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INDIVIDUAL WAVE HEIGHT DISTRIBUTIONS IN THE COASTAL ZONE: MEASUREMENTS AND SIMULATIONS AND THE EFFECT OF DIRECTIONAL SPREADING

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Abstract

Characteristics of the individual wave height distribution in shallow water have been investigated using measured wave data and results of numerical simulations using the non-hydrostatic SWASH model. It is shown that the SWASH model is capable of reproducing the temporal and spatial variation of surface elevation in a wave flume and the resulting individual wave height distributions, whereas three theoretical distributions fail to do so. SWASH was also applied to a 2D field case and was able to reproduce the individual wave height distribution. Finally, the effect of directional spreading on the individual wave height distribution was determined by performing two SWASH model runs; one with directional spreading and one with uni-directional irregular waves.. The results suggest that directional spreading increases the highest individual wave height.

Keywords: Wave height distribution, maximum wave height, directional spreading, SWASH, wave measurements, numerical experiments

1. Introduction

Many shallow water coastal processes and engineering applications require knowledge of the individual wave height distribution, which, because of depth effects, is known to deviate from the deep water Rayleigh distribution (Battjes-Groenendijk, 2000, Rattanatipitikon, 2010). Several wave height distributions have been derived on the basis of uni-directional wave flume experiments; still, they are applied to two-dimensional (2D) field situations in directionally spread seas. Salmon and Holthuijsen (2013) showed that, in addition to depth, directional spreading influences wave breaking in shallow water. Therefore, it is expected that directional spreading also influences the individual wave height distribution. Support for the hypothesis that wave height distributions may be different in the field from flume data is given by Caires and Van Gent (2012), who showed that the Battjes-Groenendijk distribution underestimates the higher wave heights in field situations. Further evidence that directional spreading affects the individual wave height distribution was obtained by Latheef and Swan (2013) who performed 2D-laboratory experiments to investigate the role of directional spreading on crest height statistics. A related problem of interest is the statistical distribution of higher representative wave heights like $H_{1/10}$ or the highest individual wave height $H_{\max}$ in the coastal zone. Estimates of these measures are now based on local depth and slope information, assuming an instantaneous adaption of the individual wave height distribution. A short-coming of commonly used methods is their omission of 2D-effects in bathymetry as well as in neglecting directional spreading.

The purpose of this study is to determine whether a numerical model can be used to study individual wave height statistics and to determine whether directional spreading may affect the individual wave height distribution. The numerical experiments are carried out with the non-hydrostatic flow model SWASH developed by Delft University of Technology (Zijlema et al., 2011). The applicability of the

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SWASH model is first verified against a one-dimensional (1D)-wave flume experiment performed by Boers (1996). Then, the applicability of the SWASH model to reproduce wave height characteristics in a 2D-field situation is investigated. Finally, the role of directional spreading on the individual wave height distribution is assessed for waves propagating over an artificial sloping bottom.

2. Individual wave height distributions

In deep water it is generally assumed that individual wave heights *H* follow a Rayleigh distribution. The probability density function *p*(*H*) can be expressed as:

$$p(H) = \frac{2H}{H_{rms}^2} \exp\left[-\left(\frac{H}{H_{rms}} \right)^2 \right] \tag{1}$$

where H_{rms} is the root-mean-square wave height.

Many studies have shown that the Rayleigh distribution is not accurate in shallow water where the highest wave heights are reduced in height by dissipation effects. Rattanapitikon (2010) provides an extensive overview of alternative formulations that have been proposed to overcome the limitations of the Rayleigh distribution. Out of many possible distributions, two ‘shallow’ water wave height distributions will be used to compare observed and computed individual wave height distributions. The first is the modified Glukhovskiy (1996) distribution as corrected by Klopman (1996) to ensure internal consistency. This distribution will be denoted by GK96 and consists of a Weibull distribution whose probability density function is given by:

$$p(H) = \frac{A\kappa H^{\kappa-1}}{H_{rms}^{\kappa}} \exp\left[-A\left(\frac{H}{H_{rms}} \right)\kappa \right] \tag{2}$$

