Stellingen behorende bij het proefschrift

"Investigation of Crack-Closure Prediction Models for Fatigue in Aluminum Alloy Sheet under Flight-Simulation Loading"

van

Utama Herawan Padmadinata
1. Crack closure is the predominant mechanism responsible for interaction effects during fatigue crack growth in aluminum alloy sheet material under variable-amplitude loading.

2. A fatigue crack growth prediction model is incorrect without considering crack closure, but the model is not necessarily correct if crack closure is included.

3. An increasing crack opening stress level during crack growth in the 2024-T3 alloy under flight-simulation loading is predicted by the CORPUS model and the modified CORPUS model. This is a major reason why these models are superior to the PREFFAS model and the ONERA model. (Chapters 8 and 9)

4. The analysis of three closure models in the present study has learned that the effect of severe downwards loads have their own affected zone independent of the overload affected zone. (Chapter 9)

5. The application of the rain-flow procedure in crack closure prediction models, as proposed for the PREFFAS model to obtain conservative results, has a small effect on the predicted crack growth under flight-simulation loading. This paradoxical result can be understood by considering the cycle-by-cycle variation of the crack opening stress level.

6. Visualization of the variation of crack opening levels in flight simulation loading is essential for understanding the crack opening stress level behaviour.
7. It is very surprising if not embarrassing that the fatigue crack closure phenomenon was not discovered earlier, since the fatigue problem was recognized in the previous century.

8. Financial investments do not solve problems if personal dedication is absent.

9. An easily accessible computerized catalogue in a university library is essential for an optimal search of the available literature.

10. Draftsman and drawing softwares have their own superiority in making figures.

11. Personal computers have opened a new era of computerization to solving problems. The risk of not understanding the physical aspects involved is still present.

12. There is some comfort for a "promovendus" in the saying of Goethe: So eine Arbeit wird eigentlich nie fertig, man muss sie für fertig erklären, wenn man nach Zeit und Umständen das Möglichste getan hat.
INVESTIGATION OF CRACK-CLOSURE PREDICTION MODELS FOR FATIGUE IN ALUMINUM ALLOY SHEET UNDER FLIGHT-SIMULATION LOADING

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus, prof.drs. P.A. Schenck, in het openbaar te verdedigen ten overstaan van een commissie aangewezen door het College van Dekanen op dinsdag 27 maart 1990 te 16.00 uur

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Utama Herawan Padmadinata

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LIST OF SYMBOLS

a  half crack length
Δa  crack growth increment
ADP plastic zone size measured from the center of specimen
     (a + plastic zone)
ADPH memorized ADP
C  constant in the Paris relation for crack growth rate;
     Dixon width correction factor (CORPUS model)
CA  constant amplitude
Cp  crack growth retardation factor used in the Wheeler model
D  plastic zone size (CORPUS model)
da/dN crack growth rate
d^d  dominant plastic zone size (CORPUS model)
D_{sn} plane strain plastic zone size (CORPUS model)
D_{ss} plane stress plastic zone size (CORPUS model)
eff effective
eq equivalent (ONERA model)
EF  sequence efficiency (PREFFAS model)
f1,f2 K_{op}/K_{max}(R) functions (ONERA model)
K  stress intensity factor
ΔK  range of stress intensity factor (= K_{max} - K_{min})
ΔK_{eff} range of effective stress intensity factor (= K_{max} - K_{op})
KH memorized stress intensity factor
m exponent in the Paris relation for crack growth rate
m  relaxation parameter (CORPUS model)
op opening
OL  overload
PPZ primary plastic zone size (CORPUS and modified CORPUS model)
R  stress ratio (= S_{min}/S_{max})
RR retardation factor (PREFFAS model)
r_p plastic zone size
S_a stress amplitude
S_{gr} ground stress
SH memorized stress
$S_{max}$  maximum stress
$S_{mf}$  mean stress in flight
$S_{op}$  opening stress
SPZ  secondary plastic zone used in the CORPUS model
$S_u$  ultimate stress
TH  threshold
t  thickness
$U$  $\Delta S_{eff}/\Delta S = \Delta K_{eff}/\Delta K$; strain energy per unit thickness
UL  underload
ULZ  underload affected zone (modified CORPUS model)
VA  variable amplitude loading
W  width of specimen
$\alpha$  load spectrum parameter (ONERA model)
$\rho$  plastic zone size (ONERA model)
$Y$  $K_{op}/K_{max}$
CHAPTER 1

INTRODUCTION

1.1 Preface

Throughout their service life machines, equipment, vehicles and buildings are subjected to loads, the majority of which vary with time. This type of load causes fatigue failures of the material. According to Ref. [1], fatigue is a failure of a metal under repeated or otherwise varying load which never reaches a level sufficient to cause failure in a single application. The failures caused by fatigue were recognised long ago, for example the failures of railway axles in 1843 [2].

Throughout the last three decades much research effort was applied to fatigue crack growth. In the past, fatigue data were presented in the form of S-N curves and Goodman diagrams. The damage was calculated by using Palmgren's [3] and Miner's [4] cumulative damage theory. In this theory, the damage caused by a cycle in a variable-amplitude load history is equal to that of a cycle of the same size under constant-amplitude loading. "Damage" related to one cycle is defined by the reciprocal 1/N obtained from the S-N curve of the material or component. The damage is assumed to accumulate in a linear manner. The theory can be classified as being the first fatigue life prediction method.

Research in fatigue crack propagation has made significant progress. A vast amount of data has been collected and the basic mechanisms have been identified. Linear elastic fracture mechanics, computers, electron microscopes, servo-hydraulic test machines and linear variable-displacement transducers became the tools for investigation of the subject. Symposia and publications on fatigue are dominated by crack propagation studies, with damage being equated to crack length and the rate of damage accumulation to the rate of crack growth.
1.2 Progress in fatigue crack growth prediction

Fatigue crack growth is affected very much by the type of load history. Two primary influences on crack growth behavior are retardations following overloads and accelerations following underloads. The sequence of overloads and underloads will also result in a different crack growth behavior. As a consequence, the experimental approach is to carry out more realistic simulations of the load histories in service, rather than to do constant-amplitude tests.

Progress of test load sequences is discussed in Ref. [5] from which the following lines are taken.

"An important improvement was the introduction of the programme fatigue test by Gassner [6] which allowed a much better adaptation to the actual load experience met in service. For aircraft, however, the mixture of stochastic and deterministic loads needs a more realistic representation in a test. In the past, many civil aircraft were fatigue tested by a simple flight-by-flight load sequence [7]. The first realistic simulation on a full scale structure was performed by Branger [8]. However, a real break-through was the introduction of electro-hydraulic servo valves in closed loop systems on general purpose fatigue machines [9]."

Joint institution activities have resulted in standardized load sequences such as TWIST, FALSTAFF and miniTWIST in Refs. [5,10 and 11] respectively. Recently two new standardized load histories have been proposed for helicopter rotor components HELIX and FELIX [12].

Progress was also made in fatigue crack growth prediction methods. As previously mentioned, there were influences of load sequences which cause accelerations and retardations on fatigue crack growth. This has led to the definition of "interaction effects", which implies that \Delta a in a load cycle will also depend on what occurred in the preceding cycles. Similarly, a load cycle will affect \Delta a in subsequent cycles. An illustration from Ref. [13] is given in Figure 1.1. Palmgren's and Miner's linear cumulative damage theory is not valid for variable-amplitude loading, because it can not account for interaction effects.
For instance, it predicts that an overload will reduce crack growth life, whereas in reality it can lead to most significant life extensions due to crack growth retardations. Trend predictions on the effect of variations of the load spectrum are also highly unreliable. Interaction effects have to be expected, because $\Delta a$ will depend on some factors, e.g. crack tip blunting, shear lip developments, crack closure, cyclic strain hardening and residual stresses around the crack tip, all factors produced by the preceding load history [14]. The understanding of interaction effects has led to better prediction models, which will be discussed in more detail in Chapters 5 to 9.

There are various types of prediction models, i.e. cycle-by-cycle and characteristic $K$ methods (surveyed in Ref. [14]). The cycle-by-cycle models are the most realistic ones from a mechanicistic point of view. Replicas from fatigue fracture surface studied in the electron microscope have revealed the now well-known striations, see for example Figure 1.2. Recently, striations in relation to interaction effects have been studied in Refs. [17,18]. Such striations clearly prove that crack extension occurred in every load cycle. This type of evidence was mainly obtained for macro-cracks, whereas for micro-cracks striations can not be observed for several reasons. However, if crack propagation occurs as a cyclic sliding-off mechanism it appears reasonable to assume that the crack growth of a micro-crack also occurs in every load cycle.

In the present study only the cycle-by-cycle type of prediction models will be considered.

1.3 The need for prediction method

The application of crack growth predictions in aeronautics has been highly stimulated by new airworthiness requirements imposed by government and military agencies. These requirements were initiated to ensure the fatigue safety and durability of a fleet of aircraft. Small cracks, either pre-existing at the time of manufacture or created by in-service conditions are assumed to grow during the life of the aircraft. The growth of the crack should be predictable to provide guidance for inspection programs, which ensure that cracks will never
propagate to failure prior to detection. This implies a damage tolerance evaluation as a part of the aircraft certification procedures [16].

1.4 Generations of prediction models

The prediction models were reviewed in Refs.[19-21]. They can be classified into three generations [20] namely,

- Yield zone models
- Crack closure models (based on the Elber crack closure concept)
- Strip yield model (based on the Dugdale strip yield model)

Short descriptions of the models will be given below, but the models will be discussed in more detail in Chapters 5 to 9.

The yield zone models are represented by Willenborg's model [22] and Wheeler's model [23]. The models are based solely on the relationship of current plasticity to the previous plastic zone size. The models were proposed mainly to simulate retardations as observed in tests. They do not include crack closure in the wake of the crack, which is physically not realistic.

Crack closure models are based on the mechanism proposed by Elber [24a,24b]. The crack growth is controlled not only by the behavior of the plastic zone but also by residual deformation left in the wake of the crack as it grows through previously deformed material. During loading and unloading the cracks are opened and closed at positive stresses. The crack grows only if the crack tip is open. An effective stress intensity factor range, \( A_K_{eff} \), is introduced which accounts for that part of the cycle during which the crack is fully open.

Prediction models based on this concept are the CORPUS model [25,26], the ONERA model [27-29] and the PREFFAS model [30-32]. The models have shown good prediction results for various types of loading, see also Refs. [33-35].

Heuler and Schütz [36] concluded that crack growth prediction models based on the crack closure concept presumably provide the most accurate and reliable results of all life prediction approaches (i.e. including the Palmgren-Miner cumulative damage model, local strain
approach, the Willenborg model and the Wheeler model). The ratios "actual life / predicted life" are between 0.5 and 2.0 in most cases. Their conclusion was also supported by Blom in Ref. [37].

The strip yield model was initially proposed by Dugdale [38]. He assumed that for a thin sheet loaded in tension, the yielding will be confined to a narrow strip lying along the crack line. Subsequent strip yield models include crack closure. For that purpose the Dugdale model must be modified to leave plastically deformed material in the wake of the advancing crack. The crack-surface displacements, which are used to calculate contact (or closure) stresses under cyclic loading, are influenced by plastic yielding at the crack tip and residual deformations left in the wake of the advancing crack. For a numerical treatment of the problem, discretized elements are used to allow a detailed analysis of the effect of individual load cycles on the crack opening load. The plastically stretched layer at the crack tip is changed to residual deformation left in the wake of the crack after further crack growth. Iterative solution procedures are to be used, which require an extensive computer capacity for a cycle-by-cycle calculation.

1.5 Aims of the present investigation

The aims of the present investigation are:

1. Analysis and evaluation of the crack closure models (2nd generation).
2. Collecting a representative data bank of flight simulation loading.
3. Completing that data bank with new experiment on variables not yet well studied empirically.
4. Improvement of models, mainly the CORPUS model.

Because the closure models (2nd generation models) developed so far were almost exclusively checked by the originators of the model, a general comparative study of these models would seem to be most opportune.
The load spectra involved are mainly representative for aircraft wing tension skins, and the materials considered are aluminum alloy sheet materials widely used in the aircraft industry, i.e. 2024-T3 and 7075-T6.
2.1 Some general comments on crack growth

For economical reasons aircraft structures are designed for a finite fatigue life. Because of the finite life it should be expected that microcracks start relatively early in the fatigue life. Figure 2.1 illustrates that in the absence of initial defects some 90 percent of the fatigue life is spent in the micro-range. In such cases the fatigue damaging process largely occurs in a very small volume of the material and it will be highly dependent on local conditions. However, in other cases, where a macrocrack has to be considered, bulk properties of the material will be involved [39]. Macocrack growth is highly significant for the damage tolerance properties of an aircraft structure.

Fatigue life can be divided into three stages [39,40]:
- crack initiation stage
- crack growth stage
- final failure.

In the crack initiation stage, the initiation process leads to a microcrack. The process is most susceptible to the influence of the local microstructure and the possibilities of dislocation movements. Grain boundaries and other dislocation obstacles can still be very significant. The same is true for intermetallic inclusions as crack nucleation sites. However, after some successful microcrack growth the influence of the local material structure is decreasing. The crack is entering the second stage, and it is then characterized by a growth direction normal to the applied maximum principal stress [40]. According to Schijve [39], a crack can be defined as a macrocrack as soon as the stress intensity factor $K$ has a real meaning for describing its growth. Fatigue is no longer a localized surface phenomenon and stress/strain distributions are significantly affected
by the crack itself. Bulk properties of the material become important, because they are representative for the crack growth resistance. Final failure occurs in the very last cycle of the life when the macrocrack is growing larger and the remaining uncracked cross section becomes smaller. Finally it will be too small to carry the maximum of the cyclic load. The final failure usually is a quasi-static failure.

Two types of cracks can be recognized, namely a through crack and a part through crack, see Figure 2.2. For part through cracks, the crack front is curved, approximately in a semi or quarter elliptical form. The stress intensity factor K varies along the elliptical crack front. In many cases it has a maximum value at the ends of the minor axis and a minimum value at the end of the major axis.

In the present study, the analysis generally applies to macrocrack growth, and the type of crack considered is the through crack in relatively thin material, i.e. sheet materials applied to aircraft structures.

2.2 Relevant aspects of fatigue macrocrack growth

A number of aspects of macrocrack growth should be summarized, because they are relevant to the assumptions made for the crack growth prediction models to be discussed later (Chapters 5 to 9). These aspects are: striations, stress ratio effects, interaction effects, environmental and frequency effects, and thickness effects. A short literature survey will be given below.

2.2.1 Striations

Fatigue fracture surfaces have been extensively studied using optical, transmission and scanning electron microscopes. The most prominent feature of fatigue fracture surfaces is that of distinct line markings, approximately parallel to one another and normal to the direction of crack growth. These are generally called striations; each striation corresponds to one load cycle, although not every load cycle need result in a striation. An example has already been presented in
Figure 1.2. The occurrence of striations has prompted prediction techniques to calculate Δa cycle-by-cycle.

2.2.2 The similarity approach and stress ratio effects

The application of fracture mechanics to fatigue crack growth is based on the similarity concept employing the stress intensity factor $K$. As long as the crack tip plastic zone is relatively small, the similarity concept implies that cracks with the same $K_{\text{applied}}$ will show the same stress and strain distribution in the crack tip area, including the same (small) plastic zone. For a cyclic load the similarity concept implies that cracks subjected to the same $K$-cycle, will have the same cyclic stress and strain field at the crack tip. As a consequence, the crack extension in that cycle, which is the crack rate per cycle (da/dN), will also be similar. In other words, it may be expected that da/dN is a function of $K_{\text{max}}$ and $K_{\text{min}}$, ignoring frequency and wave shape effects.

$$\frac{\text{da}}{\text{dN}} = f\left(K_{\text{max}}, K_{\text{min}}\right)$$

or, which is the same:

$$\frac{\text{da}}{\text{dN}} = f\left(\Delta K, R\right)$$

where $\Delta K = K_{\text{max}} - K_{\text{min}}$ \hspace{2cm} \text{(2.3)}

$$R = \frac{K_{\text{min}}}{K_{\text{max}}} = \frac{S_{\text{min}}}{S_{\text{max}}} \hspace{1cm} (R=\text{stress ratio})$$

Paris and co-workers [42-44] assumed that the $R$-effect was negligible. From test data they concluded a linear relation between log(da/dN) and log $\Delta K$, which implies:

$$\frac{\text{da}}{\text{dN}} = C \Delta K^m$$

This relation is usually referred to as the Paris relation. It implies that log da/dN is a linear function of log $\Delta K$. Paris et al. also
assumed that the R-ratio effect is negligible. Later, abundant evidence showed that Eq. (2.5) is a rather crude over-simplification of empirical observations. A qualitative picture of trends is shown in Figure 2.3. A systematic R-effect is usually observed. Moreover, the crack growth rate, da/dN is described by a sigmoidal curve in a log da/dN - log ΔK diagram. The curve is bounded by two vertical asymptotes, viz. by the threshold stress intensity range, ΔK_{TH}, at the lower end of the ΔK range, and by the critical value, ΔK_{C}, at the upper end of the ΔK range. Figure 2.3 illustrates that the power function applies in regime B (also called the Paris regime).

Sigmoidal curves for several materials are reported in Ref. [45]. The sigmoidal curves have been observed for many metals and alloys including aluminium alloys [46], carbon and low carbon steels [47-49], ferritic and perlitic steels [50,51], ultra-high strength steel [52,53], cast steel [54,55], mild steel [56], stainless steel [57] and nickel alloys [58,59]. For some metals and alloys, e.g. ferritic and perlitic steels, mild steel nickel alloy, the R-ratio effect is less pronounced or non-existent in region B [50,51,56,58]. For 2 mm 2024-T3 bare material, the R-ratio effect is shown in Figure 2.4, adopted from Ref. [61]. A similar result is shown in Figure 2.8 (the left graph) for 2 mm 2024-T3 Al clad material [39]. The two figures show the influence of the R-ratio effect on the crack growth rate. For the same ΔK, the crack grows faster for increasing R values.

In most cases fatigue involves stress fluctuations around a non-zero stationary stress, the magnitude of which would be expected to alter the extension rate of a growing crack. Generally, an increase in mean stress leads to an increase in growth rate [62-64]. It corresponds to an increasing R-ratio.

As pointed out above, the similarity approach predicts equal crack length increments if the same K_{max} and K_{min} are applicable. The approach tacitly presume that all other conditions are similar, i.e. the material, the state of stress and the crack front geometry are all similar. These conditions may well be satisfied when considering constant-amplitude loading and the same sheet material (same thickness) with through cracks with a straight crack front (mode I crack).
However, under variable-amplitude loading the conditions are not always similar, which can lead to interaction effects to be discussed later. Here it may be pointed out that a certain crack extension (Δa) in a cycle is a consequence of a ΔK cycle (crack driving force) and the resistance of the material to crack extension (crack growth resistance). The crack extension in a cycle is a consequence of the balance between the crack driving force and the crack growth resistance. The similarity approach implies that the same crack driving force will always give the same Δa if the crack growth resistance of the material is the same. The crack driving force is usually associated with a K-cycle. It may also be associated with the strain energy release rate per unit thickness of the material, because of the relation.

\[
\frac{dU}{da} = \frac{K^2}{E}
\]  

The dimension of \(dU/da\) is N/m, i.e. a force per unit length of the crack front if the crack is still in the tensile mode (mode I). The crack driving force and the crack growth resistance are depending on several circumstances and unfortunately not in the same way. That is a complicating aspect, especially for crack growth under variable-amplitude loading. The major aspects to be considered then are crack tip plasticity and crack front geometry. Several "interaction" effects can occur.

2.2.3 Interaction effects

Fatigue loads in service generally imply a randomly variable-amplitude, rather than constant-amplitude loading. Different types of load sequences are known to induce a number of different load-interaction effects, which can result in significant accelerations and retardations. As said in the introduction, interaction effects imply that the crack extension in a load cycle will depend on what occurred in the preceding cycles. The interaction effects are a major topic in fatigue crack growth prediction models. A more complete understanding of interaction effects is required to develop and evaluate prediction models. Information can be found in Refs. [14,37,65-71].
Several mechanisms have been proposed in the literature to account for interaction effects in fatigue crack growth:

- Crack closure
- Residual stress
- Crack tip blunting
- Crack front geometry
- Effect of yield stress and strain hardening

2.2.3.1 Crack Closure

Crack closure under cyclic tension was discovered by Elber in 1970 [24a,24b]. He observed that fatigue cracks are closed during unloading when the load is still in tension. During uploading the fatigue cracks are opened if the load is sufficiently high. Elber suggested that a zone with residual plastic tensile strains is left behind the crack tip, see Figure 2.5. The residual stretch between the crack faces leads to closure of the crack before complete unloading.

During increased loading, a monotonic plastic zone is formed at the crack tip. The same is true for the reversed plastic zone during unloading. Since the reversed plastic zone is much smaller than the monotonic plastic zone, the consequence is that the monotonic plastic deformations are built up in front of the crack tip, see Figure 2.5. When the crack grows, the plastic deformation is left on the crack flanks behind the propagating crack. It implies that the crack flanks are elongated normal to the crack surfaces, which has to be accommodated by the surrounding elastically stressed material, leading to crack closure at positive stress levels. This phenomenon is referred to as plasticity-induced crack closure. In fatigue crack propagation, the crack closure is related to a concept that crack propagation is governed by the portion of the stress intensity range for which the crack is fully open, see Figure 2.6. Therefore, an effective stress range is defined as:

$$4S_{eff} = S_{max} - S_{op}$$  \hspace{1cm} (2.7)

where $S_{op}$ is the crack opening stress level. In terms of the stress intensity factor it implies:
\[ \Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}} \leq \Delta K \tag{2.8} \]

Apparently the "crack driving force" is reduced from \( \Delta K \) to \( \Delta K_{\text{eff}} \) as a consequence of plastic deformation left in the wake of the crack.

Elber has defined a U-factor as:

\[ U = \frac{S_{\text{max}} - S_{\text{op}}}{S_{\text{max}} - S_{\text{min}}} = \frac{\Delta S_{\text{eff}}}{\Delta S} = \frac{\Delta K_{\text{eff}}}{\Delta K} \tag{2.9} \]

He derived \( S_{\text{op}} \) from compliance measurements on fatigue cracked specimens from a series of constant-amplitude tests on 2024-T3 sheet specimens. Stress ratios were in the range \(-0.1 < R < 0.7\). The U-values obtained are shown in Figure 2.7. Elber's empirical relation between \( U \) and \( R \) is:

\[ U = 0.5 + 0.4 R \tag{2.10} \]

This equation is also successful in correlating the results in Figure 2.8. The three curves in the left graph do coincide in a single band in the right hand graph, where \( da/dN \) is plotted as a function of \( \Delta K_{\text{eff}} \). It confirms that the same \( \Delta K_{\text{eff}} \) leads to the same crack growth rate in spite of different \( R \)-values.

Instead of the U-factor it is also instructive to consider the ratio \( \gamma \) defined as:

\[ \gamma = \frac{S_{\text{op}}}{S_{\text{max}}} = \frac{K_{\text{op}}}{K_{\text{max}}} \tag{2.11} \]

It is easily shown that:

\[ \gamma = 1 - (1-R)U \tag{2.12} \]

Substitution of Elber's U relation (Eq. 2.10) gives:

\[ \gamma = 0.5 + 0.1R + 0.4R^2 \tag{2.13} \]

Unfortunately for decreasing negative \( R \) values \( \gamma \) will increase again, i.e. \( S_{\text{op}} \) will increase again. As pointed out by Schijve [72] that is
physically unrealistic. He proposed an alternative equation to overcome this drawback:

\[ U = 0.55 + 0.33R + 0.12R^2 \] (2.14)

and with (2.12):

\[ Y = 0.45 + 0.22R + 0.21R^2 + 0.12R^3 \]

The equation was shown to agree with several crack growth data sets.

The Paris fatigue crack propagation relation (Eq. 2.5) can still be adopted after replacing \( \Delta K \) by the effective stress intensity range:

\[ \frac{da}{dN} = C (\Delta K_{\text{eff}})^m = C(U \Delta K)^m \] (2.15)

The limitations for low and high \( \Delta K_{\text{eff}} \)-values are still present, i.e. it does not account for the usually observed sigmoidal shape.

Under variable-amplitude loading, the crack closure concept may account for both crack retardation and acceleration effects, as illustrated by Figure 2.9. In the Lo-Hi sequence, the \( \Delta K_{\text{eff}} \) for the higher levels are temporarily larger than the stabilized value which causes accelerations of crack growth at the beginning of high-level cycles. In the Hi-Lo sequence, the \( \Delta K_{\text{eff}} \) for the lower levels are temporarily less than the stabilized value which causes retardations.

As said before, the above crack closure mechanism is referred to as "plasticity-induced" crack closure or the Elber mechanism. It is now recognized that other mechanisms can be involved in the development of crack closure such as roughness-induced closure and crack-filling closure [73-75]. Again it implies that the crack flanks will touch before complete unloading. It can occur as a consequence of a rough fracture surface or extra "foreign" material entering the crack.

- Roughness-induced closure.

This type of closure is caused by the mis-match of fracture surface asperities on unloading, which occurs as the result of combined mode
I and mode II propagation. Closure may then depend on the grain size of the material. A small grain size will develop a low roughness, which can have a small effect in developing crack closure. In several materials (survey in [76]) tested, under decreasing $\Delta K$-conditions to obtain a $\Delta K_{TH}$, the crack driving force is so low that the crack growth path becomes very irregular. As a consequence an irregular crack surface is obtained, which can lead to considerable roughness induced crack closure and $\gamma$-values approaching 1 (i.e. $S_{op} = S_{max}$). According to Suresh [71], this type of closure may also have pronounced effects in the post-overload zone and under plane strain conditions, when the effective $\Delta K$ is approaching the near-threshold level, or when crack growth rates are comparable to near-threshold growth rate (typically $10^{-8}$ to $10^{-6}$ mm/cycle). It presumes that a rough fracture surface is created under such conditions.

- Crack-filling closure.

This type of closure is induced by external agents which can produce corrosion products, oxides and fretting debris etc., which again reduce the effective $\Delta K$ by filling the crack.

These mechanisms may prolong retardation when they are activated by other mechanisms such as plasticity-induced crack closure, crack tip branching and residual compressive stress in the post overload growth region.

In the present report the crack closure term used in the prediction models corresponds to plasticity-induced crack closure. This limitation will be reconsidered later.

2.2.3.2 Residual stress

In the literature it is sometimes suggested that residual compressive stresses will reduce the crack growth rate as a direct effect apart from crack closure. In this respect results of Blazewicz [77] are instructive. He introduced ball impressions on 2024-T3 sheet specimens, see Figure 2.10a. This causes a residual compressive stress zone between the impressions. The test result shows that only a small delay is found during the growth through the zone between the ball impressions, whereas a significant delay occurs at a later stage, see Figure
2.10b. It shows that residual stress is active in the wake of the crack, because it promotes crack surface contact behind the crack tip. It then leads to an increase of crack closure and hence to retardation. It means that the residual stress ahead of the crack tip is relatively insignificant. This argument is in agreement with the fatigue crack propagation concept, that a crack grows when the crack is open.

2.2.3.3 Crack tip blunting

Crack tip blunting can occur if a high-overload is applied. In several literature sources, for example in Refs. [68,70,71], crack tip blunting was assumed to induce an initial acceleration directly after an overload cycle, see Figure 2.11. In Ref. [68], it was shown by metallographic and numerical methods that the crack was blunted by the overload and remained open at zero load for some distance behind the crack front. It reduces the crack opening stress level and causes the initial acceleration. This was supported by observations of larger crack opening displacements along the crack following the overload. The conclusion is that crack tip blunting causes a crack growth acceleration following an overload due to the reduction of crack closure. In other words, the crack tip blunting does not lead to acceleration during the overload cycle itself, but to an initial acceleration after the overload.

2.2.3.4 Crack front geometry

In general it is assumed in a fracture mechanics analysis that a fatigue crack is a flat mode I crack, and if it is a through crack, with a straight single-line crack front. Two phenomena can disturb this simplified picture: crack branching and shear lips.

Usually fatigue macrocracks grow perpendicular to the main principle stress (mode I). However, when the crack driving force is low (i.e. when it is difficult to extend the crack) deviations of that direction may occur on a microscopic level. It can lead to a kinked crack path and sometimes to crack branching. Such conditions can occur after an
overload as shown by Suresh [71]. For three types of cracks, namely a kinked crack, a forked crack and a double-kinked crack (see Figure 2.12) he estimated a reduction of the stress intensity factor of 15%, 35% and 25% respectively. A reduction of the crack growth rate is then due to a lower crack driving force, which will give an additional contribution to crack growth retardation.

Another complication of the crack front geometry is the occurrence of shear lips, see Figure 2.13. As pointed out by Schijve and Vogelesang [78] the occurrence of shear lips is a free-surface effect and not a direct consequence of the local plane stress condition at the surface. The occurrence of cyclic crack closure is easily observed on the shear lips because the mixed mode contact of the shear lips causes rubbing. As a consequence the black corrosion debris on the shear lips are a well-known feature a fatigue fractures in Al-alloys. As the same time it should be expected that crack closure will be more predominant at the shear lips than for the core material. It is a consequence of the larger plastic zones (plane stress) at the material surface. The trend was confirmed by investigations of Ewalds and Furnee [79], McEvily [80] and Rao and Ritchie [70]. Removal of surface material of a fatigue cracked specimen reduced $S_{op}$ [79, 80] and also reduced the post-overload retardations [70].

In Refs. [78,81], it was shown that the shear lip width is strongly dependent on the aggressiveness of the environment. A more aggressive environment gives a smaller shear lip width. On the other hand, Schijve and Arkema [83] observed that that the crack opening stress level in 2024-T3 and 7075-T6 specimens was independent of the environment, when comparing cracks grown in vacuum, air and salt water. A similar result was found by Ewalds [84]. It thus has to be concluded that crack closure is a phenomenon which occurs largely at the surface of material but it is not dependent on the presence or the absence of shear lips [85].

Another mechanism was introduced by Schijve [13], namely incompatible crack front orientation. In small amplitude load cycles, the crack front orientation will be perpendicular to the loading direction
(mode I crack), whereas high amplitude loading will lead to a 45-deg mode or a slant mode. This slant mode is incompatible with the normal mode, which will contribute to the retardation associated with small-amplitude loading following high-amplitude loading.

Another consequence of shear lips was discussed by Schijve [13]. When shear lips are present the crack front is no longer a straight line. Mixed mode cracking (I and III) does occur. That implies that the K-factor for mode I cannot give a correct indication of the crack driving force. Moreover the crack growth resistance can also be different. The similarity concept (e.g. the applicability of Eq. (2.15)) could still work for constant-amplitude loading if the shear lip width would be a unique function of $AK_{eff}$. However, for variable-amplitude loading "incompatible crack fronts" can easily occur. If high-amplitude cycles, which produce wide shear lips, are followed by low-amplitude cycles, which do not produce wide shear lips, such an incompatible situation is created. It will contribute to the retardation effect. Another incompatibility occurs if low amplitude cycles (small shear lips) are followed by high amplitude cycles. Then an acceleration will occur. Both effects were observed indeed [13].

2.2.3.5 Effects of yield stress and strain hardening

Low ductility materials with a relatively high yield stress generally show high crack growth rates. As an example the 7075-T6 Al-alloys ($S_{0.2} = 470$ MPa, $S_u = 550$ MPa) in comparison to the more ductile and lower strength 2024-T3 Al-alloys ($S_{0.2} = 320$ MPa, $S_u = 450$ MPa) has inferior crack growth properties. Two arguments may be valid. The higher $S_{0.2}$ will lead to a smaller crack tip plastic zone and thus to less crack closure and a higher $AS_{eff}$. In other words, the crack driving force will be higher. On the other hand, it also may be purely a matter of a lower crack growth resistance. Another interesting example of a yield stress effect is shown in Figure 2.14 for a steel alloy heat treated to three significantly different yield stress levels. For constant-amplitude loading (upper curve in the figure) a lower crack rate is found for the lower yield stress. In the tests with periodic overloads the retardation is much larger for the low
yield material. It suggests that more crack closure is the predominant effect rather than the lower crack growth resistance. However, a strict separation of the effects of crack driving force and crack growth resistance can not be made.

In some other investigations fatigue crack growth was studied in plastically predeformed material. The predeformation was applied to the specimens before the crack starter notch was introduced. As a result the whole specimen was strain hardened. The predeformation was applied by a tensile load in Refs. [87-89], by a compressive load in Refs. [90,91] and by cold rolling in Refs. [92,93]. Kang and Liu [87] found that a 3% tensile prestrain approximately doubled the crack growth rate. The same result was found by Schijve [88]. He also measured crack closure, which indicated a lower $S_{op}$ in the prestrained material associated with the increased yield stress. However, it could explain only half the effect of the higher crack growth rate. Consequently part of the effect should be attributed to a reduced crack growth resistance. Schulte et al. [92] found a similar effect on 7475 Al-alloys. They refer to a reduced deformation capability and an increased yield stress to explain a reduced crack closure level. They also found that the prestrain had broken the incoherent particles (intermetallic inclusions) which introduces voids. Moreover the dislocation density was significantly increased. The latter aspects were supposed to influence the crack growth resistance. It should be pointed out here that such arguments may be correct, but since such effects can not be measured separately it remains a speculative argumentation.

Arbert et al. [90] applied compressive pre-deformations on a 20MnMoNi 5.5 steel prior to a fatigue loading. They observed a decrease in fatigue crack propagation rates. Schulte et al. [91] applied about 8% compressive pre-straining on Al 2024-T3 prior to fatigue loading. The results for constant amplitude loading showed a lower crack growth rate for pre-deformed material, when compared to the non pre-deformed material, particularly at low da/dN-values. It was reported that the yield strength and the fracture strain of the pre-deformed material and the original material were only slightly different.
When considering the crack growth resistance it should be realized that the crack extension does occur in the reversed plastic zone, i.e. in material which has been subjected to large cyclic plastic strains. Under such conditions fatigue crack growth results of monotonically prestrained material can hardly be expected to give relevant information on the effect of strain hardening on the fatigue crack growth resistance. Jacoby et al. [64] wanted to study the influence of cyclic strain hardening at the crack tip. They performed constant-amplitude tests with a step-wise change of the mean stress, see Figure 2.15. Both an increased and a decreased $S_m$ were adopted. The material was 2024-T3 (1 mm thickness). As the results in Figure 2.15 show the step-wise increased $S_m$ caused an initially high crack rate, significantly larger than the stabilized crack growth rate obtained after further crack growth. The authors explain this acceleration by strain hardening arguments. Also in this case the reasoning is not free of speculation.

2.2.4 Thickness effect

The effect of sheet thickness on fatigue crack growth has been recognized for a long time. An early investigation was carried out by Schijve and Broek [94] on 2024-T3 sheet material with five different thicknesses. Illustrative results are shown in Figure 2.16, which indicates that faster fatigue crack growth is observed in thicker sheets. The trend is generally associated with the state of stress. The size of the plastic zone relative to the thickness will determine whether the crack tip state of stress is essentially plane stress, plane strain or a combination of the two. For a thick specimen, the state of stress is depicted schematically in Figure 2.17. There is a gradual transition from plane stress at the free surface to plane strain at the interior.

In the literature, the plastic zone size for two states of stress are

\[
\text{Plane stress} : \ r_p = \frac{1}{n} \left[ \frac{K}{\sigma_y} \right]^2 \quad (2.16)
\]

\[
\text{Plane strain} : \ r_p = \frac{1}{3n} \left[ \frac{K}{\sigma_y} \right]^2 \quad (2.17)
\]
The formulas suggest that the plastic zone size is three times larger in the plane stress condition. Larger plastic zones imply more deformation left in the wake of the growing crack and hence more crack closure. A faster crack growth under plane strain conditions suggests that a thick specimen will show less closure than a thin specimen because of different degrees of plasticity. This is in agreement with the thickness effect as observed in constant-amplitude tests.

Plasticity-induced crack closure in thick specimens made from BS4360 50B steel was observed by Fleck and Smith [95] and Fleck [96]. They used three methods to measure crack closure: a back face strain gauge, a crack mouth opening gauge and a novel push-rod gauge to measure the closure level at the centre of specimens, where plane strain conditions prevail. They found that the effective stress range, \( U \), was 0.8 at the centre of the thickness, whereas the \( U \)-value was 0.7 at the surface, where plane stress conditions are applicable. The plastic zone size and the residual plastic deformation in the wake of the crack tip are larger at the surface of the specimens, which causes a variation of \( S_{op} \) along the crack front. It was confirmed by tests where a reduction of \( S_{op} \) was observed after removal of surface layers of the specimen, as discussed before. The variation of \( S_{op} \) along the crack front is an unpleasant complication. Usually a single "average" value is assumed, but even then the difference between plane strain and plane stress plastic zone sizes cannot be ignored. For the CORPUS prediction model De Koning [26] assumes plane strain conditions if the ratio of plastic zone size to thickness is smaller than 0.35 and plane stress condition if the ratio exceeds 0.5 (see also Chapter 8).

Because of the thickness effect on the plastic zone size, it should be expected that the crack growth retardation behaviour under variable-amplitude loading will also be thickness dependent. Empirical evidence is documented in the literature [94, 98-100]. Illustrative single overload tests were carried out by Mills and Hertzberg [98] for three different thicknesses, see Figure 2.18. The tests were performed with a 100% overload \( (\Delta K_2/\Delta K_1 = 2) \). They observed the delay period after the overload and fatigue crack growth behaviour through the delay region, see Figures 2.18c and d. Figure 2.18c shows that the delay period is
larger for thinner material. Figure 2.18d shows that the minimum crack growth rate during the delay period is lower for a thinner sheet. Moreover the retardation can still be observed for a larger Δa after the overload. All these observations can easily be explained when considering the plastic zone size and its effect on crack closure.

A similar thickness effect is also observed in flight-simulation fatigue tests. A survey of thickness effects for various alloys and spectra was given by Schijve [99], see Table 2.1. The results confirm that retardation decreases with increasing sheet thickness.

2.2.5 Environmental and frequency effects

The effects on fatigue crack growth of aggressive environments and frequency effects in such environments are well-known. Fortunately, for fatigue crack growth in Al-alloys in humid air the frequency effect is rather limited. That even is true for fatigue crack growth under flight-simulation loading as shown by Schijve et al. [112] in tests at 10 Hz, 1 Hz and 0.1 Hz. The question may be raised whether interaction effects will occur during variable-amplitude loading in aggressive environments, in a similar way as in less aggressive environments (e.g. laboratory air). If interaction effects are predominantly controlled by plasticity-induced crack closure, it should be expected that similar interaction effects will happen. Plastic zone sizes and plane strain/stress conditions are not depending on the environment. Constant-amplitude test results of Al-alloys [83,84] had already confirmed that $S_{op}$ is not dependent on the environment (tests in vacuum, air and salt water). Chanani [113] found considerable delay periods after an overload, both for tests in air and in salt water. The materials tested were 2024-T8 and 7075-T6. Schijve et al. [97] carried out flight-simulation tests on 2024-T3 and 7075-T6 in air and in salt water. Also in salt water a similar retardation behaviour after severe flights was observed. Moreover, the thickness effect discussed before was again found in tests in salt water, for both constant-amplitude loading and flight-simulation loading. Wanhill et al. [145] observed the environmental effect in three environments i.e. dry air, normal air and salt water. The tests were carried out on 10
mm thick 2024-T3 and 7075-T6 materials. They found faster crack
growths for a more aggressive environment. The 7075-T6 material showed
a greater environmental sensitivity. Schijve [76] indicated that the
faster crack growth in the aggressive environment is partly caused by
the lower crack growth resistance in that environment. Another con-
tribution is due to a smoother fracture surface of the crack, which
implies a higher crack driving force.

2.3 Interaction effects observed in simple variable amplitude loadings

Various simple variable amplitude load sequences were adopted in the
literature, e.g. a single overload, multiple overloads, an underload,
combinations of overload and underload, Hi-Lo or Lo-Hi block loading,
see Figure 2.19.

Trends of load interaction effects observed in such tests are
summarized below.

1. Overloads induce significant crack growth retardations [89,114-
125]. Sometimes a small initial acceleration is observed. Striation
measurements indicate delayed retardation. The amount of crack
retardation increases by:

a. Increasing the magnitude of the overload [121,123,126].

Jones [89] found in Ti-6Al-4V sheet that a 20% overload had no
effect on crack growth, a 50% overload caused retardation, but
not yet crack arrest, and 70% and 100% overloads induced a
temporary crack arrest, followed by retarded crack growth.

De Koning [121] tested 7075-T6 material for six different
overload levels in the range from 70% to 166%, see Figure 2.20.
A 70% overload induced a small retardation (about 3 kc delay).
Crack arrest (delay ≥ 200 kc) occurred for an overload higher
than 160%.

A survey of crack arrest after a single overload was given by
Nelson [62]. For Ti-6Al-4V [124], 1020 Cr steel [127],
Austenitic Mn. steel [128], 2024-T3 aluminum [114,115,129] and
4340 steel [130], the overload percentage to cause crack growth
arrest were 170%, 150%, 130%, 100-150% ,and 140% respectively.
Of course such percentages depend on the magnitude of the
baseline cycle.
b. Multiple overloads.

The amount of retardation can be increased by applying multiple overloads [114,117,120]. Bathias and Vancon's test results [120] are given in Figure 2.21. The tests were carried out on 10mm thick 2024 aluminum, for 2, 10, 50 and 100 multiple overloads. Figure 2.21 shows that the delay period increases with an increasing number of overload cycles until a saturation level after some 50 to 100 overload cycles.

2. If two overloads with a number of intermediate cycles are applied the maximum retardation will depend on the interval of crack extension or the number of cycles between the two overloads [116,117,123,124]. According to Mills and Hertzberg [117], the maximum interaction between two single overloads is obtained when the increment of the crack extension between the overloads is about 0.25 of plastic zone size of the first overload, see Figure 2.22.

3. Compressive stresses in constant-amplitude loading (negative R) are considered to be hardly damaging [14,131]. However, a periodic underload combined with constant-amplitude loading will cause a faster crack growth rate [61,125]. Moreover, if a negative overload is introduced immediately after a positive overload the crack growth delay is significantly reduced [118, 132]. If a negative overload precedes a positive overload, the reduction of the delay is small if not insignificant [116,118,132,133]. Illustrative results were already presented before (Fig. 1.1). Tests with a periodic overload/underload cycles and underload/overload cycles added to constant-amplitude loading were carried out by several authors [18,61,125,134] for different R ratios. The number of intermediate cycles varied from 4 to 1000 cycles. For both types of load sequences, the test results indicated small systematic differences, where the overload/underload load sequence showed a somewhat faster crack growth rate.

4. A Hi-Lo sequence block loading produces results which are qualitatively similar to overload cycles. A Lo-Hi sequence can produce an acceleration, see also Figure 2.9 [114,115].
2.4 Flight simulation loading

Flight-simulation loadings is a load history with a random sequence of different types of flight with a random sequence of cyclic loads in each flight. It is supposed to be representative for load sequences recurring in service. It is described in more detail in Chapter 3.

A survey of variables in flight simulation fatigue tests was given by Schijve [99], from which the following lines are summarized. Several trends observed in flight-simulation tests are,

1. Effect of truncation.

   High positive loads in a flight simulation test generally have a beneficial effect on fatigue life and crack growth. Truncation of the high loads (reducing the amplitude) usually implies a shorter fatigue life and faster crack growth [34,135-137]. For FALSTAFF, which is maneuver dominated load spectrum, the life reduction due to truncating high positive peak loads is much smaller. Fatigue under a maneuver load spectrum is more similar to high-level fatigue.

2. Effect of omission low-amplitude cycles.

   The purpose of omitting of low-amplitude cycles is to save testing time. However, omission of small load cycles can affect the fatigue life in some types of loading. A tentative conclusion was given by Schijve [99] where the effect of omitting small cycles from a transport wing load spectrum has a larger effect than omitting these cycles from a maneuver wing load spectrum.

   The effect of omitting taxiing load cycles from the landing cycle was very small [138,139]. The taxi load amplitudes were relatively small and occurred at a low mean stress or a compressive mean stress.

3. Effect of the minimum stress of the (GAG) cycle (GAG = ground-air-ground cycle).

   This cycle should be considered as a periodic underload. A systematic effect is found in several tests [138,140-144]. A more severe GAG cycle causes shorter lives.
4. Effect of design stress level.

The mean stress in flight ($S_{mf}$) in a flight simulation test is related to the design stress level (e.g. for TWIST, F27). The same is true for the maximum stress ($S_{max}$) of the untruncated load spectrum (e.g. for FALSTAFF). Changing the design stress level will change all stress levels of the load spectrum in a linearly proportional way. From several test results in Ref. [99], it can be concluded that a higher design stress level gives a shorter life, as should be expected.

2.5 Closing remarks

In this chapter several aspects of fatigue crack growth have been discussed. Possible interaction mechanisms for crack growth under variable-amplitude loading were summarized as well as empirical trends observed in tests under this type of loading. The empirical trends can be understood qualitatively if crack closure is assumed to be the predominant mechanism responsible for the interaction effects. It may be stated that a prediction model for fatigue crack growth that does not account for crack closure level variations is definitely bound to be essentially incomplete and thus incorrect. Unfortunately, the reverse statement cannot be made. In other words, it is not sure that a model, which adopts crack closure as the only mechanism for interactions, is sufficiently complete for satisfactory predictions. It is obvious that a prediction model, which includes more interaction mechanisms, will become a very complex model. In addition to crack closure aspects like crack tip blunting and strain hardening in the crack tip plastic zone should then be considered. Actually, the instrumental knowledge to quantify these additional aspects is hardly available. Speculative arguments have to be used, which cannot easily be checked empirically. In view of the predominant influence of crack closure it seems to be a practical approach to analyse prediction models based on crack closure only, and to see whether they can correctly predict the empirical trends observed in flight-simulation tests. Many variable of such tests have been studied in various test series in the literature. Moreover, additional test series were carried out as part of the present investigation. A useful prediction
model should be able to give reasonable crack growth estimates for the effects of relevant variations of the flight load histories. The survey in the present chapter indicates that a prediction model must also include in an acceptable way the influence of material thickness and material yield stress, because they can have a significant effect on the crack closure level.
CHAPTER 3
FLIGHT-SIMULATION LOAD HISTORIES

3.1 Introduction

Flight-simulation testing nowadays is a generally accepted fatigue testing procedure in aeronautics. It is adopted in full-scale tests, but also for component testing and comparative testing for various purposes (evaluation of materials, fasteners, surface conditions, etc.). Flight-simulation load histories are supposed to be a realistic simulation of load histories, which can occur on an aircraft structure during its life time. A flight-simulation load history thus must consist of a flight-by-flight sequence. A sample is shown in Figure 3.1. It shows that the cyclic loads vary from flight to flight. Usually, the cyclic loads in one flight are a superposition of deterministic and stochastic loads, see Figure 3.2. The load history applied in the test is a time-condensed version of the real-time history, obtained by (1) omitting parts during which the load does not vary, (2) omitting small cycles which are supposed to be insignificant for fatigue damage, and (3) by increasing the loading frequency. For aircraft structures of Al-alloys the time compression is allowed because the fatigue behaviour of the alloys is rather insensitive to rate effects in the applicable frequency regime.

