SUPERCONDUCTING QUANTUM INTERFERENCE BASED ELECTROMECHANICAL SYSTEMS
SUPERCONDUCTING QUANTUM INTERFERENCE BASED ELECTROMECHANICAL SYSTEMS

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Mechanical sensors are essential tools for the detection of small forces. Ultimately, the random displacement of the mechanical element due to quantum fluctuations determines the minimum noise floor. To reach this limit, the detector which transduces displacement to voltage must be extremely low-noise and high-gain, with limits which are ultimately determined by quantum mechanics. This chapter introduces a vibrating dc SQUID as a displacement detector. First, an introduction to resonant micro- and nanoelectromechanical sensors is given, followed by the motivation for low-noise position detection. Backaction forces from the detector to the resonator are discussed, because they influence both the dynamics and the force noise of the resonator. Finally, the dc SQUID displacement detector is described, along with an outline of the content of this thesis.
1.1 RESONANT MICROELECTROMECHANICAL SENSORS

Mechanical sensors transduce forces into displacements. The mechanical element is usually operated at one of its resonant modes. Measurement of the deflection of the resonator enables detection of very small forces, such as those induced by gravitational waves [1] and quantum mechanical fluctuations [2–4]. Detection of shifts in the resonance frequency enables sensitive mass detection [5] and force gradient detection, which is the basis for Dynamic Force Microscopy techniques [6–8]. Figure 1.1 shows several resonators which are used in the MED group in Delft and describes their purpose.

The resonator displacement is converted, usually to an electronic signal, by a detector. In optical displacement detectors, the signal is generated by light which falls onto a photodiode. Displacement is detected by reflecting the light off of the mechanical sensor and using the photodiode to measure, for example, the position of the reflected beam [8] or the intensity of the light which is transmitted through a second, semi-transparent mirror (a Fabry-Perot cavity) [9]. Although optical detectors can achieve high displacement resolution [10, 11], the size of the resonators to be detected is limited by the wavelength of the light. Optical methods thus cannot be used on high-frequency, low-mass, nanoscale resonators, such as carbon nanotubes.

Electromechanical systems do not have this limitation, because they convert mechanical motion directly to electronic signals and vice versa [12]. Electromechanical actuation and detection can be broadly divided into capacitive, inductive and piezoelectric/resistive based mechanisms. Classical detectors based on these principles are widely used in commercial sensors. At low temperatures, mesoscopic electronics such as the Single-Electron Transistor (SET) [13, 14], the Atomic Point Contact (APC) [15] and the Quantum Point Contact (QPC) [16] can be used. These low-temperature schemes greatly increase the detector resolution with respect to classical electronic readout methods. Electromechanical resonators can also be used as memory elements in quantum information architectures [17]. This has recently been demonstrated using a piezoelectric resonator coupled to a qubit [2].

The displacement of a resonator mode can be described by a single coordinate $u$, which gives the root-mean-square displacement from the resonator equilibrium position. The equation of motion a damped resonator mode, driven by a force $F(t)$, is

$$m_0 \dddot{u} + \frac{m_0 \omega_0}{Q_0} \dot{u} + k_0 u = F(t), \quad (1.1)$$

where $m_0$, $\omega_0$, $Q_0$ and $k_0$ are respectively the intrinsic mass, the angular resonance
frequency, the quality factor and the spring constant, \( k_0 = m_0 \omega_0^2 \), of the mode. Using the equation of motion, a sinusoidal force \( F(t) = \hat{F} \sin(\omega t) \) at angular frequency \( \omega \) results in a steady-state sinusoidal motion of the resonator at the same frequency; \( u(t) = \hat{u} \sin(\omega t + \varphi) \). The amplitude \( \hat{u} \) and phase \( \varphi \) of the resonator response are given by

\[
\hat{u} = \frac{Q_0 \hat{F}}{k_0} \sqrt{\frac{Q_0^2(1 - (\omega/\omega_0)^2)^2 + (\omega/\omega_0)^2}{Q_0^2(1 - (\omega/\omega_0)^2)}} \quad \text{and} \quad \tan(\varphi) = \frac{\omega/\omega_0}{Q_0(1 - (\omega/\omega_0)^2)} .
\] (1.2)

The largest response \( \hat{u}/\hat{F} = Q_0/k_0 \) occurs at the resonance frequency and is \( Q_0 \) times larger than the non-resonant, Hooke's law response. Mechanical resonators with high resonance frequencies, typically in the MHz to GHz range, are desirable for quantum measurements because they can be cooled to the ground state in cryogenic refrigerators. High-frequency resonators are also attractive because
they increase the acquisition speed of scanning-probe and mass-detection measurements.

To achieve maximum response at high frequencies, the resonator must have a small mass and small damping (large $Q_0$). These requirements lead to the use of micro- and nanometer scale flexural resonators, i.e., resonators for which at least one dimension is very small compared to the other two. The resonators can be either machined from bulk material in a top-down process (the resonators in figure 1.1a-c) or they can be 'grown' through chemical synthesis by using a bottom-up process (the carbon nanotube in figure 1.1d). Compared to larger resonators, micro- and nanometer scale flexural resonators have a larger surface-to-volume ratio. Because of this, the damping of the small resonators is more strongly influenced by surface defects and is typically higher than that of larger resonators [12]. However, recent progress in fabrication techniques for top-down [20] and carbon nanotube [21] resonators has resulted in high-frequency, nanoscale flexural resonators with quality factors on the order of $10^6$, on-par with larger resonators.

1.2 Detector Backaction and the Quantum Limit

It is known from statistical mechanics that the expectation value for the potential energy in a harmonic oscillator at temperature $T_0$ is

$$\frac{1}{2} k_0 \langle u^2 \rangle = \frac{1}{2} \hbar \omega_0 \left( \frac{1}{2} + \frac{1}{e^{\hbar \omega_0 / k_B T_0} - 1} \right).$$  \hspace{1cm} (1.3)

At temperatures for which the thermal energy is much larger than the energy of a single phonon, i.e. $k_B T_0 \gg \hbar \omega_0$, this expression reduces to the equipartition theorem:

$$\frac{1}{2} k_0 \langle u^2 \rangle = \frac{1}{2} k_B T_0,$$ \hspace{1cm} (1.4)

and in the opposite limit, $k_B T_0 \ll \hbar \omega_0$, equation 4.1 becomes:

$$\frac{1}{2} k_0 \langle u^2 \rangle = \frac{1}{2} \hbar \omega_0.$$ \hspace{1cm} (1.5)

Thus, at absolute zero the only fluctuations that remain are the zero-point fluctuations with an average energy of half a phonon. The total force noise on the resonator is however higher than this due to backaction from the position detector. If the position detector couples linearly to the deflection of the resonator, quantum mechanical analysis shows that even an ideally coupled, quantum-limited detector adds at least another half a phonon of noise energy [22, 23]. This can be understood as the contribution of the internal degrees of freedom in the detector when
they are coupled to the resonator. The total added noise consists of the imprecision noise of the detector, which shows as a white displacement noise floor at the detector output and the back action force noise, which increases the amplitude of the Lorentzian noise peak of the resonator. Although the resolution can be made arbitrarily small by increasing the coupling between the detector and the resonator, the optimum detector-resonator coupling yields a resolution on the order of the zero-point motion of the resonator:

\[
\sqrt{\langle u^2 \rangle_{\text{QL}}} = \sqrt{\frac{\hbar \omega_0}{2k_0}}.
\]

This is the mean-square displacement associated with half a phonon and is known as the standard quantum limit. Note, that these fluctuations only become significant once the resonator is cooled to a temperature such that \( \hbar \omega_0 \ll k_B T_0 \).

Besides stochastic force noise, the position detector can also influence the dynamics of the resonator. If the detector exerts a force on the resonator which is proportional to the displacement, \( u \), then equation 1.1 shows that this force modifies the effective spring constant of the resonator. In the same way, a velocity-proportional backaction force changes the resonator damping. These dynamical backaction effects can lead to resonator cooling and heating [24] and must therefore be studied in order to examine whether a position detector can in principle be quantum-limited.

### 1.3 Introducing the Vibrating SQUID

The dc Superconducting QUantum Interference Device, or SQUID, is a highly sensitive, potentially quantum-limited, detector of magnetic flux [25–27]. The mechanical resonator is formed by suspending a part of the SQUID loop to create a microbridge (figure 1.1c). A constant magnetic field then couples the position of the bridge to the magnetic flux in the loop, which couples the SQUID and the resonator. This thesis describes the properties of this vibrating SQUID, from the first proof-of-principle measurements to exploration of the coupled SQUID-resonator dynamics.

In chapter 2, the fabrication method and the measurement principles are described. The first measurements on the SQUID position detector are shown, with a resolution of 51 times the standard quantum limit. Chapter 3 presents measurements of the dynamic back action of the SQUID on the resonator. A numerical model based on time-integration is developed and used to explain the behavior of the SQUID-resonator system. Chapter 4 gives a detailed description of the measurement setup and calibration procedures. The addition of a cryogenic amplifier stage results in an improved resolution of 4.4 times the standard quantum limit. In
chapter 5, a discrete-time quadrature feedback is developed to actively damp the resonator. The signal-to-noise ratio at base temperature enables a decrease of the mode temperature down to 14 mK.

In order to increase the coupling between the SQUID and the resonator, in chapter 6 the entire SQUID loop (figure 1.1c) is freely suspended. The resulting resonator has hybrid flexural-torsional eigenmodes. In the high flux-to-voltage regime of the dc SQUID, the two lowest modes start to oscillate spontaneously with large amplitude. These self-sustained oscillations are due to strong negative damping by the SQUID back action. We analyze the oscillator behavior and relate it to the backaction model of chapter 3. In the self-oscillating regime, we calibrate the displacement responsivity by using the periodic voltage-flux relationship of the SQUID. For this strongly coupled torsional SQUID detector, we find that the displacement resolution is a factor 1.5 below the standard quantum limit.
REFERENCES


Superconducting Quantum Interference Devices (SQUIDs) are the most sensitive detectors of magnetic flux [1] and are also used as quantum two-level systems (qubits) [2]. Recent proposals have explored a novel class of devices which incorporate micromechanical resonators into SQUIDs in order to achieve controlled entanglement of the resonator ground state and a qubit [3] as well as permitting cooling and squeezing of the resonator modes and enabling quantum limited position detection [4–10]. In spite of these intriguing possibilities, no experimental realization of an on-chip, coupled mechanical resonator-SQUID system has yet been achieved. Here, we demonstrate sensitive detection of the position of a 2 MHz flexural resonator which is embedded into the loop of a DC-SQUID. We measure the resonator's thermal motion at millikelvin temperatures, achieving an amplifier-limited displacement sensitivity of 10 fm/√Hz and a position resolution that is 36 times the quantum limit.

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DC-SQUIDs consist of a superconducting loop with two Josephson junctions [1]. The voltage across the DC-SQUID does not only depend on the current through it, but also on the magnetic flux piercing through the loop, allowing tiny changes in the magnetic field to be detected. Besides this more common application, the DC-SQUID should also be able to detect small variations in the area of its loop due to the motion of an integrated flexural resonator in the presence of a static magnetic field. Our Nb-based DC-SQUID displacement detector is based on this principle and is shown in Fig. 2.1a,b and described in Methods. Its potential displacement sensitivity can be estimated as follows: The resonator has a length of $\ell = 50 \, \mu\text{m}$ and the loop is placed in a magnetic field of $B = 0.1 \, \text{T}$ oriented as described in Fig. 2.1d. A deflection $u = 1 \, \text{fm}$ of the resonator will then result in a change in flux through the loop on the order of $B\ell u = 2.5 \, \mu\Phi_0$, where $\Phi_0 = h/2e = 2.07 \, \text{fTm}^2$ is the flux quantum. Low-temperature DC-SQUIDs have a typical flux sensitivity of $10^{-6} \Phi_0/\sqrt{\text{Hz}}$ [11], which is sufficient to reach a displacement sensitivity of $0.4 \, \text{fm/Hz}$. This places the DC-SQUID detector in the same league as other highly sensitive on-chip position detectors [12–15].

2.1 DC SQUID CHARACTERIZATION

Before using the DC-SQUID as a displacement detector, its characteristics are determined in order to find a proper bias point. Fig. 2.1d shows a schematic of the measurement setup, which is described in detail in Methods. A bias current $I_B$ is applied to the DC-SQUID and its output voltage $V$ is measured. The DC-SQUID produces a non-zero output voltage once $I_B$ exceeds the DC-SQUID’s critical current $I_C$, which depends on the magnetic flux bias $\Phi$ through the DC-SQUID loop and on the critical current of the individual junctions, $I_0$. Figure 2.2a shows that at $B = 0 \, \text{T}$ the DC-SQUID’s $V-I_B$ relationship exhibits hysteresis, indicating that the DC-SQUID is underdamped [1]. This hysteresis must be suppressed in order to operate the DC-SQUID as a sensitive linear flux detector. This is achieved by increasing $B$, which decreases the critical current of the junctions [16]. We find that for $B \geq 0.1 \, \text{T}$ the DC-SQUID is sufficiently damped such that no hysteresis is observed (Fig. 2.2a). Note that in this detection scheme the DC-SQUID is biased above the critical current. It can in principle also be used as a position detector when it is biased below the critical current, where it acts as a tunable inductor [4].

The DC-SQUID is most sensitive to changes in the magnetic flux when it is tuned to a working point with a steep slope of $V(\Phi)$. Figure 2.2b shows the relation between $V$ and the applied stripline current $I_F$ which changes $\Phi$. For a DC-SQUID, both $I_C$ and $V$ depend periodically on $\Phi$ with a period of $\Phi_0$. 
2.1. DC SQUID CHARACTERIZATION

Figure 2.1: The coupled DC-SQUID-resonator geometry and measurement setup. (a) Colorized scanning electron micrograph of a device at 80° inclination. The beam resonator (’R’) is buckled away from the substrate due to compressive strain (see Supplementary Information). The stripline (’S’) is used to change the flux bias through the DC-SQUID. The Nb-InAs weak links (’J’) are located adjacent to unused Nb side gates. (b) Colorized scanning electron micrograph of one of the 200 nm long Nb-InAs-Nb junctions; The scalebar is 1 µm. (c) A 3-dimensional image of the vibration amplitude of the driven beam resonance at 2 MHz, acquired at room temperature using dynamic force microscopy. The inset shows the static buckling profile of the beam with a maximum deflection of 1.5 µm measured using atomic force microscopy. (d) Schematic overview of the measurement circuit. A coupling field $B$ is applied parallel to the DC-SQUID-plane and perpendicular to the length direction of the resonator (marked by the red box). The sample is glued onto a piezo actuator, which is connected to a filtered voltage source for the driven measurements.
The steepest slope occurs approximately half-way between the minimum and maximum of the voltage swing. The difference between these two voltages is the peak-to-peak voltage swing $V_{pp}$. Figure 2.2c shows $V_{pp}$ as a function of $I_B$, where $V_{pp}^{max}$ is the maximum voltage swing. Numerical simulations (see the Methods section) show that this maximum occurs at $I_B = 2I_0$.

In our experiment there is a slow flux drift, which makes it difficult to maintain a constant flux bias by simply applying a constant stripline current. Therefore a feedback loop is used that maintains a constant average setpoint voltage $\langle V \rangle = V_{SP}$ by adjusting $I_F$ for a given value of $I_B$. An added advantage of using the feedback loop is that it compensates external low-frequency ($< 1$ kHz) flux noise.

Numerical analysis (see the Methods section) reveals that the ratios $I_B/I_0$ and $V_{SP}/V_{PP}^{max}$, needed to achieve the maximum value of the flux responsivity $dV/d\Phi$, are approximately constant. This allows us to apply a bias that gives maximum $dV/d\Phi$ even if $I_0$ and $V_{PP}^{max}$ are changing due to temperature variations as shown in Fig. 2.2d. In practice, values for $V_{SP}$ and $I_B$ must be chosen such that they allow stable operation of the feedback loop in addition to maximizing $dV/d\Phi$. The values $I_B = 2I_0$ and $V_{SP} = V_{PP}^{max}/2$ are found to provide a good balance between these requirements.

### 2.2 DETECTION OF THE DrIVEN MOTION

As the resonator is integrated into the DC-SQUID, the flux through the loop depends on the position $u$ of the fundamental out-of-plane mode of the beam according to $\Phi = \Phi_a + aB\ell u$, where $\Phi_a$ is the applied flux when the resonator is in its equilibrium position and $a$ is a geometrical factor that depends on the mode shape. The position $u$ is defined such that the effective spring constant of the mode is $k_R = m(2\pi f_R)^2$, where $m$ is the total mass of the beam and $f_R$ is the resonance frequency of the mode. Atomic force microscopy shows that the resonator is buckled (Fig. 2.1c inset) and we use a continuum model to find $a = 0.91$ (see the Methods section). In our configuration, the DC-SQUID functions as a linear displacement detector in the small signal limit $aB\ell u \ll \Phi_0$ with a displacement responsivity $dV/du = (dV/d\Phi)(d\Phi/du)$.

The resonance frequency $f_R$ is located by driving the resonator with a piezo actuator (Fig. 2.1d) and monitoring the resulting output voltage of the DC-SQUID. The resonance frequency is 2.0018 MHz with a Lorentzian amplitude response (Fig. 2.3a). Room-temperature dynamic force microscopy [17] confirms that this is indeed the fundamental mode (Fig. 2.1c) and it is also in good agreement with our continuum model. From a least-squares Lorentzian fit we extract a quality factor $Q = 1.8x10^4$ at $B = 100$ mT and refrigerator temperature $T = 20$ mK.
2.2. Detection of the Driven Motion

Figure 2.2: Characteristics of the DC-SQUID. (a) DC-SQUID voltage as a function of the bias current $I_B$ at 20 mK. At zero magnetic field the DC-SQUID exhibits hysteretic behavior (red curves), which indicates that the DC-SQUID is underdamped. The blue curve shows the voltage-current response at 100 mT, which is the lowest of two fields used for position detection. The increased magnetic field has suppressed the hysteresis due to a reduction of the critical current. (b) Average DC-SQUID voltage as a function of the applied stripline current at 100 mT and $I_B = 2.5 \mu$A. The peak-to-peak voltage swing is $V_{PP}$. (c) Measurements of $V_{PP}$ as a function of $I_B$ at $B = 100$ mT. The parameters $I_0$ and $V_{PP}^{max}$ obtained from this measurement are used to tune the DC-SQUID to a sensitive bias point. (d) $V_{PP}^{max}$ as a function of refrigerator temperature at two different magnetic fields.
2.3 THERMAL NOISE AND SENSITIVITY

Thermomechanical noise thermometry is used to calibrate the deflection responsivity [13–15]. Without actively driving the resonator, the noise power spectral density of the output voltage is acquired around the mechanical resonance frequency. The spectra in Fig. 2.3b show a constant background upon which a Lorentzian peak is superimposed. This peak is caused by the Brownian motion of the beam. The quality factor and resonance frequency are identical to those of the driven response (Fig. 2.3a). The voltage noise power due to the resonator $\langle V^2_R \rangle$, i.e. the area underneath the peak, is extracted from the fitted response function.