The parameters A and κ are given by:

$$A = \left[\Gamma\left(\frac{2}{\kappa} + 1 \right) \right]^{\kappa/2} \tag{3}$$

$$\kappa = \frac{2}{1 - 0.7\, H_{rms}/h} \tag{4}$$

Here, *h* is water depth and Γ() is the gamma function. For deep water $h \to \infty$ the parameter κ=2, *A*=1 and expression (2) reduces to the Rayleigh distribution. Battjes and Groenendijk (2000) analyzed a large amount of wave flume data and proposed the second shallow water formulation used here. Now, the local bottom slope plays a role too. The distribution consists of two connected Weibull distributions with an exponent of 2.0 for the lower wave heights (i.e. below a so-called transition depth H_{tr}) and an exponent of 3.6 for the higher wave heights. The probability density function of this distribution (hereafter referred to as BG2000) is given by:

$$p(H) = \begin{cases} \dfrac{2H}{H_1^2} \exp\left[-\left(\dfrac{H}{H_1} \right)^2 \right] & for \quad H < H_{tr} \\ \dfrac{2H^{2.6}}{H_2^{3.6}} \exp\left[-\left(\dfrac{H}{H_2} \right)^{3.6} \right] & for \quad H \geq H_{tr} \end{cases} \tag{5}$$

Battjes and Groenendijk (2000) define the transitional wave height H_{tr} as a function of foreshore slope α and water depth *h* according to:

$$H_{tr} = (0.35 + 5.8 \tan\alpha)h \tag{6}$$

From expression (6) it follows that steeper slopes lead to higher waves as wave breaking is slowed down. The wave heights H_1 and H_2 are chosen in such a way that the two parts of the distribution are continuous and consistency with the Rayleigh distribution for deep water. Details about the computation of the wave height H_1 and H_2 can be found in Battjes-Groenendijk (2000) and Rattanapitikon (2010).

Figure 1 shows a comparison of the probability density functions according to Rayleigh, GK96 and BG2000 assuming H_{rms}=1, h=4 m and a bottom slope of α=0.005. Both the GK96 and BG2000 model show that the probability of the largest waves is lower than the Rayleigh distribution. The discontinuity at H_{tr}=1.52 m consistent with the different slope of the two Weibull distributions.

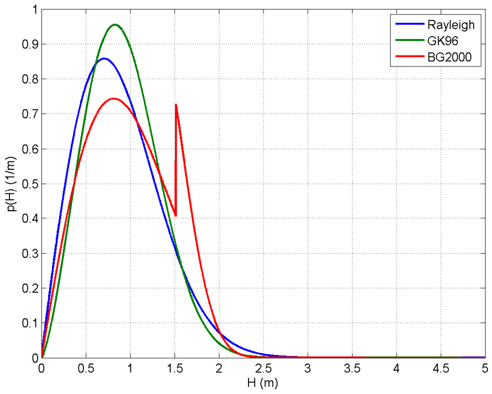

Figure 1 Comparison of theoretical individual wave height distributions (Rayleigh, GK96 and BG2000) for a root-mean square wave height H_{rms} = 15 m, a water depth of 4 m and a bottom slope of 0.005.

Measures of the characteristic wave height can be derived from each individual wave height distribution. In this study the significant wave height $H_{1/3}$ (the mean of the highest one-third), the mean of the highest 10% of all waves $H_{1/10}$ and the highest maximum individual wave height $H_{\max}$ are examined, where $H_{\max}$ is determined as having a probability of exceedance of $1/N_{\mathrm{w}}$ (N_{w} is the number of individual waves in a wave record). It is noted that various alternative definition are possible (see for instance Rattanapitikon, 2010).

3. The Boers experiment

Boers (1996) carried out wave flume experiments to determine the variation of wave height characteristics as irregular waves propagate over a barred beach. The layout of his flume experiment is shown in Figure 1. The vertical bars denote the location of the wave gauges. The high density of these gauges ensured a detailed view of the variation of these characteristics.

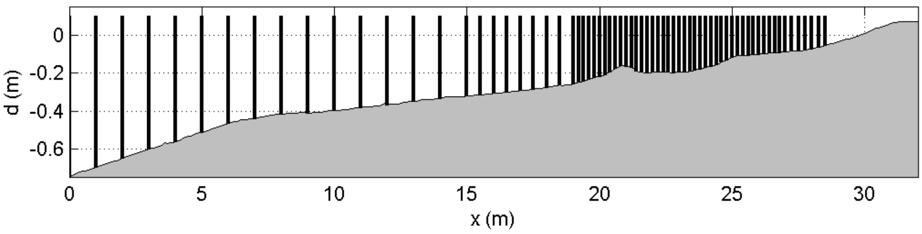

Figure 2: Depth profile as used in the Boers experiments and the location of the wave sensors.

