krstanoski, N. 2014. "Modelling Passenger Distribution on Metro Station Platform." *International Journal for Traffic and Transport Engineering* 4 (4): 456–465.
- Lam, W. H. K., Z. Y. Gao, K. S. Chan, and H. Yang. 1999. "A Stochastic User Equilibrium Assignment Model for Congested Transit Networks." *Transportation Research Part B: Methodological* 33: 351–368.
- Leurent, F. 2010. "On Seat Capacity in Traffic Assignment to a Transit Network." *Journal of Advanced Transportation* 46: 112–138.
- Liu, Z., D. Li, and X. Wang. 2016. "Passenger Distribution and Waiting Position Selection Model on Metro Station Platform." *International Conference on Civil, Structure, Environmental Engineering (I3CSEE 2016)*, Guangzhou, China.
- Márquez, L., J. V. A. Alfonso, and J. C. Poveda. 2019. "In-vehicle Crowding: Integrating Tangible Attributes, Attitudes, and Perceptions in a Choice Context Between BRT and Metro." *Transportation Research Part A: Policy and Practice* 130: 452–465.
- Munizaga, M. A., and C. Palma. 2012. "Estimation of a Disaggregate Multimodal Public Transport Origin-Destination Matrix From Passive Smartcard Data From Santiago, Chile." *Transportation Research Part C: Emerging Technologies* 24: 9–18.
- Muñoz, J. C., J. Soza-Parra, A. Didier, and C. Silva. 2018. "Alleviating a Subway Bottleneck Through a Platform Gate." *Transportation Research Part A: Policy and Practice* 116: 446–455.
- Nguyen, S., S. Pallottino, and F. Malucelli. 2001. "A Modeling Framework for Passenger Assignment on a Transport Network with Timetables." *Transportation Science* 35: 238–249.
- Nuzzolo, A., F. Russo, and U. Crisalli. 2001. "A Doubly Dynamic Schedule-based Assignment Model for Transit Networks." *Transportation Science* 35: 268–285.
- Papola, N., F. Filippi, G. Gentile, and L. Meschini. 2009. "Schedule-based Transit Assignment: A New Dynamic Equilibrium Model with Vehicle Capacity Constraints." In *Schedule-based Modeling of Transportation Networks: Theory and Applications*, edited by N. H. M. Wilson and A. Nuzzolo, 145–170. Springer, US.
- Peftitsi, S., E. Jenelius, and O. Cats. 2020. "Determinants of Passengers' Metro Car Choice Revealed Through Automated Data Sources: A Stockholm Case Study." *Transportmetrica A: Transport Science* 16 (3): 529–549.
- Poon, M. H., S. C. Wong, and C. O. Tong. 2004. "A Dynamic Schedule-based Model for Congested Transit Networks." *Transportation Research Part B: Methodological* 38: 343–368.
- Pownall, C., M. Prior, and J. Segal. 2008. "What Rail Passengers Will Do to Get a Seat." *Proceedings of European Transport Conference*, Leiden, The Netherlands.
- Ranjbari, A., M. Hickman, and Y. C. Chiu. 2020. "Network Design with Elastic Demand and Dynamic Passenger Assignment to Assess the Performance of Transit Services." *Journal of Transportation Engineering, Part A: Systems* 146 (5): 04020030.
- Raveau, S., Z. Guo, J. C. Muñoz, and N. H. M. Wilson. 2014. "A Behavioral Comparison of Route Choice on Metro Networks: Time, Transfers, Crowding, Topology and Socio-demographics." *Transportation Research Part A: Policy and Practice* 66: 185–195.

- Rexfelt, O., T. Schelenz, M. Karlsson, and A. Suescun. 2014. "Evaluating the Effects of Bus Design on Passenger Flow: Is Agent-based Simulation a Feasible Approach?" *Transportation Research Part C: Emerging Technologies* 38: 16–27.
- Schmöcker, J. D., M. G. H. Bell, and F. Kurauchi. 2008. "A Quasi-dynamic Capacity Constrained Frequency-based Transit Assignment Model." *Transportation Research Part B: Methodological* 42 (10): 925–945.
- SL. 2017. "Fakta om SL och länet 2016." *Storstockholms Lokaltrafik*. Diarienummer: S–2016–0762. (In Swedish).
- Sohn, K. 2013. "Optimizing Train-stop Positions Along a Platform to Distribute the Passenger Load More Evenly Across Individual Cars." *IEEE Transactions on Intelligent Transportation Systems* 14 (2): 994–1002.
- Spieß, H., and M. Florian. 1989. "Optimal Strategies: A New Assignment Model for Transit Networks." *Transportation Research Part B: Methodological* 23 (2): 83–102.
- Sumalee, A., Z. Tana, and W. H. K. Lam. 2009. "Dynamic Stochastic Transit Assignment with Explicit Seat Allocation Model." *Transportation Research Part B: Methodological* 43: 895–912.
- Szplett, D., and S. C. Wirasinghe. 1984. "An Investigation of Passenger Interchange and Train Standing Time At LRT Stations: (i) Alighting, Boarding and Platform Distribution of Passengers." *Journal of Advanced Transportation* 18 (1): 1–12.
- Tirachini, A., D. A. Hensher, and J. M. Rose. 2013. "Crowding in Public Transport Systems: Effects on Users, Operation and Implications for the Estimation of Demand." *Transportation Research Part A: Policy and Practice* 53: 36–52.
- Tirachini, A., R. Hurtubia, T. Dekker, and R. A. Daziano. 2017. "Estimation of Crowding Discomfort in Public Transport: Results From Santiago De Chile." *Transportation Research Part A: Policy and Practice* 103: 311–326.
- Toledo, T., O. Cats, W. Burghout, and H. N. Koutsopoulos. 2010. "Mesoscopic Simulation for Transit Operations." *Transportation Research Part C: Emerging Technologies* 18: 896–908.
- Toole, J. L., S. Colak, B. Sturt, L. P. Alexander, A. Evsukoff, and M. C. González. 2015. "The Path Most Traveled: Travel Demand Estimation Using Big Data Resources." *Transportation Research Part C: Emerging Technologies* 58: 162–177.
- TRB. 2014. "Transit Capacity and Quality of Service Manual". *Transit Cooperative Highway Research Program (TCRP) Report 165*. Transportation Research Board.
- Wahba, M., and A. Shalaby. 2005. "Multiagent Learning-Based Approach to Transit Assignment Problem." *Transportation Research Record: Journal of the Transportation Research Board* 1926: 96–105.
- Wardman, M. 2004. "Public Transport Values of Time." *Transport Policy* 11 (4): 363–377.
- Wardman, M., and G. Whelan. 2011. "Twenty Years of Rail Crowding Valuation Studies: Evidence and Lessons From British Experience." *Transport Reviews* 31 (3): 379–398.
- Xu, X.-Y., J. Liu, H.-Y. Li, and M. Jiang. 2016. "Capacity-Oriented Passenger Flow Control Under Uncertain Demand: Algorithm Development and Real-World Case Study." *Transportation Research Part E: Logistics and Transportation Review* 87: 130–148.
- Zhang, Q., B. Han, and D. Li. 2008. "Modeling and Simulation of Passenger Alighting and Boarding Movement in Beijing Metro Stations." *Transportation Research Part C: Emerging Technologies* 16: 635–649.
- Zhang, Y., E. Jenelius, and K. Kottenhoff. 2017. "Impact of Real-time Crowding Information: A Stockholm Metro Pilot Study." *Public Transport* 9 (3): 483–499.