Traffic Flow Modeling: a Genealogy

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80 years ago, Bruce Greenshields presented the first traffic flow model at the Annual Meeting of the Highway Research Board. Since then, many models and simulation tools have been developed. We show a model tree with four families of traffic flow models, all descending from Greenshields' model. The tree shows the historical development of traffic flow modeling and the relations between models. Based on the tree we discuss the main trends and future developments in traffic flow modeling and simulation.

INTRODUCTION

Traffic flow models have been applied for almost a century to describe, simulate and predict traffic. The first model showed a relation between the distance between vehicles and their speed [1]. Later, dynamics was included in the models and models were applied for predictions [2, 3]. Nowadays, traffic flow simulation tools are used for long term planning as well as for short term predictions based on actual traffic data. In the future, the models and simulation tools may be developed further to (better) support, for example, adaptive cruise control, dynamic traffic management and evacuation planning.

In this contribution we give an overview of past developments in traffic flow modeling and simulation in the form of a model tree showing the genealogy of traffic flow models, see

FIGURE 1. It shows how four families of traffic flow models have developed from one common ancestor: the fundamental diagram by Greenshields [1]. Each of the families, namely the fundamental diagram, microscopic models, mesoscopic models and macroscopic models, will be discussed in separate sections below. Finally, using the model tree, we identify the main trends and give an outlook for future developments.

FUNDAMENTAL DIAGRAM

The fundamental diagram, as it was originally introduced at the 13^{th} Annual Meeting of the Highway Research Board in 1934, relates the distance between two vehicles (spacing) to their speed [1], see FIGURE 2. However, the author, Bruce Greenshields, became famous for the fundamental he introduced one year later at the 14^{th} Annual Meeting [4]. This fundamental diagram relates the number of vehicles on one unit length of road (density) to their speed, see FIGURE 3.

Shape of the fundamental diagram

Since its first introduction the shape of the fundamental diagram has been much debated. TABLE 1 shows some of the proposed shapes [4-7]. It also shows an alternative representation of the fundamental diagram, relating the density to the flow: the number of vehicles per time unit. Del Castillo [8] recently introduced a set of requirements for the fundamental diagram. Of the fundamental diagrams in TABLE 1, the ones by Greenshields, Smulders and Daganzo, satisfy the criteria. However, it is argued that they do not represent scatter in observed density-flow (or density-speed) plots well enough.

Scatter in the fundamental diagram

Scatter in observed density-flow plots (e.g. FIGURE 4) is partly introduced by the measurement method and the aggregation of data. The remaining scatter is explained and modeled in different ways. In 1961 Edie [9] proposed a fundamental diagram with a capacity drop. The capacity drop models that the outflow out of a congested area is lower than the flow just before breakdown. Using the graphs in FIGURE 5, Edie showed that a fundamental diagram with capacity drop better represents scattered data. A few years later, in 1965, Newell [10] introduced the concept of hysteresis: in congestion, when accelerating the density-speed relation is different from the relation when decelerating. Almost a decade later, Treiterer and Myers [11] showed that hysteresis could explain much of the observed scatter, see FIGURE 6. In 1997, Kerner and Rehborn [12] take a different approach by proposing an other non-unique relation between density and flow. They argue that in congestion, traffic may be in any state in the gray area in FIGURE 7. Finally, in 2003, Chanut and Buisson [13] propose a 3-dimensional fundamental diagram. In this fundamental diagram the density of cars is taken into account separately from the density of trucks. Therefore, with the same total number of vehicles, a larger share of trucks, leads to lower speeds, see FIGURE 8.

MICROSCOPIC TRAFFIC FLOW MODELS

The three other families in the model tree include dynamics. They describe how traffic states evolve over time. The microscopic model family is the eldest of those families. Microscopic models describe and trace the behavior of individual vehicles and have evolved into car-following models, including three branches, and one separate branch including cellular automata models.

Safe-distance models

The earliest car-following model was a safe-distance model and was introduced by Pipes in 1953 [2]. In his model, vehicles adjust their speed according to a safe distance to their leader, illustrated in FIGURE 9. Safe-distance car-following models were refined by Gipps in 1981 by introducing two regimes [14]. In one regime the speed is limited by the vehicle or the (legal) speed limit and in the other regime the speed is reduced because the drivers keeps a safe distance to the leading vehicle.

A revival of safe distance models took place in the last decade, starting by Newell with a simplification of his 1961 car following model [15, 16]. This simplified car following model has been shown to be equivalent to certain models in the cellular automata branch and in the kinematic wave branch [17, 18]. The equivalence is used to develop hybrid models combining properties of microscopic and macroscopic models [19, 20].