For this study three cases of the Boers experiment are used. The characteristics of these experiments are given in Table 1. They consist of irregular waves that were generated on the basis of a JONSWAP spectrum with a peak enhancement factor $\gamma = 3.3$. In the present paper only results of experiment 1 are used. It is noted that the results for the Boers cases 2 and 3 are similar to those of experiment 1.

Table 1: Summary of wave parameters as used in the experiments by Boers (1996).

	H_s (m)	T_p (s)
1	0.15	2.0
2	0.20	2.0
3	0.10	3.5

4. The SWASH model

The SWASH model (Zijlema et al., 2011) is a is a shock-capturing hydrodynamic model for simulating non-hydrostatic free-surface flows. The model is based on the non-linear shallow water equations, including non-hydrostatic pressure, which are derived from the incompressible Reynolds Averaged Navier Stokes equations that describe conservation of mass and momentum. In principal, shock-capturing non-hydrostatic models inherently account for the energy dissipation in the surf-zone due to breaking waves. This however requires high vertical resolutions. Smit et al. (2013) proposed to use a hydrostatic pressure distribution at the front of the wave to ensure that the wave develops a steep front. It was shown that with this adaptation SWASH accurately resolves the wave-breaking with only a few vertical layers. This adaptation allows to study the wave propagation in shallow environments with only a few vertical layers.

The SWASH model version 1.10AB has been applied to the 1D Boers cases to simulate the variation of the surface elevation. To that end the measured signal near the wave maker was imposed as a time varying boundary condition. In these experiments the SWASH was applied with one layer, a spatial step of 0.02 m and a time step of 0.001 s.

Figure 3 shows a snapshot of the measured and simulated surface elevation for the first experiment at location *x*=10 m. The performance of the SWASH model is quite good although some peaks are missed, affecting the distribution of wave heights. These promising results provided the motivation to study the characteristics of the individual wave height distribution using the numerical model SWASH.

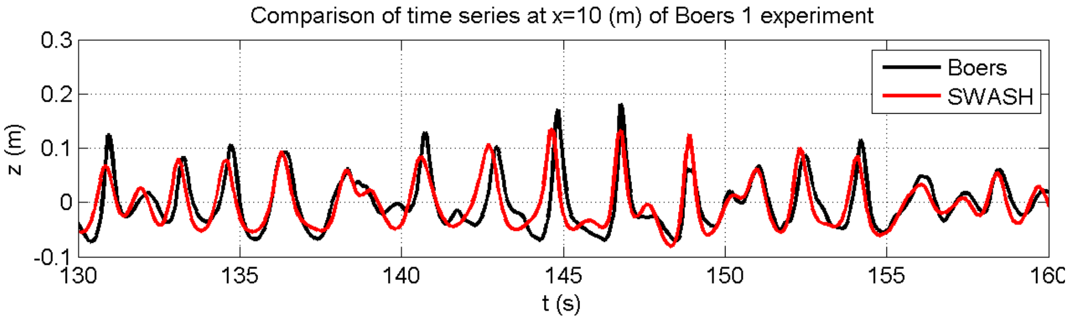

Figure 3: Comparison of observed and computed variation of surface elevation using the SWASH model for Boers experiment 1.

5. Comparison of individual wave height distributions

The results of the SWASH computations for the various Boers experiments were analyzed to compare the observed (black), computed (red) and three theoretical individual wave height distributions at various locations along the wave flume. The first bar is located at x=22 m and the surf zone start at x=25 m. Figure 4 shows the results of this comparison for 9 locations along the flume.

