Experimental studies on heat transfer in thermo-magnetic convection for para- and diamagnetic fluids

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EXPERIMENTAL STUDIES ON HEAT TRANSFER IN THERMO-MAGNETIC CONVECTION FOR PARA- AND DIAMAGNETIC FLUIDS

by

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in partial fulfilment of the requirements for the degree of

Master of Science
in Applied Physics

at the Delft University of Technology,
to be defended publicly on Wednesday 23 April 2014 at 15:30.

Faculty: Applied Sciences
Department: Chemical Engineering
Research-group: Transport Phenomena

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In industrial heat transfer processes, natural convection enters in various forms. One form of natural convection is thermo-magnetic convection. Besides gravitational force, magnetic force causes warmer fluid to rise or fall dependent on the fluids magnetic susceptibility and direction of magnetic field gradient. Magnetic susceptibility is a material property which indicates a degree of magnetization in a material. For paramagnetic fluids magnetic susceptibility depends on temperature, is positive and therefore attracted by magnetic field. Magnetic susceptibility of diamagnetic materials is independent on temperature, is negative and hence repelled by magnetic field. Magnetic force can be used to enhance or suppress gravity. This phenomena is widely investigated for many materials, magnetic field strengths and set-up geometries.

In this research thermo-magnetic convection and the effect it has on internal heat transfer is experimentally investigated for para- diamagnetic fluids. Making use of a 10 Tesla superconducting magnet, which can generate field gradients up to $870 \text{T}^2/\text{m}$, steady, oscillating and turbulent flow regimes can be observed. I performed the experiments at the AGH University of Science and Technology in Krakow, Poland. A small cubical enclosure filled with para- or diamagnetic fluid is placed at different positions in the magnet to get enhancement or suppression of internal heat transfer. Enclosure is heated from below and top is kept at constant temperature. Temperature of the fluid is measured with thermocouples at six different positions inside the enclosure. From these temperature-time measurements a power spectrum is obtained to determine the characteristic flow regime. Internal heat transfer is investigated by measuring different variables and calculate thermo-magnetic Rayleigh and Nusselt numbers.

As paramagnetic fluid a 40% water-glycerol solution is used and gadolinium nitrate is added to create a higher magnetic susceptibility. Enclosure was placed above the magnet centre which should give a magnetic force that enhances gravitational buoyancy. Temperature difference between the hot and cold plate of the enclosure is 5 and 11°C, respectively case G5A and G11A. Case G5A shows transition in the flow regime from steady to oscillating to turbulent with increasing of magnetic field. Case G11A shows turbulent regime for each measurement. Nusselt number calculations for glycerol solution measurements show an increase, up to 2.5 times, in internal heat transfer. Turbulence causes better mixing and hence better heat transfer. Relation between $Ra_{TM}$ and Nu are compared with previous (experimental) relations and show good agreement.

Pure water is used as diamagnetic fluid. First enclosure is placed below magnet centre and temperature difference is 5 and 1 °C, respectively case W5B and W1B. Here magnetic force should enhance internal heat transfer. Case W5A and W3A are measured above the magnet centre and have respectively a temperature difference of 5 and 3 °C. Case W5B and W5A both show turbulent flow regime for all measurements. Internal heat transfer is about the same for both cases but show a slight increase for W5B and decrease for W5A. There can be concluded that for turbulent flow regimes magnetic force direction has no significant influence on internal heat transfer. For smaller temperature differences, case W1B and W3A, magnetic force does influence measurements. Case W1B shows steady flow regime first and for higher magnetic field strengths fluid plumes start to rise and sink due to magnetic force. Case W3A shows a very clear transition from turbulence to oscillating flow. Small temperature differences cause large measurement errors and internal heat transfer is assumed to be constant.

Recommendations for further research is to get a better impression of fluid structures and temperatures in the enclosure. Fluid behaviour can be visualized with liquid crystals and velocity fields can be determined by using PIV on these visualizations. The velocities can be compared to simulations. To get realistic simulations, fluid properties need to be measured for different temperatures and if necessary different magnetic field strengths.
Natuurlijke convectie is een veel voorkomend fenomeen in industriële warmtetransport processen. Bij thermomagnetische convectie zorgt, naast zwaartekracht, magnetische kracht ervoor dat vloeistof stijgt of daalt. Dit hangt af van de magnetische susceptibiliteit van de vloeistof en de richting van het magnetische veld. Magnetische susceptibiliteit is een materiaaleigenschap die aangeeft in welke mate een materiaal gemagne- tiseerd wordt. Voor paramagnetische vloeistoffen hangt de susceptibiliteit af van de temperatuur, is positief en wordt aangetrokken door het magnetische veld. De susceptibiliteit van diamagnetisme vloeistoffen is temperatuur onafhankelijk, negatief en wordt afgestoten door het magnetisch veld. Magnetische kracht kan de zwaartekracht versterken of verzwakken. Dit fenomeen is veel bestudeerd voor verschillende materialen, magnetische veld sterktes en opstelling geometriën.

In dit onderzoek wordt thermomagnetische convectie en welke invloed dit heeft op interne warmte overdracht in para- en diamagnetische vloeistoffen experimenteel bestudeerd. Hierbij wordt gebruik gemaakt van een 10T supergeleidende magneet die een magnetisch veld gradiënt tot 870 T/m kan genereren. Hierdoor kunnen verschillende karakteristieke stromingen, zoals stabiel, oscillierend of turbulent onderzocht worden. Ik heb de experimenten aan de AGH Universiteit van Wetenschap en Technologie in Krakau, Polen uitgevoerd. Een kleine vierkante behuizing, gevuld met para- of diamagnetische vloeistof, is op verschillende plekken in de magneet geplaatst om versterking of verzwakking te creëren. De kubus is verwarmd vanaf de onderkant en de bovenkant wordt op constante temperatuur gehouden. Temperatuur van de vloeistof wordt op zes plekken gemeten met thermokoppels. Van deze temperatuur-tijd metingen wordt een energie spectrum verkregen en de karakteristieke stroming bepaald. De interne warmtetransport is onderzocht door verschillende variabelen te meten en daarmee het thermomagnetische Rayleigh en Nusselt nummer te berekenen.

De paramagnetische vloeistof is een 40% water-glycerol oplossing waar gadolinium nitraat aan is toegevoegd om een hogere magnetische susceptibiliteit te krijgen. De behuizing is bovenin de magneet geplaatst zodat de magnetische kracht de zwaartekracht versterkt. Temperatuurverschil tussen de boven en onder plaat is 5 en 11°C voor respectievelijk case G5A en G11A. In case G5A is voor een toenemend magneetveld de transitie van stabiel naar oscillierend naar turbulent waargenomen. Case G11A is turbulent voor alle metingen. Nusselt nummer berekeningen voor de glycerol metingen laten een verhoging, tot 2,5 keer, van de interne warmteoverdracht zien. Meer turbulentie zorgt voor betere menging en daardoor betere warmteoverdracht. De relatie tussen RTM en Nu is vergeleken met eerder verkregen relaties en laten een goede overeenkomst zien.

Puur water is gebruikt voor diamagnetische metingen. Eerst is de kubus onderin de magneet geplaatst en is er gemeten bij een temperatuurverschil van 5 en 1°C, respectievelijk case W5B en W1B. Magnetische kracht zorgt hier voor versterking van interne warmteoverdracht. Case W5A en W3A zijn gemeten met de kubus boven het magneet centrum. Het temperatuur verschil is respectievelijk 5 en 3°C. Case W5B en W5A zijn beide turbulent voor alle metingen. Interne warmteoverdracht metingen zijn ongeveer gelijk en laten een kleine stijging voor W5B en daling zien voor W5A. Hieruit wordt geconcludeerd dat voor turbulent stromingen de richting van het magnetische veld geen significante invloed heeft op de interne warmteoverdracht. Bij case W1B en W3A is het temperatuurverschil kleiner en heeft de magnetische kracht invloed op de metingen. Case W1B is eerst stabiel en als het magneetveld verhoogt wordt stijgen er warme vloeistoffellen van de bodem en koudere zakken naar beneden. In case W3A is een duidelijke overgang van turbulent naar oscillierende stroming waargenomen. Door de kleine temperatuurverschillen zijn er grote onzekerheden in de warmteoverdracht metingen en deze worden als constant beschouwd.

En vervolg onderzoek kan zorgen voor een beter inzicht in de stromingsstructuur en temperatuurprofiel in de behuizing te krijgen. Hoe de vloeistof zich gedraagt kan zichtbaar gemaakt worden met vloeibare kristallen. Vanuit deze visualisaties kan een snelheidsveld verkregen worden met PIV. Deze snelheden kunnen vergeleken worden met simulaties. Voor realistische simulaties moeten de vloeistofeigenschappen gemeten worden bij verschillende temperaturen en eventueel verschillende magneetveld sterktes.
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INTRODUCTION

Convection is a heat transfer mechanism where heat is transported with motion of the fluid. It is a combination of two other mechanisms, advection and diffusion. In diffusion, heat spreads out through the medium and in advection heat is transferred due to bulk velocity in the fluid. Motion of fluid occurs due to temperature differences. Warmer fluid is lighter than colder fluid. Therefore fluid with lower temperature is sunken by gravitational force and warmer fluid is rising. This is called natural or thermal convection. Convection is forced when fluid motion is induced by, for example a propeller or pressure difference.

A classical fluid dynamic problem is Rayleigh-Bénard convection. Here fluid is heated from below and cooled from above which generates a buoyancy driven flow. It appears on large scales in the atmosphere, oceans and stars. In industrial heat transfer processes, natural convection enters in various forms. For example in nuclear reactors, crystallization processes and in solar heating devices.

Another form of natural convection is magnetic convection. Magnetic susceptibility of a paramagnetic material depends on temperature just like density. Magnetic force causes warmer fluid to rise or fall, depending on its magnetic susceptibility and direction of the magnetic field gradient. This can cause enhancement or suppression of natural convection. Fluid motion induced by gravitational and magnetic forces is called thermo-magnetic convection.

1.1. PREVIOUS AND PRESENT WORK

First to experimental investigate thermo-magnetic convection in paramagnetic fluid is Braithwaite et al. [1]. They used an 8-T superconducting magnet to enhance or suppress buoyancy-driven convection in a cell filled with gadolinium nitrate solution (paramagnetic fluid) and examined internal heat transfer in this enclosure. Three configurations where used. First, cell is heated from below and magnetic force applied downwards. Here natural convection occurs and is enhanced by magnetic contribution. Second, cell is heated from below and magnetic force applied upwards. Here natural convection occurs but is suppressed by magnetic contribution. Third, cell is heated from above and magnetic force applied upwards. Here no natural convection occurs, but magnetic convection can be generated by magnetic force.

Since then thermo-magnetic convection was investigated both experimentally and numerically for different fluids, enclosure geometries and magnetic field orientations. For example a numerical computation of Rayleigh-Bénard convection for water in a cylindrical enclosure was done by Tagawa et al. [2]. Paramagnetic fluids where experimentally and numerically investigated by Bednarz et al. [3]. They used a side heated cubical enclosure in a horizontal orientated magnetic field. Cylindrical enclosure with paramagnetic fluid in a vertical orientated magnetic field was investigated by Wróbel et al. [4]. These studies all considered steady flow regimes. Unsteady flow regimes where analysed by Pyrda et al. [5]. They looked at cubical enclosure filled with paramagnetic fluid and used a stronger magnetic field than in previous studies.

Busse examined Rayleigh-Bénard convection experiments and obtained relations for internal heat transfer [6]. Mukhopadhyay et al. obtained similar relations for internal heat transfer in thermo-magnetic convection by performing a scaling analysis [7].
In this study both para- and diamagnetic fluids are investigated experimentally. The used cubical enclosure is heated from below. For unsteady flow regimes with relatively small temperature differences, magnetic field gradients must be high. To obtain this a 10-T superconducting magnet, which can produce field gradients up to $870 \, \text{T}^2/\text{m}$, is used. Enclosure is placed inside the magnet bore such that the magnetic force is in the same or opposite direction as gravitational force. Because paramagnetic fluids have a positive magnetic susceptibility, fluid is attracted by magnetic field. Diamagnetic fluids are repelled by magnetic field because of their negative magnetic susceptibility. To acquire characteristic flow regime of the fluid in the enclosure local temperature measurements are performed. Internal heat transfer is investigated by measuring different variables and calculate corresponding Nusselt numbers.

As guideline for this study to thermo-magnetic convection in para- and diamagnetic fluids, following research goals are selected:

**Goal 1:** Influence of magnetic force direction on heat transfer by measuring at different enclosure positions

**Goal 2:** Determine characteristic flow regime from local temperature measurements

**Goal 3:** Examine flow regime transition by measuring fluid with $\text{Pr} \approx 70$ at different magnetic field strengths

**Goal 4:** Influence of thermo-magnetic convection on internal heat transfer

**Goal 5:** Verify (theoretical) relations between heat transfer and internal forces from previous researches

I did the experimental part of this research at the AGH University of Science and Technology in Krakow Poland. AGH stand for Akademia Górniczo-Hutnicza which is polish for Academy of Mining and Metallurgy [8]. This is the biggest technical university of Poland and has almost 37,000 students and more than 4,000 staff members. For many years it has been ranked in the top of the list of institutions of higher education. The University was founded after long-lasting endeavours which had started in 1782 and was established on 20th of October 1919. It has been continuing the traditions of the Academy of Mining in Kielce (1816-1827), which was inaugurated by Stanisław Staszic. AGH’s moto is “Labore creata, labori et scientiae servio” (Created in labour, I serve labour and science). Figure 1.1 shows their symbol.

During second world war the University was occupied by the German General Government and the buildings where plundered and destroyed. Staff was able to save part of the library and the authorities created a provisional underground teaching base. After the war AGH supported development of other polish technical universities.

The AGH has 16 faculties. One of them is the Faculty of Energy and Fuels which develops very rapidly and in line with an increasing demand for energy and fuels. The Faculty was established in 1991 as the Faculty of Coal Energochemistry and Physical Chemistry of Sorbents. In 2008, the Senate of the University decided to integrate the Faculty of Fuels and Energy and the Interfaculty School of Power Engineering in order to form a large and significant faculty whose aim was to combine education and research on fuel technology and power engineering.

The Faculty of Energy and Fuels has 8 departments. I was staying at the department of Fundamental Research in Energy Engineering which has a lot of experiences with researching thermo-magnetic convection [3–5, 9, 10]. They have a 10 Tesla superconducting magnet which I used to execute experiments for my research. I got the opportunity to give a poster presentation about my experimental research at the International Conference on Magneto-Science [11]. Front of the abstract booklet, the abstract and poster are included in appendix A.
1.2. **Outline**

This thesis starts with a brief part about magnetism, chapter 2. Here an introduction into magnetization and material properties that arise from it are given. In this research considered dia- and paramagnetic materials are described and magnetic force from Landau and Lifshitz is presented in the last section.

Chapter 3 gives a theoretical analysis of the research. Conservation laws in a strong magnetic field are derived and adjusted for this experimental problem. A dimension analysis is done and some useful dimensionless number are explained. From a linear stability analysis, chapter 3.4, a possible onset for convection is given and different fluid flow regimes are described. Internal heat transfer for this research is presented in chapter 3.5. This chapter ends with a scaling analysis to determine a relation between heat transfer and thermo-magnetic convection.