The noise spectrum is measured in the temperature range $20 \text{ mK} < T < 500 \text{ mK}$. To compare the noise power at different temperatures, $\langle V^2_R \rangle$ must be corrected for differences in $dV/d\Phi$. The flux responsivity is proportional to $V^\text{max}_{pp}$ and thus has the same temperature dependence (Fig. 2.2d). This enables the introduction of the corrected voltage noise power $\langle V^2_R \rangle = \langle V^2_R \rangle \cdot (V^\text{max}_{pp}(20 \text{ mK})/V^\text{max}_{pp}(T))^2$. The corrected noise powers in Fig. 2.3c show a linear decrease with temperature down to 100 mK. The slope $S(B)$ from a linear fit to this data, combined with the equipartition theorem gives the deflection responsivity $dV/du = V^\text{max}_{pp}(T)/V^\text{max}_{pp}(20 \text{ mK}) \cdot (S(B)k_R/k_B)^{1/2}$, where $k_B$ is the Boltzmann constant and $k_R = 97 \text{ N/m}$, using $m = 6.1 \cdot 10^{-13} \text{ kg}$. The resulting displacement responsivities at 20 mK are $3.0 \times 10^{-2} \text{nV/fm}$ and $2.3 \times 10^{-2} \text{nV/fm}$ for $B = 100 \text{ mT}$ and $111 \text{ mT}$ respectively. The fact that the displacement responsivity at 111 mT is lower than at 100 mT might appear counter intuitive, but the larger flux change for the same displacement is compensated by a stronger reduction of $V^\text{max}_{pp}$, as shown Fig. 2.2d. The best displacement sensitivity coincides with the highest responsivity because the flux-based position detector is limited by the noise floor of the room temperature voltage amplifier $\sqrt{S_V}$. The highest observed sensitivity is $\sqrt{S_u} = (dV/du)^{-1} \sqrt{S_V} = 10 \text{ fm/\sqrt{Hz}}$ and occurs at 20 mK and 100 mT.

2.4 TOWARDS THE QUANTUM LIMIT

It is interesting to calculate the position resolution of the detector and compare it to the fundamental limit imposed by quantum mechanics [18].
2.4. Towards the Quantum Limit

Figure 2.3: Driven response and Brownian motion of the resonator. (a) The driven resonator response at $T=20$ mK and $B=100$ mT. Amplitude and phase data are shown in blue and black, respectively. The amplitude and phase response are well fitted by the that of a harmonic oscillator (red). The peak amplitude corresponds to approximately 20 pm deflection. (b) Noise power spectral density around the beam resonance frequency at 100 mT for two different temperatures (black, blue). The peak in the noise spectra is due to the Brownian motion of the resonator. (c) The corrected voltage noise power $\langle V_r^2 \rangle$ extracted from the thermal spectra at 100 mT (red) and 111 mT (blue). The solid lines indicate a linear least mean squares fit to the noise powers for temperatures above 100 mK. The linear relationship is predicted by the equipartition theorem and its slope is used to calibrate the deflection responsivity of the flux-based transducer. The steeper line for the 100 mT data indicates a higher responsivity than at 111 mT. The lowest achieved resonator temperature is 84 mK at a refrigerator temperature of 20 mK. This difference implies that the resonator does not thermalize at the lowest temperatures (see the Methods section)
The position resolution is $\Delta u = \sqrt{S_u \Delta f}$, where $\Delta f = \pi f_R/2Q$ is the effective noise bandwidth of the resonator. This yields $\Delta u = 133$ fm for the highest observed sensitivity of the detector. The quantum limit for the position resolution of a continuous linear detector is $\Delta u_{QL} = \sqrt{\hbar/m(2\pi f_R)} = 4$ fm for our resonator [19], so that the detector resolution is a factor $\Delta u/\Delta u_{QL} = 36$ from the quantum limit.

The resonator enters the quantum regime once it has a thermal occupation factor $N \approx T_R/T_Q < 1$, where $T_Q = h f_R/k_b$ is the quantum temperature and $T_R$ is the resonator temperature, which can be found from the voltage noise power using $T_R = \langle V_R^2 \rangle k_R/k_b \langle dV/du \rangle^2$. From the data in Fig. 2.3c, the lowest observed resonator temperature is $T_R = 84$ mK, which yields $N = 878$.

There are two major challenges for observing quantum behavior in a macroscopic mechanical resonator [20]: a quantum limited position detector [18, 19, 21] with resolution $\Delta u = \Delta u_{QL}$ and a resonator with $N < 1$. These requirements can be simultaneously met by our device configuration: The quantum mechanical ground state for a 1 GHz resonator is reached at temperatures below $T_Q = 45$ mK, which can be achieved in a dilution refrigerator. The DC-SQUID is known to be a near quantum-limited flux detector and flux sensitivities of $p_S \Phi = 0.01 \mu \Phi_0/\mu$Hz are possible [22]. For the current device $\sqrt{s_b} \sim 10 \mu \Phi_0/\sqrt{\text{Hz}}$ is limited by the room temperature amplifier. Thus, by reducing the amplifier noise floor by using for example a second DC-SQUID as an amplifier and by increasing the flux responsivity of the first DC-SQUID, the sensitivity may ultimately be improved by three orders of magnitude. Under these conditions, a 1 GHz resonator made of a 300 nm long InAs beam with $Q = 1000$ will require a magnetic field of 1 T for the detector to reach quantum limited sensitivity. Note that InAs is very suitable for such a resonator, as beams that are only tens of nanometers wide can still carry a substantial supercurrent [23]. DC-SQUID operation in the required high magnetic field may be possible by utilizing narrow and thin Nb lines [24]. With these improvements, our flux-based measurement method can potentially be extended to detect the resonator’s ground state.

### 2.5 Methods

**Device details**

The Nb of the DC-SQUID loop and the stripline is evaporated (thickness 100 nm) onto a thin InAs-AlGaSb heterostructure that has been grown epitaxially on a GaAs(111)A substrate (see the Methods section and [25]). The inner area of the loop is 40 $\mu$m x 80 $\mu$m and the line width is 4 $\mu$m. The stripline is placed 1.5 $\mu$m from the DC-SQUID and runs 70 $\mu$m parallel to the DC-SQUID loop. Weak links are formed at two points where the Nb is interrupted (Fig. 2.1b). At these points the current flows through the InAs surface layer, thus forming an SNS-type Josephson junction.
A dry-etch is used to remove the conducting InAs layer everywhere except underneath the metallized parts and at the junctions. Electrical contact to the Nb is made by evaporation of 20 nm Ti and 200 nm Au. The flexural beam resonator is made by removing the GaAs substrate underneath part of the loop with a wet-etch. The resulting beam is buckled away from the substrate due to compressive strain (Fig. 2.1c).

**Measurement Setup**

The device is mounted in a dilution refrigerator and the coarse magnetic field $B$ is applied using a superconducting solenoid magnet. The low frequency (LF) circuit is used to set the DC-SQUID to a working point by applying a bias current $I_B$ through the DC-SQUID and a current $I_F$ through the stripline. The battery powered current sources and LF voltage amplifier are optically isolated from mains operated equipment. The high frequency (HF) DC-SQUID output voltage at the resonator frequency is measured using a two-stage room temperature amplifier with 80 dB gain, 10 kΩ input impedance and equivalent input voltage noise $\sqrt{\text{S}_V} = 0.3$ nV/√Hz. The HF and LF circuits are separated by 1 kΩ resistors and 10 nF capacitors at the cryogenic stage (Fig. 2.1d).

**Heterostructure Properties**

A more detailed description of the heterostructure that forms the DC-SQUID is given here. It’s mechanical properties are needed to calculate the eigenmodes of the beam. The DC-SQUID loop consists of three different materials on a GaAs(111)A substrate, as illustrated in Fig. 2.4. First an insulating layer of Al$_{0.5}$Ga$_{0.5}$Sb is grown on the substrate using molecular beam epitaxy, followed by a layer of InAs. The GaAs substrate is used to grow a high-crystalline-quality InAs/Al$_{0.5}$Ga$_{0.5}$Sb film despite the large lattice mismatch of 7% between the substrate and InAs [27]. Finally, a layer of evaporated niobium serves as the superconductor for the DC-SQUID. At two positions in the loop, the Nb is interrupted and the supercurrent has to flow through the InAs surface layer, thereby forming two SNS-type Josephson junctions [26]. The InAs surface layer has a mobility of $8 \cdot 10^3$ cm$^2$/Vs and an electron density of $1.3 \cdot 10^{12}$ cm$^{-2}$ at 77 K.

Table SI shows the dimensions and mechanical properties of the different layers in the heterostructure (compiled from Ref. [28–31]).

**Eigenfrequencies and Modeshape**

To obtain the displacement responsivity from the temperature dependence of the thermal noise power, the relation between the displacement $u$ and the change in
The flux has to be calculated. This requires a full description of the flexural bending modes of the buckled beam. This analysis also allows the calculation of the resonance frequency, which is in excellent agreement with the value that is found experimentally.

To analyze the flexural modes of the multi-layer beam it is convenient to replace it by an effective beam which has the same mechanical properties and which has a rectangular cross-section $A = wh$. These requirements give four equations from which the values for the thickness $h$, width $w$, mass density $\rho$ and Young’s modulus $E$ of the effective beam are calculated, as shown in Table SI. The analysis in Ref. [32] can now be used to find the flexural modes using the beam equation for the effective beam:

$$\rho A \frac{\partial^2 u_{tot}}{\partial t^2} + D \frac{\partial^4 u_{tot}}{\partial x^4} - T \frac{\partial^2 u_{tot}}{\partial x^2} = 0,$$

(2.1)

together with the boundary conditions for the displacement $u_{tot}$ of a doubly-clamped beam.

<table>
<thead>
<tr>
<th>material</th>
<th>$h$ (nm)</th>
<th>$w$ (µm)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$E$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb</td>
<td>100</td>
<td>4.0</td>
<td>$8.57 \cdot 10^3$</td>
<td>104.9</td>
</tr>
<tr>
<td>InAs</td>
<td>42.5</td>
<td>4.5</td>
<td>$5.68 \cdot 10^3$</td>
<td>51.4</td>
</tr>
<tr>
<td>$\text{Al}<em>{0.5}\text{Ga}</em>{0.5}\text{Sb}$</td>
<td>350</td>
<td>4.5</td>
<td>$5.0 \cdot 10^3$</td>
<td>$\sim 60$</td>
</tr>
<tr>
<td>effective</td>
<td>512</td>
<td>4.2</td>
<td>$5.68 \cdot 10^3$</td>
<td>67.5</td>
</tr>
</tbody>
</table>

Table 2.1: Dimensions and physical properties (at room temperature) of the different layers of the SQUID loop. The values given for $\text{Al}_{0.5}\text{Ga}_{0.5}\text{Sb}$ are estimates based on the properties of GaSb and the similarity between AlAs and GaAs. The properties of the effective beam are obtained by requiring that its mechanical properties beam are identical to those of the original beam.
beam [33]. The beam has a bending rigidity \( D = EI = 3.20 \cdot 10^{-15} \) J. For a rectangular beam the moment of inertia is \( I = h^3w/12 \) [33]. Furthermore, the beam is under a tension \( T \), which is positive for tensile- and negative for compressive stress. This tension has two contributions: first the beam is compressed due to the lattice mismatch between the beam and the substrate. This is the so-called residual tension, \( T_0 \). The second contribution comes from the stretching of the beam when it is displaced. Combining both effects gives:

\[
T = T_0 + \frac{EA}{2\ell} \int_0^\ell \left( \frac{\partial u_{tot}}{\partial x} \right)^2 \, dx.
\] (2.2)

The total deflection \( u_{tot} \) is the sum of a static \( (u_{DC}) \) and oscillating part \( (u_{AC}) \), where the latter is typically much smaller than the former, so that the beam equation can be separated:

\[
D \frac{\partial^4 u_{DC}}{\partial x^4} - T_{DC} \frac{\partial^2 u_{DC}}{\partial x^2} = 0,
\] (2.3)

\[
\rho A \frac{\partial^2 u_{AC}}{\partial t^2} + D \frac{\partial^4 u_{AC}}{\partial x^4} - T_{DC} \frac{\partial^2 u_{AC}}{\partial x^2} = T_{AC} \frac{\partial^2 u_{DC}}{\partial x^2},
\] (2.4)

with:

\[
T_{DC} = T_0 + \frac{EA}{2\ell} \int_0^\ell \left( \frac{\partial u_{DC}}{\partial x} \right)^2 \, dx,
\] (2.5)

\[
T_{AC} = \frac{EA}{\ell} \int_0^\ell \frac{\partial u_{DC}}{\partial x} \frac{\partial u_{AC}}{\partial x} \, dx.
\] (2.6)

Eq. 2.3 only has a non-trivial solution \( u_{DC} = u_{\text{max}}[1 - \cos(2\pi n x/\ell)]/2 \) when \( T = n^2 T_c \), where \( T_c = -4\pi^2 D/L^2 \) is the critical tension at which the beam buckles and \( n \) is an integer. When the compressive residual tension \( T_0 \) is more negative than \( T_c \), the beam is buckled to a displacement that keeps the tension exactly at \( T_c \) (for \( n=1 \)). The value of the displacement depends on the residual tension that caused it: \( u_{\text{max}} = 2\ell / \pi ([T_c - T_0]/EA)^{1/2} \). The height profile in the inset of Fig. 2.1c shows that the beam is buckled upwards with a shape belonging to \( n = 1 \) and that it has a maximal displacement \( u_{\text{max}} = 1.5 \) \( \mu m \). From this it can be concluded that the beam was under a compressive residual strain \( \epsilon_0 = T_0/EA = -2.5 \cdot 10^{-3} \).

The eigenfrequencies of buckled beams were calculated by Nayfeh [32]. Following this analysis, the homogeneous solution to Eq. 2.4 is written as:

\[
u_{AC}^{(h)} = c_1 \sin(k_+ x/\ell) + c_2 \cos(k_+ x/\ell) + c_3 \sinh(k_- x/\ell) + c_4 \cosh(k_- x/\ell)
\] (2.7)

with:

\[
k_\pm = \left( \pm 2\pi^2 + \sqrt{(2\pi^2)^2 + \frac{m\omega^4 \ell^4}{EI}} \right)^{1/2},
\] (2.8)
while the particular solution is given by:

$$u_{AC}^{(p)} = c_5 \cos(2\pi x / \ell).$$

(2.9)

The boundary conditions provide four equations for the five coefficients \(c_i\). A fifth equation is obtained when the entire solution \(u_{AC} = u_{AC}^{(h)} + u_{AC}^{(p)}\) is inserted into Eqs. 2.4 and 2.6 as the right hand side of Eq. 2.4 depends on \(u_{AC}\) itself. These conditions can be written as:

$$
\begin{bmatrix}
0 & 1 & 0 & 1 & 1 \\
k_+ & \cos(k_+) & \sinh(k_-) & \cosh(k_-) & 1 \\
-k_+ \sin(k_+) & -k_- \cosh(k_-) & k_- \sinh(k_-) & -k_- \sinh(k_-) & 0 \\
k_+ \cos(k_+) - k_- & k_- \cosh(k_-) - k_+ & -k_+ \sinh(k_-) - k_- & k_+ \cosh(k_-) - k_- & -\frac{\omega^2 m \ell^4}{4\pi^4 E A u_{\text{max}} - 1}
\end{bmatrix}
\times
\begin{bmatrix}
c_1 \\
c_2 \\
c_3 \\
c_4 \\
c_5
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.
$$

(2.10)

Eigenmodes have some of the coefficients \(c_i\) non-zero, which can only occur at the frequencies \(\omega = \omega_n\) where the matrix in Eq. 2.10 is not invertible. The resulting eigenfrequencies are shown in Fig. 2.5. The frequency of the fundamental mode increases with increasing buckling. The second mode has an eigenfrequency \(\omega_2/2\pi = 1.44\) MHz and is independent of \(u_{\text{max}}\), as it is anti-symmetric around the node, giving \(T_{AC} = 0\). When \(u_{\text{max}}\) is increased to \(0.92\) \(\mu\)m, the two lowest modes cross and the fundamental mode is higher in frequency than the second mode. The calculated eigenfrequency of 1.93 MHz of the fundamental mode for the measured static buckling of \(u_{\text{max}} = 1.5\) \(\mu\)m matches the frequency of 2.0018 MHz observed in the experiments well, considering the uncertainty in the tabulated values for the mechanical properties of Al\(_{0.5}\)Ga\(_{0.5}\)Sb. The displacement of each mode \(n\) can be obtained by expanding the displacement in the basis of eigenmodes:

$$u_{AC}(x) = \sum_n u_{AC}^{(n)} \cdot \xi_n(x),$$

(2.11)

where the basis functions \(\xi_n\) are normalized:

$$\frac{1}{\ell} \int_0^\ell \xi_m(x) \xi_n(x) \, dx = \delta_{m,n}.$$

(2.12)

\footnote{We classify the modes by their shape and not by the ordering of eigenfrequencies. The fundamental mode is the mode without a node.}
Note that with these definitions, the effective mass and spring constant appearing in the equipartition theorem and zero-point motion are equal to the total mass $M$ and $M\omega_n^2$ respectively. Furthermore, the change in area due to the motion can be expressed as:

$$
\delta A = \int_0^\ell u_{AC}(x)\,dx = a_n \ell u_{AC}^{(n)}, \quad a_n = \frac{1}{\ell} \int_0^\ell \xi_n(x)\,dx. \tag{2.13}
$$

The numerical constant $a_n$ relates the change in flux to the displacement of the oscillating mode using $d\Phi/d\xi_{AC} \approx a_n B \ell$. Anti-symmetric modes have a vanishing value of $a_n$ and are not visible in the SQUID experiments. Furthermore, these modes cannot be excited with the piezo element and they do not appear in the dynamic force measurements either. For the fundamental mode observed in the experiments with $u_{\text{max}} = 1.5$ \textmu m, the model gives $a_0 \approx 0.91$. In the main text the symbols $u$ and $a$ are used for $a_0$ and $u_{AC}^{(0)}$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.6\textwidth]{figure2_5.png}
\caption{The calculated eigenfrequencies of the beam. The mode shapes at the position of the dots are shown. At a buckling of $u_{\text{max}} = 1.5$ \textmu m (dashed line), the fundamental flexural mode (blue) has an eigenfrequency of 1.93 MHz. The regime above 5 MHz is not accessible with the driving piezo element used in the experiments.}
\end{figure}
DC-SQUID PARAMETERS

This section gives the DC-SQUID parameters at $B = 111$ mT that are needed for the simulations described in the next section. The relevant parameters are the critical current $I_0$, the normal state resistance $R$ and capacitance $C$ of a single Josephson junction and the inductance $L$ of the DC-SQUID.

In the simulations, identical parameters for the two junctions in the DC-SQUID are assumed. We find good agreement between the measurements and the simulations, which indicates that this assumption is reasonable as asymmetries in the parameters would have caused deviations in the $V(I_B, \Phi)$ characteristics [34].

The critical currents of $I_0 = 1.2$ µA and $0.7$ µA at 100 mT and 111 mT respectively are obtained from the location of $V_{pp}^{\text{max}}$ as indicated in Fig. 2.2c. In the next section it is shown that $V_{pp}^{\text{max}} \approx I_0 R$, which allows the determination of the normal state resistance $R$ of the individual Josephson junctions from the data in Fig. 2.2c and 2.2d. At 111 mT a temperature independent value $R(111 \text{ mT}) = 30 \, \Omega$ is obtained.

The capacitance of the Josephson junctions is calculated from the hysteresis of the $V-I_B$ curves at zero magnetic field (Fig. 2.2a) by using the Stewart-McCumber parameter $\beta_c$. This number indicates whether the DC-SQUID is over- or under-damped, i.e. whether it is hysteretic or not. The resistively and capacitively shunted junction (RCSJ) model relates the ratio $i_r$ of the return current and critical current to the Stewart-McCumber parameter $\beta_c = (2 - (\pi - 2)i_r)/i_r^2$ [1]. For a critical current of 10.1 µA and a return current of 5.7 µA this gives $\beta_c = 4.3$ at $B = 0$ T. The Stewart-McCumber parameter is related to the physical parameters of the DC-SQUID according to $\beta_c = 2\pi I_C R_{SQ}^2 C_{SQ}/\Phi_0$, where $C_{SQ}$ and $R_{SQ}$ are the capacitance and the resistance of the two junctions in parallel, respectively. The slope of the $V-I_B$ curve (Fig. 2.2a) at high currents yields $R_{SQ}(0 \text{ T}) = 8.7 \, \Omega$. With the values for $\beta_c$ and $I_C$ that are mentioned above, $C_{SQ} = 1.8$ pF and thus $C = C_{SQ}/2 = 0.9$ pF is found. It is assumed that this value is valid at all temperatures and magnetic fields in the experiments. Note that this value is higher than expected from the geometry of the junction, which is likely caused by the presence of an additional conductive layer in the heterostructure.

