Optimal rerouting of short-turning trains during track obstructions

Nadjla Ghaemi · Rob M.P. Goverde · Oded Cats

Abstract Railway traffic controllers have only limited decision support to deal with disruptions. The aim of this research is to provide an algorithm to compute conflict-free routes for trains that cannot continue their planned operation due to a track obstruction. In case of complete blockage, trains need to short-turn at the closest possible station to the disturbed area and provide services in the opposite direction. In such cases, a new timetable is needed to provide a plan during disruption. In this paper a rescheduling and rerouting model is used to find feasible route plans with a focus on short-turnings. The model is applied on a corridor of the Dutch railway network. The results show that such algorithm can provide real-time solutions for traffic controllers during disruption. In addition, it is shown that rerouting the short-turned trains can decrease the delay propagation to the neighboring stations significantly.

Keywords Railway disruption · Rescheduling · Optimization

1 Introduction

In case of large disruptions (e.g. incidents, infrastructural breakdown, etc.) the traffic controllers should apply fast and proper measures, to resolve the con-
gestion and prevent delay propagation to the rest of the network. In the Dutch railway network there are predefined solutions called contingency plans to assist the traffic controllers to deal with such disrupted traffic. Each contingency plan corresponds to a specific disruption scenario in a specific location designed manually by experienced traffic controllers. The disadvantages of these plans are that they are not worked out in detail on infrastructure allocation level and cannot cover all the disruption cases throughout the network. They are constantly getting designed and updated based on the changes in timetable and infrastructure. In practice, it might happen that no suitable contingency plan is already designed for a disruption case. For such cases the traffic controllers are faced with a high workload to communicate to reach an agreement about the suitable plan. These plans being static and inflexible, even if a suitable plan is already designed, the traffic controllers still need to make some adjustment before being able to implement them. Hence, an algorithm that is able to suggest conflict free routes and short-turned trains on both sides of the disruption area is needed in practice.

To study disruptions the bathtub model is used in Figure 1. This model is divided into three phases. When a disruption happens, the traffic will be decreased (first phase). The traffic remains low during the time the problem is being handled (second phase). When the problem is solved the traffic could increase to the original timetable (third phase). The first and third phases are called transition phases, since they represent transition of the operations from the original timetable to the disruption timetable and vice versa. In transition phases the traffic is not as regular as the traffic in the second phase or undisturbed situation. To give an example for the first phase, in the disruption timetable, there might be some service cancellations due to the capacity drop. In real-time, it might be the case that they already started their operations although the disruption timetable suggests their cancellations for the second phase. Thus, those services that are meant to be cancelled in the disruption timetable should be handled in the first transition phase. One of the drawbacks of contingency plans is related to lack of providing any transition plan. The first transition phase is discussed further in the case study.
There are many approaches proposed in the literature for computing the timetable in case of disruption. To get an overview on the literature we refer to Ghaemi and Goverde (2015) where the algorithms to deal with disruptions are reviewed and classified according to the bathtub phases. In this section, we limit the literature study to the most relevant approaches for rerouting and rescheduling that consider microscopic level of infra and operational details.

Caimi et al (2011) developed a resource-constrained multicommodity flow model for rerouting and rescheduling. In this approach, the resource utilization is taken into account. Each resource represents a track section and utilization refers to the blocking times. Then, given an initial timetable, the utilization for all resources are collected. Out of these intervals, a conflict graph is produced. The vertexes of this graph represent each interval and the edges that connect these vertexes represent the conflict. The solution is called a stable set in which there is no adjacent vertexes. The difference with node packing approach used by Zwaneveld et al (1996) is that in node packing approach the conflict graph is developed based on the paths of trains. Thus, it is not known exactly where and when a conflict takes place. The main advantage of multicommodity flow is that for each clique, there will be one constraint, where in node-packing approach, there is one constraint for each edge.

Set-packing approach developed by Lusby et al (2011) incorporates the time and place dimensions to the problem formulation, by considering each resource utilization in interval time of 15 seconds. The drawback of time discretization is the possibility of missing any conflicts that might take place between two discretized time points. In addition both node-packing and set-packing approaches require pre-processing effort for computing the resource utilizations and conflict detection which eventually lead to limited rescheduling alternatives.

