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MATCH FILTERING APPROACH FOR SIGNAL ACQUISITION IN RADIO-PULSAR NAVIGATION

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Pulsars with their periodic pulses and known positions are ideal beacons for navigation. The challenge, however, is the detection of the very weak pulsar signals that are submerged in noise. Radio based approaches allow the use of advanced techniques and methods for the detection and acquisition of such weak signals. In this paper, an effective signal acquisition method based on epoch folding and matched filtering is proposed that can enable pulsar navigation on spacecraft.

Traditionally astronomers use an epoch folding algorithm to search for new pulsars which is a very time and processing power-consuming approach. Since a pulsar navigation system uses signals from known pulsars, advanced algorithms can reduce the time and processing power required for pulsar detection. Applying optimization methods on folding algorithms could lead to an increase in detection speed, however, it is not practical when taking all known signal parameters into account. In this paper a new approach is proposed to reduce the time and processing power further, considering a-priori knowledge such as pulse shape.

This approach is based on the concept of matched filtering. Matched filtering is the basic tool for extracting known wavelets from a signal that has been contaminated by noise. A matched filter is obtained by correlating the observation with a template of a known signal, to detect its presence. Such a matched filter is the optimal linear filter for maximizing the signal-to-noise-ratio (SNR) in the presence of additive stochastic noise.

After a description of the underlying theory, simulations shows that by using this method, significant increases in detection speeds are possible.

I. INTRODUCTION

Since the 1970's, reports have been surfacing on using celestial navigation systems using pulsars [1][2]. Although research on algorithms for detecting signals from X-ray pulsars for use in navigation is advanced [3][4], the use of radio pulsars for navigation is still in its infancy, as the signal-to-noise ratios are much lower [5].

Deep space navigation however would benefit greatly from a system using pulsars for time-of-arrival based navigation, as the angular accuracy of ground-based ranging is limited when the distances to the spacecraft increase [6][7]. Radio pulsars in that respect are potentially better suited to that task, as they can work with omni-directional antennas, which does not restrict their movement as much as x-ray detectors would.

Signals from radio pulsars are periodic electromagnetic pulses with a very accurate period which are transmitted by rapidly rotating neutron stars. Since they are very weak signals, detecting them requires antennas with large aperture, advanced algorithms or a combination of both. Traditionally astronomers use an epoch folding algorithm to search for new pulsars which is a time- and processing power-consuming approach [8].

Since a pulsar navigation system uses signals from known pulsars, advanced algorithms can reduce the time and processing power required for pulsar detection. Applying optimization methods on folding algorithms could lead to an increase in detection speed however, it is not practical when taking all known signal parameters into account. In this paper a new approach is proposed to reduce the time and processing power further, considering a-priori knowledge such as pulse period and pulse shape.

II. DETECTION OF PULSAR SIGNALS

A key task in (radio) pulsar-based navigation is estimation of the pulse phase. Several techniques are known for detecting the pulsar signal in the presence of noise, but most commonly the epoch folding method is used [8].

II.I Matched filter

The epoch folding technique assumes that we know the periodicity of the underlying signal, say T_0. In that case, we can reduce the effect of the additive noise component by averaging signal segments of length T_0. By doing so, the T_0-periodic signal will be added constructively, in contrast to the noise component which will be cancelled out. Indeed, let the observed signal be given by $y(t) = x(t) + n(t)$, where $x(t)$ and $n(t)$ are the periodic signal and additive zero-mean noise, respectively. We then estimate one period of $x(t)$ by

$$\begin{aligned} z_e(t) &= \frac{1}{K}\sum_{k=0}^{K-1} y(t+kT_0) \\ &= \frac{1}{K}\sum_{k=0}^{K-1} x(t+kT_0) + \frac{1}{K}\sum_{k=0}^{K-1} n(t+kT_0) \\ &= x(t) + \bar{n}(t), \quad t \in [0, T_0), \end{aligned} \tag{1}$$

so that, assuming $x(t)$ is deterministic,

$$\begin{aligned} EZ_e(t) &= E\left(\frac{1}{K}\sum_{k=0}^{K-1} Y(t+kT_0)\right) = EX(t) + E\bar{N}(t) \\ &= X(t), \quad t \in [0, T_0), \end{aligned}$$

where $E(\cdot)$ denotes the expectation and Z_e, Y, X and $\bar{N}$ are the random variables associated with z_e, y, x and $\bar{n}$, respectively.