The inductance of the DC-SQUID is estimated by finite element simulations of the electrodynamics of the superconducting loop using FastHenry[35], giving a value of $L = 175$ pH.

DC-SQUID MODEL

In this section, the flux-responsivity of the DC-SQUID is calculated using numerical simulations for the experimental conditions at a field of 111 mT. To simulate the flux responsivity of the DC-SQUID, it is modeled using the framework described in Ref [1]. This model consists of two coupled second order differential equations,
2.5. Methods

governing the time-dependence of the phase differences $\delta_{1,2}$ of the junctions in the SQUID. For a symmetric DC-SQUID they are given by:

\begin{align*}
\beta_c \ddot{\delta}_1 + \dot{\delta}_1 + \sin \delta_1 &= i/2 + j \\
\beta_c \ddot{\delta}_2 + \dot{\delta}_2 + \sin \delta_2 &= i/2 - j \\
2\pi(\phi + \beta_L j/2) &= \delta_2 - \delta_1.
\end{align*}

(2.14) (2.15) (2.16)

Here, $i = I_b/I_0$ is the normalized bias current and $j$ is the circulating current. The time is normalized using the characteristic frequency $I_0R/\Phi_0$, where $R$ is the normal-state resistance of a single junction. Furthermore, the model contains the parameter, $\beta_c$ which has been defined in the previous section and the inductive screening parameter $\beta_L = 2I_0L/\Phi_0$ [1]. Using the values of the physical parameters from the previous section, $\beta_c = 1.6$ and $\beta_L = 0.1$ are obtained for a magnetic field of $B = 111$ mT. With these values, the differential equations are integrated numerically for different values of the flux through the SQUID and the bias current. The output voltage of the DC-SQUID is calculated from the resulting time-traces $\delta_{1,2}(t)$ using $V = RI_0(\dot{\delta}_1 + \dot{\delta}_2)/2$. This voltage has high frequency components (of the order of the characteristic frequency) and an average value. The latter of the two is the voltage that is measured. By calculating the average voltage for different amounts of flux penetrating the DC-SQUID, the peak-to-peak voltage swing $V_{PP}$ can be found. The maximum voltage swing $V_{PP}^{max}$ occurs at $2I_0$ and equals $RI_0$. The responsivity $dV/d\Phi$ is found to scale linearly with $V_{PP}^{max}$. The dimensionless flux responsivity $v_\phi$ is obtained by taking the derivative of the voltage with respect to the applied flux and scaling with $V_{PP}^{max}$ and $\Phi_0$, i.e. $v_\phi = \Phi_0/V_{PP}^{max} \cdot dV/d\phi$. Fig. 2.6 shows that the highest responsivity is obtained when the DC-SQUID is biased close to the onset of the dissipative region, where $V \neq 0$.

Noise Power Saturation

At temperatures below 100 mK the noise power of the resonator is no longer proportional to the refrigerator temperature and saturates as shown in Fig 2.3c. This implies that below 100 mK, the temperature of the resonator $T_R$ is higher than the refrigerator temperature $T$. Similar behavior has been observed in other studies where it was attributed to local heating of the substrate [13, 15] or excessive force noise power $S_{det}^{F}$ coming from the position detector [14, 36]. A way to experimentally discriminate between these two effects is to change the coupling between the resonator and the detector, but in our case the coupling field cannot be varied sufficiently to distinguish between the two sources. Instead we present a quantitative analysis which shows that $S_{det}^{F}$ is not sufficiently large to cause the rise in resonator temperature. The detector force noise power is due to the Lorentz force,
\[ S_F^{\text{det}} = (aB\ell)^2 S_{IR}. \]

The current noise power through the resonator, \( S_{IR} \), is the sum of several components:

\[ S_{IR} = \frac{8k_b T_{\text{junc}}}{R} + \frac{eI_B}{2} + \frac{k_b T_{\text{amp}}}{R_{\text{amp}}}. \]  

(2.17)

The first term gives the current noise power due to the resistance of the SQUID junctions, where we define the junction temperature \( T_{\text{junc}} \) which may also be higher than \( T \) below 100 mK. The second term gives the shot-noise contribution of the bias current \( I_B \) and the last term is the contribution of the input impedance of the room temperature high-frequency amplifier \((R_{\text{amp}} = 10 \, \text{k}\Omega \text{ at } T_{\text{amp}} = 300 \, \text{K})\). Using the fluctuation-dissipation theorem, the resonator temperature becomes \( T_R = T + T_{\text{det}} \) with

\[ T_{\text{det}} = \frac{(aB\ell)^2 S_{IR} Q}{2k_b m(2\pi f_R)}. \]  

(2.18)

By entering the parameters for the measurements at 100 mT and 20 mK and assuming that the junctions are saturated like the resonator \((T_{\text{junc}} = 100 \, \text{mK})\) we arrive at \( \sqrt{S_{IR}} = 1 \, \text{pA}/\sqrt{\text{Hz}} \) and \( T_{\text{det}} = 2 \, \text{mK} \). This resulting resonator temperature is much lower than the lowest observed value of \( T_R = 84 \, \text{mK} \), which means that the difference between \( T_R \) and \( T \) is not due to back-action heating.

A more likely possibility is that the substrate temperature is higher than that of the refrigerator. Note that resistive heating of the junctions is not the main cause.
of this effect; Compared to the measurements at 100 mT, the current through the junctions was 1.7 times less at 111 mT, because of the lower critical current. The data in Fig. 2.3c does not show a lower saturation temperature in the latter case, even though the dissipation in the junction has decreased by a factor 3. The most likely cause of the increased substrate temperature is local heating due to heat transfer through the wires. This effect can be reduced by improving the thermal anchoring of the wires to the refrigerator.

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REFERENCES


SQUID BACKACTION ON THE INTEGRATED RESONATOR

We have measured the backaction of a dc superconducting quantum interference device (SQUID) position detector on an integrated 1 MHz flexural resonator. The frequency and quality factor of the micromechanical resonator can be tuned with bias current and applied magnetic flux. The backaction is caused by the Lorentz force due to the change in circulating current when the resonator displaces. The experimental features are reproduced by numerical calculations using the resistively and capacitively shunted junction (RCSJ) model.

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It has recently been demonstrated that a macroscopic mechanical resonator can be put in a quantum state [1] by coupling it to another quantum system. At the same time, linear detectors coupled to mechanical resonators are rapidly approaching the quantum limit on position detection. This limit implies that the position of the resonator cannot be measured with arbitrary accuracy, as the detector itself affects the resonator position [2]. This is an example of backaction, i.e., the influence of a measurement device on an object. Moreover, backaction does not just impose limits, it can also work to one’s advantage: Backaction can cool the resonator, squeeze its motion, and couple and synchronize multiple resonators. In experiments with micro- or nanomechanical resonators, different backaction mechanisms have been identified. When using optical interferometers [3–6] or electronic resonant circuits [7–9], backaction results from radiation pressure. In single-electron transistors (SET) [10], Cooper-pair boxes [11], carbon nanotube quantum dots [12, 13], or atomic and quantum point contacts [14, 15] backaction is due to the tunneling of electrons. Recently, we have used a dc SQUID as a sensitive detector of the position of an integrated mechanical resonator [16]. This embedded resonator-SQUID geometry enables the experimental realization of a growing number of theoretical proposals for which a good understanding of the backaction is required [17–23].

In this chapter, we present experiments that show that the dc SQUID detector exerts backaction on the resonator. By adjusting the bias conditions of the dc SQUID the frequency and damping of the mechanical resonator change. The backaction by the dc SQUID has a different origin than in the experiments mentioned above: It is due to the Lorentz force generated by the circulating current. Numerical calculations using the RCSJ model for the dc SQUID [24] reproduce the experimental features.

3.1 Device Description and Fabrication

The device (Fig. 3.1a) consists of a dc SQUID with proximity-effect-based junctions [16]. A part of one arm is underetched, forming a 1 MHz flexural resonator with length \( \ell = 50 \mu \text{m} \). In this chapter we present data on a device in an in-plane magnetic field of \( B = 100 \text{ mT} \). Measurements have been performed at several magnetic fields and an additional device; the observed backaction is similar (see the Methods section). Before using the device to detect the resonator displacement, first the dc SQUID is characterized. The output voltage of a dc SQUID depends on the magnetic flux through its loop \( \Phi \) [24] and we measure the minimum and maximum voltage (\( V_{\text{min}} \) and \( V_{\text{max}} \)) by sweeping the flux over a few flux quanta \( \Phi_0 = \hbar/2e \) with a nearby stripline (Fig. 3.1a).

Figure 3.1: (a) Schematic overview of the dc SQUID and measurement setup. The scanning electron micrograph shows the dc SQUID with the suspended resonator. The magnetic field $B$ transduces a displacement of the beam $u$ into a change in magnetic flux. A bias current $I_B$ is sent through the SQUID and its output voltage is measured. The flux $\Phi$ is fine-tuned with a stripline current $I_F$ that is controlled by a feedback circuit (dashed) that tries to keep the output voltage $V$ at $V_{SP}$. (b) The bias current dependence of the measured $V_{\text{min}}$ and $V_{\text{max}}$. (c) The amplitude (bottom) and phase (top) response of the resonator. The line is a fitted harmonic oscillator response.
This is repeated for different bias currents to obtain the current-voltage curves shown in Fig. 3.1b. The maximum critical current is $I_{c}^{\text{max}} = 2.19 \mu A$ and the normal-state resistance of the junctions is $R = 15.6 \Omega$. After this characterization, the dc SQUID is operated at a given setpoint voltage $V_{SP}$ using a feedback loop that adjusts the flux via the stripline current [16, 24]. The feedback loop is used to reduce low-frequency flux noise and flux drift and has a bandwidth of $\sim 2$ kHz, i.e., it does not respond to the 1 MHz resonator signal.

The fundamental mode of the flexural resonator is excited using a piezo element underneath the sample and the displacement of the beam is detected as follows: The in-plane magnetic field transduces a displacement of the beam $u$ into a flux change $\sim \ell Bu$, which in turn changes the voltage over the dc SQUID. This voltage is amplified using a cryogenic high-electron mobility transistor followed by a room temperature amplifier and then recorded using a network analyzer. Figure 3.1c shows the amplitude and phase of the measured response, from which the resonance frequency $f_{R}$ and quality factor $Q$ are obtained.

### 3.2 BACKACTION MEASUREMENTS

To observe backaction of the dc SQUID detector on the resonator, the frequency response is measured for different bias conditions of the dc SQUID. Figure 3.2 shows that both $f_{R}$ and $Q$ depend on the bias current. The resonance frequency saturates at $f_{0} = 1.053010$ MHz for large positive and negative bias currents. However, when decreasing the current, the resonance frequency first goes up by a few hundred Hz around $I_{c}^{\text{max}}$ and then it decreases rapidly with about $-2000$ Hz at the lowest stable setpoint voltage. This is more than $10\times$ the linewidth $f_{R}/Q = 194$ Hz of the resonance shown in Fig. 3.1c. Figure 3.2b shows that the quality factor of the resonator changes from $Q_{0} = 5300$ to less than 2000. Similar to the resonance frequency, first an increase and then a stronger decrease in $Q$ is observed when lowering the bias current.

Figure 3.3 shows that the frequency and damping can also be changed by adjusting $V_{SP}$, i.e., the flux through the SQUID loop. The shifts are largest for low setpoints and low bias currents (dark regions). The regions with a lower frequency coincide with the regions where the damping has increased. Bias points with positive frequency shifts and increases in $Q$ are indicated in white. Finally, by varying the driving power we confirm that the observed effects are not due to nonlinearities in the SQUID or in the resonator (see the Methods section).
3.2. Backaction Measurements

**Figure 3.2:** Frequency shift (a) and quality factor (b) plotted versus the normalized bias current. The voltage setpoint was halfway between $V_{\text{min}}$ and $V_{\text{max}}$ in these measurements.

**Figure 3.3:** Measured bias current and voltage setpoint dependence of the frequency shift (left) and damping (right). Points without a good lock are indicated in gray.
3.3 Modelling the Backaction

Unlike for position detectors such as the SET where the backaction originates from the Coulomb force, the backward coupling between the dc SQUID and the beam is the Lorentz force $F_L$ [18, 21]. This force is due to the current that flows through the beam in the presence of the magnetic field that couples the resonator and the SQUID. A displacement of the beam changes the flux, and this in turn changes the circulating current in the loop $J$ [24], giving a different force on the beam. In addition, motion of the resonator yields a time-varying flux through the loop, which induces an electro-motive force and thereby also generates currents that change the Lorentz force [25].

The displacement of the fundamental out-of-plane flexural mode of the beam $u$ is given by the equation of motion of a damped harmonic oscillator [26]:

$$m \ddot{u} + m \omega_0 \dot{u} / Q_0 + m \omega_0^2 u = F_d(t) + F_L(t).$$  (3.1)

The resonator has a mass $m$, (intrinsic) frequency $f_0 = \omega_0 / 2\pi$ and quality factor $Q_0$. $F_d$ is the driving force and $F_L = aB\ell (I_B / 2 + J)$ is the Lorentz force. Here, $a$ is a geometrical factor that relates the averaged spatial profile of the mode $u(x)$ to the coordinate $u$: $a = (u\ell)^{-1} \int_0^\ell u(x) \, dx \approx 0.9$ for the fundamental mode, so that also $\partial \Phi / \partial u = aB\ell$ [16, 26]. For small amplitudes and low resonator frequencies (much smaller than the characteristic SQUID frequency $\omega_c = \pi R I_{c_{\text{max}}} / \Phi_0$), the average circulating current can be expanded in the displacement and velocity $\dot{u}$ (see the Methods section):

$$J(u, \dot{u}) = J_0 + \frac{\partial J}{\partial \Phi} aB\ell u + \frac{\partial J}{\partial \dot{\Phi}} aB\ell \dot{u}. \quad (3.2)$$

Inserting Eq. (3.2) into Eq. (3.1) shows that the $\partial J / \partial \Phi$ term affects the spring constant $m \omega_R^2$ and thus the resonance frequency $f_R$, whereas the $\partial J / \partial \dot{\Phi}$ term renormalizes the damping. The shifted resonance frequency and quality factor are:

$$f_R = f_0 \left(1 - \Delta f \phi_0^2 \right)^{1/2}, \quad \Delta f = \frac{a^2 B^2 \ell^2 I_{c_{\text{max}}}}{m \omega_0^2 2 \Phi_0}, \quad (3.3)$$

$$Q = Q_0 \frac{f_R}{f_0} \frac{1}{1 - \Delta Q \phi_0^2}, \quad \Delta Q = \frac{a^2 B^2 \ell^2 Q_0}{m \omega_0 R 2\pi}. \quad (3.4)$$

Here, $\phi_0 = \partial J / \partial \Phi \times 2 \Phi_0 / I_{c_{\text{max}}}$ and $\phi_0 = \partial J / \partial \dot{\Phi} \times 2 \omega_c \Phi_0 / I_{c_{\text{max}}}$ are the scaled flux-to-current transfer functions [27]. The former indicates how much the circulating current changes when the flux through the ring is altered, whereas the latter quantifies the effect of a time-dependent flux on the circulating current. These functions were first studied in the analysis of the dynamic input impedance of tuned
SQUID amplifiers [27, 28] and are intrinsic properties of the dc SQUID (i.e., they do not depend on the properties of the resonator).

Before looking in more detail at the transfer functions, we first focus on the coupling. The dimensionless parameters $\Delta_f$ [20] and $\Delta_Q$ characterize the backaction strength. They contain the term $aB\ell$ squared as both the flux change and the Lorentz force are proportional to the magnetic field. This implies that the backaction remains the same when the direction of the magnetic field is reversed and this is what we observe experimentally. $\Delta_f$ is proportional to the critical current of the SQUID, whereas the damping induced by the dc SQUID depends on the normal-state resistance of the Josephson junctions $R$. By a careful design of the resonator and dc SQUID, the backaction strengths can be tuned over a wide range. Eqs. (3.3) and (3.4) show that the largest backaction occurs for large flux changes $aB\ell$, low spring constants $m\omega_0^2$, and large circulating currents, i.e., large $I_0$ and low $R$. For the device studied in this chapter, we estimate the values of the coupling constants to be $\Delta_f = 4.1 \times 10^{-4}$ and $\Delta_Q = 2.8 \times 10^{-4}$. Finally, note that the two coupling parameters are related by $\Delta_Q = \Delta_f \times Q_0 \omega_0 / \omega_c$. In our device $\omega_0 \ll \omega_c$, but this mismatch between the resonator frequency and the characteristic SQUID frequency is balanced by the Q-factor, so the two coupling parameters happen to be of the same order of magnitude.

So far, the analysis did not assume anything about the number of junctions, nor about their microscopic details. To obtain the transfer functions $j_\phi$ and $j_\dot{\phi}$, we model the junctions in the dc SQUID using the RCSJ model [24]. The transfer functions can be calculated analytically in certain limits [29]. However, to obtain their full bias-condition dependence, $j_\phi$ and $j_\dot{\phi}$ must be calculated numerically. This is done by simulating the dynamics of the dc SQUID in the presence of a time-varying flux (see the Methods section). Figure 3.4a shows the bias-dependence of $j_\phi$. In the region where $V = 0$, the circulating current redistributes the bias current between the two junctions such that no voltage develops. Here the circulating current is of the order of $I_c^{\text{max}} / 2$, which gives $j_\phi \sim -1$ (blue). In the dissipative region ($V \neq 0$), the circulating current is suppressed. Therefore, the circulating current changes rapidly close to the edge of the dissipative region. The orange color in Fig. 3.4a indicates that $j_\phi$ is large and positive near the critical current. The largest downward frequency shift is expected near a half-integer number of flux quanta, whereas $j_\phi$ vanishes for integer flux. With the value of the coupling parameter $\Delta_f$ and the resonance frequency $f_0$ the frequency shift is calculated as shown in Fig. 3.4c. The maximum value $j_\phi = 53$ gives a frequency shift of $-12$ kHz in the lower-left corner, which has to be compared with the experimental value of $\sim -2$ kHz. Increasing the bias current above the critical current results in a smaller $j_\phi$ (light yellow and light blue) that depends linearly on the inductive screening parameter $\beta_L$ [24] for the experimental conditions.
Figure 3.4: Surface plots with iso-voltage lines at different bias conditions of a dc SQUID. The (logarithmic) color-scale represents the calculated flux-to-current transfer functions $j_\phi$ (a) and $J_\phi$ (b). With the values for the coupling parameters $\Delta f$ and $\Delta Q$, the frequency shift (c) and quality factor change (d) are calculated. The simulation is done for the experimental conditions where the inductive screening parameter is $\beta_L = 0.21$ and the Steward-McCumber parameter is $\beta_C = 0.23$ (see [24] and the Methods section).
In this region the simulations predict both positive (blue) and negative (yellow) value for $J_\phi$. Positive and negative shifts are also observed in the experiment (Figs. 3.2 and 3.3). The largest negative value found in the simulations of that region is $J_\phi = -0.65$, which results in an increase in the resonance frequency of about 140 Hz, which is in agreement with the observed value of $\sim 200$ Hz (Fig. 3.2a). For even larger bias currents, the frequency shift vanishes and the measured frequency $f_R$ matches the intrinsic resonance frequency $f_0$. This corresponds to the flat region in Fig. 3.2a.

