CHALMERS UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF NAVAL ARCHITECTURE
AND MARINE ENGINEERING

GOTHENBURG - SWEDEN

THE INFLUENCE OF WATER DEPTH ON THE HEAVING, SWAYING, ROLLING MOTIONS AND ON THE BENDING MOMENTS OF SHIPS IN REGULAR WAVES

by

C. H. KIM

DIVISION OF SHIP HYDROMECHANICS REPORT NO. 43B-45B

Göteborg, November 1969

Hydrodynamic Forces and Moments for Heaving, Swaying, and Rolling Cylinders on Water of Finite Depth

By Cheung H. Kim[1]

The potentials for forced heaving, swaying, and rolling cylinders on the calm-water surface of deep water used by Grim and Tamura are extended to be applied for the oscillations of cylinders in shallow water by Thorne's method. Firstly, the hydrodynamic forces and moments on oscillating Lewis cylinders are calculated by Grim's method; secondly, the exciting forces and moments on a fixed cylinder in a transverse incident wave system are calculated according to the Haskind-Newman method; and, finally, the influence of the shallow-water effect on the forces is illustrated graphically and discussed.

Introduction

We consider two problems: (*a*) the problem of forced oscillations in heave sway and roll of a cylinder on the calm water surface of shallow water, and (*b*) the problem of a fixed cylinder in a transverse incident wave system of shallow water. Our problems are to determine the hydrodynamic forces and moments on the cylinder in both cases and to determine the influence of the shallow-water effect on the forces and moments. There are two reports on the first problem, by Yu—Ursell [5][2] and Wang [8], and two reports on the second problem, by Haskind [6] and Newman [7].

Firstly, the potentials of the forced oscillations on the calm water surface of deep water used by Grim and Tamura [1,2] are extended to those of shallow water by Thorne's method [3]. There are two reports on such extensions [4, 5].

Secondly, by making use of the potential of shallow-water oscillation, the hydrodynamic forces and moments are calculated.

Thirdly, the exciting forces and moments on a fixed cylinder in beam waves are calculated by using the preceding calculations for forced oscillations according to the Haskind—Newman method [6,7]; alternatively, the exciting forces and moments can be calculated by Grim's method [1,2], and a comparison of the results of both methods shows that they are in good agreement.

[1] Staff Scientist, Davidson Laboratory, Stevens Institute of Technology, Hoboken, N. J. This work was done at the Institute of Ship Hydromechanics, Chalmers University of Technology, Gothenburg, Sweden.

[2] Numbers in brackets designated References at end of paper.

Manuscript received at SNAME Headquarters May 1, 1968. Revised manuscript received January 8, 1969.

Nomenclature

a = amplitude of oscillation
$\bar{A}$ = wave amplitude ratio
A_n = source intensity in complex number
B = beam
C = added mass coefficient
E = exciting force and moment
F = force
g = gravity constant
h = water depth
$\bar{h}$ = wave amplitude
H = half beam draft ratio or subscript designating heave
i = $\sqrt{-1}$ or subscript designating imaginary part
Im = imaginary part of
I'' = added moment of inertia
l = moment arm
m'' = added mass
M = moment
n = integer
N = damping coefficient
r = subscript designating real part or $\sqrt{x^2 + y^2}$
Re = real part of
R = subscript designating roll
S = subscript designating sway or sectional area
t = time
T = draft
U = velocity amplitude of sway
V = velocity amplitude of heave
x,y = rectangular coordinate
α = phase lag or lead
β = polar coordinate $\tan^{-1}(x/y)$ or fullness coefficient of cylinder section
δ = damping parameter
η = wave elevation
ν = deep-water wave number (ω^2/g)
ν_0 = shallow-water wave number
ρ = density of water
φ, Φ = velocity potential
ψ, Ψ = stream function
ω = circular frequency
Ω = angular velocity amplitude of roll
∞ = subscript designating infinite depth

Finally, some calculated results are represented in conventional dimensionless forms including the depth parameter as functions of dimensionless frequency.

Extension of Potentials

First of all, we define our rectangular coordinate system O-xy: the x-axis, on the calm water level, is pointing to the right and the y-axis, in the centerline of the cylinder at rest, is pointing downward. In the following discussion the irrotational motion of incompressible inviscid fluid and the linearized boundary conditions and linear hydrodynamic pressure are assumed.

To begin with, the potentials for oscillations in deep water used by the authors [1] and [2] must be reproduced. They are generally represented in the following form

$$\Phi_\infty + i\Psi_\infty = \begin{bmatrix} V \\ U \\ \Omega \end{bmatrix} e^{i\omega t} \sum_{n=0}^{\infty} A_n(\varphi_{n\infty} + i\psi_{n\infty}) \qquad (1)$$

where V, U, Ω = linear velocity amplitude of heave and sway and angular velocity amplitude of roll; A_n = complex source intensities to be determined by the boundary condition on the cylinder surface; $\varphi_{n\infty} = \varphi_{nr\infty} + i\,\varphi_{ni\infty}$ = partial velocity potential which satisfies the law of continuity everywhere in the liquid, the free surface condition at $y = 0$, the condition of rest of water at $y \to \infty$ and radiation condition at $|x| \to \infty$; $\psi_{n\infty} = \psi_{nr\infty} + i\psi_{ni\infty}$ = partial stream function.

The partial potentials are for heave:

$$\left.\begin{aligned} \varphi_{0r\infty} + i\varphi_{0i\infty} &= \int_0^\infty \frac{e^{-ky}}{k-\nu}\cos kx\,dk - i\pi e^{-\nu y}\cos \nu x \\ \varphi_{nr\infty} + i\varphi_{ni\infty} &= (-1)^n\left[\frac{\cos 2n\beta}{r^{2n}} + \frac{\nu}{(2n-1)}\,\frac{\cos(2n-1)\beta}{r^{2n-1}}\right] + i\cdot 0 \end{aligned}\right\} \qquad (2)$$

and for sway and roll:

$$\left.\begin{aligned} \varphi_{0r\infty} + i\varphi_{0i\infty} &= \frac{\sin\beta}{r} + \nu\int_0^\infty \frac{e^{-ky}}{k-\nu}\sin kx\,dk - i\pi\nu e^{-\nu y}\sin \nu x \\ \varphi_{nr\infty} + i\varphi_{ni\infty} &= \left[-\frac{\nu}{2n}\,\frac{\sin 2n\beta}{r^{2n}} - \frac{\sin(2n+1)\beta}{r^{2n+1}}\right] + i\cdot 0 \end{aligned}\right\} \qquad (3)$$

where

$$\nu = \omega^2/g,\ r = \sqrt{x^2+y^2},\ \beta = \tan^{-1}(x/y)$$

and the wave source integral $\int^\infty \frac{e^{ik(x+iy)}}{k-\nu}dk$ expanded by Grim may be seen in [9].

The foregoing potentials for infinitely deep water are extended to those of limited water depth by making use of Thorne's method [3,4,5]. We set the complete potential for the oscillations in shallow water, which are to be extended, in the following form.

$$\Phi_h + i\Psi_h = \begin{bmatrix} V \\ U \\ \Omega \end{bmatrix} e^{i\omega t} \sum_{n=0}^{\infty} A_n(\varphi_{nh} + i\psi_{nh}) \qquad (4)$$

where the subscript h designates the potential for limited depth of water and $\varphi_{nh} = \varphi_{nrh} + i\varphi_{nih}$ must satisfy the following conditions:

(1) $\Delta\varphi_{nh} = 0$ everywhere in the liquid:

(2) $\nu\varphi_{nh} + \dfrac{\partial}{\partial y}\varphi_{nh} = 0$ at $y = 0$;

(3) $\dfrac{\partial\varphi_{nh}}{\partial y} = 0$ at $y = h$;

(4) $\lim_{x\to\infty}\sqrt{x}\left(\dfrac{\partial\varphi_{nh}}{\partial x} + i\nu_0\varphi_{nh}\right) = 0$, $x = \sqrt{x^2}$, $\nu = \nu_0 \tanh \nu_0 h$, where ν_0 = shallow-water wave number.

Since the potential $\varphi_{n\infty}$ does not satisfy the bottom condition (3), it is to be extended, by introducing an adjusting potential $\varphi_{nr\,ad.}$, to the corresponding potential of finite water depth, i.e.

$$\varphi_{nh} = (\varphi_{nr\infty} + \varphi_{nr\,ad.}) + i\varphi_{nih}$$

The adjusting potentials $\varphi_{nr\,ad.}$ for heave and sway and roll are assumed in the following forms respectively.

For heave:

$$\varphi_{nr\,ad.} = \int_0^\infty [\alpha_n(k)\sinh ky + \beta_n(k) \times \cosh k(h-y)]\cos kx\,dk$$

For sway and roll:

$$\varphi_{nr\,ad.} = \int_0^\infty [\gamma_n(k)\sinh ky + \delta_n(k)\cosh k(h-y)] \times \sin kx\,dk$$

where h = depth of water; $\alpha_n(k)$, $\beta_n(k)$, $\gamma_n(k)$, $\delta_n(k)$ = unknown constants.

Firstly, the unknown constants $\beta_n(k)$ and $\delta_n(k)$ are eliminated by satisfying the free-surface condition (2) by the potentials $\varphi_{nr\,ad.}$ alone. Secondly, the unknown constants $\alpha_n(k)$ and $\gamma_n(k)$ are determined by fulfilling the bottom condition (3) by the potentials $(\varphi_{nr\infty} + \varphi_{nr\,ad.})$. Thus we determine the required adjusting potential $\varphi_{nr\,ad.}$ and the potential φ_{nrh} in the following forms:

For heave:

$$\varphi_{0r\,ad.} = \int_0^\infty \frac{e^{-kh}}{k-\nu}\,\frac{\nu\sinh ky - k\cosh ky}{\nu\cosh kh - k\sinh kh} \times \cos kx\,dk \qquad (5)$$

$$\varphi_{nr\,ad.} = \frac{(-1)^n}{(2n-1)!} \int_0^\infty k^{2(n-1)}(k+\nu)e^{-kh} \times \frac{\nu \sinh ky - k \cosh ky}{\nu \cosh kh - k \sinh kh} \cos kx\, dk \qquad (5) \text{ cont'd}$$

$$\left.\begin{aligned} \varphi_{0rh} &= -\int_0^\infty \frac{\cosh k(h-y)}{\nu \cosh kh - k \sinh kh} \cos kx\, dk \\ \varphi_{nrh} &= \frac{(-1)^n}{(2n-1)!} \int_0^\infty k^{2(n-1)} (\nu^2 - k^2) \times \frac{\cosh k(h-y)}{\nu \cosh kh - k \sinh kh} \cos kx\, dk \end{aligned}\right\} \qquad (6)$$

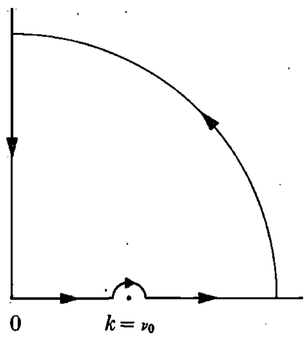
For sway and roll:

$$\left.\begin{aligned} \varphi_{0r\,ad.} &= \int_0^\infty \frac{k}{k-\nu} e^{-kh} \frac{\nu \sinh ky - k \cosh ky}{\nu \cosh kh - k \sinh kh} \times \sin kx\, dk \\ \varphi_{nr\,ad.} &= -\frac{1}{(2n)!} \int_0^\infty k^{2n-1}(k+\nu)e^{-kh} \times \frac{\nu \sinh ky - k \cosh ky}{\nu \cosh kh - k \sinh kh} \sin kx\, dk \end{aligned}\right\} \qquad (7)$$

$$\left.\begin{aligned} \varphi_{0rh} &= -\int_0^\infty \frac{k \cosh k(h-y)}{\nu \cosh kh - k \sinh kh} \sin kx\, dk \\ \varphi_{nrh} &= -\frac{1}{(2n)!} \int_0^\infty k^{2n-1}(\nu^2 - k^2) \times \frac{\cosh k(h-y)}{\nu \cosh kh - k \sinh kh} \sin kx\, dk \end{aligned}\right\} \qquad (8)$$

Finally, the potential φ_{nih} is determined by satisfying the radiation condition (4). Before fulfilling the condition, we need the asymptotic expression of the potential φ_{nrh} for $x \rightarrow \pm\infty$.

For this purpose we consider the complex-contour integral of the function of the form $F(k)e^{ikx}$, which may generally represent the potentials φ_{nrh} in k-complex plane. As the contour we take the quadrant $Re(k) > 0$, $Im(k) > 0$, which passes over the singular point $k = \nu_0 (\nu_0 > 0)$, ν_0 being the only positive root of the equation $\nu \cosh \nu_0 h - \nu_0 \sinh \nu_0 h = 0$.



Taking the radius of the quadrant infinitely large and the indentation radius infinitely small, we find:

For heave:

$$\varphi_{nrh} = Re[\pi i\, Res(\nu_0)]$$

For sway and roll:

$$\varphi_{nrh} = Im[\pi i\, Res(\nu_0)]$$

By limiting process, the residues of the functions $F(k)e^{ikx}$ at $k = \nu_0$, i.e. $Res\ (\nu_0)$, are calculated, and thus the asymptotic expressions for $x \rightarrow \pm\infty$ are

For heave:

$$\left.\begin{aligned} \varphi_{0rh} &\backsim -\frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \sin \nu_0|x| \\ \varphi_{nrh} &\backsim -\frac{(-1)^n}{(2n-1)!} \nu_0^{2n}(1 - \tanh^2 \nu_0 h) \times \frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \sin \nu_0|x| \end{aligned}\right\} \qquad (9)$$

For sway and roll:

$$\left.\begin{aligned} \varphi_{0rh} &\backsim \pm \nu_0 \frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \times \cos \nu_0|x| \\ \varphi_{nrh} &\backsim \pm \frac{1}{(2n)!} \nu_0^{2n+1}(\tanh^2 \nu_0 h - 1) \times \frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \cos \nu_0|x| \end{aligned}\right\} \qquad (10)$$

We now return to the radiation condition (4) and substitute the foregoing potentials in the condition and thus obtain the potential φ_{nih} in the following forms:

For heave:

$$\left.\begin{aligned} \varphi_{0ih} &= -\frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \times \cos \nu_0|x| \\ \varphi_{nih} &= -\frac{(-1)^n}{(2n-1)!} \nu_0^{2n}(1 - \tanh^2 \nu_0 h) \times \frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \cos \nu_0|x| \end{aligned}\right\} \qquad (11)$$

For sway and roll:

$$\left.\begin{aligned} \varphi_{0ih} &= \mp \nu_0 \frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \times \sin \nu_0|x| \\ \varphi_{nih} &= \mp \frac{1}{(2n)!} \nu_0^{2n+1}(\tanh^2 \nu_0 h - 1) \times \frac{2\pi \cosh^2 \nu_0 h}{2\nu_0 h + \sinh 2\nu_0 h} \frac{\cosh \nu_0(h-y)}{\cosh \nu_0 h} \sin \nu_0|x| \end{aligned}\right\} \qquad (12)$$

where $\mp$ corresponds to $x \rightarrow \pm\infty$

The extension is now complete. The two sets of the equations (6) and (11) and (8) and (12) are the extended potentials φ_{nh} for heave and sway or roll respectively. Since the adjusting potentials $\varphi_{nr\ ad.}(\psi_{nr\ ad.})$ are obtained in the form of Cauchy integrals, they are expanded in series to be used for numerical calculation [5] (see Appendix).

Hydrodynamic Forces and Moments

The condition of zero relative normal velocity to be satisfied on the cylinder surface for heave, sway, and roll may be written in the following form

$$Re\left[e^{i\omega t}\sum_{n=0}^{\infty} A_n\psi_{nh}\right] = \begin{bmatrix} x \\ y \\ \frac{1}{2}(x^2+y^2) \end{bmatrix}\cos\omega t \qquad (13)$$

where the stream functions $\sum_{n=0}^{\infty} A_n\psi_{nh}$ depend on the mode of motion. This condition determines the unknown source intensities and consequently the velocity potential Φ_h, designated in advance in equation (4).

Having obtained the velocity potential Φ_h, one may integrate the hydrodynamic pressure distribution $Re\left[\rho\frac{\partial\Phi_h}{\partial t}\right]$ on the submerged cylinder surface and determine the hydrodynamic forces and moments:

$$\begin{bmatrix} \text{vertical forces in heaving} \\ \text{horizontal forces in swaying and rolling} \\ \text{longitudinal moments in swaying and rolling} \end{bmatrix}$$

$$= -\rho\omega\begin{bmatrix} V \\ U,\Omega \\ U,\Omega \end{bmatrix}\left\{Re\left(\sum_{n=0}^{\infty} A_n\int_S \varphi_{nh}\begin{bmatrix} dx \\ dy \\ xdx+ydy \end{bmatrix}\right)\sin\omega t + Im\left(\sum_{n=0}^{\infty} A_n\int_S \varphi_{nh}\begin{bmatrix} dx \\ dy \\ xdx+ydy \end{bmatrix}\right)\cos\omega t\right\} \qquad (14)$$

where the constants A_n depend on each mode of motion. The forces (moments) consist of inertial and damping parts, both of which are in phase with the acceleration and velocity of oscillation. They are called respectively hydrodynamic inertial forces (moments) and hydrodynamic damping forces (moments).

From the inertial and damping forces (moments) we define the added mass and added moment of inertia and damping force (moment) coefficients. These are represented in dimensionless forms. In case of sway and roll, it is necessary, in addition to the foregoing, to define the added moment arm and damping moment arm, which relate the moment and force of the inertial and damping parts respectively. For convenience the above mentioned may be represented in tabular form, Table 1.

It is customary to represent the damping force and moment coefficients N_H, N_S, and N_R by wave amplitude ratios $\bar{A}_H$, $\bar{A}_S$, and $\bar{A}_R$, which will be discussed in the following section.

Table 1 (B = Beam; T = Draft)

	Heave	Sway	Roll
inertial force	F_{Hr}	F_{Sr}	F_{Rr}
damping force	F_{Hi}	F_{Si}	F_{Ri}
inertial moment		M_{Sr}	M_{Rr}
damping moment		M_{Si}	M_{Ri}
added mass	$m_H'' = \frac{F_{Hr}}{\omega V}$	$m_S'' = \frac{F_{Sr}}{\omega U}$	
added moment of inertia			$I'' = \frac{M_{Rr}}{\omega\Omega}$
damping force coefficient	$N_H = \frac{F_{Hi}}{V}$	$N_S = \frac{F_{Si}}{U}$	
damping moment coefficient			$N_R = \frac{M_{Ri}}{\Omega}$
added mass coefficient	$C_H = \frac{m_H''}{\rho\frac{\pi}{8}B^2}$	$C_S = \frac{m_S''}{\rho\frac{\pi}{2}T^2}$	
added moment of inertia coefficient			$C_R = \frac{I''}{\rho\frac{\pi}{8}T^4}$
damping force parameter	$\delta_H = \frac{N_H}{\rho\omega\frac{\pi}{8}B^2}$	$\delta_S = \frac{N_S}{\rho\omega\frac{\pi}{2}T^2}$	
damping moment parameter			$\delta_R = \frac{N_R}{\rho\omega\frac{\pi}{8}T^4}$
added moment arm		$\frac{l_{Sr}}{T} = \frac{M_{Sr}}{F_{Sr}T}$	$\frac{l_{Rr}}{T} = \frac{M_{Rr}}{F_{Rr}T}$
damping moment arm		$\frac{l_{Si}}{T} = \frac{M_{Si}}{F_{Si}T}$	$\frac{l_{Ri}}{T} = \frac{M_{Ri}}{F_{Si}T}$

Damping and Exciting Forces and Moments

The asymptotic expressions of the velocity potentials $\Phi_h(x \to \pm\infty)$ for heave, sway, and roll are easily obtained by combining equations (9) and (11) and (10) and (12). They are, omitting the subscript h, as follows:

For heave:

$$\Phi_{X\to\pm\infty} = -iV\sum_{n=0}^{\infty} A_n H_n \frac{\cosh\nu_0(h-y)}{\cosh\nu_0 h}e^{i(-\nu_0|x|+\omega t)} \qquad (15)$$

where

$$H_0 = \frac{2\pi\cosh^2\nu_0 h}{2\nu_0 h+\sinh 2\nu_0 h}$$

$$H_n = \frac{(-1)^n}{(2n-1)!}\nu_0^{2n}(1-\tanh^2\nu_0 h)H_0$$

For sway and roll:

$$\begin{bmatrix}\Phi_S \\ \Phi_R\end{bmatrix}_{x\to\pm\infty} = \pm\begin{bmatrix} U \\ \Omega \end{bmatrix}\sum_{n=0}^{\infty} A_n H_n' \frac{\cosh\nu_0(h-y)}{\cosh\nu_0 h}e^{i(-\nu_0|x|+\omega t)} \qquad (16)$$

where

$$H_0' = \nu_0\frac{2\pi\cosh\nu_0 h}{2\nu_0 h+\sinh 2\nu_0 h}$$

$$H_n' = \frac{1}{(2n)!}\nu_0^{2n}(\tanh^2\nu_0 h - 1)H_0'$$

and

$\pm$: corresponds to $x \to \pm\infty$

A_n: source intensities, depending on sway and roll

The waves generated by the oscillations progressing at far distance from the body

$$\eta = -\frac{1}{g}\frac{\partial\Phi}{\partial t} \text{ at } y = 0$$

are obtained from the foregoing asymptotic expressions in complex numbers, i.e.