Stimulus-response models

The second branch of car-following models consists of stimulus-response models. The model tree shows a rapid development of these models in around 1960 [21-24]. The authors propose that acceleration of drivers can be modeled as a reaction to three stimuli:

1. Own current speed
2. Distance to leader
3. Relative speed with respect to leader

A lot of effort has been put into calibrating and validating stimulus-response models. However, in 1999 Brackstone and McDonald [25] concluded that the models were used less frequently because of contradictory findings on parameter values. Nevertheless, new stimulus-response models have been developed since, including the optimal velocity model [26] and the intelligent driver model [27]. Again, it is argued that it is often difficult, if at all possible, to find good parameter values [28]. The authors (Wilson and Ward) argue that researchers should focus on a small subset of stimulus-response models with good qualitative properties. Wilson also proposes a framework to assess the models with respect to qualitative properties [29].

Action point models

Action point models form the third, and last, branch of car-following models. They were first introduced by Wiedemann in 1974 [30]. For these models, it is assumed that drivers only react if the change is large enough for them to be perceived. In contrast to other car-following models, this implies that driving behavior is only influenced by other vehicles if headways are small and if changes in relative velocity or headways are large enough to be perceived.

Cellular-automata models

Cellular-automata models are usually categorized in the microscopic model family, as a branch separate from the car-following models. In cellular-automata models, the movement of individual vehicles is described and traced, just like in other microscopic models. In contrast to car-following models, space and sometimes time is discretized as well. The first model in this branch stems from 1986 [31] but the model introduced in 1992 by Nagel and Schreckenberg [32] is regarded as the prototype cellular-automata model. The road is discretized into cells and each time step each vehicle is advanced zero, one or even more cells, according to a certain algorithm. Some of the most popular cellular automata models are compared in [33].

MESOSCOPIC MODELS

Mesoscopic models fill the gap between microscopic models that model and trace the behavior of individual vehicles and macroscopic models that describe traffic as a continuum flow. Mesoscopic models describe vehicle movements in aggregate terms such as probability distributions. However, behavioral rules are defined for individual vehicles. The family of mesoscopic models includes headway distribution models [34, 35] and cluster models [36]. However, the oldest and most extended and popular branch within this family consists of gas-kinetic traffic flow models.

Gas-kinetic traffic flow models were first introduced in the early 1960's [37, 38]. It describes traffic flow in a way similar to how gas is modeled in gas kinetic models. The movement of vehicles (or molecules in a gas) are not modeled individually. Instead, distributions of density and speeds are used to calculate, and lead to expected densities and speeds. A first revival of the branch took place in the mid and late 1970's with an improved model [39] and a continuum gas-kinetic model [40]. A second revival of gas-kinetic traffic flow models took place from the mid 1990's. The older models were extended and generalized [41, 42] and more continuum models were derived [43-46].

MACROSCOPIC MODELS

The fourth and last families in the model tree consists of macroscopic models. They describe traffic as if it were a continuum flow. Only aggregated variables such as (average) density, (average) flow and (average) speed are considered. The family consists of two major branches: kinematic wave models and higher-order models. In order to include differences between types of vehicles (e.g. passenger cars and trucks), multi-class versions of both types of macroscopic models are developed as well.

Kinematic wave models

The prototype macroscopic model is a kinematic wave model introduced in the mid 1950's by Lighthill [3] and, independently, Richards [47]. This model, also known as the LWR model, has received much attention and critique. The main critique is that vehicles are assumed to attain a new speeds immediately after a change in the density. This implies infinite acceleration or deceleration. The issue has mainly been dealt with in higher order macroscopic models (see next section), but also by relatively recent variants of the LWR model including bounded acceleration [48, 49]. In the original LWR model, the transition from free flow to congestion regime (breakdown) always happens at the same density and without capacity drop. This is considered as a second major drawback. It was addressed by introducing lane changes [50, 51] and by introducing breakdown probabilities [52].

LWR models are often used for simulations studies as they are relatively simple and computations can be done fast. Therefore, space and time are discretized into spatial cells of typically 200 meters long and time steps of 0.5 to several seconds. Densities in each cell are computed using the old densities and the flow into and out of the cell each time step. This approach is used in the cell transmission model introduced by Daganzo in 1994 [7] and the Godunov scheme [53]. More advanced and accurate simulation methods have been introduced in the past few years [18, 54].

Furthermore, since 2001, many multi-class kinematic wave models have been proposed [13, 55-62]. They address the issue of breakdown taking place at various densities place by introducing multiple vehicle classes. This model approach also allows for different speeds and other distinctive features for each class. As discussed in the section on fundamental diagrams, multi-class models can reproduce scattered fundamental diagrams. Multi-class models often include different fundamental diagrams for different classes, see Table 2. Furthermore, the speed does not depend on the total number of vehicles per time unit (density) but most models apply an ’effective density’ to which some classes contribute more than other classes. For example, trucks are supposed to have a higher impact on traffic flow than passenger cars. Therefore, relatively few trucks can create a breakdown, while many passenger cars are needed to do the same. Multi-class kinematic wave models were generalized in the Fastlane model [61, 63], which can also be used to assess multi-class kinematic wave models [63, 64].