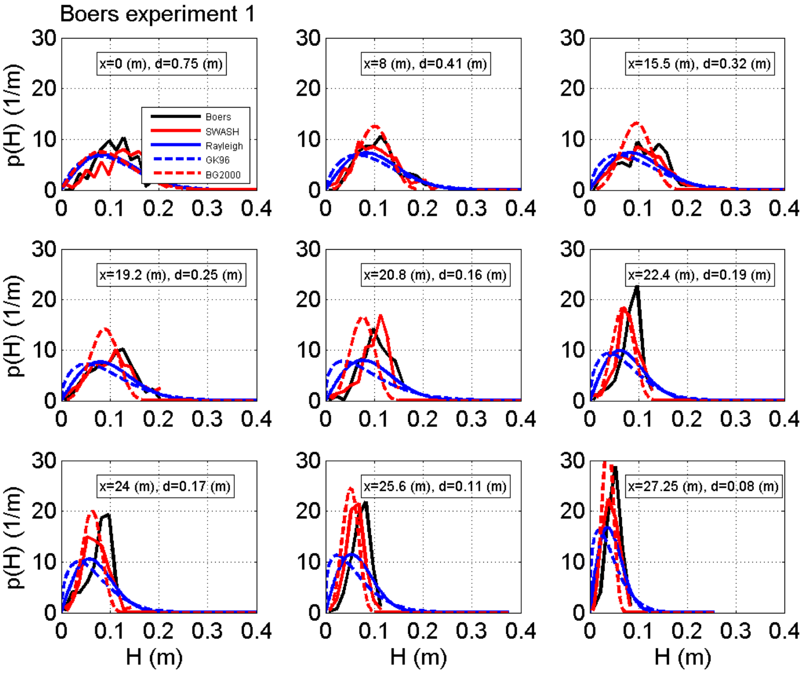

Figure 4: Comparison of observed, computed and theoretical individual wave height distributions at 9 locations along the flume for Boers experiment 1.

The results indicate that SWASH is quite capable to catch the spatially varying shape of the individual wave height distribution as the waves enter shallow water. It can also be clearly seen that the Rayleigh and BG96 distribution overestimate the density of the higher wave heights. The BG2000 formulation performs better than Rayleigh and BG96, but is still not able to reproduce the measured distribution. Based on the observed, computed and theoretical wave height distributions, the spatial variation of the wave parameters $H_{1/3}$, $H_{1/10}$ and $H_{\max}$ was computed using a zero-up crossing method. Figure 5 shows the results of this analysis for Boers experiment 1. The results show that SWASH is quite capable to catch the spatial variation of these characteristic wave heights. BG2000 consistently underestimates these values, as was also noted by Mai et al. (2010), and Caires and Van Gent (2012). The Rayleigh and BG96 distribution show a consistent over-estimation. The spike in the observed maximum wave height was shown to be an outlier and was removed in subsequent analyses

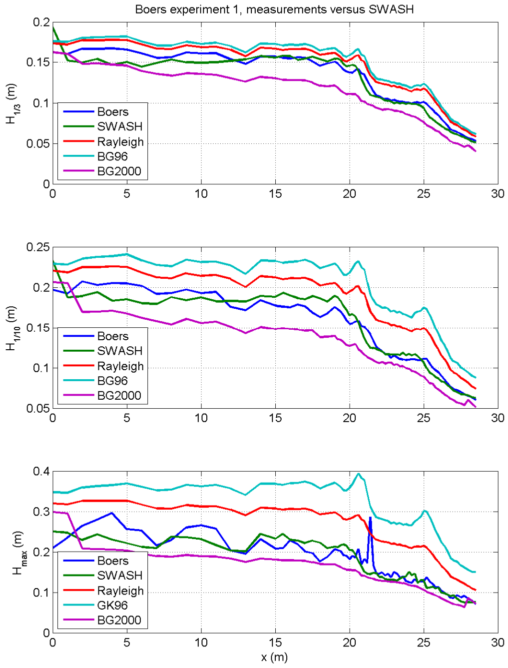

Figure 5: Spatial variation of observed, computed and theoretical characteristic wave heights for the Boers 1 experiment.

The above data can be considered as a single realization of a random process; the input signal was generated from a spectrum using a random generation to obtain the Fourier coefficients of the input signal. As can be seen in Figure 3, the SWASH model closely followed the measured surface elevation. To test the robustness of this approach and to study the statistical variation of the wave parameters $H_{1/3}$, $H_{1/10}$ and H_{max} the SWASH model was rerun 10 times using the same target input spectrum but with different seeds for the random generator. The results for the Boers 1 experiment are shown in Figure 6 as a function of location along the flume.