The change in damping is determined by $J_\phi$ as indicated by Eq. (3.4). Its dependence on the bias conditions is shown in Fig. 3.4b. Well inside the experimentally inaccessible non-dissipative region $J_\phi \sim +1$ and the backaction results in a small increase in $Q$. In the opposite limit of large bias currents $J_\phi = -\pi$ (light blue). This value combined with the small value of $\Delta Q$ implies that the quality factor in the flat region in Fig. 3.2b is close to the intrinsic Q-factor, $Q_0$. In this region the dynamics of the junctions is not significant and the small additional damping is due to the current induced by the time-varying flux $\dot{\Phi}/2R$, which is dissipated in the normal-state resistance of the junctions [29]. This contribution is well-known from magnetomotive readout of mechanical resonators [25]. When lowering the bias current, $J_\phi$ changes sign and rises to about $+500$. This reduces the damping and might even lead to instability ($Q < 0$) if $\Delta Q$ is large enough. This decrease of damping corresponds to the bumps in Fig. 3.2b. The largest observed quality factor $Q = 5800$ corresponds to $J_\phi = +400$, which is in reasonable agreement with the simulations. Close to the critical current, $J_\phi$ goes to large negative values leading to an enhanced dissipation. Figure 3.4d shows that the calculated Q-factor is indeed lowest near the critical current. In summary, our model shows that although the coupling strength is small, the dynamics of the dc SQUID greatly enhances the backaction.

### 3.4 Outlook

Various interesting effects can be observed when the backaction is strong. If the resonator and dc SQUID are strongly coupled, the resonator temperature is set by the effective bath temperature [10, 14] of the dc SQUID. The increased damping cools the resonator, but the shot noise in the bias current leads to an increase in the force noise on the resonator, heating it. The question whether the resonator temperature is above or below the environmental temperature should be addressed in future research. Furthermore, the dependence of the resonator frequency and quality factor on the bias conditions allows parametric excitation of the mechanical resonator by either modulating the flux or the bias current. This enables squeezing of the thermomechanical noise of the resonator [20, 30]. Finally, if the dc SQUID
contains multiple, nearly identical mechanical resonators, these are coupled to each other by the backaction. This, in turn, can synchronize their motion and might lead to frequency entrainment if higher order terms in Eq. (3.2) become significant [31]. These examples are only a few intriguing possibilities of the rich physics connected to the backaction that we have described in this chapter.

3.5 METHODS

TEMPERATURE AND POWER DEPENDENCE

Figures 3.5a and b show the temperature dependence of the resonance frequency and quality factor. These values are measured at large bias currents where the backaction is negligible. The frequency change due to temperature is small compared to the observed backaction (see main text). The intrinsic quality factor decreases significantly with increasing temperature. This rules out that the observed frequency shift and Q-factor change are caused by heating of the resonator due to Joule heating in the junctions: We observe an increase in quality factor with increasing bias current and voltage setpoint (i.e. dissipation), but a decrease in quality factor with increasing temperature. An increased damping at higher temperatures is seen more often in micro- or nanomechanical resonators [32–35].

The observed frequency shift and change in damping do not depend on the driving power. As shown in Figure 3.5c, the measured resonator response stays the same in all panels. Although the driving power is changed by three orders of magnitude, the only effect is that the signal-to-noise ratio becomes better when increasing the power.

For the highest driving power ($P_d = -75$ dBm) and highest Q-factor ($Q \sim 5800$) the amplitude of the resonator motion is $u_{\text{max}} = 20$ pm, as determined using the calibrated displacement responsivity [16]. The flux through the dc SQUID is then modulated with an amplitude $aB\ell u_{\text{max}} = 0.02 \Phi_0$. So even for the largest resonator motion, the change in flux is much smaller than a flux quantum. Exactly on resonance, the piezo motion is $Q$ times smaller than $u_{\text{max}}$, about 3 fm. The driving force $F_d$ is then given by the resonator mass $m$ times the acceleration of the piezo element, $F_d = m\omega^2 u_p \approx m\omega_0^2 u_p$. The measurements shown in the main text are done with a driving power of $-80$ dBm ($F_d = 48$ fN).

DEVICE B

All effects that we have observed in the device that we discuss in the main text have also been measured in a second device. The resonator in this dc SQUID operates around 2 MHz. The maximum critical current of device B is $I_{c}^{\text{max}} = 2I_0 = 2.4$ $\mu$A at $B = 115$ mT and $I_{c}^{\text{max}} = 1.0$ $\mu$A at $B = 130$ mT.
Figure 3.5: Temperature dependence of the intrinsic quality factor $Q_0$ (a) and resonator frequency $f_0$ (b). (c) Colormap plot of the oscillator response at $B = 100 \text{ mT}$ at different driving powers $P_d$. This measurement is done with a voltage setpoint halfway between $V_{\text{min}}$ and $V_{\text{max}}$. 
In the measurements on device B, the feedback loop could not maintain the SQUID voltage for low voltage setpoints. Also, a less sensitive, room temperature amplifier was used for the resonator signal. Its lower gain resulted in an increased scatter in the data and the inability to explore the region with the highest backaction, i.e., at low bias current and low voltage setpoints. The measurements in Fig. 3.6 show qualitatively the same backaction as the data presented in the main text.

**The RCSJ Model for the DC SQUID**

To calculate the backaction, the dc SQUID is modelled using the resistively- and capacitively-shunted junction (RCSJ) model. This widely-used model is discussed in detail in Ref. [24]. The introduction to this model presented here is largely based on this review. A current $I_B$ is sent through the SQUID and the circulating current
3.5. Methods

$J$ redistributes the current over the two junctions, which we assume to be identical. In the RCSJ model, the two junctions (labelled with $i = 1, 2$) are modelled as a resistor ($R$), capacitor ($C$) and an “ideal” Josephson junction with critical current $I_0$ in parallel. The voltage over each junction is related to the time derivative of the phase difference $\delta_i$ of the superconducting wave function: $V_i = \Phi_0 \frac{\dot{\delta_i}}{2\pi}$, where $\Phi_0 = h/2e = 2.05 \times 10^{-15} \text{Tm}^2$ is the flux quantum. Current conservation yields two second-order differential equations, governing the time-dependence of the phase differences $\delta_{1,2}$ of the junctions:

$$\frac{\Phi_0}{2\pi} C \ddot{\delta}_1 + \frac{\Phi_0}{2\pi R} \dot{\delta}_1 + I_0 \sin \delta_1 = \frac{1}{2} I_B + J,$$

(3.5)

$$\frac{\Phi_0}{2\pi} C \ddot{\delta}_2 + \frac{\Phi_0}{2\pi R} \dot{\delta}_2 + I_0 \sin \delta_2 = \frac{1}{2} I_B - J.$$

(3.6)

These equations are coupled to each other by the amount of flux piercing the loop:

$$\delta_2 - \delta_1 = 2\pi \cdot \Phi_{\text{tot}} / \Phi_0.$$  

(3.7)

The total flux $\Phi_{\text{tot}}$ has two contributions: the externally applied flux $\Phi$ (which also includes the flux due to the resonator displacement) and the flux due to the circulating current flowing through the inductance of the loop $L$, i.e., $\Phi_{\text{tot}} = \Phi + LJ$.

The equations are scaled to simplify their numerical integration. This yields:

$$\beta_C \ddot{\delta}_1 + \dot{\delta}_1 + \sin \delta_1 = \frac{1}{2} I_B + J,$$

(3.8)

$$\beta_C \ddot{\delta}_2 + \dot{\delta}_2 + \sin \delta_2 = \frac{1}{2} I_B - J,$$

(3.9)

$$2\pi (\phi + \beta_L J / 2) = \delta_2 - \delta_1.$$  

(3.10)

The bias current and circulating current are normalized using the critical current: $I_B = I_B / I_0$ and $J = J / I_0$. Furthermore, time is scaled using the characteristic frequency $\omega_c = 2\pi R I_0 / \Phi_0$, fluxes using the flux quantum, i.e., $\phi = \Phi / \Phi_0$, and voltages using the characteristic voltage $RI_0$ so that $v = V / RI_0 = (\dot{\delta}_1 + \dot{\delta}_2) / 2$. The parameter $\beta_L = 2I_0 L / \Phi_0$ and $\beta_C = 2\pi I_0 R^2 C / \Phi_0$ are the inductive screening parameter and the Stewart-McCumber number respectively. The inductive screening parameter indicates how much a change in flux is screened by the circulating current $J$ flowing through the self-inductance of the loop $L$, whereas $\beta_C$ indicates the importance of inertial terms due to the junction capacitance $C$.

The three equations are integrated numerically for different bias conditions, i.e., different values for the bias current $I_B$ and for the flux $\Phi$ through the SQUID. Figure 3.7a shows typical examples of calculated time-traces of the circulating current $J$ and voltage $v$. Both are rapidly oscillating at a frequency of 0.69 $\omega_c$, which is the Josephson frequency that equals the average value of the voltage $\overline{v}$ [24].
maximum current that the dc SQUID can carry without generating a voltage equals the sum of the critical currents of the two junctions: \( I_{c}^{\text{max}} = 2I_{0} \).

The critical current \( (I_{0} = 1.1 \mu\text{A}) \) and the normal-state resistance \( (R = 15.6\Omega) \) of the junctions are estimated from the IV-characteristics (Fig. 3.1b). Using finite-element simulations we estimate \( L = 175\text{pH} \) for our device. Finally, the capacitance \( C = 0.6\text{pF} \) is obtained from the position of the LC resonance in the dc SQUID [24].

**Calculation of the Transfer Functions**

When the resonator moves, the flux through the dc SQUID loop is altered, which in turn changes the average circulating current \( J \). In principle, \( J(t) \) could depend on all the past displacements, \( u(t') \) for \( t' < t \). However, the dc SQUID reacts at a frequency \( \sim \omega_{c}/2\pi \sim 8\text{GHz} \) that is much faster than that of the resonator (1 MHz), so the circulating current \( J(t) \) is expected to depend on the instantaneous displacement \( u(t) \). For small amplitudes \( u \ll \Phi_{0}/aB\ell \) the response is linear and gives a contribution \( J_{1}(t) = c_{1}u(t) \). Another contribution comes from the velocity of the resonator, \( \dot{u} \), which causes a time-varying flux. This generates an electromotive force in the SQUID loop (Faraday’s induction law), which also changes the circulating current by \( J_{2}(t) = c_{2}\dot{u}(t) \). Combining these two effects gives \( J(t) = J(u, \dot{u}) = c_{1}u(t) + c_{2}\dot{u}(t) \). The values of the parameters \( c_{1} \) and \( c_{2} \) depend on the dynamics of the dc SQUID and are \( c_{1} = \partial J/\partial u \) and \( c_{2} = \partial J/\partial \dot{u} \). As discussed in the main text, these quantities are related to the intrinsic flux-to-current transfer functions, \( \alpha_{\Phi} = \Phi_{0}/(aB\ell I_{0}) \times \partial J/\partial u \) and \( \alpha_{\dot{\Phi}} = \omega_{c}\Phi_{0}/(aB\ell I_{0}) \times \partial J/\partial \dot{u} \).

In principle \( \alpha_{\Phi} \) can be obtained by calculating the average circulating current at different fluxes and then numerically differentiating this to obtain \( \alpha_{\Phi} = \partial J/\partial \Phi \). However, for the velocity-dependent transfer function this is not possible. Our method for calculating these transfer functions \( \alpha_{\Phi} \) and \( \alpha_{\dot{\Phi}} \) works as follows: We calculate the steady-state response of the circulating current with a small modulation added to the applied flux, \( \phi \to \phi + \Phi_{\text{mod}} \cos(\omega_{\text{mod}}t) \). Figure 3.7b shows that this modulates the circulating current \( J(t) \). Figure 3.7c shows the Fourier transform of the circulating current and the voltage. In the spectrum of both \( J \) and \( V \) a peak appears at the modulation frequency. The real part of the peak corresponds to the derivative \( J_{\Phi} = \text{Re}[J_{\text{mod}}/\Phi_{\text{mod}}] \), while \( \text{Im}[J_{\text{mod}}/\Phi_{\text{mod}}] = -\omega_{\text{mod}}J_{\Phi} \) and similar for \( \nu_{\text{mod}} \). The frequency dependence in Fig 3.7d, shows a constant \( \text{Re}[J_{\text{mod}}] \) and a linearly increasing \( \text{Im}[J_{\text{mod}}] \) as indicated by the dotted lines. The transfer functions do not depend on the modulation frequency, provided that it is sufficiently low. This confirms that the circulating current only depends on the instantaneous displacement and velocity as was postulated at the beginning of this Section.
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Figure 3.7: (a) Calculated time trace of the circulating current $j(t)$ and voltage $v(t)$ in the absence of noise. (b) Circulating current for a small ($\phi_{\text{mod}} = 0.01$) modulation of the flux with $\omega_{\text{mod}} = 0.02$. The time-averaged value of the circulating current $\overline{j}(t)$ has a phase shift with respect to the modulation $\phi(t)$ as indicated by the orange lines. (c) Absolute value of the Fourier transform of the time traces of the SQUID voltage and circulating current shown in (b). (d) The modulation frequency dependence of the real (dark gray) and imaginary (light gray) parts of the transfer function. The lines are a guide to the eye. These simulations were done for the dc SQUID parameters from the main text ($\beta_L = 0.21$ and $\beta_C = 0.23$) at $I_B = 2$ and $\phi = 0.25$. 
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REFERENCES


Superconducting quantum interference devices (SQUIDs) can detect tiny amounts of magnetic flux and are also used to study macroscopic quantum effects. We employ a dc SQUID as a linear detector of the displacement of an embedded micromechanical resonator with a femtometer sensitivity. We discuss the measurement method, including operation in high magnetic field and a cryogenic amplification scheme which allows us to reach a resolution which is only a factor 4.4 above the standard quantum limit.

Parts of this chapter have been published in Comptes Rendus du Physique.
4.1 Introduction

4.1.1 Mesoscopic Mechanics

Mechanical resonators are used as sensitive detectors of forces. Although they are typically macroscopic structures consisting of many atoms and thus have many degrees of freedom, the eigenmodes with the lowest energies can be engineered to be almost completely isolated from all other modes, even at the micro- and nanometer scale [1]. These modes then act as one-dimensional harmonic oscillators. They can store mechanical energy for long times and react with a large change in oscillation amplitude or resonance frequency to any applied force. The fundamental limits to the noise added by these resonant mechanical force detectors to the force to be measured was explored early on by the gravitational wave community [2].

The mean-square displacement due to stochastic force noise is related to the potential energy of the resonator \( \frac{1}{2} k_R \langle u^2 \rangle \), where \( k_R \) is the spring constant of the resonator mode and \( u \) is the displacement of the resonator with respect to its equilibrium position. It is known from statistical mechanics that the expectation value for the potential energy in a harmonic oscillator with frequency \( f_R \) at temperature \( T_R \) is

\[
\frac{1}{2} k_R \langle u^2 \rangle = \frac{1}{2} h f_R \left( \frac{1}{2} + \frac{1}{e^{\frac{h f_R}{k_B T_R}} - 1} \right). \tag{4.1}
\]

At temperatures for which the thermal energy is much larger than the energy of a single phonon, i.e. \( k_B T_R \gg h f_R \), this expression reduces to the equipartition theorem

\[
\frac{1}{2} k_R \langle u^2 \rangle = \frac{1}{2} k_B T_R. \tag{4.2}
\]

At absolute zero the only fluctuations that remain are the zero-point fluctuations with an average energy of half a phonon. In order to use the resonator as a force sensor its displacement must be detected, typically by conversion to an output voltage. If the position detector couples linearly to the deflection of the resonator, quantum mechanical analysis shows that even an ideally coupled, quantum limited detector adds at least another half a phonon of noise energy [2, 3]. This can be understood as the contribution of the internal degrees of freedom in the detector when they are coupled to the resonator. The total added noise consists of the imprecision noise of the detector, which shows as a white displacement noise floor at the detector output and the back action force noise, which increases the amplitude of the Lorentzian noise peak of the resonator. In this chapter only the detector resolution due to measurement imprecision is discussed, back action noise is not included. Although the resolution can be made arbitrarily small by increasing the
coupling between the detector and the resonator, the optimum coupling will yield a resolution on the order of the zero-point motion of the resonator:

\[ \sqrt{\langle u^2 \rangle_{QL}} = \sqrt{\frac{hf_R}{2k_R}}. \]  

(4.3)

This is the mean-square displacement associated with half a phonon phonon and is known as the standard quantum limit. Note that these fluctuations only become significant once the resonator is cooled to a temperature such that \( hf_R \ll k_B T \).

Position measurements of micro and nanomechanical in the quantum regime are of interest not only because they reveal the quantum nature of a massive, macroscopic degree of freedom; A ground-state cooled resonator can also be prepared in a non-classical state and subsequently used to transfer that state to for example a quantum information element [4]. Resonators which are coupled to mesoscopic electronics allow investigation of the interaction between their respective degrees of freedom. The resulting interesting physics has been investigated for devices such as the Single-Electron Transistor (SET) [5, 6], the Atomic Point Contact (APC) [7] and the Quantum Point Contact (QPC) [8]. Although most of these device have the potential for resolution beyond the quantum limit, optomechanical [9] and electromechanical [10] cavity resonators are among the first micro-scale systems to reach this milestone.

We use a Superconducting Quantum Interference Device (SQUID) to detect the position of an integrated microresonator. SQUIDs are an important class of mesoscopic detectors which are primarily used for the detection of tiny magnetic fields [11]. We use a magnetic field to couple the position of the integrated resonator to the flux that threads the loop. In previous work, we achieved a position resolution of a factor 36 above the quantum limit for linear position detection [12]. In this chapter we focus on the position detection in more detail. We explain the measurement setup and electronics in more detail and we report on improvements which have resulted in an order-of-magnitude improvement in the resolution to 4.4 times the standard quantum limit.

### 4.1.2 The dc SQUID Position Detector

We start with a short explanation of the operating principle of the dc SQUID (figure 4.1). A more comprehensive description can be found in [11]. The basic element of the dc SQUID is the Josephson junction, through which supercurrent can flow up to a maximum (critical) value \( I_0 \).
Figure 4.1: Colored SEM image of the dc SQUID, which consists of a 40x80 µm niobium loop with 4.5 µm wide arms (red), interrupted by two 300 nm long indium arsenide junctions (marked X), contacted by gold pads (yellow) and situated on an insulating AlGaSb-on-GaAs substrate (blue) [13]. The mechanical resonator has length \( \ell = 50 \) µm. It is defined by two windows which are etched through the AlGaSb into the GaAs (dark blue) and it is suspended by removing the underlying GaAs. The resonator has a fundamental mode which moves perpendicular to the substrate plane with a coordinate \( u \), whose displacement is equal to the root-mean-squared displacement of the beam averaged over the beam's length.
For bias currents $I_B$ beyond $I_0$, a voltage develops across the junction due to a quasiparticle current which runs through the normal-state resistance $R$ of the junction\(^1\).

A dc SQUID consists of two Josephson junctions in parallel inside a superconducting loop which is contacted by two leads. Due to the condition of phase continuity of the Cooper pair wave function around the loop, a supercurrent $J$ will run around the loop in response to an applied magnetic flux $\Phi$ through the inner part of the SQUID loop. The result is that the dc SQUID acts as a single Josephson junction with a critical current $I_C$ which can be tuned between the sum of the critical current of the two junctions, $I_C^{\text{MAX}} = I_{0,1} + I_{0,2}$ and their difference $I_C^{\text{MIN}} = |I_{0,1} - I_{0,2}|$ by changing $\Phi$ by half a flux quantum, $\Phi_0 = h/2e = 2 \text{ fT m}^2$. $\Phi_0$ is a tiny amount of magnet flux and this the reason why the SQUID has a high flux sensitivity. If the SQUID is biased above $I_C$, a voltage $V_{SQ}$ develops which also depends on $\Phi$, making the SQUID a linear flux detector for flux signals which are much smaller than $\Phi_0$. The flux responsivity is then defined as $V_\Phi = \frac{\partial V_{SQ}}{\partial \Phi}$, which is tunable by $\Phi$ and $I_B$. $\Phi$ is controlled by applying a current $I_F$ through a stripline next to the SQUID (see figure 4.1).

