STELLINGEN

behorende bij het proefschrift

Generation and Transport of
Subaqueous Fluid Mud Layers

van

Thijs van Kessel

Delft, 16 september 1997
1. Het vermelden van de viscoseiteit van slib is alleen zinvol indien tevens de condities worden vermeld waaronder deze is gemeten.

2. Het gezegde 'van veel slaag wordt men hard' is ook van toepassing op zachte sliblagen, indien de belasting dermate langzaam wordt opgelegd dat gedraaide condities prevaleren.

3. De sterkte van vers geconsolideerde sliblagen neemt proportioneel toe met de effectieve spanning.

4. De stromingsweerstand in het laminaire regime van materialen met een zwichtspanning is niet slechts afhankelijk van het Reynoldsgetal.

5. Modellering van slibtransport wordt bemoeilijkt door het optreden van discontinuïteiten in erosie en depositie.

6. Baanbrekend onderzoek kan niet worden gecreëerd per ambtelijk decreet.

7. Indien aan universiteiten alleen nog plaats is voor toponderzoek, zal dit niet meer dan middelmatig zijn.
8. Vervuiling van slib moet bij de bron worden aangepakt en niet bij de monding.

9. Het ethische dilemma of het is toegestaan een van de straat opgeraapte papiertje elders weer neer te gooien, speelt ook bij baggerwerkzaamheden een grote rol.

10. Bij het voeren van mobiliteitsbeleid moet men er zich rekenschap van geven dat automobilisten zich veelal door irrationele motieven laten leiden.

11. Door de voortschrijdende popularisering van de luchtvaart zal de statuskloof tussen de beroepen piloot en buschauffeur onherroepelijk vernauwen.

12. De vooruitgang die in de afgelopen jaren is geboekt bij het fabriceren van beeldschermen die het oog niet vermoeien, wordt vaak volledig teniet gedaan door de beelden die hierop worden vertoond.

13. De terugverdientijd van spaarlampen is binnenshuis aanmerkelijk langer dan de consument wordt voorgespiegeld, aangezien de warmteproductie door gloeilampen tijdens koude winteravonden in dit geval niet als een verliespost mag worden gezien.
Generation and transport of subaqueous fluid mud layers

*Ontstaan en transport van vloeibare siblagen onder water*
Generation and transport of subaqueous fluid mud layers

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
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Front cover: mudhoppers on a mudbank near the coast of Brunei, Borneo
photo by Thijs van Kessel

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RAADSEL

Vroeger schreef ik aan een zwaar bureau
Lichtzinnige gedichten;
Nu, met een plankje op mijn knie,
Een lijvigen roman.
Ben ik vooruitgegaan? Wie
Kan mij zeggen, of ik het ben
Dan wel de materie
Die ten slotte moest zwichten?

J. Slauerhoff
Abstract

Siltation of harbours and coastal zones is often undesirable as it diminishes navigability. Besides, deposited mud is often polluted, because it specifically adsorbs contaminants from the water. For managers of harbours and coastal zones it is therefore important to know prior to execution whether planned projects will diminish or increase siltation. The performance of the current cohesive sediment transport models, which are being used as a tool for making predictions, is sometimes disappointing and therefore needs further improvement. The shortcomings are caused to a great extent by an insufficient knowledge of mechanisms underlying cohesive sediment transport.

Of these mechanisms some are studied in this thesis, particularly the generation and transport of concentrated fluid mud layers near the sea bed. For this purpose, first the material properties such as strength and viscosity of mud both in fluid form and solid form are determined, using rheological instruments and a miniature sounding probe. Subsequently, the liquefaction of deposited mud by waves is studied with laboratory experiments, as well as the resulting flow on a sloping bed caused by gravity. Mathematical models aiming at describing these phenomena are compared with the experiments. Finally, these models are used to evaluate the behaviour of mud under natural conditions. In support of this, also some field observations reported in the literature are analyzed.

The conclusion is that mud layers that have been deposited under quiet conditions may suddenly become liquid under storm conditions and can subsequently be transported. The likelihood of this type of behaviour depends on the strength of the deposits, which is related to the degree of consolidation after deposition. Transport of fluid mud layers may result in sudden changes in bathymetry. Especially in navigation channels this phenomenon may occur, as flow of fluid mud on adjacent slopes is enhanced by gravity. This should be well taken into account when modelling. Rheological properties such as strength and viscosity should not be assumed to be constant, but should be set to be dependent on the actual stress condition and the stress history.
'Ontstaan en transport van vloeibare sliblagen onder water'  door Thijs van Kessel

Samenvatting

Aanslibbing van havens en kustgebieden is vaak ongewenst omdat hierdoor de bevaarbaarheid verminderd. Bovendien is het afgezette slib meestal vervuild, doordat het verontreinigingen uit het water specifiek aan zich bindt. Het is voor haven- en kustbeheerders daarom van belang om al voor de uitvoering van projecten te weten of deze de aanslibbing zullen doen toe- dan wel afnemen. De prestaties van de huidige slibstransportmodellen, die als hulpmiddel bij het doen van voorspellingen worden gebruikt, laten nog te wensen over. De tekortkomin- gen worden in belangrijke mate veroorzaakt door een onvoldoende kennis van de mechanismen die aan slibtransport ten grondslag liggen.

Van deze mechanismen worden er in dit proefschrift enkele nader bestudeerd, met name het ontstaan en transport van geconcentreerde, vloeibare sliblagen nabij de bodem. Hiertoe worden allereerst de materiaaleigenschappen zoals sterkte en viscositeit van slib in zowel vaste als vloeibare vorm bepaald met behulp van reologische instrumenten en een kleinschalig sonderingsapparaat. Vervolgens wordt met behulp van laboratoriumexperimenten het verwekingsgedrag van geconsolideerd slib onder golven bestudeerd, alsmede de resulterende afstroming op een hellende bodem onder invloed van de zwaartekracht. Mathematische modellen die deze verschijnselen beogen te beschrijven, worden getoetst aan deze experimenten. Tenslotte worden deze modellen gebruikt om het gedrag van slib onder natuurlijke condities te evalueren. Ter ondersteuning worden ook enige veldwaarnemingen uit de literatuur geanalyseerd.

De conclusie is dat sliblagen die onder rustige condities zijn afgezet, onder stormcondities plotseling weer vloeibaar kunnen worden en vervolgens worden ge- transporteerd. De waarschijnlijkheid van verweking is afhankelijk van de sterkte van de afgezette lagen, die weer gerelateerd is aan de mate van consolidatie die plaatsvindt na depositie. Transport van vloeibare sliblagen kan leiden tot plotseling veranderingen in bathymetrie. Vooral in geulen kan dit verschijnsel optreden, omdat afstroming van vloeibaar slib langs aanpalende hellingen wordt versterkt onder invloed van de zwaartekracht. Bij het modelleren moet hiermee rekening worden gehouden. Reologische eigenschappen als sterkte en viscositeit mogen niet als constant worden beschouwd, maar moeten afhankelijk worden gesteld van de belasting en de belastingsgeschiedenis.
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Chapter 1

Introduction

The bathymetry of estuaries and navigation channels changes with time as a result of erosion and deposition of sediment, which can be characterized by its grain size distribution. Fine-grained sediment, mainly consisting of clay particles—which have particle sizes smaller than a few $\mu$m—is distinguished from silt and sand because of its sticky or cohesive properties. At locations not very much exposed to the influence of waves and current, often a net deposition of cohesive sediment or mud is observed, as opposed to the situation along the coast, where erosion and deposition of sand occur predominantly.

Siltation of navigation channels is undesirable for at least two reasons. First, it results in a decrease in navigable depth, limiting the accessibility to ports. Maintenance dredging is then necessary to keep these channels at their desired depths. Secondly, cohesive sediment is often heavily contaminated with compounds like pesticides, heavy metals and PCB, due to its very large specific surface area. The chemical constitution of this surface also enhances the adsorption of contaminants. So even if siltation does not hinder navigation, sanitary dredging may be necessary if high demands are made upon water and channel bottom quality.

An additional problem is how to dispose of polluted dredged material. As heavily polluted spoil cannot be dumped at sea, cleaning, combustion or disposal under well-controlled conditions are the only alternatives left, all incurring great expense.

It goes without saying that managers of ports and waterways are eager to limit unwanted siltation. The impact of measures to this effect can be assessed partly using cohesive sediment transport models as a tool to predict erosion, transport and deposition of mud, given the movement of water and boundary conditions.

Until now, these models have fallen short in predictive value; a further refinement is therefore needed [96]. Because of ever-increasing computing power, such refinements can be implemented without an excessive increase in processing time. The shortcomings of most current cohesive sediment transport models are
caused by insufficient description of the relevant physical processes themselves rather than by their numerical implementation [107]. If these models are to be improved, first priority should therefore be given to the study of physical processes governing cohesive sediment transport.

Classical transport models focus on the movement of fine particles in the water column. However, mud can be present in three manifestations:

- in suspended, diluted form in the water column;
- in aggregated, highly concentrated but still fluid form near the bottom;
- in solid, consolidated form at the bottom, of which it is part.

By definition, the contribution of solid mud to the total sediment transport is zero, but the contribution of fluid mud layers near the bottom may be significant. In order to assess this possibility, the mechanisms leading to the formation of fluid mud layers have to be identified, as well as the conditions under which these mechanisms operate. Together with the modelling of the transport properties of fluid mud this is the main purpose of this thesis.

Chapter 2 of this thesis is a literature review of processes that play a prominent part in cohesive sediment transport. Processes such as flocculation, sedimentation, deposition, consolidation, erosion, liquefaction, etc. are discussed.

The rheological properties of fluid mud are the subject of Chapter 3, as they are indispensable—combined with knowledge of the external forces—for the calculation of transport rates. These properties are rather complex, since they depend on the stress history of the material. A model to describe them is presented and calibrated with a number of rheological experiments.

Fluid mud layers can develop either by fast deposition of suspended sediment or by failure of soft, freshly deposited mud layers at the sea bed. Failure will occur if the hydrodynamic forces inside the mud layer exceed its yield strength. For adequate modelling, it is essential to know when a fluid mud layer may be considered a permanent part of the sea bed and when such a layer is only temporarily immobilized, and may liquefy under extreme conditions, such as storms, and subsequently be transported. Determination of strength and its evolution is therefore important and will be dealt with in Chapter 4.

Not only the yield strength of deposited mud, but also the forces acting on it have to be known in order to assess whether a mud layer will fail or not. The influence of surface waves is focussed on in this study, as they may generate significant stresses inside a mud layer. A model to calculate these stresses is presented in Chapter 5. Predictions that can now be made about the occurrence of failure are tested with laboratory experiments. The liquefaction process is also studied in some detail.
In Chapter 6, additional laboratory experiments are discussed, regarding fluid mud flow down an incline. These experiments are used to check how well flow can be predicted using simple models, partly based on rheological experiments. Moreover, entrainment of the overlying water and the transition between laminar and turbulent flow are studied. The effect of gravity is explicitly taken into account, as it is one of the main driving forces for fluid mud transport on the slopes of navigation channels, for instance.

Results from previous chapters are integrated in Chapter 7, where their implications for cohesive sediment transport in the field are discussed. It is indicated under which circumstances a significant contribution of fluid mud transport to the total mud transport is to be expected. As an illustration, some field observations reported in the literature are analysed too. Recommendations are made to improve cohesive sediment transport models.

Finally, overall conclusions are drawn in Chapter 8.
Chapter 2

Process Description

In order to be able to model transport of cohesive sediments in coastal areas, the processes involved have to be well understood. First the properties of cohesive sediments will be discussed, followed by the processes involved in cohesive sediment transport. Processes both in the water column and in layers deposited on the sea bed are important. Also the exchange of sediment between water and sea bed has to be considered. In Figure 2.1 these processes are depicted schematically. They are concisely discussed in this chapter; for a more comprehensive discussion the reader is referred to the literature [79, 80, 31, 67, 42, 82, 77, 78, 98].

2.1 Clay properties

In Table 2.1 a classification of sediments is given according to their mean size [65]. The limits between sand, silt and clay are somewhat arbitrary; often slightly different limits are given. Qualitatively they all agree.

The properties of clay particles are significantly different from the properties of sand and silt. Clay minerals have a size of a few μm and smaller, which has several consequences for their behaviour. The permeability of beds consisting mainly of clay particles is very low. Therefore consolidation of these beds as well as the increase in bed strength proceeds very slowly. Freshly deposited clay layers may therefore be easily re-eroded before the consolidation process has finished. Another important consequence of the small particle size of clay is

<table>
<thead>
<tr>
<th>material</th>
<th>d(μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sand</td>
<td>&gt; 60</td>
</tr>
<tr>
<td>silt</td>
<td>2 &lt; d &lt; 60</td>
</tr>
<tr>
<td>clay</td>
<td>&lt; 2</td>
</tr>
</tbody>
</table>

Table 2.1: Classification of sediments according to their mean particle size d [65]
its large specific surface area \( s \), which amounts to \( 10^4 \text{ m}^2 \text{ kg}^{-1} \) and more. It is even larger than expected when particles are assumed to be spherical, in which case the specific surface area \( s \) is calculated from \( s = 6/(\rho d) \), where \( d \) is the particle diameter and \( \rho \) its density, as clay particles have a plate-like rather than a spherical form. The surfaces of clay minerals are charged—either negatively or positively—as a result of differences in electron affinity of the atoms of which clay particles consist. The distance between particles is small enough for electrostatic attraction of surfaces with opposite charges to occur. The thus formed bonds result in cohesion. Van der Waals forces also contribute to cohesion of particles if they are sufficiently closely packed.

The electrolyte concentration has an important impact on the electrostatic interaction. If it is low, the repulsive energy barrier of the equally charged edges or faces may be too high for the particles to coagulate. At sufficiently high electrolyte concentrations, however, coagulation is enhanced.

Cohesion is one of the key factors that cause the difference in behaviour between sediments (mainly) consisting of clay and sediments with a much larger average particle size. Cohesion as described so far is the purely physical interaction of inorganic compounds. However, often a significant amount of organic material is present in natural clayey sediments, which is generally attended with biological activity. Excreational products such as extracellular polymers may form chemical bonds with sediment particles, thus appreciably enhancing cohesion [10, 97].

Figure 2.1: Schematic representation of cohesive sediment processes [79]; see also Figure 2.2
2.2. PROCESSES IN THE WATER COLUMN

Cohesion is also much enhanced when particles have been subjected to high effective stresses during consolidation, for example. They are then packed more closely together, increasing particle interactions. This effect is (partly) irreversible, and after removal of the effective stress an increase in strength compared to the value before consolidation is observed. The sediment bed then is said to be 'overconsolidated'.

These are examples of genuine cohesion; i.e. a strength at zero normal stress. Cohesion may also be apparent, caused by the behaviour of water in very fine-grained sediments. If the water table falls below a clayey sediment bed, e.g. a tidal mudflat during ebb tide, pore water is retained above the water table by capillary tension. Negative pore pressures then result, and a finite effective stress at zero normal stress exists. Capillary forces therefore contribute to the cohesion of partly saturated fine-grained sediments. For these sediments this effect is often more important than genuine cohesion.

Because of the large specific surface area of fine-grained sediments and their chemical composition, pollutants such as heavy metals and pesticides are easily adsorbed. Pollutants therefore accumulate in cohesive sediments; generally much higher concentrations are encountered than in the water column. In industrial areas sea beds consisting of cohesive sediments are often heavily polluted, in which case disposal of dredged material is only feasible after expensive treatment or in special depots under well-controlled conditions.

Another difference between cohesive and non-cohesive sediments is the plastic behaviour of the former sediment at the appropriate water content. The water contents between which soil behaves as a plastic material are given by the Attenberg limits. The plastic limit is defined as the water content of the soil at which threads of soils with a diameter of 3 mm begin to crumble; the liquid limit is defined as the water content at which a certain number of blows is required to close a specific width groove for a specific length in a standard liquid limit device [64]. The plasticity index is defined as the difference between liquid and plastic limits.

2.2 Processes in the water column

Sediment present in the water column settles under the influence of gravity, as the density of sediment particles exceeds the density of water [63]. The settling velocity $v_s$ of a spherical particle can be calculated from the well-known Stokes’ law given its diameter $d_p$, density $\rho_p$ and given the density $\rho_w$ and viscosity $\mu_w$ of the ambient water:

$$v_s = \frac{(\rho_p - \rho_w)gd_p^2}{18\mu_w} \quad (Re \leq 1) \quad (2.1)$$
where \( g \) is the acceleration due to gravity. The settling velocity of particles smaller than approximately 1 \( \mu \text{m} \) is negligible even in still water, as Brownian diffusion will counteract the developing concentration gradients during settling. If the water is turbulent, settling will be negligible at a much higher particle diameter, as turbulent diffusion is more effective than Brownian diffusion. From a balance between downward particle convection (caused by settling) and upward particle diffusion (caused by concentration gradients), an equilibrium vertical concentration profile can be calculated, e.g. [121].

When cohesive sediment particles collide—as a result of differential settling or velocity gradients in the ambient water—they tend to aggregate [68, 66]. Sediment flocs with a diameter of up to \( 10^{-3} \) m are often observed in coastal water, consisting of primary particles with a size in the order of a few \( \mu \text{m} \) [66]. These sediment flocs often show a fractal structure originating from their growth mechanism [55, 44]. The excess density of fractal aggregates \( \Delta \rho_a \) is related to the excess density of the primary particle \( \Delta \rho_p = \rho_p - \rho_w \) according to [55]

\[
\Delta \rho_a = \Delta \rho_p \left( \frac{d_p}{d_a} \right)^{3-D}
\]

(2.2)

where \( d_a \) is the diameter of the floc and \( D \) its fractal dimension, which generally ranges between 1.4–2.2. From (2.2) it is clear that large flocs have only a small excess density and therefore mainly consist of pore fluid. Assuming \( D = 2 \), combining (2.1) and (2.2) leads to a linear relationship between floc size and settling velocity. When subjected to weak external forces, flocs may restructure and become more compacted, which is equivalent with a higher fractal dimension. The maximal floc size depends on the level of turbulence within the ambient fluid; if the (shear) forces are too high, the floc will break up [66]. In shallow-water areas the time needed for deposition may be short compared to the time needed for aggregate growth, in which case the residence time of flocs in the water column is the limiting factor [128].

Aggregate growth is enhanced by an increase in particle concentration, as the collision rate increases. Therefore, the settling velocity will also increase with concentration. However, at higher concentrations—generally near the sediment bed—the phenomenon of hindered settling is observed [63]. In consideration of continuity, a downward volume flux of aggregates must be balanced with an identical upward volume flux of water. If the sediment concentration is sufficiently high, a considerable counter-current is thus created, which reduces the effective settling velocity. The maximal settling velocity generally occurs at a concentration of only a few kg m\(^{-3}\). At such a low solid concentration, already an appreciable volume concentration of flocs can exist because of the tenuous fractal aggregate structure. The concentration at which the particle settling flux
2.2. PROCESSES IN THE WATER COLUMN

is maximal is only slightly higher, and amounts to about 10 kg m\(^{-3}\), depending on sediment type.

At concentrations over approximately 1 kg m\(^{-3}\), stratification effects may become important and density-driven flows may be observed. Stratification has a damping influence on turbulence [43], characterized by the gradient Richardson number \(R_i\), given by

\[
R_i = \frac{-g \frac{\partial \rho}{\partial z}}{\rho(\frac{\partial u}{\partial z})^2}
\] (2.3)

where \(\partial \rho/\partial z\) is the vertical density gradient and \(\partial u/\partial z\) the velocity gradient. For \(R_i >\) about 0.2, turbulence is damped and the stratification is stable [36]. Stratified flow may result in a near-bed velocity of which the direction differs from the depth-averaged velocity. This may have an important implication for sediment transport, as the near-bottom sediment concentration is often much higher than in the upper part of the water column.

Whereas the turbulent (eddy) viscosity decreases in stratified flow, the ‘molecular’ viscosity—which is several orders of magnitude smaller—increases because of the presence of sediment particles enhancing the transfer of momentum [27]. The viscosity of a dilute suspension of spherical particles can be estimated using Einstein’s formula:

\[
\mu = \mu_w (1 + 2.5 \phi)
\] (2.4)

where \(\mu\) is the viscosity of the suspension, \(\mu_w\) the viscosity of the ambient fluid (water) and \(\phi\) the volume fraction of the particles. Equation (2.4) is valid for \(\phi < 0.01\) for non-deformable monodisperse spheres, which is not exactly a good representation of a cohesive sediment suspension. An empirical relationship for the viscosity of cohesive sediment suspensions is [122]

\[
\mu = \mu_w (1 + c_3 \phi^{c_4})
\] (2.5)

where \(c_3\) and \(c_4\) are empirical constants that have to be determined from rheological experiments [19]. The exponent \(c_4\) has a value in the range 1–2, and \(c_3\) may be of the order of \(10^3\), more than two orders of magnitude larger than in (2.4). This may be explained by substituting \(\phi = \phi_p\) in (2.5), where \(\phi_p\) is the volumetric solids concentration, whereas \(\phi = \phi_a\) in (2.4), where \(\phi_a\) is the volumetric concentration of the particle aggregates. This concentration is much larger because of the large quantity of captured pore fluid caused by fractal growth. It is reported that at \(\phi_p = 0.08\) already a network structure may be present throughout the fluid, and therefore \(\phi_a = 1\) (gel point). Suspensions with this floe concentration show a yield strength or yield stress \(\tau_y\), i.e. a shear stress below which the material does not flow. Even at concentrations well below the gel point, the viscosity of suspensions of cohesive sediments depends on the shear rate (non-Newtonian behaviour), as aggregates deform and break-up under shear [120]. The rheology
of cohesive sediment suspensions will be discussed in a more comprehensive form in Chapter 3, which is focused on concentrations beyond the gel point. Suspensions with a concentration of only a few kg m\(^{-3}\) can be well modelled using a slightly increased viscosity compared to water, assuming Newtonian behaviour. In most flow situations encountered in the field, the turbulent eddy viscosity is much more important for these low-concentration suspensions.

As a result of settling, bed deposition occurs (Figure 2.1). The deposition process is most efficient at low turbulence intensities [45], during slack tide, for example. The shear rate is highest near the bed, therefore large flocs may break up close to the bottom and may not reach the bed. The sediment deposition rate \(D\) can be calculated according to [59]

\[
D = v_d C \left(1 - \frac{\tau_b}{\tau_d}\right) \quad (0 \leq \tau_b \leq \tau_d)
\]  

(2.6)

where \(C\) is the sediment mass concentration, \(\tau_b\) the bed shear stress and \(\tau_d\) the critical shear stress for deposition above which no deposition occurs [124]. Eq. (2.6) is an empirical relationship valid for monodisperse suspensions; in reality coarse-grained sediments may deposit whereas fine-grained sediments remain in suspension. Therefore, \(\tau_d\) has to be adapted for each sediment class. The accumulation of sediment on the bed results in an increase in bed level.

### 2.3 Processes in the bed

After deposition the sediment tends to get more compacted as it is slowly buried by more recent deposits. The compaction process is known as consolidation. During consolidation, sediment particles that were fluid-supported before deposition, become gradually supported by the grain matrix: an effective stress, which is the difference between total and pore water pressure, develops. Outflow of water from the bed is an essential part of this process. Consolidation ends when the pressure of the pore water becomes equal to the hydrostatic pressure and pore water outflow stops. The increase in effective stress is attended with an approximately proportional increase in yield strength.

The effective stress \(\sigma'\) represents the stress carried by the grain skeleton. It is defined as the difference between total pressure \(p_t\) and pore water pressure \(p_w\):

\[
\sigma' = p_t - p_w
\]

In a sediment suspension, in which particles are supported by the water phase, the effective stress is zero, whereas the pore water pressure equals the total pressure at all levels:

\[
p_w = p_t = \int_h^0 \rho g dz
\]

(2.7)

where \(z\) is the vertical co-ordinate and \(h\) the thickness of the layer considered. The excess pore water pressure \(p_e\) is defined as the difference between pore water
2.3. PROCESSES IN THE BED

pressure and hydrostatic pressure \( p_h \): \( p_e = p_w - p_h \). For a suspension it equals \( p_e = \int_0^h (\rho - \rho_w) g dz \). As a result of consolidation, the excess pore pressure decreases, whereas the total pressure remains unchanged. During consolidation \( p_e + \sigma' = \int_0^h (\rho - \rho_w) g dz \), and afterwards \( p_e = 0 \); \( p_w = p_h = \int_0^h \rho_w g dz \) and \( \sigma' = \int_0^h (\rho - \rho_w) g dz \). In short, a decrease in excess pore pressure leads to an increase in effective stress.

The time needed for consolidation is strongly dependent on the permeability of the sediment \( k \) and the thickness of the sediment layer \( h \). For primary consolidation, it can be estimated with [65]

\[
t_{92\%} = \frac{\rho_w g m_w h^2}{k}
\]

(2.8)

where \( m_w \) is the soil compressibility and \( t_{92\%} \) the time at which consolidation is 92% complete, often taken as the ‘end’ of consolidation. In (2.8) the compressibility of pore water has been neglected and deformations are assumed to be infinitesimal. Owing to the much lower permeability of cohesive sediment beds compared to sand beds, the consolidation time is several orders of magnitude longer. With consolidation theory [38, 39] the dissipation of pore pressure, the generation of effective stress, the course of concentration distribution etc. can be calculated for a given relationship between \( \sigma' \) and \( e \) and \( k \) and \( e \), where \( e \) is the void ratio defined by \( e = (1 - \phi_p)/\phi_p \). The consolidation process is not discussed in detail here; for further information the reader is referred to the literature [38, 39, 54, 118, 113, 84].

Complications regarding the description of bed properties result from chemical activity and biological activity. These have at least two important consequences:

1. the formation of gas, by which the compressibility of the soil is much increased,

2. a change—either an increase or a decrease—in bonding strength.

The former is caused by either aerobic or anaerobic oxidation of organic material present in the sediment. The latter may be caused by changes in redox potential, pH and the generation of extra-cellular polymers (ECP) by sediment fauna, for example [109]. Also the reworking of sediment by worms and other animals may affect the sediment properties (‘bioturbation’). It is difficult to include all these effects in a consolidation model predicting properties such as bulk density and strength profiles.

If the gain in strength as a result of the consolidation process is so large that the deposited sediment will resist even the largest forces acting on it given the tidal and wave regimes, the sediment becomes a permanent part of the sea bed.
However, if the gain in strength is not that large and the deposition therefore is only temporary, the thus immobilized sediment should remain a part of the sediment budget and deposition should not be considered as a permanent sink.

Several mechanisms of erosion of a sediment bed may occur; a distinction is made between surface erosion and bulk erosion. As can be conjectured from the term 'surface erosion', surface erosion takes place at the surface of the sediment bed. The driving force for this type of erosion is the bed shear stress \( \tau_b \) caused by the motion of the overlying water. The stability of non-cohesive particles like sand and gravel can be estimated with the well-known Shields' diagram. Before a cohesive sediment particle can be eroded, the bonding between the particle and its neighbours has to be broken. As sediment particles at the surface are bonded to fewer particles than in the bulk, the critical shear stress for erosion \( \tau_c \) of these particles is lower than the yield strength. For a perfectly homogeneous material a ratio of approximately 2/3 between \( \tau_c \) and \( \tau_y \) is to be expected, whereas for an inhomogeneous material this ratio may be locally much smaller [126]. The critical shear stress for erosion tends to increase with decreasing elevation in the bed, because deeper sediment layers are more compacted as a result of the consolidation process and have stronger interparticle bonding [76, 4]. The surface erosion rate \( E \) per unit of area can be estimated empirically with [57, 94, 93]

\[
E = \frac{dm}{dt} = m_\varepsilon (\tau_b - \tau_c)
\]

where \( m_\varepsilon \) is the erosion constant which is strongly dependent on sediment type, bed density, etc. If surface erosion occurs, the bed level will decrease gradually.