The load history in a flight-simulation test must be representative for a certain aircraft structure and an assumed utilization of the aircraft. As a consequence the load history applied in a full-scale fatigue test is essentially depending on the aircraft type to be tested. Some standardized flight-simulation histories have been developed for general purposes, usually for comparative test programs, or test series to study the effects of the variables of flight-simulation load history. The flight-simulation load histories to be discussed below are partly associated with specific aircraft (F27, CN-235) and for another part with standardized load histories (TWIST and FALSTAFF). This selection was made because these well defined load histories were available, as well as results from fatigue crack growth
test series. The results include the effects of several variables, e.g. the effects of changing the design stress level, truncating high loads and omitting small load cycles. It would be most profitable if a fatigue crack growth model can predict such effects. Some more comments on practically relevant aspects of this problem are presented in the last section (3.3) of this chapter.

3.2 Characteristics of flight-simulation load histories

A flight-simulation load history is characterized by a variety of flights, the sequence of those flights, the sequence of loads in each flight, and load spectra. A summary of data on the load histories to be discussed is given in Table 3.1. More information on the load spectra is presented in Tables 3.2 to 3.5 and in Figures 3.3 to 3.6. The random flight sequences are shown in Figures 3.7 to 3.10 and illustrative samples of the flight-profiles are given in Figures 3.11 to 3.17. The flight profiles shown were generated on a PC with an algorithm also used for the prediction models to be discussed later. The square wave shape shows the maxima and minima in a better way than the analogue signal, which controls the load in a flight-simulation test.

The load spectra are based on statistical information of loads in service. The spectra are presented for a fixed number of flights, the block size, see Table 3.1. If one block of flights has been applied in a test, it is repeated again and again until the end of the test. The consequence of the application of similar blocks of flight in a test is that the maximum load occurring in a block is also the maximum load in the whole test. It is by definition the truncation level of the load spectrum. Although higher load levels may statistically occur in the aircraft life, they do not occur in flight simulation tests characterized by similar blocks of flights. There is a good reason to truncate the rarely occurring extremely high loads to a common truncation level. The extremely high loads can introduce favourable residual stress systems in all kinds of notched elements, i.e. compressive residual stresses at the notch root. As a result flattered fatigue results are possible, which will not apply to aircraft that do not
meet those high loads. To obtain "conservative" fatigue results, truncation must be advised. In [149] Schijve proposed that a truncation level should not exceed the load level which is statistically expected to be equalled or exceeded 10 times in the aircraft life. Quite often lower truncation levels are adopted, which is especially true if the block size is relatively small.

Most flight-simulation tests are carried out with a random sequence of different types of flights, which can vary from "light" flights to most "severe" flights. It can also include training flights. The randomness is supposed to be realistic for service conditions. Flight sequences are shown in Figures 3.7 to 3.10. The same philosophy applies to the loads in each flight. These loads are also supposed to occur in a random sequence, which is quite evident for gust loads. Obviously the distribution functions of the various types of flights and of the loads in flight must agree with the load statistics applicable to either a specific aircraft model or to some assumed aircraft utilization in standardized load histories. More information on the various flight-simulation load histories is given below.

3.2.1 TWIST

The TWIST flight-simulation load history (TWIST = Transport Wing Standard) was the first standardized flight-simulation history developed. It was published in 1973. It is well described and quantitatively defined in Ref. [5]. The load spectrum was obtained as an average of several wing load spectra for civil transport aircraft. Such spectra are dominated by gust loads and the spectrum is usually referred to as a gust spectrum. In one block 4000 flights of 10 different types (A to J in Table 3.2) occur in a random sequence (see Fig. 3.7). The random sequence is obtained by a random number selection. Strictly speaking it is a pseudo random selection. The severity of the flights vary from light (type J, nice weather) to highly severe (type A, severe storm). The load spectrum of each type of flight can be obtained from Table 3.2. The overall load spectrum for one block of 4000 flights is given in Figure 3.3. The continuous
load spectrum is approximated by a stepped function (10 amplitudes) for practical reasons.

The amplitudes ($S_m$) are related to the mean stress in flight ($S_{mf}$). In practice $S_{mf}$ will depend on the ultimate design stress level. The ground stress ($S_{gr}$), i.e. the minimum stress of the ground-air-ground cycle (GAG cycle) was standardized at $S_{gr} = -0.50 S_{mf}$. Samples of flights are presented in Figure 3.11. The stepped values of $S_m / S_{mf}$ are easily recognized. Each upward gust is followed by a downward gust, although not necessarily of the same magnitude. The selection of the amplitude sequence is again a randomly selected sequence. Because the random selection is made for all flights, flights of the same type will generally have different load amplitude sequences, although the amplitude spectrum for each type of flight is always the same. The different sequences are illustrated for flight type G in Figure 3.12 of a miniTWIST spectrum.

3.2.2 MiniTWIST

MiniTWIST is described in Refs. [5,11]. It was generated because experiments with TWIST were rather time consuming due to the large average number of 100 cycles per flight. MiniTWIST was derived from TWIST by omitting small amplitude cycles from the load spectrum, see Table 3.2 and Figure 3.3. The average number of cycles per flight was thus reduced to 15. That is a rather drastic reduction, which makes the testing time per flight considerably shorter. It was hoped that the fatigue damaging effect would still be about the same. Unfortunately comparative tests have shown that fatigue lives in flights are larger for miniTWIST and crack growths rates (mm/flight) are lower [99].

Because the same random number generators were used for TWIST and miniTWIST to select the sequence of flight types, the sequence of flights (Fig. 3.7) is the same for the two load histories. Samples of flight profiles of miniTWIST flights are shown in Figure 3.13 in comparison to profiles of the same type of flight for TWIST. The reduction of cycles per flight is evident from this figure. A comparison of flight types G and H in Figures 3.11 and 3.13 illustrates the large difference in numbers of cycles per flight.
3.2.3 F-27

The F-27 flight-simulation load history [150] was generated by similar procedures adopted for TWIST and miniTWIST. The load spectrum (gust dominated) is shown in Table 3.3 and Figure 3.4. It is based on data for the Fokker F-27. The load history was primarily generated for comparative studies, i.e. to study the effects of \( S_{mf} \), \( S_{mr} \), and \( S_{gr} \), and more and less severe gust spectra, which will be discussed later. The flight type sequence in one block is shown in Figure 3.8, and samples of flight profiles are given in Figure 3.14.

3.2.4 CN-235

The load spectrum was derived from the wing bending moment history of the full-scale fatigue test on the CN-235. This 40/44-seater aircraft was designed and is now produced by Airtech (Aircraft Technology Industry), a joint venture between the Indonesian aircraft industry Nusantara (IPTN) and CASA from Spain. Data on the flight types are given in Table 3.4. The load spectrum is shown in Figure 3.5. In one block of 1000 flights 10 different flight types occur on a random sequence as shown in Figure 3.9. The flight profiles are shown in Figure 3.15. In contrast with the previous load histories all flights of the same type always have the same sequence of peak loads. This procedure is also used by Boeing.

For TWIST, miniTWIST and the F-27 load spectrum the mean stress in flight \( (S_{mf}) \) is adopted as the characteristic stress level. However for the CN-235 spectrum due to its origin, 128 equidistant load levels were defined. The maximum level 128 corresponds to the maximum stress occurring once in the most severe flight (type A). The minimum level 1 corresponds to the lowest minimum stress in the same flight. Level 128 thus corresponds to \( S_{max} \) of the stress spectrum. Zero stress \( (S=0) \) corresponds to (the non-existent) load level 19.7. The stress levels corresponding to the other load levels can then be calculated, see the conversion relation in Table 3.4. The flight profiles in Figure 3.15 do not suggest such a strict constant mean stress in flight as for (mini) TWIST and F-27 flight profiles. The calculated mean stress in
flight corresponds to load level 63.76. In terms of stress ratio's it implies: \( S_{a,\text{max}}/S_{\text{mf}} = 1.46 \) and \( S_{\text{gr}}/S_{\text{mf}} = -0.31 \).

3.2.5 FALSTAFF

FALSTAFF is a standardized flight simulation load sequence for fighter aircraft wings. It was derived from four different aircraft types as flown by different air forces. Each aircraft operation considered comprised a mixture of exercises rather than flying one particular mission-type all the time. This implies that the total data sample represents a variety of exercises, such as: high/low navigation, instrument flying, combat patrol, tactical weapon training, conventional weapon training, close air support, forward area control, etc. [10]. As a consequence of the various mixtures all 200 flights of one block are different. The block size of 200 flights is significantly smaller than for the transport aircraft. However, the design life of a military aircraft is also much shorter, i.e. it can be 4000 flying hours, which should be compared to e.g. 60,000 flights for a commercial transport aircraft. Because the fatigue loads are largely due to manoeuvres, the spectrum is referred to as a manoeuvre spectrum. Figure 3.10 shows the numbers of cycles per flight and the maximum and the minimum stress occurring in each flight. It confirms that all flights are different. The differences are more obvious from the flight profiles presented in Figure 3.16 for three flights with the same numbers of cycles per flight.

Similar to the procedure for the CN-235 load spectrum, the FALSTAFF spectrum is characterized by equidistant load levels. There are 32 load levels, see Table 3.5. Level 32 corresponds to \( S_{\text{max}} \) of the load spectrum, which is the characteristic stress of the spectrum. Level 1 is the lowest negative manoeuvre load, for which \( S_{\text{min}}/S_{\text{max}} = -0.2667 \). With these two reference load levels all the other ones can be converted into \( S/S_{\text{max}} \) values, see the relation in Table 3.5.
3.2.6 MiniFALSTAFF

The miniFALSTAFF load history is a simple derivative of FALSTAFF, obtained by removing small load variations of the FALSTAFF load history. Load variations, counted by the rainflow method [151], were removed if the range was equal or smaller than three load level intervals. The rainflow counting method is explained by Figure 3.18. As a result the average number of cycles per flight is reduced to about 50%. The overall load spectrum is shown in Figure 3.6, with numerical data in Table 3.5.

The much smaller number of cycles per flight is shown by Figure 3.10 although the maximum stress and the minimum stress of each flight remain the same. The removal of small load variations is also illustrated by the flight profiles in Figure 3.17. This figure should be compared to Figure 3.16 which shows the same flights for FALSTAFF. The comparison shows that many small load variations have been omitted indeed*

3.3 Variables of flight-simulation histories

The load histories of aircraft components depend on the type of aircraft, its utilization in service and also on the type of component considered. It is evident that load histories for wing structures, fuselage, tail planes and undercarriages can be highly different. It might be expected that the fatigue load history of a fuselage is relatively simple. The dominant role of the pressurization cycle suggests that the fatigue load is approximately constant-amplitude loading with a zero minimum stress (R=0). In reality it may be more complex due to the biaxial loading, fuselage bending and torsion due to manoeuvres and gusts, whereas the crack growth phenomenon is complicated by stress biaxiality and bulge-out of crack edges. For wing structures the tension skin (lower wing skin) is fatigue critical

* The omission was not applied to the taxiloards. Although that is not logical, it was not done because France investigators, who initiated miniFALSTAFF did not do it.
for several types of aircraft. The fatigue load of a wing structure is
definitely complex, because of the superposition of deterministic and
stochastic loads as indicated before (Figure 3.2). The safety of the
aircraft is depending on fatigue crack growth in wing structures.
Nowadays the aircraft industry must show that the so-called damage
tolerance properties do satisfy the official requirements [82,152].
That implies that it must be known how fast fatigue cracks will grow.
Of course the best proof of the crack growth behaviour is obtained in
a flight-simulation full-scale fatigue test. However, it will be
obvious that it is highly desirable to have predictions on fatigue
crack growth in the design development phase of the aircraft. More-
over, a full-scale test covers only one load-history. Although that
history will be the best estimate which could be made, it will not
apply to all aircraft of that type used by various operators all over
the world. Figure 3.19 shows a comparison between estimated gust
spectra for the Fokker F-28 and measured data obtained in service
years after the full-scale fatigue test was completed. Apparently,
operators B and E encounter a less severe gust spectrum than the
original design spectra. A less severe, gust spectrum does not always
lead to much slower fatigue crack growth as shown in test series by De
Jonge et al. [153]. Anyhow, it is evident that prediction for devia-
ting load spectra can be desirable. This is also true if the weight of
an aircaft is increased, which can lead to higher design stress
levels. If reliable predictions can be made on the effect of the
design stress level a new costly full-scale fatigue test may not be
necessary.

Two other aspects of flight-simulation tests were mentioned before,
viz. truncation of high load amplitudes and omission of small cycles.
Both aspects are illustrated by Figure 3.20. The different shapes of
the gust spectrum and the manoeuvre spectrum are obvious. Actually the
truncation and the omission of small cycles are also modifications of
the load spectrum. Again quantitative information and possibilities
for prediction are desirable. In order to check whether prediction
models can account for such spectrum variations representative ex-
perimenatal data must be available. In the following chapter a "data
bank" of such data is described. It includes several test series
carried out as part of the present investigation.
CHAPTER 4
RESULTS FROM FLIGHT-SIMULATION TESTS

In Section 4.1 the flight-simulation tests carried out as part of the present investigation are described. Flight simulation test results selected from various literature sources are presented in Section 4.2.

4.1 Experimental program

Flight-simulation crack growth tests were carried out in an Amsler closed loop fatigue machine with a maximum capacity of 250 kN. The crack growth was measured by the d.c. electrical potential drop method as described in Ref. [154]. To allow compressive loads during the tests the specimens were enclosed in anti-buckling guides.

The material was aluminum alloy 2024-T3 Alclad with a nominal thickness of 2 mm. The dimensions of the specimen and the static properties are presented in Figure 4.1. The tests were carried out in a laboratory air environment at room temperature, all in duplicate. In the case of some scatter a third similar test was performed.

Two load spectra were adopted in these test series viz. miniTWIST and the CN 235 spectrum as described in the previous chapter. The variables of the load spectra used in the tests are listed in Table 4.1.

4.1.1 Experiments with the miniTWIST load history

Because of the random selection of gust amplitudes in a flight the positive and negative gusts occur in some random sequence depending on the random number generator. As suggested by the results shown in Figure 1.1 it may be important whether the large gust loads occur in a positive/negative sequence (small crack growth delay in Fig. 1.1) or in a negative/positive sequence (large delay in Fig. 1.1). If such a highly different behaviour occurred in a flight-simulation test, crack growth would heavily depend on the selected random sequence. Its pseudo random character could then be significant, because the same
sequence is applied in all blocks. This aspect did not get much attention in the literature. The sequence of the most severe amplitude levels (levels I, II and III in Table 3.2), which occur only in the three most severe types of flight (A, B and C), is indicated in Figure 4.2 in column "Original miniTWIST". In the tests load amplitude levels I and II were truncated to level III. As a consequence load amplitude level III occurs 8 times in one flight A, one flight B and three flights C. Figure 4.2 shows that the high upwards and downwards gusts occur in a typically random sequence. Two modified sequences were adopted, obtained by rearranging the positions of the severe gust loads. In one case the highest upward gusts were replaced to occur before the highest downward gusts, and in the other case the highest upward gusts were replaced to occur after the highest downward gusts, see the last two columns in Figure 4.2. The three different load sequences are indicated as:
- Original miniTWIST
- Modified miniTWIST max-min
- Modified miniTWIST min-max.

The tests were carried out at $S_{mf} = 85$ MPa. The truncation at level III corresponds to $S_a/S_{mf} = 1.3$. For the high gusts it implies $S_{max} = 2.3 S_{mf}$ and $S_{min} = -0.3 S_{mf}$. Because $S_{gr} = -0.5 S_{mf}$ negative gusts are not the most severe downwards loads, and the position of the most severe negative gusts in a flight may be less significant. For that reason a second test series was carried out with $S_{gr} = 0$. The most severe downward gusts ($S_{min} = -0.3 S_{mf}$) then may have a more predominant effect.

Test results have been presented in more detail in Refs. [155, 156]. Average results are presented here in Table 4.2 (crack growth lives) and in Figure 4.3 (crack growth rates). It appears that the three sequences give almost the same results. Moreover, the small differences do not show an easily recognized systematic trend. Some comments on the results are made later.

The crack growth increment ($\Delta a$) in flight A (the most severe flight) is relatively large. It was tried to determine $\Delta a$ in that flight in
two ways. First \( a_a \) was determined by extrapolating the crack length before flight A (\( a_1 \), see Figure 4.4) from flight nos. 1000, 1200, 1400 and 1600 and after flight A (\( a_2 \)) from flight nos. 1800, 2000, 2200 and 2400. The crack growth increment in flight A is equal to \( a_2 - a_1 \). The crack growth increments in flight A for the normal ground stress level \( (S_{gr} = -0.5 \, S_{mf}) \) and for light ground stress level \( (S_{gr} = 0 \, MPa) \) are presented in Figure 4.5. The figure indicates higher crack growth increments for the more severe ground stress level.

In the second test series, a second method was introduced for comparison. It was possible to measure manually the crack length with the electric potential drop method within a range of 4 flights. The crack lengths were measured at the end of flight numbers 1652 and 1656. In other words, the crack growth increments are for 4 flights i.e. flight types D(1653), J(1654), J(1655) and A(1656). It might be expected that the crack increment contributions of flight types D and two flights J are very small compared to flight A. Crack growth increments were measured with the two methods are presented in Figure 4.6. The results of both methods are in good agreement.

4.1.2 CN 235 flight simulation tests

Tests were carried out at two stress levels, viz. \( S_{max} = 200 \) and 162 MPa respectively. The fatigue lives in Table 4.3 and the crack growth rate in Figure 4.7 confirm a large effect of the stress level as should be expected. The test results are reported in more detail in [157]. Measurements of the crack growth increment in flight A were improved by modifying the computer program. Potential drop measurements can now be made at the beginning and at the end of a flight. This has been done only for \( S_{max} = 200 \, MPa \), due to limitations of the potential drop method accuracy. Results are given in Figure 4.8. Recently more refined fractographic measurements have been made by Partl [158] by using a scanning electron microscope. Because in the CN 235 load history all flights of the same type have always the same profile, Partl managed to recognize several types of flights on the fatigue fracture surface and to measure the crack lengths increments in those flights. Average results are presented in Figure 4.9.
As part of the fractographic examination tests were also carried out with programmed sequences of the flights in one block. Two programmed sequences adopted are shown in Figure 4.10. The first one is a simple LoHiLo sequence, with an increasing/decreasing sequence of flight severity. The second one is a modified LoHiLo sequence which allowed to measure fractographically as in low severity flights. The measurements were made for $S_{max} = 162$ MPa only. As shown by the results in Table 4.3 and Figure 4.11 the crack growth life is longer for the programmed flight sequences. It confirms that realistic random sequences should be preferred.

In another tests series the effect of the truncation level was studied. The tests were carried out, by Pratomo [159] as part of his master thesis project. Three truncation levels were used which correspond to the maximum load in flights of type A, B and C respectively (load levels 128, 117 and 103 in Table 3.4). The same procedure was adopted for truncating the severe negative gust loads (load levels 1, 18 and 26 in Table 3.4). The truncation levels are indicated in the load spectrum in Figure 3.5. Average crack growth lives are presented in Table 4.3. A significant truncation level is observed, i.e. shorter lives after truncating high gust loads. It is also shown by the crack growth curves in Figure 4.12.

Pratomo also carried out tests with and without taxiing loads. The number of taxiing loads is 6 to 8 per flight, see the flight profiles in Figure 3.15. The taxi loads are small cycles occurring under compressive loads. They hardly can be damaging, and the tests indeed indicated a negligible effect of omitting the taxi load cycles.

4.2 Flight-simulation test results from other sources

In addition to the results of the previous section, several flight simulation test results have been collected from the literature. The purpose is to collect a sufficient set of representative flight-simulation test data to evaluate crack growth prediction models. The load spectra involved have been described in Chapter 3. In addition results will be presented from tests with highly simplified flight simulation loading, the so-called F 4 load sequence and load sequences
adopted by Misawa and Schijve [61]. The various spectra were used in extensive test series to study the effects of spectrum variables. The results of such test series are required to examine the prediction capabilities of cycle-by-cycle crack growth models. The models will be studied to see whether they can account for variations such as:

- severity of gust loads (F-27 tests)
- severity of the ground-air-ground cycle (GAG) levels (F-27, mini-TWIST and Misawa/Schijve tests)
- variation of the design stress level (F-27, CN 235, FALSTAFF and miniFALSTAFF tests)
- truncation levels (TWIST tests, CN 235 tests)
- influence of manoeuvre spectrum (FALSTAFF and miniFALSTAFF tests)
- omission of small cycles (miniFALSTAFF tests)
- load interaction effects (Misawa/Schijve tests)
- influence of load sequence in flight (miniTWIST, Misawa/Schijve tests)
- influence of flight sequence (CN 235 tests)
- influence of the number of cycles in flight (Misawa/Schijve tests).

In the various test series, three different materials were used namely 2024-T3 Al clad, 2024-T3 bare and 7075-T6 Clad. The nominal thickness was 2mm for all materials. The tests were all performed in normal laboratory air at room temperature. Flight simulation test variables are presented in Table 4.4. More details are presented in the following sections.

4.2.1 F-27 and F 4 flight-simulation tests

Test results were reported by Van der Linden [150] for 2024-T3 Al clad (2 mm) and by De Jonge [160] for 7075-T6 Clad (2 mm). The width of the central cracked specimens (similar to the specimen shown in Fig. 4.1) was \( W = 160 \text{ mm} \). The central notch width was \( 2a_0 = 7 \text{ mm} \).

The load spectrum and the random sequence of flights described in Section 3.2.3 was used as a basis for comparative tests. This load history is referred to as case NN (Normal gust spectrum, Normal ground stress level). Variables studied were the gust spectrum severity, the ground stress \( (S_{gr}) \) severity and the mean stress in flight \( (S_{mf}) \).
- Gust spectrum severity.

A severe gust spectrum and a light gust spectrum were obtained by increasing and decreasing the $S_a/S_{mf}$-values of the normal gust spectrum, see Table 4.5. It should be pointed out that it did not affect the number of cycles per flight, neither the sequence of the gust loads in a flight. The spectra are shown in Figure 4.13.

- Ground load level.

Several severities were used (three for 2024-T3 and four for 7075-T6) with $S_{gr}/S_{mf}$ varying from -0.5 (severe) to +0.125 (light), see Table 4.6. The table also indicates the letter codes used for the various gust spectrum/ground stress level severities.

- Mean stress in flight.

The above severities were studied at $S_{mf} = 100$ MPa and $S_{mf} = 70$ MPa for 2024-T3 and 7075-T6 respectively. For the case NN different $S_{mf}$ values were adopted, viz. 70, 90, 100 and 110 MPa for 2024-T3 and 70, 80 and 90 for 7075-T6.

The F 4 load sequence is a simple one-amplitude load in flight, see Figure 4.14. In the fifties this type of flight-simulation load sequence was applied in full-scale tests (e.g. Comet, Caravelle, F-27). It is now considered to be an unrealistic service simulation, which probably gives rather conservative results.

The test results are presented as crack growth lives in Table 4.7 and Figures 4.15 and 4.16. Crack growth rates are given in Tables 4.8 and 4.9. The results in Figure 4.15 show several systematic trends: shorter crack growth lives for more severe gust spectra and for more severe ground stress levels (i.e. more negative $S_{gr}$). The trends are qualitatively similar for both alloys, but it is remarkable that the effects are more obvious for the 2024-T3 alloys. Figure 4.16 shows the well-known effect of $S_{mf}$ on crack growth life. The result for the F 4 load history is considerably below the F-27 result, although it was supposed to be equally damaging according to a linear damage calculation. Apparently that gives a conservative result.
4.2.2 TWIST flight-simulation tests

The tests were carried out by ProvoKluit [161] on central cracked specimens (width $W = 100$ mm) of two materials, 2024-T3 Al clad and 7075-T6 Clad (thickness $= 2$ mm). The central notch was a saw cut with a total length $2a_o = 3$ mm.

The test variable was the truncation level. For 2024-T3 Al clad material, the tests were carried out for truncation levels II, III, IV and V, while for 7075-T6 Clad material the tests were truncated at levels I, II, III, IV and V, see Figure 3.3.

The mean stress in flight was 70 MPa for all tests. The load spectrum and the random sequence of flights and loads in flight are described in Section 3.2.2.

The test results are presented in crack propagation life and crack propagation diagrams, see Table 4.10 and Figures 4.9 and 4.10 respectively. The results show that a lower truncation level leads to shorter lives for both alloys. However, the effect of truncation level is small for 7075-T6, whereas it is rather obvious for the 2024-T3 alloy.

4.2.3 FALSTAFF and miniFALSTAFF flight simulation tests

The tests were carried out by Vlutters [162] on central cracked specimens (width $W = 100$ mm) of 2024-T3 bare material (thickness $t = 2$ mm).

The central notch was a saw cut with a total length $2a_o = 3$ mm.

The purpose of the tests was to check the effect of the stress level and to compare crack growth under FALSTAFF and miniFALSTAFF. The tests were carried out for two maximum stress values: 202.5 MPa and 247.5 MPa for both FALSTAFF and miniFALSTAFF.

The test results are presented in crack propagation life from a crack length of 4 mm until 19 mm and crack propagation rate diagrams, see Table 4.11 and Figure 4.19 respectively. The influence of the design stress level is evident. An increase of maximum stress level decreases
the crack growth life. The difference between FALSTAFF and miniFALSTAFF results is negligible. This is in agreement with other test result reported in Ref. [163].

4.2.4 *Simplified flight-simulation loading tests*

The load sequences used in this tests series are shown in Figure 4.20 adopted from Ref. [61]. During one test all flights are equal. Three types of flight load sequence were adopted:
- Type I: all gust cycles in flight are equal
- Type II: similar to type I with a single overload at the beginning of the flight
- Type III: similar to type I with the overload at the end of the flight.

The mean stress in flight was 80 MPa and the stress amplitude was 40 MPa. The other variables were:
- The number of cycles per flight, \( m \). Two values adopted are 5 and 100 cycles.
- The minimum of the ground stress values used are 0, -40, and -80 MPa.
- The overload stresses are 160 and 200 MPa.

The purpose of the tests was also to study the effect of periodic overloads and underloads.

The material was 2024-T3 bare with a nominal thickness of 2 mm. The width of the central cracked specimens was 100 mm, and the central notch \( 2a_o = 3 \) mm. Because the results for types II and III were not significantly different (see Table 4.12) average results of II and III are presented as crack propagation lives and crack propagation rate diagrams in Figures 4.22 to 4.24.

Some conclusions were drawn by Misawa and Schijve [61]:
- The ground stress level had a small effect on the crack growth life under the type I load sequence, while for types II and III a moderate effect was observed. Crack growth was faster for lower \( S_{gr} \) values.
- The effect of the overload in each flight was large. An increased $S_{0L}$ made the life shorter for $m = 5$, and much larger for $m = 100$, see Figure 4.22.

- Crack growth was very much similar for types II and III load sequences.
CHAPTER 5
PREDICTION MODELS

In the present chapter the yield zone models and the strip yield models are reviewed. The crack closure models are discussed in Chapters 6, 7 and 8. The yield zone models are represented by the Wheeler model [23] and the Willenborg model [22], while the strip yield model is represented by Newman's model [164,165]. The models are briefly described and some comments are given.

5.1 Yield zone models

5.1.1 Wheeler model

The Wheeler model predicts retardation by reducing the crack growth rate within the plastic zone created by an overload, see Figure 5.1. The amount of retardation depends on the ratio of instantaneous plastic zone size \( r_p,i \) and the remaining distance of the crack tip to the plastic zone boundary \( \lambda \). This ratio was adopted by Wheeler to define a crack growth retardation factor, \( C_p \), as follows

\[
C_p = \begin{cases} 
(r_p,i / \lambda_i)^m & \text{for } (r_p,i/\lambda_i) < 1 \\
1 & \text{for } (r_p,i/\lambda_i) > 1 
\end{cases} \tag{5.1}
\]

The Wheeler exponent \( m \) is still to be determined.

The plastic zone sizes \( r_{p,OL} \) and \( r_{p,i} \) are calculated using the simple plastic zone size equation:

\[
r_p = \alpha \left( \frac{K}{\sigma_y} \right)^2 \tag{5.2}
\]

The constant \( \alpha \) accounts for the state of stress (\( \alpha = 1/\pi \) for plane stress, \( \alpha = 1/3\pi \) for plane strain).
λ_i = r_{p,OL} - Δa_i \quad (5.3)

where Δa_i is the crack increment after the overload, see Figure 5.1.

The crack growth rate is then obtained by multiplying the non-interaction crack growth rate with the retardation factor C_p.

\[
\frac{da}{dN} = C_{p,i} (\frac{da}{dN})_{CA}
\]

where \((\frac{da}{dN})_{CA} = C \Delta K^m\) \quad (5.5)

The Wheeler exponent, m is not a material constant, and unfortunately it is also dependent on the type of variable-amplitude loading. It has to be empirically determined, using material and the load spectrum for which the prediction has to be made. The exponent m is thus an empirical data-fitting parameter. The model had some success in predicting crack growth under various spectra, but many spectrum tests are required for each material in order to determine the Wheeler exponent m. The model excludes the possibility of accelerated crack growth. These limitations imply that the model can not have any general applicability. It can not be applied to explore the significance of spectrum variations, such as truncation and omission of small cycles.

Modifications of the Wheeler model were proposed by Pinckert [166], by Broek and Smith [167] and by Gray and Gallagher [168]. The modified models in Refs. [166,167] still use the exponent m for matching the prediction to experimental data. Gray and Gallagher [168] eliminate the dependency on data fitting by calculating the exponent m as shown below.

Eq. (5.3) can be rewritten as

\[
λ_i = r_{p,OL} (1 - \frac{Δa_i}{r_{p,OL}})
\]
Since $\lambda_i$ can be considered as the limit of plastic zone size in order to have retardation, $\lambda_i$ can be correlated with the stress intensity factor in Eq. (5.2). Substituting Eq. (5.2) to Eq. (5.3) gives

$$\alpha \left( \frac{K_{\text{max}}}{\sigma_y} \right)^2 = \alpha \left( \frac{K_{\text{max},0L}}{\sigma_y} \right)^2 \left( 1 - \frac{\Delta \lambda_i}{r_{p,0L}} \right)$$

or

$$K_{\text{max}}^* = K_{\text{max},0L} \left( 1 - \frac{\Delta \lambda_i}{r_{p,0L}} \right)^{1/2}$$  \hspace{1cm} (5.6)

where $K_{\text{max}}^*$ is the stress intensity factor related to $\lambda_i$.

The Wheeler reduction $C_p$ in Eq. (5.1) can be written as a function of stress intensity factor by substituting Eq. (5.2) into Eq. (5.1)

$$C_p = \left( \frac{K_{\text{max}}^*}{K_{\text{max}}} \right)^{2m} \quad \text{if } K_{\text{max}} < K_{\text{max}}^*$$

$$C_p = 1 \quad \text{if } K_{\text{max}} \geq K_{\text{max}}^*$$  \hspace{1cm} (5.7)

The crack growth rate can be expressed in terms of an effective stress intensity range, $\Delta K_e$, which is not the same as that used in Elber's closure concept.

With Eq. (5.4),

$$\frac{da}{dN} = C_c \Delta K_e^n = C_p \Delta K^n$$

or

$$\Delta K_e = C_p^{1/n} \Delta K$$  \hspace{1cm} (5.8)

Eq. (5.8) is then combined with Eq. (5.7), which gives

$$\Delta K_e = \left( \frac{K_{\text{max}}}{K_{\text{max}}^*} \right)^{2m/n} \Delta K \quad \text{if } K_{\text{max}} < K_{\text{max}}^*$$  \hspace{1cm} (5.9)

$$\Delta K_e = \Delta K \quad \text{if } K_{\text{max}} \geq K_{\text{max}}^*$$  \hspace{1cm} (5.10)
Test evidence indicates that there is a limit value of the overload ratio $S (S = K_{\text{max,OL}}/K_{\text{max}})$ where crack arrest occurs after a single overload. It implies that after an overload, $\Delta a = 0$ when $K_{\text{max,OL}}/K_{\text{max}} = S$. Substituting these relations into Eqs. (5.6) and (5.9) yields

$$\Delta K_e \text{ (at arrest)} = \left[ \frac{S}{S} \right]^{2m/n} \Delta K$$

(5.11)

The arresting of crack growth indicates that the effective stress intensity range is equal to or less than the threshold stress intensity range,

$$\Delta K_e \text{ (at arrest)} = \Delta K_{TH}$$

(5.12)

For simplification, $\Delta K_{TH}$ was assumed to be independent on stress ratio $R$. Eq. (5.11) and Eq. (5.12) can be combined and gives

$$m = \frac{n}{2} \log \left( \frac{\Delta K}{\Delta K_{TH}} \right)$$

(5.13)

The above descriptions show that $m$ is no longer a data fitting parameter, but that it can be calculated as a function of some known parameters. The parameters are supposed to be material dependent. The prediction results were reported to be good within a factor of two for predicting delay periods following a single overload for the 2024-T3 Al-alloy, 4340 steel and Ti-6Al-4V. However, as to the original Wheeler model, it still does not include the possibility of accelerated crack growth. Although the current plastic zone size in relation to the largest plastic zone of the preceding load history is part of the model, it is still difficult to see that the model agrees with the present understanding of interaction effects.

5.1.2 Willenborg model

The Willenborg model also includes some assumptions to account for retardation within the plastic zone of an overload. The model does not include a data-fitting parameter. The model is formulated as described below.
A required maximum stress intensity factor, $K_{\text{max,req}}$ is defined, which will produce a plastic zone just large enough to touch the border of the overload plastic zone, see Figure 5.2. It should be noted that the approach starts in a similar way as for the Wheeler model, compare also Figures 5.1 and 5.2 and Eq. (5.3) and the following equation (5.14). The definition of $r_{p,\text{req}}$ implies:

$$a_i + r_{p,\text{req}} = a_{OL} + r_{p,OL}$$  \hspace{1cm} (5.14)

With Eq. (5.2)

$$a (K_{\text{max,req}}/\sigma_y)^2 = a_{OL} + r_{p,OL} - a_i$$  \hspace{1cm} (5.15)

or

$$K_{\text{max,req}} = K_{\text{max,OL}} \sqrt{1 - (a_i - a_{OL})/r_{p,OL}}$$  \hspace{1cm} (5.16)

The difference between $K_{\text{max,req}}$ and the actual $K_{\text{max},i}$ is then defined as $K_{\text{red}}$:

$$K_{\text{red}} = K_{\text{max,req}} - K_{\text{max},i}$$  \hspace{1cm} (5.17)

Subsequently $K_{\text{max},i}$ and $K_{\text{min},i}$ of the current cycle are reduced by $K_{\text{red}}$ to obtain the effective values

$$K_{\text{max,eff},i} = K_{\text{max},i} - K_{\text{red}}$$

$$K_{\text{min,eff},i} = K_{\text{min},i} - K_{\text{red}}$$  \hspace{1cm} (5.18)

$$R_{\text{eff},i} = K_{\text{min,eff},i} / K_{\text{max,eff},i}$$

The model neglects negative stresses. The effective range of the stress intensity factor can be reviewed as follows,

if $K_{\text{max,eff},i}$ and $K_{\text{min,eff},i} > 0$ then $\Delta K_{\text{eff},i} = \Delta K_i$

if $K_{\text{min,eff},i} < 0$ then $\Delta K_{\text{eff},i} = K_{\text{max,eff},i}$

if $K_{\text{max,eff},i} < 0$ then $\Delta K_{\text{eff},i} = 0$  \hspace{1cm} (5.19)
By knowing the $\Delta K_{\text{eff},i}$ and $R_{\text{eff},i}$ the crack growth in cycle (i) can be calculated using the Paris relation or the Forman relation.

As a consequence, the reduction of the $K$ values in Eq. (5.18) does not affect $\Delta K_{\text{eff},i}$ as long as $K_{\text{min,eff},i} > 0$, but it does decrease the $R$ ratio. As soon as $K_{\text{min,eff},i} < 0$ the negative part is truncated. According to Eqs. (5.16 and 5.17) $K_{\text{max,req}}$ and $K_{\text{red}}$ reach a maximum value if $(a_i - a_o)$ approaches zero. This means that maximum retardation occurs immediately after the overload. Delayed retardation and also acceleration are not predicted. Actually, the reduction of $K_{\text{max},i}$ and $K_{\text{min},i}$ is a rather strange procedure, which becomes more obvious after the substitution of Eqs. (5.16 and 5.17) into Eq. (5.18) to obtain

$$K_{\text{max,eff},i} = 2K_{\text{max},i} - K_{\text{max,OL}} [1 - (a_i - a_{OL})/r_{p,OL}]^{1/2}$$

The last term is a function of the overload cycle and the current crack length. Strangely enough it implies a reduction not of $K_{\text{max},i}$ but of $2K_{\text{max},i}$.

An improvement of the model was proposed by Chang et al. [169]. They apparently recognized some limitations of the Willenborg model. Extensions introduced by Chang et al. were related to the effects of high compressive loads (underloads) and crack growth arrest caused by overloads. If an overload is followed by an underload the crack growth retardation effect of the overload is reduced. The Willenborg model was modified to account for this effect. Crack growth arrest after an overload was also accounted for by introducing a threshold $\Delta K$. Moreover the favourable effect of overloads was limited by a so-called shut-off overload ratio. The modification of the model implies some new material constants. The physical meaning and consequences of the assumptions made are not easily understood. After all, the modified model still does not predict delayed retardation, neither can it account for accelerated crack growth. It does not include the crack closure concept. Also here it must be concluded that the Willenborg model and the modified model do not agree with the present understanding of interaction effects.
5.2 Strip yield models

Strip yield models were proposed by Newman [164,165], Seeger [170] and De Koning [171]. The models are based on the Dugdale model for calculating the plastic zone size [38]. In order to understand how the strip yield models predict crack growth, the Newman model is summarized below. However, prior to this the Dugdale strip yield model must be discussed first.

The aim of the model of Dugdale [38] was to estimate the size of the plastic zone \( r_p \) at the tip of a crack. Dugdale assumes that yielding occurs in a narrow strip ahead of the crack tip, see Figure 5.3a. The material response to plastic deformation is rigid-perfectly plastic, which leads to a constant stress (yield stress) in the plastic zone. Dugdale assumes that the situation in Figure 5.3a is equivalent to the stress system in Figure 5.3b where a larger fictitious crack length \( a^* \) = \( a + r_p \) is present with crack edge loading between \( a \) and \( a^* \) with a stress \( \sigma = \sigma_y \). The equivalence also requires that the stress at the fictitious crack tip is non-singular (i.e. equal to \( \sigma_y \)). In other words the singularity has to be removed, which implies that:

\[
K_s + K_{\sigma_y r_p} = 0
\]

For an infinite sheet with a central crack it leads to:

\[
S \sqrt{\pi a^*} - (2/\pi) \sigma_y \sqrt{\pi a^* \arccos (a/a^*)} = 0
\]  

(5.31)

With \( a^* = a + r_p \) the value of \( r_p \) can now be calculated from Eq. (5.31) which leads to:

\[
r_p = \frac{a}{\sec(\pi/2 \frac{\sigma_y}{S})^2 - 1} \approx \frac{a^2}{8 (S/\sigma_y)^2} = \frac{\pi}{8} \left( \frac{K}{\sigma_y} \right)^2
\]

(5.32)

The approximate value in (5.32) may be compared to the Irwin plane stress estimate:

\[
r_p = \frac{1}{\pi} \left( \frac{K}{\sigma_y} \right)^2
\]

(5.33)
which is indeed of a similar order of magnitude. However, the remarkable aspect to be noted here is that a plastic zone size in the Dugdale model is calculated by a superposition (Figure 5.3b and Eq. (5.30)) of two essentially elastic K-solutions. This also allows the calculation of the crack opening displacement (COD) for the fictitious crack (a'). The assumption made in Newman's strip yield model is that COD in the plastic zone (a' x a') is accommodated by plastic elongation in the plastic strip. The COD is assumed to become the vertical length of plastic material, see Figure 5.3c. In other words, the elastic crack opening of the fictitious crack tip is supposed to be equal to the plastic deformation in the real crack tip zone. It also implies that an initially thin strip with zero height is transformed into plastically deformed material with a finite height. Also these above aspects are remarkable.

The major reason for the previous modelling is that the crack now will grow into a plastic zone with a quantified plastic elongation. The result is that plastically deformed material is left on the edges of the crack, i.e. in the wake of the crack. Upon unloading, that should lead to crack closure at a positive stress, as illustrated by the graphs from Newman shown in Figure 5.4.

For numerical calculations the plastic zone is supposed to consist of 10 vertical bar elements with a width of 0.01, 0.01, 0.02, 0.04, 0.06, 0.09, 0.12, 0.15, 0.2 and 0.3 times r_p respectively. The lower values apply to bar elements near the real crack tip. The bar elements will yield in tension if the load on the bar element (σ_j) is ασ_y where α is a plastic constraint factor (α = 3 for pure plane strain and α = 1 for pure plane stress). The elements will yield in compression if σ_j = -σ_y (no constraint factor applied). Under cyclic load the problem is to determine whether bar elements will deform plastically and whether opposite bar elements in the wake of the crack will lose contact (opening) or touch again (closing). The conditions, which govern these questions, are based on the compatibility of the elastic crack edge (vertical) displacements (V_i) and the length of the bar element (L_i).

The compatibility implies:

\[ V_i = L_i \]  

(5.34)
The elastic crack edge displacements are calculated from:

\[ V_i = S f(x_i) + \sum_{j=1}^{n} \sigma_j g(x_i, x_j) \]  \hspace{1cm} (5.35)

The first contribution \( S f(x_i) \) comes from the remote loading by the nominal stress \( S \), while \( f(x_i) \) accounts for the geometry. For an infinite sheet and plane stress \( f(x_i) \) leads to the well-known elliptical crack opening:

\[ f(x_i) = \frac{2}{E} \sqrt{(a^2 - x_i^2)} \]  \hspace{1cm} (5.36)

The \( \sigma_j \) contributions are due to the loads on the bar elements. Newman gives the equations for \( g(x_i, x_j) \). Both for \( f(x_i) \) and \( g(x_i, x_j) \) he includes finite width corrections for a centrally cracked sheet specimen.

The major problem then is to calculate the loads \( \sigma_j \) of the bar elements, because these loads depend on the crack edge displacements \( V_i \). To do this a set of linear equations must be solved by a Gauss-Seidel iterative method. The problem is complicated by constraints on the model. The yielding status of the bar elements has to be considered. In the plastic zone if \( \sigma_j > \sigma_y \) then yielding in tension occurs and \( \sigma_j = \sigma_y \). If \( \sigma_j < \sigma_y \) yielding in compression occurs and \( \sigma_j = -\sigma_y \). In the wake of the crack only yielding in compression is possible (\( \sigma_j = -\sigma_y \)). Also in the wake, if \( \sigma_j > 0 \) then \( \sigma_j = 0 \) because opening does occur.

Apparently a cycle-by-cycle calculation of \( S_{op} \) would require most extensive computer time, while reassessments of the number and location of bar elements would also be problematic. To avoid these problems Newman assumes that \( S_{op} \) remains constant during a small crack extension arbitrarily defined as:

\[ \Delta a = 0.05 r_{p, max} \]  \hspace{1cm} (5.37)

Crack growth is then calculated cycle by cycle with:
\[ \frac{da}{dN} = f (\Delta K_{\text{eff}}) \] (5.38)

The calculation is continued either until a crack extension equal to \( \Delta a \) in Eq. (5.37) has been reached or until 300 cycles have been applied. When using the model the constant \( S_{\text{op}} \) is calculated by considering the load cycle with highest \( S_{\text{max}} \) in the period considered and the lowest \( S_{\text{min}} \) before and after the occurrence of that \( S_{\text{max}} \) value. The procedures are then repeated for the next interval \( \Delta a \). Newman was aware of the fact that the cycle definition for calculating \( S_{\text{op}} \) can imply that certain sequence effects will get lost.

5.3 Comparison of different generations of crack growth prediction models

In Section 1.4 it was said that there are three generations of cycle-by-cycle prediction models.

1. Yield zone models
2. Crack closure models
3. Strip yield models

As pointed out in Section 5.1, the yield zone models are considered to be too primitive to agree with the present understanding of crack growth interaction effects under variable-amplitude loading. More specifically, crack closure is not included in these models whereas empirical evidence clearly shows that it does occur and can cause significant interaction effects.

The second generation of models, the main subject of this thesis, is essentially based on the occurrence of crack closure. Later discussions will show that crack closure is introduced in the models using intuitive arguments based on empirical observations. They have a hypothetical character, which is supported by qualitatively physical reasons.

The third generation, the strip yield models, possess the appeal of being the most sophisticated models. One good argument in their favour is that the models do not rely on assumptions about the magnitude of crack closure. The models predict the amount of crack closure. Several
good crack growth predictions were mentioned by Newman [164,165] for simple load histories. Good predictions under standardized sequences i.e. TWIST and FALSTAFF were reported in Refs. [172-174]. Recently Wang and Blom [172] compared predictions for the single overload case (Figure 5.5) for the Willenborg model, the Wheeler model and a strip yield model. Only the last one predicts the initial acceleration and delayed crack growth in agreement with the test results.

Sometimes, the better performance seems to be associated with the large computer capacity required, which now may be a problem for general application, but which may be expected to be less important in the future. At the same time, the recapitulation of Newman's model in the previous section should make it clear that the physical arguments are not as obvious as might be hoped. Remarkable features have been indicated. The plastic zone size follows from an elastic analysis. Plastic deformations follow from an elastic analysis of the crack opening of a fictitious crack tip. The plastic material behavior is rather simple, i.e. rigid-perfectly plastic. The plastic constraint factor is different for tension and compression, and it is difficult to understand why that should be so. Finally, the significance of the simplifications necessary in view of the computer time required is not easily evaluated.

