Measuring the Locations of the Disc Edge Vortices and Flapping Angles of the TU-LR Helicopter Model

Memorandum M-846

B.S.M. Renier

This memorandum has been prepared in the framework of a joint NLR/TUD project

TU Delft
Delft University of Technology

July 1998
Title: Measuring the Locations of the Disc Edge Vortices and Flapping Angles of the TU-LR Helicopter Model

Author(s): B.S.M. Renier

Abstract: An experimental investigation was conducted in the 2.24m diameter Open Tunnel Facility of the Delft University of Technology to locate the disc edge vortices of a rotor at two advance ratios: \( \mu = 0.091 \) (\( \text{CT} = 0.0055 \)) and \( \mu = 0.121 \) (\( \text{CT} = 0.0062 \)). A two-bladed 1.4 m diameter teeter rotor was used for this experiment. The rotor had rectangular untwisted blades with a NACA 0015 airfoil section and was driven by a 840 kW electromotor. Total pressure measurements were conducted in three planes perpendicular to the freestream flow to locate the disc edge vortices at both the retreating and advancing side. Smoke visualisation was used for a qualitative investigation of the rotor wake flow. The results were compared to the analytical theory of Roos. The correlation showed large deviations between theory and the experiment. Besides the wake measurements, the flapping angles of the rotor at advance ratios between 0 and 0.15 were determined optically. The results of this experiment were compared to the theories of Ypma, Chen and Bramwell. The theories using non-uniform inflow gave the best results. Finally, shake tests were performed to determine the ground resonance characteristics of the test set-up. Coleman diagrams proved that ground resonance could happen at rotorspeeds above the nominal rotor RPM.

Keyword(s): Windtunnel experiment, helicopter model, rotor disc edge vortices, flapping angles, ground resonance

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<td>Date</td>
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<td>Approved</td>
<td>Th. van Holten</td>
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Acknowledgements

The work I have done would not have been possible without the co-operation of many people. In random order, I would like to thank:

K.P. Jessurun from the Institute of Wind Energy, for bringing a lot of literature to my attention. His theoretical and experimental expertise was a well of knowledge to me.

The tunnel engineer S. Toet, his great assistance which was indispensable during the wind tunnel tests and troubleshooting. The preliminary design of the test set-up is his idea.

Th. van Holten, my graduation professor, for the contacts with the Institute for Wind Energy and the National Aerospace Laboratory and for having patience with me.

M.D. Pavel, Phd student at our department, for helping me on the literature on the theory of ground resonance and the flapping angles of a rotor.

J.H. Weerheim, test assistant and ‘strain gauge’-man, who configured the strain gauges on the measuring tube. The measures to reduce the vibration levels were mainly his ideas. Much the test instrumentation was provided by him.

F.G.C. Oostrum, for digitising the tapes with the flow visualisation.

J.D. Stegeman J. Takens of the NLR for providing respectively the ground resonance evaluation program plus shake test method and delivering the information on the vortex indicator.

The work shop people of the Institute of Wind Energy and the Department of Aerospace Engineering for manufacturing parts and lending equipment.

And finally, I own a lot to my parents and sister who have supported me in many ways throughout my studies.

Delft, May 1998

Bram S.M. Renier
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Summary

Two experimental investigations were conducted in the 2.24 m diameter Low-speed Open Tunnel Facility of the IVW. The first objective was to record the variation of the flapping angles with forward speed. The second objective to locate the main rotor disc edge vortices of a helicopter in forward flight. Besides the experiments in the windtunnel, the ground resonance behaviour of the structure was analysed.

A Vario radio controlled helicopter model 'Silence' was purchased for the experiment. The model has a two-bladed semi-rigid teetering rotor with untwisted wooden blades of rectangular planform with a NACA 0015 symmetrical airfoil section. The rotor is driven by a 840 kW electromotor.

To record the coning and flapping angles of the blades of the rotor at three azimuth stations: $\psi = 270^\circ$, $\psi = 90^\circ$ and $\psi = 30^\circ$, strobe light and camera are used. This data provides information on the tip path plane at advance ratios of $\mu = 0$ to $\mu = 0.15$ in increments of $\Delta \mu = 0.03$. The experimental results are correlated with Chen [1980], Bramwell [1976] and Ypma [1996].

The information on the attitude of the Tip Path Plane is also needed to correlate the experimental results of the vortex locations with the analytical theory of Roos [1996].

Total pressure measurements in the rotor wake are used to determine axial and lateral co-ordinates of both the retreating and advancing side vortex at advance ratios of $\mu = 0.091$ and $\mu = 0.121$ in three planes behind the rotor: $x/R = 1$, $x/R = 2$ and $x/R = 3$. The test results are correlated with the theory of Roos [1996].

A smoke devise is used to visualise the wake flow and to get a qualitative impression of the location and behaviour of the vortices.

Finally, support shake tests are conducted to investigate the ground resonance of the set-up.

The results of the flapping angles correlate well with all three theories. The method to measure the flapping angles proved to work well.

The results show that it's possible to get a reasonable impression of the location of the disc edge vortices with total pressure measurements but there's a sometimes a large deviation between the theory and the experimental results. The Coleman diagrams showed that ground resonance could take place at higher Rpm's than the operating RPM.
# Symbols

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<th>Description</th>
<th>Unit</th>
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<td>( \alpha )</td>
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</tr>
<tr>
<td>( \alpha )</td>
<td>Nondimensional damping coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( \alpha_d )</td>
<td>Disc angle of attack</td>
<td>[rad] or [degrees]</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Flapping angle</td>
<td>[rad] or [degrees]</td>
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<td>( \beta )</td>
<td>Nondimensional damping coefficient</td>
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<tr>
<td>( \chi )</td>
<td>Wake skew angle</td>
<td>[deg or rad]</td>
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<tr>
<td>( \delta_3 )</td>
<td>Pitch-flap coupling angle</td>
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<td>Non-dimensional flapping hinge spring constant</td>
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<tr>
<td>( \gamma )</td>
<td>Circulation per unit of length / vorticity</td>
<td>[m/s]</td>
</tr>
<tr>
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<td>Lock number ( \rho c_p c R^2 / I )</td>
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<td>Constant in wake circulation distribution</td>
<td>[m^{3/2}/s]</td>
</tr>
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<td>Constant in disc edge vortex circulation distribution</td>
<td>[m^{3/2}/s]</td>
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<td>Lateral non-dimensional inflow velocity</td>
<td>[-]</td>
</tr>
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</tr>
<tr>
<td>( \nu )</td>
<td>Nondimensional frequency</td>
<td>[-]</td>
</tr>
<tr>
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<td>Nondimensional nonrotating eigenfrequency</td>
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<td>( \rho )</td>
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<td>Rotor solidity ( Nc / \pi R )</td>
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<tr>
<td>( \Delta )</td>
<td>Difference</td>
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<td>Blade circulation</td>
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<td>Unit</td>
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</tr>
<tr>
<td>$\Gamma_0$</td>
<td>azimuth independent circulation</td>
<td>m$^2$/s</td>
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<tr>
<td>$\Gamma_1$</td>
<td>azimuth dependent circulation</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>rotor rotational velocity</td>
<td>rad/s</td>
</tr>
<tr>
<td>$Z$</td>
<td>length of vortex sheet that has rolled up into disc edge vortex during unit time</td>
<td>m</td>
</tr>
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<td>$a_0$</td>
<td>coning angle</td>
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<td>$a_1$</td>
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<td>rad</td>
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<td>$A$</td>
<td>area of the rotordisc</td>
<td>m$^2$</td>
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<td>$A_t$</td>
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<td>rad or [degrees]</td>
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<td>$A_f$</td>
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<td>V</td>
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<td>$A_s$</td>
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<td>longitudinal cyclic angle</td>
<td>rad or [degrees]</td>
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<td>$c$</td>
<td>blade chord</td>
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<td>$C_L$</td>
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<td>$C_x$</td>
<td>damping coefficient of the support in x-direction</td>
<td>kg/s</td>
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<tr>
<td>$C_y$</td>
<td>damping coefficient of the support in y-direction</td>
<td>kg/s</td>
</tr>
<tr>
<td>$e$</td>
<td>flapping hinge offset</td>
<td>m</td>
</tr>
<tr>
<td>$e$</td>
<td>longitudinal co-ordinate where vortex is fully formed</td>
<td>-</td>
</tr>
<tr>
<td>$EI$</td>
<td>blade stiffness (for flapping)</td>
<td>Nm$^2$</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>height above the no feather plane</td>
<td>m</td>
</tr>
<tr>
<td>$I_B$</td>
<td>blade moment of inertia around flapping hinge</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>$I_L$</td>
<td>blade moment of inertia around lead-lag hinge</td>
<td>kgm$^2$</td>
</tr>
<tr>
<td>$i_m$</td>
<td>rotor shaft tilt</td>
<td>-</td>
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<tr>
<td>$K_B$</td>
<td>flapping hinge spring constant</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$K_L$</td>
<td>lead-lag hinge spring constant</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>$K_x$</td>
<td>cosine component of first harmonic inflow</td>
<td>-</td>
</tr>
<tr>
<td>$K_y$</td>
<td>sine component of first harmonic inflow</td>
<td>-</td>
</tr>
<tr>
<td>$M_x$</td>
<td>equivalent mass of the support in x-direction</td>
<td>kg</td>
</tr>
<tr>
<td>$M_y$</td>
<td>equivalent mass of the support in y-direction</td>
<td>kg</td>
</tr>
<tr>
<td>$P$</td>
<td>period</td>
<td>s</td>
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<tr>
<td>$q_0$</td>
<td>free-stream dynamic pressure</td>
<td>N/m$^2$</td>
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Symbols

r radial location [m]
R rotor radius [m]
$S_\zeta$ first moment of blade lag inertia [kgm]
T rotor thrust [N]
t time [s]
V velocity [m/s]
$V_\infty$ free-stream velocity [m/s]
v$_m$ mean inflow velocity [m/s]
v$_i$ inflow velocity [m/s]
x downstream co-ordinate [m]
y lateral co-ordinate [m]
z axial co-ordinate [m]

subscripts

$\beta$ concerning flapping motion
$\zeta$ concerning lead-lag motion
adv advancing side
c center of vortex
cg center of gravity
d disc
r in the rotating system
retr retreating side of the rotor

abbreviations

BVI Blade vortex interaction
D.L. disc loading
DPI Digital Pressure Indicator
DUT Delft University of Technology
FFT Discrete Fourier Transform
HP Hub Plane
I.V.W. Instituut voor Windenergie (Institute for Wind Energy)
L.E. leading edge
LDV Laser Doppler Velocimetry
LR Luchtvaart- en Ruimtevaarttechniek (Aerospace Engineering)
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<td>N.L.R.</td>
<td>Nationaal Luchtvaart- en Ruimtevaartlaboratorium (National Aerospace Laboratory)</td>
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<tr>
<td>NFP</td>
<td>No Feather Plane</td>
</tr>
<tr>
<td>O.J.F.</td>
<td>Open Jet Facility</td>
</tr>
<tr>
<td>RPM</td>
<td>Rotations per Minute</td>
</tr>
<tr>
<td>SP</td>
<td>Shaft Plane</td>
</tr>
<tr>
<td>T.E.</td>
<td>trailing edge</td>
</tr>
<tr>
<td>TPP</td>
<td>Tip Path Plane</td>
</tr>
<tr>
<td>TU</td>
<td>Technische Universiteit</td>
</tr>
<tr>
<td>VTR</td>
<td>Video Tape Recorder</td>
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</table>
1 Introduction

In forward flight, the individual rotor blade vortices roll up in two discrete rotor disc edge vortices similar to tip vortices of a fixed wing plane. These vortex structures might cause directional stability problems when they interfere with the tail rotor and fin.