Experimental method is explained in chapter 4. First the set-up and its components are described. Fluid properties of the two working fluids are given in chapter 4.2. Finally measurement procedure and how the obtained data is processed is described in the last two sections.

In chapter 5, measurement results are analysed and discussed. First the temperature-time series obtained from thermocouples in the fluid and second the internal heat transfer in the enclosure are studied. Both sections start with paramagnetic and then diamagnetic results. The heat transfer chapter includes a section about heat loss measurements.

Conclusions of the research questions are stated in chapter 6 and chapter 7 contains recommendations for potential further research.
In this chapter different types of magnetism are explained. For this research dia- and paramagnetism are most important. Other types like ferro-magnetism shall not be treated. In addition magnetic force will be described.

Total magnetic (dipole) moment of a material is given by the total of individual orbital and spin magnetic moments. Orbital magnetic moment are caused by movement of electric charges. Spin magnetic moment comes from the intrinsic magnetism of elementary particles. Magnetic dipoles are randomly orientated in such a way that the net magnetic moment is practically zero. By placing material in an external magnetic field, magnetic moments align or anti-align with respect to the field direction. This is called magnetization or magnetic polarization, denoted as \( \vec{M} \) and is defined by total number \( N \) of magnetic moments \( \vec{m} \) per volume \( V \).

\[
\vec{M} = \frac{N}{V} \vec{m}
\]  

(2.1)

Magnetic induction \( \vec{B} \) in a material is given by the sum of the applied external magnetic field \( \vec{H} \) and the magnetization \( \vec{M} \) in the material and multiplied is with \( \mu_0 = 4\pi \cdot 10^7 \text{ kg m s}^{-2} \text{ A}^{-2} \), the magnetic permeability of vacuum.

\[
\vec{B} = \mu_0 (\vec{H} + \vec{M})
\]  

(2.2)

Dipoles in materials can respond in different ways to the external magnetic field. Para- and diamagnetic materials behave linear and their magnetization is proportional to the external field and their volume magnetic susceptibility \( \chi \).

\[
\vec{M} = \chi \vec{H}
\]  

(2.3)

Volume magnetic susceptibility is a dimensionless proportionality constant. This constant indicates the degree of magnetization of a material and is equal to the material density \( \rho \) multiplied with its mass magnetic susceptibility \( \chi_g \).

\[
\chi = \rho \chi_g
\]  

(2.4)

Related to magnetic susceptibility is the magnetic permeability. This is a material property which describes how the external magnetic field deforms in the material and is given by \( \mu_m = \mu_0 (1 + \rho \chi_g) \). Substituting equation 2.3 and 2.4 in equation 2.2 gives magnetic induction as a function of the magnetic permeability.

\[
\vec{B} = \mu_0 (1 + \rho \chi_g) \vec{H} = \mu_m \vec{H}
\]  

(2.5)

2.1. CLASSIFICATION OF MAGNETIC MATERIAL

Based on their magnetic susceptibility, materials can be divided into three classes, dia-, para and collective magnetism (ferro and antiferro-magnetic). Figure 2.1 shows each material and its class [13]. In this research two types of magnetic material are investigated. Pure water (H\(_2\)O) as diamagnetic material. To create a paramagnetic material some gadolinium nitrate (Gd(NO\(_3\))\(_3\)) is added to an initial diamagnetic water-glycerol (C\(_3\)H\(_8\)O\(_3\)) solution.
2.1.1. DIAMAGNETISM

In diamagnetic materials, external magnetic field induces dipoles which are opposite to the field due to Lenz’s law. Therefore the magnetic susceptibility is negative, figure 2.2. When the external field is removed, dipoles get randomly oriented again. This effect happens in every material but is a very weak property, $\chi \sim O(10^{-6})$. So only in absence of para- and collective magnetism, diamagnetism is relevant. Magnetic susceptibility for diamagnets is not dependent on temperature, figure 2.2. Superconductors are ideal diamagnets, they repel the external field completely $\chi = -1$ [12].

$$\chi_{para} = \text{const.} < 0$$ (2.6)

2.1.2. PARAMAGNETISM

Paramagnetism occurs when there are permanent magnetic moments present in the material. These permanent moments align parallel with external field direction, figure 2.2. The total of the permanent moment is much larger, $\chi \sim O(10^{-4})$, than the moments produced by Lenz’s law. Therefore the material is attracted by the external field and the magnetic susceptibility is positive, figure 2.2. Thermal fluctuations can influence the orientation of the aligned dipoles. For high temperatures, approximately room temperature, the Curie law, equation 2.7 becomes valid [14]. $C$ is a constant depending on the material and $T_K$ is the absolute temperature in Kelvin.

$$\chi_{dia} = \frac{C}{T_K}$$ (2.7)

---

Figure 2.1: Periodic table of elements coloured according to the type of magnetism they show at room temperature [13]

Figure 2.2: Temperature dependence of volume magnetic susceptibility (left) and magnetization versus external magnetic field (right) for para- (red) and diamagnetic (green) materials
2.2. MAGNETIC FORCE

When fluid is placed in a force field a body force will act on the fluid\[15\]. Most well known body force is gravity, others are Coriolis, centrifugal, electrostatic, electromagnetic and magnetic force. First four can be neglected in this research because the frame of reference is fixed and no electric field is present. Because magnetic field of the earth is order of 10-100 µT, the magnetic force is negligible as well. However in this experiment magnetic field can be up to 10 Tesla. Therefore magnetic and gravitational force need to be included in the momentum equation.

Magnetic force is given by Landau and Lifshitz \[16\]:

\[
\vec{f}_m = \frac{1}{2}\nabla \left( H^2 \rho \left( \frac{\partial \mu_m}{\partial \rho} \right) \right) - \frac{1}{2} H^2 \nabla \mu_m + \mu_m (\vec{J} \times \vec{H})
\]

(2.8)

In this research fluids are assumed to be non-conducting \[17\]. For a non-conducting fluid \(\sigma = 0\) hence \(\vec{J} = \sigma \left[ \vec{E} + \mu_m (\vec{u} \times \vec{H}) \right] = 0\) and the last term on the right hand side vanishes. Magnetic permeability \(\mu_m\) can be assumed to be constant and equal to \(\mu_0\) because magnetic susceptibility is \(\ll 1\).

Final result of the magnetic force is:

\[
\vec{f}_m = \rho \chi g \frac{\nabla B^2}{2\mu_0}
\]

(2.9)

This force and the gravitational force are the driving mechanisms behind thermo-magnetic convection. Depending on their direction, they can enhance or suppress each other.
THEORETICAL ANALYSIS

In this chapter a theoretical analysis of the experimental situation is presented. Derivation of the equations is based on the method used in Deen and Chandrasekhar [18][19]. First a set of equations and associated boundary conditions valid for the experimental conditions are obtained. These governing equations are made dimensionless and are simplified using dimensionless numbers. A linear stability analysis, based on Pellows, is carried out to determine a critical Rayleigh number as the onset for convection [20].

3.1. CONSERVATION LAWS IN A MAGNETIC FIELD

By adjusting the conservation laws (mass, momentum and energy), a set of equations for this specific problem is obtained. A detailed description of the derivation is given in appendix B, here only a few steps are shown.

The fluids used in this experiment are assumed to be non-conducting, incompressible and viscous. In the momentum equation the magnetic force \( \vec{f}_m \) needs to be included as a body force since magnetic field can increase to 10 Tesla. Magnetic force for this research is derived in chapter 2.2 and given by equation 2.9. Its sign depends on the material type (para- or diamagnetic) and magnetic field direction.

\[
\vec{f}_m = \rho \chi g \nabla B^2 / 2\mu_0
\]

(3.1)

Buoyancy occurs when temperature differences in a fluid induces density differences and for paramagnetic fluids also magnetic susceptibility differences. Density and susceptibility variations are approximated with a Taylor expansion at \( T_0 \). Using \( \rho_0 \) and \( \chi_{g0} \) density and magnetic susceptibility at \( T = T_0 \), which is the reference (mean) temperature. Because temperature differences are small, Taylor expansion can also be applied to density of water.

\[
\rho - \rho_0 = \frac{\partial \rho}{\partial T} (T - T_0) = -\rho_0 \beta (T - T_0)
\]

(3.2)

\[
\rho \chi_g - \rho_0 \chi_{g0} = \frac{\partial (\rho \chi_g)}{\partial T} (T - T_0) = -\rho_0 \chi_{g0} \left( \beta + \frac{1}{T_{K0}} \right) (T - T_0)
\]

(3.3)

With \( \beta \) the thermal expansion coefficient and \( T_{K0} \) absolute reference temperature in Kelvin. Magnetic susceptibility of diamagnetic material is temperature independent therefore \( \rho \chi_g - \rho_0 \chi_{g0} = (\rho - \rho_0) \chi_{g0} \).

Making use of the dynamic pressure \( P_d \), Boussinesq approximation and other assumptions stated in appendix B, the conservation laws reduce to the following set of equations:

\[
\nabla \cdot \vec{u} = 0
\]

(3.4)

\[
\frac{D}{Dt} \vec{u} = -\frac{1}{\rho_0} \nabla P_d - \beta (T - T_0) \vec{g} - \chi_{g0} \left( \beta + \frac{1}{T_{K0}} \right) (T - T_0) \frac{\nabla B^2}{2\mu_0} + \nu \nabla^2 \vec{u}
\]

(3.5)

\[
\frac{D}{Dt} T = \alpha \nabla^2 T
\]

(3.6)
It is convenient to introduce the magnetization number $\gamma$ which is a vector because it can have different values in each direction, depending on the magnetic field gradient.

\[ \vec{\gamma} = \chi g \frac{\nabla B^2}{g\mu_0} \]  

(3.7)

Moment equation can be rewritten:

\[ \frac{D}{Dt} \vec{\mu} = -\frac{1}{\rho_0} \nabla P_d - \beta \left( 1 + \frac{1}{\beta T_0} \right) \left( T - T_0 \right) \vec{g} + v \nabla^2 \vec{u} \]  

(3.8)

### 3.2. Problem Description

To solve the set of equations (3.4, 3.6 and 3.8), boundary conditions are required. These conditions are obtained from the problems geometry and settings. Figure 3.1 provides a sketch of the situation.

The non-conducting, incompressible and viscous fluid is kept between two horizontal plates placed at distance $l = 0.032$ m from each other. In this experiment magnetic field gradients in the $x$ and $y$ direction are negligible and both the gravitational and magnetic force are aligned in the $z$-direction. Both plates are kept at constant temperature. Lower plate has temperature $T_h$ and upper plate temperature $T_c < T_h$. At $z = 0$, $T = T_0$ and temperature difference between heating and cooling plate is $\Delta T = T_h - T_c$. Because the fluid is incompressible and viscous, the non-slip condition is valid [15]. Meaning that $\vec{u} = 0$ at the boundaries. Horizontal velocities are zero at the surfaces for every $x$ and $y$. Substituting $u = v = 0$ in equation 3.4 results in $\frac{\partial w}{\partial z} = 0$ at $z = \pm \frac{1}{2} l$. Dynamic pressure $P_d$ is set zero at $z = -\frac{1}{2} l$.

This results in the following boundary conditions:

\[ T \left( \pm \frac{1}{2} \right) = T_0 \pm \frac{1}{2} \Delta T, \quad T(0) = T_0 \]  

(3.9)

\[ P_d \left( -\frac{1}{2} \right) = 0 \]  

(3.10)

\[ w(\pm \frac{1}{2} l) = \frac{\partial w}{\partial z} (\pm \frac{1}{2} l) = 0 \]  

(3.11)

---

**Figure 3.1:** Schematic representation of forces, lengths and temperatures of the problem
3.3. **Non-dimensionlization**

In this research different configurations are examined and it is convenient to write the governing equation set and boundary conditions in a dimensionless form. Dimensionless derivatives, time, velocities and pressure are denoted with an accent ‘. Dimensionless temperature is denoted by \( \Theta \). Scaling the parameters with constant system characteristics ends up in equation 3.12 to 3.14. Details of this non-dimensionalization are given in appendix C.

\[
\nabla' \cdot \vec{u}' = 0 \quad (3.12)
\]

\[
\frac{D}{Dt'} \vec{u}' = -\nabla' p' \frac{v}{\alpha} \frac{\beta g}{v a} \Delta T l^3 \left(1 + \frac{1}{1 + \frac{1}{\beta T_{k0}}} \right) \hat{z} + \frac{v}{a} \nabla'^2 \vec{u}' \quad (3.13)
\]

\[
\frac{D}{Dt'} \Theta = \nabla'^2 \Theta \quad (3.14)
\]

With dimensionless boundary conditions:

\[
\Theta(\pm \frac{1}{2}) = \pm \frac{1}{2}, \quad \Theta(0) = 0, \quad p'(\frac{-1}{2}) = 0 \quad (3.15)
\]

\[
w' \hat{z} \left( \pm \frac{1}{2} \right) = \frac{\partial w'}{\partial z'} \left( \pm \frac{1}{2} \right) = 0 \quad (3.16)
\]

### 3.3.1. **Dimensionless Numbers**

Equation 3.13 contains a few dimensionless groups of variables. These groups are so called dimensionless numbers and define products or ratios between physical phenomena.

The rate of viscous momentum transfer relative to conductive heat transfer is given in equation 3.17 and called the Prandtl number \([15]\). Here \( v \) is kinematic viscosity and \( \alpha \) thermal diffusivity, which are both fluid properties. This makes Pr a fluid property as well.

\[
Pr = \frac{v}{\alpha} \quad (3.17)
\]

Thermal Rayleigh number, equation 3.18, is defined by the ratio between buoyancy and viscous forces \([15]\). It represents an onset for convection. In this research buoyancy due to magnetic forces should be included. Ratio between magnetic buoyancy and viscous forces is given by magnetic Rayleigh number whose sign depends on material type and magnetic field orientation. Add thermal and magnetic Rayleigh number to get the thermomagnetic Rayleigh number \([4]\).

\[
Ra_T = \frac{\beta g}{\nu a} \Delta T l^3 \quad (3.18)
\]

\[
Ra_M = \frac{\beta g}{\nu a} \Delta T l^3 \left(1 + \frac{1}{1 + \frac{1}{\beta T_{k0}}} \right) = Ra_T \frac{\gamma}{2} \left(1 + \frac{1}{1 + \frac{1}{\beta T_{k0}}} \right) \quad (3.19)
\]

\[
Ra_{TM} = Ra_T + Ra_M = Ra_T \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{1 + \frac{1}{\beta T_{k0}}} \right) \right) \quad (3.20)
\]

Substituting \( Pr \) and \( Ra_{TM} \) in equation 3.13 gives:

\[
\frac{D}{Dt'} \vec{u}' = -\nabla' p' - Pr Ra_{TM} \hat{z} + Pr \nabla'^2 \vec{u}' \quad (3.21)
\]
3.4. **LINEAR STABILITY ANALYSIS**

When fluid is heated from below and cooled from above and there is an uniform temperature gradient, the fluid is stable. Viscosity of the fluid opposes (upward) motion due to body forces (buoyancy) and heat transfer is mainly in the form of conduction. When the temperature gradient increases and the Rayleigh number reaches a critical value, the fluid becomes unstable and convection starts [21]. Viscous forces are no longer able to suppress buoyancy and fluid becomes unstable.