Higher-order Models

Higher order models form the other main branch in the family of macroscopic traffic flow models. They were first introduced by Payne in 1971 [65]. Higher order models include an equation to account for the acceleration and deceleration towards the equilibrium speed prescribed by the fundamental relation. This way, they address the issue of infinite acceleration/deceleration in the LWR model. However, also this type of models received much critique. In 1995, Daganzo initiated an ongoing discussion on whether or not higher order models are flawed because they are not anisotropic and on whether traffic flow models ought to be anisotropic [66, 67]. The most important implication of a traffic flow model that is not anisotropic is that, in the model and the simulation, vehicles do not only react on their leader but also on their follower which results in vehicles driving backward in certain situations. Since the start of this discussion, many anisotropic models have been developed [68-70], including a multi-class higher order model [71].

Other recent models in the higher-order branch include the generalized higher-order model by Lebacque et. al. [72] and a hybrid model that couples a higher-order model with a microscopic version of it [73, 74].

DISCUSSION AND OUTLOOK

The model tree is used to identify recent trends and provide outlooks for the future.

Trends

We identify four main trends in the model tree:

1. Certain branches converge to a generalized model. Del Castillo develops a framework that includes most fundamental diagrams [8], many car-following models are generalized in Wilson's model [29], Hoogendoorn and Bovy generalize gas-kinetic models [42], a generalized multi-class kinematic wave model is proposed by Van Wageningen-Kessels et. al. [61, 63] and a generalized higher-order model is proposed by Lebacque et. al. [72].
2. The LWR model is extended and adapted to better reproduce observations. Zhang proposes a model that includes hysteresis [68], Lebacque includes bounded acceleration and deceleration [48] and multi-class models are introduced by Wong and Wong [55] and many other authors thereafter.

3. Multi-class versions of different types of models are introduced. Hoogendoorn introduces a multi-class gas-kinetic model [44], a multi-class higher order model is introduced by Bagnerini and Rascle [71] and, again, multi-class kinematic wave models are introduced by Wong and Wong [55] and many other authors thereafter.
4. Hybrid models are introduced to combine the advantages of both microscopic and macroscopic models. Bourrel and Lesort [19] and Leclercq [20] apply the LWR model for hybridization, a higher-order model is combined with a car-following model by Moutari and Rascle [74].

Outlook

From the model tree, we see that the cellular automata and the mesoscopic model families do not receive much research attention recently. Cellular automata models are used in simulations, but less often than microscopic and macroscopic models. Mesoscopic models are often hard to discretize and are therefore seldom applied in simulation tools. Therefore, we expect that future traffic flow modeling and simulation will focus on new and improved car-following and macroscopic models. Especially for the macroscopic models and simulation tools, good fundamental diagrams will be needed as well.

Furthermore, we expect the other trends discussed above to set in. The development of generalized models as described in the first trend, is valuable to assess models and to select qualitatively appropriate models. Future developments include even more generalized models and assessment of existing and new models. This way, it can be prevented that qualitatively inferior models, which inevitably lead to quantitatively poor results, are applied in simulations.

Furthermore, it is prevented that resources are spend in quantitatively calibrating models that will give qualitatively undesirable results.

Microscopic models and simulation tools predict traffic in more detail than macroscopic models. Therefore, they are well suited for adaptive cruise control and similar applications where it is necessary to predict the behavior of individual vehicles. However, in many applications the details are less important and fast computations achieved with macroscopic models are necessary. This includes dynamic traffic management for large areas and evacuation optimization. For these applications more realistic macroscopic models as described in the second and third trend are valuable. Finally, some applications require on the one hand detail and accuracy in a small area and on the other hand fast computations to make predictions over a longer time horizon. These applications benefit from the fourth trend in which hybrid models are developed. Detailed predictions can be made, for example for a small urban area, and the less detailed prediction for the larger surrounding area allow for fast computations.