Roos [1996] has derived an analytical model to determine the location and the strength of the main rotor disc edge vortices. However, few experimental data on this matter is available. One of the few experiments is conducted at the NASA Langley Research Center [Ghee, 1996]. The analysis of Roos matched the results of this experiment reasonably well. Still the Faculty of Aerospace Engineering of the Delft University of Technology and the National Aerospace Laboratory NLR agreed to conduct their own experimental investigation to check the validity of the theory [Roos & Melkert, 1996].

A ready-to-build model was bought to reduce development costs. A few changes are made to make it suitable for wind tunnel testing. The measuring equipment is developed at the Departement of Aerospace engineering itself. Much time is spent to reduce vibration levels probably caused by ground resonance. In August 1997, the first smoke tests are conducted in the Open Jet Facility of the Institute for Wind Energy. These proved that the test set-up could be used for further testing. However the rotor RPM in this stage was still limited to 550 RPM. The set-up was eventually modified till the rotor was able to increase its normal operating 1000 RPM. For the tests a RPM of 900 was maintained because the rotor runs smoother at this speed.

For the correlation of the vortex theory, the attitude of the tip path plane must be known. This is a good opportunity to check the work of Ypma [1996] on the flapping angles of a rotor.
2   The experiment set-up

This chapter describes briefly the lay-out of the test. The major characteristics of the tunnel facility, the helicopter model and the measuring device are treated below.

2.1   The tunnel facility

The experiments are conducted in the open jet facility of the Institute for Wind Energy (IVW) at Delft University of Technology. This 2.24 m diameter open low-speed facility is able to reach tunnel speeds up to 14.5 m/s. This tunnel meets the need for the experiment. Since the region of interest is the low speed area, no large tunnel speeds are required: an advance ratio of $\mu = 0.15$ requires a tunnel speed of about 10 m/s with a rotor tipspeed of 66 m/s. Figure 1 shows a drawing of the tunnel. The major tunnel parameters can be found in figure 2.

The traverse unit consists of a T-shape construction (figure 3). A horizontal and a vertical spindle allow movement in vertical (z-axis) and lateral (y-axis) direction correct to a millimetre. This movement is computer controlled, figure 4 shows the control screen. The longitudinal movement (x co-ordinate) of the T-shape is possible by moving the structure on a rail.

The data acquisition consists of high-speed voltmeter with a sample frequency of 100 kHz, a 24 channel multiplexer and a counter. This system can be operated in 2 ways: to measure and save the voltage signal (-10 < V < +10) or measure the pulse signal of an instrument.

2.2   The helicopter model

The helicopter model is a modified radio controlled Vario Silence helicopter. The model has a two bladed teetering rotor. The untwisted wooden blades have the symmetrical NACA 0015 profile. The blades are clipped to bring the diameter of the rotor to 1.4 meter (instead of 1.5 m) to prevent interaction of the disc edge vortices with the tunnel flow boundary layer. The blade roots are bolted in a hub fork. A single bolt serves as lagging hinge. Lag damping is provided by friction between the fork and the blade root. This friction can be adjusted by tightening or loosening of the bolt. The teeter hinge has a spring. The rotor has no pitch flap coupling ($\delta_p = 0$). The moments of inertia of the rotor and the hinge spring constant are determined experimentally. The rotor parameters can be found in figure 6.

The rotor is driven by a Graupner Ultra 2000-7 electromotor. An electromotor was preferred for two reasons: 1) there are no exhaust gasses and 2) an electromotor provides a much more stationary RPM. The motor is able to deliver 840W at a direct current of 24V. A Delta feed supplies the power for the motor instead of the battery that normally feeds a free-flying helicopter because the latter has an endurance of only 7.5 minutes before it has to be recharged. The motor is air cooled by a fan. The transmitter and receiver are powered by their own batteries. The transmitter can not be powered by an external feed since the recharge port is cut off when the transmitter is switched on. The receiver and servo’s feed has been converted for external feed but this caused malfunctioning of the servo’s. Therefore receiver and servo’s were reconverted for battery feed. This leads to the necessity of recharging receiver and
transmitter in-between measurements. Since this equipment doesn’t require much power, measuring sessions never had to be stopped because of low battery voltage. The helicopter model can be seen in figure 5.

The tail section and skids of the helicopter were removed for this experiment. Two aluminium plates augment the stiffness of the fibreglass framework: one is mounted vertical and one horizontal. The model is bolted to the measuring tube via the aluminium ground plate.

2.3 The force-measuring tube

The primary input parameters of the analytical model of Roos are the advance ratio $\mu$, the rotor thrust coefficient $C_T$, and the position of the tip path plane. If these parameters are known together with rotor parameters, the vortex position and strength can be calculated. To measure the lift of the rotor, a force measuring tube is manufactured.

The model is fixed to a friction line hinge. This hinge enables us to tilt the model nose-up or nose-down in the direction of the free-flow. A rod runs down from this hinge through two bronze membranes that seal a hollow steel tube. Strain gauges are fitted to the lower membrane. The outer tube has flanges to which the struts of the tripod are bolted. Rotorlift will deform both membranes. The membranes are arranged in such way that only forces vertical to the membrane are registered. The force-strain relationship is linear throughout the measuring region, although the force measuring tube is sensitive to heat changes. If the outer tube is heated, it will expand and cause the membranes to bend inwards. The strain-gauges on the lower membrane register this as a pulling force. Therefore, the tube is calibrated with a zero-measure at the beginning of each run.
3 Measuring of the flapping angles

The importance of measuring the flapping angles is twofold: 1) to check the graduation work of F. Ypma [1996] and 2) the rotor attitude must be known to correct the theoretical vortex positions. Roos [1996] described the vortex positions relative to a flat rotor plane. The centre of the hub is taken as origin for measuring the vortex locations. Since the disc edge vortices are shed at the rotor rim, the location of the rim relative to the hub must be known to correct the vortex location.

3.1 Flapping angles test procedure

For measurement of the flapping angles, the method of Toet and Jessurun (1996) to determine the life twist of a blade, is used. Basically, it’s the same method as often used for blade tracking. A retro-reflective tape is fixed at the tip. The tape is illuminated by strobe light when the blade passes the desired azimuth position. This strobe light is triggered to the RPM-counter unit. The RPM-counter is a light sensitive (an inductive can be used as well) element which sends a light beam to the retro-reflective tape on the lower side of the blade root and counts the reflected signals. The blade position is recorded with a Sony Hi-8 camera and later projected on a drawing paper where the locations are copied.

First, the blade movement of the retreating blade is registered ($\psi = 270^\circ$). The rotor is spun up to 900 RPM at flat pitch ($\theta_0 = 0$). Since the blades aren’t twisted and the airfoil is symmetric, the rotor produces no lift and the tip path plane is flat (no coning). This is set as the reference plane from which every vertical displacement is measured. The tunnel speeds are increased in increments of 2 m/s ($\Delta \mu = 0.03$) from 0 m/s to 10 m/s ($\mu = 0.152$). Tip displacement at azimuth positions $\Psi = 90^\circ$ and $\Psi = 30^\circ$ are determined via a mirror since the camera and strobe light were fixed at the azimuth position $\Psi = 270^\circ$. Figure 7 shows the test set-up for recording the vertical displacements at $\Psi = 90^\circ$ via the mirror. This method is almost the same as $\psi = 270^\circ$, except for the VTR looking past the rotor plane in the mirror. The VTR is mounted on a tripod that rests on a table to get the lens as high as the flat pitch plane (no feather plane). The positions of the camera, strobe light and mirror can be seen in figure 8.

Later the video is projected onto a drawing board. Per azimuth angle, at every different tunnel speed, the video is paused and a line is drawn along the projected cord while the leading and trailing edge are marked. After an azimuth session is finished, a ruler is filmed at the cord position in order to be able to determine the magnifying factor $K$. This must be done since the distance between the camera and blade tip at different azimuth positions isn’t equal. Another way to determine is the magnifying factor is to simply measure the projected cord of a stationary blade and divide this by the real length of the cord. The stationary blade must be positioned in the line of sight of the camera to minimise deviation.
From the perpendicular distance of both the leading edge (L.E.) and trailing edge (T.E.) of the profile to the flat-pitch cord line (the reference line), the real vertical displacement of the quarter cord point can be determined via figure 9:

\[ \Delta h = \frac{(TE + 0.75(LE - TE))}{K} \]

This method can also be used to determine the live twist of the blades. The difference in vertical displacement at every azimuth projection must then be corrected for cyclic input and shaft angle.

The major disadvantage of this method is that it does not provide real time information on the tip path plane attitude. This wasn’t however an issue in this project, the information is needed for data processing later on.

**Accuracy**

The rotorspeed deviates less than 0.5% of the nominal tunnel speed of 900 RPM. The thrust T [N] varies with an average of 2.3% with a peak deviation of 4.6%. The thrust coefficient \( C_T \) has a mean deviation of 2% with a peak of 4%. The average advance ratio deviation is 2.1% with one peak deviation of 11.7% (!). The power has a mean deviation of 0.8% with a peak of 2.5%. The power is the registered electrical power corrected for gearbox losses.

Reflection angles via the mirrors are kept small to minimise deviations. It’s hard to estimate the deviations of this optical method. The vertical displacement of the tip is often very small, magnifying the tip makes it easier to measure the leading and trailing edge movement accurately. The tip is always illuminated at the desired azimuth position but the camera does not always take a shot of it at exact the right time because the camera records at a frequency of 24 Hz while the rotor blades move around at a frequency of 15 Hz. So horizontal displacement of the tip is caused by two effects: camerashot leads or lags and lead-lag movement of the blade around the lead-lag hinge. Only the reflections that are almost on a vertical line are used to determine the vertical displacements.

To gain insight in the variation of this method, the line through the projected cord is drawn twice in three cases (same azimuth angle, different advance ratio). The difference is a measure for the variation of the test data. The highest variation between two measurements at one advance ratio and azimuth position is 8.5 mm. This is the equivalent of a \( \Delta \beta = 0.7^\circ \).

### 3.2 Data processing

To determine the flapping angles out of the vertical displacements a Matlab program is written.

The tip path plane motion of a rotor blade as a function of the azimuth angle is generally described by the equation:

\[ \beta = a_0 - a_1 \cos \psi - b_1 \sin \psi \]
Measuring the flapping angles

for the height $h$ of the tip to a reference plane ($\beta = 0^\circ$):

$$h = R \sin \beta = \beta = \arcsin(h/R)$$

This is equal to:

$$\arcsin(h / R) = a_0 - a_1 \cos \psi - b_1 \sin \psi$$

For the three azimuth positions (see figure 10):

$$\arcsin(h_1 / R) = a_0 - a_1 \cos \psi_1 - b_1 \sin \psi_1$$
$$\arcsin(h_2 / R) = a_0 - a_1 \cos \psi_2 - b_1 \sin \psi_2$$
$$\arcsin(h_3 / R) = a_0 - a_1 \cos \psi_3 - b_1 \sin \psi_3$$

In matrix notation:

$$\begin{pmatrix}
\arcsin(h_1 / R) \\
\arcsin(h_2 / R) \\
\arcsin(h_3 / R)
\end{pmatrix} =
\begin{pmatrix}
1 & -\cos \psi_1 & -\sin \psi_1 \\
1 & -\cos \psi_2 & -\sin \psi_2 \\
1 & -\cos \psi_3 & -\sin \psi_3
\end{pmatrix}
\begin{pmatrix}
a_0 \\
a_1 \\
b_1
\end{pmatrix}$$

$$\begin{pmatrix}
a_0 \\
a_1 \\
b_1
\end{pmatrix} =
\begin{pmatrix}
1 & -\cos \psi_1 & -\sin \psi_1 \\
1 & -\cos \psi_2 & -\sin \psi_2 \\
1 & -\cos \psi_3 & -\sin \psi_3
\end{pmatrix}^{-1}
\begin{pmatrix}
\arcsin(h_1 / R) \\
\arcsin(h_2 / R) \\
\arcsin(h_3 / R)
\end{pmatrix}$$