Pellegrini et al (2014) proposed a Mixed Integer Linear Programming formulation for rescheduling and rerouting trains in complex junctions. The advantage of this formulation is that there is no need for pre-processing the resource utilization to detect conflicts. Thus this formulation offers more scheduling alternatives. In this approach the conflicts are avoided by computing an order variable that prevents simultaneous resource utilization.

Our rerouting approach is based on the model developed by Pellegrini et al (2014) with the focus of short-turning services. The remaining of the paper is structured as follow. Section 2 provides the terminology and assumptions. Section 3 presents the mathematical formulation of the rerouting and rescheduling model. The case study is discussed in Section 4. Finally, the conclusions are presented in Section 5.

2 Terminology and assumptions

In this section, the terminology and assumptions that will be used throughout the rest of paper are defined. Each pair of consecutive stations are connected by routes. Each route consists of an ordered group of block sections. Each
block section is an ordered group of track sections. Different routes can have common block sections and different block sections can also have common track sections. In other words, each route consists of a specific ordered group of track sections. Note that a group of track sections can be a part of two different routes in opposite directions depending on the order.

In this approach, the headway is formulated based on blocking time theory, which is the commonly accepted method to detect conflicts and compute capacity consumption (UIC, 2013). The blocking time of each track section starts from the sight distance before the train has entered the first track section of the previous block (Pachl, 2014). The end of blocking time depends on the location of the track section. If the track section is located in an interlocking area, the end of blocking time is after the time needed for running, clearing and releasing the track section. If the track section is located in an open track, then the end of blocking time is after the time needed for running, clearing and releasing the last track section of a block that shares one or more track sections with its block.

In our approach, it is assumed that the trains are allowed to wait only in the stations until there is a conflict-free route available, excluding any possibilities for trains to stop between stations behind red signals. The disruption duration is assumed to be known and all the trains running towards the disrupted area in this period are assumed to be short-turned. The delays are computed as the difference between the arrival time at the last station of the route as stated in the original timetable and the arrival time computed by the model.

3 Mathematical formulation

In this section, the mathematical model is explained. In this formulation the following notation is used.

\( V_{} \): The set of all trains.

\( R_{} \): The set of all routes.

\( B_{} \): The set of all block sections.

\( S_{} \): The set of all track sections.

\( S_{v} \): The set of all track sections that can be used by train \( v \).

\( R_{v} \): The set of all routes of train \( v \).

\( \tau_{d}^{v} \): The scheduled departure time of train \( v \).

\( \tau_{a}^{v} \): The scheduled arrival time of train \( v \).

\( s_{D} \): The virtual track section that represents the end of the route.

\( s_{O} \): The virtual track section that represents the start of the route.

\( p_{r,s} \): The previous track section of \( s \) along the route \( r \).

\( f_{r,s} \): The following track section of \( s \) along the route \( r \).

\( l_{r,s} \): The last track section of the block including track section \( s \) along route \( r \).

\( t_{v,r,s}^{run} \): The running time of train \( v \) on the track section \( s \) along the route \( r \).

\( S_{r} \): The ordered set of all track sections of route \( r \).

\( B_{r} \): The ordered set of all block sections composing route \( r \).
$S_{r,b}$: The ordered set of all track sections composing block section $b$ along route $r$.

$T(v,v')$: If $T(v,v') = 1$ train $v$ results from the (short) turning of train $v'$ and otherwise $T(v,v') = 0$.

$\theta_v^{\min}$: The minimum short-turning time needed for the short-turning of train $v'$ to train $v$.

$P$: Platform track sections are listed in the set.

$q_{r,s}$: The first track section of the previous block is referred as the reservation track section for the track section $s$ along the route $r$.

$t^*$: The switching time needed for release and setup the track section.

$t_{c,v,r,s}^e$: The clearing time of train $v$ of track section $s$ along the route $r$.

$M$: The large constant.

3.1 Decision variables

The decision variables are defined as follows.