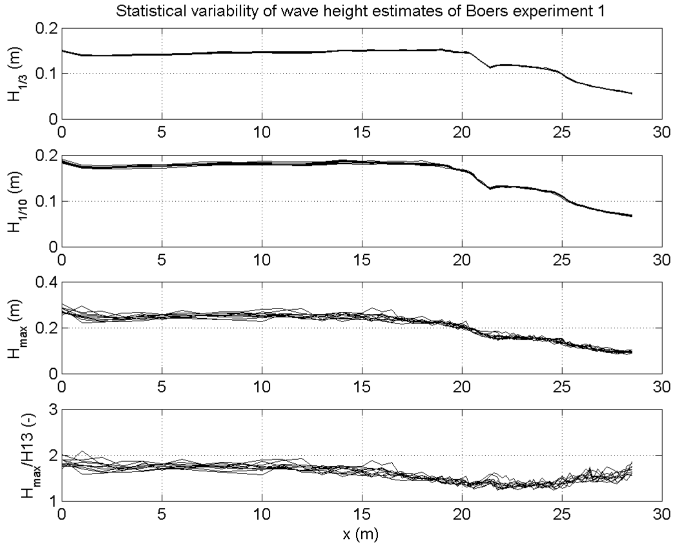

Figure 6: Spatial variation of wave height estimates $H_{1/3}$, $H_{1/10}$ and H_{max} and the ratio of H_{max} over $H_{1/3}$ as computed by SWASH for the target spectrum for Boers experiment 1.

The results in Figure 6 show that the significant wave height estimates have a very small variation. This variation is about 4% for the $H_{1/10}$ estimates up to 10% for the $H_{\max}$ estimates. This number is of practical interest for design studies based on theoretically determined expected values for these parameters. The lowest panel in Figure 6 shows the ratio between the maximum individual wave height $H_{\max}$ and the significant wave height $H_{1/3}$. The most striking observation is that this ratio is generally lower than the commonly used factor of 1.8. This implies that at least for this Boers experiment, the traditional methods significantly overestimate the $H_{\max}$ values. In the data processing no distinction was yet made between bounded-long waves and free short waves. This choice may have affected the results of the analysis of the individual wave height distribution as short waves may be modulated by depth variations on the long infra-gravity wave time scale.

6. 2-D field cases

The previous sections are based on results obtained for uni-directional irregular waves. To determine the role of 2-D effects, bathymetry as well as directional spreading the SWASH model was applied to a field case and subsequently to an artificial simple geometry. The field case is taken from Ruessink et al. (2001) which was undertaken at a beach near the town of Egmond, the Netherlands. Several bidirectional current meters and pressure sensors were deployed at the experimental site, which is characterized by a double bar (Figure 7). These instruments gathered data for approximately 34min per hour at a sampling rate of 2 or 4Hz. A wave buoy located at 1 6m depth 5 km offshore measured offshore wave conditions. A wide range of conditions were measured during this field experiment, which spanned two months in 1998 starting at October 15. Here we focus on a high-tide, mild wave energy case collected on October 24 starting at 1800 (case 9330). For this particular case the wave buoy measured a significant wave height of 2.7 m and a peak period of 7.5 s. Spatially extensive surveys of the local bathymetry were conducted every few days.

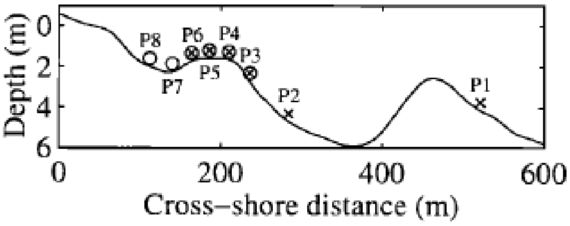

Figure 7: Depth relative to mean sea level versus cross-shore distance on October 16, 1998 at Egmond and location of current meters (circles) and pressure sensors (crosses). Source: Ruessink et al. (2001).

The wave buoy and the most seaward located gauge are not suitable to serve as a seaward boundary condition in SWASH because the gauge is located too far from the coast (computationally expensive) and in shallow water (non-linear waves), respectively. To circumvent this problem, the seaward SWASH boundary location is chosen at a depth of 10m. We use SWAN Booij et al. (1999) to compute the wave conditions at both boundary locations. Computations were performed with SWAN model version 40.91A in stationary mode. All deep water physics and shallow water physics were included and were applied with their default settings. The directional spectrum computed with SWAN is used to force the wavemaker in SWASH. For the nearshore Egmond area a rectangular schematization was made of 1000 m by 1000 m with a spatial resolution of 1 m. The bottom is schematized with the alongshore averaged bathymetry, which is obtained from cross-shore smoothed (5 m boxcar filter) local bathymetric data, which was measured at October 24, 1998.