The input is a matrix with the advance ratio and the respective displacements at the three azimuth angles. The output are plots of $a_\alpha$, $a_\mu$, and $b_\mu$ as a function of the advance ratio $\mu$. Figure 12 shows the result for the data acquired with the TU-LR helicopter model. Notice that the Tip Path Plane (TPP) makes an angle with the shaft plane: $a_\alpha \neq 0$ and $b_\mu \neq 0$ at $\mu = 0$. This means that the control plane isn’t perfectly perpendicular to the rotor shaft (or parallel to the hub plane). This is probably caused during the blade pitch setting: the rotor track is balanced with a blade pitch indicator. This is in fact a combination of a protractor and level that can be pinched on the blade tip. So every blade can be given a pitch angle relative to the direction of the earth’s gravity. The fact that the scale of the protractor is divided in whole degrees and the fact that it is hard to position shaft perfectly aligned with the earth’s gravity may have caused the difference between the No Feather Plane (NFP) and the Shaft Plane (SP). In a previous paragraph, it is shown that the deviation of the method used can be as high as $0.7^\circ$. The deviations shown here are: $0.56^\circ$ for $a_\alpha$ and $0.32^\circ$ for $b_\mu$. 
According to the sign convention in figures 11b and 11c, the relations between the NFP feather plane and the SP are:

\[ a_i = B_i + a_n \]
\[ b_i = -(A_i - b_n) \]

\[ B_i = \text{longitudinal cyclic + fud} \]
\[ A_i = \text{lateral cyclic + right} \]

For correlation with the theories the assumption \( a_i \) and \( b_i = 0 \) when \( \mu = 0 \) is made. This leads to:

\[ B_i = -a_n \]
\[ A_i = b_n \]

and following apparent cyclic control angles:

\[ B_i = -0.56^\circ \]
\[ A_i = 0.32^\circ \]

### 3.3 The flapping angles theories

In the following chapter the test results will be correlated with three theories: 1) the formulas of Bramwell [1976], 2) those of Chen [1980] and 3) the prediction of Ypma [1996]. First, a brief summary of each of the methods is given. Except for Chen, the other theories needed some correction because of the teetering rotor system used in the experiment.

To correlate the test data, rotor parameters as the hinge spring constant, the blade stiffness and the rotor flapping moment of inertia are determined experimentally.

The only parameters monitored during the test besides the tip path plane are the tunnel speed and the rotor thrust. Hence, other parameters needed for the correlation, the (non-dimensional) induced velocities \( \lambda \) and wake skew angles \( \chi \) are deduced from these two. In fact, the experiments are not compared with pure theory since some experimental data is used in the formulas. To check the validity of the theory, one should calculate the thrust coefficient in stead of using the experimental result.
3.3.1 The flapping angles via Bramwell (1976)

Bramwell [1976] derived following closed form formulas for the flapping angles with respect to plane perpendicular to the shaft (using Glauert’s inflow model):

\[
a_0 = \frac{\gamma}{8(1+\varepsilon)} \left[ \theta_0 (1+\mu^2) + \frac{4}{3} \lambda \right]
\]

\[
a_1 = \frac{2\mu (4\theta_0 / 3 + \lambda)}{1 - \mu^2 / 2} + \frac{8}{\gamma} \frac{\varepsilon}{1 - \mu^2 / 2} b_1
\]

\[
b_1 = \frac{4\mu a_0 / 3 + K\lambda}{1 + \mu^2 / 2} - \frac{8}{\gamma} \frac{\varepsilon}{1 - \mu^2 / 2} a_1
\]

with the trust coefficient written as:

\[
t_\varepsilon = \frac{a}{4} \left[ \theta_0 (1+3\mu^2 / 2) + \lambda \right]
\]

where:

\[
\varepsilon = \bar{K} \beta,
\]

\[
\mu = \mu_s,
\]

\[
\lambda = \mu \alpha_s - \lambda_0,
\]

\[
\theta_0 = \theta_{0.75},
\]

\[
K = K_c,
\]

\[
t_\varepsilon = C_T / \sigma
\]

The implicit formulas from above can be written in a matrix:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \varepsilon \\
0 & 1 & 8 / \gamma & 1 - \mu^2 \\
-4/3 \mu & 8 / 1 + \mu^2 & \gamma & 1 - \mu^2 \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
b_1 \\
\end{bmatrix}
= \begin{bmatrix}
\gamma / 8 & 1 + \varepsilon & [\theta_0 (1+\mu^2) + \frac{4}{3} \lambda] \\
2\mu (\theta_0 + \lambda) / (1 - \mu^2 / 2) \\
\frac{K\lambda}{1 + \mu^2 / 2} \\
\end{bmatrix}
\]
According to Bramwell the influence of the lateral flapping $b_i$ on the longitudinal flapping $a_i$ (third element in the second row) is the same as vice versa (second element in the third row):

$$ \frac{8}{\gamma} \frac{\varepsilon}{1 - \mu^2} $$

Replacing ‘$\varepsilon$’ with ‘$v^2 - 1$’:

with $v^2 = v_{a_i}^2 + \alpha$

because a teetering rotor behaves like an articulated rotor with a centrally springed hinge for the longitudinal and lateral flapping, while the coning of the rotor can better be modelled as if the rotor has flexible blades.

<table>
<thead>
<tr>
<th></th>
<th>$v_{a_i}^2$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>central flapping hinge</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>central flapping hinge with $K_\beta/(\Omega^2)$</td>
<td>$1+3\varepsilon/2(1-\varepsilon)$</td>
<td>1</td>
</tr>
<tr>
<td>flapping hinge with offset</td>
<td>0</td>
<td>$1+3\varepsilon/2(1-\varepsilon)$</td>
</tr>
<tr>
<td>flapping hinge with offset and spring</td>
<td>$K_\beta/(\Omega^2)$</td>
<td>1.188</td>
</tr>
<tr>
<td>flexible blade</td>
<td>12.37EI/mR$^2$</td>
<td>1.188</td>
</tr>
</tbody>
</table>

(source: Van Holten, 1994)

To calculate $a_i$ and $b_i$, the second row of the table above must be used, for the calculation of $a_i$ the last row is used. The table is only valid for one blade per hinge. A teetering rotor has two interconnected blades flapping around one hinge with one spring. Hence, for a teeter rotor the non-rotating part must be divided by 2:

$$ v_{a_i}^2 = K_\beta/(2\Omega^2). $$

### 3.3.2 The flapping angles via Chen (1980)

Chen [1980] derived a theory for the longitudinal and lateral flapping angles, respectively $a_i$ and $b_i$ of a teetering rotor with uniform inflow and no tip loss. The coning angle $a_i$ is considered as a constant but isn’t quantified.

He also states that the quality of his approximation of the teetering rotor has not been evaluated. The formulas are simplified to the steady state by setting all velocities and accelerations to zero:

$$ \ddot{a}_i = \ddot{b}_i = \dot{a}_i = \dot{b}_i = p_w = q_w = \dot{p}_w = \dot{q}_w = 0 $$
\[
\begin{align*}
\begin{bmatrix}
P^2 - \frac{1 + \frac{\gamma K_i \mu^2}{16}}{} & \frac{\gamma}{8} \left(1 + \frac{\mu^2}{2}\right) \\
-\frac{\gamma}{8} \left(1 - \frac{\mu^2}{2}\right) & P^2 - 1 + \frac{3\gamma K_i \mu^2}{16}
\end{bmatrix}
\begin{bmatrix}
a_1 \\
b_1
\end{bmatrix}
= \\
\begin{bmatrix}
0 & 0 & \frac{\gamma}{8} \left(1 + \frac{\mu^2}{2}\right) \\
-\frac{\gamma \mu}{3} & -\frac{\gamma \mu}{4} & 0 & \frac{\gamma}{8} \left(1 + \frac{3\mu^2}{2}\right)
\end{bmatrix}
\begin{bmatrix}
\theta_o \\
\theta_i \\
A_{ic} \\
B_{ic}
\end{bmatrix}
+ \\
\begin{bmatrix}
0 \\
-\frac{\gamma \mu}{4}
\end{bmatrix}
\lambda
\end{align*}
\]

Where \( P^2 = 1 + \frac{K_b}{\epsilon_0 \Omega^2} + \frac{\gamma K_i}{8} \)

In order make a comparison, the induced velocity \( v_i \) has first to be derived from the thrust coefficient \( C_t \).

In these formulas

- \( P \) \quad ratio of flapping frequencies
- \( a_i \) \quad longitudinal first harmonic flapping angle
- \( b_i \) \quad lateral first harmonic flapping angle
- \( A_{ic} \) \quad lateral cyclic pitch
- \( B_{ic} \) \quad longitudinal cyclic pitch
- \( K_i \) \quad pitch flap coupling ratio, \( \equiv \tan \delta \)
- \( K_b \) \quad flapping hinge restraint (spring)
- \( \lambda \) \quad inflow ratio, \( \frac{V \sin \alpha - v_i}{\Omega R} \)
- \( \mu \) \quad advance ratio, \( \frac{V \cos \alpha}{\Omega R} \)
- \( \theta_o \) \quad blade root collective pitch (measured from hub plane)
- \( \theta_i \) \quad total twist

The lateral flapping angle has been corrected for non-uniform inflow. A static first harmonic inflow model has been assumed:

\[
v_i = v_0 \left(1 + K_c r_o \cos \psi + K_s r_o \sin \psi \right)
\]
in which \( K_e = \tan(\chi/2) \) and \( K_f = 0 \) (Coleman inflow model). This leads to a correction term for the lateral flapping angle:

\[
\Delta b_l = \frac{\tan\left(\frac{\chi}{2}\right) \lambda_l}{1 + \frac{\mu^2}{2}}
\]

### 3.3.3 The flapping angles via Ypma (1996)

The reduced matrix version (setting \( \mu_y = \bar{\rho} = \bar{q} = \bar{r} = 0 \)) of Ypma [1996]:

\[
\frac{\gamma}{8} \begin{bmatrix}
\frac{8}{\gamma} (1 + \bar{K}_\beta) & 0 & 0 \\
0 & 1 - \frac{1}{2} \mu_x^2 & -\frac{8}{\gamma} \bar{K}_\beta \\
\frac{4}{3} \mu_x & -\frac{8}{\gamma} \bar{K}_\beta & -1 - \frac{1}{2} \mu_x^2
\end{bmatrix}
\begin{bmatrix}
a_0 \\
\frac{a_1}{b_1}
\end{bmatrix} =
\]

\[
\left[ \begin{array}{c}
\frac{\theta_0 (1 + \mu_x^2) + \frac{4}{3} \theta_u \mu_x + \theta_{wu} \left( \frac{4}{5} + \frac{2}{3} \mu_x^2 \right) - \frac{2}{3} \mu_x \lambda_u + \frac{4}{3} \mu_x - \frac{4}{3} \lambda_0}{3} \\
\frac{8}{3} \theta_0 \mu_x + \theta_u \left( 1 + \frac{3}{2} \mu_x^2 \right) + 2 \theta_{wu} \mu_x + 2 \mu_x \mu_z - 2 \lambda_0 - \lambda_u \theta_{wu} \left( 1 + \frac{1}{2} \mu_x^2 \right) - \lambda_{wu}
\end{array} \right]
\]

with \( C_T = \frac{1}{4} c_u \sigma \left[ \frac{\theta_0 (\frac{3}{2} + \mu_x^2) + \theta_u (\frac{3}{2} + \mu_x^2) + \frac{1}{2} \theta_u \mu_x - \frac{1}{2} \lambda_0 + \frac{1}{2} \mu_x \lambda_u \right] \)

Some simplifications and changes are made to make these expressions suitable to correlate them with the experimental data.