Pellew and Southwell examined this instability analytically for fluid heated from below for an enclosure with free and/or rigid boundaries [20]. In appendix D a similar analysis is performed for a situation where a magnetic field is present. Starting with conservation equations for initial steady state, functions for temperature and pressure which are only depending on vertical direction are derived. Instabilities can be caused by small disturbances in pressure, temperature or velocity. These variables will be represented as static function plus a perturbation term. After substitution perturbation functions in the conservation equations and eliminating pressure term, only two equations with two unknowns remain, temperature and velocity perturbation term. By using separation of variables and assuming neutrally stable situation ($\frac{\partial}{\partial t} = 0$) a general solution for the $z$ depending velocity perturbation $W$ is obtained. General solution is separated in an even and odd part. A non-trivial solution for the even part results in a critical thermo-magnetic Rayleigh number:

$$Ra_{TM} = 1707.8 \quad (3.22)$$

This value is equal to the critical Rayleigh number $Ra_T$, in absence of magnetic force. Meaning that an initial unstable situation ($Ra_T > Ra_{TM}$) can be stabilized by choosing magnetic field in such a way that $Ra_{TM} \leq 1707.8$. Vice versa the onset of convection can be accelerated by choosing a proper magnetic field.

### 3.4.1. **CHARACTERISTIC FLOW REGIMES**

When $Ra_{TM}$ is lower than the critical value there is no convection and heat transfer is solely conductive. As the Rayleigh number increases the fluid goes from steady to a transient, oscillating or even turbulent flow regime [9]. In steady regime the temperature profile in the fluid is linear. With increasing Rayleigh number the profile flattens in the middle and has higher gradients at the boundaries. Development of the temperature profile is given in figure 3.2. In transition/oscillatory flow regime, plumes of hot fluid rise from the bottom layer and simultaneously cold plumes fall from the top creating convection cells. The core of the fluid is practically at the average temperature $T_0 = \frac{1}{2}(T_h + T_c)$. At the top and bottom thermal conduction layers $\delta_T$ arise.

In turbulent flow regime horizontal velocity fluctuations cause instabilities in the convection cells, hence the rising and falling plumes move no longer in a structured way. The turbulent core is at average temperature and turbulent thermal boundary layers have temperature difference of approximately $\frac{1}{2} \Delta T$. Turbulent flow contains eddies of different sizes $\ell$. Eddies of the largest scale $\ell_0$ (almost equal to enclosure dimension $L$), contain kinetic energy from the convective motion in the fluid (energy-containing range). These large eddies are unstable and break up into smaller and smaller ones, $\ell_{EI} > \ell > \ell_{DI}$ (inertial sub-range) until they are stable. Energy of stable smallest eddies (Kolmogorov length $\eta$) is dissipated by viscosity (dissipation range). This is called energy cascade and is schematically presented in figure 3.3 [22]. In the energy spectrum of turbulence the inertial sub-range has a characteristic power-law behaviour called the Kolmogorov -5/3 spectrum [22]. In an energy spectrum plotted in log-log scale the inertial sub-range has a slope of -5/3.
3.5. Heat Transfer

A measure for heat transfer in a fluid is given by the dimensionless Nusselt number, equation 3.23 [10]. This number is the ratio of net convective $Q_{\text{net conv}}$ to net conductive heat transfer $Q_{\text{net cond}}$.

$$\text{Nu} = \frac{\dot{Q}_{\text{net conv}}}{\dot{Q}_{\text{net cond}}}$$  \hspace{1cm} (3.23)

Net convective and net conductive heat fluxes are estimated according to a method investigated by Ozoe and Churchill and are given by the convective or conductive heat flux ($\dot{Q}_{\text{conv}}$ or $\dot{Q}_{\text{cond}}$) minus the heat loss ($\dot{Q}_{\text{loss}}$) [23]:

$$\dot{Q}_{\text{net conv}} = \dot{Q}_{\text{conv}} - \dot{Q}_{\text{loss}}$$  \hspace{1cm} (3.24)

$$\dot{Q}_{\text{net cond}} = \dot{Q}_{\text{cond}} - \dot{Q}_{\text{loss}}$$  \hspace{1cm} (3.25)

When heat loss is purely conductive it can be presented as follows:

$$\dot{Q}_{\text{loss}} = \dot{Q}_{\text{cond}} - \dot{Q}_{\text{thr cond}}$$  \hspace{1cm} (3.26)

Theoretical conductive heat flux $\dot{Q}_{\text{thr cond}}$ is calculated with Fourier’s law based on a conductive area of size $l^2$ Hanjalic et al. [24]. In equation 3.27 $\lambda$ is thermal conductivity of the fluid and $\Delta T$ temperature difference between the hot and cold plate.

$$\dot{Q}_{\text{thr cond}} = l\lambda\Delta T$$  \hspace{1cm} (3.27)

Substitute $\dot{Q}_{\text{loss}}$ in 3.25 gives:

$$\dot{Q}_{\text{net cond}} = \dot{Q}_{\text{cond}} - \dot{Q}_{\text{cond}} + \dot{Q}_{\text{thr cond}} = \dot{Q}_{\text{thr cond}} = l\lambda\Delta T$$  \hspace{1cm} (3.28)

Convective heat rate is equal to the heating plate power (voltage $U$ times current $I$):

$$\dot{Q}_{\text{conv}} = \dot{Q}_P = UI$$  \hspace{1cm} (3.29)

Nusselt number becomes:

$$\text{Nu} = \frac{UI - \dot{Q}_{\text{loss}}}{l\lambda\Delta T}$$  \hspace{1cm} (3.30)
3.5.1. **Scaling Analysis**

Influence of magnetic and gravitational forces on internal heat transfer can be examined by plotting Nusselt number as a function of the thermo-magnetic Rayleigh number. By performing a scaling analysis a relation between the integral heat transfer and internal forces is determined.

Mukhopadhyay *et al.* and Fornalik have done a 2-D scaling analysis for a square enclosure which is heated from the side. This analysis is presented in appendix E and relation derived is [7] [10]:

\[
\text{Nu} \sim \text{Ra}_{TM}^{1/4}
\]  

(3.31)

Bejan proposes a very simple scaling analysis to determine the relation between Nusselt and thermal Rayleigh number (\(\text{Ra}_T\)) in turbulent regime [25]. Similar analysis can be performed for convection due to thermo-magnetic buoyancy. Now thermo-magnetic Rayleigh number \(\text{Ra}_{TM}\) is used. In the enclosure with height \(H\), turbulent core is sandwiched between thermal boundary layers \(\delta_T\), figure 3.4. Those boundary layers become unstable when Rayleigh number based on \(\delta_T\), \(\text{Ra}_{TM,\delta_T}\), > 1707.8, hence is of order \(10^3\):

\[
\text{Ra}_{TM,\delta_T} \sim 10^3
\]  

(3.32)

This is the Rayleigh number based on conduction. To get \(\text{Ra}_{TM}\) based on convection, multiplying equation 3.32 with the cubed enclosure height, \(H^3\).

\[
\text{Ra}_{TM}\delta_T H^3 \sim 10^3 \delta_T^3 H^3
\]  

(3.33)

Now a scale for \(\text{Ra}_{TM}\) can be found:

\[
\text{Ra}_{TM} \sim 10^3 \left(\frac{H}{\delta_T}\right)^3
\]  

(3.34)

The Nusselt number is the ratio of convection to conduction (equation 3.23), which can be represented by a convection length \(H\) and a conduction length \(\delta_T\). Nusselt can be approximated with \(H/\delta_T\) and equation 3.34 becomes:

\[
\text{Ra}_{TM} \sim 10^3 \text{Nu}^3
\]  

(3.35)

Final relation between Nusselt and thermo-magnetic Rayleigh number is:

\[
\text{Nu} \sim \text{Ra}_{TM}^{1/3}
\]  

(3.36)

Busse [6] compared experimental results of similar heat transfer measurements and found that for fluids with \(\text{Pr}>1\) the \(\text{Nu} \text{Ra}_{TM}\) relation is:

\[
\text{Nu} \sim \text{Ra}_{TM}^{0.28}
\]  

(3.37)

![Figure 3.4: Turbulent core and thermal boundary layer (left) and turbulent temperature profile (right) from Bejan [25]](image-url)
In this chapter the experimental method is presented. First the used set-up and its main components are described. Second part concerns properties of the dia- and paramagnetic working fluids. Next the experimental procedure is presented and last part is about data processing and uncertainties.

4.1. **Experimental set-up**

The experimental set-up components are schematically presented in figure 4.1. It consists of a cubical enclosure which is placed in the bore of a 10 Tesla superconducting magnet. For consistent and stable positioning, the enclosure is placed in an aluminium frame which fits the bore perfectly. A thermostatic bath is used to keep one side of the enclosure at constant temperature. Opposite enclosure side is electrically heated with a nichrome wire connected to a power supply. This supply generates a controlled constant heat flux which is monitored with multi-meters. Temperatures of the cooled and heated enclosure walls and the fluid temperature are measured with thermocouples. Additionally the temperature in the room and magnet bore are measured. To minimize convective flow due to temperature difference between room (±21°C) and bore temperature (±18°C), lower part of the magnet bore is closed with foam. Recorded temperatures are stored in a data logger which is connected to a computer.

![Figure 4.1: Schematic presentation of all the components of the set-up and how they are connected and controlled](image-url)
4.1.1. **SUPERCONDUCTING MAGNET**

The used magnet is a superconducting helium-free magnet, model HF10-100-VHT-B. Figure 4.2a shows a picture of the magnet. It is made by a Japanese company called Sumitomo Heavy Industries. It can provide a magnetic field up to 10 Tesla. Diameter and length of the magnets bore is 0.1 and 0.45 m respectively. Coils of the magnet are placed in such a way that the centre of the magnetic field is at 0.19 m from the top of the bore. At this magnetic field centre $z$ is defined as zero. Maximum magnetic field value is denoted with $|b_0|$ and used to specify field strength.

![Picture of the magnet, note the levitating wrench](a) Picture of the magnet, note the levitating wrench

![Calculated magnetic field for $|b_0| = 1$ T by Wróbel et al. with the different cube placement positions](b) Calculated magnetic field for $|b_0| = 1$ T by Wróbel et al. with the different cube placement positions

Figure 4.2: Picture (left) and schematic (right) presentation of the 10 T superconducting magnet

Making use of the Biot-Savart law, magnetic field in the whole magnet bore is numerically calculated by Wróbel et al. and presented in figure 4.2b [4]. From these calculations the magnetic field gradient inside the magnet and hence inside the enclosure can be derived. Cubical enclosure is placed in the highest field gradient, about 9 cm below or above the centre. Position above the centre is position A and below is position B, see figure 4.2b.

Figure 4.3 shows vertical profile of the calculated magnetic field strength and the magnetic field gradient for $|b_0|$ is 1, 5 and 10 Tesla. The vertical axis shows the distance from the centre of the magnetic field, $z = 0$. Magnetic field gradient in the enclosure is estimated as $\nabla B^2 = \pm 8.5 |b_0|^2 \text{T}^2 \text{m}^{-1}$. In position A the gradient is negative and in position B positive.

![Figure 4.3: Calculated vertical profile of the magnetic field (left) and field gradient (right) in the bore centre for $|b_0| = 1, 5, 10$ T](Figure 4.3: Calculated vertical profile of the magnetic field (left) and field gradient (right) in the bore centre for $|b_0| = 1, 5, 10$ T)
4.1.2. **Cubical enclosure**

Figure 4.4 and 4.5 show schematic representation and pictures of the in and outside of the cubical enclosure. The aluminium frame is visible in figure 4.4. The cube has sides of \( l = 0.032 \text{ m} \). The vertical walls are made of Plexiglass. To receive uniform heat distribution, horizontal plates are made of copper, figure 4.5a. The coolant plate is held at 18 °C. Heating plate temperature varies from 19 to 29 °C. Heating is done from below, except not in heat loss measurements. Temperature of the copper plates are measured with two T-type thermocouples, one placed in the middle and one at the side of the plate. Temperature of the fluid inside the enclosure is measured with six K-type thermocouples. The names and positions of the thermocouples are given in table 4.1. They all have a different colour that is used to represent each thermocouple throughout this research. Figure 4.5b is a picture of the Plexiglass side walls of the cube and shows the thermocouples that are used to measure fluid temperature. They are placed 6 mm inside the fluid.

<table>
<thead>
<tr>
<th>TC1</th>
<th>TC2</th>
<th>TC3</th>
<th>TC4</th>
<th>TC5</th>
<th>TC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x ) [m]</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>( y ) [m]</td>
<td>-0.008</td>
<td>0</td>
<td>0.008</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( z ) [m]</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.008</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Cubical enclosure with the heating and cooling plates and thermocouple positions

(b) Measurement distance inside the enclosure

Figure 4.4: Picture of the enclosure in its aluminium frame

Figure 4.5: Schematic representation and picture of cubical enclosure and thermocouples
4.2. **WORKING FLUIDS**

Two different fluids were investigated, one diamagnetic and one paramagnetic fluid. Pure water was used as diamagnetic fluid. Kenjereš et al. created a stability diagram from numerical simulations, showing steady, oscillatory and turbulent flow regimes, figure 4.6 [9]. This diagram is for a cubical enclosure heated from below, cooled from above, filled with a paramagnetic fluid. Temperature difference between hot and cold wall is 5°C. Magnetic force has the same direction as gravity. To investigate stability transitions, a fluid with Prandtl 70 is wanted. This resulted in a 40% glycerol-water solution which was made paramagnetic by adding 0.8 mole per kg solution of gadolinium nitrate hexahydrate \((\text{Gd(NO}_3)_3 \times 6\text{H}_2\text{O})\).

![Stability diagram from Kenjereš et al. [9]](image)

Properties of both fluids (at \(T = 18^\circ\text{C}\)) are presented in table 4.2. Density measurement where done with a pycnometer (Brand 25 ml). Kinematic viscosity is measured with an Ubbelohde viscometer. Magnetic susceptibility is measured with a Magnetic Susceptibility Balance MSB (Sherwood Scientific, AUTO). Thermal conductivity and specific heat for glycerol solution where approximated from measurements done by Pyrda et al. [5]. For water thermal conductivity and specific heat where obtained from Janssen and Warmoeskerken [26]. From these results thermal diffusivity, expansion coefficient and the Prandtl number are calculated.

Table 4.2: Fluid properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Glycerol solution</td>
<td>Pure water</td>
<td></td>
</tr>
<tr>
<td>Fluid Properties at (T = 18^\circ\text{C})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>(\rho)</td>
<td>1391</td>
<td>998</td>
<td>kg m(^{-3})</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>(\nu)</td>
<td>(6.97 \cdot 10^{-6})</td>
<td>(1.04 \cdot 10^{-6})</td>
<td>m(^2) s(^{-1})</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>(\lambda)</td>
<td>0.40</td>
<td>0.61</td>
<td>W K(^{-1}) m(^{-1})</td>
</tr>
<tr>
<td>Specific heat</td>
<td>(c_p)</td>
<td>2990</td>
<td>4185</td>
<td>J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>Thermal diffusivity</td>
<td>(\alpha)</td>
<td>(9.62 \cdot 10^{-8})</td>
<td>(1.46 \cdot 10^{-7})</td>
<td>m(^2) s(^{-1})</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>(\beta)</td>
<td>(4.60 \cdot 10^{-4})</td>
<td>(2.24 \cdot 10^{-4})</td>
<td>K(^{-1})</td>
</tr>
<tr>
<td>Mass magnetic susceptibility</td>
<td>(\chi_g)</td>
<td>(2.52 \cdot 10^{-7})</td>
<td>(-9.08 \cdot 10^{-9})</td>
<td>m(^3) kg(^{-1})</td>
</tr>
<tr>
<td>Volume magnetic susceptibility</td>
<td>(\chi)</td>
<td>(3.51 \cdot 10^{-4})</td>
<td>(-9.06 \cdot 10^{-6})</td>
<td>-</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>(\text{Pr})</td>
<td>72</td>
<td>7</td>
<td>-</td>
</tr>
</tbody>
</table>
4.3. MEASUREMENT PROCEDURE

First heat loss of the set-up must be estimated to be able to calculate the internal heat transfer $\text{Nu}$, equation 3.30. This is done with an air filled cube placed inside the magnet bore. To get a purely conducting state, cube is heated from above and cooled from below. Heat loss is calculated with equation 3.26. To investigate influence of magnetic field, measurements are done for several field strengths and increasing heating power.

