INVESTIGATION OF
FLASHING INDUCED INSTABILITIES
IN BWRS
- SIMULATIONS AND EXPERIMENTS -

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Master's thesis

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Abstract

Flashing induced instabilities in a BWR test-facility (CIRCUS) have been investigated. A steady-state model of the thermo-hydraulic behavior of a simplified natural circulation cooled BWR setup has been derived. An asymmetric power distribution in the core leads to asymmetric boiling boundaries in the core and small differences in flow-rate in the core-channels. However, the natural circulation is mainly driven by density differences between the riser section and the downcomer section, keeping the differences in flow-rate small.

The time dependent flow-rate in the core-channels has been measured by means of LDA during unstable conditions of the CIRCUS facility, a steam-water simulator operating at 1.2 bar. Various asymmetric power distributions (leading to a difference in specific energy at the core exit up to 1 MJ/kg) have been studied under several core inlet temperatures, ranging from 98°C – 102°C. Increasing inlet temperature and total power leads to a destabilizing of CIRCUS at the particular points measured. Increasing asymmetry in power distribution leads to increasing flow-rate differences between the two core channels measured. The steady-state simulations overestimate the experiments, but the trends in flow-rate at increasing inlet temperature and total power are similar.
# Symbols

## Roman symbols

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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<td>$a$</td>
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</tr>
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<td>( R )</td>
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<tr>
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<tr>
<td>( v )</td>
<td>velocity</td>
<td>( ms^{-1} )</td>
</tr>
<tr>
<td>( v' )</td>
<td>velocity after trend removal</td>
<td>( ms^{-1} )</td>
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<tr>
<td>( \bar{v} )</td>
<td>average velocity</td>
<td>( ms^{-1} )</td>
</tr>
<tr>
<td>( V )</td>
<td>volume</td>
<td>( m^3 )</td>
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<tr>
<td>( w )</td>
<td>energy per second falling on detector</td>
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<tr>
<td>( w )</td>
<td>work</td>
<td>( W )</td>
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<td>distances at which cone of interference intersects tube wall</td>
<td>((m, m))</td>
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<td>( W_{Hen} )</td>
<td>Hanning window</td>
<td>—</td>
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<td>quality</td>
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Greek symbols

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<td>angle</td>
<td>( rad )</td>
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<tr>
<td>( \alpha )</td>
<td>void fraction</td>
<td>—</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>angle between directions of light wave and moving object</td>
<td>( rad )</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>idem for scattered light towards detector and moving object</td>
<td>( rad )</td>
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<td>( \gamma )</td>
<td>angle</td>
<td>( rad )</td>
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<td>(( rad, rad ))</td>
</tr>
<tr>
<td>(( \gamma_{w,1}, \gamma_{w,2} ))</td>
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<td>(( rad, rad ))</td>
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<tr>
<td>( \gamma_{max} )</td>
<td>maximum angle for detectable interference pattern</td>
<td>( rad )</td>
</tr>
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</table>
δ width of slit in annular flow channel $m$

$\delta F_w$ deviation between focal point of lens in water and axis tube $m$

$\delta h$ enthalpy iteration step $Jkg^{-1}$

$\delta v$ velocity fluctuation $ms^{-1}$

$\delta z$ spatial iteration step $m$

$\delta \Phi_m$ flow-rate iteration step $kgs^{-1}$

$\Delta f$ filter range $Hz$

$\Delta p$ pressure difference $Pa$

$\Delta t$ time interval $s$

$\Delta t_s$ sampling time $s$

$\Delta z$ difference in height $m$

ε minimum tolerance for iteration convergence $Jkg^{-1}, Pa$

η dynamic viscosity $kgm^{-1}s^{-1}$

η quantum efficiency $-$

θ angle $rad$

θ$_B$ Bragg angle $rad$

λ mean rate of occurrence of an event $Hz$

λ$_0$ wavelength of laser light $m$

Λ acoustic wavelength $m$

ρ density $kgm^{-3}$

ρ$_m$ density of homogeneous mixture $kgm^{-3}$

σ$_a$ standard deviation of Fourier series coefficient $ms^{-1}$

σ$_v$ standard deviation of velocity $ms^{-1}$

τ travelling time $s$

τ$_w$ wall shear-stress $kgm^{-1}s^{-2}$

Φ$_m$ mass-flow-rate $kgs^{-1}$

Φ$_{mg}$ gas mass-flow-rate $kgs^{-1}$

Φ$_v$ volumetric flow-rate $m^3s^{-1}$

Φ$_{vav}$ period averaged volumetric flow-rate $m^3s^{-1}$

### Subscripts

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<td>(channel) 1,2 related quantity</td>
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<tr>
<td>1φ</td>
<td>one-phase related quantity</td>
</tr>
<tr>
<td>2φ</td>
<td>two-phase related quantity</td>
</tr>
<tr>
<td>a</td>
<td>quantity in air</td>
</tr>
<tr>
<td>acc</td>
<td>acceleration related quantity</td>
</tr>
<tr>
<td>b</td>
<td>iteration step parameter</td>
</tr>
<tr>
<td>bb</td>
<td>boiling boundary</td>
</tr>
<tr>
<td>fb</td>
<td>flashing boundary</td>
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<tr>
<td>bent</td>
<td>bent related quantity</td>
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<tr>
<td>c</td>
<td>core related quantity</td>
</tr>
<tr>
<td>ce</td>
<td>core-exit related quantity</td>
</tr>
<tr>
<td>ci</td>
<td>core-inlet related quantity</td>
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</table>
$d$  downcomer related quantity
$d_1$  first section of downcomer related quantity
$d_2$  second section of downcomer related quantity
$di$  downcomer-inlet related quantity
$eq$  equivalent related quantity
$eq$  quantity under equilibrium conditions
$f$  quantity at fluid saturation conditions
$f_{ric}$  friction related quantity
$fuel$  fuel-assembly related quantity
$g$  quantity at gas saturation conditions
$grav$  gravitation related quantity
$h$  heat-exchanger related quantity
$i$  index
$j$  index
$k$  index
$l$  liquid related quantity
$max$  maximum of a quantity
$min$  minimum of a quantity
$n$  index
$r$  riser related quantity
$re$  riser-exit related quantity
$valve$  valve-assembly related quantity
$w$  quantity in water

Miscellaneous

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<tr>
<td>CIRCUS</td>
<td>CIRCulation during Start-up</td>
</tr>
<tr>
<td>D</td>
<td>Detector</td>
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<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>O</td>
<td>Origin</td>
</tr>
<tr>
<td>P</td>
<td>Particle</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>S</td>
<td>Source</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>TBD</td>
<td>Time Between Data</td>
</tr>
<tr>
<td>W</td>
<td>Wall</td>
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Chapter 1

Introduction

1.1 Background of the project

Many countries in the world today operate nuclear power plants to provide the necessary electricity. The most commonly used nuclear reactors, the so-called Light Water Reactors (LWR), can be divided into two main groups: the Pressurised Water Reactor (PWR) and the Boiling Water Reactor (BWR). A schematic representation of a PWR and a BWR is given in figure 1.1.

Figure 1.1: A schematic representation of a PWR (a) and a BWR (b) respectively.

A Pressurised Water Reactor has two separate loops: the primary and secondary loop. The pressure of the coolant in the primary loop is set high enough (150 bar) to prevent the coolant from boiling. Heat is transported from the primary, high pressure loop to the secondary, low pressure loop. The heat from the primary loop is fed to the secondary loop by means of a heat exchanger. In the secondary loop the pressure is lower than the pressure in the primary loop, in order to turn the coolant into steam. The steam is used to drive the steam turbines. These turbines, in turn, drive an electric generator to generate the wanted electricity. A condenser
condenses steam back to water. Feed-water pumps are used to recirculate the water through both loops.

The configuration of a Boiling Water Reactor looks similar to the configuration of a Pressurised Water Reactor, but there is a main difference: a BWR has only one loop of relatively low pressure (75 bar). Again heat is produced in the reactor core. This heat directly turns the coolant (water) into steam. This steam is used to drive the turbines. The coolant is recirculated by a feed-water pump. The focus in this thesis will be on two-phase flow phenomena in this type of nuclear reactors. A further detailed description of a Boiling Water Reactor will be given.

![Diagram of BWR recirculating loop](image)

**Figure 1.2: The configuration of the BWR recirculating loop.**

The nuclear fission reactions take place inside more than 700 fuel bundles, located in the reactor core. A fuel bundle itself consists of a lattice of typically 8 × 8 fuel rods. These rods are rather thin, approximately 1 cm in diameter, and long, about 4 meters in length. Each individual rod has a cladding of zircaloy, which confines the radioactive products released by the uranium-dioxide ($UO_2$) fuel. The heat produced by nuclear fission is removed by the coolant (ordinary water $H_2O$).

The principle of operation of a BWR design is clarified in figure 1.2. In this figure the one-phase coolant enters the core where nuclear fission reactions take place. Normally, the coolant enters the core several degrees below saturation temperature. As the coolant flows upwards along the the fuel assemblies, the temperature increases, and the coolant starts to boil. At the core outlet, about 10% of the coolant is turned into steam. This steam-water mixture flows further upwards through the steam separators, where steam is separated from water. The steam is directed via steam dryers to a cascade of turbines. The turbines drive electric generators which
produces the needed electricity. The steam is condensed and enters the reactor vessel at the feed-water inlet. The water, which is separated from the steam, flows downwards in the periphery (the so-called downcomer) of the vessel and mixes with the steam-condensate.

The thermal power production of a BWR is typically 3600 MWth. A typical efficiency of a BWR is 33%, leading to an electric output of 1200 MWe. This huge amount of energy is produced in a relatively small core volume of about 65 m$^3$. This leads to an average core power density of 55 kW/l. Therefore, core cooling should be guaranteed under all circumstances, to avoid excessively high fuel temperatures.

Core cooling in a typical BWR presented in figure 1.2 is partly caused by density differences between the low density core region (steam is present) and the high density downcomer region (only one-phase coolant is present). This density difference causes a hydrostatic pressure difference between the downcomer region and the core region. This pressure difference gives rise to a 'spontaneous' circulation of the coolant through the loop. This natural circulation is limited because of frictional pressure losses in the loop (wall friction, restrictions, etcetera). At a certain power level, the natural circulation no longer increases but saturates. Therefore, pumps are used in all BWRs to force the coolant through the core to guarantee adequate cooling.

Passive safety is one of the main concepts in the newer designs of nuclear reactors. A passive safe design does not have to rely on active components (recirculating pumps) or by active controllers (reactor operators), this design is governed by natural occurring processes. Core cooling through natural circulation leads to an increase of the safety of a BWR and to lower operational costs.

The Dutch nuclear power plant called 'Dodewaard' is a prototype of a Boiling Water Reactor making use of the natural circulation principle. The reactor operated from 1968 until it was shut down in 1997. A cut-away view of the Dodewaard vessel can be seen in figure 1.3. The main difference between the Dodewaard BWR and a conventional designed BWR is that an unheated riser is placed on top of the core region. This riser section lengthens the region with two-phase flow, giving an increase of the pressure difference between the downcomer region and the core-riser region. In this way it is made possible to achieve higher flow-rates needed during normal operation and the need for recirculating pumps is eliminated. Also the steam separators are not present in this design. Steam separation takes place at the free water surface. Not all steam escapes at the surface, about 20% is dragged into the downcomer channel by the down falling liquid. This phenomenon is called carry under. Carry under has a negative effect on the natural circulation flow rate, since it reduces the pressure difference between the downcomer and core-riser.

After unexpected large amplitude power oscillations observed in the Caorso (1984) and the LaSalle (1988) plants, much research has been performed on the dynamic stability of conventional Boiling Water Reactors. The instability issue in natural circulation cooled BWRs is more complicated since the flow-rate is not an independent variable anymore (as is the case in forced circulation cooled BWRs). Also many measurements were taken from the Dodewaard reactor under start up conditions.
by [Van der Hagen and Stekelenburg, 1997]. From these measurements it turned out that the physics of a natural circulation loop, operating at moderate power is not well understood. So-called flashing induced instabilities can occur at these conditions, which are encountered during the start-up process of a natural circulation cooled BWR.

The stability of a Boiling Water Reactor at start-up conditions is the starting point for this project. In the next section the objectives to be achieved in this project are described in view of unstable conditions of a Boiling Water Reactor. A BWR becomes unstable if small perturbations in the flow-rate grow at a certain frequency. These oscillations in flow-rate saturate due to non-linear effects (such as friction).

### 1.2 Objectives and tasks

The following objectives are pursued:

- Understanding in the physics of flow hydrodynamics at low power, low pressure conditions in a Boiling Water Reactor;

- Acquire knowledge in the division of the total flow-rate over asymmetrically heated parallel boiling channels (by means of interpreting the results obtained from the measurements);
1.3 Outline

- Provide experimental information about flow-rate behaviour in heated boiling channels for future experiments (symmetric power distribution) and simulations (asymmetric power distribution).

These objectives are reached by making use of the following tasks:

- Modelling a steady-state model which simulates natural circulation in a Boiling Water Reactor;
- Design and test an Laser Doppler Anemometry (LDA) setup by making use of the forward scattering reference beam technique;
- Measure simultaneously the velocity (flow-rate) of the coolant in two boiling channels in the core section of an unstable Boiling Water Reactor, with different power settings in the heated rods (by means of LDA).

1.3 Outline

The thesis is structured as follows:

Two types of instabilities occurring in BWRs are described in chapter 2. A physical explanation is given for these instabilities which can be mapped in a so-called stability map. Some physical properties turn out to be important for the occurrence of these instabilities. By combining these properties into two dimensionless numbers, the stability map can be built.

In order to understand the natural circulation process, a steady-state model has been developed in chapter 3. The model has been derived using a very simplified representation of a Boiling Water Reactor loop consisting of two pairs of parallel fuel channels which can be heated separately. Some results of the simulations are presented and discussed in chapter 4.

Chapter 5 focuses on an experimental non-intrusive measurement method which is used for studying differences in flow-rate behaviour in two fuel channels which have different power settings. This method is called Laser Doppler Anemometry (LDA). By making use of the principle of the so-called Doppler shift, it is possible to measure the velocity at a point in the flow.

To study the instabilities of a BWR, an experimental facility called CIRCUS (CIRCulation during Start-up) has previously been constructed. It is described in chapter 6. To measure velocities in two core-channels simultaneously, two equal LDA-setups have been built and one of them is described in this chapter.

Several measurements have been performed on the CIRCUS-facility during different conditions of the reactor. In chapter 7 the processing of data to time-series of the velocity and the analysis of this time-series are handled.

Chapter 8 gives an overview of all measurements performed and its results of the flow-rates in the two channels.

Finally, in chapter 9 conclusions are drawn. Also some recommendations for improvement and future work are given.
Chapter 2

The stability of a Boiling Water Reactor

2.1 Classification of BWR instabilities

This section discusses the physical mechanisms that may cause instabilities in BWRs. More detailed information about the stability of a BWR under start-up conditions can be read in [Van Bragt, 1998].

The most important instabilities in BWRs are purely thermohydraulic and coupled neutronic-thermohydraulic instabilities, respectively. These instabilities are basically induced by two-phase flow 'density waves' in the BWR. Following [Van Bragt, 1998], the mechanism causing density-wave oscillations is depicted, in figure 2.1.

![Figure 2.1: Illustration of the local pressure drop delay introduced by the density-wave mechanism.](image)

In the figure a boiling channel is taken with a constant pressure difference between core-inlet and core-outlet. If the inlet flow is reduced at constant heating power, this will lead to an increased production of voids in the channel. These voids travel upwards as a packet of lower density. This travelling 'density wave' causes changes in the local pressure drop (or gradient) at higher positions. These changes are delayed with respect to the change of the flow rate, because of the finite velocity of the
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Propagating wave. Especially at higher positions, where the volumetric amount of steam is high, these frictional pressure drops becomes very large. Hence, if the flow-rate is perturbed sinusoidally, as can be seen in figure 2.1, the total pressure drop will also change sinusoidally, but with a certain time delay. If the total pressure drop is delayed exactly 180° with respect to the flow-rate, as in figure 2.1, a decrease in inlet flow results in an increase of total pressure drop (and vice versa). So, perturbations of the inlet flow thus receive a positive feedback and the oscillations will grow, the system gets unstable.

This type of instability is known in literature as the so-called Type-II instability which has been studied extensively in the past ([Bouré et al., 1971], [Fukuda and Kobori, 1979]). Van Bragt developed an analytical model, which predicts this kind of instability, also measurements performed at the Dodewaard reactor show this Type-II instability, [Van Bragt, 1998]. According to the explanation given in the previous paragraph, this instability is mainly caused by friction. The Type-II instability can occur during low-flow/high-power conditions.

The second type of instability, only relevant for natural circulation cooled BWRs, is called the Type-I instability. Instead of the frictional pressure drop being the cause for the Type-II instability, the gravitational (hydrostatic) pressure drop is the cause for the Type-I instability. The Type-I instability mechanism becomes dominant in natural circulation reactors operating at low-power/low-pressure conditions, e.g. during the reactor startup. Under these circumstances, the flow quality (or mass percentage of steam), \( x \), defined as:

\[
x \equiv \frac{\Phi_m}{\Phi} \tag{2.1}
\]
where $\Phi_m$ is the vapour mass flow and $\Phi_m$ is the total mass flow, becomes very small. But under low-pressure conditions, the void-fraction ($\alpha$) strongly depends on the flow quality, see figure 2.2. The void fraction is defined as:

$$\alpha \equiv \frac{V_g}{V} \quad (2.2)$$

where $V_g$ is the gas volume in the total volume, $V$.

So, a small quality at the exit of the core corresponds to a high void fraction under low-pressure conditions. During these conditions, a small decrease in inlet flow leads to a strong increase of void production. In a natural circulation reactor, a low-density wave will travel through the riser enhancing the driving force, leading to an increase of inlet flow. Then the opposite process occurs, the void fraction in the riser decreases, leading to a decrease of the inlet flow again. This completes a cycle of a Type-I oscillation. Again, these oscillations will grow if a perturbation of the inlet flow receives a positive feedback from the total pressure drop.

The Type-I and Type-II instabilities are purely thermohydraulic instabilities. Instabilities can also be caused by coupled neutronic-thermohydraulic processes. The coolant in BWRs acts as a neutron moderator. After each fission, high-energy neutrons are released. These neutrons lose their kinetic energy as they collide with the coolant nuclei (the moderator) and thus have an increased probability of causing a new fission. If the moderator density decreases, due to void production, the neutrons will leave the core or will be absorbed. This leads to a decrease of the fission rate. This negative feedback mechanism leads to a very stable BWR system. However, at certain frequencies, the negative feedback in a BWR can have a destabilizing effect, in particular when the feedback processes occur with a certain time delay. Coupled neutronic-thermohydraulic instabilities will not occur in the CIRCUS-facility, since no nuclear fission takes place in the reactor core, instead the rods are electrically heated.

2.2 'Flashing-induced' Type-I oscillations

Usually, theoretical models assume constant physical properties in the flow loop. This assumption is true for normal operating pressures (~ 75 bar) and leads to accurate results. At high pressures, the saturation temperature, $T_{\text{sat}}$, is approximately constant in the flow loop. But at low-pressures, the saturation temperature is strongly dependent on the pressure level, see figure 2.3 for an illustration.

Figure 2.3a illustrates how the temperature of the liquid, $T_l$, increases in the core section due to heating and reaches the saturation temperature, $T_{\text{sat}}$. The fluid starts to boil at that position in the core where the temperature of the liquid is equal to the saturation temperature. At higher positions in the core and riser, the temperature of the two-phase mixture remains constant.