Considering the body fixed axis as the shaft axis system:

- \( \mu_y = \mu \)
- \( \mu_z = 0 \)
- \( \theta_u = \theta_w = \theta_{wu} = 0 \) \( \text{no longitudinal or lateral cyclic nor twist} \)
- \( \bar{K}_\beta = K_p/(2I, \Omega^2) \) \( \text{note again the factor 2 in the denominator because there are two interconnected blades which flap around one hinge and spring} \).
The 'Stiffness coefficients' on the first row in the first column of the first matrix, the $a_y$ related term, should be treated as rigid rotor coefficients:

\[ 1 + \frac{\bar{K}_p}{mR^I} = \frac{v^I}{v} = 1.188 + 12.37EI/mR^I \]

\[ \lambda_u = K\lambda_0 \]

\[ \lambda_u = K\lambda_0 \]

Using Drees' [1949] inflow model:

\[ v_i = v_0(1+K_r\cos\psi + K_r\sin\psi) \]

with

\[ K_r = \frac{4}{3}(1-1.8\mu^2)\tan(\chi/2) \]

\[ K_u = -2\mu \]

\[ \tan\chi = \mu/\lambda_0 \]

### 3.4 The results

In the tests the relation between the three constants $a_y$, $a_i$, and $b_i$ of the static first harmonic model of the tip path plane and the advance ratio is recorded visually. In this paragraph, the results are compared to the theory and vice versa. To compare the test results, the experimentally found values for $a_i$ and $b_i$ are shifted to put the values for $\mu = 0$ in the origin. The coning angle $a_y$ has been left unaltered.

The coning angle $a_y$ (figure 13 & 14)

The coning angle of the rotor is directly related to the rotor thrust and the induced velocity. Since the collective pitch is kept constant $\theta_0 = 8.5^\circ$ during the test, the rotor thrust increases when the tunnel speed increases. On the other hand, the induced velocity decreases with increasing forward speed when no fuselage drag has to be overcome. The theory predicts a slight increase in coning angle while the experiment shows a constant. Chen also assumes the coning angle of a teetering rotor to be constant but he doesn't quantify it.

Figure 13 shows that the results for the coning angle of Ypma and Bramwell are almost identical. This also shows in the table with the results (figure 14). The maximum deviation between the experiment and the theoretical results is less than half a degree. Some deviation might be contributed to the fact that the thrust coefficient is the average of the three $C_T$'s measured during the sessions (three azimuth positions).
The longitudinal flapping angle $a$, (figure 15 & 16)

The longitudinal flapping $a$, angle shows an almost linear relationship with the advance ratio $\mu$. All theories seem to slightly underestimate the experimental data. Maximum deviation is again approximately half a degree.

The lateral flapping angle $b$, (figure 17 & 18)

Figure 17 reveals the difference between the three theories and the test data. The solid line represents Chen’s theory with uniform inflow. This method shows the worst correlation with experimental data. This proves that to predict the lateral flapping angle $b$, the inflow should be regarded as non-uniform. The corrected Chen formulas seem to correlate well at low advance ratios but at higher advance ratios the curve keeps on increasing while the test data show a decline after $\mu = 0.12$.

Bramwell’s approach underestimates the test data in this region of advance ratios but intersects the test data in the end. Apparently overestimating the lateral angle $b$, at higher advance ratios.

While Ypma’s theory overestimates the test data, it’s the only theory that seems to follow the regressing trend of the tunnel measurements. The deviation seems to be a constant: approximately $0.4^\circ$ for advance ratios higher than 0.
4 The main rotor disc edge vortices

This chapter starts with a brief review of the analytical model for the location and vortex strength of Roos [1996]. Next the test procedure is explained. Finally, the test results are correlated with the test data.

4.1 The analytical model of Roos

Roos graduated on the subject "Main Rotor Disc Edge Vortices" in March 1996. His aim was to derive an analytical model which describes the strength and location of the rotor disc edge vortices. A brief summary of his theory is inserted in this report because this might explain the differences between the theoretical results and the experimental data.

Roos [1996] first determined the strength of the blade circulation using a constant circulation along the span but varying with azimuth. This is the circulation distribution proposed by Meijer Drees [1949]:

$$\Gamma_b = \Gamma_0 + \Gamma_1 \sin \psi$$

He then used the Kutta-Joukowski law to determine the average lift on a blade during one revolution in terms of $\Gamma_0$ and $\Gamma_1$. Setting this equal to the average lift coefficient of a rotor, he got the first relationship between $\Gamma_0$ and $\Gamma_1$ as a function of thrust, tip speed and advance ratio. The assumption of a zero roll moment is the second condition used to determine the magnitude of $\Gamma_0$ and $\Gamma_1$ (Assuming zero pitch moment results in a matter of course).

Using Ormiston's [1972] flat wake actuator disc theory, the rotor wake is redefined as an equivalent wing wake. Using this theory brings along some controversy: Ormiston himself states that his theory isn't valid for advance ratios lower than 0.15 while Heyson [1956] says predicting induced velocities with a cylindrical wake is only valid for advance ratios from 0.09 to 0.15.

For the circulation distribution along an equivalent wing:

$$\Gamma = \frac{b}{\pi \mu} \Gamma_0 \left( \sqrt{1 - \bar{y}^2} - \frac{1}{2} \mu \bar{y} \ln \left( \frac{1 + \sqrt{1 - \bar{y}^2}}{|\bar{y}|} \right) \right)$$

with

$$\Gamma_0 = \frac{2T}{b \rho R V_T \left( 1 - \frac{1}{2} \mu^2 \right)}$$
The resultant spanwise vorticity distribution is:

\[
\gamma = \frac{\partial \Gamma}{\partial y} = \frac{b}{\pi \mu R} \Gamma_0 \left[ \frac{1 - \frac{1}{3} \mu \bar{y} - \frac{1}{3} \mu \sqrt{1 - \bar{y}^2}}{\sqrt{1 - \bar{y}^2}} \ln \left( \frac{1 + \sqrt{1 - \bar{y}^2}}{\bar{y}} \right) \right]
\]

A typical vorticity distribution over a rotor (figure 19) shows a higher circulation level on the retreating side than on the advancing side. In order to get zero roll moment around the X-axis more circulation is needed on the retreating side because the local velocity of a blade element on the retreating side is lower than on the advancing side. The vertical dashed line is the location of the highest local circulation. According to the theory, all the circulation left of this line contributes to the circulation of the retreating side vortex. The circulation right of the line contributes to the advancing side vortex. The \(y\) co-ordinate of the vortex asymptotes is the ‘centre of gravity’ of each region.

Heyson [1956] agrees that this is true when one looks at each blade separately but found through experiment that the summation of vorticity behind the rotor disc should be equal on both sides.

The rolling up of the equivalent wake is described with the theory of Kaden (see Roos [1996]). The major drawback of the theory of Kaden is that it’s only valid for the initial stages of the rolling up process. Kaden derived a theory to describe the rolling up process of semi-infinite vortex sheets. His formula for the circulation is as follows:

\[
\Gamma = 2\kappa \sqrt{\zeta} = 2\sqrt{R} \sqrt{\zeta}
\]

In which:

- \(\kappa\) measure for the intensity of the vortex sheet, \([m^{3/2}/s]\)
- \(\zeta\) distance from the edge of the semi infinite vortex sheet, \([m]\)

Using \(\Gamma = 2\kappa \sqrt{\zeta} = 2\sqrt{R} \sqrt{\zeta}\) and

\[
\Gamma_0 = \frac{2T}{b \rho R V_r (1 - \frac{1}{3} \mu^2)}
\]

the expression for the \(\kappa\)-factors becomes

for the advancing side:

\[
\kappa_{adv} = \frac{1}{2} \sqrt{2} \frac{b}{\pi \mu \sqrt{R}} \Gamma_0 \left( 1 - \frac{3}{2} \mu \right)
\]
for the retreating side:

$$\kappa_{ret} = \frac{1}{2} \sqrt{2} \frac{b}{\pi \mu \sqrt{R}} \Gamma_0 \left(1 + \frac{3}{2} \mu \right)$$

This shows an asymmetry in the vortex sheets intensity.

Kaden further deduced that the part of the vortex sheet that has rolled up into an individual vortex during time \( t \) equals:

$$Z = \left(\frac{9}{2\pi^2 \kappa t}\right)^{\frac{1}{3}} \quad \text{with} \quad \bar{x} = \frac{Vt}{R} \quad \text{or} \quad t = \frac{R \bar{x}}{V}$$

$$\bar{Z}_{adv} = R^{-\frac{1}{3}} \left(\frac{9}{2\pi^2 \kappa_{adv}} \frac{\bar{x}}{V}\right)^{\frac{1}{3}}$$

$$\bar{Z}_{ret} = R^{-\frac{1}{3}} \left(\frac{9}{2\pi^2 \kappa_{ret}} \frac{\bar{x}}{V}\right)^{\frac{1}{3}}$$

The non-dimensional \( y \) and \( z \) co-ordinate of the vortex core relative to the vortex sheet are according to Kaden:

$$\bar{y} = 0.57 \bar{Z}$$
$$\bar{z} = 0.88 \bar{Z}$$

In the rotor co-ordinate system of Roos this leads to:

$$\bar{y}_{adv} = 1 - 0.57 \bar{Z}_{adv}, \quad \bar{z}_{adv} = 0.88 \bar{Z}_{adv}$$
$$\bar{y}_{ret} = 0.57 \bar{Z}_{adv} - 1, \quad \bar{z}_{ret} = 0.88 \bar{Z}_{ret}$$

The \( x, y \) and \( z \) co-ordinates of the vortex core before they reach their asymptotes are now known.

To determine the part of the vortex sheet which rolls up in the retreating vortex, respectively the advancing vortex, one has to find the lateral co-ordinate of the maximum circulation \( y_{max} \). This co-ordinate divides the circulation in two, each contributing to its own vortex.
The $y$ co-ordinate of the asymptotes to which the vortices converge are the centre of gravity of the two regions:

\[
\bar{y}_{\text{cer}} = \frac{\int_{-1}^{y_{(r=r_{\text{max})}}} \frac{\partial \Gamma}{\partial \bar{y}} \bar{y} d\bar{y}}{\int_{-1}^{y_{(r=r_{\text{max})}}} \frac{\partial \Gamma}{\partial \bar{y}} d\bar{y}}
\]

\[
\bar{y}_{\text{cer}} = \frac{\int_{-1}^{1} \frac{\partial \Gamma}{\partial \bar{y}} \bar{y} d\bar{y}}{\int_{-1}^{1} \frac{\partial \Gamma}{\partial \bar{y}} d\bar{y}}
\]

Now the $x$ co-ordinate where the vortices reach their asymptote can simply be calculated out of the equality of the expression for the $y_d$ and the $y_c$. The $z$ co-ordinate needs a correction for the downward motion of the wake. Roos found for the downward velocities of the vortex cores before the vortex is fully formed:

\[
v_{\text{cer}} = \frac{C_t}{2\mu} \left[ 1 + \frac{4}{3} (1 - 1.8\mu^2) \sqrt{1 - \bar{y}_{\text{cer}}^2} - 2\mu \bar{y}_{\text{cer}} \right] V_i - V\alpha_d
\]

\[
v_{\text{cer}} = \frac{C_t}{2\mu} \left[ 1 + \frac{4}{3} (1 - 1.8\mu^2) \sqrt{1 - \bar{y}_{\text{cer}}^2} - 2\mu \bar{y}_{\text{cer}} \right] V_i - V\alpha_d
\]

Note that the angle of attack $\alpha_c$ of the rotor disc (hence the attitude of the tip path plane) is required to compute the downward velocity:

\[
\alpha_c = -\theta_c + i_n - a_i
\]

Where

- $\theta_c = \text{longitudinal cyclic pitch, + fwd}$
- $i_n = \text{incidence of the main rotor shaft (usually with respect to the body axis system, here with respect to the free flow direction)}$
- $a_i = \text{the longitudinal flapping angle}$

After the vortices are fully formed the Biot-Savart law is used:

\[
v_c = \frac{\Gamma_{\text{max}}}{2\pi (\bar{y}_{\text{cer}} - \bar{y}_{\text{cer}})}
\]
The main rotor disc edge vortices

The value of $\Gamma_{\text{max}}$ can be calculated by putting equation for the spanwise vorticity distribution equal to zero, iteratively determine $y_{\Gamma=\Gamma_{\text{max}}}$ and fill in this value in the expression for $\Gamma$ to compute $\Gamma_{\text{net}}$.