In contrast to surface erosion, failure of a sediment layer may result in a sudden change in bed level. Failure occurs if the deviator stress—which is the maximal shear stress—locally exceeds the yield strength of the bed. Shear stresses inside the bed are generated by pressure gradients on the bed surface and earthquakes, for example [90, 6]. Under extreme conditions the shear stresses thus generated are much higher than the bed shear stress caused by velocity gradients near the bed. The effects of earthquakes on the stability of freshly deposited sediment layers are not considered herein.

Pressure gradients at the bed surface are caused by hydraulic gradients, waves and turbulence, for example. Hydraulic gradients are generally weak, and typically range between 0.01 and 1 Pa m\(^{-1}\). These gradients are far too small to cause failure of a consolidated mud bed, but may cause a net transport of fluid mud, which has a low strength. As shown by De Wit [131] and in Chapter 5, pressure gradients caused by (surface) waves are much higher, and may be up to a few thousand Pa m\(^{-1}\) during storm periods. These gradients are sufficient to cause failure of freshly deposited mud layers. The penetration depth of wave-induced shear stresses inside the bed is of the same order as the wave-length.
2.3. PROCESSES IN THE BED

Turbulent pressure gradients may be of the same order of magnitude as wave-induced pressure gradients, but have a much smaller penetration depth as their length-scale is smaller than for most surface waves. In this case, only failure of the upper layer of the bed is possible. Only the largest turbulent structures, which are of the same order of magnitude as the water depth in shallow water areas, have a larger penetration depth. However, the pressure gradients generated by these structures are generally small compared to those generated by waves.

Water pressure gradients have several effects on a sediment bed, depending on whether the situation considered is drained or undrained. If water pressure changes are very slow, pore water flow is not limited by the finite permeability of sediment. In this case the bed is fully drained, and the pore pressure equals the hydrostatic pressure. Forces exerted by the water pressure gradients on the sediment bed are negligible. If pressure changes are fast, for example caused by waves, pore water flow is inhibited by the finite permeability of sediment. In this case stresses inside the bed are generated by the pressure gradients on the bed surface [137, 136, 111, 112, 71, 25]. Normal stresses are well absorbed by the pore water, but the shear stresses are fully carried by the grain skeleton. If these stresses exceed the yield strength of the bed, failure occurs. Depending on the packing of the sediment, two possibilities exist:

1. positive excess pore pressure is generated, or
2. negative excess pore pressure is generated.

The first possibility occurs when the sediment is loosely packed. Upon failure, the particles partly lose contact as bonds break-up, resulting in the reported positive excess pore pressure. An increase in pore pressure is attended with a decrease in effective stress, which causes a sudden loss of strength [81]. The original solid-like behaviour of the bed then changes into a more fluid-like behaviour, referred to as 'liquefaction' [100, 131].

The second possibility occurs if the sediment is densely packed. In this case, the void ratio tends to increase during rearrangement of the particles upon failure, resulting in the reported negative excess pore pressure. A decrease in pore pressure is attended with an increase in effective stress, which causes an increase in strength. This is a stable situation, and failure will only occur if the shear stress is further increased, or if the pore pressure is allowed to dissipate, in which case failure is not undrained anymore. The transition between loosely and densely packed sediments is given by the critical state line [51], defined as the line in the $p-e$-plane at which no volume expansion or contraction is observed under shear. Here $p = \frac{1}{2}(\sigma_{xx} + \sigma_{zz})$ and $e$ is the void ratio. Sediments that tend to expand upon failure are in a state below this line and are stable, whereas sediments in
a state above this line are potentially unstable and prone to liquefaction. Undrained failure of closely packed sediments by wave-induced pressure gradients is unlikely, as the yield strength of these sediments is high. Undrained failure of freshly deposited, loosely packed sediments is much more likely and therefore is the subject of further discussion in Chapter 5. As no water flow occurs in the fully undrained case, the bed may be modelled using a one-phase approximation.

Until now, only the two extremes—fully undrained and fully drained—were considered. Often the behaviour of a sediment bed is partially drained, some pore water flow then occurs, but the excess pore pressure is not dissipated completely. This has a stabilizing influence on loose sediments, as effective stresses are higher than in the undrained case. If the build-up of pore pressure caused by the wave-induced oscillatory shear stress is balanced by the pore pressure dissipation caused by pore water flow, no failure occurs. After removal of the oscillatory stress and dissipation of the pore pressure, the sediment bed has been densified compared to the initial situation [34].

For dense sediments below the critical state line, however, (partially) drained behaviour has a negative influence on the yield strength, as negative excess pore pressures are partially dissipated. Although failure is unlikely in the undrained case, it may occur if shear stresses are applied for a sufficiently long time. If the stress is removed before failure occurs, the sediment bed has become more loosely packed, after dissipation of the negative pore pressure, compared to the initial situation.

Under waves and assuming partially drained behaviour, there will be a net flow of water into the bed between the passage of a wave crest and a wave trough, and a net flow of water out of the bed between the passage of a wave trough and a wave crest. This effect has a destabilizing influence on sediment particles under wave troughs, thereby enhancing erosion. This erosion mechanism may be important for sediments with an intermediate permeability (e.g. sand), but is of negligible importance for mud beds.

2.4 Fluid mud

In this thesis, fluid mud is defined as a suspension of clay-sized particles up to 60 \( \mu m \) in water with a concentration that is sufficiently high to prevent lutocline formation when it is left at rest, and in which the effective stress is negligible compared to the excess pore pressure. It is important to note that sediment particles in fluid mud are supported by pore water for the major part, accounting for its low yield strength. If, as a result of consolidation, effective stresses become of the same order of magnitude as the excess pore pressures, mud may not be termed fluid anymore.

Processes involved in fluid mud transport are schematically indicated in Fi-
2.4. **FLUID MUD**

Figure 2.2. Fluid mud can either be generated by failure of loosely packed cohesive sediment beds, or by deposition if the flux of settling particles towards the bed exceeds the consolidation rate of the bed and a gradual build-up of unconsolidated sediment takes place. Fluid mud generated by the former mechanism has a concentration close to or even equal to the original bed concentration [37], whereas the concentration of fluid mud generated by deposition tends to be lower. The former mechanism is more important in areas with a relatively high sediment concentration in the water column resulting in a sufficiently large flux, whereas the latter mechanism is more important in areas with a low sediment concentration in the water column. In that case a large amount of sediment is suddenly mobilized, which had been slowly accumulating on the sea bed.

The clue to the existence of fluid mud layers is their low permeability, which makes support of sediment particles by pore water possible for a prolonged period of time. When fluid mud is left at rest, it will slowly consolidate and eventually form a consolidated sea bed. However, when fluid mud is mildly agitated, for example by waves or tidal flow, consolidation will slow down [133]. Fluid mud layers have been reported to be permanently present for a period of several years, for instance at Emden harbour [135]. Fluid mud is bound to disappear when it is left at rest, but it will also disappear when it is agitated too vehemently. The interface between fluid mud and overlying water, which is stable when at rest and at mild shear because of the high local density gradient, then becomes unstable and interfacial mixing will occur. Interfacial stability can be expressed in terms of the Richardson number $R_i$, defined in (2.3) [56, 18].

Fluid mud is easily displaced under the influence of external forces, for example pressure gradients or gravity, as its strength is low [24, 138, 101]. The displacement of fluid mud may result in very high transport rates compared to sediment transport in the water column [102], as the sediment concentration in fluid mud generated by liquefaction is in the order of a few hundred kg m$^{-3}$, whereas the sediment concentration in the water column is in the order of a few tenths of a kg m$^{-3}$, at the most, leaving a gap of approximately a factor 1000. In order to calculate the transport rate of fluid mud its flow velocity, and as a consequence the forces applied to the mud as well as the constitutive behaviour of the mud have to be known.

These forces can be divided into volume forces—of which only gravity is relevant in this case—and surface forces. The gravity force acting on a fluid mud layer can be easily calculated given its (excess) density, thickness and given the bathymetry of the sea bed. For example in cases of siltation of navigation channels caused by slicing of fluid mud layers on adjacent slopes, it is important to take this into account [107]. The surface forces—normal stress (pressure) and shear stress—can be derived from a hydrodynamical model of the overlying water layer. As was mentioned before, pressure gradients are generated by hydraulic
gradients, surface waves and turbulence. Shear stresses are generated by velocity gradients near the bed and turbulence.

A problem arises if the thickness and expanse of fluid mud is so large that the hydrodynamics of the water layer are influenced. For example, waves may be significantly dampened when propagating over a fluid mud layer, as wave energy is easily dissipated in fluid mud because of its high viscosity [139, 132, 72, 83, 75, 74, 47, 48]. Also, turbulence may be dampened by the density gradients near the interface between fluid mud and water, resulting in a much reduced bottom friction coefficient. The propagation velocity of tidal waves in estuaries with large amounts of cohesive sediments can often only be modelled well when assuming a bottom friction coefficient that would be unrealistically low if no dampening of turbulence near the bottom would occur [127]. If water motion is influenced by fluid mud motion, two coupled problems exist, that should be solved simultaneously. Alternatively, an iterative approach may be adopted.

If the forces exerted on the mud layer are known, its transport rate can be calculated assuming a certain rheological behaviour, i.e. the relation between stress and strain or strain rate. The rheology of fluid mud is rather complex, as its viscosity and yield strength are functions of both its shear rate and its shear rate history [125, 92, 22, 117, 15, 14, 89, 88, 123, 35]. In Chapter 3 the rheology of fluid mud is discussed comprehensively; here only a few remarks suffice. As the sediment concentration of fluid mud is high, its viscosity will be much higher than that of water. A viscosity of 1 Pa s—1000 times the viscosity of water—is not exceptional. Laminar motion tends to prevail, in which case the effective viscosity is determined solely by the rheological properties of fluid mud instead of the properties of the flow. Fluid mud exhibits a shear-thinning behaviour, which means that its effective viscosity decreases with increasing shear rate.
Chapter 3

Rheology of fluid mud

The rheological behaviour of fluid mud determines its transport rate given the forces acting on it. Knowledge of this behaviour is therefore essential to model the (laminar) flow of fluid mud. This chapter\footnote{Submitted in adapted form to J. Hydr. Res. under the title ‘Rheology of cohesive sediments: comparison between a natural and an artificial mud’ by Thijs van Kessel and C. Blom} deals with the rheology of concentrated cohesive sediment suspensions. A one-phase approximation is adopted; therefore experiments have to be carried out in such a way that sedimentation and consolidation effects are negligible.

In §3.1 rheological models are discussed in general; in §3.2 a constitutive model for cohesive sediments is presented; in §3.3 methods for measuring rheological properties and sources of error are discussed concisely; in §3.4 the instruments and materials used are discussed; in §3.5 experimental results regarding cohesive sediments are presented and discussed; finally, conclusions are drawn in §3.6.

3.1 Rheological models

3.1.1 Balance equations

In order to describe the motion of a fluid, balance equations have to be formulated. In principle, the density $\rho$, the velocity vector $\mathbf{v}$ and the total energy $e$ can be calculated from the balances of mass, momentum and energy. If one assumes the equation of momentum to be independent of the thermodynamic state of the material, the equations of continuity and momentum are independent of the equations of energy and can be solved separately. The assumption of incompressible flow—which is a good approximation for fluid mud—leads to the following equation of continuity: $\nabla \cdot \mathbf{v} = 0$. The only balance equation left that is needed to solve $\mathbf{v}$ is the momentum balance:

$$\frac{D\rho \mathbf{v}}{Dt} = -\nabla p - \nabla \cdot \mathbf{\sigma} + \rho g$$

(3.1)
where $t$ is time, $p$ is pressure, $\sigma$ is the stress tensor and $g$ is gravity. A constitutive equation is needed for the closure of (3.1), i.e. a relationship between velocity gradients and stress tensor. Such equation can be derived from rheological measurements.

### 3.1.2 Constitutive equations

The constitutive behaviour of materials is basically determined by microscopic internal mechanisms like spring action (elasticity), friction (viscosity) and inertia. If the latter is neglected, materials are called visco-elastic. Visco-elastic materials are therefore characterized by two independent moduli, the modulus of elasticity $G$ (Pa), and the modulus of viscosity $\eta$ (Pa s). Those materials, also called ‘simple materials’, have a characteristic time ($[\eta/G] = s$), but no characteristic length scale, which means that the stress tensor of visco-elastic materials at a certain position is determined by the deformation history of that position only. The quotient of characteristic time to observation time is the Deborah number $De$; materials with $De \rightarrow 0$ behave viscously, whereas materials with $De \rightarrow \infty$ show elastic behaviour. In materials for which inertia may not be neglected, the stress tensor at a certain position is determined by the deformation history of a finite volume around that position. Therefore these materials also have a characteristic length scale. Inertia can be important under extreme conditions, e.g. abrupt loading. Only simple materials are considered herein, in which inertia is negligible.

### 3.1.3 Viscous models

Flows with $De \rightarrow 0$ can be well described with a viscous model. The material is now only characterized by its viscosity $\eta$. For a Newtonian fluid $\eta$ is a constant by definition, yielding as a constitutive equation for incompressible flow $\tau = 2\eta D$, where $\tau$ is the deviatoric part of the stress tensor $\sigma$ and $D$ is the symmetric rate of deformation tensor with Cartesian components $D_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$. Pure-viscous fluids are Newtonian by definition.

However, the viscosity of a lot of materials, such as polymers and suspensions, is dependent on the shear rate (non-Newtonian behaviour). In this case, the dependency of $\eta$ on the stress tensor invariants has to be determined: $\eta = f(I_{D}, II_{D}, III_{D})$, where $I_{D}$, $II_{D}$ and $III_{D}$ are the first, second and third invariants of the rate of deformation tensor, respectively. If the flow is assumed to be incompressible, $I_{D} = 0$. For simple shear also $III_{D} = 0$ and $II_{D} = 2\dot{\gamma}^2$, where $\dot{\gamma}$ is the shear rate. With classical viscometry, the effective viscosity can therefore only be determined as a function of the shear rate $\dot{\gamma} = (\frac{1}{2}II_{D}^{\frac{1}{2}})$, as simple shear flow is present in the rheometer. From this follows $\tau = 2\eta(II_{D})D$ or, in simple
3.1. RHEOLOGICAL MODELS

shear flow,

$$\tau = \eta(\dot{\gamma})\dot{\gamma}$$  \hspace{1cm} (3.2)

This relationship contains the maximum information that can be obtained from classical viscosimetry. It can only be used in flows that do not deviate much from simple shear flows.

A power-law dependency of $\eta$ as a function of $\dot{\gamma}$ is often observed over several decades: $\eta = c_1 \dot{\gamma}^{c_2}$, where $c_1$ and $c_2$ are model constants. Due to its simplicity and wide range of applicability this model is popular in engineering practice.

In principle, the stress response of a material to a certain deformation or deformation rate is not only a function of that deformation but also of the deformation history. This should be taken into account if the time for memory decay of the material is of the same order as the observation time or larger. For suspensions of cohesive sediments this is generally the case. This phenomenon—called ‘thixotropy’—is addressed in greater detail in §3.2.

3.1.4 Visco-plastic models

For flows defined by (3.2), it is assumed that the shear rate is a continuous function of the shear stress as it approaches zero. This leads to the definition of a viscous material as a material that flows when a deviatoric stress—whatever small—is applied to it. However, this is not true for all materials. The shear rate may also be a discontinuous function of the deviatoric stress, e.g. if a shear rate only develops for deviatoric stresses exceeding a certain critical value, the ‘yield stress’ $\tau_y$. This type of material can be characterized as visco-plastic. In order to take this into account, (3.2) is sometimes rewritten for simple shear flow as:

$$\tau = \eta_\infty \left(1 + \left(\frac{\tau_y}{\eta_\infty}\dot{\gamma}\right)^\frac{1}{m}\right)^m \dot{\gamma}$$  \hspace{1cm} (3.3)

where $\eta_\infty$ is the effective viscosity at infinite shear rate, $\tau$ the scalar shear stress and $m$ a flow behaviour index. Substitution of $m = 1$ leads to the well-known Bingham plastic model, whereas $m = 2$ leads to the Casson model. Equation (3.3) is partly based a yield criterion introduced by Maxwell and Von Mises:

$$II\tau - 2\tau_y^2 \geq 0$$  \hspace{1cm} (3.4)

This criterion is sufficient for simple shear flows for which the third stress tensor invariant vanishes. With this criterion it is assumed that yielding is only dependent on the deviatoric stresses and independent of pressure. For granular materials this is not true [50]. In Chapter 4 this point is discussed in detail.

It is important to notice that the concept of yield stress is dependent on the time-scale considered; it only exist for large Deborah numbers, that is, if
the intrinsic response time of the material is much larger than the observation time. A material that seems not to flow if a certain shear stress is applied, will eventually flow if the shear stress is applied for a sufficiently long duration. The time needed to observe flow is strongly dependent on the accuracy of the instrument. However, if one is not interested in long time scales or extremely low shear rates, yield stress is a valid concept.

3.1.5 Differential visco-elastic models

The models described in the previous sections only incorporate viscous and plastic behaviour. However, as non-Newtonian behaviour is coupled with memory effects, in principle also elastic behaviour should be incorporated into a model describing non-Newtonian flow accurately. For flows in which deformations are large and frequencies low, visco-elastic models of the differential type are most suitable. By definition, a model is of the differential type if the stress tensor $\tau$ can be expressed as a function of a finite number of material fluxes of the deformation gradient.

To take non-Newtonian behaviour into account, three material functions can be defined for stationary flow:

\[
\begin{align*}
\tau_{12} = \tau_{21} &= \eta(\dot{\gamma}) \dot{\gamma} \\
N_1 &= \sigma_{11} - \sigma_{22} = \psi_1(\dot{\gamma}^2) \dot{\gamma}^2 \\
N_2 &= \sigma_{22} - \sigma_{33} = \psi_2(\dot{\gamma}^2) \dot{\gamma}^2
\end{align*}
\] (3.5)

where $\eta$ is the previously defined effective viscosity and $\psi_1$ and $\psi_2$ are the first and second normal stress coefficients. Here, the $x_1$-axis represents the direction of flow, $x_2$ the direction of velocity gradients and $x_3$ the indifferent direction. In addition to the shear stress function, now also the first and second normal stress functions are taken into account to improve the material description. If normal stresses are measured during stationary simple shear experiments, the information obtained can at best be used to quantify the generalized model of a second order fluid of the differential type:

\[
\tau = \eta A_1 - \frac{1}{2} \psi_1 A_2 + (\psi_1 + \psi_2) A_1^2
\] (3.6)

where $A_2 = (\partial/\partial t) A_1$ and $A_1 = 2D$. The generalized Newtonian model (3.2) presented in §3.1.3 is a special case of (3.6).

Normal stress measurements are not easy to perform. Especially the second normal stress coefficient is difficult to measure; even its sign is often wrong. From the normal stress measurements the characteristic time $t_{(n)}$ of a material can be calculated according to:

\[
t_{(n)} = \lim_{\dot{\gamma} \to 0} \frac{N_1(\dot{\gamma})}{\tau(\dot{\gamma}) \dot{\gamma}} = \frac{\psi_{1,0}}{\eta_0}
\] (3.7)
where the subscript 0 refers to the situation where $\dot{\gamma} \to 0$.

### 3.1.6 Integral visco-elastic models

If deformations are small and frequencies high, models of the integral type are more suitable than models of the differential type. The former type of models is derived from the Boltzmann superposition principle for linear systems, leading to a convolution integral. The constitutive equation for incompressible, linear visco-elastic fluids can be written as [60]

$$\tau = 2\eta_s D(t) + 2 \int_0^\infty G(s) D(t-s) ds$$  \hspace{1cm} (3.8)

where $G$ is the shear relaxation modulus and $\eta_s$ the viscosity of the fluid in which the particles are suspended. From (3.8) discretised models such as the Maxwell and Kelvin-Voigt models can be derived.

The shear relaxation modulus can be determined by applying a sinusoidal strain $\gamma(t) = \gamma^0 \sin \omega t$, and measuring the resulting shear stress response $\tau(t)$, which can be written as

$$\tau(t) = \tau^0 \sin(\omega t + \delta) = \gamma^0 (G' \sin \omega t + G'' \cos \omega t),$$  \hspace{1cm} (3.9)

where the storage modulus $G'$ is defined as $\tau^0 / \gamma^0 \cos \delta$ and the loss modulus $G''$ as $\tau^0 / \gamma^0 \sin \delta$. The parameter $\delta$ is the loss angle, equal to zero for elastic materials and equal to $\pi/4$ for viscous materials. For intermediate angles the material behaves visco-elastic. The storage and loss moduli are constants only at sufficiently low strains, where the behaviour of the material is linear. At higher strains the structure breaks down and $G'$ and $G''$ decrease. The sample is then affected by the experiments, whereas this is not the case in the linear range. Varying the applied strain therefore gives an important indication of the range of structural integrity.

Apart from the strain also the oscillation frequency can be varied. Assuming a strain amplitude in the linear range, a predominantly elastic behaviour is generally observed at high frequencies, whereas a more viscous behaviour is observed at low frequencies. The frequency of transition between these regimes reveals the time-scale of the relaxation mechanism.

Experiments with sinusoidal strains are relevant for cohesive sediments, because under field conditions the same type of loading is caused by surface waves. It can be assessed whether the response is linear by varying the strain amplitude, and the theory of elasticity can be used to model the results. If the response is non-linear, which occurs at high deformations, higher-order models are needed. However, such models are not dealt with herein.
3.2 Rheological model for cohesive sediments

To describe the rheological measurements reported in §3.5, a thixotropic model proposed by Toorman [117] is adopted. This model incorporates two independent time-scales: one for structural changes within the particle aggregates responding to changes in shear rate, and another for changes in the interaction between the aggregates. The former structure is referred to as the ‘dynamic’ structure, whereas the latter, resulting in a network structure throughout the material, is called the ‘static’ structure. As will be shown from the experiments, the build-up of the static structure takes much more time than the build-up of the dynamic structure, legitimating the use of two structural parameters. The static structure is broken up already at very low shear rates, whereas the dynamic structure is still partly existent at \( \dot{\gamma} = 20 \text{ s}^{-1} \) for China clay, for example.

In this way, structural interactions and changes within the fluid mud can be taken into account on a macroscopic level. The intensity of those interactions and their response to changes in shear rate can only be quantified from data obtained from rheological experiments. It would be convenient to calculate these properties from a microscopic analysis taking into account all particle interactions. However, ‘computational rheology’ is still in its stage of infancy for cohesive sediment systems, as these are very complex and have, in the case of natural muds, an ill-defined composition.

The rheological model proposed by Toorman [117] for simple shear flow is described by

\[
\tau = \lambda_S(\tau_y,S - \tau_y,D) + \lambda_D\dot{\tau}_y,D + (\eta_\infty + c_{1,D}\lambda_D + (c_{2,D}/c_{3,D})\lambda_{D,e}\tau_y,D)\dot{\gamma}, \tag{3.10}
\]

where \( \lambda \) is a structural parameter which equals zero when the aggregate structure is completely broken up and equals unity when this structure is completely intact. Subscripts \( S \) and \( D \) refer to the static and dynamic structure, respectively, and the subscript \( e \) refers to an equilibrium or steady-state situation. The symbols \( c_1, c_2 \) and \( c_3 \) are model constants. If the static structural parameter is dropped and the yield stress made independent of the remaining structural parameter, the better-known Moore [91] model is obtained.

The rate equation for the structural parameters, assuming first-order kinetics, is given by

\[
\frac{d\lambda_i}{dt} = c_{3,i}(1 - \lambda_i) - c_{2,i}\lambda_i\dot{\gamma}, \tag{3.11}
\]

where \( i = D, S \). At equilibrium, when \( d\lambda_i/dt = 0 \), \( \lambda_{i,e} = 1/(1 + (c_{2,i}/c_{3,i})\dot{\gamma}) \) applies. Under equilibrium conditions (3.10) changes into

\[
\tau_e = \lambda_{S,e}(\tau_y,S - \tau_y,D) + \lambda_{D,e}\tau_y,D + (\eta_\infty + (c_{1,D} + (c_{2,D}/c_{3,D})\tau_y,D)\lambda_{D,e})\dot{\gamma}. \tag{3.12}
\]
This model is able to reproduce a minimum in the equilibrium flow curve, as is sometimes observed for cohesive sediments (§3.5.1). This is another argument for the use of two independent structural parameters. If $\lambda_{S,e}$ is dropped, the minimum disappears. However, the minimum can also be reproduced with (3.10) in which $\lambda_S$ has been set to zero, assuming disequilibrium during the determination of a flow curve. Disequilibrium is easily achieved if the shear rate is increased or decreased too fast.

The Worrall and Tuliani [134] model, which has been reported to reproduce experimental flow curves of clay suspensions quite well, can be obtained from (3.12) by taking $\lambda_{S,e} = 0$ and dropping $\lambda_{D,e}$ multiplying $\tau_{y,D}$, resulting in

$$\tau_e = \tau_{y,D} + [\eta_\infty + (c_{1,D} + (c_{2,D}/c_{3,D})\tau_{y,D})\lambda_{D,e}][\dot{\gamma}]$$ (3.13)

If all structural changes are neglected, also taking $\lambda_{D,e} = 0$, the well-known Bingham plastic model is found: $\tau = \tau_0 + \eta_\infty \dot{\gamma}$, where $\tau_0 = \tau_{y,D}$.

From the equilibrium flow curves, which have to be determined experimentally, all parameters used in the rheological models described above can be determined, except for the parameters $c_{2,i}$ and $c_{3,i}$, which define the time-scale for structural changes. Only the ratio $c_{2,i}/c_{3,i}$ of these parameters is found, but not the parameters themselves. Additional experiments are necessary, for example experiments in which stepwise changes in shear rate are applied. The shear stress response of the material then reveals the time-scale for structural changes. If both the highest and lowest shear rate are sufficiently high ($\dot{\gamma} > 5$ s$^{-1}$, in the experiments to be described in §3.5), $\lambda_S$ may be assumed to be zero, and the shear stress response results from changes in $\lambda_D$ only. This response can then be written as

$$\tau(t) = a_3 + a_2 \exp(-a_1 t),$$ (3.14)

assuming a step in shear rate at $t = 0$. In this equation

$$\begin{align*}
a_1 & = c_{3,D} + c_{2,D}\dot{\gamma}_\infty \\
a_2 & = (\tau_D + c_{1,D}\dot{\gamma}_\infty)(\lambda_{D0} - \lambda_{D\infty}) \\
a_3 & = \lambda_{D\infty}\tau_D + (\eta_\infty + (c_{2,D}/c_{3,D})\lambda_{D\infty}\tau_D + c_{1,D}\lambda_{D,\infty})\dot{\gamma}_\infty
\end{align*}$$ (3.15)

where $\lambda_{D,0}$ and $\lambda_{D,\infty}$ are the equilibrium structural parameters just before and long after the applied step, respectively, and $\dot{\gamma}_\infty$ is the shear rate after the applied step. Together with the already known ratio $c_{2,D}/c_{3,D}$ the parameters $c_{2,D}$ and $c_{3,D}$ can now be calculated separately, and the time-scale for structural changes is obtained. For this calculation the parameters $a_2$ and $a_3$ are not essential, they are only mentioned for the sake of completeness.

Now the only unknown parameters left in the rheological model are $c_{2,S}$ and $c_{3,S}$, the ratio $c_{2,S}/c_{3,S}$ of which is known from the equilibrium flow curve. The parameter $c_{3,S}$ can be obtained from yield stress recovery experiments as follows.
The sample is sheared at a very high shear rate ($\dot{\gamma} \geq 100$ s\(^{-1}\)) for some time, breaking down all structure. After that the shearing is stopped, resulting in a slow recovery of the static structure and a fast recovery of the dynamic structure. After a certain time the yield stress of the sample is measured by slowly increasing the applied torque from zero until the material starts to flow. This procedure is repeated several times for different recovery times, thereby obtaining the yield stress as a function of recovery time. From (3.10) and (3.11) it then follows that

$$\tau(t) = \tau_y(t) = (1 - \exp(-c_{3,y}t)) (\tau_S - \tau_D) + \tau_D,$$

(3.16)
as $\dot{\gamma} = 0$ at the onset of motion, and assuming a fast recovery of the dynamic structure ($\lambda_D = 1$). The value for $c_{3,y}$ and therefore also $c_{2,y}$ follows from regression of this equation to experimental results. Summarizing, all coefficients in the rheological model used to describe the simple shear flow behaviour of cohesive sediments can be obtained from their equilibrium flow curves, yield stress recovery experiments and steps-in-shear-rate experiments.