As will be shown later, the cycle-by-cycle variation of $S_{op}$ occurring in the second generation models can easily be visualized by the computer. At the moment this is not true for the strip yield models. A more indirect verification is possible with empirical macro crack growth curves. Although results as shown in Figure 5.5 are promising, the ultimate goal is to arrive at predictions for the truly random service load histories. There are still difficulties to be solved. If a model becomes very complicated there is a risk that it can be understood by a few people only and that it remains a "black box" for the people who want to use it.
CHAPTER 6
CRACK-CLOSURE MODELS

6.1 Introduction

As previously said (Section 1.4), the crack growth prediction models for variable amplitude loading can be categorized in three generations:

1. Yield zone models
2. Crack closure models
3. Strip yield models

It was pointed out that the mechanistic concepts of the yield zone models are oversimplified. A major shortcoming is that they do not include crack closure (Elber's mechanism) in an acceptable way and this has unacceptable consequences.

The second generation of models, i.e. the crack closure models PREFFAS, ONERA and CORPUS do include crack closure as the basic mechanism to account for interaction effects. They also account for plane strain and plane stress differences. The cycle-by-cycle crack growth calculation requires a cycle-by-cycle calculation of crack growth and readjustments of the crack opening stress level ($S_{op}$). Although the calculation efforts may be more substantial than for the first generation of yield zone models, the predictions can still be made in a relatively short time on small computers, if compared to the third generation of prediction models, i.e. the strip yield models.

The crack closure models in the literature were checked almost exclusively by the authors of the model. Good predictions have been reported. However, the understanding of how the models account for important variables of flight-simulation tests has not been evaluated. Illustrations of the variation of $S_{op}$ during flight-simulation tests are not presented, although it would improve the understanding of how a model works. Such illustrations are given in this thesis. The models
are not similar and comparisons of the models, as well as a study of their limitations, is very much needed. The same is true for application of the models to a more selective data bank of test results. These arguments are the backbone of the present study. The three models will be discussed and analysed for the above purposes in this chapter (PREPPAS model) and in Chapters 7 (ONERA model) and 8 (CORPUS model).

The three models have the same basic aspects in common. The crack opening stress level \( S_{op} \) and the corresponding \( K_{op} \) must be calculated cycle-by-cycle which leads to an effective stress intensity range for the subsequent cycle \( (i) \):

\[
\Delta K_{eff,i} = K_{max,i} - K_{op,i}
\]

Crack growth data obtained under constant-amplitude loading are then adopted in the following format (see discussion in Section 2.2.3.1)

\[
da/dN = f(\Delta K_{eff})
\]

It is thus assumed that the same \( \Delta K_{eff} \) under constant-amplitude loading and under variable-amplitude loading will produce the same crack extension (similarity concept):

\[
\Delta a = da/dN
\]

In all three models it is recognized that \( K_{op} \) will heavily depend on high positive peak loads and high negative peak loads. The obvious reason is that the high positive loads will produce the larger (monotonic) plastic zones at the crack tip, while the high negative loads can lead to significant reversed plasticity (see Figure 6.1) and reductions of \( K_{op} \). It should not be expected that every \( K_{max,i} \) and \( K_{min,i} \) in a new cycle \( (i) \) will always change \( K_{op} \). A number of relatively high historical peak values (\( KH_{max} \) and \( KH_{min} \), \( H \) from historical) of the preceding load history will have a predominant influence of the current \( K_{op} \) level. For all three models it leads to a convergent series of historical load levels, see Figure 6.2. These load
levels have to be memorized because they are significant for $K_{op}$ in each new cycle. The models are significantly different in the assumptions made on the cycle-by-cycle evolution of the historical stress levels. The PREFFAS model is the simplest of the three models, although it was the latest to be published. The CORPUS model is the most sophisticated model.

Different methods to check prediction models were discussed by Schijve [20]. For practical purposes, the ratio between prediction and test is sufficient. Further, he also mentioned that prediction to test result ratios in the range from 1/2 to 2 are still of an acceptable quality. But even a ratio value equal to one does not prove that the prediction model is good, because it is possible that the prediction is underestimated in the beginning and overestimated later on. In the interest of more confidence, the crack growth rate as a function of crack length and crack growth increment in each flight or every load cycle can be used to check the model.

A comparison between test result and the non-interaction prediction gives an indication of the occurrence of significant interaction effects. A non-interaction prediction can easily be programmed by calculating for each cycle the $S_{op}$ level from the $S_{max}$ and the preceding $S_{min}$. That will give $S_{op}$ and $\Delta S_{eff} = S_{max} - S_{op}$ as it is assumed to occur in constant-amplitude tests. The corresponding crack extension per cycle follows from the $da/dN = AK_{eff}$ relation. In a computer program the stationary crack concept is applied for fast calculation. The stationary crack concept is also used in the PREFFAS model, which will be described later. Schijve's effective stress range $U (= \Delta S_{eff}/\Delta S = AK_{eff}/AK)$ Eq. (2.14) is adopted in the non-interaction calculations, where

$$U = 0.55 + 0.33R + 0.12R^2$$

The Paris relation is used:

$$da/dN = C AK_{eff}^m$$
The C and m parameters used in the relation are

a. For 2024-T3 Al clad material derived from Ref. [148].
   \[ C = 3.57 \times 10^{-11} \]
   \[ m = 4.36 \]

b. For 2024-T3 bare material derived from Ref. [61].
   \[ C = 1.61 \times 10^{-11} \]
   \[ m = 4.15 \]

c. For 7075-T6 Clad material derived from Ref. [148].
   \[ C = 3.9 \times 10^{-10} \]
   \[ m = 3.52 \]

The above C values are for crack growth rate in m/cycle and \( \Delta K_{\text{eff}} \) in MPa/\( \sqrt{\text{m}} \).

The non-interaction prediction results for different flight-simulation tests presented in Chapter 4 are shown in Figure 6.3. The results show that the predicted crack growth lives are in most cases considerably lower than the test results. It implies that significant retardation effects must have occurred. The question then is whether the prediction models to be discussed will produce better results. The PREFFAS model is discussed below. The ONERA model and the CORPUS model are evaluated in Chapters 7 and 8 respectively.

6.2 The PREFFAS model

The PREFFAS model was proposed by Aliaga, Davy and Schaff [30-32] (PREFFAS = PREvision de la Fissuration en Fatigue, AeroSpatiale). Apparently the authors wanted a simple model with a few crack growth calibration tests to characterize the material response and its sensitivity to overload effects.
The following aspects will be discussed:
- the cycle-by-cycle variation of $K_{op}$ under variable-amplitude loading
- rain flow effect
- crack growth calculation procedures
- parameters determination
- material and thickness effects
- negative loads

6.2.1 $K_{op}$ in the PREFFAS model

The load history will be considered as cycles of the stress intensity factor $K$. Each cycle (cycle number $i$) is supposed to start with a maximum $K_{max,i}$ followed by a minimum $K_{min,i}$.

The model is based on Elber's crack closure concept, including the Paris type crack growth equation. In cycle $(i)$:

$$A_{ai} = C (K_{max,i} - K_{op,i})^m = C K_{eff,i}^m$$  \hspace{1cm} (6.1)

where $C$ and $m$ are material parameters, $K_{op,i}$ is the crack opening stress for cycle $i$, which depends on the previous $K$ history. An essential part of the PREFFAS model is the question how $K_{op,i}$ depends on the previous $K$ history. All previous $K_{max}$ values can induce plastic zones which cause crack closure later on. The effect of $K_{max,j}$ in cycle $j$ on $K_{op,i}$ will depend on $K_{min}$ values occurring after that maximum and before cycle $i$, because of reversed plasticity. The maximum reversed plasticity should be considered. It occurs in cycle $k$ with $K_{min,k}$ the lowest minimum between $K_{max,j}$ and $K_{max,i}$ ($j < k < i$, see Fig. 6.1). As a consequence, if cycle $i$ occurs $K_{max,j}$ and $K_{min,k}$ must be considered to calculate $K_{op,i}$. If this value associated with $K_{max,j}$ is denoted as $K_{op,i,j}$, each previous maximum can yield a different $K_{op}$. According to the PREFFAS model the maximum $K_{op}$ should be used for cycle $i$:

$$K_{op,i} = \text{maximum of } K_{op,i,j} \text{ for } j=1 \text{ to } i-1$$  \hspace{1cm} (6.2)
For the calculation of $K_{op,i,j}$ the empirical Elber approach is adopted:

$$K_{\text{max},j} - K_{op,i,j} = U (K_{\text{max},j} - K_{\text{min},k}) \quad (6.3)$$

where $U$ is a linear function of the stress ratio $R$ with

$$R = \frac{K_{\text{min},k}}{K_{\text{max},j}} \quad (6.4)$$

$$U = A + BR \quad (6.5)$$

$A$ and $B$ are material parameters.

The assumption of the maximum $K_{op}$ in Eq. (6.2) implies that other combinations of monotonic plastic zones and reversed plasticity, which will lead to a lower $K_{op}$, are dominated by the highest $K_{op}$. Although the assumption is perhaps not unreasonable, it disregards effects of plastic zone sizes and the change of the location of new plastic zones due to intermediate crack growth. The CORPUS model, to be discussed later, is more sophisticated in this respect.

The question arises as to whether all previous $K_{\text{max}}$ and $K_{\text{min}}$ values have to be remembered to determine $K_{op}$ according to Eq. (6.3). This is not necessary. Only a limited number of historical $K$-values ($KH$-values) have to be remembered. This can be explained by considering Figures 6.1 and 6.2. A $K_{\text{max}}$ value, which is larger than all previous $K_{\text{max}}$ values, will erase all history effects of those previous cycles. This implies that the series of $KH_{\text{max}}$ values must be a decreasing series.

By definition $K_{op,i,j}$ requires selection of the lowest $K_{\text{min}}$ between cycles $(j)$ and $(i)$. As a consequence $K_{\text{min}}$ values to be remembered must form a series of increasing values. The $K$ values histories are then characterized by decreasing $K_{\text{max}}$ values and increasing $K_{\text{min}}$ values to determine the $K_{op}$ level in cycle $(i)$, see Figure 6.2. Such $K$ values are labelled as history values $KH_{\text{max}}$ and $KH_{\text{min}}$. The $K_{op,i}$ is determined by a $KH_{\text{max}}$ value and the subsequent $KH_{\text{min}}$ value, which form a
pair. Each pair determines a "historical" \( K_{op} \) level (\( KH_{op} \)). The \( K_{op} \) history with a rank number \( p \) is defined as:

\[
KH_{op,p} = KH_{max,p} - U (KH_{max,p} - KH_{min,p})
\]  \( (6.6) \)

Each pair \( KH_{max,p} \) and \( KH_{min,p} \) will cause its own opening level \( KH_{op,p} \) if they fulfill the following requirements:

\[
KH_{max,p} < KH_{max,p-1}
\]  \( (6.7a) \)

\[
KH_{min,p} > KH_{min,p-1}
\]  \( (6.7b) \)

\[
KH_{op,p} > KH_{op,p-1}
\]  \( (6.7c) \)

These requirements lead to a limited number of historical \( K \) values.

The crack increment in Eq. (6.1) can be written as:

\[
\Delta a_i = C (K_{max,i} - KH_{op,r})^m
\]  \( (6.8) \)

where \( r \) is the rank of the latest history values, which gives the highest opening level.

The \( KH \) series must be updated to satisfy the requirements in Eq. (6.7). This occurs if:

\[
KH_{max,i} > KH_{max,r} \quad \text{(an overload effect)}
\]  \( (6.9a) \)

\[
KH_{min,i+1} < KH_{min,r} \quad \text{(an underload effect)}
\]  \( (6.9b) \)

\[
KH_{op,i+1} > KH_{op,r} \quad \text{(an increase of opening load)}
\]  \( (6.9c) \)

6.2.2 The rain-flow effect

The rain-flow counting method (also range pair count method) was introduced for the statistical analysis of load time histories \([177]\). It recognizes, see Figure 6.4, that a large load variation \( AD \) interrupted by a small one \((BC)\) cannot be considered as two smaller load variations \( AB \) and \( CD \). This is especially true if the corresponding damage is a power law function of the load variation. For fatigue
crack growth the Paris equation (Eq. 6.1) with \( m = 3 \) to 4 large variations of \( \Delta K_{\text{eff}} \) are particularly damaging. According to a rain-flow analysis the load variation AD in Figure 6.4 should be considered as the large variation AD and a small variation BC. The authors of the PREFFAS model have introduced the rain-flow concept in order to avoid unconservative predictions. However, instead of considering \( K_{\text{max}} \) and \( K_{\text{min}} \) values the relevant levels are \( K_{\text{max}} \) values and \( K_{\text{op}} \) values instead of \( K_{\text{min}} \) values. This is a logical consequence of the crack closure concept, i.e. only those K-variations during which the crack is open are relevant. The consequence for the K variation AD in Figure 6.5a is that the crack-open K variation becomes A'B C'D in Figure 6.5b. According to the rain-flow approach this variation should be replaced by A'D and the cycle C'B' (Figure 6.5c). This is what occurs in the PREFFAS model. The rain-flow count had to be made because the interruption BC' satisfied the two requirements for such a count:

\[
\begin{align*}
K_A' &< K_C' \quad (6.10a) \\
K_D' &> K_B \\ 
\end{align*}
\]

In a cycle-by-cycle calculation it is not known during cycle 1 whether it will be involved in rain-flow effects later on. In Figure 6.5 when calculating \( \Delta a \) for cycle BC it is not yet known whether \( K_D' > K_B \) in the next cycle. If this applies, the \( \Delta a \) calculation for previous cycles must be corrected. In Figure 6.5 it implies that \( \Delta a \) calculated for cycle BC:

\[
\Delta a = C \left( K_B - K_A' \right)^m
\]

must be subtracted, while two new \( \Delta a 's \) for cycles A'D and C'B' have to be added:

\[
\Delta a = C \left( K_D - K_A' \right)^m + C \left( K_B - K_C' \right)^m - C \left( K_B - K_A' \right)^m \quad (6.11)
\]

It is possible that for high \( K_D \) values more previous cycles are
affected by rain-flow counts, see Figure 6.6a. The generalization of Eq. (6.11) leads to

$$\Delta a_i = C (K_{\max, i} - KH_{op,p})^m + C \sum_{j=p+1}^{r} (K_{\max,j} - KH_{op,j})^m$$

$$- C \sum_{j=p+1}^{r} (K_{\max,j} - KH_{op,j-1})^m$$

(6.12)

The application of this equation is illustrated by Figure 6.6b, where the three contributions occurring in the equation are shown. Figure 6.6b illustrates one inherent feature of the rain-flow procedure. The effective $K$ range in cycle $i$ (dotted bar) starts at the $KH_{op,i}$ level which occurred possibly many cycles before. Due to intermediate crack extension this feature is physically debatable, but it is generally accepted since better solutions are not available, whereas ignoring the rain-flow effect might well be unconservative.

6.2.3 Crack growth calculation

In the PREFFAS model, the crack length for the calculations of $K$ values is assumed to be constant within a block of the flight load spectrum. It implies that the stress intensity factor $K$ is only a function of the stress history without an influence of the crack length variation. This leads to a rather simple solution for crack growth calculations as shown below.

If $\Delta a$ is the crack growth increment in one block of the load spectrum, it is:

$$\Delta a = \sum_{i=1}^{n} \Delta a_i = C \sum_{i=1}^{n} \Delta K_{eff,i}^m$$

(6.13)

where $n$ is the number of load cycles in one block.

$$\Delta K_{eff,i} = f(a) \Delta S_{eff,i} \sqrt{n/a}$$

(6.14)
where \( f(a) \) is the geometry correction factor. For a CCT specimen the finite width correction factor is:

\[
f(a) = \sqrt{\sec(\pi a/W)}
\] (6.15)

Due to the assumption that the crack length \( a \) is constant in one block, Eq. (6.13) can be written as

\[
\Delta a = C \left[ f(a) \sqrt{\pi a} \right]^m \sum_{i=1}^{n} \Delta S_{eff,i}^m
\] (6.16)

For subsequent calculations Aliaga et al. [32] have defined the sequence efficiency (EF) as

\[
EF = \sum_{i=1}^{n} \Delta S_{eff,i}^m
\] (6.17)

Eq. (6.16) can then be written as:

\[
\Delta a = C \left[ f(a) \sqrt{\pi a} \right]^m EF
\] (6.18)

The average crack growth rate per cycle is

\[
\left[ \frac{da}{dN} \right]_{av} = \frac{\Delta a}{n} = C \left[ f(a) \sqrt{\pi a} \right]^m \frac{EF}{n}
\] (6.19)

After substitution of Eq. (6.15) it can be rewritten as

\[
dN = \frac{n}{\pi C EF} \left[ \cos \left( \frac{\pi a}{W} \right) \right]^{m/2} da
\] (6.20)

Integration gives the fatigue life

\[
N = \frac{n}{\pi C EF} \frac{1}{W(m/2 - 1)} \int_{\gamma_o}^{\gamma_f} \left( \frac{\cos \gamma}{\gamma} \right)^{m/2} d\gamma
\] (6.21)

where \( \gamma = \pi a/W \) and \( N \) is the fatigue crack growth life from an initial crack length \( a_o \) to a final crack length \( a_f \). The integral is
independent of the load history. On the other hand the sequence efficiency $EF$ is independent of the crack length. As a result the two variables (load history and crack length) have been separated in Eq. (6.21). Consequently the calculation effort can be relatively small. The $EF$ value has to be calculated by considering the crack opening history and the rain-flow approach described in Sections 6.2.1 and 6.2.2. In principle it should be calculated only once, i.e. for one block. However, in view of an initialization effect on $S_{op}$ it must be done for two blocks to obtain a stationary $EF$ value, where the maximum load affects all load cycles. The different behaviour in the first and the second block is illustrated by Figure 6.13. After the most severe flight (No. 1653, type A) in the first block the effect of $S_{max}$ of the spectrum is present. As a result the $S_{op}$ in flight 2229 (type C) is already the same in the first and the second block, which is not true for flight 106 (type B), occurring before flight A. In view of the $S_{max}$ effect the $EF$ value for the second block should be used in Eq. (6.21). The $EF$ calculation is described in Appendix A.

A remarkable aspect of the PREFFAS model is that the assumption of $K$ being independent of the crack length in one block has led to simple calculation procedures. Obviously, the assumption is only reasonable if $R$ in one block is small. A stationary flight simulation load history with a limited block size is a prerequisite.

6.2.4 Parameters determination

The above descriptions indicate that four material parameters are needed in the calculation, i.e. $C$ and $m$ in Eq. (6.1) which are the constants in the Paris relation, and $A$ and $B$ in Eq. (6.5), the two constants to determine the effective stress range $U$. According to Aliaga et al. [30,31], their experience has shown that the relationship

$$A + B = 1$$  \hspace{1cm} (6.21a)

can be used for aluminum alloys and steels used in aircraft structures.
It then is a characteristic requirement of the PREFFAS model that two types of crack growth tests have to be done on specimens of the material considered including the material thickness. The purpose is to determine the material parameters. The two types of tests are a constant-amplitude test with $R = 0.1$ and a constant amplitude test with periodic overloads every 1000 cycles, (see Figure 6.7). The overload factor $(K_{OL}/K_{max})$ is selected to be 1.7, because it is supposed to be realistic especially for wing spectra. Moreover, experience has shown that it gives good prediction results. The thickness of the material used in the tests should be the same as that used for the prediction problem, since thickness influences the retardation.

Results of such calibration tests as shown in Figure 6.7 confirm that the constant amplitude results are in agreement with the Paris Eq.(6.1). The two constants $C$ and $m$ can thus be obtained.

$$\frac{da}{dN} = C \Delta K_{eff}^m = C (U \Delta K)^m = (CU^m) \Delta K^m = C_1 \Delta K^m$$

and

$$C = C_1 / U^m$$ (6.22)

where $C_1$ is the $C$ parameter obtained from $da/dN = \Delta K$ of the constant amplitude diagram and $m$ is the slope of this diagram.

In view of $A + B = 1$ (Eq. (6.21a)) there is still one missing constant which should follow from the retardation in the tests with overload cycles. The block size for these tests is 1001 cycles, a block size which can also be adopted for the constant amplitude tests. According to Eq. (6.19) the model then predicts a retardation factor $RR$:

$$RR = \frac{(da/dN)_{CA}}{(da/dN)_{CA+OL}} = \frac{EF_{CA}}{EF_{CA+OL}}$$ (6.23)

For the constant amplitude tests $EF$ follows directly from Eq. (6.17)

$$EF_{CA} = 1001 \times \Delta S_{eff}^m$$ (6.24)

with $\Delta S_{eff} = U \Delta S$, $\Delta S = 0.9 S_{max}$, $U = A + BR$ and $R = 0.1$. 
For the tests with periodic overload cycles $S_{op}$ will be constant and equal to $S_{op}$ of the overload cycle, which implies:

$$S_{op} = 1.7 S_{\text{max}} - U \cdot \Delta S$$

with $\Delta S = 1.6 S_{\text{max}}$, $U = A + B R_{OL}$ and $R_{OL} = 1/17$ (Figure 6.7).

The same $S_{op}$ applies to the constant amplitude cycles in these tests and $EF_{CA+OL}$ can then be calculated. Substitutions lead to:

$$RR = \frac{1001 (0.9 - 0.81 B)^m}{[(1 - 16/17 B)]^m + 1000 [(1 -16/17 B) 1.6 - 0.7]^m} \quad (6.25)$$

Because $RR$ follows from the test results ($RR \times 10$ in Figure 6.7) B can be solved from Eq. (6.25). (It leads to $B = 0.42$, see Figure 6.8 for a graphical solution of Eq. (6.25). Thus $A = 0.58$)

It was pointed out before that predictions with the PREPPAS model are made relatively easily because of the separation of the two variables, load history and crack length. There is still another noteworthy consequence. If all stress levels of a load history are linearly increased, for instance by a factor $q$, it follows from Eq. (6.17) that $EF$ will increase by a factor $q^m$. According to Eq. (6.19) the average crack growth rate under stationary variable-amplitude loading will also increase by a factor $q^m$. If a stationary, variable amplitude load history is characterized by a design stress level $S$ the consequence is that

$$(da/dN)_{VA} + S^m \quad (6.26)$$

or

$$(da/dN)_{VA} = \text{constant} \cdot S^m \quad (6.27)$$

The important consequence is that the slope factor $m$ of the Paris relation, obtained under constant-amplitude loading, should also apply to variable-amplitude load histories. If predictions are obtained for one stress level they can easily be translated to another stress level. For the integrated fatigue life it is similarly found that:
crack growth life $+ S^{-m}$ \hspace{1cm} (6.28)

Such simple relations are not obtained with the ONERA and the CORPUS model.

6.2.5 Material and thickness effects

For different materials and thickness, estimated B values given by Aliaga et al. [31] are shown below.

<table>
<thead>
<tr>
<th>Material thickness</th>
<th>B values $(A + B = 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3 (and T351)</td>
<td>~0.45</td>
</tr>
<tr>
<td>7075/7010/7050 (T6... and T7...)</td>
<td>~0.35 ~0.3</td>
</tr>
<tr>
<td>steel ($S_u = 1000$ MPa)</td>
<td>~0.25</td>
</tr>
</tbody>
</table>

A large B value implies more crack closure and more retardation. The data indicate a lower B value for thicker material, which correlates with less crack closure and less retardation.

6.2.6 Negative loads

In the PREFFAS model negative loads are set equal to zero. It is not made clear by the authors why that is done. It should be noted here that cracks are closed under compressive load, because in the PREFFAS model cracks are already closed at positive stress levels. The reason why it is still necessary to introduce a limitation on compressive loads can be understood from Figure 6.9. In the PREFFAS model the linear Elber type relation is adopted for $U$:

$$ U = \Delta S_{\text{eff}} / \Delta S = A + B R $$

with $\Delta S_{\text{eff}} = S_{\text{max}} - S_{\text{op}}$, $\Delta S = S_{\text{max}} - S_{\text{min}}$, $R = S_{\text{min}} / S_{\text{max}}$ and $A + B = 1$ we obtain:

$$ \gamma = S_{\text{op}} / S_{\text{max}} = R + B (1 - R)^2 \hspace{1cm} (6.29) $$
For three B values from a previous table \( \gamma \) is plotted in Figure 6.9. Obviously for negative \( R \) values (i.e. negative \( S_{\min} \)) a strange behaviour occurs, \( S_{\text{op}} \) increases again. That is physically unacceptable as previously noted in Chapter 2. The introduction of \( S_{\min} = 0 \) if \( S_{\min} < 0 \) can then be understood. The consequence for \( S_{\text{op}} \) is indicated in Figure 6.10 by the horizontal lines for \( R < 0 \).

6.3 Prediction results

Fatigue crack growth life can be calculated with Eq. (6.21)

\[
N = \frac{n}{\pi C_{\text{EF}} W^{(m/2 - 1)}} \int_{\gamma_0}^{\gamma_f} \left[ \frac{\cos \gamma}{\gamma} \right]^{m/2} d\gamma
\]

The integral can be calculated numerically, which requires as input data the exponent \( m \) and the initial and final crack lengths. As previously mentioned, some parameters, which depend on the material, are needed to apply the model. It is not correct to adopt the constants of a \( da/dN-\Delta K_{\text{eff}} \) relation as available in the literature. For the calculation of \( \Delta K_{\text{eff}} \) a \( U(R) \) relation was used, and that relation should be consistent with the prediction model to be checked. It implies that it should be based on the same \( U(R) \) relation, which is used for PREFFAS is Eq. (6.5).

The parameters used in the present calculation for different material are:

a. 2 mm 2024-T3 Alclad material.

The constant amplitude data \( (R = 0.1) \) are adopted from Ref. [150]. It gives \( m = 3.7 \) and \( C_1 = 1.5364 \times 10^{-11} \) (\( da/dN \) in \( \text{m/cycle} \) and \( \Delta K \) in MPa\(\sqrt{\text{m}} \)). Parameters A and B are interpolated for 2mm thickness from data given by Aliaga et al. [31]. This gives \( A = 0.56 \) and \( B = 0.44 \). The C value can be determined from Eq. (6.22),

\( C = 9.92 \times 10^{-11} \).

b. 2 mm 2024-T3 bare material.

As for 2024-T3 Alclad, the \( A \) and \( B \) values used are \( A = 0.56 \) and \( B = 0.44 \). The \( C \) and \( m \) parameters are adopted from Ref. [35], with a modification due to different \( A \) and \( B \) values. The result gives
\[ C = 1.707 \times 10^{-10} \text{ and } m = 3.2. \]
c. \( 2 \text{ mm } 7075-T6 \) Clad material.

The constant-amplitude data are adopted from Ref. [148]. The data are adopted for \( R = 0.23 \), which gives \( m = 3.7 \) and \( C = 3.49 \times 10^{-11} \). Parameters \( A \) and \( B \) are adopted from data given by Aliaga et al. [31] i.e. \( A = 0.65 \), \( B = 0.35 \). The \( C \) value is then \( C = 1.12 \times 10^{-10} \).

Comparisons between tests and predictions for all data are presented in Figure 6.11. In general, the prediction results are good. Almost all data are in the range between 0.5 to 2.

Crack propagation life prediction results for different load spectra and ratios between prediction and test result are presented in Tables 6.1 to 6.7. A summary of the average ratios between prediction and test result is presented in Table 6.8 and Figure 6.12.

It was already shown before that \( S_{op} \) according to the PREFFAS model does not change very much, see Figure 6.13b (2nd block). During less severe flights the maxima are insufficiently high to raise \( S_{op} \). The \( S_{op} \) level is then determined by \( S_{max} \) of flight A and \( S = 0 \) because negative loads in the PREFFAS model are truncated to zero. In view of the \( R = 0 \) condition it follows from Eq. (6.29) that:

\[ S_{op} = B S_{max} \quad (6.30) \]

In view of the limited variations of \( S_{op} \) it is of some interest to calculate predicted crack growth lives if it is assumed that \( S_{op} \) is always constant, and equal to \( S_{op} \) as obtained from the maximum and the minimum stress of the load spectrum. Actually, that is the basic assumption of the simplified model proposed by Schijve [101]. Predictions with a constant \( S_{op} \) according to Eq. (6.30) have been made for a few test series. The results in Table 6.9 indicate an approximately 1\% smaller predicted crack growth life. It emphasizes the highly predominant effect of \( S_{max} \) of the spectrum on the PREFFAS predictions.
6.4 Discussion of the PREFFAS model

6.4.1 Prediction results

For ten different load spectra, the prediction and test ratios are quite acceptable from an accuracy point of view. That is a surprising result in view of the relative simplicity of the model. Moreover, the computational efforts are very limited indeed. However, it still has to be checked whether systematic, empirical trends are indicated by the PREFFAS model.

a. Effect of gust load severity

Figure 6.14a for 2024-T3 shows a moderate effect of the spectrum severity predicted by PREFFAS, whereas the test results indicate a larger effect. Figure 6.14b for 7075-T6 shows a larger effect of the spectrum severity predicted by PREFFAS and for this alloy the trend is in qualitative agreement with the test results. The same conclusions can be drawn for Tables 6.3 and 6.4 when considering the ratio's test/prediction. If the ratio remains approximately constant for spectrum variations, the empirical trend is correctly predicted in a relative way. For the 2024-T3 alloy the ratio is anyhow varying significantly more than for the 7075-T6 alloy as shown by the standard deviation

2024-T3: average ratio ± standard deviation = 1.41 ± 0.85
7075-T6: average ratio ± standard deviation = 1.17 ± 0.31.

b. Effect of ground stress level ($S_{gr}$)

PREFFAS does not predict any effect of $S_{gr}$ if $S_{gr} < 0$ because compressive loads are truncated to zero. Even for a positive $S_{gr}$ the effect is still absent, because $S_{min}$ of the most severe downward gust is negative and thus truncated to zero. The dominant $S_{min}$ value remains zero. Figure 6.14 shows that the empirical trend is a systematic effect of $S_{gr}$ with shorter crack growth lives for a more negative $S_{gr}$. The same trend was found for the miniTWIST test series (see Table 6.1), where fatigue lives were approximately doubled when $S_{gr}/S_{mf}$ was increased from -0.5 to 0. PREFFAS again predict no effect.
The absence of effects of negative loads is certainly a weak point of PREFFAS, which is discussed in more detail in Section 6.4.2.

c. Effect of design stress level

Variations of the design stress level were applied in the F-27, CN 235, FALSTAFF and miniFALSTAFF tests. The results are shown in Figures 6.15a to 6.15d respectively. The trend of increasing life for a lower design stress level is obviously predicted. The prediction results vary with the design stress level in agreement with equation (6.28), i.e. crack growth life + S^m. The results for 7075-T6 Clad (Fig. 6.15b) do agree quantitatively with the predicted effect. For the 2024-T3 alloy the effect of the design stress level is larger than predicted.

d. Load sequence effects

Load sequence variations are applied in the miniTWIST and the Misawa/Schijve tests. The test results did not show a significant effect of the load sequence. The prediction results are also quite similar for the original, modified max-min and modified min-max of the miniTWIST spectrum, (see Table 6.1). However, these predictions are also influenced by truncation of compressive loads at zero stress level. An underload-overload sequence and an overload-underload sequence do not make much differences, since the underload level is equal to the ground stress level (= zero stress level). Under simplified flight simulation loading tests [61] (see Figure 6.16, Misawa/Schijve tests), different predictions are obtained for types II and III, especially for m = 100, see Figure 6.17. Type II leads to a longer predicted life, due to the sequence of loads. The sequence is underload directly followed by an overload, see Figure 6.16. After an overload, the crack opening stress level is determined by the overload stress and the minimum stress of the small cycles. For type III loading, the crack opening stress level is determined by the overload and the underload stresses, where the underload is more compressive than the minimum stress of the small cycles. As a consequence, the crack opening stress level is higher for type II, which leads to a longer prediction life. For m = 5, the crack extension in one period is mostly produced by the overload cycle. Therefore, the crack opening stress level difference is not significant.
Flight sequence reorderings were applied in the CN 235 tests. The test results (Table 6.2) show that programmed sequences II and I lead to a longer life in the order of 1.3 to 1.46 compared to the random sequence. Prediction results show exactly the same fatigue life, i.e., a flight sequence effect is not predicted.

e. Effect of truncation

Truncation effects were observed in tests with the TWIST and the CN 235 spectra. The trends of the truncation effect are correctly predicted by the model. A higher truncation level leads to a longer life, see Figure 6.18. It is noteworthy that the smaller truncation effect for the 7075-T6 alloy is also indicated by the PREFFAS predictions.

f. Effect of omitting small load variations

This effect was studied in comparative tests between FALSTAFF and miniFALSTAFF. The tests indicated a negligible difference. PREFFAS also predicts a negligible difference (Table 6.6), although the absolute accuracy was not very good (see Figure 6.15d).

6.4.2 Effect of negative loads and the $S_{op}$ function $U(R)$

The parameters A and B are empirically obtained from constant amplitude tests with $R = 0.1$ and tests with periodical overloads every 1000 cycles ($K_{OL}/K_{max}$ = 1.7, see Figure 6.7). This procedure can be considered as a kind of a calibration tests to adjust the crack opening stress level in the model. The calibration results will depend on the type of material and the material thickness (plastic zone size retardation effect in the test with overloads in relation to material thickness). Unfortunately, the $U(R)$ relation ($U = A + BR$) leads to strange results for negative $R$-values as discussed before (Figure 6.9). Most probably, Aliaga et al. truncated all negative loads to zero for that reason (Figure 6.10). As pointed out before this is a weak point of PREFFAS, because it then cannot account for the $S_{gr}$-effect as discussed in the previous section. It might be suggested that this problem can easily be removed by adopting a $U(R)$ relation which can also describe negative $R$-effects, e.g., Eq. (2.14). Negative loads need not be truncated anymore to zero. Unfortunately, the simple linear $U(R)$ relation is essential for the empirical determination of
the material constants A and B. Perhaps, this problem could be overcome by another plausible assumption. However, it still can be of little help if $S_{gr}$ is not the most negative load. Again the F-27 test series are illustrative. For severe, normal and light gust spectra and ground stress level the most negative loads are:

<table>
<thead>
<tr>
<th></th>
<th>$S_{min}/S_{mf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>negative gust</td>
</tr>
<tr>
<td>Severe</td>
<td>- 0.39</td>
</tr>
<tr>
<td>Normal</td>
<td>- 0.25</td>
</tr>
<tr>
<td>Light</td>
<td>- 0.11</td>
</tr>
</tbody>
</table>

It is easily observed that for a severe or a normal gust spectrum with a normal or light ground stress, the most negative load is a gust load and not $S_{gr}$. As long as a variation of $S_{gr}$ still leaves the situation $S_{gust,min} < S_{gr}$ the dominant $SH_{min}$ value remains the same. As a consequence the dominant $S_{op}$ also remains unaffected, and the variation of $S_{gr}$ will not (or hardly) affect the predicted crack growth life. This consequence does not agree with the test results.

The above discussion shows that an improvement of the PREFFAS model with respect to negative loads is not as easy as one might think. There is a related aspect to be revealed here. The most negative gust in the most severe flight occurs only once in a block. If PREFFAS did not truncate that load to zero, an occasionally occurring high negative load would have a large effect on the dominating $S_{op}$-value. Because its influence should come from reversed plasticity such a large, and ever lasting effect may not be realistic. Its negative effect will be relaxed after some crack growth. It will turn out in the following chapter that such a relaxation is introduced in the ONERA model.

6.4.3 Effect of rain-flow approach

The effect of the rain-flow approach is checked by comparing the predictions with predictions obtained without the rain-flow procedures. Calculations are made for different spectra and different
materials. The prediction results are presented in Table 6.10. The table shows the predicted life and the number of cycles to be "rain-flowed". Almost in all cases the effect of the rain-flow procedure is very small, also because the number of rain-flow counts is very small. Figure 6.19 shows two minifALSTAFF flights, during which 3 rain-flow counts were made (indicated by arrows). The (perhaps unexpected) low number of rain-flow counts is a consequence of the fact that rain-flow counts in PREPPAS are not based on relative positions of $S_{\text{min}}$ and $S_{\text{max}}$ levels, but on relative positions of $S_{\text{op}}$ and $S_{\text{max}}$ levels. This was explained before (Fig. 6.5). Relatively high $S_{\text{op}}$-levels will drastically reduce the number of rain-flow counts. That applies to the F-27 test series on 7075-T6 material, and to TWIST tests if the truncation level is low. Even if the number of rain-flow counts seems to be fairly high, the number is still rather small as compared to the total number of cycles (compare the last two columns of Table 6.10).

The addition of the rain-flow procedure to the PREPPAS model might well suggest a significant improvement, which can also be motivated by being more conservative. The surprising result here is that the introduction of the rain-flow procedure had only a small effect on the prediction, which in most cases was even negligible.

6.4.4 The influence of the assumption of a stationary crack length in each period

In the PREPPAS model, the load history effect can be separated from the crack length influence to calculate the fatigue life. It implies that $K$ in one block is independent of the crack length. The calculation effort is then relatively small, which saves considerable calculation time. In order to know the influence of this assumption, some calculations have been made for different spectra with cycle-by-cycle crack increments. The prediction results are presented in Table 6.11, in comparison to the previous results based on a stationary crack length. It turns out that the assumption of a stationary crack has only a rather small effect on the predicted life. Under such conditions, the "stationary assumption" is acceptable. The larger differences are in the order of 1.5 to 15 percent. The block size and the
spectrum severity will influence the stationary character of the crack growth. Larger crack extensions per block will undermine the stationarity concept. The crack then has grown to a substantially extent into the plastic zone of the most severe load, or perhaps already outside that plastic zone. Its crack growth retarding effect will have decreased. This is not accounted for in PREFFAS due to the "stationary crack length" assumption.

The calculation time to be saved will be large especially for short block spectra and long predicted lives, e.g. under FALSTAFF, miniFALSTAFF and the CN 235 spectra. For FALSTAFF and miniFALSTAFF, the stationary assumption requires only two percent of the calculation time of the cycle-by-cycle prediction, for the CN 235 spectrum it is about ten percent. For a large block size (TWIST) and a short prediction life there is hardly any saving time, because the stationary crack prediction must be calculated for two blocks of loads, whereas the predicted life may be of a similar order of magnitude. With the present development of computers, computer time is not a very significant criteria.

6.5 Summary and conclusions on the PREFFAS model

The PREFFAS model is a simple model. The model is based on the Elber crack closure concept. The crack closure level is evaluated cycle-by-cycle. During one block of flights the crack length is assumed to be stationary. The model includes a "rain-flow" approach. Conclusions are given below.

1. The model is easy to use, but it requires some "calibration" experiments for determining the parameters used in the calculation.
2. Cycle-by-cycle calculations with a growing crack as a basis for comparison have shown that the "stationary crack" assumption is quite acceptable. As a consequence of the assumption a separation in the calculations is possible between the variables of the load history on one hand, and the crack length and geometry effects on the other hand. It can save much computer time. If a crack extension in one block of flights becomes of a similar magnitude or
larger than the plastic zone of the maximum load of the spectrum, the stationary crack length concept may become debatable. Plastic zone sizes do not occur in the PREFFAS model.

3. A noteworthy result is that the variation of the stress opening level \((S_{op})\) under a flight-simulation loading is rather limited. There is one predominant \(S_{op}\) level, determined by the maximum stress \((S_{\text{max}})\) and the minimum stress \((S_{\text{min}})\) of the load spectrum. In the analysis it became clear that such a predominant effect of \(S_{\text{min}}\), which should be a consequence of reversed plasticity, is not a realistic proposition, if that \(S_{\text{min}}\) is due to a rarely occurring peak load (in the most severe flight only).

4. In the PREFFAS model compressive stresses are truncated to zero \((S=0)\). That is a weak point. It predicts that the effect of the ground stress level \((S_{\text{gr}})\) will be absent for most spectra, which systematically disagrees with test results. It is recognized that the truncation could not be avoided in the model because of the linear (Elber type) \(U(R)\) relation. In view of the third conclusion a simple and realistic improvement of the PREFFAS model will be problematic.

5. Another noteworthy result was the almost negligible effect of the rain-flow procedure. It can be understood by considering the relatively high predominant \(S_{op}\) and the fact the rain-flow procedure is applied to \(S_{\text{max}}/S_{op}\) time series instead of \(S_{\text{max}}/S_{\text{min}}\) series.
7.1 Introduction

The ONERA model was proposed by Baudin and Robert [28]. The model is based on a stress history dependent concept of crack growth thresholds, which essentially is a crack closure concept according to Elber's mechanism. The crack opening level (in terms of Refs. [28] and [29]: threshold level) is recalculated cycle-by-cycle. Baudin and Robert assume that the crack opening level is limited by two extreme conditions i.e. the constant-amplitude case and the single overload case. For variable amplitude loading, the crack opening level is somewhere between these two cases, depending on a load spectrum parameter α. The ONERA model includes considerations on plane stress and plane strain conditions and the effect of material thickness. The variation of the plastic zone size is essential for the interaction effects in the ONERA model.

7.2 Predicted crack growth

The stress intensity factor in each cycle is defined by a maximum \( K_{\text{max},i} \) followed by a minimum \( K_{\text{min},i} \). After cycle (i) the crack opening stress level is \( K_{\text{op},i} \), which is applicable to the following cycle (i+1). The crack opening K level \( K_{\text{op},i} \) is recalculated cycle-by-cycle. It depends on so called equivalent \( K_{\text{max}} \) and \( K_{\text{min}} \) values, where \( K_{\text{max.eq},i} \) is affected by high loads and \( K_{\text{min.eq},i} \) by low loads. The equivalent K values account for load history effects. The values must also be calculated from cycle to cycle. The crack extension in cycle (i) \( \Delta a_i \) depends on \( \Delta K_{\text{eff},i} \) according to the Paris relation:

\[
\Delta a_i = C \cdot \Delta K_{\text{eff},i}^m \quad (7.1)
\]

where

\[
\Delta K_{\text{eff},i} = K_{\text{max},i} - K_{\text{op},(i-1)} \quad (7.2)
\]
Again the behavior under variable amplitude loading is correlated to crack growth data obtained under constant amplitude loading (similarity approach).

7.3 Influence of an overload

Each cycle can affect the $K_{op}$ stress level for subsequent load cycles. $K_{max,i}$ will have an effect only if it will cause a new monotonic plastic zone. Suppose that the last overload (OL) occurred at $a = a_{OL}$ (see Figure 7.1) and caused a plastic zone with a size $r_{OL}$. The $K_{max,i}$ of a later cycle at $a = a_i$ will cause a new monotonic plastic zone if its plastic zone (size $\rho$) will penetrate the elastic material:

$$a_i + \rho_i > a_{OL} + r_{OL}$$  \hspace{1cm} (7.3)

or with $\rho_{eq} = (a_{OL} + r_{OL}) - a_i$  \hspace{1cm} (7.4)

$$\rho_i > \rho_{eq}$$

The $K_{max}$ value required to cause such a zone will be labelled as $K_{max, eq}$ equivalent, while the corresponding plastic zone size is $\rho_{eq}$. If new monotonic plasticity does not occur, $K_{max, eq}$ becomes smaller when the crack is growing (increasing $a_i$ in Eq. (7.4)) due to a reduction of $\rho_{eq}$.

The plastic zone size is calculated using Irwin's formula for plane stress condition:

$$\rho = \frac{1}{\pi} \left[ \frac{K_{max, eq}}{\sigma_y} \right]^2$$  \hspace{1cm} (7.5)

which implies

$$K_{max, eq} = \sigma_y \sqrt{\pi \rho_{eq}}$$  \hspace{1cm} (7.6)

with $\rho_{eq}$ according to Eq. (7.4).
It is well known that the size of the plastic zone strongly depends on the state of stress near the crack tip. Two extreme conditions are the states of plane strain and plane stress. In the ONERA model, the ratio between plane stress and plane strain plastic zone size is equal to 6. At the material surface there is always a plane stress zone. In the model, it is assumed that the size of the plastic zone decreases into the thickness direction following a 45 degree straight line. Different situations are possible as indicated in Figure 7.2. In the left figure there is still core material in full plane strain. In the right figure there is no full plane strain material left. The middle figure shows the transition between the other two situations. It occurs at \( \rho = 0.6 t \).

In other words, the transition depends on the sheet thickness. The model adopts the average plastic zone \( \rho_m \) which leads to:

(a) \[ \rho_m = \frac{\rho}{6} \left(1 + \frac{25}{6} \frac{t}{\rho}\right) \quad \text{if} \quad \rho < 0.6 \ t \quad (7.7) \]

(b) \[ \rho_m = \rho - \frac{t}{4} \quad \text{if} \quad \rho \geq 0.6 \ t \quad (7.8) \]

where \( t \) is the sheet thickness and \( \rho \) is calculated with Eq. (7.5). \( \rho_m \) is calculated only if a new monotonic plastic zone is created, i.e. when \( K_{\text{max}} > K_{\text{max.eq}} \). The decrease of the plastic zone size is related to \( \rho_m \). If there is a new primary plastic zone (overload cycle), its effective size according to the model is immediately reduced by the crack extension (\( \Delta a_i \)) in that cycle.

\[ \rho_{m,i} = \rho_{m,\text{OL}} - \Delta a_i \quad (7.9) \]

If there is no new primary plastic zone, but still crack extension, the same reduction applies:

\[ \rho_{m,i} = \rho_{m,(i-1)} - \Delta a_i \quad (7.10) \]

The minimum \( K_{\text{max}} \) value to create a new plastic zone in the following cycle, which is by definition \( K_{\text{max,eq,i}} \), can now be calculated cycle-by-cycle from \( \rho_{eq,i} \), which is derived from \( \rho_{m,i} \) by inversion of Equations (7.7) and (7.8).
(a) \( \rho_{eq,i} = 0.12 \ t \left[ \sqrt{1 + 100 \frac{\rho_{m,i}}{t}} \right] - 1 \)  \hspace{1cm} (7.11)

(b) \( \rho_{eq,i} = \rho_{m,i} + t/4 \)  \hspace{1cm} (7.12)

Then, according to Eq. (7.6):

\[
K_{max.eq,i} = \sigma_y \sqrt{n} \ \rho_{eq,i} 
\]  \hspace{1cm} (7.13)

Obviously the sheet thickness \( t \) does affect \( K_{max.eq,i} \) and as a consequence \( K_{op,i} \). A thickness depending plane strain/plane stress transition is thus included in the model, as well as an effect of the yield limit \( (\sigma_y \text{ in Eq. (7.6)}) \).

7.4 Evolution of the minimum equivalent stress intensity factor

Similarly to \( K_{max.eq} \), \( K_{min.eq} \) also changes from cycle to cycle depending on cyclic plasticity at the crack tip. Obviously the \( K_{op} \) level will be reduced if \( K_{min,i} \) is sufficiently low.