Working fluid is injected in the cube with a syringe and a thin needle through a filling hole. When all air-bubbles are removed from the cube, filling hole is closed with (non-conducting) silicon paste. Enclosure is place inside the magnet bore at position A or B, see figure 4.2b. Cooling plate temperature is set at 18 °C and heating plate temperature is set by changing heating power. To get a Rayleigh-Bénard configuration the heating plate is the bottom plate.

Before starting measurements, enclosure is placed in position, cooling is turned on and the magnet bore is closed with foam. Heating power is set such that the heating plate gets approximately the preferred temperature. After about twelve hours the system is in a steady state. Heating plate temperature is checked and adjusted if needed. When the system is in a steady state again, measurements can start. Thermocouple temperatures are recorded for at least 14 minutes to get enough data for Fourier analysis. Heating power input is monitored and listed.

After first reference run the magnet is turned on and set to the required field. If necessary heating plate power is adjusted to keep right temperature difference. After 30-90 minutes system reached steady state again and temperature recordings can start. This is repeated with increased magnetic field until it reaches 10 Tesla.

4.4. DATA PROCESSING

Temperature difference between the heating and cooling plate $\Delta T$, bore temperature $T_b$, heating power $P$ and magnetic field $|b_0|$ are recorded and used to calculate heat loss, Nusselt number and Rayleigh numbers.

Thermal and thermo-magnetic Rayleigh number are calculated with equation 4.1 to 4.3.

$$\text{Ra}_T = \frac{\beta g \Delta T l^3}{v \alpha}$$ (4.1)

$$\text{Ra}_M = \frac{\beta}{v \alpha} \chi g_0 \frac{\nabla B^2}{2 \mu_0} \Delta T l^3 \left( 1 + \frac{1}{\beta T_K} \right) = \text{Ra}_T \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_K} \right) \right)$$ (4.2)

$$\text{Ra}_T M = \text{Ra}_T + \text{Ra}_M$$ (4.3)

Where $\beta_{TM} = \beta$ for diamagnetic fluids and $\gamma = \chi g_0 \frac{\nabla B^2}{2 \mu_0}$.

Nusselt number is given by equation 3.30. Include the heat loss that is estimated in chapter 5.2.1 (by equation 5.2) in equation 3.30 gives:

$$\text{Nu} = \frac{UI - 0.10 \Delta T_b}{I \Delta T}$$ (4.4)

4.4.1. UNCERTAINTY ANALYSIS

The accuracy in the measured parameters is: $\delta U = 0.01 \text{ V}$, $\delta I = 0.002 \text{ A}$, $\delta \Delta T = 0.14^\circ \text{C}$, $\delta \Delta T_b = 0.22^\circ \text{C}$ and $\delta l = 0.001 \text{ m}$. The relative uncertainty in the calculated values is derived with the following function.

$$\frac{\delta f(x_1, \ldots, x_n)}{f(x_1, \ldots, x_n)} = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \delta x_i \right)^2}$$ (4.5)

The error in the Nusselt number is:

$$\delta \text{Nu} = \sqrt{\frac{(I \delta U)^2 + (-0.10 \delta \Delta T_b)^2 + (-\text{Nu} \delta \Delta T)^2}{I \Delta T}}$$ (4.6)
RESULTS AND DISCUSSION

In this chapter the obtained data is processed and analysed. Heat loss is estimated to get a realistic heat transfer calculation. After this estimation, measurements with the different fluids for all the configurations presented in table 5.1 are processed. Maximum magnetic field strength \(|b_0|\) is used to identify different measurement configurations, although the magnetic field gradient is the main driving force behind magnetic convection.

For each measurement maximum magnetic induction \(|b_0|\), temperature difference \(\Delta T = T_h - T_c\), bore temperature \(T_b\) and heating power \(P\) where listed. With these values heat loss \(\dot{Q}_{\text{loss}}\), Nusselt number \(Nu\), thermal and thermo-magnetic Rayleigh number \(Ra\) where calculated. Results are presented in tables in appendix F.

Observed from the measurements is that the required energy to keep constant temperature difference between the hot and cold plate needed to be larger with a stronger magnetic field. However with the pure water experiments, heating power remains practically constant for each case. Remarkable is that bore temperature \(T_b\) changes with magnetic field strength and has a different starting temperature for each case. Assumed will be that environmental temperature and hence magnet bore temperature influence measurements.

In section 5.1 measured temperature-time series of the six thermocouples in the fluid and their corresponding power spectrum are presented and analysed. Starting with para- and then diamagnetic fluid. In the second part internal heat transfer is treated, again first para- and second diamagnetic fluid.

| Fluid          | Position | \(\Delta T\) | \(|b_0|\) | Case |
|----------------|----------|--------------|----------|------|
| Glycerol solution | A        | 5            | 0-6      | G5A  |
|                 | A        | 11           | 0-6      | G11A |
| Pure water      | B        | 5            | 0-10     | W5B  |
|                 | B        | 1            | 5-10     | W1B  |
|                 | A        | 5            | 0-10     | W5A  |
|                 | A        | 3            | 0-10     | W3A  |
5.1. **Temperature-time series and power spectra**

Temperatures of the heating plate, cooling plate and six positions inside the cube were recorded. These signals were smoothed with a smoothing filter and plotted versus the measurement-time. The measurement time was 14 minutes, for clearance also a zoom of the first 3 minutes is shown. The fluid inside the cube can show different characteristics flow regimes; steady, oscillating or turbulent. To verify this regime, power spectrum of the fluid temperature signals is obtained by using Fourier analysis [27]. For oscillating regime, spectrum shows one or more oscillation peaks. For turbulent regime, the slope of the power spectrum is -5/3, see chapter 3.4.1. This slope is represented with a red dashed line in the spectra. For steady regime the spectrum is flat.

### 5.1.1. Glycerol solution (paramagnetic)

For all glycerol solution experiments the cube was placed above the centre of the magnet, position A. First series has a temperature difference of 5 °C and second series has a difference of 11 °C, respectively case G5A and case G11A.

In both case G5A and case G11A magnetic field gradient has the same direction as gravity. In the lower part of the cube, where the fluid is heated, gravity causes the fluid to rise. The fluid is paramagnetic hence magnetic force also cause the warmer fluid to rise. In the upper part of the cube, where fluid is cooled again, forces get opposite direction and cause the fluid to sink. Magnetic force enhances gravitational force. Left scheme in figure 5.1 represents forces for both cases.

**Figure 5.1: Gravitational and magnetic force directions in cubical enclosure for paramagnetic fluid in position A (left) and for diamagnetic fluids in position A and B (right)**

#### CASE G5A

In this case temperature difference between hot and cold plate is \(\Delta T = 5 \, ^\circ\text{C}\). Figure 5.2 shows on the top and left smoothed temperature signals of the six thermocouples placed inside the cubical enclosure for magnetic induction of 0, 1, 2 and 6 Tesla. On the right side corresponding power spectra of the time series are presented. Spectrum of \(|b_0| = 1 \, \text{T}\) is zoomed in on small frequency range. Spectra for magnetic induction of 2 and 6 Tesla are plotted in log-log scale.

Temperature in the time series varies from 20 to 21 °C which is as expected because the mean temperature is \(T_0 = \frac{1}{2} (23 + 18) = 20.5^\circ\text{C}\). Signal amplitudes are around 0.1 degree and practically zero for \(|b_0| = 0 \, \text{T}\).

In figure 5.2a, where \(|b_0| = 0 \, \text{T}\), temperature signals are spaced and almost flat. TC3 is lower than the others, this is the thermocouple in the lower right corner. In this figure magnetic field is zero, therefore convection...
is only caused by gravity. Thermocouple 1 increases about 0.5 degrees in 14 minutes. When magnetic field is increasing, figure 5.2d, 5.2g and 5.2j, TC1 decreases again and the signals become unsteady. At \(|b_0| = 1\) T fluid starts oscillating. Thermocouples in the corners, TC1 and TC3, measure lower temperatures. Remaining thermocouples measure same temperature with little oscillations. This can indicate that magnetic force causes warm fluid to rise in the middle and sink at corners of the enclosure. With increasing magnetic field to 2 Tesla, figure 5.2g, thermocouples have approximately the same temperature. However there are no periodic oscillations observed. Figure 5.2j shows signals at \(|b_0| = 6\) T. Measured temperatures are higher and the same, except TC3 is still lower.

Power spectra in figure 5.2c and 5.2f confirm steady and oscillating flow regimes for \(|b_0|\) is 0 and 1 T. Dominant frequencies are 0.06 and 0.11 Hz. These frequencies indicate time scales for fluid movement. Fluid circulates in different patterns through the enclosure in approximately 16.67 and 9.09 seconds. Figure 5.2i show spectrum for \(|b_0| = 2\) T. The red dashed \(-\frac{5}{3}\) slope fitted through spectrum suggest turbulent flow regime. Due to turbulence, mixing in the fluid is better which increases heat transfer in the fluid. This makes the temperature distribution in the cube more homogeneous and explains why the signals become more alike and higher with increasing magnetic field. Figure 5.2j show the power spectra for 6 T. Here the turbulence slope (inertial sub-range) is shifted to higher frequencies. Meaning that energy dissipation happens earlier, hence eddies destabilize faster.

According to the simulations in stability diagram of chapter 4.2, figure 4.6, the flow regime for \(Pr \approx 70\) and \(|b_0| = 0\) T should be oscillating. For \(|b_0| = 2\) T it is steady and for \(|b_0| \geq 4\) T turbulent. In case G5A, the flow regime is steady at 0 T, oscillating at 1 T and already turbulent at 2 T. The sequence of steady and oscillating regime is reversed and turbulence starts about 2 Tesla earlier. Simulations for \(Pr = 60\) are more in agreement with measurements because oscillatory is followed by turbulent regime.

(a) Thermocouple signals of G5A for \(|b_0| = 0\) T

(b) Zoom of TC signals of G5A for \(|b_0| = 0\) T

(c) Power spectrum of G5A for \(|b_0| = 0\) T

Figure 5.2: Time series and power spectrum of case G5A for \(|b_0| = 0, 1, 2, 6\) T
Figure 5.2: Time series and power spectrum of case G5A for $|b_0| = 0, 1, 2, 6$ T (continued)
Figure 5.2: Time series and power spectrum of case G5A for $|b_0| = 0, 1, 2, 6 \text{T}$ (continued)
**CASE G11A**

Case G11A only differs from case G5A by the hot plate temperature. Here $\Delta T = 11^\circ C$. Figure 5.3 shows the smoothed time series of the six thermocouples placed in the fluid (top and left) and corresponding power spectra in log-log scale (right) for magnetic induction of 0, 2, 4 and 6 Tesla.

Mean temperature is case G11A is $T_0 = \frac{1}{2} (29 + 18) = 23.5^\circ C$. This corresponds with the temperature variations in the thermocouple signals. Amplitudes of the signals decrease with increasing of the magnetic force from 0.5 to 0.1 degree.

In the temperature-time series, top and left in figure 5.3, thermocouples signals become more alike with increasing of the magnetic field. In figure 5.3a sequence of the thermocouples is similar to case G5A at $|b_0| = 0$ T, which indicates consistency in the measurements.

From the spectrum where convection is only caused by gravity, figure 5.3c, it is visible that there is already turbulence. In the power spectra, figure 5.3c to 5.3l, turbulence slope shifts to the higher frequencies just like case G5A at 6 Tesla. Eddies in the fluid become unstable earlier. Viscosity of the glycerol solution decreases with higher temperatures, this might cause the shifting of the internal sub-range.

![Figure 5.3: Time series and power spectrum of case G11A for $|b_0| = 0, 2, 4, 6$ T](image-url)
Figure 5.3: Time series and power spectrum of case G11A for $|b_0| = 0, 2, 4, 6$ T (continued)
Figure 5.3: Time series and power spectrum of case G11A for $|b_0| = 0, 2, 4, 6 \, T$ (continued)
DISCUSSION PARAMAGNETIC FLUID

During the glycerol solution measurements there where some complications with the set-up. When the magnetic field became larger than 6 Tesla, the cubical enclosure started leaking. Due to this leakage, air bubbles came in the enclosure and the fluid wasn’t completely connected to the upper cooling plate. Now the fluid was heated from the lower side and not cooled from the top anymore. This causes the fluid temperature to rise rapidly. Because of this leakage, measurements for $|b_0| \geq 7$ are unreliable and only results for magnetic field from 0 to 6 Tesla are analysed. Figure 5.4 shows the difference between the measured signals of 4 (left) and 9 Tesla (right). At 9 T the signals are steady and about 3 degrees higher than for 4 Tesla.

![Figure 5.4: Difference between temperature signal for 4 (left) and 9 T (right) shows the effect of leakage in cubical enclosure](image)

CONCLUSION PARAMAGNETIC FLUID

As expected for both glycerol cases fluid temperature varies around mean temperature $T_0$. It can be concluded that with increasing magnetic field, characteristic flow regime becomes more turbulent and inertial sub-range of the energy spectrum shifts because eddies destabilize sooner. Energy is dissipated easier and more power is needed to keep the preferred temperature difference between the plates.
5.1.2. **Pure Water (Diamagnetic)**

For the pure water experiments the cube was first placed at position B, below the centre of the magnet. Measurements where done with temperature differences between hot and cold plate of 5 °C and 1 °C, respectively case W5B and W1B. The enclosure was also placed at position A, above centre of magnet. Here temperature difference was 5 °C and 3 °C respectively case W5A and W3A.

For each series, magnetic field was changed from 0 to 10 Tesla. There was no leakage observed during changing of the magnetic field.

When the cube is placed below the centre of the magnet, magnetic force is directed opposite to the gravitational force. Pure water is diamagnetic and repelled by magnetic force. On the bottom warm fluid is lifted by gravitational force. However the field gradient is directed in opposite direction, fluid is also lifted by magnetic force because of the negative magnetic susceptibility.

When the cube is placed above the magnet centre, field gradient has the same direction as gravity. Warm fluid is lifted by gravity but because of the negative magnetic susceptibility of water it is lowered by the magnetic force. In case W5A and W3A magnetic force suppresses gravitational force. When the magnetic force is large enough it can suppress gravity totally and even cause fluid to move in opposite direction.

Direction of the forces for the pure water cases are presented on the right in figure 5.1.

**Case W5B**

The temperature difference between the hot and cold plate is 5 °C. Top and left in figure 5.5 temperature signals obtained from the six thermocouples placed inside the fluid for magnetic field of 0, 3, 6 and 10 Tesla are shown. Next to the zoom in of temperature-time series, corresponding power spectrum are given. In this case the temperature difference and force directions are equal to case G5A, only the fluid properties differ.