For heave:

$$\eta_H = -\nu a_H \sum_{n=0}^{\infty} A_n H_n e^{i(-\nu_0|x|+\omega t)}$$

For sway and roll:

$$\eta_S = \mp i\nu a_S \sum_{n=0}^{\infty} A_n H_n' e^{i(-\nu_0|x|+\omega t)}$$

$$\eta_R = \mp i\nu a_R/B/2 \sum_{n=0}^{\infty} A_n H_n' e^{i(-\nu_0|x|+\omega t)} \quad (17)$$

where

a_H, a_S = linear amplitude of heave and sway

$$a_R = \frac{B}{2}\cdot\Omega$$

$\mp$ = corresponds to $x \to \pm\infty$

Consequently, the wave amplitude ratios for heave, sway, and roll are written in the following form:

$$\left.\begin{aligned} \bar{A}_H &= \nu\left|\sum_{n=0}^{\infty} A_n H_n\right| \\ \bar{A}_S &= \nu\left|\sum_{n=0}^{\infty} A_n H_n'\right| \\ \bar{A}_R &= \nu\left|\sum_{n=0}^{\infty} A_n H_n'\right| \Big/ B/2 \end{aligned}\right\} \quad (18)$$

where A_n depend on the mode of motion.

In addition, by equating the mean power dissipated by the damping force in the forced oscillation and the mean power used in forming the progressive wave system in both positive and negative directions of the x-axis, we obtain the relation between damping coefficient N and wave amplitude ratio $\bar{A}$ in the following form:

$$\begin{bmatrix} N_H \\ N_S \\ N_R \end{bmatrix} = \begin{bmatrix} \bar{A}_H^2 \\ \bar{A}_S^2 \\ \bar{A}_R^2 (B/2)^2 \end{bmatrix} \frac{\rho g}{\omega\nu_0}\left(1 + \frac{2\nu_0 h}{\sinh 2\nu_0 h}\right) \quad (19)$$

Next we consider the vertical and horizontal exciting forces and longitudinal exciting moment on a fixed cylinder in a transverse incident wave system. Haskind's theory [6] stated that the exciting forces and moments depend only on the asymptotic behavior of the potential for forced oscillations of the body with unit velocity amplitude. Newman [7] extended the theory and further established the formulae relating between exciting forces and moments and wave amplitude ratios $\bar{A}$ in the two-dimensional case. The exciting force (moment) in our case is represented by the following contour integral [7]

$$E = i\omega\rho e^{i\omega t}\int_0^h \left[\varphi_I \frac{\partial\varphi}{\partial x} - \varphi\frac{\partial\varphi_I}{\partial x}\right]_{x\to-\infty}^{x\to+\infty} dy \quad (20)$$

where

φ_I = potential of transverse wave system
φ = asymptotic potential of unit velocity amplitude

We take two incident waves of the following form, each of which is oncoming from the positive and negative ends of the x-axis:

$$\varphi_I = i\frac{g\bar{h}}{\omega}\frac{\cosh\nu_0(h-y)}{\cosh\nu_0 h}e^{\pm i\nu_0 x} \quad (21)$$

Each asymptotic potential φ is obtained from the potentials Φ_H, Φ_S, and Φ_R by taking unit velocity amplitude [see equations (15) and (16)]. Substituting the above defined potentials in equation (20) and executing the integral we obtain the heave-exciting and sway-exciting forces, E_H and E_S, and the roll-exciting moment E_R:

$$\begin{bmatrix} E_H \\ E_S \\ E_R \end{bmatrix} = \rho g\bar{h}e^{i\omega t}\begin{bmatrix} \sum_{n=0}^{\infty} A_n H_n \\ \pm i \sum_{n=0}^{\infty} A_n H_n' \\ \pm i \sum_{n=0}^{\infty} A_n H_n' \end{bmatrix}\frac{2\nu_0 h + \sinh 2\nu_0 h}{2\cosh^2 \nu_0 h} \quad (22)$$

where $\pm$ correspond to the waves which are oncoming from the positive and negative ends of the x-axis; A_n depend on the mode of motion. Making use of the relation $\nu = \nu_0 \tanh \nu_0 h$ and the formula of the wave amplitude ratios (18), we obtain the magnitude of E in conventional dimensionless forms:

$$\left.\begin{aligned} \frac{|E_H|}{\rho g\bar{h}B} &= \frac{\bar{A}_H}{\nu_0 B}\left(1 + \frac{2\nu_0 h}{\sinh 2\nu_0 h}\right) \\ \frac{|E_S|}{\rho g\nu_0\bar{h}S} &= \frac{\bar{A}_S}{\nu_0^2 S}\left(1 + \frac{2\nu_0 h}{\sinh 2\nu_0 h}\right) \\ \frac{|E_R|}{\rho g\nu_0\bar{h}ST} &= \frac{\bar{A}_R H}{\nu_0^2 S}\left(1 + \frac{2\nu_0 h}{\sinh 2\nu_0 h}\right) \end{aligned}\right\} \quad (23)$$

where

S = submerged section area
H = half beam draft ratio
T = draft

The phase differences between the exciting forces (moments) and both oncoming waves from right and left are obtained by comparing the phase angles of the wave elevations $Re\left[-\frac{i\omega}{g}\varphi_I e^{i\omega t}\right]_{y=0}$ with those of exciting forces (moments) $Re[E_H, E_S, E_R]$. These are for heave always α_H; while for sway and roll they are $\alpha_S \pm (\pi/2)$ and $\alpha_R \pm (\pi/2)$, where

$$\alpha_H = arg\left(\sum_{n=0}^{\infty} A_n H_n\right) \tag{24}$$

$$\begin{bmatrix}\alpha_S\\ \alpha_R\end{bmatrix} = arg\left(\sum_{n=0}^{\infty} A_n H_n'\right) \tag{25}$$

It should be noted that the minus sign of phase difference means the phase lag and vice versa. The value of A_n for sway and the value of A_n for roll should be different, but the numerical computation revealed that, for our low-frequency range, $\alpha_S = \alpha_R$. This is because the wave pressure distribution on a fixed cylinder consists of symmetrical and asymmetrical parts with respect to the y-axis: the symmetrical part contributes to heave-exciting force, and the asymmetrical part contributes to both sway- and roll-exciting forces; and both of the latter take the maximum values at the same instant for the low-frequency range. However, it should be noted that, since the y-axis is positive downward and the positive sense of moment is chosen to follow the right-handed system, then, when the sway-exciting force becomes positive maximum, the roll-exciting moment takes a negative maximum value. It is sufficient, therefore, to calculate heave- and sway-exciting forces together with the roll-exciting moment arm.

$$\frac{l_E}{T} = \frac{\bar{A}_R}{\bar{A}_S} H \tag{26}$$

Numerical Calculation and Discussion

By the transformation $x + iy = e^{i\Theta} + ae^{-i\Theta} + be^{-i3\Theta}$ the stream functions in the boundary condition [equation (13)] are represented as functions of three variables Θ, a, and b. The functions are expanded in finite trigonometric series on the cylinder contour, namely in even sine series for heave:

$$\psi_{nh}(a,b,\Theta) = b_{n0}\left(\frac{\pi}{2} - \Theta\right) + \sum_{m=1}^{4} b_{nm} \sin 2m\Theta$$

and in odd sine series for sway and roll:

$$\psi_{nh}(a,b,\Theta) = \sum_{m=1}^{4} c_{nm} \sin (2m - 1)\Theta$$

The boundary conditions are thus reduced to ten and eight simultaneous linear equation systems respectively, from which the source intensities are determined.

The adjusting potentials or stream functions are given in infinite trigonometric series (in the appendix), and in our calculation the first nine terms were taken. In the numerical integration of the singular functions $G_{2s+1}(\nu h)$ and $F_{2s+1}(\nu h)$ the limits were taken at $2\rho = 0$ and 30 and the increment of (2ρ) was 0.5 (see Fig. 2). The calculated forces and moments are represented as the functions of dimensionless frequency in the figures. The numerical calculations were carried out by the IBM 360 computer at Chalmers University.

Our discussion will be primarily concerned with the influence of the finite-depth effect upon the hydrodynamic behavior of forces and moments in the very low frequency range for both heave and sway.

Wave Length

To begin with, we consider the lengths of the waves generated by the oscillations on the surface of a limited water depth. The formula for shallow water wave number, $\nu = \nu_0 \tanh \nu_0 h$, reveals that in the low frequency range the relation $\nu_0 > \nu$ is valid. This explains that at the same frequency of oscillation the generated wave length of shallow water is shorter than that of deep water. It should be noted that the approximate relation $\nu_0/\nu \approx 1/(\nu_0 h)$ is valid at $\nu \to 0$ (Fig. 1).

Source Intensities

In our discussion we are obliged to depend on the calculated source strengths. For vanishing frequency $\nu \to 0$ we need to consider only the significant source intensity A_0. From Fig. 3 we see that firstly at $\nu = 0$ the non-dimensional source intensities for heave $A_{0r}\pi/B$ and $A_{0i}\pi/B$ approach unit and zero respectively for any depth of water; secondly, in the vicinity of $\nu \to 0$, the intensity $A_{0i}\pi/B$ for finite depth becomes higher than that of infinite depth as the depth decreases, while $A_{0r}\pi/B$ for finite depth decreases first and then increases as ν increases. Although Fig. 3 shows the calculated result of a circular cylinder, the foregoing statements are valid for any form of cylinder. As to the swaying oscillation, it is known from several calculations that the intensities A_0 for finite depth are generally higher than those of infinite depth when $\nu \to 0$.

Added Mass Coefficients

One may obtain the approximate formula of the added mass coefficient for heaving oscillation of a circular cylinder in the following form

$$\underset{\substack{\nu\to 0\\ h=h}}{C_H} \approx \underset{\substack{\nu\to 0\\ h\to\infty}}{C_H} + \frac{8}{\pi^2} G_{2s+1}(\nu h)\Big|_{\substack{s=0\\ \nu\to 0}}$$

$$\underset{\substack{\nu\to 0\\ h\to\infty}}{C_H} \approx \frac{8}{\pi^2}\left[-\ln\left(\frac{\nu B}{2}\right) - 0.577\right]$$

From the above formulae we see that the influence of the shallow-water effect on the behavior of added mass coefficients as $\nu \to 0$ may be explained by the behavior of the singular function $G_{2s-1}(\nu h)\Big|_{\substack{s=0\\ \to 0}}$ (see Figs. 4, 5, 6, and 7 in connection with 2). It is stated that C_H for finite depth is higher than that of infinite depth of water at $\nu \to 0$ and this is valid for any form of cylinder. Let us observe for a moment the added mass coefficient at high frequency. Figs. 5 and 7 illustrate the added mass co-