The SWASH model was applied with a spatial resolution of 1 m in both directions and with one vertical layer. Data were extracted at the location of the wave sensors (Figure 7) and compared with the SWASH model results. In this analysis the possible effect of infra-gravity waves on the individual wave height distribution has not specifically been considered. In view of the focus of this paper, only results of the observed and computed individual wave height distribution are shown in Figure 8 (sensor 2 was out of order in this experiment). The results show that SWASH is quite capable to reproduce the observed individual wave height distribution, except at sensor 4, which is located on top of the first underwater bar. The reason for this mismatch is still to be determined.

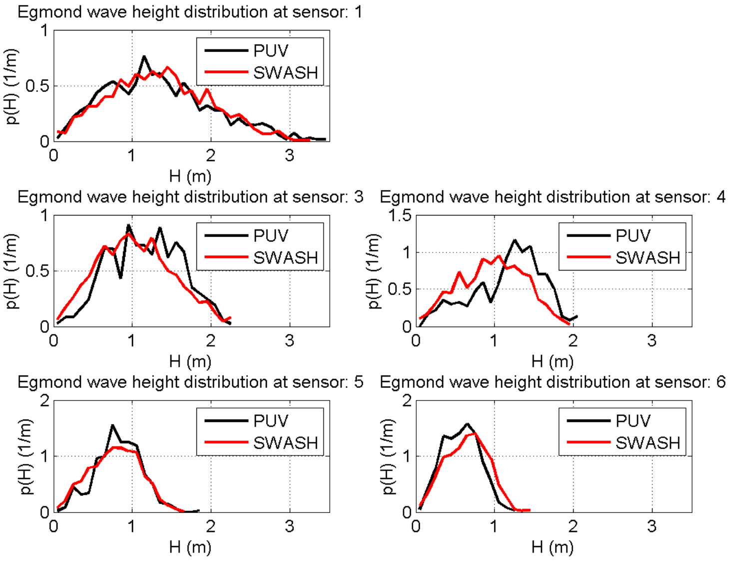

Figure 7: Comparison of observed (red) and computed (black) individual wave height distributions for Egmond case 9330 starting on October 24, 1998 at 1800 hours.

The above results gave trust in the capabilities of the SWASH model to simulate wave height characteristics in 2D-field situations with directionally spread waves. Despite these promising results further tests are needed to determine the optimal spatial resolution and number of layers in the SWASH model application.

7. Directional spreading

To determine the possible role of directional spreading on the individual wave height characteristics a simple academic SWASH application was setup. The setup consists of a planar underwater slope of 6/500. The along-shore dimension was 800 m and the cross-shore dimension 500 m. The input wave condition consisted of irregular waves with a significant wave height of 1 m and a peak period of 10 s. The along-shore spatial step was 1 m and in cross-shore direction 0.5 m. Two SWASH computations were carried out to determine the role of the directional spreading; one with uni-directional waves and one with a direction spreading of 20°. The time step was 0.02 s and the duration was 90 minutes to obtain at least 1000 individual waves for the analysis. Figure 8 shows a snapshot of the spatial variation of surface elevation for the case with a directional spreading of 20°. The results show that the crests are generally higher than the depths of the troughs, refraction diminishes the directional spreading as the waves approach the coast, wave heights become lower due to breaking in shallower water and disappear completely at the coast line.

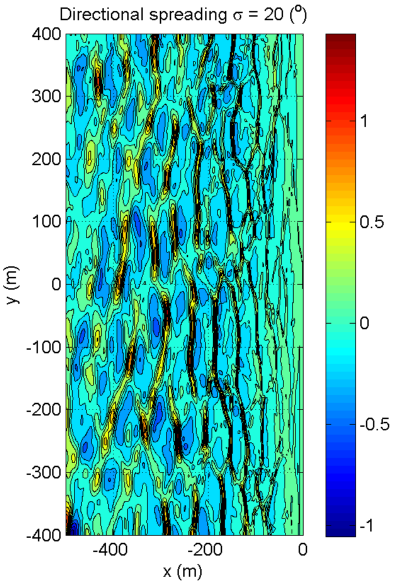

Figure 8: Snapshot of surface elevation in meters for the case with a directional spreading of 20°, an incident significant wave height of 1 m and a peak period of 10 s.