### 3.3 Rheometry

Instruments for rheological measurements can be divided into two categories, i.e. controlled stress and controlled shear (rate) instruments. With the first a torque is exerted on the bob of the instrument, which results in shear stresses in the sample. The resulting rotation angle or, when the material flows, rotation speed is measured. This technique is the most suitable one if fluids with a yield stress have to be measured, as the structure of the sample stays intact at stresses lower than the yield stress. With the second category the rotation speed is controlled and the resulting torque at the bob is measured. This technique is more suitable for oscillation experiments and experiments in which a step in shear rate is applied, as the inertial time-scale for this type of instrument is much less than for controlled stress instruments [58].

The yield stress of a material can be measured in several ways [46, 114]. The most direct one is to increase the applied stress with a controlled stress rheometer until the material starts to flow. A second less direct way is to extrapolate the flow curve backwards to zero shear. The yield stress obtained in this way is strongly dependent on the range of the lowest shear rates in the flow curve. If for a visco-plastic material only measurements at high shear rates ($> 10$ s\(^{-1}\)) are available, the extrapolated yield stress is likely to be too high. A third way to determine the yield stress is to measure the peak value of the shear stress at a low shear rate (e.g. $10^{-2}$ s\(^{-1}\)). A fourth possibility to measure the yield stress is by shearing the material at a constant shear rate until equilibrium is achieved and subsequently stop the rotation suddenly. The residual stress then
3.3. RHEOMETRY

is defined as the yield stress corresponding to the residual structure. Repeating this procedure at different pre-shear rates and extrapolating to zero shear gives the yield stress corresponding to a structure that is intact. For the extrapolation the same problem arises as for extrapolation from the flow curve. Throughout this chapter, 'yield stress' is taken to refer to the yield stress as obtained from controlled stress rheometry, unless stated otherwise.

This yield stress is the undrained yield strength. An undrained test is defined as a test during which no flow of pore water with respect to sediment particles occurs. A one-phase approximation then is justified. For short stress loading times this is a good approximation, as the permeability of concentrated sediment layers is low. The difference between drained and undrained yield strengths is discussed in Chapter 4.

Several sources of error may arise when these experiments are performed, especially with cohesive sediments. The most important ones will be dealt with in this section. For more information the reader is referred to the literature [92, 19, 114].

The first source of error is the determination of the shear rate distribution within the sample. For Newtonian fluids this is straightforward, as the shear rate distribution is solely determined by the rotation speed of the instrument and the dimensions of the measuring device used, and independent of the viscosity. However, in order to calculate the shear rate distribution within a non-Newtonian material, the rheological behaviour of this material must be known, which unfortunately is generally unknown prior to experiments. Therefore, in order to determine the rheological properties of cohesive sediments, these properties have already to be known. This problem can be solved with an iteration procedure [115]. For axially symmetric Couette flow and assuming a Bingham plastic behaviour, for example, it can be shown that \( \dot{\gamma} \sim \Omega^{\frac{1}{2}} \) at low shear rates, when partial plug flow behaviour is observed in the gap, whereas \( \dot{\gamma} \sim \Omega \) at high shear rates, when the fluid is sheared throughout the gap (Figure 3.1). Here \( \Omega \) is the rotation speed of the cup.

For the clay suspensions used in this study the transition between these two regimes lies approximately at \( \dot{\gamma} = 10 \text{ s}^{-1} \) for the Couette geometry used. At low shear rates the accuracy of the measurements will decrease because the material is sheared only within a small zone of the instrument gap, a zone that may well be in the order of the size of the aggregates present in the material. This type of behaviour, caused by a difference in shear stress between cup and bob, occurs if the shear stress at the cup becomes lower than the yield stress. The shear rate in the sheared zone is not known exactly and has to be determined iteratively. Therefore results obtained with a Couette geometry at low shear rates have to be assessed with caution; another configuration might be more suitable. For the cone-plate geometry, for example, the shear rate is constant throughout the gap.
Figure 3.1: Shear rate at \( r = r_c \) as a function of rotation speed within a concentric cylinder (Couette) geometry for a Bingham-plastic fluid; cup radius \( r_c = 0.017 \text{ m} \); bob radius \( r_b = 0.016 \text{ m} \); \( r_y - r_b \) represents the thickness of the sheared zone; \( \tau_y = 4.7 \text{ Pa} \); \( \eta_{\infty} = 0.01 \text{ Pa s} \)

for cone top angles \( \theta < 4^\circ \) and is given by \( \dot{\gamma} = \Omega/\theta \). Thus at low shear rates, this geometry is to be preferred compared with the Couette geometry.

The second source of error emerging during rheometrical experiments is sedimentation, which after some time results in an inhomogeneous sample, making interpretation of the experimental results difficult. To avoid this, the time-scale of the experiments should be smaller than the time-scale of sedimentation. At the high sediment concentrations used in the experiments described in §3.5, sedimentation is very slow because of hindered settling. This makes an investigation of structural changes at long time-scales possible. Sample inhomogeneity may also be caused by shear-induced particle migration, which is also most important at low sediment concentrations. It can be suppressed by using a geometry in which the shear rate is constant, for example a cone-plate geometry.

A third source of error that may occur is wall-slip. For geometrical reasons, a particle can never be at a smaller distance to a wall than its radius, resulting in a lower particle concentration at the wall and therefore a lower viscosity. For polydisperse systems wall-slip decreases. Wall-slip may be avoided by using rough walls, with a roughness equal to the mean particle diameter of the sample. The magnitude of wall slip can be assessed by measuring at two different gap-widths. Correction for this error then is possible [35]. Wall-slip should not be
confused with the partial plug flow behaviour in concentric geometries at low shear rates as was described above.

A fourth source of error is sample injection, which may partially destroy the sample structure. This is no problem for equilibrium flow curve measurements, as the 'dynamic' structure described in the previous section recovers fast and an equilibrium structure is obtained within a short time. However, for yield stress measurements and oscillation experiments at low strains, where the 'static' structure is concerned, recovery is very slow. This explains the scatter often observed in yield stress measurements. Sample destruction can be minimized by inserting a vane into the sample container in which the sample is at equilibrium. A vane has the great advantage that the location of the yield surface created during experiments differs from that of the yield surface created by the immersion of the vane, so that the yield stress of the undisturbed structure can be measured. Another possibility, which was made use of in the reported experiments, is to wait after sample injection until the structure has recovered.

3.4 Instrumentation and Materials

3.4.1 Rheological instruments used

Rheometrics RMS–800

The Rheometrics RMS–800 is a rheometer in which the angular velocity of the bob is controlled and the resulting torque at the bob is measured. For the present experiments a concentric cylinder geometry was used, with dimensions given in Table 3.1. A sketch of this geometry is shown in Figure 3.2. In order to reduce undesired friction at the bottom of the bob, air is locked in at this place. The measurements with the Rheometrics consisted of pre-shearing at 100 s⁻¹ to get a reproducible and identical structural state for every measurement and subsequently decreasing the shear rate in a single step to a value in the range between 10⁻⁴ and 10 s⁻¹, depending on the experiment.

Carrimed CSL

The Carrimed CSL is a rheometer in which the torque of the bob or cone is controlled and the resulting angular velocity at the bob or cone is measured. This instrument is therefore suitable for yield stress measurements. For the present experiments a cone-plate geometry and a double concentric cylinder geometry were used, with dimensions given in Table 3.1. A sketch of these geometries is given in Figure 3.2. The instrument was used for two types of experiments, namely equilibrium flow curve and yield stress recovery experiments. The exact
Table 3.1: Configuration dimensions; DC = double concentric geometry; CP = cone-plate geometry; E1, 1/30 and 1/60 are labels to distinguish between the cone-plate geometries used

<table>
<thead>
<tr>
<th>distance (mm)</th>
<th>Couette</th>
<th>DC</th>
<th>CP E1</th>
<th>CP 1/30</th>
<th>CP 1/60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>32</td>
<td>36.5</td>
<td>60</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>$d_2$</td>
<td>34</td>
<td>47.5</td>
<td>3</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>$d_3 / \theta$</td>
<td>16</td>
<td>40</td>
<td>1°</td>
<td>2.5°</td>
<td>1°</td>
</tr>
<tr>
<td>$d_4$</td>
<td>32</td>
<td>41</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d_5$</td>
<td>-</td>
<td>56.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>sample vol. (ml)</td>
<td>20</td>
<td>6</td>
<td>3</td>
<td>1.5</td>
<td>2</td>
</tr>
</tbody>
</table>

procedure followed is explained in §3.5. Torques were varied in the range of 0–2000 $\mu$Nm. The instrument was equipped with a vapour lock in order to reduce evaporation of the sample. This is very important as the yield stress recovery experiments conducted took up to 12 hours and the viscosity and yield stress of highly concentrated suspensions are very sensitive to small changes in water content.

Bohlin VOR

The Bohlin VOR is a rheometer in which the angular velocity of the plate or cup is controlled and the resulting torque at the cone or bob is measured. For the present experiments use has been made of two cone-plate geometries, with dimensions given in Table 3.1. A sketch of these geometries is shown in Figure 3.2. The Bohlin VOR was also equipped with a vapour lock to suppress evaporation. Two types of experiments were performed with this instrument: flow curve measurements and harmonic oscillation experiments to investigate the visco-elastic behaviour of clay suspensions at small deformation. Shear rates were varied between approximately $10^{-2}$ and 100 s$^{-1}$ for the flow curve experiments; frequencies between 0.01 and 10 Hz and strains between 0.2 and 5% for the oscillation experiments. Measurements with both the Carrimed and Bohlin were carried out at Twente University of Technology.

3.4.2 Sample preparation

China clay

China clay (supplied by Johnson Matthey B.V., product code RM-225, Kaolinite powder) was mixed with tap water and 0.5% NaCl. Salt was added to increase flocculation and to eliminate the possible influence of small quantities of
other chemicals on the characteristics of the clay [131]. The mixture was allowed to stand for 3 weeks in order to reach an equilibrium situation, as the suspension properties change in the first few weeks because of ion-exchange processes between water and sediment. The particle size distribution and chemical properties of China clay are given in [131]; its average particle size is about 4 μm. Mass concentrations of the suspensions used ranged between 419 and 570 kg m⁻³.

Caland Channel mud

Caland Channel mud was dredged from the Caland channel near the port of Rotterdam, the Netherlands. To get the desired concentrations it was diluted with water also originating from the Caland channel. The particle size distribution and chemical properties of Caland Channel mud are given in [131]; its average particle size is about 4 μm. The small sand fraction was removed by sieving the mud with a 64 μm sieve because sand particles may affect the experiments by getting stuck in the gap of the measuring geometry. Mass concentrations of the suspensions used ranged between 260 and 372 kg m⁻³.

Sample injection

Prior to taking samples, the suspension bottles were mounted in a slowly rotating device to ensure homogeneous suspensions. Samples were taken with a syringe and injected into the rheometer. The diameter of the nozzle of the syringe was larger than the gap width of the geometries used (∼ 10⁻³ m). The maximal
shear rate in the sample as a result of injection into the rheometer is estimated at 100 s$^{-1}$.

3.5 Results and Discussion

3.5.1 Flow curves

Flow curves were measured with both a rotation speed controlled instrument (Bohlin vOR) and a stress controlled instrument (Carrimed csl). In all but a few cases a cone-plate configuration was used. Flow curves with the Bohlin were obtained by increasing the shear rate in 18 steps from 0.01 to 100 s$^{-1}$ and subsequently decreasing it from 100 back to 0.01 s$^{-1}$; the time interval between the steps was 20 s. With the Carrimed csl flow curves were measured by increasing the applied shear stress in steps and recording the shear rate when a stationary state was reached.

The first experiment with Caland Channel mud with the Bohlin is remarkable (Figure 3.3). The shear stress at low shear rates is much higher for the upward than for the downward curve, and shows a minimum at higher shear rates. This suggests a structural collapse that is irreversible at short time scales, which is confirmed by subsequent experiments with the same sample, where this behaviour is not observed, as the static structure has not yet recovered. This observation can be well reproduced by the equilibrium model presented in (3.12), in which $\lambda_S$ has been set to zero for the downward curve, as the static structure has been broken up at high shear and recovery of this structure takes much more time than the experimental time. For China clay this type of behaviour is not observed (Figure 3.4). The contribution of the static yield stress is much less and no minimum in the flow curve is observed.

A comparison of results obtained with the Bohlin and Carrimed is presented in Figure 3.5 for China clay and in Figure 3.6 for Caland Channel mud. At high shear rates the reproducibility is good, at low shear rates the scatter is large as the shear stress tends to decrease as the sample is sheared several times. This once again points to the destruction of the static structure that may be still partly existent after injection of the sample. Also hysteresis effects are observed at low shear rates; the upward curves lie above to downward ones, so no equilibrium state has been reached.

An important difference between the Carrimed and Bohlin flow curves is the absence of a minimum with the Carrimed. This is caused by the fact that the shear stress is applied and not the rotation speed, resulting—at the moment that the yield stress is exceeded—in a sudden step from zero shear rate to a shear rate at which the decrease of the yield stress caused by structural breakdown is just compensated by the viscous contribution to the shear stress. For China clay
3.5. RESULTS AND DISCUSSION

Figure 3.3: Flow curves for Caland Channel mud; $C = 260$ kg m$^{-3}$. Bohlin VOR; cone-plate geometry CP 1/60

Figure 3.4: Flow curves for China clay; $C = 513$ kg m$^{-3}$. Bohlin VOR; cone-plate geometry CP 1/60
Figure 3.5: Flow curves for China clay; $C = 563$ kg m$^{-3}$. Comparison between Carrimed CSL and Bohlin VOR; cone-plate geometry

Figure 3.6: Flow curves for Caland Channel mud; $C = 260$ kg m$^{-3}$. Comparison between Carrimed CSL ‘○’ and Bohlin VOR (all other markers); cone-plate geometry
3.5. RESULTS AND DISCUSSION

This effect is only small; at a shear rate of approximately 2 s\(^{-1}\) equilibrium is achieved. For Caland Channel mud, however, the shear rate suddenly increases to a value larger than 20 s\(^{-1}\), which was to be expected from the flow curve obtained with the Bohlin (Figure 3.6).

The curves obtained with the Carrimed lie slightly above those from the Bohlin and consequently the results are not completely independent of the configuration used. One should therefore be cautious when comparing results obtained with different instruments or even geometries.

Flow curves obtained with the Carrimed are shown on linear scales in Figure 3.7 for China clay and in Figure 3.9 for Caland Channel mud. A magnification of Figure 3.7 for low \(\dot{\gamma}\) is presented in Figure 3.8 to show the yield stress behaviour of China clay better. The reproducibility of these measurements is satisfactory, especially at intermediate shear rates. At low torque the material does not flow at all, which proves the presence of a real yield stress.

The value of this yield stress is not the same for all experiments, which might be caused by minor differences in sample, sample injection and sample injection time. A new sample was not used in each experiment, in which case also the rest time between the experiments and the stress history of the sample during previous experiments is important. At higher shear rates this becomes less important as the structure will be broken up for the larger part and equilibrium is reached within a short time.

Structural break-up is a fast process and the structure of the sheared material will be quickly in equilibrium with the higher shear rate, whereas static structural recovery, which prevails at low or zero shear rates, is a slow process so that measurements at low shear rates are not likely to be at equilibrium. Generally speaking, the lower the shear rate, the more the structure is likely to deviate from equilibrium, which explains the relatively high scatter at low shear rates.

In Figure 3.7 and Figure 3.9 the theoretical equilibrium flow curves based on (3.12) and with the coefficients given in Table 3.2 are shown. As it is only possible to obtain from the equilibrium flow curve the ratio between \(c_{2,D}\) and \(c_{3,D}\), the rate of structural decay and recovery cannot be estimated from it. Transient experiments are needed, for example those described in §3.5.2. However, the fact that equilibrium flow curves can be measured at the time-scale of minutes means that the time-scales of dynamic structural decay and recovery cannot be larger than that.

For one experiment with China clay a double concentric cylinder geometry was used. The agreement with the results obtained with the cone-plate geometry is good in the range 1–50 s\(^{-1}\). (Figure 3.7). This is another indication that the results are reliable at intermediate shear rates. The yield stress, however, is twice as high, which may be explained by a better grip on the sample as a result of the much larger contact surface (Figure 3.7). Wall slip may therefore have affected
Figure 3.7: Flow curves for China clay; $C = 467 \text{ kg m}^{-3}$; range in shear rate 0–100 s$^{-1}$; with Carrimed CSL; all experiments with cone-plate geometry except for ‘○ cylinder’.

Table 3.2: Non-linear regression of (3.12) to experimental results (Figure 3.7 for China clay and Figures 3.6 and 3.9 for Caland Channel mud)

<table>
<thead>
<tr>
<th>parameter</th>
<th>China clay 467 kg m$^{-3}$</th>
<th>Caland Channel mud 260 kg m$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{y,S}$ (Pa)</td>
<td>0.6 ± 0.2</td>
<td>6.7 ± 1</td>
</tr>
<tr>
<td>$\tau_{y,D}$ (Pa)</td>
<td>0 ± 0.1</td>
<td>1.0 ± 0.2</td>
</tr>
<tr>
<td>$\eta_{\infty}$ (Pa s)</td>
<td>0.0066 ± 0.0004</td>
<td>0.0136 ± 0.001</td>
</tr>
<tr>
<td>$c_{1,D}$ (Pa s)</td>
<td>0.86 ± 0.05</td>
<td>1.6 ± 0.3</td>
</tr>
<tr>
<td>$c_{2,D}/c_{3,D}$ (s)</td>
<td>0.081 ± 0.005</td>
<td>0.23 ± 0.04</td>
</tr>
<tr>
<td>$c_{2,S}/c_{3,S}$ (s)</td>
<td>~ 10</td>
<td>4 ± 3</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.98</td>
<td>0.79</td>
</tr>
</tbody>
</table>
the measurements with the cone-plate geometry. The change in slope at a shear rate of approximately 20 s\(^{-1}\) is remarkable (Figure 3.7). This transition is also found with the double concentric cylinder geometry, and it is therefore highly unlikely that this result is caused by an anomaly of the instrument instead of a transition in the internal structure of the sample. It is reproduced by the equilibrium model (3.12) and can be explained by break-up of the dynamic structure. At higher shear rates the reproducibility becomes less satisfactory, which might be explained by sample distortion by centripetal accelerations. The cone-plate configuration is not designed for shear rates over 100 s\(^{-1}\).

The main difference between the flow curves for China clay and Caland Channel mud lies in the shear-thinning behaviour of the former and the true Bingham plastic behaviour of the latter at high shear rates (Figure 3.9). For Caland Channel mud the slope of the flow curve (the 'slope viscosity') is constant in the range of shear rate covered in the experiments, whereas for China clay the slope viscosity is only constant for \(\dot{\gamma} > 20\) s\(^{-1}\); for \(\dot{\gamma} < 20\) s\(^{-1}\) China clay is strongly shear-thinning. At the same volumetric concentration of the particles, Caland Channel mud shows a higher yield strength and a higher viscosity.

Measurements were performed at two different concentrations for each type of sediment. This is insufficient to model the concentration-dependency of the
yield stress and viscosity accurately. Previous experiments by other authors [122, 130] show $\tau_y \sim N^3$ and $\eta_\infty \sim N^{1.7}$, for China clay, with $N$ the volumetric concentration, which is not contradicted by the present experiments.

It can be concluded that for Caland Channel mud the bonds between the particles are stronger than for kaolinite, which may be caused by the presence of organic materials and differences in specific surface area, cation exchange capacity and sodium adsorption ratio, for example. The measurements also suggest that Caland channel aggregates are more resistant to break-up than China clay aggregates, as no shear-thinning behaviour is observed for this mud.

### 3.5.2 Steps in shear rate

In order to investigate the time-scale for structural changes in cohesive sediment suspensions, experiments were performed during which the shear rate was changed in steps, and the resulting (time-dependent) stress response was recorded. These experiments were carried out with the Rheometrics RMS–800. Samples were pre-sheared at $100 \text{ s}^{-1}$ until a stationary shear stress was reached in order to get a reproducible starting situation. Then the shear rate was decreased abruptly to $10, \ 1, \ 0.1, \ \ldots, \ 10^{-4} \text{ s}^{-1}$. The resulting shear stress response in time gives information about the response of the material to the altered flow regime.
3.5. RESULTS AND DISCUSSION

As noted in §3.5.1, the equilibrium flow curve alone is insufficient to determine all parameters necessary to describe thixotropic flow.

First the influence of settling has to be quantified as it causes inhomogeneity in the sample which can have a large impact on the shear stress. The time-scale of settling should therefore be much larger than the time-scale of the structural changes one is interested in. Experiments not presented herein showed that at the sediment concentrations used sedimentation effects were absent for tens of minutes, which makes the assessment of structural recovery within this period possible.

Two additional sources of error may occur during transient experiments. The response time of the instrument should be much smaller than the stress response of the material, and the inertia of the sample should be negligible. As the samples tested have a yield strength of approximately 1 Pa, inertia effects may only be important for time scales < 0.1 s for the Couette configuration used. Also the response time of the Rheometrics RMS-800 is estimated at \( t < 0.1 \) s. The experiments with steps in shear rate are therefore able to monitor structural changes with time-scales in the range \( 0.1-10^3 \) s.

Specific responses to steps in shear rate are presented in Figure 3.10 for China clay and in Figure 3.11 for Caland Channel mud. The difference in stress response between China clay (a gradual transition) and Caland Channel mud (overshoot behaviour) is remarkable. The latter behaviour can be reproduced by (3.14) and (3.15) derived from the thixotropic model (3.10). A sudden increase in shear rate results in a stress response above the equilibrium value, as the structure needs time to adapt itself to the changed shear rate and will initially have a structure in equilibrium with a lower shear rate. Similarly, a sudden decrease in shear rate will result in a stress response initially below the equilibrium value. The most important result from these experiments is that for Caland Channel mud the time-scale of the structural changes, which could not be deduced from the equilibrium flow curves, is estimated at approximately 5 s. Values for the coefficients \( c_{2,D} \) and \( c_{3,D} \) can be obtained with non-linear regression of (3.14) to the experimental data. They can be calculated from Table 3.3 together with (3.14) and Table 3.2. For Caland Channel mud \( c_{2,D} = 0.22 \) and \( c_{3,D} = 0.20 \) s\(^{-1}\).

The response of China clay to steps in shear rate can be explained by its visco-elastic behaviour, which is attended with stress relaxation [30]. The specific time for this phenomenon is approximately 27 s at a shear rate of 20 s\(^{-1}\) and 3 s at a shear rate of 100 s\(^{-1}\), as can be derived from Figure 3.10. Unfortunately, the thixotropic model used is unable to reproduce this phenomenon, as it does not incorporate elastic effects. Gently tapping the plate at rest results in a much shorter stress relaxation time, and is recommended to ensure zero stress at the start of the next experiment.

Another interesting effect is the quasi-elastic behaviour of Caland Channel
**Table 3.3:** Non-linear regression of (3.14) to experimental results (Figure 3.11); step in shear rate from 100 s\(^{-1}\) to 10 s\(^{-1}\); Parameters \(a_i\) are defined in (3.15)

<table>
<thead>
<tr>
<th>parameter</th>
<th>Caland Channel mud</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1) (Pa)</td>
<td>2.2 ± 0.2</td>
</tr>
<tr>
<td>(a_2) (Pa)</td>
<td>-5.33 ± 0.02</td>
</tr>
<tr>
<td>(a_3) (s(^{-1}))</td>
<td>12.9 ± 0.1</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.934</td>
</tr>
</tbody>
</table>

**Figure 3.10:** Stress response after step in shear rate 100–20 s\(^{-1}\) at \(t=150\) s. China clay, 570 kg m\(^{-3}\); Rheometrics RMS–800; Couette geometry.
3.5. RESULTS AND DISCUSSION

![Graph showing stress response over time](image)

**Figure 3.11**: Stress response after step in shear rate 100–1–100 s⁻¹ at t=60, 180 s. Caland Channel mud, 372 kg m⁻³; Rheometrics RMS-800; Couette geometry.

mud at strains lower than 50% for shear rates < 10⁻¹ s⁻¹ (Figure 3.12). At strains of approximately 50% the mud seems to yield, independent of the applied shear (strain) rate. This behaviour, which has been reported previously [46, 92], might be explained by a competition between build-up and destruction of the aggregate structure. At these low shear rates only a small zone within the gap, decreasing in time because of structural recovery, is likely to be sheared and therefore the strain will be larger than 50% locally near the bob. Thus the observed behaviour is quasi-elastic and not real-elastic, as harmonic oscillation experiments show that particle interactions may only stay intact at strains smaller than 1% (§3.5.4). It can be concluded that strains of 50% and higher are much too high for elastic behaviour. An exact explanation is cumbersome, as even the continuum approach might not be valid within the small sheared zone.

### 3.5.3 Yield stress recovery experiments

In order to examine the time-scale of build-up of the static yield stress, measurements were performed with the Carrimed CST rheometer. Again, both China clay and Caland Channel mud were tested. Only high sediment concentrations were used, so that settling could be neglected. A cone-plate configuration was used for most experiments with China clay and Caland Channel mud.

The samples were pre-sheared at an initial torque of 1000 μNm during 60
Figure 3.12: Strain-stress relationship of Caland Channel mud after step in shear rate from 100 s\(^{-1}\) to a rate varying from 10\(^{-4}\) to 0.1 s\(^{-1}\) as indicated; \(C = 372\) g l\(^{-1}\); measured with Rheometrics RMS-800; Couette configuration.

s, which resulted in \(\dot{\gamma} > 100\) s\(^{-1}\), higher than during injection. In this way the influence of sample injection on the measurements was minimized and an identical and reproducible starting situation was ensured for each experiment. This is very important, bearing in mind the large scatter for the yield stress for the flow-curve experiment described in §3.5.1. After pre-shearing the sample was allowed to rest, so that structural recovery could take place. Rest times ranged from several minutes up to 6 hours. After that a gradually increasing torque was exerted on the sample, until the shear rate became non-zero because the yield stress was attained.

In Figure 3.13 the yield stress of China clay is given as a function of rest time. Notwithstanding considerable scatter still present, a clear and positive correlation can be observed, which leads to the conclusion that the yield stress definitely increases with the rest time of the sample. Adopting the approach of Toorman [117], in which the yield stress is split in two parts, \(\tau_y(t)\) can be expressed by (3.16) assuming \(\lambda_0\) to be unity, as the shear stress increase during the experiments considered here takes place after a waiting period of at least some minutes.

Values for the parameters found after non-linear single response regression of (3.16) to the experimental result are given in Table 3.4. Unfortunately, the
3.5. RESULTS AND DISCUSSION

Figure 3.13: Yield stress build-up as a function of rest time for China clay; $C = 467$ and $419$ kg m$^{-3}$; with Carrimed CSL; cone-plate geometry; Regression of (3.16) to experimental data

maximal rest time is too short to observe the asymptotic value of $\tau_y$ for $t \to \infty$, and the value found for the time-scale of recovery ($\sim 10^5$ s) is therefore uncertain.

Figure 3.14 shows the yield stress build-up for sieved Caland Channel mud at 260 kg m$^{-3}$. Considering the much lower concentration compared to China clay, the yield stress of Caland Channel mud is markedly higher, as was to be expected due to the presence of organic material, the larger specific surface area and other factors. Again an increase in yield stress as a function of the rest time can be observed. Results of regression of (3.16) to the experimental results are presented in Table 3.4. As mentioned before, sedimentation was negligible because of the high sediment concentrations used. The yield stress increase only showed a correlation with rest time after pre-shearing; no correlation was found with the residence time in the rheometer.