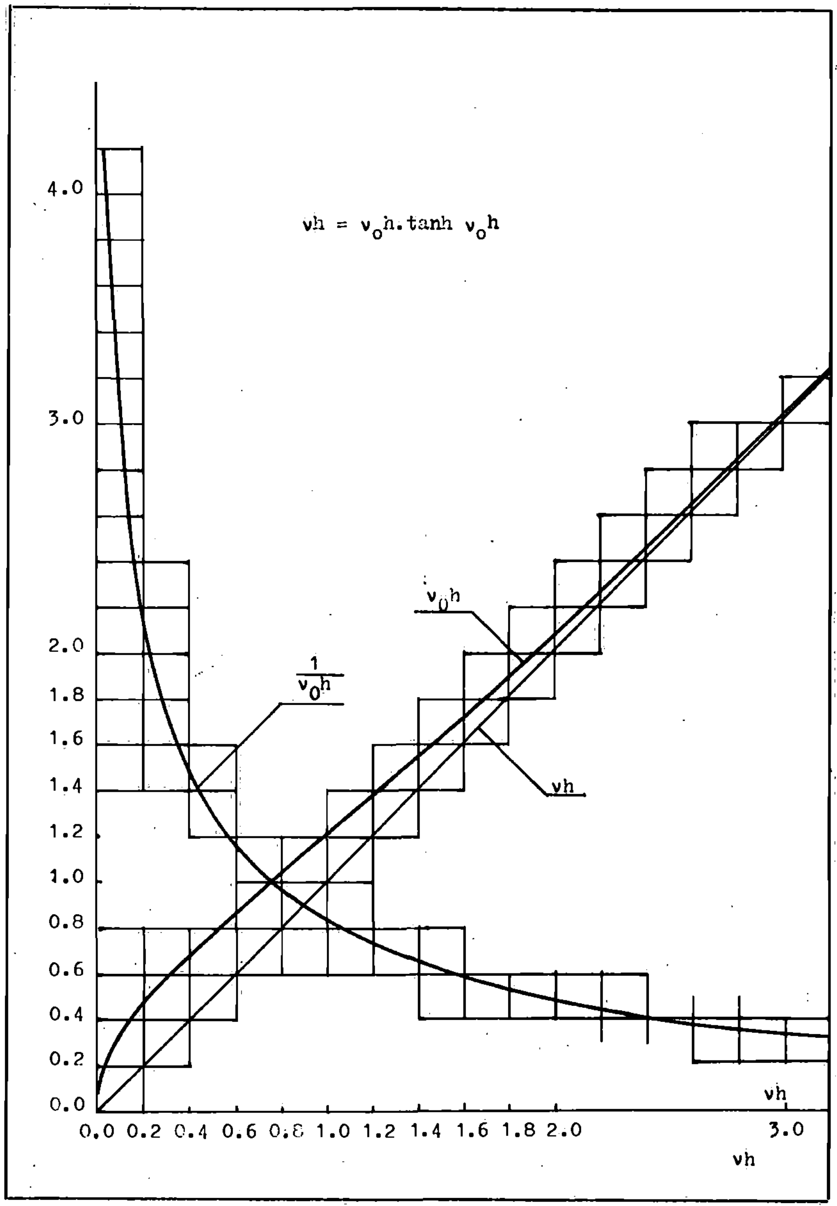


Fig. 1 Shallow-water wave number ν_0 as function of frequency ν and depth h

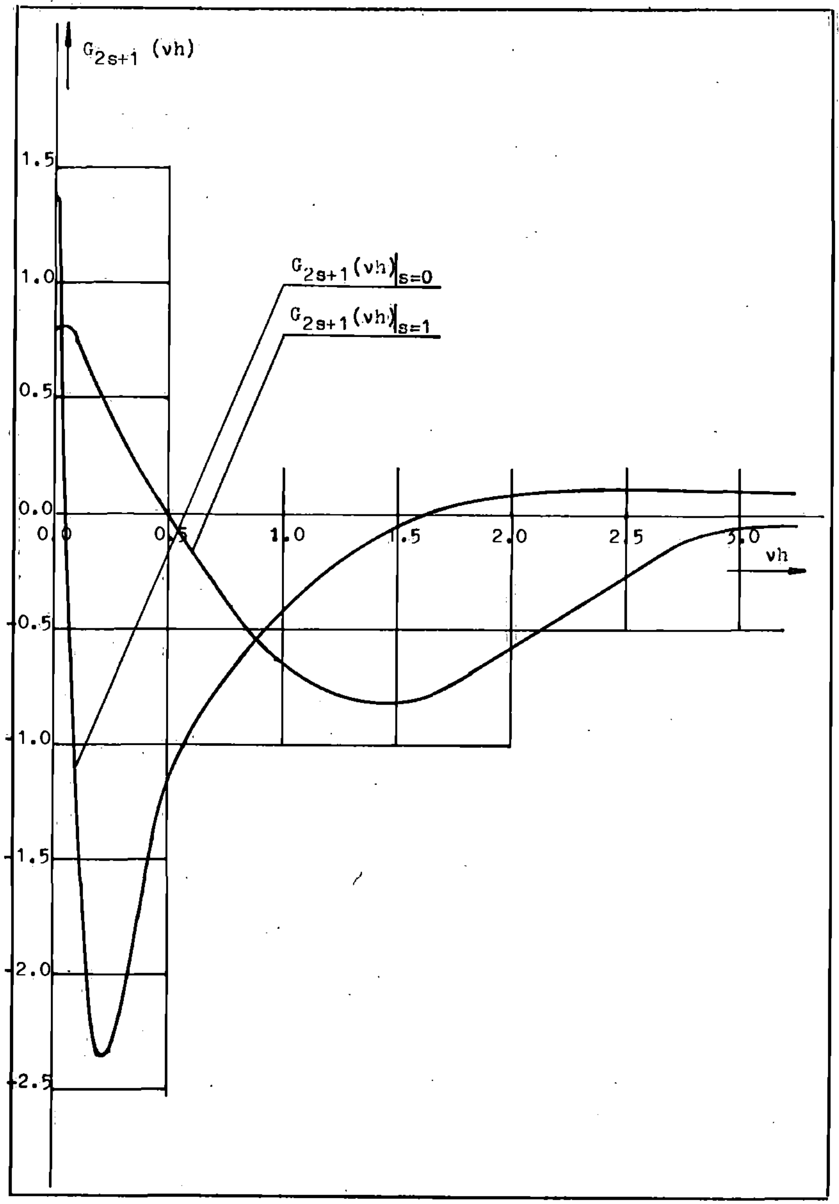


Fig. 2 Curves of the singular integral $G_{2s+1}(\nu h)$

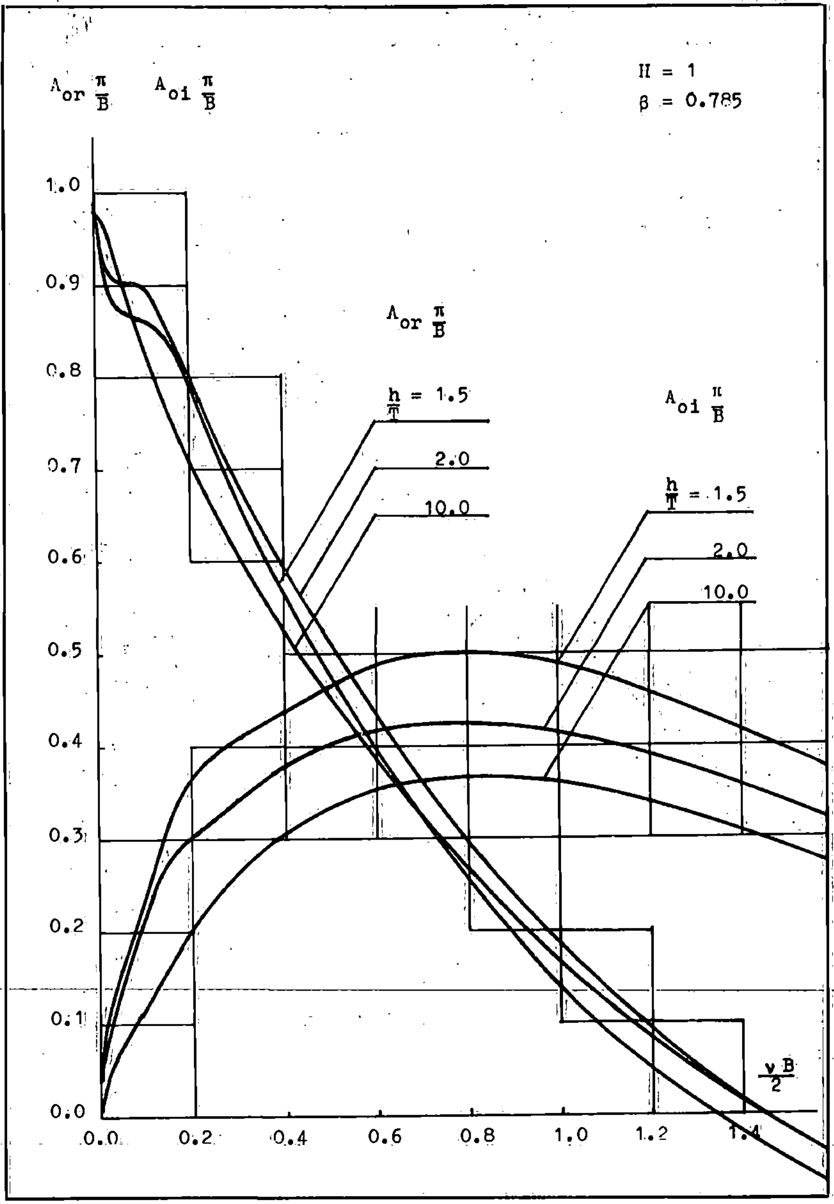


Fig. 3 Source intensities for heave

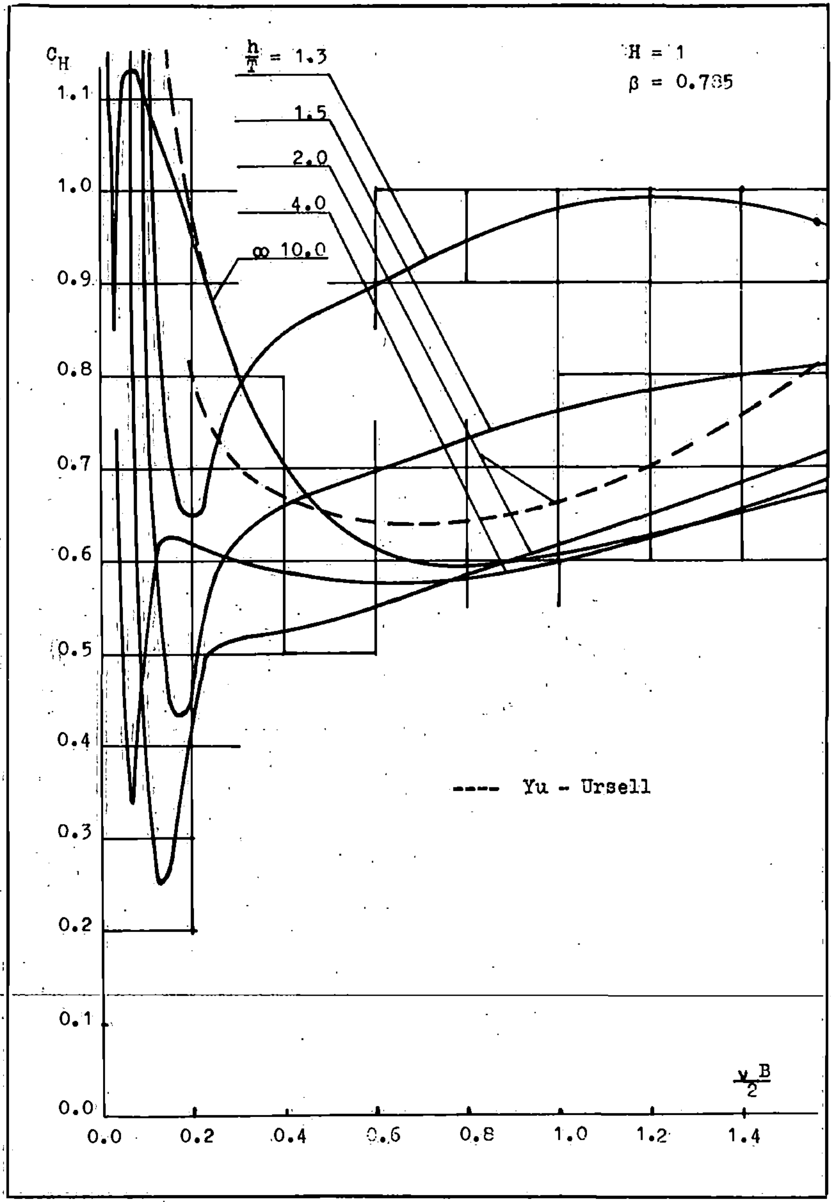


Fig. 4 Added mass coefficient for heave

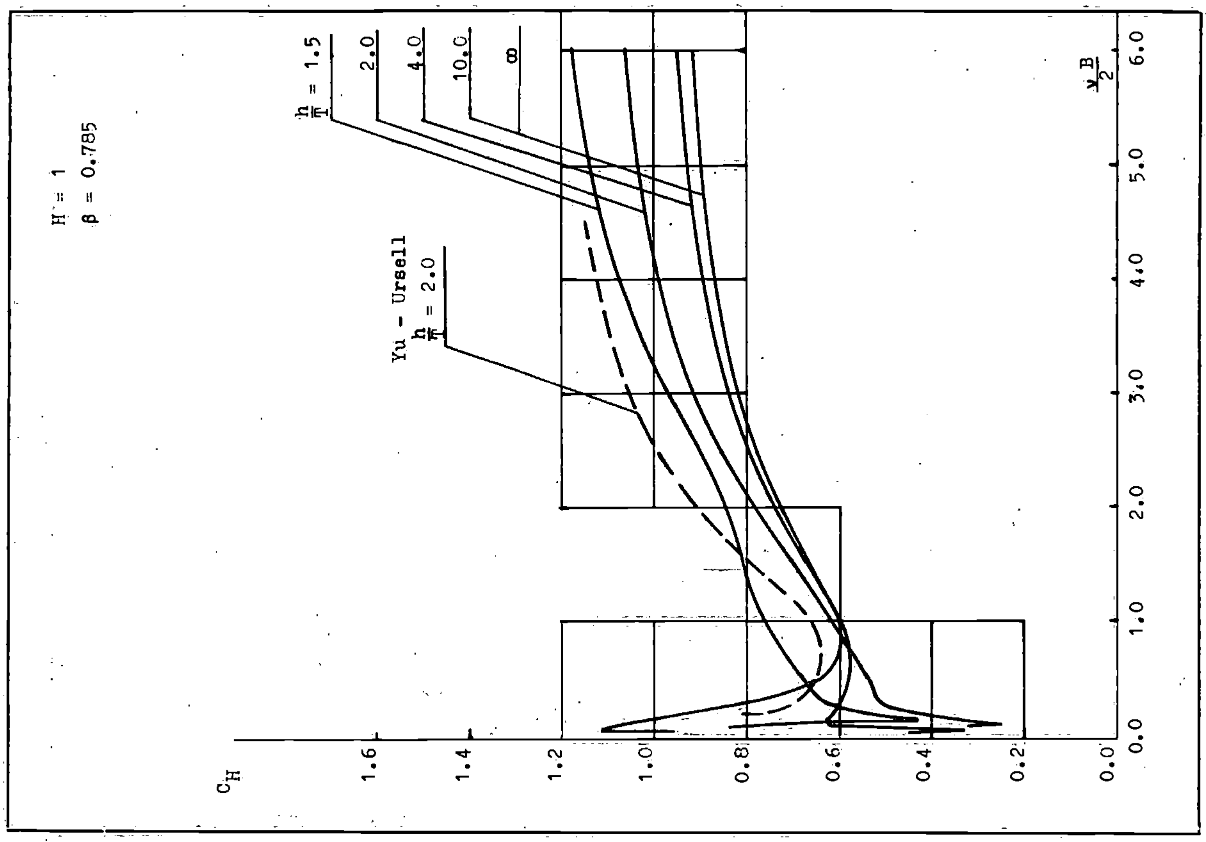


Fig. 5 Added mass coefficient for heave

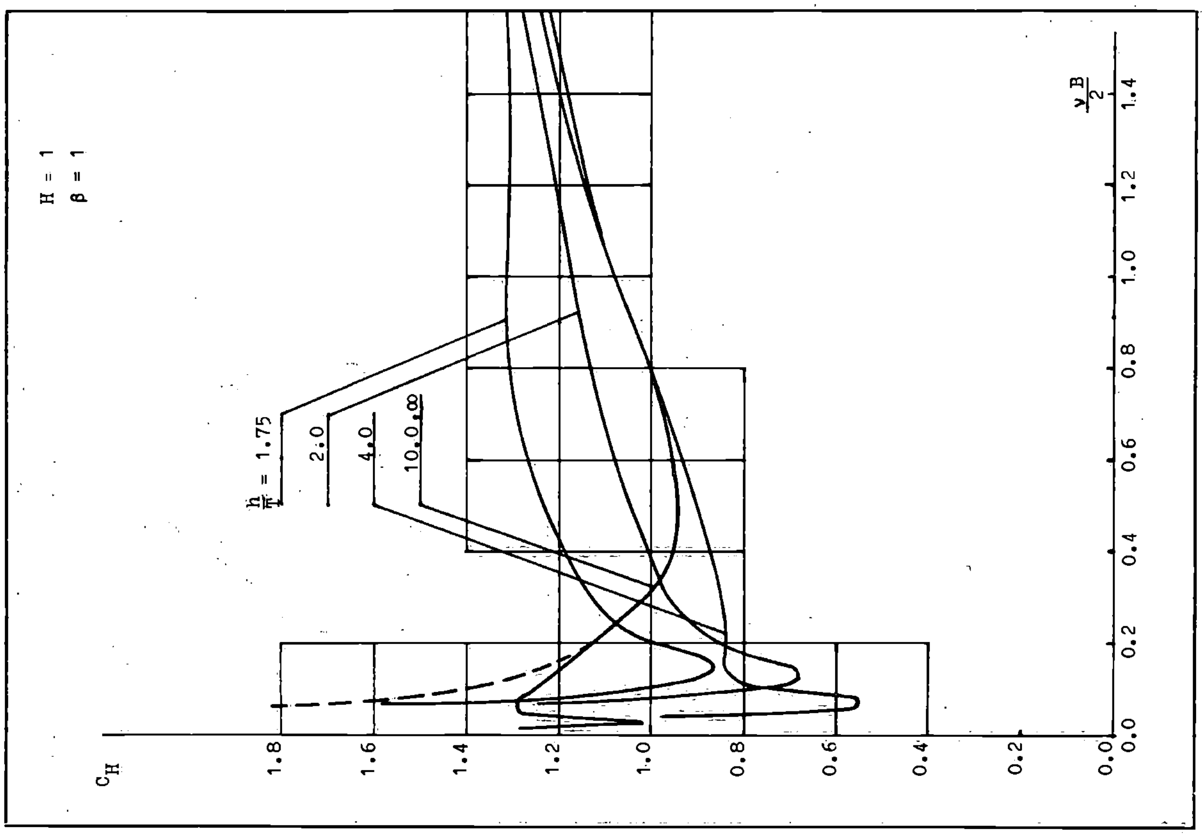


Fig. 6 Added mass coefficient for heave

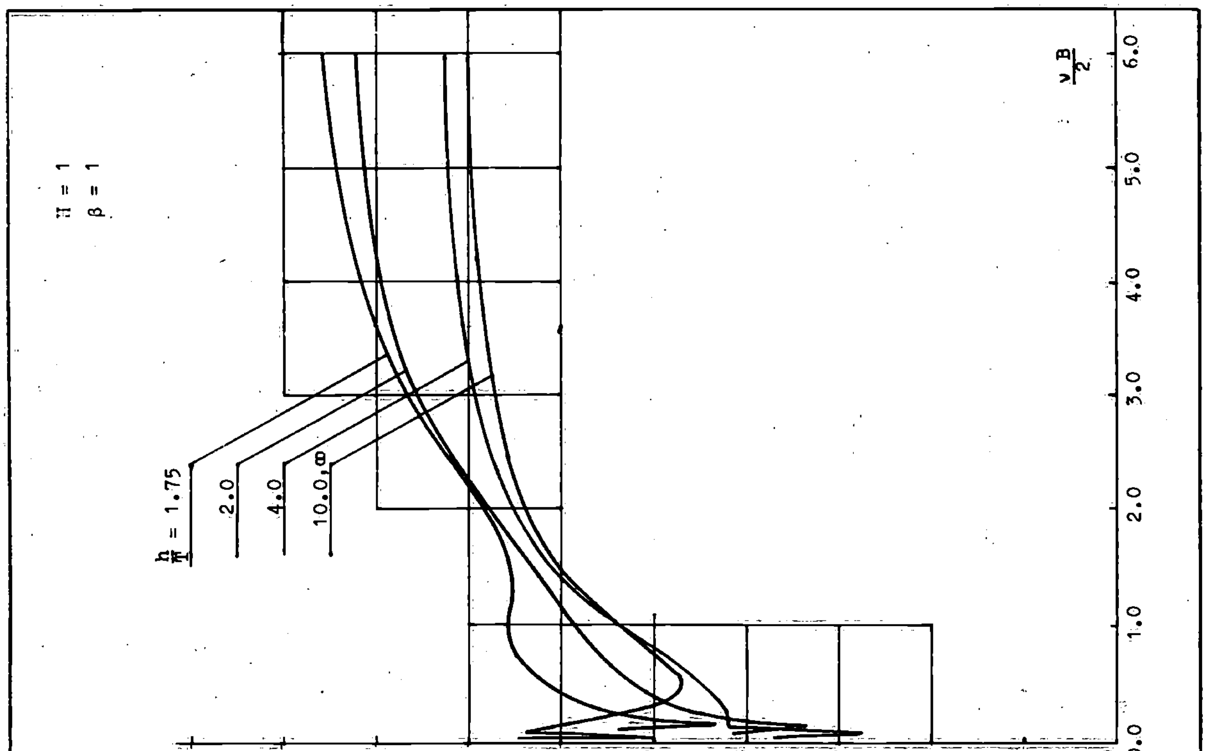


Fig. 7 Added mass coefficient for heave

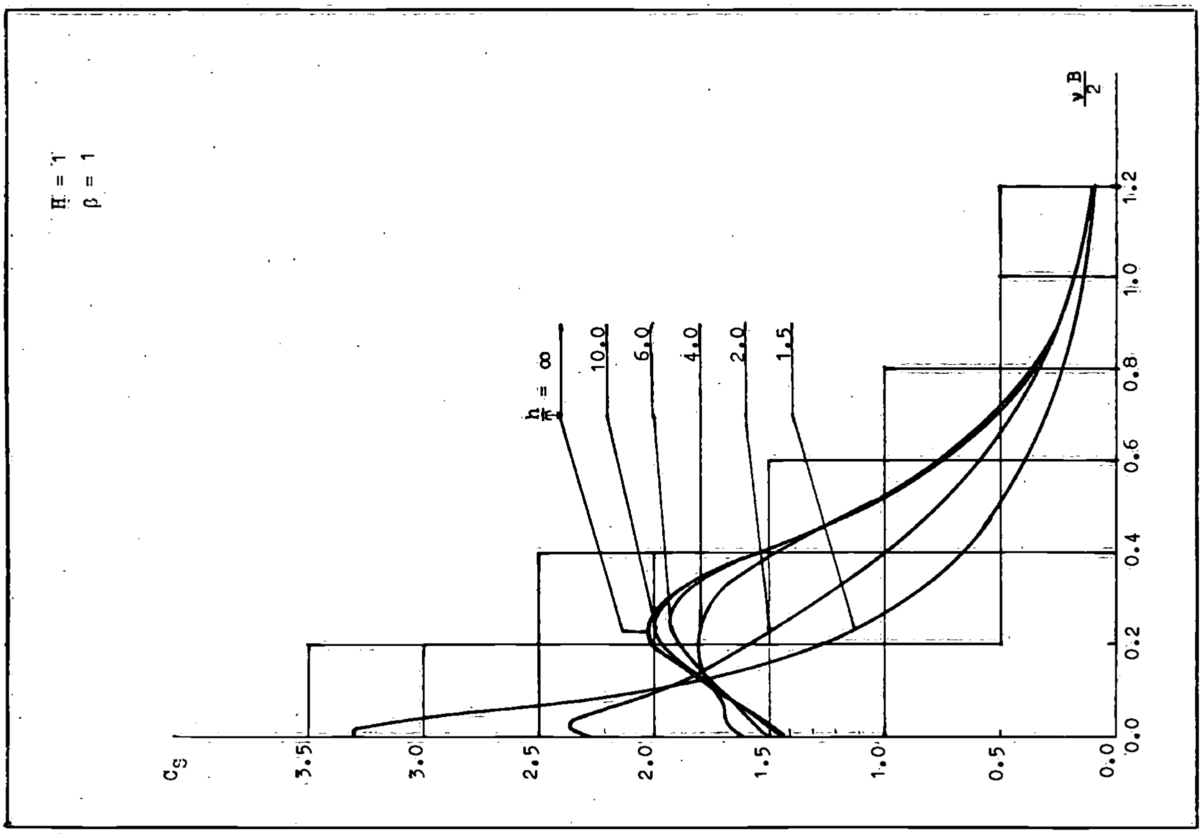


Fig. 8 Added mass coefficient for sway

efficients C_H at a wide frequency range. We see from them that at high frequency the values of C_H for finite depth are generally higher than those of deep water. and from Fig. 5 that the values of C_H tend to approach the known values of C_H for $\nu \rightarrow \infty$, as given by Havelock [11]; i.e.

$$\underset{h=h}{C_H} \approx \underset{h\rightarrow\infty}{C_H}\left[1 + \frac{1}{2}\left(\frac{T}{h}\right)^2\right]$$

It appears that at infinitely high frequency the value of C_H by Yu–Ursell tends to be higher than that given by Havelock ($C_H = 1.125$), while the author's C_H apparently approaches that by Havelock. It seems that Yu–Ursell's numerical calculation might not be accurate enough in comparison with the author's.

Fig. 7 also illustrates the tendency of C_H in the very high frequency range, which seems reasonable. Now we come to the discussion on the added mass coefficient C_S in swaying oscillation. For infinite depth of water at $\nu \rightarrow 0$ the exact formula of C_S is

$$C_S = \frac{(1 - a)^2 + 3b^2}{(1 - a + b)^2}$$

This value is also obtained numerically by applying the potential of infinite water depth at the vanishing frequency. i.e., $A_{0r}\sin\beta/r$.

For the finite depth of water at $\nu \rightarrow \infty$, on the other hand, the velocity potential is approximately written by

$$A_{0r}\left[\frac{\sin\beta}{r} + \frac{G_{2s+1}(\nu h)|_{s=1}}{h^2}\cdot r\cdot\sin\beta\right]$$

and since the source intensity A_{0r} for finite water depth is in general relatively higher than that of infinite depth and since we see the behavior of the singular function $G_{2s+1}(\nu h)|_{s=1}$ at $\nu \rightarrow 0$ (see Fig. 2), we come to the conclusion that C_S for finite depth is higher than that of infinite depth when $\nu \rightarrow 0$ (see Fig. 8) and the behavior of C_S in the vicinity of $\nu \rightarrow 0$ is caused by the contribution of the singular function $G_{2s+1}(\nu h)|_{s=1}$.

Wave Amplitude Ratio

The influence of the shallow-water effect on the wave amplitude ratio $\bar{A}_H$ and $\bar{A}_S$ at $\nu \rightarrow 0$ may be easily discussed by observing the asymptotic formula for $\nu \rightarrow 0$, namely, for heave:

$$\underset{h\rightarrow\infty}{\bar{A}_H} = 2\left(\frac{\nu B}{2}\right); \underset{h=h}{\bar{A}_H} = \left(\frac{1}{\nu_0 h}\right)_{\nu\rightarrow 0}\cdot\left(\frac{\nu B}{2}\right)$$

and for sway:

$$\underset{h\rightarrow\infty}{\bar{A}_S} = \frac{4\pi}{B^2}\sqrt{A_{0r}^2 + \underset{\nu\rightarrow 0}{A_{0i}^2}}\cdot\left(\frac{\nu B}{2}\right)^2; \quad \underset{h=h}{\bar{A}_S} = \frac{\pi}{Bh}\sqrt{A_{0r}^2 + \underset{\nu\rightarrow 0}{A_{0i}^2}}\cdot\left(\frac{\nu B}{2}\right)$$

The slopes of the curves $\underset{h\rightarrow\infty}{\bar{A}_H}$ and $\underset{h=h}{\bar{A}_H}$ at the origin are respectively 2 and $\left(\frac{1}{\nu_0 h}\right)_{\nu\rightarrow 0}$. The value of the latter is infinitely large (Fig. 1). It is obvious therefore that the amplitude ratio for shallow water is higher than that of deep water at $\nu \rightarrow 0$ (Figs. 10, 11, 12). Further, it is stated that the that of amplitude ratio for shallow water is higher than deep water throughout the whole frequency range (Fig. 11). This is due to the contribution of the higher-order potentials to forming the progressive wave system [equation (18)].

As to the amplitude ratio for sway, it is easily observed from the above formulae that, at $\nu \rightarrow 0$, $\underset{h\rightarrow\infty}{\bar{A}_S}$ is a second-order parabola with zero slope, while $\underset{h=h}{\bar{A}_S}$ is a straight line with finite positive slope; thus both curves must cross each other (Figs. 13, 14). It should be noted that the coefficients for roll, C_R and $\bar{A}_R$, must have similar characters to those of sway, for both oscillations are essentially similar from the point of view of antisymmetry about the y-axis (Figs. 9, 14).

Added Moment and Damping Moment Arm

The added moment arms l_{Sr}/T and l_{Rr}/T for shallow water are generally shorter than those of deep water at $\nu \rightarrow 0$, but at the other low-frequency range in the vicinity of $\nu \rightarrow 0$ the opposite phenomena appear (Figs. 15, 16). The damping moment arms l_{Si}/T and l_{Ri}/T and the exciting moment arm l_E/T coincide with each other and decrease as the depth of water decreases (Fig. 17).

Exciting Forces

In the preceding section the discussion was confined to the Haskind—Newman method. We can also easily calculate the exciting forces and moments by Grim's method. By applying the latter method the same calculations were carried out and both results were compared. It was found that they were in good agreement. In this report the results by the Haskind—Newman method were represented in Figs. 18, 19, and 20. It is obvious from the wave amplitude ratios $\bar{A}_H$ (Figs. 10–14) that the heave exciting forces of shallow water are generally higher than those of deep water. Similarly, the explanation of the behavior of the sway-exciting force may be obtained from the behavior of the wave amplitude ratio for sway, $\bar{A}_S$ [equation (23)]. The exciting moment arm l_E/T is represented in Fig. 17.