Hence the axial $z$ co-ordinate becomes:

$$z_{\text{adv}} = 0.88\bar{Z}_{\text{adv}} - v_{\text{adv}} \frac{x}{V} \quad \text{for} \quad \bar{x} \leq \bar{e}_{\text{adv}}$$

$$z_{\text{rer}} = 0.88\bar{Z}_{\text{rer}} - v_{\text{rer}} \frac{x}{V} \quad \text{for} \quad \bar{x} \leq \bar{e}_{\text{rer}}$$

and

$$z_{\text{adv}} = 0.88\bar{Z}_{\text{adv}} - v_{\text{adv}} \frac{x}{V} \quad \text{for} \quad \bar{x} > \bar{e}_{\text{adv}}$$

$$z_{\text{rer}} = 0.88\bar{Z}_{\text{rer}} - v_{\text{rer}} \frac{x}{V} \quad \text{for} \quad \bar{x} > \bar{e}_{\text{rer}}$$

One has to take care when using these formulas that the lateral $y$ co-ordinate expression changes when the asymptote is reached and the expression for the axial $z$ co-ordinate changes when both (!) vortices are fully formed. Roos suggests with the formulas to change the expression for the downward velocity if each vortex separately is fully formed while he says in the text that the axial velocity changes when both vortices are fully formed. The latter definition is used for correlation with the experimental data.

Roos assumes a flat rotor disc with no coning or lateral flapping. If the vortices are assumed to leave the rotor disc at the respectively azimuth positions $\psi = 90^\circ$ and $\psi=270^\circ$, they can be corrected as follows:

$$\left( \frac{\Delta z}{R} \right) = \sin(a_0 - a_1 \cos \psi - b_1 \sin \psi)$$

Which results in a correction of:

$$\left( \frac{\Delta z}{R} \right)_{\text{adv}} = \sin(a_0 - a_1)$$

$$\left( \frac{\Delta z}{R} \right)_{\text{rer}} = \sin(a_0 + a_1)$$
4.2 Methods to locate disc edge vortices

Five methods to conduct experimental research on the properties of vortices are considered. The major advantages and disadvantages are considered. Finally, the chosen method is explained.

4.2.1 The methods considered

In the literature on measuring the vortex properties, principally following methods are used:

1. hot wire anemometry
2. total pressure measurements
3. laser sheet visualisation
4. laser Doppler velocimetry
5. vortex indicator

Each of the methods is concisely described how it could be used for this project.

*Hot wire anemometry*

*Method*

Heyson [1956] conducted many hot wire measurements to determine the downwash velocities. This was done in various vertical planes in the rotor wake. At a certain x and z position the velocities were measured in ‘spanwise’ direction. The influence of the disc edge vortices is clearly visible in the radical change of downward velocity near the tip to an upwash when moving outboard. This can give a hint of the spanwise location of the vortex. To determine the axial co-ordinate of the vortex, the velocity distribution in y-direction must also be determined.

*Advantages:*

This method is frequently used by the Institute for Wind Energy in their experimental research. A lot of in-house expertise is available.

The traverse unit at the Open Jet Facility is very well suited to carry light test apparatus such as a hot wire probe.

Hot wire measurements can be used not only for locating the vortices but also for determining the vortex strength.

*Disadvantages:*

A traverse is needed: this disturbs the airflow and hence vortex location.

Many data points are required since the wake in a certain vertical plane in the wake has to be traversed in y-direction and z-direction.
Total pressure measurements

Method

At various vertical planes perpendicular to the free flow, the total pressure in streamwise direction is measured in a vertical-horizontal grid. Near the core of the vortex, the total pressure drops. The point with the lowest underpressure gives the location of the vortex.

The benefits and disadvantages are very much equal to those of a hot wire anemometry.

Advantages:
The total pressure probe is very straight forward to use. It’s small and light.

Disadvantages:
Again, a traverse is needed. This method is not suited to determine the (local) vortex strength. The pressure drop is no measure for the local circulation.

Laser sheet visualisation

Method:

This method is extensively described by Ghee [1996]. A vertical light sheet is placed perpendicular to the flow at different x/R positions (positive pointing downstream). A videotape recorder is set downstream at a fixed position to record the illuminated vortex slice. This is done twice, once for the advancing side and once for the retreating side. A reference grid is placed to determine the location. Laser sheets are also placed in vertical planes parallel to the flow.

This method is also used for experimental investigation of vortex flows over delta wings by Verhaagen [1983] of the Low Speed Facility of the University of Technology in Delft.

Advantages:
No traverse required. Relative simple method.

Disadvantages:
The vortices need to roll up tight in order to create a vortex core void which is clearly visible.

Highly specialised apparatus is required.
Smoke has to be inserted upstream of the model, this disturbs the airflow in front of the model.

Laser velocimetry

Method:

Measurements are performed in grid pattern similar to the total pressure and hot wire methods. Radical changes in downward and lateral velocities are an indication of the vortex location.

Advantages:
This method can provide a three dimensional velocity distribution in the wake.

The airflow is not disturbed by a traversing unit.
Disadvantages:

Many data points are needed to get an idea of the flow field behind the rotor.
The test set-up uses a lot of complex and expensive apparatus.

Vortex Indicator

Method:

A vortex generator has been developed and tested by the National Aerospace Laboratory NLR. The testing is carried out on a fixed wing by Massee [1950]. At various z-positions (vertical) at a certain distance behind the wing, the vortex indicator was traversed spanwise. Near the vortex core the rotational velocity of the indicator shows a distinct peak. To determine a circulation strength, the vortex indicator has to be calibrated.

Advantages:

The vortex indicator is a relative simple device to determine the enclosed vorticity.
The rotational speed is proportional to the circulation enclosed by the vane wheel.

Disadvantages:

Again the wake must be traversed in a vertical plane perpendicular to the free-flow.
The proportionality factor circulation/rotation depends on the vorticity distribution.
This makes it difficult to calibrate the device.

The vortex indicator is rather large and heavy which requires a rigid traverse unit.

Overall remarks

The further the vortex has travelled in the wake the weaker he gets and the more prone he is to 'wandering'. This makes it very difficult to pinpoint the location of the vortex. Time averages of the data are most likely to give a clue of the location but has the disadvantage of losing information like e.g. the periodicity of the phenomenon.

The method chosen is a compromise between the available test apparatus and the desired accuracy.

4.2.2 The applied method

From the methods described above, two were directly at our disposal at the Open Jet Facility of the Institute for Wind Energy: 1) the hot wire anemometry and 2) the vortex indicator of the NLR. The Low Speed Wind Tunnel of the Department of Aerospace Engineering has expertise in laser sheet visualisation and laser velocimetry.

Since time was limited, moving complex and sensitive test equipment for laser Doppler velocimetry or laser sheet visualisation from the Low Speed Wind Tunnel to the Open Jet Facility was out of the question.

The vortex indicator is a rather heavy and relatively large instrument. The horizontal arm of the traverse at the Open Jet Facility is not very rigid but the traverse auto-corrects for the deflection caused by the weight of apparatus fixed at the end. An adapter should have been constructed in order to fix the vortex indicator to the
traverse. The only information found on this instrument consists of the NLR report cited above. No users manual is available.

The hot wire anemometry was very attractive to use since the Open Jet Facility frequently uses this method. The only disadvantage is the fact that the hot wire probe only has one wire. Such a probe gives an absolute number of the speed in a plane perpendicular to the wire, but can’t provide the direction of the flow.

Finally, the decision was made to use the total pressure probe. This probe was borrowed from the Low Speed Wind Tunnel. It is a very light and small probe, the method is very simple.

The total pressure probe is connected to a digital pressure indicator (DPI), see figure 22. This indicator shows the difference between the ambient pressure and the pressure in the flow at a frequency of 0.5 Hz. The indication is in mbar (hPa) with maximum pressure of 7000 N/m². The output of the DPI is mV/mbar. The readings never exceeded ± 100 mbar (0.1 V). This proved to be too small, in connection with noise on the signal, for the data acquisition unit. Hence, the readings had to be recorded by hand.

4.3 Vortex test procedure

The advancing and retreating side vortex co-ordinates are determined at three positions in the rotor wake: x/R = 1, x/R = 2 and x/R = 3. This is done at two tunnel speeds: v_tunnel = 6 m/s and v_tunnel = 8 m/s. At a tip speed of 66 m/s, this corresponds with an advance ratio of respectively: µ = 0.091 and µ = 0.121.

The shaft angle of attack i_\alpha and the control plane \theta_\alpha and \theta_\alpha are maintained at 0°. The collective angle is kept at a constant pitch: \theta_0 = 8.5°.

Since collective is kept constant while the tunnel speed was varied, the thrust isn’t kept constant. In this case, it was easier to monitor the thrust by computer and leave the controls untouched. The first disadvantage of the primary collective pitch is that it’s only adjustable in increments of 0.7°. For more refined steering in collective pitch, the trim knob must be used. The scaling of the trim on the transmitter however is very rudimentary. The second disadvantage is that when the collective pitch is changed, the rotor RPM changes too. To keep the thrust at a desired level, both collective pitch and power changes are required at the same time. For proper propagation of the tests, the controls are fixed.

The traverse unit is first positioned at the advancing side, outside the tunnel jet wake. The rotor hub is set as origin of the axis system in which the locations are determined. The sign convention of the axis system is the same as used by Roos: The positive X-axis points downstream, the positive Y-axis to the left (looking downstream) and the positive Z points upwards.

The first measurements at the advancing side vortex at x/R = 1.0 is in increments of 0.1R (70 mm). However, to reduce the number of data points, the measuring procedure is changed. To get a first impression of the location of the vortex, two tufts are used. One tuft is fixed to a rod and manually brought in the vortex region. The other tuft is fixed at the end of the horizontal arm of the traverse (figure 21). When the vortex is roughly located manually, the traverse is directed to the spot until the tuft of the traverse starts rotating. The distance between the traverse tuft and the total pressure probe is known. This is used to put the total pressure probe on the spot where the tuft
The main rotor disc edge vortices

was. Since this position is a good estimation of the vortex position, the mesh size is divided in half: 35 mm or 5% R. From this starting point the lowest pressure is sought in a spiral mode. After this point is found, a new grid pattern with half the mesh size is laid around the point with the lowest total pressure. The traverse is then moved 12 or 13 mm (respectively: 2.42% and 2.57% R) in the desired direction. Measurements are always carried through until the grid is rectangular. After the traverse is moved to another grid point, readings are paused before starting since the long tubes running from the probe to the indicator cause a time lag in the readings.

The advancing vortex at $x/R = 1.0$ and at $\mu = 0.09$ is first located. Then the co-ordinates at the other $x$-positions are sought. While the traverse is still at the $x/R = 3$ position, the tunnel speed is increased to 8 m/s and the procedure is repeated backwards.

After the advancing side is finished, the traverse is moved to the left side where the same sequence is applied again.

In total 12 positions are determined, per advance ratio at 3 $x/R$ positions for both advancing and retreating side.

Since the flow isn't steady, the readings on the DPI tend to fluctuate. At a certain probe position, the estimated average value of one point or the longest displayed value is written down. This means that a time averaged location of the vortex is recorded. Instead of using the average value seen on the DPI, the lowest pressure of all readings at one point can be used as criterion too. Moving away from the rotor the large pressure gradients soon disappear and the latter method could prove very useful since in this region there is more fluctuation and the pressure gradients are small.

From the contour plots figure 24a and 24b can be seen that the pressure gradient decreases with increasing distance from the rotor disc. The vortex gets weaker and moves closer to the converging boundary layer of the tunnel jet. The results at $x/R = 3.0$ already are in the boundary layer of the tunnel jet and are subsequently less reliable.