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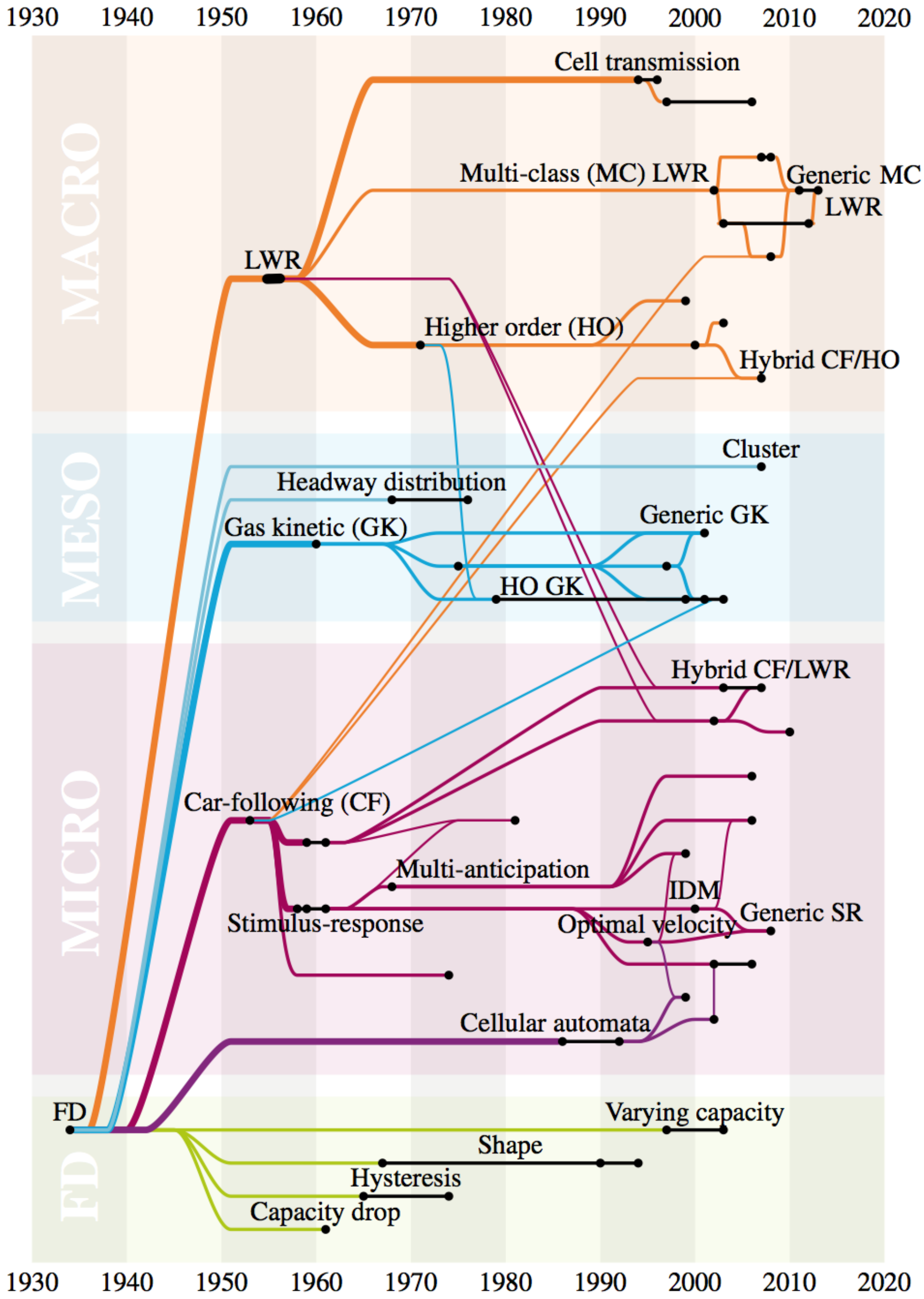


FIGURE 1 Genealogy of traffic flow models. Black dots indicate models, black lines between dots indicate that the same (or a very similar) model has been proposed multiple times, colored lines indicate descent. A full (and much larger) version of the genealogy can be found in [63, 75].