Variations in the fluid temperature lie between 20 and 21 °C, with a few peaks from the lowest thermocouples to 21.5 °C. These variation are in agreement with the mean temperature, $T_0 = 20.5^\circ$C. Amplitudes are about 1 degree for all different $|b_0|$ measurements.

Visible from figure 5.5, temperature-time series and its power spectra, is that there is no relevant change with increasing magnetic field. At $|b_0| = 0$ T, characteristic flow regime is turbulent and remains turbulent. It seems that magnetic force has no influence on the fluid temperature and flow regime. This must be attributed to the small magnetic susceptibility of water.
Figure 5.5: Time series and power spectrum of case W5B for $|b_0| = 0, 3, 6, 10$ T
(g) Thermocouple signals of W5B for $|b_0| = 6 \text{ T}$

(h) Zoom of TC signals of W5B for $|b_0| = 6 \text{ T}$

(i) Power spectrum of W5B for $|b_0| = 6 \text{ T}$

(j) Thermocouple signals of W5B for $|b_0| = 10 \text{ T}$

(k) Zoom of TC signals of W5B for $|b_0| = 10 \text{ T}$

(l) Power spectrum of W5B for $|b_0| = 10 \text{ T}$

Figure 5.5: Time series and power spectrum of case W5B for $|b_0| = 0, 3, 6, 10 \text{ T}$ (continued)
**CASE W1B**

In case W1B, temperature difference between hot and cold plate is a little lower than 1 degree, $\Delta T = 0.8^\circ C$. Figure 5.7 shows six temperature-time series on the top left and its corresponding power spectrum on the right for magnetic field of 5, 7 and 10 Tesla. Measurements for $|b_0| \leq 4$ T where performed with a slightly higher heating power, nevertheless measurements where all steady. Figure 5.6 shows steady flow regime for slightly higher heating power, which results in $\Delta T = 1^\circ C$, for $|b_0| = 4$ T.

Mean temperature is $T_0 = \frac{1}{2} (18.8 + 18) = 18.4^\circ C$ and that is in agreement with the measurements. Figure 5.7a shows steady temperature-time series. With increasing magnetic field, figure 5.7d and 5.7g, temperature signals show some humps and dents. This is caused by fluid plumes that rise or fall due to buoyancy forces. Magnetic field enhances this buoyancy effect.

Corresponding with the steady flow regime at magnetic field of 5 T, power spectrum in figure 5.7c is practically flat. Spectra show some turbulence with increasing magnetic field, figure 5.7f and 5.7i. This comes from the velocity fluctuations which are cause by rising and falling of the fluid blobs.

![Figure 5.6: Previous steady flow regime measurement for $|b_0| = 4$ T](image_url)

Figure 5.6: Previous steady flow regime measurement for $|b_0| = 4$ T
Figure 5.7: Time series and power spectrum of case W1B for $|b_0| = 5, 7, 10$ T
(g) Thermocouple signals of W1B for $|b_0| = 10$ T

(h) Zoom of TC signals of W1B for $|b_0| = 10$ T

(i) Power spectrum of W1B for $|b_0| = 10$ T

Figure 5.7: Time series and power spectrum of case W1B for $|b_0| = 5, 7, 10$ T (continued)
**CASE W5A**

Temperature difference between the hot and cold plat is just like in case G5A and W5B $\Delta T = 5 \, ^\circ C$. Figure 5.8 shows temperature-time series (top and left) and its corresponding power spectrum (right) for magnetic field of 0, 3, 6 and 10 Tesla. The only difference with case W5B is the position of the enclosure, therefore magnetic force should work suppressing instead of enhancing.

As expected from the mean temperature $T_0 = 20.5 \, ^\circ C$, temperature variations in the fluid are between 20 and 21 $^\circ C$. Similar to case W5B there are a few peaks to 21.5 $^\circ C$. Amplitudes are about 1 degree for all different $|b_0|$ measurements.

Time-series and power spectra in figure 5.8 shows practically no difference with case W5B, figure 5.5. There is no significant change in the temperature signals, figure 5.8a, 5.8d, 5.8g and 5.8j, when magnetic field increases and power spectra, figure 5.8c to 5.8l, show that the characteristic flow regime stays turbulent for all measurements. By comparing both cases it can be concluded that magnetic force has no visible influence on the fluid when flow regime is turbulent.
(d) Thermocouple signals of W5A for $|b_0| = 3$ T

(e) Zoom of TC signals of W5A for $|b_0| = 3$ T

(f) Power spectrum of W5A for $|b_0| = 3$ T

(g) Thermocouple signals of W5A for $|b_0| = 6$ T

(h) Zoom of TC signals of W5A for $|b_0| = 6$ T

(i) Power spectrum of W5A for $|b_0| = 6$ T

Figure 5.8: Time series and power spectrum of case W5A for $|b_0| = 0, 3, 6, 10$ T (continued)
(j) Thermocouple signals of W5A for \(|b_0| = 10 \, \text{T}\)

(k) Zoom of TC signals of W5A for \(|b_0| = 10 \, \text{T}\)

(l) Power spectrum of W5A for \(|b_0| = 10 \, \text{T}\)

Figure 5.8: Time series and power spectrum of case W5A for \(|b_0| = 0, 3, 6, 10 \, \text{T}\) (continued)
CASE W3A
The temperature difference between the hot and cold plate is 3 °C. Top and left in figure 5.9, temperature-time series of the six thermocouples inside the fluid are shown and right their corresponding power spectrum for magnetic field of 0, 4, 7 and 10 Tesla. Power spectra of 0 and 4 Tesla are in log-log scale and spectra of 7 and 10 Tesla are in normal scale and for clearness only a small frequency range is shown.

Mean temperature in case W3A is \( T_0 = \frac{1}{2} (21 + 18) = 19.5^\circ C \). Temperature variations are between 19.3 and 19.9 °C. TC2 has a few higher peaks and its amplitude is about 0.3 degrees. TC1 has an amplitude of 0.5 °C and remaining thermocouples have one of 0.1 degree. Higher amplitudes in the left part of the cube can indicate that hot fluid is rising at this side and sinking on the right and suggest clockwise rotations.

The time series, figure 5.9a, 5.9d, 5.9g and 5.9j, show a very clear transition from turbulent to oscillating flow regime with increasing of the magnetic induction. In turbulent regime, fluid plumes rise and fall arbitrary. With increasing \(|b_0|\), magnetic force causes the fluid plumes to oscillate in a structured way. The dominant oscillating frequency are 0.05, 0.1 and 0.15 Hz. Different frequencies suggest different rotation scales.

CONCLUSION DIAMAGNETIC FLUID
From the measurement result can be concluded that case W5B and W5A are practically equal. Each measurement shows turbulence, independent on enclosure position or magnetic induction. When temperature difference \( \Delta T \) decreases, besides turbulence, steady and oscillating flow regimes were observed. Here magnetic force enhanced (case W1B) or suppressed (case W3A) gravitational force. For all case fluid temperature varied around mean temperature \( T_0 \).

(a) Thermocouple signals of W3A for \( |b_0| = 0 \) T

(b) Zoom of TC signals of W3A for \( |b_0| = 0 \) T

(c) Power spectrum of W3A for \( |b_0| = 0 \) T

Figure 5.9: Time series and power spectrum of case W3A for \( |b_0| = 0, 4, 7, 10 \) T
Figure 5.9: Time series and power spectrum of case W3A for $|b_0| = 0, 4, 7, 10$ T (continued)
Figure 5.9: Time series and power spectrum of case W3A for $|b_0| = 0, 4, 7, 10$ T (continued)
5.2. **INTERNAL HEAT TRANSFER**

In this part the effect of the magnetic field on the internal heat transfer is evaluated. Heat transfer in the enclosure is calculated with the Nusselt number from chapter 3.5, equation 3.30.

\[
\text{Nu} = \frac{UI - \dot{Q}_{\text{loss}}}{l\lambda\Delta T} \tag{5.1}
\]

In this equation heat losses through the enclosure are unknown and need to be estimated first. In part 5.2.2, internal heat transfer for enclosure filled with paramagnetic fluid (glycerol solution) is calculated and analyzed. In the next section 5.2.3 diamagnetic fluid (pure water) is examined. All the measurements and calculations are presented in appendix F.

5.2.1. **HEAT LOSS**

For the heat loss estimation an air filled cube is placed in the magnet. To create a conductive state the enclosure is heated from above and cooled from below at \( T_c = 18 \, ^\circ\text{C} \). Magnetic field \(|b_0|\) is set to 0, 5 and 10 Tesla. For each field strength, heating power \( P \) is increased from 0.06 to 0.9 W in five steps. Temperature of heating plate \( T_h \), cooling plate \( T_c \) and bore \( T_b \) are noted and heat loss is calculated with equation 3.26. Measurements are presented in table 5.2.

Heat loss is influenced by magnet bore temperature \( T_b \) which depends on the magnetic field strength. When heat loss is plotted as a function of \( \Delta T_b = T_h - T_{\text{bore}} \), figure 5.10 following relation can be determined.

\[
\dot{Q}_{\text{loss}} = 0.10\Delta T_b \tag{5.2}
\]

| \(|b_0| \) [T] | \( P \) [W] | \( \Delta T \) ['C] | \( \Delta T_b \) ['C] |
|---|---|---|---|
| 0 | 0.06 | 2.6 | -1.1 |
| 0 | 0.21 | 5.6 | 1.6 |
| 0 | 0.48 | 9.1 | 4.8 |
| 0 | 0.93 | 15.4 | 9.8 |
| 5 | 0.06 | 2.6 | 0.2 |
| 5 | 0.21 | 5.2 | 2.1 |
| 5 | 0.49 | 9.0 | 5.1 |
| 5 | 0.92 | 14.1 | 9.3 |
| 10 | 0.06 | 2.1 | 0.9 |
| 10 | 0.20 | 3.8 | 2.4 |
| 10 | 0.49 | 7.9 | 5.5 |
| 10 | 0.92 | 12.5 | 9.4 |

Figure 5.10: Heat loss estimation

Substitute estimated heat loss in equation 5.1 gives the following equation for the internal heat transfer:

\[
\text{Nu} = \frac{UI - 0.10\Delta T_b}{l\lambda\Delta T} \tag{5.3}
\]

Equation 4.6 from chapter 4.4.1 is used to calculate the error in Nusselt number:

\[
\delta\text{Nu} = \sqrt{\frac{(I \delta U)^2 + (-0.10 \delta\Delta T)^2 + (-\text{Nu} \delta\Delta T)^2}{l\lambda\Delta T}} \tag{5.4}
\]
5.2.2. **GLYCEROL SOLUTION (PARAMAGNETIC)**

Figure 5.11 and 5.12 show internal heat transfer in the cubical enclosure as a function of the magnetic induction for case G5A and G11A. In both cases Nusselt number is increasing with increasing magnetic field because the magnetic force works enhancing, which can be seen in left in figure 5.1.

For case G5A (figure 5.11) Nusselt number is almost 22 at $|b_0| = 6$ T and about 2.3 times higher than Nu at 0 Tesla. For case G11A Nusselt number is even 2.5 times higher at 6 Tesla. Comparing internal heat transfer of case G5A and G11A shows that internal heat transfer is about 1.4 times larger in case G11A which can be addressed to the bigger temperature difference $\Delta T$ between the plates.

From the scaling analysis in chapter 3.5.1 relations between $\text{Nu}$ and $\text{Ra}_{\text{Tm}}$ are derived, equation 3.31 and 3.36. To verify these relations, Nusselt is plotted versus absolute value of thermo-magnetic Rayleigh number in log-log scale in figure 5.13. The red dashed line is fitted through the measurement points and has a slope of 0.30. This is approximately equal to the experimental relation for Rayleigh-Bernard convection given by Busse [6], see chapter 3.5.1.

Orange and yellow thick dashed lines represent scaling analysis relations with respectively a slope of $1/3$ and $1/4$. Thin solid lines show 10 % error in the slopes to check if the measurement points lie within this range. From this error can be seen that measurements fit better to the turbulent scaling slope ($1/3$). This makes good sense because the thermocouples signals, figure 5.2 and 5.3 show turbulent regimes for almost all glycerol solution measurements. Also the other scaling analysis is performed for side heating.

Performed scaling analysis for turbulence is very simple, for example side wall heat losses and frictions are not taken into account. This is a possible explanation for the difference between measured and scaled slope.
Figure 5.11: Nu as a function of $|b_0|$ for case G5A

Figure 5.12: Nu as a function of $|b_0|$ for case G11A
Figure 5.13: Log-log plot of Nu versus Ra_{TM} for Glycerol solution
5.2.3. Pure Water (Diamagnetic)

Figure 5.14 to 5.17 show Nusselt number versus magnetic field strength $|b_0|$ for all the pure water cases. Errors in Nu are larger compared to the errors in the glycerol solution calculations. This is caused mostly by the smaller temperature differences and constant heating power that makes variations in Nusselt small.

In figure 5.14 a decrease in Nu of $12.0 - 10.4 = 1.6$ between 0 and 2 Tesla is observed. After this, Nusselt number increases again to 11.3. In case W5A, figure 5.16, this in- and decrease is reversed and varies between 11.5 and 12.0. Turning point is around $|b_0| = 4$ T. Magnetic force should respectively amplify and weaken internal heat transfer for case W5B and W5A. Whichever is the case for larger magnetic fields, $|b_0| \geq 4$ T. Case W5B and W5A have approximately the same Nusselt numbers which confirms the assumption that magnetic force has practically no influence on turbulent diamagnetic fluid. Difference between values of case W5B and W5A can be explained by different bore temperature which can be seen in appendix F.

As expected, Nusselt numbers are lower when the temperature difference is lower, as in case W1B and W3A respectively figure 5.15 and 5.17. There is less heat and therefore less heat transfer. Case W1B has large error-bars because of small $\Delta T$, however it has a slight heat transfer increase of $6.9 - 6.0 = 0.9$ with higher magnetic field. This confirms the presumption of enhancing heat transfer due to magnetic force. Case W3A shows a ‘M’ shape in- and decreasing pattern of internal heat transfer and it alters between 9.4 and 9.9.

In figure 5.18, Nusselt number Nu is plotted as a function of thermo-magnetic Rayleigh number $R_{\text{TM}}$ for all pure water cases. Because of the different enclosure positions $R_{\text{TM}}$ increases (position A) or decreases (position B) with larger magnetic force. This in- or decreasing of $R_{\text{TM}}$ and starting points ($|b_0| = 0$ T) are indicated with arrows. Starting points move to larger negative values of $R_{\text{TM}}$ with higher $\Delta T$. Remarkable are the starting points of cases with $\Delta T = 5$°C, they lay within each others error and can be considered equal. Which makes sense because in absence of magnetic force, $\Delta T = 5$°C cases are practically identical. When the enclosure is in position A, thermo-magnetic Rayleigh number changes from negative to positive. When $R_{\text{TM}} = 0$, magnetic and gravitational force are cancelled out and heat transfer would be only conductive. However smallest value is $R_{\text{TM}} = -0.3 \cdot 10^6$, which is much larger than zero.
Figure 5.14: Nu as a function of $|b_0|$ for case W5B

Figure 5.15: Nu as a function of $|b_0|$ for case W1B
Figure 5.16: Nu as a function of $|b_0|$ for case W5A

Figure 5.17: Nu as a function of $|b_0|$ for case W3A
Figure 5.18: Nu versus Ra_{TM} for all pure water cases
5.2.4. **Heat Transfer Conclusions**

In figure 5.19 Nusselt number versus thermo-magnetic Rayleigh number for all the glycerol solution and pure water measurements are plotted. \( Ra_{TM} \) grows with larger magnetic force for case W5A and W3A and for the other cases \( Ra_{TM} \) is negative and decreases. Absolute Rayleigh number for the glycerol solution are much higher compared to pure water measurements, even though maximum magnetic field strength is \(|b_0| = 6 \) T. This can be ascribed to the difference in mass magnetic susceptibility \( \chi_g \).