Minimum Depth

To find the minimum allowable depth parameter h/T for which our solution converges, the calculation has been carried out for several widely varying forms of cylinders. From the comparison of the results it was concluded that:

1 For deep-draft cylinders the minimum parameters are very low.

2 For shallow-draft cylinders the values of minimum parameter are relatively high.

3 The influence of fullness coefficient β on the mini-

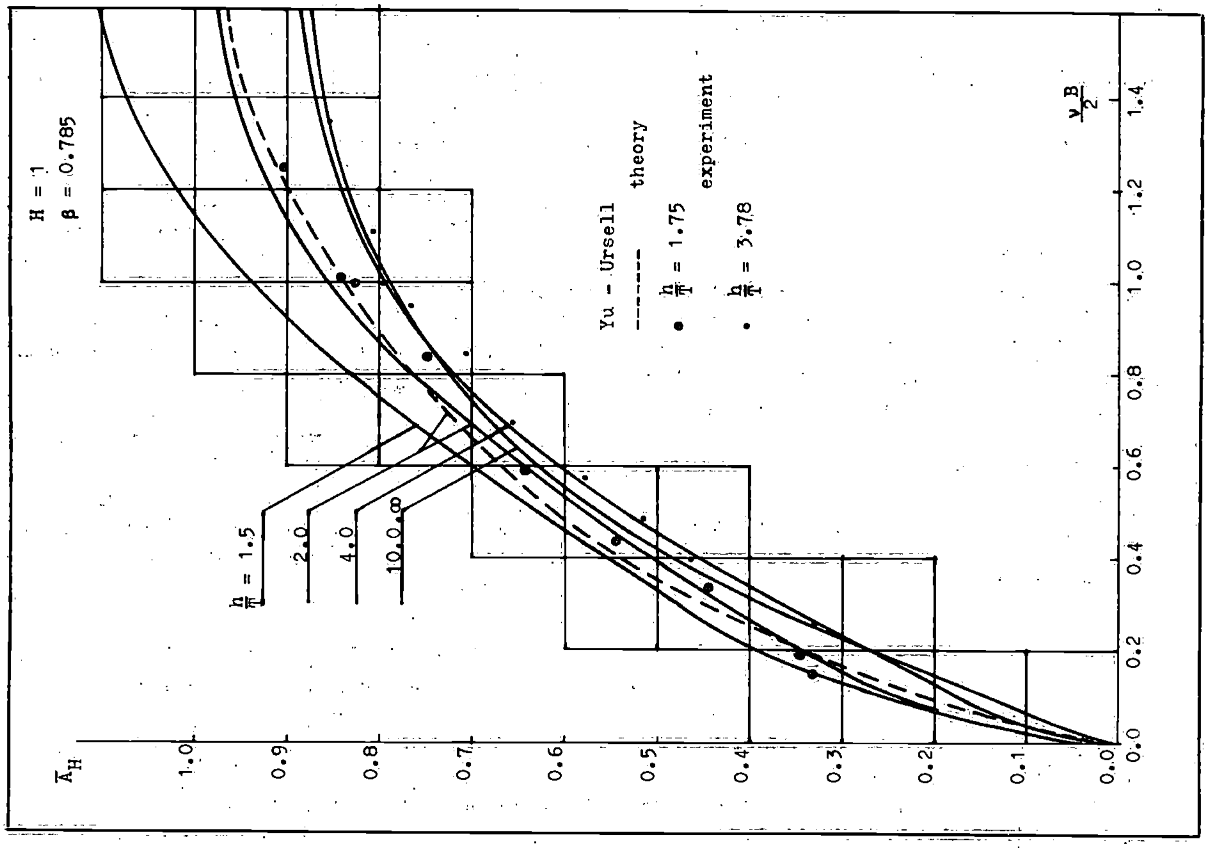


Fig. 10 Wave amplitude ratio for heave

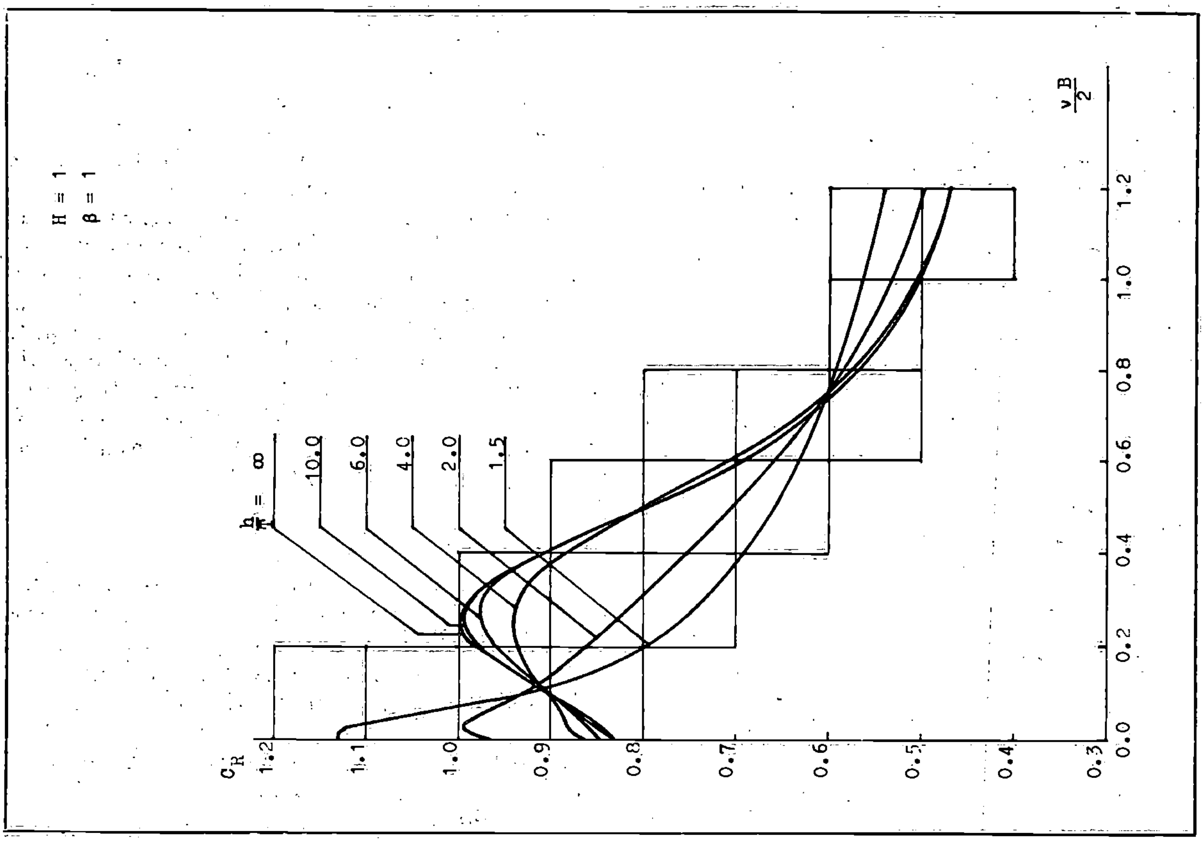


Fig. 9 Added moment of inertia for roll

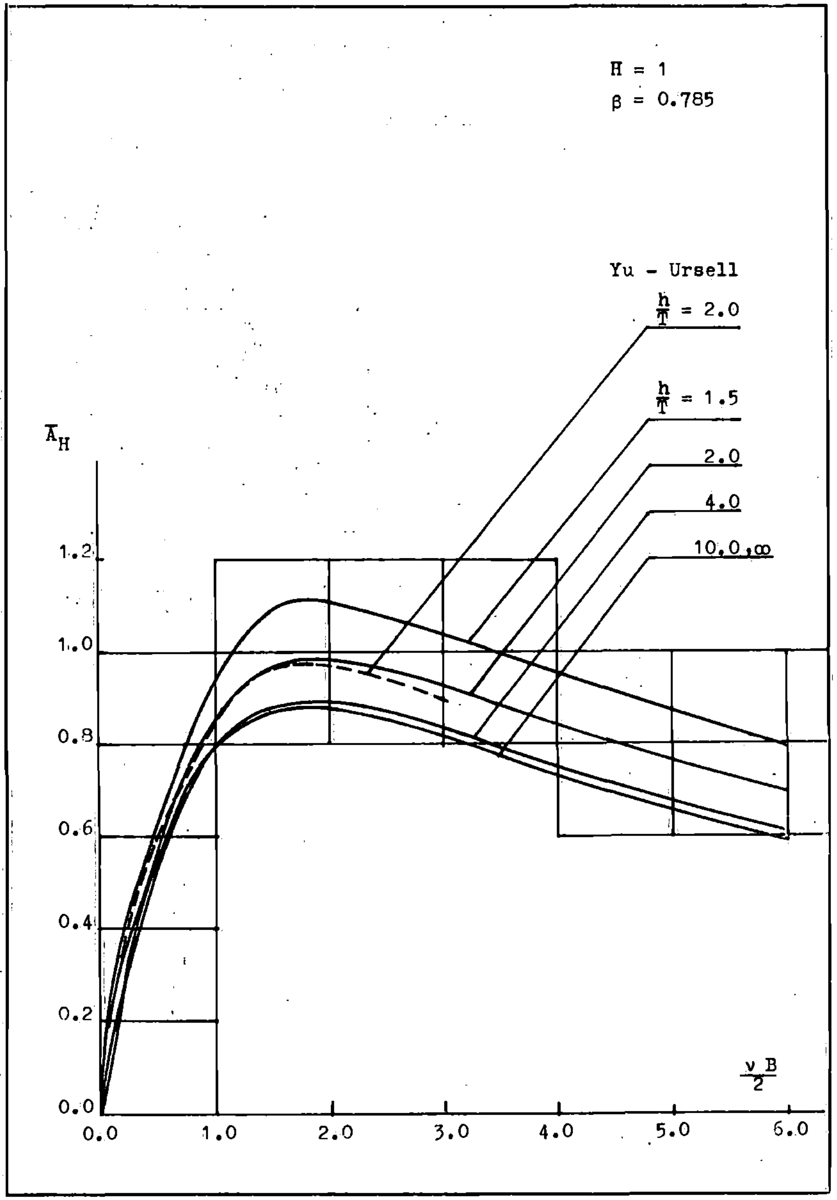


Fig. 11 Wave amplitude ratio for heave

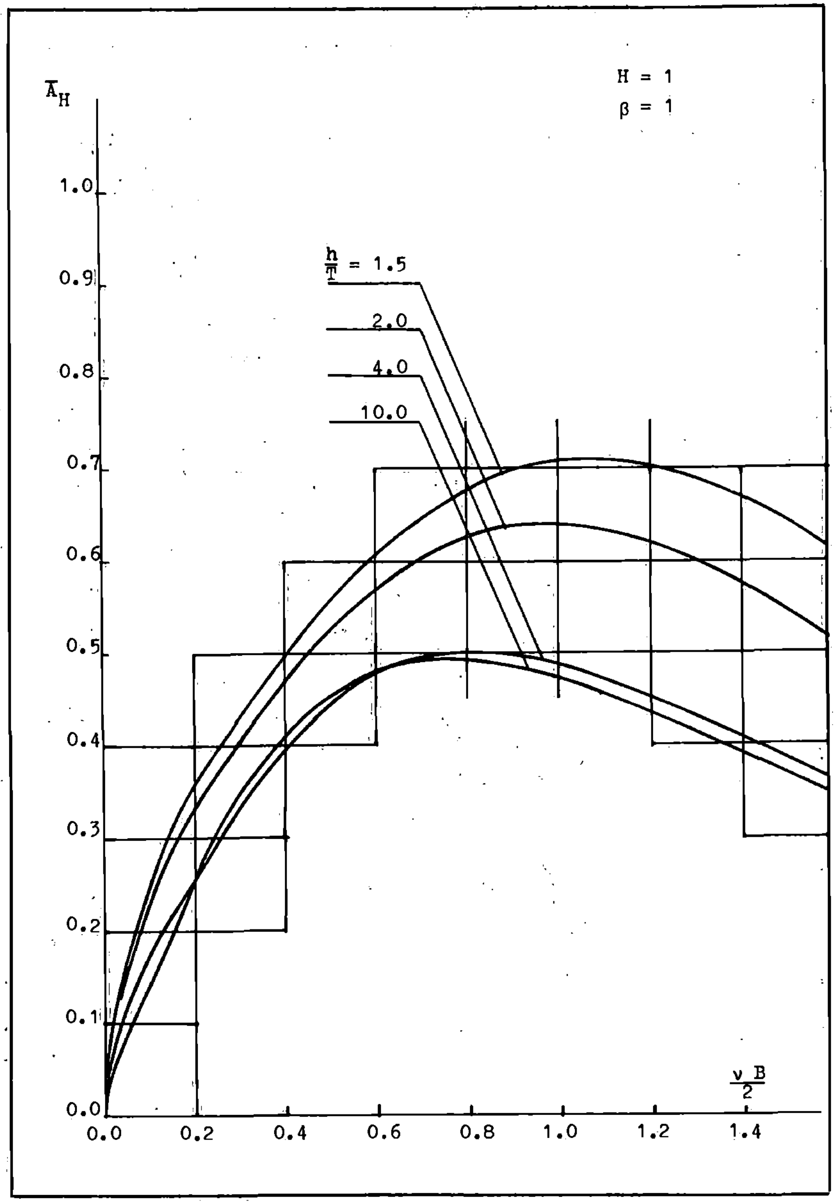


Fig. 12 Wave amplitude ratio for heave

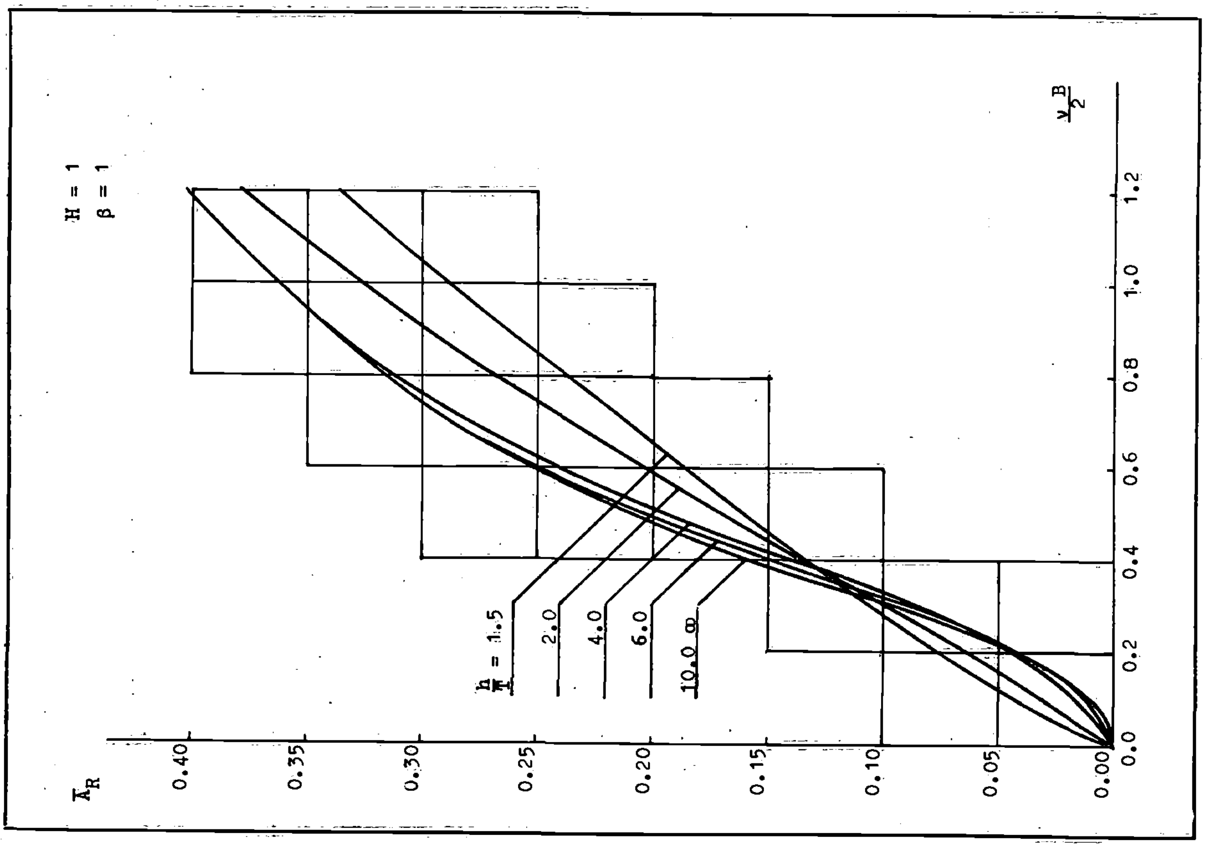


Fig. 14 Wave amplitude ratio for roll

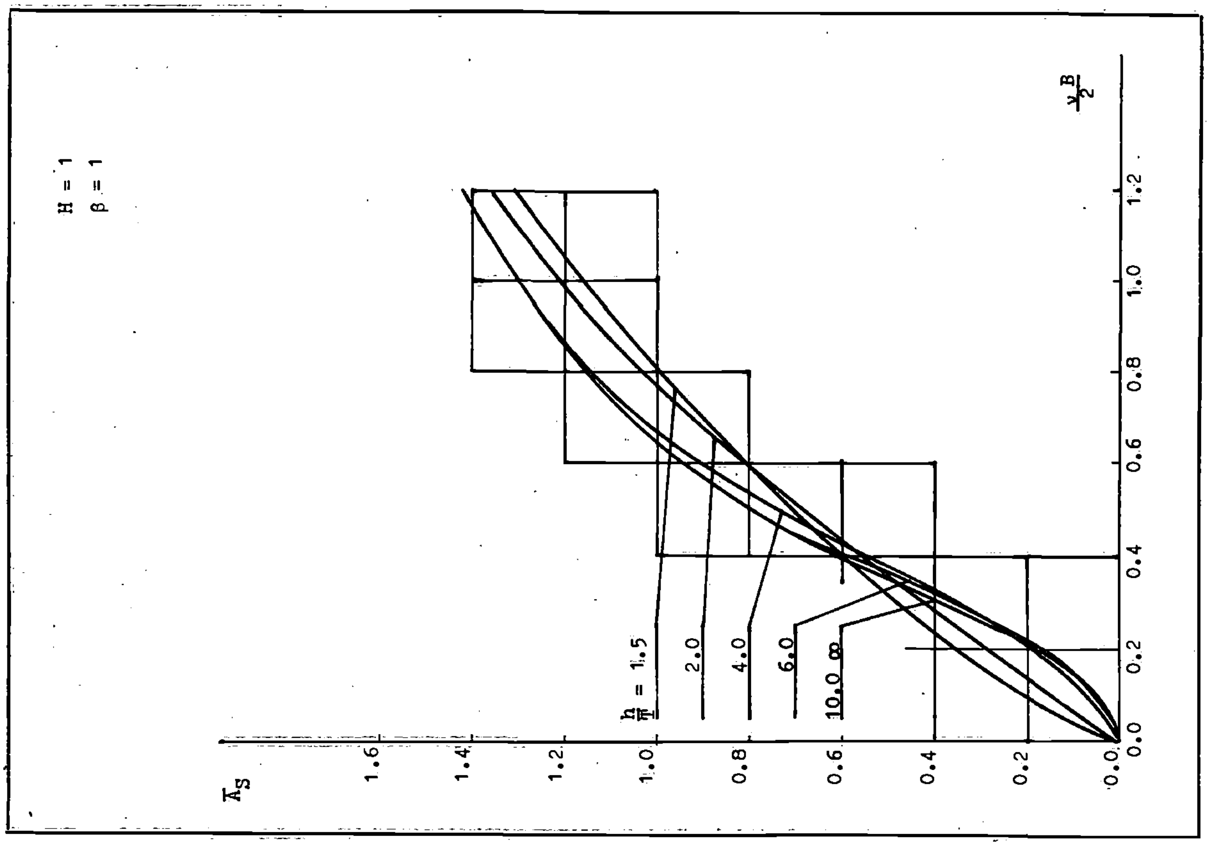


Fig. 13 Wave amplitude ratio for sway

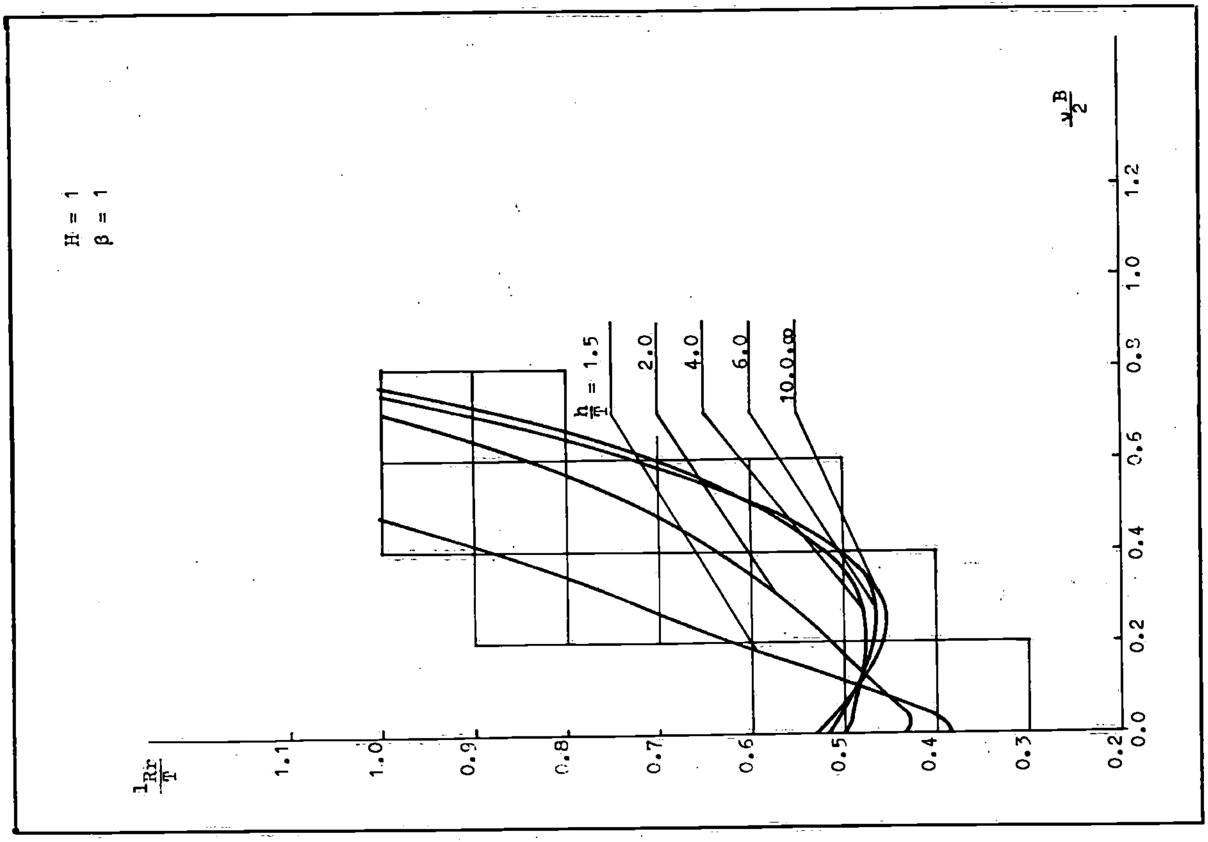


Fig. 16 Added moment arm for roll

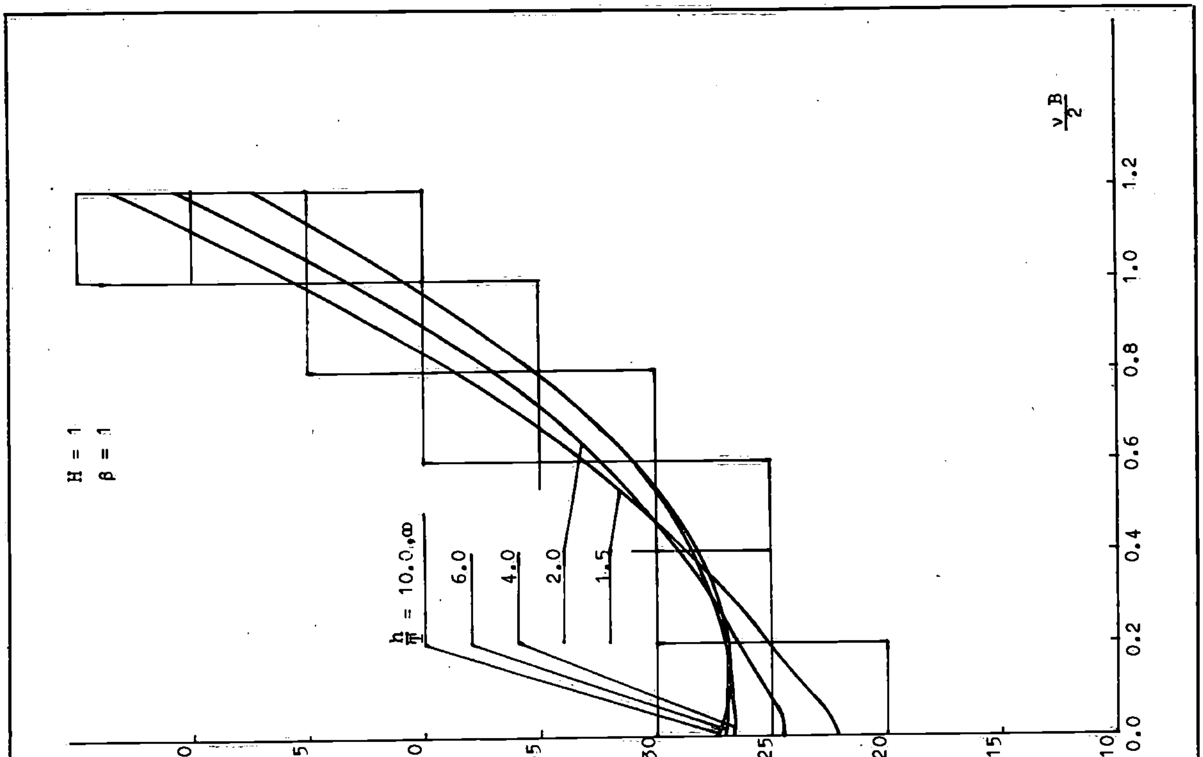


Fig. 15 Added moment arm for sway

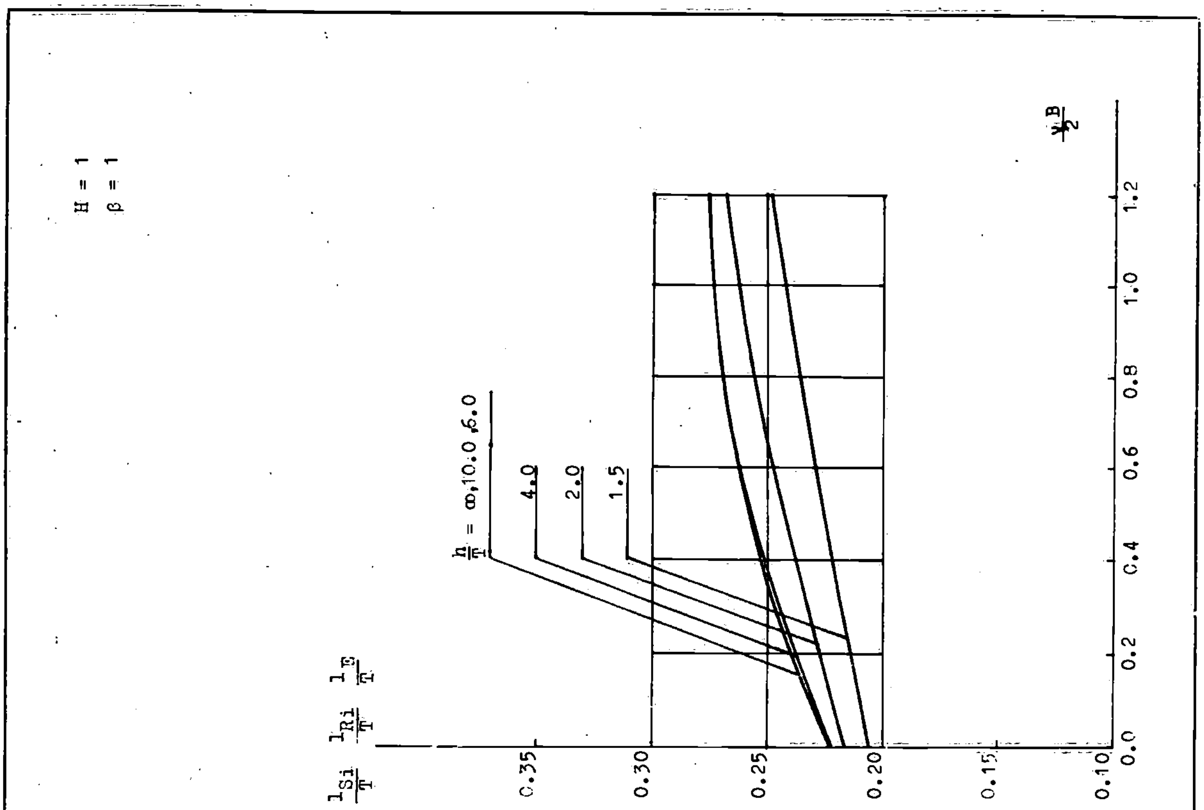


Fig. 17 Damping moment arm for sway and roll and exciting moment arm

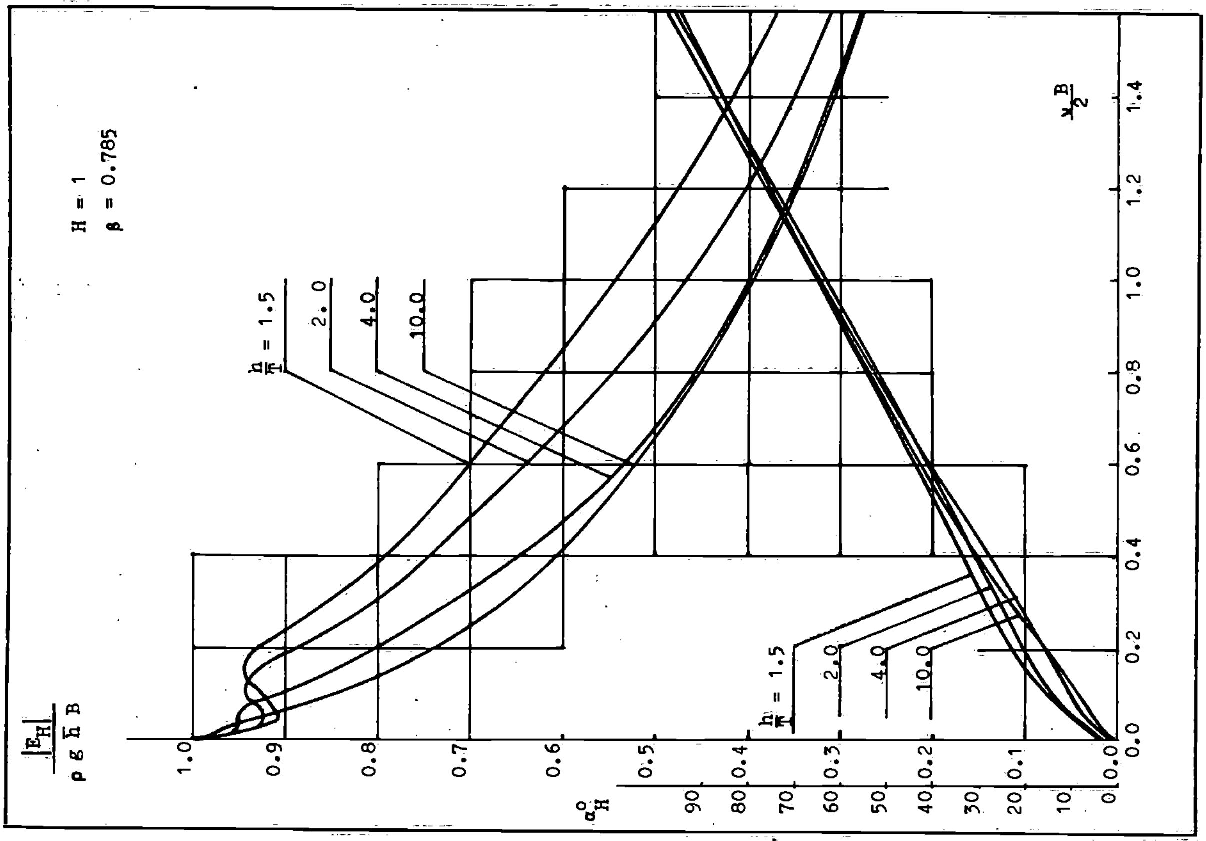


Fig. 18 Heave exciting force coefficient

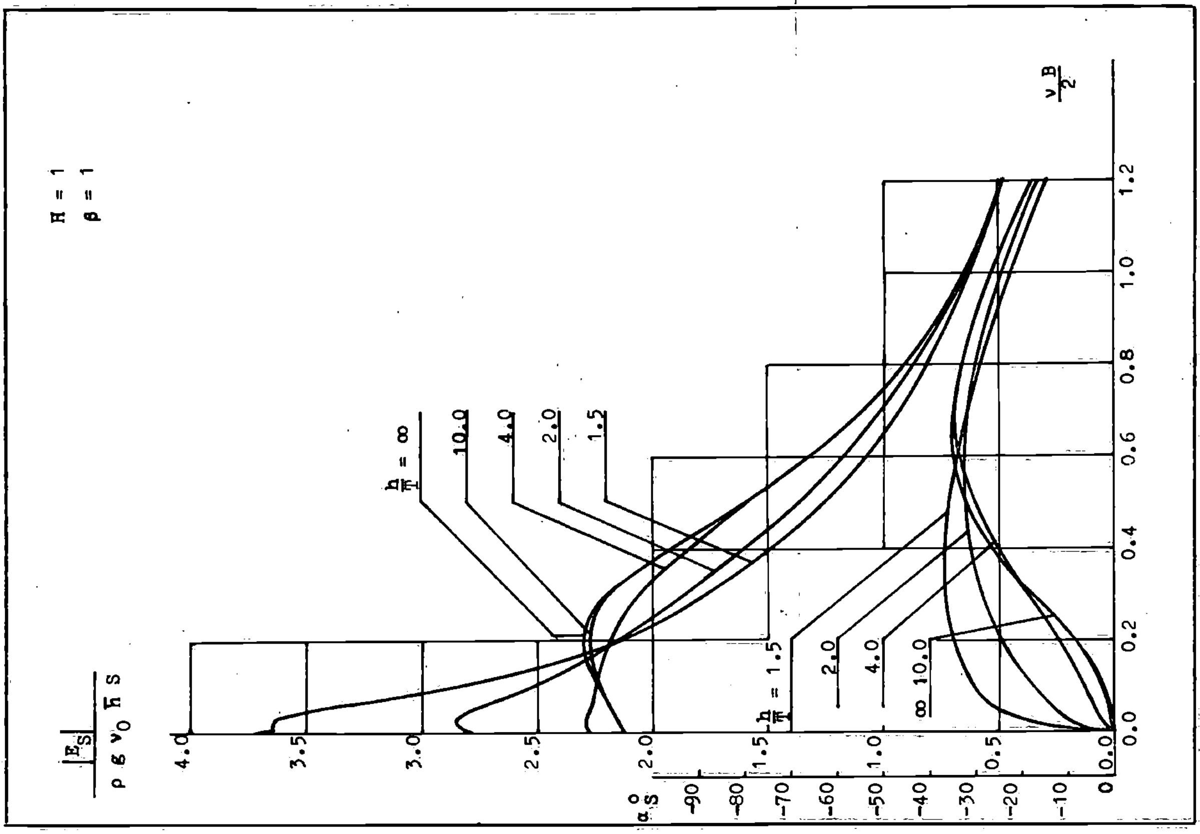


Fig. 20 Sway exciting force coefficient

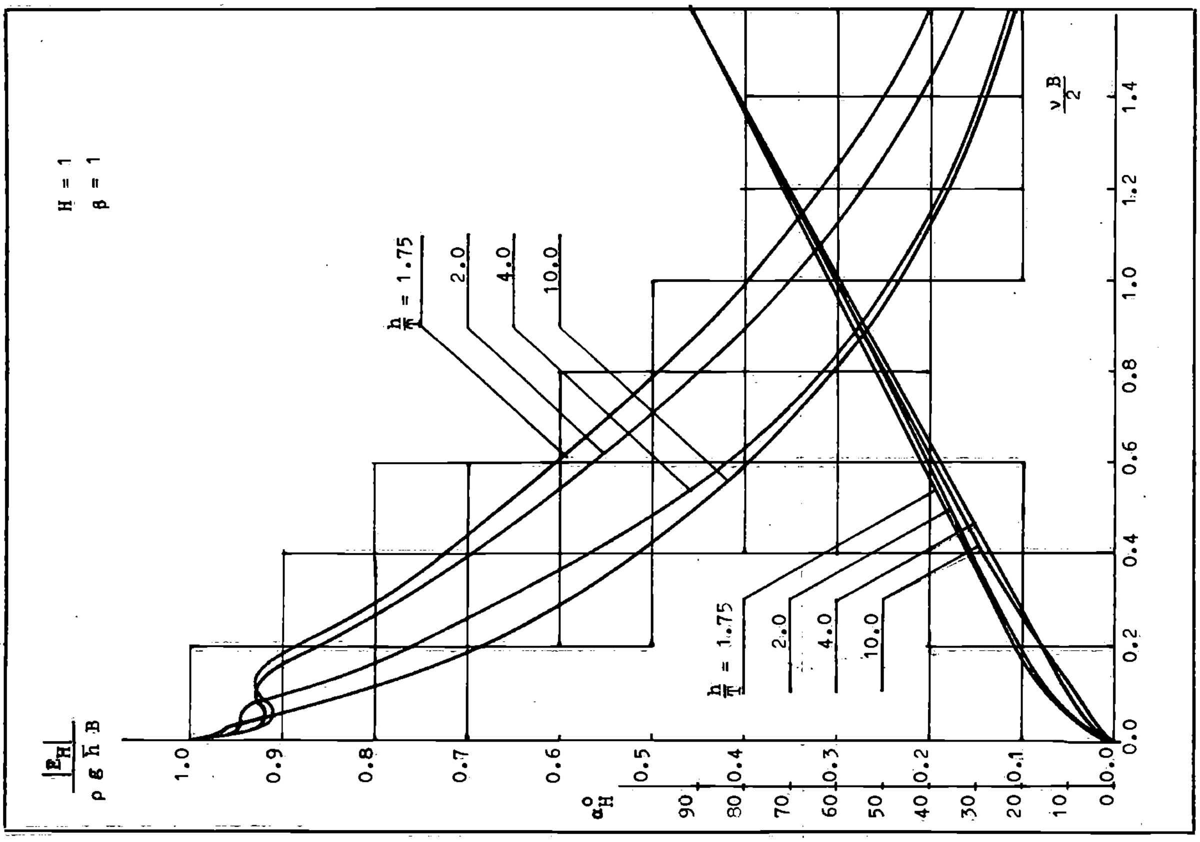


Fig. 19 Heave exciting force coefficient

mum allowable depth is also significant when the shallow-draft cylinders are considered.

4 The minimum parameters for sway and roll are lower than those of heave.

Conclusion

In general, the influence of the shallow-water effect on the hydrodynamic forces and moments is remarkable. The curve of the added mass coefficient $C(C_H, C_S, C_R)$ for finite depth generally crosses the curve of C for infinite depth twice, and thus the two crossing points divide the frequency range into three parts. The values of $C_{h=h}$ are first higher, then lower, and then higher again than those of $C_{h\to\infty}$ at the first and second and third frequency ranges respectively. As to the wave damping or wave amplitude ratio $\bar{A}$, $\bar{A}_{H\, h=h}$ is generally higher than $\bar{A}_{H\, h\to\infty}$ throughout the whole frequency range, while $(\bar{A}_S$ or $\bar{A}_R)_{h=h}$ is higher and then lower than $(\bar{A}_S$ or $\bar{A}_R)_{h\to\infty}$ at $\nu \to 0$ and in the vicinity of $\nu \to 0$ respectively.

Acknowledgments

The author expresses sincerely his gratitude to Professor Falkemo, the head of the Institute of Ship Hydromechanics, who supported this work.

Above all, the author is deeply indebted to Professor Grim for furnishing his unpublished computer program for heave and pitch of ships in regular waves, and to Professor Porter, who allowed the author to copy the work of Yu and Ursell from his own library and willingly helped him by sending the recent report on a heave theory of a sphere in shallow water by Dr. Wang.