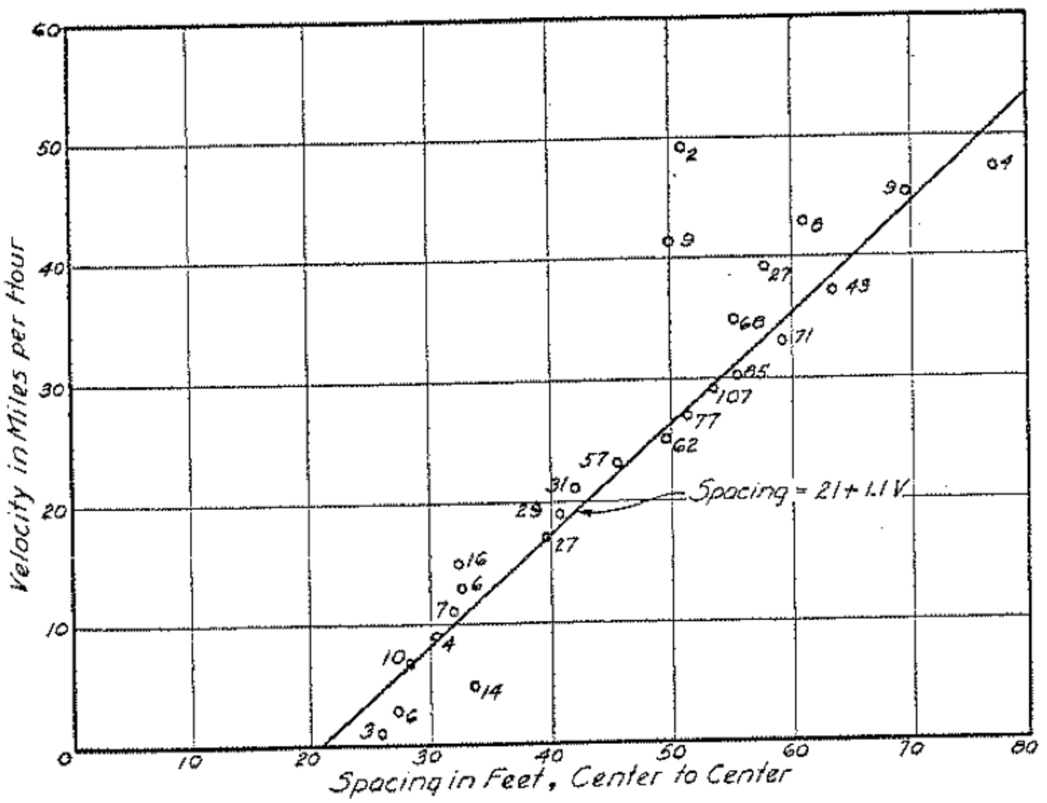


FIGURE 2 Greenshields' original fundamental diagram (1934), showing a linear relation between spacing and speed. Picture from [1].

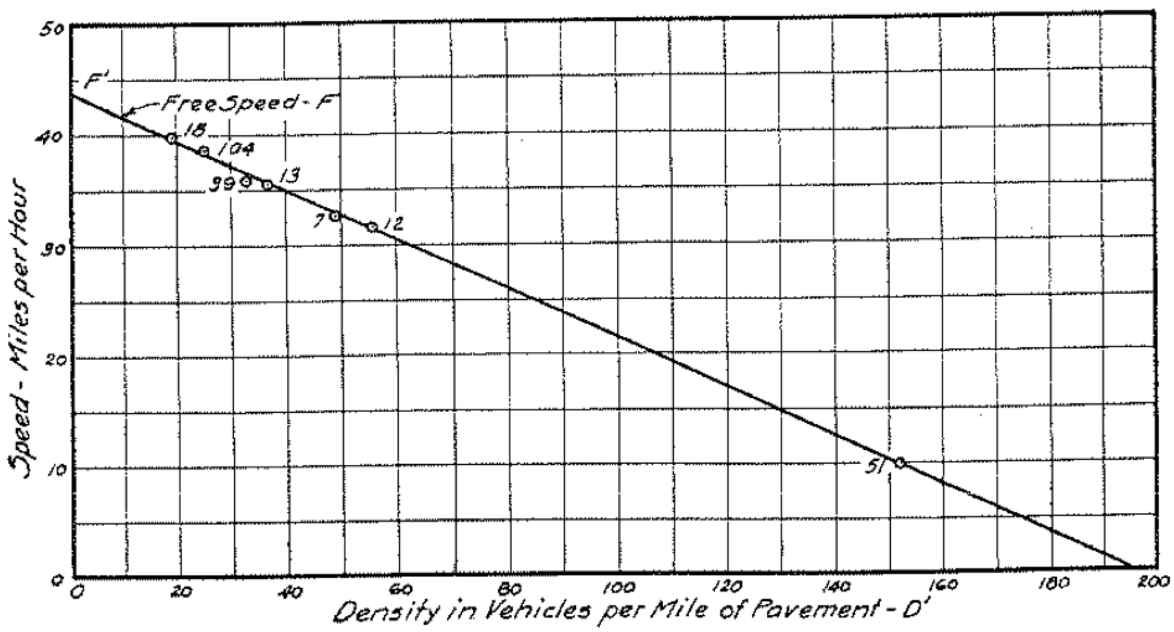
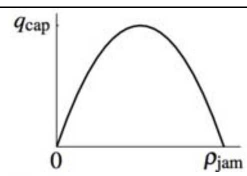
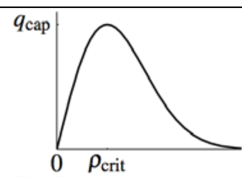
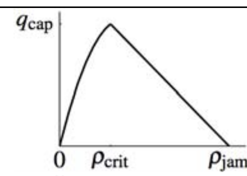
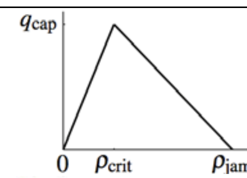
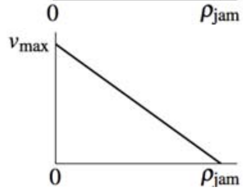
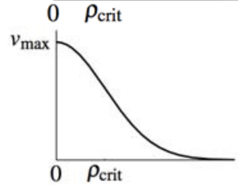
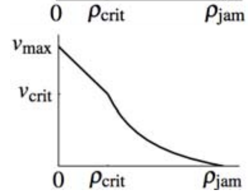
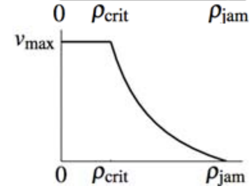


FIGURE 3 Greenshields' fundamental diagram (1935), showing a linear relation between density and speed. Picture from [4].

TABLE 1 Different shapes of fundamental diagrams, in density-flow and in density-speed plane.