The model considers five different load cycle cases shown in Figure 7.3. In cases 2 and 3,

\[
K_{min,i} < K_{min.eq,(i-1)} \]  \hspace{1cm} (7.14)

The assumption made is that \( K_{min.eq} \) is reduced to the applied \( K_{min} \) level i.e:

\[
K_{min.eq,i} = K_{min,i} \]  \hspace{1cm} (7.15)

The same equation is assumed to be applicable in case 4, where a new monotonic plastic zone is created. That erases the prehistory and a new \( K_{min.eq} \) has to be set (Eq. (7.15)). In case 1, \( K_{max,i} \) does not exceed \( K_{op} \) and \( K_{min,i} \) is not below \( K_{min.eq} \). There will be no influence of crack tip plasticity and thus no change of \( K_{min.eq} \). However in the more general case 5 there is plasticity in going to \( K_{max,i} \) (although no primary plasticity) and in the reversal of the load going to \( K_{min,i} \). It then may be expected that both \( K_{max,i} \) and \( K_{min,i} \) will cause some change of \( K_{min.eq,i} \).
\[ \Delta K_{\text{min.eq},i} = K_{\text{min.eq},i} - K_{\text{min.eq},(i-1)} \]  \hspace{1cm} (7.16)

The model assumes that the change depends on the ratio

\[ \text{ratio} = \frac{K_{\text{max},i}}{K_{\text{max.eq},i}} \] (<1) \hspace{1cm} (7.17)

and on the difference,

\[ \text{diff} = K_{\text{min},i} - K_{\text{min.eq},(i-1)} \] (>0) \hspace{1cm} (7.18)

The model assumes:

\[ \Delta K_{\text{min.eq},i} = \text{diff} \cdot \text{ratio}^\beta \] \hspace{1cm} (7.19)

where the exponent \( \beta \) is still thickness dependent, for which it is assumed that:

\[ \beta = 2 + t/2 \] \hspace{1cm} (t in mm) \hspace{1cm} (7.20)

Equation (7.19) implies that \( \Delta K_{\text{min.eq},i} \) is linearly proportional to \( \text{diff} \) and a power function of \( \text{ratio} \). Although that may be reasonable, physical arguments to justify the equation are not offered, neither are they offered for Eq. (7.20).

It should be noted that one limiting condition is satisfied by Eq. (7.19). If \( K_{\text{max},i} \) is equal to \( K_{\text{max.eq},i} \) then the ratio in Eq. (7.17) is equal to one and \( \text{diff} \) in Eq. (7.18) according to Eq. (7.19) becomes equal to \( \Delta K_{\text{min.eq},i} \) in Eq. (7.20). As a consequence, \( K_{\text{min.eq},i} \) is equal to \( K_{\text{min},i} \) in agreement with case 4.

7.5 Crack opening K level

As shown above \( K_{\text{max.eq},i} \) and \( K_{\text{min.eq},i} \) depend on the previous load history including the last cycle (i). The crack opening K level after cycle (i) depends on these two equivalent stress levels.

By definition:
\[ R_{eq,i} = \frac{K_{\text{min.eq,i}}}{K_{\text{max.eq,i}}} \]  

(7.21)

For constant amplitude loading the Elber crack closure concept was previously written as:

\[ \Delta K_{\text{eff}} = U \Delta K \]

where \( U \) is a function of the stress ratio \( R \). It can also be written as:

\[ K_{\text{op}} = K_{\text{max}} - \Delta K_{\text{eff}} = K_{\text{max}} \{1 - U(1-R)\} = Y K_{\text{max}} \]

where \( Y \) is again a function of \( R \).

It is postulated in the model that

\[ K_{\text{op},i} = K_{\text{max.eq,i}} \left[ \alpha f_1(R_{eq,i}) + (1 - \alpha) f_2(R_{eq,i}) \right] \]  

(7.22)

with \( 0 < \alpha < 1 \). Apparently the \( Y \) function is split up in two crack opening functions \( f_1 \) and \( f_2 \), each with a different weight factor, \( \alpha \) and \( 1 - \alpha \) respectively. \( f_1 \) is related to the overload case and \( f_2 \) is related to the constant amplitude case. The crack opening functions were obtained empirically by applying constant amplitude loading after some different overload-underload combinations. Crack arrest occurs if the maximum load of the constant amplitude load cycles is smaller than the crack opening \( K \) level, \( K_{\text{op}} \). Two cases were considered: (1) a single overload cycle (Figure 7.4a) and (2) a large number of overload cycles (Figure 7.4b). In both cases \( K_{\text{op}} \) after the overload cycle \( i \) is a function of \( K_{OL} \) and \( K_{UL} \) \((R = K_{UL}/K_{OL})\).

Case 1 : A single overload

\[ K_{\text{op}}(1) = K_{OL} \cdot f_1(R) \]  

(7.23)

Case 2 : A large number of overload cycles, corresponding to constant amplitude loading
In the second case the opening level is higher as a result of a multiple overload effect.

\[ K_{op}(2) > K_{op}(1) \rightarrow f_2(R) > f_1(R) \]

According to Ref. 27, the empirical functions \( f_1 \) and \( f_2 \) for 2024-T3 are:

\[ f_1 = 0.377 + 0.623 \frac{R}{R_{eq}} \quad \text{if} \quad R_{eq} > -0.0624 \]  
(7.25a)

or

\[ f_1 = 0.35 + 0.19 \frac{R}{R_{eq}} \quad \text{if} \quad R_{eq} < -0.0624 \]  
(7.25b)

For the constant amplitude case:

\[ f_2 = \frac{1}{1.9 - 0.9 \frac{R}{R_{eq}}} \]  
(7.26)

A diagram with the two functions is presented in Figure 7.5. The intersection of \( f_1 \) and \( f_2 \) at \(-0.5\) is not discussed by Baudin and Robert. It is physically strange. However, the significance of \( f_2 < f_1 \) at high \( R \) values is probably not of great practical interest.

According to Eq. (7.22) the model assumes that in general \( K_{op,1} \) is a mixture of the two cases, where the factor \( \alpha \) determines the ratio between the two cases \((0 \leq \alpha \leq 1)\).

7.6 Determination of the loading parameter, \( \alpha \)

The loading parameter \( \alpha \) varies between 0 for constant amplitude loading and 1 for a single overload cycle. In Ref. 28, the loading parameter \( \alpha \) was adjusted for matching predictions to crack propagation test results obtained under FALSTAFF and MiniTWIST spectra. Apparently it has led to a simple formula, which accounts for the type of load spectrum.
\[
\alpha = 1 - \frac{1}{M} \sum_j m(j) \cdot S_{\text{max}}(j) \tag{7.27}
\]

where the peak stress \( S_{\text{max}}(j) \) occurs \( m(j) \) times in the load spectrum. \( M \) is the total number of cycles in the load spectrum \( (M = \sum m(j)) \) and \( S \) is the largest \( S_{\text{max}} \) value, see Figure 7.6. In Eq. (7.27) \( \sum m(j) \) \( S_{\text{max}}(j) / M \) represents the "centre of gravity" of the load spectrum expressed in terms of \( S_{\text{max}} \) values of the in-flight loads. It implies that \( \alpha \) accounts for the shape of the load spectrum.

Loading parameters for different spectra were given in Ref. 28. Some of them are given below.

<table>
<thead>
<tr>
<th>Type of spectrum</th>
<th>Version</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FALSTAFF</td>
<td>Basic</td>
<td>0.68</td>
</tr>
<tr>
<td>TWIST</td>
<td>Basic</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>Truncated at level III</td>
<td>0.46</td>
</tr>
<tr>
<td>MiniTWIST</td>
<td>Basic</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>Truncated at level III</td>
<td>0.42</td>
</tr>
<tr>
<td>F-27</td>
<td>All versions</td>
<td>0.42</td>
</tr>
<tr>
<td>Boeing</td>
<td>Transport A-SS</td>
<td>0.56</td>
</tr>
</tbody>
</table>

It is easily seen that for constant amplitude loading (all \( S_{\text{max},i} \) are equal) Eq. (7.27) leads to \( \alpha = 0 \). Unfortunately, for the single overload case Eq. (7.22) leads to \( \alpha = 1 - S_{\text{max}}/S_{\text{OL}} \) instead of \( \alpha = 1 \). That result is not consistent with Eq. (7.22). Apparently the equation may be incorrect for such an exceptional load spectrum.

7.7 Prediction results

In the ONERA model, there are some parameters needed to apply the model. These parameters are the \( C \) and \( m \) constants in the Paris equation, the loading parameter \( \alpha \) and the yield stress \( \sigma_y \). The \( C \) and \( m \)
constants and the yield stress are material dependent, whereas the loading parameter $\alpha$ is load spectrum dependent. The $\alpha$ values used in the present calculation are presented in Table 7.1. The material parameters are given below.

a. 2 mm 2024-T3 Al clad material.

The parameters were given by Baudin and Robert in Ref. [28].

\[
\begin{align*}
\sigma_y &= 340 \text{ MPa} \\
C &= 8.92 \times 10^{-10} \\
m &= 3
\end{align*}
\]

b. 2 mm 2024-T3 bare material.

The C and m values are adopted from Ref. [175]. The values are derived from ten $R$ values of constant amplitude tests reported in Ref. [61]. The parameters are

\[
\begin{align*}
\sigma_y &= 394 \text{ MPa} \\
C &= 3.5884 \times 10^{-11} \\
m &= 4.0443
\end{align*}
\]

c. 2 mm 7075-T6 Clad material.

The C and m values are derived from three $R$ values ($R = 0.52$, 0.23 and $-0.05$) of constant amplitude tests reported in Ref. [148].

\[
\begin{align*}
\sigma_y &= 490 \text{ MPa} \\
C &= 8.3715 \times 10^{-10} \\
m &= 3.4467
\end{align*}
\]

The above C values are for crack growth rate in m/cycle and $\Delta K_{eff}$ in MPa/m.

A flow diagram of the ONERA model is presented in Figure 7.7. Predicted crack propagation lives results for different load spectra are presented in Tables 6.1 to 6.7, as well as the ratios between predictions and tests. A summary of the $N_p/N_T$ values is presented in Table 6.8 and Figure 6.7. Comparisons between tests and predictions for all data are presented in Figure 7.8.

In general, the prediction results are good. Only for the TWIST spectrum, the predictions are unconservative, the $N_p/N_T$ values are between 3.03 to 5.16, see Table 6.8. Also for the ONERA model it will be checked whether empirical trends are indicated by the model.
a. Effect of gust spectrum severity

The gust load severity effects can be observed in the F-27 tests. Crack propagation lives for the two Al-alloys (2024-T3 Alclad and 7075-T6 Clad) are presented in Figure 7.9a and b. The figures show the influence of gust spectrum severity and the ground stress level. The effect of the gust severity is qualitatively predicted by the model. A more severe gust spectrum leads to a shorter life. Figures 7.9a and b also show that the predictions are apparently better for 2024-T3 Alclad than for 7075-T6 Clad material. The predictions were done with the same f1 and f2 functions, although the empirical functions were proposed for 2024-T3 only. Similar functions were not available for the other Al-alloy. This may influence the prediction results, as pointed out by the authors of the ONERA model in Ref. [28]. According to the PREFFAS model, 7075-T6 material produces less closure.

Crack growth rates for different gust spectrum severities, but the same ground stress level (normal) (cases SN, NN and LN) are presented in Figures 7.10a and b for the 2024-T3 and 7075-T6 material respectively. The figures show that the model predicts a lower crack growth rate in the beginning and a faster growth rate later on. The predictions approximately follow a log linear increasing crack growth rate for increasing crack length.

b. Effect of ground stress level

Different ground stress levels were applied in miniTWIST, the F-27 spectrum and in the Misawa/Schijve tests. The predicted trend is qualitatively correct. A lower ground stress level leads to a shorter life, see Figures 7.9 and 7.11. The crack growth rates for the F-27 spectrum (cases NS, NN and NL) and the two aluminium alloys are presented in Figure 7.12. A similar trend as shown in Figure 7.10 is found again, with unconservative growth rate predictions in the beginning, followed by conservative predictions for longer cracks.

c. Effect of design stress level

Different design stress levels were applied in the CN 235, F-27, FALSTAFF and miniFALSTAFF tests. The effect of the design stress level on the crack growth prediction and the test results are shown in Figures 7.13a to 7.13c for the F-27, CN 235 and FALSTAFF
spectra. The trend of increasing life for a lower design stress level is obviously well predicted. The slope factor of the curves can be defined as \(-d(\log N)/d(\log S)\) (\(N = \text{life}\), \(S = \text{design stress level}\)). The slope factors of the predictions and the test results for different spectra, as compared to the exponent \(m\) of the Paris relation are presented in Table 7.2.

For the 2024-T3 alloy the slope factors of the test results are higher than the predicted values and the exponent \(m\). For the 7075-T6 alloy the values are quite similar. For both types of alloys the predicted slope values and \(m\) are of the same order of magnitude.

Crack growth rates for the F-27 and FALSTAFF spectra are presented in Figures 7.14a to 7.14c. Similar trends as in the previous figures are observed again, with lower crack growth rates in the beginning followed by faster crack growth rates. For the NN 70 case for the F-27 spectrum with 2024-T3 material, the crack growth rate in the beginning is of the same order of magnitude as the test result, followed by conservative predictions. This leads to a ratio 0.4 between prediction and test.

d. Effect of load sequence

Load sequence variations are applied in miniTWIST and the Misawa/Schijve tests. The test results did not show a significant effect of the load sequence. Under the miniTWIST spectrum, the prediction results are also quite similar for the three types of sequences, see Table 6.1. Under the Misawa/Schijve type of loads, the model predicts a systematic shorter fatigue life for sequence II than for sequence III, see Figure 7.11. Figure 7.15 shows that the ONERA model predicts a different \(K_{op}\) behaviour for the two sequences. The typically increasing \(K_{op}\) is shown by sequence III, but it does not apply to sequence II. As a consequence the \(\Delta K_{eff}\) ranges are also different. The largest \(\Delta K_{eff}\) is found for the OL cycle, but it is larger for sequence II. In view of the exponent \(m\) in the Paris relation this cycle has a predominant damaging effect, and as a result the crack growth life is shorter for sequence II.

Different flight sequences were applied in the CN 235 tests. Although the test results show a longer life for programmed sequences II and I (in the order of a factor 1.3 to 1.46, see Table
6.2) than for the random sequence, the prediction results do not show any difference.

e. Crack increment in the most severe flight

Crack increments in the most severe flight were measured in mini-TWIST and CN 235 tests. The crack increments during flight A are presented in Figure 7.16, both predicted values and test results. The figure shows that the crack increments during flight A are significantly larger than predicted. For a longer crack length, the difference is increasing. Such a large discrepancy is apparently possible when the overall crack growth life prediction is fairly accurate, see the life ratios in Tables 6.1 and 6.2. The discrepancy can be a consequence of using the Paris relation. For high $\Delta K_{eff}$ values, the crack growth rate does not follow a log linear relation. It will asymptotically approach the limit $K_{max} = K_C$. Other mechanisms such as ductile tearing (stable crack extension) can enhance the crack growth increment at high $K_{max}$, which is not included in the model prediction.

f. Effect of truncation

Effects of truncation were observed in the TWIST and in the CN 235 tests. The predicted trend is qualitatively correct, see Figure 7.17. A spectrum, which is truncated at a lower maximum stress, gives a shorter life. Also the much smaller truncation effect for the 7075-T6 alloy is predicted. The quantitative accuracy is apparently poor in 2 of the 3 cases in Figure 7.17. Crack growth rates for the TWIST and the CN 235 spectra are presented in Figure 7.18. The truncation effect is predicted better for the CN 235 spectrum than for the TWIST spectrum. The effect of the truncation level on $S_{op}$ during flights of type B and C is shown in Figure 7.19. Lower $S_{op}$ levels for the lower truncation level can be observed.

A further discussion about the effect of truncating high loads is given in Section 7.8.

g. Effect of omitting small load variations

The trend of the omission effect can be observed in the miniFALSTAFF tests as compared to the FALSTAFF tests. The prediction results show a correct trend. Predicted life differences between FALSTAFF and miniFALSTAFF are not significant, see Table 6.6.
7.8 Discussion

7.8.1 Crack opening K level determination

The crack opening K level in the ONERA model is determined between two conditions, namely the single overload case and the constant amplitude case (Figure 7.4). The constant amplitude case produces higher K_{op} levels due to some kind of a multiple OL effect. The equation for the intermediate K_{op} level (Eq. (7.22)) is then adjusted by using the loading parameter \( \alpha \), where \( \alpha \) is depending on the shape of the load spectrum (\( \alpha = 0 \) is for the constant amplitude case and \( \alpha = 1 \) is for the single overload case). This implies that a small \( \alpha \) value indicates a high crack opening level.

Three aspects related to the crack opening K level will be discussed below, namely - Application of the \( \alpha \) parameter

- The physical meaning of \( K_{\text{max.eq}} \), \( K_{\text{min.eq}} \)
- The transient phenomenon.

Application of \( \alpha \) parameter

As previously mentioned, the loading parameter \( \alpha \) determines the crack opening stress level, where \( \alpha \) is depending on the shape of the load spectrum. Truncation of high loads reduces \( \alpha \), see Table 7.1. As a consequence the term between square brackets in Eq. (7.22)

\[
[\alpha f_1 + (1 - \alpha) f_2]
\]

will increase, and \( f_2 \) will become more dominant in the determination of \( K_{op} \). In other words, the response of the spectrum moves in the direction of constant amplitude loading. Actually that should be expected if the extreme loads of the spectrum are truncated to a lower value. A lower \( \alpha \) value also means that \( K_{op}/K_{\text{max.eq}} \) obtained with Eq. (7.22) will increase. That does not necessarily mean that \( K_{op} \) will also increase, because the truncation will reduce \( K_{\text{max.eq}} \). Indications on the absolute change of \( K_{op} \) can be obtained by substituting numerical values in Eq. (7.22). This has been done in Table 7.3 for the TWIST spectrum and two \( R_{eq} \) values. It shows that \( K_{op}/K_{\text{max}} \) for
decreasing α does indeed increase if R = 0, whereas it remains constant for R = 0.5. The latter unexpected result is associated with the strange intersection of the f1 and f2 function at R ~ 0.5 as mentioned before (Section 7.5). The last two columns of Table 7.3 indicate that that K_{op} is decreasing for decreasing α values, even if K_{op}/K_{max} is increasing. Apparently the concept of the spectrum shape factor α (Eq. (7.27)) is a plausible interpolation between spectrum shapes (including CA-loading), which leads to changes of S_{op} in agreement with expectations and the trend of test results. Another illustration is given in Figures 7.19a and b, which indicate lower S_{op} values predicted by ONERA for a lower truncation level.

Omission of small cycles also affects the spectrum shape and thus the value of α. It reduces α, see Table 7.1 for FALSTAFF/mini-FALSTAFF and TWIST/miniTWIST. Omission of small cycles implies a limitation on the variation of amplitudes. It thus is also a kind of moving towards more constant amplitudes. A lower α represents this change of the spectrum shape. In this case the maximum amplitudes are not affected and lower α values should then be associated with higher S_{op} values. Whether that really occurs depends on R-values, and as shown by Table 7.3 it does not occur for R ~ 0.5 due to the strange intersection of f1 and f2 as discussed before. Calculations indicate that the differences between FALSTAFF and miniFALSTAFF are insignificant.

As previously mentioned a higher crack opening level is related to constant amplitude loading. The higher crack opening level is the result of some kind of multiple overload interaction effects. However, a multiple overload effect between the most severe flights in successive blocks does not occur according to the ONERA model. The block size (number of flights in one block) does not affect α, because it does not affect the spectrum shape. As a consequence, the predicted crack closure behavior in different blocks is rather similar, as shown by Figure 7.20 for the F-27 load history. In this graph S_{op} of each cycle has been plotted and as a result it illustrates the variation of S_{op} within a flight (see also Figure 7.19). It also shows the decreasing trend of S_{op} after the most severe flight type A. That is a consequence of crack growth into the plastic zone of the most severe
upward gust in flight type A, which implies a decreasing $p_{\text{eq}}$ (Figure 7.1) and thus a decreasing $K_{\text{max.eq}}$ and $S_{\text{op}}$. The decrease is partly restored by other severe flights (types B, C and D) which create new plastic zones.

Because the crack closure levels in the ONERA model are highly depending on $\alpha$ (shape of the spectrum) and stress ratios, it might well be expected that the design stress level will have a minor effect on the predicted crack closure behaviour. This is also illustrated by Figure 7.21 by comparing the same flight for different design stress levels. The conclusion than should be that the predicted slope factor of the flight-simulation histories should be of a comparable magnitude as the exponent $m$ of the Paris relation. This is confirmed by the data in Table 7.2. It should be recalled that according of the PREFFAS model the predicted slope factors are exactly equal to $m$. Because the ONERA model is not based on a "stationary crack" concept the exact equality does not apply here, but the similar magnitude is maintained. The significant higher slope factors of the test results for the 7024 alloy (Table 7.2) can be due to a multiple overload interaction between high loads occurring in different, but successive blocks. The similar slope factor values for the 7075 alloy may indicate that such a multiple overload interaction is less important for this alloy.

The physical meaning of $K_{\text{max.eq}}$, $K_{\text{min.eq}}$ and the transient phenomenon

The $K_{\text{op}}$ level after each cycle is calculated from $K_{\text{max.eq}}$ and $K_{\text{min.eq}}$ (see Eq. (7.22)). The flight profiles in Figures 7.19 and 7.21 illustrate how $K_{\text{op}}$ is following the change of $K_{\text{max.eq}}$ and $K_{\text{min.eq}}$. The flight profiles also show that $K_{\text{max.eq}}$ is increased by high loads. $K_{\text{max.eq}}$ decreases when the crack is growing as discussed in Section 7.3. This cannot be seen in the flight profiles, because the crack growth during one flight is too small to reveal such a decrease. However, the decrease of $K_{\text{max.eq}}$ and the related decrease of $K_{\text{op}}$ do account for a decay of the retardation after crack growth. At the same time, an overload with a significant increase of $K_{\text{max.eq}}$, and thus also $K_{\text{op}}$, is immediately followed by a maximum retardation. In other words, delayed retardation is not possible in the ONERA model.
As pointed out before, the transient decrease of the crack growth rate in the first part of a flight-simulation test can last for several blocks. It will be clear from the previous paragraph that such a transient phenomenon cannot be predicted by the ONERA model. In the first block the most severe flight will set a high $K_{\text{max.eq}}$ with an immediate retardation effect. In the second block the most severe flight will increase $K_{\text{max.eq}}$ but its effect will be independent of the load history in the previous block, according to the model. Although $K_{\text{max.eq}}$ clearly reflects the predominant effect of the maximum load level of the spectrum, the model is unable to cope with the transient phenomenon.

The $K_{\text{min.eq}}$ level in the ONERA model shows a transient behavior in a rather short time, see the flight profiles in Figures 7.19 and 7.21. After a severe downward load, e.g. the ground load, $K_{\text{min.eq}}$ rapidly increases especially if some high loads follow. The model says that negative loads do reduce the retardation of high loads, but effectively for a very short time only (less than one flight). The $K_{\text{min.eq}}$ effect must be associated with reversed plasticity. Reversed plastic zones are significantly smaller than the monotonic plastic zones, which should imply a faster decay of the $K_{\text{min.eq}}$ effect. Although the $K_{\text{min.eq}}$ procedure of the model seems to be plausible of a physical point of view, it is more difficult to see whether Eq. (7.19) is a reasonable assumption.

Eq. (7.19): $\Delta K_{\text{min.eq,i}} = \text{diff} \cdot \text{ratio}^\beta$

\[
\text{with diff} = K_{\text{min,i}} - K_{\text{min.eq,i}}
\]

\[
\text{ratio} = K_{\text{max,i}}/K_{\text{max.eq,i}}
\]

\[
\beta = 2 + t/2
\]

It is difficult to understand why $K_{\text{min,i}}$ should have such a prominent effect on relaxation of $K_{\text{min.eq}}$. Also the thickness effect is not easily understood. Both reversed plasticity and monotonic plasticity, still contained within the largest plastic zone, might be expected to follow a plane strain behavior, i.e. independent of the material thickness.
7.8.2 Influence of input parameters

The material parameters such as $C$, $m$ and the yield stress, do influence the prediction results. Some material data are provided by the authors of the model to run the model. Material data given by Baudin and Robert are presented in Table 7.4.

For constant amplitude loading $\alpha = 0$ and according to Eq. (7.22),

$$K_{\text{op}} = K_{\text{max}} \cdot f_2(R)$$

(7.28)

Furthermore:

$$\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}} = K_{\text{max}} (1 - f_2(R))$$

(7.29)

Baudin and Robert start from empirical data for $R = 0$

$$\frac{da}{dn} = C_o \cdot K_{\text{max}}$$

(7.30)

with $C_o = 0.95 \cdot 10^{-7}$ and $m = 3$ for 2024-T3 material.

For $R = 0$, Eq. (7.29) reads:

$$\Delta K_{\text{eff}} = K_{\text{max}} (1 - f_2(0))$$

Substitution in Eq. (7.30) gives

$$\frac{da}{dn} = \frac{C_o}{(1 - f_2(0))^m} \Delta K_{\text{eff}}^m = C \Delta K_{\text{eff}}^m$$

(7.31)

where $C = C_o \left[ (1 - f_2(0)) \right]^{-m}$

This is the procedure mentioned by Baudin and Robert in Ref. [27] to determine the $C$ and $m$ values. In the present study, the $C$ and $m$ values are determined based on more than one $R$ value. For example, for 2024-T3 bare and 7075-T6 Clad $C$ and $m$ are determined based on crack growth data for ten and three $R$ values respectively. Eq. (7.29) is applied to the $\frac{da}{dn} - \Delta K_{\text{eff}}$ data for all $R$ value. The $C$ and $m$ values are then derived from these data by using linear regression. If $C$ and $m$ are obtained from data for a limited number of $R$-values the values of $C$ and $m$ can be different. This issue will be illustrated by a numerical example.
For 7075-T6 material, two sets \( C \) and \( m \) parameters are derived from the results for a different number of \( R \) values adopted from Ref. [148]. One set used in the present calculation is \( C = 8.3715 \times 10^{-10} \) and \( m = 3.4467 \) (based on three \( R \) values: 0.52, 0.23 and -0.05). The data are shown in Figure 7.22a. The other set based on two \( R \) values: 0.52 and 0.23, gives \( C = 1.6695 \times 10^{-9} \) and \( m = 3.1447 \), see Figure 7.22b. These two sets of \( C \) and \( m \) values give different prediction results, see Table 7.5 for the F-27 and TWIST spectra. Shorter predicted lives are obtained if \( C \) and \( m \) are based on two \( R \) values (0.52 and 0.23). The exercise illustrates that the predictions are indeed rather sensitive to the material constants \( C \) and \( m \) of the Paris equation, while these constants are sensitive to the amount of CA data used. One might expect that a broad \( R \) range coverage is advisable for this purpose.

7.8.3 Effect of plane strain and plane stress considerations and effect of thickness

The ONERA model includes the effect of the state of stress in determining the average plastic zone size \( \rho_m \), see Eqs. (7.7) and (7.8). Equation (7.7) is applicable if there is still a core region in the plate where plane strain applies (left part of Fig. 7.2). Equation (7.8) must be applied if that region has vanished (right part of Fig. 7.2). The first case will be labelled as the partly plane strain case, the second are as the plane stress case. The partly plane strain case applies if \( \rho < 0.6 \, t \), where \( \rho \) is the plastic zone size of the material surface (Irwin's plane stress equation). The significance of the transition from partly plane strain to plane stress will now be analysed by comparing \( \rho_{eq} \) for the two cases. A crack will be considered, which is growing into the primary plastic zone of an overload (plastic zone size \( \rho_{OL} \)). If the crack has grown into the overload plastic zone with a crack extension \( \Delta a \), \( \rho_{eq} \) has decreased because \( \rho_m \) in Eqs. (7.11) and (7.12) must be reduced with \( \Delta a \). The two equations then become

\[
\rho_{eq} = 0.12 \, t \left[ \left(1 + 100 \frac{\rho_{OL,m} - \Delta a}{t} \right)^{1/2} - 1 \right]
\]

\[
\rho_{eq} = \rho_{OL,m} - \Delta a + t/4
\]
The crack extension into the plastic zone will be written as $\Delta a = \phi \rho_{OL}$ ($\phi < 1$). Substitution of Eqs. (7.7) and (7.8), applied to the overload plastic zone, into the above equations leads to:

Partly plane strain:

$$\rho_{eq} = 0.12 t \left[ \left\{ 1 + 100 \frac{\rho_{OL}}{t} \left( \frac{1}{6} + 25/36 \frac{\rho_{OL}}{t} - \phi \right) \right\}^{1/2} - 1 \right]$$  (7.32)

Plane stress: $\rho_{eq} = \rho_{OL} (1 - \phi)$  \hspace{1cm} (7.33)

As a numerical example the two $\rho_{eq}$ functions are plotted in Figure 7.23 for $\phi = 0.1$ and $\phi = 0.2$ (i.e. $\Delta a/\rho_{OL} = 0.1$ and 0.2) and $t = 2$ mm. The full line represents $\rho_{eq} = \rho_{OL}$. For the plane stress case parallel lines are found, just below the $\rho_{eq} = \rho_{OL}$. Note that in the plane stress case this result is independent of the sheet thickness (Eq. 7.33). However, for the partly plane strain case $\rho_{eq}$ is significantly smaller, and even more so for increasing $\Delta a$ and smaller $\rho_{OL}$. Because low $\rho_{eq}$ values imply low $K_{max,eq}$ and thus low $K_{op}$, the ONERA model correctly predicts lower $S_{op}$ values for plane strain conditions.

In order to explore the meaning of the above analysis, predictions have been made, assuming that plane stress does apply always. In other words Eqs. (7.7) and (7.11) are ignored. The application of Eqs. (7.8) and (7.12) is then equivalent to assuming a constant plastic zone size (plane stress) over the full thickness of the material. Calculations have been made for the F-27 spectrum and the materials 2024-T3 Alclad and 7075-T6. The prediction results are presented in Table 7.6, besides the results of the basic model. The results show that the predicted life differences are very small for 2024-T3 Alclad material. It shows that the plane stress condition is dominant in the 2024-T3 Alclad predictions. For 7075-T6 Clad material, the pure plane stress assumption predicts longer lives. It implies that including the partly plane strain case in the model leads to a shorter lives, which should be expected. This supports the indication given in Figure 7.23.

In order to see how the ONERA model accounts for the thickness effect, predictions were made for different thicknesses. The CN 235 spectrum with $S_{max} = 200$ MPa and 2024-T3 Alclad were adopted for this purpose. The predictions were carried out for three different conditions, i.e.
(1) for the basic ONERA model (2) for a constant, exponent $\beta$ equal to 3 (which applies to $t = 2 \text{ mm}$ only, see Eq. (7.20)), and (3) for plane stress conditions being always applicable with $\beta$ values according to the model rule (Eq. (7.20)). The basic ONERA model is used as the reference. The second and the third condition are adopted to evaluate the effect of $\beta$ in accounting for the thickness effect and once more the influence of the plane strain assumption (Eqs. (7.7) and (7.11)). The prediction results are presented in Table 7.7. Apparently the ONERA model predicts a significant thickness effect in agreement with trends reported in the literature [94]. If plane stress is assumed to apply always almost the same results are obtained. However, if $\beta$ is kept constant, the prediction life does not significantly change. This implies that the exponent $\beta$ is dominant in accounting for the thickness effect, rather than the plane strain/plane stress transition. The $\beta$ influence occurs through its effect on the "relaxation" of $K_{\text{min,eq}}$ (Eq. 7.19). Profiles of two similar flights for $t = 2 \text{ mm}$ and $t = 8 \text{ mm}$ respectively are presented in Figure 7.24. It clearly illustrates a slower increase of $K_{\text{min,eq}}$, and thus of $K_{\text{op}}$, for the larger thickness. Although the lower $S_{\text{op}}$ for the thicker sheet seems to be acceptable, it is strange that the lower $S_{\text{op}}$ is a consequence of a longer lasting "memory" for severe negative loads.

7.9 Summary and conclusions on the ONERA model

The ONERA model is based on the crack closure concept proposed by Elber. The crack closure level depends on the stress history. It is recalculated cycle-by-cycle. The crack opening level is limited by two extreme conditions namely the constant amplitude case and the single overload case. The crack opening level is determined by introducing a loading parameter $\alpha$ depending on the shape of the load spectrum. The model mechanism is simple. A unique feature is that it includes a relaxation of $K_{\text{min,eq}}$, i.e. the negative effect of severe downwards load is vanishing in a relatively small number of cycles. The model prediction results are summarized below.
1. In general, the predicted trends are qualitatively correct. This applies to results found in tests for variables such as gust load severity, severity of the ground stress levels, variations of design stress level, omission of small cycles can be good predicted. However, the quantitative agreement in several cases is limited.

2. Under simple variable-amplitude load sequences, a systematic difference was predicted between underload-overload and overload-underload sequences, which was not found in the tests. The predicted difference between the two sequences can be understood by considering the model mechanism.

3. The effect of flight sequence reordering in the CN 235 test series is not predicted by the model. Predictions show a similar fatigue crack growth life for the random sequence and two programmed flight sequences, whereas the test results show that programmed sequence cause a longer life.

4. Crack increment in the most severe flight are predicted too small as compared to the test results. This may due to using the Paris relation in calculating the crack growth increment. For a real growing fatigue crack, other mechanism such as ductile tearing (stable crack extension), may also contribute to crack growth, but that is not included in the prediction model.

5. Approximately log linearly increasing crack growth rates are observed for different type of spectra. This is caused by a constant a value and a relatively constant K ratios (K_{\text{min.eq}}/K_{\text{max.eq}}) in the same flight for different blocks.

6. Input data play an important role to give good prediction results. A different R values adopted in constant amplitude data results a different material parameter. A broad R range coverage is advisable for this purpose.

7. The crack opening functions f1 and f2 are quite dominant in determining the prediction results. The f1 and f2 used in the present calculations are based on results for 2024-T3 material. The f1 and f2 might be expected to be different for different materials.

8. The state of stress (plane strain or plane stress) influences the prediction results. Plane strain leads to a shorter predicted life, whereas plane stress leads to a longer predicted life.
9. The exponent $\beta$ plays an important role in accounting for the thickness effect and in the relaxation of the $K_{\text{min.eq}}$. It is difficult to understand this role from a physical point of view.

10. The effect of severe downward loads is vanishing rapidly, practically within one flight (relaxation of $K_{\text{min.eq}}$). The $K_{op}/K_{max}$ ratio does not increase for longer cracks. A multiple overload interaction between severe flights of successive blocks does not occur.
CHAPTER 8
THE CORPUS MODEL

8.1 Introduction

The CORPUS model was proposed by De Koning [26] in 1981. An earlier version of the model was published in Ref. [25]. In the CORPUS model, overload interaction effects play an important role. It was mentioned in Chapter 2 that the number of overloads and the distance between two overloads have a significant influence on the crack growth retardation (see Figures 2.21 and 2.22). Interactions between two or more overloads cause an increasing crack opening stress level, which will augment the crack growth retardation. This "multiple overload effect" is included in the CORPUS model. It is one of the major differences with the two previous models (PREFFAS and ONERA). The CORPUS model also includes other characteristic features, such as considerations of plane stress and plane strain conditions for the plastic zone size, a correction for the effect of high loads on the crack opening stress levels and the recognition of primary and secondary plastic zones. Crack growth increments are calculated cycle-by-cycle. The model was applied by De Koning et al for 2mm aluminum sheet materials i.e. 2024-T3 Al clad and 7075-T6 Clad under F-27 spectra [26]. It will be applied here on all data collected in Chapter 4.

8.2 The crack opening level

As a consequence of the crack closure model, the crack extension in every cycle is determined by the effective stress range. In cycle (i) \( \Delta a_i \) is

\[
\Delta a_i = f(\Delta K_{\text{eff},i}) \tag{8.1}
\]

where

\[
\Delta K_{\text{eff},i} = C \Delta S_{\text{eff},i} \sqrt{\pi a} \tag{8.2}
\]
The cycle (i) is defined by a maximum $S_{\text{max},i}$ followed by a minimum $S_{\text{min},i}$. The effective stress range is determined as

$$\Delta S_{\text{eff},i} = S_{\text{max},i} - S_{\text{min},i-1} \quad \text{if } \sigma_{\text{op}} < S_{\text{min},i-1}$$

$$\Delta S_{\text{eff},i} = S_{\text{max},i} - \sigma_{\text{op}} \quad \text{if } S_{\text{min},i} < \sigma_{\text{op}} < S_{\text{max},i} \quad (8.3)$$

$$\Delta S_{\text{eff},i} = 0 \quad \text{if } \sigma_{\text{op}} \geq S_{\text{max},i}$$

where $\sigma_{\text{op}}$ is the maximum crack opening stress level, which will be described below. (Note: The first condition of Eq. (8.3) seems logical. However, as a consequence of the hump mechanism described below, it is impossible that $\sigma_{\text{op}} \leq S_{\text{min},i-1}$ in the CORPUS model).

The CORPUS model is basically associated with plastic deformation left in the wake of the crack (Elber mechanism). It is assumed that each cycle will leave its own plastic deformation, including its reversed plastic deformation during unloading. When the crack is growing plastic deformation of previous cycles will be left in the wake of the crack. According to the CORPUS model the wake of the crack is covered with "humps", where each hump is associated with a previous cycle. The hump is created during uploading, while reversed flow during unloading will reduce the hump. In other words, both $S_{\text{max}}$ and $S_{\text{min}}$ will affect the hump. Figure 8.1 shows a schematic picture of a crack with three humps in the wake of the crack. A crack is supposed to be still closed as long as there is contact between humps. A crack is considered to be just opened during uploading when the last hump looses contact. In Figure 8.1 it implies that

$$\sigma_{\text{op}} = S_{\text{op}}^3 \quad \text{or} \quad \sigma_{\text{op}} = \max S_{\text{op}}^n \quad (8.4)$$

Each hump will have its own effect on crack opening and closure. Consider a hump created in cycle number $n$ at a crack length $a = a^n$ and an associated plastic zone size $D^n$, see Figure 8.2. In the CORPUS model the crack opening stress $S_{\text{op}}^n$, if it depends on this hump only, is supposed to be constant as long as the crack tip is still in that zone, see Figure 8.2. In reality a decreasing function will apply, but for simplicity a block function is adopted. In terms of the CORPUS
model a peak load will activate a "delay switch" which is turned off if the crack has grown through the plastic zone of the peak load.

\[ S_{op}^n = g(S_{\max}^n, S_{\min}^n) \quad \text{if } a^n \leq a \leq a^n + b^n \quad (8.5) \]

\[ S_{op}^n = 0 \quad \text{if } a > a^n + b^n \quad (8.6) \]

where \( b^n \) is the retardation region (plastic zone) and \( g(S_{\max}^n, S_{\min}^n) \) is the hump opening function. Based on empirical evidence the function was defined as:

\[ g(S_{\max}^n, S_{\min}^n) = S_{\max}^n (-0.4R^4 + 0.9R^3 - 0.15R^2 + 0.2R + 0.45) \quad \text{if } R > 0 \quad (8.7) \]

\[ g(S_{\max}^n, S_{\min}^n) = S_{\max}^n (0.1R^2 + 0.2R + 0.45) \quad \text{if } -0.5 \leq R \leq 0 \quad (8.8) \]

where \( R \) is the stress ratio \( (S_{\min}^n/S_{\max}^n) \). The same function was supposed to be applicable to both 2024-T3 and 7075-T6.

Newman [164] demonstrated that the opening stress level does not only depend on \( S_{\max}^n \) and \( S_{\min}^n \), but also on the maximum stress in relation to the yield stress, i.e. on \( S_{\max}^n/\sigma \). In Newman's analysis elastic-perfectly-plastic material behavior was assumed with a kind of an average yield stress \( \sigma = (\sigma_{0.2} + \sigma_u)/2 \). De Koning [26] defined a correction function \( h \) which is a good fit to Newman's results. The correction function \( h \) is

\[ h = 1 - 0.2 \left( 1 - R^n \right)^3 \frac{(S_{\max}^n/1.15 \sigma_y)^3}{S_{\max}^n} \quad (8.9) \]

The corrected \( S_{op}^n \) level then is

\[ S_{op}^n = g(S_{\max}^n, S_{\min}^n) \cdot h \quad (8.10) \]

A graphical presentation for five \( K \)-values is given in Figure 8.3 (Note: in [26] De Koning's equations were slightly different from Eqs.
(8.8) and (8.9), but according to him the present ones should be used). \( S^0_{\text{op},i} \), due to the hump created by cycle \( n \) depends on \( S^{\text{max}}_n \) and \( S^{\text{min}}_n \) (Eqs. 8.5, 8.7 and 8.8). However application of more severe underloads later on will influence the hump opening stress level due to a further reduction of the hump. If in cycle \( i \) a lower minimum occurs (\( S^{\text{min},i}_n < S^0_n \)) then \( S^0_{\text{op}} \) must be recalculated with Eq. (8.10), which implies:

\[
S^0_{\text{op},i} = g(S^{\text{max}}_n, S^{\text{min},i}_n) \cdot h^i
\]

If there is another still lower minimum stress \( S^{\text{min},j}_n \) the hump opening stress level becomes

\[
S^0_{\text{op},j} = g(S^{\text{max}}_n, S^{\text{min},j}_n) \cdot h^j
\]

which is lower than \( S^0_{\text{op},i} \). The value of \( S^{\text{min},i}_n \) can then be erased from the memory of the material, and it is replaced by \( S^{\text{min},j}_n \).

De Koning [26] gave a schematic illustration of crack opening for a load sequence presented in Figure 8.4. In cycle 1, the hump is created by application of \( S^{\text{max}}_n \) and subsequently flattened by \( S^{\text{min}}_n \). The opening stress of the hump \( S^1_{\text{op}} \) is determined by Eq. (8.10),

\[
S^1_{\text{op}} = g(S^{1\text{max}}_n, S^{1\text{min}}_n) \cdot h^1
\]

In the next cycle, the effective stress range \( \Delta \sigma_{\text{eff}} \) is equal to

\[
\Delta \sigma_{\text{eff}} = S^2_{\text{max}} - S^1_{\text{op}}
\]

The hump is created by application of \( S^{2\text{max}}_n \) and flattened by \( S^{2\text{min}}_n \). The opening stress is

\[
S^2_{\text{op}} = g(S^{2\text{max}}_n, S^{2\text{min}}_n) \cdot h^2
\]

The first hump is not changed, because \( S^{2\text{min}}_n \) is equal to \( S^{1\text{min}}_n \). According to Eq. (8.4), the crack opening stress must be selected from the hump that last lost contact. Since \( S^{1\text{op}} > S^{2\text{op}} \), the first hump is the
last one to lose contact. $S_{op}^1$ then determines the crack opening stress. In the third cycle, the effective stress range $\Delta \sigma_{eff}$ is equal to

$$\Delta \sigma_{eff} = S_{max}^3 - S_{op}^1$$

In this cycle $S_{min}^3$ is lower than $S_{min}^1$ and $S_{min}^2$, therefore the first and the second hump are also further flattened. The hump opening stresses are equal to

$$S_{op}^1 = g(S_{max}^1, S_{min}^3) \cdot h^1$$

$$S_{op}^2 = g(S_{max}^2, S_{min}^3) \cdot h^2$$

$$S_{op}^3 = g(S_{max}^3, S_{min}^3) \cdot h^3$$

The crack is fully opened at the highest opening stress level, which corresponds to $S_{op}^1$. In the next cycle, $S_{max}^4$ is greater than all previous $S_{max}$. In this case, the hump opening stress level is always determined by $S_{max}^4$ and $S_{min}^4$. Whatever the value is of later $S_{min}$ levels, the hump of $S_{max}^4$ will always be larger than the hump of $S_{max}^4$. In other words $S_{min}^1$ can be erased in the material memory. Similarly, $S_{min}^3$ also becomes irrelevant for future hump considerations. The memory aspects and the multiple-overload effect will be addressed in Sections 8.4 and 8.5.

8.3 Retardation regions

As discussed in Chapter 2 the plastic zone size depends on the state of stress at the crack tip, plane strain, plane stress or a transition between those two conditions. A characteristic feature of the CORPUS model is that it also distinguishes between a plastic zone developing into virgin (elastic) material, and a plastic zone induced in material, that was already plastically deformed before. These two types are called primary and secondary plastic zones respectively. De Koning derived a special equation for the primary plastic zone. It was based on the Irwin type equation, but it was modified to account for large zones if the stress level approaches the net section yield.
limit. Moreover, De Koning used the Dixon width correction for centrally cracked specimens. That has led to a fairly complicated equation for the primary plastic zone size:

\[
D = \frac{1 - \gamma \left( \frac{S_{\text{max}}}{\sigma_y} \right)^2 + \left( \frac{a}{b} \right)^2 - \sqrt{1 - \gamma \left( \frac{S_{\text{max}}}{\sigma_y} \right)^2 + \left( \frac{a}{b} \right)^2 - 4 \left( \frac{a}{b} \right)^2}}{2 \left( \frac{a}{b} \right)^2} - 1
\]

(8.11)

where \( b \) is the semi width of the specimen and \( \gamma \) depends on the state of stress. For plane stress \( \gamma = 1/1.32 \) and for plane strain \( \gamma = 1/9. \)

For 7075-T6 material with a high yield stress, the plastic zone size equation was obtained from Eq. (8.11) by a second order approximation:

\[
D = a \gamma \left( \frac{S_{\text{max}}}{\sigma_y} \right)^2 \left[ 1 + \left( \frac{a}{b} \right)^2 + \gamma \left( \frac{S_{\text{max}}}{\sigma_y} \right)^2 \right]
\]

(8.12)

In the CORPUS model the crack tip state of stress depends on the size of the plastic zone relative to the thickness. First, the plastic zone size is calculated under plane stress condition \((D_{ss})\). If \( D_{ss} \geq 0.5t \) (\( t = \) sheet thickness) then the plastic zone is supposed to be in plane stress. If \( D_{ss} \leq 0.35t \), the plastic zone is assumed to be in plane strain, size \( D_{sn} \). During the transition from plane strain to plane stress \((0.35t < D_{ss} < 0.5t)\):

\[
D = D_{sn} + 2D_{ss} \left[ \left( \frac{D_{ss}}{t} - .35 \right) / .15 \right]^4 \left( D_{ss} - D_{sn} \right) / t
\]

(8.13)

Secondary plastic zones are always assumed to develop under plane strain conditions. The size of a secondary plastic zone is:

\[
D = \frac{1}{9n} \left( \frac{\Delta K_{\text{eff}}}{2\sigma_y} \right)^2
\]

(8.14)

This equation is based on the Irwin plastic zone size approximation with \( \gamma = 1/9 \) for plane strain (see above) and \( 2\sigma_y \) instead \( \sigma_y \) in view of reversed plastic deformation present in the primary plastic zone.
The latter argument is debatable because reversed plasticity does occur in a smaller part of the primary plastic zone only. It should be noted that the width correction factor applied in the CORPUS model is the Dixon correction factor:

\[ C = \frac{1}{\sqrt{1 - \left(\frac{a}{b}\right)^2}} \quad (8.15) \]

8.4 Modelling of the material memory

During crack growth under variable amplitude loading various primary and secondary plastic zones will be created. If they can affect \( S_{op} \) in later cycles they must be stored in the material memory. In Figure 8.5 a schematic picture is shown with two primary plastic zones and two secondary plastic zones. The second primary plastic zone (PPZ) was formed at \( a = a_2 \). It became a PPZ because it penetrated elastic material, that means:

\[ a_2 + dp_2 > a_1 + dp_1 \quad (8.16a) \]

At \( a = a_3 \) a primary plastic zone was not formed because its size \( dp_3 \), calculated with Eq. (8.11) was not large enough to penetrate elastic material. In other words:

\[ a_3 + dp_3 < a_2 + dp_2 \quad (8.16b) \]

The load cycle occurring at \( a = a_3 \) thus created a secondary plastic zone with a size according to Eq. (8.14).