This work has been financially supported by the Swedish Council for Applied Research.

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Appendix

Adjusting Potentials and Stream Functions for Heave

$$\varphi_{0r\ ad.} = \sum_{s=0}^{\infty} \frac{1}{(2s)!} \frac{G_{2s+1}(\nu h)}{h^{2s}} r^{2s} \cos 2s\beta - \nu \sum_{s=0}^{\infty} \frac{1}{(2s+1)!} \frac{G_{2s+1}(\nu h)}{h^{2s}} r^{2s+1} \cos (2s+1)\beta$$

$$\psi_{0r\ ad.} = \nu \sum_{s=0}^{\infty} \frac{1}{(2s+1)!} \frac{G_{2s+1}(\nu h)}{h^{2s}} r^{2s+1} \sin (2s+1)\beta - \sum_{s=0}^{\infty} \frac{1}{(2s+2)!} \frac{G_{2(s+1)+1}(\nu h)}{h^{2s+2}} r^{2s+2} \sin (2s+2)\beta$$

$$\varphi_{nr\ ad.} = -\frac{(-1)^n}{(2n-1)!} \sum_{s=0}^{\infty} \frac{1}{(2s)!} \frac{F_{2n+2s-1}(\nu h)}{h^{2n+2s}} r^{2s} \cos 2s\beta + \frac{(-1)^n}{(2n-1)!} \nu \sum_{s=0}^{\infty} \frac{1}{(2s+1)!} \frac{F_{2n+2s-1}(\nu h)}{h^{2n+2s}} r^{2s+1} \times \cos (2s+1)\beta$$

$$\psi_{nr\ ad.} = -\frac{(-1)^n}{(2n-1)!} \nu \sum_{s=0}^{\infty} \frac{1}{(2s+1)!} \frac{F_{2n+2s-1}(\nu h)}{h^{2n+2s}} \times r^{2s+1} \sin (2s+1)\beta + \frac{(-1)^n}{(2n-1)!} \sum_{s=0}^{\infty} \frac{1}{(2s+2)!} \times \frac{F_{2n+2s+1}(\nu h)}{h^{2n+2s+2}} r^{2s+2} \sin (2s+2)\beta$$

Adjusting potentials and stream functions for sway and roll

$$\varphi_{0r\ ad.} = -\nu \sum_{s=0}^{\infty} \frac{G_{2s+3}(\nu h)}{(2s+2)!\, h^{2s+2}} r^{2s+2} \sin (2s+2)\beta + \sum_{s=0}^{\infty} \frac{G_{2s+3}(\nu h)}{(2s+1)!\, h^{2s+2}} r^{2s+1} \sin (2s+1)\beta$$

$$\psi_{0r\,ad.} = -\nu \sum_{s=0}^{\infty} \frac{G_{2s+1}(\nu h)}{(2s)!\ h^{2s}} r^{2s} \cos 2s\beta + \sum_{s=0}^{\infty} \frac{G_{2s+3}(\nu h)}{(2s+1)!\ h^{2s+2}} r^{2s+1} \cos (2s+1)\beta$$

$$\varphi_{nr\,ad.} = -\frac{\nu}{(2n)!} \sum_{s=0}^{\infty} \frac{F_{2n+2s+1}(\nu h)}{(2s+2)!\ h^{2n+2s+2}} r^{2s+2} \times \sin (2s+2)\beta + \frac{1}{(2n)!} \sum_{s=0}^{\infty} \frac{F_{2n+2s+1}(\nu h)}{(2s+1)!\ h^{2n+2s+2}} \times r^{2s+1} \sin (2s+1)\beta$$

$$\psi_{nr\,ad.} = -\frac{\nu}{(2n)!} \sum_{s=0}^{\infty} \frac{F_{2n+2s-1}(\nu h)}{(2s)!\ h^{2n+2s}} r^{2s} \cos 2s\beta + \frac{1}{(2n)!} \sum_{s=0}^{\infty} \frac{F_{2n+2s+1}(\nu h)}{(2s+1)!\ h^{2n+2s+2}} r^{2s+1} \cos (2s+1)\beta$$

$$G_{2s+1}(v) = \oint_0^{\infty} \frac{e^{-u} u^{2s+1}}{(v-u)(v \cosh u - u \sinh u)}\, du$$

$$F_{2s+1}(v) = \oint_0^{\infty} \frac{u^{2s+1}(u+v)e^{-u}}{v \cosh u - u \sinh u}\, du$$

The expansion of the foregoing Caucy integrals for numerical calculations are obtained from Reference [5].

The Influence of Water Depth on the Heaving and Pitching Motions of a Ship Moving in Longitudinal Regular Head Waves

Cheung H. Kim

Abstract

The heaving and pitching motions of a Series 60 model of $_B = 0.7$ moving in longitudinal regular head waves of shal-
ɔw water are calculated by Watanabe's strip method [1], [2],
3]. The results are represented in Figures and the shallow
ater effect is discussed.