Density-flow				
Density-speed				
Shape	Parabolic	Bell	Parabolic-linear	Bi-linear
Author	Greenshields	Drake	Smulders	Daganzo
Year	1934	1967	1990	1994
Reference	[4]	[5]	[6]	[7]

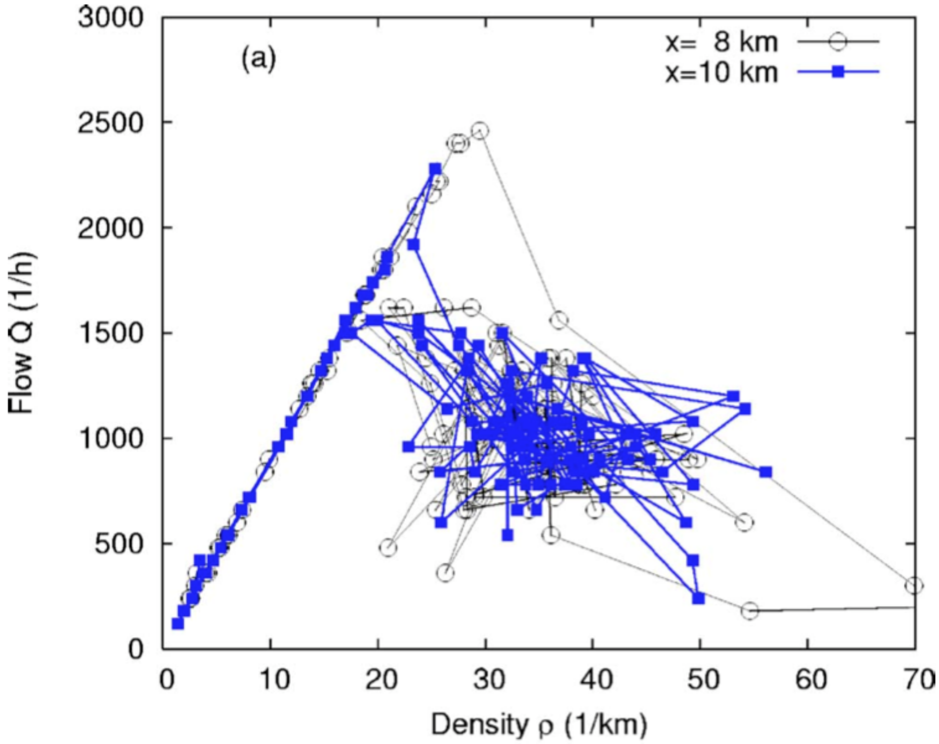


FIGURE 4 Scatter in an observed density-flow plot. Picture adapted from [76].