From analysis of internal heat transfer for glycerol solution (paramagnetic fluid) can be seen that Nu increased between 2.3 and 2.5 times for magnetic field of 6 Tesla. Magnetic force does enhance gravitational force which results in better mixing of the fluid and hence better heat transfer. When \( \Delta T \) is higher Nusselt number is higher as well.

From pure water (diamagnetic fluid) cases can be concluded that magnetic force hardly influences internal heat transfer. Especially for large (\( > 3 \)) temperature difference between heating and cooling plate. Nusselt number are assumed to be constant and are, as in glycerol solution cases, higher for larger \( \Delta T \).
Figure 5.19: Nu versus $\text{Ra}_{TM}$ for all glycerol and water cases
Aim of this research is to investigate thermo-magnetic convection in para- and diamagnetic fluids. This is realized by placing a small cubical enclosure filled with fluid in a 10 Tesla superconducting magnet. As a guideline for this study in the introduction of this thesis 5 research goals are given. In this chapter conclusions for each goal are provided.

**Goal 1:** Influence of magnetic force direction on heat transfer by measuring at different enclosure positions

For paramagnetic fluids cubical enclosure was only placed in one position. For diamagnetic fluid two positions where examined. From these measurements can be concluded that enclosure position and hence magnetic force direction does influence thermo-magnetic convection by enhancement or suppression. However for turbulent flow (large temperature difference) magnetic force is not able to dominate thermal effects.

**Goal 2:** Determine characteristic flow regime from local temperature measurements

Local fluid temperatures measured with six thermocouples placed inside cubical enclosure. Characteristic fluid flow regimes like steady, oscillating and turbulence were observed from temperature-time measurements. Corresponding power spectra, obtained with FFT, were examined to validate these observed flow regimes.

**Goal 3:** Examine flow regime transition by measuring fluid with $Pr \approx 70$ at different magnetic field strengths

Transition between flow regimes is seen in case G5A, W1B and W3A. For paramagnetic case G5A transition from steady to oscillation to turbulent is observed. Difference in this sequence compared to numerical simulations are the first two regimes. Also transition happen at lower magnetic field strength. In diamagnetic case W1B transition from steady to unsteady is visible. Transition from turbulent to oscillating is observed in case W3A.

**Goal 4:** Influence of thermo-magnetic convection on internal heat transfer

From measured heating and cooling plate temperatures, heating plate power and bore temperature Nusselt number is calculated. Nu is a measure for internal heat transfer. For paramagnetic fluids a strong increase in Nu is observed at magnetic field of 6 Tesla, about 2.3 times higher than for 0 Tesla. Turbulent thermo-magnetic convection causes better mixing and less heat is needed to keep the wanted heating plate temperature. With diamagnetic fluid measurements little expected in- and decrease of Nu is seen. However due to large errors and small magnetic field influence no strong conclusions can be formulated.

**Goal 5:** Verify (theoretical) relations between heat transfer and internal forces from previous researches

Experimental relation between heat transfer and forces obtained from paramagnetic measurements is $Nu \sim Ra_{TM}^{0.30}$. This is in good agreement with previous experimental relation ($Ra_{TM}^{0.28}$) and scaling analysis for turbulent motion ($Ra_{TM}^{1/3}$). No relation for diamagnetic measurements is derived due to large errors and small variations in Nusselt number.
In this thesis thermo-magnetic convection in para- and diamagnetic fluids is experimentally investigated. This chapter gives several recommendations and advice to obtain results for more profound research with the used 10 Tesla superconducting magnet.

- Measure glycerol solution in position B to study suppression of thermo-magnetic convection.
- To observe transition between flow regimes in more detail, measure glycerol solution with $\Delta T = 5^\circ C$ in smaller $|b_0|$ steps (0.5 or 0.1 T).
- For diamagnetic transitions examine pure water in position B with small $\Delta T$ ($\sim 2^\circ C$)
- To investigate relation between $\text{Nu}$ and $Ra_{TM}$ for diamagnetic fluids a strong diamagnetic solution can be used.
- Interesting would be zero-gravity case, magnetic force opposes gravity exactly and $Ra_{TM} = 0$.

- To get a better impression of fluid structures and temperatures in the enclosure, fluid behaviour can be visualized with liquid crystals [3][10].
- By using PIV [28] on visualized fluid structures a velocity vector field can be obtained.
- To get realistic numerical simulations fluid properties need to be measured for different temperatures and if necessary magnetic field strengths.
- Velocities en temperatures coming from the visualizations can be compared to simulations.
This appendix contains:

- Front of abstract booklet
- Abstract
- Poster
A cubical enclosure represents a geometrical idealization for various engineering fluid flows. Additionally, the cubical enclosure with varying temperatures on opposite walls (heated and cooled horizontal walls, Rayleigh-Benard configuration) can be found in many real systems, both on macro and micro scales— everywhere where temperature difference between the walls occurs in a closed space. Some applications include systems for storage of solar energy and cooling of electronical devices. In nature, the thermal streams that arise in lakes and reservoirs can also be represented by such closed domain.

In the engineering applications the natural convection is characterized by high values of non-dimensional parameters (the Rayleigh number) describing ratio between gravity force and thermal diffusivity. It means that for study of real flows the large cavity, high temperature gradients or the fluids of special properties are necessary. With an application of high magnetic gradients and common fluids it is possible to obtain flows and heat transfer, which are inaccessible in normal conditions. Moreover, the experimental data and signal analysis can be a source of additional information on the flow stability. Due to this such phenomena as transition between laminar and turbulent flow can be studied in details.

In the present work we want to show and describe the analysis of weakly magnetic fluids (paramagnetic and diamagnetic) in Rayleigh-Benard configuration (Fig. 1) placed in the strong magnetic gradient. This gradient generates the magnetic force supporting the gravitational one, what results in enhanced convection. The transition between different flow types is determined. From the other side, for the unstable in the small temperature gradients fluid (like water) the location of transition was searched in the system, in which magnetic gradient generated the magnetic force opposing the gravitational one and finally suppressing the convection. To obtain the information on flow type and its changes the Fast Fourier Transform was applied. The exemplary analysis is shown in Fig. 2.
Described analysis and results enriched the collection of experimental data, which can be used for validation of numerical models. They also show the possibility of obtaining the high values of Rayleigh number in simple geometry and conditions.

Fig. 2. (a) Temperature signal (oscillatory flow), (b) its FFT analysis

Fig. 1. Schematic view of cubical enclosure

Acknowledgements
The present work was supported by the Polish Ministry of Science (Grant AGH No. 11.11.210.198)

http://www.kpce.agh.edu.pl/
http://cheme.nl/tp/
http://www.researchgate.net/profile/Elzbieta_Fornalik-Wajs/
http://www.researchgate.net/profile/Sasa_Kenjeres/
Stability of thermo-magnetic convection in magnetic fluids

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Introduction

- Forced and natural (thermo-gravitational) convection
- Published in different applications (crystal growth)
- Thermo-convection
- Magnetic field is exposed to a strong non-uniform magnetic field
- Fluid attracted or repulsed by magnetic force.

Fluid properties at T = 18 °C

<table>
<thead>
<tr>
<th>Property</th>
<th>Paramagnetic</th>
<th>Diamagnetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% Glycerol solution</td>
<td>0.8 kg/m cryd</td>
<td>0.8 kg/m cryd</td>
</tr>
<tr>
<td>0.8 kg/m cryd</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>( \rho )</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>( \nu )</td>
<td>( \nu )</td>
</tr>
<tr>
<td>Thermal conductivity ( \lambda )</td>
<td>( \lambda )</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Heat capacity ( c_p )</td>
<td>( c_p )</td>
<td>( c_p )</td>
</tr>
<tr>
<td>Thermal diffusivity ( \alpha )</td>
<td>( \alpha )</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>( \beta )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Mass magnetic susceptibility</td>
<td>( \chi_m )</td>
<td>( \chi_m )</td>
</tr>
<tr>
<td>Volume magnetic susceptibility</td>
<td>( \chi_v )</td>
<td>( \chi_v )</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>( Pr )</td>
<td>( Pr )</td>
</tr>
</tbody>
</table>

Heat transfer

\[
\frac{\rho}{\nu} \frac{\Delta T}{\Delta T} = \frac{P - \dot{Q}_{\text{loss}}}{\Delta T} = 4095 \frac{T_b}{\Delta T} = \frac{c_p g T^3}{\nu \alpha}
\]

Magnetic Rayleigh number

\[
Ra_M = Ra_T \frac{\gamma}{2} \left[ 1 + \frac{1}{\beta T_0} \right]
\]

Rayleigh number for TM convection

\[
Ra_{TM} = Ra_T + Ra_M
\]

Scaling analysis for cube heated from below

\[
\text{Nu} \sim Ra_{TM}^{0.2}
\]

Conclusion

- Experiments confirmed theoretically estimated scaling of heat transfer: \( \text{Nu} \sim Ra_{TM}^{0.2} \)
- Targeted control of heat transfer (enhancement or suppression) possible
CONSERVATION LAWS IN MAGNETIC FIELD

In this appendix the conservation equations are adjusted by including magnetic field term and simplified to fit this research.

Starting with conservation equations from Kundu and Cohen [15].

Conservation of mass is:
\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \quad (B.1)
\]

Conservation of momentum is:
\[
\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial P}{\partial x_i} + f_i + \frac{\partial \sigma_{ij}}{\partial x_j} \quad (B.2)
\]

The forces are given by:
\[
f_i = f_{g_i} + f_{m_i} \quad (B.3)
\]

With \( f_{g_i} = \rho g_i \), the gravitational force and \( f_{m_i} \) the magnetic force from chapter 2.2, equation 2.9.

The deviatoric stress tensor is given by:
\[
\sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \quad (B.4)
\]

Conservation of momentum becomes:
\[
\rho \frac{D u_i}{D t} = -\frac{\partial P}{\partial x_i} + \rho g_i + \frac{1}{2 \mu_0 \rho} \frac{\partial \chi g \partial B^2}{\partial x_i} + \frac{\partial}{\partial x_j} \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \right) \quad (B.5)
\]

Conservation of energy:
\[
\rho c_p \left( \frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} \right) - p \frac{\partial u_j}{\partial x_j} + \Phi \quad (B.6)
\]

With viscous energy dissipation
\[
\Phi = 2 \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 - \frac{2}{3} \mu \left( \frac{\partial u_k}{\partial x_k} \right)^2 \quad (B.7)
\]

EQUATION OF STATE
To solve these equations, an equation of state is necessary [18]. Here it is the momentum equation at \( T_0 \), at this temperature the density is \( \rho_0 \), mass magnetic susceptibility is \( \chi g_0 \), pressure is \( P_0 \) and the velocity is zero. This is called conducting state.
\[
0 = -\frac{\partial P_0}{\partial x_i} + \rho_0 g_i + \frac{\partial \chi g \partial B^2}{2 \mu_0 \partial x_i} \quad (B.8)
\]
It is desired to work with the dynamic pressure, \( P_d \) which is given by:

\[
\frac{\partial P_d}{\partial x_i} = \frac{\partial P}{\partial x_i} - \rho_0 g_i - \frac{\rho_0 \chi_{\text{Bo}}}{2 \mu_0} \frac{\partial B^2}{\partial x_i}
\]  
(B.9)

The conservation of momentum can be rewritten with the dynamic pressure. Now two buoyancy terms become visible.

\[
\rho \frac{Du_i}{Dt} = -\frac{\partial P_d}{\partial x_i} + (\rho - \rho_0) g_i + (\rho \chi_g - \rho_0 \chi_{\text{Bo}}) \frac{1}{2 \mu_0} \frac{\partial B^2}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\zeta}{3} \frac{\partial u_k}{\partial x_k} \right)
\]  
(B.10)

**BOUSSINESQ APPROXIMATION**

Because of the very small temperature variations, \( \mu, c_p \) and \( \lambda \) can assumed to be constant. According to the Boussinesq approximation density variations can be ignored everywhere but not in the buoyancy terms [18]. Here density is approximated with the Taylor expansion at \( T_0 \). Although density of water is not linear dependent on temperature it can be approximated with the Taylor expansion because of the small temperature differences.

\[
\rho = \rho_0 + \left\{ \frac{\partial \rho}{\partial T} \right\}_{T_0} (T - T_0), \quad \frac{\partial \rho}{\partial T}_{T_0} = -\rho_0 \beta
\]  
(B.11)

\[
\rho - \rho_0 = -\rho_0 \beta (T - T_0)
\]  
(B.12)

Where \( \beta \) is the (assumed constant) thermal expansion coefficient and \( T_0 \) the temperature at which \( \rho = \rho_0 \).