Figure 2.3b illustrates the effect of decreasing saturation temperature due to decreasing hydrostatic pressure along the flow path. Boiling is reached at a lower
Chapter 2. The stability of a Boiling Water Reactor

(a) high pressure

(b) low pressure

(c) low pressure & low power

Figure 2.3: Influence of the pressure dependence on the saturation temperature, $T_{\text{sat}}$, on the steam production in a natural circulation BWR. (a): At high pressures, $T_{\text{sat}}$ is approximately constant. The fluid starts to boil in the core. (b): At low pressures and high power, $T_{\text{sat}}$ decreases along the flow path due to decreasing hydrostatic pressure. The fluid starts to boil at a lower position in the core with respect to (a). (c): At low pressures and low power, the fluid suddenly starts to boil in the unheated riser: 'flashing'.

This effect becomes extremely important if the power in the core is low. The core is not able to turn water into steam, since the boiling point is not reached. In the riser the temperature of the one-phase fluid remains constant. But, the saturation temperature keeps decreasing along the riser flow path. At a specific position in the riser, the temperature of the fluid reaches the saturation temperature. This leads to a sudden void production without adding any heat to the fluid. This is illustrated in figure 2.3c. From this point on, the term 'flashing' refers to sudden void production in the riser. Flashing is not likely to occur at high pressures, since the saturation temperature is approximately independent of the axial position. Void production in the riser directly affects the gravitational pressure drop over this section, leading to an increase of the driving force. The flow-rate will increase leading to a decrease of the temperature at the exit of the core and thus in the riser. If this decrease is strong enough, the liquid will no longer reach the saturation temperature. The void-production in the riser dies out and the flow-rate decreases again. This process repeats itself, leading to self-sustained flow-oscillations. Hence, it can be expected that 'flashing' contributes to (and amplifies) the Type-I mechanism. Thus, flow oscillations caused by flashing are called: 'flashing-induced' Type-I oscillations.

The flashing phenomenon has been studied by for instance [Furuya et al., 1995],

position with respect to figure 2.3a. The (saturation) temperature keeps decreasing at higher positions, leading an increase of steam production. This effect is known as void 'flashing'.
2.3. The stability map

[Van Bragt, 1998] and [Manera et al., 2000]. Van Bragt developed an analytical model including flashing-induced instabilities. Manera reports experiments done on the CIRCUS-facility showing unstable flow oscillations, due to the flashing-induced Type-I oscillations.

2.3 The stability map

Mapping of the Type-I and Type-II instabilities mentioned in the previous sections, can be done in the so-called Zuber-Subcooling plane (introduced by Ishii and Zuber [Ishii and Zuber, 1970]). This plane is widely used to represent the stability of BWRs. The stability map is built by two dimensionless numbers: the Zuber number, \( N_{zu} \), and the subcooling number, \( N_{sub} \). The Zuber number is defined as:

\[
N_{zu} = \frac{P_c}{\Phi_m h_{ev,ci}} \cdot \frac{\rho_{f,ci} - \rho_{g,ci}}{\rho_{g,ci}}
\]

where \( P_c \) is the power produced in the core, \( h_{ev,ci} \) the heat of evaporation at the core inlet, \( \rho_{f,ci} \) the density of the fluid at saturation at the core inlet and \( \rho_{g,ci} \) the density of the vapour at saturation at the core inlet. The Zuber number is proportional to the power and inversely proportional to the flow-rate \( \Phi_m \), and is taken with respect to the properties at the core-inlet.

The subcooling number, with respect to the riser exit, is defined as:

\[
N_{sub,re} = \frac{h_{f,re} - h_{l,ci}}{h_{ev,ci}} \cdot \frac{\rho_{f,ci} - \rho_{g,ci}}{\rho_{g,ci}}
\]

where \( h_{f,re} \) is the enthalpy of the fluid at saturation at the riser exit and \( h_{l,ci} \) is the enthalpy of the one-phase liquid at the core inlet. Figure 2.4 gives a typical stability map.

The dashed line in figure 2.4 represents the bisectrice of the Zuber-subcooling plane and divides the one-phase region from the two phase region, as the following discussion makes clear. The core produces a power \( P_c \) which is removed by the coolant, which flows upward along the riser (in the riser no heat is added, the enthalpy of the coolant remains constant). A simple heat balance results in:

\[
P_c = \Phi_m (h_{re} - h_{ci})
\]

The coolant enters the core several degrees below saturation temperature and thus the coolant there is one-phase: \( h_{l,ci} \). Assuming that the coolant partly is turned into steam, a two-phase mixture exits the riser. The enthalpy \( h_{re} \) can be written as:

\[
h_{re} = x_{re} h_{g,re} + (1 - x_{re}) h_{f,re} \Rightarrow h_{re} = x_{re} h_{ev,re} + h_{f,re}
\]
where $x_{re}$ is the quality (or the fraction of steam) at the riser exit and $h_{ev, re}$ is the heat of evaporation at the riser exit which is equal to: $h_{g, re} - h_{f, re}$.

Inserting eq. 2.6 into eq. 2.5 and using definitions 2.3 and 2.4 leads to:

$$x_{re} = \frac{N_{zu} - N_{sub, re}}{\frac{h_{ev, re}}{h_{ev, ci}} \cdot \frac{p_{liq} - p_{g, ci}}{p_{g, ci}}} \tag{2.7}$$

If one-phase liquid exits the riser, $x_{re}$ is equal to zero leading to ($N_{zu} = N_{sub, re}$); the bisectrice of the Zuber-subcooling plane divides the one-phase region and the two-phase region.

The dotted line in figure 2.4 represents the stability boundary between two-phase stable and unstable flow. Two regions of unstable two-phase flow are recognised in the map. The first region is found between the bisectrice and the stability boundary (at low Zuber number). Section 2.1 showed that the Type-I instability occurs at low-power/low-pressure conditions. Low-power corresponds to a low Zuber number. Hence, this region is governed by Type-I instabilities and is marked as 'I' in figure 2.4. Section 2.2 showed that flashing in the riser induces the Type-I instability. This leads to a larger Type-I region in the stability map, the stability boundary moves to the solid line. The second region is found to the right of the stability boundary (at high Zuber number). Section 2.1 showed that the Type-II instability occurs at low-flow/high-power conditions. This corresponds to a high Zuber number. Hence, this region is governed by Type-II instabilities and is marked as 'II' in figure 2.1.
Chapter 3

Steady-state model

3.1 Introduction

This chapter describes the modelling of the steady-state behaviour of a Boiling Water Reactor (BWR). As explained in previous chapters, the natural circulation cooled BWR is being driven by density differences between downcomer section and core-riser section. The model presented here describes a BWR which consists of two coolant channels which have different power and friction settings. These channels are connected to a combined riser. The steam-water mixture is cooled by a heat exchanger at the top of the BWR. The fluid is transported through the downcomer to the core-section. The model takes into account the two-phase regions in the core and riser sections (including the pressure dependence of saturated properties). The model calculates the flow in the loop under the condition that the total pressure-drop over the loop equals zero. The model derived in this chapter has been made according to the experimental facility CIRCUS (CIRCulation during Start-up), chapter 6. It should be mentioned that the BWR modelled is very simplified. A schematic setup of the natural circulation loop which has been modelled is shown in figure 3.1. In the next section the basic principle for this particular steady-state model is described.

3.2 One-dimensional steady homogeneous equilibrium flow

The basic equations for steady one-dimensional homogeneous equilibrium flow in a tube are:

\[ M = \text{constant} \]  
\[ M \frac{dv}{dz} = -A \frac{d\rho}{dz} - \pi D_{w} - A_{ \rho_{m} g \cos \theta} \]  
\[ \frac{d\xi}{dz} - \frac{dv}{dz} = M \frac{d}{dz} \left( h + \frac{v^{2}}{2} + gz \right) \]

where \( M \) is total mass, \( v \) the velocity of the flow, \( A \) the area of the tube, \( p \) the
Chapter 3. Steady-state model

Figure 3.1: Schematic overview of the natural circulation loop which has been modelled. The picture is to scale.

pressure, $D$ the diameter of the tube, $\tau_w$ the average wall shear stress, $g$ the gravitational acceleration, $\theta$ the angle of the tube with the vertical, $e$ the energy, $w$ the work, $h$ the enthalpy and $z$ the vertical coordinate. $\rho_m$ is called the density of the mixture and is defined as:

$$\rho_m \equiv \alpha \rho_g + (1 - \alpha) \rho_f$$

(3.4)

where $\rho_g$ is the density of the vapour, $\rho_f$ the density of the liquid (at saturation) and $\alpha$ the void-fraction (i.e. ratio of vapour-volume to total volume, see eq. 2.2).

The momentum equation (eq. 3.2) is often rewritten as an explicit equation for the pressure gradient:

$$\frac{dp}{dz} = -\frac{\pi D}{A} \tau_w - \frac{M}{A} \frac{dv}{dz} - \rho_m g \cos \theta$$

(3.5)

The three terms on the right side can be regarded as frictional, accelerational and gravitational components. So the total pressure gradient is the sum of the components, as follows:

$$\frac{dp}{dz} = \left(\frac{dp}{dz}\right)_{\text{fric}} + \left(\frac{dp}{dz}\right)_{\text{acc}} + \left(\frac{dp}{dz}\right)_{\text{grav}}$$

(3.6)
3.3 Natural circulation flow-rate

The natural circulation flow-rate can be calculated if the total pressure drop over a closed loop (as depicted in figure 3.1) equals zero:

$$\int \frac{dp(\Phi_m)}{dz} dz = 0$$

(3.7)

The pressure drop is a function of flow-rate. The closed loop is divided into four sections: the downcomer, the core, the riser and the heat-exchanger. Eq. 3.7 is approximated by calculating the pressure drops in every section:

$$\Delta p_d + \Delta p_c + \Delta p_r + \Delta p_h = 0$$

(3.8)

where $\Delta p_d$ is the total (one-phase) pressure drop in the downcomer, $\Delta p_c$ is the total (one-phase and two-phase) pressure drop in the core, $\Delta p_r$ is total (one-phase and two-phase) pressure drop in the riser and $\Delta p_h$ is the total (one-phase and two-phase) pressure drop in the heat-exchanger.

By making use of an iterative procedure, in which the flow-rate is changed (see appendix B), eq. 3.8 is solved. The solution leads to a natural circulation flow-rate under the conditions which can be set, such as: the powers in the two core-channels ($P_1$ and $P_2$), the friction in the two core-channels ($K_1$ and $K_2$) the core inlet-temperature (proportional to the core inlet enthalpy, $h_d$, which is constant in the isolated downcomer if gravity is neglected) and a fixed pressure at the exit of the heat-exchanger. The pressure is fixed at this position, because at this point in the CIRCUS-facility (described in chapter 6) a steam-dome is present. In the steam-dome, liquid is in equilibrium with the steam at a constant temperature. Therefore, the pressure in the steam-dome is constant.

To make sure that only liquid exits the heat-exchanger the enthalpy of the liquid is set lower than the saturation enthalpy of the liquid:

$$h_d < h_{f,di}$$

(3.9)

where $h_{f,di}$ is the liquid saturation enthalpy at the inlet of the downcomer.

The liquid and vapour saturation density ($\rho_f$ and $\rho_g$) and enthalpy ($h_f$ and $h_g$) depend on the local pressure. The liquid density and enthalpy are approximated (according to [Moran and Shapiro, 1993]) by:

$$\rho_l(T,p) \approx \rho_f(T)$$

(3.10)

$$h_l(T,p) \approx h_f(T)$$

(3.11)

Tabled values (given by [Moran and Shapiro, 1993]) and fits performed on these values are given in appendix A.
The next sections handle the total pressure drops in the downcomer, core, riser and heat-exchanger, respectively. The pressure drops are calculated with a known flow-rate. Several iterative procedures are needed to calculate the boiling boundaries in the core and riser and the pressure at the exit of the core and riser.

### 3.4 The downcomer section

The downcomer starts after the heat-exchanger and ends at the inlet of the valve assembly. In the downcomer a contraction is present. The one-phase pressure drop is divided in a gravitational, frictional and accelerational component, respectively:

\[
\begin{align*}
\Delta p_{d,grav} & = -\rho g (L_{valve} + L_{fuel} + L_r) \\
\Delta p_{d,frie} & = 4f \left( \frac{L_{d1}}{D_{d1}} \right) \frac{\Phi_m^2}{2\rho_l A_{d1}^2} + 4f \left( \frac{L_{d2}}{D_{d2}} \right) \frac{\Phi_m^2}{2\rho_l A_{d2}^2} \\
\Delta p_{d,acc} & = 0.45 \left( 1 - \frac{A_{d2}}{A_{d1}} \right) \frac{\Phi_m^2}{2\rho_l A_{d2}^2}
\end{align*}
\]  

(3.12)\, \quad (3.13)\, \quad (3.14)

where \((L_{valve} + L_{fuel} + L_r)\) is the total height in which \(L_{valve}\) stands for the length of the valve-assembly, \(L_{fuel}\) for the length of the fuel assembly and \(L_r\) for the length of the riser). \(L_{d1}\) is the length of the first section of the downcomer from the outlet of the heat exchanger to the contraction, with diameter \(D_{d1}\) and area \(A_{d1}\). \(L_{d2}\) is the length of the second section of the downcomer from the contraction to the inlet of the core-section, with diameter \(D_{d2}\) and area \(A_{d2}\). The sum of \(L_{d1}\) and \(L_{d2}\) minus the sum of \(L_{valve}, L_{fuel}\) and \(L_r\) is equal to the horizontal parts of the downcomer section. The acceleration term has been derived from [Janssen and Warmaoeskerken, 1997]. \(4f\) denotes for the friction factor which depends on the Reynolds number \(Re\) as ([Todreas and Kazimi, 1990]):

\[
\begin{align*}
4f & = 0.316 \cdot Re^{-0.25} \quad \text{if } Re < 3 \cdot 10^4 \\
4f & = 0.184 \cdot Re^{-0.20} \quad \text{if } Re > 3 \cdot 10^4
\end{align*}
\]  

(3.15)

The laminar friction factor (for \(Re\) smaller than about 2000) has not been taken into account in these correlations (see also the remark based on this factor in section 3.8).

By summing those three terms (= total pressure drop in the downcomer, \(\Delta p_d\)), the local pressure at the core inlet, \(p_{ci}\), is known:

\[
p_{ci} = p_{di} - \Delta p_d
\]  

(3.16)

### 3.5 The core section

The core section consists of a valve assembly and a fuel assembly (two heated channels). It is possible to set different power and friction values to those channels. With
known flow at the inlet \( (\Phi_m) \), friction settings in channel 1 \( (K_1) \) and channel 2 \( (K_2) \), power settings in channel 1 \( (P_1) \) and channel 2 \( (P_2) \) and pressure at the inlet of the core-section \( (p_d) \), it is possible to calculate the flow in channel 1 \( (\Phi_{m1}) \), the flow in channel 2 \( (\Phi_{m2}) \) and the boiling boundaries in channel 1 \( (z_{1,bb}) \) and channel 2 \( (z_{2,bb}) \) respectively. Because the channels are connected to each other at the inlet and the exit of the core-section, the pressure-drop over channel 1, \( \Delta p_1 \), equals that of channel 2, \( \Delta p_2 \):

\[
\Delta p_1(\Phi_{m1}) = \Delta p_2(\Phi_{m2}) \tag{3.17}
\]

### 3.5.1 The valve-assembly

The valve assembly is placed below the fuel assembly. In this region it is possible to adjust the friction of each channel by changing the valve-setting. The pressure drop in the valve-assembly is divided into two components: a gravitational and a frictional component:

\[
(\Delta p_{\text{valve,grav}})_\phi = \rho g L_{\text{valve}}
\]

\[
(\Delta p_{\text{valve,fric}})_\phi = 4f \left( \frac{L_{\text{valve}}}{D_{\text{valve}}} \right) + K_{\text{valve}} \frac{\Phi_m}{2\rho_l A_{\text{valve}}^2} \tag{3.19}
\]

where \( A_{\text{valve}} \) is the area of the assembly channel with diameter \( D_{\text{valve}} \). \( K_{\text{valve}} \) is the friction-factor of the valve. The sum of those two components gives the one-phase pressure drop in the valve assembly, \( (\Delta p_{\text{valve}})_\phi \). The frictional component consists of two terms due to wall friction and valve friction.

### 3.5.2 The one-phase region in the fuel assembly

The fluid entering the fuel-assembly is heated by electrical rods. A heat-balance over a core channel, assuming a uniform heat-flux, results in the following enthalpy-equation:

\[
h_j(z) = \left( \frac{P_j}{\Phi_{m,j}} \right) \left( \frac{z}{L_{\text{fuel}}} \right) + h_d, \quad j = 1, 2 \tag{3.20}
\]

where \( h_j(z) \) is the enthalpy at height \( z \), \( P_j \) the power of the rod, \( \Phi_{m,j} \) the mass-flow and \( L_{\text{fuel}} \) is the length of the core-section. The location \( z = 0 \) is at the inlet of the fuel assembly.

With increasing height \( z \), the enthalpy of the fluid increases (eq. 3.20) and the saturation enthalpy decreases (due to decreasing hydrostatic pressure). At height \( z_{bb} \), the enthalpy of the fluid is equal to the saturation enthalpy. Therefore, \( z_{bb} \) is called the boiling boundary. An iterative procedure is used to calculate the boiling boundary (see appendix B). If the power and friction settings in the two channels
are different, this will result in different boiling boundary heights as is made clear in figure 3.2.

The liquid saturation enthalpy depends on local pressure, which is calculated as follows:

\[ (\Delta p_{\text{fuel, grav}})_{\phi} = \rho g \Delta z \]  \hspace{1cm} (3.21)

\[ (\Delta p_{\text{fuel, fric}})_{\phi} = 4f \left( \frac{\Delta z}{2\delta_{\text{fuel}}} \right) \frac{\Phi_m^2}{2\rho_l A_f^2} \]  \hspace{1cm} (3.22)

where \( \Delta z \) is the difference in height, \( \delta_{\text{fuel}} \) the distance between the rod and the wall of the flow-channel (\( 2\delta_{\text{fuel}} \) is the hydraulic diameter, [Janssen and Warmoeskerken, 1997]) and \( A_{\text{fuel}} \) is the area of the annular flow channel.

The sum of these two terms gives the one-phase pressure drop in the channel, \((\Delta p_{\text{fuel}})_{\phi}\). The local pressure in a core channel is calculated as:

\[ p(z) = p_{ci} - (\Delta p_{\text{valve}})_{\phi} - (\Delta p_{\text{fuel}})_{\phi} \]  \hspace{1cm} (3.23)

where \( p_{ci} \) has been calculated by making use of eq. 3.16.

If the enthalpy of the fluid at the exit of a core-channel is lower than the calculated liquid saturation enthalpy, there is no boiling boundary present.

3.5.3 The two-phase region in the fuel assembly

The transition from one-phase to two-phase flow is not so well defined as is assumed in the previous section. This is made clear in figure 3.3.
3.5. The core section

First, the thermodynamic equilibrium quality, $x_{eq}$, of the flow is introduced:

$$x_{eq}(z) = \frac{h(z) - h_f}{h_{ev}}$$  \hspace{1cm} (3.24)

where $h_{ev}$ is the heat of evaporation. The enthalpy $h(z)$ is calculated by making use of eq. 3.20. This quality is called thermodynamic equilibrium quality because it is defined using the thermodynamic concept of the enthalpy of the flow.