The time series of surface elevation of the SWASH computations were analyzed to determine the individual wave height distribution for the cases with a directional spreading of 0° and 20° as shown in Figure 9. The results show that directional spreading affects the distribution by creating a wider distribution with a slightly larger amount of higher waves. The aggregated results are presented in Figure 10 showing the spatial variation of the wave parameters $H_{1/3}$, $H_{1/10}$ and H_{max}. These results suggest that directional spreading decreases the amount of breaking, leading to higher significant wave heights $H_{1/3}$, $H_{1/10}$ and H_{max}.

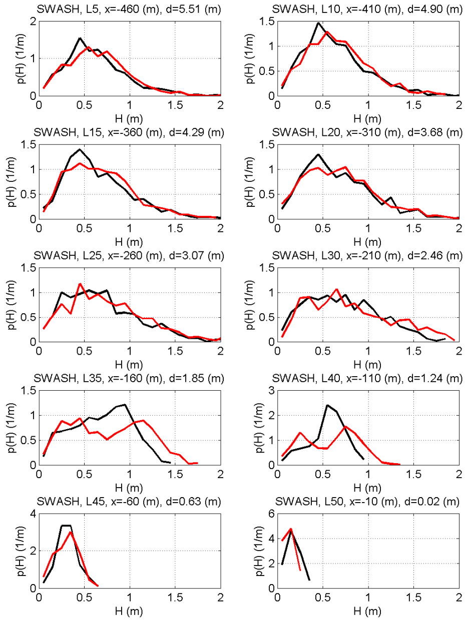

Figure 9: Comparison of computed individual wave height distribution for the cases with a directional spreading of 0° (black) and 20° (red).