3.5.4 Oscillation experiments

Oscillation frequencies applied were in the range of 0.01–10 Hz, well covering the range of surface wave frequencies in the coastal zone. The frequency was first increased in steps from 0.01 to 10 Hz and subsequently decreased from 10 to 0.01 Hz in order to check for the presence of hysteresis, which points to structural changes. The strain amplitude was also varied to investigate the range
Figure 3.14: Yield stress build-up as a function of rest time for Caland Channel mud; $C = 260 \text{ kg m}^{-3}$; with Carrimed csl; cone-plate geometry; Regression of (3.16) to experimental data

Table 3.4: Non-linear regression of (3.16) to experimental results (Figures 3.13 and 3.14)

<table>
<thead>
<tr>
<th>parameter</th>
<th>China clay 467 kg m$^{-3}$</th>
<th>China clay 419 kg m$^{-3}$</th>
<th>Caland Channel mud 260 kg m$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{y,s}$ (Pa)</td>
<td>4 ± 1</td>
<td>2.2 ± 0.4</td>
<td>14 ± 1</td>
</tr>
<tr>
<td>$\tau_{y,d}$ (Pa)</td>
<td>0.9 ± 0.2</td>
<td>0.5 ± 0.2</td>
<td>6 ± 1</td>
</tr>
<tr>
<td>$c_{3,s}$ (s$^{-1}$)</td>
<td>$10 \pm 6 \times 10^{-5}$</td>
<td>$4 \pm 2 \times 10^{-4}$</td>
<td>$2.4 \pm 1.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.717</td>
<td>0.858</td>
<td>0.739</td>
</tr>
</tbody>
</table>
of linearity. Measurements were performed with the Bohlin VOR rheometer using a cone-plate geometry with a vapour lock. At frequencies over 10 Hz instrument inertia interfered with the measurements. The rheometer was therefore not used at these frequencies. The minimal strain needed to get a reliable torque response was about 0.005, fortunately just in the linear range of the samples used. Both China clay and Caland Channel mud were tested at two different concentrations. Results for China clay are shown in Figure 3.15, from which it can be concluded that its behaviour is only linear visco-elastic at strain amplitudes at 0.005 and below. At these strains the values for \( G' \) and \( G'' \) become strain-independent and no hysteresis is observed; structural decay (thixotropy) during the experiment can be ruled out. At low strains \( G'' \) is markedly smaller than \( G' \) which indicates a predominantly elastic behaviour. At low frequencies an increase in \( G''/G' \) points to a more viscous and less elastic behaviour. Higher strains (amplitude 5%) result in the decay of the structure during a measurement, because values of \( G' \) and \( G'' \) for the downward frequency sweep are clearly below the values for the upward frequency sweep. As the ratio between \( G'' \) and \( G' \) now is much larger, the material now behaves predominantly viscous. No observable transitions take place in the frequency range studied.

For Caland Channel mud (Figure 3.16) the general picture is the same as for China clay (Figure 3.15). Values for \( G' \) and \( G'' \) are higher at comparable volumetric concentrations, and the range of linearity is more than twice as large (up to strains of 0.01). This indicates once more that the particle bonding is stronger and acts over a longer distance.

An estimate of the internal time-scale \( t_n \) of the materials tested can be calculated from \( t_n = \lim_{f \to 0} (1/2\pi f)(G'/G'') \) [60]. As the experimental values for \( t_n \) do not approach a constant value at low frequencies, but instead strongly increase with decreasing \( f \), no extrapolation to \( f = 0 \) is possible. The only thing that can be concluded is that the characteristic time is much larger than \( 10^2 \) s; the Deborah number is much larger than unity during these measurements and a yield stress is present. Because of instrument limitations the visco-elastic properties could not be determined at frequencies over 10 Hz. As the characteristic time-scale of the materials tested is estimated to be much larger than \( 10^2 \) s, this is not a serious restriction.

3.6 Conclusions and recommendations

Flow curves of China clay and Caland Channel mud were shown to be at equilibrium for shear rates larger than about 5 s\(^{-1}\). For these shear rates results are well reproducible, but for lower shear rates differences in stress history of the samples caused by sample injection and previous experiments affected the reproducibility. This can be explained by the slow recovery rate of the static structure.
Figure 3.15: Oscillation experiment with China clay; $C = 589 \text{ kg m}^{-3}$; with Bohlin VOR; cone-plate geometry CP 1/60; percentages refer to strain amplitudes

Figure 3.16: Oscillation experiment with Caland Channel mud; $C = 260 \text{ kg m}^{-3}$; with Bohlin VOR; cone-plate geometry CP 1/60; percentages refer to strain amplitudes
3.6. CONCLUSIONS AND RECOMMENDATIONS

Also partial plug flow may affect experiments at low shear rates, especially if a Couette configuration is used.

The often used Bingham plastic rheological model is only suitable for describing the measurements at shear rates over 20 s\(^{-1}\), but fails to describe the structural changes at lower shear rates. The Bingham model therefore should not be used for typical field situations, where the shear rates are small.

The flow curves could be successfully reproduced with a thixotropic model proposed by Toorman [117], which takes into account both short-term and long-term structural changes. Structural recovery at large time-scales might be explained by gel-formation, i.e. aggregate bonding throughout the sample, whereas short term structural recovery might be explained by the growth or change of shape of particle aggregates without all aggregates being interconnected. The latter effect is more pronounced for China clay than for Caland Channel mud. Caland Channel mud is much more cohesive than China clay, resulting in a higher effective viscosity at the same sediment concentrations.

The specific time-scales of the structural changes were determined from transient experiments. Short-term or dynamic structural changes in aggregate structure have a time-scale in the order of seconds, whereas the long-term yield stress recovery has a time-scale in the order of \(10^4 \text{ - } 10^5\) s. This is an important observation, as it confirms the expectation that a deposited mud layer builds up strength during slack tide. China clay showed stress relaxation at short time scales, which cannot be reproduced by the thixotropic model used, as this does not include elastic effects. However, the response of Caland Channel mud to steps in shear rates is well reproduced by the model.

Yield stress measurements are best performed using controlled stress instruments. It is very important to use only samples with an easily reproducible and identical stress history to limit scatter in the experimental results. The yield stress of Caland Channel mud is much higher than that of China clay at the same sediment concentration because of its more cohesive nature. This should be borne in mind when conducting laboratory experiments with artificial muds. Such experiments are suitable to examine processes which are important during deposition, (bulk) erosion and flow of cohesive sediment layers, for example, but unsuitable to obtain quantitative results which can be directly applied to the field situation.

The measurements described in this chapter are less suitable for consolidated sediments, in which significant effective stresses exist. If a sample of such a compacted layer is remoulded by injection into a rheometer and subsequently allowed to recover, the yield strength will be lower than its original value because of the (nearly) absence of effective stress in the rheometer. The only way to determine its original strength is by means of an in situ test, that is, a vane test or a sounding test (Chapter 4), for example.
From oscillation experiments it can be concluded that the behaviour of the mudd tested is visco-elastic at strain amplitudes below 0.01 for Caland Channel mud and 0.005 for China clay. At these strains $G''$ is markedly smaller than $G'$ which indicates a predominantly elastic behaviour. At low frequencies a decrease in $G'$ might point to creep effects. High strains result in the decay of the structure during a measurement and a strong increase in the ratio $G''/G'$. Apparently, only short-range particle interactions exist.

In order to abandon the semi-black-box approach with one or more structural parameters, the structural changes should be observed that occur in a suspension of cohesive sediments during a rheological experiment. This requires special techniques, of which Nuclear Magnetic Resonance (NMR) is an example. A complication from a practical point of view is the large variability, both in space and time, in structure and composition (physico-chemical, organic, biological) of these sediments.

Results from this chapter concerning the oscillation experiments at low strains are used in Chapter 5 and those concerning the equilibrium flow curves in Chapter 6.
Chapter 4

Yield strength of mud

4.1 Introduction

In the previous chapter measurement of the yield strength and viscosity of fluid mud was discussed. Whereas viscosity is an important property of flowing mud, yield strength determines whether mud will flow or not—given the forces acting on it. If one wants to calculate at which magnitude of wave and current forces cohesive sediment starts to erode, its strength is of major importance. Strength of freshly deposited sediment is also important for the determination of navigable depth, a decisive parameter for dredging operations.

However, yield stress measurements as described in the previous chapter are only suitable for mud samples of which the maximal effective stress during its stress history is much smaller than its cohesion at zero effective stress. As was explained in §2.3, effective stress—defined as the difference between total stress and pore pressure—is representative of the load carried by the grain matrix. Samples with a negligible effective stress compared to their yield strength have a spatially evenly distributed strength, at least if particles and particle properties are evenly distributed. Homogeneity is a prerequisite for rheological measurements.

If strength is much enhanced by compaction caused by high effective stresses, rheological experiments as described in the previous chapter are less suitable because of several reasons:

- The effective stress tends to increase with depth; concurrently also the strength will increase. Only small samples may then be considered as homogeneous, which makes a lot of sampling and measuring necessary when strength gradients are to be measured.

- During injection into a rheometer, a sample will be remoulded. Although strength may partly recover with time, it will not reach its original level, because the effective stress within a rheometer is generally negligible compared to the original situation.

Therefore, in situ determination of strength is preferable for these samples.
CHAPTER 4. YIELD STRENGTH OF MUD

Summarizing, determination of yield strength by measuring samples in a rheometer is a suitable technique for fluid mud—in which effective stresses are negligible—and for the uppermost layer of a cohesive sediment bed, where the yield strength substantially exceeds the effective stress. For consolidating and consolidated beds, a less disturbing technique is preferable that does not affect effective stress too much.

Measurement of bed strength is not straightforward. The method to be used depends in the first place on the order of magnitude of the bed strength. Sediment layers that are buried or have been buried generally have a strength in the order of a few kPa and more. They can be tested with classical geotechnical methods, for example with a triaxial test. Weaker beds generally cannot be tested with this method, as it is intended for much higher stresses. Another disadvantage of the triaxial test is that an average strength of the sample as a whole is obtained, and not the strength profile within the sample.

Measurement of bed strength is also possible with a vane test, during which the torque needed to rotate a vane immersed in the sediment is measured. This torque $T$ can easily be related to strength $c$ via $T = \pi c (d^2 h/2 + d^3/6)$, where $d$ is the overall vane width and $h$ is the vane immersion depth. By increasing the immersion depth, a strength profile is obtained. However, in order to measure bed strengths of about 1 Pa accurately, air bearings are necessary. This makes the instrument expensive.

Another possibility is the fall cone test. In this test a fall cone is dropped from a certain height above the sediment bed, and its penetration depth is measured. From this depth the bed strength can be calculated. Zreik et al. [140] analysed cone penetration in soft marine muds extensively. However, no strength profile is obtained and the shear rates during this short-lived test (duration approximately 1 s) are high.

Other methods are static stability tests on inclined planes [21] and methods based on static equilibrium of an immersed body [92]. With the latter method it is assumed that the buoyant weight of the body is carried by the vertical component of the yield stress $\tau_y$ acting over the body surface. For spheres, for example, this leads to a criterion for incipient motion of $Y_G \equiv \tau_y / gd (\rho_s - \rho) = 2/3\pi \approx 0.212$, where $Y_G$ is the gravity yield group, $\rho_s$ and $\rho$ the densities of the sphere and surrounding material, respectively, and $d$ the sphere diameter. However, once the motion of the body ceases, neither the normal nor the shear stress distributions on the surface are known. Pressure may not be hydrostatic and so the buoyant weight of the sphere may not be relevant; the shear stress acting on the surface may not equal the yield stress everywhere [16]. Experimental values for $Y_G$ in the range of 0.04–0.2 have been reported. When the force exerted on a body moving through a visco-plastic fluid is measured, another problem arises. The flow field around the body can be very complex, as the object may be
enclosed by a fluid envelope within which no shearing occurs. The size of this
envelope depends on the yield strength of the fluid and the velocity of the object
relative to the fluid and at present cannot be predicted accurately.

In this chapter, a method is described, based on the principle of sounding,
to measure the strength profile of an undisturbed cohesive sediment bed in a
laboratory. The strength profile is measured by recording the force exerted on
a probe slowly penetrating into a sediment bed that has been consolidated in a
settling column. It provides a simple and accurate way of measuring bed strength
and is easily achievable in the laboratory. The only instruments needed are an
accurate balance to measure the force and a traversing system to control the
movement of the sample column with respect to the balance. Measuring probes
different shape are easily constructed.

In §4.2 the experimental set-up is presented and explained. In §4.3 the elabo-
rations needed to obtain bed strength from the measured force are presented. In
§4.4 experimental results are presented and discussed. The sounding instrument
has been tested on consolidated beds consisting of both an artificial mud (China
clay) and a natural mud (Caledon Channel mud). In §4.5 conclusions are drawn
and some recommendations are made.

4.2 Sounding instrument

The experimental set-up used for the sounding tests consists of a sensitive balance
to measure the vertical force exerted on a small probe penetrating into a sub-
merged sediment layer, and an automatic traversing unit to control the motion
of the settling column containing the sediment layer with respect to the pene-
tration probe and the balance. A sketch of the experimental set-up is presented
in Figure 4.1.

Measurements were performed by slowly immersing the probe into the sedi-
ment and recording the vertical force acting on the probe and, if measured, the
total and pore pressures at its tip. The diameter of the columns used was 8 cm.
Traversing speeds were in the order of 0.1 mm s⁻¹ for most experiments. In some
experiments the traversing speed was varied in order to investigate its influence
on the force response. At the start of each experiment the probe was positioned
in the cylinder with its tip above the interface between water and sediment. The
top of the probe was also immersed in the water in order to minimize the change
in water level, now only resulting from the immersion of the suspension wire.
The experiments were stopped when the tip of the probe reached the bottom of
the column. In some experiments the vertical force was also measured during

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1Submitted in adapted form to *Géotechnique* under the title ‘Miniature Sounding Test on
Soft Saturated Cohesive Soils’ by Thijs van Kessel and Henri L. Fontijn
the subsequent traversing in the reversed direction.

After each experiment the bed concentration profile was measured by slowly inserting a conductivity probe into the sediment bed and measuring the conductivity at several positions. More information about the conductivity probe and the calculation of sediment concentration from conductivity can be found in De Wit [131], p. 110.

The balance used to measure the vertical force is a Sartorius Research R 200 D, which has a weighing capacity of 2 N and an accuracy of 3 μN. The accuracy for forces up to 412 mN is 0.5 μN. With this balance accurate measurements of yield stresses as low as 1 Pa are possible with penetration probes with a surface area of only a few cm². The traversing unit used is an Elmo motion control unit equipped with a stepping electric motor, resulting in a velocity range of $2.5 \times 10^{-4} - 10$ mm s⁻¹ and an accuracy of vertical position of 0.1 mm. Both traversing unit and balance are controlled with a PC.

Two plates, a rod and a disk were used as penetration probes. A sketch of these geometries is shown in Figure 4.2, and their dimensions are given in Table 4.1. By using different probes, an estimate could be obtained of the relative contributions of the shaft surface and the tip surface of the probes to the bearing capacity.

The tip of one plate was equipped with a pore water and a total pressure transducer (Druck DPCR 81, used with a Druck DPI 260 pressure indicator,
4.2. SOUNDING INSTRUMENT

![Sketch of measuring probes used](image)

**Figure 4.2:** Sketch of measuring probes used

<table>
<thead>
<tr>
<th>probe</th>
<th>( D ) (mm)</th>
<th>( B ) (mm)</th>
<th>( L ) (mm)</th>
<th>mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rod</td>
<td>5</td>
<td>-</td>
<td>150</td>
<td>25.26</td>
</tr>
<tr>
<td>plate 1</td>
<td>1</td>
<td>20</td>
<td>150</td>
<td>19.22</td>
</tr>
<tr>
<td>plate 2</td>
<td>10</td>
<td>21</td>
<td>150</td>
<td>84.8</td>
</tr>
<tr>
<td>disk</td>
<td>20</td>
<td>-</td>
<td>3</td>
<td>7.71</td>
</tr>
</tbody>
</table>

**Table 4.1:** Dimensions of measuring probes used (Figure 4.2); plate 2 is fitted with total and pore pressure transducers
range 0–75 mbar). From the pressure recordings the effective stress profile in the
sediment bed can be evaluated.

Sediments tested with the sounding instrument are China clay, an artificial
clay mainly consisting of kaolinite, and Caland Channel mud, a natural mud dred-
ged from the harbour of Rotterdam. The properties of these sediments are descri-
based by De Wit [131]. Initial concentrations prior to sedimentation/consolidation
ranged between 60 and 275 kg m$^{-3}$ by mass; the columns were filled with sedi-
ment suspension up to a height of 30 cm. China clay suspensions were prepared by
adding tap water with 0.5% NaCl to the dry clay powder and subsequent mixing.
Caland Channel mud was available as wet, but highly concentrated
dredged material; suspensions were prepared by adding water also originating
from the Caland Channel to the concentrated slurry and subsequent mixing.

4.3 Calculation of strength

In this section, the calculation is discussed of the bed strength profile from the
vertical force measured with the balance during penetration. However, first the
difference between undrained and drained shear strengths is addressed, as drai-
nage has important consequences for the experimental results.

4.3.1 Drained and undrained shear strengths

The shear strength, yield strength or yield stress of a material is defined as the
smallest shear stress at which the material fails, i.e. the stress at which interpar-
ticle bonds are broken up. A failure criterion that is often adopted because of its
simplicity is the Mohr-Coulomb criterion, in which the shear strength $\tau_y$ of the
material is assumed to depend on its cohesion $c$, normal stress $\sigma_n$ and an angle
of internal friction $\phi$ via

$$\tau_y = c + \sigma_n \tan \phi \quad (4.1)$$

The cohesion of materials mainly consisting of sand is low and may generally be
neglected compared to the contribution of internal friction. However, for cohesive
materials such as clay, cohesion is important by definition. Under saturated and
undrained conditions, which are likely to occur at short time-scales because of
the low permeability of sediment layers that consist of clay, an increase in normal
stress does not result in an increase in yield strength, as the normal stress increase
is absorbed by the pore water and not by the grain matrix. Therefore $\phi = 0$ for
saturated clays under undrained conditions, resulting in $\tau_y = c$. The material
behaves purely plastic in this case.

Under drained conditions, when the pore water pressures are allowed to dis-
sipate during a measurement, an approach in terms of total stress is not allowed
any more. In this case a failure criterion in terms of effective stress is more appropriate, changing the Mohr-Coulomb criterion into \( \tau_y = c' + \sigma'_n \tan \phi' \), where primes refer to effective stress. Now \( \phi' \neq 0 \) also in the case of cohesive sediments.

In the following section an undrained condition during the sounding test is assumed. Also yield strength measurements with a vane, which can be used to compare with the sounding experiments, usually take place under undrained conditions. Total and pore water pressure measurements during a sounding test can validate the total stress approach. If necessary, an analysis in terms of effective stress is then also possible.

In order to assess whether a test is drained or undrained, the Fourier number \( Fo = D_l t / l^2 \) has to be known, where \( D_l \) is the pore water diffusion coefficient (related to permeability), \( l \) the bed height and \( t \) time. If \( Fo \ll 1 \) then the test is undrained, if \( Fo \gg 1 \) then the test is drained. If \( Fo \approx 1 \) then partially drained conditions prevail. A simple rule of thumb is that the test is undrained if the test time is much shorter than the consolidation time of the sample, otherwise it is drained or at least partially drained. In view of this, undrained conditions are to be expected for the fast tests described in §4.4, as the experimental time, which is in the order of minutes, is much smaller than the consolidation time of the order of hours or even days.

### 4.3.2 Force balance

The force balance of a prismatic probe that is is completely immersed in water and only partially immersed in the sediment bed can be written as (Figure 4.3):

\[
F_{\text{tot}} = -F_z - F_{h,u} + F_{h,l} + F_q + F_w
\]  \hspace{1cm} (4.2)

In (4.2) the contribution of the wire from which the probe is suspended has been neglected, as it is very small compared to the other contributions. Note that the vertical force is defined positive in upward direction. In (4.2) the following contributions are represented:

- The gravity force: \( F_z = m_p g \), where \( m_p \) is the mass of the probe.

- The hydrostatic force acting on the upper side of the probe:

\[
F_{h,u} = \int_{L}^{h_w+z_i} A_p \rho_w g dz,
\]  \hspace{1cm} (4.3)

where \( L \) is the length of the probe, \( A_p \) its cross-sectional area, \( h_w \) is the depth of the water layer on top of the sediment bed, \( z \) is the vertical coordinate and \( z_i \) is the height of the interface between sediment and water. The position \( z = 0 \) represents the tip of the probe.
CHAPTER 4. YIELD STRENGTH OF MUD

\[ F_{tot} \]

\[ \frac{\rho_w}{\text{water}} \]

\[ F_{h,u} \]

\[ h_w \]

\[ \text{probe} \]

\[ \rho \]

\[ F_w \]

\[ F_s \]

\[ z = z_i \]

\[ z = 0 \]

\[ F_{h,i} \]

\[ F_q \]

\[ L \]

\textbf{Figure 4.3: Definition sketch; co-ordinate system moves with probe}

- The hydrostatic force acting on the lower side of the probe:

\[ F_{h,i} = \int_0^{z_i} A_p \rho g dz + \int_{z_i}^{z_i+h_w} A_p \rho_w g dz, \tag{4.4} \]

where \( \rho \) is the bulk density of the sediment.

- The normal force acting on the lower side of the probe as a result of the bed strength \( \tau_y \):

\[ F_q = \tau_y s_c N_c A_p, \tag{4.5} \]

where \( s_c \) is a shape factor and \( N_c \) is a bearing capacity factor. Eq. (4.5) applies to undrained conditions only and is discussed in more detail in §4.3.3.

- The shear force acting on the side area of the probe as a result of the bed strength:

\[ F_w = \int_0^{z_i} \alpha \tau_y O_p dz, \tag{4.6} \]

where \( O_p \) is the perimeter length of the probe and \( \alpha \) is the shaft adhesion factor. \( F_w \) is discussed in more detail in §4.3.4.

Note that of these contributions, only \( F_q \) and \( F_w \) are representative of the bed strength. To single out these contributions, the force measured at the reference position \( z_i = 0 \), when the lower side of the probe is located at an infinitesimal
small distance from the interface between sediment and water, is subtracted from the force at the position \( z_i = z_1 \), where \( z_1 \) is the penetration depth of the probe. After some elaborations the following expression can be derived:

\[
W_{exp} = F_{int}(z_i = z_1) - F_{int}(z_i = 0) = \int_0^{z_1} A_p(\rho - \rho_w)g dz + \tau_y s_c N_c A_p + \int_0^{z_1} \alpha \tau_y O_p dz
\]  

(4.7)

With (4.7) the yield strength profile can be calculated from the experimental data obtained with the balance, which was set to zero at \( z_i = 0 \). If evaluation of (4.7) does not lead to an explicit expression for \( \tau_y \), it must be obtained iteratively. The expression \((F_{h,i} - F_{h,u} + F_y)/A_p\) is often referred to as the bearing capacity factor \( q_f \).

### 4.3.3 Bearing capacity

Normal vertical stresses act on the horizontal surfaces at the top and the bottom of the probe. If the traversing speed \( u_t \) is sufficiently low, dynamic forces will be small compared to static forces. At \( u_t = 0.1 \text{ mm s}^{-1} \), for example, the dynamic pressure is \( 10^{-5} \text{ Pa} \), at least 5 orders of magnitude smaller than the yield strength of the materials tested.

Therefore, a hydrostatic pressure can be assumed at the top of the probe, which is always in the water layer overlying the sediment bed. At the tip, however, also the presence of the bed material contributes to the normal stresses acting on the tip surface. At low traversing speed, these forces will equal the bearing capacity of the footing. The problem is how to relate the normal forces acting at the slowly moving tip to the yield strength of the material tested.

Using plasticity theory, Prandtl derived an exact expression for the bearing capacity \( q_f \) of a strip footing of infinite length on the surface of a semi-infinite, isotropic weightless soil for the undrained condition \( (\phi = 0) \), given by \( q_f = (2 + \pi) c \approx 5.14 c \), where \( q_f \) is the bearing capacity (Figure 4.4).

If the footing is not located on the surface of the soil, but at a depth \( z_1 \) below the interface, as is the case during a sounding test, another term has to be included representing the surcharge pressure. Based on similar considerations, Terzaghi developed an expression for the bearing capacity of the soil under a shallow footing, in the undrained case given by [65]

\[
q_f = c s_c N_c + \int_0^{z_1} \gamma d\gamma
\]  

(4.8)

where \( c = \tau_y \) and \( \gamma \) is the specific volumetric weight given in this specific case by \( \gamma = (\rho - \rho_w)g \). \( N_c \) is a dimensionless bearing capacity factor and \( s_c \) is a dimensionless shape factor depending only on the footing geometry used.
Figure 4.4: Assumed failure patterns under deep foundations

According to Skempton [103], the shape factor $s_c$ is also dependent on the ratio between depth ($z$) and breadth ($D$, see Figure 4.2) of the footing. Skempton proposed for undrained conditions for a rectangular footing

$$ s_c = \left(1 + 0.2 \frac{D}{B}\right) \left(1 + 0.2 \frac{z}{D}\right) \text{ if } \frac{z}{D} < 2.5 $$

$$ s_c = 1.5 \left(1 + 0.2 \frac{D}{B}\right) \text{ if } \frac{z}{D} > 2.5. $$

(4.9)

where $D < B$. The values proposed by Skempton are generally accepted and give as a simple relation between undrained shear strength and bearing capacity $q_f = 9c + \int_0^z \gamma dz$ for shallow circular footings and $(z/D) > 2.5$.

Values for shape and bearing capacity factors of shallow footings for several geometries are given in Table 4.2. Alternative proposals for shape factors have been presented, but these are not significantly different [28, 41]. The cone factor $N_c$ tends to increase markedly with overconsolidation ratio (OCR), which is defined as the ratio between the highest former stress and the present stress. For OCR > 20, $N_c$ has approximately doubled for China clay [26].

It is important to notice that the effect of soil compressibility is not incorporated in the bearing capacity theory, which will lead to errors if compressibility effects are important [7]. An analysis based on the cavity expansion theory is then more appropriate. Throughout this report the sediment bed is considered to be incompressible. For saturated sediments (without air!) under undrained conditions this is a very good approximation. However, for slow, drained tests se-
Table 4.2: Values for shape and bearing capacity factors of shallow footings proposed by [110] used in (4.8) and (4.10)

<table>
<thead>
<tr>
<th>(un)drained:</th>
<th>( N_\gamma )</th>
<th>( N_c )</th>
<th>( N_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0 )</td>
<td>0</td>
<td>5.7</td>
<td>1</td>
</tr>
<tr>
<td>( \phi \neq 0 ) [85]</td>
<td>( 1.5(N_q - 1) \tan \phi )</td>
<td>( (N_q - 1) \cot \phi )</td>
<td>( \tan^2(\frac{\phi}{2} + \frac{\pi}{2}) \times \exp(\pi \tan \phi) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>probe:</th>
<th>( s_\gamma )</th>
<th>( s_c )</th>
<th>( s_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>strip</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>square</td>
<td>0.8</td>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td>circle</td>
<td>0.6</td>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td>rectangle</td>
<td>( 1 - 0.2(D/B) )</td>
<td>( 1 + 0.2(D/B) )</td>
<td>1</td>
</tr>
</tbody>
</table>

diment particles will get more closely packed under the weight of the penetrating probe and compressibility effects become important.

4.3.4 Shear stress at the shaft

The value of \( \alpha \) in (4.6), which quantifies the contribution of the shear force acting on the side area of the probe during penetration of the bed, needs some attention. Its absolute value may range from 0.3 to 1; for very weak cohesive soil samples it is likely to be close to unity [99]. By repeating a sounding test several times with probes with a different ratio between tip and shaft area, the value of \( \alpha \) can be estimated. For the tests described in §4.4, \( \alpha \) was assumed to be unity, as an acceptable collapse of experimental data was thus obtained.

In (4.6) the viscous shear stress exerted by water on the shaft has been neglected. For the typical values of \( \Delta z = 0.05 \text{ m} \), \( \tau_y = 10 \text{ Pa} \), \( L = 0.15 \text{ m} \), traversing speed \( u_t = 0.1 \text{ mm s}^{-1} \), column diameter \( d_c = 0.08 \text{ m} \) and probe diameter \( D = 0.005 \text{ m} \), the force resulting from this shear stress amounts to 2 nN, whereas the force calculated with (4.6) amounts to 8 mN, 4 \( \times \) 10^6 times larger.