Introduction

By applying Watanabe's strip theory some important hydro-
ynamic forces and moments acting on a Series 60 model of
$_B = 0.7$ moving in a longitudinal head wave system of shal-
ɔw water are calculated. The calculated results are represen-
ed in non-dimensional forms and shown in Figs. A.

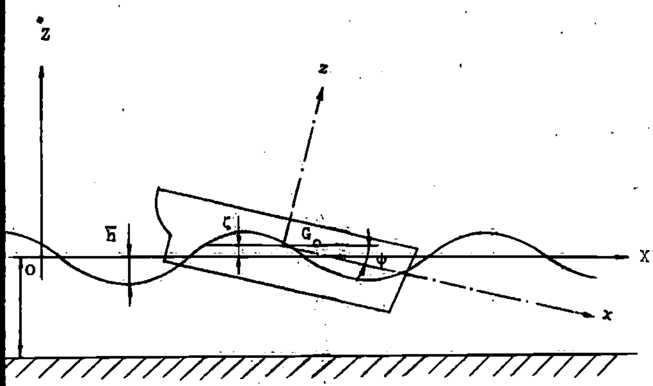
Subsequently, heaving and pitching motions of the ship are
alculated and the results are presented in Figures B.

The influence of shallow water depth on the hydrodynamic
ɔrces and motions as shown in the figures are discussed.

It is revealed by the calculations that the heaving and pitch-
ıg motions are remarkably damped by the shallow water
ffect.

Definition of Ship Motions and Waves

The coordinate systems here utilized are space- and body-
ɔordinate system 0—XYZ and G_0-xyz respectively. X-axis lies
n the undisturbed water surface and Z-axis points vertically
pward. x-axis is longitudinal passing through the center of
ravity G_0 of the ship, while y- and z-axis point port and up-
ard, respectively. The coordinate system Go-xyz coincides
ith the system 0—XYZ at the initial rest condition. We follow
ıe convention of right-handed coordinate system.



Assuming only heaving and pitching motions of a ship
the speed V in a longitudinally oncoming wave system, we
escribe the surface wave as follows

$$\zeta_W = \bar{h} \cos (\nu_0 x + \omega_{et}) , \qquad (1)$$

here

- $\bar{h}$ wave amplitude
- ν_0 shallow water wave number, i. e. ($\omega^2/g = \nu_0 \tanh \nu_0 h$)
- ω_e circular frequency of encounter i. e. ($\omega + \nu_0 V$) .

The heaving and pitching motions of the ship corresponding to the wave defined above are then expressed by

$$\begin{aligned} \zeta &= \zeta_a \cos (\omega_e t + \varepsilon_{\zeta w}) \\ \psi &= \psi_a \cos (\omega_e t + \varepsilon_{\psi w}) \end{aligned} \qquad (2)$$

respectively, where ζ_a, ψ_a are heave and pitch amplitudes and $\varepsilon_{\zeta w}$, $\varepsilon_{\psi w}$ phase differences between heave and wave and pitch and wave, respectively.

The Coupled Equations and Coefficients

The coupled equations of heave and pitch of a ship moving in longitudinal regular waves [1], [2] are written in the form

$$\begin{aligned} a\ddot{\zeta} + b\dot{\zeta} + c\zeta - d\ddot{\psi} - e\dot{\psi} - g\psi &= F_a \cos (\omega_e t + \varepsilon_{FW}) \\ A\ddot{\psi} + B\dot{\psi} + C\psi - D\ddot{\zeta} - E\dot{\zeta} - G\zeta &= M_a \cos (\omega_e t + \varepsilon_{FW}) \end{aligned} \qquad (3)$$

The coefficients on the left-hand sides of the above equations are

$$\begin{aligned}
a &= \varrho\nabla + \int_L m'' \, dx \\
b &= \int_L N \, dx \\
c &= 2\varrho g \int_L y_w \, dx \\
d &= \int_L m'' x \, dx + \varrho \int_L S_w x \, dx \\
e &= \int_L N x \, dx - V \int_L m'' \, dx \\
g &= 2\varrho g \int_L y_w x \, dx - V \int_L N \, dx \\
A &= I_{yy} + \int_L m'' x^2 \, dx \\
B &= \int_L N x^2 \, dx \\
C &= 2\varrho g \int_L y_w x^2 \, dx - VE \\
D &= \int_L m'' x \, dx + \varrho \int_L S_w x \, dx \\
E &= \int_L N x \, dx + V \int_L m'' \, dx \\
G &= 2\varrho g \int_L y_w x \, dx
\end{aligned}$$

where ϱ water density

- g gravity constant
- ∇ displacement volume
- S_w sectional area under calm water level
- y_w half-breadth of a section on the calm water-line
- I_{yy} longitudinal moment of inertia of the ship's mass about y-axis
- m'' sectional added mass of unit thickness for heave
- N sectional heave damping coefficient of unit thickness

The exciting forces and moments on the right-hand sides of the equations (3) are represented in the form

$$\begin{aligned} F_a \begin{Bmatrix} \cos \varepsilon_{FW} \\ \sin \varepsilon_{FW} \end{Bmatrix} &= 2\varrho g \bar{h} \int_L y_w \frac{\cosh \nu_0 (h - \bar{T}_m)}{\cosh \nu_0 h} \begin{Bmatrix} \cos \nu_0 x \\ \sin \nu_0 x \end{Bmatrix} dx \\ &- \omega\bar{h} (\omega + \nu_0 V) \int_L m'' \frac{\cosh \nu_0 (h - \bar{T}_m)}{\cosh \nu_0 h} \begin{Bmatrix} \cos \nu_0 x \\ \sin \nu_0 x \end{Bmatrix} dx \end{aligned}$$

$$\mp \omega \bar{h} \int_L N \frac{\cosh \nu_0 (h - \bar{T}_m)}{\cosh \nu_0 h} \left\{ \frac{\sin \nu_0 x}{\cos \nu_0 x} \right\} dx$$

$$M_a \left\{ \begin{matrix} \cos \varepsilon_{MW} \\ \sin \varepsilon_{MW} \end{matrix} \right\}$$

$$= \bar{h} \int_L (\omega^2 m'' - 2 \varrho g y_w) \times \frac{\cosh \nu_0 (h - \bar{T}_m)}{\cosh \nu_0 h} \left\{ \begin{matrix} \cos \nu_0 x \\ \sin \nu_0 x \end{matrix} \right\} dx$$

$$\pm \omega \bar{h} \int_L \left(N - V \frac{dm''}{dx} \right) \times \frac{\cosh \nu_0 (h - \bar{T}_m)}{\cosh \nu_0 h} \left\{ \begin{matrix} \sin \nu_0 x \\ \cos \nu_0 x \end{matrix} \right\} dx$$

where

- h water depth
- $\bar{T}_m$ mean draft of a section
- ε_{FW}, ε_{MW} phase differences between exciting force and wave and exciting moment and wave, respectively.

Sectional values of added mass and damping coefficient m'' and N for heave are obtained from [3]. In the case of deep water these values are obtained from [8]. If $h \to \infty$ then ν_0 and $\frac{\cosh \nu_0 (h - \bar{T}_m)}{\cosh \nu_0 h}$ are replaced by $\nu = \omega^2/g$ and $e^{-\nu \bar{T}_m}$, respectively.

Dimensionless Representation

In representing the calculated results, the following dimensionless forms are used:

- $\frac{h}{T}$ depth parameter
- $\frac{\lambda}{L}$ wave length to ship length ratio
- $\frac{V}{\sqrt{g L}} = F_n$ Froude number
- $\omega_e \sqrt{L/g}$ frequency of encounter
- $\frac{a}{\varrho \nabla}$ virtual mass coefficient
- $\frac{b \sqrt{L g}}{\varrho g \nabla}$ heave damping coefficient
- $\frac{A}{\varrho \nabla L^2}$ virtual inertia coefficient
- $\frac{B \sqrt{L g}}{\varrho g \nabla L^2}$ pitch damping coefficient
- $\frac{F_a}{\varrho g A_w \bar{h}}$ exciting force coefficient
- $\frac{M_a}{\varrho g I_W \nu_0 \bar{h}}$ exciting moment coefficient
- $\frac{\zeta_a}{\bar{h}}$ heave amplitude ratio
- $\frac{\psi_a}{\nu_0 \bar{h}}$ pitch amplitude ratio

By assuming that C.G. lies at midship the absolute bow and stern motions and relative bow motion are expressed in non-dimensional forms as follows

$$\left\{ \begin{matrix} \frac{\zeta_{AB}}{\bar{h}} \\ \frac{\zeta_{AS}}{\bar{h}} \end{matrix} \right\} = \left\{ \left(\frac{\zeta_a}{\bar{h}} \cos \varepsilon_{\zeta W} \mp \frac{\psi_a L}{2 \bar{h}} \cos \varepsilon_{\psi W} \right)^2 + \left(\frac{\zeta_a}{\bar{h}} \sin \varepsilon_{\zeta W} \mp \frac{\psi_a L}{2 \bar{h}} \sin \varepsilon_{\psi W} \right)^2 \right\}^{1/2}$$

$$\zeta_{RB} = \left\{ \left[\frac{\zeta_a}{\bar{h}} \cos \varepsilon_{\zeta W} - \frac{\psi_a L}{2 \bar{h}} \cos \varepsilon_{\psi W} - \cos \left(\frac{\pi L}{\lambda} \right) \right]^2 + \left[\frac{\zeta_a}{\bar{h}} \sin \varepsilon_{\zeta W} - \frac{\psi_a L}{2 \bar{h}} \sin \varepsilon_{\psi W} - \sin \left(\frac{\pi L}{\lambda} \right) \right]^2 \right\}^{1/2}$$

where

- λ wave length
- L length between perpendiculars
- A_w waterplane area
- I_W moment of waterplane area about y-axis.

Calculation and Discussion

For the numerical calculations we adopt a Series 60 mode of $C_B = 0.7$ having following particulars.

Length between perpendiculars	3.000	m
Displacement volume	0.1537	m³
Draft	0.171	m
Beam	0.428	m
Radius of gyration	0.750	m

Station	B (x)	H (x)	β (x)
1	0.0830	0.2425	0.8386
3	0.2803	0.8186	0.8716
5	0.4001	1.1685	0.9301
7	0.4280	1.2498	0.9761
9	0.4280	1.2498	0.9860
11	0.4280	1.2498	0.9850
13	0.4280	1.2498	0.9633
15	0.4113	1.2010	0.8660
17	0.3372	0.9848	0.6794
19	0.1575	0.4599	0.3751

where

- B (x) beam of a section
- H (x) half-beam draft ratio of a section
- β (x) fullness coefficient of a section

The calculations are carried out for the following speeds waves and depths.

F_n = 0.0, 0.1 and 0.2.

λ/L = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.3, 1.5, 1.7 and 2.0.

h/T = ∞, 10.0, 4.0, 2.5, 2.0 and 1.5.

In the calculation the following assumptions are made:

1. Although trim and parallel sinkage exist they are no considered.
2. The center of gravity lies at L/2.

The Virtual Mass, Virtual Inertia, Heave and Pitch Damping Coefficients are represented as functions of frequency for different depth parameter h/T at $F_n = 0.0$. They are illustrated in Figures A—1 to A—4. It is seen from Fig. A—1 that at high frequency range the added mass for the depth h = 1.5 T is approximately twice as large as that for h/T = ∞, 10.0, 4.0, and 2.5. This suggests that the natural heaving period of a ship in shallow water will be longer than that in deep water, provided that damping is comparatively small. From Fig. A—3 we observe that heave damping coefficients increase noticeable as the depth decreases and at the depth of h/T = 1.5 they are nearly twice as big as those for h/T = ∞ and 10.0.

The Exciting Forces and Moments acting on the restrained ship moving at the velocity of $F_n = 0.2$ are illustrated in Fig. A—5 and A—6 as functions of wave length to ship length ratio. It is found that these forces and moments generally increase as the depth decreases.

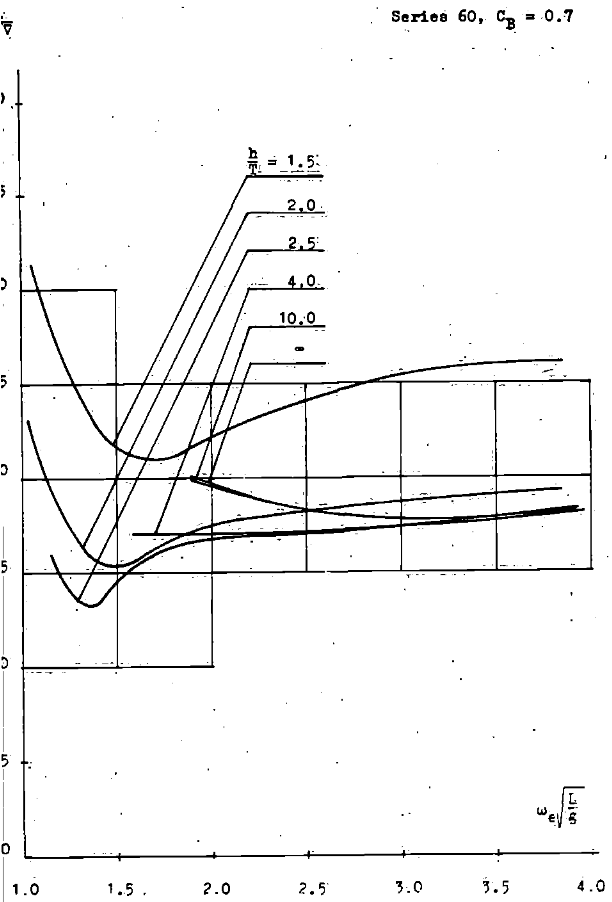


Fig. A-1 Virtuel Mass Coefficient at $F_n = 0.0$

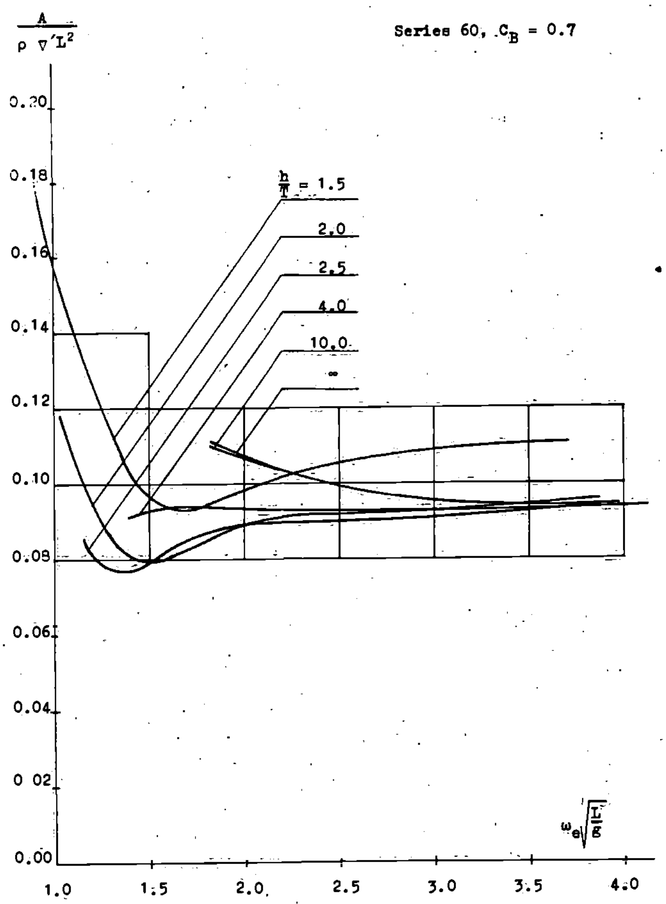


Fig. A-2 Virtuel Inertia Coefficient at $F_n = 0.0$

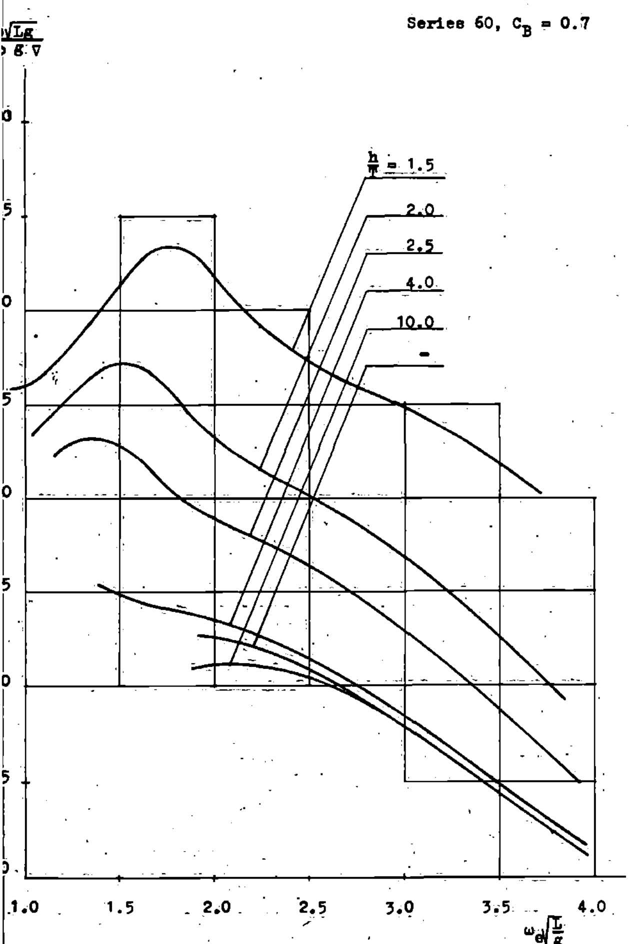


Fig. A-3 Heave Damping Coefficient at $F_n = 0.0$

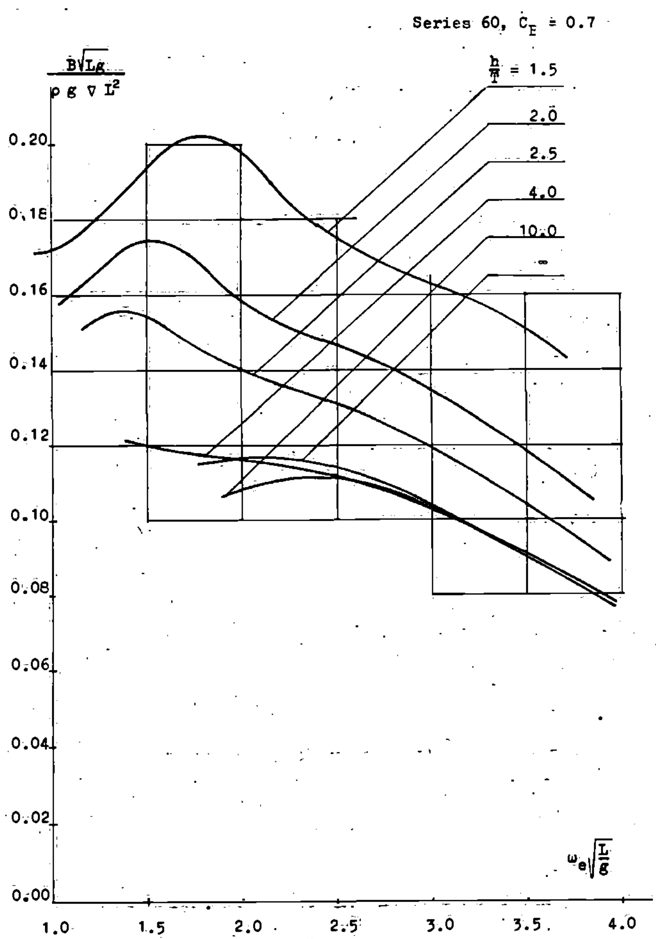


Fig. A-4 Pitch Damping Coefficient at $F_n = 0.0$

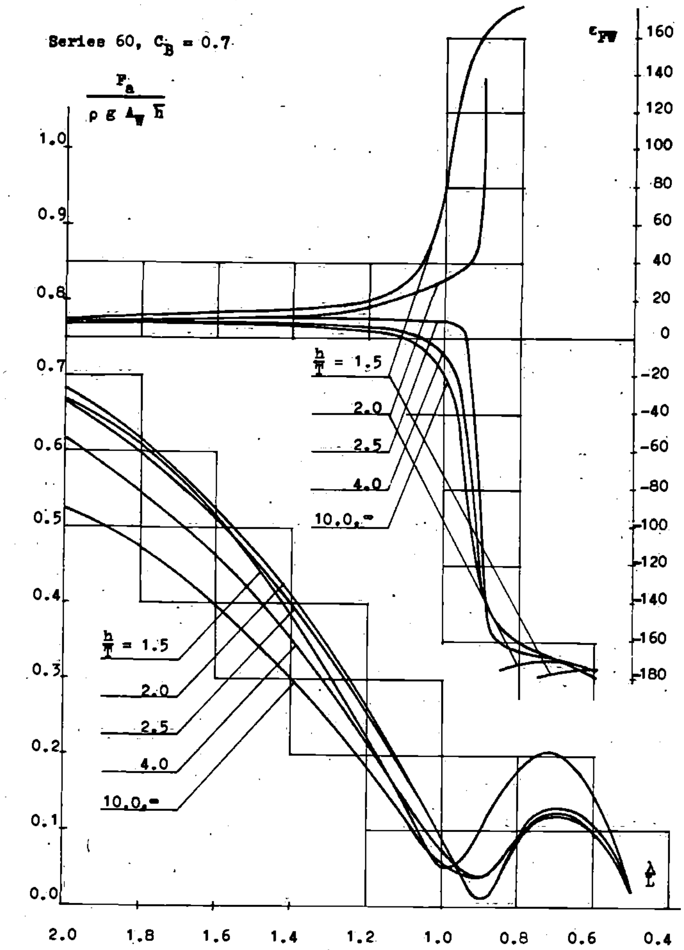


Fig. A-5 Exciting Force Coefficient on the Restrained Ship Moving at $F_n = 0.20$