Figure 4.2 shows the relationship between $V_{SQ}$, $I_B$ and $I_F$ as measured for the device in this chapter. The device behavior is consistent with the resistively shunted Josephson junction (RCSJ) model \([11]\). The largest values for $V_\Phi$ are obtained for $I_B < I_C^{\text{MAX}}$ and $V_{SQ} \to 0$ V (figure 4.2b). In fact, in a noiseless SQUID model, $V_\Phi$ goes to infinity for $V_{SQ} \to 0$. Flux noise however strongly decreases the gain in this region, and this is why the maximum gain is found above $V_{SQ} = 0$.

A description of the device is given in figure 4.1. In order to use the dc SQUID as a displacement detector, a section of the loop forms a freely suspended beam and the SQUID is placed in a magnetic field $B$ which is parallel to the substrate plane and perpendicular to the length direction of the beam. In this way, $\Phi$ changes when the beam displaces. The displacement $u$ of the fundamental eigenmode is defined as the root-mean-square (RMS) displacement of the modeshape $u(x)$ along the length $\ell$ of the beam,

$$ u = \left( \ell^{-1} \int_0^\ell u(x)^2 \, dx \right)^{1/2}. \quad (4.4) $$

With this definition, the spring constant of the mode is $k_R = m(2\pi f_R)^2$, where $m$ is the total mass of the beam. The displacement $u$ causes a change in flux of $aBlu$,

\(^1\)The device in this chapter is overdamped, which means that the $V_{SQ}$-$I_B$ curves are non-hysteretic. This is a necessary condition for using the SQUID as a continuous linear flux detector above its critical current, i.e. in the dissipative regime \([11]\).
where
\[ a = (u\ell)^{-1} \int_{0}^{\ell} u(x) \, dx \] (4.5)
is the normalized change in loop area. For the fundamental mode of the buckled beam, \( a \approx 0.9 \) [12]. If the SQUID is biased above \( I_C \), it can thus act as a linear transducer for the motion of the beam with a responsivity
\[ \frac{\partial V_{SQ}}{\partial u} = G_{HF} V_\Phi a B \ell, \] (4.6)
where \( G_{HF} \) is the gain of the high-frequency voltage amplifier which is discussed in the next section. In order to have a large responsivity, it would seem that the largest possible \( B \) field is preferable. This is however not true, because the critical current of the junctions and thus also the flux gain \( V_\Phi \) decrease rapidly for fields on the order of 100 mT due to flux vortex formation [12]. All the measurements in this...
chapter are performed at $B = 60$ mT, where the optimal displacement responsivity was found for this device.

The motion of the resonator can be detected either by measuring the thermal and/or quantum mechanical position fluctuations using a spectrum analyzer or by measuring the response of the resonator due to a driving force using a network analyzer. To perform the driven measurement with the network analyzer, the resonator is mounted on a piezo actuator which moves the entire substrate perpendicular to the substrate plane, thereby actuating the resonator. Using a driven measurement, the fundamental resonance frequency $f_R$ of the beam is found at 2.1385 MHz. The beam has a calculated total mass $m = 6 \times 10^{-13}$ kg [12], which gives a spring constant $k_R = 110$ N/m. The resonator has a quality factor $Q_R = 3 \times 10^4$ at 15 mK. Note that at this temperature, the resonator is far in the limit $k_B T_R / \hbar f_R > 100$, where equipartition applies (equation 4.2).

Due to the applied magnetic field, the circulating current $J$ exerts a Lorentz force on the resonator which is proportional to its displacement and its velocity. This back-action causes a tunable shift in the resonance frequency and damping rate of the resonator, as we have measured recently [14]. Back-action may also enable cooling below the environmental temperature [15]. For the device in this chapter, the back-action is disregarded, as it is weak and the experiments are conducted at a SQUID bias where its effects can not be detected.

## 4.2 Measurement Setup

Figure 4.3 shows the physical setup and figure 4.4 shows a schematic of the measurement electronics. In this section we address the two main measurement issues, which are the low-noise amplification of the dc SQUID output voltage and the stabilization of the magnetic flux in the SQUID loop while operating it in a relatively large magnetic field.

### 4.2.1 Vibration Control

Optimization of displacement detection requires a constant average magnetic flux in the SQUID loop. This is a highly nontrivial task since small vibrations of the superconducting magnet with respect to the SQUID can already cause changes in the magnetic flux through the SQUID loop of the order of $\Phi_0$. To clarify this point, we must examine the setup in more detail. The device is connected to a cold finger which in turn is connected to the mixing chamber of a dilution refrigerator. Figure 4.3 shows the physical situation. The dilution unit is mounted at the end of a 5 cm diameter dipstick which is hanging in a dewar of liquid helium. The dipstick is supported only at the neck of the dewar and is therefore able to vibrate in directions perpendicular to its length. The superconducting magnet on the other hand
The physical setup consists of the SQUID which is mounted on a dipstick-sized dilution refrigerator and placed in a helium dewar with a superconducting magnet to provide the magnetic field $B$. The SQUID is connected to custom-built, battery fed control and readout electronics which are optically isolated from the mains-fed measurement equipment. Mechanical vibrations cause magnetic flux interference by changing the angle of the dipstick and thus the SQUID with respect to the magnet. To minimize this interference, the dewar is placed on a mechanically damped platform and has only flexible connections to the gas handling system and the mains-fed electronics. The remaining flux interference is compensated by operating the stripline in a feedback loop which maintains a constant average output voltage across the SQUID.
is suspended separately in the dewar. Considering the inner area of the SQUID loop, 40x80 (µm)², a free length of 1 m for the dipstick and a typical coupling field of 60 mT, a simple geometrical calculation yields a flux change of 1 Φ₀ for every 10 µm of lateral displacement of the end of the dipstick. Because of this extreme sensitivity to vibrations, the dewar is placed on a damped platform. It damps most strongly in the direction of the pumping lines coming from the gas handling system, as they are the strongest source of vibrations. Nevertheless, the remaining RMS amplitude of the vibration-induced flux interference is around 0.5 Φ₀ up to 1 kHz.

In order to further stabilize the amount of magnetic flux in the loop, we employ a proportional-integral feedback loop which controls the current through the stripline to maintain a constant voltage setpoint, \( V_{SP} \), across the SQUID. Before operating the feedback loop, all electronic interference which is not related to the magnetic flux must be removed, as it will be coupled back into the SQUID by the feedback loop. The feedback loop operates at frequencies up to 1 kHz and thus does not influence the high frequency oscillations due to the micromechanical resonator (MHz range). The optimized setup stabilizes the flux to a standard deviation of 0.01 Φ₀.

### 4.2.2 Cryogenic High Frequency Amplifier

The measurement imprecision is determined by the total noise that is added to the displacement of the resonator by all the amplification stages. Ideally the first transducer, the dc SQUID in this case, should be the dominant noise source. Note that the dc SQUID can in principle be a quantum limited flux [16] and position [17] detector.

In previous work [12] the first voltage amplifier was at room temperature and its noise was dominant, with an equivalent input noise level of 0.30 nV Hz at 2 MHz at the amplifier input. As this level is partly determined by Johnson noise of the transistor channel, it should decrease when the operating temperature of the amplifier is lowered. Another important factor is the signal loss from the SQUID to the amplifier due to the parasitic capacitance of the coaxial cable which was several meters in length. The loss of gain was estimated to be almost an order of magnitude, which means that the actual equivalent input noise level at the SQUID was around 3 nV/Hz\(^{1/2}\). This loss can be decreased by mounting the amplifier closer to the SQUID. For these two reasons, we have mounted a High Electron Mobility (HEMT) amplifier on the 1 K stage of the dilution refrigerator, using a phosphor-bronze core coaxial cable with a length of 0.4 m to connect the HEMT to the SQUID (figure 4.4).

The HEMT is used as a common-source, high-input-impedance amplifier [18]. The variable resistor in figure 4.4 is used to set the bias current through the HEMT.
FIGURE 4.4: Schematic of the amplification stages and control electronics at the various temperatures. The dc SQUID is connected to the low frequency biasing electronics on the left side via 10 kΩ resistors and is connected to the high frequency amplifier through capacitors to prevent dc currents from running between the low (<10kHz) and high frequency (MHz) sides. The first high frequency amplification stage is a High Mobility Electron Transistor (HEMT), mounted on the 1 K plate of the refrigerator. A 10 kΩ bleed resistor connects the gate to ground. A room temperature voltage source and tunable resistor are used to bias the transistor in a common source configuration. The amplified voltage is measured using a second, room temperature amplifier connected to the drain of the HEMT, with a bias-tee to separate the dc bias and the high frequency signal.

The amplifier has a maximum signal-to-noise ratio when it is tuned to its maximum gain point, which is found for a bias current of 0.8 mA. At room temperature the output of the HEMT is further amplified by a high-impedance JFET amplifier with a gain of $1.7 \times 10^3 \, V/V$ at 2 MHz. When the dilution unit is in operation and the HEMT is cold (1-4 K) it has a gain of 10 V/V, which was determined by replacing the SQUID with a 1 kΩ resistor and measuring its Johnson noise as a function of temperature. The SNS junctions of the SQUID can not be used for this purpose, even though they also produce Johnson noise; They have a normal-state resistance $R_{on}$ on the order of 10 Ω and at millikelvin temperatures, their contribution to the noise is much smaller than that of the HEMT amplifier. The total gain of the high-frequency amplifiers is thus $G_{HF} = 1.7 \times 10^4 \, V/V$.

The noise power at the output of the JFET amplifier is $S_{V_n}^{1/2} = 4.5 \, \mu V/Hz^{1/2}$, with the dominant noise contribution coming from the HEMT. This gives an equivalent input noise of 0.26 nV/Hz$^{1/2}$ at the SQUID, over an order of magnitude better than the previous configuration [12]. Surprisingly, the noise floor at the HEMT input is similar to the same HEMT at room temperature. The major improvement is thus due to a reduction of capacitive losses.
4.3 Motion detection

4.3.1 DC SQUID measurements

During acquisition of the mechanical resonator signal, the SQUID is operated in a feedback loop, which means that the voltage setpoint $V_{SP}$ is specified instead of $\Phi$. In order to find the optimum gain point and to measure other properties such as the mechanical resonance frequency, we must first determine the values in $V_{SP}$-$I_B$ space for which the feedback loop produces a stable lock. This is done by taking the minimum and maximum voltage, $V_{MIN}$ and $V_{MAX}$, as a function of $I_B$ from measurements like the one shown in figure 4.2. The feedback loop is operated within this region (between the red lines in figure 4.5a), and at each bias point, the flux gain is measured in addition to either the driven response or the noise spectrum of the resonator.

The flux gain $V_{\Phi}$ is measured by applying a small sinusoidal current signal to the stripline at 5 kHz. The corresponding response of the SQUID is measured using a lock-in amplifier which is connected to the low frequency side of the measurement circuit (figure 4.4). $V_{\Phi}$ is related to the measured amplitude $V_{LIA}$ according to

$$V_{LIA} = \frac{V_{IF}M_{STRIP}G_{LF}}{A_{IF}R_{IF}} V_{\Phi},$$  \hspace{1cm} (4.7)$$

where $M_{STRIP} = 16 \text{ pH}$ is the mutual inductance between the stripline and the SQUID loop (figure 4.2b). $V_{IF} = 0.85 \text{ V}$ is the RMS output voltage of the lock-in amplifier which is attenuated by a factor $A_{IF} = 4 \text{ kV/V}$ by the input circuit of the battery-fed electronics (figure 4.3) and applied to a 1 k$\Omega$ resistor, $R_{IF}$, which is
4. IMPROVED MEASUREMENT SETUP

placed in series with the stripline. This couples a small flux signal of 6.5 m\(\Phi_0\) into the loop. The SQUID output voltage is then amplified by the low frequency amplifier by a factor \(G_{LF} = 10\, \text{kV/V}\). Since only \(V_\Phi\) depends on the SQUID bias parameters (and temperature), \(V_\Phi\) is determined by measuring \(V_{LIA}\) and using equation 4.7. Figure 4.5a shows \(V_{LIA}\) as a function of SQUID bias. The highest flux gain occurs at low voltage setpoints, as expected from figure 4.2. By using the parameters mentioned above and the maximum \(V_{LIA} = 3\, \text{mV}\) from figure 4.5, the maximum flux gain is found to be \(V_\Phi = 176\, \mu\text{A}/\Phi_0\).

The minimum and maximum critical currents, \(I_C^{\text{MIN}}\) and \(I_C^{\text{MAX}}\) decrease for increasing temperatures (4.5b). For the temperature-dependance measurements, it is therefore important to track the critical currents and the voltage swing in order to keep the setpoint of the feedback loop within the stable region. The flux gain \(V_\Phi\) depends on \(I_C\) and thus also decreases with increasing temperature. As the value of \(V_\Phi\) is needed to calibrate the displacement sensitivity, \(V_{LIA}\) is measured with each resonator spectrum. This calibration is described in the next section.

4.3.2 DISPLACEMENT CALIBRATION

A reference is needed in order to calibrate the responsivity \(\partial V_{SQ}/\partial u\) of the combined dc SQUID position detector and amplifier chain. Thermal energy is a natural choice because the equipartition theorem relates the mean-square position fluctuations of the resonator \(\langle u^2 \rangle\) to its temperature \(T_R\). By using equation 4.2, the integrated voltage noise power due to the motion of the resonator becomes

\[
\langle V_{R}^2 \rangle = \left(\frac{\partial V_{SQ}}{\partial u}\right)^2 \langle u^2 \rangle = \left(\frac{\partial V_{SQ}}{\partial u}\right)^2 k_B T_R / k_R.
\]  

(4.8)

The total voltage noise has an additional white noise floor \(S_{V_{n}}\) due to the detector and the amplifiers. When this noise is referred back to the resonator displacement, i.e.

\[
S_{u_{n}} = S_{V_{n}} \left(\frac{\partial V}{\partial u}\right)^{-2},
\]  

(4.9)

it is known as the displacement sensitivity or imprecision noise. The imprecision noise is related to the displacement resolution \(\sqrt{\langle u_{n}^2 \rangle}\) of the detector according to [6]

\[
\langle u_{n}^2 \rangle = S_{u_{n}} \frac{\pi f_R}{2 Q_R}.
\]  

(4.10)

In order to examine the quantum limit for linear detection, the resolution should be tunable to be less than \(\sqrt{\langle u^2 \rangle_{QL}}\) (equation 4.3).
4.3. Motion detection

Figure 4.6: (a) Normalized voltage noise spectra at 15 mK (blue) and 420 mK (red). The Lorentzian peaks are due to the Brownian motion of the resonator and the integrated area under the curves is proportional to the root-mean-square motion of the resonator which is in turn proportional to the substrate temperature according to the equipartition theorem. The area under the red curve is clearly larger than under the blue curve. The measurement sensitivity is determined by the flat noise floor away from the resonance peak and is also lower at decreased temperature. (b) Normalized integrated noise power plotted as a function of refrigerator temperature. The noise power decreases linearly with temperature up to a saturation temperature of 70 mK (black vertical line). A linear fit of the data above the saturation temperature (blue line) yields a slope which is used to calculate the displacement-to-voltage gain of the SQUID position detector. This is in turn used to convert the measured noise spectra from voltage noise to displacement noise.

Figure 4.6a shows the voltage noise spectrum around the resonance frequency, measured at two different temperatures. As was mentioned in the previous subsection, the flux gain of the SQUID changes with temperature and therefore, to compare displacement noise spectra at different temperatures, they must be normalized by $V_\Phi$. For this reason $V_{LIA}$, which is proportional to $V_\Phi$, is measured after each spectrum and the spectra are normalized by $V_{LIA}$. As can be seen in figure 4.6a, the integrated area under the spectra, $\langle V_R^2 \rangle / V_{LIA}^2$, is larger for the higher temperature, as is expected from equation 4.8. Another observation is that the normalized noise floor is lowest at low temperatures. This is because the dominant noise source is the HEMT amplifier and although its noise does not depend on the bath temperature $T_B$, the flux gain $V_\Phi$ decreases with increasing $T_B$, thereby increasing the noise level when it is referred back to the resonator displacement.

Although one could calculate $\frac{\partial V_{SO}}{\partial u}$ from a single spectrum by using equation 4.8, this assumes that the temperature of the resonator $T_R$ is the same as the temperature of the bath $T_B$, which is measured by a separate thermometer. At temperatures below 1 K, this point is not trivial and many groups observe a saturation of $T_R$, where it no longer follows $T_B$ for varying reasons, such as on-chip power dissipation and back-action of the displacement detector [7, 15]. The responsivity can
be more accurately determined by measuring $\langle V_R^2 \rangle / V_{LIA}^2$ as a function of $T_B$. From equation 4.8, a linear relationship between these two quantities is expected as long as $T_R = T_B$. Then, $\frac{\partial V_{SQ}}{\partial u}$ can be determined by calculating the slope of $\frac{d\langle V_R^2 \rangle / V_{LIA}^2}{dT_B}$ and using

$$\frac{\partial V}{\partial u}(T_B) = \left[ \frac{k_R}{k_B} \left( \frac{d\langle V_R^2 \rangle / V_{LIA}^2}{dT_B} \right) \right] V_{LIA}(T_B).$$

(4.11)

Here, $\frac{\partial V}{\partial u}$ at temperature $T_B$ is stated explicitly as consisting of the parameters within the square root, which are approximately temperature independent and $V_{LIA}$, which depends strongly on $T_B$. By combining equations 4.3, 4.9, 4.10 and 4.11, the position resolution with respect to the quantum limit can be expressed in terms of only measured quantities and natural constants:

$$\sqrt{\frac{\langle u_n^2 \rangle}{\langle u_{QL}^2 \rangle}} = \sqrt{\frac{k_B S_{Vn}}{2\hbar Q_R} \left[ \frac{d\langle V_R^2 \rangle / V_{LIA}^2}{dT_B} \right]^{-1} \left( \frac{1}{V_{LIA}(T_B)} \right)}.$$  

(4.12)

Note that $Q_R$ and $V_{LIA}$ both depend on temperature and will be maximal at the lowest bath temperatures $T_B$. This is where the best resolution is achieved.

Figure 4.6b shows the thermal calibration for the dc SQUID detector. The optimum bias point is determined by finding the maximum gain in figure 4.5. The bias is defined in relative terms: $I_B = I_{C\text{MIN}} + 0.8(I_{C\text{MAX}} - I_{C\text{MIN}})$ and $V_{SB} = V_{\text{MIN}} + 0.3(V_{\text{MAX}} - V_{\text{MIN}})$ because these values change with temperature and must be tracked, as was explained in the previous subsection. For this purpose, the SQUID is calibrated at each temperature point. From a linear fit to the data in figure 4.6b, the slope $\frac{d\langle V_R^2 \rangle / V_{LIA}^2}{dT_B} = 0.25 \text{ K}^{-1}$ is obtained. The resonator temperature saturates around $T_R = 70 \text{ mK}$, which is similar to what was observed in previous work [12]. At $T_B = 15 \text{ mK}$, we measure $V_{LIA} = 3 \text{ mV}$ and $Q_R = 3\times10^4$. Combined with the output noise floor $\sqrt{S_{Vn}} = 4.5 \mu\text{A/Hz}^{1/2}$ which was mentioned in section 2 and the spring constant $k_R = 110 \text{ N/m}$ from section 1, this yields $\frac{\partial V}{\partial u} = 4.2 \mu\text{V/fm}$ and $\sqrt{\frac{\langle u_n^2 \rangle}{\langle u_{QL}^2 \rangle}} = 4.4$ or in absolute terms, $\sqrt{S_{u_n}} = 1 \text{ fm/Hz}^{1/2}$. As discussed in section 2, this order-of-magnitude magnitude improvement over previous work [12] is achieved by placing the HEMT amplifier closer to the SQUID at the 1 K stage of the dilution refrigerator.

A further improvement of the displacement resolution is possible by increasing the flux responsivity of the SQUID and by using an amplifier with an even lower noise floor, such as a second SQUID. With these improvements, one should be able to push the SQUID displacement sensitivity beyond the standard quantum limit, where detector back-action becomes the dominant source of fluctuations. Since
the back-action in this SQUID-based detection scheme is tunable [14], it is an interesting system to study in more detail. For example, it may become possible to go from a regime where the SQUID is cooling the resonator to a regime where the resonator oscillation is amplified, possibly to the point of self-sustained oscillation [19]. Because of their small mass and low dissipation, embedded micro- and nanomechanical resonators are an excellent platform to study these effects.