Another definition of quality is the flow quality, $x$, defined as the ratio of vapour mass-flow to total mass-flow (eq. 2.1). Note that $0 \leq x \leq 1$, which is not true for $x_{eq}$.

Figure 3.3 shows that at negative $x_{eq}$, small bubbles are formed at the tube wall and escape from the wall, the quality $x$ increases. This point is called the departure point. The boiling boundary is reached at $x_{eq} = 0$ (where $x > 0$). At the equilibrium point $x_{eq}$ becomes equal to $x$.

In the present model, it is assumed that the departure point and equilibrium point are equal to the boiling boundary.

The void-fraction (defined in eq. 2.2) and flow-quality are closely related. The simplest model which relates $\alpha$ to $x$ is the homogeneous equilibrium model (HEM). HEM assumes homogeneously mixed phases with the same velocity. Slip-velocity is neglected. The void-quality relation according to HEM is ([Todreas and Kazimi, 1990]):
This relation is graphically illustrated in figure 2.2. The two-phase pressure drop consists of three components: gravitation, friction and acceleration. The pressure drop due to gravitation is complicated due to the fact that $\alpha \neq 0$:

$$
(\Delta p_{\text{fuel,grav}})_{2\phi} = \int_{z_{bb}}^{L_{\text{fuel}}} [(1 - \alpha(z))\rho_f g dz + \alpha(z)\rho_g] dz
$$

The pressure drop due to friction is similar to the expression for the one-phase pressure drop (eq. 3.22) except for the presence of a two-phase multiplier, $\phi(x)$:

$$
(\Delta p_{\text{fuel,fric}})_{2\phi} = 4f \left( \frac{L_{\text{fuel}} - z_{bb}}{2\delta_{\text{fuel}}} \right) \frac{\Phi_m^2}{2\rho_f A_{\text{fuel}}^2} \phi(x)
$$

The two-phase multiplier, $\phi$, is a function of the flow quality ([Todreas and Kazimi, 1990]). It can be estimated by:

$$
\phi(x) = 1 + \left( \frac{\rho_f}{\rho_g} - 1 \right) x
$$

Since the quality is a function of height, the two-phase multiplier is also a function of height. But because $\phi$ is linear in $x$, a constant multiplier based on the average quality in the two-phase region is used. The pressure drop due to acceleration is calculated by considering the momentum of the flow entering the two-phase region and leaving the channel:

$$
(\Delta p_{\text{fuel,acc}})_{2\phi} = \frac{(x_{ce} \Phi_m)^2}{\alpha_{ce} \rho_f A_{\text{fuel}}^2} + \frac{((1 - x_{ce}) \Phi_m)^2}{(1 - \alpha_{ce}) \rho_f A_{\text{fuel}}^2} - \frac{\Phi_m^2}{\rho_f A_{\text{fuel}}^2}
$$

where the quality, $x_{ce}$, and void-fraction, $\alpha_{ce}$, are the values at the exit of the core-section. The acceleration pressure drop is caused by the effect that the flow speeds up as vapour is produced. The total two-phase pressure drop in the fuel assembly, $(\Delta p_{\text{fuel}})_{2\phi}$, is equal to the sum of the three components.

The problem arises here that $\alpha_{ce}$ depends on $x_{ce}$, which in turn depends on $h_{f,ce}$, which in turn depends on the local pressure at the exit of the core-section. But the local pressure at the exit of the core-section is not known. An iterative process is used to calculate the liquid saturation enthalpy at the exit of the core (see appendix B).

The pressure at the exit of the fuel assembly is calculated as:

$$
p_{ce} = p_{ci} - (\Delta p_{\text{value}})_{1\phi} - (\Delta p_{\text{fuel}})_{1\phi} - (\Delta p_{\text{fuel}})_{2\phi}
$$

The sum of the last three components is equal to the total pressure drop in the core section, $\Delta p_c$.  

$$
\alpha(z) = \frac{x(z)}{x(z) + \frac{\rho_g}{\rho_f} (1 - x(z))}
$$

(3.25)
3.5.4 Coupling channel 1 to channel 2

Channel 1 and channel 2 are coupled by the fact that the pressure drop in channel 1 is equal to the pressure drop in channel 2 (see eq. 3.17). According to this condition, the total flow \( \Phi_m \) is spread over the two channels: \( \Phi_{m1} \) and \( \Phi_{m2} \). The sum of the flows in the channels should equal the total flow, according to the conservation of mass:

\[
\Phi_m = \Phi_{m1} + \Phi_{m2}
\]  

(3.31)

By changing \( \Phi_{m1} \) at constant flow-rate \( \Phi_m \) (iterative procedure, appendix B), the condition of equal pressure drops can be found. If convergence is reached, this results in known \( \Phi_{m1}, \Phi_{m2}, z_{1,bb} \) and \( z_{2,bb} \). Also the pressure at the exit of the fuel assembly \( p_{ce} \) and the saturation enthalpy at the exit of the core-section are known.

3.6 The riser section

The coolant exits both channels and is completely mixed at the inlet of the riser. A heat-balance results in the following expression for the enthalpy of the fluid in the riser, \( h_r \)

\[
h_r = \frac{\Phi_{m1}h_{ce,1} + \Phi_{m2}h_{ce,2}}{\Phi_m}
\]  

(3.32)

where \( h_{ce,1}, h_{ce,2} \) are the enthalpies of the coolant at the exit of the core-section in channel 1 and channel 2, respectively. Because the riser is isolated and there is no external heat-source, the enthalpy of the coolant in the riser remains constant. Due to the mixing it could be possible that when there is a boiling boundary present in one of the channels, the coolant at the inlet of the riser does not boil anymore, as the enthalpy is smaller than the saturation enthalpy at the inlet of the riser. This saturation enthalpy is equal to the saturation enthalpy at the exit of the core-section calculated in the previous section.

In a similar way as is described in subsection 3.5.2, a transition from one-phase to two phase flow may be found. The point at which this transition takes place is called the flashing boundary, \( z_{fb} \), since no heat is added to the coolant. This is made clear in figure 3.4.

Considering the situation \( h_r < h_{f,ce} \), the two components for the one-phase pressure drop (gravitational and frictional) are:

\[
\begin{align*}
(\Delta p_{r,grav})_1\phi &= \rho g \Delta z \\
(\Delta p_{r,frie})_1\phi &= 4f \left( \frac{\Delta z}{D_r} \right) \frac{\Phi_m^2}{2\rho_l A_f^2}
\end{align*}
\]  

(3.33)  

(3.34)
where \( A_r \) is the area of the riser with diameter \( D_r \). The sum of the last two components is equal to the total pressure drop in the one-phase region, \((\Delta p_r)_{1\phi}\). The local pressure in the riser is calculated as:

\[
p(z) = p_{ci} - \Delta p_c - (\Delta p_r)_{1\phi}
\]

A similar iterative procedure is used to find the (bulk) flashing boundary as is described in subsection 3.5.2.

The two-phase pressure drop components are (gravitation, friction and acceleration):

\[
(\Delta p_{r,grav})_{2\phi} = \int_{z_{f,b}}^{z_{riser}} [(1 - \alpha(z))\rho_f g dz + \alpha(z)\rho_g g] dz
\]

\[
(\Delta p_{r,fri})_{2\phi} = 4f \left( \frac{L_r - z_{f,b}}{D_r} \right) \frac{\Phi_m^2}{2\rho_f A_r^2} \phi(x)
\]

\[
(\Delta p_{r,acc})_{2\phi} = \frac{(x_{re}\Phi_m)^2}{\alpha_{re}\rho_g A_r^2} + \frac{(1 - x_{re})\Phi_m^2}{(1 - \alpha_{re})\rho_f A_r^2} - \frac{\Phi_m^2}{\rho_f A_r^2}
\]

Since in the equations the values of \( x_{re} \) and \( \alpha_{re} \) at the exit of the riser-section are needed, which are not known, those values are iteratively calculated (similar to discussion in subsection 3.5.3). The local pressure at the exit of the riser-section, \( p_{re} \), is calculated as:

\[
p_{re} = p_{ci} - \Delta p_c - (\Delta p_r)_{1\phi} - (\Delta p_r)_{2\phi}
\]
where \((\Delta p_r)_{2\phi}\) is the sum of eqns. 3.36, 3.37 and 3.38. The sum of the last two terms on the right hand side is equal to the total pressure drop in the riser, \(\Delta p_r\).

### 3.7 The heat-exchanger

In the heat-exchanger the water-vapour mixture is cooled until the coolant-enthalpy reaches the specified set-point, \(h_d\). If the heat-flux is uniform over the total length of the heat-exchanger, \(L_h\), a heat balance over the heat-exchanger results in:

\[
h_h(z) = \left(1 - \frac{z}{L_h}\right) h_d + \left(\frac{z}{L_h}\right) h_r
\]  

(3.40)

where \(z\) is a distance \((z = 0\) at entrance heat-exchanger). The saturation enthalpy at the inlet of the heat-exchanger (equal to the saturation enthalpy at the exit of the riser \(h_{f,re}\)) and the saturation enthalpy at the exit of the heat-exchanger (equal to the saturation enthalpy at the inlet of the downcomer \(h_{f,di}\)) are both known. A linear relation for the saturation enthalpy in the heat-exchanger is proposed:

\[
h_f(z) = \left(1 - \frac{z}{L_h}\right) h_{f,di} + \left(\frac{z}{L_h}\right) h_{f,re}
\]  

(3.41)

Inserting eq. 3.41 into eq. 3.40 leads to:

\[
z_{bb} = \frac{(h_{f,di} - h_d)}{(h_{f,di} - h_d) - (h_{f,re} - h_r)} L_h
\]  

(3.42)

Where \(z_{bb}\) is the boundary between two-phase and one-phase flow in the heat exchanger. If \(z_{bb} < 0\) the fluid in the heat-exchanger is totally one-phase. Only two components in the pressure drop are important: the friction and the acceleration component. The last component is equal to zero if \(z_{bb} \leq 0\). Thus:

\[
(\Delta p_{h,fric})_{(1\phi+2\phi)} = 4f \left(\frac{z_{bb}}{D_{d1}}\right) \frac{\Phi_m^2}{2\rho_f A_{d1}^2} + 4f \left(\frac{L_h - z_{bb}}{D_{d1}}\right) \frac{\Phi_m^2}{2\rho_f A_{d1}^2} \phi(x_{re})
\]  

(3.43)

\[
(\Delta p_{h,acc})_{2\phi} = \frac{(x_{re}\Phi_m)^2}{\alpha_{re} \rho_f g A_{d1}^2} + \frac{(1-x_{re})\Phi_m^2}{(1-\alpha_{re})\rho_f A_{d1}^2} - \frac{\Phi_m^2}{\rho_f A_{d1}^2}
\]  

(3.44)

Here \(x_{re}\) is the quality at the riser exit and \(\alpha_{re}\) the void-fraction at the riser exit. The sum of the last two equations gives the total pressure drop in the heat-exchanger, \(\Delta p_h\).
3.8 Some remarks

- The model starts with a very small start-value for the mass-flow. This means that the flow in the entire loop is laminar. The value for the friction factor calculated by eq. 3.15 is not the true value. For laminar flow the following equation should be used:

\[ 4f_{\text{laminar}} = \frac{64}{\text{Re}} \]  

Especially low calculated natural circulation flow-rates could be overestimated since a turbulent friction factor is used. It should be noted that the friction factors used are suitable for tubular flows. The core-channels are annular; a tubular friction factor is assumed;

- The \( K_{\text{value}} \)-factor is constant for \( \text{Re} > 10^5 \). For \( \text{Re} < 10^5 \) this factor also depends on the geometry. Measurements on CIRCUS were done to determine the \( K_{\text{value}} \)-factor. In the present model also for \( \text{Re} < 10^5 \) a constant \( K_{\text{value}} \)-factor is assumed;

- The same reasoning as the previous comment is true for the hydraulic diameter, \( 2\delta_{\text{fuel}} \);

- Some bents are located in the CIRCUS natural circulation loop. They also contribute to the pressure drop:

\[ \Delta p_{\text{fric,bent}} = K_{\text{bent}} \frac{\dot{m}^2}{2\rho_f A^2} \]  

where \( K_{\text{bent}} \) is the bent-friction factor;

- In the present model a pressure drop is calculated for a contraction in the downcomer section. There are other contractions present in the loop (downcomer-valve assembly, valve assembly-fuel assembly, fuel assembly-riser and riser-downcomer). The accompanying accelerational pressure drops are not incorporated;

- In this model a solution is found for the natural circulation mass-flow rate. It could be possible that more solutions exist at higher mass-flow rates. Multiple solutions were not found.
Chapter 4

Simulations

This chapter gives some results on simulations done, with the model described in the previous chapter. The actual core-section of the experimental setup CIRCUS (described in chapter 6) consists of four boiling channels. They are treated as two pairs: One boiling channel in the model is changed to two boiling channels with identical settings. This is achieved by changing eq. 3.31 (the sum of the flow-rates in channels 1 and 2 is equal to the total flow-rate) to:

\[ \Phi_m = 2\Phi_{m1} + 2\Phi_{m2} \]  

A power ratio, \( R \), is defined as being the ratio of the varying power in channel 2 divided by the constant maximum power in channel 1:

\[ R = \frac{P}{P_{\text{max}}} \]  

An increase of power ratio leads to a more symmetric power distribution in the core-section and an increase of the total power input.

Four cases are considered:

- Case 1 (asymmetric): \( P_1 = 2.8 \text{ kW} \) and \( P_2 = 0.0 \text{ kW} \) \( \rightarrow R = 0 \);
- Case 2 (asymmetric): \( P_1 = 2.8 \text{ kW} \) and \( P_2 = 1.4 \text{ kW} \) \( \rightarrow R = \frac{1}{2} \);
- Case 3 (symmetric): \( P_1 = 2.8 \text{ kW} \) and \( P_2 = 2.8 \text{ kW} \) \( \rightarrow R = 1 \);
- Case 4 (symmetric): \( P_1 = 1.4 \text{ kW} \) and \( P_2 = 1.4 \text{ kW} \) \( \rightarrow R = 1 \).

Each case considers two settings of the valves: low friction (all valves open, \( K_{\text{value}} = 10 \)) and high friction (all valves are almost closed, \( K_{\text{value}} = 500 \)). The simulations are run with varying inlet temperature: \( T_{\text{inlet}} = 98^\circ \text{C} - 102^\circ \text{C} \). The pressure at the top of the loop is fixed at 1.2 bar. These settings are chosen in order to compare the steady-state solutions to the measurements performed (see chapter 8). Case 4 (symmetric case) is compared to case 1 (asymmetric case), because the total power input is the same and also to case 3 (symmetric case, twice total power input as in case 4).
4.1 Case 1

Figure 4.1: The natural circulation flow-rates, ((a),(b)), and corresponding boiling boundaries, ((c),(d)), are plotted against inlet temperature with all valves open, ((a),(c)), and all valves almost closed ((b),(d)), $R = 0$. The horizontal solid line in (c),(d) divides the core (below solid line) from the riser (above solid line). Note that the total flow-rate is twice the sum of the flow-rates $\Phi_m$ and $\Phi_m$, see eq. 4.1.

In figure 4.1 the results of flow-rate and boiling boundaries are plotted against inlet temperature, at $R = 0$ (asymmetric case).

The following observations were obtained from figures 4.1a,c (low friction):

- Increasing inlet temperature leads to increasing flow-rate and decreasing flashing boundary in the riser.

- No boiling boundaries are present in the core.

- The flow-rate in channel 1 is slightly higher than the flow-rate in channel 2.
4.2. Case 2

The following observations were obtained from figures 4.1b,d (high friction):

- The same trends in flow-rate are observed as in the low friction case, but because of a higher friction the natural circulation flow-rate is lower.

- Because of a lower flow-rate, the position of the flashing boundary is lower than in the low friction case and it reaches the inlet of the riser at about $T_{inlet} \sim 101.0\,^\circ C$.

- A boiling boundary is present in the heated channel 1 and its position decreases slightly with increasing inlet temperature.

- No boiling boundary is present in channel 2.

- The natural circulation flow-rate remains approximately constant when the flashing boundary reaches the inlet of the riser.

- The difference in flow-rate between channels 1 and 2 is higher than in the low friction case.

4.2 Case 2

In figure 4.2 the results of flow-rate and boiling boundaries are plotted against inlet temperature, at $R = \frac{1}{2}$ (asymmetric case).

The following observations were obtained from figures 4.2a,c (low friction):

- The same trends in flow-rate and flashing boundary are observed as in figures 4.1a,c, but the calculated flow-rate at a certain inlet temperature is higher and the flashing boundary is lower.

- No boiling boundaries are present in the core.

The following observations were obtained from figures 4.2b,d (high friction):

- The same trends in flow-rate and flashing boundary are observed as in figures 4.1b,d, but the flashing boundary reaches the inlet of the riser at a lower inlet temperature, $T_{inlet} \sim 99.0\,^\circ C$.

- As the flashing boundary reaches the inlet of the riser, the natural circulation flow-rate saturates.

- The boiling boundary in channel 1 decreases with increasing inlet temperature and at $T_{inlet} \sim 101.2\,^\circ C$, a boiling boundary in channel 2 appears.

- A small difference between flow-rates in channels 1 and 2 can be observed, but the differences are smaller than the corresponding ones in figure 4.1b.
Chapter 4. Simulations

Figure 4.2: The natural circulation flow-rates, ((a),(b)), and corresponding boiling boundaries, ((c),(d)), are plotted against inlet temperature with all valves open, ((a),(c)), and all valves almost closed ((b),(d)), $R = \frac{1}{2}$. The horizontal solid line in (c),(d) divides the core (below solid line) from the riser (above solid line). Note that the total flow-rate is twice the sum of the flow-rates $\Phi_{m1}$ and $\Phi_{m2}$, see eq. 4.1.

4.3 Case 3

In figure 4.3 the results of flow-rate and boiling boundaries are plotted against inlet temperature, at $R = 1$ (symmetrical case).

The following observations were obtained from figures 4.3a,c (low friction):

- The same trends in flow-rate and flashing boundary are observed as in figures 4.1a,c and figures 4.2a,c, but the calculated flow-rate at a certain inlet temperature is higher and the flashing boundary is lower and reaches the inlet
4.3. Case 3

![Graphs showing flow rates and boiling boundaries](image)

(a) Low friction.  
(b) High friction.

(c) Low friction.  
(d) High friction.

Figure 4.3: The natural circulation flow-rates, ((a), (b)), and corresponding boiling boundaries, ((c), (d)), are plotted against inlet temperature with all valves open, ((a), (c)), and all valves almost closed ((b), (d)), $R = 1$. The horizontal solid line in (c), (d) divides the core (below solid line) from the riser (above solid line). Note that the total flow-rate is twice the sum of the flow-rates $\Phi_{m1}$ and $\Phi_{m2}$, see eq. 4.1.

- As the flashing boundary reaches the inlet of the riser, boiling boundaries in the core appear and the natural circulation flow-rate saturates.
- There are no differences in flow-rate between channels 1 and 2.

The following observations were obtained from figures 4.3b,d (high friction):

- The coolant in the total riser section is flashing, increasing inlet temperature leads to a decrease of boiling boundaries in both channels.
The natural circulation flow-rate slightly increases with increasing inlet temperature, no differences between flow-rates in channels 1 and 2.