$x_{r,v}$: If route $r$ is selected for train $v$ then $x_{r,v} = 1$ and otherwise $x_{r,v} = 0$.

$y_{v,v',s}$: If train $v$ uses track section $s$ before train $v'$, then $y_{v,v',s} = 1$ and otherwise $y_{v,v',s} = 0$.

$e_{v,r,s}$: The time when train $v$ enters track section $s$ along the route $r$.

$b_{v,s}^e$: The start of blocking time of track section $s$ for train $v$.

$b_{v,s}^e$: The end of blocking time of track section $s$ for train $v$.

$d_v$: The delay of train $v$ at the last station.

3.2 Objective function and constraints

The objective is to minimize the sum of delays of all trains at their last stations.

$$\min \sum_{v \in V} d_v$$ (1)

For each train, exactly one route should be assigned.

$$\sum_{r \in R_v} x_{r,v} = 1 \quad \forall v \in V$$ (2)

The entrance time for all the track sections related to the unselected routes should be zero.

$$e_{v,r,s} \leq M x_{r,v} \quad \forall v \in V, \ r \in R_v, \ s \in S_r$$ (3)

The entrance time of train $v$ to the first track section $s_Q$ along the route $r$ should at least be equal to the scheduled departure time $\tau_v^d$. 
\[
\sum_{r \in R_v} e_{v,r,s} \geq \tau^d_v \quad \forall v \in V
\] (4)

Excluding any possibility for delays, the entrance time for each track section along the route can be computed by adding the entrance time to the previous track section and the running time on the previous track section.

\[
e_{v,r,s} = e_{v,r,p}, + t_{r,v}^{un} x_{r,v} \quad \forall v \in V, \; r \in R_v, \; s \in S_r
\] (5)

The delay of each train is measured as the difference between the entrance time of the final track section \(e_{v,r,s}^D\) and the scheduled arrival time \(\tau^a_v\).

\[
d_v = \sum_{r \in R_v} e_{v,r,s}^D - \tau^a_v \quad \forall v \in V
\] (6)

The departure time of the short-turned trains should respect the minimum short-turning time.

\[
\sum_{r \in R_v} e_{v,r,s} \geq \sum_{r \in R_v'} e_{v',r,s} + \theta_{v,v'}^{min} \quad \forall v, v' \in V : T(v, v') = 1
\] (7)

To ensure that the short-turned trains are departing from the same platform that is used for the arrival, the following constraint is considered.

\[
\sum_{r \in R_v, s \in S_r} x_r = \sum_{r \in R_v', s \in S_r} x_{r,v'} \quad \forall v, v' \in V : T(v, v') = 1, \; s \in P
\] (8)

To model the capacity, the blocking time for each track section is considered. The start and end time of each block is computed by \(b^s_{v,s}\) and \(b^e_{v,s}\). Each block start \(t'\) before the train enters the reservation track section \(q_{r,s}\) of track section \(s\). This equality holds for all the trains and track sections, except the short-turned trains that are departing.

\[
b^s_{v,s} = \sum_{r \in R_v, s \in S_r} (e_{v,r,q_{r,s}} - t' x_{r,v})
\]
\[
\forall v \in V, \; s \in S_v : (\exists v \in V : T(v, v') = 1) \vee (\forall r \in R_v : q_{r,s} \neq f_{r,s})
\] (9)

In case of short-turning the relevant track section which is a platform track should be kept occupied. Hence, the start blocking time of the block section for the departing should take place before the end blocking time for the arriving. It also implies that the previous constraint should be relaxed with inequality for the platform track section and departing short-turned train.

\[
\sum_{r \in R_v, s \in S_r : q_{r,s} = s_O} b^s_{v,s} < \sum_{r \in R_v', s \in S_r : f_{r,s} = s_D} b^e_{v',s} \quad \forall v, v' \in V : T(v, v') = 1
\] (10)
\[ b_{v,s}^e \leq \sum_{r \in R_v : s \in S_v} (e_{v,r,q_{r,s}} - t'_{r,v}) \quad \forall v \in V, s \in S_v : (\exists v \in V : T(v, v') = 1), (\exists r \in R_v : q_{r,s} = f_{r,s,0}) \] (11)