Also the viscous contribution to the shear stress exerted by the sediment bed on the shaft has to be negligible when compared to the static contribution caused by the yield strength. For \( u_t = 0.1 \text{ mm s}^{-1} \), \( \dot{\gamma} \) is estimated at 0.1 s\(^{-1}\). At this shear rate the viscous contribution is very small (cf. Figure 3.4); for higher shear rates, however, it may become significant.

The sign of \( \alpha \) and therefore \( F_w \) needs special attention. If the probe is moving downwards, the shear stress will only act upwards if the settling or consolidation velocity is smaller than the traversing speed. Otherwise the shear stress may act downwards, which is known as 'negative skin friction' and is represented by
a negative value for $\alpha$. If experiments are started with the tip of the probe above the interface between sediment and water, and the traversing speed is kept constant, negative skin friction is impossible, as the interface will never be reached as long as the traversing speed is smaller than the consolidation speed. However, if the probe is halted at a certain position inside the sediment bed, negative skin friction may develop. In this case, the value of the shear stress is uncertain and may lie between plus and minus the yield strength. If the probe is moving upwards the shear stress will always act downwards.

4.3.5 Drained test

In the case of a slow, drained test, during which dissipation of excess pore pressures is allowed, an analysis in terms of effective stress is necessary. In this case, bearing capacity is calculated with:

$$q_f = \frac{1}{2} \gamma D s_r N_r + c' s_c N_c + s_q N_q \int_0^l \gamma dz$$

(4.10)

where $N_r$, $N_c$ and $N_q$ are bearing capacity factors dependent on $\phi'$ [41, 86]; $s_r$, $s_c$ and $s_q$ are shape factors (Table 4.2). $D$ is the breadth or diameter of the footing.

The assumption of incompressibility is doubtful in the drained case. The freshly deposited, uncompacted grain skeleton is easily compressed by the penetrating probe if pore water is allowed to flow freely and no excess pore pressures develop. The sediment below the probe will consolidate under the extra load of the probe, and its strength will increase. This strength is no longer representative of the strength of the original, undisturbed sample.

4.3.6 Wall effects

The diameter of the settling columns used for the sounding tests should be sufficiently large to avoid wall effects. A prerequisite is that the increase in fluid level because of the immersion of the probe should be small compared to the total fluid volume in the cylinder, which for circular rods results in the criterion $d_s^2 / d_c^2 h_w \ll 1$, where $l$ is the length of the immersed part of the rod, $d_s$ is its shaft diameter, $d_c$ is the diameter of the settling column and $h_w$ is the height of the fluid in the column. The ratio should be preferably smaller than 0.01, which in the limiting case $l = h_w$ results in $d_s / d_c < 0.1$.

From a geotechnical point of view this criterion is certainly sufficient, as can be concluded from an analysis of the size of pressure bulbs under and next to a probe in a sediment bed [104]. An important result from this analysis is that the determination of the shear strength of a material within a distance less than twice the diameter of the probe from the bottom of the cylinder is inaccurate, as the pressure distribution is then influenced by the bottom.
4.4 EXPERIMENTAL RESULTS AND DISCUSSION

Atapattu et al. [5] studied the effect of cylindrical walls on the creeping terminal velocity \( v_t \) of spheres in viscoplastic media. Their analysis leads to a criterion for the maximal value of \( d/d_e \) that is less strict than the criterion based on fluid level increase.

Based on these criteria and on experiments that will not be discussed here, wall effects may be neglected for all geometries used for the experiments described in §4.4, except for the plate equipped with pressure transducers.

4.4 Experimental results and discussion

In this section experimental results are presented and discussed. First the reproducibility of the measurements is dealt with, subsequently the influence of the penetration probe used, the traversing speed, the sediment type and the sediment consolidation time. The yield stress is calculated from the vertical force as described in the previous section. Unless indicated otherwise, undrained behaviour has been assumed.

4.4.1 Reproducibility

In Figure 4.5 results of three tests on identically prepared beds are presented, which were obtained using a plate geometry. The beds were prepared from sedimentation of a suspension of China clay in tap water (with 0.5\% NaCl) with a concentration of 275 kg m\(^{-3}\) and an initial thickness of 0.07 m. The consolidation time was approximately 3 days. Agreement is fair, the measurements are reproducible within an error band of approximately 2 Pa for these experiments. If one bed is tested several times, reproducibility is only good if the measuring probe penetrates at a position in the bed that is not disturbed by the previous experiment(s), as bed strength is negatively affected by disturbance of the sample.

4.4.2 Influence of probe shape

In Figure 4.6 experimental results for the measuring probes discussed in §4.2 are shown. The beds were prepared from sedimentation of a suspension of China clay in tap water (with 0.5\% NaCl) with a concentration of 275 kg m\(^{-3}\) and an initial height of 0.23 m. The consolidation time was approximately 10 days.

It is clear that with the bearing capacity theory a good collapse of results is obtained, even without adapting the coefficients for shaft adhesion \( \alpha \) and cone penetration \( N_c \), which were set at the theoretically predicted values 1.0 and 5.14, respectively. The error band is not larger than for duplicate measurements
with the same probe, therefore the testing method is found to be insensitive to geometrical variations.

The disk is only suitable to determine the yield strength at the water-sediment interface properly, as its resistance is so high that it gets stuck before it reaches the bottom of the sediment bed in all but the weakest samples. Moreover, its non-prismatic geometry makes accurate calculations difficult at \( z \) greater than about 0.5 cm.

Results for the plate equipped with pressure transducers are not shown in Figure 4.6. As the volume of this probe is not very small with respect to the bed volume, deviation from the results obtained with the other probes is to be expected. Also wall effects may become important. However, this probe is primarily intended to compare and relate pressure data with strength. Whether this is the strength of an undisturbed or partially disturbed bed is not of primary importance.

### 4.4.3 Bulk density profiles

In order to calculate strength profiles, the bulk density of the bed has to be known. The average bulk density of the bed, \( \rho_{av} \), is easily calculated from the initial height of the suspension \( h_i \), the initial suspension concentration \( C_i \) by mass and the actual height \( h \):

\[
\rho_{av} = \left[ \left( h_i / h \right) C_i / \rho_c \right] \left( \rho_c - \rho_w \right) + \rho_w,
\]  

\[(4.11)\]
4.4. EXPERIMENTAL RESULTS AND DISCUSSION

![Graph showing yield strength profiles for different probe shapes](image)

**Figure 4.6: Yield strength profiles for different probe shapes**

where \( \rho_c \) is the dry density of the clay particles, usually around 2,700 kg m\(^{-3}\). However, the bulk density varies in the vertical. Lower layers are generally more compacted than upper layers, resulting in a higher bulk density. If accurate calculations of the yield strength are desirable, this should be taken into account.

Bulk density profiles were measured using a conductivity probe as mentioned in §4.2. Examples of these profiles are shown in Figures 4.7 en 4.8 for China clay and Caland Channel mud respectively. For China clay the initial sediment concentration was 250 kg m\(^{-3}\) and the initial height was 0.30 m. For Caland Channel mud these values are 60 kg m\(^{-3}\) and 0.225 m, respectively. Sediment beds in columns 1, 2 and 3 were prepared in an identical way. A trend of increasing density with increasing depth can be clearly observed. Measurements in the upper and lower few millimetres of the bed are inaccurate, as the sensor then is only partly immersed in the bed or is influenced by the bottom. The bulk density of Caland Channel mud is markedly lower than that of China clay. Because of the stronger interparticle bonds (cohesion), the consolidation process stops at a higher void ratio and lower density at the same effective stress. This is in agreement with yield strength measurements, as shown in Figure 4.18, for example.

From the density and yield strength profiles a plot of the yield strength versus the density can be constructed. An example is shown in Figure 4.9, where a clear, positive correlation between density and yield strength can be observed. Applying the relationship between yield strength and concentration based on
CHAPTER 4. YIELD STRENGTH OF MUD

Figure 4.7: Density profile for China clay

Figure 4.8: Density profiles for Caland Channel mud
4.4. EXPERIMENTAL RESULTS AND DISCUSSION

fractal theory [55]:

$$
\tau_y \sim (\rho - \rho_w)^{\frac{2}{D}}
$$

(4.12)

a good agreement is obtained between experimental data and theoretical curves. The fractal dimension of China clay is thus estimated at $D = 2.68$, whereas for Caland Channel mud $D = 2.26$. This once more points to the more cohesive properties of Caland Channel mud compared to China clay: for (cohesionless) sand the fractal dimension $D = 3$. On the basis of this well-formed correlation, one might wonder if it is possible to calculate yield strength directly from the easily measured density profile without the bustle of measuring the force on a probe or the torque on a vane.

Although there often is a positive correlation between yield strength and density, no unique relation between these two parameters exists. Different sediment types with the same density may have totally different yield strengths, as a comparison between China clay and Caland Channel mud illustrates. But even if only one sediment type is considered, the relation between density and yield strength is not unique. By disturbance of a freshly consolidated sediment bed excess pore pressures are generated, as the loosely packed particles tend to get more closely packed. However, as the permeability of cohesive sediment beds is low, initially no flow of water occurs and therefore no bulk density changes are observed. Opposite to this, the disturbance does affect the yield strength, which shows that even for one sediment type at a certain density, the bed strength may vary. The bed may 'liquefy' if it is disturbed, for example by hydrodynamic forces. In Chapter 5 this process will be dealt with.

4.4.4 Influence of traversing speed

Traversing speed is an essential parameter in the sounding process. If it is very high, the viscous contribution to the vertical force may be important (§4.3.4), if it is very small, the test may be drained or partially drained instead of undrained. The latter effect can be assessed with pore pressure measurements.

In Figure 4.10 the influence of the traversing speed $u_t$ on the forces acting on the measuring probe during a sounding test is shown. Traversing speeds ranged from $10^{-3}$ to $10$ mm s$^{-1}$. The beds were prepared from sedimentation of a suspension of China clay in tap water (with 0.5% NaCl) with a concentration of 275 kg m$^{-3}$ and an initial height of 0.23 m. The consolidation time was approximately 7 days. The plate equipped with pressure transducers was used for these experiments in order to be able to discern between drained and undrained tests.

From Figure 4.10 it is clear that the absolute value of the force decreases with increasing traversing speed for $u_t < 0.02$ mm s$^{-1}$, whereas in the range 0.1 mm
Figure 4.9: Yield strength versus density for freshly deposited China clay and Caland Channel mud. Theoretical curves are discussed in the text.

Figure 4.10: Influence of traversing speed on vertical force for China clay.
4.4. EXPERIMENTAL RESULTS AND DISCUSSION

Figure 4.11: Excess pore pressure as a function of traversing speed for China clay

$s^{-1} < u_t < 2 \text{ mm s}^{-1}$ the force much less and not monotonously dependent of the traversing speed.

The decrease for $u_t < 0.02 \text{ mm s}^{-1}$ can be explained by the transition from drained to undrained behaviour. This can be concluded from Figure 4.11, where the excess pore pressure is shown for sounding tests at several traversing speeds. The excess pore pressure $p_{exc}$ is found by subtracting the hydrostatic pressure $p_{hydr} = \rho_w g z$ from the pore pressure measured at the tip of the measuring probe. The interface between sediment bed and overlying water is defined as reference level $z = 0$, where $p_{hydr} = p_{exc} = 0$. From Figure 4.11 it is clear that the pore pressure is nearly hydrostatic at $u_t = 10^{-3}\text{ mm s}^{-1}$, the sounding test is therefore drained. In this case the yield strength should be calculated from the force with an effective stress analysis. At $u_t > 0.1 \text{ mm s}^{-1}$ the excess pore pressure is nearly independent of traversing speed and the sounding test may be assumed to be fully undrained for the samples considered. An analysis in terms of total stress is now allowed.

The much higher strength in the drained experiment compared to the undrained experiments is caused by consolidation effects in the bed under the load at the tip of the probe. In the drained case excess pore pressures are allowed to dissipate, causing an increase in effective vertical stress, as is illustrated in Figure 4.12. Therefore, in the drained case the sample is much affected by the measurement and no undisturbed strength is obtained. At relatively high traver-
Figure 4.12: Effective vertical stress during an experiment with China clay

...
Figure 4.13: Influence of traversing speed on vertical force for Caland Channel mud

Figure 4.14: Influence of traversing speed on vertical force for China clay; focussed on traversing speeds of 0.01–1 mm s$^{-1}$
transducers. From (4.8) one can estimate the undrained yield strength with 
\[ \tau_y = c = \sigma' / s_c N_c. \]

In Figure 4.15 the strength calculated in this way is compared with the 
strength calculated from vertical force for several traversing speeds. Caland 
Channel mud was used as the bed material. Qualitative agreement between 
the two totally independent methods is excellent, quantitative agreement is fair. 
Differences are attributed mainly to the much lower accuracy of the pressure 
transducers compared to the accuracy of the balance. Compare Figure 4.15 with 
Figure 4.13, which is based on the same experimental data.

### 4.4.6 Negative skin friction

The effect of negative skin friction can easily be studied by reversing the traversing 
direction. The force acting on the shaft of the probe now also reverses in 
direction. If the contribution of the force at the tip is small, which is the case for 
the plate geometry, the sum of forces for the upward and downward movement 
should approximately be zero. This is shown in Figure 4.16, where the force 
response to traversing in both directions is presented. The deviation from zero 
should represent two times the tip resistance, as it acts in the same direction for 
both traversing directions. In the upper part of the bed the agreement is good, 
whereas in the lower part the agreement is worse. In this way tip resistance and
4.4. EXPERIMENTAL RESULTS AND DISCUSSION

![Graph showing skin friction data](image)

**Figure 4.16:** Skin friction, ‘up’ and ‘down’ refer to the motion of the plate geometry relative to the settling column

Skin friction at the shaft can also be used to obtain a time-scale for particle settling and stress relaxation. Just prior to the moment when movement of the probe into the sediment is stopped, the shear stress at the shaft will act in the upward direction. However, because of stress relaxation and particle sedimentation, the shear stress at the shaft will decrease and may even become negative, depending on the ratio between shaft and tip area. This is illustrated in Figure 4.17, where the vertical force is displayed acting on a plate inside a bed of China clay after cessation of motion at $t = 1000$ s ($z = 0.10$ m). The response is fitted with the function $W = a + b \exp (-c(t - t_0))$, where $a = 19.3$ mN, $b = -35.87$ mN, $c = 0.0020$ s$^{-1}$ and $t_0 = 1000$ s. Stress relaxation effects where also observed during rheological experiments, cf. Figure 3.10.

4.4.7 Yield strength at sediment-water interface

Measurement of the yield strength at the interface between sediment and water is important as it determines the true cohesion of the material at zero effective stress. It is also essential if sounding test results are to be compared with rheological measurements. However, the accuracy of the sounding test at the sediment-water interface is less than at a lower level inside the sediment bed, owing to the following effects:
**Figure 4.17:** Stress relaxation at shaft after cessation of motion at $t = 1000$ s

- The yield strength at the interface is small for freshly deposited layers, especially for China clay. Absolute errors of 1 Pa, to an order of magnitude, then are significant.

- The density of the bed at the interface, which has to be known to be able to calculate yield strength from vertical force, is difficult to determine with a conductivity probe.

- Surface tension forces are not accounted for.

In Table 4.3 the interface yield strength of China clay after 26 days of consolidation from a suspension with an initial concentration of 275 kg m$^{-3}$ and an initial height of 0.23 m is presented in duplicate for the rod and disk geometries. The plate geometry is unsuitable for interface measurements because of its tiny $D/B$-ratio (Table 4.1). Agreement between the duplicate experiments is satisfactory. However, a significant discrepancy between plate and disk geometries can be observed, which may be a consequence of the last two points itemized above. Agreement between these results and controlled stress (CS) rheological measurements is surprisingly good. The results for the disk geometry are more reliable than those for the rod geometry, as the tip area of the former is much larger.

It is desirable to compare sounding test results with rheological measurements, not only at the interface but also inside the bed where the effective stress
4.4. EXPERIMENTAL RESULTS AND DISCUSSION

Table 4.3: Yield strength at sediment-water interface for China clay

<table>
<thead>
<tr>
<th>probe</th>
<th>$\tau_y$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rod</td>
<td>5.3</td>
</tr>
<tr>
<td>rod</td>
<td>5.9</td>
</tr>
<tr>
<td>disk</td>
<td>3.7</td>
</tr>
<tr>
<td>disk</td>
<td>3.6</td>
</tr>
<tr>
<td>average</td>
<td>4.6</td>
</tr>
<tr>
<td>CS rheometry</td>
<td>$\sim 4$</td>
</tr>
</tbody>
</table>

is not equal to zero. With rheological measurements the yield strength is measured directly, whereas with the sounding test the yield stress has to be calculated from the measured forces using bearing capacity theory. However, strength profiles in sediment beds can not easily be measured with standard rheological instruments, as was discussed in §4.1.

4.4.8 Influence of effective stress

Initial effective stress for fully consolidated beds can be related to the vertical co-ordinate $z$ by $\sigma' = \int_0^z (\rho - \rho_w)gdz$. The density profile can be measured with a conductivity probe as described in §4.4.3. The initial effective stress should not be confused with the effective stress acting at the tip of a probe during penetration. For undrained tests the latter will generally be smaller than the former, as excess pore pressures are generated by the penetration of the probe. For drained tests, however, the latter tends to be larger than the former, as excess pore pressures are allowed to dissipate and consolidation effects under the weight of the probe may occur.

In Figure 4.18 the yield strengths of Caland Channel mud and China clay are shown as functions of initial vertical effective stress $\sigma_{v,0}$. A continuous increase in yield strength with effective stress can be observed. The dependency of the yield strength of China clay on $z$ and thus on $\sigma_{v,0}$ is less pronounced than for Caland Channel mud. This may be explained by the differences in the ratio between vertical and horizontal effective stress. Mohr's circle shows that the maximal shear stress within a material equals half the difference between the major and minor principal stresses, being the vertical and horizontal stresses in the case of axi-symmetric consolidation columns. If it is supposed that consolidation proceeds as long as the gravity-induced stresses in the bed exceed the bed strength, the maximal shear stress in a fully consolidated bed will approximately equal the yield strength. For beds with a small cohesion like China clay, the
bed will be supported by the column walls for the major part and the difference between major and minor principal stress will be small. A theoretical maximum for the yield stress is found if the horizontal stress is zero. The yield stress then equals half the vertical effective stress, as is approximately true for Caland Channel mud in Figure 4.18. This maximum is only valid for freshly deposited layers which have not yet been exposed to higher effective stresses in the past than the present effective stress. Also cohesion effects—which are responsible for strength at zero effective stress—are not taken into account in this argument. For (partially) drained conditions during a sounding test, the yield strength may exceed the theoretical maximum, as the penetration of the probe into the bed then results in a vertical effective stress higher than the initial vertical effective stress.

In contrast with freshly deposited beds, the yield strength of overconsolidated beds may show a better correlation with the highest effective stress in its stress history than with the actual effective stress. Overconsolidation is caused by erosion, for example, as sediment that carried the weight of overlying layers becomes exposed.
4.4. EXPERIMENTAL RESULTS AND DISCUSSION

![Graph showing the dependency of yield strength on consolidation time and vertical co-ordinate for China clay](image)

**Figure 4.19**: Dependency of yield strength on consolidation time and vertical co-ordinate for China clay

4.4.9 Influence of consolidation time

One of the features of the measuring method under consideration is that it can be used to measure strength development in consolidating beds. Strength development can be partly explained by the increase in effective stress during consolidation. Results are presented in Figure 4.19, where the strength profiles for China clay after 3, 6, 8 and 10 days of consolidation are shown. The initial sediment concentration was 275 kg m\(^{-3}\) and the initial height was 0.225 m. A plate was used as measuring probe. From this figure it is clear that for a sediment bed consisting of China clay with a height of approximately 0.10 m, primary consolidation ends after about one week, after which the height of the sediment bed remains constant. Evaluation of strength increase is also possible on much shorter timescales, as illustrated in Figure 4.20. The initial sediment concentration was 60 kg m\(^{-3}\) and the initial height was 0.30 m. A significant difference exists between strength at 1 hour after the start of sedimentation/consolidation and strength after 3 hours. The strength increase appears to be most pronounced near the bottom of the cylinder, where the largest decrease of pore pressure is to be expected. The characters 'a' and 'b' refer to the two columns used. Reproducibility is good, except for the lowest 2 cm after 6 days of consolidation.

For Caland Channel mud similar experiments were carried out; results are presented in Figure 4.21. The initial sediment concentration and height were the
same as for the experiments with China clay. The results from the sounding test appear sensible once more and reproducibility is good. Bed strengths from 1 Pa can be measured accurately with the plate geometry.

4.5 Conclusions and recommendations

The sounding probe and test described provide a simple and fairly accurate method to measure yield strength and yield strength profiles of sediment beds in a laboratory. There are no theoretical impediments to use a modified version of the sounding instrument for field experiments. Yield strengths down to 1 Pa can be measured fairly accurately. A good agreement is obtained between plate, rod and disk geometries. The method can be used, for example, to measure the evolution of yield strength with time during the consolidation process.

If the penetration probe is equipped with pore and total pressure transducers, valuable information on quantities such as effective stress and excess pore pressure is obtained in addition to the force acting on the probe. The transition between drained and undrained behaviour can be clearly derived from excess pore pressures.

The bed strength can be calculated independently of the effective stresses acting on the tip of the probe during penetration. Qualitative agreement between this method and that based on a force balance is good; quantitative agreement
Figure 4.21: Dependency of yield strength on consolidation time for Caland Channel mud

is only fair, because of the lower accuracy of the pressure sensors.

Comparison of the true cohesion intercept at zero effective stress with the yield strength showed satisfactory agreement. For this comparison, the former value was determined with the sounding test, whereas the latter value was determined with controlled stress rheometry. Additional comparisons with vane tests are still to be made. Some preliminary results suggest that the yield strength obtained with the sounding test show a satisfactory agreement with the residual yield strength measured with a vane.

A clear and approximately linear relationship between vertical effective stress—which is calculated from the density profile of a consolidated bed—and shear strength is observed. However, a priori estimation of the yield strength from the effective stress is generally not possible, because additionally the ratio of horizontal to vertical effective stress has to be known. Also the cohesion of the particles is important; especially at low effective stress levels it may be dominant. Moreover, cohesion may increase by ageing. Last but not least, sediment beds may have a much higher strength than would follow from their actual state of effective stress as a result of their effective stress history. For example, if a sediment layer is re-exposed after having been buried in the past, it has been exposed to much higher effective stresses than the prevailing stress.

The measuring probe equipped with pressure transducers used in this study was quite bulky compared to the dimensions of the settling columns. However, if
a more compact instrument should be designed, preferably also equipped with a miniature acoustic probe to measure bed concentrations, a very powerful instrument would be obtained. The combined measurement of force, pore pressure, total pressure and concentration as a function of the vertical co-ordinate, makes a thorough analysis of bed strength and strength evolution during consolidation possible.
Chapter 5

Wave-induced liquefaction and flow

5.1 Introduction

In the previous chapters, the constitutive behaviour of cohesive sediments both prior to and after failure was discussed. The present chapter\(^1\) is focussed on the process of failure of loosely packed sediment layers owing to wave action. As was demonstrated in Chapter 4, a positive excess pore pressure is observed upon failure of loosely packed sediment because the grain matrix tends to collapse when subjected to shear. As a result, the volume of the grain matrix decreases. Sediment particles become fluid-supported for the major part, the effective stress within the matrix concurrently decreases, and a transition from a predominantly solid-like behaviour into a more fluid-like behaviour is observed. This process is often referred to as 'liquefaction'. It should be remarked that liquefaction as described here is undrained: no significant upward pore water flow is needed to support sediment particles, as in the case of fluidisation of fine sands. After liquefaction, mud may stay liquid for a prolonged period of time because of its low permeability.

This failure mechanism should be well distinguished from (drained) surface erosion, which will occur if the bed shear stress exerted by current or by orbital wave motion exceeds the critical shear strength for erosion [79]. For the experiments reported in this chapter, the former mechanism prevailed, as the frictional bed shear stress remained much smaller than shear stress inside the bed generated by wave-induced pressure gradients. This is demonstrated in Section 5.4.

Liquefied mud is easily transported when exposed to net hydrodynamic or gravity forces. For example, sliding of fluid mud layers into navigation channels might explain part of the high siltation rates often observed, especially after storm periods. In order to assess this possibility, wave-induced liquefaction of a mud bed, and the subsequent downslope transport of the resulting fluid mud, have been investigated both experimentally and theoretically.

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\(^1\)Submitted in adapted form to Coastal Engineering under the title 'Wave-induced liquefaction and flow of subaqueous mud layers' by Thijs van Kessel and C. Kranenburg
The organisation of this chapter is as follows. In the next section models describing liquefaction and subsequent transport downslope are presented. In Section 5.3 an experimental set-up is presented that has been used to examine the liquefaction of a sloping cohesive sediment bed by waves. Both an artificial mud and a natural mud have been tested. In Section 5.4 results from these experiments are presented. A comparison is made between the observed and predicted wave heights at the onset of liquefaction in Section 5.4.1; Section 5.4.2 is devoted to the onset of motion of fluid mud just after liquefaction. As the experiments were carried out in a tilted flume, a significant shear stress inside the bed was generated by gravity force. This caused the mud to flow downslope after liquefaction. The experiments are compared with a mathematical model of this gravity flow. Finally, conclusions are drawn in Section 5.5.

5.2 Modelling of liquefaction and transport

In §5.2.1 a model is presented to calculate the stresses inside a sediment layer generated by waves. With this model, together with the strength profile of the exposed sediment bed—which was measured as described in Chapter 4—the wave conditions at which liquefaction occurs can be predicted. A model for the subsequent downslope flow of liquefied mud under waves is discussed in §5.2.2

5.2.1 Calculation of wave-induced stresses

Failure of a sediment layer under waves will occur if the deviator stress generated by wave-induced pressure gradients on the bed surface exceeds the yield strength of the sediment. For the calculation of this deviator stress a number of models have been proposed. An overview of these models is presented by De Wit [131].

The choice of the most suitable model depends on the situation considered. First of all, the period of (wave) loading has to be compared to the time scale for pore water flow, which is strongly related to the permeability of the sediment bed. If the period is sufficiently long to expect pore water flow, a two-phase approximation should be adopted; otherwise the bed may be assumed to be impermeable and a one-phase approximation is justified. Two-phase modelling is complicated, as compressibility of the grain matrix has to be taken into account. In principle, also consolidation effects (either negative or positive) should be incorporated into a two-phase model. An example of a relatively simple two-phase model is the poro-elastic model [137, 105] in which the grain matrix is assumed to be isotropic and to behave as a linear elastic medium; pore water flow is modelled with Darcy's law. Compressibility is taken into account, but consolidation is not.
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For the experiments presented in this chapter, the bed is assumed to be impermeable, as the wave period is negligible compared to the time-scale of significant pore water flow. In addition, the bed is assumed to be incompressible. The assumption of incompressibility is a good approximation for fully saturated impermeable sediment beds, which have a negligible gas content.

As was shown in Chapter 3, the constitutive behaviour of mud at small strains is well represented by a linear visco-elastic model of the integral type. Examples of this model are the Maxwell model and the Kelvin-Voigt model. The visco-elastic model applies prior to failure only, when the wave-induced deviator stress is substantially smaller than the bed shear strength and the bed response is linear. If this model is used to calculate the wave-induced deviator stress at the onset of liquefaction, an upper limit of the deviator stress will be obtained, as non-linear effects are not taken into account.

A drawback of this model is that mud properties are taken as constant both in space and time, which they are definitely not. In Chapter 4 a clear increase in bed strength with depth was observed; a similar increase in $G'$ is to be expected. Therefore, in order to calculate the response of a mud bed to waves, the bed is often subdivided in several layers, each having own values of $G'$ and $G''$. Values of $G'$ and $G''$ can also be taken dependent on the strain amplitude [37, 17]. Solutions for these layered models can often be obtained numerically only.