4.4 Case 4

The following observation was obtained if a comparison is made between case 4 and case 3 (figure 4.4 and figure 4.3, respectively). These two cases are both symmetric, but the power in case 4 is half of that of case 3:

- A higher total power input leads to higher flow-rates and lower boiling/flash boundaries, as expected.

The following observations were obtained if a comparison is made between case 4 and case 1 (figure 4.4 and figure 4.1, respectively). These two cases have the same total power, but case 1 is asymmetric whereas case 4 is symmetric:

- The total flow-rate in the symmetric case 4 is about 6% lower than the flow-rate in the asymmetric case 1.
- The flashing boundary in the symmetric case 4 is shifted upwards with respect to the flashing boundary in the asymmetric case 1. At low friction the upward shift of the flashing boundary is about 1.5%. At high friction the upward shift of the flashing boundary is about 9%.
- The asymmetric case 1 (high friction) shows for all temperatures a boiling boundary in the heated channel while in the asymmetric case 4 (high friction) boiling boundaries in both channels appear at inlet temperature 101.2°C.

4.5 Conclusions

The simulations done, show some important characteristics of a natural circulation cooled BWR at steady-state conditions. The following conclusions can be drawn from these characteristics:

- If a flashing boundary in the riser is present, an increase of inlet temperature leads to an increase of the natural circulation flow-rate as expected.
- An increase of inlet temperature leads to an increase of the temperature at the outlet of the core; saturation conditions are reached at lower heights in the riser (a decrease of the flashing boundary).
- The area of a core-channel is a factor 8.5 smaller than the area of the riser. If a boiling boundary enters the core-section, the increase of the natural circulation flow-rate saturates due to the large increase of two-phase friction pressure drop.
4.5. Conclusions

Figure 4.4: The natural circulation flow-rates, ((a),(b)), and corresponding boiling boundaries, ((c),(d)), are plotted against inlet temperature with all valves open, ((a),(c)), and all valves almost closed ((b),(d)), $R = 1$. The horizontal solid line in (c),(d) divides the core (below solid line) from the riser (above solid line). Note that the total flow-rate is twice the sum of the flow-rates $\Phi_{m1}$ and $\Phi_{m2}$, see eq. 4.1.

- An increase of the power ratio (at constant maximum power in channel 1, see eq. 4.2) leads to a higher power input. A higher power input leads to a downward shift of the boiling boundaries and an increase of flow-rate (at certain range of inlet temperatures) as the simulations show.

- The simulations of asymmetric cases show differences between the flow-rates in the two boiling channels, but these differences remain small. Natural circulation is mainly driven by density differences between riser and downcomer. The reason is that the friction in the riser is smaller with respect to the friction in the core (see third point).
• An asymmetric power distribution in the core may lead to a boiling boundary in channel 1 and no boiling in channel 2. At the exit of the core, the coolant exiting channels 1 and 2 are mixed. Because of mixing, two situations are possible: the coolant at the inlet of the riser is boiling or non-boiling depending on the conditions of the coolant exiting channels 1 and 2. For example, figure 4.2d shows the two situations. If $T_{inlet} < 101.2^\circ C$, the mixed coolant (two-phase in channel 1 and one-phase in channel 2) leads to non-boiling coolant at the inlet of the riser. Again saturation conditions are reached at higher positions in the riser. If $T_{inlet} > 101.2^\circ C$, the mixed coolant leads to boiling coolant at the inlet of the riser.

• An increase of power ratio (at constant maximum power in channel 1, see eq. 4.2) leads to a more symmetric power distribution in the core: the asymmetry in flow-rate and boiling boundaries disappears as expected.

• A higher friction (due to closing the valves) leads to a decrease of flow-rate (as expected) and a downward shift of the boiling boundaries (at certain inlet temperature). And due to lower flow-rates, at low $R$ larger flow-rate differences are expected.

• Comparing the symmetric case 4 and asymmetric case 1 (total power input for both cases is the same), reveals that the symmetric case 4 has a slightly lower total flow-rate and higher flashing boundary for low and high frictions. For high friction the following explanation is given. The asymmetric case 1 shows a boiling boundary in the heated channel, the two-phase flow in this channel contributes to the driving force leading to a higher total flow-rate and a lower flashing boundary.
Chapter 5

Laser Doppler Anemometry

5.1 Introduction

In any form of wave propagation, frequency changes can occur due to movement of source, receiver, propagating medium, or intervening reflector or scatterer. These shifts are generally called 'Doppler' shifts after the Austrian physicist, Christian Johann Doppler (1803-1853), who first considered the phenomenon in 1842. The Doppler shift, familiar in acoustics, is due to the relative motion of source and receiver and this type of shift is also well known for electro-magnetic radiation, including light. The technique of using the Doppler shift of laser light to determine velocities in water flow is called as Laser Doppler Anemometry, or abbreviated to LDA.

The Doppler-shift is determined of light scattered by a small particle that moves with the flow. Hence, the Doppler shift is a measure for the velocity of the particle and thus for the flow-velocity. Since the velocities commonly encountered are very small compared with the velocity of light, the corresponding Doppler shifts are small. This shift (typically $10^5$ Hz) is very small compared to the frequency of light of $10^{14}$ Hz. It goes beyond the resolution of detectors and therefore, direct optical spectroscopy is not a practical method of measurement. The only technique suitable for measuring very small Doppler shifts uses the principle of heterodyning or 'optical beating' of two frequencies in a device having a non-linear response. There are various ways to implement the heterodyning principle, see [Drain, 1980]. This principle will later be discussed briefly.

The advantages of the LDA-technique are:

- The technique is non-intrusive, which means that the flow is not disturbed during the measurement;
- The Doppler frequency is a measure for the velocity component in a direction that is determined by the geometry of the optical arrangement;
- The technique is directionally sensitive which means that it is able to measure flow reversal;
• The LDA-technique is ideally suited for measuring in turbulent flows. It gives accurate information about mean velocities and Reynolds stresses;

• There is a linear relationship between the Doppler frequency and the velocity.

There are also some disadvantages:

• Because the particles are randomly distributed in the flow, it is not possible to use directly many standard data-processing methods like Fast Fourier Transform (FFT);

• LDA measures the velocity of the particle, a relationship between the particle velocity and flow velocity must be found;

• The measuring technique is very complex, for example aligning the different components like: lens, beam splitter and Bragg-cells;

• There are numerous noise sources present, more than in the hot-wire method.

In this chapter the following topics will be discussed briefly: the Doppler shift, the principle of optical 'beating' in the reference beam technique, the control or measurement volume, measuring positive/negative velocities, noise, signal to noise ratio, Mie scattering and the Doppler signal. The chapter ends with some conclusions with respect to the environment in which the measurements are done.

5.2 The Doppler shift

In figure 5.1 we consider light of frequency $f_0$ from a source S scattered by a particle P. The scattered light is detected by detector D. Because the particle is moving with velocity $\vec{v}$, it will 'detect' light with a shifted frequency with respect to the frequency of the light source S. The particle will scatter the frequency shifted light in every direction, also in the direction of the detector. But because of the motion of the object, the detector will receive a second shifted frequency with respect to the frequency of the light source S. So the detector receives a frequency which is Doppler shifted ($f_{\text{detector}} = f_0 + f_D$) due to light scattering on a moving object. This Doppler shift, $f_D$, is given by [Drain, 1980] as:

$$f_D = \frac{f_0 v}{c} (\cos \beta_1 + \cos \beta_2)$$

(5.1)

where $c$ is the velocity of light, $\beta_1$ is the angle between the direction of the light wave and the direction of motion of the object, $\beta_2$ is the angle between the direction in which the scattered frequency is received by the detector and the direction of motion of the object. This equation can be rewritten to a more convenient equation by making use of trigonometrical transformations and in terms of the wavelength $\lambda_0$: 
where $\theta$ is the angle between the direction of the original light wave and the direction of scattering to the detector, $|\vec{v}| \cos \alpha$ is the component of the velocity in the direction perpendicular on the bisectrice of the angle of the light wave and the scattered light to the detector.

![Figure 5.1: Schematic illustration of the Doppler shift and the corresponding angles](image)

**Figure 5.1: Schematic illustration of the Doppler shift and the corresponding angles**

### 5.3 Optical 'beating' in the reference beam technique

#### 5.3.1 Optical 'beating'

Because the velocities are normally much smaller than the velocity of light, the Doppler shift in frequency will be very small compared to the frequency of light. Therefore, a technique must be used which solves this problem: heterodyning or optical 'beating'. The essence of heterodyning is that when two coherent light waves with slightly different frequencies are mixed on the surface of a detector, the output signal oscillates with the difference frequency. In this way it will be possible to detect the (small) Doppler frequencies.

#### 5.3.2 Reference beam technique

One technique suitable to implement this principle is called the reference beam technique. This technique is illustrated in figure 5.2. A laser beam is divided in two beams by a beam-splitter. One beam enters the flow channel, the so-called scatter beam. Particles which are present in the flow will scatter light if they pass the scatter beam. This light is scattered in every direction. Because the particles are moving, the scattered frequency will be Doppler shifted. The second beam, the so-called reference beam, is directed via e.g. a glass-fiber to the detector aperture.
A small part of the scattered light will also be detected by the detector. Because the scattered light is shifted in frequency, optical 'beating' takes place at the detector surface. The output of the detector then contains a signal of the difference frequency (or Doppler frequency) between the two beams.

5.3.3 Interference between two coherent beams

The question now arises at which position the velocity of the flow is obtained. Light is scattered on every place of the laser beam which travels through the flow. If the scattered light comes from the same direction as the reference beam, interference (in time) between the two coherent light waves takes place at the detector surface. This interference pattern is caused by optical mixing. However, if the direction of the scattered light makes an angle with the direction of the reference beam, there will also be a spatial wavelength of the interference pattern on the detector surface. If the angle gets larger, the wavelength of the spatial interference pattern gets smaller and more dark and light interference spots appear on the detector surface. The detector acts as an integrator over its sensitive area and the resulting output is a constant signal. According to [Drain, 1980], the maximum angle, $\gamma_{\text{max}}$, at which the detector can detect separate dark and light fringes is:

$$\gamma_{\text{max}} = \frac{\lambda_0}{2d_{\text{det}}}$$

(5.3)

where $d_{\text{det}}$ is the diameter of the detector.

In figure 5.3 two coherent beams intersect each other with angle $\gamma$. The two beams will form an interference pattern of dark and light fringes. The spacing between two fringes is given by:

$$d_f = \frac{\lambda_0}{2\sin\frac{\gamma}{2}}$$

(5.4)

in which $d_f$ is called the fringe spacing. Note that this is not the fringe spacing as meant in the so-called differential Doppler technique, another LDA-technique,
5.4 The measurement volume

Figure 5.3: Illustration of the interference condition

see [Drain, 1980]. Now for the detector to detect separate dark and light fringes, the maximum diameter of the detector should be half of the fringe spacing. As \( \sin \gamma \approx \gamma \), for small \( \gamma \), this will lead to eq. 5.3. As a result, the detector selects a small cone around the reference beam in which the scattered light interferes with the light of the reference beam; detectable optical mixing takes place. If the two beams are properly aligned, the cone will intersect the scatter beam. So, the part of the scatter beam which intersects this cone will scatter light in the direction of the detector which interferes with the light of the reference beam. This part of the scatter beam is called the measurement volume. More about the measurement volume follows in section 5.4.

5.3.4 A practical reference beam set-up

A better way to make use of this technique is made clear in figure 5.4. In this arrangement also the laser beam is split into two beams. Those beams are focused by a lens. The detector is placed in the reference beam. Light scattered from the scatter beam which is within the cone of interference causes the 'detectable' optical 'beating' at the detector surface. The advantage of this set up is that the measurement volume is positioned at the location of the focal point of the lens. At that place the intensity of the scattering beam is very high. The set-up shown in figure 5.2 is not practical, it only makes clear that it is not necessary for the reference beam to cross the scatter beam to obtain the Doppler shifted frequency.

5.4 The measurement volume

The part of the scatter beam which intersects the 'cone of interference' forms the measurement volume. A particle moving in the measurement volume scatters light in the direction of the detector, the frequency shifted light yields optical beating with the un-shifted frequency of the reference beam. The width of the measurement
volume, $d_{cv}$, is just the diameter of a Gaussian beam, $d_b$, focused in water by a thin lens (focal length in air, $F_a$), propagating only through flat surfaces:

$$d_{cv} = \sqrt{\frac{n_w}{n_a} \frac{4F_a \lambda_0}{\pi d_b}}$$  (5.5)

where $n_a$, $n_w$ are the refractive indexes of air and water, respectively.

The determination of the length of the measurement volume, $l_{cv}$, is more difficult since the directions of the cone of interference and the reference beam are changed at the air/water transition (e.g. the wall of the tube), see figure 5.5. The derivation of the length of the measurement volume is done in appendix C. The following assumptions are made: the tubular flow channel has negligible wall thickness, the beam enters the tube perpendicular to the tube wall and the scatter beam intersects the middle of the cone. The result for this derivation is:
5.5 Positive versus negative velocities

\[
\frac{\delta F_w \tan \theta_w}{2} = \frac{\delta F_w \tan \frac{\theta_w}{2} + \frac{1}{2} D \tan \gamma_{w,1} - w_1}{\tan \gamma_{w,1} + \tan \frac{\theta_w}{2}} \quad \text{and} \quad \frac{\delta F_w \tan \frac{\theta_w}{2} + \frac{1}{2} D \tan \gamma_{w,2} - w_2}{\tan \gamma_{w,2} + \tan \frac{\theta_w}{2}}
\]

(5.6)

where \(\delta F_w\) is the deviation of the focal point in water from the middle of the tube, \(\theta_w\) is the angle between scatter and reference beam in water, \(w_1\) and \(w_2\) are the heights at which the cone intersects the wall of the tube with angles perpendicular on the wall \(\gamma_{w,1}\) and \(\gamma_{w,2}\) (in water), respectively.

**Table 5.1: Representative numbers used to determine the length of the control volume.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{det})</td>
<td>1.0 mm</td>
<td>(\gamma_{w,1})</td>
</tr>
<tr>
<td>(d_b)</td>
<td>1.0 mm</td>
<td>(w_1)</td>
</tr>
<tr>
<td>(D)</td>
<td>10.0 mm</td>
<td>(w_2)</td>
</tr>
<tr>
<td>(F_a)</td>
<td>100.0 mm</td>
<td>(\gamma_{w,1})</td>
</tr>
<tr>
<td>(\lambda_0)</td>
<td>532.0 nm</td>
<td>(\gamma_{w,2})</td>
</tr>
<tr>
<td>(\eta_n)</td>
<td>1.33</td>
<td>(\frac{\theta_w}{2})</td>
</tr>
</tbody>
</table>

Now, if the representative numbers given in table 5.1 are used, this will lead to the following dimensions of the measurement volume (assuming that the focal point in water is positioned in the middle of the tube, \(\delta F_w = 0\)):

\[
d_{cv} \approx 0.08 \text{mm}; \quad l_{cv} \approx 0.85 \text{mm}
\]

This means that the length of the measurement volume is on the order of a millimeter and the diameter of the volume is a factor ten smaller than its length. The form of the measurement volume is cylindrical.

**5.5 Positive versus negative velocities**

The Doppler frequency shift can be positive or negative depending on the value of \(\alpha\), see eq 5.2. However, a photo multiplier cannot distinguish between positive and negative values of \(f_D\). To get rid of this problem a common method is used: frequency shifting. In that case the frequency of one of the laser beams is shifted by a constant frequency, \(f_{shift}\). This can be done by an acousto-optic Bragg cell. Due to this shift in frequency, the Doppler frequency will be:

\[
f = f_{shift} + \frac{2 \sin \frac{\theta}{2} |\vec{v}| \cos \alpha}{\lambda_0}
\]

(5.7)

If the shift-frequency is chosen larger than the Doppler frequency that corresponds to the largest negative anticipated velocity in the flow, each value of \(|f_D|\) is uniquely
related to one velocity value and hence the directional ambiguity is removed. The factor \( \frac{\lambda_0}{2 \sin \frac{\theta}{2}} \) relates velocity to a frequency and hence is called conversion factor, \( d_{\text{conv}} \).

### 5.6 Noise sources; signal to noise ratio

If one obtains a signal from the LDA-system, this signal will be affected by noise. Noise is created by various sources present in the LDA-system. The most important ones are [Van Maanen, 1999]:

- Modulation noise of the laser;
- Optical path difference of the incoming laser beams;
- Phase front distortion by particles in the incoming beams;
- Distortion of the laser beams in the measurement volume due to particles;
- Quantisation noise of the detector current;
- Electronic, thermal noise from electronic circuits;
- Unwanted reflections from outside the measurement volume (wall, lens, etc.).

The optical path difference does not add a serious amount of noise if the optical path difference is small compared to the coherence length of the laser. The phase front distortion will become more important if the diameter of the particles gets larger (if for example large seeding particles are used). The influence of unwanted reflections can be reduced if, for example, pinholes are used to block them from reaching the detector’s surface. The quantisation noise has been studied intensively and has been worked out by [Drain, 1980].

Photo emission of electrons (in e.g. a detector) is a random process, which follows the laws of probability. Therefore, the output current of the detector contains an irregular component called noise. The signal to noise ratio compares the mean square signal current to the mean square noise current. The signal to noise ratio with respect to quantum noise is specified as follows ([Drain, 1980]):

\[
\text{SNR} = \frac{\eta I_1 I_2}{h f \Delta f (I_1 + I_2)}
\]  

(5.8)

where \( \eta \) is the quantum efficiency, \( I_1 \) the intensity of the scattered light, \( I_2 \) the intensity of the reference beam, \( h \) is the constant of Planck and \( \Delta f \) the filter bandwidth. Eq. 5.8 shows that as the intensity of the reference beam increases \( (I_2) \) the signal to noise ratio saturates at:

\[
\text{SNR}_{I_2 \to \infty} = \frac{\eta I_1}{h f \Delta f}
\]  

(5.9)
Eq. 5.9 shows that a high intensity of the scattered light and a small filter bandwidth optimises the signal to noise ratio. It should be noted that the intensity of the scattered light and the intensity of the reference beam should be in the same order to obtain a high signal to noise ratio. In order to achieve this, the intensity of the scatter beam should be higher than the intensity of the reference beam.

5.7 Mie Scattering

Closely related to the Doppler approach, in which light is represented as a continuous field, is the Mie scattering theory. In 1908 Gustav Mie [Durst et al., 1976] derived an analytical solution from Maxwell’s equations for electro-magnetic waves. This solution results in scattering properties for spherical particles larger than the wavelength of the incident beam, which is the case in LDA. The remarkable result of this complicated theory is that the intensity of light scattered in the forward direction is considerably higher than the intensity of light scattered in other directions. This can be seen in figure 5.6, in which a logarithmic intensity distribution is given as function of the scatter-direction. This result is counter intuitive; the idea of higher intensity scattered backwards seems more plausible.

The big advantage of this result is that measuring in the forward direction results in a high intensity of the scattered light. Therefore, the signal to noise ratio will be higher and the detector will also pick up light scattered from very small particles (in the order of the wavelength of the incident beam). In this way it is not really necessary to add seeding to the flow to increase the scattered light intensity. In CIRCUS this will be a big advantage because of the following items:

- Seeding can act as nucleation sites, triggering voids production, changing the stability of the system;
- Seeding can disappear in the dead corners of the system, therefore, constantly seeding has to be added to keep the data-rate high;
• The seeding particles should resist the high temperatures in the system;
• The density of the particles should approximately be the same to the density of water (which is not constant!) to avoid buoyancy of the particles.