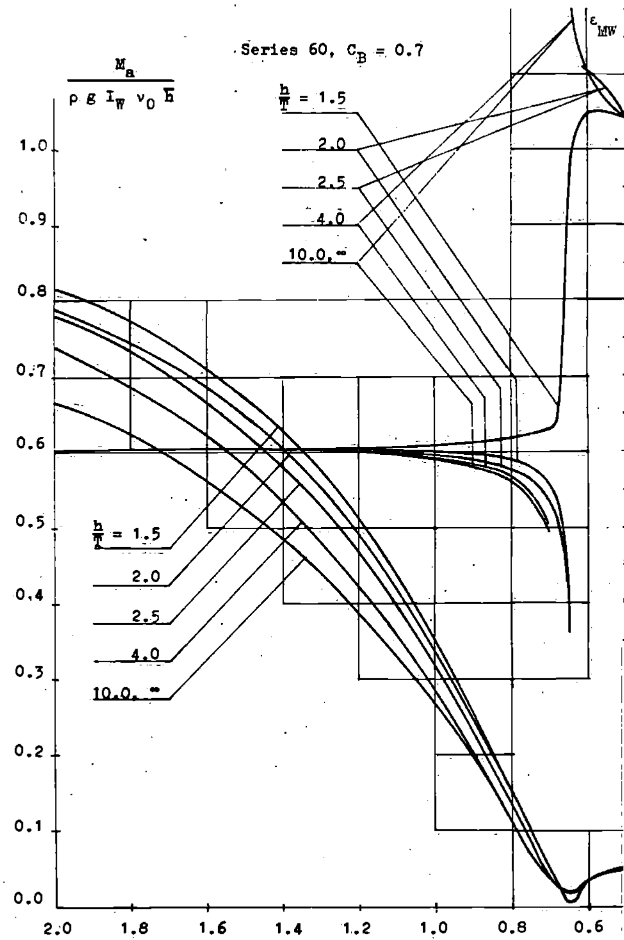


Fig. A-6 Exciting Moment Coefficient on the Restrained S Moving at $F_n = 0.20$

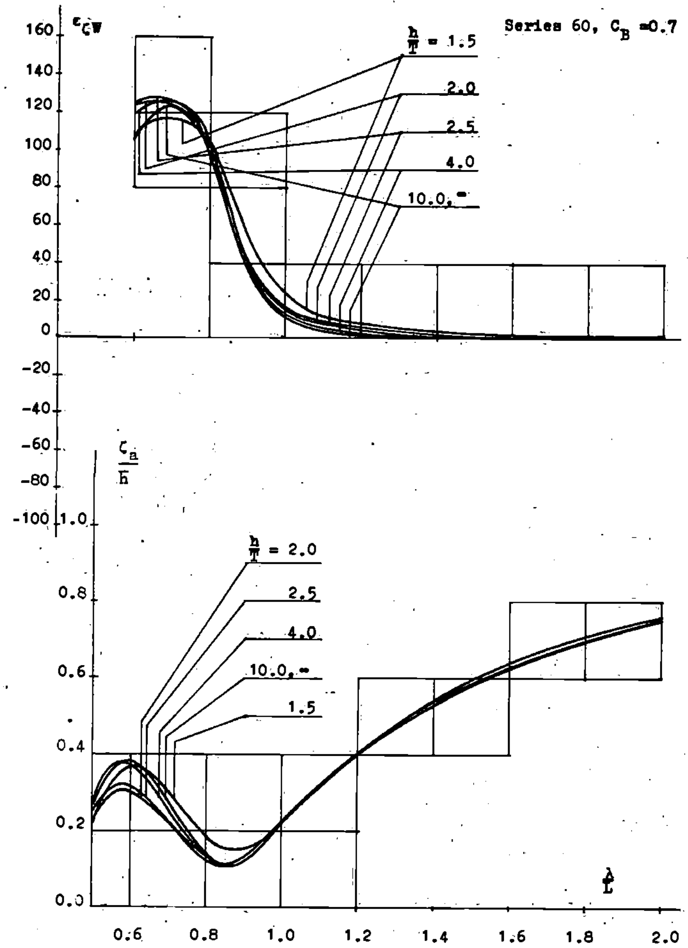


Fig. B-1 Heave Amplitude Ratio at $F_n = 0.0$

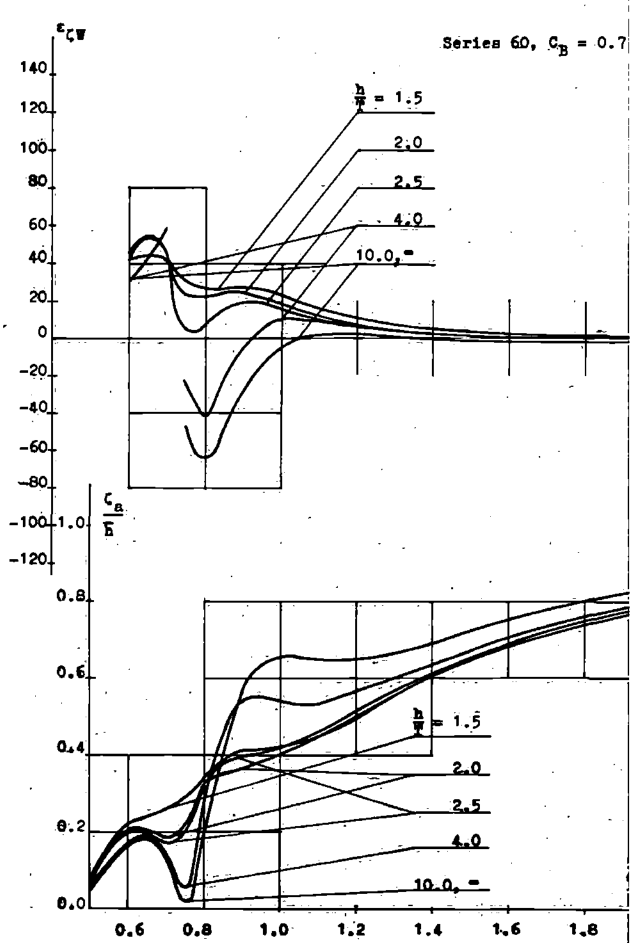


Fig. B-2 Heave Amplitude Ratio at $F_n = 0.10$

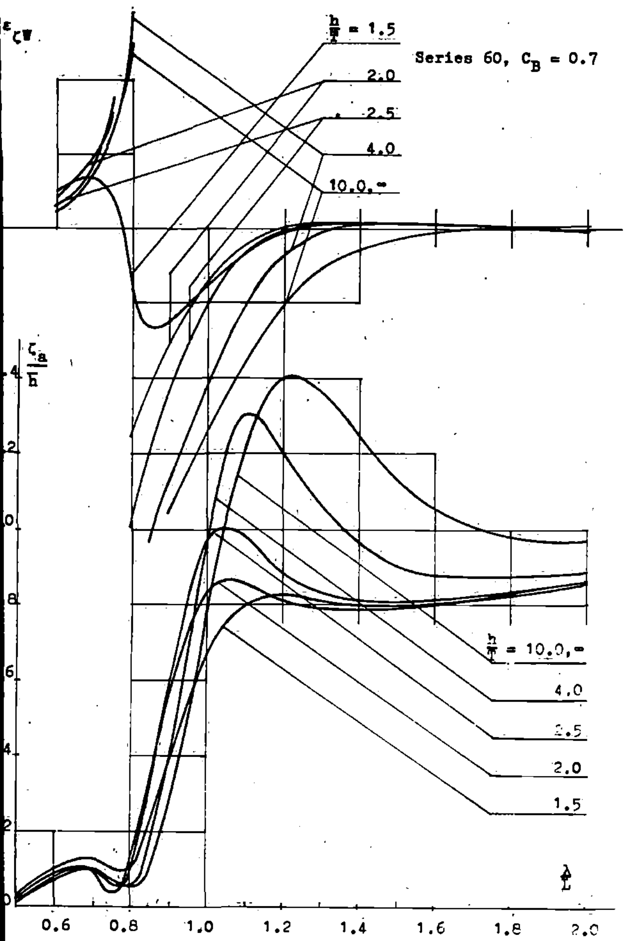


Fig. B-3 Heave Amplitude Ratio at F_n = 0.20

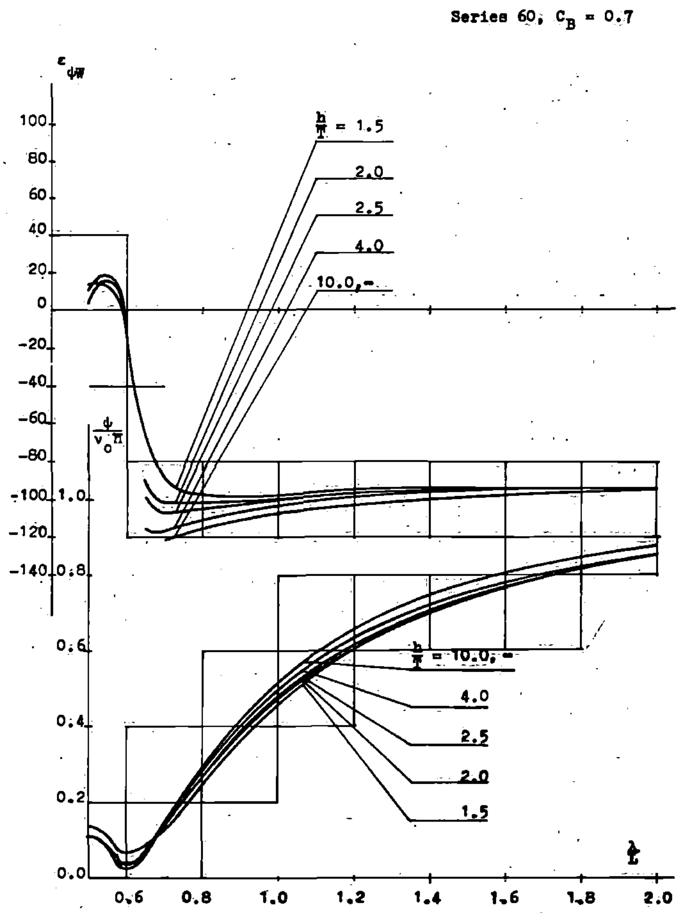


Fig. B-4 Pitch Amplitude Ratio at F_n = 0.0

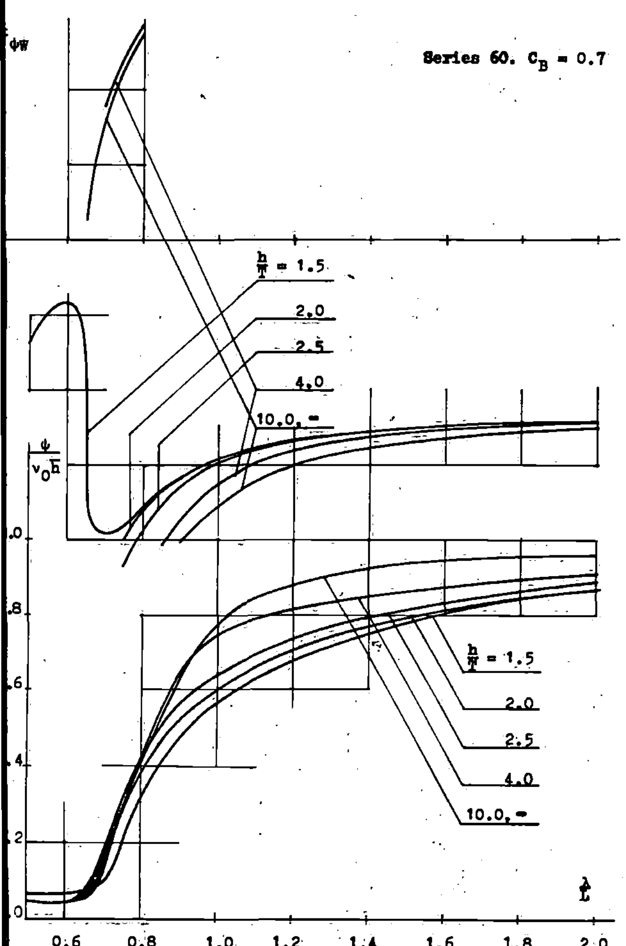


Fig. B-5 Pitch Amplitude Ratio at F_n = 0.1

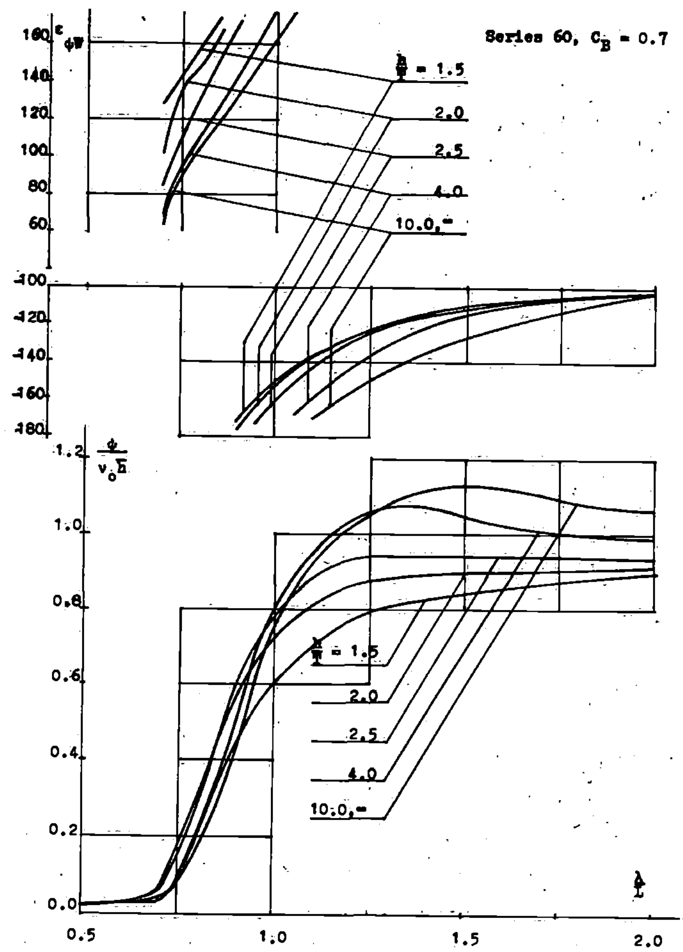


Fig. B-6 Pitch Amplitude Ratio at F_n = 0.2

The Heave and Pitch Amplitudes together with the phase differences with respect to the waves are illustrated in Figs. B. In general the motions are remarkably damped as the depth decreases. This tendency is more significant as the Froude Number increases.

Although our theory cannot solve the motion problem in very shallow water, it will be quite useful to consider underkeel clearences. Provided that this theoretical calculation is proved to be reasonable in an experimental study, this method will be a routine technique for further research on ship's behaviour in restricted waters.

Acknowledgements

This research was carried out with financial support of Chalmers University of Technology.

The author expresses sincerely his thanks to Prof. C. Falkemo, Head of the Division, for his constant support.

The numerical calculations were carried out by the computer IBM 360 at Chalmers University.

(Received 5th July 1968)

Nomenclature

a, b, c, d, e, g	coefficients of heave equation
A, B, C, D, E, G	coefficients of pitch equation
A_w	waterplane area
B (x)	beam of a section
F_a	exciting force amplitude
g	gravity constant
G_0	center of gravity (C. G.)
h	water depth
$\overline{h}$	wave amplitude
H	half-beam draft ratio
I_w	moment of waterplane area
I_{yy}	moment of inertia of the ship about y-axis
L	length between perpendiculars
m″	sectional added mass of unit thickness for heave
M_a	exciting moment amplitude
N	sectional heave damping coefficient
S_w	sectional area under calm water surface
t	time
T	draft
T_m	mean draft
V	ship velocity
∇	displacement volyme
w	suffix designating wave
x, y, z	body coordinates
X, Y, Z	space coordinates
β	section fullness coefficient
$\varepsilon_{\zeta W}$	phase difference between heave and wave
$\varepsilon_{\psi W}$	phase difference between pitch and wave
$\varepsilon_{\psi\zeta}$	phase difference between pitch and heave
ε_{FW}	phase difference between exciting force and wave
ε_{MW}	phase difference between exciting moment and wave
ζ	heave at time t
ζ_a	heave amplitude
ζ_W	wave elevation at time t
ζ_{AB}	amplitude of absolut bow motion
ζ_{AS}	amplitude of absolut stern motion
ζ_{RB}	Amplitude of relativ bow motion
λ	wave length
ν	ω^2/g
ν_0	shallow water wave number
ϱ	water density
ψ	pitch at time t
ψ_a	pitch amplitude
ω	circular frequency
ω_e	circular frequency of encounter

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C. H. Kim

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REPRINTED FROM

EUROPEAN SHIPBUILDING
JOURNAL OF THE
SHIP TECHNICAL SOCIETY

NO. 1 - FEBRUARY 1969 - VOL. XVIII

THE INFLUENCE OF WATER DEPTH ON THE MIDSHIP BENDING MOMENTS OF A SHIP MOVING IN LONGITUDINAL REGULAR HEAD WAVES

By

C. H. Kim *)

Abstract

The heaving and pitching motions and the midship bending moments of a T-2 tanker model moving in longitudinal regular head waves of shallow water are calculated by Watanabe's strip theory [1], [2], [3], [4], [5]. The results are represented in figures and the depth effect is discussed.

Nomenclature

a, b, c, d, e, g	coefficients of heave equation
A,B,C,D,E,G	coefficients of pitch equation
A_W	waterplane area
B (x)	beam of a section
C_m	midship bending moment coefficient
F_a	exciting force amplitude
g	gravity constant
G_0	center of gravity (C. G.)
h	water depth
$\bar{h}$	wave amplitude
H	half-beam draft ratio
i	longitudinal rad. of gyration in % of L
I_W	moment of waterplane area
I_{yy}	moment of inertia of the ship about y-axis
L	length between perpendiculars
m_0	midship bending moment at time t
m_a	amplitude of midship bending moment
M_a	exciting moment amplitude
N	sectional heave damping coefficient
t	time
T	draft
$\bar{T}$	mean draft of a section
V	ship velocity
∇	displacement volume
W	suffix designating wave
x, y, z	body coordinates
X,Y,Z	space coordinates
β	fullness coefficient of a section
$\varepsilon_{\zeta W}$	phase difference between have and wave.
$\varepsilon_{\psi W}$	phase difference between pitch and wave.
$\varepsilon_{\psi\zeta}$	phase difference between pitch and heave.
ε_{FW}	phase difference between exciting force and wave.
ε_{MW}	phase difference between exciting moment and wave.
ε_{mW}	phase difference between midship bending moment and wave.
ζ	heave at time t
ζ_a	heave amplitude
ζ_W	wave elevation at time t
λ	wave length
ν	wave number (ω^2/g)
ν_0	shallow water wave number
ρ	water density
ψ	pitch at time t
ψ_a	pitch amplitude
ω	circular frequency
ω_e	circular frequency of encounter

Introduction

By applying Watanabe's strip method [1], [2], [3], [4], [5], the heaving and pitching motions as well as the midship bending moments of a T-2 tanker model moving in regular head waves of shallow water are calculated and the effects of water depth are discussed.

It is revealed by the calculation that the midship bending moments are increased, while the motions heave and pitch are remarkably damped by the depth effect.

Definition of ship motions and waves

The coordinate systems here utilized given in Fig. 1 are space- and bodycoordinate system O-

*) Chalmers University of Technology, Department of Naval Architecture and Marine Engineering, Gothenburg, Sweden.

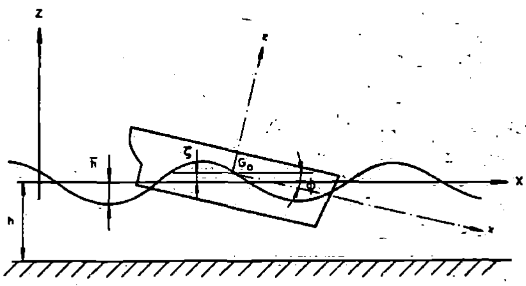

Fig. 1. The space and body coordinate system.

XYZ and G_0-xyz respectively. X-axis lies on the undisturbed water surface and Z-axis points vertically upward. X-axis is longitudinal passing through the center of gravity G_0 of the ship, while y- and z-axis point port and upward, respectively. The coordinate system G_0-xyz coincides with the system O-XYZ at the initial rest condition. We follow the convention of right-handed coordinate system.

Assuming only heaving and pitching motions of a ship at the speed V in a longitudinally approaching wave system, we describe the surface wave as follows

$$\zeta_W = \bar{h} \cos (\nu_0 x + \omega_e t) \quad \ldots\ldots (1)$$

where $\bar{h}$ wave amplitude

ν_0 shallow water wave number, i. e. $\left(\frac{\omega^2}{g} = \nu_0 \tanh \nu_0 h\right)$

ω_e circular frequency of encounter i. e. $(\omega + \nu_0 V)$

The heaving and pitching motions of the ship corresponding to the wave defined above are then expressed by

$$\left.\begin{aligned} \zeta &= \zeta_a \cos (\omega_e t + \varepsilon_{\zeta W}) \\ \psi &= \psi_a \cos (\omega_e t + \varepsilon_{\psi W}) \end{aligned}\right\} \quad \ldots\ldots (2)$$

respectively, where ζ_a, ψ_a are heave and pitch amplitudes and $\varepsilon_{\zeta W}$, $\varepsilon_{\psi W}$ phase differences between heave and wave and pitch and wave, respectively.