Because magnetic susceptibility is temperature dependent for paramagnetic fluids, the term \( \rho \chi_g - \rho_0 \chi_{\text{Bo}} \) is approximated with Taylor expansion as well. Mass magnetic susceptibility for paramagnetics is given by Curie’s law [14]. Important here is to use the absolute temperature (\( T_K = 273 + T \) and \( T_{K_0} = 273 + T_0 \)).

\[
\chi_g = \frac{C}{T_K}, \quad \chi_{\text{Bo}}(T_0) = \frac{C}{T_{K_0}} = \chi_{\text{Bo}0}, \quad C = \chi_{\text{Bo}0}(T_{K_0})
\]  
(B.13)

Again use Taylor expansion at \( T_0 \):

\[
\rho \chi_g - \rho_0 \chi_{\text{Bo}0} = \left\{ \frac{\partial (\rho \chi_g)}{\partial T} \right\}_{T_0} (T - T_0) = \left\{ \rho \frac{\partial \chi_g}{\partial T} + \chi_g \frac{\partial \rho}{\partial T} \right\}_{T_0} (T - T_0)
\]

\[
= \left[ -\rho_0 \frac{\chi_{\text{Bo}0}}{T_{K_0}} - \chi_{\text{Bo}0} \rho_0 \beta \right] (T - T_0) = -\rho_0 \chi_{\text{Bo}0} \left( \beta + T_{K_0}^{-1} \right) (T - T_0)
\]  
(B.14)

For diamagnetic fluids the magnetic susceptibility is independent of temperature, \( \chi_g = \chi_{\text{Bo}0} \). This results in:

\[
\rho \chi_g - \rho_0 \chi_{\text{Bo}0} = (\rho - \rho_0) \chi_{\text{Bo}0} = -\rho_0 \chi_{\text{Bo}0} \beta (T - T_0)
\]  
(B.15)

Use these approximations in the conservation equations. The continuity equation becomes:

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  
(B.16)

Use this result and previous assumptions in the momentum equation. The shear stress becomes:

\[
\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\zeta}{3} \frac{\partial u_k}{\partial x_k} \right) = \mu \left( \frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial^2 u_j}{\partial x_i \partial x_j} \right) = \mu \frac{\partial^2 u_i}{\partial x_j^2}
\]  
(B.17)

With kinematic viscosity \( \nu = \frac{\mu}{\rho_0} \), the momentum equation becomes:

\[
\frac{Du_i}{Dt} = -\frac{1}{\rho_0} \frac{\partial P_d}{\partial x_i} - \beta (T - T_0) g_i - \chi_{\text{Bo}0} \left( \beta + T_{K_0}^{-1} \right) (T - T_0) \frac{1}{2 \mu_0} \frac{\partial B^2}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2}
\]  
(B.18)
In the energy equation a scaling analysis is performed to check the order of the different terms. First use assumptions for constants, Boussinesq approximation, continuity equation and use thermal diffusion coefficient $\alpha = \frac{1}{\rho_0 c_p}$.

\[
\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2} + \frac{1}{\rho_0 c_p} \Phi \tag{B.19}
\]

Use $l = 0.032$ m as length scale, $t = \frac{l^2}{\alpha}$ as time scale and $T = \Delta T$ as temperature scale. The orders for paramagnetic fluid are:

\[
\frac{\partial T}{\partial t} \sim u_j \frac{\partial T}{\partial x_j} \sim \alpha \frac{\partial^2 T}{\partial x_j^2} \sim \alpha \frac{\Delta T}{l^2} = \mathcal{O}(10^{-3}), \quad \frac{1}{\rho_0 c_p} \Phi \sim \frac{\mu}{\rho_0 c_p} \left( \frac{\alpha}{l^2} \right)^2 = \mathcal{O}(10^{-17}) \tag{B.20}
\]

The order of the viscous dissipation is much lower than the others, which results in:

\[
\frac{DT}{Dt} = \alpha \frac{\partial^2 T}{\partial x_j^2} \tag{B.21}
\]

Final set of equations in vector form is:

\[
\nabla \cdot \vec{u} = 0 \tag{B.22}
\]

\[
\frac{D}{Dt} \vec{u} = -\frac{1}{\rho_0} \nabla P_d - \beta (T - T_0) \vec{g} - \chi_{\vec{E}0} \left( \beta + T_{E0}^{-1} \right) (T - T_0) \frac{1}{2\mu_0} \frac{\partial B^2}{\partial x_i} + \nu \nabla^2 \vec{u} \tag{B.23}
\]

\[
\frac{DT}{Dt} = \alpha \nabla^2 T \tag{B.24}
\]
In this appendix the conservation equations in a magnetic field are non-dimensionalized [18].

For this research the following set of equations and boundary conditions are derived in appendix B and chapter 3.2.

\[ \nabla \cdot \vec{u} = 0 \quad (C.1) \]

\[ \frac{D}{Dt} \vec{u} = -\frac{1}{\rho_0} \nabla P_d - \beta g \left( 1 + \frac{Y}{2} \left( 1 + \frac{1}{\beta T_{K_0}} \right) \right) (T - T_0) \hat{z} + \nu \nabla^2 \vec{u} \quad (C.2) \]

\[ \frac{DT}{Dt} = \alpha \nabla^2 T \quad (C.3) \]

Boundary conditions:

\[ T \left( \pm \frac{1}{2} l \right) = T_0 \pm \frac{1}{2} \Delta T, \quad T(0) = T_0 \quad (C.4) \]

\[ P_d \left( \frac{1}{2} l \right) = P_0 \left( -\frac{1}{2} l \right) = 0 \quad (C.5) \]

\[ w \left( \pm \frac{1}{2} l \right) = \frac{\partial w}{\partial z} \left( \pm \frac{1}{2} l \right) = 0 \quad (C.6) \]

These can be made dimensionless by using following dimensionless parameters.

\[ t' = \frac{t}{t_0}, \quad x' = \frac{x}{x_0}, \quad u' = \frac{\vec{u}}{u_0}, \quad p' = \frac{P_d}{p_0}, \quad \Theta = \frac{T - T_0}{\Delta T} \quad (C.7) \]

With

\[ t_0 = \frac{l^2}{a}, \quad x_0 = l, \quad u_0 = \frac{a}{l}, \quad p_0 = \rho_0 \frac{a^2}{l^2}, \quad T_0 = \frac{1}{2} (T_h + T_c), \quad \Delta T = T_h - T_c \quad (C.8) \]

Where \( l = 0.032 \text{ m} \) the characteristic length, \( a \) thermal diffusivity and \( \rho_0 \) the density at \( T_0 \).

Derivatives become:

\[ \frac{D}{Dt} = \frac{a}{l^2} \frac{D}{Dt'}, \quad \nabla = \frac{1}{l} \nabla', \quad \nabla^2 = \frac{1}{l^2} \nabla'^2 \quad (C.9) \]

These parameters and derivatives can be substituted into the equations and boundary conditions.

Equation C.1 becomes:

\[ \frac{1}{l} \nabla' \cdot \frac{a}{l} \vec{u} = \frac{a}{l^2} \nabla' \cdot \vec{u} = 0 \quad \Rightarrow \quad \nabla' \cdot \vec{u}' = 0 \quad (C.10) \]
Different terms of equation C.2 become:

\[
\frac{D}{Dt} \vec{u} = \frac{a^2}{l^3} \frac{D}{Dt} \vec{u}'
\]

(C.11)

\[
\frac{1}{p_0} \nabla P_d = \frac{a^2}{l^3} \nabla' p'
\]

(C.12)

\[(T - T_0) \hat{z} = \Delta T \Theta \hat{z}\]

(C.13)

\[
\nu \nabla^2 \vec{u} = \frac{\nu}{l^3} \nabla'^2 \vec{u}'
\]

(C.14)

Divide each term by \(\frac{a^2}{l^3}\) gives:

\[
\frac{D}{Dt}' \vec{u}' = -\nabla' p' - \frac{\beta g}{\alpha} \Delta T \Theta \hat{z} + \frac{\nu}{\alpha} \nabla'^2 \vec{u}'
\]

(C.15)

Because \(T_0\) is a constant, equation C.3 becomes:

\[
\frac{\alpha}{l^2} \frac{D}{Dt} (\Delta T \Theta + T_0) = \frac{\alpha}{l^2} \nabla'^2 (\Delta T \Theta + T_0) \Rightarrow \frac{D}{Dt}' \Theta = \nabla'^2 \Theta
\]

(C.16)

Boundary conditions become:

\[
T(\pm \frac{1}{2}l) - T_0 = \pm \frac{1}{2} \Delta T = \Delta T \Theta (\pm \frac{1}{2}) \Rightarrow \Theta (\pm \frac{1}{2}) = \pm \frac{1}{2}
\]

(C.17)

\[
T(0) - T_0 = 0 = \Delta T \Theta (0) \Rightarrow \Theta (0) = 0
\]

(C.18)

\[
P_d(-\frac{1}{2}l) = \rho_0 \frac{a^2}{l^2} p'(-\frac{1}{2}) = 0 \Rightarrow p'(-\frac{1}{2}) = 0
\]

(C.19)

\[
\frac{\alpha}{l} w'(\pm \frac{1}{2}l) = \frac{\alpha}{l^2} \frac{\partial w'}{\partial z'} (\pm \frac{1}{2}l) = 0 \Rightarrow w'(\pm \frac{1}{2}) = \frac{\partial w'}{\partial z'} (\pm \frac{1}{2}) = 0
\]

(C.20)

For convenience buoyancy term of the momentum equation is multiplied with '\(\gamma\)' and final non-dimensional set of equations is:

\[
\nabla' \cdot \vec{u}' = 0
\]

(C.21)

\[
\frac{D}{Dt}' \vec{u}' = -\nabla' p' - \frac{\beta g}{\alpha} \Delta T l^3 \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T K_0}\right)\right) \Theta \hat{z} + \frac{\nu}{\alpha} \nabla'^2 \vec{u}'
\]

(C.22)

\[
\frac{D}{Dt}' \Theta = \nabla'^2 \Theta
\]

(C.23)

With non-dimensional boundary conditions:

\[
\Theta (\pm \frac{1}{2}) = \pm \frac{1}{2}, \quad \Theta (0) = 0, \quad p'(-\frac{1}{2}) = 0
\]

(C.24)

\[
w'(\pm \frac{1}{2}) = \frac{\partial w'}{\partial z'} (\pm \frac{1}{2}) = 0
\]

(C.25)
LINEAR STABILITY ANALYSIS

In this appendix a linear stability analysis for thermo-magnetic convection in an enclosure with rigid boundaries heated from below based on the method from Pellew and Southwell [20] (also used in Deen [18] and Chandrasekhar [19]) is performed.

Starting point are the conservation equations from appendix B and the boundary conditions from chapter 3.2.

\[ \nabla \cdot \mathbf{u} = 0 \quad (D.1) \]
\[ \frac{D}{Dt} \mathbf{u} = -\frac{1}{\rho_0} \nabla p_d - \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) (T - T_0) \hat{z} + \nu \nabla^2 \mathbf{u} \quad (D.2) \]
\[ \frac{dT}{Dt} = \alpha \nabla^2 T \quad (D.3) \]

\[ T(0) = T_0 \quad , \quad T(\pm \frac{1}{2}l) = T_0 \pm \frac{1}{2} \Delta T \quad (D.4) \]
\[ p_d(-\frac{1}{2}l) = 0 \quad (D.5) \]
\[ w(\pm \frac{1}{2}l) = \frac{\partial w}{\partial z}(\pm \frac{1}{2}l) = 0 \quad (D.6) \]

In the initial steady state heat transfer is purely conductive. In such a stable situation there is no motion in the fluid. The steady momentum and energy equation (D.7 and D.8) are used to determine a static temperature \( T_s \) and pressure \( P_s \) function only depending on vertical \( (z) \) direction.

\[ 0 = \alpha \frac{d^2}{dz^2} T \quad (D.7) \]
\[ 0 = -\frac{1}{\rho_0} \frac{d}{dz} p_d - \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) (T_s - T_0) \quad (D.8) \]

Solving steady energy equation D.7 with boundary conditions D.4 gives equation D.9 as the steady temperature function.

\[ T_s(z) = T_0 - \frac{\Delta T}{l} z \quad (D.9) \]

Substitute equation D.9 in steady momentum equation D.8 and use boundary condition D.5 to solve the steady pressure function, equation D.10.

\[ P_s(z) = \frac{1}{2} \rho_0 \beta g \frac{\Delta T}{l} \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) \left[ z^2 - \left( \frac{l}{2} \right)^2 \right] \quad (D.10) \]
Instabilities can be caused by small disturbances in pressure $P$, temperature $T$ or velocity $\vec{u}$. These variables will be represented as a static function (respectively $P_0$, $T_0$, and $\vec{u} = 0$) plus a perturbation term (respectively $p$, $\theta$ and $\vec{u}$).

\[
P(x, y, z, t) = P_0(x, y, z) + p(x, y, z, t) \tag{D.11}
\]

\[
T(x, y, z, t) = T_0(x, y, z) + \theta(x, y, z, t) \tag{D.12}
\]

\[
\vec{u}(x, y, z, t) = 0 + \vec{u}(x, y, z, t) \tag{D.13}
\]

Perturbation functions can be substituted into the conservation equations. Perturbations are small and the double perturbation terms can be neglected because for example $u^2 \ll u$. The continuity equation D.1 remains the same and the different terms of the momentum equation D.2 become:

\[
\frac{D}{Dt} \vec{u} = \frac{\partial}{\partial t} \vec{u} + \vec{u} \cdot \nabla \vec{u} \approx \frac{\partial}{\partial t} \vec{u} \tag{D.14}
\]

\[
\nabla p' = \frac{d}{dz} P_0 + \nabla p = \rho_0 \beta \frac{\Delta T}{T} \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) z + \nabla p \tag{D.15}
\]

\[
T - T_0 = T_s + \theta - T_0 = -\frac{\Delta T}{l} z + \theta \tag{D.16}
\]

Static pressure and temperature terms cancel, viscous part does not change and final result for the momentum equation is:

\[
\frac{\partial}{\partial t} \vec{u} = -\frac{1}{\rho_0} \nabla p - \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) \theta \hat{z} + \nu \nabla^2 \vec{u} \tag{D.17}
\]

Various terms of the energy equation D.3 become:

\[
\frac{\partial}{\partial t} \theta = \frac{\partial}{\partial t} \theta_s + \frac{\partial}{\partial t} \theta + w \frac{d}{dz} T_s + \vec{u} \cdot \nabla T \approx \frac{\partial}{\partial t} \theta - \frac{\Delta T}{l} w \tag{D.18}
\]

\[
a \nabla T = a \frac{d^2}{dz^2} T_s + a \nabla^2 \theta = a \nabla^2 \theta \tag{D.19}
\]

Which results in:

\[
\frac{\partial}{\partial t} \theta = \frac{\Delta T}{l} w + a \nabla^2 \theta \tag{D.20}
\]

There are now 5 equations and 5 unknowns left, $p$, $\theta$ and $\vec{u}$. To reduce this number, the curl of equation D.17 is taken.

\[
\nabla \times \left[ \frac{\partial}{\partial t} \vec{u} \right] = -\nabla \times \left[ \frac{1}{\rho_0} \nabla p \right] - \nabla \times \left[ \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) \theta \hat{z} \right] + \nabla \times [\nu \nabla^2 \vec{u}] \tag{D.21}
\]

Use $\nabla \times \nabla p = 0$ to eliminate $p$ and use the vorticity $\vec{\omega} = \nabla \times \vec{u}$.

\[
\frac{\partial}{\partial t} \vec{\omega} = -\beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) \left( \frac{\partial \theta}{\partial y} \hat{x} - \frac{\partial \theta}{\partial x} \hat{y} \right) + \nu \nabla^2 \vec{\omega} \tag{D.22}
\]

Take the curl again

\[
\nabla \times \left[ \frac{\partial}{\partial t} \vec{\omega} \right] = -\nabla \times \left[ \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) \left( \frac{\partial \theta}{\partial y} \hat{x} - \frac{\partial \theta}{\partial x} \hat{y} \right) \right] + \nabla \times [\nu \nabla^2 \vec{\omega}] \tag{D.23}
\]

Use continuity equation D.1 and use $\nabla \times \vec{\omega} = \nabla \times (\nabla \times \vec{u}) = \nabla (\nabla \cdot \vec{u}) - \nabla^2 \vec{u} = -\nabla^2 \vec{u}$ to get:

\[
\frac{\partial}{\partial t} \nabla^2 \vec{u} = -\beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) \left( \frac{\partial^2 \theta}{\partial x \partial z} \hat{x} - \frac{\partial^2 \theta}{\partial y \partial z} \hat{y} + \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \hat{z} \right) + \nu \nabla^4 \vec{u} \tag{D.24}
\]
Because equation D.20 is independent of \( u \) and \( v \), the number of unknowns can be reduced to 2 when equation D.20 and the \( z \)-component of equation D.24 are considered. The two resulting equations only depending on \( w \) and \( \theta \) with \( \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) are:

\[
\begin{align*}
\left( \frac{\partial}{\partial t} - \alpha \nabla^2 \right) \theta &= \frac{\Delta T}{l} w \\
\left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) \nabla^2 w &= -\beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_{\infty}} \right) \right) \nabla_1^2 \theta
\end{align*}
\]  

(D.25)  

(D.26)