4.4 The results

To get an impression of the pressure distribution and more important: the pressure gradients in the vicinity of a disc vortex, isobar plots, are produced in Matlab (see figures 24a and 24b). These should be used with care. Only the points on the grid (axis lines) were input, the rest is produced with the contour function of Matlab. This may explain the strange plots at the advancing side at $\mu = 0.091$. Not all the axis have the same scale because the grid best suited to produce the isobars (mostly the rough grid, sometimes the refined grid) is used. The contour plot of the advancing side vortex at $\mu = 0.091$ and $x/R = 1.0$ has the largest mesh width (0.1 R). This width is only used once.

The plots show that the closer to the rotor disc, the larger the gradient between the isobars become. For instance, in the first plot the difference between the lowest and the highest pressure is 90 hPa. The pressure difference on the $x/R = 3.0$ is respectively 12 and 20 hPa for the retreating and the advancing side. Movement of the vortex near the rotor disc will result in a clear difference in pressure readings. Further downstream, the gradient is lower, this means that in a larger region the pressure will vary between almost the same pressures. The minimum total pressure becomes harder to find.

For most drawings of the contour plots, the data points of the 'rough' grid (5% R) are used. The lowest pressure point (the cross in the lowest pressure isobar) is the result of
the fine mesh (2.5% R) measurements. If measurements are good, the lowest pressure point lies within the lowest pressure isobar contour. If not, they are not further than 2.5% R away from the contour with the lowest pressure. The reason data points are out of the lowest contour is twofold. On the one hand, for some of the plots only 12 data points are used to develop the contour plots (the number of data points is the product of the number of horizontal and vertical axis grid lines). Subsequently these plots are merely a rough estimate of the real pressure distribution near a disc edge vortex. A typical illustration of such a point is the vortex location at μ = 0.091, the advancing side vortex at x/R = 2 and μ = 0.091. On the other hand, in the regions directly behind the rotor, with the high pressure gradients, shifting of the vortex in one direction can cause the pressure of a point in the grid to change drastically. Typical for this case is the advancing side vortex at μ = 0.121 and x/R = 1.0. During the recording of the pressure at a certain grid point, a fluctuation between readings of about 20 hPa occurs frequently. For points near the rotor disc, this could result in a displacement of about 0.02 R. Further downstream, a fluctuation of 20hPa results in a position shift of about 0.05 R. Again the remark must be made that few data points are used for the contour plots. Consequently, these are rough estimates of the position error.

Another Matlab program is written to give a three-dimensional insight in the location of the measured vortex positions and the vortex positions according to Roos (figures 25 and 26). The lines running from the rotor disc represent the vortex locations via Roos. The stars and 'x'-marks represent the experimental data points. The circles around the experimental data have a radius of 0.05R. The plots also show the tip path plane (the reason why an oval can be seen in the views from aside and behind) and the diameter of the jet boundary at the x/R positions where the vortex co-ordinates are determined. Apparently, the jet diameter shrinks so fast that the experimentally found co-ordinates at x/R = 3 lie outside the undisturbed jet (outside the smallest circle, as seen from behind).

The results show (figure 23) that the theoretical data deviate a lot from the experimental data. Especially the axial z co-ordinate of the retreating side vortex at μ = 0.121. The lateral y co-ordinate of the advancing side vortex also shows an average deviation of over 10%. This means that assuming that the lateral vortex asymptote is the c.g. of the circulation distribution doesn't work for the advancing side disc edge vortex location. However, the retreating side disc edge lateral vortex co-ordinate is very well predicted by this theory.

Concluding, figure 23c shows that the lateral co-ordinate of the advancing side and the axial co-ordinate of the retreating side deviate the most. The other co-ordinates have an average deviation of about 6% R.
4.5 Flow visualisation

To get a visual impression of the vortex flow coming from the rotor disc edge, smoke tests are conducted. These tests are done at the same advance ratios as the measurements: $\mu = 0.091$ and $\mu = 0.121$. The camera positions are respectively downstream, then from aside and later from above to get a qualitative view much the same way as the Matlab plots in figure 25-26. The results can be seen in figures 27-29.

The smoke generator was constantly moved around to get the smoke wrapped in the vortex structure. The further the smoke is injected relative to the vortex core, the larger the resulting smoke cone. This can clearly be seen in the difference between 27c and 27d. Injecting smoke above or below the rotor changes the pattern entirely.

The amount of smoke is not constant. This gives the false impression that a vortex is weaker when it’s less visible.

The pictures taken downstream show a tighter roll up of the advancing vortex (27a/b vs. 27c/d). More conclusions can not be drawn since these plots show smoke at different x/R positions. Laser sheet visualisation would make it possible to evaluate the vortex position and structure at a certain distance behind the rotor.

The pictures with the side views show that the smoke rapidly dissipates after a distance of about three rotor radii in the wake. The smoke stays within a certain conus with a well outlined upper and lower boundary, figure 28 b shows this very clearly. This is also described by Ghee [1996]. Figure 28a seems to show a bend in the downward displacement. This can also be seen in the Matlab plot, figure 25b.

The plots with the shots from above show no distinct difference between the retreating or advancing side. What’s interesting is the place where the smoke leaves the rotor disc. In some cases (29a and 29c/d) the smoke seems to follow the disc edge up from either 270° or 90° azimuth to a certain azimuth downstream before leaving the rotor disc.

During these smoke visualisation tests, the disturbance of the airflow caused by the smoke injector could be heard as a change in noise level. Apparently, the smoke injector works as a vortex generator causing BVI noise. This proves that care should be taken when smoke tests are conducted for data acquisition.

In the video, the unsteady motion of the vortices is very clear. This motion of the vortex could be partly explained by the existence of a second counter-rotating vortex observed by Ghee [1996]. This could cause the main vortex core to perform a rolling manoeuvre around the ‘cg’ of disc edge vortex and the counter rotating vortex.
5 The vibration characteristics of the support

Many vibration problems submerged during the trial runs of the test set-up which limited rotor RPM. All problems were dealt with until a satisfactory high RPM could be reached. Finally the rotor could run as fast as the free flying model (1030 RPM). Due to the limit of the tunnel speed and smooth running of the model at 900 RPM, this RPM was maintained during the measurements.

5.1 Main vibration causes and their solution.

A lot of vibration problems are encountered. The five most important and the their solutions are treated below.

1. Mass Unbalance
2. Blade out of track
3. Electromagnetic interference
4. Insufficient stiffness of the inner rod of the load measuring tube.
5. Insufficient stiffness of the tripod

Mass Unbalance

During the first run trials, a rather high vibration level was encountered. It turned out that the one of the blades was crooked. A new set of wooden blades was purchased. These blades are clipped by 4 cm each to reduce the diameter to 1.4 m. This has to be done because the tunnel has a diameter of 2.24 m. Considering a the boundary layer expansion of 7°, the diameter of the undisturbed jet has shrunk to a mere 1.4 m (the diameter of a clipped rotor) in 3.4 meters behind the tunnel exit. Despite this measure, the co-ordinates of the vortices at downstream co-ordinate x/R = 3.0 are located in the jet boundary layer.

Mass balances are prone to happen during clipping. Another problem with wood is that is tends to absorb water. The effects of rotor imperfections on fuselage vibrations is treated by Kidd, Spivey & Lawrence [1967]. The rotor has to be balanced both statically and dynamically. In short, this means that in the case of wooden blades, the mass unbalance is cut from a strip of tape, the same tape that covers the wooden blades. This tape is fixed on the lightest blade in such way that the rotor, with the axis horizontal, has no tendency to tip one way or another. However, this extra thickness causes extra drag that causes the thicker blade to lag more around the lead-lag hinge, shifting the rotor c.g. out of the axis of rotation. Generally, this kind of unbalance is not so severe since the drag-force is much smaller than the centrifugal force acting on the blade. The way Klöppel and Marsch [1984] deal with unbalance problems and aspects of balancing a two-bladed tail rotor is equally applicable to the TU-LR rotor.
Blade out of track

After the rotor is balanced, it has to be tracked. This is done by spinning up the rotor and illuminating both blades with a strobe light at a certain azimuth position. One of the blades is marked (in this case one blade tip has red tape wrapped around it). Aerodynamic differences between the two blades in lift, drag and moment coefficients can be caused during manufacturing and handling. Especially the trailing edge of the blades is very vulnerable since a lighter and softer kind of wood is used. This can cause the trailing edge to act as an trim tab forcing one blade in a higher track than the other. This problem can be by solved by adjusting the length of a pitch link.

Electromagnetic interference

Another problem encountered during testing is electromagnetic interference of the electromotor with the wires running from the receiver to the servo's causing the latter to perform a periodic motion while they should stay at a fixed position. This problem was solved by wrapping the wires in aluminium film (see figure 5). This is one of the few drawbacks of using a radio-controlled model.

The inner rod

The model is bolted to some sort of adapter. This adapter is screwed on the inner rod of the load measuring device. The inner rod is linked to the outer tube by two membranes. These membranes can only carry an in-plane load. Moments and loads perpendicular to the membranes can easily deform them. In the case the inner rod is too flexible the membranes act as hinges allowing the inner rod to bend when a moment is applied on it. The model is then able to swing on top of its support. Thickening the inner rod solved this problem.

Insufficient stiffness of the tripod

The load measuring device is bolted via its flanges onto three steel struts. First, we had the tube pointing upwards (the flanges at a low position). This was done to get the rotor at least as high as the tunnel centreline. This means that the force of a mass unbalance at the rotor hub has a large arm (height of the model + length of the tube) relative to the tripod connection. While running the model in this configuration, the struts could be seen shaking distinctly. The rotor could not run harder than 600 RPM. In August 1997, a first smoke visualisation test was conducted at the Open Jet Facility of the Institute for Wind Energy in this set-up. Modifications to the support are treated in the paragraph 5.2 where the first shake test is described.

5.2 The first shake test

The first shake test dates back from August 1997. In this stage, the eigenfrequencies of the support had to be known because of the RPM-limiting vibrations which occurred. Especially, if the frequency region where the support resonated was small or not. If this were true, the rotor could be accelerated rapidly through this RPM region. The test set-up was simple: a rope was attached to the rotor shaft just below the hub. This rope was fitted to a woofer which is converted to actuator. The frequency of excitation was slowly increased until the support was seen shaking in its eigenfrequency. Since the
The vibration characteristics of the support

deflections were small and the scale of the excitor was in increments of 5 Hz, the result can only be regarded as a rough estimate. First, the load was applied along the lateral axis of the model (in y-direction). This showed an increase in the vibration level at 10 Hz (the equivalent of 600 RPM). When the excitations were over 10 Hz, the vibration level reduced rapidly. When the load was applied along the longitudinal axis, the structure started to shake again at 10 Hz. The vibration did not clearly decrease before 14 Hz (840 RPM) was reached. Because of the relative large frequency range the structure could be seen shaken in its eigenfrequency, the decision was made not to try to run the rotor through this region before modifications are made to the support.

To be sure the vibrations could be contributed to the support the model was put on its skids and the rotor was spun up easily to 700 RPM (the region were eigenfrequencies of the support were determined) while no excessive vibrations were noticed.

The aim was to try to increase the rotor RPM prior to the measuring sessions. To succeed, following two measures were taken.

First, the measuring tube is mounted inverted on its tripod. The membranes are switched too to put the strain-gauges again on the lower membrane because this membrane is less loaded then the upper membrane. This measure reduces the moment arm of vibration force caused an unbalance in the rotor.

Second, the struts are connected by a triangle to stiffen the support. Another measure taken is putting rubber anti-slip mats under the sheet metal base of the struts. This increases the friction surface of each strut while it also provides some damping.

5.3 The ground resonance theory

Coleman & Feingold [1957] derived a theory to describe the ground resonance. They also proposed a method of testing this theory. It is actually Brooks [Coleman & Feingold 1957] who treated the matter on the two-bladed rotors in the same report. Basically, the theory relies on the energy method. The blades have kinetic energy due to rotation and lead lag movement and the movement of the hub. The potential energy consists of spring energy in the pylon and the lead-lag hinge springs. Damping is provided by the damping in the pylon and the lead-lag dampers. Every location is given in the non-rotating axis system. Finally the equations of motion of the two lead-lag angles and the x and y co-ordinates of the rotor hub are determined with Lagrange. From these equations the characteristic equation is deduced. The equations are derived for an isotropic support.