Even in demi-water, very small (dust)-particles are present and will scatter light in the forward direction. The big disadvantage of forward scattering is the difficult alignment. In most back-scatter set-ups the light source and detector are coupled, thereby offering a friendly way of traversing. In the forward direction this is not possible.

5.8 The Doppler signal

In the previous sections it is made clear that the detector experiences a difference frequency due to optical mixing of the reference beam and the scattered light. The output signal of the detector looks like the signal depicted in figure 5.7.

![Doppler signal, without noise](image)

Figure 5.7: Doppler signal, without noise

The frequency of the signal is equal to the Doppler frequency plus the shift frequency, see eq. 5.7. This frequency is modulated. This effect is caused by the particles which are travelling through the Gaussian scatter beam (a beam with a Gaussian intensity distribution). This passing of particles through the measurement volume is a random process. If it is assumed that the particles are randomly distributed across the fluid, the process of measuring the velocity of the passing particles is a Poisson process.

5.8.1 Poisson distribution

Poisson processes describe the pattern of occurrences of random events. An example is the emission of α-particles from a radio-active substance. A Poisson process can be described as follows: let \( N(t) \) be the number of events in a given space of time \( (0, t) \); the probability of finding \( i \) events is:

\[
P[N(t) = i] = e^{-\lambda t} \frac{(\lambda t)^i}{i!}, \quad i = 0, 1, \ldots
\]  

(5.10)

where \( \lambda \) is the mean rate of occurrence of an event. The collecting of LDA-data is also a Poisson process. Another way to describe the time interval distribution is by looking at the rate of occurrence of a certain time interval between two events.
instead of by looking at the rate of occurrence of data in a time interval. In that case the time interval distribution becomes exponential.

\[ P(\Delta t) = f_{\text{data}}e^{-f_{\text{data}}\Delta t} \]  

(5.11)

where \( \Delta t \) is a time interval between two Doppler signals (time between data), \( f_{\text{data}} \) is the data rate. Now by plotting the time interval distribution on lin-log scale, the time interval distribution becomes a straight line. This is true if the time between data is Poisson distributed. This is graphically made clear in figure 5.8.

\[ f_{\text{data}} = 1000 \text{ Hz} \]

Figure 5.8: Time interval distribution

5.8.2 Problems

There are four basic problems which affect the time interval distribution:

- Not constant flow-rate;
- High seeding density;
- Noise;
- Signal processor.

If the flow is not constant, \( f_{\text{data}} \) will not be constant. The data-rate is dependent on the velocity and position of the measured frequency in the filter range. If the velocity is high, more particles pass the measurement volume in a unit time; the data-rate will be higher. If the measured frequency is near to the filter range, the frequency is less detectable; this leads to a decrease in the average data-rate. The
variable average data-rate leads to a deformation of the time interval distribution. The time interval distribution on log-scale will not be a straight line.

If the seeding density is too high, the chance of having more than one particle present in the measurement volume is higher. More particles in the measurement volume scatter light to the detector with frequencies which are not exactly the same, due to a very small range of scattering angles possible in the measurement volume. The signal processor may have problems processing this signal. In the forward scatter mode, the intensity of the scattered light is high. Therefore, the detector can even catch very small particles present in the flow. Even in demi-water there are many dust particles present, in the forward mode the data-rate will be high. The signal is almost continuous.

In a measuring system, always noise is present. See section 5.6 about noise. Noise directly affects the Doppler signal. This influences the signal processing done by the particular signal processor used in the experimental set-up, see subsection 6.2.2. Also processing of data done by the signal processor affects the time distribution. In subsection 6.2.2 this will be explained.

5.9 Summary

In this chapter, the Laser Doppler Anemometry method is described. This method is necessary to measure, non-intrusively, the velocity in a flow channel around a boiling rod. The method has been described for the reference beam technique. The main reason that the reference beam technique is used is that the use of seeding should be avoided. The problems of using seeding in CIRCUS have been discussed in section 5.6. One other thing is that demi-water is used in CIRCUS, which is very clean. The only way to collect LDA-data from the flow channel is by making using of the reference beam technique in forward mode. This results in the following advantages:

- High scatter intensity, large signal to noise ratio;
- The detector is able to detect very small particles (dust particles);
- High data-rate;
- Reference beam technique itself positively affects the signal to noise ratio;
- No seeding needed.

There are also some disadvantages to this technique:

- Difficult alignment, bad alignment affects the signal to noise ratio;
- High data-rate, noise and signal processor itself confuses the signal processing done by the signal processor (more in subsection 6.2.2). The time interval distribution will be affected.
Chapter 6

The experimental facility

To study the stability under low pressure, low power conditions of a BWR, a facility has been built at the Kramers laboratory, Delft. In the next section, this facility, called the CIRCUS-facility, will be discussed. This project is focused on LDA-experiments in two boiling channels simultaneously. Therefore, in section 6.2 the LDA experimental facility is described.

6.1 CIRCUS experimental facility

The experimental test facility CIRCUS has been built at the Delft University of Technology to study the two-phase flow dynamics during start-up conditions of innovative Boiling Water Reactors based on natural-circulation cooling. The reason that CIRCUS was built is that flashing related instabilities during start-up conditions were predicted and measured at the 'Dodewaard' reactor by [Van der Hagen and Stekelenburg, 1997]. The goal of CIRCUS is to study these flashing-related instabilities. A full-scale model of a section of the 'Dodewaard' reactor was built. CIRCUS means 'CIRCUlation during Start-up'. A schematic representation of CIRCUS is given in figure 6.1.

CIRCUS is a type of a Boiling Water Reactor. The coolant (several degrees below saturation-temperature) removes the heat in the core-section and part of the coolant is turned into steam. The steam-water mixture enters the riser-section (chimney). The mixture then is condensed in the heat exchanger at the top. The one-phase coolant enters via the downcomer the core-section again. Instead of circulating the coolant by a pump, the coolant is circulated by natural circulation.

The core-section of CIRCUS consists of eight glass tubes. Four of them are boiling channels, the other ones are called bypass-channels. The boiling channels contain electrically heated rods, placed concentrically inside. The size and geometry of these rods are approximately the same as in 'Dodewaard'. The inner diameter is chosen so that the available flow area is equal to the flow area per fuel rod in 'Dodewaard'. The bypass channels are installed to simulate the effect of coolant bypassing the fuel-bundles. The inner cross-section represents the amount of bypass area per fuel rod in 'Dodewaard'.
Figure 6.1: Schematic view of the CIRCUS-facility. The main instrumentation is also shown. A horizontal cross-section of the core (A - - - A) can be seen in figure 6.2.
CIRCUS contains four heating rods, therefore, the inner diameter of the riser is chosen so that its cross-section represents four times the riser area per fuel rod in 'Dodewaard'. The same scaling is done for the steam-dome present after the buffer vessel at the top of the facility and for the cross-section of the downcomer.

To get some insight in the main characteristics of the facility, see table 6.1.

Table 6.1: Main characteristics of the CIRCUS-facility

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power range per rod</td>
<td>0-3 kW</td>
</tr>
<tr>
<td>Pressure range</td>
<td>1-5 bar</td>
</tr>
<tr>
<td>Fuel channel diameter</td>
<td>20.4 mm</td>
</tr>
<tr>
<td>Fuel rod diameter</td>
<td>12.5 mm</td>
</tr>
<tr>
<td>'Pyrex' glass refractive index</td>
<td>1.490</td>
</tr>
<tr>
<td>Thickness 'Pyrex' glass</td>
<td>2.5 mm</td>
</tr>
<tr>
<td>Bypass channel diameter</td>
<td>10 mm</td>
</tr>
<tr>
<td>Fuel channel length</td>
<td>1.95 m</td>
</tr>
<tr>
<td>Riser diameter</td>
<td>47 mm</td>
</tr>
<tr>
<td>Riser length</td>
<td>2.78 m</td>
</tr>
<tr>
<td>Downcomer diameter</td>
<td>50 mm</td>
</tr>
<tr>
<td>Number of boiling channels</td>
<td>4</td>
</tr>
<tr>
<td>Number of bypass channels</td>
<td>4</td>
</tr>
</tbody>
</table>

Below the core-section, a valve-assembly is present. In this assembly, eight separately adjustable valves are present to set the friction in every channel individually.

The core- and riser-section are made of glass, called 'Pyrex'. In this way it is possible to make visual observation of the flow during experiments. The facility contains a steam-dome to control the pressure in the system during operation. A pressure-vessel is present to control the pressure during warming up (when no steam is present in the steam-dome). A concentric pipe heat exchanger placed at the top removes the heat inserted in the core-section. Two buffer-vessels are placed to make the temperature uniform at the inlet of the core-section. A pump is present to circulate the coolant when the system is not operating.

6.1.1 Instrumentation

In the facility several instruments are present to register physical properties in the system. Besides thermocouples, flow meters and pressure sensors, the following special instrumentation is present:

- Double LDA-setup; measuring local velocity fluctuations in two channels simultaneously;
- Gamma-transmission system; measuring chordal void-fractions at different heights in the riser-section;
- Wire-mesh sensor; measuring void-fraction distribution (2D) at the top of the riser-section.
6.2 LDA experimental facility

In figure 6.2 the experimental facility is shown to obtain the velocity of the flow at a specific point.

Two LDA-setups are present to measure simultaneously velocities in two different boiling channels. In the middle of these channels, a heated rod is present. The coolant flows upwards along the core channel through a small slit around the heated rod. To avoid most of the negative effect of breaking of the laser light when entering the glass boiling channel, a flat-surface glass mall has been designed. The two reference beams from the two setups are directed via two separate boiling channels to the detectors. The signals from the two detectors are processed by two Intelligent Flow Analysers (IFA-550) from TSI. The resulting data-file from the IFA-550 then is ready for processing by a computer, a 486/66MHz PC. The LDA-setup is described in more detail in subsection 6.2.1. In subsection 6.2.2 a short description of the IFA-550 is given.

6.2.1 LDA-setup

In figure 6.3 a schematic overview is given for one separate LDA-setup. Two of these setups are used.

In this setup a diode pumped green crystal laser is used. This laser emits green light at TEM00 mode. Some specifications are given in table 6.2.
Table 6.2: Specifications of the laser.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>~ 30 mW</td>
</tr>
<tr>
<td>Wavelength ($\lambda_0$)</td>
<td>532 nm</td>
</tr>
<tr>
<td>Coherence length</td>
<td>&gt; 100 m</td>
</tr>
<tr>
<td>Beam diameter ($d_b$)</td>
<td>0.36 mm</td>
</tr>
<tr>
<td>Beam divergence</td>
<td>2 mrad</td>
</tr>
</tbody>
</table>

The beam diameter, $d_b$, is defined as that contour corresponding to an intensity amplitude $\frac{1}{e}$ of its maximum value. The laser beam is split by a 40%/60% beamsplitter. This means that 40% of the light is transmitted and 60% is refracted. The beam with the higher intensity is used as scatter beam referring to the signal to noise ratio, eq 5.9.

Both beams are directed through a Bragg-cell (one cell per beam). The working of the particular Bragg cell used is explained in the instruction manual, [IntraAction, 2000]. Here it will be discussed briefly.

In a Bragg cell, an acoustic wave with wavelength $\Lambda$ causes refractive index changes of an optically transparent medium. This periodic variation produces a grating capable of diffracting an incident laser beam. The line spacing is equal to $\Lambda$. If the incident laser beam enters the sound field at the Bragg angle, maximum diffraction efficiency occurs. The Bragg angle is defined as:

$$\theta_B = \frac{\lambda_0}{2\Lambda}$$  \hspace{1cm} (6.1)
where $\theta_B$ is the Bragg angle. The symmetrical diffracted beam is frequency modulated with the frequency corresponding to the acoustic wavelength $\Lambda$. This is the first-order beam. Depending upon design, up to 90% of the incident beam can be diffracted in the first order. The angular deviation between the first order and zeroth order is twice the Bragg angle. For one Bragg cell the modulation frequency was set to 40 MHz. The modulation frequency for the second Bragg cell was made variable. Now by setting this frequency for example on 40.5 MHz a shift frequency between the two beams is obtained of 500 KHz. The first order beams are directed via a pinhole to a lens. Table 6.3 gives the settings of both LDA-setups.

<table>
<thead>
<tr>
<th>Shift-frequency ($f_{shift}$)</th>
<th>900.000 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal point in air ($F_a$)</td>
<td>100.0 ± 2.0 mm</td>
</tr>
<tr>
<td>Beam spacing ($d_{beam}$)</td>
<td>25.0 ± 1.0 mm</td>
</tr>
<tr>
<td>Angle between scatter and reference beam in air ($\theta_a$)</td>
<td>14.25 ± 0.31°</td>
</tr>
<tr>
<td>Angle between scatter and reference beam in water ($\theta_w$)</td>
<td>10.71 ± 0.31°</td>
</tr>
<tr>
<td>Conversion factor ($d_{conv}$)</td>
<td>2.83 ± 0.16 (\mu m)</td>
</tr>
</tbody>
</table>

### 6.2.2 IFA 550

The rather complicated working of the signal analyser IFA-550 is beyond the scope of this thesis. But some points are interesting to be acquainted with.

The IFA-550 is designed for detecting separate Doppler bursts. A Doppler burst is caused by a single particle. The shape of such a burst is Gaussian and the frequency corresponds to the Doppler frequency-shift. Separate Doppler bursts occur if only one particle is present in the measurement volume and when the seeding number density is very low. The IFA is commonly used for processing signals provided by back-scatter techniques.

The processor operates in the following way. When the signal is higher than a certain threshold level, the processor counts eight cycles, or zero-crossings, and registers the time interval associated with these eight cycles. This time interval is validated as the time interval in which a Doppler burst occurred. The reciprocal of this time interval is proportional to the Doppler frequency.

A problem arises when there are much more cycles present in a Doppler burst. This happens if the shift frequency is too high with respect to the Doppler-frequency. When the processor validated a Doppler burst after eight cycles, the signal is still higher than the threshold level. Again, after a very short time (in the order of the dead time of the processor), the processor starts to count eight cycles. In this way one single burst can be multiple validated.

Another physical phenomenon which causes 'multiple validation' is noise, described in section 5.6. Noise affects the signal in such a way that it can happen that the signal decreases for a short time below the threshold level. When the signal again is higher
than the threshold level, the processor will validate this signal as a second burst. Multiple validation affects the time distribution explained in section 5.8. Especially the short time intervals will be over represented. In the time distribution, a peak is observed at small time intervals. If one is interested in turbulent aspects of the flow, 'multiple validation' destroys this information. For determining the average velocity, 'multiple validation' is not a big problem, since large time-scales give enough information. More about data processing with respect to this project can be read in chapter 7.

In newer versions of the IFA it is possible to set the option 'single measurement per burst'. In this way it is possible to solve the first problem (many cycles per burst), but not the second (noise). The IFA-550 does not have this option.

The IFA-550 contains a filter-bank, to filter the Doppler signal with the wanted frequency range. This filters part of the noise out, but not the noise in the same frequency range as the Doppler frequency. If the velocity is not constant, the filter range has to be chosen in a way, that the maximum and minimum velocity can be obtained. The filter range may not be chosen too large, since it influences the signal to noise ratio in a negative way. The larger the filter range, the lower the signal to noise ratio gets, see eq. 5.9. Test measurements showed that the maximum Doppler frequency shift corresponding to the maximum velocity was around 500.0 kHz. Therefore, the shift frequency was set at 900.000 kHz. Together with a downward Doppler shift the filter range was set at 300.0 kHz - 1.0 MHz. No large negative velocities were expected in the particular measurements done, see chapter 8.

The reference beam technique is because of its high scattering properties more a frequency tracking technique rather than a separate burst detecting technique. But because of the interest in average velocity time-series and friendly way of using the IFA, the IFA is used as the signal processor instead of the more commonly used frequency-trackers.
Chapter 7

Data processing and analysis

In this chapter the treatment of data is discussed. In section 7.1, the data-processing from raw-data files provided by the IFA-550 to velocity files will be described. In section 7.2, the velocity signal of a measurement will be analysed.

7.1 Data processing

The IFA-550 software produces a raw binary data file after a measurement. This file contains all the data points gathered during the measurement from both processors. For each data point the processor number, the number of cycles, the burst time and the time between data are stored. After converting the raw-data file to ASCII data, the data is ready for further processing. The burst time is stored in nanoseconds, the time between data in microseconds. To obtain a time-series of the velocities, the following computations are done:

\[ t_{i+1} = t_i + 1.0 \cdot 10^{-6} TBD_i \]  \hspace{1cm} (7.1)

\[ f_i = \frac{N_{cycles}}{1.0 \cdot 10^{-9} t_{burst,i}} \]  \hspace{1cm} (7.2)

\[ v_i = d_{conv}(f_i - f_{shift}) \]  \hspace{1cm} (7.3)

\[ d_{conv} = \frac{\lambda_0}{2\sin\frac{\theta_x}{2}} \]  \hspace{1cm} (7.4)

in which \( t_i \) is the time in seconds at the \( i \)-th data point, \( TBD_i \) is the time between data point \( i + 1 \) and data point \( i \) in microseconds, \( f_i \) is the measured frequency in Hertz of the \( i \)-th data point, \( N_{cycles} \) is the number of counted cycles, \( t_{burst,i} \) is the
burst time in nanoseconds or the time in which $N_{\text{cycles}}$ were counted at the $i$-th data point, $v_i$ is the computed velocity at the $i$-th data point and $d_{\text{conv}}$ is a conversion factor in meters. The variable $i$ runs from 1 to $N$, in which $N$ is the total number of data points. Note that the signal processor counts 8 cycles to validate a 'burst', so $N_{\text{cycles}}$ is equal to 8. Eq. 7.3 is extracted from eq. 5.7.

Each point registers the number of the processor, $N_{\text{chan}}$. Regardless of the processor number, the time series is built by eq. 7.1. By making use of eqns. 7.1-7.4 two data files are written for both processors in the way made clear in figure 7.1.

**Figure 7.1:** Schematic representation of data processing to time series of velocity for both processors.

At this point, the two data sets written in the so-called file 1 and file 2 are ready for further data analysis.
7.2 Data analysis

In the chapters about LDA and the experimental set-up, some problems were discussed when collecting LDA-data. The major problem which affects the LDA-data is called multiple validation caused by noise and the signal processor. In this chapter, a measurement performed in a boiling channel is studied. The measurement was performed with a core inlet temperature of $\approx 100^\circ$ C and the power of the rod was $\approx 2.8$ kW.

The data will be analysed in different ways. First, the randomly sampled data will be analysed by making use of the time-series of the velocity signal (subsection 7.2.1) and the distribution of the time between data intervals (subsection 7.2.2). It will be shown that the velocity signal is not constant, but changes periodically in time. It is proven that multiple validation is present in the signal. Also attention is paid to the shape of the time between data distribution. The velocity signal is studied for the highest important frequency present by making use of the Fast Fourier Transform (FFT). This method has been described in detail by [Van der Hagen, 1995], and is discussed briefly in subsection 7.2.4. The Fast Fourier Transform 'asks' for evenly sampled sampled data. The velocity signal is randomly sampled and has to be re-sampled on equidistant time-intervals. In subsection 7.2.3, the re-sampling of the velocity-signal is explained. The result of the FFT is used to re-sample the time-series on equidistant time-intervals with the highest time interval possible to filter the unwanted noise out of the velocity signal. The resulting velocity signal is given in subsection 7.2.5 as well as a measure of the standard deviation of the velocity.
7.2.1 Time-series of velocity

In figure 7.2a the time-series of the measured velocity is plotted. It is clear that the velocity is not constant, but changes periodically. The period of the oscillation is called $T_{\text{flash}}$, because the oscillation is caused by the flashing phenomenon. The data is randomly collected. Figures 7.2b,c show an enlargement of the time-series around 25 and 86 seconds, respectively. In these figures it can be seen that the signal is full of 'clusters' of points, with very small time intervals. These clusters are caused by multiple validation as explained in subsection 6.2.2.