Fortunately not all plastic zones can affect \( S_{op} \), because their effect on \( S_{op} \) is overruled by other plastic zones, or the delay switch was turned off. As a consequence only a limited number of plastic zones must be stored in the material memory. Aspects of primary plastic zones will be discussed first with reference to the two zones shown in Figure 8.5.
If the crack is growing it will reach a size $a = a_1 + dp_1$. At that moment the "delay switch" is turned off and the first PPZ can be removed from the memory.

The second PPZ was formed at $a = a_2$. If $S_{\text{max}}^2$ exceeds $S_{\text{max}}^1$ it will create a larger hump, and the hump of the first PPZ can never become dominant afterwards. The first PPZ can thus be erased in the material memory. This was discussed before (Figure 8.4). Actually the hump criterion can also be formulated in the same way as it was done for the PREFFAS model in Section 6.2.1: Relevant $S_{\text{op}}$ values from preceding cycles are calculated from $S_{\text{max}}$ values of those cycles, and from the lowest $S_{\text{min}}$ value occurring between those cycles and the current one (The CORPUS model was published before the PREFFAS model). However, there is also an important difference. The CORPUS concept is based on a growing crack. The "stationary crack" concept of PREFFAS is not used. That implies that the second PPZ can extend beyond the first PPZ without erasing the first PPZ. As soon as the delay switch of the first PPZ is turned off ($a = a_1 + dp_1$), the second PPZ becomes the dominant one.

From the previous paragraph it follows that the $S_{\text{max}}$ values associated with primary plastic zones must form a series of decreasing values. These "history" values have to be stored in the material memory. The values are labelled as $S_{\text{max}}^\text{SH}$. In a similar way, the arguments on creating humps and reducing humps by downward loads, imply that there is one $S_{\text{min}}^\text{SH}$ value associated with each PPZ, and secondly the $S_{\text{min}}^\text{SH}$ values must form a series of increasing, or at least equal, $S_{\text{min}}$ levels. An obvious example of equal $S_{\text{min}}^\text{SH}$ values will occur in a flight simulation history, if the ground stress ($S_{\text{gr}}$) is the most severe downward load. $S_{\text{gr}}$ will be the $S_{\text{min}}^\text{SH}$ of the PPZ's. The PREFFAS model was characterized by a decreasing series of $S_{\text{max}}^\text{SH}$ values, an increasing series of $S_{\text{min}}^\text{SH}$ values (equal $S_{\text{min}}^\text{SH}$ do not occur in the PREFFAS model as a consequence of the stationary crack length concept) and an increasing series of associated $S_{\text{op}}^\text{SH}$ values. The latter characteristic does not apply to the CORPUS model. Each pair of $S_{\text{max}}^\text{SH}$ and $S_{\text{min}}^\text{SH}$ of a plastic zone is connected with an $S_{\text{op}}^\text{SH}$ value, but the $S_{\text{op}}^\text{SH}$ values of successive plastic zones do not necessarily form a series of increasing (or decreasing) values. For a crack tip located
in overlapping plastic zones (Figure 8.5), the $SH_{op}$ values of the zones have to be checked to find the maximum value. In addition to $SH_{max}$, $SH_{min}$ and $SH_{op}$ also the plastic zone size and its location (a, dp) must be stored in the material memory. As an illustration Figure 8.6 shows series of five PPZ's induced by flights of types A, B, C, D and E. These zones with the associated SH stress levels are kept in the material memory for some time, until a new flight A erases this information.

If a PPZ is not formed, but the crack is still opened, a secondary plastic zone (SPZ) is formed. That may occur in many cycles. However, only the "historic" SPZ's have to be stored in the material memory. A SPZ is a historic SPZ if it can have an effect on $S_{op}$ in subsequent cycles. As a consequence the above argumentation on PPZ's, based on the hump and the delay switch concepts is also applicable on the historic SPZ's. That implies that again the $SH_{max}$ and the $SH_{min}$ values form a decreasing and an increasing series of stress levels respectively, whereas again no specific sequence applies to the associated $SH_{op}$ values. Moreover, in view of the hump concept the combined series of $SH_{max}$ and $SH_{min}$ values of both PPZ's and SPZ's together must form such series as schematically shown in Figure 8.7. With respect to $SH_{op}$ of the SPZ's there is a limitation. If in Figure 8.5 $SH_{op}$ of zone 3 is lower than the $SH_{op}$ levels of the two PPZ's (zones 1 and 2), there is no need to store $SH$ levels of zone 3 in the material memory. SPZ's are by definition embedded in PPZ's. If those PPZ's have a higher $SH_{op}$ than a SPZ the hump of the SPZ will always remain less significant than the humps of the PPZ's. In other words, the $SH_{op}$ values of the SPZ's must all be higher than the lowest $SH_{op}$ of the PPZ's.

The above characteristic features are a logical result of the hump concept and the "delay switch" concept. Some consequences are recapitulated below:

1. A PPZ is formed only if plastic deformation enters the elastic material. More PPZ's can exist at the same time. In random flight simulation tests PPZ's will be formed by the more severe upward loads. Lower upward loads will form SPZ's.
2. SPZ's must be stored in the material memory when they can have an
effect on $S_{op}$ in subsequent cycles. The number of such historical
SPZ's will be limited because they are easily erased either by high
$S_{max}$ peaks, low $S_{min}$ peaks, or crack growth (delay switch turned
off).

Several examples of erasing plastic zones, or decreasing $SH_{min}$ and
$SH_{op}$, are schematically illustrated in Figure 8.7. In flight-
simulation histories the ground stress level will usually erase all
SPZ's. However, in flight new historical SPZ's can easily be formed
again, because the high $S_{min}$ values imply high $S_{op}$ values.

Requirements to form a historic SPZ are more easily satisfied then.
The existence of PPZ's and SPZ's is one reason why the CORPUS model is
a more sophisticated model than the PREFFAS and the ONERA model. At
the same time it implies that the computer program is more complex. A
survey of the computer program is given in Appendix C.

8.5 Interaction of overloads

As already mentioned in the introduction of this chapter, overload
interaction effects play an important role in the CORPUS model.
Interaction between overloads with overlapping primary plastic zones
cause an increase of crack opening levels, which will give more the
crack growth retardation.

The hump opening stress level given in Eq. (8.5) and specified in Eqs.
(8.7) and (8.8) is valid for a single overload and underload
combination. If interactions between overloads occur, then the hump
opening stress becomes higher than the $S_{op}^n$ according to the
equations. The opening stress is increased after each overload until
it has reached a certain upperbound. De Koning has defined the
upperbound as:

$$g(S_{max}, S_{min}) + m_{st} [S_{max} - g(S_{max}, S_{min})]$$  (8.17)
The stationary parameter \( m_{st}^n \) is depending on the crack growth increment \( \Delta a \) between the overloads and the plastic zone size of the overload \( D^n \). In the CORPUS model the following relations are adopted:

\[
\begin{align*}
    m_{st}^n &= 0.1 + 0.2 \frac{\Delta a}{D^n} & \text{if } 0 < \frac{\Delta a}{D^n} \leq 0.25 \\
    m_{st}^n &= 0.15 & \text{if } 0.25 < \frac{\Delta a}{D^n} \leq 1 \quad (8.18) \\
    m_{st}^n &= 0 & \text{if } \frac{\Delta a}{D^n} > 1
\end{align*}
\]

A graphical representation is given in Figure 8.8. The first condition of Eq. (8.18) implies that the overload interaction increases when \( \Delta a \) is increasing until \( \Delta a/D^n = 0.25 \). De Koning introduced this trend in view of some empirical evidence. The last condition of Eq. (8.18) implies that the crack has grown completely through the overload plastic zone. Then the delay switch is turned off and at the same time a later overload interaction is eliminated \( m_{st}^n = 0 \).

**Constant amplitude case**

Under CA loading the plastic zone will extend in each cycle into elastic material. It implies that each cycle is an overload for the previous cycle. According to the CORPUS concept a multiple overload interaction will occur, which will raise \( S_{op} \). It may be noted that the ONERA model also adopts a higher \( S_{op} \) for CA loading than for a single load cycle of the same magnitude \( f_2 > f_1 \) in Section 7.5.

For the constant amplitude case, \( \Delta a \) is small compared to \( D^n \). This implies that \( \Delta a/D^n \) goes to zero. According to Eqs. (8.17) and 8.18) the stationary crack opening stress for constant amplitude loading is equal to

\[
S_{op} = g(S_{max}^n, S_{min}^n) + 0.1 \left[ S_{max}^n - g(S_{max}^n, S_{min}^n) \right] \quad (8.19)
\]

This equation has to be used to calculate the \( \Delta K_{eff} \) values for the Paris relation. It easily follows from Eq. (8.19) that:

\[
\Delta S_{eff}^{*} = 0.9 \Delta S_{eff}
\]
where $\Delta S_{\text{eff}}^{\ast}$ is the effective stress range corrected for the multiple overload interaction effect and $\Delta S_{\text{eff}}$ is the uncorrected value. As a consequence:

$$\Delta K_{\text{eff}}^{\ast} = 0.9 \Delta K_{\text{eff}}$$

If the Paris relation for uncorrected $\Delta K_{\text{eff}}$ values is:

$$\frac{da}{dN} = C \Delta K_{\text{eff}}^m$$

the same relation for corrected $\Delta K_{\text{eff}}$ values is:

$$\frac{da}{dN} = C (\Delta K_{\text{eff}}^{\ast}/0.9)^m = C^{\ast} \Delta K_{\text{eff}}^m$$

with $C^{\ast} = (0.9)^{-m} C$. In the Paris relation only $C$ is affected but $m$ remains the same. As an example, for 2024-T3 bare material $m = 4.1$ and thus $C^{\ast} = 1.540 C$. This implies that for the same $\Delta K_{\text{eff}}$, the crack growth rate becomes larger. The influence of these parameters will be shown in the prediction results later on.

**Variable-amplitude case**

In general, a large number of repeated overloads is required to reach the stationary condition of $S_{\text{op}}^n$. The $S_{\text{op}}^n$ is "relaxed" (terminology of De Koning) to the stationary value in a step by step manner, starting with the single overload value. To describe the relaxation process a parameter $m^n$ was introduced, as shown in the equation below.

$$S_{\text{op}}^n = g(S_{\text{max}}^n, S_{\text{min}}^n) + m^n \left[ S_{\text{max}}^n - g(S_{\text{max}}^n, S_{\text{min}}^n) \right] \quad (8.20)$$

For $m^n = 0$, $S_{\text{op}}^n$ is equal to $g(S_{\text{max}}^n, S_{\text{min}}^n)$ for the single overload-underload case, and for $m^n = m^n_{\text{st}}$ the $S_{\text{op}}^n$ is equal to the stationary level. The value of $m^n$ is updated each time when a new overload of level $n$ is applied. The update equation is:

$$m^n_{\text{new}} = m^n_{\text{old}} + \delta (m^n_{\text{st}} - m^n_{\text{old}}) \frac{\Delta a}{D^d} \quad (8.21)$$
where $\delta$ is the relaxation factor, $\Delta a$ is the crack extension between the overloads and $D^d$ is the plastic zone size of the dominant hump of preceding overloads.

The stationary condition was first considered for interaction effects of similar overloads in the plane stress condition. In a more general load sequence, interactions of overloads with different load levels must be considered, and the plastic zone must also be considered under different conditions. To account for these effects, De Koning applied corrections on the relaxation factor,

$$\delta = 0.28 \delta_1 \delta_2 \tag{8.22}$$

The constant 0.28 is the relaxation factor adopted for 2024-T3 material, $\delta_1$ accounts for the differences in overload levels, and $\delta_2$ accounts for the effect of reduced overload interaction for plane strain.

The formulation for $\delta_1$ and $\delta_2$ are

$$\delta_1 = \left[ \frac{4 D^d D^n}{(D^d + D^n)^2} \right]^{1/4} \quad \text{if } D^n \leq D^d \quad \tag{8.23a}$$

or

$$\delta_1 = \frac{D^d}{D^n} \quad \text{if } D^n > D^d \quad \tag{8.23b}$$

and:

$$\delta_2 = \frac{D^d}{D^{d_{ss}}} \quad 0 \leq \delta_2 \leq 1 \quad \tag{8.24}$$

$D^d$ is the plastic zone size associated with the dominant hump, which provides the highest crack opening level selected from the primary plastic zone. $D^d$ and $D^{d_{ss}}$ are the actual plastic zone and its value for the hypothetical case of pure plane stress (Eq.8.11) with $\gamma = 1/1.32$ respectively for the dominant hump. $D^n$ is the current plastic zone size. $m^n_{st}$ in Eq. (8.21) is determined according to Eq. (8.18) by using $D^d$ instead of $D^n$. The interaction factor $\delta_1$ is plotted in Figure 8.9. The graph shows that the interaction is smaller if $D^n$ and $D^d$ are significantly different, whereas the interaction is larger if the two sizes are closely together. Apparently De Koning thought that
Eq. (8.23a) did not decrease sufficiently for $D^n/D^d > 1$ and he then adopted Eq. (8.23b). Intelligent guesses are evidently involved. The $m^n_{D^d}$ in Eq. (8.21) is replaced by $(m^d_{old} \cdot \delta)$ if that is larger than $m_{old}$. This also enhances interaction effects related to the dominant hump if different load levels interact.

As an illustration the predicted variation of $S_{op}$ during an F-27 load history is plotted in Figure 8.10. The step by step increase of $S_{op}$ occurs during the most severe flight (type A, No. 1653 in a block of 2500 flights). In this graph $S_{op}$ is plotted for every cycle. The dots are caused by higher $S_{op}$-values which are reduced by a negative gust load or a ground-to-air cycle. That occurs more frequently in the beginning of this life when $S_{op}$ of the PPZ is still low. Later on it occurs during the more severe loads only. Figure 8.11 shows the behavior during some severe flights.

In Ref. [26], De Koning did not apply interaction of overloads for 7075-T6 material.

Finally, it should be noted that the interaction effects of multiple overloads are applied only for peak loads which produce a primary plastic zone.

The flow diagram of the model is presented in Appendix B.

8.6 Prediction results

De Koning [26] has provided material parameters for 2024-T3 Al clad and 7075-T6 Clad materials to run the CORPUS model. The material parameters for 2024-T3 bare are determined with the same procedure as for 2024-T3 Al clad, i.e. with considering the stationary condition (multiple overloads).

The parameters used in the present calculations are given below.

a. 2 mm 2024-T3 Al clad material.

\[
\begin{align*}
\sigma_y &= 360 \text{ MPa} \\
K_{IC} &= 65 \text{ MPa} \sqrt{m}
\end{align*}
\]
\[ C = 1.26 \times 10^{-10} \quad \text{(da/dN in m/cycle)} \]
\[ m = 3.7 \]
\[ \delta = 0.28 \delta_1 \delta_2 \]

b. 2 mm 7075-T6 Clad material.
\[ \sigma_y = 500 \, \text{MPa} \]
\[ K_{IC} = 70 \, \text{MPa m}^{1/2} \]
\[ C = 1.6 \times 10^{-9} \quad \text{(da/dN in m/cycle)} \]
\[ m = 2.5 \]
\[ \delta = 0 \quad \text{(no interaction of overloads)} \]

c. 2 mm 2024-T3 bare material.
\[ \sigma_y = 394 \, \text{MPa} \]
\[ C = 2.9443 \times 10^{-11} \quad \text{(da/dN in m/cycle)} \]
\[ m = 4.1 \]
\[ \delta = 0.28 \delta_1 \delta_2 \]

Comparison between tests and prediction results for all data is presented in Figure 8.12. In general, the prediction results are good. Almost all data are in the range between 0.5 to 2. Most deviations of the 1:1 relation are on the conservative side.

Predicted crack propagation lives and the ratios between prediction and test result are presented in Tables 6.1 to 6.7. Averages of the prediction to test ratios for different spectra are given in Table 6.8 and Figure 6.12.

In crack closure models, the crack opening stress level in every cycle is the most essential part of the model. The CORRUPUS model mechanism has been discussed in the previous sections. Some samples of the crack opening stress histories under F-27 spectrum are presented in Figure 8.11. The figures show the variation of the crack opening stress levels in flights A, B, C and I for case NN 100 in the first and the second block. Flight B (flight number 106) occurs before flight A (flight number 1653), and a flight C (flight number 2229) occurs after flight A. The flight I is shown to indicate the crack opening stress level produced by flight A. Some features to be observed in the figures are:
a. A severe gust induces a high crack opening stress level, see crack opening stress levels after the most severe gust in flight A. This level is the highest opening level produced by the highest memorized S_{\text{max}} (SH_{\text{max}}) and the lowest memorized S_{\text{min}} (SH_{\text{min}}) afterwards. It may be labelled as the dominant S_{\text{op}} level. That level occurs in the subsequent flight (flight 1654), while it also can be recognized in much later flights, see flights 2606 and 2229.

b. A lower S_{\text{max}} can still produce a high S_{\text{op}} which is higher than the dominant level, see for example after the severe gusts in flight B and C. However, after a severe downward load the dominant S_{\text{op}} level mentioned in the previous paragraph returns again.

c. An increasing crack opening stress level, due to overloads interaction can be seen in flight I in the first and the second block (flight 1654 and flight 4154). It is more explicitely illustrated by Figure 8.10 which shows the S_{\text{op}} development during a full predicted life.

General trends of the prediction results are presented below.

a. Effect of gust load severity

The gust load severity effects are indicated in the F-27 tests. Crack propagation lives for two Al-alloys (2024-T3 Alclad and 7075-T6 Clad) are presented in Figure 8.13, which shows the influence of the gust spectrum severity and the ground stress level. The effect of the gust spectrum severity is well predicted for 7075-T6 Clad material. For the 2024-T3 material, the prediction is not good in some cases, especially if the ground stress is positive (light ground stress level). In the latter case, the most severe downward load is a gust load. Although it is a rarely occurring load (once in 2500 flights), it has a predominant effect on the crack opening stress level. The ratios between predicted life and test result are 0.62, 0.42 and 0.33 for the cases LL 100, NL 100 and SL 100 respectively. The crack growth rate diagram is presented in Figure 8.14, which shows higher predicted crack growth rates than the test results.
More comments related to the model predictions for 7075-T6 and 2024-T3 materials will be given later.

b. Effect of ground stress level

The effect of the ground stress levels can be observed in tests with miniTWIST and the F-27 spectrum, and in the Misawa/Schijve tests. Under the F-27 spectrum for 7075-T6 material, the effect of variations of the ground stress level are predicted correctly. Crack growth rates for cases NS, NN, NL*, NL are presented in Figure 8.15. For 2024-T3 material, the predictions are good if the ground stress level is more compressive than the most severe gust loads, see Figure 8.13a. In load conditions where the most severe downward gust is lower than the ground stress the predictions are not good, see the results for the light ground case, i.e. \( S_g / S_{mf} = 0.125 \). (prediction/test ratios 0.6, 0.4 and 0.3, Table 6.3). The crack growth rate results for these three cases are presented in Figure 8.14. It confirms that predicted growth rates overestimate the test results. Apparently the large effect of a rarely occurring negative load as predicted by CORPUS does not agree with the experimental trend.

Under miniTWIST, the tests were carried out for normal ground stress (\( S_g = -0.5 S_{mf} \)) and for a light ground stress (\( S_g = 0 \)). The prediction results are shown in Figure 8.16a for the original flight sequence. The prediction is good for the normal ground stress, but it is more conservative for the light ground case. The ratios between predictions and test results are 0.78 and 0.58 for the normal and light ground cases respectively. Again in the light ground case, the most severe gust load is more compressive than the ground loads, with a similar consequence as found for the F-27 spectrum.

The prediction results are good for Misawa and Schijve tests, see Figure 8.16b (for \( m = 100 \) and \( S_{OL} = 200 \) MPa) and Table 6.14. The ratios between prediction and test result are in the range between 0.71 to 1.28 for \( m = 5 \) and between 0.47 to 1.78 for \( m = 100 \). For \( m = 5 \), the prediction results are better, because crack extension is dominated by the overload cycle.
c. Effect of design stress level

Different design stress levels were applied in the CN 235, F-27, FALSTAFF and miniFALSTAFF tests. The effect of design stress level on the fatigue crack growth prediction life is shown in Figure 8.17 for the CN 235, F-27 (for 2024-T3 Alclad and 7075-T6 Clad materials) and FALSTAFF spectra. The prediction results agree rather well with the test results.

Again the slope factor is defined as \(-d\log N/d\log S\) (N = life, S = design stress level). Slope factors are collected in Table 8.1. Apart from the 7075-T6 result, the slope factors are quite similar for the test results and the predictions. Such an agreement was not found for the ONERA model (Table 7.2). More, the slope factors are significantly larger than the exponent of the Paris relation (again 7075-T6 is the exception). Also this trend is in contrast to the findings for the ONERA model. Crack growth rates for the F-27 spectrum with two aluminum alloys are presented in Figure 8.18.

d. Effect of load sequence

Load sequence variations were applied in the Misawa/Schijve tests and the tests with the miniTWIST spectrum. The test results do not show significant effects of the load sequence.

For the Misawa/Schijve tests, the predictions indicate a significant sequence effect, especially for \(m = 100\), see Figure 8.16b. Although CORPUS does predict the effect of \(S_{gr}\) in the Misawa/Schijve tests reasonably well. Figure 8.16b also indicates that the results for sequence II are overpredicted and for sequence III underpredicted. The tests indicate similar results for the two sequences, whereas CORPUS predicts systematically larger lives for sequence II. The flight profiles in Figure 8.19 confirm that the difference is associated with a different development of \(S_{op}\). In the CORPUS model, the crack opening stress level after an overload is determined by the overload and the lowest minimum stress after the overload. For load sequence type II, it implies that \(S_{op}\) is determined by \(S_{OL}\) and the minimum stress of the small cycles (\(S_{min} = 40\) MPa), whereas in type III, the crack opening stress levels are determined by \(S_{OL}\) and the ground stress which is lower than \(S_{min}\). Therefore, the crack opening stress level in load sequence type II is higher than in type III, which leads to a longer life.
Under the miniTWIST load spectrum, the predicted differences between the original, modified max-min and modified min-max sequences are not significant. This is valid for both normal ground stress level \((S_g = -0.5 \, S_{mg})\) and light ground stress level \((S_g = 0\) MPa). Under the normal ground stress, the dominant crack opening level is determined by the most severe gust and the ground stress level. In this case, the ground stress level is more compressive than the most severe downward gust load. Therefore, the sequence effect is not pronounced. Under the light ground case, the most severe compression gust is more compressive than the ground stress level. CORPUS then predicts a sequence effect, since the crack opening levels are determined by the maximum stress and the subsequent minimum stress. In the modified max-min sequence, the crack opening stress levels are determined by the most severe upward gust \(S_{max} = 195.5\) MPa and the subsequent most severe downward gust in the same flight \(S_{min} = -25.5\) MPa. However in the min-max load sequence, the crack opening stress levels are determined by the most severe upward gust \(S_{max} = 195.5\) MPa and the ground stress level \((S = 0)\). The two different \(S_{min}\) values result in a small crack opening level difference, which leads to a still fairly small different fatigue life prediction (Table 6.1).

Different flight sequences were also applied in the CN 235 tests [158]. By rearranging the flight sequence into programmed sequence II and programmed sequence III, the fatigue life became longer in the order of 1.46 and 1.3 as compared to the original sequence. The prediction results do not show any differences.

e. Crack increment in the most severe flight

Crack increments in flight A were measured during miniTWIST and CN 235 tests. Comparisons between predictions and test results are presented in Figures 8.20a - 8.20c. The figures show that the crack increments during flight A are larger in the tests than predicted. For longer crack lengths, the difference increases. This can be caused by several aspects i.e. the application of the Paris relation to calculate the crack increment, but it can also be influenced by other mechanisms such as ductile tearing, crack tip blunting and crack tip strain hardening, which are not included in the model.
f. Effect of truncation

Effects of truncation can be observed in the TWIST and CN 235 tests. Comparisons between predictions and test results are presented in Figures 8.21a - 8.21c. The trend is correctly predicted. Crack growth rates for the TWIST and the CN 235 spectra are presented in Figure 8.22.

g. Effect of omitting small load variations

The effect of omitting small cycles was examined in the miniFALSTAFF tests. The prediction results are good. Predicted life differences between FALSTAFF and miniFALSTAFF are very small, see Table 6.6.

8.7 Discussion

8.7.1 Aspects of the multiple overload effect

De Koning introduced the multiple overload effect in CORPUS because constant-amplitude tests with OL's had shown that such an effect does occur. In terms of the hump concept it may be argued that two large humps will be more effective in raising \( S_{op} \) than a single one, since reducing the height of two humps at \( S_{min} \) is more difficult. The consequences in the CORPUS predictions are of great interest.

1. As shown before (Figure 8.10) \( S_{op} \) during a flight simulation prediction increases step by step due to the multiple overload effect in agreement with the step by step increase of the load interaction factor \( m \) in Eq. (8.21). The latter equation can be rewritten as:

\[
\Delta m = \delta(m_{st} - m) \Delta a/D^d
\]  

(8.25)

If \( m \) is approaching \( m_{st} \) the \( \Delta m \) steps will become smaller. This asymptotic behavior is clearly reflected in Figure 8.10, where \( m \) was increased every time that the most severe flight occurred (type A). The \( m \)-values after each flight were:

0.0180  0.0366  0.0522  0.0655  0.0772  0.0876  0.0969

The stationary value (\( m_{st} = 0.15 \)) is still larger than the last value. Nevertheless the increasing OL interaction and the
associated increase of $S_{op}$ imply that $da/dN$ is considerably lower than the predicted value without OL interaction, see Figure 8.23. The difference increases when the crack is growing.

2. The introduction of the OL interaction has to be applied to the CA tests, where the stationary situation will be reached rather early in the life. As a result of the smaller $\Delta K_{eff}$ the C factor in the Paris relation had to be increased as discussed before (Section 8.5). For the prediction of crack growth under flight simulation loading it implies that higher crack growth rates will be predicted, as long as the OL interaction is still limited, i.e. in the beginning of the life.

The consequence of the two above aspects is that CORPUS initially predicts higher crack growth rates and later on lower crack growth rates, as compared to CORPUS prediction without the multiple OL effect, see Figure 8.23. This agrees better with the observed trends on the crack growth rate development during flight simulation loading. Apparently the multiple OL effect is sufficiently meaningful to be introduced in crack growth prediction models. The conclusion is confirmed by some comparative calculations of predictions made with and without applying the multiple OL interaction, see Table 8.2. The influence of $S_{mf}$ is reasonably well predicted with the OL interactions included. Without these effects large deviations between prediction and test result are found.

It may be noted that the PREFFAS and the ONERA model do not include a multiple OL effect. The PREFFAS model predicts the same $S_{op}$ behavior for all successive blocks of flights. For the ONERA model a slight decrease of $S_{op}$ levels was observed towards the end of the crack growth life (Section 7.8.4), contrary to the increase predicted by CORPUS.

8.7.2 The significance of the secondary plastic zones (SPZ)

In the discussion on Figure 8.10 it was pointed out that SPZ's do affect $S_{op}$ for a very short time only. This behavior was further illustrated by referring to the flight profiles in Figure 8.11. It was considered to be of some interest to obtain CORPUS predictions with
fully ignoring the creation and existence of SPZ's. Such results are presented in Table 8.3. for the F-27 spectrum (three $S_{mf}$ values) and the TWIST spectrum (4 truncation levels). A remarkably small influence on the predicted crack growth lives is found for the F-27 spectrum, in agreement with the above arguments. However, for the TWIST spectrum there is some effect of ignoring SPZ's. The SPZ's may be still more significant if the load spectrum contains many small cycles with a high mean stress. In the present data bank of flight-simulation tests such spectra do not occur. The large numbers of small cycles occur around the mean stress in flight. Because the PPZ's induce relatively high $S_{op}$ levels (see Figure 8.10) the small cycles in general can not lead to humps with a still higher $S_{op}$. In such cases the SPZ's are irrelevant indeed for $S_{op}$.

8.7.3 Effect of a severe compressive gust load

The prediction results show conservative results, if the most severe negative gust is more compressive than than the ground load. In these cases, the severe compressive gust governs the crack opening stress level within the retardation region created by the related $S_{max}$. Since no other mechanism in the model can eliminate the influence of the low gust load, the crack opening stress level is kept at a relatively low level, although the severe gust occurs only in a small number of flights, usually only once in the most severe flight. Therefore, low prediction lives are obtained in such cases. In the ONERA model the effect of the most severe negative gust load is rapidly relaxed, see $K_{min.eq}$ in Figure 7.21, but this does not occur in the CORPUS model.

8.7.4 Introduction of the rain-flow procedure

In Refs.[25, 26] De Koning did not introduce a rain-flow count procedure in the CORPUS model. Later he has introduced such a procedure, which implies that $S_{op}$ and $S_{max}$ values are considered to see whether rain-flow count situations (see Figure 6.4) do occur. If so, a rain-flow procedure can be introduced similar to the procedure described for PREFFAS in Section 6.2.2 De Koning (private communication) noticed that the effect on the prediction was very small, which is also true
for the PREFFAS model. Here some comparative predictions were made for the F-27 spectrum, see the results in Table 8.4. The results show a difference of 0 to 1% with the normal CORPUS predictions without the rain-flow procedure. All CORPUS predictions presented in this document were made without including the rain-flow procedure.

8.7.5 Effect of the yield stress

The yield stress affects the plastic zone sizes and the transition of plane strain to plane stress. It should be recognized that $\sigma_y$ also affects the multiple-overload process because in Eq.(8.25) $m$ is an inverse function of the dominant plastic zone size. Larger plastic zone sizes imply smaller $\Delta m$ values. The "relaxation" of $S_{op}$ to the stationary multiple-overload value will occur more slowly. Calculations to explore the $\sigma_y$ effect have been made for the CN 235 spectrum. The results are:

<table>
<thead>
<tr>
<th>CN 235 spectrum</th>
<th>Crack growth life ($N_{6-30}$ flights)</th>
<th>$\sigma_y = 360$ MPa</th>
<th>$\sigma_y = 340$ MPa</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{max} = 162$ MPa</td>
<td>60713</td>
<td>54591</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>$S_{max} = 200$ MPa</td>
<td>19593</td>
<td>16836</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>Trunc. at flight B</td>
<td>10570</td>
<td>9561</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>Trunc. at flight C</td>
<td>4077</td>
<td>3972</td>
<td>0.97</td>
<td></td>
</tr>
</tbody>
</table>

The table indicates a systematic effect: shorter crack growth lives for a lower yield stress. This is generally considered to be an incorrect trend because more ductile Al alloys with a lower yield stress usually give larger lives. The incorrectly predicted trend is due to the plastic zone size effect as noted above. Although this seems to be unfavorable result for the CORPUS model, it should be realized that a lower yield stress is generally related with another Paris relation with lower crack growth rates for the same $\Delta K_{eff}$. Moreover, in Eq.(8.22) for the relation factor $\delta (\delta = 0.28 \delta_1 \delta_2)$ the constant 0.28 was adopted by De Koning for 2024-T3 Alclad. The
constant may well be material dependent, and more in particular, yield stress dependent.
The present conclusion should be that the CORPUS model is not suitable for indicating the yield stress effect per se. At the same time it indicates that good predictions require relevant crack growth data for the material under consideration.

It may be recalled that De Koning approximated the fairly complex equation for the plastic zone size in 2024-T3 material (Eq. 8.11) by a somewhat less complex equation (Eq. 8.12). In order to see whether this might have a significant influence, the latter equation was also applied to same 2024-T3 predictions, see Table 8.5. Apparently the effect on the predictions is rather small.

8.7.6 Comparison between 2024-T3 Alclad and 7075-T6 Clad

According to the De Koning it is not necessary to introduce the multiple overload effect in CORPUS for predictions of results for 7075-T6 sheet material. Due to the high yield stress plastic zone sizes are small as compared to plastic zones in 2024-T3 material. As a consequence overlapping PPZ's will occur less frequently, which in the concept of CORPUS excludes multiple overload effects. For 7075-T6 De Koning also omitted the effect of $S_{\text{max}}/\sigma_{\text{yield}}$ on $S_{\text{op}}$ (Eq. 8.9). Actually it is better to say that De Koning on one hand introduced the multiple overload effect and the $S_{\text{max}}/\sigma_{\text{yield}}$ effect, because there was evidence which indicated these effects to be present in the more ductile 2024-T3 alloy. On the other hand, agreement with the empirical trends was hard to obtain, whereas for the 7075-T6 alloy it was unnecessary. A proof of the latter statement is given by the results in Table 8.6 for predictions without and with the overload interaction. The predictions with the overload interactions included are not much different from predictions without the overload interaction effect. Predictions are quite good for both cases, while the scatter of the prediction to test ratio's is somewhat larger if the overload interactions are included.
For the 2024-T3 alloy the ground stress effect in the F-27 spectrum test is poorly predicted by CORPUS as discussed before. The reason is that a rarely occurring severe negative gust overrules the effects of the ground stress level \( S_{gr} \) with respect to the dominant \( S_{op} \). However, for the 7075-T6 material such a predominant effect does not occur very long, due to small plastic zone sizes (delay switches turned off).

8.8 Summary and conclusions on the CORPUS model

The CORPUS model is an interesting model because of its relation to the physical world of crack growth under variable amplitude loading. Concepts adopted are related to crack closure, the plastic zone size, the location of the crack tip in plastic zones, the humps of plastically deformed material and the boundaries of retardation regions (delay switches) associated with the end of dominant plastic zones. Although De Koning announces his model as a "simple crack closure model" it requires a fairly thorough analysis to understand the crack growth behavior as controlled by the CORPUS model.

An interesting feature of the CORPUS model is the multiple overload (OL) interaction. It predicts that \( S_{op} \) will slowly increase during a flight simulation load history, until it asymptotically reaches a stationary value. Under constant amplitude (CA) loading, each load cycle includes an overload. The stationary \( S_{op} \) level is reached rather soon. As a consequence CORPUS predicts higher crack growth rates in the beginning and lower crack growth later on, if compared to predictions without multiple OL interactions. As a result an improved agreement is obtained for the crack growth rate development in flight simulation tests.

Another noteworthy feature of the CORPUS model is the differentiation between primary plastic zones (PPZ's, penetrating elastic material) and secondary plastic zones (SPZ's, embedded inside PPZ's). The PPZ's have a large effect on the \( S_{op} \) development. The SPZ's have a temporarily effect which is almost negligible for some load histories but not for all histories. However, by introducing SPZ's it is
recognized that there are many cycles which contribute to crack
growth, but which have a limited effect on the dominant $S_{op}$.

The CORPUS model includes a transition from plane strain to plane
stress, which affects the sizes of the plastic zones. It does not
affect the $S_{op}(R)$ function, which is still a limitation. It also
includes an effect of $S_{max}/\sigma_{yield}$ on the $S_{op}(R)$ function.
Conclusions drawn from the predictions and the comparison with test
results are summarized below.

1. The predictions in general are quite good. The ratios of prediction
crack growth life and test result are in the range between 0.5 and
2 for almost all data observed in the present study.

2. Predictions on the crack growth rate as a function of the crack
length are also reasonably good.

3. Some shortcomings were also observed.
   (a) Crack extensions in the most severe flights were
underpredicted.
   (b) The CORPUS model gives too much weight to a rarely occurring
negative load if that load is more compressive than the
frequently occurring ground stress level. The prediction is
then inaccurate but conservative.
   (c) In some test series with simple load sequences, a load sequence
effect was predicted although it did not occur in the test
series. In another test series a flight sequence effect was
observed, but it was not predicted.
   (d) CORPUS predicts an increasing crack growth rate for a lower
yield stress if the other material constants are not changed.
The latter condition is not realistic, but it indicates that
relevant CA crack growth rates are essential for good
predictions.

4. CORPUS does not apply the multiple OL effect on the 7075 alloy.
This seems to be inconsistent. If it is introduced for the 7075
alloy the effect on the predictions is rather small. It indicates
that the multiple OL effect is not so important for this alloy,
which is due to the much smaller plastic zone sizes.
CHAPTER 9
SUMMARY OF THE THREE CLOSURE MODELS AND AN IMPROVEMENT OF THE CORPUS MODEL

9.1 Introduction

Crack closure models represented by the PREFFAS model, the ONERA model and the CORPUS model were described in the three previous chapters. The characteristics and the limitations of the models were also discussed. Crack closure is now widely accepted to be the most important mechanism to explain interaction effects. Unfortunately accurate measurements of crack closure are still difficult. Moreover, for variable-amplitude loading measurements were made only for a limited number of loading types. This makes it difficult to model fatigue crack growth. In addition, fatigue crack growth is a complex process, where a number of variables are involved.

The three models predict different developments of the crack opening stress level. An illustration is given in Figure 9.1, which shows the variation of $S_{op}$ during the same flight as predicted by the three models. Differences are easily recognized. In addition, the picture for PREFFAS will not change from one block (2500 flights) to another one, whereas for CORPUS the $S_{op}$ levels will increase because of the multiple overload interactions. The three models sometimes give similar prediction of trends, but not in all cases. In general the better predictions were obtained with CORPUS. There are some shortcomings, which were found for all three models: (1) The crack extension in the most severe flight is underestimated (2) The trend of some sequence effects was incorrectly predicted. Anyhow, the analysis in the previous chapters has led to a better understanding of the crack closure behavior predicted by the models. Such an understanding is essential for an improvement of the CORPUS model proposed later in this chapter. For that purpose some highlights of the models will be summarized first.
9.2 Some important features of the three crack closure models

9.2.1 The significance of the maximum load of the load spectrum

In the PREFFAS model the maximum stress ($S_{\text{max}}$) and the minimum stress ($S_{\text{min}}$) of the load spectrum have been shown to be highly significant for the average stress level during a flight-simulation history. Crack growth lives have been calculated with the assumption that $S_{\text{op}}$ is constant during the full life, and $S_{\text{op}}$ derived from $S_{\text{max}}$ and $S_{\text{min}}$ of the load spectrum only. The predicted lives are marginally different from the true PREFFAS predictions, see Table 6.9.

$S_{\text{max}}$ and $S_{\text{min}}$ of the load spectrum are also very important for the CORPUS model because they determine the $S_{\text{op}}$ level of the dominant primary plastic zone (PPZ). However, as a result of the multiple overload (OL) interaction there is no tendency to a constant $S_{\text{op}}$ during the flight simulation history. The dominant $S_{\text{op}}$ level will slowly increase during the crack growth life, see Figure 8.10. The comparison between predicted and empirical crack rate data has shown that a much better agreement is then obtained. Moreover, the multiple OL interaction appears to be a logical consequence of the crack closure mechanism (more plastic deformation in the wake of the crack).

The ONERA model shows a different behavior, i.e. decreasing $S_{\text{op}}$-levels (see Figure 7.20.), when the crack is growing into the plastic zone. The ONERA model is presented in terms of $K_{\text{max.eq}}$ and $K_{\text{min.eq}}$ to determine $K_{\text{op}}$. However, adopting the terminology of the CORPUS model, case 4 of the ONERA model can be interpreted as the occurrence of a PPZ. There is no feature in the ONERA model which corresponds to the SPZ. An apparent difference with the CORPUS model is the above mentioned decrease of $S_{\text{op}}$. It does not occur in the CORPUS model because De Koning for reasons of simplicity adopted a block function for the $S_{\text{op}}$ level (Figure 8.2).

With respect to the model prediction of a single OL added to constant-amplitude (CA) loading, it is noteworthy that none of the three models predicts delayed retardation. They all predict an immediate maximum
retardation, contrary to fractographic observations e.g. [17,18]. It then may be questioned whether the failure to predict delayed retardation after a single OL is practically relevant to the flight simulation load histories considered in this report.

The multiple OL interaction of the CORPUS model does not occur in the ONERA model. If there are multiple overloads they have a weak effect in the ONERA model because they have a small effect on \( \alpha \) (Eq. 7.22), but \( \alpha \) actually is an empirical constant to account for the spectrum shape. It does not lead to an increase of \( S_{op} \) during the flight simulation load history.

9.2.2 The significance of the minimum load of the load spectrum

The effect of minimum stress (\( S_{min} \)) of the load spectrum was already indicated in the previous section for the PREFFAS model. The effect of \( S_{min} \) seems to be somewhat similar for the CORPUS model, but there is still an important difference. In the PREFFAS model all compressive loads are clipped to zero. As a result the PREFFAS model predicts that the effect of different compressive ground stress levels adopted in the F-27 spectrum tests is absent, whereas a significant effect is shown by the test results (Tables 6.4 and 6.5). Compressive loads are significant in the CORPUS model for determining the dominant \( S_{op} \) level. However, as pointed out in Chapter 8 a rarely occurring negative gust load can overrule the effect of less negative, but frequently occurring ground stress (\( S_{gr} \)) applications (once in each flight). As a consequence, the CORPUS model does not correctly predict the effect of \( S_{gr} \) in some cases (see Figure 8.13a).

In the ONERA model the effect of a rarely occurring, severely negative gust load is rather limited, because of the fast "relaxation" of \( K_{min,eq} \), see Figure 7.21. A fast relaxation also occurs after the application of \( S_{gr} \). However, this load is applied in each flight, and as a consequence the ONERA model does predict an \( S_{gr} \) effect, see Figure 7.9, although not very accurately. At the same time it is questionable whether the material has "forgotten" the effect of a very severe negative gust load so fast as predicted by the ONERA model.
(almost in the same flight). According to the CORPUS model the material will "remember" that severe negative gust load as long as the corresponding PPZ does exist. That memory feature seems to be unrealistic as well.

9.2.3 Sequence effect

The three models predict a sequence effect when comparing the loading types II and III used in the Misawa/Schijve tests. However the sequence effect was not found in the tests. The sequence of underload-overload in type II and overload-underload in type III do influence the prediction results. The PREFFAS and the CORPUS model predict that the type III load sequence should give a shorter life than the type II load sequence, see Figures 6.17 and 8.16. The crack opening levels after the underload are relatively low in type III, which leads to shorter lives. In the type II load sequence, the crack opening levels are relatively high after the overload due to higher minimum loads (see Figure 8.19). After the overload the preceding underload does not any longer affect \( S_{op} \). In the ONERA model, the crack opening stress level for the type II load sequence is determined in a similar way as in the PREFFAS and in the CORPUS model. However, for the type III load sequence, the ONERA model shows an increasing \( K_{\text{min.eq}} \) level (Figure 7.15) and thus also an increasing \( K_{\text{op}} \) level. Because the increase occurs rapidly, the \( K_{\text{op}} \) is not too much different from that in type II, but the \( K_{\text{op}} \) level for the overload is significantly higher. As a consequence the overload is less damaging and the crack growth life is higher than for type II. The trend is the opposite of the prediction by the PREFFAS and the CORPUS model.

9.2.4 Retardation regions

In the ONERA model and the CORPUS model the plastic zone is the region in which crack growth retardation can occur. This argument does not occur in the PREFFAS model, due to the assumption of the stationary crack length. Physically it seems unacceptable that plastic zone sizes are irrelevant in a prediction model. It might be less relevant if all PPZ's of a growing crack are overlapping , and that might occur for
"short" spectra for which the PREFFAS model was developed. Actually, for the longer spectra considered here such an overlapping does frequently occur, but it cannot be guaranteed.

A transition from plane strain to plane stress is included in the ONERA model and the CORPUS model. It is not introduced in the PREFFAS model, but that model requires empirical input data for the material and thickness for which predictions have to be made. It should be pointed out that the transition from plane strain to plane stress in the other two models affects the predicted plastic zone sizes only. It does not affect the \( S_{op} (R) \) relation. Actually that is a shortcoming. This problem is considered in strip yield models [164] where plastic restraint factors are introduced. However, it appears that the procedures are more or less data fitting procedures, rather than a rational approach.

9.2.5 Material crack growth data

An inherent feature of the cycle-by-cycle crack growth prediction models is the similarity approach. Crack growth data obtained under CA loading are applied to variable amplitude loading. The similarity implies that similar \( \Delta K_{eff} \) values will lead to similar \( \Delta a \) values. In order to computerize the calculations the most simple \( da/dN - \Delta K_{eff} \) relation is adopted in all three models, which is the Paris relation.

\[
da/dN = C \Delta K_{eff}^m \tag{9.1}
\]

where

\[
\Delta K_{eff} = U(R) \Delta K
\]
or

\[
\Delta K_{eff} = \gamma(R) K_{\text{max}}
\]

The three models handle the determination of \( U(R) \) or \( \gamma(R) \) in a different way. In the PREFFAS model \( U(R) \) must be obtained from tests on the material and the material thickness under consideration. In the ONERA model a \( \gamma(R) \) function is assumed (Eq. 7.26) based on empirical evidence, and \( C \) and \( m \) are also obtained empirically. For the CORPUS model a similar procedure is adopted, \( (\gamma(R) \) in Eqs. 8.7 and 8.8), but
there is one additional aspect. The multiple OL interaction is supposed to be applicable to the CA tests as well (each cycle is an OL and that rapidly leads to a "stationary" higher $S_{op}$). That must be taken into account when using the CA data.

Apparently the three models deal with CA data in a different way. In view of the accuracy of predictions it appears to be more important whether the data are coming from a material, that may be considered to be representative for the material, for which predictions have to be made. In this respect the "calibration tests" required for the PREFFAS model may well be recommended for any model application.

9.3 An improvement of the CORPUS model

The prediction results show that the CORPUS model gives the best results of the three models. Predicted crack growth rates are in good agreement with test results. Even a decreasing crack growth rate is indicated, as it is usually found in the transient phase of fatigue crack growth, see Figure 8.22. However, conservative predictions are obtained if the most severe gust is more compressive than the ground load, e.g. in the F-27 and miniTWIST spectra test variations. As previously mentioned the conservative prediction is influenced by the severe compressive gust which governs the crack opening level in the plastic zone region created by the maximum overload. As a consequence the underload effect of the severe negative gust is lasting very long because it is coupled to relatively large PPZ's. In most cases it implies that it will last almost during the full flight-simulation history. That appears to be unrealistic because it is thus masking ground stress level effects, which were clearly shown by test results, both in tests with the F-27 and the miniTWIST spectrum. The rapid relaxation of the underload effect predicted by the ONERA model also appears to be unrealistic. An improvement of the CORPUS model proposed below will be an "uncoupling" of underload effects from PPZ dimensions.