For the present experiments, the deviator stress in the bed exerted by waves prior to liquefaction is calculated using an analytical model yielding the stresses in a slab of a homogeneous, incompressible, linearly elastic solid on a no-slip base. Pore-water flow is neglected because of the low permeability of the mud layer and the relatively rapid fluctuations of wave-loading. The viscous part of the shear stress is neglected, as prior to liquefaction it is much smaller than the elastic part (Figure 5.8) as a result of consolidation. For the bed the long-wave approximation is used, as the thickness of the sediment layer was much smaller than the wave-length. The equations of equilibrium of the elastic model, which is a special case of the Yamamoto et al. [137] model, then become, for the effects of waves only and neglecting changes in water depth,

$$G'\frac{\partial^2 u_s}{\partial z^2} = \frac{\partial p}{\partial x}$$  \hspace{1cm} (5.1)

$$G'\frac{\partial^2 w_s}{\partial z^2} = \frac{\partial p}{\partial z}$$  \hspace{1cm} (5.2)

$$\frac{\partial u_s}{\partial x} + \frac{\partial w_s}{\partial z} = 0$$  \hspace{1cm} (5.3)

with boundary conditions at the interface between sediment and water ($z = 0$):

$$\tau_{xz} = 0 \Leftrightarrow \frac{\partial u_s}{\partial z} + \frac{\partial w_s}{\partial x} = 0$$  \hspace{1cm} (5.4)
\[ p = p_0 \exp(i(kx - \omega t)) \]  
(5.5)

and at the bottom of the sediment layer \((z = D)\):

\[ u_s = 0 \]  
(5.6)

\[ w_s = 0 \]  
(5.7)

In these equations \(x\) and \(z\) are the co-ordinates parallel and normal to the bed, respectively, \(u_s\) and \(w_s\) are the soil displacements in \(x\) and \(z\) directions, \(p\) is the pressure and \(p_0\) the pressure amplitude, \(D\) is the thickness of the sediment bed, \(G\) is its shear modulus, \(\tau_{xz}\) the shear stress, \(k\) the wave number, \(\omega\) the wave frequency and \(t\) time.

The solution of these equations is given by

\[ u_s(z, t) = \frac{i}{kG} \left( b_0 + b_1 \exp(kz) + b_2 \exp(-kz) \right) \exp(i(kx - \omega t)) \]  
(5.8)

\[ w_s(z, t) = \frac{1}{kG} \left( c + b_0 k z + b_1 \exp(kz) - b_2 \exp(-kz) \right) \exp(i(kx - \omega t)) \]  
(5.9)

\[ p(z, t) = \left( b_1 \exp(kz) + b_2 \exp(-kz) \right) \exp(i(kx - \omega t)) \]  
(5.10)

with

\[ b_0 = -\left( \frac{\exp(kD) + a_1 \exp(-kD)}{a_1 + 1} \right) p_0 \]  
(5.11)

\[ b_1 = \frac{1}{a_1 + 1} p_0 \]  
(5.12)

\[ b_2 = \frac{a_1}{a_1 + 1} p_0 \]  
(5.13)

\[ c = 2 \left( \frac{a_1 - 1}{a_1 + 1} \right) p_0 \]  
(5.14)

where

\[ a_1 = \frac{\frac{1}{2} \exp(kD)(1 - kD) - 1}{\frac{1}{2} \exp(-kD)(1 + kD) - 1} \]  
(5.15)

The maximal shear stress or deviator stress \(\tau_{dev}\) in the bed is calculated from

\[ \tau_{dev}(z, t) = \sqrt{\tau_{xz}^2 + \frac{1}{4} (\sigma_x - \sigma_z)^2} \]  
(5.16)

where

\[ \tau_{xz} = i \left( c + b_0 k z + 2 (b_1 \exp(kz) - b_2 \exp(-kz)) \right) \exp(i(kx - \omega t)) + \tau_{xz}^* \]  
(5.17)

\[ \sigma_x = - \left( 2b_0 + b_1 \exp(kz) + b_2 \exp(-kz) \right) \exp(i(kx - \omega t)) + \sigma_x^* \]  
(5.18)
5.2. MODELLING OF LIQUEFACTION AND TRANSPORT

\[ \sigma_z = (2b_0 + 3 (b_1 \exp(kz) + b_2 \exp(-kz))) \exp(i(kz - \omega t)) + \sigma_z^s \]  
(5.19)

The superscript \( s \) refers to the stresses caused by gravity and slope. For mild slopes, \( \tau_x^s = \Delta \rho g z \sin \theta \), whereas \( \sigma_z^s \approx \sigma_x^s \) just prior to liquefaction. From this analytical model the shear stresses inside the sediment bed can be calculated given the wave conditions.

5.2.2 Calculation of transport of fluid mud downslope

After liquefaction, the bed starts to flow if subjected to a net force, which in the present study is the gravity force. As failure of the bed is undrained, the sediment concentration of the liquefied layer equals the original bed concentration, which generally is a few hundreds kg m\(^{-2}\). Because of its high viscosity, the flow of these layers tends to be laminar. Mixing with overlying water hardly occurs, because of the large density gradient stabilizing the interface.

An analytical model to simulate this laminar flow is presented here in order to extrapolate laboratory results to field situations. With this model the velocity \( u(z, t) \) resulting from wave action and gravity force is calculated numerically. This velocity in the fluid mud layer is parallel to the bed, and is a function of time and the co-ordinate normal to the bed. The model is based on the following equation of motion:

\[ \rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + (\rho - \rho_w)g \sin \theta \]
(5.20)

where \( \theta \) is the tilting angle of the sediment bed. The convective terms that should appear in (5.20) have been neglected, as for the present experiments they are very small compared to the other terms. Additionally, it has been assumed that the motion of the overlying water layer is not influenced by the mud layer. The pressure gradient at the bed caused by wave action is approximated by:

\[ \frac{\partial p}{\partial x} \approx -k \frac{-\rho_w g a}{\cosh kh} \cos(\omega t - kz) \]
(5.21)

where \( h \) is the water depth.

For the prediction of \( u(z, t) \), use is made of the thixotropic model presented in Chapter 3 with coefficients obtained from the rheological measurements. According to this model [116], the shear stress is given by:

\[ \tau_{zz} = \text{sgn} \left( \frac{\partial u}{\partial z} \right) \lambda \tau_y + \left( \eta_{\infty} + \frac{c_1}{1 + \beta \left| \frac{\partial u}{\partial z} \right|} \right) \frac{\partial u}{\partial z} \quad (|\tau_{zz}| > \lambda \tau_y) \]

\[ \frac{\partial u}{\partial z} = 0 \quad (|\tau_{zz}| \leq \lambda \tau_y) \]
(5.22)
where \( \lambda \) is a structure parameter, here decreasing with time from 1 to 0 as a result of structural break-up of the sediment caused by wave action [91]:

\[
\frac{d\lambda}{dt} = -c_3 \lambda \left| \frac{\partial u}{\partial z} \right|
\]  

(5.24)

Structural recovery is not taken into account for the present experiments. Values for the model coefficients \( \tau_y, \eta_{\infty}, c_1, \beta \) and \( c_3 \) for the artificial mud, which were determined independently from rheological experiments, are listed in Table 5.4.

Boundary conditions are:

\[
\begin{align*}
  z &= D_1 & u &= 0 \\
  z &= 0 & \frac{du}{dz} &= 0
\end{align*}
\]

(5.25)

Here \( z = D_1 \) is the level of the interface between non-liquefied and liquefied mud determined from observations and \( z = 0 \) is the interface between liquefied mud and water. The initial condition is:

\[
u(z, 0) = 0
\]

(5.26)

These equations, which describe the motion of the mud layer after liquefaction, were solved numerically.

### 5.3 Experimental procedure

The experimental set-up built to study the wave-induced liquefaction of an inclined mud bed is shown in Figure 5.1. For the first set of experiments China clay was used as an artificial mud, the properties of which are described in De Wit [131]. An additional experiment was made with natural mud dredged from the entrance of the Caland-Beer Channel, a navigation channel in the port of Rotterdam. Some properties of this mud are given in Table 5.1; its particle size distribution is shown in Figure 5.2.

For the first set of experiments, sediment beds were prepared by sedimentation and consolidation of a suspension of China clay in tap water in which 0.5% NaCl had been dissolved. The initial sediment concentration was 275 kg m\(^{-3}\). The suspension was mixed for 2 weeks in order to reach physicochemical equilibrium. China clay—mainly consisting of kaolinite—was used as an artificial mud because of its reproducible properties and easy handling. After 1 week of consolidation the bed height remained constant and the bed was tilted to its desired angle (0.05 rad). The bed height after consolidation was 0.12 m, its width 0.65 m and its length 4.67 m.

At measuring station 3 (Figure 5.1), four pore pressure transducers were mounted prior to consolidation—at \( z = 0, 0.02, 0.06 \) and 0.10 m from the bed.
5.3. EXPERIMENTAL PROCEDURE

![Diagram of experimental setup]

**Figure 5.1**: Experimental set-up (not to scale), lengths in m; measuring stations 1, 2, and 3 are shown

surface—to observe the liquefaction behaviour. The transducer at $z = 0$ cm was used as a reference. The sediment concentration profile in the bed was measured with a conductivity probe. Velocities both in the bed (after liquefaction) and above the bed were measured with two electromagnetic flow meters (EMF) mounted on traversing units to obtain vertical profiles. These traversing units were located at measuring stations 1, 2 and 3. Sediment concentrations in the water column were measured with turbidity meters in order to estimate the amount of interfacial mixing. Also three wave height meters were installed.

Details of these instruments are discussed in De Wit [131]. The signals of these instruments were logged onto a PC.

For the second experiment, with natural mud, some changes in set-up and instruments were made. As the amount of mud available was limited, the length of the sediment compartment was reduced to 3.15 m, and the bed height after consolidation was reduced to 0.10 m. Only two pore pressure transducers were used, which were installed—prior to consolidation—at measuring station 3 at 0.02 and 0.06 m from the bed surface after consolidation. Two out of three turbidity meters were replaced with instruments equipped with optical fibres, which make high-frequency in situ measurements possible [29]. No traversing unit was used at station 1 because of the shorter length of the mud compartment. All other instruments were the same as for the first set of experiments with artificial mud.

The experiments were started by generating waves, the amplitude of which was gradually increased. The wave period was set at 1.65 s. Only waves were used to load the sediment bed; current was absent. The wave conditions during the experiments are shown in Table 5.2.
Figure 5.2: Particle size distribution of mud dredged from the Caland-Beer Channel

Table 5.1: Properties of mud dredged from the Caland-Beer Channel

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>density of solids (kg m$^{-3}$)</td>
<td>2,467.1 ± 2.3</td>
</tr>
<tr>
<td>loss on ignition (LOI)</td>
<td>24.65 %</td>
</tr>
<tr>
<td>sodium adsorption ratio (SAR, mmol l$^{-1}$)</td>
<td>9.3</td>
</tr>
<tr>
<td>cation exchange capacity (CEC, cmol kg$^{-1}$)</td>
<td>21.7</td>
</tr>
</tbody>
</table>

Table 5.2: Wave conditions during liquefaction experiments

<table>
<thead>
<tr>
<th>natural mud</th>
<th>artificial mud</th>
</tr>
</thead>
<tbody>
<tr>
<td>time (s)</td>
<td>wave amplitude (m)</td>
</tr>
<tr>
<td>0-420</td>
<td>0.004</td>
</tr>
<tr>
<td>420-1,910</td>
<td>0.009</td>
</tr>
<tr>
<td>1,910-3,340</td>
<td>0.014</td>
</tr>
<tr>
<td>3,340-5,190</td>
<td>0.019</td>
</tr>
<tr>
<td>5,190-6,410</td>
<td>0.024</td>
</tr>
<tr>
<td>6,410-7,060</td>
<td>0.034</td>
</tr>
<tr>
<td>7,060-7,750</td>
<td>0.044</td>
</tr>
<tr>
<td>7,750-8,640</td>
<td>0.055</td>
</tr>
</tbody>
</table>
5.4 Results and discussion

5.4.1 Liquefaction

Artificial mud During the experiments with artificial mud, the wave amplitude was increased as indicated in Table 5.2. In Figure 5.3 the evolution of wave-averaged pore pressures in the bed during the experiment is presented. At the initial wave amplitude of $5 \times 10^{-3}$ m no changes in pore pressure are observed; the wave-induced stresses in the bed are still below the yield value. However, when the wave amplitude is increased to $a = 8 \times 10^{-3}$ m, an increase in pore pressure is observed, which is taken as evidence for the occurrence of liquefaction. This is consistent with Figure 5.4: at $a = 5 \times 10^{-3}$ m the wave-induced deviator stress is less than the yield strength at all depths, but at $a = 8 \times 10^{-3}$ m the deviator stress exceeds the yield stress in the upper 8 cm of the bed. The deviator stress was calculated from Section 5.2.1, the yield strength profile was measured with a sounding test (Chapter 4).

The maximum shear stress at the bed surface caused by the oscillatory motion of the waves at the onset of liquefaction can be estimated with $\tau_{max} = \frac{1}{2} \rho \omega f_w \bar{u}_0^2$, where $\bar{u}_0$ is the velocity amplitude near the bottom and $f_w$ is the friction factor. Using linear wave theory, $\bar{u}_0$ is estimated at $\omega a / \sinh kh = 0.04$ m s$^{-1}$ for $\omega = 3.8$ s$^{-1}$, $a = 8 \times 10^{-3}$ m, $k = 2.8$ m$^{-1}$ and $h = 0.3$ m. The friction factor is estimated
at $f_w = 0.07$, assuming the bottom roughness to be $10^{-3}$ m [49]. With these values $\tau_{\text{max}}$ becomes less than 0.1 Pa, which is an order of magnitude smaller than the shear stress generated by the pressure gradients and the yield strength of the bed (Figure 5.4). It is therefore concluded that the latter effect is dominant for the liquefaction process, as was remarked previously [106, 131].

The distinct pore pressure increase observed when the amplitude was increased from 5 to $8 \times 10^{-3}$ m is taken to signify the fact that the collapsing grain skeleton now is partially supported by the pore water. At $a = 1.0 \times 10^{-2}$ m the pore pressure suddenly collapses, as the liquefied mud is transported downslope and replaced by water which has a lower density. A layer of stronger mud then becomes exposed, which would flow only after an increase in wave amplitude to $2.7 \times 10^{-2}$ m.

After liquefaction and erosion of all overlying sediment, the pore pressure at a certain position should equal its value at the start of the experiment, being the hydrostatic pressure. For $z = 2$ cm this is true, however, for $z = 6$ cm and $z = 10$ cm pore pressure values after liquefaction and erosion are clearly smaller than their initial values. This is ascribed to the presence of excess pore pressures at the start of an experiment, i.e. a not fully consolidated bed.

The wave-averaged pore pressure relative to that at $z = 0$ equals $\rho g z$ if the overlying bed is totally liquefied but still present, and equals $\rho_w g z$ after all overlying sediment is eroded. Therefore, the maximal pore pressure change at a
5.4. RESULTS AND DISCUSSION

<table>
<thead>
<tr>
<th>$z$ (cm)</th>
<th>$(\Delta p)_{\text{max \ measured}}$ (Pa)</th>
<th>$(\Delta p)_{\text{max \ calculated}}$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$7 \times 10$</td>
<td>$7 \times 10$</td>
</tr>
<tr>
<td>6</td>
<td>$2.0 \times 10^2$</td>
<td>$2.0 \times 10^2$</td>
</tr>
<tr>
<td>10</td>
<td>$2.9 \times 10^2$</td>
<td>$3.3 \times 10^2$</td>
</tr>
</tbody>
</table>

Table 5.3: Measured and calculated maximal pressure changes

position can be estimated from $\Delta p_{\text{max}} = (\rho - \rho_a)gz$. The calculated values of $\Delta p_{\text{max}}$ agree well with the measured values from Figure 5.3 (Table 5.3), only at $z = 10$ cm a significant difference exists. This difference is caused by the thin layer of mud above $z = 10$ cm still present at the end of the experiment.

The erosion process could also be observed with velocity measurements. For each wave amplitude velocity profiles were measured both above the bed and in the bed. In Figure 5.5 profiles of velocity amplitude for wave amplitude $a = 1.0 \times 10^{-2}$ m are presented, which were taken just prior to erosion of the liquefied bed. In Figure 5.6 profiles are shown for $a = 3.6 \times 10^{-2}$ m, when all but the lowest few centimetres of the bed had been removed. The differences in Figure 5.6 between the profiles at positions 1, 2 and 3 are caused by the presence of a sill close to position 1. This sill had a height equal to the initial bed height (0.12 m) and was necessary to support the bed during consolidation and to create a smooth transition between flume bottom and sediment layer.

Slope stability One experiment with artificial mud was made without waves. The bed slope was gradually increased until the bed suddenly failed. As failure of the bed was not expected for the maximal slope achievable in the experimental set-up used, water overlying the sediment bed was removed after consolidation in order to increase the effect of gravity. Because of the very low permeability of the mud bed, the destabilizing effect of pore water flow after the removal of water is assumed to be negligible.

Failure occurred at a bed slope of 0.0867 rad at a level of $z = 0.09$ m. The yield strength at this level was calculated with a slip circle analysis [65], taking into account the sill at the lower end of the mud compartment. With $\rho = 1300$ kg m$^{-3}$, $\tau_y$ is estimated at 63 Pa, which is larger than $\tau_y = 35$ Pa at $z = 0.09$ m, as was determined with the sounding test (Figure 5.4).

An explanation for this difference may be the static nature of the slope stability test and the dynamic nature of the sounding test. After failure, the mud bed flowed downslope at once, within one or two seconds. This points to metastability at slopes milder than the slope of failure. As wave loading is also a dynamic
Figure 5.5: Average velocity amplitudes for $a = 1.0 \times 10^{-2} \text{ m}$

Figure 5.6: Average velocity amplitudes for $a = 3.6 \times 10^{-2} \text{ m}$
process, it is not astonishing that the sounding test gives a better estimation of the deviator stress at the onset of liquefaction than the slope stability test. The latter test may show a better agreement with a vane test, however.

**Natural mud** The experiment with natural mud showed no significantly different behaviour. Prior to liquefaction, the elastic deformation of the natural mud bed owing to the surface waves could be more clearly observed visually than for the artificial mud bed, as the strain amplitude was higher. This is in agreement with rheological measurements, which showed natural mud to behave still linearly elastic at higher strain amplitudes than artificial mud (Chapter 3).

Liquefaction was observed at \( a = 0.019 \text{ m} \). If the wave-induced deviator stress inside the bed at this amplitude is compared with the yield strength profile of the bed, the former exceeds the latter in the upper part of the bed, as is shown in Figure 5.7. At \( a = 0.014 \text{ m} \), when no liquefaction was yet observed, the deviator stress does not significantly exceed the yield strength, taking into account the moderate accuracy of the sounding test used to measure the bed strength profile, and the assumptions made to calculate the stress distribution. This confirms the statement made for artificial mud, i.e. that the wave conditions at which undrained failure of a cohesive sediment bed occurs can be well estimated if its yield strength profile is known, for example by means of a sounding test (Chapter 4).

Comparison of Figure 5.4 with Figure 5.7 reveals that the yield strength profile of the natural mud markedly resembles the yield strength profile of the artificial mud, whereas the sediment concentration in the bed of the former is substantially lower. This can be partially explained by the higher cohesion of natural mud, as a result of which consolidation stops at a higher void ratio. Also the more uniform strength profile of natural mud compared to that of artificial mud can be attributed to its higher cohesion.

The process of liquefaction of natural mud by oscillatory loading was also studied in a rheometer (Physica UDS-200). A sample from a mud bed that was prepared in an identical way as for the flume experiment, was inserted into the Couette geometry of the rheometer. By this action, the sample was disturbed; therefore only qualitative conclusions should be drawn from the experiment described here. During the experiment, an oscillatory stress was applied for 1800 s, after which the stress amplitude was increased. This procedure was repeated until the stress amplitude was reached at which the yield strength was exceeded and failure occurred.

In Figure 5.8 results from this experiment are displayed for two 1800 s-runs, one prior to liquefaction (\( \dot{\tau} = 0.2 \text{ Pa} \)) and the run in which liquefaction occurred (\( \dot{\tau} = 1 \text{ Pa} \)). The response at a stress amplitude of \( \dot{\tau} = 0.2 \text{ Pa} \) is clearly linear. In this region, the storage modulus \( G' \) is much higher than the loss modulus
CHAPTER 5. WAVE-INDUCED LIQUEFACTION AND FLOW

Figure 5.7: Yield strength, deviator stress and concentration of a bed of natural mud dredged from the Caland-Beer Channel, obtained from sedimentation of a suspension of 164 kg m\(^{-3}\)

\(G''\), pointing to predominantly elastic behaviour; the strain amplitude remains below 0.01. At a stress amplitude of 0.2 Pa, \(G'\) and \(G''\) remain nearly constant in time; the slight increase of \(G'\) may be explained by structural recovery after sample insertion. At a stress amplitude of 1 Pa, however, the material suddenly fails. \(G'\) decreases dramatically; the peak in \(G''\) upon failure is caused by the energy dissipation needed to break up the structure. After failure, \(G''\) decreases as the viscosity of the material is much lower now. The observed sudden loss of strength, and the undrained transition from predominantly elastic to predominantly viscous behaviour is characteristic of liquefaction.

5.4.2 Transport of fluid mud downslope

Artificial mud Results of the calculations according to §5.2.2 are shown in Figure 5.9 for China clay; the parameter values used in the calculations were determined independently from rheological experiments, and are not fitted values. The velocity amplitude \(\hat{u}\) increases with time as the structural parameter \(\lambda\) decreases in consequence of structural break-up. Calculated velocities are comparable with experimental values, which also indicated initial fluid mud flow velocities of only a few cm s\(^{-1}\). Unfortunately, the accuracy of the flow meters is low at these velocities, and therefore only a rough comparison is possible. The model
Figure 5.8: Storage modulus, loss modulus and strain amplitude of natural mud during cyclic loading with two stress amplitudes $\tau$; $f = 0.7$ Hz

shows that the combination of wave-loading and the shear-thinning behaviour of mud results in a decrease of its effective viscosity, which markedly enhances transport under the influence of net forces, such as slopes. In Figure 5.9 negative velocities do not occur as the difference between wave-induced shear stress and shear stress due to gravity and slope does not exceed the yield stress.

Even at the largest wave amplitude, $a = 0.042$ m, not all sediment was liquefied. After the experiments a layer of 0.02-0.03 m was still present in the upper part of the tilting flume, which result is not influenced by the sill at the lower end. This can be attributed to two factors. First, the yield strength close to the fixed bottom is the highest as the mud has been most compacted by the weight of the overlying sediment (Figure 5.4), and secondly, the (high) pressure gradients at the end of the experiments are much less effective, as the bed becomes thinner and thinner, which leads to lower shear stresses inside the bed.

Natural mud In Figure 5.10 sediment concentrations in the water column at 0.03, 0.05 and 0.08 m above the sediment bed are shown. Prior to liquefaction, concentrations are virtually zero. However, at $t = 3340$ s, when the wave amplitude is increased to 0.019 m, the sediment concentration suddenly increases. At the same moment—the onset of liquefaction—the wave-averaged pore pressure at 0.02 m below the bed surface shows a distinct increase (not shown herein).
Figure 5.9: Velocity profile $u(z,t)$ resulting from wave-induced liquefaction of China clay on a slope. $D_1 = 0.04$ m, $\rho = 1346$ kg m$^{-3}$, $\theta = 0.05$ rad, $h = 0.30$ m, $a = 10$ mm, $k = 2.49$ m$^{-1}$, $\omega = 3.93$ s$^{-1}$; rheological coefficients used are listed in Table 5.4.

Table 5.4: Calibrated values for China clay of the parameters in (5.22) based on Figure 3.8 and $\tau_y$ shown in Figure 5.4.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_y$</td>
<td>$3.4 + 235z$</td>
<td>Pa (z in m)</td>
</tr>
<tr>
<td>$\eta_\infty$</td>
<td>0.0066</td>
<td>Pa s</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.86</td>
<td>Pa s</td>
</tr>
<tr>
<td>$\beta$</td>
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<td>s</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$10^{-2}$</td>
<td>-</td>
</tr>
</tbody>
</table>
5.4 RESULTS AND DISCUSSION

![Graph of sediment concentration over time](image)

**Figure 5.10:** Evolution of sediment concentration in time at several positions above the bed; natural mud

The sediment concentration gradually increases as the wave amplitude is further increased; the concentration close to the bed is always the highest. Turbulence is generated in the wave boundary layer near the bed. In the upper part of the water column the turbulence intensity is much smaller, as no flow is present. This explains the limited exchange of sediment with the water layer above the boundary layer, where the concentration remains small. It is estimated that even at the end of the experiment, the amount of suspended sediment is less than 10% of the amount originally present in the consolidated bed. After liquefaction, visual observations showed that most of the sediment was transported downslope as fluid mud.

During the experiment, the velocity inside the bed at \( z = 0.05 \text{ m} \) below the interface with the water was measured continuously (Figure 5.11). The maximal velocity that is possible if the bed still behaves elastic is determined as follows. Assuming elastic behaviour to be possible for \( \gamma \leq 0.01 \), as was observed with rheological measurements, the maximal horizontal displacement \( \eta \) at \( z = 0.05 \text{ m} \) is \( \pm 5 \times 10^{-4} \text{ m} \). The displacement at this position can be expressed as \( \eta = \hat{\eta} \sin \omega t \), where \( \hat{\eta} = 5 \times 10^{-4} \text{ m} \). The horizontal velocity then is \( u_t = (d\eta/dt) = \omega \hat{\eta} \cos \omega t \), where \( \omega \hat{\eta} = \hat{u} \) is the velocity amplitude, estimated at \( 2 \times 10^{-3} \text{ m s}^{-1} \). Only velocity amplitudes well above this value are indicative of a liquefied bed. This lower boundary is shown in Figure 5.11, together with the upper boundary calculated with linear short wave theory, assuming the mud
Figure 5.11: Evolution of wave-averaged velocity and velocity amplitude inside bed consisting of natural mud; wave conditions are shown in Table 5.2. Note velocity amplitude scales on both vertical axes. Horizontal lines indicate velocity amplitudes of completely liquefied mud modelled as water.

bed to be replaced with water. At \( t = 3340 \text{ s} \), when liquefaction occurred for the first time, an increase in velocity amplitude is observed. The amplitude now does much exceed \( 2 \times 10^{-3} \text{ m s}^{-1} \), which confirms that the bed starts to liquefy. Although liquefaction starts, no mass flow is yet observed, which is confirmed by visual observations. At \( t = 6410 \text{ s} \), when the wave amplitude was increased to \( a = 0.034 \text{ m} \), sudden mass flow occurs and the bed is eroded. Both the velocity amplitude and the wave-averaged velocity suddenly increase as the mud is removed.

5.5 Conclusions

Undrained failure of freshly deposited mud layers will be caused by pressure gradients on the bed surface, if the resulting shear stresses inside the layer locally exceed the yield strength. If the yield strength profile is known, failure can be well predicted using an elastic model to calculate shear stresses in the mud layer. For loosely packed layers, which have not yet been subjected to much higher effective stresses than the actual effective stress, positive excess pore pressures are generated upon failure and liquefaction occurs. The strength of the layer is then much reduced, and a transition from predominantly elastic to predomi-
nantly viscous behaviour takes place. Mud displacements and velocities during liquefaction are very difficult to model, as the constitutive properties of mud then change rapidly and non-linearly in space and time.

After break-up of the grain matrix, fluid mud is formed. Modelling fluid mud flow is more feasible: constitutive properties are still dependent on shear rate and time, but sudden transitions like those during liquefaction do not occur. Flow of fluid mud generated by liquefaction may result in very high transport rates in a short period of time, as the concentration of the liquefied mud equals the original bed concentration (generally a few hundreds kg m\(^{-3}\)). This may partly explain the large mud accumulations in depressions of the bed and navigation channels, which are often observed after storm periods.