To determine the support mass, damping and stiffness, Coleman and Feingold [1957] advised to do shake tests at two different weights. The results are so-called equivalent mass, damping and stiffness. ‘Equivalent’ means that the equivalent system has the same kinetic and potential energy at the hub as the real system.

Johnson [1980] first derives the equations of motion in the non-rotating axis system and later transformates the equations to the rotating system to get rid of the periodic coefficients. However he only treats the ground resonance due to an a isotropic support. Since the TU-LR model is slightly asymmetric, these formulas had to be changed to cover the anisotropic support as well.
The vibration characteristics of the support

The differential equations describing the ground resonance of a two-bladed rotor in the rotating frame with anisotropic support are:

\[
\begin{bmatrix}
I_\zeta^* & -S_\zeta^* & 0 & \zeta_1 \\
-S_\zeta^* & M_\zeta^* & 0 & y_r \\
0 & 0 & M_x^* & x_r
\end{bmatrix}
\begin{bmatrix}
\zeta_1 \\
y_r \\
x_r
\end{bmatrix}
\begin{bmatrix}
I_\zeta^*C_\zeta^* & 0 & -2S_\zeta^* \\
0 & M_\zeta^*C_\gamma^* & 2M_\gamma^* \\
2S_\zeta^* & -2M_\zeta^* & M_x^*C_x^*
\end{bmatrix}
\begin{bmatrix}
\zeta_1 \\
y_r \\
x_r
\end{bmatrix}
\]

With \( \zeta \) describing the the lead-lag motion and the \( x, y \), describing the movement of the rotorhub.

The "*"-superscript stands for non-dimensional:

\( S_\zeta^* = S_\zeta R/I_b \); \( I_\zeta^* = I_\zeta/I_b \); \( C_\zeta^* = C_\zeta/(I_\zeta\Omega) \); \( \nu_\gamma^* = \nu_\gamma/\Omega \); \( \omega_\zeta^* = \omega_\zeta/\Omega \); \( M_x^* = M_x R^2/(n I_b) \);

\( C_x^* = C_x/(M_\zeta \Omega) \).

Note: the denominator of the equivalent mass is 'n I_b' for a two-bladed rotor while for a rotor system with more than two blades the denominator is 'n/2 I_b'.

5.4 The second shake test

During the tests a accelerometer is fitted near the upper bearing. This accelerometer is connected to a FFT-unit (Discrete Fourier Transform). Figure 30 shows the display of this unit. Despite the picture being blurry some typical features are shown. On the horizontal axis the vibration frequency is shown. On the vertical the amplitude is shown. The first number on the horizontal axis is 20 the last number 200 (logarithmic scaling). The first peak, marked by a vertical line, represents the 1-P vibration of 15 Hz. The second peak is the notorious 2-P vibration of the two-bladed rotor. With the wind tunnel off, the 1-P vibration has a larger amplitude than the 2-P. Probably the most interesting is the 6-P vibration which is higher than the 1-P(!). Apparently, each of the struts is excited twice during rotation of the rotor. Obviously, an unbalance shifts the c.g. of the rotor out of the axis of rotation and shakes a strut once when it passes over it and once when it passes 180° further. Before the support was stiffened this structure, this 6-P vibration was probably even higher.

Stegeman [1998] of the NLR provided a method, merely based on the methods described by Johnson [1980] and Coleman [1957], to experimentally determine the eigenfrequencies, damping and equivalent mass of a structure relevant for ground resonance research.

The support can be modelled as a simple spring-mass-damper unit:

\[ M_x \ddot{x} + C_x \dot{x} + K_x x = f(t) \]
The vibration characteristics of the support

To determine the equivalent mass according to Coleman [1957], a vibration test should be done once with and once without extra mass added to the support. This mass should be high enough to spot a clear difference in frequency between both tests. With both frequencies resulting from these tests, the equivalent mass can be calculated the following way:

\[
\omega_{x_1} = \sqrt{\frac{K_x}{M_x}}
\]
\[
\omega_{x_1} = \sqrt{\frac{K_x}{M_x + \Delta M}}
\]
\[
M_x = \frac{\Delta M\omega_{x_1}^2}{(\omega_{x_1}^2 - \omega_{x_2}^2)} \quad [kg]
\]

Damping, stiffness and frequencies are:

\[
\beta_x = \frac{\ln \left( \frac{A_1 - A_e}{A_2 - A_e} \right)}{2\pi} \quad [-]
\]

\[
A_1 \quad \text{first maximum in amplitude}
\]
\[
A_2 \quad \text{amplitude after one period}
\]
\[
A_e \quad \text{amplitude in the end}
\]

\[
C_x = 2\omega x M, \beta \quad \text{[kg/s]}
\]
\[
P_x = (T_x - T_e) \quad \text{[T]}
\]
\[
\omega_x = 2\pi P_x \quad \text{[rad/s]}
\]

For the vibration tests, the support is excited by applying a horizontal load to the rotor hub. Figure 32 shows how the load is applied: a weight (0.5 kg) pulls the hub via a rope and pulley. To apply the load, the rope is cut. The extra mass consisting of 4 blocks of 1 kg is put on the model to determine the second frequency \( \omega_x \). This frequency is only used to determine the equivalent mass. An accelerometer is attached to the hub (figure 31) which registers the acceleration in the direction of the applied load. The signal is sent to a computer and gives the time history of the voltage. Unfortunately, in most cases, the signal reached its limit of ±10V which made the signal unsuitable for integration to a displacement time history. (This gives an integrated offset in velocity and displacement.)
The vibration characteristics of the support

In total four test are done: two in x-direction and two in the y-direction. In each direction with and without 4 kg extra mass. When the support is considered isotropic, one direction could suffice. Since the struts aren’t perfectly on a 120° angle with each other and the body-axis of the model is not parallel to one of the medians, the structural characteristics had to be determined in two directions.

Figures 34-37 show the output of the support shake tests. The influence of adding weight on the frequency can be clearly seen in both the lateral and for/aft shake tests. In the plots were no weight is added (figures 35 and 37), the points chosen to determine the damping $\beta$, are marked as amplitudes $A_1$ and $A_2$. Another striking feature of the Volt-time histories are the overall high frequencies. This is almost in contradiction with the frequencies found during the first shake test. The modifications would have made the support two to three times as rigid as before! This makes the results suspicious. Maybe the eigenfrequency of the rotor shaft has been determined, instead of the eigenfrequency of the total support. The next time a shake test is done both the load and the accelerometer should be applied to a rigid part of the model like the upper bearing. If the load is not directed through the cg of the support, a rotational vibration will result besides the normal response. If the accelerometer is not mounted on the axis of rotation the rotational vibration will be recorded too.

To determine the lead lag damping, no additional mass was needed since the blade inertia around the lead-lag hinge was determined. The lead-lag equation is given by:

$$I_\zeta \dddot{\zeta} + C_\zeta \dot{\zeta} + K_\zeta \zeta = f(t)$$

$$2\alpha = \frac{C_\zeta}{I_\zeta}$$

$$\frac{A_1}{A_2} = e^{\alpha(t_2-t_1)}$$

$$\downarrow$$

$$C_\zeta = 2 \frac{\ln \left( \frac{A_1}{A_2} \right)}{(t_2-t_1)} I_\zeta$$

To determine the damping coefficient $\alpha$ in this case a load was applied to the tip the same way as a load was applied to the support. Since the lead-lag hinge doesn’t have a hinge spring, the blade was forced to its equilibrium position by rubber bands (see figure 33). When the load was applied the lead-lag motion was rapidly damped but the support was excited. Nevertheless, the time history of the acceleration (figure 38) only shows noise in the beginning of the plot, the following part is very smooth.

The parameters found with this experimental method can be found in figure 39.
5.5 The Result of the shake tests: Coleman diagrams

The experimental determined parameters together with other rotor characteristics:

- blade number \( n \)
- lead-lag hinge offset \( e \)
- rotor polar moment of inertia \( I \)
- blade first moment \( S \)
- RPM

serve as input parameters for the ground resonance program of the NLR.

The output of the program are two Coleman diagrams: one with the imaginary parts of the characteristic equations and one with the real parts. A typical solution for a four-bladed rotor is given in figure 40. The regressing and progressing lead-lag modes can be clearly seen in the diagram with the imaginary part. The points where the support frequencies intersect the regressing lead-lag frequency, are the critical RPM’s. In the real part diagrams these regions look like bumps in the straight line. As long as these bumps stay below zero and out of the operating range of the rotor (usually such an RPM should be below 90% or above 110% of the normal operating range) no diverging resonance develops. The solid vertical line represents the normal operating speed of the rotor.

The Coleman diagrams of the two-bladed TU-LR helicopter rotor are given in figure 41. The first thing that strikes is all the support frequencies are high. Intersecting of the regressing mode with the one of the support frequencies only happens at very high frequency. However according to the vibration becomes unstable much earlier: just past the nominal rotor speed (the solid vertical line).

Compared to the example helicopter, the real part diagram of the TU-LR model shows the same ‘bumps’ but features an extra hoop. These bumps do not occur at the intersection point of the regressing lead-lag mode and the support frequencies as in the case of a n>2 blade rotor but at the intersection point of the regressing modes with the lead-lag eigenfrequency of the blade. The second critical region, the oval in the real part plot of figure 40, occurs when the eigenfrequencies of the support are equal to the rotor RPM.

The real parts of both regions exceed the zero-line in the real part region, the resonance would become unstable at higher RPM’s. Fortunately, both eigenfrequencies develop above the nominal rotor speed.
6 Conclusions

In this report three main subjects are treated: the measuring of the flapping angles of a two-bladed rotor, locating the disc edge vortices and an investigation in the ground resonance behaviour of the support. The data on the flapping angles is used in two ways: 1) to correlate the theory of Ypma [1996] with and 2) to correct the results of the theory of Roos [1996]. Since vibration reduction was a big issue prior to conducting wind tunnel tests with the TU-LR helicopter model, the ground resonance behaviour of the support is also investigated.

The flapping angles concluded

1. Measuring the flapping angles by recording the tip path plane, is a simple but yet a rather accurate way to determine the flapping angles.

2. The coning angle $a_c$ according to the theories of Bramwell [1976] and Ypma [1996] correlate well with the experimental data. However, the test data show a more constant value for $a_c$ while Bramwell and Ypma produce almost identical increasing values of $a_c$ with increasing advance ratio $\mu$.

3. The experimental and theoretical values of the longitudinal flapping angle $a_l$ correlate well too. Variations between the theories are small. Although, the theoretical results underestimate the experimental data slightly.

4. The lateral flapping angle $a_l$ really shows the mutual differences between the theories. At these low advance ratios, the theories of Bramwell and Chen (non-uniform) deviate the least. But the theory of Ypma shows the best trend.

5. Finally, the uniform inflow approximation of Chen proves that a reasonable calculation of the lateral flapping angle requires the assumption of non-uniform inflow.

The disc edge vortices concluded

1. The results show a considerable difference between experiment and theory. Especially, the lateral co-ordinate of the advancing side vortices and the axial co-ordinate of the retreating side vortex show This is partly because it is a) difficult to locate the vortices experimentally and b) it's hard to derive a simple theory that describes an unsteady phenomenon well.

2. It is difficult to get an accurate impression of the location experimentally. Near the rotor disc, the total pressure drop near the vortex core is very high. On the one hand this makes it easy to pinpoint the location but on the other hand a small change in vortex core location changes the pressure reading at a certain grid point drastically. Further downstream the total pressure drop is much less while position shift of the vortex give almost the same average readings of total pressure in a large part of the measured mesh.

3. The theory possibly needs more refinement to become a useful tool in determining the vortex location.
Conclusions

The vibration tests concluded

1. The second shake test revealed rather high eigenfrequencies of the support.

2. Coleman diagrams are constructed with the experimental data, they show a second instability region in the case of the two-bladed TU-LR rotor.