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REFERENCES


Feedback Cooling of the Resonator

We have employed a feedback cooling scheme, which combines high-frequency mixing with digital signal processing. The frequency and damping rate of a 2 MHz micromechanical resonator embedded in a dc SQUID are adjusted with the feedback, and active cooling to a temperature of 14.3 mK is demonstrated. This technique can be applied to GHz resonators and allows for flexible control strategies.

Parts of this chapter have been published in Applied Physics Letters (2011) [1].
Mechanical systems in the quantum regime [2–4] can be used to answer fundamental questions about quantum measurement, decoherence, and the validity of quantum mechanics in macroscopic objects. This requires a mechanical resonator which is cooled to such a low temperature that it is in its ground state for most of the time. In the past few years tremendous progress has been made in actively cooling resonators [4], mainly by using sideband cooling [3, 5, 6] and active feedback cooling [7–9]. The latter technique has mainly been applied to low-frequency (kHz) resonators combined with optical detection. The largest cooling factors have been obtained using velocity-proportional feedback, i.e., by feeding back the differentiated displacement signal. However, at higher frequencies, delays in the feedback system seriously degrade the cooling performance. Here, we demonstrate a feedback cooling technique [10] with a nearly unlimited bandwidth, based on fast digital signal processing (DSP) in combination with single-sideband mixing. A 2 MHz micromechanical resonator with inductive readout is cooled to 14.3 mK using this scheme.

5.1 Device Characterization

Figure 5.1a shows the device, which consists of a dc SQUID with a part of its loop suspended. This forms a 50 µm long flexural resonator with its fundamental mode around \( f_0 \sim 2 \text{MHz} \). The chip is glued onto a piezo element for feedback and actuation, and cooled in a dilution refrigerator with a minimum bath temperature of 15 mK. By applying an in-plane magnetic field \( B \) (green), a displacement of the beam \( u \) changes the amount of flux through the dc SQUID loop. When a bias current \( I_B \) is applied, a change in flux results in a change in the SQUID voltage \( V \). This way, the dc SQUID is a sensitive displacement detector [11]. In all measurements presented here, the same working point for the dc SQUID is used, to avoid backaction-induced changes in frequency and damping [12].

The thermal noise of the resonator is used to calibrate the dc SQUID detector. Figure 5.1b shows the displacement noise spectrum \( S_{uu} \) measured at two different cryostat temperatures \( T \). The thermal motion of the resonator shows up as a peak on top of the imprecision noise floor \( S_{uu} \). The cryogenic-amplifier-limited displacement noise is 2 fm/√Hz at \( T = 15 \text{mK} \). The area under the peak is the amplitude of the Brownian motion of the resonator squared. When the temperature of the refrigerator is increased to \( T = 0.3 \text{K} \), the spectrum changes: Firstly, the noise floor is higher due to a decrease in the transduction, \( dV/du \), as the critical current decreases with increasing temperature [11]. Secondly, the peak is higher and wider (the intrinsic damping rate \( \gamma_0 \) increases with temperature), indicating that the thermal motion is larger at higher temperatures. The resonator temperature \( T_0 = k_0 \langle u^2 \rangle / k_B \) \( (k_0 = 110 \text{N/m} \) is the spring constant) is plotted against \( T \) in Fig.
FIGURE 5.1: (a) Schematic overview of the SQUID detector (red) with the integrated flexural resonator. (b) Displacement noise spectra without feedback. (c) The resonator temperature extracted from the thermal noise spectra plotted against the mixing chamber temperature. (d) Generic linear system representation (e.g. Ref. [13]) of feedback cooling. (e) The feedback filter consists of a digital signal processor with a single-sideband mixer at the input and output.
5.1c: It follows the cryostat temperature for $T > 50\,\text{mK}$ (solid line) and saturates below this value.

### 5.2 Feedback Setup

To further lower the resonator temperature, active feedback is employed, where the displacement of the resonator is fed back to it to damp its thermal motion. Fig. 5.1d illustrates the generic process [4]: The thermal force noise $F_{th} \equiv k_0 f_{th}$ drives the resonator whose response is $H_R = f_0^2/(f_0^2 - f^2 + if\gamma_0/2\pi)$. Note, that $f_{th}$ and the other signals are scaled to have the unit of position. The force results in a displacement which is measured by the dc SQUID detector, and imprecision noise $u_n$ is added to its output $v$. This signal is fed to the feedback filter with transfer function $G_{fb}$. The actuation $a$ is multiplied by $A$, which consists of the SQUID transduction, an attenuation (-40 dB) and the piezo responsivity. Finally, the resulting piezo displacement $u_p$ exerts an inertial force on the resonator. Note, that in practise crosstalk ($X$) exists between the applied feedback and the detector output, which modifies the system response.

To fully characterize the linear system an ac signal is applied to $a$ (see Fig. 5.1d) and the response at $v$ is measured at the same frequency, while sweeping the driving frequency across the resonance. In this case, the feedback $G_{fb}$ is disabled. From this network-analyzer measurements the elements $A = 1.94 \times 10^{-4} \exp(-0.73i)$, and $X = 0.26 \exp(2.56i)$ of the linear systems are obtained as well as the parameters of $H_R$: $f_0$ and $\gamma_0$. The non-zero phase of $A$ is due to the time it takes for the signal to travel through the whole system. If an analog differentiator would be used for $G_{fb}$, this delay causes the feedback to not be purely velocity proportional thus degrading the cooling performance. The DSP-based feedback presented here can compensate for this effect as demonstrated below.

Our implementation of the feedback filter $G_{fb}$ is shown in Fig. 5.1e. The high-frequency input signal $v$ is split and both branches are mixed with local oscillator (LO) signals with a 90° phase difference between them. This IQ mixer gives both quadratures $v_s$ and $v_c$ of the input signal. The LO frequency is $f_{LO} = 2.0492\,\text{MHz}$ so the down-mixed signals oscillate at $f_R - f_{LO} = 8.9\,\text{kHz}$. They are digitized and the DSP (Adwin Pro II at a sampling rate $f_s = 820\,\text{kS/s}$) applies the following transformation to the input signals to generate two output signals $a_c$ and $a_s$:

$$
\begin{pmatrix}
  a_c \\
  a_s
\end{pmatrix} = g_{fb} \begin{pmatrix}
  \cos\theta_{fb} & -\sin\theta_{fb} \\
  \sin\theta_{fb} & \cos\theta_{fb}
\end{pmatrix} \begin{pmatrix}
  v_c \\
  v_s
\end{pmatrix}
$$

These quadratures are then up-converted by the LO frequency with a second IQ mixer.
Figure 5.2: Feedback phase dependence of the resonator frequency (a), damping rate (b), and resonator temperature (c) for $g/f_b = 0.1$. The dashed lines indicate their measured values in the absence of feedback, ($f_0$, $\gamma_0$, and $T_0$ resp.); the solid line is the phase dependence calculated using independent measurements.
The final result is a signal $a$ at the original frequency that is phase-shifted by the feedback phase $\theta_{fb}$ and multiplied by the feedback gain $g_{fb}$, i.e. $G_{fb} = g_{fb} \exp(i\theta_{fb})$. The only frequency requirement for this mixing scheme is that the quadratures do not change faster than the sampling rate, which is equivalent to $\gamma_R/2\pi \lesssim f_s/2$. The operation is thus not limited to resonators with frequencies within the bandwidth of the DSP, allowing feedback cooling of radio and microwave frequency resonators. Note, that the imprecision noise floor is still determined by the cryogenic amplifier; the contributions from the mixers and discretization are negligible.

The feedback modifies the resonator response from $H_R$ to its closed-loop form $H'_R$ [4]:

$$H'_R = \frac{f_0^2}{f_0^2 - f^2 + i f \gamma_0/2\pi - f_0^2 G'_{fb} A},$$

where $G'_{fb} = g'_{fb} \exp(i\theta'_{fb}) = G_{fb}/(1 - XG_{fb})$ is the feedback filter modified by the crosstalk. The real part of $G'_{fb} A$ modifies the resonance frequency from $f_0$ to $f_R \approx f_0(1 - \text{Re}[G'_{fb} A])/2$, whereas the imaginary part changes the damping from $\gamma_0$ to $\gamma_R \approx \gamma_0 - 2\pi f_0 \text{Im}[G'_{fb} A]$. Both the frequency shift and the change in damping depend periodically on the phase of $G'_{fb} A$ and the maximum frequency shift is half the maximum damping rate change.

### 5.3 Results

In order to achieve optimal cooling the feedback phase is varied for a fixed feedback gain as shown in Fig. 5.2. The feedback gain is chosen sufficiently small so that $G'_{fb} \approx G_{fb}$. At every point the thermal noise spectra are measured and fitted to obtain the resonance frequency (top), the damping rate (center), and the resonator temperature (bottom). The resonance frequency and damping rate show the expected sinusoidal dependence on the feedback phase. The amplitude of the frequency shift is half of that of the change in damping, consistent with the discussion above. The phase where the damping is maximized, coincides with the lowest resonator temperature and zero frequency shift. At this phase the system delay is compensated and a pure velocity-proportional feedback is applied to the resonator (i.e. $\angle A G_{fb} = -\pi/2$). The phase dependencies can also be calculated without any free parameters by using the values from the network characterization (Fig. 5.1d). Figure 2 show that these are in good agreement with the feedback results.

To further cool the resonator, the feedback gain is increased at the optimal phase as indicated in Fig. 5.3. First the resonator temperature decreases rapidly with increasing gain due to the increased damping rate. However, by increasing the gain further, more of the imprecision noise $u_n$ is fed back as force noise.
5.3. Results

Figure 5.3: Resonator temperature as a function of feedback gain, showing both the feedback measurements (symbols) and calculations for pure velocity-proportional feedback (solid line). The inset shows the root-mean-squared value of the actuation signal ($\sqrt{\langle a^2_c \rangle + \langle a^2_s \rangle}$) as a function of gain for different filter bandwidths.

This causes a steady increase in $T_R$ for large $g_{fb}$. The minimum temperature that can be reached is set by $S_{uu_uu}$ and the solid line shows the predicted curve for velocity-proportional feedback [8, 14] calculated with the experimental parameters. The achieved minimum of 14.3 mK is close to the predicted lowest temperature of 14.0 mK. Note, that a temperature of 14.3 mK corresponds to an average thermal phonon occupation of $\hat{n} \approx k_B T_R / h f_R = 138$ for a 2 MHz resonator. The heterodyne DSP-based technique employed in this work thus successfully reaches the lowest temperature possible for the standard fully-analog approach, but now applied to a high-frequency resonator.

Another advantage of DSP-based feedback is that the transformation of $\nu_s$ and $\nu_c$ to $a_c$ and $a_s$ can be designed with almost arbitrary transfer characteristics, allowing implementation of optimal control strategies [15]. In the measurements in
Fig. 5.3, the input signal is digitally filtered using a Fourier transform filter which is centered around $f_0$ and a tunable filter bandwidth $\Delta f$. The filter reduces the bandwidth of the feedback which prevents excess signal output outside the resonator bandwidth that may overload the detector or the amplifiers. The inset of Fig. 5.3 shows the root-mean-square output voltage as a function of $g_{fb}$ for three values of $\Delta f$. For the full bandwidth ($f_s/2 = 410$ kHz) an instability occurs around $g_{fb} = 0.23$, which affects the cooling. The 10 kHz bandwidth, which is used for the cooling curve of Fig. 5.3, has two orders of magnitude less actuation compared to the full bandwidth, enabling efficient cooling without affecting the closed-loop response as long as $\Delta f \gg \gamma_R/2\pi$. This again illustrates the versatility of our digital quadrature feedback cooling platform.

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REFERENCES


We study the resonant behavior and backaction of a torsional mechanical resonator which contains a dc SQUID displacement detector. We find that, when biased in the high flux-to-voltage gain regime, the Lorentz force backaction causes self-sustained oscillation of the resonator. Temporal evolution of the resonator motion is studied to distinguish self-sustained oscillations from thermal motion. Compared to devices in the previous chapters, the torsion resonator is coupled more strongly to the SQUID. We show that this increased coupling boosts the displacement resolution below the standard quantum limit.
6.1 Introduction

Backaction effects play an important role in detectors which probe mechanical displacements. Thus, the detector exerts a force on the motion and this can in turn lead to frequency shifts of the mechanical resonator and changes in the damping [1]. The interaction can be so strong that the damping becomes negative, leading to stable periodic motion with a high amplitude [2–4]. These self-sustained oscillations are distinct from the forced motion of a harmonic oscillator because the oscillation period does not depend on the frequency of the source-signal. Resonators in self-sustained oscillation typically produce signals with a spectral linewidth which is narrower than the intrinsic linewidth of the resonator from which they are built. The reduced linewidth enhances the sensitivity of resonance-based force-gradient sensors [5]. On the other hand, these oscillations can be unwanted in experiments which require observation of very small displacements, such as those induced by gravitational waves [6] and quantum mechanical zero-point fluctuations [7–9].

To achieve self-sustained oscillations, the feedback amplifier can either be engineered on purpose or the feedback can be an intrinsic effect in the device. An example of an engineered oscillator is an electronic feedback amplifier which is fed by a dc current source and which drives a GHz-range electromechanical oscillation [10]. Another example is an optomechanical oscillator which is sustained by photons which impart their momentum to the 10 MHz mechanical resonator at a rate of 100 THz [11]. In micro- and nanomechanical systems, self-sustained oscillation has been engineered on-purpose by classical feedback effects such as periodic thermal contraction of the support beams [12] and oscillation due to Coulomb forces during field emission of carbon nanotubes [13].

The detector-resonator interaction for optomechanical cavities in the visible and microwave range has been extensively studied [2, 14, 15]. In contrast, the backaction of mesoscopic electronic displacement detectors remains largely unexplored. Although the use of mesoscopic electronics such as the Atomic Point Contact (APC) [16] and the Single Electron Transistor (SET) [17, 18] greatly enhances the displacement-to-voltage gain, it also increases backaction. Despite considerable theoretical interest [19–21], direct observation of self-sustained oscillations have not been reported in these mesoscopic electromechanical devices.

The displacement detector in this chapter consists of a mechanical resonator which is part of the loop of a dc SQUID. The dc SQUID is a potentially quantum-limited mesoscopic detector of magnetic flux which is capable of detecting tiny fractions of a flux quantum [22–24]. Resonator motion is inductively coupled to the flux in the SQUID by placing the device in a constant magnetic field. The output of the SQUID then is sensitive to the deflection of the resonator. The inductive coupling between SQUID and resonator also leads to backaction on the resonator.
This backaction can be understood as follows: The displacement of the resonator results in a change in the current which flows around the loop. Due to the applied magnetic field, this current exerts a Lorentz force on the resonator which changes the effective spring constant and the effective damping [25].

In previous work, the resonator consisted of a beam-shaped section of the SQUID loop. In this chapter we introduce a torsional resonator geometry in which the entire SQUID loop is suspended. We show that this geometry increases the coupling between the SQUID and the resonator such that the total damping can become negative. Once this condition is reached, the measured oscillation amplitude of the resonator grows strongly and settles into a stable limit cycle. In this regime, the detector resolution is found to be a factor two below the standard quantum limit.

6.2 Setup and Device Characterization

The dc SQUID (figure 6.1a) is fabricated from an InAs-AlGaSb heterostructure which is epitaxially grown on a GaAs [111a] substrate. The SQUID ring (95x40 µm²) is formed by evaporation of 100 nm of Niobium on the InAs surface. A 250 nm long gap in each of the 4.5 µm wide arms forces current to flow through the 42.5 nm thick InAs layer, thereby forming Superconductor-Normal metal-Superconductor (SNS) type Josephson junctions. Titanium-gold contacts are evaporated on the leads to connect to off-chip electronics. Around the SQUID, the InAs and AlGaSb layers are removed by a BCl₃ reactive ion etch in order to define the resonator and to provide electronic insulation. The GaAs underneath the resonator is removed by an isotropic ammonia wet etch so that the SQUID is almost completely suspended: only the current leads anchor the SQUID to the substrate. This geometry allows the SQUID loop to perform flexural and torsional motion. We refer to the SQUID resonator in this chapter as the torsional SQUID to distinguish it from our previous work, where torsional motion was not important. The substrate with the SQUID is glued on top of a piezoelectric transducer which is used to excite the eigenmodes of the resonator. This ensemble is mounted inside a dilution refrigerator and is operated at its base temperature of 20 mK in high vacuum.

To detect the deformation of the suspended SQUID loop, a magnetic field is applied (figure 6.1a,d). This field transduces the change in loop area due to mechanical deformation into a change in magnetic flux. In figure 6.1b, the deformation of the loop is represented by an in-plane displacement of a section of the loop, in a magnetic field which is perpendicular to the substrate. In the case of the torsional SQUID, the entire loop is moving out-of-plane and the magnetic field is oriented parallel to the substrate.
6. SELF-SUSTAINED ELECTROMECHANICAL OSCILLATION

Figure 6.1: Device characteristics. (a) Colorized Scanning Electron Microscope image of the suspended SQUID (red) at an angle of 80 degrees to the substrate (blue). The 95x40 $\mu$m$^2$ SQUID is current biased and the voltage is measured across the gold leads (yellow, from the upper right pad to the lower left pad). The Josephson junctions are located in the loop. (b) Schematic overview of the geometry and parameters of the SQUID. The resonator is represented by the deformed section of the loop marked in red. $B$, $I_B$ and $\Phi_a$ are respectively the applied magnetic field, bias current and magnetic flux. The two junctions have phase $\delta_{1,2}$ of the superconducting order parameter. $u$ is the deflection of the resonator coordinate from its equilibrium position, as defined in the main text. $J$ is the circulating current around the loop and $F_L$ is the Lorentz force on the resonator. The voltage across the loop is measured using a low- ($< 1$ kHz) and a high-frequency amplifier ($> 1$ kHz). The arrows point in the positive direction of the respective variables. (c) Measured SQUID voltage as a function of bias current (main) and applied magnetic flux (inset). The red lines show the minimum and maximum voltage at each current. These are obtained by sweeping the current $I_{\text{coil}}$ through the coil to induce several flux quanta in the loop (inset) and extracting the extreme values from the resulting trace (green dots). The blue dots mark the bias at which the resonator measurements of figure 2 are taken. (d) Finite-element simulation of the two lowest mechanical resonance modes for a geometrically symmetrical SQUID. Red denotes high displacement amplitude while blue denotes a small displacement. The mode shapes are dominated by the coupled flexural motion of the long arms. The out-of-substrate motion of the arms causes a change in the magnetic flux in the loop due to the magnetic field which is oriented parallel to the substrate plane and perpendicular to the arms.
The displacement $u$ of a given resonance mode is defined as the root-mean-squared displacement of that mode. This definition ensures that the spring constant equals $k_R = m_R \omega_R^2$, where $m_R$ is the total mass of the resonator and $\omega_R$ is the resonance frequency (see chapter 2). The change in magnetic flux for a given displacement is then found by calculating the change in area of the inner loop in the plane perpendicular to the magnetic field direction. This results in a flux change $\partial \Phi/\partial u = aB\ell$ per unit deflection. Here, $\ell$ is the length of the resonator, perpendicular to the magnetic field direction. $a$ is a geometrical factor which depends on the shape of the resonance mode and the orientation of the magnetic field. For the two lowest modes, we estimate $a \approx 1$.