The combination of waves and a slope turns out to be very effective for the transport of fluid mud. Waves of sufficient amplitude cause liquefaction of mud layers formed from deposition and, after liquefaction, cause a decrease in effective viscosity of the mud because of its shear thinning behaviour. Subsequently, the slope provides the opportunity for net transport because of gravity.

The transport mechanism described in this chapter may therefore be important for the redistribution of cohesive sediments in depositional areas, where layers of freshly deposited, still consolidating mud are present. In erosional areas, however, where the sea bed has generally been exposed in the past to a much higher effective stress than the present effective stress—because it had previously been buried by sediment layers, for example—liquefaction is much less likely to occur. The sea bed in these areas tends to be overconsolidated and will have a high strength, reflecting its stress history. For those beds gradual, drained surface erosion will prevail.
Chapter 6

Steady flow of fluid mud on slopes

6.1 Introduction

In the previous chapter wave-induced liquefaction of a cohesive sediment bed and its subsequent onset of motion downslope were discussed. As a result of break-up of the static structure within the sediment, fluid mud is formed. In the present chapter\(^1\), the flow of fluid mud on a slope is discussed both from experimental and theoretical points of view.

Sediment particles in fluid mud may be supported by turbulence as in turbidity currents, or by interactions between particles and pore water under excess pressure, because of the low permeability of fluid mud. The former mechanism is possible only at sufficiently low concentrations; at high concentrations particle interactions cause the effective viscosity of the suspension to increase so that laminar flow conditions may prevail. The transition between these two regimes is dependent on the effective Reynolds number and is dealt with in §6.2.

Many experimental studies have been performed on turbidity currents, but mostly at low concentrations that allow the Boussinesq approximation and the assumption of Newtonian flow behaviour [95, 3]. Middleton [87] and Edwards [32] presented reviews on turbidity currents, while Brørs [11] elaborated Reynolds stress turbulence models for low concentration turbidity currents. Another Reynolds stress model concerning fine-sediment transport was presented by Teisson et al. [108]. Experiments with high sediment concentrations and laminar flow conditions have been performed by Ali and Georgiadis [1], Crapper and Ali [23] and Kusuda et al. [61, 62]. They generated gravity currents by the settling of a homogeneous suspension on an inclined bed. Coussot [20] studied the flow of concentrated mud on an inclined plane without the presence of an overlying water layer.

In this chapter the subaqueous flow of fluid mud on mild slopes is examined. The main purpose is to analyse, analytically as well as experimentally, the distinction between laminar and turbulent flows and the transition of one flow type

\(^1\) published previously in adapted form in *J. Hydr. Eng.* 122(12):710-717, 1996 under the title ‘Gravity current of fluid mud on sloping bed’ by Thijs van Kessel and C. Kranenburg
to another. The motive for this choice is that the type of flow highly affects bed friction and entrainment of overlying water. In addition, rates of entrainment of overlying water for turbulent flow and friction factors at the bottom of the suspension layer are obtained and compared with data from the literature. Under field conditions deposition of suspended sediment from the fluid mud layer and erosion of bed material may play a significant role. However, to single out the analysis of the flow type, these processes are not considered herein.

In §6.2 a laminar flow model using the Bingham plastic rheological model and a turbulent flow model are presented, and the laminar-turbulent transition is discussed. Sediment settling can be neglected, because for the experiments made the residence time of sediment in the measurement section of the flume was only short, about 30 s. With a settling velocity less than $10^{-4} \text{ m s}^{-1}$ due to hindered-settling ($C > 10 \text{ kg m}^{-3}$) this is a valid assumption. In §6.3 the experimental set-up and procedure are explained, together with the characteristics of the experiments. In §6.4 experimental results are presented, compared with analytical results and discussed; conclusions are drawn in §6.5.

6.2 Flow modelling

6.2.1 Laminar flow

For laminar flow the rheological behaviour of the suspension is modelled as a Bingham plastic and the flow is assumed to be steady and uniform. In this case, the pressure field is considered to be hydrostatic. The only shear stress that has to be taken into account then is the shear stress acting in planes parallel to the bed (simple shear). The equation of momentum in the $x$-direction for the steady and uniform flow of a gravity current is (Figure 6.1)

$$\frac{\partial \tau_{xz}}{\partial z} + (\rho - \rho_w) g \sin \theta = 0$$

(6.1)

where $\rho$ is the bulk density of the suspension, $\rho_w$ is the density of water, $u$ is the local velocity in the $x$-direction, $\tau_{xz}$ is the shear stress, $x$ is the co-ordinate parallel to the bed, $z$ is the co-ordinate normal to the bed, $g$ is the gravitational acceleration and $\theta$ the angle of inclination of the bed.

The interfacial shear stress $\tau_i$ between the mud layer and the overlying water can be estimated from $\tau_i = c_D \rho_w U^2$, where $U \approx 0.1 \text{ m s}^{-1}$ is the maximum velocity in the water layer and $c_D \approx 0.001$ is the drag coefficient in the experiments reported in §6.4. It follows that $\tau_i < 0.1 \text{ Pa}$, much smaller than the bottom shear stress as will be shown in §6.4.3, and $\tau_i$ can therefore be neglected. If $\tau_i$ is not negligible, then not taking into account $\tau_i$ will result in an overestimation of the flow velocity of the mud layer.
6.2. FLOW MODELLING

Neglecting the interfacial shear stress at the suspension-clear water interface \((z = H)\), where \(H\) is the depth of the suspension layer), and assuming a constant density \(\rho\) (see §6.4), integrating (6.1) gives

\[
\tau_{xz} = \left(1 - \frac{z}{H}\right)\tau_b
\]  

(6.2)

where \(\tau_b\) is the bed shear stress, which can be derived from (6.1) and (6.2),

\[
\tau_b = (\rho - \rho_w)gH\sin\theta
\]  

(6.3)

The shear stress is expressed as a function of the shear rate assuming a Bingham plastic rheological behaviour:

\[
\tau_{xz} = \tau_y \text{sgn} \left(\frac{\partial u}{\partial z}\right) + \mu \frac{\partial u}{\partial z}, \quad \text{if} \quad |\tau_{xz}| \geq \tau_y
\]  

\[
\mu \frac{\partial u}{\partial z} = 0, \quad \text{if} \quad |\tau_{xz}| < \tau_y
\]  

(6.4)

where \(\tau_y\) is the yield stress, \(\mu\) is the dynamic viscosity and \(\partial u/\partial z\) is the shear rate. The boundary where \(|\tau_{xz}| = \tau_y\) is the yield surface, above which \(|\tau_{xz}| < \tau_y\) the suspension moves as a solid plug, and below which flow occurs with deformation.

As was remarked in Chapter 3, the Bingham model is only suitable to describe the constitutive behaviour of China clay at shear rates exceeding approximately 20 s\(^{-1}\). For the experiments reported in this chapter, the maximal shear rate, which for laminar flow is estimated from \(\partial u/\partial z = (\tau_b - \tau_y)/\mu\), always exceeds 100 s\(^{-1}\). Use of the Bingham plastic model is therefore legitimate.
Table 6.1: Values of the coefficients used in the empirical relations (6.5) and (6.6) for China clay in water at 25 °C

<table>
<thead>
<tr>
<th>coefficient</th>
<th>Wan [122] values used in this chapter</th>
<th>De Wit [130]</th>
<th>units</th>
</tr>
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<td>968</td>
</tr>
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<td>3</td>
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</tr>
<tr>
<td>$c_3$</td>
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<td>206</td>
<td>933</td>
</tr>
<tr>
<td>$c_4$</td>
<td>1.68</td>
<td>1.68</td>
<td>1.98</td>
</tr>
</tbody>
</table>

From measurement on the rheology of China clay performed by De Wit [130] and Wan [122], the following empirical correlations for $\tau_y$ and $\mu$ as functions of the volumetric concentration of clay $N$ were derived:

$$\tau_y = c_1 N^{c_2}$$  \hspace{1cm} (6.5)

$$\mu = \mu_w (1 + c_3 N^{c_4})$$  \hspace{1cm} (6.6)

where $c_1$–$c_4$ are coefficients and $\mu_w$ is the viscosity of water. The volumetric concentration $N$ is related to the mass concentration $C$ by $N = C/\rho_s$, where $\rho_s$ is the density of clay ($\rho_s \approx 2,590$ kg m$^{-3}$ for China clay). $C$ can be calculated from $\rho$ according to

$$C = \rho_s \left( \frac{\rho - \rho_w}{\rho_s - \rho_w} \right)$$  \hspace{1cm} (6.7)

Approximate values of the coefficients $c_1$–$c_4$ are given in Table 6.1. Differences between the values from Wan [122] and De Wit [130] may be attributed to different experimental conditions. Wan [122] used tap water, whereas De Wit [130] used water with a salinity of 5 ppt. Also differences in clay properties and the rheological instruments used may be important. Of course, substituting $N = 0$ gives the values of $\tau_y$ and $\mu$ for pure water.

The yield stresses obtained from these data are rather high, as they are extrapolated from high shear rates using controlled shear rate rheometers. The yield stress of a material can be determined more accurately with controlled shear stress instruments. As some experiments of this type showed (Chapter 3), the yield stress of China clay is lower than expected from the controlled shear rate experiments performed by Wan [122] and De Wit [130]. Wan's results were adopted, but $c_1$ was reduced by a factor of 0.65. Rheological measurements on cohesive sediment suspensions are not easy to make [92]. Settling and wall slip may cause inaccuracies, so that the constants shown in Table 6.1 should be handled with caution.
6.2. FLOW MODELLING

Together with (6.4) the velocity profile can be calculated from (6.2), which gives

\[ u = \frac{\tau_b H}{2\mu} \left( 2\xi \frac{z}{H} - \left( \frac{z}{H} \right)^2 \right) \quad \text{if} \quad 0 < \frac{z}{H} < \xi \]

\[ u = u_p = \frac{\tau_b H}{2\mu} \xi^2 \quad \text{if} \quad \xi < \frac{z}{H} < 1 \]  

(6.8)

where \( \xi = h_0/H \), \( h_0 \) is the height of the yield surface and \( u_p \), the flow velocity of the solid plug above the plane \( z/H = \xi \). This result is similar to that of Liu and Mei [69]. The local flow rate is given by

\[ q = \int_0^H u \, dz = \frac{\tau_b H^2}{2\mu} \xi^2 \left( 1 - \frac{1}{3} \xi \right) \]  

(6.9)

The layer-averaged velocity \( U \) is given by \( U = q/H \). At the yield surface \( (z = h_0) \) the shear stress should be equal to the yield stress. From (6.2) it then follows that

\[ \xi = 1 - \frac{\tau_v}{\tau_b} \]  

(6.10)

Equations (6.4), (6.9) and (6.10) form a system from which the flow parameters \( H, \tau_b \) and \( \xi \) can be calculated for given \( q, \theta \) and \( \rho \). In the experiments, \( q, \theta \) and \( \rho \) were known a priori. In §6.4 experimental values for \( U \) and \( H \) are compared with calculated values. In a field situation \( q \) is usually unknown, however, when \( \theta, \rho, H \) and the rheological behaviour are known from measurements, \( q \) can be estimated.

The approximations in these equations are only valid when the flow is laminar and the density is constant. The influence of the overlying water is limited to the effect of reducing the effective gravitational force with a factor \( (\rho - \rho_w)/\rho \).

6.2.2 Laminar-turbulent transition

When the suspended sediment concentration is sufficiently low, the mud flow can become turbulent. The onset of turbulence in the flow of a Bingham plastic is often expressed in terms of an effective Reynolds number \( Re^e \) [69], defined by

\[ \frac{1}{Re^e} = \frac{1}{Re^\mu} + \frac{1}{Re^\tau} \]  

(6.11)

where

\[ Re^\mu = \frac{4\bar{\rho}UH}{\mu}; \quad Re^\tau = \frac{8\bar{\rho}U^2}{\tau_y} \]  

(6.12)
Here $\overline{\rho}$ is the layer-averaged density.

Equation (6.11) takes into account the influence of a yield stress on the laminar-turbulent transition. Using the analysis developed in §6.2.1, $Re^\mu$ and $Re^\tau$ can be expressed as functions of $\rho$ and $\theta$. Liu and Mei [69] have proposed the following, approximate criterion for transition from laminar to turbulent flow: $Re^\mu > (2 - 3) \times 10^3$. A more accurate criterion is given by Hanks [40]. However, as at high concentrations the effective Reynolds number is mainly determined by the 'cohesive' Reynolds number $Re^\tau$, its value becomes less accurate due to inaccuracies in the values for the Bingham yield stress. Therefore the criterion proposed by Liu and Mei [69] is sufficient in this case.

### 6.2.3 Turbulent flow

To avoid detailed turbulence modelling, an approach using layer-averaged quantities is more appropriate [33]. As opposed to laminar flow, turbulent flow is always non-uniform because of entrainment of overlying water. The integral momentum balance reads:

$$\frac{d\overline{\rho}U^2 H}{dx} + \tau_b - S_1(\overline{\rho} - \rho_w)gH \sin \theta + \frac{1}{2} \rho w \cos \theta \frac{dS_2(\overline{\rho} - \rho_w)H^2}{dx} = 0 \quad (6.13)$$

The total mass balance is

$$\frac{d\overline{\rho}UH}{dx} = \rho_w w_e \quad (6.14)$$

where $w_e$ is the entrainment velocity; the sediment mass balance is

$$\frac{d(\overline{\rho} - \rho_w)UH}{dx} = 0 \quad (6.15)$$

The bottom shear stress $\tau_b$ is assumed to be given by a quadratic friction law,

$$\tau_b = c_D \overline{\rho} U^2 \quad (6.16)$$

where $c_D$ is the drag coefficient. The shape factors $S_1$ and $S_2$ are given by

$$S_1(\overline{\rho} - \rho_w)gH = \int_0^\infty (\rho - \rho_w)g dz \quad (6.17)$$

$$S_2(\overline{\rho} - \rho_w)gH^2 = \int_0^\infty 2(\rho - \rho_w)gz dz \quad (6.18)$$

For laminar flow with constant $\rho$, these shape factors are equal to unity. The layer-averaged values $U$ and $\overline{\rho}$, and the equivalent layer depth $H$ are defined as follows [95]):

$$UH = \int_0^\infty u dz \quad (6.19)$$
6.3. EXPERIMENTAL SET-UP

\[ \bar{p} U H = \int_{0}^{\infty} \rho u dz \quad (6.20) \]

\[ \bar{p} U^2 H = \int_{0}^{\infty} \rho u^2 dz \quad (6.21) \]

The values of these integrals and the shape factors were evaluated using the measured concentration and velocity profiles. The only unknown in the momentum balance (6.13) now is the shear stress \( \tau_b \) or drag coefficient \( c_D \), which was calculated from the measurements.

Equations (6.14) and (6.15) can easily be combined to give:

\[ \frac{dU H}{dx} = E_w U \quad (6.22) \]

where \( E_w = \frac{u}{U} \) is a dimensionless entrainment rate. Following Ellison and Turner [33] and many others, this entrainment rate is assumed to be a function of the overall Richardson number \( Ri \), defined by

\[ Ri = \frac{(\bar{p} - \rho_w) g H \cos \theta}{\bar{p} U^2} \sim Fr^{-2} \quad (6.23) \]

There is no consensus about the exact form of the entrainment law [36]. \( E_w \) becomes independent of \( Ri \) at \( \tilde{Ri} < 0.5 \), but as \( \tilde{Ri} \) increases, several power-law dependencies have been observed. Various authors have proposed empirical relations [18, 2].

At low concentrations and therefore turbulent flow, the Richardson number is approximately independent of the concentration. This can be shown by neglecting the first and the last term on the left hand side of (6.13), i.e. gravity force and inertia. One obtains, using (6.16)

\[ Ri \approx \frac{c_D}{S_1 \tan \theta} \quad (6.24) \]

It is stressed that (6.24) is only an approximate equation, in which the influence of entrainment has been neglected. At high concentrations however, \( Ri \) is a function of the concentration, as can be derived from the laminar flow model proposed in §6.2.1. This is caused by the dependency of \( \mu \) and \( \tau_b \) on the sediment concentration.

6.3 Experimental set-up

Experiments were performed in a laboratory flume, 13.75 m long, 0.50 m wide and 0.72 m high. Sloping bottoms were installed in the flume, first a slope of 1:42.6 and a length of 8.75 m, and after that a slope of 1:18.5 and a length of 8.35
Suspending of China clay in tap water were used as artificial muds. Bulk densities ranged from 1050 to 1230 kg m\(^{-3}\). The suspensions were prepared in a mixing tank with a content of 3.5 m\(^3\). Mixing was performed by a rotating grid and a circulation pump. By opening a valve, the same pump could be used to feed the sediment to a small inflow compartment of the flume (see Figure 6.2), which was emptied beforehand using a vacuum tube. The flow rate was kept constant and measured with a Foxboro electromagnetic flow meter, which was suitable for sludge flows. Flow rates were about 4 \(\times 10^{-3}\) m\(^3\) s\(^{-1}\). The main part of the flume, which was filled with tap water, was separated from the inflow compartment by a movable weir. When the hydrostatic pressure of the suspension at the bottom of the inflow compartment was equal to that of the tap water in the flume, the weir was lifted to the desired height (\(\approx 5 \times 10^{-2}\) m). By doing so, a gravity current was generated for 300-600 s, until the supply of suspension in the mixing tank was nearly depleted. At the end of the flume the gravity current was caught in a deep compartment and drained off through an adjustable valve. The water level in the flume was kept constant by means of an overflow weir. Thus entrainment of water into the gravity current was compensated by supplying tap water. The drained suspension was stored in a large settling tank, from which it could be pumped back into the mixing tank for the next experiment.

Measurements were performed at two positions in the flume, at 1.27 m and 5.43 m from the weir. Velocities were measured with an electromagnetic velocity
6.3. EXPERIMENTAL SET-UP

<table>
<thead>
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<th>experiment number</th>
<th>initial bulk density (kg m(^{-3}))</th>
<th>bed slope ((\cdot))</th>
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<td>1229</td>
<td>1:18.5</td>
<td>4</td>
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</tbody>
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Table 6.2: Experimental conditions

meter, concentrations with an ultrasonic high-concentration meter (UHCM) developed by Delft Hydraulics. The accuracy of the electromagnetic velocity meters was determined by De Wit [129], who found that the operation of these instruments was unaffected at high sediment concentrations. As measurements closer than 2 cm from the bottom gave unreliable results, the instruments were not used at such positions. The specifications of the UHCM can be found in its manual [8]. Because only one UHCM was available, concentrations at the upstream position were obtained by sampling with a peristaltic pump and subsequent gravimetric analysis. The concentration measurements obtained with the UHCM agreed well with the concentrations obtained with the sampling method. The instruments were attached to automatic traversing units, by which the distance of the instruments from the bottom was adjusted. In this way, concentration and velocity profiles were obtained. The instruments were placed in the center-line of the flume. As the aspect ratio of the gravity current was about 0.1, side-wall effects were assumed to be of minor importance. Measurements started well after the head of the gravity current had passed, so that the flow could be assumed to be steady. Data were collected on a personal computer. An overview of the experimental conditions is given in Table 6.2.

Low concentrations resulted in overall Richardson numbers below unity and
therefore supercritical flow conditions, whereas high concentrations resulted in overall Richardson numbers greater than unity and therefore subcritical flow conditions. If the flow is subcritical, it will be affected by the downstream boundary condition. This could be observed at the end of the slope, where the mud layer depth increased in time to create a pressure gradient in the horizontal part of the flume to overcome friction. However, the second measuring position was located far enough upstream not to be affected by this backwater effect.

6.4 Results and discussion

6.4.1 Velocity and concentration profiles

Concentration and velocity profiles were obtained from the experiments. In Figure 6.3 and 6.4 results of two selected experiments are represented, one with a relatively low bulk density \( \rho = 1055 \text{ kg m}^{-3} \), and another with a relatively high bulk density \( \rho = 1200 \text{ kg m}^{-3} \). The low-concentration profile is smoothed by turbulent mixing, whereas the high-concentration profile shows a very strong velocity gradient near the upper interface. The velocity profile for \( \rho = 1200 \text{ kg m}^{-3} \) also shows a plug-like behaviour below the interface between the mud layer and clear water, which points to a significant yield strength within the mud, whereas the velocity profile for \( \rho = 1055 \text{ kg m}^{-3} \) is smoothed.

Figures 6.5 and 6.6 show a similarity collapse of bulk density and velocity profiles, normalized as:

\[
\eta = \frac{z}{H}, \quad \zeta_p = \frac{\rho(z) - \rho_w}{\bar{\rho}}, \quad \zeta_u = \frac{u(z)}{U}
\]  \hspace{1cm} (6.25)

For bulk densities of 1150 kg m\(^{-3}\) and below (Figure 6.5), these profiles are very similar to those of Parker et al [95], for example. The concentration gradually decreases from the bottom upward without any lutocline. There is a turbulent interface between the suspension layer and the overlying water (Figure 6.7). At bulk densities over 1150 kg m\(^{-3}\), however, a step in concentration exists (Figure 6.6). During the experiments this step could be visually observed as a distinct and smooth interface. A video image of this phenomenon is reproduced in Figure 6.8. At high concentrations also a partially plug-like velocity profile could be observed (Figures 6.4 and 6.6). This result is in agreement with the assumption of Bingham flow behaviour. It is noticeable that the velocity just above this interface, where the concentration is nearly zero, is nonzero due to drag forces. As shown below, this has consequences for the interpretation of the entrainment coefficient.
6.4. RESULTS AND DISCUSSION

Figure 6.3: Selected concentration profiles at position 1

Figure 6.4: Selected velocity profiles at position 1
Figure 6.5: Similarity collapse, low concentration

Figure 6.6: Similarity collapse, high concentration
Figure 6.7: Gravity current $\rho = 1100 \text{ kg m}^{-3}$; slope 1:42.6

Figure 6.8: Gravity current $\rho = 1230 \text{ kg m}^{-3}$; slope 1:42.6
6.4.2 Laminar flow versus turbulent flow

The observed velocity and density profiles, the observation of a clear interface between the suspension layer and the overlying water and the low entrainment rates at high concentrations suggest that the assumptions underlying the calculation of the laminar flow (§6.2.1) are correct for bulk densities \( > 1150 \text{ kg m}^{-3} \). In Figure 6.9 the theoretical bulk velocity \( U \) and the suspension layer height \( H \) are plotted as functions of the bulk density for the slope 1:42.6, together with experimental results. At high concentrations, at which the theoretical values for \( U \) and \( H \) are obtained from the laminar flow model proposed in §6.2.1 assuming uniform flow, the agreement is satisfactory. Discrepancies may be caused by experimental errors, the assumptions made when deriving the laminar flow model, the application of the Bingham plastic rheological model which only gives an approximate description of the actual rheological properties of mud, especially at low shear rates [120] and inaccuracies in the assumed rheological data, which are difficult to obtain. The agreement for the slope 1:18.5 is similar.

At low concentrations, at which the theoretical values for \( U \) and \( H \) are estimated using (6.23) and (6.24), the agreement with measured values shown in Figure 6.9 is also satisfactory. The drag coefficient \( c_D \) was given the value proposed by Brørs [11], \( c_D = 0.0035 \).

An estimate of the sediment concentration at which the laminar-turbulent transition takes place can be obtained from Figure 6.10, which confirms that...
Figure 6.10: Dependence of effective Reynolds number on volumetric concentration $N$; rheological coefficients used to be found in Table 6.1; transition range of $Re^e$ according to Liu and Mei [69]

the occurrence of laminar or turbulent flow is determined by the effective Reynolds number. Measured turbulence intensities (RMS $u$-velocities) and visual observations were used to determine if the flow was laminar or turbulent. The agreement between the transition criterion proposed by Liu and Mei [69] and the experimental results is good, the flow regimes of all experiments were properly predicted by the criterion $Re^{e_{cr}} \approx (2 - 3) \times 10^3$. With the analysis of §6.2, a theoretical effective Reynolds number could be calculated using theoretical values for $U$ and $H$ and a measured value for $\rho$. Values for $U$ and $H$ were calculated in the same way as for Figure 6.9 for both laminar and turbulent flows; also the rheological data used were identical. The agreement between this calculated Reynolds number and the effective Reynolds number based on measured $\rho$, $U$ and $H$ values and the same rheological data is satisfactory, as was to be expected from Figure 6.9. From Figure 6.10 it also follows that for these experiments the transition takes place at about $\rho \approx 1140 \text{ kg m}^{-3}$ ($N \approx 0.085$). For bulk densities over 1140 kg m$^{-3}$, the turbulence intensities are nearly zero. Of course, this transition is strongly dependent on the rheological behaviour of the material considered, which is embodied in the effective Reynolds number criterion.
6.4.3 Entrainment

From the experimental velocity- and concentration profiles, the layer-averaged values $U$, $\rho$ and $H$ can be calculated using Equations (6.19)–(6.21). Subsequently, the entrainment coefficient can be evaluated using (6.22). It is important to notice that, at constant velocity $U$, the entrainment coefficient will be approximately $1 \times 10^{-3}$ if in the experiments the difference in suspension layer heights at the two measuring positions is 0.5 cm. This is just below the resolution of the measurements ($\approx 1$ cm), so that entrainment coefficients close to $1 \times 10^{-3}$ and less are inaccurate. One should be cautious when integrating (6.17)–(6.21), as the way it is done affects the definition of the entrainment rate. When integration is performed up to the zero velocity level ($u = 0$), the entrainment rate includes the influence of the drag of water above the gravity current. When integration is performed up to the zero concentration level ($c = 0$), only interfacial mixing is taken into account. At low concentrations the levels of zero velocity and zero concentration nearly coincide (Figure 6.5); both ways of integration then give nearly the same results. At high concentrations, however, an appreciable amount of water is dragged along without being mixed. Integration is performed in these cases up to the level of zero concentration (the interface), so that only water that passes through the interface contributes to the calculated entrainment rate.

In principle the interfacial shear stress should now be taken into account, because the velocity gradient at the interface is nonzero. However, from Figure 6.6 it can be concluded that at high concentrations the velocity gradient in the sediment layer near the interface is small compared to that near the bottom. Therefore it is reasonable to take only the bottom shear stress into account. In these cases large velocity gradients can be observed just above the interface in the overlying water, where the effective viscosity is much smaller.

The entrainment can be predicted theoretically by considering the turbulent kinetic energy equation. When the entrainment model developed by Kranenburg and Winterwerp [56] is applied to the present experimental conditions, the following equation can be derived:

$$E_w \approx \frac{2cD}{4Ri - 1 + \sqrt{(4Ri - 1)^2 + 25cD(2 + 25cD)}}$$  (6.26)

This result is compared to the experimental results in Figure 6.11. Entrainment rates below $1 \times 10^{-3}$ could not be measured properly due to experimental limitations. No entrainment rates above this value were observed for $Ri > 2$. Because these results cover a rather limited range, experimental results concerning gravity currents obtained by other authors using salt water and fresh water have also been plotted. The agreement between the entrainment model of Kranenburg and Winterwerp [56] and the present measurements is fair, considering the
experimental errors that may have been present. The results indicate that if the fluid mud is in a fully fluidized state, i.e. there is no interconnected aggregate structure throughout the mud, the entrainment process is the same as in salt water/fresh water systems. However, more accurate measurements are needed; measuring entrainment rates, though, was not the primary goal of this work.

High bulk Richardson numbers occurred in experiments with high suspension concentrations. This is shown in Figure 6.12 for the slope 1:42.6, together with the theoretically predicted values of $Ri$ based on laminar Bingham flow for $\rho > 1140 \text{ kg m}^{-3}$ and turbulent flow for $\rho < 1140 \text{ kg m}^{-3}$, equivalent with Figures 6.9 and 6.10. The agreement is good, also for the slope 1:18.6 (not shown). It can therefore be concluded that at high concentrations interfacial mixing is negligible.