In practice we also have knowledge of the shape of the periodic signal, something that is not exploited in the epoch folding method. We can, however, include this a priori knowledge about the signal in our detection scheme, thereby improving the detection of the pulsar signal.

One way of doing so is the use of so-called matched filters; it is the basic tool for extracting known wavelets from a signal that has been contaminated by noise. A matched filter is obtained by correlating the observation with a known signal, sometimes called a template, to detect the presence of the template. The matched filter is the optimal linear filter for maximizing the signal-to-noise-ratio (SNR) in the presence of additive stochastic noise. Examples of matched filters can be found in digital communications, where the communication system sends binary messages across a noisy channel, and a matched filter is used to detect the transmitted pulse pattern in the noisy received signal. For the situation at hand, the output of the matched filter is given by

$$z_m(t) = \int y(s)h(t-s)ds = z_x(t) + z_n(t),$$

where $z_x(t)$ and $z_n(t)$ are the output contributions due to the desired signal $x(t)$ and the noise $n(t)$, respectively. Assuming that $n(t)$ is stationary, having a power spectral density σ_n^2, the average output power of $n(t)$ at the output of the filter is given by

$$\sigma_n^2 \int_{-\infty}^{\infty} \left|\hat{h}(f)\right|^2 df,$$

where $\hat{\ }$ denotes Fourier transformation. The desired signal at time instant τ at the output of the filter is given by

$$z_x(\tau) = \int_{-\infty}^{\infty} \hat{x}(f)\hat{h}(f)e^{j2\pi f\tau}df,$$

so that the SNR is given by

$$\rho(\tau) = \frac{\left|\int_{-\infty}^{\infty} \hat{x}(f)\hat{h}(f)e^{j2\pi f\tau}df\right|^2}{\sigma_n^2 \int_{-\infty}^{\infty}\left|\hat{h}(f)\right|^2 df}.$$

Applying the Cauchy-Schwartz inequality, given by

$$\left|\int f(x)g(x)dx\right|^2 \leq \int |f(x)|^2 \int |g(x)|^2,$$

to the numerator of the expression for ρ, we obtain

$$\rho(\tau) \le \sigma_n^{-2} \int_{-\infty}^{\infty} |\hat{x}(f)|^2 df ,$$

where we have equality if and only if $\hat{h}(f) = c\hat{x}^*(f)e^{-j2\pi f\tau}$, or equivalently, when

$$h(t) = cx^*(\tau - t),$$

where c is an arbitrary constant. As a consequence, the output of the matched filter is given by

$$\begin{aligned} z_m(t) &= c\int_{-\infty}^{\infty} y(s)h(t-s)ds \\ &= c\int_{-\infty}^{\infty} \big(x(s)+n(s)\big)x^*(\tau - t + s)ds. \end{aligned}$$

In other words, the matched filter computes the cross-correlation between the noisy signal and the clean input signal.

For the situation at hand, where we assume that the support of y is KT_0, an arbitrary multiple of the pulse period T_0, we have

$$z_m(t) = \frac{c}{K}\int_0^{KT_0} y(s)h(t-s)ds,$$

where the normalization by K is introduced for convenience (as we will see later). Since $x(t)$ is T_0-periodic, we have

$$\begin{aligned} z_m(t) &= \frac{c}{K}\sum_{k=0}^{K-1}\int_0^{T_0} y(s+kT_0)x^*(\tau - t + s + kT_0)ds \\ &= c\int_0^{T_0}\left(\frac{1}{K}\sum_{k=0}^{K-1} y(s+kT_0)\right)x^*(\tau - t + s)ds \\ &= c\int_0^{T_0} z_e(s)x^*(\tau - t + s)ds. \end{aligned}$$