In interlocking area each blocking time ends after the train passes all track sections from the reservation track section \( q_{r,s} \) to the end of the considered track section \( s \) plus the time needed for clearance and release. The following constraint represents the end of block.

\[ b_{v,s}^e = \sum_{r \in R_v : s \in S_v} (e_{v,r,s} + (t'_{r,v} + t_{r,v} + t^s)x_{r,v}) \quad \forall v \in V, s \in S_v \] (12)

In open track the blocking times end when the train exits the last track section of the block. The following constraint makes sure that the end of blocking time is after the entrance time to the first track section of the following block plus the time needed for clearance and release.

\[ b_{v,s}^e \geq \sum_{r \in R_v : s \in S_v} (e_{v,r,f_{r,s}} + (t'_{r,v} + t_{r,v} + t^s)x_{r,v}) \quad \forall v \in V, s \in S_v \] (13)

To exclude the possibility of blocking a track section by more than one train, an order is defined for each common track section that may be used by each pair of trains.

\[ y_{v,v',s} + y_{v',v,s} = 1 \quad \forall v, v' \in V, s \in S_v \cap S_{v'} \] (14)

Through this order variable, any block overlap is prevented. Since in case of short-turning the track sections might commonly be occupied by train \( v \) or \( v' \) the following constraint is defined for the track sections where there is no short-turning train. Constraint (15) ensures that the start blocking time of the second train occurs after the end blocking time of the first train.

\[ b_{v',s}^e \leq b_{v,s}^e + My_{v,v',s} \quad \forall v, v' \in V : s \in S_v \cap S_{v'} : T(v, v') = 0 \] (15)

4 Case Study

The case assumes that there is a disruption near a small station in the Dutch railway network. The track between station Oss (O) and Den Bosch (Ht) is completely obstructed, which prevents train operations to continue after Oss or reach Oss from Den Bosch. Figure 2 shows the itinerary from station Nijmegen through Nijmegen Dukenburg (Nmd), Wychen (Wc), Ravenstein (Rvs), Oss (O) and further towards Den Bosch.
In the original timetable, two train lines (3601 intercity and 4401 sprinter) run from station Nijmegen to Den Bosch on one track, and train lines 3600 and 4400 run in the opposite direction on the other track. Due to the obstruction, trains 3601 and 4401 should be short-turned in station Oss and they should continue running back towards station Nijmegen. This short-turning implies a changed station track utilization with adjusted routes and platform track allocations that needs to be checked for conflicts, acceptable track occupation and fit in the new timetable with preferably all short-turned trains running in the original opposite train paths. Note that the running and blocking times change due to the changed routes. Likewise, the platform track occupation time of a short-turning train also takes longer than the minimum dwell time for a continuing train.

In this case the disruption occurs at 6:00 and it is assumed to last for over an hour. In this period, two intercity trains and two sprinter trains are planned to depart from station Nijmegen towards Oss and further. However, due to the blockage, they need to short-turn in station Oss. It would be ideal to replace the cancelled services on the opposite direction by these short-turned trains. However, the original timetable presented in Table 1 shows that train SP 4417 arrives at station Oss at 6:14. The train SP 4416 departs from Oss towards Nijmegen at 6:14. Thus, there is not enough time for short-turning. The same applies to the IC 3617 train. Hence they are short-turned as a replacement for the services that depart later from station Oss. The train SP 4419 arrives at 6:44 to station Oss and still cannot continue the route towards Den Bosch. So it needs to short-turn as well. The last train IC 3619 arrives at 7:03 at Oss. Since the disruption ends around 7:00, IC 3619 can proceed towards Den Bosch and does not need to short-turn. Figure 3 shows the track layout in station Oss.
The routes that are used by IC and SP trains are shown in Figure 4. As it is shown in this figure there is a single track between Wychen and Ravenstein. The blue dashed lines represent the routes for sprinters and the pink dashed dotted lines represent the routes for intercity trains. R1 to R4 are routes from station Nijmegen to Oss and R5 to R8 are routes from station Oss to Nijmegen. In the original timetable, both SP and IC trains run on the upper track from Nijmegen to Oss and on the lower track from Oss to Nijmegen. Since due to disruption, trains need to short-turn, and eventually run on the lower track towards Nijmegen, alternative routes are defined that start from the same platform in Nijmegen and end on the lower track in station Oss (R2 for IC and R4 for SP as alternatives).