An inconsistency of the CORPUS model will be removed. CORPUS does not apply the multiple overload interactions to 7075-T6, although PPZ's
are recognized as crack growth retardation regions. It is then more consistent to apply the multiple overload interaction effect as well.

9.3.1 Modification of the CORPUS model

a. Underload affected zone

In addition to the overload affected zone (PPZ) an underload affected zone (ULZ) is introduced. It should be associated with compressive loads and reversed plastic deformation induced by those underloads. Because it should account for reversed plasticity not covered by the original CORPUS, the K range to be considered is \( K_{\text{op}} - K_{\text{min}} \). After the crack is closed going from \( K_{\text{max}} \to K_{\text{min}} \), the singularity at the crack tip itself is removed at the moment that \( K = K_{\text{op}} \). It requires a further downward load to close the crack tip. It then may be expected that severe downward loads will be able to induce reversed plasticity. It is assumed that this reversed plasticity in the ULZ occurs under plane strain conditions. The size of the ULZ is then approximated by the Irwin type equation:

\[
D_u = \frac{1}{9\pi} \left( \frac{K_{\text{op}} - K_{\text{min}}}{2\sigma_y} \right)^2
\]  \hspace{1cm} (9.2)

An underload affected zone can overlap with another underload affected zone. A more severe underload overrules the previous lighter underload. As a consequence, the series of underload affected zones is characterized by an increasing series of \( S_{\text{min}} \) values. The most severe underload is considered to be the dominant one. It is used to determine the \( S_{\text{op}} \) level. It should be noted that reversed plasticity considered above can affect \( S_{\text{op}} \) of later overloads.

b. Selection of crack opening level in every cycle

Two different crack opening occurrences are defined in the modified CORPUS model, i.e. (1) crack opening related to the history stress levels (\( \text{SH}_{\text{max}} \) and \( \text{SH}_{\text{min}} \)) and (2) crack opening related to the local stress levels (\( S_{\text{max}}, S_{\text{min}} \)). The history values are related to
overloads \( S_{\text{max}} \) which produce primary plastic zones and to the
dominant underload \( S_{\text{min}} \) of the underload affected zone where the
-crack tip is present. The overload history is the same as in the
CORPUS model. It is characterized by a decreasing series of \( S_{\text{max}} \)
values. As a consequence of the concept of the underload affected
zone, the \( S_{\text{max}} \) must be combined with the dominant \( S_{\text{min}} \) i.e. the
lowest \( S_{\text{min}} \) because of the maximum of reversed plasticity. This
implies that the highest \( S_{\text{max}} \) is always the dominant one, which is
combined with the lowest \( S_{\text{min}} \) to calculate \( S_{\text{op}} \) (i.e. \( S_{\text{op}} \)).
The local value is related to loads in the present cycle (i). The
-crack opening level is determined from \( S_{\text{max},i} \) and the successive
\( S_{\text{min},i} \). The local \( S_{\text{op}} \) is applied in the next cycle only, and only if
it exceeds the above \( S_{\text{op}} \). The procedure of the local \( S_{\text{op}} \) is much
similar to the concept of the secondary plastic zones of the original
CORPUS model. It is a simplification because no historic stress levels
and plastic zone sizes must be memorized. In view of the limited
significance of SPZ's in the CORPUS model (Section 8.7.2) such a
simplification can be justified.

A flow diagram with a few comments is given in Figure 9.2. Actually
the flow diagram is simpler than for the original CORPUS.

c. 7075-T6 sheet material

As pointed out before there is no obvious reason to have a different
model for the two Al-alloys, 2024-T3 and 7075-T6. In the modified
CORPUS the same model is applied to both alloys. The multiple overload
interaction relaxation factor \( \delta \) (Eq. 8.22) contains a material
constant 0.28 for 2024-T3 material. Another value will be adopted for
7075-T6 as discussed later.

9.3.2 Prediction results

The material input data are in principle the same as for the CORPUS
model. Only for the 7075-T6 Clad material, the \( C \) and \( m \) parameters are
determined by considering the stationary condition (multiple overload
interaction, Section 8.5). The parameters used in the calculation are given below.

a. 2 mm 2024-T3 Alclad material.
   \[ \sigma_y = 360 \text{ MPa} \]
   \[ C = 1.26 \times 10^{-10} \]
   \[ m = 3.7 \]
   \[ \delta = 0.28 \delta_1 \delta_2 \]

b. 2 mm 2024-T3 bare material.
   \[ \sigma_y = 394 \text{ MPa} \]
   \[ C = 2.9443 \times 10^{-11} \]
   \[ m = 4.1 \]
   \[ \delta = 0.28 \delta_1 \delta_2 \]

c. 2 mm 7075-T6 Clad material.
   \[ \sigma_y = 500 \text{ Mpa} \]
   \[ C = 2.08 \times 10^{-9} \]
   \[ m = 2.5 \]
   \[ \delta = 0.15 \delta_1 \delta_2 \]

The above C values are valid for crack growth rates in \( \text{m/cycle} \) and \( AK_{eff} \) in \( \text{MPa}\sqrt{\text{m}} \). Predicted crack propagation lives for different load spectra are presented in Tables 9.1 to 9.7, besides the basic CORPUS prediction results. The ratios between predictions and tests are presented in the same tables. A summary of the average \( N_p/N_T \) values is presented in Table 9.8 and Figure 9.3. Comparisons between tests and predictions for all data are presented in Figure 9.4.

On the average the differences between the predictions of the modified and the original CORPUS model are very limited. However, the scatter of \( N_p/N_T \) values is less for the modified CORPUS model.

<table>
<thead>
<tr>
<th></th>
<th>CORPUS</th>
<th>modified CORPUS</th>
</tr>
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<tbody>
<tr>
<td>( N_p/N_T )</td>
<td>0.33 - 1.78</td>
<td>0.44 - 1.25</td>
</tr>
<tr>
<td>mean ( N_p/N_T )</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.275</td>
<td>0.182</td>
</tr>
</tbody>
</table>
The modified CORPUS model predicts better for cases where the most severe gust is more compressive than the ground stress level, i.e. for the F-27 spectrum tests (2024-T3) for cases SL, SN, NL and LL and for the miniTWIST data with $S_{gr} = 0$. For 7075-T6 material, the CORPUS model on the average predicts slightly better than the modified CORPUS model. The trends are given below.

a. Effect of gust spectrum severity

The gust load severity effect in the F-27 tests is shown by Figures 9.5a and b for the two alloys (2024-T3 Al clad and 7075-T6 Clad). The figures show that the influence of the gust spectrum severity is qualitatively predicted by the model. A more severe gust spectrum leads to a shorter life. The figures also show that the predictions are apparently better for 2024-T3 Al clad than for 7075-T6 Clad material. The average ratios between prediction and test are ~1 and 0.75 for the two alloys respectively.

Crack growth rates for the SN, NN, and LN cases (F-27 spectrum) for both alloys are presented in Figure 9.6. For 2024-T3 Al clad, the predicted crack growth rates are in excellent agreement with the test results. For 7075-T6 Clad material, slightly faster crack growth rates are predicted for cases NN 70 and LN 70.

b. Effect of ground stress level

Different ground stress levels were applied in the miniTWIST, in the F-27 spectrum and in the Misawa/Schijve tests. The predicted trend is qualitatively correct. A lower ground stress level leads to a shorter life, see Figures 9.5 and 9.7, with load sequence type I of the Misawa/Schijve tests as an exception.

Predicted crack growth rates for different ground stress levels under normal gust severity are presented in Figures 9.8a and b for the F-27 spectrum and the two aluminum alloys. A remarkable performance is found in NL* 70 case using 7075-T6 Clad material. The crack growth rate increases until a crack length $a = 15$ mm, it then decreases until $a = 20$ mm and afterwards increases again. It is quite helpful for understanding this behavior to make a plot of the variation of $S_{op}$ during the entire life time. The plot is shown in Figure 9.9 together with the corresponding crack growth curve. In the first three blocks of 2500 flights PPZ's of the severe
flights do not overlap. The delay switches of the severe flight A is turned off before the next severe flight B is applied. When the crack becomes larger the PPZ size also increases and in the 4th block the overlapping does occur, and thus multiple overload interaction as well. The curve in Figure 9.9b shows that crack growth afterwards is more retarded. In this graph adph is also plotted, where adph is the outer boundary of the last created PPZ. In the first part of the crack growth curve the difference between a and adph is very small, which plots the two points as a single curve. After the 4th flight A adph remains larger than a and the crack tip is growing in PPZ's until the end of the life. The apparently abrupt transition from "no overlap" to "overlapping" is a consequence of the block function assumed in CORPUS for Sop in the retardation region (plastic zone, Figure 8.2). A more gradually decaying function would remove the abruptness. Nevertheless, a crack growth curve of the type shown in Figure 9.9b was observed in flight simulation tests reported in [97]. The curve is reproduced here in Figure 9.10. Apparently after 4 severe flights a consistently more effective retardation was observed. Such evidence supports the multiple overload interaction under flight simulation loading.

c. Effect of design stress level
Different design stress levels were applied in the F-27, CN 235, FALSTAFF and miniFALSTAFF tests. The effect of the design stress level on the crack growth lives is shown in Figures 9.11a to 9.11d for the F-27, CN 235 and FALSTAFF spectra. For 2024-T3 material the slopes of the curves are very much similar for the predictions and the test results (see also Table 8.1). The effect of the design stress level is apparently well predicted. For 7075-T6 a difference between the predicted slope and the empirical one is more evident, although it is slightly better than for the original CORPUS (Table 8.1).
Crack growth rates for the F-27 and FALSTAFF spectra are presented in Figures 9.12a to 9.12c. The figures show that the predictions are qualitatively good.
d. Effect of load sequence

Load sequence variations were applied in the miniTWIST and in the Misawa/Schijve tests. The modified CORPUS model does predict a negligible effect of the sequence for both test loads, see Tables 9.1, 9.7a and 9.7b, and Figure 9.7. That is in good agreement with the test results, which indicate insignificant effects only. The predictions are obviously better than for the original CORPUS model, which predicts significant differences between load sequences type II and type III of the Misawa/Schijve tests (Figure 8.16). For the miniTWIST tests with $S_{gr} = 0$ the original CORPUS model also predicts some effect of the sequence in disagreement with the test results.

A different flight sequence was applied in the CN 235 tests. The prediction results (Table 9.2) show that the original and the modified CORPUS model are unable to predict the effect of programmed flight sequences as applied in the tests. The predicted lives for the two programmed sequences are in the same order of magnitude as for the random sequence, whereas the test results showed longer lives for the programmed sequences.

e. Crack increment in the most severe flight

The crack increments in the most severe flight were measured in the miniTWIST and the CN 235 tests. The crack increments during flight A are presented in Figure 9.13. Similar as for the other models, the crack increments during flight A are significantly larger than predicted. For a longer crack length, the difference becomes larger. As mentioned in the previous chapters, underestimates during severe flights can be due to using the Paris relation. Furthermore, the prediction model does not include other mechanism such as ductile tearing, which can enhance the crack growth increment at high $K_{max}$.

f. Effect of truncation

Truncation of high loads was applied in the TWIST and in the CN 235 tests. The predicted trend is qualitatively correct, see Figure 9.14. A lower truncation level leads to a shorter predicted life. Crack growth rates for different truncation levels are presented in Figures 9.15a and b for the TWIST spectrum truncated at levels II and IV and the CN 235 spectrum truncated at the maximum load in
flight types A and C for 2024-T3 Alclad material. Both Figures show a similar trend. The predicted crack growth rates at lower truncation level (lower maximum stress) are faster than the tests results. This implies that the model predicts not enough retardation. For a higher truncation level, overestimation in the beginning and underestimation at a later stage are observed.

g. Effect of omitting small load variations

The effect of omitting small load variations can be observed by comparing results for the miniFALSTAFF spectrum and the FALSTAFF spectrum. Test results indicated that the fatigue life of both spectra are quite identical. The model predicts a similar trend as found in the test result, see Table 9.6.

9.4 Discussion

9.4.1 The underload affected zone

The introduction of the underload affected zone (ULZ) has improved the prediction results for load histories where the most severe compressive gust is more compressive than the ground stress level. Another improvement is found for the prediction of sequence effects, especially for the simple load sequences applied in the Misawa/Schijve tests.

The introduction of the ULZ implies that \( S_{op} \) for a short time will be as low as in the original CORPUS model. That will last for a certain number of flights until the delay switch of the ULZ is switched off. The number is significantly smaller than the block size of a flight spectrum. Table 9.10 presents the numbers of flights for the F-27 spectrum for case SL (severe gust spectrum, light ground stress) after the most severe flight A. For that case the original CORPUS maintains the low \( S_{op} \) during the entire life, whereas the modified CORPUS model for same 20 flights only. It is also illustrated by Figure 9.16, which shows the variation of \( S_{op} \) during one block of 2500 flights of the same flight load history. \( S_{op} \) is temporarily at the low level due to severe negative gusts, always for a relatively short period, due to the limited size of the ULZ's. Most of the time it is higher than in the original CORPUS model. That has a significant influence on crack
growth, which is further illustrated by Figure 9.17. In this figure a comparison is made between crack growth rate predictions of the original and the modified CORPUS model. The comparison is made for two cases (F-27 spectrum, cases NL and SL), for which CORPUS gave poor predictions. The poor results were previously associated with the unrealistically long lasting effect of a severe negative gust combined with a light \( S_{gr} \) (positive). Those cases were a major reason for introducing the modified CORPUS model. The graphs in Figure 9.17 clearly confirm that the CORPUS predictions in these cases are significantly too high, whereas the modified CORPUS predictions agree very well with the test results. The graphs also show that very high crack rates are not very well predicted, probably for similar reasons why \( S_{gr} \) in the most severe flight is underpredicted.

The CORPUS model predicted a significant sequence effect for the Misawa/Schijve tests (Figure 8.16), which did not occur in the tests. The discrepancy was previously associated with the different effect of the underload \( S_{gr}\) in the two sequences II and III. In the modified CORPUS model the sequence effect is negligible (Tables 9.7a and b). Due to the interaction of the ULZ and the multiple overload interaction of the overloads the same constant \( S_{op} \) is approached for both sequences.

In view of the temporary effect of the ULZ's it may be expected that the modified CORPUS model predictions will be sensitive for the size of the ULZ. Some calculations were made to explore this effect. The following equation borrowed from [176] was adopted.

\[
D_{rev} = 0.033 \left( \frac{K_{max} - K_{min}}{\sigma_y} \right)^2
\]  

(9.3)

A comparison is made in Table 9.9 between the predictions with this equation and the predictions with Eq. (9.2). The effect of the larger ULZ is very small for most F-27 spectrum variants, except for those where the negative gust is more compressive than \( S_{gr} \). This should be expected because in those cases temporarily lower \( S_{op} \) values occur, due to the ULZ effect. Table 9.10 shows that the periods with a lower
$S_{\text{op}}$, although still small, have increased considerably. In other words, for the load histories with significantly improved predictions the size of the ULZ is of prominent importance.

9.4.2 Effect of local $S_{\text{op}}$ value

As pointed out before, the crack opening stress in every cycle is the highest value of the history $S_{\text{op}}$ value and the local $S_{\text{op}}$ value. The local value of a cycle depends on $S_{\text{max}}$ and $S_{\text{min}}$ in the preceding cycle, whereas the history value depends on $S_{\text{max}}$ and $S_{\text{min}}$. The $S_{\text{min}}$ value in a simple load sequence like the F4 spectrum (Figure 4.14) or loading type I of the Misawa/Schijve tests, is equal to the ground stress level. Such load sequences are constant-amplitude loading with periodic underloads. The history value is then determined by $S_{\text{max}}$ and $S_{\text{cr}}$. This value is relatively low compared to the local value which is determined by $S_{\text{max}}$ and $S_{\text{min}}$ of the small cycles. For such a case, the $S_{\text{op}}$ is determined by the local value. In flight simulation load histories, a similar condition can occur, but they will be less important for similar reasons as for the limited significance of secondary plastic zones in the CORPUS model. However, the prediction of constant-amplitude loading with periodic underloads would be rather poor if the local $S_{\text{op}}$ values are disregarded.

9.4.3 Predictions for 7075-T6 material

In the modified CORPUS model the overload interaction is also applied to 7075-T6. As a consequence, the constant-amplitude input data have to be corrected, see Section 8.6. The $C$ and $m$ parameters are $2.08 \times 10^{-9}$ and 2.5 respectively. Without considering the correction, the $C$ value is $1.6 \times 10^{-9}$, while $m$ is the same.

In the relaxation factor (Eq.8.22) the constant 0.28 should be adjusted to account for the material effect. The constant used for the predictions for 2024-T3 is 0.28 as proposed by De Koning. For 7075-T6 calculations were made with several values. The predictions for $\delta = 0.15 \delta_1 \delta_2$ and $\delta = 0.28 \delta_1 \delta_2$ are compared in Table 9.11 (F-27 spectrum). The results for the constant 0.15 are better, especially the standard deviation is significantly smaller. The constant 0.15 was
then adopted for the predictions with the modified CORPUS model for 7075-T6 material.

The effects of the gust severity and the ground stress for 7075-T6 material (F-27 spectrum) is shown in Figure 9.5b. A comparison with the predictions by the original CORPUS model (Figure 8.13 and Table 9.4) shows a significant improvement for the results of the $S_{gr}$ effect. However, for the light gust spectrum the modified CORPUS model predicts about 25% too low, where the CORPUS model was fairly accurate.

In Section 9.3.2 the transition from non-overlapping plastic zones to overlapping plastic zones was already discussed for 7075-T6 material. The increasing retardation at a larger crack length can be observed in several $da/dN - a$ graphs (Figures 9.6 and 9.8) both in the test results and the predictions. For the case NL (Figure 9.8b, lower right hand graph) this effect is overemphasized in the predictions. However, in general the results indicate that a multiple overload interaction has to be included for 7075-T6 material.

9.5 Summary and conclusions on the modified CORPUS model

The modified CORPUS model is, to a large extent, similar to the CORPUS model. The multiple overload interaction is maintained as an important feature, which can lead to an increasing $S_{op}$ in the ductile 2024-T3 alloy during the crack growth life. As a consequence a good agreement between predicted crack growth rates and test results can be obtained. Also the dependence of $S_{op}$ on $S_{max}$, R and $\sigma_{yield}$, as well as the equations for calculating plastic zone sizes and the transition from plane strain to plane stress, are similar. The most important difference between CORPUS and modified CORPUS is the introduction of the underload affected zone (ULZ) introduced in the modified CORPUS model. That has a significant effect on the memory of the material for effects of severe downward loads. Another difference is related to the effect of secondary plastic zone, which leads to a somewhat simpler algorithm for the modified CORPUS model without a significant changing of the basic idea of this type of zones. Furthermore, the different
CORPUS rules for the more ductile 2024-T3 alloy and the low ductility 7075-T6 alloy have been dropped. Physically there is no reason why a single model can not apply to both types of alloys.

The prediction results of the modified CORPUS model are summarized below.

1. On the average the predictions are fairly accurate. That is also true for the CORPUS model. However, the scatter of \( N_P/N_T \) values is significantly lower for the modified CORPUS (standard deviation 0.182 for modified CORPUS, 0.275 for CORPUS). The improvement is mainly due to better predictions for cases related directly or indirectly (sequence effects) to effects of severe downward loads.

2. Empirical trends are correctly predicted for the effects of (1) design stress level, (2) truncation of high loads, (3) omission of small cycles, (4) different ground stress levels and (5) sequence effects. The original CORPUS correctly predicts the first three trends, but not the last two trends.

3. The crack extension in the most severe flight is poorly predicted. That applies to the CORPUS, PREFFAS and ONERA models as well.

4. The application of the multiple overload interaction to the 7075-T6 alloy appears to be justified.
CHAPTER 10
SUMMARY AND CONCLUDING REMARKS

10.1 Introduction

The main theme of the present study is the evaluation of existing fatigue crack growth prediction models, and the aim to come up with an improved model if possible. The background of the problem setting is the need for crack growth prediction methods to be used in the aircraft industry. Predictions have to be made as part of the aircraft design process and later for aircraft certification purposes. It can be most helpful if predictions can indicate the trend of the effects of design variables. However, to be really meaningful, predictions should quantitatively have a reasonable accuracy. The non-interaction predictions are unreliable in both a qualitative and a quantitative sense. Non-interaction predictions on spectrum variations can predict longer lives, where shorter lives are found in experiments. Quantitatively, non-interaction predictions can lead to gross underestimates of the crack growth life. There is indeed a good case for developing a reliable fatigue crack growth prediction model. As pointed out in Chapter 5 the cycle-by-cycle crack closure models are candidates to meet the requirements. The PREFFAS model, the ONERA model and the CORPUS model are considered to be potential candidates. A review of the literature learned: (1) the originators of the models claim accurate predictions, (2) each model was checked almost exclusively by its own originators only, and (3) it was not easy to understand how $S_{op}$ according to the prediction models varied from cycle to cycle under realistic flight-simulation load histories. The last aspect implies that understanding the operation of a model becomes problematic. Moreover, detailed comparisons between predicted crack growth rates and empirical growth rates were seldom presented. A critical analysis of the model conceptions and extensive comparison of the models were not found in the literature. The major goal of the present investigation is to fill this gap and to improve the understanding about how the models are working. As said in Chapter 5,
there is a risk that the models are understood by a few people only, whereas the models remain a "black box" for people who want to use it. It is thought that the understanding would be served by presenting flight profiles with indications of $S_{op}$ in every cycle. Such profiles are almost completely absent in the literature. If computer programs are written for a cycle-by-cycle prediction model it is relatively simple to extend the program for showing such profiles. All calculations and flight profiles shown in the present thesis were obtained on personal computers, with programs written in turbobasic. Another goal of the present investigation is to see whether an improved understanding of the crack closure models can lead to the concept of an improved prediction model. As a result a modified CORPUS model has been postulated.

The present investigation is summarized below by surveying the coverage first and a recapitulation of the major findings afterwards.

10.2 Coverage of the investigation

Crack closure models have been considered only. The older yield zone models are thought to be physically unrealistic. The recent strip yield models still require an extensive computer capacity. Moreover, there are still some fundamental conception problems involved (Chapter 5).

Results of flight-simulation fatigue tests have been collected to check the reliability of the prediction models. The flight-simulation load histories are restricted to load spectra for aircraft wings. It should be recognized that such load histories are characterized by random sequences of different flights and random sequences of loads in each flight. The variability of load amplitudes is large. The load spectra considered are: the F-27 spectrum (Fokker Friendship), the CN-235 spectrum (Indonesian Spanish transport aircraft), the TWIST spectrum (standardized spectrum for transport aircraft wings), and the FALSTAFF spectrum (standardized spectrum for fighter aircraft wings). Shorter versions of the last two spectra were also included (miniTWIST and miniFALSTAFF). Each spectrum is characterized by a block of
flights (2500, 1000, 4000 and 200 flights for the four spectra respectively). After a block has been completed the same block of flights is repeated. Finally highly simplified flight simulations have been considered also. They can be described as constant-amplitude load cycles with periodic overloads and/or underloads. In this type of load histories all flights have the same flight load profile. Such sequences were adopted because of their fundamental significance for overload/underload and underload/overload combinations. In order to understand the predictions and test results the flight-simulation load histories under the above spectra are described in detail (Chapter 3).

The test results were collected from the literature. New flight-simulation tests were carried out to extend the data bank. The CN-235 and the miniTWIST spectrum have been used in these tests. In the miniTWIST experiments different sequences of severe upwards and downwards gusts were introduced because no data on this issue are available in the literature. The more important variables of the flight-simulation tests are (1) the design stress level, (2) the severity of gust spectra, (3) the ground stress level, (4) the truncation level, (5) the omission of small cycles, (6) the sequence of flight loads and (7) the sequence of flight types. The majority of tests was carried out on 2024-T3 sheet material and another part on 7075-T6 sheet material. The databank is described in Chapter 4.

10.3 The three crack growth models (Chapters 6 to 8)

1. Several trends are qualitatively predicted by all three models. This applies to the effect of the design stress level, the severity of gust spectra, the truncation of high loads, and the omission of small cycles. The quantitative accuracy is better for the CORPUS model.

2. The effect of the most severe negative loads of the load spectrum is not well accounted for by any of the three models. In the PREPPAS model compressive loads are clipped to zero. It excludes any effect of negative ground stresses, which does not agree with the empirical results. In the ONERA model severe downward loads have a significant effect in reducing $S_{op}$ levels. However, the
effect is decaying very rapidly. Within one flight it has practically vanished. Such a "short material memory" seems to be unrealistic. In the CORBUS model a severe negative gust load has a long lasting effect if it is more compressive than the ground stress. Due to overlapping plastic zones the effect can remain during the entire crack growth life. Such a long lasting memory also seems to be unrealistic. In such cases the CORBUS predictions are poor (although conservative).

3. The sequence effect in the simple flight-simulation is poorly predicted by all three models, i.e. the models predict significantly different results for tests with underload/overload cycles and tests with overload/underload cycles, whereas the tests indicated small and unsystematic differences. Moreover the ONERA model predicted a different sequence sensitivity than the other two models. For the ONERA model the misprediction is associated with the rapidly vanishing effect of underloads. For the other two models the misprediction is due to the predominant effect of the overload, which eliminate the underload effect for the underload/overload combination and not for the overload/underload combination.

4. A multiple overload interaction is present in the CORBUS model only. This interaction can lead to increasing $S_{op}$ values from block to block, i.e. increasing during the crack growth life time. As a result the more detailed comparison between predicted and measured crack growth rates is significantly better for the CORBUS model.

5. The rainflow procedure must be applied in the PREFFAS model. It was unexpected that omission of the rainflow procedure predicted a rather small or negligible increase of the crack growth life. However, where it is realized that the rainflow procedure must be applied on the history of successive $S_{op}, S_{max}$ sequences (instead of $S_{min}, S_{max}$ sequences) it is easily understood that the effect is small, because in most cycles $S_{op}$ is considerable higher than $S_{min}$. The rainflow procedure has been applied to a number of CORBUS predictions and the same minor effect is observed.

6. The PREFFAS model is the most simple model which also requires considerably less computer time. However, computer time is not a significant criterion for the quality of the model since the
computer time for the other models is far from excessive. The PREFFAS model predicts a constant $S_{op}$ during almost all cycles. That dominant $S_{op}$ level is a function of the maximum and the minimum stress of the load spectrum. Adopting that constant level for all cycles leads to a further simplification and only about 1% shorter predicted crack growth lives.

7. All three models fail to predict the crack extension in the most severe flight of the load history. That crack extension is significantly underestimated. It can partly be attributed to adopting the Paris relation for describing the material crack growth resistance under constant amplitude loading. At the same time it is realized that other crack growth mechanisms may occur at high $K_{max}$ values (e.g. ductile tearing).

8. The thickness effect and the plane strain/plane stress transition are covered in different ways in the three models. In the PREFFAS model they are not covered at all, but there is a requirement for "calibration tests" with constant-amplitude loading ($R = 0.1$) and the same loading with periodic overloads on the material with the thickness for which predictions have to be made. In the ONERA model and the CORPUS model both aspects are included. The thickness effect in the ONERA model, although predicting the correct trend, is primarily obtained by the influence on the vanishing memory for the effect of downward loads. That seems to be an incorrect concept. In the ONERA and the CORPUS model the transition from plane strain to plane stress is reflected in the calculation of plastic sizes. In that way it does affect crack growth retardation. However, it does not affect the crack opening functions, which remains a difficult issue.

9. Indications are obtained that the predictions are sensitive to the constant-amplitude data adopted for the prediction. The quality of the basic material crack growth data is an essential link to obtain satisfactory predictions for variable-amplitude load sequences. For practical applications some "calibration tests" seem to be highly advisable.
10.4 The modified CORPUS model (Chapter 9)

10. The analysis of the three models has learned that the effect of severe downwards loads is a weak point of these models. That was the starting point for the concept of the "underload effected zone" introduced in the modified CORPUS model. Shortcomings of the predictions summarized in conclusions 2 and 3 have been removed by this concept. The effects of the ground stress and the sequence effect are now correctly predicted. In most cases the modified CORPUS predictions are not much different from the original CORPUS predictions, except in those cases were the CORPUS predictions are poor. As a result the average ratio of predicted crack growth life to test life (83 test series) is practically the same (0.89 for CORPUS, 0.87 for modified CORPUS), but the larger deviations are much smaller for the modified CORPUS model (standard deviation 0.275 for CORPUS 0.182 for modified CORPUS).

11. Two additional features of the modified CORPUS model are somewhat different from the original model, i.e. a simpler treatment of secondary plastic zones and a similar application to 2024-T3 and 7075-T6, which are different for the CORPUS model.

12. The incorrect prediction of crack extension is the most severe flight applies to the modified CORPUS model as well.

10.5 Closing remarks

The accuracy obtained with the modified CORPUS model seems to be quite acceptable for practical applications. The investigation at the same time has revealed several points were further study is desirable. That applies to the crack extension in the severe flight. There is more however. The retardation function is CORPUS was assumed to be a block function, which is a rather drastically simplifying assumption. More appropriate functions can be meaningful for prediction models. The effect of sheet thickness was not explicitly investigated in the present study and that still should be done. It may be pointed out that the relatively ductile 2024-T3 alloy is a "difficult" material for prediction models, especially if the sheet thickness is low. Interaction effects can be most significant then. Another limitation
of the present investigation is that the data bank was limited to results of central cracked specimens. This is the only type of specimens for which so many data are available. For practical applications prediction studies on stringer stiffened skins (locally decreasing $K$ values) are a most relevant topic.
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Appendix A

Calculation of the sequence efficiency of the PREFFAS model

The sequence efficiency EF is the summation of the effective stress ranges of all load cycles in one block of load spectrum. This was expressed in Eq. (6.17)

\[
EF = \sum_{i=1}^{n} \Delta S_{eff,i}^m
\]  

(6.17)

The effective stress range is calculated cycle-by-cycle.

\[
\Delta S_{eff,i}^m = (S_{max,i}^m - SH_{op,r}^m)^m
\]  

(A.1)

If \( S_{max,i}^m \) is smaller than \( SH_{op,r}^m \), then \( \Delta S_{eff,i}^m \) is set equal to zero. If \( S_{max,i}^m \) is larger than \( SH_{max,r}^m \), then the rain-flow effect is included in the \( \Delta S_{eff,i}^m \) calculation.

\[
(\Delta S_{eff,i}^m)^m = (S_{max,i}^m - SH_{op,(r-\ell)}^m)^m + \sum_{j=r-\ell}^{r} (SH_{max,j}^m - SH_{op,j}^m)^m 
- \sum_{j=r-\ell}^{r} (SH_{max,j}^m - SH_{op,j-1}^m)^m
\]  

(A.2)

where \( \ell \) is the number of \( SH_{max}^m \)-values exceeded by \( S_{max,i}^m \)

After calculation of \( \Delta S_{eff,i}^m \), the history values are checked against several cases based on requirements in Eqs. (6.7a) to (6.7c). Resetting of the history values is applied for cases 1, 2 and 3.

Case 1 (see Figure A.1):

\( S_{max,i}^m > SH_{max,r}^m \)

\( \ell \) pairs of \( SH_{max,r}^m \) and \( SH_{min}^m \) levels are overruled.

Resetting: \( p = p - \ell \)

\[
SH_{max,r}^m = S_{max,i}^m
\]

\[
SH_{min,r}^m = S_{min,i}^m
\]

\[
SH_{op,r}^m = S_{op,i}
\]
Case 2 (see Figure A.2): \[ S_{\text{min},i} < S_{\text{min},r} \]

(l-1) \( S_{\text{max}} \) levels and \( l \) \( S_{\text{min}} \) levels are overruled.

Resetting: \( r = r - (l - 1) \)

- \( S_{\text{max},r} \) unchanged
- \( S_{\text{min},r} = S_{\text{min},i} \)
- \( S_{\text{op},r} \) to be calculated from \( S_{\text{max},r} \) and \( S_{\text{min},i} \)

Case 3 (see Figure A.3): \[ S_{\text{op},i} > S_{\text{op},r} \]

One set of new history value is added.

Resetting: \( r = r + 1 \)

- \( S_{\text{max},r} = S_{\text{max},i} \)
- \( S_{\text{min},r} = S_{\text{min},i} \)
- \( S_{\text{op},r} = S_{\text{op},i} \)

A flow diagram of the sequence efficiency calculation is given in Figure A.4.
Three cases which modify the series of history stress levels.
Figure A.4: Flow diagram of the PREFFAS model.
Appendix B

The flow diagram of the CORPUS model

The CORPUS model was described and analysed in Chapter 8. The model is the most complicated one if compared to the PREPFAAS model and the ONERA model. This will easily be recognized if the flow diagram in Figure B1 is compared to the flow diagram if the ONERA model in Figure 7.7. Several comments will be made below to further explain the diagram. Figure 8.7 is helpful to follow the flow diagram and the comments below.

PPZ = primary plastic zone (1), N1 = number of PPZ's
SPZ = secondary plastic zone (2), N2 = number of SPZ's.

In the cycle-by-cycle calculation aspects of PPZ's are considered first, followed by aspects of SPZ. In the flow diagram several steps can be recognized.

1. Read cycle $i(S_{\text{max},i}, S_{\text{min},i})$ and calculate its $S_{\text{op},i}$.
2. If $S_{\text{max}} < S_{\text{op},i-1}$ the crack is not opened: $\Delta a = 0$. Consequences of a low $S_{\text{min},i}$ must be considered (go to B).
3. Calculate $\Delta a$ with the Paris relation. $\Delta S_{\text{eff}} = S_{\text{max},i} - S_{\text{op},i-1}$.
4. Check if life is completed. Criterion $a = a_{\text{max}}$ or $K_{\text{max},i} > K_{\text{IC}}$. The first criterion is adopted in this report.
5. Calculate the plastic zone size. $D_{\text{SS}}$ is the plane stress plastic zone size. The ratio $D/D_{\text{SS}}$ indicates how far the plane stress situation has been approached. The ratio is used to calculate the multiple overload factor $\delta_2$ (Eq. 8.24).

Consider aspects of PPZ's

6. If $a_i > \text{ADPH (1)}$ a delay switch of a PPZ must be switched off. Memory values of remaining PPZ's must be renumbered.
7. If $S_{\text{max},i}$ exceeds a SH (1) value a new PPZ will be created and an old one will be eliminated. All SPZ's should be erased.
8. If $\text{ADP}_i (= a_i + d_p_i$, see Fig. 8.5) exceeds the limit of the last created PPZ a new PPZ will be created (penetration of plasticity in elastic material). The overload interaction parameter $m$ must be
calculated, which will effect subsequent $S_{op}$ levels. The highest $S_{op}$ of the PPZ's must be selected. If in the previous step ($S_{max,i} > SH_{max}$ (N1), comment 5) a new PPZ was created the present inequality (ADP$_i > ADPH$ (N1)) will be automatically satisfied and the corresponding calculations also be made.

9. If $S_{min,i}$ is lower than an $SH_{min}$ value a resetting of $SH_{min}$ and $SH_{op}$ values is necessary. That is also necessary for the SPZ's, however it will lead to $S_{op}$ values of the SPZ's which are lower than the $SH_{op}$ of the PPZ's. As a consequence they are no longer relevant for the selection of the highest $S_{op}$ and can thus be erased for the memory.

10. If a new PPZ has been created there are no SPZ's to be considered any further. Thus goto A for next cycle.

Consider aspects of SPZ's

11. Here N2=0 is possible only if a PPZ was formed in the previous cycle and not in the present cycle. As a consequence N2=0 implies that a new SPZ will be created.

12. Check delay switches of SPZ's.

13. If $S_{max,i} > SH_{max}$ (2) one or more SPZ's will be overruled and can be erased from the memory.

14. If $S_{min,i} < SH_{min}$ (2) then resetting of $SH_{min}$ (2) and reduction of $SH_{op}$ (2) must be done.

15. A new SPZ will be present if its $S_{op}$ is larger than the maximum $S_{op}$ of the PPZ.

16. The last step decides on the valid $S_{op}$ for the next cycle, which is the highest $S_{op}$ of PPZ's or SPZ's if present.

17. Goto next cycle.
Figure B-1a: Flow diagram of the CORPUS model.
Figure B-1b: Flow diagram of the CORPUS model.
Figure B-1c: Flow diagram of the CORPUS model.
Table 2.1: The effect of material thickness on crack growth in flight-simulation tests [99].

<table>
<thead>
<tr>
<th>spectrum</th>
<th>material</th>
<th>thickness ( t (\text{mm}) )</th>
<th>crack growth interval ( a_1 + a_2 (\text{mm}) )</th>
<th>( S_{\text{max}} ) of PST (MPa)</th>
<th>life ratio's (*)</th>
<th>comments</th>
<th>CA tests</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>gusts</td>
<td>TWIST</td>
<td>7975-T6</td>
<td>2.8 9.3</td>
<td>10 20</td>
<td>161</td>
<td>1.2</td>
<td>1.1 *</td>
<td>'t Hart et al. [102]</td>
</tr>
<tr>
<td></td>
<td>TWIST</td>
<td>2024-T3, 7075-T6</td>
<td>2 10</td>
<td>10 20</td>
<td>126.5</td>
<td>6.2</td>
<td>5.3  3.3</td>
<td>Schijve et al. [97]</td>
</tr>
<tr>
<td></td>
<td>TWIST</td>
<td>2024-T3, 7075-T6</td>
<td>1.6 3.1</td>
<td>6 * failure ( W = 160 )</td>
<td>182</td>
<td>1.46 (I)</td>
<td>1.35 (III)</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>TWIST</td>
<td>7075-T7351, 7075-T73651, 7010-T7651, 7010-T73651</td>
<td>1.4 2.4 1.8 2.8</td>
<td>2 5 10 15</td>
<td>8 * 30</td>
<td>126.5</td>
<td>1.2  3.2  1.5  2.4</td>
<td>Wanhill [103]</td>
</tr>
<tr>
<td>mini</td>
<td>TWIST</td>
<td>2024-T3, 2324-T39</td>
<td>2 10</td>
<td>4 * failure ( W = 100 )</td>
<td>161</td>
<td>1.4</td>
<td>3.9  1.7  1.7</td>
<td>Wanhill et al. [104, 105]</td>
</tr>
<tr>
<td>F27</td>
<td>TWIST</td>
<td>2024-T3</td>
<td>2 4</td>
<td>3.5 * failure ( W = 160 )</td>
<td>211</td>
<td>1.7</td>
<td>1.3  1.3  1.3</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>F27</td>
<td>7075-T7351, 71-6A1-HV</td>
<td>2.5 6.9</td>
<td>3.5 * failure ( W = 160 )</td>
<td>239</td>
<td>1.7</td>
<td>yes  Van der Linden/Nedervooren [107]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F4</td>
<td>7075-T6</td>
<td>2 3.8 6.4 12.7 1 0.5</td>
<td>2.5 * failure ( W = 160 )</td>
<td>211</td>
<td>1.7</td>
<td>yes  Sippel/Weisgerber [108]</td>
<td></td>
</tr>
<tr>
<td>FALSTAFF</td>
<td>103 (-2014)</td>
<td>3 6</td>
<td>10 * failure ( W = 100 )</td>
<td>77 1.5</td>
<td>yes  Sharp/Masterton [110]</td>
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<td>2024-T3</td>
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<td>5 * failure ( W = 160 )</td>
<td>156</td>
<td>2.8</td>
<td>1.7</td>
<td>yes  Palmberg [111]</td>
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</tr>
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</table>

\(*\) life ratio = \( \frac{\text{crack growth life for } t_{\text{min}}}{\text{crack growth life for } t_{\text{max}}} \)
<table>
<thead>
<tr>
<th>Predominate flight loads</th>
<th>Load-time history</th>
<th>Block size (flights)</th>
<th>Number of flight types</th>
<th>Flight cycles in 1 block</th>
<th>Average number of cycles/flight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gusts</td>
<td>TWIST</td>
<td>4000</td>
<td>10</td>
<td>39,8665</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>miniTWIST</td>
<td>4000</td>
<td>10</td>
<td>58,442</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>F-27</td>
<td>2500</td>
<td>9</td>
<td>27,907</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>CN-235</td>
<td>1000</td>
<td>10</td>
<td>19,805</td>
<td>20</td>
</tr>
<tr>
<td>Manoeuvres</td>
<td>PALSTAFF</td>
<td>200</td>
<td>(b)</td>
<td>17,983</td>
<td>90</td>
</tr>
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<td></td>
<td>miniPALSTAFF</td>
<td>200</td>
<td></td>
<td>9006</td>
<td>45</td>
</tr>
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</table>

(a) excluding taxicycles
(b) all flights are different

Table 3.1 Survey of flight-simulation load-time histories
<table>
<thead>
<tr>
<th>Flight type</th>
<th>Number of flights in one block</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>Total number of cycles per flight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>18</td>
<td>64</td>
<td>112</td>
<td>391</td>
<td>900</td>
<td>(0)</td>
<td>1500 (600)</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>11</td>
<td>39</td>
<td>76</td>
<td>366</td>
<td>899</td>
<td>(0)</td>
<td>1400 (520)</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>22</td>
<td>61</td>
<td>44</td>
<td>208</td>
<td>680</td>
<td>(0)</td>
<td>1250 (380)</td>
</tr>
<tr>
<td>D</td>
<td>24</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>14</td>
<td>24</td>
<td>165</td>
<td>603</td>
<td>603</td>
<td>(0)</td>
<td>800 (200)</td>
</tr>
<tr>
<td>E</td>
<td>60</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>24</td>
<td>19</td>
<td>115</td>
<td>512</td>
<td>650</td>
<td>(0)</td>
<td>650 (130)</td>
</tr>
<tr>
<td>F</td>
<td>181</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>16</td>
<td>70</td>
<td>412</td>
<td>490</td>
<td>(80)</td>
<td>250 (40)</td>
</tr>
<tr>
<td>G</td>
<td>420</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>16</td>
<td>70</td>
<td>412</td>
<td>490</td>
<td>(80)</td>
<td>250 (40)</td>
</tr>
<tr>
<td>H</td>
<td>1,090</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>26</td>
<td>78</td>
<td>230</td>
<td>1030</td>
<td>5200</td>
<td>40000</td>
<td></td>
<td>398665 (58442)</td>
</tr>
<tr>
<td>I</td>
<td>2,211</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>26</td>
<td>78</td>
<td>230</td>
<td>1030</td>
<td>5200</td>
<td>40000</td>
<td></td>
<td>398665 (58442)</td>
</tr>
<tr>
<td>J</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$S_a/S_m = -0.50$

Number of cycles for miniTWIST are given in brackets.

Average number of cycles per flight = 100 (15)

Table 3.2 TWIST flight-simulation load spectrum.
<table>
<thead>
<tr>
<th>Flight type code</th>
<th>Number of flights in one block</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>Total number of cycles per flight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>28</td>
<td>47</td>
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<td>27</td>
<td>44</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>25</td>
<td>40</td>
<td>23</td>
<td>36</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>24</td>
<td>32</td>
<td>20</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>E</td>
<td>11</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>18</td>
<td>22</td>
<td>14</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>F</td>
<td>27</td>
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<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>12</td>
<td>14</td>
<td>9</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>G</td>
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<td>1</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>13</td>
<td>14</td>
<td>9</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>H</td>
<td>184</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>17</td>
<td>14</td>
<td>9</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>I</td>
<td>2200</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>17</td>
<td>14</td>
<td>9</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>cycles per block of 2500 flights</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>15</td>
<td>46</td>
<td>114</td>
<td>364</td>
<td>2728</td>
<td>24630</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cumulative number of cycles</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>25</td>
<td>71</td>
<td>185</td>
<td>549</td>
<td>3277</td>
<td>27907</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average number of cycles per flight = 11
$S_{gr}/S_{mf} = -0.234$ for 2024-T3 and $S_{gr}/S_{mf} = -0.254$ for 7075-T6

Table 3.3: F-27 flight-simulation load spectrum (Case: normal gusts, normal $S_{gr}$).
<table>
<thead>
<tr>
<th>level number of maxima</th>
<th>level number of maxima</th>
<th>level number of minima</th>
<th>level number of minima</th>
</tr>
</thead>
<tbody>
<tr>
<td>128 1</td>
<td>82 1398</td>
<td>1 1</td>
<td>48 1181</td>
</tr>
<tr>
<td>117 6</td>
<td>81 261</td>
<td>12 2</td>
<td>49 3522</td>
</tr>
<tr>
<td>113 4</td>
<td>80 3600</td>
<td>18 4</td>
<td>50 40</td>
</tr>
<tr>
<td>111 4</td>
<td>78 2504</td>
<td>20 3</td>
<td>51 1440</td>
</tr>
<tr>
<td>105 5</td>
<td>77 158</td>
<td>25 1</td>
<td>52 210</td>
</tr>
<tr>
<td>103 14</td>
<td>75 1805</td>
<td>26 14</td>
<td>54 1</td>
</tr>
<tr>
<td>100 6</td>
<td>74 6</td>
<td>28 9</td>
<td>55 1</td>
</tr>
<tr>
<td>98 23</td>
<td>73 84</td>
<td>31 23</td>
<td>56 15</td>
</tr>
<tr>
<td>97 3</td>
<td>72 22</td>
<td>33 17</td>
<td>57 25</td>
</tr>
<tr>
<td>96 29</td>
<td>71 273</td>
<td>36 82</td>
<td>58 514</td>
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<tr>
<td>95 1</td>
<td>70 93</td>
<td>37 39</td>
<td>59 6</td>
</tr>
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<td>93 90</td>
<td>69 2320</td>
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</tr>
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<td>92 12</td>
<td>68 157</td>
<td>39 6</td>
<td>61 307</td>
</tr>
<tr>
<td>90 275</td>
<td>67 983</td>
<td>41 431</td>
<td>62 3070</td>
</tr>
<tr>
<td>88 423</td>
<td>66 3</td>
<td>42 1038</td>
<td>63 192</td>
</tr>
<tr>
<td>87 81</td>
<td>64 1137</td>
<td>43 275</td>
<td>64 2144</td>
</tr>
<tr>
<td>86 3</td>
<td>63 250</td>
<td>44 285</td>
<td>67 41</td>
</tr>
<tr>
<td>85 865</td>
<td>62 563</td>
<td>45 12</td>
<td></td>
</tr>
<tr>
<td>84 30</td>
<td>55 927</td>
<td>46 1427</td>
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</tr>
<tr>
<td>83 1314</td>
<td>54 73</td>
<td>47 1799</td>
<td></td>
</tr>
</tbody>
</table>

(a) Level 128 is $S_{\text{max}}$ of spectrum

Level 1 corresponds to $-0.1728 S_{\text{max}}$

Level x corresponds to $[0.009235 \times (x-1) -0.1728] S_{\text{max}}$

$S=0$ corresponds to level 19.7, $S_{\text{mf}}$ corresponds to level 63.76

Table 3.4: CN-235 flight-simulation load spectrum for one block of 1000 flights.
<table>
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<tr>
<th>level (a)</th>
<th>numbers</th>
<th>cumulative numbers</th>
<th>numbers</th>
<th>cumulative numbers</th>
</tr>
</thead>
<tbody>
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<td>minima</td>
<td>maxima</td>
<td>minima</td>
</tr>
<tr>
<td>32</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>31</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>24</td>
<td>193</td>
<td>2</td>
<td>183</td>
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<td>23</td>
<td>233</td>
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<td>213</td>
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<td>22</td>
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<td>21</td>
<td>533</td>
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<td>484</td>
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<td>23</td>
<td>540</td>
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<td>37</td>
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<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Level 32 corresponds to $S_{max}$ of spectrum, level 1 corresponds to $-0.2667 S_{max}$

S=0 corresponds to level 7.5269

Level x corresponds to $[0.04086 (x-1) -0.2667] S_{max}$

Table 3.5: FALSTAFF and miniFALSTAFF flight-simulation load spectrum.