6.4.4 Drag coefficient

The drag coefficient was calculated from the momentum balance (6.13). Values for $U$, $\rho$ and $H$ were obtained by integrating (6.19)–(6.21) from the bottom to the interface ($z = H$), or, at lower concentrations, up to the level where $c \approx 0.1c_{\text{max}}$. The results are plotted in Figure 6.13 as a function of the effective Reynolds number. At high effective Reynolds numbers, the drag coefficient seems to approach a constant value, as was to be expected. The value obtained by Brørs [11] ($c_D = 0.0035$) is a good approximation. At lower Reynolds numbers, where the
flow is laminar, the agreement between the experimental and theoretical values is also fair. Transition between a constant and $Re^\theta$-dependent drag coefficient takes place near $Re_{\text{crit}}^\theta \approx (2 - 3) \times 10^3$, which is in agreement with the laminar-turbulent transition discussed in §6.2.2. The drag coefficient can be calculated theoretically from (6.9) and (6.16):

$$c_D = \frac{8}{Re^\mu \left( \xi^2 - \frac{1}{3} \xi^3 \right)} \quad (6.27)$$

where $\xi$ is calculated from

$$\frac{Re^\mu}{Re^\tau} = \frac{1 - \xi}{\left( \xi^2 - \frac{1}{3} \xi^3 \right)} \quad (6.28)$$

The quotient of $(Re^\mu)^2$ and $2Re^\tau$ is often designated as the Hedström number.

### 6.5 Conclusions

Experiments have been performed on gravity currents down an incline, which consisted of suspensions of China clay at various concentrations. At bulk densities over approximately 1160 kg m$^{-3}$, a distinct interface was observed between the suspension layer and the overlying water. At lower bulk densities the transition was more gradual. This change in behaviour coincides with the turbulent-laminar transition. This transition could be well predicted with a criterion in
6.5. **CONCLUSIONS**

![Graph showing drag coefficient as a function of the effective Reynolds number](image)

**Figure 6.13:** *Drag coefficient as a function of the effective Reynolds number*

Terms of the effective Reynolds number, proposed in the literature. The measurements confirmed that the laminar-turbulent transition takes place at about $Re^e \approx (2 - 3) \times 10^3$.

Using analytical models, flow parameters such as $H$, $U$, $Re^e$ and $Ri$ could be satisfactorily predicted for both laminar and turbulent flow. Some discrepancies for the laminar case can be attributed to the application of the Bingham plastic rheological model, which only gives an approximate description of the actual rheological properties of mud, and to the values assigned to its parameters $\tau_y$ and $\mu$ as functions of the volumetric sediment concentration.

The Richardson number was found to be only a weak function of the bulk density for turbulent flow (its mean value being about 0.2), but is strongly dependent on the concentration for laminar flow. Mixing is of minor importance in laminar flow. Although experimental entrainment rates in turbulent flow showed considerable scatter, they seem to be in line with data from the literature on the entrainment of salt water/fresh water.

The bottom friction drag coefficient $c_D$ could be satisfactorily predicted. At high Reynolds numbers, $c_D$ seems to be a constant close to 0.0035, which value is based on calculations by Brørs [11]. Under laminar conditions its dependency on $Re^e$ could be modelled using the Bingham plastic analytical model. It should be taken into account that $c_D$ is not solely dependent on the Reynolds number anymore, but also on $\xi$ (6.27). The transition of $c_D$ from $Re^e$-dependent to $Re^e$-independent takes place near $Re^e = 2 \times 10^3$, in agreement with the laminar-
turbulent transition.
Chapter 7

Implications for field situations

7.1 Introduction

In the previous chapters the strength evolution of deposited mud layers, the liquefaction of these layers by waves, the rheology of fluid mud and finally the transport of fluid mud down an incline were all discussed in detail. This chapter aims to discuss the implications of the above-mentioned processes for the transport of cohesive sediments in field conditions. As fluid mud has a concentration of up to a few hundred kg m$^{-3}$, high transport rates are to be expected when it is displaced. In addition to the laboratory experiments described previously, some field observations will also be used, as the field is our best laboratory—unfortunately sometimes difficult of access and with conditions that cannot be controlled.

Some field observations are discussed in the next section. It is by no means intended to present a comprehensive overview of field experiments; just a few observations are shown for illustration. Section 7.3 gives an estimate of the wave-induced stresses inside a sediment bed under specific field conditions. In Section 7.4 recommendations are made for improving cohesive sediment transport models. Finally, conclusions are drawn in Section 7.5.

7.2 Field observations

Fluid mud is observed in many estuaries around the world [31]. In these areas, its occurrence is the rule rather than the exception.

Echo sounding is a suitable technique to detect fluid mud layers. A sound with a frequency of 33 kHz tends to be reflected at the sea bed, whereas a sound with a frequency of 210 kHz is reflected at the lutocline between water and fluid mud. This is illustrated in Figure 7.1, where an echo sounding of the access channel of the Barito river, Kalimantan, is displayed, together with a vertical density profile. A layer of fluid mud with a density of approximately 1,150 kg m$^{-3}$ is clearly observed in the access channel. Figure 7.1 suggests that on the western 1:50 slope of the navigation channel, a fluid mud layer with a thickness of
approximately 1 m is present. This layer will slide downwards if its strength does not exceed 1 kPa, approximately. For unconsolidated layers, in which effective stresses are ill-developed, this condition is likely to be fulfilled.

A disadvantage of the observation of fluid mud by echo sounding is that the 210 kHz echo is only reflected if the concentration gradient at the interface between fluid mud and water is sufficiently large. When the interface is destabilized, for example as a result of breaking internal waves or turbulence caused by tidal forces, mixing occurs and the concentration gradient at the interface decreases. Fluid mud may then not be visible anymore, although it may still be present. In addition, the location of the 33 kHz echo with respect to the density profile is not unambiguous. Therefore it is preferable to measure vertical sediment concentration profiles in addition to an echo sounding survey. Especially near-bottom concentrations are important, as it is here that fluid mud layers may be present.

Strength profiles are equally important, as they are essential to estimate whether undrained failure of mud layers caused by hydrodynamic or gravity forces is likely to occur. Whereas concentration profiles are frequently measured, reports about strength profiles are scarce. For example, if the strength profile of the fluid mud in the case of Figure 7.1 would have been known, a better assessment of the likelihood of mud sliding could be made. In Figure 7.2 the strength profile is shown of a fluid mud layer at the port of Zeebrugge. Note that the yield strength at a level of more than 1 m below the fluid mud surface is below 50 Pa, which points to low effective stresses and an unconsolidated state.

In Figure 7.3 a plot is displayed of the bathymetric changes observed in several navigation channels of Kumamoto harbour, Japan [52]. It is clear that
nearby all changes occur during two sedimentation events, which were triggered by storms. Sudden changes in bed level of nearly one meter within a few hours cannot be explained by sedimentation from the water column; however, they can be explained by the sliding of fluid mud layers generated by waves into the navigation channel. In this way sediment that has accumulated in a large area during calm weather may, when mobilized by storms, be collected into trenches. This mechanism was also discussed by Teisson [107], without which he was not able to properly model the siltation of the fairway of St. Nazaire harbour.

A similar event was observed on the Amazon continental shelf (Figure 7.4): a bed level increase of 0.5 m within 12 hours was recorded [13]. Note that the vertical axis of Figure 7.4 represents the distance from the sensor to the sea floor, i.e. a sudden decrease in distance means a sudden increase in bed level. Bacchione et al. [13] attributed the observed sudden accumulation to the downslope migration of fluid mud. It is estimated that more than 90% of the suspended sediment on the Amazon continental shelf is present as fluid mud [53].

'Events' as presented here are difficult to observe from survey vessels, as they mostly occur during storm conditions and in a short period of time. Measurements in storm conditions are notoriously difficult or expensive, as a result of which they are rare. Therefore, observations of sedimentation events are rare too, which does not mean that the events themselves are rare or that their impact on bathymetry is small.

One well-monitored event is a gravity current in the Haringvliet, the Nether-
Figure 7.3: Time variation of deposition heights in three trenches in Kumamoto harbour [52]

Figure 7.4: Bed level changes on the Amazon continental shelf [13]
Figure 7.5: Gravity current of fluid mud in Haringvliet, the Netherlands [9]

lands, which was artificially triggered by water injection dredging. The purpose of this operation was to investigate the feasibility of subaqueous transport of dredged material from dredging location to dump site through a natural, sloping connecting channel. Although no rheological data are available, the flow is assumed to be turbulent, as an excess density of only 35 kg m\(^{-3}\) is insufficient to affect viscosity and yield strength very much. Anyhow, the yield strength has to be well below \(\sim 0.3\) Pa, otherwise no flow at all would occur at the mild slope of approximately \(2 \times 10^{-3}\).

Density and velocity profiles measured in this channel are shown in Figure 7.5. Calculating the current velocity by equating (6.23) to (6.24) and assuming \(c_D = 5 \times 10^{-3}\) and, on the basis of Figure 7.5, \(H = 0.5\) m and \(\Delta \rho = 35\) kg m\(^{-3}\), gives \(U = 0.26\) m s\(^{-1}\), which is in good agreement with the observations.

The bulk Richardson number is estimated at approximately 3, leading to an entrainment rate \(E_w \approx 4 \times 10^{-4}\) (6.26). As the length of the connecting channel
was only $10^3$ m and entrainment was counteracted by settling, just over 50% of
the dredged material reached the dump site. Would the connecting channel have
been significantly longer, a less favourable result might have been obtained i.e.
most of the sediment would not have reached the dump site.

The excess density of the gravity current was relatively low as a result of
the dredging method, i.e. injection of water, which forced drainage. A gravity
current triggered by (undrained) liquefaction of a mud layer by waves may have
a substantially higher excess density. As a result, its bulk Richardson number
will increase (Figure 6.12), and mud may be transported over a much larger dis-
tance without being mixed with water. Depending on its rheological properties,
laminar flow conditions may now prevail.

It is important to notice that the dominant erosion and deposition mech-
nisms on tidal flats are significantly different from the mechanisms in navigation
channels. The difference in forcing between quiet and storm periods is much
less on tidal flats, for breaking waves more often occur in very shallow water.
Another main difference is that these flats dry once or twice a day, enhancing
consolidation. Moreover, the upper sediment layer is affected by biological ac-
tivity. As a result, accumulation of unconsolidated mud layers of a significant
height that are prone to liquefaction is unlikely on tidal flats. Gradual rather
than sudden bed level changes are therefore to be expected at these locations.

7.3 Wave-induced stresses under field conditions

In §5.2.1 the wave-induced stresses were calculated for a thin layer of mud overly-
ing a rigid bed—a calculation specific to the laboratory experiments performed.
This model is applicable to field situations in which a thin layer of soft mud is
overlying a stiff, overconsolidated sea bed. For a homogeneous elastic bed of infi-
nite thickness, a model proposed by Mallard and Dalrymple [73] is more suitable.
In their model, the influence of the deformable bed on wave propagation is taken
into account. This results in an adapted dispersion relation:

$$\omega^2 = g \frac{\frac{F'}{F' - \tanh kh} - 1}{F' \tanh kh}$$

(7.1)

where $h$ is the water depth and

$$F' = \frac{Gk^2}{\rho \omega^2 (1 - \nu)}$$

(7.2)

in which $\nu$ is the Poisson ratio. The maximal deviator stress generated by waves
inside an elastic bed is expressed as

$$\hat{\tau}_{max} = \hat{\rho}k z \exp(-kz)$$

(7.3)
7.3. WAVE-INDUCED STRESSES UNDER FIELD CONDITIONS

where

\[
\hat{p} = \frac{\rho g a}{\cosh kh} \left[ \frac{F'}{F' - \tanh kh} \right]
\]  

(7.4)

From this model it follows that the pressure gradients exerted on a deformable bed tend to be higher than those on a rigid bed, enhancing failure.

Assuming a soft mud bed to have a shear modulus of \(10^5\) Pa, \(T = 6\) s and \(h = 15\) m, the pressure gradients show an increase of 53% compared to the rigid bed solution. For an even softer layer, strains become much larger than 1% under these conditions and a viscous rather than an elastic response is to be expected. In these situations the Mallard and Dalrymple model [73] does not apply.

From Chapter 4 it is conjectured that the yield strength of fluid mud generally does not exceed \(10^2\) Pa and is more or less evenly distributed across the layer. This is in agreement with Figure 7.2, at least regarding the first observation. Effective stress is negligible, as sediment particles are pore water supported. As a result of consolidation, effective stresses do develop and the yield strength strongly increases. The yield strength now also shows a clear increase with effective stress and therefore depth. Based on this relationship, a yield stress profile can be estimated for beds with a thickness much larger than that of beds tested in the laboratory.

In Figure 7.6 the thus estimated yield strength profile and the maximal deviator stresses generated inside a stiff, consolidated mud bed under various wave conditions are displayed, calculated with the model of Mallard and Dalrymple [73]. From this Figure it is clear that for this type of bed, undrained failure is only to be expected under extreme wave conditions. As the increase in yield strength with depth generally amounts to more than 1 kPa m\(^{-1}\) for fully consolidated mud layers, it is concluded that undrained failure may occur only in the upper few meters of these layers. If the bed is significantly overconsolidated, failure is unlikely altogether. These beds can only be eroded gradually under drained conditions, allowing the dissipation of negative excess pore pressures. Contrary to fully consolidated mud layers, fluid mud and partially consolidated mud is easily affected by waves, resulting in a (further) loss of strength. It may subsequently be transported by waves (Stokes' drift), current and gravity force. The transport rate will be strongly dependent on the rheological properties of the mud.

Figure 7.6 also shows, for one wave condition, the deviator stress calculated with the model presented in §5.2.1 assuming the bed thickness to be 1 m. The deviator stress thus calculated does not deviate much from the value calculated with the Mallard and Dalrymple model [73]. The only important difference is the deviator stress at the bed surface: it is non-zero for the former model (finite bed thickness) and zero for the latter model (infinite bed thickness).

In areas of erosion, where sediment at the bed surface tends to be significantly
overconsolidated because of the weight of previously eroded layers, the likelihood of liquefaction is small, whereas in areas of deposition, where fresh deposits are present, it is high, depending on the water depth. If the water is very deep, waves generate a negligible stress inside the bed, even during severe storms. However, if the water is very shallow, wave breaking will occur already under moderate wave conditions; sediment is left little time to accumulate. At intermediate depths, only storm waves will generate a significant shear stress, and liquefaction of sediments deposited under normal conditions is likely. During storms the average wavelength and period tend to increase, as a result of which the influence of the waves reaches deeper. These considerations apply to wave dominated areas; in tide dominated areas liquefaction is improbable and surface erosion is dominant.

7.4 Modelling cohesive sediment transport

Based on the previous observations, it is conjectured that a good cohesive sediment transport model consists of at least three parts: a suspended sediment module, a fluid mud module and a bottom module (Figure 7.7).
7.4. MODELLING COHESIVE SEDIMENT TRANSPORT

Figure 7.7: Cohesive sediment transport modules

water module  In the hydrodynamic module processes—as already described in Chapter 2—such as flocculation, (hindered) settling, stratification, turbulence damping etcetera should be included. In most 'classical' cohesive sediment transport models these processes are indeed included.

fluid mud module  This module is intended to describe the transport of fluid mud. It should at least include:

- generation of fluid mud either by deposition of particles from the water column or by liquefaction of (partially) consolidated layers;
- transport caused by gravity and hydrodynamic forces;
- disappearance of fluid mud either by entrainment of particles at the interface with water or by consolidation as a result of which fluid mud becomes solid.

Material properties such as strength, viscosity and permeability have to be known in detail in order to make accurate modelling possible.

bottom module  Although no sediment transport occurs in this layer, it is still taken into account, as it acts as a source of suspended sediment or may become fluid again. By bookkeeping its stress history, the onset of liquefaction can be estimated. Drained and undrained erosion should be well distinguished. The former proceeds slowly and generally leads to suspended sediment transport, whereas the latter leads to the sudden formation of fluid mud. Consolidation (either negative or positive) as a result of oscillatory shear (strain softening and hardening) should be included in this module. Linking the state of consolidation to the evolution of strength is a major challenge.

The boundaries between these three states are not very strict; especially the transition between fluid mud and (soft) bottom is difficult to capture. For example, it can be defined as the level at which effective stresses are not negligible anymore compared to yield strength. The transition between fluid mud and
suspended sediment is not straightforward either. Fluid mud may be defined as a suspension in which the volumetric concentration of fractal aggregates equals or is close to unity (Chapter 2). Note that the volumetric particle concentration is much less than unity. In fluid mud, sediment is supported by upward intergranular flow rather than by turbulence.

It is important to model the exchange processes between the modules properly. As the modules are interdependent, an iteration procedure will be necessary to solve the complete model. For example, if fluid mud is present, wave damping will occur and the bottom shear stress tends to decrease.

A cohesive sediment transport model consisting only of a suspended sediment module can only be successfully applied in areas where deposited sediment remains at its location of deposition and is not remobilized and subsequently transported elsewhere. But even if a full 3-D modular approach is adopted, the predictive value of the models may be limited, as the material properties are not exactly known. Nevertheless, these models are valuable as a tool to get insight into dominant transport mechanisms.

7.5 Conclusions

In order to improve cohesive sediment transport modelling, attention must be paid to processes near the bed and inside. The transition from fluid into solid and vice versa should be included, as was touched upon in the previous section.

In view of this, measuring near-bed concentration profiles should not be neglected during field surveys. Although fluid mud displacements are difficult to monitor due to instrumental limitations and because they mostly occur during storm periods, the sensitivity of deposits to liquefaction can be well assessed in calm weather by measuring strength profiles. Sediment layers with a high liquefaction potential are most likely to be found in areas where sedimentation prevails and with an intermediate depth—where under normal circumstances hydrodynamic forces are low, but under storm conditions forces are high.

In this way, an estimate is obtained of the amount of cohesive sediment present in the area under consideration that may be suddenly mobilized during storms and transported into trenches, for example. In this respect, it is also important to measure the rheological properties of mud. As they are strongly dependent on the shear rate and history, these should be accurately indicated. Reporting the viscosity of fluid mud without such specifications—as unfortunately is often done—is utterly useless and should be avoided.
Chapter 8

Conclusions and Recommendations

8.1 Conclusions

From this study, the following conclusions are drawn:

Rheology

• The rheological properties of fluid mud cannot be characterized completely with an equilibrium flow curve. Additional experiments are necessary to quantify its time-dependent properties. It is important to make sure that equilibrium is actually achieved during equilibrium flow curve measurements.

• As fluid mud is strongly shear-thinning, especially at shear rates below ±10 s\(^{-1}\), modelling fluid mud flow assuming a constant effective viscosity leads to an overestimation of transport if small hydrodynamic forces are present, and to an underestimation if forces are large. No accurate modelling is then possible.

• During rheological experiments, undrained behaviour is assumed, i.e. pore water flow is neglected and a one-phase approximation is adopted. However, at large time-scales pore water flow does occur, leading to consolidation effects. A one-phase approximation then leads to erroneous results.

Strength

• The yield strength distribution of a cohesive sediment layer can be measured reasonably accurately using a miniature sounding test. In a freshly consolidated layer, yield stress increases approximately linearly with effective stress and depth. Measurement of yield stress during consolidation shows that strength develops from the bottom upward. By decreasing the penetration speed of the measuring device, a transition from undrained to drained behaviour is observed.
• Undrained yield strength depends on the time-scale considered. For a fast loading it tends to be higher than for a slow loading.

• Because of the more cohesive properties of the natural muds used in this study compared to the artificial mud used (China clay), their strength is substantially larger at equal particle fractions. However, this effect is largely counterbalanced by consolidation, which for artificial mud proceeds to a higher particle fraction.

Liquefaction

• Freshly consolidated mud has a tenuous card-house structure in which positive excess pore water pressure is generated upon failure. The shear stresses inside the bed generated by waves during storms are sufficiently high for failure to be possible.

• However, for compacted, overconsolidated layers this type of behaviour is unlikely, or even impossible if negative excess pore pressures are generated upon failure. Overconsolidation develops for example as a result of creep, long-term loading or erosion of overlying sediment layers.

• Undrained failure occurs when the yield strength of the material is exceeded. A sudden loss in strength is observed without any change in concentration, and concurrently a transition takes place from elastic to viscous behaviour. This phenomenon, which is also clearly observed during oscillation experiments by increasing the strain or stress amplitude, is referred to as 'liquefaction'. The evolution of this process can be monitored effectively with pore pressure transducers.

• The onset of liquefaction can be well predicted by combining an analytical mathematical model to calculate wave-induced shear stresses inside a mud layer with the measured yield strength profile of this layer.

Transport

• Upon liquefaction fluid mud is generated, in which effective stress is low compared to excess pore pressure. Notwithstanding this, fluid mud has a small but finite yield strength. Sediment particles are predominantly supported by pore water. The properties of fluid mud are much more homogeneous than those of consolidated mud. Fluid mud can also be generated if the suspended sediment flux towards the sea bed exceeds the particle settling flux in fluid mud.
8.2. RECOMMENDATIONS

- When fluid mud is displaced by waves, current or gravity, high transport rates are to be expected in view of its high sediment concentration, which can be as high as a few hundred kg m\(^{-3}\).

- Transport may be laminar due to a high sediment concentration, which equals the original bed concentration in case of undrained failure. This can be assessed with a transition criterion based on an effective Reynolds number, taking into account the static contribution of the yield stress.

- Laminar motion of fluid mud cannot be modelled accurately without a thorough knowledge of its rheological properties. A mathematical flow model for transport of fluid mud on slopes under waves, which includes these properties, shows that waves can significantly enhance transport, as they cause the effective viscosity of mud to decrease and the yield strength to be exceeded earlier on milder slopes. The contribution of waves to transport is strongly non-linear.

- Although fluid mud is bound to disappear as a result of consolidation, it may be quite persistent due to its low permeability.

- Large sediment displacements in a short period of time are possible under extreme conditions. Sediment that has been accumulating on the sea bed during calm weather may be suddenly mobilized and redistributed, particularly in trenches because of gravity.

8.2 Recommendations

- The balance of pore pressure generation by shear and pore pressure dissipation by consolidation determines whether the strength of the material will increase or decrease. More attention should therefore be paid to consolidation under oscillatory loading, by varying the stress amplitude of loading on a large time-scale.

- Even if all physical processes were perfectly known, predictions of cohesive sediment transport would still have a limited validity in view of the many non-linear interactions inhibiting long-term forecasts. It is worth investigating what is maximally achievable in this respect.

- For cohesive sediment transport models to be successful, it is important to incorporate fluid mud dynamics. The possibility of a sudden mobilization of previously deposited mud layers should also be included. In order to achieve this, a bottom module for bookkeeping the effective stress history may be essential.
• Determination of bed strength profiles during field surveys should be given more priority. In this way a better estimation can be made with regard to the amount of sediment that may be mobilized and redistributed in storm conditions.

• Bottom and fluid mud modules should be included into cohesive sediment transport models. With respect to this it is desirable to establish a relation between consolidation and evolution of strength.
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List of Symbols

Roman symbols

\( a \)  acceleration, amplitude \\
\( a_i \)  model constant \\
\( A \)  area \\
\( A_2 \)  \( = (\partial / \partial t)A_1 \) \\
\( A_1 \)  \( = 2D \) \\
\( b \)  model constant \\
\( B \)  breadth \\
\( c \)  cohesion, constant \\
\( c_i \)  coefficient \\
\( C \)  concentration \\
\( d \)  diameter \\
\( D \)  fractal dimension, diameter, diffusion coefficient, thickness, deposition rate \\
\( D_e \)  symmetric rate of deformation tensor \\
\( E \)  Deborah number \\
\( e \)  energy, void ratio \\
\( E \)  surface erosion rate \\
\( E_w = w_e/U \)  dimensionless entrainment rate \\
\( f \)  frequency \\
\( f_w \)  friction faction \\
\( F_o \)  Fourier number \\
\( F_r \)  internal Froude number \\
\( g \)  gravity \\
\( g \)  gravity \\
\( G \)  shear modulus \\
\( G' \)  storage modulus \\
\( G'' \)  loss modulus \\
\( h \)  depth, height, thickness \\
\( H \)  depth, height \\
\( H_e \)  Hedström number \\
\( k \)  permeability, wave number \\
\( l \)  height, length
LIST OF SYMBOLS

$L$
flow behaviour index
$m$
flow behaviour index
$m_e$
erosion constant
$m_v$
soil compressibility
$N$
volumetric concentration
$N_c$, $N_q$, $N_γ$
bearing capacity factors
$O_s$
perimeter length
$p$
pressure
$q$
the local flow rate
$q_f$
bearing capacity
$r$
radius
$Re^e$
effective Reynolds number
$Re^μ$
viscous part of $Re^e$
$Re^π$
plastic part of $Re^e$, ‘cohesive’ Reynolds number
$R_i$
Richardson number
$s$
specific surface area
$s_c$, $s_q$, $s_γ$
shape factors
$S_1$, $S_2$
shape factors
$t$
time
$T$
torque, period
$u = \Omega r_c$
rotation speed
$u$
local velocity in $x$-direction
$u_x$
soil displacement in $x$ direction
$\partial u/\partial z$
velocity gradient
$U$
layer-averaged velocity
$v$
velocity
$\mathbf{v}$
velocity vector
$w_e$
entrainment velocity
$w_z$
soil displacement in $z$ direction
$x$
coordinate parallel to the bed
$x_{i,j}$
cartesian coordinates
$Y_G$
gravity yield group
$z$
coordinate normal to the bed

Greek symbols

$\alpha$
shaft adhesion factor
$\beta$
rheological coefficient
$\gamma$
strain, specific volumetric weight
$\dot{\gamma}$
 shear rate
LIST OF SYMBOLS

δ  loss angle  
Δρ  excess density  
ΔWs  vertical force  
ζρ, ζu, η  dimensionless variables  
η  viscosity, displacement  
θ  angle  
λ  strucutral parameter  
μ  dynamic viscosity  
ξ  = h₀/H  
ρ  density  
σ  stress  
σ'  effective stress  
σ  stress tensor  
τ  scalar shear stress  
τ  deviatoric part of the stress tensor  
φ  volume fraction, angle of internal friction  
ψ₁  first normal stress coefficient  
ψ₂  second normal stress coefficient  
ω  wave frequency, angular velocity  
Ω  rotation speed

Additional symbols

∇  nabla operator  
I_D  first stress tensor invariant  
II_D  second stress tensor invariant  
III_D  third stress tensor invariant

Subscripts

a  aggregate  
b  bed, bob  
c  clay, cup, critical  
d  deposition  
D  drag, dynamic  
e  equilibrium, erosion, excess  
h  hydrostatic  
i  initial, interface
LIST OF SYMBOLS

\begin{align*}
n & \quad \text{natural, normal} \\
p & \quad \text{particle} \\
s & \quad \text{settling, shaft, solid} \\
S & \quad \text{static} \\
t & \quad \text{total} \\
v & \quad \text{vertical} \\
w & \quad \text{water} \\
0 & \quad t, \dot{\gamma} \to \text{zero} \\
\infty & \quad t, \dot{\gamma} \to \text{infinity}
\end{align*}
Dankwoord

Graag neem ik de gelegenheid te baat om mijn erkentelijkheid uit te spreken voor alle hulp en ondersteuning—groot dan wel klein—die ik tijdens het uitvoeren van mijn onderzoek en het schrijven van mijn dissertatie heb ontvangen. Personen die hieraan hebben bijgedragen zijn:


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Curriculum Vitae


In het kader van deze studie volbracht hij onder meer een stage bij het—toenmalig aldus genaamde—Koninklijke/Shell Laboratorium te Amsterdam, alwaar hij proceskundige aspecten van de omzetting van aardgas in was- en olieprodukten in beschouwing nam. Na voltooiing van zijn afstudeerwerk, dat hij bij de sectie Reactorkunde onder supervisie van prof.dr.ir. G.B. Marin verrichtte op het gebied van Chemical Vapour Deposition, werd hem in 1993 de ingenieursbul overhandigd. Aan dit diploma is het predikaat ‘met lof’ toegekend.

Van 1993 tot 1997 was hij werkzaam bij de sectie Vloeistofmechanica van de Technische Universiteit Delft, waar hij zowel experimenteel als theoretisch onderzoek verrichtte naar het ontstaan en transport van vloeibare sliblagen onder water. Dit onderzoek vindt zijn neerslag in dit proefschrift.