Based on the original timetable, the trains SP 4418 and IC 3618 cannot arrive to Oss due to the obstruction. However the arrival time of train SP 4420 is at 7:14. Thus, this train might arrive to the station Oss. The next intercity train IC 3620 arrives at Oss and can continue towards Nijmegen. One scenario is defined for the case where SP 4420 cannot arrive to Oss, and another scenario is defined for the case where it can arrive to Oss. To show the support provided to traffic controllers, the results of the optimization in case alternative routes are included (i.e., with rerouting) are compared with the results in case no alternative routes are available (i.e., without rerouting) for each scenario. The parameters that are used in this case study are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t' )</td>
<td>2</td>
</tr>
<tr>
<td>( t^s )</td>
<td>3</td>
</tr>
<tr>
<td>( t'_{v,r,s} )</td>
<td>10</td>
</tr>
<tr>
<td>( p_{v,w} )</td>
<td>480</td>
</tr>
</tbody>
</table>

Table 2 Parameters values considered in this case study
4.1 Disruption timetable

In this scenario, it is assumed there is no traffic from the disrupted area towards Oss. In other words, train SP 4420 cannot arrive to station Oss. The train SP 4417 short-turns as SP 4418, train IC 3617 as IC 3618 and SP 4419 as SP 4420. Train IC 3620 is the first train arriving to station Oss from the obstructed area on the planned route (r7) and IC 3619 is able to continue towards Den Bosch on the planned route (r1). In this scenario, there is no services arriving to Oss from Den Bosch, thus both track sections in Oss are available for short-turning. Consequently there is a fast transition to the disruption timetable by short-turning the trains and no capacity is consumed by any train that is operating based on the original timetable. For this reason, the computed timetable is called disruption timetable.

Figure 5 to 8 show the blocking stairways of the selected routes in case rerouting is possible. The original timetable is shown by red dash-dotted lines for IC trains and red dotted lines for sprinters. The realized trajectories are shown by blue lines passing through the blocks. The results show that trains SP 4417 and SP 4419 short-turn on the lower track (see Figures 5 & 6), whereas train IC 3617 short-turns on the upper track (see Figures 7 & 8). The computed disruption timetable is shown in Table 3.

In case rerouting is not applied, then all trains use the planned platform track for short-turning and the delay increases. Figure 9 shows that IC 3617 waits around 17 minutes for SP 4418 to depart from Oss, so that by the time it arrives to Oss, there is no conflict. Train SP 4419 is delayed around 26 minutes and in this case there is no need for short-turning since it arrives at Oss after
Fig. 5  The blocking stairways of r4 based on disruption timetable: Sprinter short-turnings

Fig. 6  The blocking stairways of r8 based on disruption timetable: Sprinter short-turnings
Fig. 7  The blocking stairways of r1 based on disruption timetable: Intercity short-turning

Fig. 8  The blocking stairways of r5 based on disruption timetable: Intercity short-turning
Table 3 The computed disruption timetable between Oss and Nijmegen

<table>
<thead>
<tr>
<th>Train#</th>
<th>Route</th>
<th>Departure (O)</th>
<th>Arrival (Nm)</th>
<th>Delay in Oss (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP 4417</td>
<td>r4</td>
<td>5:54</td>
<td>6:13</td>
<td>0</td>
</tr>
<tr>
<td>IC 3617</td>
<td>r1</td>
<td>6:18</td>
<td>6:32</td>
<td>0</td>
</tr>
<tr>
<td>SP 4419</td>
<td>r4</td>
<td>6:29</td>
<td>6:48</td>
<td>309.63</td>
</tr>
<tr>
<td>IC 3619</td>
<td>r1</td>
<td>6:49</td>
<td>7:03</td>
<td>61.58</td>
</tr>
</tbody>
</table>