Load levels and number of peaks in one block of flights.
Table 4.1: Survey of tests and tests variables applied in miniTWIST and CN 235 flight simulation tests.

Material 2024-T3 Alclad (2 mm), $S_{mf} = 85$ MPa

<table>
<thead>
<tr>
<th>miniTWIST spectrum</th>
<th>Normal ground stress ($S_{gr} = -0.50 S_{mf}$)</th>
<th>Light ground stress ($S_{gr} = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{6+30}$ (flights) ratio</td>
<td>$N_{12+30}$ (flights) ratio</td>
</tr>
<tr>
<td>Original</td>
<td>12700 1</td>
<td>6300 1</td>
</tr>
<tr>
<td>Modified max-min</td>
<td>12625 0.99</td>
<td>6350 1.01</td>
</tr>
<tr>
<td>Modified min-max</td>
<td>14560 1.25</td>
<td>7400 1.17</td>
</tr>
</tbody>
</table>

$N_{6+30} = $ crack growth life from $a = 6$ mm to $a = 30$ mm

Table 4.2: Crack growth life in miniTWIST flight-simulation tests. Effects of rearranged severe gusts and ground stress level.
Table 4.3: Crack growth life in CN-235 flight-simulation tests. Effects of stress level, truncation level and flight sequence.
<table>
<thead>
<tr>
<th>spectrum</th>
<th>material (*)</th>
<th>width of specimen (mm)</th>
<th>test variables</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-27</td>
<td>2024-T3 Alclad</td>
<td>160</td>
<td>12 combinations with different</td>
<td>[150]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- gust load severities</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- ground load levels</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- design stress levels</td>
<td></td>
</tr>
<tr>
<td>P 4</td>
<td>2024 T3 Alclad</td>
<td>160</td>
<td>1 type of load</td>
<td>[150]</td>
</tr>
<tr>
<td>$S_{mf}=91$ MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-27</td>
<td>7075-T6 Clad</td>
<td>160</td>
<td>13 combinations with different</td>
<td>[160]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- gust load severities</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- ground load levels</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- design stress levels</td>
<td></td>
</tr>
<tr>
<td>P 4</td>
<td>7075-T6 Clad</td>
<td>160</td>
<td>1 type of load</td>
<td>[160]</td>
</tr>
<tr>
<td>$S_{mf}=63.7$ MPa</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TWIST</td>
<td>2024-T3 Alclad</td>
<td>100</td>
<td>4 different truncation levels</td>
<td>[161]</td>
</tr>
<tr>
<td>$S_{mf}=70$ MPa</td>
<td></td>
<td></td>
<td>- trunc. levels II,III,IV and V</td>
<td></td>
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<tr>
<td>TWIST</td>
<td>7075-T6 Clad</td>
<td>100</td>
<td>5 different truncation levels</td>
<td>[161]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- trunc. levels I,II,III,IV and V</td>
<td></td>
</tr>
<tr>
<td>FALSTAFF</td>
<td>2024-T3 bare</td>
<td>100</td>
<td>2 design stress levels</td>
<td>[162]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- $S_{max}=247.5$ MPa</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- $S_{max}=202.5$ MPa</td>
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</tr>
<tr>
<td>mini-FALSTAFF</td>
<td>2024-T3 bare</td>
<td>100</td>
<td>2 design stress levels</td>
<td>[162]</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>- $S_{max}=247.5$ MPa</td>
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<tr>
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<td></td>
<td></td>
<td>- $S_{max}=202.5$ MPa</td>
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<tr>
<td>Simplified flight sim-</td>
<td>2024-T3 bare</td>
<td>100</td>
<td>30 combinations with different</td>
<td>[61]</td>
</tr>
<tr>
<td>ulation loading</td>
<td></td>
<td></td>
<td>- types of loading</td>
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<td></td>
<td>- numbers of cycle in one flight</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- ground load levels</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- overload values</td>
<td></td>
</tr>
</tbody>
</table>

(*) Thickness 2 mm

Table 4.4: Flight simulation test variables studied in other source.
<table>
<thead>
<tr>
<th>flight type code</th>
<th>Basic-programme</th>
<th>Severe gust</th>
<th>Light gust</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.25</td>
<td>1.39</td>
<td>1.11</td>
</tr>
<tr>
<td>9</td>
<td>1.15</td>
<td>1.28</td>
<td>1.02</td>
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<td>8</td>
<td>1.05</td>
<td>1.18</td>
<td>0.92</td>
</tr>
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<td>7</td>
<td>0.95</td>
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<tr>
<td>6</td>
<td>0.85</td>
<td>0.97</td>
<td>0.73</td>
</tr>
<tr>
<td>5</td>
<td>0.75</td>
<td>0.87</td>
<td>0.63</td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td>0.76</td>
<td>0.54</td>
</tr>
<tr>
<td>3</td>
<td>0.55</td>
<td>0.65</td>
<td>0.45</td>
</tr>
<tr>
<td>2</td>
<td>0.425</td>
<td>0.515</td>
<td>0.335</td>
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<tr>
<td>1</td>
<td>0.30</td>
<td>0.39</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 4.5: The gust amplitude for the basic programme and the derived severe gust and light gust versions, F-27 Spectrum.

<table>
<thead>
<tr>
<th>$S_{gr}/S_{mf}$</th>
<th>Gust severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3</td>
<td>light normal severe</td>
</tr>
<tr>
<td>7075-T6</td>
<td>light normal severe</td>
</tr>
<tr>
<td>light</td>
<td>+0.125</td>
</tr>
<tr>
<td>light*</td>
<td>+0.125</td>
</tr>
<tr>
<td>normal</td>
<td>-0.234</td>
</tr>
<tr>
<td>severe</td>
<td>-0.5</td>
</tr>
<tr>
<td>light</td>
<td>-0.125</td>
</tr>
<tr>
<td>light*</td>
<td>-0.254</td>
</tr>
<tr>
<td>normal</td>
<td>-0.5</td>
</tr>
<tr>
<td>severe</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

(*) Applied to 7075-T6 only

Table 4.6: Letter codes of the gust spectrum and ground load severities, based on the F-27 spectrum.
<table>
<thead>
<tr>
<th>$S_{mf}$ (MPa)</th>
<th>ground load level $S_{gr}/S_{mf}$</th>
<th>gust load severity</th>
<th>light (code)</th>
<th>normal (code)</th>
<th>severe (code)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>light $-0.125$</td>
<td>72830 (LL)</td>
<td>44000 (NL)</td>
<td>23500 (SL)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>normal $-0.234$</td>
<td>31180 (LN)</td>
<td>18250 (NN)</td>
<td>11500 (SN)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>severe $-0.5$</td>
<td>19000 (LS)</td>
<td>10250 (NS)</td>
<td>5375 (SS)</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>normal $-0.234$</td>
<td></td>
<td>123000 (NN)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td></td>
<td></td>
<td>36300 (NN)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td>18250 (NN)</td>
<td></td>
<td></td>
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<tr>
<td>110</td>
<td></td>
<td></td>
<td>10860 (NN)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>91</td>
<td>light $-0.052$</td>
<td></td>
<td>F 4 spectrum, life = 6340</td>
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<td></td>
</tr>
</tbody>
</table>

(*) Crack growth life from $a = 3.5$ mm to $a = 25$ mm.

Table 4.7a: Crack growth life in F-27 flight-simulation tests. Effect of ground and gust load severities and stress level (material 2024-T3 Al clad).

<table>
<thead>
<tr>
<th>$S_{mf}$ (MPa)</th>
<th>ground load level $S_{gr}/S_{mf}$</th>
<th>gust load severity</th>
<th>light (code)</th>
<th>normal (code)</th>
<th>severe (code)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>light $+0.125$</td>
<td>27653 (LL*)</td>
<td>24555 (NL)</td>
<td>17488 (SL*)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>light* $-0.125$</td>
<td>25304 (LN*)</td>
<td>16933 (NL*)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>normal $-0.254$</td>
<td>21434 (LS)</td>
<td>16523 (NN)</td>
<td>9745 (SN)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>severe $-0.5$</td>
<td></td>
<td>12450 (NS)</td>
<td>8724 (SS)</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>normal $-0.254$</td>
<td></td>
<td>16523 (NN)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
<td>10685 (NN)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td></td>
<td></td>
<td>6980 (NN)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>63.7</td>
<td>light $-0.052$</td>
<td></td>
<td>F 4 spectrum, life = 10553</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(**) Not applied in the tests on 2024-T3 Al clad.

(****) Crack growth life from $a = 3.5$ mm to failure.

Table 4.7b: Crack growth life in F-27 flight simulation tests. Effect of ground and gust load severities and stress level (material 7075-T6 Clad).
<table>
<thead>
<tr>
<th>a (mm)</th>
<th>basic program with $S_{mf}$ variations</th>
<th>variations on basic program with $S_{mf} = 100$ MPa</th>
<th>$F^{-1}$ test with $S_{mf} = 91$ MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NN70</td>
<td>NN90</td>
<td>NN100</td>
</tr>
<tr>
<td>3.75</td>
<td>0.10</td>
<td>0.33</td>
<td>0.41</td>
</tr>
<tr>
<td>4.5</td>
<td>0.10</td>
<td>0.33</td>
<td>0.50</td>
</tr>
<tr>
<td>5.5</td>
<td>0.11</td>
<td>0.32</td>
<td>0.53</td>
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<tr>
<td>6.5</td>
<td>0.13</td>
<td>0.38</td>
<td>0.62</td>
</tr>
<tr>
<td>7.5</td>
<td>0.15</td>
<td>0.36</td>
<td>0.62</td>
</tr>
<tr>
<td>8.5</td>
<td>0.17</td>
<td>0.41</td>
<td>0.67</td>
</tr>
<tr>
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<td>0.41</td>
<td>0.77</td>
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<td>0.16</td>
<td>0.55</td>
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</tr>
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<td>0.18</td>
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<td>1.67</td>
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<td>2.9</td>
</tr>
<tr>
<td>22.5</td>
<td>0.26</td>
<td>1.92</td>
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</tr>
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<td>0.83</td>
<td>8.3</td>
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</tbody>
</table>

Table 4.8: Average crack propagation rates under F-27 load spectrum for 2024-T3 Al clad material [150].
<table>
<thead>
<tr>
<th>( a ) (mm)</th>
<th>LL*</th>
<th>LN</th>
<th>LS</th>
<th>NL*</th>
<th>NN</th>
<th>NS</th>
<th>SL*</th>
<th>SN</th>
<th>SS</th>
<th>NN 80</th>
<th>NN 90</th>
<th>F-4</th>
<th>NL</th>
<th>SL</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.75</td>
<td>0.25</td>
<td>0.20</td>
<td>0.26</td>
<td>0.34</td>
<td>0.37</td>
<td>0.43</td>
<td>0.41</td>
<td>0.65</td>
<td>0.55</td>
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<td>0.87</td>
<td>0.36</td>
<td>0.18</td>
<td>0.33</td>
</tr>
<tr>
<td>4.5</td>
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<td>0.40</td>
<td>0.49</td>
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<td>0.71</td>
<td>0.89</td>
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</tr>
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<td>0.77</td>
<td>0.95</td>
<td>1.18</td>
<td>1.05</td>
<td>1.43</td>
<td>1.74</td>
<td>2.08</td>
<td>2.35</td>
<td>1.74</td>
<td>2.50</td>
<td>1.48</td>
<td>1.0</td>
<td>1.42</td>
</tr>
<tr>
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<td>2.27</td>
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<td>2.86</td>
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<td>25.0</td>
<td>5.0</td>
<td>8.33</td>
</tr>
</tbody>
</table>

Table 4.9: Average crack propagation rates under F-27 load spectrum for 7075-T6 Clad material [160].
<table>
<thead>
<tr>
<th>Material</th>
<th>Truncation level</th>
<th>Crack growth life (flights)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3 Al clad</td>
<td>II</td>
<td>$N_{4+24}$ 64870</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>21750</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>12630</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>8400</td>
</tr>
<tr>
<td>7075-T6 Clad</td>
<td>I</td>
<td>$N_{3.5+25}$ 4388</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>3960</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>3400</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>3016</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>3560</td>
</tr>
</tbody>
</table>

Table 4.10: Crack growth lives under TWIST with different truncation levels [161].

<table>
<thead>
<tr>
<th>Type of Loading</th>
<th>Fatigue life $N_{4+19}$ in flights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{\text{max}} = 202.5$ MPa</td>
</tr>
<tr>
<td></td>
<td>$S_{\text{max}} = 247.5$ MPa</td>
</tr>
<tr>
<td>FALSTAFF miniFALSTAFF</td>
<td>11307</td>
</tr>
<tr>
<td></td>
<td>11763</td>
</tr>
<tr>
<td></td>
<td>3892</td>
</tr>
<tr>
<td></td>
<td>3928</td>
</tr>
</tbody>
</table>

Material 2024-T3 bare, 2 mm

Table 4.11: Crack propagation lives for FALSTAFF and miniFALSTAFF at two stress levels [162].
<table>
<thead>
<tr>
<th>type of sequence</th>
<th>$S_{OL}$ MPa</th>
<th>$S_{gr}$ MPa</th>
<th>crack growth life (kc) $m = 5$</th>
<th>crack growth life (kc) $m = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>48.4</td>
<td>63.4 (a)</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td>48.8</td>
<td></td>
<td>59.8</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td>42.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>200</td>
<td>21.3</td>
<td>264.6 (b)</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>0</td>
<td>44.9</td>
<td>195 (c)</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>-40</td>
<td>16.6</td>
<td>212.7</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>37.7</td>
<td>145</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>-80</td>
<td>13.8</td>
<td>138.5</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>31</td>
<td>133</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>200</td>
<td>21.6</td>
<td>302 (b)</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>0</td>
<td>46.3</td>
<td>172.5 (c)</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>-40</td>
<td>16.4</td>
<td>209</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>36</td>
<td>134.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>-80</td>
<td>14.5</td>
<td>130.5</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>31.1</td>
<td>121.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Type I crack growth life from $a = 4$ mm to $a = 30$ mm.
(b) Types II and III for $S_{OL} = 200$ MPa crack growth life from $a = 4$ mm to $a = 20$ mm.
(c) Types II and III for $S_{OL} = 160$ MPa crack growth life from $a = 4$ mm to $a = 25$ mm.

Table 4.12: Crack growth life under load sequences adopted by Misawa and Schijve [61].
Effects of different types of load sequences, ground and overload levels.
MiniTWIST spectrum

Material: 2024-T3 Alclad, $S_{mf} = 85$ MPa

$N_{6+30}$ (flights)

<table>
<thead>
<tr>
<th>Load (1) history</th>
<th>test</th>
<th>predictions non-</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
<th>prediction/test non-</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>interaction</td>
<td></td>
<td></td>
<td></td>
<td>interaction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>12700</td>
<td>2005</td>
<td>14099</td>
<td>18739</td>
<td>11059</td>
<td>0.16</td>
<td>1.1</td>
<td>1.5</td>
<td>0.9</td>
</tr>
<tr>
<td>Max-min</td>
<td>12625</td>
<td>2004</td>
<td>14087</td>
<td>18651</td>
<td>10790</td>
<td>0.16</td>
<td>1.1</td>
<td>1.5</td>
<td>0.9</td>
</tr>
<tr>
<td>Min-max</td>
<td>14560</td>
<td>2004</td>
<td>14065</td>
<td>19278</td>
<td>10434</td>
<td>0.14</td>
<td>1.0</td>
<td>1.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

$\sigma_{gr} = 0$

| Normal           | 27800 | 2212             | 14099   | 43709 | 13653  | 0.08                 | 0.5     | 1.6   | 0.5    |
| Max-min          | 26250 | 2211             | 14087   | 43401 | 12597  | 0.08                 | 0.5     | 1.6   | 0.5    |
| Min-max          | 26600 | 2211             | 14065   | 45557 | 15568  | 0.08                 | 0.5     | 1.7   | 0.6    |

$N_{12+30}$ (flights)

| Normal           | 6300  |                 | 5319    | 7501  | 6328   | 0.8                  | 1.2     | 1.0   |
| Max-min          | 6350  |                 | 5314    | 7472  | 6289   | 0.8                  | 1.2     | 1.0   |
| Min-max          | 7400  |                 | 5306    | 7739  | 5363   | 0.7                  | 1.0     | 0.7   |

$\sigma_{gr} = 0$

| Normal           | 12600 |                 | 5319    | 17595 | 7507   | 0.4                  | 1.4     | 0.6   |
| Max-min          | 11400 |                 | 5314    | 17676 | 6881   | 0.5                  | 1.5     | 0.6   |
| Min-max          | 11900 |                 | 5306    | 18701 | 7897   | 0.4                  | 1.6     | 0.7   |

(1) Load history defined in Section 4.1.1.

Table 6.1: Crack growth lives, tests and prediction results for miniTWIST spectrum.
<table>
<thead>
<tr>
<th>Load history S&lt;sub&gt;max&lt;/sub&gt;</th>
<th>S&lt;sub&gt;a,max&lt;/sub&gt;/S&lt;sub&gt;mf&lt;/sub&gt;</th>
<th>sequence</th>
<th>N&lt;sub&gt;6.30&lt;/sub&gt; (flights) non-interaction</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
<th>prediction/test non-interaction</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>162</td>
<td>1.46</td>
<td>random</td>
<td>92250</td>
<td>7573</td>
<td>58386</td>
<td>50591</td>
<td>60713</td>
<td>0.08</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>200</td>
<td>1.46</td>
<td></td>
<td>26000</td>
<td>3022</td>
<td>26772</td>
<td>22978</td>
<td>19593</td>
<td>0.12</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>1.21</td>
<td>0.89</td>
<td></td>
<td>18750</td>
<td>3026</td>
<td>10766</td>
<td>16087</td>
<td>10570</td>
<td>0.16</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>162</td>
<td>1.46</td>
<td>program I</td>
<td>6700</td>
<td>3050</td>
<td>4464</td>
<td>9225</td>
<td>4077</td>
<td>0.46</td>
<td>0.7</td>
<td>1.4</td>
</tr>
<tr>
<td>1.46</td>
<td></td>
<td>program II</td>
<td>134700</td>
<td>7573</td>
<td>58386</td>
<td>48515</td>
<td>59504</td>
<td>0.06</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>120000</td>
<td></td>
<td></td>
<td>134700</td>
<td>7573</td>
<td>58386</td>
<td>49444</td>
<td>59636</td>
<td>0.06</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 6.2: Crack growth lives, tests and prediction results for CN 235 spectrum.
### F-27 and F 4 spectra

**Material:** 2024-T3 Al clad  
N3,5.25 (flights)

<table>
<thead>
<tr>
<th>spectrum variant (1)</th>
<th>$S_{mf}$ (MPa)</th>
<th>test</th>
<th>predictions</th>
<th>prediction/test</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>non- PREFFAS ONERA CORPUS</td>
<td>non- PREFFAS ONERA CORPUS</td>
<td></td>
</tr>
<tr>
<td>NN 110 90 70</td>
<td>10860 36300 123000</td>
<td>2227 5342 15973</td>
<td>17400 36556 92653</td>
<td>11287 23038 52485</td>
<td>7765 39381 107092</td>
</tr>
<tr>
<td>LL 100 31180 19000</td>
<td>72830 6642 5847</td>
<td>8604 34159 34159</td>
<td>123466 26463 15689</td>
<td>45106 32739 19030</td>
<td>0.12 0.21 0.31</td>
</tr>
<tr>
<td>LN 100 31180 19000</td>
<td>72830 6642 5847</td>
<td>8604 34159 34159</td>
<td>123466 26463 15689</td>
<td>45106 32739 19030</td>
<td>0.12 0.21 0.31</td>
</tr>
<tr>
<td>LS 100 31180 19000</td>
<td>72830 6642 5847</td>
<td>8604 34159 34159</td>
<td>123466 26463 15689</td>
<td>45106 32739 19030</td>
<td>0.12 0.21 0.31</td>
</tr>
<tr>
<td>NL 100 44000 18250 10250</td>
<td>3956 3373 3097</td>
<td>24753 24753 24753</td>
<td>53616 15958 10044</td>
<td>18669 18319 8669</td>
<td>0.09 0.18 0.30</td>
</tr>
<tr>
<td>NN 100 44000 18250 10250</td>
<td>3956 3373 3097</td>
<td>24753 24753 24753</td>
<td>53616 15958 10044</td>
<td>18669 18319 8669</td>
<td>0.09 0.18 0.30</td>
</tr>
<tr>
<td>NS 100 44000 18250 10250</td>
<td>3956 3373 3097</td>
<td>24753 24753 24753</td>
<td>53616 15958 10044</td>
<td>18669 18319 8669</td>
<td>0.09 0.18 0.30</td>
</tr>
<tr>
<td>SL 100 23500 11500 5375</td>
<td>2024 1822 1717</td>
<td>18480 18480 18480</td>
<td>20089 9010 6254</td>
<td>7731 7329 5493</td>
<td>0.09 0.16 0.32</td>
</tr>
<tr>
<td>SN 100 23500 11500 5375</td>
<td>2024 1822 1717</td>
<td>18480 18480 18480</td>
<td>20089 9010 6254</td>
<td>7731 7329 5493</td>
<td>0.09 0.16 0.32</td>
</tr>
<tr>
<td>SS 100 23500 11500 5375</td>
<td>2024 1822 1717</td>
<td>18480 18480 18480</td>
<td>20089 9010 6254</td>
<td>7731 7329 5493</td>
<td>0.09 0.16 0.32</td>
</tr>
<tr>
<td>F 4 91 6340 5801 5199</td>
<td>6999 6082 6848</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) N = normal, S = severe, L = light  
First capital refers to gust spectrum, second one to $S_{gr}$.

Table 6.3: Crack growth lives, tests and prediction results for  
F-27 and F 4 spectra (2024-T3 Al clad material).
## F-27 and F 4 spectra

**Material:** 7075-T6 Clad  
**N:** 3.5x35 (flights)

<table>
<thead>
<tr>
<th>Spectrum Variant (1)</th>
<th>$S_{mf}$ (MPa)</th>
<th>Test</th>
<th>Predictions non-interaction</th>
<th>Prediction/Test non-interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>90</td>
<td>6980</td>
<td>2871</td>
<td>7274</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>10685</td>
<td>4348</td>
<td>11247</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>16523</td>
<td>6958</td>
<td>18434</td>
</tr>
<tr>
<td>LL*</td>
<td>70</td>
<td>27653</td>
<td>13610</td>
<td>39675</td>
</tr>
<tr>
<td>LN</td>
<td>25304</td>
<td>13040</td>
<td>39675</td>
<td>24150</td>
</tr>
<tr>
<td>LS</td>
<td>21434</td>
<td>12007</td>
<td>39675</td>
<td>15601</td>
</tr>
<tr>
<td>NL</td>
<td>70</td>
<td>24555</td>
<td>7715</td>
<td>18434</td>
</tr>
<tr>
<td>NL*</td>
<td>16953</td>
<td>7149</td>
<td>18434</td>
<td>17064</td>
</tr>
<tr>
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</tr>
<tr>
<td>NS</td>
<td>12850</td>
<td>6586</td>
<td>18434</td>
<td>9255</td>
</tr>
<tr>
<td>SL</td>
<td>70</td>
<td>17488</td>
<td>4383</td>
<td>12591</td>
</tr>
<tr>
<td>SL*</td>
<td>11752</td>
<td>4162</td>
<td>12591</td>
<td>8529</td>
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<tr>
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<td>9745</td>
<td>4083</td>
<td>12591</td>
<td>7100</td>
</tr>
<tr>
<td>SS</td>
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<td>3927</td>
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</tr>
<tr>
<td>F 4</td>
<td>63.7</td>
<td>10553</td>
<td>10618</td>
<td>17622</td>
</tr>
</tbody>
</table>

(1) N = normal, S = severe, L = light, L* = not so light  
First capital refers to gust spectrum, second one to $S_{gr}$.

Table 6.4: Crack growth lives, tests and prediction results for F-27 and F 4 spectra (7075-T6 Clad material).
**TWIST spectrum**

a. Material: 2024-T3 Alclad  
$N_{4+24}$ (flights)

<table>
<thead>
<tr>
<th>Trunc. level</th>
<th>test</th>
<th>predictions non-interaction</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
<th>prediction/test non-interaction</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>64870</td>
<td>4710</td>
<td>77175</td>
<td>100584</td>
<td>53656</td>
<td>0.07</td>
<td>1.2</td>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
<td>III</td>
<td>21750</td>
<td>4709</td>
<td>25195</td>
<td>64496</td>
<td>19691</td>
<td>0.22</td>
<td>1.2</td>
<td>3</td>
<td>0.9</td>
</tr>
<tr>
<td>IV</td>
<td>12630</td>
<td>4711</td>
<td>12000</td>
<td>45969</td>
<td>7976</td>
<td>0.37</td>
<td>1.0</td>
<td>3.6</td>
<td>0.6</td>
</tr>
<tr>
<td>V</td>
<td>8400</td>
<td>4716</td>
<td>6701</td>
<td>33656</td>
<td>5431</td>
<td>0.56</td>
<td>0.8</td>
<td>4</td>
<td>0.6</td>
</tr>
</tbody>
</table>

b. Material: 7075-T6 Clad  
$N_{3.5+25}$ (flights)

<table>
<thead>
<tr>
<th>Trunc. level</th>
<th>test</th>
<th>predictions non-interaction</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
<th>prediction/test non-interaction</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4388</td>
<td>1882</td>
<td>8495</td>
<td>23099</td>
<td>3287</td>
<td>0.43</td>
<td>1.9</td>
<td>5.3</td>
<td>0.7</td>
</tr>
<tr>
<td>II</td>
<td>3960</td>
<td>1881</td>
<td>6190</td>
<td>21381</td>
<td>2816</td>
<td>0.48</td>
<td>1.6</td>
<td>5.4</td>
<td>0.7</td>
</tr>
<tr>
<td>III</td>
<td>3400</td>
<td>1882</td>
<td>4620</td>
<td>19179</td>
<td>2542</td>
<td>0.55</td>
<td>1.4</td>
<td>5.6</td>
<td>0.7</td>
</tr>
<tr>
<td>IV</td>
<td>3016</td>
<td>1882</td>
<td>4368</td>
<td>17055</td>
<td>2434</td>
<td>0.62</td>
<td>1.4</td>
<td>5.7</td>
<td>0.8</td>
</tr>
<tr>
<td>V</td>
<td>3560</td>
<td>1883</td>
<td>4240</td>
<td>14138</td>
<td>2399</td>
<td>0.53</td>
<td>1.2</td>
<td>4</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 6.5: Crack growth lives, tests and prediction results for TWIST spectrum.
**FALSTAFF and miniFALSTAFF spectra**

Material: 2024-T3 bare
$N_{4,19}$ (flights)

<table>
<thead>
<tr>
<th>spectrum</th>
<th>$S_{\text{max}}$ (MPa)</th>
<th>test</th>
<th>predictions non-interaction</th>
<th>prediction/test non-interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>PREFFAS</td>
<td>ONERA</td>
</tr>
<tr>
<td>FALSTAFF</td>
<td>202.5</td>
<td>11307</td>
<td>4688</td>
<td>16869</td>
</tr>
<tr>
<td></td>
<td>247.5</td>
<td>3892</td>
<td>2038</td>
<td>8876</td>
</tr>
<tr>
<td>mini-FALSTAFF</td>
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Table 6.6: Crack growth lives, tests and prediction results for FALSTAFF and miniFALSTAFF spectra.
### Misawa and Schijve tests

Material: 2024-T3 bare  
\( m = 5 \) (cycles/flight)

<table>
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<tr>
<th>type</th>
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<th>( S_{OL} ) (MPa)</th>
<th>test (kc)</th>
<th>predictions non-interaction</th>
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<th>ONERA</th>
<th>CORPUS</th>
<th>prediction/test non-interaction</th>
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<th>CORPUS</th>
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- Type I, \( N_{4+30} \) (kc)  
- Types II and III, \( S_{OL} = 200 \) MPa, \( N_{4+20} \) (kc)  
- Types II and III, \( S_{OL} = 160 \) MPa, \( N_{4+25} \) (kc)

Table 6.7a: Crack growth lives, tests and prediction results for Misawa and Schijve tests \( (m = 5) \).
Misawa and Schijve tests

Material: 2024-T3 bare  
\( m = 100 \) (cycles/flight)

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<th>type</th>
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<th>test (kc)</th>
<th>predictions non-interaction</th>
<th>( \text{PREFFAS} )</th>
<th>( \text{ONERA} )</th>
<th>( \text{CORPUS} )</th>
<th>prediction/test non-interaction</th>
<th>( \text{PREFFAS} )</th>
<th>( \text{ONERA} )</th>
<th>( \text{CORPUS} )</th>
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<td>1.9</td>
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<td>0.8</td>
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</table>

- Type I, \( N_{4,30} \) (kc)
- Types II and III, \( S_{OL} = 200 \) MPa, \( N_{4,20} \) (kc)
- Types II and III, \( S_{OL} = 160 \) MPa, \( N_{4,25} \) (kc)

Table 6.7b: Crack growth lives, tests and prediction results for Misawa and Schijve tests \( (m = 100) \).
<table>
<thead>
<tr>
<th>Type of spectrum</th>
<th>non-int.</th>
<th>PREFFAS</th>
<th>ONERA</th>
<th>CORPUS</th>
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<tbody>
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<td>MiniTWIST ( S_{gr} = -0.5 \ S_{mf} )</td>
<td>0.15</td>
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<td>0.6</td>
</tr>
<tr>
<td>F-27 (2024-T3)</td>
<td>0.19</td>
<td>1.4</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>F-27 (7075-T6)</td>
<td>0.43</td>
<td>1.2</td>
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<td>1.1</td>
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<td>F 4 (7075-T6)</td>
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<td>5.2</td>
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<td>1.6</td>
<td>1.3</td>
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<td>Misawa/Schijve (m=100)</td>
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Table 6.8: Average ratios between prediction and test.

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<th>Predicted crack growth life</th>
<th>Difference</th>
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<td>PREFFAS</td>
<td>Constant ( S_{op} )</td>
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<td>( N_{6+30} ) (flights)</td>
<td>-1.1%</td>
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<td>26719</td>
<td>26415</td>
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<td></td>
<td>58270</td>
<td>57607</td>
</tr>
<tr>
<td>FALSTAFF</td>
<td>247.5</td>
<td>( N_{4+19} ) (flights)</td>
<td>-1.0%</td>
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<td>8876</td>
<td>8788</td>
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<td>16702</td>
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Table 6.9: Comparison between PREFFAS predictions and constant \( S_{op} \) predictions.
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<th>number of cycles</th>
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Table 6.10: The influence of rain-flow approach on prediction life.
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</tr>
<tr>
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<td></td>
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</tr>
<tr>
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<td>26719</td>
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<td>(= 162 , \text{MPa})</td>
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<td>56535</td>
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</tr>
<tr>
<td>2. F-27 spectrum, material 2024-T3 Al clad</td>
<td></td>
<td></td>
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<tr>
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<td>91378</td>
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<tr>
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<td>23606</td>
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<td>12258</td>
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<td>trunc. level IV</td>
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<td>4431</td>
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<td>trunc. level V</td>
<td>4240</td>
<td>4303</td>
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<td>5. FALSTAFF spectrum</td>
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<tr>
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<td>8876</td>
<td>8901</td>
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<td>(= 202.5 , \text{MPa})</td>
<td>16869</td>
<td>16905</td>
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<td>6. MiniFALSTAFF spectrum</td>
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<td>8951</td>
<td>8973</td>
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<tr>
<td>(= 202.5 , \text{MPa})</td>
<td>17012</td>
<td>17031</td>
<td>0.1</td>
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Table 6.11: The influence of cycle-by-cycle crack increment calculations.
<table>
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<tr>
<th>Load spectrum</th>
<th>Type of load</th>
<th>α value</th>
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<td>MiniTWIST</td>
<td>Truncation level II</td>
<td>0.462</td>
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<tr>
<td></td>
<td>Truncation level III</td>
<td>0.416</td>
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<td>All variations</td>
<td>0.49</td>
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<td></td>
<td>Truncated at B</td>
<td>0.430</td>
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<td></td>
<td>Truncated at C</td>
<td>0.334</td>
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<td>F-27</td>
<td>All variations</td>
<td>0.42</td>
</tr>
<tr>
<td>F-4</td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>TWIST</td>
<td>Truncation level I</td>
<td>0.523</td>
</tr>
<tr>
<td></td>
<td>Truncation level II</td>
<td>0.504</td>
</tr>
<tr>
<td></td>
<td>Truncation level III</td>
<td>0.461</td>
</tr>
<tr>
<td></td>
<td>Truncation level IV</td>
<td>0.423</td>
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<td></td>
<td>Truncation level V</td>
<td>0.378</td>
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<tr>
<td>FALSTAFF</td>
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<td>0.68</td>
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<tr>
<td>MiniFALSTAFF</td>
<td></td>
<td>0.62</td>
</tr>
<tr>
<td>Misawa and Schijve tests</td>
<td></td>
<td></td>
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<tr>
<td>m = 5 and m = 100</td>
<td>Type I</td>
<td>0.2</td>
</tr>
<tr>
<td>m = 5, SOL = 160 MPa</td>
<td>Types II and III</td>
<td>0.32</td>
</tr>
<tr>
<td>m = 5, SOL = 200 MPa</td>
<td>Types II and III</td>
<td>0.32</td>
</tr>
<tr>
<td>m = 100, SOL = 160 MPa</td>
<td>Types II and III</td>
<td>0.25</td>
</tr>
<tr>
<td>m = 100, SOL = 200 MPa</td>
<td>Types II and III</td>
<td>0.40</td>
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</tbody>
</table>

Table 7.1: Loading parameter α values used in the present calculations.
### Table 7.2: The slope factors of the predictions and the test results for different spectra, as compared to the exponent \( m \) of the Paris relation.

\[
f_1(R) = 0.377 + 0.623R \quad (R>0) \quad (7.25a) \\
f_2(R) = 1/(1.9 - 0.9R) \quad (7.26) \\
K_{op} = K_{max} [\alpha f_1(R) + (1-\alpha) f_2(R)] \quad (7.22)
\]

<table>
<thead>
<tr>
<th>Spectrum Truncation level</th>
<th>( K_{max}/K_{mf} )</th>
<th>( \alpha )</th>
<th>( K_{op}/K_{max} )</th>
<th>( K_{op}/K_{mf} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td>R=0</td>
<td>R=0.5</td>
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<td>TWIST I</td>
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<td>0.449</td>
<td>0.689</td>
</tr>
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<td>0.50</td>
<td>0.452</td>
<td>0.689</td>
</tr>
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<td>III</td>
<td>2.3</td>
<td>0.46</td>
<td>0.458</td>
<td>0.689</td>
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<tr>
<td>V</td>
<td>1.995</td>
<td>0.38</td>
<td>0.470</td>
<td>0.689</td>
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</table>

### Table 7.3: The effect of truncation on \( K_{op} \) (ONERA model)

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness [mm]</th>
<th>Yield Stress [MPa]</th>
<th>( m )</th>
<th>( C ) ( \times 10^{-10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3</td>
<td>2</td>
<td>340</td>
<td>3</td>
<td>0.95</td>
</tr>
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<td>1.62</td>
<td>340</td>
<td>3</td>
<td>0.83</td>
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<tr>
<td>2024-T3*</td>
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<td>340</td>
<td>3</td>
<td>0.40</td>
</tr>
<tr>
<td>2024-T351</td>
<td>10 and 12</td>
<td>340</td>
<td>3</td>
<td>0.95</td>
</tr>
<tr>
<td>2214-T651</td>
<td>10</td>
<td>428</td>
<td>3</td>
<td>1.25</td>
</tr>
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<td>2219-Y851</td>
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<td>360</td>
<td>3</td>
<td>1.14</td>
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<td>7475-T7351</td>
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<td>400</td>
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<td>0.78</td>
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</table>

* specimens machined from a thick sheet.
** \( C \) values \( m \) and MPa are the units adopted.

### Table 7.4: Material data given by Baudin and Robert in Ref. [28].
<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Crack growth life (flights)</th>
<th></th>
<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Test</td>
<td>Predictions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1st</td>
<td>2nd</td>
<td>Ratio</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1st/2nd</td>
</tr>
<tr>
<td>F-27 spectrum</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NN 70</td>
<td>16523</td>
<td>13755</td>
<td>10249</td>
<td>1.34</td>
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<tr>
<td>NN 80</td>
<td>10685</td>
<td>8350</td>
<td>6547</td>
<td>1.28</td>
</tr>
<tr>
<td>NN 90</td>
<td>6980</td>
<td>5386</td>
<td>4456</td>
<td>1.21</td>
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<td>NL 70</td>
<td>24555</td>
<td>41363</td>
<td>23931</td>
<td>1.73</td>
</tr>
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<td>12450</td>
<td>9212</td>
<td>7533</td>
<td>1.22</td>
</tr>
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<td>25332</td>
<td>18370</td>
<td>1.38</td>
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<td>15555</td>
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</tr>
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<td>7268</td>
<td>5739</td>
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<tr>
<td></td>
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<td>1st/2nd</td>
<td></td>
<td>av. 1.35</td>
</tr>
<tr>
<td>TWIST spectrum</td>
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</tr>
<tr>
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<td></td>
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<td>1st/2nd</td>
<td></td>
<td>av. 1.68</td>
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</tbody>
</table>

Note: 1st calculations are based on R = 0.52, 0.23 and -0.05 data.
2nd calculations are based on R = 0.52 and 0.23 data.
See Figure 7.22a and b.

Table 7.5: Effects of C and m parameters based on data for 3 and for 2 R values for 7075-T6 Clad material (ONERA-model).
<table>
<thead>
<tr>
<th>Material</th>
<th>Codes</th>
<th>Basic model (I) flights</th>
<th>Pure p.stress predictions (II) flights</th>
<th>Ratio II/I</th>
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<td>SS 100</td>
<td>6237</td>
<td>6238</td>
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<td>13755</td>
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</table>

Table 7.6: Pure plane stress predictions under F-27 spectrum loading.

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<th>Thickness (mm)</th>
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<td>Basic ONERA</td>
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<td>22889</td>
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<td>4</td>
<td>11831</td>
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<td>6</td>
<td>7320</td>
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<tr>
<td>8</td>
<td>5313</td>
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Table 7.7: Prediction lives for different material thickness under CN 235 spectrum, $S_{max} = 200$ MPa.
### Spectrum

<table>
<thead>
<tr>
<th>materials:</th>
<th>F-27</th>
<th>F-27</th>
<th>CN-235</th>
<th>FALSTAFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3</td>
<td>Alclad</td>
<td>7075-T6</td>
<td>Alclad</td>
<td>bare</td>
</tr>
<tr>
<td>2024-T3</td>
<td>Clad</td>
<td>2024-T3</td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Slope factors</th>
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<th>F-27</th>
<th>CN-235</th>
<th>FALSTAFF</th>
</tr>
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<tr>
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<td>3.43</td>
<td>6.01</td>
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<td>predictions</td>
<td>CORPUS(*)</td>
<td>5.81</td>
<td>2.27</td>
<td>5.37</td>
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<td>modified CORPUS(*)</td>
<td>5.91</td>
<td>2.50</td>
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<td></td>
<td>m (Paris relation)</td>
<td>3.7</td>
<td>2.5</td>
<td>3.7</td>
</tr>
</tbody>
</table>

(*) The slope factor is calculated from the results for the highest and the lowest stress level.

Table 8.1: Slope factors (-d(log N)/d(log S). Comparisons with predictions and the Paris constant m.

Material: 2024-T3 Alclad, crack growth life $N_{3.5+25}$

<table>
<thead>
<tr>
<th>F-27 spectrum Code (Table 4.6)</th>
<th>$S_{mf}$ (MPa)</th>
<th>test (flights)</th>
<th>Overload interactions in prediction</th>
<th>prediction (flights)</th>
<th>prediction/test</th>
</tr>
</thead>
<tbody>
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<td>No</td>
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<td>64496</td>
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</table>

Table 8.2: CORPUS predictions for 2024-T3 Alclad for the F-27 spectrum Comparison between predictions with and without overload interaction effects.
Material: 2024-T3 Al clad

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>(S_e) (MPa)</th>
<th>Truncation level</th>
<th>Crack growth life (flights) (N_{3.5,25}) (flights)</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-27 (case NN)</td>
<td>110 90 70</td>
<td>I</td>
<td>7765 39381 107092</td>
<td>7365 38677 105791</td>
</tr>
<tr>
<td>TWIST</td>
<td>70</td>
<td>II, III, IV, V</td>
<td>53656 19691 7976 5431</td>
<td>36168 17653 5277 4014</td>
</tr>
</tbody>
</table>

\(N_{3.5,25}\) for F-27 spectrum, \(N_{4,24}\) for TWIST spectrum.

Table 8.3: The effect of ignoring secondary plastic zones (SPZ's) on CORPUS predictions.

Material: 2024-T3 Al clad

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>(S_e) (MPa)</th>
<th>Crack growth life (N_{3.5,25}) (flights)</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-27 (case NN)</td>
<td>110 90 70</td>
<td>7765 39381 107092</td>
<td>7687 39186 106935</td>
</tr>
</tbody>
</table>

Table 8.4: Effect of including the rain-flow procedure on CORPUS predictions.

Material: 2024-T3 Al clad

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>(S_e) (MPa)</th>
<th>Crack growth life (N_{3.5,25}) (flights)</th>
<th>Simpler plastic zone Eq.</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-27 (case NN)</td>
<td>110 90 70</td>
<td>7765 39381 107092</td>
<td>8115 39231 107743</td>
<td>1.05 1.00 1.01</td>
</tr>
</tbody>
</table>

Table 8.5: Effect of adopting a more simple equation (Eq. 8.12) for the calculation of the plastic zone size.
<table>
<thead>
<tr>
<th>F-27 spectrum Code (Table 4.6)</th>
<th>$S_{mf}$ (MPa)</th>
<th>Test result</th>
<th>Overload interactions</th>
<th>prediction/test result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>predictions</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>NN</td>
<td>90</td>
<td>6980</td>
<td>8587</td>
<td>9713</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>10685</td>
<td>11271</td>
<td>12540</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>16523</td>
<td>15255</td>
<td>15499</td>
</tr>
<tr>
<td>LL*</td>
<td>70</td>
<td>27653</td>
<td>25882</td>
<td>26880</td>
</tr>
<tr>
<td>LN</td>
<td></td>
<td>25304</td>
<td>24065</td>
<td>22397</td>
</tr>
<tr>
<td>LS</td>
<td></td>
<td>21434</td>
<td>22061</td>
<td>18218</td>
</tr>
<tr>
<td>NL</td>
<td>70</td>
<td>24555</td>
<td>17528</td>
<td>20110</td>
</tr>
<tr>
<td>NL*</td>
<td></td>
<td>16953</td>
<td>16179</td>
<td>16344</td>
</tr>
<tr>
<td>NN</td>
<td></td>
<td>16523</td>
<td>15255</td>
<td>15499</td>
</tr>
<tr>
<td>NS</td>
<td></td>
<td>12450</td>
<td>13184</td>
<td>11586</td>
</tr>
<tr>
<td>SL</td>
<td>70</td>
<td>17488</td>
<td>12612</td>
<td>14587</td>
</tr>
<tr>
<td>SL*</td>
<td></td>
<td>11752</td>
<td>11252</td>
<td>12323</td>
</tr>
<tr>
<td>SN</td>
<td></td>
<td>9745</td>
<td>10794</td>
<td>11571</td>
</tr>
<tr>
<td>SS</td>
<td></td>
<td>8732</td>
<td>9632</td>
<td>9232</td>
</tr>
</tbody>
</table>

Average: 0.98       1.00
Standard deviation: 0.141  0.160

**Table 8.6:** CORPUS predictions for 7075-T6 Clad material for F-27 spectrum variants. Comparison between predictions without and with overload interactions.
### MiniTWIST spectrum

Material: 2024-T3 Alclad

$N_{6,30}$ (flights)

<table>
<thead>
<tr>
<th>Load (1) history</th>
<th>test</th>
<th>predictions</th>
<th>prediction/test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CORPUS</td>
<td>Modified CORPUS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>12700</td>
<td>11059</td>
<td>9653</td>
</tr>
<tr>
<td>Max-min</td>
<td>12625</td>
<td>10790</td>
<td>9653</td>
</tr>
<tr>
<td>Min-max</td>
<td>14560</td>
<td>10434</td>
<td>9656</td>
</tr>
</tbody>
</table>

- **normal $S_{gr} (= -0.5 S_{mf})$**

- **$S_{gr} = 0$**

| Normal           | 27800 | 13653  | 20731           | 0.49   | 0.75            |
| Max-min          | 26250 | 12597  | 20702           | 0.48   | 0.79            |
| Min-max          | 26600 | 15568  | 20834           | 0.59   | 0.78            |

$N_{12,30}$

- **normal $S_{gr} (= -0.5 S_{mf})$**

| Normal           | 6300  | 6328   | 5558            | 1.00   | 0.88            |
| Max-min          | 6350  | 6289   | 5568            | 0.99   | 0.88            |
| Min-max          | 7400  | 6363   | 5561            | 0.72   | 0.75            |

- **$S_{gr} = 0$**

| Normal           | 12600 | 7507   | 11488           | 0.60   | 0.91            |
| Max-min          | 11400 | 6881   | 11459           | 0.60   | 1.01            |
| Min-max          | 11900 | 7897   | 11574           | 0.66   | 0.97            |

(1) Load history defined in section 4.1.1

Table 9.1: Crack growth lives, tests and prediction results for miniTWIST spectrum
Material: 2024-T3 Alclad
N_{6+30} (flights)

<table>
<thead>
<tr>
<th>S_{max} (MPa)</th>
<th>S_{a,max} / S_{mf}</th>
<th>sequence</th>
<th>test</th>
<th>predictions CORPUS</th>
<th>Modified CORPUS</th>
<th>prediction/test CORPUS</th>
<th>Modified CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>162</td>
<td>1.46</td>
<td>random</td>
<td>92250</td>
<td>60713</td>
<td>70372</td>
<td>0.66</td>
<td>0.76</td>
</tr>
<tr>
<td>200</td>
<td>1.46</td>
<td></td>
<td>26000</td>
<td>19593</td>
<td>23087</td>
<td>0.75</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>1.21</td>
<td></td>
<td>18750</td>
<td>10570</td>
<td>10520</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>0.89</td>
<td></td>
<td>6700</td>
<td>4077</td>
<td>4939</td>
<td>0.61</td>
<td>0.74</td>
</tr>
<tr>
<td>162</td>
<td>1.46</td>
<td>program I</td>
<td>134700</td>
<td>59504</td>
<td>69073</td>
<td>0.44</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>1.46</td>
<td>program II</td>
<td>120000</td>
<td>59636</td>
<td>69435</td>
<td>0.50</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 9.2: Crack growth lives, test and prediction results for CN 235 spectrum.
F-27 and F 4 spectra

Material: 2024-T3 Al clad
N 5-25 (flights)

| spectrum variant (1) | $S_{mf}$ (MPa) | test | predictions | | | prediction/test |
|----------------------|----------------|------|-------------|---|---------------|
|                      |                |      | CORPUS      | Modified | CORPUS | Modified |
| NN                   | 110            | 10860| 7765        | 7935     | 0.72   | 0.73     |
|                     | 90             | 36300| 39381       | 40407    | 1.08   | 1.11     |
|                     | 70             | 123000| 107092      | 114583   | 0.87   | 0.93     |
| LL                   | 100            | 72830| 45106       | 73981    | 0.62   | 1.02     |
| LN                   |                | 31180| 32739       | 32342    | 1.05   | 1.04     |
| LS                   |                | 19000| 19030       | 18448    | 1.00   | 0.97     |
| NL                   | 100            | 44000| 18669       | 44737    | 0.42   | 1.02     |
| NN                   |                | 18250| 18319       | 18781    | 1.00   | 1.03     |
| NS                   |                | 10250| 8669        | 8567     | 0.85   | 0.84     |
| SL                   | 100            | 23500| 7731        | 24791    | 0.33   | 1.05     |
| SN                   |                | 11500| 7329        | 10426    | 0.64   | 0.91     |
| SS                   |                | 5375 | 5493        | 5313     | 1.02   | 0.99     |
| F 4                  | 91             | 6340 | 6848        | 6848     | 1.08   | 1.08     |

(1) N = normal, S = severe, L = light
First capital refers to gust spectrum, second one to $S_{gr}$.