3. The ground resonance behaviour becomes unstable at RPM's past the normal operating RPM.
7 Recommendations

The recommendations are split in the three subjects treated in this report. They are mainly points of further investigation and some practical clues for future test set-ups.

Measuring rotor flapping angles

1. Considering the good results of the described visual method to visually determine the static flapping angles, this simple method is highly recommended.

2. Ypma [1996] suggests to conduct experiments to investigate the influence of helicopter angular velocities on the blade flapping motion. The model must be able to pivot. This implies a modification of the support.

Measuring vortex locations and strength

1. The preconceived object of this investigation was to experimentally determine the location and strength of the disc edge vortices. Considering the tight schedule and limits of the total pressure measurements, the strength of the vortices is not treated in this investigation. To determine the influence of these vortices on the performance of a helicopter, knowledge of the vortex strength is equally important.

2. The apparatus used should provide a signal which can be by computer analysed. This helps to analyse data objectively.

3. To be able to determine a movement of the vortex a wake rake could be used to measure the pressures at different points at the same time.

4. In close co-operation with the low speed facility of the Department of Aerospace Engineering, other methods of quantifying the wake can be considered: laser sheet visualisation or laser velocimetry.

Ground resonance investigation

1. When future shake tests are performed, the best results can be obtained by a combination of shake test 1 and 2 described in this report. The model should be excited by an actuator (shake test one). With this actuator the frequency can then be changed from zero to over the nominal rotor speed. This actuator should apply the load to a stiff part of the structure (for instance: the crossing of the vertical back plate and the upper bearing). The g’s recorded by the accelerometer (same method as shake test 2) can then be plotted against the excitation frequency. The excitation should then be repeated for heavier model. The resulting eigenfrequencies can then be used to determine equivalent masses and damping of the support for computing of the Coleman diagrams. The shake test method described by Peterson and Hoque [1994] can serve as an example.

2. The accelerometer(s) should be located at several positions to prevent positioning it in a knot. The output signal must stay within the limits of ±10V.

3. The method to determine the lag damping worked well but better results can be obtained with the hub disassembled from the support and fixed firmly. This counteracts the excitation of a support which influences the result.
References


Jessurun K.P., 'Model rotor boundary layer flow visualisation with and without partial stall and tripwires', Twenty First European Rotorcraft Forum, Saint-Petersburg, Russia, August 30-September 1, 1995.


References


Roos J.P., 'Main Rotor Disc Edge Vortices, an analytical model', Department of Aerospace Engineering, Delft University of Technology, March 1996.


Figures
Figure 1: The Open Jet Facility of the Institute for Wind Energy

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Height tunnel centreline</td>
<td>2.33 m</td>
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<tr>
<td>Maximum tunnel speed</td>
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<tr>
<td>Turbulence fraction</td>
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<tr>
<td>Boundary layer angle</td>
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<tr>
<td>Power electromotor</td>
<td>45 kW</td>
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Figure 2  The primary tunnel parameters of the Open Jet Facility of the Institute for Wind Energy.
Figure 3: The Traverse Unit

Figure 4: the Traverse Control Screen
Figure 5: The TU-LR helicopter model without fairing (during the tests in August '97)

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<th>Parameter</th>
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<td>teetering</td>
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<td>Cord c [mm]</td>
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<td>Solidity σ [-]</td>
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<td>Hinge spring constant Kₗₜ [Nm/rad]</td>
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<td>Blade flapping stiffness EI [Nm²]</td>
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Figure 6: The Rotor parameters of the TU-LR helicopter model
Figure 7: Test set-up for measuring the vertical displacement of the TPP at $\psi=90^\circ$ and $\psi=270^\circ$.

Figure 8: Test set-up for measuring the vertical displacement of the TPP at $\psi = 30^\circ$. 
$\Delta h_{0.25c}$

$\Delta L.E.$

$\theta_0 = 8.5^0$

$\Delta T.E.$

$\theta_0 = 0$

Figure 9: Relation between $\Delta L.E$, $\Delta T.E$ and $\Delta h$.

Figure 10: The definition of $h_1$, $h_2$, and $h_3$. 
Figure 11a: Cyclic stick sign convention

Figure 11b: Relation and sign convention of $B_e$, $a_1$, and $a_u$

Figure 11c: Relation and sign convention of $A_e$, $b_1$, and $b_u$

Figure 11d: Longitudinal flapping and coning of the rotor

Figure 11e: Lateral flapping and coning of the rotor

(Courtesy of M. Pavel)
Figure 12: The flapping angles derived from the vertical displacements at $\psi=30^\circ$, $\psi=90^\circ$ and $\psi=270^\circ$. 
Coning angle $a_0$

Figure 13: Correlation of the coning angle $a_0$ of the TU-LR model data with the theories of Ypma and Bramwell.

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<th>windtunnel, 1998</th>
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<th>Bramwell, 1976</th>
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<td>0.04</td>
<td>0.02</td>
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<td>1.95</td>
<td>1.13</td>
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<td>0.1208</td>
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<td>1.92</td>
<td>1.29</td>
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<tr>
<td>0.1513</td>
<td>1.38</td>
<td>1.76</td>
<td>1.42</td>
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Figure 14: The coning angle values as a function of the advance ratio $\mu$
Figure 15: Correlation of the longitudinal flapping angle $a_1$ of the TU-LR model data with the theories of Chen, Ypma and Bramwell.

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<th></th>
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<td>4.26</td>
<td>3.67</td>
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Figure 16: The longitudinal flapping angle values as a function of the advance ratio $\mu$
Figure 17: Correlation of the lateral flapping angle $b_1$ of the TU-LR model data with the theories of Chen, Ypma and Bramwell.

Figure 18: The lateral flapping angle values as a function of the advance ratio $\mu$
Figure 19: The spanwise vorticity distribution \( \gamma \) (circular wing analogy)

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<th>( \mu = 0.12 )</th>
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<td>Rotorspeed ( \Omega ) [RPM]</td>
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<td>900</td>
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<tr>
<td>Tip Speed ( V_{w} ) [m/s]</td>
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<td>66</td>
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<tr>
<td>Thrust coefficient ( C_{T} )</td>
<td>0.0055</td>
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<td>Power ( P ) [W]</td>
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<td>217.5</td>
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<tr>
<td>Shaft AOA ( \alpha_{s} ) [deg.]</td>
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<td>0</td>
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<td>8.5</td>
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<td>-0.56</td>
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<tr>
<td>Lateral flapping ( b_{f} ) [deg.]</td>
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Figure 20: The Test Conditions
Figure 21: Measuring the Total Pressure at the Retreating side with a total pressure probe (note the tuft at the end of the rod).

Figure 22: The Digital Pressure Indicator (on top of the digistrobe)
### Analytical Theory of Roos

**\( \mu = 0.091 \)**

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<th>( z/R )</th>
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**\( \mu = 0.121 \)**

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*Figure 23a: The vortex locations according to the analytical theory of Roos*

### TU-LR Experiment

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*Figure 23b: The vortex locations determined experimentally with the TU-LR model*
### Deviation

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**μ = 0.121**

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<td>1</td>
<td>0.0545</td>
<td>0.2398</td>
<td>-0.0888</td>
<td>0.0722</td>
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<tr>
<td>2</td>
<td>0.0045</td>
<td>0.2571</td>
<td>-0.1639</td>
<td>0.0830</td>
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<tr>
<td>3</td>
<td>-0.0712</td>
<td>0.2860</td>
<td>-0.1623</td>
<td>0.0855</td>
</tr>
</tbody>
</table>

*Figure 23c: The deviation of the analytical model from the experimental data*
Figure 24a: Isobars at $\mu = 0.091$ and $C_r = 0.0055$
Figure 24b: Isobars at $\mu = 0.121$ and $C_r = 0.0062$
Figuur 25a: The Lateral position of the disc edge vortices for $\mu = 0.091$, test results: x-marks (retr.) and stars (adv.), theory: lines (retr. side: dashed, adv. side: solid).

Figuur 25b: The axial location of the rotordisc disc edge vortices for $\mu = 0.091$. 
Figuur 25c: The axial and lateral position of the vortices for \( \mu = 0.091 \) (viewed from downstream)

Figuur 25d: 3-dimensional view of the location of the rotor disc edge vortices for \( \mu = 0.091 \)
Figuur 26a  The Lateral position of the disc edge vortices for $\mu = 0.121$, test results: x-marks (retr.) and stars (adv.), theory: lines (retr. side: dashed, adv. side: solid).

Figuur 26b  The axial location of the rotordisc disc edge vortices for $\mu = 0.121$. 
Figuur 26c: The axial and lateral position of the vortices for $\mu = 0.121$ (viewed from downstream)

Figuur 26d: 3-dimensional view of the location of the rotor disc edge vortices for $\mu = 0.121$
Figure 27a: advancing side,
\[ \mu = 0.091, C_\tau = 0.0055 \]

Figure 27b: advancing side,
\[ \mu = 0.121, C_\tau = 0.0062 \]

Figure 27c: retreating side,
\[ \mu = 0.091, C_\tau = 0.0055 \]

Figure 27d: retreating side,
\[ \mu = 0.121, C_\tau = 0.0062 \]
Figure 28a: advancing side, \[ \mu = 0.091, C_r = 0.0055 \]

Figure 28b: advancing side, \[ \mu = 0.121, C_r = 0.0062 \]

Figure 28c: retreating side, \[ \mu = 0.091, C_r = 0.0055 \]

Figure 28d: retreating side, \[ \mu = 0.121, C_r = 0.0062 \]
Figure 29a: advancing side,

\[ \mu = 0.091, \ C_r = 0.0055 \]

Figure 29b: advancing side,

\[ \mu = 0.121, \ C_r = 0.0062 \]

Figure 29c: retreating side,

\[ \mu = 0.091, \ C_r = 0.0055 \]

Figure 29d: retreating side,

\[ \mu = 0.121, \ C_r = 0.0062 \]
Figure 30: FFT of the two-bladed TU-LR model with the tunnel running.

Figure 31: Hub mounted accelerometer
Figure 32: The forlaff test set-up with 4 kg added

Figure 33: Test set-up for measuring the lead-lag damping
Support Vibration Test, for/aft + 4 kg

Figure 34: Support response to an impulse load in x-direction, extra mass of 4 kg added.

Support Vibration Test, for/aft

Figure 35: Support response to an impulse load in x-direction
Support Vibration Test, Lateral + 4 kg

Figure 36: Support response to an impulse load in y-direction, extra mass of 4 kg added.

Support Vibration Test, Lateral

Figure 37: Support response to an impulse load in y-direction
Blade Lag Damping Test #1

Figure 38: Lead-Lag Response of a Blade to an impulse load

<table>
<thead>
<tr>
<th>Parameter</th>
<th>x-direction</th>
<th>y-direction</th>
<th>( \zeta )-direction</th>
<th>Units</th>
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<tbody>
<tr>
<td>Frequency</td>
<td>202</td>
<td>133.68</td>
<td>48.7</td>
<td>rad/s</td>
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<tr>
<td>Effective mass</td>
<td>3.585</td>
<td>6.093</td>
<td>X</td>
<td>kg</td>
</tr>
<tr>
<td>Damping</td>
<td>128.6</td>
<td>208.0</td>
<td>0.212</td>
<td>Ns/m [Nms/ rad]</td>
</tr>
</tbody>
</table>

Figure 39: The experimentally determined structural parameters of the support.
Example helicopter rotor

Rotor speed [Hz]

Imaginary part [Hz]

\( \omega_x = -20.00 \text{ rad/s} \)
\( \omega_y = -21.00 \text{ rad/s} \)

Example helicopter rotor

Rotor speed [Hz]

Real part [Hz]

\( \omega_x = -20.00 \text{ rad/s} \)
\( \omega_y = -21.00 \text{ rad/s} \)

Figuur 40: Typical Coleman diagrams
Figur 41: The Coleman diagrams of the TU-LR model