![Diagram of velocity time-series with $T_{\text{flash}} \sim 41$ s](image)

(a) Time-series of velocity, 1% of points is plotted here.

(b) Enlargement time series, in 'valley'.

(c) Idem, at 'peak'.

Figure 7.2: Time-series of the velocity. This time-series is measured in a boiling channel with a power of 2.8 kW and inlet temperature of 100° C. The oscillation period of flashing is about 41 seconds.
7.2. Data analysis

7.2.2 Histogram of time between data

The passing of particles through the measurement volume is a random process. Therefore, as stated in section 5.8.1, the velocity signal is randomly sampled. As a result, the time between data (or TBD) becomes exponentially distributed. If the velocity is constant, the probability density function of the time between data, plotted on a semi logarithmic scale, becomes a straight line (see eq. 5.11). The slope of the line is equal to the data-rate, \( f_{\text{data}} \).

However, during the measurement, the velocity is not constant as can be seen in figure 7.2a. If the velocity changes in time, \( f_{\text{data}} \) will change. If the velocity increases, more particles will travel through the measurement volume, leading to an increase of \( f_{\text{data}} \). On the other hand, the illumination time decreases, leading to a decrease of \( f_{\text{data}} \). But more important during the measurements is the following: the measured frequency corresponding to the highest velocity is close to the border of the filter-range. This leads to a decrease of \( f_{\text{data}} \), because this frequency is partly filtered. Measurements showed that \( f_{\text{data}} \) was smaller at higher velocities.

![Figure 7.3: Simplified form of the velocity-signal, \( \alpha \) is the fraction with respect to the period in which the velocity is high (\( v_{\text{max}} \)), (1 - \( \alpha \)) is the fraction with respect to the period in which the velocity is low (\( v_{\text{min}} \)), time (t) is made dimensionless with flashing period (\( T_{\text{flash}} \)).](image)

In figure 7.4, the histogram of the time between data is plotted. The shape of the histogram is curved. To understand this, consider the signal depicted in figure 7.3. In this figure, the velocity-signal is simplified to a block-signal. During \( \alpha T_{\text{flash}} \) seconds, the velocity is high, \( v_{\text{max}} \), and during \( (1 - \alpha)T_{\text{flash}} \) seconds, the velocity is low, \( v_{\text{min}} \). The velocities \( v_{\text{min}} \) and \( v_{\text{max}} \) correspond to \( f_{\text{data},\text{min}} \) and \( f_{\text{data},\text{max}} \). Both processes are Poisson processes and are coupled in the following way:

\[
P[\Delta t] = \alpha f_{\text{data},\text{max}} e^{f_{\text{data},\text{max}}\Delta t} + (1 - \alpha) f_{\text{data},\text{min}} e^{f_{\text{data},\text{min}}\Delta t}
\]

(7.5)

For the coupled processes as stated in eq 7.5, it is assumed that both processes are uncorrelated. First, the time between data is fitted on a single Poisson process
Figure 7.4: Probability density function of the time between data on semi logarithmic scale. Binwidth = 0.1 ms.

(α = 1 and \( f_{data,\max} \rightarrow f_{data} \)). Then the time between data is fitted to the model assumed in eq. 7.5. Fit 1 and Fit 2 lead to the following results:

\[
\begin{align*}
&\text{fit 1: } f_{data} \approx 778 \text{ Hz} \\
&\text{fit 2: } \begin{cases} f_{data,\max} & \approx 600 \text{ Hz} \\ f_{data,\min} & \approx 1180 \text{ Hz} \\ \alpha & \approx 0.11 \end{cases}
\end{align*}
\]

It can be seen that the fit for the coupled Poisson processes (fit 2) fits better to the results than that for the single Poisson process (fit 1). The average data-rate at \( v_{\max} \) is a factor two smaller than the average data-rate at \( v_{\min} \). The factor \( \alpha \) calculated, means that about 10% of the oscillation period, the velocity is high (flashing) and 90% of the period the velocity is low. If \( \alpha \) is calculated by dividing the time interval where the velocity is between (95% - 100%) \( v_{\max} \), to the oscillation period, the result is around 10%. Despite the rough approximation of combined Poisson processes in eq. 7.5, it gives a reasonable explanation for the curved shape of the time between data distribution (compared to fit 1 performed on the measured data), see figure 7.4.

Figure 7.4 also makes clear that the time intervals smaller than 0.4 ms are over represented. About 70% of all the points are in these bins! This is caused by multiple validation, as could also be seen in the time-series of the velocity. To remove these clusters, the data should be re-sampled with a time interval higher than 0.4 ms. The way in which this is done, is described in the next subsection. The time-series of the velocity and the histogram again make clear, that the signal processor is not usable if one is interested in turbulent scales. Multiple validation cannot be prevented, since the signal is 'continuous' and noisy. The signal processor
7.2. Data analysis

designed for detecting separate 'bursts' is not suitable for such a signal. But as can be seen in figure 7.2a: the average velocity is easy to follow!

7.2.3 Re-sampling the velocity signal

The previous subsection showed a huge amount of multiple validation present in the signal. To get rid of the clusters of points in the signal, caused by this effect, re-sampling of the signal is done. Figure 7.4 showed that the largest time interval present in the measurement is about 10 ms. As will be shown in the next subsection, the fastest changes in velocity are a factor ten slower than the largest time scale. Because of the interest in the average velocity and not the time scales, re-sampling on equidistant time intervals will be done. A large time interval with respect to the randomly sampled time intervals will give a lot of points in the bins, giving better statistics.

The conclusion from the previous subsection was that it is easy to follow the slowly changing average velocity. Now, by re-sampling the time-series, it is possible to filter the clusters and also the noise out of the signal. The minimum time interval to filter the clusters is about 0.4 ms. In this subsection the re-sampling routine is described.

\[ f_s > 2f_{max} \]  

(7.6)

where \( f_{max} \) is the highest physical frequency present. This is called the Nyquist-criterion. More about this criterion in the next subsection. All data-points in a bin are averaged in velocity:

\[ \Delta t_s \]

Figure 7.5: Illustration of the re-sampling routine of a random sampled velocity-signal.

The time-axis is divided in bins, the binwidth is equal to the time interval. The time is fixed in the middle of the bin (for bin number \( j: t_j \)). This can be seen in figure 7.5. If the time interval is taken small enough, the velocity is assumed constant in every bin. The time interval \( (\Delta t_s = \frac{1}{f_s}) \) is small enough if:

\[ f_s > 2f_{max} \]  

(7.6)
Chapter 7. Data processing and analysis

\[ \bar{v}_j = \frac{1}{N_j} \sum_{i=1}^{N_j} v_{i,j}, \quad 1 \leq j \leq t_{\text{max}f_s} = J \]  

(7.7)

where \( t_{\text{max}f_s} \) is the total time times the sampling frequency, or the total number of time bins \( (J) \), \( v_{i,j} \) is the \( i \)-th velocity point in bin \( j \), \( N_j \) is the total number of velocity points in bin \( j \). For all velocity points in bin \( j \) the deviation from the average velocity can be calculated as:

\[ \delta v_{i,j} = v_{i,j} - \bar{v}_j \]  

(7.8)

The standard deviation of the velocity, \( \sigma_v \), is calculated as:

\[ \sigma_v^2 = \frac{1}{J} \sum_{j=1}^{J} \frac{\sum_{i=1}^{N_j} \delta v_{i,j}^2}{N_j - 1} \]  

(7.9)

where \( \sigma_v \) is the standard deviation of the velocity and \( J \) is the total number of bins. Also, the points can be averaged in time:

\[ \bar{t}_j = \frac{1}{N_j} \sum_{i=1}^{N_j} t_{i,j} \]  

(7.10)

in this way it is possible to calculate the deviation of \( \bar{t}_j \) with respect to the fixed time-point \( t_{ji} \): \( \delta \bar{t}_j \).

7.2.4 The Fast Fourier Transform

In order to obtain a frequency spectrum of the velocity signal, the Fast Fourier Transform algorithm is used. This algorithm is a computationally efficient algorithm for calculating the Discrete Fourier Transform (the DFT).

The DFT, for a \( N \)-point signal \( v'[n] \) (in which the trend is removed: \( v'[n] = v[n] - \bar{v}[n] \)), is computed in the following way:

\[ a_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} v'[n] \cdot e^{-j \frac{2\pi}{N} kn} \]  

(7.11)

where \( 0 \leq n \leq N - 1, 0 \leq k \leq N - 1, a_k \) is the \( k \)-th Fourier series coefficient. For each \( k \) the evaluation of \( a_k \) requires \( N \) (complex) multiplications and additions, and hence the evaluation of the complete set of \( a_k \) requires \( N^2 \) operations. According to [Van der Hagen, 1995], the number of multiplications in FFT is: \( \frac{N\log_2 N}{2} \), which is substantially smaller than the \( N^2 \) operations required for the DFT.
As $v'[n]$ is a real time sequence, the power spectrum will be symmetric around $\frac{N}{2}$:

$$|a_k|^2 = |a_{N-k}|^2$$  \hspace{1cm} (7.12)

The frequency resolution of the spectrum is:

$$f_{res} = \frac{1}{N} f_s$$  \hspace{1cm} (7.13)

in which $f_s$ is the sample frequency of the evenly sampled data. The Fourier series coefficient $a_k$ then corresponds to frequency $f_k$ as:

$$f_k = kf_{res}$$  \hspace{1cm} (7.14)

The FFT generates a spectrum of $\frac{N}{2}$ frequencies out of $N$ data-points. The maximum frequency is half of the sampling frequency. This frequency is also called the Nyquist frequency, $f_{Nyq}$. This frequency is very important, since frequencies higher than the Nyquist frequencies are folded into the frequency range $0 - f_{Nyq}$. This effect is called aliasing. To overcome this problem is to use a low-pass filter, which removes all components above the Nyquist frequency before sampling.

The FFT-algorithm is based upon the assumption that the measured time sequence is repeated throughout time. This does not cause a problem for frequencies with a period that exactly matches the time window. Other frequencies, however, will exhibit distortions at the edges of the sequence. These sharp edges in the time domain will result in additional frequency components in the frequency domain. This is called spectral leakage, causing side lobes around the main lobes in the frequency spectrum. This problem can be solved by weighting the data. Data-points at the edge of a sequence are weighted less than points in the middle of the sequence. An often used window is called the bell-shaped Hanning window:

$$W_{Hann}[n] = \frac{1 - \cos \frac{2\pi n}{N}}{2}$$  \hspace{1cm} (7.15)

The loss by making use of such a window is that the main lob becomes wider, see [Van der Hagen, 1995].

In order to increase the accuracy of spectral estimates, the total sequence of the measurement data is divided into so-called blocks. The number of points in a block remains a power of two. For each block, spectra are calculated and thereafter averaged. The number of blocks is $M$. One can use the data available more efficiently by using overlapping blocks. Usually the blocks overlap each other with 50%. This is made clear in figure 7.6. So the number of FFT's to be computed is $2M - 1$. The accuracy of the spectral estimate will increase, see [Van der Hagen, 1995]:
where $\sigma_k$ is the standard deviation of the Fourier series coefficient $a_k$.

\[
\sigma_k^2 \approx \frac{1.2}{2M-1} a_k^2 \tag{7.16}
\]

Figure 7.6: Illustration of splitting data sequence in overlapping blocks.

The cost of splitting the data-sequence in blocks is that the frequency resolution of the frequency spectrum decreases:

\[
f_{res} \approx \frac{M}{N} f_s \tag{7.17}
\]

7.2.4.1 Applying FFT to time-series of velocity

In table 7.1, the values are placed to calculate the FFT.

| Table 7.1: Values used for calculating the FFT of the velocity signal. |
|----------------------|---|
| $\Delta t_s$        | 0.4 ms |
| $N$                  | 524288 |
| $M$                  | 4     |

A time interval of 0.4 ms leads to a sampling frequency of 2500 Hz and the Nyquist frequency at 1250 Hz. The total number of overlapping blocks used is: $2 \cdot 4 - 1 = 7$. According to eq 7.17, the frequency resolution will be 0.019 Hz (~ 52.4 seconds per block). The standard deviation of the Fourier series coefficient $a_k$ will be $0.41 \cdot a_k$, according to eq 7.16. Taking these values, the FFT is computed and its power spectrum is plotted in figure 7.7.

The power spectrum plotted in figure 7.7b shows that all interesting physical frequencies are below 1 Hz. The dominant frequency caused by flashing is calculated from the FFT at 0.0285 Hz (~ 35 s). Also a peak is observed around the 0.5 Hz.
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This frequency is caused by oscillations which can be observed in figure 7.2a, after the flashing peak. Because of the short measured time interval, the frequency resolution is rough and the error in the peaks is about 41%.

In order to get a more accurate estimate of the power spectrum, the same point is measured again for a longer time interval. Table 7.2 gives the values used for calculating the FFT.

Table 7.2: Values used for calculating the FFT of the velocity signal.

| $\Delta t_s$ | 20 ms |
| $N$ | 458752 |
| Segments | 7 |
| $M$ | 56 |

Figure 7.7b suggests that the highest physical frequency is below the 1.0 Hz. This leads to a conclusion that a sampling frequency of 2.0 Hz would give all information of the signal. Because of the roughness of the frequency resolution and the pretty large error in the amplitude of the Fourier series coefficient, it is chosen to set the sampling frequency at 50 Hz (time interval is 20 ms). In this way it is possible to catch frequencies below 25 Hz. All 458752 measurement points are gathered in approximately 9175 seconds. Unfortunately, the processor sets a maximum measuring time, about 1800 seconds. Therefore, the measurement has been repeated in 7 segments. Each segment contains 65536 points (measurement time per segment is $65536 \cdot 20 \cdot 10^{-3} = 1310.72$ seconds). If each segment is divided in 8 blocks, 15 overlapping blocks can be made for each segment. The total number of blocks is $7 \cdot 8 = 56$, as is written in the table. The total number of 50% overlapping blocks will be $7 \cdot 15 = 105$. The number of points in each block is a power of two: $2^{13}$. This
all leads to a frequency resolution of 6.104 mHz (~ 163.8 seconds per block) and an error of the Fourier series coefficient of 10.7%. Figure 7.8 gives the power spectrum as a result of the FFT.

The power spectrum plotted in figure 7.8 gives a clear conclusion that no important frequency is present in the velocity signal for frequencies higher than 1.0 Hz. Figure 7.8b shows the dominant frequency caused by flashing and its higher harmonics. The peak for the highest frequency present in the signal lies at 0.4 – 0.5 Hz. This peak corresponds to oscillations in the flow, starting after flashing and ending before flashing starts again. At higher frequencies, only noise is present.

7.2.5 Conclusions and results

The previous subsections lead to the following conclusions:

- Time-series of velocity and the histogram of the time between data show a huge amount of points which are affected by multiple validation;
- The shape of the histogram is curved, due to a non-constant velocity;
- The signal has to be re-sampled to filter the clusters caused by multiple validation, the minimum time interval should higher than 0.4 ms;
- Re-sampling should be done at the largest time interval possible to achieve a better accuracy of the velocity;
- The power spectrum of the signal shows that the highest frequency of interest is less than 1.0 Hz.
The measurement analysed in this section has been chosen as a representative measurement for all the measurements done. The dominant frequency due to flashing present in the particular measurement analysed, is about the maximum dominant frequency measured at all set-points (see chapter 8). All set-points show that the oscillations after the flashing peak remain in the range of 0.4 – 0.5 Hz.

Therefore, it is assumed that for all measurements, the highest frequency will remain below 1.0 Hz. The measurements are re-sampled at 10.0 Hz, setting the highest detectable frequency at 5.0 Hz (a factor 10 higher than the highest frequency peak). In this way it is possible to detect all interesting frequencies and to get an accurate estimate of the standard deviation.

In figure 7.9 the resulting velocity signal is plotted.

![Figure 7.9: Time-series of velocity, the signal has been re-sampled at $f_s = 10$ Hz.](image)

Figure 7.10a shows the PDF of the velocity. This PDF gives a measure of the shape of the signal. It can be seen that the velocity most of the time is around $v = 34$ cm/s. But also relatively more velocity-points were gathered around $v = 75$ cm/s, corresponding to the peaks in the velocity signal (see figure 7.9).

Figure 7.10b shows the PDF of the deviation of the calculated mean time to the fixed time for all bins. The PDF seems to be normal distributed around $\delta t_j = 0$. If the PDF was not (normal) distributed around $\delta t_j = 0$, it is not justified to take a fixed time in the middle of a bin.

Figure 7.10c shows the PDF of the deviation of the velocity to the mean-velocity for all bins. The shape of the PDF is Gaussian, as expected (because the binwidth is small compared to the change of the velocity). From this PDF, the standard deviation of the velocity can be calculated. The result is: $\sigma_v = 3.54$ cm/s. Other measurements shows similar results, $\sigma_v$ varied for 3.0 cm/s to 5.0 cm/s. As a measure for the accuracy of the velocity, the standard deviation is taken as: $\sigma_v \sim 4.0$ cm/s.
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Figure 7.10:

Figure 7.10d shows the PDF of the standard deviation for all bins. The shape is approximately Gaussian, but the PDF is biased towards higher standard deviations. It can be seen that all standard deviations are distributed around $\sigma_v = 3.54$ cm/s.
Chapter 8

Results

8.1 Laminar flow scan

In order to test the LDA-setup described in section 6.2.1, a scan is made through the annular flow channel as can be seen in figure 8.1a. The settings are made such, that the flow is laminar. According to [Bird et al., 1960], the axial flow in an annular channel is laminar if the Reynolds number (Re) is smaller than about 2000, in which the Reynolds number is defined as:

\[
Re = \frac{2(R_2 - R_1) \bar{u}}{\eta} = \frac{\bar{u}(2\delta)}{\eta} \tag{8.1}
\]

where \( R_1 \) is the inner diameter of the annular flow channel (i.e. half of the diameter of the fuel rod, according to table 6.1: \( 2R_1 = 12.5 \text{ mm} \)), \( R_2 \) is the outer diameter of the annular flow channel (i.e. half of the diameter of the flow channel, according to table 6.1: \( 2R_2 = 20.4 \text{ mm} \)), \( 2\delta \) is the hydraulic diameter of the annular flow channel ([Janssen and Warmoeskerken, 1997]) and \( \eta \) is the dynamic viscosity of the fluid.

The results are compared to the theoretical velocity profile which is approximately parabolic ([Bird et al., 1960]):

\[
\frac{v(r)}{v_{max}} \propto 1 - \left( \frac{r}{R_2} \right)^2 + \frac{1 - \left( \frac{R_1}{R_2} \right)^2}{\ln \left( \frac{R_2}{R_1} \right)} \ln \left( \frac{r}{R_2} \right) \tag{8.2}
\]

where and \( v_{max} \) is the maximum velocity. This radial profile has to be converted to the profile corresponding to the direction of the flow scan:

\[
r^2 = x^2 + y^2 \tag{8.3}
\]

where \( x \) is the distance perpendicular on the direction of the scan and \( y \) is the position along the scanning line, see figure 8.1. Because of rotational symmetry, the velocity profile is symmetric around \( y = 0 \).
Figure 8.1: A velocity profile is measured in an annular flow channel in the way made clear in (a). The results are presented in (b) together with a fit to the theoretical velocity profile.