To determine the working point of the SQUID, its electronic transport characteristics are first measured. Figure 6.1c shows voltage-current and voltage-flux traces for the SQUID at an applied magnetic field of $B = +80$ mT which is used throughout this work. The magnetic field is applied by a superconducting solenoid magnet, while the magnetic flux through the loop is fine-tuned with a small coil near the device. The SQUID output voltage is zero at low-bias currents and behaves Ohmically at bias current well above the critical current. The voltage is most sensitive to the magnetic flux in the transition region. The inset of figure 6.1c shows the output voltage of the SQUID when the magnetic flux is swept by applying a current through the small coil. The voltage is periodic with a period of one flux quantum $\Phi_0 = h/2e = 2fTm^2$. The SQUID parameters are extracted from the transport curves. At $B = +80$ mT, the maximum critical current is $I_c^{\text{max}} = 1.1 \mu A$ (0.55 $\mu A$ per junction) and the normal-state resistance of the two junctions in parallel is $R = 8 \Omega$. Stable operation is achieved by operating the SQUID in a low-frequency feedback loop (1 kHz) which adjusts the applied flux to maintain a constant setpoint voltage $V_{SP}$. The high-frequency resonator voltage and the low-frequency voltage are measured by two separate amplifiers (figure 6.1b). A detailed description of the measurement electronics is found in reference [26]. Note, that for a given bias current and voltage setpoint, the feedback loop can be set to lock either to the upward or the downward flank of the $V-\Phi$ curve (inset of figure 6.1c). These are denoted $V_{\Phi} \equiv \partial V/\partial \Phi > 0$ and $V_{\Phi} < 0$, respectively.

### 6.3 Resonator Characterization

The resonator displacement is detected by measuring the SQUID voltage at the resonator frequency. This voltage is measured using a High-Electron-Mobility-Transistor (HEMT) amplifier at the 1 K stage of the dilution refrigerator [26]. The eigenmodes of the torsional SQUID are found by driving the piezo actuator with

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1For two dissimilar arms, the geometrical factor is effectively that of a single beam resonator, as described in chapter 2, for which the geometrical factor was calculated to be 0.91.
a sinusoidal voltage and measuring the SQUID voltage at the same frequency. A frequency sweep up to the maximum piezo frequency of 2.5 MHz reveals many eigenmodes, with the lowest modes at 109 kHz and 129 kHz. The quality factor is extracted from a fit to the frequency response. For the lowest modes the intrinsic quality factor $Q_0$ is around 7000. Without piezo drive, the thermal motion of the resonator modes is measured with a spectrum analyzer. The thermal spectrum contains clear peaks for the 109 kHz and 129 kHz modes.

Finite-element simulations show the nature of the eigenmodes to be a combination of flexural and torsional motion (figure 6.1d). The lowest two modes at 109 and 129 kHz are dominated by the flexural motion of the long arms; The lowest mode exhibits in-phase motion of the two arms. The simulations suggest that the in-phase mode is not detectable because there is no net change of the loop area in the direction of the applied magnetic field (figure 6.1d). A small asymmetry between the two arms, however, makes this mode detectable.

### 6.4 Self-sustained oscillation measurements

Voltage noise spectra of the two lowest modes are shown in figure 6.2a and b. The spectra are measured on the positive flank (blue data) and the negative flank (red data) of the $V - \Phi$ curve (figure 6.1c). Compared to the spectra on the negative flank, the spectra on the positive flank are much sharper and the total power (the area under the resonance peaks) increases strongly, indicating oscillations with high amplitude. Figures 6.2 c and d show respectively the power and the quality factor as a function of the bias current. The high amplitudes for the 109 kHz mode occur near the maximum critical current at positive bias current (see figure 6.1c), while the high amplitudes for the 129 kHz mode occur at negative bias current. Thus, the modes do not oscillate with high amplitude simultaneously and apparently the large motion of one mode disrupts the backaction on the other mode. This competition is also seen between cantilever modes in optomechanical cavities [15] and between cavity modes in lasers [27].
6.4. Self-sustained oscillation measurements

Figure 6.2: Spectral measurements. (a) Power spectral density plots of the 109 kHz mode at $I_{\text{bias}} = +1.4\, \mu A$. The blue spectrum is due to thermal motion of the mode, measured on the positive flank $V_\Phi > 0$. On the negative flank $V_\Phi < 0$, the noise peak (in red) sharpens and increases in amplitude by several orders of magnitude. This is the backaction-induced Self-Sustained Oscillation (SSO) regime. The frequency shift between the blue and red spectra is also an effect of the SQUID backaction and is described in the supplement. (b) The spectra of the 129 kHz mode are measured at $I_{\text{bias}} = -1.4\, \mu A$. The behavior is similar to the 109 kHz mode, with SSO occurring on the same flank $V_\Phi < 0$. The amplitude of the SSO peak is however sharper and several orders of magnitude larger than that of the 109 kHz mode. (c) The integrated noise power of the resonance peaks as a function of bias current. The 129 kHz points (red) are measured at negative bias current. The data points for the spectra of figures a and b are marked by the green window. On the positive flank $V_\Phi > 0$, no SSO is observed and the noise power is due to thermal motion (blue). Only the 109 kHz thermal data is shown, as the 129 kHz data is nearly identical. The SSO power is largest at low-bias current and decreases steadily until no signature of SSO can be detected (hollow points). (d) Quality factor obtained from fits to the noise spectra. The thermal spectra on the positive flank $V_\Phi > 0$ have the lowest Q values, while the 129 kHz mode reaches values greater than $10^5$ in the SSO regime. The determination of the line width is limited to approximately 1 Hz by the frequency stability of the oscillations.
Next, we study the resonator behavior in the time domain. We have recorded the waveforms (voltage versus time) at the resonance frequency. Figure 6.3a shows the measured and processed $^2$ time record of the SQUID voltage after switching the SQUID from the positive flank to the negative flank of the $V - \Phi$ curve. The oscillation amplitude increases until it saturates after 20 milliseconds $^3$ (figure 6.3a). Once the SQUID is switched back to the positive flank, an exponential ring-down is observed. The ring-down is consistent with that of an unforced resonator with a dissipation that corresponds to the quality factor of the 129 kHz mode $^4$.

Next, we examine the time it takes for the resonance frequency to shift after the SQUID switches flanks and compare it to the ring-up and down times. The resonance frequency shifts with SQUID bias due to dynamical backaction [28]. Figure 6.3b shows the frequency as a function of time. We have also measured the frequency shift as a function of bias current and $V\text{-}\Phi$ flank and the data is shown in the supplement. The instantaneous frequency is extracted from the waveform by filtering out the harmonics of the 129 kHz signal and locating the times at which the resulting waveform crosses the zero volt axis. The frequency is then calculated by taking the reciprocal of the time between two adjacent zero-crossings. In figure 6.3b, the frequency shifts by 1 kHz within a few milliseconds of switching the flanks. The step response of the frequency is thus much faster than the amplitude step response. This also implies that the time it takes to switch the SQUID between the positive and negative flanks is less than a few milliseconds.

The individual oscillations in the saturated range of figure 6.3a are shown in figure 6.3c. For comparison, the much smaller waveform at the positive flank of the $V - \Phi$ curve is also shown. Note that on this timescale, both waveforms seem to have a constant amplitude and phase. This is expected even for random thermal motion because the intrinsic quality factor of the 129 kHz mode (7000) is such that the amplitude and phase is randomized on a timescale much larger than the one used in figure 6.3c (many thousands of oscillations). A larger timescale must thus be examined in order to conclusively distinguish the self-sustained oscillations from the thermal motion.

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$^2$The mean of the measured waveform has been subtracted. Additionally, the waveform has been filtered such that it only contains the signal in a bandwidth of 1 kHz around the 129 kHz resonance peak and its harmonics. Note that the full-width-half-maximum of the mechanical signal is less than 20 Hz.

$^3$The amplitude saturates because the positive feedback of the SQUID to the resonator becomes nonlinear beyond some oscillation amplitude, which decreases the backaction at the resonance frequency. The shape of the ring-up fits well to the step-response of a harmonic oscillator (leftmost blue line in figure 6.3a).

$^4$The waveforms are measured at a bias current of -1.1 $\mu$A, or $-I_c^{\text{max}}$. The quality factor obtained from the ring down is around 1500, which is consistent with the value from the spectrum measurements of figure 6.2d (a value between the second and the third blue open circle from the left).
6.4. SELF-SUSTAINED OSCILLATION MEASUREMENTS

Figure 6.3: Time-resolved measurements of the SQUID voltage at $I_B = -I_c^{\text{max}}$. (a) Measured oscillator waveform during startup and ring-down (red). The startup follows the step response of a negative-feedback amplifier (left blue line) and the ring-down follows the exponential decay which is expected for a harmonic oscillator with $Q = 7000$ (right blue line). (b) Oscillation frequency during startup and ring-down. When the $V_\Phi$ flank is switched, the frequency shifts to its new value within a few milliseconds. In contrast, the amplitude of the oscillator as seen in figure a reacts much more slowly. (c) Waveform measurement of the 129 kHz self-sustained oscillations at $I_B = -I_c^{\text{max}}$ (red) and of the thermal motion on the other $V_\Phi$ flank (blue). The oscillator waveform contains higher harmonics due to the nonlinear response of the SQUID detector. The ratio between the harmonics is used to calibrate the displacement of the resonator mode. (d) Simulation of the oscillator waveform, with the peak-voltage and peak-flux oscillation amplitude as free parameters. The flux oscillation amplitude is found to be $0.3 \Phi_0$ and the peak-peak voltage amplitude is $0.75 \text{ V}$. 
The waveform behavior on a longer time scale is shown in figure 6.4. Figure 6.4a shows the phase plot of the low-amplitude waveform of figure 6.3c over a period of 2000 seconds. To extract the amplitude and the phase of the measured waveform, the time record is multiplied by pure cosine and sine waveforms at the oscillation frequency. The resulting time series are passed through a moving-average filter to eliminate spurious frequency components. This yields the in-phase and quadrature (90 degrees out-of-phase) amplitudes of the waveform. A noiseless oscillator has a phase plot which consists of a single point, or in other words, it is a stable phasor. Amplitude noise causes broadening in the direction of the phasor, while phase noise causes broadening in the tangential direction. If the resonator displacements are due to thermal force noise, the phase plot consists of a Gaussian distribution which is centered around zero amplitude. The phase plot in 6.4a has such a distribution, confirming that the blue waveform of figure 6.3c is due to Brownian motion. We have verified that the other blue spectra in figure 6.2 have the same characteristic Gaussian distribution and we therefore refer to these as thermal spectra.

In contrast, the high-amplitude waveform of figure 6.3c has a ring-shaped distribution around zero. We see that the mean amplitude is non-zero and that the noise in the amplitude direction is much smaller than the mean value. This behavior is expected for harmonic oscillators which experience nonlinear negative feedback, because they tend towards a stable limit cycle [29]: The oscillation amplitude grows until the SQUID can no longer increase its backaction on the resonator, causing the amplitude to settle on a steady-state value. Any perturbation in the amplitude results in a forcing back to the steady-state. The phase of the oscillator has no such restriction and is therefore allowed to drift freely, leading to a ring-shaped distribution in phase plots such as the one in figure 6.4b. The phase plot of the high-amplitude waveform is a clear signature of self-sustained oscillation.

To calibrate the resonator displacement, one generally measures thermal spectra to determine the displacement-to-voltage responsivity $\frac{\partial V}{\partial u}$ [26, 30]. However, the occurrence of self-sustained oscillations in the high-gain region near the critical current indicates strong backaction and prevents calibration by this method. Instead, we use the self-sustained oscillations to calibrate the detector. The responsivity, $\frac{\partial V}{\partial u} = (\frac{\partial V}{\partial \Phi})(\frac{\partial \Phi}{\partial u})$, where $\frac{\partial V}{\partial \Phi}$ is the flux-to-voltage gain and $\frac{\partial \Phi}{\partial u} = aB\ell = 3.7 \mu \Phi_0/fm$ is the displacement-to-flux transduction factor. We know the parameters of $\frac{\partial \Phi}{\partial u}$ to good precision and so the gain $\frac{\partial V}{\partial \Phi}$ is left to be determined.
Figure 6.4: Phase plots of the 129 kHz waveforms of figure 6.3c, measured over a period of 2000 seconds. Colors indicate the probability of observing a given amplitude and phase, where yellow and red indicate respectively high and low probability. White indicates zero incidence. (a) Phase plot of the low-amplitude waveform (bottom graph is a horizontal line trace through zero). The amplitude has a two-dimensional Gaussian distribution around zero. This is a signature of random, thermal motion. (b) Phase plot of the high-amplitude waveform. The amplitude of the oscillation is well defined and phase noise dominates. This is a signature of steady-state oscillation. This phase plot is clearly distinct from that of thermal motion. Note that the horizontal scale of figure 6.4b is almost an order of magnitude larger than that of figure 6.4a, consistent with the high- and low-amplitude waveforms shown in figure 6.3c.
The calibration of $\partial V / \partial \Phi$ is achieved by matching a simulated waveform to the measured oscillator waveform of figure 6.3c. For this, a model is needed which describes the amplified SQUID voltage as a function of flux. For an ideal SQUID, the voltage is [22]:

$$V = \text{sgn}(I_b)V_{pp} \sqrt{\left(\frac{I_{\text{bias}}}{I_{\text{max}}^c}\right)^2 - \cos^2\left(\frac{\pi \Phi}{\Phi_0}\right)},$$

(6.1)

where $\text{sgn}(x)$ is the sign function and where the flux $\Phi$ is allowed to oscillate in time, $\Phi = \Phi_a + \Phi_x \sin(\omega_x t)$. The peak-peak voltage, $V_{pp}$, can be directly determined from the measured waveform in figure 6.3c and is found to be $V_{pp} = 0.75$ V. The applied flux is $\Phi_a = (\pi/4 - 0.2) \Phi_0$ and the applied bias current is $I_b = -I_{\text{max}}^c$. The simulated waveform is generated from the equation for $V$, with the inclusion of a $1 \mu s$ delay time due to the digital sampling of the signal processor (ADWIN PRO II acquisition system). The flux oscillation amplitude is found by setting the oscillation frequency $\omega_x$ to the oscillator frequency and adjusting the amplitude $\Phi_x$ to match the measured waveform. From this procedure, we find $\Phi_x = 0.3 \Phi_0$. The simulated waveform is shown in figure 6.3d and matches the measured data well.

With equation 6.1, the flux responsivity can be expressed as

$$\frac{\partial V}{\partial \Phi} = \text{sgn}(I_b) \left(\frac{\pi V_{pp}}{2 \Phi_0} \sqrt{\frac{I_{\text{bias}}}{I_{\text{max}}^c}}^2 - \cos^2\left(\frac{\pi \Phi_a}{\Phi_0}\right)\right),$$

(6.2)

With the parameters as determined in this paragraph, we find $\partial V / \partial \Phi = 1.7 \mu V/\mu \Phi_0$, so that the displacement-to-voltage responsivity equals $\partial V / \partial u = 6.1 \mu V/fm$.

We now calculate the detector resolution. By using the responsivity, the background noise power density at the output of the voltage amplifier, $S_V$, is converted to the displacement noise, $\sqrt{S_u} = \sqrt{S_V} / (\partial V / \partial u)$. The detector resolution is obtained by multiplication of $S_u$ with the effective bandwidth of the resonator, $\Delta f = (\pi/2)(f_R/Q_0)$, where $f_R$ is the resonance frequency. The output noise floor of the HEMT amplifier is $\sqrt{S_V} = 4.1 \mu V/\sqrt{Hz}$. The displacement sensitivity is thus $\sqrt{S_u} = 0.67$ fm/$\sqrt{Hz}$. With $Q_0 = 7000$, the intrinsic bandwidth of the mode is $\Delta f = 29$ Hz, so that the SQUID detector has a resolution of $u_n = S_{uu} \Delta f = 3.6$ fm. This resolution should be compared to the standard quantum limit for continuous linear position detection, $u_{\text{SQL}} = \sqrt{\hbar f_R/2k_R}$. For the 129 kHz mode, with a spring constant $k_R = 1.5$ N/m, the standard quantum limit for the displacement resolution is $u_{\text{SQL}} = 5.3$ fm. The energy resolution of the detector is thus a factor $(u_{\text{SQL}}/u_n)^2 = 2.2$ below the standard quantum limit.

\textsuperscript{5}The resolution which is calculated here is known as the imprecision noise due to the detector [25].
6.5 BACKACTION ANALYSIS

To quantify the backaction in the torsional SQUID, we use the analysis presented in chapter 3. In that chapter, the dynamical backaction on the resonator is modeled as a change in the effective damping and resonance frequency of the mechanical resonator due to the current which circulates around the SQUID loop. The total damping of the resonator was written as

$$\gamma_0 \left(1 - J_\phi \Delta Q\right), \text{ with } \Delta Q = \frac{(aB\ell)^2 f_R Q_0}{k_R R}, \quad (6.3)$$

where $\gamma_0$ is the intrinsic resonator damping. The dimensionless gain factor $J_\phi$ is tunable by the applied bias. Other than this, $J_\phi$ only depends the inductance and capacitance of the SQUID, which are the same as for the device in chapter 3. Its maximum value was calculated to be 500. The dimensionless coupling parameter $\Delta Q$ is independent of the bias of the SQUID but depends on the parameters of both the SQUID and the resonator. Negative damping occurs when the product $J_\phi \Delta Q$ is greater than one. In chapter 3, the coupling parameter $\Delta Q = 2.8 \times 10^{-4}$, which means that the maximum product $\max(J_\phi \Delta Q) = 0.14$ was too small to achieve negative damping. The increased length of the torsion SQUID resonator (95 micron versus 50 micron) and its decreased spring constant (1.5 N/m versus 100 N/m), result in $\Delta Q = 2.2 \times 10^{-3}$. This coupling is almost an order of magnitude larger than

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.5.png}
\caption{Backaction-induced frequency shift as a function of SQUID bias current, extracted from the spectra of figure 2 of the main text. Only the 109 kHz thermal data is shown, as the 129 kHz data is nearly identical.}
\end{figure}
that of the beam-SQUID and is sufficiently strong to induce negative damping, in agreement with our observations.

In addition to the observed resonator damping shift (figure 2), the SQUID back-action also shifts the resonance frequency. Figure 5 shows the frequency shift as a function of applied bias current. The trend is similar to what was observed in chapter 3 for the SQUIDs with the embedded MHz beam resonators. The frequency shift relative to the intrinsic resonance frequency is stronger for the torsion resonator than for the beam resonator. This is consistent with the increased coupling parameter; $\Delta f = 1 \times 10^{-2}$ for the torsion resonator versus $\Delta f = 4 \times 10^{-4}$ for the beam resonator of chapter 3.

6.6 Conclusion

We have demonstrated a torsional SQUID with an increased coupling between resonator and dc SQUID compared to the beam SQUIDs. This increases the displacement resolution below the standard quantum limit. The backaction causes negative damping, which leads to self-sustained oscillation. This is, to our knowledge, the first time that intrinsic self-sustained oscillation is observed in a mesoscopic electromechanical device. This type of device may serve as an on-chip clock element, which requires only a dc current and flux input.

For the detection of small forces, the resolution and the backaction must be carefully balanced to reach optimum sensitivity [1]. The self-sustained oscillator itself may enable study of frequency entrainment [31, 32] by coupling multiple torsion SQUIDs together in an array configuration [33]. Finally, the increased quality factor and large amplitude enables sensitive force gradient detection [5].

Acknowledgements

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References


SUMMARY

SQUID-BASED ELECTROMECHANICAL SYSTEMS

This thesis describes the use of a dc Superconducting QUantum Interference Device, or SQUID, as a detector of tiny displacements of a micromechanical resonator. The goal is to achieve a detector resolution which is smaller than the quantum mechanical ground state fluctuations of the mechanical resonator, the so-called standard quantum limit. Mechanical sensors are essential tools for detection of small forces, such as those induced by gravitational waves and quantum mechanical fluctuations. Detection of shifts in the mechanical resonance frequency enables sensitive mass detection and force gradient detection, which is the basis for Dynamic Force Microscopy techniques.