The coupled equations and coefficients

The coupled equations of heave and pitch of a ship moving in longitudinal regular waves [1], [2] are written in form:

The coefficients on the left-hand sides of the above equations are:

$$a = \rho \nabla + \int_L m'' \, dx$$

$$b = \int_L N \, dx$$

$$c = 2\rho g \int_L y_W \, dx$$

$$d = \int_L m'' x \, dx$$

$$e = \int_L N x \, dx - V \int_L m'' \, dx$$

$$g = 2\rho g \int_L y_W x \, dx - V \int_L N \, dx$$

$$A = I_{yy} + \int_L m'' x^2 \, dx$$

$$B = \int_L N x^2 \, dx$$

$$C = 2\rho g \int_L y_W x^2 \, dx - VE$$

$$D = \int_L m'' x \, dx$$

$$E = \int_L N x \, dx + V \int_L m'' \, dx$$

$$G = 2\rho g \int_L y_W x \, dx$$

where ρ water density

g gravity constant

∇ displacement volume

y_W half-breadth of a section on the calm waterline

I_{yy} longitudinal moment of inertia of the ship's mass about G_0-y-axis

m'' sectional added mass of unit thickness for heave

N sectional heave damping coefficient of unit thickness

The exciting forces and moments on the right-hand sides of the equations (3) are represented in the form

$$\left.\begin{aligned} a\ddot{\zeta} + b\dot{\zeta} + c\zeta - d\ddot{\psi} - e\dot{\psi} - g\psi &= F_a \cos (\omega_e t + \varepsilon_{FW}) \\ A\ddot{\psi} + B\dot{\psi} + C\psi - D\ddot{\zeta} - E\dot{\zeta} - G\zeta &= M_a \cos (\omega_e t + \varepsilon_{FW}) \end{aligned}\right\} \quad \ldots\ldots (3)$$

$$F_a \begin{Bmatrix} \cos \varepsilon_{FW} \\ \sin \varepsilon_{FW} \end{Bmatrix} = 2\rho g \bar{h} \int_L y_W \frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h} \begin{Bmatrix} \cos \nu_0 x \\ \sin \nu_0 x \end{Bmatrix} dx$$

$$- \omega \bar{h} (\omega + \nu_0 V) \int_L m'' \frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h} \begin{Bmatrix} \cos \nu_0 x \\ \sin \nu_0 x \end{Bmatrix} dx$$

$$\mp \omega \bar{h} \int_L N \frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h} \begin{Bmatrix} \sin \nu_0 x \\ \cos \nu_0 x \end{Bmatrix} dx$$

$$M_a \begin{Bmatrix} \cos \varepsilon_{MW} \\ \sin \varepsilon_{MW} \end{Bmatrix} = \bar{h} \int_L (\omega^2 m'' - 2\rho g y_W)\, x \frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h} \begin{Bmatrix} \cos \nu_0 x \\ \sin \nu_0 x \end{Bmatrix} dx$$

$$\pm \omega \bar{h} \int_L \left(N - V \frac{dm''}{dx}\right) x \frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h} \begin{Bmatrix} \sin \nu_0 x \\ \cos \nu_0 x \end{Bmatrix} dx$$

where h — water depth

$\bar{T}$ — mean draft of a section

ε_{FW}, ε_{MW} — phase differences between exciting force and wave and exciting moment and wave, respectively

Sectional values of added mass and damping coefficient m'' and N for heave are obtained from [6]. In the case of deep water these values are obtained from [12]. If $h \to \infty$ then ν_0 and $\frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h}$ are replaced by ν and $e_{-\nu T}$, respectively.

The midship bending moments

The preceeding discussions are on the calculations of the heaving and pitching motions of a ship moving in regular head waves of shallow water. By making use of the motions calculated above we obtain the midship bending moments in the following form:

$$m_0 = m_a \cos (\omega_e t + \varepsilon_{mW}) \quad \ldots\ldots (4)$$

where m_a and ε_{mW} are the midship bending moment amplitude, and the phase difference between wave and bending moment respectively. Assuming that C. G. lies at the midship section the sine and cosine components of the amplitude are written as follows:

$$\begin{Bmatrix} m_a \cos \varepsilon_{mW} \\ m_a \sin \varepsilon_{mW} \end{Bmatrix} = \begin{Bmatrix} \zeta_a \cos \varepsilon_{\zeta W} \\ \zeta_a \sin \varepsilon_{\zeta W} \end{Bmatrix} \left[- 2\rho g \int y_W x dx + \omega_e^2 \left(\int m'' x dx + \int \frac{W}{g} x dx \right) \right]$$

$$+ \begin{Bmatrix} \zeta_a \sin \varepsilon_{\zeta W} \\ \zeta_a \cos \varepsilon_{\zeta W} \end{Bmatrix} [\pm \omega_e (\int N x dx + V \int m'' dx)]$$

$$+ \begin{Bmatrix} \psi_a \cos \varepsilon_{\psi W} \\ \psi_a \sin \varepsilon_{\psi W} \end{Bmatrix} \left[2\rho g \int y_W x^2 dx - V \int N x dx - V^2 \int m'' dx - \omega_e^2 \left(\int m'' x^2 dx + \int \frac{W}{g} x^2 dx \right) \right]$$

$$+ \begin{Bmatrix} \psi_a \sin \varepsilon_{\psi W} \\ \psi_a \cos \varepsilon_{\psi W} \end{Bmatrix} [\mp \int N x^2 dx]$$

$$+ \bar{h} \int (-\omega^2 m'' + 2\rho g y_W)\, x \frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h} \begin{Bmatrix} \cos \nu_0 x \\ \sin \nu_0 x \end{Bmatrix} dx$$

$$\mp \omega h \int \left(N - V \frac{dm''}{dx}\right) x \frac{\cosh \nu_0 (h - \bar{T})}{\cosh \nu_0 h} \begin{Bmatrix} \sin \nu_0 x \\ \cos \nu_0 x \end{Bmatrix} dx$$

where the integral is taken either between —L/2 and 0 or between +L/2 and 0 and $\frac{w}{g}$ designates mass per unit length along the ship length.

Dimensionless representation

In representing the calculated results, the following non-dimensional forms are used:

h/T	depth parameter
λ/L	wave length to ship length ratio
$V/\sqrt{gL}$	Froude Number
$\zeta_a/\bar{h}$	heave amplitude ratio
$\psi_a/\nu_0\bar{h}$	pitch amplitude ratio
$C_m = m_a/\rho g\bar{h}BL^2$	midship bending moment coefficient

where λ is a wave length.

Calculation and discussion

For the numerical calculations we adopt a model of T-2 tanker having the following particulars.

Length between perpendiculars	(L)	3.066 m
Beam	(B)	0.415 m
Draft	(T)	0.183 m
Displacement volume	(△)	0.1725 m³
Block coefficient	(C_B)	0.741
Radius of gyration	(% of L)	0.23

Station	B(x)	β(x)	H(x)	w/g
1	0.168	0.442	0.459	6.15
3	0.351	0.749	0.959	12.62
5	0.406	0.911	1.115	20.01
7	0.415	0.960	1.134	24.65
9	0.415	0.980	1.134	22.82
11	0.415	0.980	1.134	24.71
13	0.415	0.980	1.134	24.74
15	0.397	0.961	1.085	21.79
17	0.311	0.871	0.850	13.27
19	0.109	0.837	0.298	5.00

where B(x) beam of a section
β(x) fullness coefficient of a section
H(x) half-beam draft ratio
w/g mass in kg per L/10 (see Fig. 2)

The calculations are carried out for the following speeds, waves and depths.

F_n = 0.0, 0.1, 0.2
λ/L = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.3, 1.5, 1.7, 2.0.
h/T = ∞, 10.0, 4.0, 2.5, and 1.5

Fig. 2. Mass distribution of T-2 tanker model.

In the calculation the following assumptions are made:

1. Although trim and parallel sinkage are produced they are not considered.
2. C. G. lies at midship section.

The Heave and Pitch Amplitudes together with the phase differences with respect to the waves are illustrated in Fig. 3—8. In general the motions are remarkably dampened as the depth decreases. This tendency is more significant as the Froude Number increases.

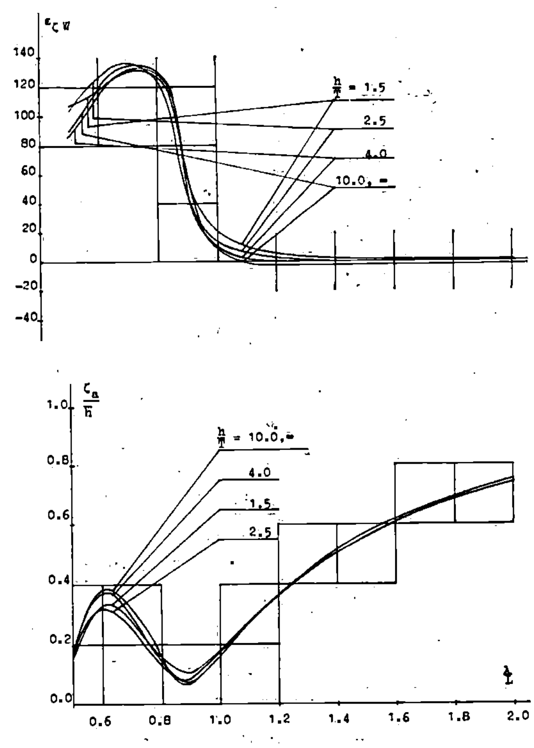

Fig. 3. Heave amplitude ratio at Fn = 0.0.

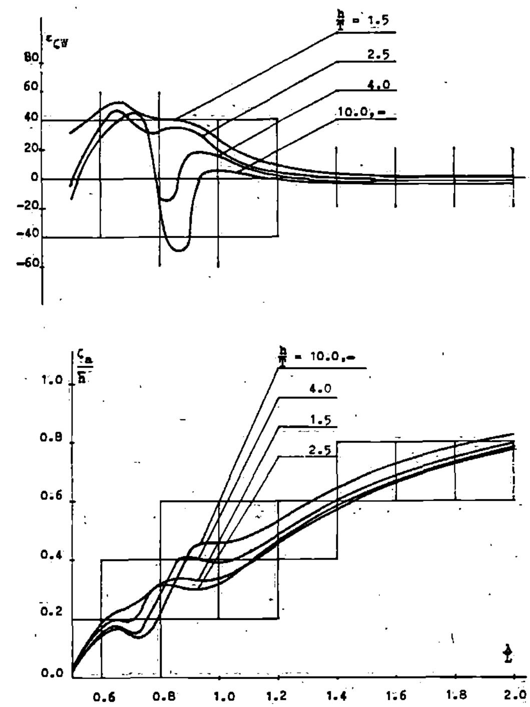

Fig. 4. Heave amplitude ratio at Fn = 0.1.

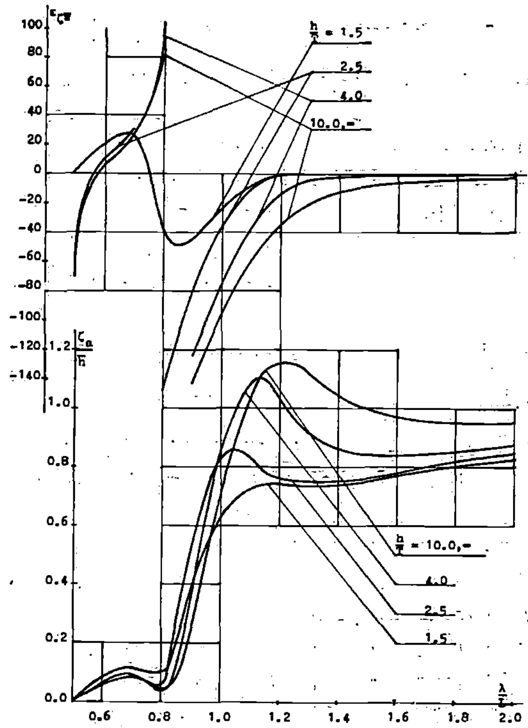

Fig. 5. Heave amplitude ratio at Fn = 0.2.

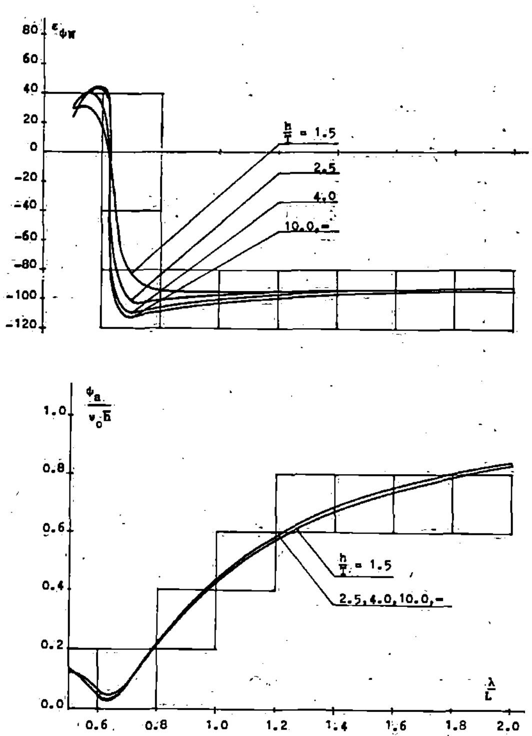

Fig. 6. Pitch amplitude ratio at Fn = 0.0.

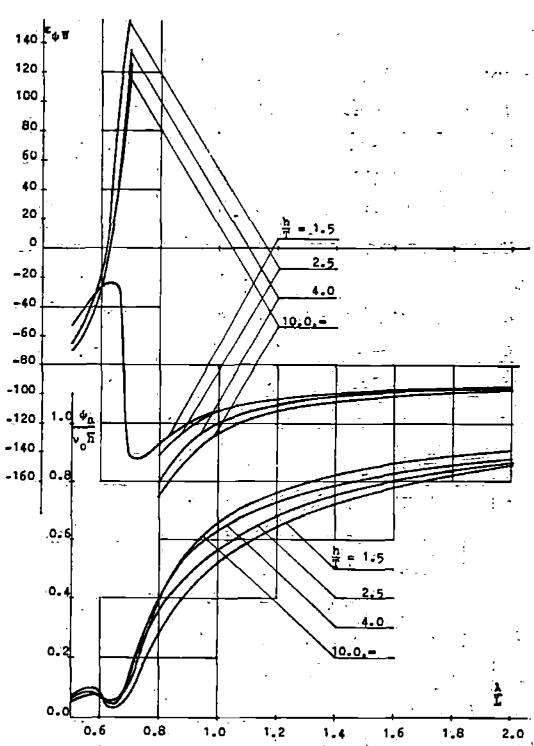

Fig. 7. Pitch amplitude ratio at Fn = 0.1.

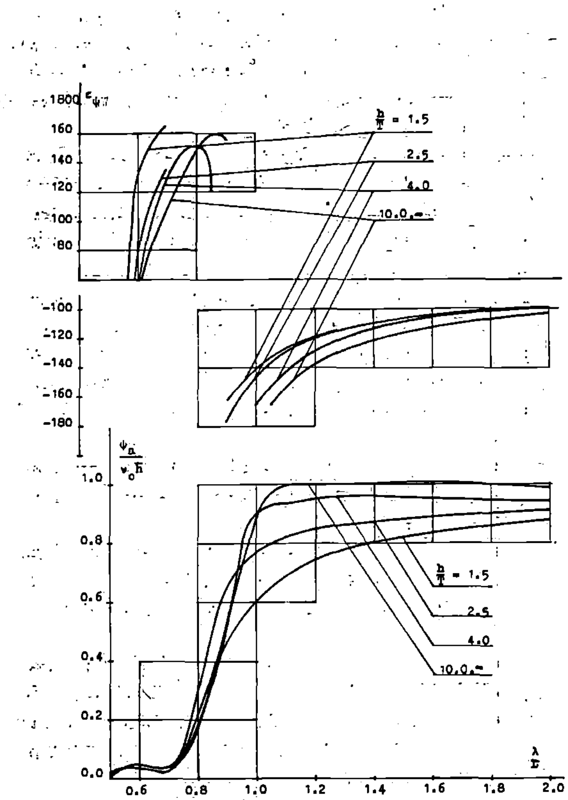

Fig. 8. Pitch amplitude ratio at Fn = 0.2.

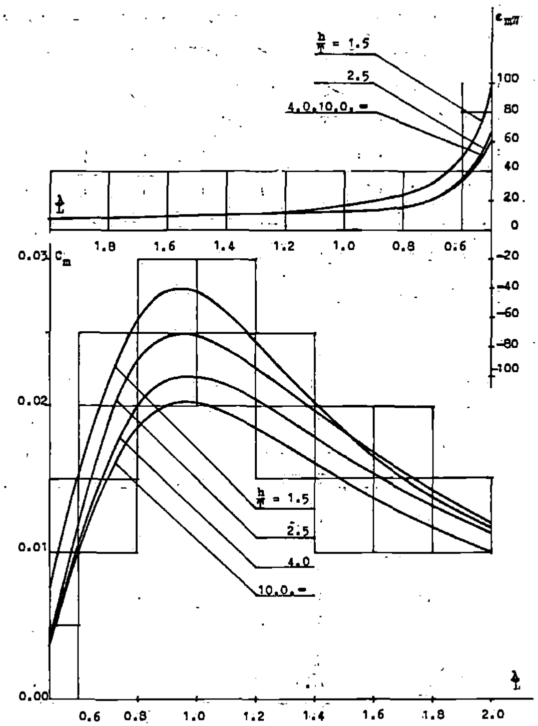

Fig. 9. Midship bending moment at Fn = 0.0.

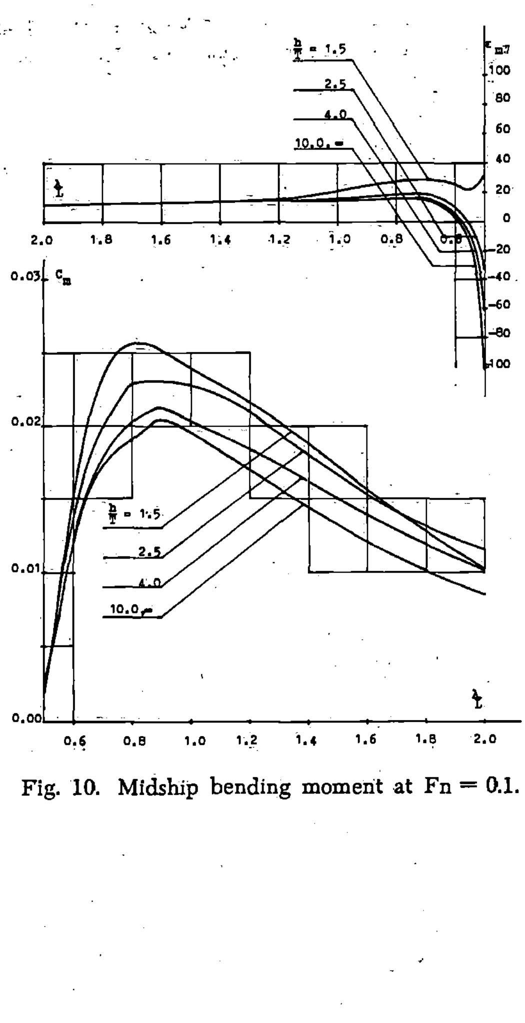

Fig. 10. Midship bending moment at Fn = 0.1.

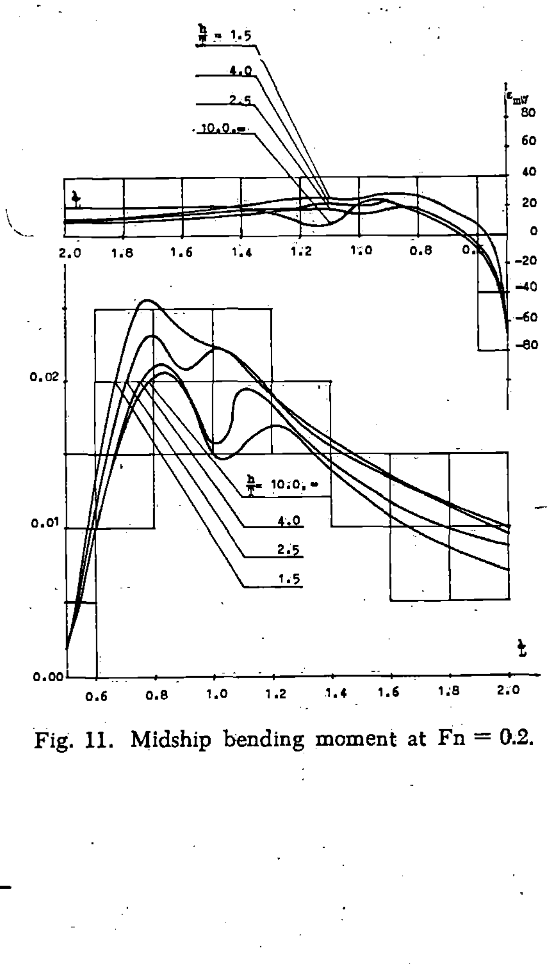

Fig. 11. Midship bending moment at Fn = 0.2.

The Midship Bending Moments together with the phase difference with respect to the waves are illustrated in Fig. 9—11. The bending moments are generally increased as the depth decreases. This tendency is quite opposite to the above-mentioned motions. Probably it is partly caused by the decrease of inertial bending moments due to the dampened motions. The double peaks are nearing each other as the depth decreases. This is probably caused by the delayed position of the peaks of heaving motions.

Acknowledements

The author expresses his thanks to Prf. Falkemo, Head of the Division, for his constant support. He wishes to thank Mr. Bennet for his kind advice on the wave bending moments and also to Mr. Sulz for his excellent assistance.

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