Figure 8.1b shows that the fit with the theoretical profile correctly follows all the points within the errorbars except for the two points at the left-side and the two points at the right-side. The reason is that the measurement volume slightly slides into the wall. The wall scatters continuously light to the detector, the difference frequency is equal to the shift frequency (which matches velocity zero). Therefore, the left and right point erroneously give velocity zero. This effect is called the 'wall scattering' effect. According to the fit, the velocity should be zero at $y \approx 7$ mm. The point most to the right is at $y = 5.75 \pm 0.20$ mm. A measure for the length of the measurement volume can be obtained: $l_{cv} \lesssim 1.25$ mm. This result is in good agreement with the result calculated in section 5.4.

Velocity bias introduces an error in the measured velocity. Velocity bias occurs close to the wall were the velocity gradient is high. If the measurement volume is positioned close to the wall the range of measured velocities is high compared to a positioned measurement volume in the middle of the flow channel where the range of measured velocities is small. A higher velocity results in more scattering particles travelling through the measurement volume compared to a lower velocity. Thus, if the measurement volume is positioned close to the wall (where a large velocity gradient is present) the average velocity measured in the measurement volume will
8.2 Calibration velocity to flow meter

In order to compare the results obtained from the LDA-measurements with measurements provided by the flow-meter present in the CIRCUS facility, the velocity is calibrated to the flow-rate in the particular flow channel. For this purpose, all channels in the core section were closed except for the particular channel in which the calibration is performed. The velocity is calibrated to the flow-rate measured by the flow-meter positioned in the bottom part in the CIRCUS-facility (see figure 6.1). The calibration has been performed for both LDA-setups which measure in two different channels. The results are shown in figure 8.2.

The results appear to lie on a straight line. This seems to be strange since a transition from laminar to turbulent flow could be expected. As stated in the previous section, the transition to turbulent flow in an annular tube takes place at Re > 2000, which corresponds to a mean velocity of 0.25 m/s. From the results this transition could not be seen. A possible explanation could be that for annular flow in a small slit, the transition from laminar to turbulent flow is very small. The results are linearly fitted, the fit parameters give the conversion factors from calculated velocity (out of measured frequency) to measured flow-rate in liters per second:

![Figure 8.2: Calibration curves for the two flow channels in which is measured. The measurements are linearly fitted.](image-url)
\[ c_1 = 0.150 \pm 0.002 \ (10^{-3} \text{m}^2) \]
\[ c_2 = 0.136 \pm 0.001 \ (10^{-3} \text{m}^2) \]

The conversion factors are different, this could be related to different measurement positions in the two flow channels. Both calibration curves show an offset. The linear fit resulted in the following offset numbers:

\[ \text{offset}_1 = -0.0124 \pm 0.0032 \ (l/s) \]
\[ \text{offset}_2 = -0.0105 \pm 0.0021 \ (l/s) \]

A test measurement showed that the flow-meter has a little offset, but this (also negative) offset is a factor ten smaller than the offset calculated here. A possible explanation is that the flow-meter has a lower-limit. This means that if the flow-rate is less than this limit, the flow-meter gives output zero flow. The flow-rates can be calculated as follows:

\[ \Phi_{v_1} = c_1 v_1 + \text{offset}_1 \]
\[ \Phi_{v_2} = c_2 v_2 + \text{offset}_2 \]

where \( \Phi_{v_1} \) and \( \Phi_{v_2} \) are the flow-rates in channel 1 and channel 2 respectively in liters per second.

### 8.3 Overview of measurements performed

The LDA-measurements in two boiling channels have been performed under several conditions. Two parameters are varied: the inlet temperature of the coolant, \( T_{\text{inlet}} \) and the power of two boiling channels. The power of two boiling channels (out of four boiling channels) is set to the maximum. The power of the other two boiling channels is varied. The ratio of the varying power to the maximum power, \( R \) gives a value for the asymmetric power distribution in the core:

\[ R \equiv \frac{P}{P_{\text{max}}} \]

An increase of \( R \) leads to a symmetric power distribution in the core and an increase of the total power input in the core. The measurements were performed with five different power settings. The maximum power is equal to: \( P_{\text{max}} = 2.7806 \pm 0.0004 \ kW \). The five power settings and the results for \( R \) are:
8.3. Overview of measurements performed

Figure 8.3: Schematic view of the core section. Four boiling channels are present here, two of them are heated with maximum power, the other two are heated with five different power settings. In two channels LDA-measurements are done. One LDA-setup is present in the boiling channel with maximum heating power (channel 1) and the other LDA-setup is present in the boiling channel with different power settings (channel 2). The coolant enters the core section with temperature $T_{\text{inlet}}$.

\[
P = 0.00547 \pm 0.00017 \text{ W} \quad \rightarrow \quad R = (0.197 \pm 0.06) \cdot 10^{-6} \sim 0
\]

\[
P = 0.70055 \pm 0.00008 \text{ kW} \quad \rightarrow \quad R = 0.25200 \pm 0.00005 \sim \frac{1}{4}
\]

\[
P = 1.40345 \pm 0.00015 \text{ kW} \quad \rightarrow \quad R = 0.50473 \pm 0.00010 \sim \frac{1}{2}
\]

\[
P = 2.10284 \pm 0.00017 \text{ kW} \quad \rightarrow \quad R = 0.75479 \pm 0.00002 \sim \frac{3}{4}
\]

\[
P = 2.78814 \pm 0.00020 \text{ kW} \quad \rightarrow \quad R = 1.00270 \pm 0.00002 \sim 1
\]

$R = 1$ corresponds to a symmetric power distribution in the core, $R = 0$ corresponds to the most asymmetric power distribution in the core: two channels are not heated, the other two are heated with maximum power.

For each $R$, measurements were performed at five different inlet temperatures, varying between $98^\circ \text{C}$ and $102^\circ \text{C}$. The results are shown in table 8.1.

The total number of measurements is 25. The flow is measured in two boiling channels: one boiling channel with constant maximum power (channel 1) and one boiling channel with varying power (channel 2). In figure 8.3 a 2D-view of the core makes clear how the measurements are performed.
Table 8.1: Inlet temperatures for all R.

<table>
<thead>
<tr>
<th>$T_{\text{inlet}, R=0}$ ($^\circ$C)</th>
<th>$T_{\text{inlet}, R=\frac{1}{4}}$ ($^\circ$C)</th>
<th>$T_{\text{inlet}, R=\frac{1}{2}}$ ($^\circ$C)</th>
<th>$T_{\text{inlet}, R=\frac{3}{4}}$ ($^\circ$C)</th>
<th>$T_{\text{inlet}, R=1}$ ($^\circ$C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.011 ± 0.005</td>
<td>97.951 ± 0.003</td>
<td>98.161 ± 0.010</td>
<td>97.781 ± 0.008</td>
<td>98.467 ± 0.011</td>
</tr>
<tr>
<td>99.063 ± 0.004</td>
<td>98.844 ± 0.003</td>
<td>98.746 ± 0.011</td>
<td>99.020 ± 0.005</td>
<td>99.182 ± 0.006</td>
</tr>
<tr>
<td>100.156 ± 0.009</td>
<td>99.909 ± 0.002</td>
<td>99.852 ± 0.008</td>
<td>100.324 ± 0.011</td>
<td>99.959 ± 0.005</td>
</tr>
<tr>
<td>101.007 ± 0.006</td>
<td>101.068 ± 0.005</td>
<td>101.480 ± 0.021</td>
<td>100.951 ± 0.003</td>
<td>101.144 ± 0.003</td>
</tr>
<tr>
<td>102.335 ± 0.006</td>
<td>102.062 ± 0.004</td>
<td>102.082 ± 0.003</td>
<td>102.236 ± 0.003</td>
<td>102.254 ± 0.005</td>
</tr>
</tbody>
</table>

8.4 Mechanism causing instability revealed

In section 2.1 it is explained that a Boiling Water Reactor becomes unstable if the total channel pressure drop is out of phase with the flow-rate. In this case a decrease in flow results in an increase of total pressure drop and vice versa. Out of all measurements performed, the same point as is studied in chapter 7 is taken as an example. The inlet temperature is 100°C. The power distribution in the core is symmetric, $R = 1$.

In figure 8.4 the flow-rate in a boiling channel and the total pressure drop over the core and riser is plotted against time. The figure reveals that the variations of the total pressure drop are exactly 180° out of phase with respect to the flow-rate variations.

Figure 8.4: The flow-rate in a boiling channel and the total pressure drop over the core and riser plotted in arbitrary units against time.
8.5 Time-series of flow-rate

A selection of all flow-rates measured at different inlet temperatures and power distributions is plotted in figure 8.5 and figure 8.6. Some important results can be obtained from the time-series.

Figure 8.5 shows the time-series of the flow-rate at three different inlet temperatures when the power distribution in the core is made most unequal \((R = 0)\).

- The left three graphs (corresponding to channel 1 with maximum power) and the right three graphs (corresponding to channel 2 with zero power) show the same trend: with increasing inlet temperature, the system gets more unstable. This means that the flow-rates at the flashing peaks are increasing with increasing inlet temperature.

- The average flow-rate increases with increasing inlet temperature, leading to a decrease of the transit-time of the perturbations. The result is a decrease of the flashing period.

- The upper two graphs show that the system is stable: the flow-rate is constant. There is no void production, only one-phase flow.

- Between the flashing peaks, the flow-rate is almost constant. The time between the flashing peaks (in which the flow-rate is constant) is called the incubation time. During the incubation time, the flow-rate is relatively low, leading to an increase of the core-outlet temperature. The temperature in the riser increases until the temperature at a point in the riser reaches saturation temperature. Void-flashing starts.

- Comparing the flow-rates in channel 1 and channel 2, shows that the minimum flow-rate in channel 1 is higher than in channel 2. The difference gets larger with increasing inlet temperature. The minimum flow-rate in channel 1 increases with increasing inlet temperature while the minimum flow-rate in channel 2 decreases with increasing inlet temperature. Figure 8.5f shows that the minimum flow-rate in channel 2 is approximately zero. Natural circulation is caused by a density difference between heated channel 1 and the downcomer. The unheated channel 2 does not contribute to natural circulation.

- If the temperature in the riser is high enough, flashing starts. The driving force is now caused by a density difference between the riser and the downcomer, therefore, also the flow-rate in channel 2 increases. The peaks in channel 2 appear to be higher than the peaks in channel 1. The reason for this effect is explained in section 8.8.
Figure 8.5: Time-series of the flow-rate in both channels at three different inlet temperatures; $R = 0$. The left graphs correspond to channel 1, the right graphs correspond to channel 2. The dashed lines represent the results of the steady-state simulations.
Figure 8.6: Time-series of the flow-rate in both channels at three different inlet temperatures; $R = 1$. The left graphs correspond to channel 1, the right graphs correspond to channel 2. The dashed lines represent the results of the steady-state simulations.
Figure 8.6 shows the time-series of the flow-rate at three different inlet temperatures when the power distribution in the core is made equal ($R = 1$).

- Again the graphs show that the system gets more unstable with increasing inlet temperature.
- The minimum flow-rate slightly increases with increasing inlet temperature.
- Both channels behave in the same way (the flow-rates are in-phase). Again the peaks in channel 2 are slightly higher than the peaks in channel 1.

Finally, some remarks are given based on both figures.

- Figures 8.5 and 8.6 show the 'fast' flow oscillations with respect to the 'slow' flashing oscillation. These 'fast' oscillations happen when flashing in the riser is dying out.
- Comparing figures 8.5 and 8.6 reveals that at about the same inlet temperature (constant subcooling), the system is more unstable at $R = 1$. The total power at $R = 1$ is twice the total power at $R = 0$.
- The results of the simulations (indicated by the dashed lines in figures 8.5 and 8.6) seem to overestimate the experiments with about 0.04 l/s. However, the simulations show the same trends: increasing flow-rate with increasing inlet temperature and small differences in flow-rate between the two channels at decreasing $R$.

### 8.6 Results for all measurements

The previous section showed three important things:

- The system gets more unstable with increasing inlet temperature;
- The system gets more unstable with increasing $R$;
- The difference in flow-rate between channel 1 and channel 2 gets larger with decreasing $R$.

But, the previous section showed only a selection. Therefore, all measurements are summarised in figure 8.7. This figure shows for all $R$ the maximum flow-rate (figures 8.7a,b), the minimum flow-rate (figures 8.7c,d) and the average flow-rate (figures 8.7e,f). The left subfigures correspond to channel 1 and the right subfigures correspond to channel 2.

Figures 8.7a,b give the following results:

- The maximum flow-rate corresponding to the flashing peaks increases with increasing inlet temperature;
- The maximum flow-rate in channel 2 is higher than in channel 1, for all measurements except at low inlet temperature and low $R$. 

Figure 8.7: Flow-rate results for all measurements performed. The maximum flow-rate ((a), (b)), the minimum flow-rate ((c), (d)) and the average flow-rate over a flashing period ((e), (f)) are plotted as function of the inlet temperature. Left graphs correspond to channel 1, right graphs correspond to channel 2. The results are plotted for all R: $R = 0$ ($+$), $R = \frac{1}{4}$ ($-$ $x$ $-$), $R = \frac{1}{2}$ ($\bullet$ $\square$ $\cdot$ $\cdot$ $\cdot$), $R = \frac{3}{4}$ ($-$ $\blacksquare$ $-$), $R = 1$ ($--$ $\circ$ $--$).
Figures 8.7c,d give the following results:

- The minimum flow-rate in channel 1 shows a small increase with increasing inlet temperature;
- An obvious trend in channel 1 could not be seen if $R$ is changed;
- Channel 2 shows big changes in minimum flow-rate. If channel 2 is not heated ($R = 0$), the minimum flow-rate decreases with increasing inlet temperature, the difference between the minimum flow-rate in channel 1 (maximum heated) and channel 2 gets larger;
- The difference between minimum flow-rate in channel 1 and channel 2 disappears if $R \to 1$. If the power distribution is equal between the two channels ($R = 1$), the minimum flow-rate in both channels show the same trend: a small increase with increasing inlet temperature.

Figures 8.7e,f give the following results:

- The average flow-rate in both channels increases with increasing inlet temperature;
- At higher $R$, the rate of increase in flow-rate in channel 2 is higher with respect to increasing inlet temperature;
- With increasing $R$, the average flow-rate increases in both channels, but the the rate of increase in channel 2 is higher than in channel 1;
- For $R = \frac{3}{4}$ and $R = 1$ the average flow-rate in channel 2 is higher than in channel 1 at high inlet temperatures ($T_{inlet} \sim 102^\circ C$).

Figure 8.8 shows the period of flashing as function of inlet temperature and $R$. The result from this figure is that an increase of inlet temperature and an increase of $R$ leads to a decrease of the flashing period.

From the previous results obtained from figure 8.7 and figure 8.8, the following conclusions can be drawn:

- Increasing inlet temperature and power (increasing $R$) leads to destabilization of the Boiling Water Reactor; the flashing peaks increase (see figures 8.7a,b) and the flashing period decreases (see figure 8.8);
- Decreasing $R$ and increasing inlet temperature leads to an increase of the difference between the minimum flow-rates between channel 1 and channel 2. During the incubation time, channel 1 induces natural circulation and channel 2 does not contribute to the natural circulation. Since channel 1 and channel 2 are connected at the bottom and the top, the pressure drop over channel 1 and channel 2 is the same. It seems that the pressure drop only due to gravitation in channel 2 is equal to the pressure drop in channel 1 caused
Comparison travelling time and flashing period

The travelling time of flow and enthalpy perturbations in the core-section can be approximated as:

\[ \tau_c = \frac{V_c}{\Phi_v} \]  

(8.6)

where \( V_c \) is the volume of the core and \( \Phi_v \) is the time averaged inlet flow-rate. However, if the power distribution in the core is asymmetric, the travelling time will be different in the two asymmetric channels. Anirumi ([Anirumi et al., 1979]) proposed an average travelling time. This proposition is not suitable; if the velocity in one channel is zero, the travelling time is infinite, leading to an infinite equivalent travelling time over the two channels. The equivalent travelling time should be equal to the travelling time in the other channel with a certain velocity.
Figure 8.9: Flashing period as function of equivalent one-phase travelling time in the core-section (a) and riser-section (b).

Manera proposes an equivalent core travelling time as [Manera et al., 2001]:

\[ \tau_{c,eq} = \frac{\tau_1 \tau_2 (\tau_1 + \tau_2)}{\tau_1^2 + \tau_2^2} \]  
(8.7)

where \( \tau_{c,eq} \) is the equivalent core travelling time, \( \tau_1 \) and \( \tau_2 \) are the travelling times in channels 1 and 2, respectively.

Also a travelling time in the riser-section can be calculated as:

\[ \tau_r = \frac{V_r}{2\Phi_{v1} + 2\Phi_{v2}} \]  
(8.8)

where \( \tau_r \) is the riser travelling time, \( \Phi_{v1} \) and \( \Phi_{v2} \) are the average flow-rates in channel 1 and 2, respectively. It is assumed that the other two core channels behave in the same way as the two channels in which is measured, leading to an average total flow-rate of twice the sum of the average flow-rates in the two core-channels measured.

Figure 8.9a,b gives the results of the travelling time in core- and riser-section, respectively. Both figures show that the points fall more or less on the same line. In the core-section, the oscillation period and the travelling time differ one order of magnitude. In the riser-section, the oscillation period and the travelling time differs a factor two. The travelling time of enthalpy perturbations in the riser-section gives thus the major contribution to the oscillation period in case of flashing-induced oscillations.
8.8 Comparison time-series flow-rate and temperature at exit core

Section 8.5 and section 8.6 showed that the peaks (maximum flow-rate) in channel 2 were higher than the peaks in channel 1. It seems that during flashing, the velocity in channel 2 is higher than the velocity in channel 1. To give an explanation for this, the outlet temperature at the exit of channels 1 and 2 are plotted against time in figure 8.10. A comparison with figures 8.6c,d and figure 8.10 reveals that at low flow-rates the coolant is boiling at the outlet of both channels (the temperature is equal to saturation temperature). If flashing starts, the velocity in both channels increases and thus the temperature in both channels decreases. At the exit of channel 2 the temperature drop is higher with respect to the temperature drop at the exit of channel 1. This result is consistent with the results of the flow-rate in both channels. The peaks in channel 2 are higher with respect to the peaks in channel 1. A simple energy balance says that the temperature drop at the exit of a channel increases with increasing flow-rate. It seems that the area of channel 1 is not exactly equal to the area of channel 2. This results in different friction factors for both channels, resulting in different velocities and temperature drops at the exit of both channels.