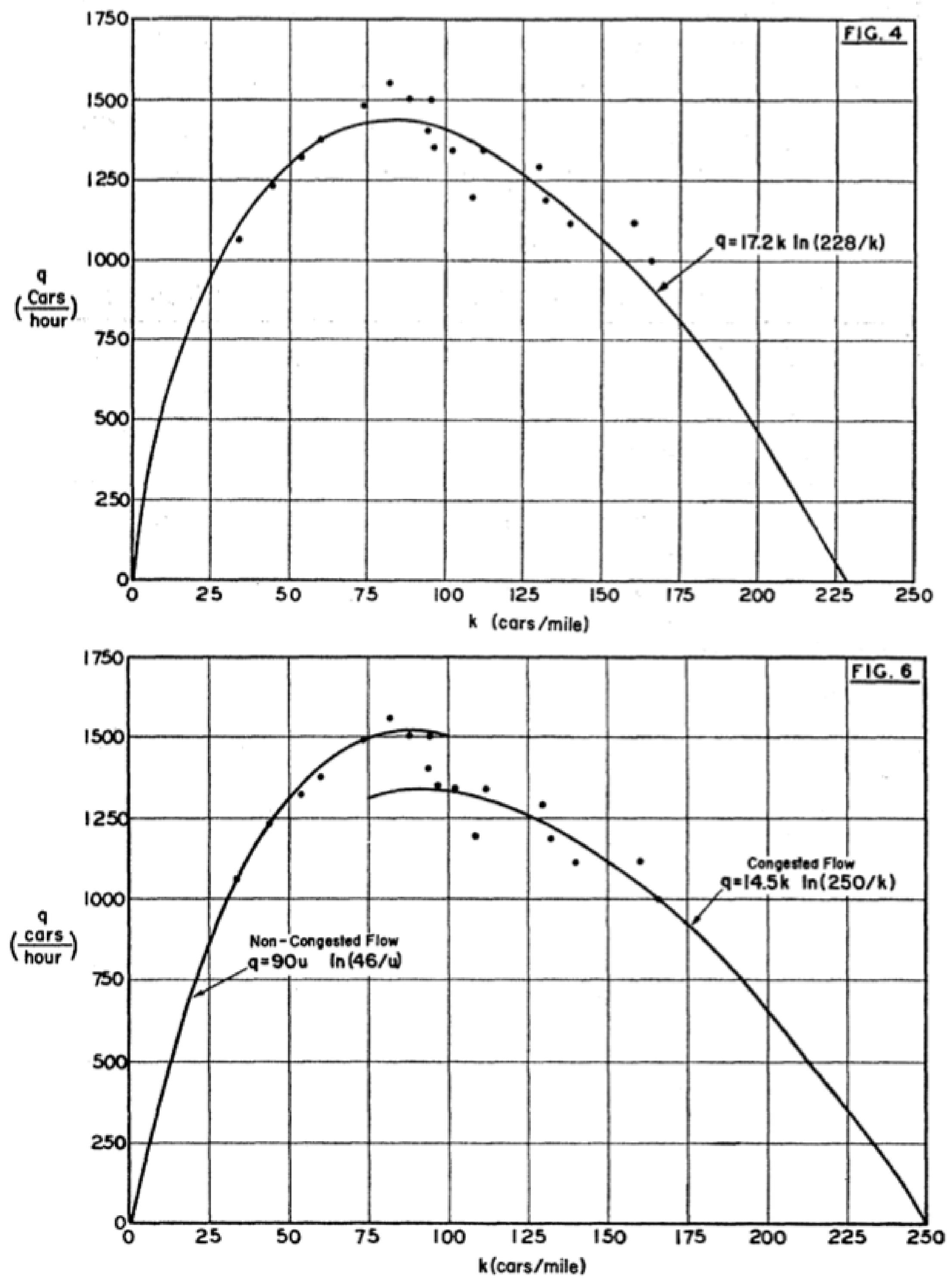


FIGURE 5 Fundamental diagram without (left) and with (right) capacity drop. The graph shows a better fit with the data of the fundamental diagram with capacity drop. Pictures from [9].

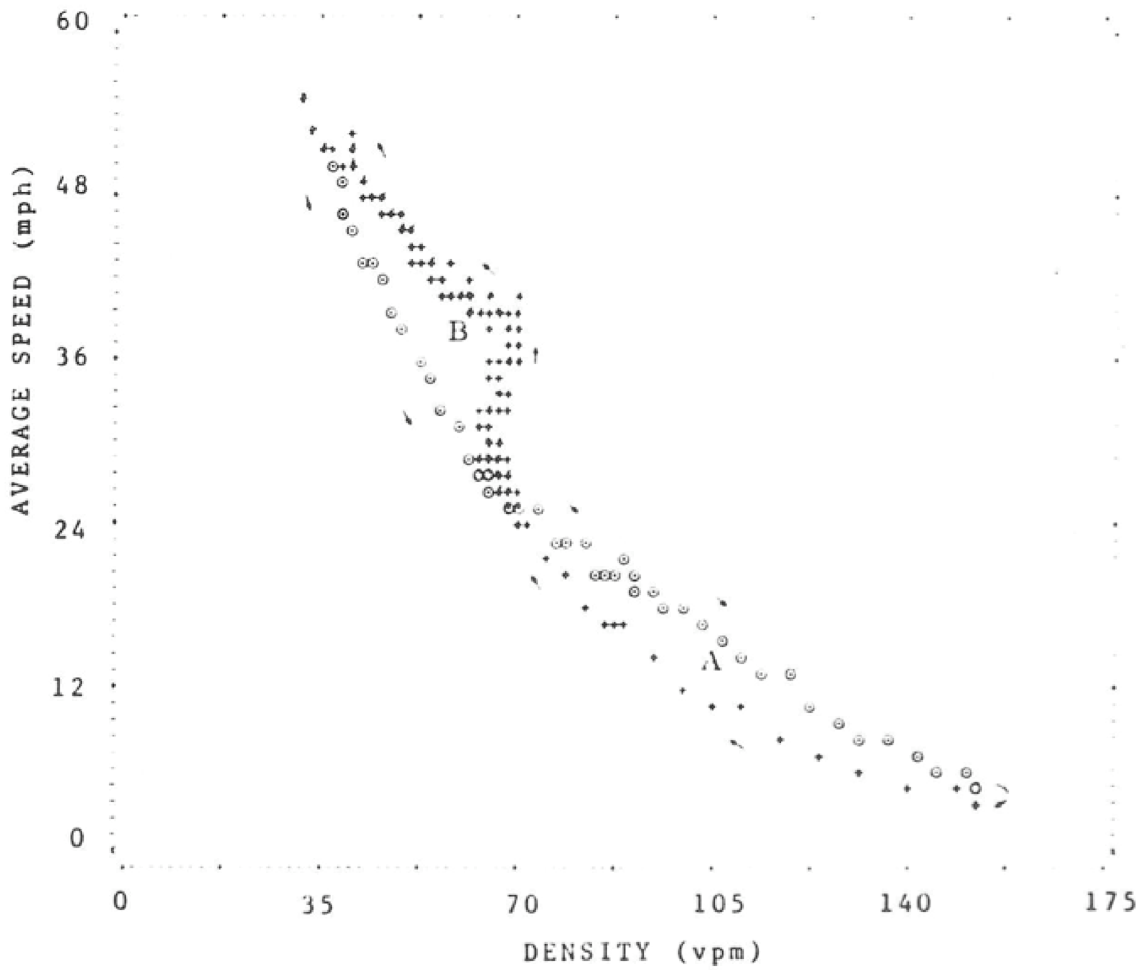


FIGURE 6 Observed densities and speeds showing the hysteresis phenomenon. At relatively low densities, speeds are higher when accelerating (diamonds) than when decelerating (circles), at relatively high densities, it is the other way around. Picture from [11].