Boundary conditions for temperature perturbation are obtained from the assumption that the temperature is constant at the surfaces and therefore \( \theta = 0 \). Boundary conditions for velocity perturbations are given in equation D.6. However, equation D.29 is a 6th order equation two more boundary conditions for \( w \) are required. Using boundary conditions of \( \theta \) in equation D.26 gives two more conditions for \( w \). The boundary conditions for velocity and temperature perturbations at \( z = \pm \frac{1}{2} l \) are:

\[
\begin{align*}
\theta &= 0 \\
w &= \frac{\partial w}{\partial z} = \left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) \nabla^2 w = 0
\end{align*}
\]  

(D.27)  

(D.28)

Substituting equation D.25 into equation D.26 to eliminate \( \theta \) results in an equation for \( w \).

\[
\left[ \left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) \left( \frac{\partial}{\partial t} - \alpha \nabla^2 \right) \nabla^2 + \beta g \frac{\Delta T}{l} \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_{\infty}} \right) \right) \nabla_1^2 \right] w = 0
\]  

(D.29)

By eliminating \( w \) a similar equation for \( \theta \) is obtained. Because they are similar only one needs to be solved. Equation D.29 can be solved using separation of variables. Assume \( w \) can be represented as a linear superposition of functions of the form:

\[
w(x, y, z, t) = W(z) e^{i(c_x x + c_y y)} \alpha t
\]  

(D.30)

Here \( s \) is a real constant and the disturbance will grow or decay with time rather than oscillate. The stability of the system with respect to all disturbances can be examined by letting the particular wave number, \( c^2 = c_x^2 + c_y^2 \), vary from 0 to \( \infty \). It is not necessary to know the contribution of individual modes to any particular disturbance. The derivatives of \( w \), with \( \frac{\partial w}{\partial z} = D^2 \), are evaluated as follows:

\[
\frac{\partial}{\partial t} w = sw, \quad \nabla_1^2 w = -c^2 w, \quad \nabla^2 w = -c^2 w + D^2 w
\]  

(D.31)

Substituting function D.30 and derivatives D.31 into equation D.29 gives:

\[
\left[ \left( \frac{s^2}{\nu} + c^2 - D^2 \right) \left( \frac{s^2}{\alpha} + c^2 - D^2 \right) (D^2 - c^2) - c^2 \frac{\beta g \Delta T}{\nu \alpha} \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_{\infty}} \right) \right) \right] W = 0
\]  

(D.32)

Substituting following dimensionless quantities, \( \zeta' = \tilde{\xi}, \ D' = l D, \ \lambda = l \lambda, \ \sigma = \frac{l \sigma}{\alpha} \) together with \( \text{Pr} \) and \( \text{Ra}_{TM} \) from chapter 3.3.1 into equation D.32 results in:

\[
(\text{Pr}^{-1} \sigma + l^2 - D^2) \left( \sigma + l^2 - D^2 \right) (D^2 - l^2) W = l^2 \text{Ra}_{TM} W
\]  

(D.33)

and the boundary conditions D.27 and D.28 at \( z' = \pm \frac{1}{2} \) become:

\[
W = D' W = \left( \text{Pr}^{-1} \sigma + l^2 - D^2 \right) (D^2 - l^2) W = 0
\]  

(D.34)

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Limiting conditions for stability are obtained when all time variations are made zero, hence $\frac{\partial}{\partial t} = 0 \Rightarrow s = 0 \Rightarrow \sigma = 0$. Include this stability condition into equation D.33 and D.34 and substitute $\lambda^2 \text{Ra}_{TM}$ with $-\lambda^6 \varepsilon^3$ (with $\varepsilon > 0$) to get:

$$\left[ (D^2 - \lambda^2)^3 + \lambda^6 \varepsilon^3 \right] W = 0$$  \hspace{1cm} (D.35)

$$W = D'W = (D^2 - \lambda^2)^3 W = 0$$  \hspace{1cm} (D.36)

General solution for this homogeneous differential equation is $W = A e^{2qz'}$ and can be separated in an even and odd solution. The even solution is:

$$W = A_0 \cos (2q_0 z') + A \cosh (2qz') + \bar{A} \cosh (2\bar{q}z')$$  \hspace{1cm} (D.37)

Where $\pm (2i q_0)^2$, $\pm (2q)^2$ and $\pm (2\bar{q})^2$ the roots of equation D.35. With $q_0 = \frac{1}{2} \lambda \sqrt{1 - 1 - i \varepsilon + \frac{1}{2} \varepsilon (1 + \sqrt{3})}$ and $\bar{q}$ is the complex conjugated of $q$.

Boundary conditions for the even solution at $z' = \frac{1}{2}$ are

$$\begin{bmatrix}
\cos q_0 & \cosh q & \cosh \bar{q} \\
-q_0 \sin q_0 & q \sinh q & \bar{q} \sinh \bar{q} \\
4q_0^2 + \lambda^2 \cos q_0 & (4q^2 - \lambda^2) \cosh q & (4\bar{q}^2 - \lambda^2) \cosh \bar{q}
\end{bmatrix}
\begin{bmatrix}
A_0 \\
A \\
\bar{A}
\end{bmatrix} = 0$$  \hspace{1cm} (D.38)

Which can be simplified to:

$$\begin{bmatrix}
\cos q_0 & \cosh q & \cosh \bar{q} \\
-q_0 \sin q_0 & q \sinh q & \bar{q} \sinh \bar{q} \\
\frac{1}{2} (i\sqrt{3} + 1) \cosh q & \frac{1}{2} (i\sqrt{3} - 1) \cosh \bar{q}
\end{bmatrix}
\begin{bmatrix}
A_0 \\
A \\
\bar{A}
\end{bmatrix} = 0$$  \hspace{1cm} (D.39)

For a non-trivial solution the determinant must be zero:

$$\begin{vmatrix}
1 & 1 & 1 \\
-q_0 \tan q_0 & q \tanh q & \bar{q} \tanh \bar{q} \\
0 & \sqrt{3} - i & \sqrt{3} + i
\end{vmatrix} = 0$$  \hspace{1cm} (D.40)

or

$$2q_0 \tan q_0 = \left( 1 - i \sqrt{3} \right) q \tanh q + \left( 1 + i \sqrt{3} \right) \bar{q} \tanh \bar{q}$$  \hspace{1cm} (D.41)

This can be solved for $\lambda$ and $\varepsilon$. Minimum value of $\text{Ra}_{TM}$ is found when $\lambda \approx 3.13$. Figure D.1 show Pellew's solution. He uses $a$ for $\lambda$ and $\lambda$ for $\varepsilon$.

Final result for the critical value of the thermal magnetic Rayleigh number is: $\text{Ra}_{TM_c} = 1707.8$. 

Figure D.1: Solution for critical Rayleigh number from Pellew and Southwell [20]
In this appendix a 2-D scaling analysis for a square enclosure heated from the side based on the analysis from Mukhopadhyay et. al and Fornalik is performed [7] [10].

Starting point are the conservation equations in the $x$ and $z$-direction from appendix B:

2-D continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} = 0 \quad (E.1)$$

Momentum equation in $x$-direction:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = \frac{1}{\rho_0} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (E.2)$$

Momentum equation in $z$-direction:

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = \frac{1}{\rho_0} \frac{\partial P}{\partial z} - \beta g \left( 1 + \gamma \left( 1 + \frac{1}{\beta T_{K_0}} \right) \right) \left( T - T_0 \right) + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (E.3)$$

2-D energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (E.4)$$

Pressure term in the momentum equation can be eliminated by differentiating $x$-direction with respect to $z$ and vice versa. Subtracting differentiated equations gives:

$$\frac{\partial}{\partial z} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} \right] - \frac{\partial}{\partial x} \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} \right]$$

$$= - \frac{1}{\rho_0} \frac{\partial P_d}{\partial x} + \nu \frac{\partial}{\partial z} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right] + \frac{1}{\rho_0} \frac{\partial P_d}{\partial z} - \nu \frac{\partial}{\partial x} \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right] - \frac{\partial}{\partial x} \left[ \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_{K_0}} \right) \right) \left( T - T_0 \right) \right]$$

$$\Rightarrow$$

$$\frac{\partial}{\partial z} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \right] + \frac{\partial}{\partial z} \left[ \frac{\partial w}{\partial t} + w \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z} \right] - \frac{\partial}{\partial x} \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \right] - \frac{\partial}{\partial x} \left[ \frac{\partial w}{\partial t} + w \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z} \right]$$

$$= \nu \left[ \frac{\partial^2 u}{\partial z^2} + \frac{\partial^3 u}{\partial x \partial z^2} - \frac{\partial^3 w}{\partial x \partial z^2} \right] - \frac{\partial}{\partial x} \left[ \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_{K_0}} \right) \right) \left( T - T_0 \right) \right]$$

$$\quad (E.5)$$
Assume that velocity has the same order in $x$ and $z$-direction ($u \sim w$), that velocity gradients in $z$-direction are much smaller than gradients in $x$-direction ($\frac{\partial}{\partial z} \ll \frac{\partial}{\partial x}$) and that temperature depends on $x$ and $z$. Use equation E.1 and neglect squared velocity terms.

\[
\begin{align*}
\frac{\partial}{\partial z} \left( \frac{\partial u}{\partial t} \right) + \frac{\partial}{\partial z} \left( \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial z} \right) - \frac{\partial}{\partial x} \frac{\partial w}{\partial t} - \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial z} \right) = \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^3 u}{\partial x^3} - \frac{\partial^3 w}{\partial x^2 \partial z^2} - \frac{\partial^2 w}{\partial x^2 \partial z^2} \right) - \frac{\partial}{\partial x} \left( \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) (T - T_0) \right)
\end{align*}
\] (E.6)

This results in a scaling for three mechanisms:

\[
\begin{align*}
\frac{-\partial}{\partial x} \frac{\partial w}{\partial t} & = \nu \frac{\partial^3 w}{\partial x^3} \quad \text{(inertia)} \\
\frac{\partial}{\partial x} \frac{\partial w}{\partial t} & = \nu \frac{\partial^3 w}{\partial x^3} - \frac{\partial}{\partial x} \left( \beta g \left( 1 + \frac{\gamma}{2} \left( 1 + \frac{1}{\beta T_0} \right) \right) (T - T_0) \right) \quad \text{(viscous)} \\
\frac{\partial}{\partial x} \left( \frac{\partial w}{\partial t} \right) & = \nu \frac{\partial^3 w}{\partial x^3} \quad \text{(buoyancy)}
\end{align*}
\] (E.7)

In the energy equation the following mechanisms are present:

\[
\begin{align*}
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} = & \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad \text{(conduction)}
\end{align*}
\] (E.8)

Immediately after $t = 0$ there is no fluid motion and energy equation is balanced between thermal inertia and conduction normal to the vertical walls. A thermal boundary layer $\delta_T \ll l$ appears.

Scale the energy equation with the following scales:

\[
\begin{align*}
\partial & \sim \Delta T, \quad \frac{\partial}{\partial t} \sim t, \quad \frac{\partial}{\partial x} \sim \delta_T, \quad \frac{\partial}{\partial z} \sim l.
\end{align*}
\]

Inertia:

\[
\frac{\partial T}{\partial t} \sim \frac{\Delta T}{t}
\] (E.9)

Convection:

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} \sim \frac{\Delta T}{\delta_T}
\] (E.10)

Conduction:

\[
\alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \sim \alpha \frac{\Delta T}{\delta_T^2}
\] (E.11)

Thermal boundary layer scale is derived from the inertia-conduction balance:

\[
\frac{\Delta T}{t} \sim \alpha \frac{\Delta T}{\delta_T^2} \Rightarrow \delta_T \sim \sqrt{\alpha t}
\] (E.12)

Thermal boundary layer is growing with a certain velocity. This velocity scale can be determined from the momentum equations. The momentum equation can be scaled with the same scales and use $u \sim w \sim v$.

Inertia:

\[
\frac{\partial}{\partial x} \frac{\partial w}{\partial t} \sim \frac{\nu}{t \delta_T}
\] (E.13)

Viscous:

\[
\nu \frac{\partial^3 w}{\partial x^3} \sim \frac{\nu}{\delta_T^3}
\] (E.14)

Buoyancy:

\[
\frac{\partial}{\partial x} (T - T_0) \sim \frac{\Delta T}{\delta_T}
\] (E.15)
Scaled momentum equation is:
\[ \frac{\nu}{\delta_T}, \frac{\nu}{\delta_T^3} \sim \beta g \frac{\Delta T}{\delta_T} \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T_0}\right)\right) \]  \hspace{1cm} (E.16)

Check scale by dividing by viscous term and use equation E.12:
Inertia:
\[ \frac{\nu}{\delta_T^2} = \frac{\nu}{\nu} = \frac{a}{\nu} = Pr^{-1} \leq 1 \]  \hspace{1cm} (E.17)

Viscous:
\[ \frac{\nu}{\delta_T^3} \sim 1 \]  \hspace{1cm} (E.18)

And buoyancy term should scale with order 1:
\[ \beta g \frac{\Delta T}{\delta_T} \frac{\delta T^3}{\delta T^3} \sim \mathcal{O}(1) \]  \hspace{1cm} (E.19)

Scale for the velocity is:
\[ \nu \sim \beta g \frac{\Delta T}{\delta_T} \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T_0}\right)\right) \frac{\delta T^2}{\nu} \]  \hspace{1cm} (E.20)

After a certain time the conductive boundary layer stops growing and part of the heat input is carried away by convection with velocity \( \nu \).
When time increases to a final time \( t_f \), inertial effects decrease and conduction in the boundary layer is balanced with convection in the cube.
\[ \frac{\nu}{\delta T} \sim \frac{a}{\delta T} \Rightarrow \nu \sim \frac{a l}{\delta T^2} \]  \hspace{1cm} (E.21)

Combining velocity scales from energy and momentum equations and use \( \delta_T \sim \sqrt{\alpha t_f} \):
\[ \frac{a l}{\delta T^2} \sim \beta g \frac{\Delta T}{\nu} \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T_0}\right)\right) \frac{\delta T^2}{\nu} \]
\[ \frac{a l}{a t_f} \sim \beta g \frac{\Delta T}{\nu} \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T_0}\right)\right) \frac{a t_f}{\nu} \]
\[ \frac{l}{t_f} \sim \frac{t_f^2}{l^2} \frac{\beta g}{a^2 v} \frac{\Delta T}{\nu} \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T_0}\right)\right) \]
\[ t_f^2 \sim \frac{a^2}{l^2} \frac{\beta g}{a^2 v} \frac{\Delta T l^3}{\nu} \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T_0}\right)\right) \]
\[ t_f \sim \frac{l^2}{a} \frac{\beta g}{a v} \frac{\Delta T}{\nu} \left(1 + \frac{\gamma}{2} \left(1 + \frac{1}{\beta T_0}\right)\right)^{-1/2} \frac{l^2}{a} \]  \hspace{1cm} (E.22)

This gives a final (conductive) boundary layer \( \delta_{T_f} \) of:
\[ \delta_{T_f} = \sqrt{a t_f} = LR_{TM}^{-1/4} \]  \hspace{1cm} (E.23)

Nusselt number represent the ratio of convective to conductive heat transfer. Conductive layer is \( \delta_{T_f} \) and the convective layer is \( l \). Dividing gives a scale for Nusselt:
\[ Nu \sim \frac{l}{\delta_{T_f}} = R_{TM}^{1/4} \]  \hspace{1cm} (E.24)
MEASUREMENT RESULTS
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