We conclude that we can implement the matched filter as the correlation of one pulse period and the estimate $z_e(t)$ obtained by epoch folding. As a consequence, we have that

$$\begin{aligned} z_m(t) &= c\int_0^{T_0} x(s)x^*(\tau - t + s)ds \\ &\quad + c\int_0^{T_0} \bar{n}(s)x^*(\tau - t + s)ds. \end{aligned} \qquad (2)$$

The expected value of the filter's output is given by

$$\begin{aligned} EZ_m(t) &= c\int_0^{T_0} EX(s)X^*(\tau - t + s)ds \\ &\quad + c\int_0^{T_0} E\bar{N}(s)X^*(\tau - t + s)ds \\ &= c\int_0^{T_0} R_{XX}(\tau - t)ds = cT_0R_{XX}(-t), \end{aligned}$$

assuming that the noise n (and thus $\bar{n}$) and the signal x are uncorrelated. Assuming that the pulse to be detected, say $p(t)$, in the periodic signal $x(t)$ has support W, we then can choose τ to make the impulse response of the matched filter h causal. That is, we can choose $\tau = t_{onset} + W$. In this case we have $h(t) = p(w - t)$ for $t = 0, \ldots, w$. Since $|R_{XX}(t)| \le R_{XX}(0)$, with equality iff the underlying signal is periodic, the location of the maximum output of the matched filter corresponds to $\tau = t_{onset} + W$, from which t_{onset} is trivially obtained.

II.II Discrete-time implementation

In practice, the matched filtering will be implemented on a DSP and, as a consequence, we have to sample the data before processing. Let f_s denote the sampling frequency and $N = f_sT_0$ the number of samples per period T_0. The integrals in (2) are then implemented as

$$\begin{aligned} z_m(m) &= c\frac{T_0}{N}\sum_{l=0}^{N-1} x(l)x^*(\tau - m + l) \\ &\quad + c\frac{T_0}{N}\sum_{l=0}^{N-1} \bar{n}(l)x^*(\tau - m + l). \end{aligned} \qquad (3)$$

However, since

$$\lim_{N\to\infty} \frac{1}{N}\sum_{l=0}^{N-1} \bar{n}(l)x^*(\tau - m + l) = R_{\bar{N}X}(\tau - m) = 0,$$

we have that $\lim_{N\to\infty} z_m(m) = cT_0R_{XX}(\tau - m)$ for all m. This relation holds *independent* of the value of K, the number of periods used for epoch folding. Even when $K = 1$ (no epoch folding at all), we can have $z_m(m) \approx cT_0R_{XX}(\tau - m)$, assuming N (and thus f_s) is sufficiently large. Indeed, for $K = 1$ we have that $\bar{n} = n$ and $R_{\bar{N}X}(\tau - m) = R_{NN}(\tau - m) = 0$.

By inspection of (1) we see that, by the central limit theorem, the variance of $\bar{n}$ decreases by $1/K$, assuming that certain conditions on the individual variance of n are satisfied, due to the averaging over K periods. Similarly, the implementation of the correlation (3) can be interpreted as an additional averaging over N realizations of the noise process and will, therefore, reduce the variance of $\bar{n}$ by an additional factor N. Hence, the product of $KN = Kf_sT_0$ determines the detection performance of the matched filtering approach. As a consequence, we can trade-off between number of periods and sampling frequency; the higher the sampling frequency, the smaller K can be chosen to obtain a certain detection performance. In fact, at least in theory, we can obtain any arbitrary detection performance for the case where $K = 1$, leading to a situation where we can detect the pulsar position in only T_0 seconds

III. SIMULATION RESULTS

In the next section, we will apply both matched filtering and epoch folding for pulsar signal acquisition on simulated data using a template of B0329+54 pulsar which is one of the strongest pulsars visible in the northern hemisphere [9]. Important to note that -different than in [7]- we apply simulated data, thus not collected with actual hardware.
Note that in these simulations, signal distortions, such as dispersion, are not taken into account.