Table 9.3: Crack growth lives, tests and prediction results for F-27 and F 4 spectra (2024-T3 Al clad material).
### F-27 and F 4 Spectra

**Material: 7075-T6 Alclad**

N_{3.5+35} (flights)

<table>
<thead>
<tr>
<th>Spectrum Variant (1)</th>
<th>S_{mf} (MPa)</th>
<th>Test</th>
<th>Predictions \hspace{1cm} Base</th>
<th>Prediction/Test \hspace{1cm} Base</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>CORPUS</td>
<td>Modified CORPUS</td>
</tr>
<tr>
<td>NN</td>
<td>90</td>
<td>6980</td>
<td>8514</td>
<td>7510</td>
</tr>
<tr>
<td></td>
<td>80</td>
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<td>14091</td>
</tr>
<tr>
<td>LL*</td>
<td>70</td>
<td>27653</td>
<td>25882</td>
<td>21449</td>
</tr>
<tr>
<td>LN</td>
<td>70</td>
<td>25304</td>
<td>23869</td>
<td>18872</td>
</tr>
<tr>
<td>LS</td>
<td>70</td>
<td>21434</td>
<td>22061</td>
<td>16394</td>
</tr>
<tr>
<td>NL</td>
<td>70</td>
<td>24555</td>
<td>17528</td>
<td>23682</td>
</tr>
<tr>
<td>NL*</td>
<td>70</td>
<td>16953</td>
<td>16179</td>
<td>14762</td>
</tr>
<tr>
<td>NN</td>
<td>70</td>
<td>16523</td>
<td>15072</td>
<td>14091</td>
</tr>
<tr>
<td>NS</td>
<td>70</td>
<td>12450</td>
<td>13184</td>
<td>10324</td>
</tr>
<tr>
<td>SL</td>
<td>70</td>
<td>17488</td>
<td>12612</td>
<td>18220</td>
</tr>
<tr>
<td>SL*</td>
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<td>11752</td>
<td>11252</td>
<td>11594</td>
</tr>
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<td>SN</td>
<td>70</td>
<td>9745</td>
<td>10699</td>
<td>10375</td>
</tr>
<tr>
<td>SS</td>
<td>70</td>
<td>8724</td>
<td>9632</td>
<td>7773</td>
</tr>
<tr>
<td>F 4</td>
<td>63.7</td>
<td>10553</td>
<td>13394</td>
<td>13153</td>
</tr>
</tbody>
</table>

(1) N = normal, S = severe, L = light, L* = not so light

First capital refers to gust spectrum, second one to S_gr.

**Table 9.4**: Crack growth lives, tests and prediction results for F-27 and F 4 spectra (7075-T6 clad material), predictions of the modified CORPUS model.
TWIST spectrum

a. Material: 2024-T3 Alclad
   $N_{6.24}$ (flights)

<table>
<thead>
<tr>
<th>Trunc. level</th>
<th>test</th>
<th>predictions CORPUS</th>
<th>Modified CORPUS</th>
<th>prediction/test CORPUS</th>
<th>Modified CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>64870</td>
<td>53656</td>
<td>47671</td>
<td>0.82</td>
<td>0.73</td>
</tr>
<tr>
<td>III</td>
<td>21750</td>
<td>19691</td>
<td>18856</td>
<td>0.91</td>
<td>0.87</td>
</tr>
<tr>
<td>IV</td>
<td>12630</td>
<td>7976</td>
<td>7506</td>
<td>0.63</td>
<td>0.59</td>
</tr>
<tr>
<td>V</td>
<td>8400</td>
<td>5431</td>
<td>4868</td>
<td>0.65</td>
<td>0.58</td>
</tr>
</tbody>
</table>

b. Material: 7075-T6 Clad
   $N_{3.5.25}$ (flights)

<table>
<thead>
<tr>
<th>Trunc. level</th>
<th>test</th>
<th>predictions CORPUS</th>
<th>Modified CORPUS</th>
<th>prediction/test CORPUS</th>
<th>Modified CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4388</td>
<td>3287</td>
<td>3440</td>
<td>0.75</td>
<td>0.78</td>
</tr>
<tr>
<td>II</td>
<td>3960</td>
<td>2816</td>
<td>2626</td>
<td>0.71</td>
<td>0.66</td>
</tr>
<tr>
<td>III</td>
<td>3400</td>
<td>2542</td>
<td>2399</td>
<td>0.75</td>
<td>0.71</td>
</tr>
<tr>
<td>IV</td>
<td>3016</td>
<td>2434</td>
<td>2305</td>
<td>0.81</td>
<td>0.76</td>
</tr>
<tr>
<td>V</td>
<td>3560</td>
<td>2399</td>
<td>2221</td>
<td>0.67</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Table 9.5: Crack growth lives, test and prediction results for TWIST spectrum, predictions of the modified CORPUS model.
FALSTAFF and miniFALSTAFF spectra

Material: 2024-T3 bare
N_{4,19} (flights)

<table>
<thead>
<tr>
<th>spectrum</th>
<th>S_{max} (MPa)</th>
<th>test</th>
<th>predictions</th>
<th>prediction/test</th>
<th>predictions</th>
<th>prediction/test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>CORPUS</td>
<td>Modified CORPUS</td>
<td>CORPUS</td>
<td>Modified CORPUS</td>
</tr>
<tr>
<td>FALSTAFF</td>
<td>202.5</td>
<td>11307</td>
<td>10360</td>
<td>12006</td>
<td>0.92</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>247.5</td>
<td>3892</td>
<td>3552</td>
<td>4530</td>
<td>0.91</td>
<td>1.16</td>
</tr>
<tr>
<td>Mini-FALSTAFF</td>
<td>202.5</td>
<td>11763</td>
<td>10340</td>
<td>11991</td>
<td>0.88</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>247.5</td>
<td>3928</td>
<td>3538</td>
<td>4526</td>
<td>0.90</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Table 9.6: Crack growth lives, tests and prediction results for FALSTAFF and miniFALSTAFF spectra, predictions of modified CORPUS model.
### Misawa and Schijve tests

**Material:** 2024-T3 bare  
**m = 5** (cycles/flight)

<table>
<thead>
<tr>
<th>type</th>
<th>$S_{gr}$ (MPa)</th>
<th>$S_{OL}$ (MPa)</th>
<th>test (kc)</th>
<th>predictions CORPUS</th>
<th>Modified CORPUS</th>
<th>prediction/test CORPUS</th>
<th>Modified CORPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td></td>
<td>48.4</td>
<td>61.9</td>
<td>61.9</td>
<td>1.27</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td></td>
<td>48.8</td>
<td>53.4</td>
<td>53.4</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td></td>
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<td>48.1</td>
<td>48.1</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>200</td>
<td>21.3</td>
<td>18.7</td>
<td>18.6</td>
<td>0.88</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>44.9</td>
<td>48.0</td>
<td>43.4</td>
<td>1.07</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td></td>
<td>16.6</td>
<td>14.0</td>
<td>13.8</td>
<td>0.84</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>37.7</td>
<td>35.5</td>
<td>29.7</td>
<td>0.94</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td></td>
<td>13.8</td>
<td>11.2</td>
<td>10.8</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>31</td>
<td>28.5</td>
<td>23.0</td>
<td>0.92</td>
<td>0.74</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>200</td>
<td>21.6</td>
<td>18.5</td>
<td>18.5</td>
<td>0.86</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>46.3</td>
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<td>42.1</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td></td>
<td>16.4</td>
<td>13.7</td>
<td>13.7</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>36</td>
<td>28.2</td>
<td>28.2</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td></td>
<td>14.5</td>
<td>10.8</td>
<td>10.8</td>
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</tr>
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<td>160</td>
<td>31.1</td>
<td>22</td>
<td>22</td>
<td>0.71</td>
<td>0.71</td>
</tr>
</tbody>
</table>

- Type I, $N_{4,30}$ (kc)  
- Types II and III, $S_{OL} = 200$ MPa, $N_{4,20}$ (kc)  
- Types II and III, $S_{OL} = 160$ MPa, $N_{4,25}$ (kc)

Table 9.7a: Crack growth lives, tests and prediction results for the Misawa/Schijve tests ($m = 5$), predictions of the modified CORPUS model.
Misawa and Schijve tests

Material: 2024-T3 bare
m = 100 (cycles/flight)

<table>
<thead>
<tr>
<th>Type</th>
<th>$S_{gr}$ (MPa)</th>
<th>$S_{OL}$ (MPa)</th>
<th>Test (kc)</th>
<th>Predictions</th>
<th></th>
<th>Prediction/Tests</th>
</tr>
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<tbody>
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<td></td>
<td>Corps</td>
<td>Modified Corps</td>
<td>Corps</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>63.4</td>
<td>58</td>
<td>72.3</td>
<td>72.3</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td>59.8</td>
<td></td>
<td>71.6</td>
<td>71.6</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td></td>
<td></td>
<td>71.1</td>
<td>71.1</td>
<td>1.19</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
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<td>195</td>
<td>349.5</td>
<td>281</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td>200</td>
<td>212.7</td>
<td>284.5</td>
<td>150.2</td>
<td>1.46</td>
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<td></td>
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<td>160</td>
<td>145</td>
<td>266.8</td>
<td>165</td>
<td>1.25</td>
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<tr>
<td></td>
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<td>257.7</td>
<td>84.7</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>215.2</td>
<td>105.2</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>236.9</td>
<td>59.1</td>
<td>1.78</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>200</td>
<td>302</td>
<td>280.6</td>
<td>280.6</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>-40</td>
<td>160</td>
<td>172.5</td>
<td>141.9</td>
<td>147.9</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td>200</td>
<td>209</td>
<td>164.6</td>
<td>164.6</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>160</td>
<td>134.5</td>
<td>80.3</td>
<td>83.7</td>
<td>0.60</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>104.8</td>
<td>104.8</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>57</td>
<td>58.6</td>
<td>0.47</td>
</tr>
</tbody>
</table>

- Type I, $N_{420}$ (kc)
- Types II and III, $S_{OL} = 200$ MPa, $N_{420}$ (kc)
- Types II and III, $S_{OL} = 160$ MPa, $N_{420}$ (kc)

Table 9.7b: Crack growth lives, tests and prediction results for the Misawa/Schijve tests ($m = 100$), predictions of the modified CORPUS model.
<table>
<thead>
<tr>
<th>Type of spectrum</th>
<th>CORPUS $N_p/N_T$</th>
<th>Modified CORPUS $N_p/N_T$ average</th>
</tr>
</thead>
<tbody>
<tr>
<td>MiniTWIST ($S_{gr} = -0.5 S_{af}$)</td>
<td>0.82</td>
<td>0.73</td>
</tr>
<tr>
<td>MiniTWIST ($S_{gr} = 0$)</td>
<td>0.52</td>
<td>0.77</td>
</tr>
<tr>
<td>CN 235</td>
<td>0.59</td>
<td>0.67</td>
</tr>
<tr>
<td>F-27 (2024-T3)</td>
<td>0.80</td>
<td>0.97</td>
</tr>
<tr>
<td>F-27 (7075-T6)</td>
<td>0.98</td>
<td>0.92</td>
</tr>
<tr>
<td>F 4 (2024-T3)</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>F 4 (7075-T6)</td>
<td>1.27</td>
<td>1.25</td>
</tr>
<tr>
<td>TWIST (2024-T3)</td>
<td>0.75</td>
<td>0.69</td>
</tr>
<tr>
<td>TWIST (7075-T6)</td>
<td>0.74</td>
<td>0.71</td>
</tr>
<tr>
<td>FALSTAFF</td>
<td>0.92</td>
<td>1.11</td>
</tr>
<tr>
<td>MiniFALSTAFF</td>
<td>0.89</td>
<td>1.09</td>
</tr>
<tr>
<td>Misawa/Schijve ($m = 5$)</td>
<td>0.92</td>
<td>0.89</td>
</tr>
<tr>
<td>Misawa/Schijve ($m = 100$)</td>
<td>1.14</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Overall average $N_p/N_T$  | 0.88             | 0.90                              |
Standard deviation          | 0.21             | 0.19                              |

Table 9.8: Average ratios between prediction and test.
F-27 spectrum

Material: 2024-T3 Alclad

Material: 2024-T3 Alclad

<table>
<thead>
<tr>
<th>spectrum variant (1)</th>
<th>S_{mf} (MPa)</th>
<th>test (flights)</th>
<th>predictions, Drev. using equation (9.2) (I)</th>
<th>predictions, Drev. using equation (9.3) (II)</th>
<th>ratio II/I</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>110</td>
<td>10860</td>
<td>7935</td>
<td>7914</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>36300</td>
<td>40407</td>
<td>40197</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>123000</td>
<td>114583</td>
<td>113647</td>
<td>0.99</td>
</tr>
<tr>
<td>LL</td>
<td>100</td>
<td>72830</td>
<td>73981</td>
<td>64923</td>
<td>0.88</td>
</tr>
<tr>
<td>LN</td>
<td>100</td>
<td>31180</td>
<td>32342</td>
<td>32339</td>
<td>1</td>
</tr>
<tr>
<td>LS</td>
<td>100</td>
<td>19000</td>
<td>18448</td>
<td>18448</td>
<td>1</td>
</tr>
<tr>
<td>NL</td>
<td>100</td>
<td>44000</td>
<td>44737</td>
<td>34248</td>
<td>0.77</td>
</tr>
<tr>
<td>NN</td>
<td>100</td>
<td>18250</td>
<td>18781</td>
<td>18743</td>
<td>1</td>
</tr>
<tr>
<td>NS</td>
<td>100</td>
<td>10250</td>
<td>8567</td>
<td>8564</td>
<td>1</td>
</tr>
<tr>
<td>SL</td>
<td>100</td>
<td>23500</td>
<td>24791</td>
<td>17488</td>
<td>0.71</td>
</tr>
<tr>
<td>SN</td>
<td>100</td>
<td>11500</td>
<td>10426</td>
<td>10194</td>
<td>0.98</td>
</tr>
<tr>
<td>SS</td>
<td>100</td>
<td>5375</td>
<td>5313</td>
<td>5303</td>
<td>1</td>
</tr>
</tbody>
</table>

(1) N = normal, S = severe, L = light
First capital refers to gust spectrum, second one to S_{gr}.

Table 9.9: Predicted crack growth lives based on two different equations for the underload affected zone.
<table>
<thead>
<tr>
<th>Block number</th>
<th>ULZ effective after flight A (No. 1653)</th>
<th>Eq. (9.2)</th>
<th>Eq. (9.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>until flight</td>
<td>period (flights)</td>
<td>until flight</td>
</tr>
<tr>
<td>1</td>
<td>1674</td>
<td>21</td>
<td>1812</td>
</tr>
<tr>
<td>2</td>
<td>1674</td>
<td>21</td>
<td>1808</td>
</tr>
<tr>
<td>3</td>
<td>1674</td>
<td>21</td>
<td>1801</td>
</tr>
<tr>
<td>4</td>
<td>1673</td>
<td>20</td>
<td>1791</td>
</tr>
<tr>
<td>5</td>
<td>1671</td>
<td>18</td>
<td>1778</td>
</tr>
<tr>
<td>6</td>
<td>1668</td>
<td>15</td>
<td>1759</td>
</tr>
<tr>
<td>7</td>
<td>1666</td>
<td>13</td>
<td>1739</td>
</tr>
</tbody>
</table>

ULZ = underload effected zone

Table 9.10: End of minimum gust effect after flight A calculated by using Eqs. (9.2) and (9.3) for case SL of the F-27 flight load history.
Material: 7075-T6 Clad
N 3.5×35 or failure (flights)

<table>
<thead>
<tr>
<th>spectrum variant</th>
<th>S_{af} (MPa)</th>
<th>test (flights)</th>
<th>prediction (N_p/N_T) based on constant value (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.28</td>
</tr>
<tr>
<td>NN</td>
<td>90</td>
<td>6980</td>
<td>7510 (1.08)</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>10685</td>
<td>10617 (0.99)</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>16523</td>
<td>14091 (0.85)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9142 (1.31)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>13483 (1.26)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15222 (0.92)</td>
</tr>
<tr>
<td>LL*</td>
<td>70</td>
<td>27653</td>
<td>21449 (0.78)</td>
</tr>
<tr>
<td>LN</td>
<td></td>
<td>25304</td>
<td>18872 (0.75)</td>
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<tr>
<td></td>
<td></td>
<td>21434</td>
<td>16394 (0.74)</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>25727 (0.93)</td>
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<td></td>
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<td>20729 (0.82)</td>
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<td></td>
<td></td>
<td>17749 (0.83)</td>
</tr>
<tr>
<td>LS</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>70</td>
<td>24555</td>
<td>23682 (0.96)</td>
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<tr>
<td>NL*</td>
<td></td>
<td>16953</td>
<td>14762 (0.91)</td>
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<tr>
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<td>14091 (0.85)</td>
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<tr>
<td>NS</td>
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<td>12450</td>
<td>10324 (0.83)</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>39221 (1.60)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>17500 (1.03)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15222 (0.92)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11195 (0.90)</td>
</tr>
<tr>
<td>SL</td>
<td>70</td>
<td>17488</td>
<td>18220 (1.04)</td>
</tr>
<tr>
<td>SL*</td>
<td></td>
<td>11752</td>
<td>11594 (0.99)</td>
</tr>
<tr>
<td>SN</td>
<td></td>
<td>9745</td>
<td>10375 (1.06)</td>
</tr>
<tr>
<td>SS</td>
<td></td>
<td>8724</td>
<td>7773 (0.89)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>25830 (1.48)</td>
</tr>
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<td></td>
<td></td>
<td>13878 (1.18)</td>
</tr>
<tr>
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<td></td>
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<td>12606 (1.29)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8998 (1.03)</td>
</tr>
</tbody>
</table>

Average N_p/N_T ratio: 0.91, 1.12
Standard deviation: 0.12, 0.25

(1) constant in Eq. (8.22)

Table 9.11: Predicted crack growth lives based on two different constants in the relaxation factor, Eq. (8.22).
Figure 1.1: Interaction effects in fatigue crack growth [13].

Figure 1.2: Striations on a fatigue fracture surface, which illustrates crack extension in every cycle [15].
Figure 2.1: Different regimes of fatigue crack growth [39].

Figure 2.2: Different types of cracks starting from a hole [41].
Figure 2.3: Effect of the load ratio on crack growth [45].
Figure 2.4: R-ratio effect for 2 mm 2024-T3 bare sheet [61].
Figure 2.5: Development of a monotonic plastic zone and a reversed plastic zone in fatigue crack propagation.

Figure 2.6: Crack opening stress level during load cycles governs the effective stress range [39].
Figure 2.7: Relationship between effective stress range ratio and stress ratio for 2024-T3 material, Elber’s result [24a].

Figure 2.8: Coinciding curves for different R values by applying $\Delta K_{\text{eff}}$ [39].
Figure 2.9: The crack closure concept accounts for crack acceleration and retardation effects [62].

Figure 2.10: Influence of ball impressions on fatigue crack propagation [39].
Figure 2.11: Initial acceleration following an overload cycle.

(a) kinked crack

(b) forked crack

(c) doubly-kinked crack

Figure 2.12: Three types of crack branching after an overload [71].
Figure 2.13: The occurrence of shear lips on the fracture surface of fatigue crack.

Material: HP-9Ni-4Co-30C, t=9 mm (0.34C, 7.5Ni, 1.1Cr, 1.1Mo, 4.5Co)
Material heat treated to three different strength levels.

<table>
<thead>
<tr>
<th>$q_{o2}$ (MPa)</th>
<th>$K_{IC}$ (MPa$\sqrt{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>675</td>
<td>invalid</td>
</tr>
<tr>
<td>1235</td>
<td>143</td>
</tr>
<tr>
<td>1400</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 2.14: Crack growth rates for a constant $\Delta K$ (R = 0.1) in an alloy steel [86]. Effect of periodic peak loads for three different strength levels.
Figure 2.15: Crack propagation behavior due to an increase and a decrease of mean stress [64].

Figure 2.16: Influence of sheet thickness on crack growth under constant-amplitude loading. Material 2024-T3 aluminum [105].
Figure 2.17: Variation of plastic zone size along the crack front [146,147].

Figure 2.18: Influence of sheet thickness on crack growth retardation following an overload cycle using 2024-T3 aluminum [98].
Figure 2.19: Different types of simple variable amplitude load sequences

- Single overload
- Multiple overloads
- Underload
- Combinations of overload and underload
- Lo-Hi and Hi-Lo block loading
Figure 2.20: Delay period ($n_D$) as a function of the overload level in 7075-T6 aluminum [121].
Figure 2.21: Delay period ($n_D$) as a function of multiple overloads [120].
Figure 2.22: Crack growth delay after two overloads as affected by the distance between the overloads. Results of Mills and Hertzberg [117].
10 different types of flight, A to K
A = severe storm
K = very nice weather
S_{mf} = mean stress in flight

flight type B

Figure 3.1: Sample of a history applied in flight-simulation tests according to the F-28 gust spectrum for the wing [148].
Figure 3.2: Flight-simulation of the wing bending moment (schematic) [41].
Figure 3.3: Standardized symmetrical gust spectra for TWIST and miniTWIST with 10 gust amplitude levels ($S_{a} = -0.5 S_{mf}$) (Table 3.2).

Figure 3.4: Symmetrical gust spectrum for F-27 with 10 $S_{a}$-levels (Table 3.3) $S_{gr} = -0.254 S_{mf}$. 
Figure 3.5: Approximately symmetric gust spectrum for CN 235 (Table 3.4) \( S_{gr} / S_{mf} = -0.31 \).

Figure 3.6: Standardized non-symmetrical load spectra for FALSTAFF and miniFALSTAFF (Table 3.5).
TWIST and miniTWIST, flight sequence in one block of 4000 flights.

Numbers of most severe flights:
Type A: 1656, type B: 2856, type C: 501, 2936, 3841
Type D: 106, 412, 684, 1099, 1653, 2682, 3360, 3538, 3898

Figure 3.7: The random sequence of flights in one block of 4000 flights for the TWIST and the miniTWIST spectrum.
F-27, flight sequence in one block of 2500 flights.

Numbers of most severe flights:

- type A: 1653, type B: 106, type C: 684, 2229
- type D: 168, 1099, 2458, 2493
- type E: 239, 965, 1071, 1121, 1211, 1378, 1465, 1851, 2324, 2365, 2434

Figure 3.8: The random sequence of flights in one block of 2500 flights of the F-27 load spectrum.
CN-235, flight sequence in one block of 1000 flights.

Numbers of most severe flights:
- type A: 591
- type B: 239, 921
- type C: 106, 169, 412, 501, 684, 831

Figure 3.9: The random sequence of flights in one block of 1000 flights of the CN 235 load spectrum.
Figure 3.10: Numbers of cycles per flight and maximum and minimum stress in each flight for the 200 flights in one block of FALSTAFF and miniFALSTAFF (for $S_{max} = 247.5$ MPa).
Figure 3.11: Flight profiles (TWIST) flights G and H.
Figure 3.12: Flight profiles (miniTWIST), same type of flight but different random sequences.
Figure 3.13: Flight profiles for different flights of miniTWIST spectrum.
Figure 3.14: Flight profiles for different flights of F-27 spectrum.
Figure 3.15 : Flight profiles for different flights of CN 235 spectrum.
Figure 3.16: Samples of load sequences in flight for the FALSTAFF load history.
Figure 3.17: Samples of load sequences in flight for the miniFALSTAFF load spectrum. The same FALSTAFF flights are shown in Fig. 3.14.
Figure 3.18: Rainflow count procedure

Figure 3.19: Measured load spectra (3 operators) compared to 2 design load spectra [153]. (Fokker data, Fatigue meter readings)
TOW = Take-off weight, N.M. = nautical miles (flight distance)
Figure 3.20: Truncation of high loads and omission of small cycles. Different effects on two load spectra [41].
Figure 4.1: Dimensions of the specimens and material properties used in miniTWIST and CN 235 tests
Position of high gust loads in flight A, B, C

<table>
<thead>
<tr>
<th>Flight type (No.)</th>
<th>Original miniTWIST</th>
<th>Modified miniTWIST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High positive gusts before high negative gusts</td>
<td>High positive gusts after high negative gusts</td>
</tr>
<tr>
<td>C1 (501)</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
<tr>
<td>A (1656)</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
<tr>
<td>B (2856)</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
<tr>
<td>C2 (2936)</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
<tr>
<td>C3 (3841)</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
</tbody>
</table>

Sequence of high gust indicated by +/-(max-min)

<table>
<thead>
<tr>
<th>C1</th>
<th>A</th>
<th>B</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram]</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
</tbody>
</table>

Figure 4.2: Exchange of the severe gusts in flights A, B, and C in miniTWIST tests.
Figure 4.3: Crack growth rate test results under miniTWIST and modified miniTWIST for normal and light ground stress levels.
Figure 4.4: Extrapolating procedure to determine the crack growth increment in flight A in the miniTWIST tests
Figure 4.5: Crack growth increments in flight A for normal and light ground stress levels in the miniTWIST tests.
Figure 4.6: Crack increment in flight A measured with two methods for miniTWIST tests with light ground stress level.
Figure 4.7: Fatigue crack growth rate for two $S_{\text{max}}$ values and different truncation levels under CN 235 load spectrum.
Figure 4.8: Crack growth increment in flight A under CN 235 load spectrum for $S_{\text{max}} = 200$ MPa
Figure 4.9: Crack growth increment in different flight under CN 235 local spectrum for $S_{\text{max}} = 162$ MPa. Results of Pärtl and Schijve [158].
Figure 4.10: Rearranging of flight sequence in CN 235 tests carried out by Pártl and Schijve [158]
Figure 4.11: Crack growth curves for different flight sequences. Results of Pártl and Schijve [158].

Figure 4.12: Crack growth life for three different truncation levels under CN 235 load spectrum. Results of Pratomo [159].
Figure 4.13: Severe and light gust spectra from the normal F-27 gust spectrum [150]

Figure 4.14: The F-4 load sequence. Gust cycles with constant amplitude. All flights are equal [150, 160]
Figure 4.15: The effects of the gust spectrum severity and the ground stress level on crack growth life in flight-simulation tests (F27 spectrum). Result of Van der Linden and De Jonge [150,160].

Figure 4.16: Effect of $S_{mf}$ on crack growth life 2024-T3 and 7075-T6, F27 spectrum. Results of Van der Linden and De Jonge [150,160].
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b) The three contributions of Eq.(6.12)

Figure 6.6: The effective K-range in cycle i depend on an opening K-level occurring many cycles before.
Figure 6.7: Constant amplitude tests with and without periodic overloads as a calibration for $U = A + BR$. 

\[ \frac{da}{dN} = 2.1 \times 10^{-7} \Delta K_{\text{eff}}^{1.2} \]

$\Delta K_{\text{eff}} = U \Delta K$

$U = 0.58 + 0.42 R$

2024-T351, t=6 mm

CT specimens
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Figure 6.15: Effect of design stress level on fatigue life predicted by the PREFFAS model
(c) CN-235 spectrum (2024-T3 Alclad)

(d) FALSTAFF spectrum (2024-T3 bare)

Figure 6.15: continued
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Case (a): $p < 0.6t$

Case (b): $p > 0.6t$

Figure 7.2: Plane strain and plane stress plastic zone diagrams (ONERA model)
Figure 7.3: Five types of load cycle cases (ONERA model)

Case (1):
Fig. a

Case (2):
Fig. b

Figure 7.4: Definition of $K_{UL}$ after one overload cycle and after many cycles (ONERA model)
Figure 7.5: $f_1$ and $f_2$ functions (ONERA model)

\[ \frac{K_{op}}{K_{max,eq}} \]

\[ R_{eq} \]

\[ f_2 \]

\[ f_1 \]

Figure 7.6: Definition of load spectrum parameter $\alpha$

\[ \alpha = 1 - \frac{\sum m(i) S_{max}(i)}{M S^*} \text{ with } M = \sum m(i) \]
Figure 7.7: Flow diagram of the ONERA model
Figure 7.8: Comparison between tests and predictions for all data (ONERA model)
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Figure 7.11a: miniTWIST

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c. CN 235 spectrum (2024-T3 Al clad)

d. FALSTAFF spectrum (2024-T3 bare)

Figure 7.13: Effect of design stress levels. Predictions of ONERA model.
Figure 7.14a,b: Crack growth rates for the F-27 spectrum, materials: 2024-T3 Alclad and 7075-T6 Clad. Effect of design stress level. Predictions of ONERA model.
Figure 7.14c: Crack growth rates for the PALSTAFF spectrum, 2024-T3 bare material. Effect of design stress level. Predictions of ONERA model.
Flight 5  Sequence II  
\[ a = 4.817 \text{ mm} \]  
(Misawa/Schijve tests, ONERA-model)

Flight 5  Sequence III  
\[ a = 4.088 \text{ mm} \]  
(Misawa/Schijve tests, ONERA-model)

Figure 7.15: Predicted crack opening K levels variations during types II and III load sequences applied in the Misawa/Schijve tests.
Figure 7.16: Crack growth increment in flight A, tests and predictions (ONERA model)

- a. MiniTWIST spectrum $S_{gr} = -0.5, S_{mf} = 0$
- b. CN 235 spectrum $S_{max} = 200$ MPa
Figure 7.17: Effect truncation of high loads under TWIST and CN 235 spectra (ONERA model).
Figure 7.18: Crack growth rates for TWIST and CN 235 spectra. Effect of truncation level (ONERA model).
Figure 7.19: Crack opening levels for the CN 235 spectrum with different truncation levels.
Figure 7.20: Development of $S_{op}$ during an F27 load history with block of 2500 flights (case NN, $S_{mf} = 100$ MPa). $S_{op}$ of each cycle has been plotted.
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Figure 7.22b: Based on R values 0.52 and 0.23

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Figure 7.23: Effect of plane stress and plane strain assumptions on $\rho_{eq}$ ($t = 2$ mm).
Figure 8.1: The opening behaviour of a crack tip in the case of 3 significant humps on the crack surface [26]

Figure 8.2: The hump opening behaviour assumed in the CORPUS model [26]
Figure 8.3: The effect of $S_{\text{max}}$, $S_{\text{min}}$ and $\sigma_{\text{yield}}$ on $S_{\text{op}}$ (CORPUS model)

Figure 8.4: Example of hump- and crack opening behaviour [26]
Figure 8.5: Overlapping plastic zones in the CORPUS model

Figure 8.7: Series of decreasing $SH_{\text{max}}$ values and increasing $SH_{\text{min}}$ values in the CORPUS model. Consequences of some $S_{\text{max}}$ and $S_{\text{min}}$ values if they do occur (PPZ = primary plastic zone, SPZ = secondary plastic zone).
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Figure 8.11a: Crack opening stress histories predicted by the CORPUS model for NN 100 case.
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Figure 8.13a: 2024-T3 Alclad

Figure 8.13b: 7075-T6 Clad

Figure 8.13: Effects of gust spectrum severity and ground stress under F-27 flight simulation loading. Predictions of the CORPUS model.
Figure 8.14: Crack growth rate diagram for cases SL 100, NL 100, and LL 100 under F-27 spectrum, tests and predictions.
Figure 8.15: Crack growth rates for the F-27 spectrum with 7075-T6 Clad material. Effect of ground stress level.
Figure 8.16a: MiniTWIST

Figure 8.16b: Misawa/Schijve tests ($S_{OL} = 200$ MPa, 100 cycles per flight.

Figure 8.16: Effects of around stress level, tests and predictions (CORPUS model)
Figure 8.17 a, b: Effect of design stress levels. Predictions of CORPUS model.
c : CN 235 spectrum (2024-T3 Alclad)

d : FALSTAFF spectrum (2024-T3 bare)

Figure 8.17 c, d : Effects of design stress levels.
Figure 8.18: Crack growth rates for the F-27 spectrum, materials: 2024 T3 clad and 7075-T6 clad. Effect of design stress level (CORPUS model).
Figure 8.19: Different $S_{op}$ levels during Sequence II and III of the Misawa/Schijve tests
a. MiniTWIST spectrum with $S_{gr} = -0.5 S_{mf}$
b. MiniTWIST spectrum $S_{gr} = 0$
c. CN 235 spectrum $S_{\text{max}} = 200$ MPa

Figure 8.20 : Crack increments in flight A. Predictions of ONERA model
Figure 8.21: Effect of truncation under TWIST and CN 235 spectra (CORPUS model).
Figure 8.22: Crack growth rates for TWIST and CN 235 spectra. Effect of truncation level (CORPUS model).
Figure 8.23: Comparison between CORPUS predictions with and without overload interactions. F-27 spectrum results.
Figure 9.1: Variation of $S_{op}$ during a flight of the F-27 spectrum (case NS). Comparison between the three model predictions.
Figure 9.2: Flow diagram of modified CORPUS model.
If $S_{\min} \leq SH_{\min}$, overrule higher $SH_{\min}$ and reset $SH_{\min}$ history.

If $ARP > ARPH$, then:
- If $new SH_{\min}$, calculate local $S_{op}$, select maximum $S_{op}$ from local value and history value, and select $S_{op,max}$.

If $a > ARPH$, then:
- If $reset SH_{\min}$, calculate local $S_{op}$, select maximum $S_{op}$ from local value and history value, and select $S_{op,max}$.

Goto A.
Figure 9.3: Average ratios between predictions and test results, predicted by the CORPUS and the modified CORPUS model.
Figure 9.4: Comparison between test results and the modified CORPUS model prediction results for all data.
Figure 9.5: Effects of gust spectrum severity and ground stress under F-27 flight-simulation loading. Predictions of the modified CORPUS model.
Figure 9.6: Crack growth rates for the F-27 spectrum. Effect of gust spectrum severity.
Figure 9.7a: MiniTWIST

Figure 9.7b: Misawa/Schijve test loading ($S_{OL} = 200$ MPa, 100 cycles/flight)

Figure 9.7: Effect of ground stress level, tests and predictions.
(Modified CORPUS model)
Figure 9.8a: Crack growth rates for the F-27 spectrum and 2024-T3 Alclad material. Effect of ground stress level. Predictions of the modified CORPUS model.
Figure 9.8b: Crack growth rates for the F-27 spectrum and 7075-T6 clad material.

Effect of ground stress level.
7075-T6 clad
F27 spectrum, case NL

Fig. 9.9a: $S_{op}$

7075-T6 clad
F27 spectrum, case NL

Fig. 9.9b: crack growth curve

Figure 9.9: Non overlapping plastic zones until the 4th severe flight and its effect on $S_{op}$ and crack growth retardation (modified CORPUS model).
Figure 9.10: Crack growth curve from [97]. More retardation becomes evident after four severe flights.
Figure 9.11: Effect of design stress level.
Predictions of the modified CORPUS model.
Figure 9.11c: CN 235 spectrum (2024-T3 Alclad)

Figure 9.11d: FALSTAFF spectrum (2024-T3 bare)

Figure 9.11: Effect of design stress level.
Predictions of the modified CORPUS model.
Fig. 9.12a : 2024-T3 material

Fig. 9.12b : 7075-T6 material

Figure 9.12c: Crack growth rates for the FALSTAFF spectrum. Effect of design stress level, predictions of the modified CORPUS model. Material 2024-T3 bare.
Figure 9.13: Crack growth increments in flight A, tests and predictions.
(Modified CORPUS model)
Figure 9.14: Effect of truncation of high loads under TWIST and CN 235 spectra, predictions of the modified CORPUS model.
Figure 9.15: Crack growth rates for TWIST and CN 235 spectra. Effect of truncation level. (Modified CORPUS model)
Figure 9.17: Comparison between predicted crack growth rates and test results for two F-27 spectrum cases.

\[ S_{\text{min, gust}} = -25 \text{ MPa} \]
\[ S_{\text{gr}} = 12.5 \text{ MPa} \]

\[ S_{\text{min, gust}} = -39 \text{ MPa} \]
\[ S_{\text{gr}} = 12.5 \text{ MPa} \]
SAMENVATTING

Voor de veiligheid van de vliegtuigconstructie is het optreden van het vermoeiingsverschijnsel van grote betekenis. Vermoeingsscheuren kunnen in de praktijk nooit geheel worden vermeden. Het is dan van groot belang om te weten hoe snel die scheuren zullen groeien. Het probleem dat centraal staat in dit proefschrift is het voorspellen van de groei van vermoeingsscheuren in plaatmateriaal van aluminium legeringen onder een wisselende belasting met een variërend karakter. Daarbij gaat het om zgn. vluchtsimulatiebelastingen, d.w.z. belastingen, die een simulatie zijn van de krachten die in de praktijk op een vleugel van een vliegtuig inwerken. In de literatuur zijn diverse modellen voorgesteld, waarmee de groei van vermoeingsscheuren onder dergelijke vluchtsimulatie belastingen voorspeld kan worden. Deze modellen kunnen worden onderverdeeld in plastische-zone modellen (1e generatie), scheursluitmodellen (2e generatie) en 'stripvloei' (strip yield) modellen (3e generatie). De eerste groep is fysisch niet aanvaardbaar. De derde generatie, die ogenschijnlijk het meest belovend is, heeft toch nog essentiële bezwaren. Dit proefschrift heeft betrekking op de scheursluitmodellen. Onder scheursluiting wordt het verschijnsel verstaan, dat scheuren 'dichtgaan' terwijl er nog een trekspanning op het materiaal staat. Scheursluiting is het gevolg van plastische deformatie, ontstaan bij de scheurtip, die na scheurgroei achter blijft in het zog van de scheur. Bij een wisselende belasting gaat de scheur pas open bij een positieve belasting, d.w.z. bij een trekspanning $S_{op}$. Als gevolg van de vluchtsimulatie belasting kan $S_{op}$ van wisseling tot wisseling veranderen. Die verandering moet worden voorspeld. Alleen als de spanning groter is dan $S_{op}$ treedt scheurgroei op. De scheuruitbreiding wordt bepaald door het effectieve spanningsinterval $\Delta S_{eff} = S_{max} - S_{op}$.

In de literatuur zijn drie modellen gepubliceerd onder de namen PREFFAS, ONERA en CORPUS. Deze drie modellen worden beschreven en geanalyseerd m.b.t. hun fysische geloofwaardigheid. Voorts worden de modellen getoetst aan een groot aantal schergroeiigegevens verkregen onder vlucht simulatiebelasting. De proefresultaten konden voor een groot deel aan de literatuur ontleed worden. Er zijn echter aanvullende proeven uitgevoerd om in lacunes in het gegevensbestand te
voorzien. Op basis van de verkregen inzichten is een gemodificeerd CORPUS model voorgesteld.

Bij de vluchtsimulatieproeven is gebruik gemaakt van de volgende belastingsspectra: F 27 spectrum (Fokker Friendship), CN-235 spectrum (Indonesisch/Spaans transportvliegtuig), TWIST spectrum (gestandaardiseerd spectrum voor vleugels van verkeersvliegtuigen) en het FALSTAFF spectrum (gestandaardiseerd spectrum voor vleugels van jagers). Verkorte versies van de laatste twee spectra (miniTWIST en miniFALSTAFF) en enige sterk vereenvoudigde vluchtsimulatie belastingen zijn eveneens toegepast.

Bij bovengenoemde spectra zijn weer verschillende variabelen van de vluchtsimulatie onderzocht, zoals het beknotten van zeer hoge belastingen (levensduurverkortend!), en het weglaten van talrijke zeer kleine belastingswisselingen, het ontwerpspanningsniveau, de volgorde van optredende belastingen en de volgorde van verschillende vluchttypes.

Een dergelijke omvangrijke en vergelijkende analyse van scheurgroei-modellen vond nog niet eerder plaats. Bovendien is de vergelijking tussen voorspelling en proefresultaat niet beperkt tot scheurgroeilevensduren, maar is ook de scheurgroeisnelheid als functie van de scheurlengte in beschouwing genomen. Voorts is het voorspelde verloop van $S_{op}$ tijdens de levensduur in detail zichtbaar gemaakt waardoor een beter inzicht over de werking van scheurgroei modellen verkregen kan worden. De belangrijkste resultaten kunnen als volgt worden samengevat.


2. Het ONERA model kent aan negatieve belasting een nadrukkelijke betekenis toe. De invloed van die belastingen heeft echter een uiterst tijdelijk karakter (niet langer dan één vlucht). Eveneens irreël is de invloed van de plaatdikte op het voorspelde resultaat.

3. Het CORPUS model is het meest geavanceerde model en geeft ook de meest accurate voorspellingen. In tegenstelling tot het ONERA model
is de invloed van een zeldzaam optredende zware drukbelasting zeer langdurig en dat kan tot onbevredigende scheurgroeivoorspellingen leiden.

4. Het voorgestelde gemodificeerde CORPUS model heeft het hiervoor genoemde bezwaar niet. De modificatie is gebaseerd op het optreden van omgekeerde plastische vervorming bij neerwaartse belastingen, en de invloed daarvan op $S_{op}$. Hierdoor worden nauwkeuriger scheurgroeivoorspellingen verkregen. Het gemiddelde resultaat voor de verhouding van voorspelde levensduur tot proefresultaat voor 83 proevenseries was 0.87 met een standaard deviatie van 0.18. Voor voorspellingen betreffende vermoeiing mag dat een goed resultaat worden genoemd.

5. In het CORPUS model is een interactiemechanisme tussen zware vluchten opgenomen. Dat is in het gemodificeerde model overgenomen. Als gevolg daarvan wordt bij de meer gedetailleerde vergelijking van voorspelde scheurgroeisnelheden en proefresultaten een goede overeenstemming gevonden. Die overeenstemming is duidelijk minder goed voor het PREFPAS en het ONERA model.

De voorspellingen zijn afhankelijk van de materiaalgegevens, terwijl ook enige veronderstellingen in de modellen een niet altijd even duidelijke invloed op het resultaat kunnen hebben. Ook daarover is enig onderzoek verricht. Het huidige onderzoek heeft vooral het karakter van een analyserende verkenning, terwijl een nieuw model is voorgesteld. Er is echter nog behoefte aan voortgezet onderzoek.
ABOUT THE AUTHOR

Utama Herawan Padmadinata was born in Purwakarta, Indonesia, in 1953. In 1972 he started his mechanical engineering study at the Bandung Institute of Technology (I.T.B.), Indonesia. He received his ingenieurs degree in 1976. After graduating he worked as a mechanical designer for two years. Since 1978 he joined the Laboratory for Strength of Materials Components and Structure (LUK-BPPT) in Jakarta. He is engaged in material research. During the years 1982 and 1983 he worked in the Indonesian Aircraft Industry (IPTN) as a visiting employe in the Stress Department.

In December 1985, he started his research work in Delft in the Production and Materials Section (B2) of the Faculty of Aerospace Engineering, Delft University of Technology in order to obtain the doctor's degree of the university. The subject of the research was fatigue crack growth prediction under flight-simulation loading. It was carried out under the supervision of Professor J. Schijve.
INVESTIGATION OF CRACK-CLOSURE PREDICTION MODELS FOR FATIGUE IN ALUMINUM ALLOY SHEET UNDER FLIGHT-SIMULATION LOADING

Utama Herawan Padmadinata

Errata

p86 line 18  
overload case Eq. (7.22)  
should read  
overload case Eq. (7.27)

p139 line 6 from the bottom  
for same 20 flights ...  
for some 20 flights ...

p246 the title of Figure 3.17, the last line  
The same ... in Fig.3.14  
The same ... in Fig. 3.16

p310 after Figure 7.23 should be Figure 7.24, see enclosure

p328 the title of Figure 8.20  
Predictions of ONERA model  
Predictions of CORPUS model
ONERA model
Flight 2668 (D)  F27 spectrum(NN)  t = 2 mm
a = 4.28 mm  Smf = 100 MPa

ONERA model
Flight 2668 (D)  F27 spectrum(NN)  t = 8 mm
a = 7.07 mm  Smf = 100 MPa

Figure 7.24: The influence of the exponent $b$ in accounting for the thickness effect (ONERA model).