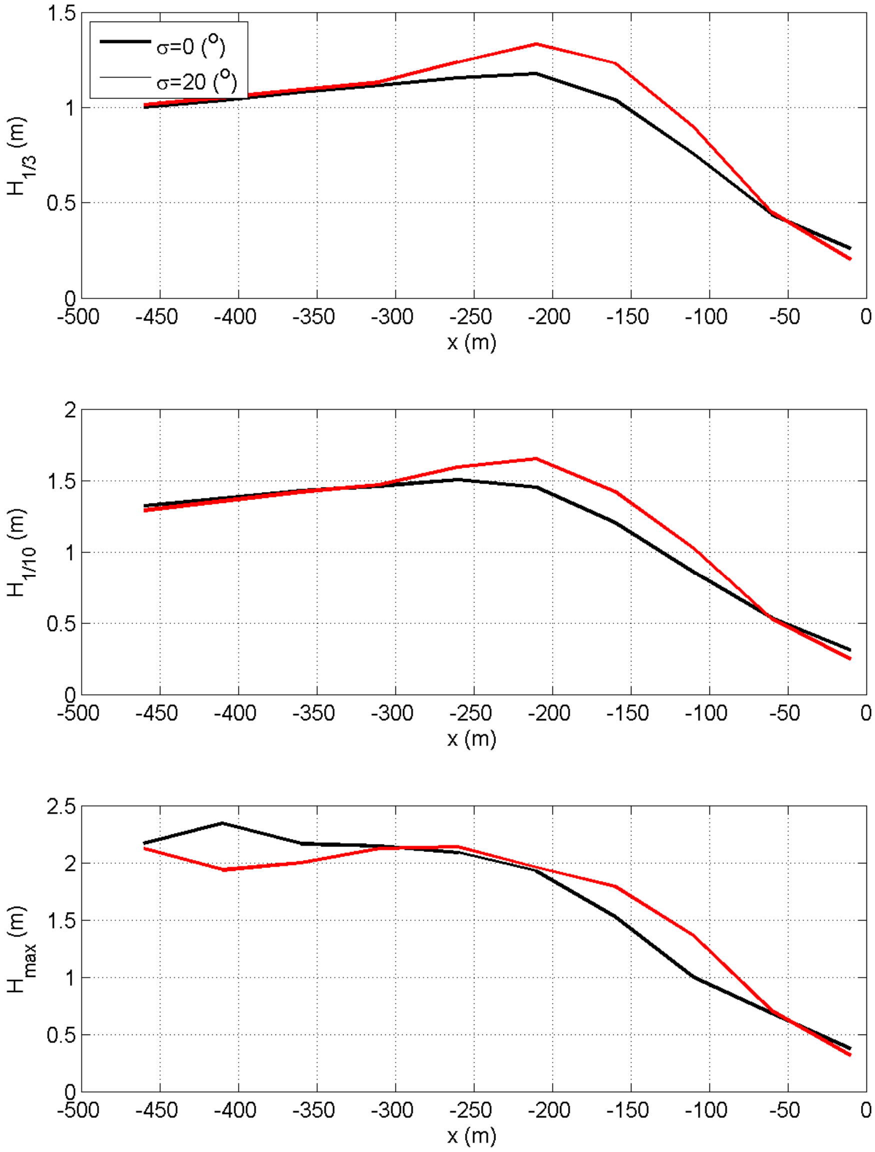

Figure 10: Spatial variation of wave parameters $H_{1/3}$, H_{10} and H_{max} for the case of uni-directional waves (black) and directionally spread waves (red).

8. Discussion

The determination of characteristics of the individual wave height distribution in shallow water is usually based on applying theoretical distributions. This approach suffers from a number of shortcomings. Firstly, the coefficients of these distributions are usually tuned on the basis of 1D-wave flume experiments. Secondly, when applying such a distribution shallow water effects are usually accounted for by incorporating local depth information, sometimes supplemented with an estimate of the bottom slope in the direction of dominant wave propagation. This assumption implicitly assumes that the wave height characteristics are in instantaneous equilibrium with local bathymetric features. This assumption can be challenged as previous history and 2D effect are neglected. Lastly, these theoretical distributions only provide information about the expected value of a certain wave height characteristic without any information on its statistical distribution.

It is noted that in the data processing no distinction was made between bounded-long waves and free short waves. This choice may affect the individual wave height distribution as short waves may be modulated by depth variations on the long infra-gravity wave time scale.

The present approach of applying a numerical wave model provides a solution to these deficiencies as it enables scientists and engineers to derive characteristics of the individual wave height distribution for any location of interest. A practical setup to achieve this is apply spectral wave modeling, e.g. by using the WaveWatch III or the SWAN model, to transform offshore conditions to nearshore locations to provide (quasi) 2D-wave spectra. These spectra can then be used as a boundary condition for the SWASH model. By performing a series of simulations using the same input spectrum but with different seeds, a set of realizations can be generated from which the required characteristics (spreading) can be derived.

9. Conclusions

From the results of this study the following conclusions can be formulated:

1. The SWASH model is capable of simulating the spatial and temporal variation of surface elevation of uni-directional wave flume experiments.
2. Consequently, characteristics of the individual wave height distribution are also well reproduced.
3. The Rayleigh distribution is clearly not valid in shallow water, as well as the GK96 distribution. The BG2000 formulation performs slightly better although.
4. The BG2000 distribution under-predicts the maximum individual wave height.
5. For the Boers experiment the ratio of $H_{\max}/H_{1/3}$ is significantly lower than commonly accepted value of 1.8.
6. Repeated simulations based on the same spectrum, produce nearly identical results for the significant wave height $H_{1/3}$, a 4% narrow band for the $H_{1/10}$ estimates and a 10% band of $H_{\max}$ estimates.
7. The SWASH model fairly well reproduces the shape of the individual wave height distribution for a 2D-field cases observed at the Egmond measurement location.
8. The results for the academic field case suggest that directional spreading affects the individual wave height distribution by lowering the dissipation leading to slightly higher maximum wave heights.

10. Recommendations

The present analysis has been performed on a limited data-set of only one wave flume experiment and one field experiment. It is therefore recommended to extend the set of laboratory experiment and field cases to provide a broader basis for the above formulated conclusions. The field case needs further attention with respect to the spatial resolution and the number of vertical layers as these model settings are expected to influence wave breaking in 2D cases in the SWASH model. In addition the effect of bound long waves on the individual wave height distributions should be assessed. The present analysis compared measurement and computational results with only two ‘shallow’ water theoretical wave height distributions. It is advised to consider also other distribution models. Finally, the present analysis could be extended with crest height statistics.

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