Figure 8.10: The temperatures at the outlet of channels 1 and 2 (measured by thermocouples) are plotted against time; $T_{\text{inlet}} \sim 100^\circ$, $R = 1$. 
8.9 Comparison flow-rate measured by LDA and flow-meter

In order to get insight into the accuracy of the LDA-results, the LDA measurements are compared with the total flow-rate measured by the flow-meter in the downcomer channel in the bottom part of the CIRCUS-facility. To calculate the total flow-rate from the LDA measurements, it is assumed that two channels with identical settings have equal flow-rates:

\[ \Phi_v = 2\Phi_{v1} + 2\Phi_{v2} \]  

(8.9)

Figure 8.11a gives the results for the measurement with the most asymmetric power distribution \((R = 0)\) in the core and inlet temperature of the coolant at 100°C. Figure 8.11b gives the results for the measurement with symmetric power distribution \((R = 1)\) in the core and inlet temperature of the coolant at 100°C. From these figures the following results are obtained:

- The flow-meter has a slow time response, the peaks are delayed;
- The flow-meter is not able to follow the 'fast' flow oscillations after the flashing peaks;
- The LDA measurements seem to underestimate the total flow-rate by about 0.04 l/s.

It should be noted that the small differences between the flow-rate measured by LDA and flow meter are caused by the assumption that the flow-rates in the two channels in which is not measured behave in the same way as the two channels in which is measured. Although all valves were open during the measurements, it could be possible that the wall friction in the four channels are different due to small differences of the channel flow-area's (see section 8.8).
Figure 8.11: The calculated total flow-rate measured by means of LDA and the total flow-rate measured by means of the flow-meter are plotted against time.

(a) $T_{\text{inlet}} \sim 100^\circ C$, $R = 0$.

(b) $T_{\text{inlet}} \sim 100^\circ C$, $R = 1$.
Chapter 9

Conclusions and recommendations

In this chapter, conclusions are drawn from this research and recommendations for further research are presented.

9.1 Summary and conclusions

- A steady-state model of the core coolant loop of a natural circulation cooled BWR has been developed, applicable for low power, low pressure conditions. The model calculates the natural circulation flow-rate and boiling boundaries in core- and riser-section. The power distribution and the friction in the core can be set asymmetric. The model predicts an increase of natural circulation flow-rate and a decrease of boiling/flashing boundaries if the inlet temperature or the total power input is increased. If the power distribution is set asymmetric, asymmetry is observed in flow-rates and boiling boundaries in the boiling channels. Increasing friction leads to a decrease of natural circulation and boiling/flashing boundaries as expected. No multiple solutions in flow-rate were found.

- Two experimental LDA-setups have been developed. Each setup measures velocities at a point in a core-boiling channel of the CIRCUS facility. A laminar flow-scan is made through the channel in order to test the accuracy of the system. The laminar flow-scan performed correctly fitted to the theoretical profile (within the error bars), except for the points measured close to the wall of the annular tube (because the measurement volume slides into the wall). The velocity measured by both LDA-setups are calibrated to the flow-rate measured by a flow-meter present in the bottom part of the CIRCUS-facility.

- The LDA-measurements were randomly sampled. A detailed analysis of one measurement showed that the LDA-data is full of clusters of points. A PDF of the time between data showed a huge contribution of small time intervals (below 0.4 ms). The PDF together with the time-series of the velocity lead to
the conclusion that the signal processor is mislead due to the 'continuous' LDA signal and added many 'ghost' points to the data-set (multiple validation). By making use of the FFT-routine, it is shown that all interesting physical frequencies remain below 1 Hz. The data-set is evenly re-sampled with a sampling frequency of 10 Hz. As a measure of accuracy, the standard deviation of the velocity has been calculated around 4.0 cm/s.

- The CIRCUS measurements reveal that increasing inlet temperature or total input power leads to an increase of the amplitude of the flashing peaks; the system gets more unstable. Also the average flow-rate increases, leading to a decrease of the travelling-time of perturbations; the flashing period decreases. If the power distribution is made more asymmetric, asymmetries in minimum flow-rate between two channels are observed. The most asymmetric power distribution (maximum power in two core-channels and zero power in the other two core-channels) shows that the non-boiling channels (not heated) are acting as bypasses.

- A comparison between simulations and measurements shows that the simulations seem to overestimate the measurements by about 0.04 (l/s). This may be due to the simplified setup that is modelled. No buffer vessels are present and some objects causing friction are not taken into account (transition from downcomer channel to four boiling channels, transition from four boiling channels to riser channel, etcetera). Also the two-phase pressure drops due to friction may be underestimated, leading to higher flow-rates. Nevertheless, the trends of the measurements and simulations are the same (increasing inlet temperature or power leads to an increase of flow-rate).

- The travelling time of flow and enthalpy perturbations in the riser-section are dominating factors for the oscillation period of flashing-induced oscillations.

- The difference in peak height between the two channels may be caused by a channel-friction difference. The time-series of the channel outlet-temperature of both channels are consistent with the time-series of the channel flow-rate.

9.2 Recommendations

- A calibration curve was acquired in order to convert the measured velocity at a point in the annular flow channel to the flow-rate in that particular channel. To understand that no transition from laminar to turbulent flow was observed and that an offset was present, more attention should be paid to the calibration curve. More points should be measured at low velocities and one should get more insight into the transition from laminar to turbulent flow in an annular flow channel.

- In order to adjust the asymmetry of the power distribution, also the total power input was changed. The total power input directly affects the stability of the CIRCUS-facility. To study only asymmetries of the power distribution,
the powers of all boiling channels should be adjusted to keep the total power input constant.

- During the experiments, the power in a pair of channels was changed and the power in the other pair of channels was kept constant. A study in asymmetries in parallel boiling channels can be extended in which all channels have different power settings. This can be achieved in two ways. First, another two LDA-setups should be constructed. By making use of four LDA-setups the flow-rates in the four boiling channels can be measured simultaneously. This is a very expensive solution. Secondly, two sets of measurements (first set in two boiling channels, the second set in the other two boiling channels) are correlated to each other by making use of the total flow-rate signal. However, the results may show similar observations and trends.

- In the present CIRCUS facility, the flow through separate core channels is combined in a single riser. For future research it can be interesting to place individual risers on top of each core channel. Each core/riser section gives its own contribution to the stability of CIRCUS as a whole. If each core channel has its own individual riser, the study mentioned in the previous recommendation will become of more interest.

- The present steady-state model developed, should be extended to a dynamic model.
Appendix A

Tabled values of liquid and saturated properties

A.1 Liquid properties in one-phase region

In the one-phase region, the liquid density and enthalpy may be approximated as:

\[ \rho_l(T, p) \approx \rho_f(T) \]  \hspace{1cm} (A.1)

\[ h_l(T, p) \approx h_f(T) \]  \hspace{1cm} (A.2)

Table A.1 gives the values of the liquid density and enthalpy as function of temperature, extracted from [Moran and Shapiro, 1993].

<table>
<thead>
<tr>
<th>( T ) (°C)</th>
<th>( h_l ) (kJkg(^{-1}))</th>
<th>( \rho_l ) (kgm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0</td>
<td>209.33</td>
<td>988.045</td>
</tr>
<tr>
<td>60.0</td>
<td>251.13</td>
<td>983.091</td>
</tr>
<tr>
<td>70.0</td>
<td>292.98</td>
<td>977.708</td>
</tr>
<tr>
<td>80.0</td>
<td>334.91</td>
<td>971.723</td>
</tr>
<tr>
<td>90.0</td>
<td>376.92</td>
<td>965.251</td>
</tr>
<tr>
<td>100.0</td>
<td>419.04</td>
<td>958.313</td>
</tr>
<tr>
<td>110.0</td>
<td>461.30</td>
<td>950.932</td>
</tr>
<tr>
<td>120.0</td>
<td>503.71</td>
<td>943.129</td>
</tr>
<tr>
<td>130.0</td>
<td>546.31</td>
<td>934.842</td>
</tr>
<tr>
<td>140.0</td>
<td>589.13</td>
<td>926.183</td>
</tr>
<tr>
<td>150.0</td>
<td>632.20</td>
<td>917.011</td>
</tr>
</tbody>
</table>

The liquid density is parabolically fitted to the liquid enthalpy, in order to include the resulting relation in the model:

\[ \rho_l = 0.123 \cdot 10^{-3} h_l^2 - 64.93 \cdot 10^{-3} h_l + 1007.16 \]  \hspace{1cm} (A.3)
Appendix A. Tabled values of liquid and saturated properties

The dimensions in this relation are, respectively: $[\rho] = kgm^{-3}$, $[h] = kJkg^{-1}$. This relation is only used for calculating the one-phase pressure drops in the loop.

A.2 Saturated properties in two-phase region

In the two-phase region the saturated density and enthalpy properties are calculated from the local pressure.

Table A.2 gives the values of the saturated properties, which are tabled in [Moran and Shapiro, 1993].

\[
\begin{array}{|c|c|c|c|c|}
\hline
p \text{ (bar)} & \rho_f \text{ (kgm}^{-3}\text{)} & \rho_g \text{ (kgm}^{-3}\text{)} & h_f \text{ (kJkg}^{-1}\text{)} & h_g \text{ (kJkg}^{-1}\text{)} \\
\hline
1.0 & 958.59 & 0.59 & 417.46 & 2675.50 \\
1.5 & 949.85 & 0.86 & 467.11 & 2693.30 \\
2.0 & 942.95 & 1.13 & 504.70 & 2706.70 \\
\hline
\end{array}
\]

Table A.2: Liquid/vapour saturation data

The properties are parabolically fitted, in order to include the resulting relations in the model:

\[
\begin{align*}
\rho_f & = 3.68p^2 - 26.68p + 981.59 \\
\rho_g & = -0.014p^2 + 0.581p + 0.023 \\
h_f & = -24.12p^2 + 159.6p + 281.98 \\
h_g & = -10.0p^2 + 61.2p + 2624.3
\end{align*}
\]

The dimensions in the relations are, respectively: $[\rho] = kgm^{-3}$, $[h] = kJkg^{-1}$ and $[p] = bar$.

The relation for the heat of evaporation, $h_{ev}$, can easily be deduced by taking the difference between the relations of $h_g$ and $h_f$. 
Appendix B

Iterations used for steady-state model

B.1 Iteration for finding boiling/flashing boundary

In the core-section the enthalpy of the fluid increases along the flow-path according to equation 3.20. In the riser-section the enthalpy of the fluid remains constant along the flow path. The saturation enthalpy can be calculated from eqns. 3.23 and A.6. At each iteration step, the enthalpy of the fluid is compared to the saturation enthalpy. The iteration process continues according to the following expressions:

\[
\begin{align*}
(h(z))_b & \leq (h_f)_b \Rightarrow (\Delta z)_{b+1} = (\Delta z)_b + \delta z \\
(h(z))_b & > (h_f)_b \Rightarrow (\Delta z)_{b+1} = (\Delta z)_{b-1} + (0.1)^i \delta z, \quad i = i + 1
\end{align*}
\] (B.1)

where \(\delta z\) is a step in height. This process is converged if \(|h(z))_b - (h_f)_b| < \varepsilon_z\). When convergence is reached, \(\Delta z\) will be equal to \(z_{bb}\).

B.2 Iteration for saturation enthalpy at exit core/riser

In order to start the iteration-process, a guess-value for the saturation enthalpy at the exit of core/riser is taken, \((h_f)_1\), assuming that no boiling/flashing boundary is present. By making use of the one-phase pressure drops, the guess saturation enthalpy at the exit of core/riser can be evaluated directly. From the guess-value of the saturation enthalpy, the quality and void-fraction at the exit of core/riser are calculated. At this point, the two-phase pressure drops are calculated, leading to a new evaluation of the pressure (and saturation enthalpy) at the exit of the core/riser, \(h_{f,calc}\). This iteration process continues according to the following expressions:
Appendix B. Iterations used for steady-state model

\[ \frac{(h_f)_b - (h_{f,calc})_b}{(h_f)_{b-1} - (h_{f,calc})_{b-1}} > 0 \Rightarrow (h_f)_{b+1} = (h_f)_b + \delta h \]  
\[ \frac{(h_f)_b - (h_{f,calc})_b}{(h_f)_{b-1} - (h_{f,calc})_{b-1}} \leq 0 \Rightarrow (h_f)_{b+1} = (h_f)_{b-1} + (0.1)^i \delta h, \quad i = i + 1 \]  

(B.2)

where \( \delta h \) is a step in enthalpy. This process is converged if \( |(h_{sat})_b - (h_{sat,calc})_b| < \epsilon_h \).

B.3 Iteration for condition \( \Delta p_1 = \Delta p_2 \)

A small start-value for \( \Phi_{m1} \) is taken: \( \Phi_{m1, start} \). The total pressure drop in both core-channels are calculated (in which two iterations are incorporated for the boiling boundary and saturation enthalpy at exit core). This iteration process continues according to the following expressions:

\[ \frac{(\Delta p_1 - \Delta p_2)_b}{(\Delta p_1 - \Delta p_2)_{b-1}} > 0 \Rightarrow (\Phi_{m1})_{b+1} = (\Phi_{m1})_b + \delta \Phi_{m1} \]  
\[ \frac{(\Delta p_1 - \Delta p_2)_b}{(\Delta p_1 - \Delta p_2)_{b-1}} \leq 0 \Rightarrow (\Phi_{m1})_{b+1} = (\Phi_{m1})_{b-1} + (0.1)^i \delta \Phi_{m1}, \quad i = i + 1 \]  

(B.3)

where \( \delta \Phi_{m1} \) is a step in mass-flow in channel 1. This process is converged if \( |(\Delta p_1 - \Delta p_2)_b| < \epsilon_{\Phi_m} \).

B.4 Iteration for natural circulation flow-rate

A small start-value for the mass-flow is taken: \( \Phi_{m,start} \). The total pressure drops in every section are calculated and summed (iterations for boiling boundaries, saturation enthalpy at exit core/riser and the condition \( \Delta p_1 = \Delta p_2 \) are incorporated).

The flow-rate is raised until the sum of all pressure drops equals to zero (within a certain tolerance value):

\[ \frac{(\Delta p_d + \Delta p_c + \Delta p_r + \Delta p_h)_b}{(\Delta p_d + \Delta p_c + \Delta p_r + \Delta p_h)_{b-1}} > 0 \Rightarrow (\Phi_{m})_{b+1} = (\Phi_{m})_b + \delta \Phi_m \]  
\[ \frac{(\Delta p_d + \Delta p_c + \Delta p_r + \Delta p_h)_b}{(\Delta p_d + \Delta p_c + \Delta p_r + \Delta p_h)_{b-1}} \leq 0 \Rightarrow (\Phi_{m})_{b+1} = (\Phi_{m})_{b-1} + (0.1)^i \delta \Phi_m, \quad i = i + 1 \]  

(B.4)

where \( \delta \Phi_m \) is a flow-rate step. This process is converged if \( |(\Delta p_d + \Delta p_c + \Delta p_r + \Delta p_h)_b| < \epsilon_{\Phi_m} \).
Appendix C

Derivation of the length of the LDA control volume

In this Appendix, the length of the LDA control volume is derived. The result of the derivation is used in section 5.4. In figure C.1 a schematic representation of the length of the control volume is given.

The scatter beam and reference beam are focused in water at the focal point in water, $F_w$, which is not the same as the focal point in air, $F_a$. The reference beam exits the flow channel and is directed to the detector. The angles in water $\gamma_{w,1}$ and $\gamma_{w,2}$ are linked by the formula of Snellius to the angles in air $\gamma_{a,1}$ and $\gamma_{a,2}$ as follows:

$$\frac{\sin \gamma_{a,1}}{\sin \gamma_{w,1}} = \frac{\sin \gamma_{a,2}}{\sin \gamma_{w,2}} = \frac{n_w}{n_a}$$

(C.1)

The angles $\gamma_{a,1}$ and $\gamma_{a,2}$ are linked to $\frac{\delta a}{2}$ together with $\gamma_{\text{max}}$, given in eq. 5.3, as:
Appendix C. Derivation of the length of the LDA control volume

\[ \gamma_{a,1} = \frac{\theta_a}{2} + \gamma_{\text{max}} \]  
\[ \gamma_{a,2} = \frac{\theta_a}{2} - \gamma_{\text{max}} \]  

The upper and lower line \( y(x)_{a,1} \) and \( y(x)_{a,2} \), respectively, around the laserbeam which falls on the detector’s eye, represent the cone of interference (which is described in section 5.3.3). These lines can be represented as:

\[ y(x)_{a,1} = (x - OD)\tan \gamma_{a,1} + L_{\text{DET},F_a}\sin \theta_a - WD\tan \gamma_{a,1} \]  
\[ y(x)_{a,2} = (x - OD)\tan \gamma_{a,2} + L_{\text{DET},F_a}\sin \theta_a - WD\tan \gamma_{a,2} \]

where \( OW \) is the distance between the axis of the tube and the wall (which is equal to \( \frac{1}{2}D \)), \( OD \) is the distance between the axis of the tube and the projection of the detector’s eye on the bisectrice of the two laser beams and \( WD \) is the distance from the wall to the projection of the detector’s eye on the bisectrice of the two laser beams.

\( WD \) can be calculated as follows:

\[ WD = F_aD - F_aW \approx F_aD - \frac{n_a}{n_w} F_w W = L_{\text{DET},F_a}\cos \frac{\theta_a}{2} - \frac{n_a}{n_w} (OW - OF_w) \]

where \( F_aD \) is the distance between the focal point in air and the projection of the detector’s eye on the bisectrice of the two laser beams, \( F_aW \) is the distance between the focal point in air and the wall of the tube, \( F_w W \) is the distance between the focal point in water and the wall of the tube and \( OF_w \) is the distance between the axis of the tube and the focal point in water (\( OF_w \) is equal to \( \delta F_w \)).

Also in water, the upper and lower line around the laser beam can be described as follows:

\[ y(x)_{w,1} = (x - OW)\tan \gamma_{w,1} + w_1 \]  
\[ y(x)_{w,2} = (x - OW)\tan \gamma_{w,2} + w_2 \]

where \( y(x)_{w,1} \) represents the lower line in water as function of \( x \) and \( y(x)_{w,2} \) represents the upper line in water as function of \( x \).

The scatter beam (i.e the beam which scatters light to the detector and does not fall in the detector’s eye) can be represented as follows:

\[ y(x)_{w,3} = -(x - OF_w)\tan \theta_w \]
where $y(x)_{w,3}$ represents the scatter beam in water as function of $x$. All angles are defined in figure C.1.

$y(x)_{w,1}$ and $y(x)_{w,2}$ intersect $y(x)_{w,3}$ at $x$-coordinates $x_{w,1}$ and $x_{w,2}$ respectively:

$$x_{w,1} = \frac{OF \tan \frac{\theta_w}{2} + OW \tan \gamma_{w,1} - w_1}{\tan \gamma_{w,1} + \tan \frac{\theta_w}{2}} \quad \text{(C.8)}$$

$$x_{w,2} = \frac{OF \tan \frac{\theta_w}{2} + OW \tan \gamma_{w,2} - w_2}{\tan \gamma_{w,2} + \tan \frac{\theta_w}{2}} \quad \text{(C.9)}$$

The length of the control volume, $l_{cv}$ times the cosine of $\frac{\theta_w}{2}$ is equal to the difference of $x_{w,1}$ and $x_{w,2}$:

$$l_{cv} \cos \frac{\theta_w}{2} = \frac{OF \tan \frac{\theta_w}{2} + OW \tan \gamma_{w,1} - w_1}{\tan \gamma_{w,1} + \tan \frac{\theta_w}{2}} - \frac{OF \tan \frac{\theta_w}{2} + OW \tan \gamma_{w,2} - w_2}{\tan \gamma_{w,2} + \tan \frac{\theta_w}{2}} \quad \text{(C.10)}$$

If $OF$ and $OW$ are replaced by $\delta F_w$ and $\frac{1}{2}D$ respectively, the final result given in equation 5.6 is obtained.
Bibliography