Mechanical resonators transduce forces into displacements and the displacement is in turn converted to an electronic signal by a detector. In contrast to optical displacement readout, electromechanical systems convert mechanical motion directly to electronic signals and vice versa. Mesoscopic electronics such as the Single-Electron Transistor and the Atomic Point Contact can be used to greatly increase the detector resolution with respect to classical electronic readout methods. Another important application for mesoscopic electromechanical devices is as memory elements in quantum information architectures.

The mechanical resonator produces a minimum amount of displacement noise when it is cooled to its quantum mechanical ground state. To reach this regime, a gigahertz resonator must be cooled to millikelvin temperatures, which is possible in cryogenic refrigerators. Megahertz resonators require microkelvin temperatures, which requires additional cooling by either active or passive feedback. The displacement detector itself also exerts a backaction force on the mechanical resonator. Quantum-limited linear displacement detectors add a minimum backaction noise to the zero-point fluctuations of the resonator which is set by the Heisenberg uncertainty principle. Most detectors will add more than this amount of noise. The output noise of the detector together with the backaction noise determine detector performance and must be carefully studied.

This thesis presents the first measurements of a displacement detector for micro- and nanomechanical resonators based on the dc SQUID. SQUIDs are highly sensitive, potentially quantum-limited, detectors of magnetic flux. The mechanical resonator is formed by suspending a part of the SQUID loop to create a micro-bridge. A constant magnetic field couples the position of the bridge to the mag-
Summary

netic flux in the loop, which couples the SQUID and the resonator. In chapter 2, the fabrication method and the measurement principles are described. The first measurements on the SQUID position detector are shown, with a resolution of 51 times the standard quantum limit for a 2 MHz resonator.

Besides backaction force noise, displacement detectors can also cause dynamic backaction. This can cause a shift in both the resonance frequency and the dissipation of the mechanical resonator. Chapter 3 presents measurements of the dynamic back action of the SQUID. The measured resonator frequency and dissipation depend on the SQUID bias current and applied magnetic flux. The backaction is caused by the Lorentz force due to the displacement-dependent current which circulates around the SQUID loop. Numerical calculations using the resistively and capacitively shunted junction (RCSJ) model are used to explain the behavior of the SQUID-resonator system.

Chapter 4 presents the results of several improvements in the experimental setup, including a low-noise cryogenic amplifier. The chapter gives a detailed description of the measurement setup and calibration procedures. With the improved setup, we achieve a resolution of 4.4 times the standard quantum limit, over an order of magnitude lower than that of chapter 2.

Resonators in the MHz range require either feedback or backaction based cooling to reach the quantum regime. Active feedback involves measuring the resonator displacement and applying a velocity-proportional feedback force. The minimum possible temperature is determined by the ratio of the peak resonator noise to the output noise floor of the displacement detector. Chapter 5 presents a flexible, high-frequency compatible feedback circuit which is used to actively damp the mechanical resonator. Due to the excellent noise properties of the SQUID, we achieve a reduction of the resonator temperature 40 mK to 14 mK.

In order to increase the coupling between the SQUID and the resonator, in chapter 6 the entire SQUID loop is freely suspended. The resulting resonator has hybrid flexural-torsional eigenmodes. In the high flux-to-voltage regime of the dc SQUID, the two lowest modes start to oscillate spontaneously with large amplitude. These self-sustained oscillations are due to strong negative damping by the SQUID back action. We analyze the oscillator behavior and relate it to the backaction model of chapter 3. In the self-oscillating regime, we calibrate the displacement responsivity by using the periodic voltage-flux relationship of the SQUID. For this strongly coupled torsional SQUID detector, we find that the displacement resolution is a factor 1.5 below the standard quantum limit. This result shows that the dc SQUID is an excellent displacement detector for micro- and nanomechanical resonators, but also that the SQUID-resonator interaction strongly influences the resonator dynamics.
SAMENVATTING

SQUID-GEBASEERDE ELECTROMECHANISCHE SYSTEMEN

Dit proefschrift beschrijft het gebruik van een Supergeleidende QUantum Interference Device, ofwel een SQUID als een detector voor minime beweging van een micromechanische resonator. Het doel is om een detector resolutie te bereiken die kleiner is dan de kwantum mechanische nulpunt fluctuaties van de mechanische resonator, de zogenaamde standaard kwantum limiet. Mechanische sensoren zijn essentiële instrumenten voor de detectie van kleine krachten, veroorzaakt door bijvoorbeeld gravitatiegolven of kwantum mechanische fluctuaties. Detectie van verschuivingen in de mechanische resonantiefrequentie maakt gevoelige massa detectie mogelijk en ook krachtsgradiënt detectie, die de basis vormt voor 'Dynamic Force Microscopy' technieken.

Mechanische resonatoren transduceren krachten naar verplaatsingen en vervolgens wordt de verplaatsing omgezet in een elektronisch signaal door middel van een detector. In tegenstelling tot optische verplaatsingsuitlezing, zetten electromechanische systemen mechanische beweging rechtstreeks om naar elektronische signalen en omgekeerd. Mesoscopische elektronica zoals de 'Single-Electron Transistor' en de 'Atomic Point Contact' kan worden ingezet om de detector resolutie te vergroten ten opzichte van klassieke elektronische uitlees methoden. Een andere belangrijke toepassing voor mesoscopische elektromechanische systemen is als geheugen elementen in kwantum informatie architectuur.

De mechanische resonator produceert een minimale hoeveelheid ruis als deze wordt afgekoeld naar zijn kwantummechanische grondtoestand. Om dit regime te bereiken moet een gigahertz resonator worden afgekoeld tot millikelvin temperaturen, wat mogelijk is in cryogene koelsystemen. Megahertz resonatoren vereisen microkelvin temperaturen, en die kunnen worden bereikt door extra koeling via actieve of passieve terugkoppeling. De verplaatsingsdetector zelf oefent ook een terugkoppelingskracht uit op de mechanische resonator. Kwantumgelimiteerde lineaire verplaatsing detectoren voegen terugkoppelingsruis toe waarvan het minimum wordt bepaald door het Heisenberg onzekerheidsprincipe. De meeste detectoren voegen meer ruis toe dan dit minimum. De uitgangsruis en de terugkoppelingsruis bepalen samen hoe goed de detector presteert, en deze moeten daarom zorgvuldig worden bestudeerd.
Dit proefschrift presenteert de eerste metingen van een verplaatsingsdetector voor micro- en nanomechanische resonatoren die gebaseerd is op de dc SQUID. SQUIDs zijn zeer gevoelige, mogelijk kwantumgelimiteerde, detectoren voor magnetische flux. De mechanische resonator wordt gevormd door van een deel van de SQUID-lus een microbrug te maken. Een constant magnetiveld koppelt de positie van de brug aan de magnetische flux in de lus en koppelt dus de SQUID en de microbrug-resonator. In hoofdstuk 2 worden de fabricagemethode en de meetprincipes beschreven. De eerste metingen aan de SQUID worden getoond, met een resolutie van 51 keer de kwantum limiet voor een 2 MHz resonator.

Naast terugkoppelingsruis, kunnen verplaatsingsdetectoren ook dynamische effecten veroorzaken. Dit kan zorgen voor een verandering in zowel de resonantiefrequentie als de dissipatie van de mechanische resonator. Hoofdstuk 3 toont metingen van de dynamische terugkoppeling van de SQUID. De gemeten resonator frequentie en dissipatie zijn afhankelijk van de instelstroom en toegepaste magnetische flux van de SQUID. De terugkoppeling wordt veroorzaakt door de Lorentzkracht vanwege de verplaatsingsafhankelijke stroom die circuleert rond de SQUID lus. Numerieke berekeningen gebaseerd op het RCSJ model worden gebruikt om het gedrag van het SQUID-resonator systeem te verklaren.

In hoofdstuk 4 wordt de meetopstelling van hoofdstuk 2 verbeterd door onder andere de toevoeging van een ruisarme cryogene versterker. Het hoofdstuk geeft een gedetailleerde beschrijving van de meetopstelling en de kalibratie procedures. De verbeteringen zorgen voor een resolutie van 4.4 maal de standaard kwantum limiet.

Resonatoren in het MHz-gebied vereisen extra koeling via terugkoppeling om het kwantumregime te bereiken. Actieve terugkoppeling betreft het meten van de resonator verplaatsing en het toepassen van een terugkoppelingsschakelaar die evenredig is met de snelheid van de resonator. De minimale temperatuur die kan worden bereikt wordt bepaald door de verhouding van de piek-ruis van de resonator tot de uitgangsruis van de verplaatsing detector. Hoofdstuk 5 presenteert een flexibel terugkoppelingsschakelaar dat kan werken op hoge frequenties en dat gebruikt wordt om de mechanische resonator actief te koelen. Vanwege de excellente ruis-eigenschappen van de SQUID, bereiken we een reductie van de resonator temperatuur van 40 mK tot 14 mK.

Om de koppeling tussen de SQUID en de resonator sterker te maken wordt in hoofdstuk 6 de hele SQUID lus vrij opgehangen. De resulterende resonator heeft hybride buiging-torsie resonanties. Wanneer de SQUID is ingesteld voor hoge verstergking gaan de laagste twee resonanties spontaan oscilleren met een grote amplitude. Deze zelfdrijvende oscillaties worden veroorzaakt door sterke negatieve demping vanwege de dynamische terugkoppeling van de SQUID. We analyseren het gedrag van de oscillator en relateren dit aan het model van hoofdstuk 3. In het
zelfoscillerende gebied, kalibreren wij de verplaatsingsgevoeligheid door gebruik te maken van de periodieke spanning-flux relatie van de SQUID. Voor deze sterk gekoppelde torsie-SQUID detector is de resolutie een factor 1.5 onder de standaard kwantum limiet. Dit resultaat laat zien dat de dc SQUID een zeer gevoelige detector is voor de verplaatsing van micro- en nanomechanische resonatoren, maar dat tegelijkertijd de SQUID-resonator interactie de resonator dynamica sterk beïnvloed.
CURRICULUM VITÆ

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LIST OF PUBLICATIONS


8. B.H. Schneider, S. Etaki, H.S.J. van der Zant and G.A. Steele *Detecting nanomechanical motion with a suspended carbon nanotube SQUID*, Submitted


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You can't stop the waves, but you can learn to surf.

Jon Kabat-Zinn

I first became interested in nanotechnology back in high school. A book called ‘engines of creation’ described amazing possibilities, one of which was nanomechanics. During my undergraduate studies I learned to temper my expectations with some real physics, but the fascination remained. When I was looking for a place to do my undergraduate research project, I naturally gravitated towards the quantum transport group of Leo Kouwenhoven. That is where I met my supervisor, Herre van der Zant. He had just started to work on the mechanics of nanotubes and nanowires, and I liked both the project and the person. I enjoyed the work and after my undergraduate research and an internship at NTT in Japan, I was hired as a graduate student in his newly founded group, MED. Herre, thank you for giving me the freedom to invent a completely new device. I have learned so much from your guidance and your example and I have tried, though not always successfully, to follow your advice. I have noticed that I now also use your technique of push-and-pull on students, pushing them forward when they are stuck and pulling them back when they become overconfident. I have watched you build your own group into an incredible place with an open and informal atmosphere, and a feeling of true community among the students. I hope to build that kind of place some day.

Between my undergraduate and my graduate project, I was granted an internship position on nanomechanics at NTT basic research in Japan. During my graduate research, I returned two more times, for a total stay of more than 18 months. This is where I fabricated the SQUID devices, based on NTT’s fabrication technology. I worked in the nanostructure group of Hiroshi Yamaguchi. Hiroshi, thank you so much for letting me be part of your group. Your enthusiasm and insight have always been a great motivation to me. Your group has grown successful in the field of nanomechanics and I am sure this will only increase. I love Japan and I have enjoyed my stay enormously. This was in no small part due to my office mate, Imran Mahboob. We started out together and when I was deciding on the best device to fabricate, he gave me some golden advice: ‘KISS: Keep It Simple, Stupid!’ Imran, thank you for all the fun we had exploring Tokyo. But most of all, thank you
for showing me the meaning of the word ‘professional’. And of course, congratulations on becoming NTT staff. My day-to-day mentor was Kenji Yamazaki, my e-beam sensei. Thank you for teaching me to use the clean-room equipment and for taking care of me when I first arrived. I will never forget your face when I said or did something inappropriate. I hope it was not too difficult. For the Niobium-on-semiconductor fabrication technique, a special thanks to Tatsushi Akazaki and his spintronics group.

During my time in Japan I met many more great people. Starting with the group members: Inokuma-san (Kanji, Kanji, Kanji), Onomitsu-san (thank you for growing the substrate), Okamoto-san (thank you for teaching me Niobium evaporation), Toru Yamaguchi-san (my fellow graduate student), Hayashi-san and Nagase-san. A big thank you to Rika Murayama for taking care of the foreigners, Yayoi Narumi for the Taiko, and Tamayo Iwamoto for the pink coffee cup. After working hours, I explored Japan with the community of gaijin PhD students and postdocs. There was always someone to have dinner or go out with. For the many good times, I thank: Nicolas Clement (zat is massif!), Alexander Kasper (used to love her), Wouter Naber (fellow-TU-Delfter), Tobias Bergsten (curly), Neill Lambert (little princess makeup kit), Jonas Rundquist (poor mice), Simon Perraud (measurement-beard), Kristina and Kei Takashina (pink big pig), Remi Riviere (Braddu Pittu), Michel Pioro Ladriere, Claire Iglesias, Cheng-Yao Chen and all the other friends with whom I have shared the island.

Between my internship and my second visit to Japan, before my work on the SQUID, Herre introduced me to Tjerk Oosterkamp. Tjerk, thank you for inviting me to work in the interface physics group in Leiden on motion detection of carbon nanotubes. I am sorry I could not get a solid result out of the project. It was however a useful experience, as I used the feedback principles I learned from the STM setup to operate the SQUID later on. I thank Erwin Heeres (SEM-king) for teaching me how to etch Tungsten tips and for mounting nanotubes on them. This is where I learned not to drink coffee before doing delicate operations. I also enjoyed working in the lab with Allard Katan, Anne France Beker, Merlijn van Spengen and the other group members.

And so, we arrive back in Delft. I’ll start by thanking Sami Sapmaz, the PhD student who supervised my undergraduate project. You taught me how to do microfabrication and how to operate the Delft electronics. Your skill and calm still inspire me today. The samples would also not exist without the TU Delft cleanroom-team: Emile van der Drift, thank you for running the nanofacility all these years. Thank you Arnold van Run and Anja van Langen for the e-beam, Roel Matterne for the wet bench, Hozanna Miro for the SEM and Marco van der Krogt, Marc Zuiddam and Gilles Gobreau for the etchers. Outside of the clean-room, there is also a lot of equipment to break, and that is when I shout ’Bram!’ (and he says ’NO’).
Thank you Bram and Remco, for the Helium, the evaporators and the nice conversations throughout the years. And then there is the other pillar of the Nanoscience department, Raymond Schouten. Without your custom electronics, and clear explanations, I would not have been able to achieve these results. For the electronics, I also thank the DEMO/CEO team led by Jack Maat.

When MED was born, there were only empty offices. Our group technician, Mascha van Oossanen, made sure that the measurement setups were installed and working properly. Mascha, thank you for all the work and good times and for trying to teach me the difference between clockwise and counter-clockwise.

In some ways, a group is only as good as its secretaries, and luckily we have some very good ones: Irma Peterse, Maria Roodenburg and Monique Vernhout have always taken care of all those students who never turn in their declarations on time. Thank you!

The weekly nanomechanics meetings would not be complete without the theorists. Yaroslav Blanter, thank you for your input on the backaction and oscillation parts of the SQUID project and thank you for taking part in my committee. François Konschelle, thank you for your work on self-sustained oscillations. Although our conversations were sometimes quite intense, they always left me with new insights and ideas about the complicated SQUID dynamics. I thank Jos Thijssen for help with the numerical integration of the SQUID differential equations. For useful conversations on the complicated phenomena in superconductors and Josephson junctions, I thank Alberto Morpurgo and Teun Klapwijk. And Teun, thank you for being part of my committee.

In my time at MED, I have enjoyed the company of many group members. Benoit, you were the MED NEMS pioneer. Even though we were from different parts of the country (hey moppie), we had some good times at MED. The combination of you and Christian in one lab was priceless. Christian, you were a good office mate and if the mess on my desk bothered you, I am sorry. Gijs was the group's first optomechanics guy, stuck in a small locked basement room. Gijs, thank you for giving me the door-code, so we could locate lost equipment. But seriously, you're a good guy and I hope you have found a nice position. And then there is Edgar. My aula-food partner, who stayed in the lab almost as much as I did. I could always count on you for existential dinner conversation or to shout random remarks into offices on the way back to the lab. Although both you and the lab have become more serious, the spirit of mischief can still be felt in MED.

Next comes Jos, our resident calculation-guru. Thank you for teaching me Fortran and for all the interesting conversations, academic and otherwise. I am proud to be the first to cause an overflow on your supercomputer. I learned Matlab from a different source: Menno, thank you for joining the SQUID project. Your work on the simulations and the measurements has greatly increased the quality of the
work. Except for some minor differences in approach (Labview as a programming language, box of trash, etc.) I think we made a great team. But most of all, thank you for your rendition of AC/DC.

We also had a number of good post-docs around. Andreas, thank you for setting up the Frosti and for your funny stories. Bo, thank you for the deep-fried ADWIN and congratulations on the great position. Anne and Diana, thanks for bringing a feminine touch to MED (pink diamonds!). And Warner, thank you for your enthusiasm for physics and 8-bit music. Our discussions on measurements could last for hours, far beyond the point where everyone else walked away and went home. I expect to see you head your own group in the near future. You deserve it.

The second generation of PhD students had it easier in the sense that the measurement setups were mostly built. On the other hand, they had big shoes to fill. To my other room mate, Alexander, I say thank you for showing me some Russian rock and for enjoying the White Stripes with me. We had good fun during the APS March meeting, visiting music clubs and avoiding the freaks (and, ehm, attending talks). That meeting is also where I experienced Chris in a social setting for the first time. Chris, thank you for trying to arrange cheese and wine. Good luck with the last stretch of your PhD. Last but not least on the molecule side, there was Ferry. You always kept saying you were 'just a chemist, playing a physicist', but in the end you were carrying a large part of the molecule group. I remember our time outside of work fondly (karaoke!). I also enjoyed deflating junior PhD students together, and noticing that we were sounding like Herre! I know you and Patricia will do brilliantly at MIT. See you around.

The second wave of NEMS PhD students consisted of Abdulaziz, Hidde, Harold, Ben. Unfortunately, Abdulaziz went back home before I really got to know him. Hidde, on the other hand, I got to know quite well. Thank you for training my sense of sarcasm and for not saying no when I showed up with yet another question about nonlinearity. And thank you for 22 tracks, which I still listen to regularly. Harold, I will never forget the look on your face when Edgar vented the desiccator while you were operating the dilution fridge. Now that you know what you’re doing, I hope you never again lose your giant smile. And Ben, the new SQUID-guy: Thank you for fabricating the carbon nanotube SQUIDs. That was not a trivial task, and you will use the superconducting nanotubes for many interesting experiments. And who knows, maybe you will point a laser at the nanotubes at some point. Just don’t tell Gary...

The third wave of MED people included a Spanish/Italian invasion and the arrival of two assistant professors, Gary and Sander. I wish Sander and his students Anna and Matt a lot of success and fun with the low-temperature STM. Gary, thanks for all the insightful conversations. Your NEMS group is growing at an impressive rate and I am sure that many good results will follow. To Andres (life qual-
ity!), Enrique, Carlos, Ricardo, Michaele (cooome oon!), Ronald, Mickael, Venkatesh (hang on!) and Sal: Thanks for the good times, both at work and outside. I know you will all do good work for the greater glory of the glorious leader. Thanks also go to my students, Joris and Abah and to the other MED master and bachelor students that I have had research and coffee talks with: Ahmet, Elham, Raj, Max, Andre, Dapeng, Hari, Arjan, Alex and Adje.

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And so, we reach the end of a long story. Let’s see what the next chapter brings.

Samir Etaki
Delft, April 2012