Table 4 Total delay and computation time for disruption timetable

<table>
<thead>
<tr>
<th>Opt. result</th>
<th>Total delay (s)</th>
<th>Computation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rerouting</td>
<td>371.22</td>
<td>4.94</td>
</tr>
<tr>
<td>Without rerouting</td>
<td>2748.73</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 4 shows the total delay in the cases where rerouting is included and excluded. The results show that around 40 minutes out of approximately 46 minutes of delay can be avoided by deploying the optimal rerouting solution. The computation time is shorter in case the possibility of rerouting is excluded.

4.2 Transition timetable

In this scenario, it is assumed that train SP 4420 is arriving to Oss from Den Bosch and the scheduled departure from Oss is at 7:14. To keep the service numbers consistent, this train is called transition train (T train). Hence, the short-turned train will still be called SP 4420. Note that in this scenario train SP 4419 does not replace the T train, so it short-turns earlier with respect to the minimum short-turning time. Since the scheduled arrival time of train SP 4419 is 6:44, the departure time of SP 4420 from Oss is set to 6:52 in order to respect the minimum short-turning time of 8 minutes (in line with Chu and Oetting (2013)). The arrival time at Nijmegen is set to 7:22. The computed transition timetable is shown in Table 5.

Figure 10 shows the results in case rerouting is possible. Train SP 4419 is delayed because both tracks are occupied in Oss. Thus, SP 4419 waits till SP 4418 leaves the station Oss towards Nijmegen. The figure also shows that train IC 3618 is delayed because of a conflict with train SP 4420. The reason why SP 4420 is delayed is the initial delay of SP 4419. In other words, the estimated departure and arrival times were not accurate.

Table 6 compares the total delay in the cases where rerouting is included and excluded. The results show that around 41 minutes out of approximately
**Fig. 9** Blocking stairways r1 based on disruption timetable without rerouting

<table>
<thead>
<tr>
<th>Train#</th>
<th>Route</th>
<th>Departure (Nm)</th>
<th>Arrival (O)</th>
<th>Delay in Oss (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP 4417</td>
<td>r3</td>
<td>5:55</td>
<td>6:13</td>
<td>0</td>
</tr>
<tr>
<td>IC 3617</td>
<td>r2</td>
<td>6:18</td>
<td>6:32</td>
<td>0</td>
</tr>
<tr>
<td>SP 4419</td>
<td>r3</td>
<td>6:32</td>
<td>6:49</td>
<td>369.47</td>
</tr>
<tr>
<td>IC 3619</td>
<td>r1</td>
<td>6:48</td>
<td>7:02</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 5** The computed transition timetable between Oss and Nijmegen

55 minutes of delay can be avoided by deploying the optimal rerouting solution. The computation time is less than a minute in both cases.

<table>
<thead>
<tr>
<th>Opt. result</th>
<th>Total delay (s)</th>
<th>Computation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rerouting</td>
<td>788.97</td>
<td>23.75</td>
</tr>
<tr>
<td>Without rerouting</td>
<td>3285.33</td>
<td>14.01</td>
</tr>
</tbody>
</table>

**Table 6** Total delay and computation time for transition timetable
5 Conclusion

In this paper, a rerouting and rescheduling optimization model is applied on a part of Dutch railway network for a case of complete track obstruction. The
focus is on rerouting the short-turning trains. It is shown that by means of rerouting the delay will be decreased significantly. The computation time seems to be promising for real-time application. The algorithm provides support to traffic controllers in case of disruption (even if a transition phase between original and disruption timetable is needed) by computing conflict-free routes for all trains. In this paper, the short-turning trains and their schedules were predefined. The next step is to extend the model to detect the short-turning trains and compute their schedules, particularly in cases where a transition phase is necessary. In addition, the algorithm should be further developed to allow short-turning only for those trains whose computed arrival times to the short-turning station are within the disruption period, disregarding the trains that arrive after the obstruction is over.

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