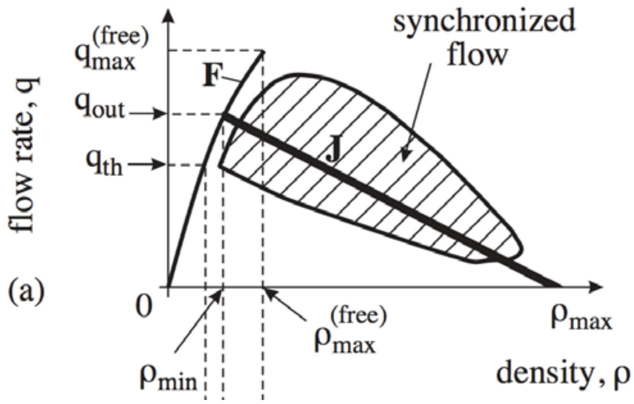


FIGURE 7 Fundamental diagram with infinitely many admissible states in the congestion branch (shaded area). Picture from [77]. For detailed explanation of the labels, we refer to [77].

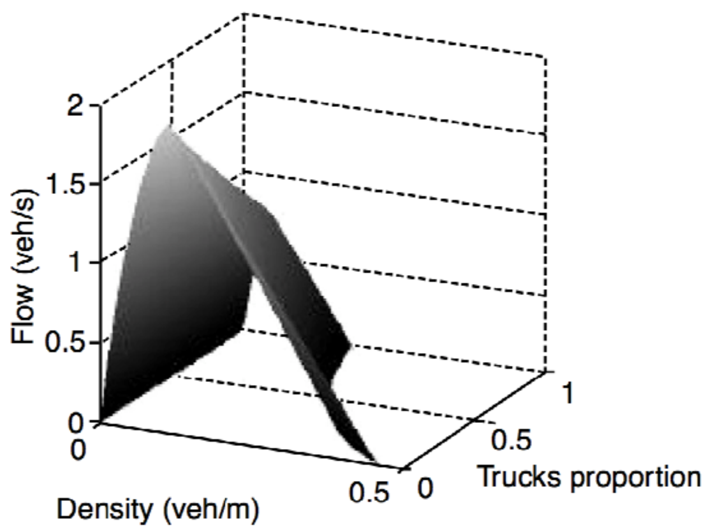


FIGURE 8 3 dimensional fundamental diagram where a high trucks proportion leads to lower speeds and flows. Picture from [13].

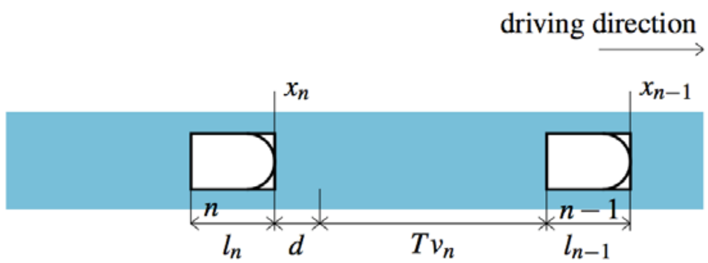
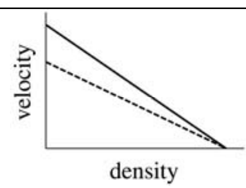
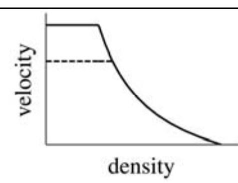
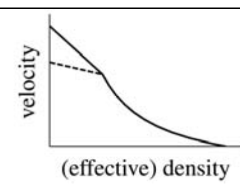


FIGURE 9 Variables and parameters in Pipes' safe distance model. The position x_n of the *n*-th vehicle is determined the position x_{n-1} of it leader and the safe distance between them. The safe distance is a constant distance determined by the distance at standstill *d*, the vehicle length l_{n-1} and a variable safe stopping distance Tv_n with *T* time safe time headway and v_n the speed.

Table 2 Fundamental diagrams of multi-class kinematic wave models.

(Effective) density-speed. Solid line: cars, broken line: trucks.			
Reference multi-class	Benzoni-Gavage and Colombo [56]	Logghe and Immers [59]	Chanut and Buisson [13], Van Wageningen-Kessels et. al. [61]
Author mixed class	Greenshields [4]	Daganzo [7]	Smulders [6]