In order to show the detection performance as a function of K and f_s, we consider the detection of the pulsar B0329+54 having a periodicity of $T_0 = 0.7145$ s, shown in Figure 1.

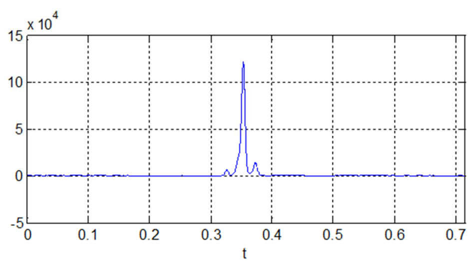

Figure 1: Pulsar B0329+54

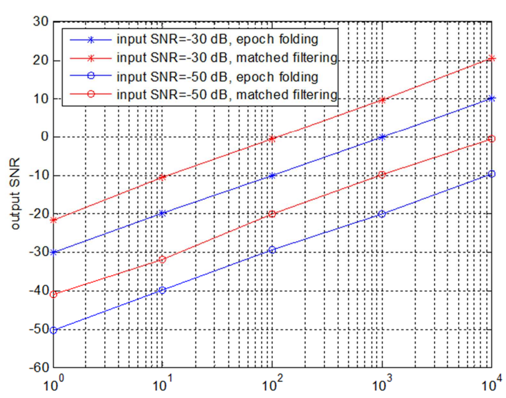

Figure 2: Output SNR as a function of the number periods K for a bandwidth of 2 kHz

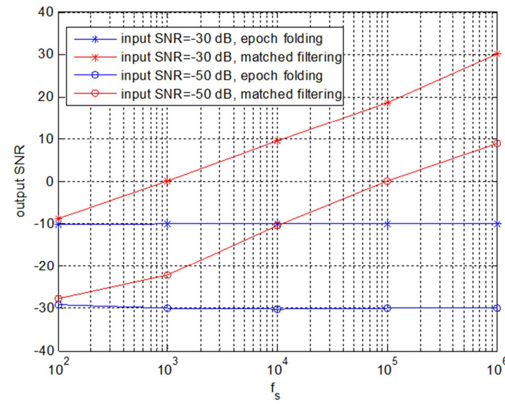

Figure 1: Output SNR as a function of the sampling frequency f_s

Figure 2 shows the detection performance as a function of K, where we measured the performance in terms of output SNR defined as the ratio of the desired signal and the noise signal. In case of epoch folding, the required signal is the pulsar itself, see (1), wheras the required signal with matched filtering is the correlation of the pulsar signal with itself, see (2). The sampling frequency is fixed to 1 KHz. As expected, the performance increases as a function of K.

Figure 3 shows similar results, but this time as a function of the sampling frequency f_s, where we fixed $K = 100$. As can be seen, increasing the sampling frequency does not affect the performance of epoch folding. Indeed, its performance is determined by the number of periods K used for averaging, which is fixed in this experiment. With matched filtering, however, the performance is proportional to the product $KN = Kf_sT_0$, and will, therefore, increase with increasing sampling frequency.

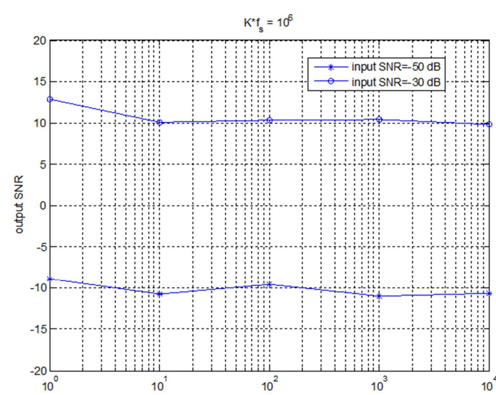

Figure 4: Output SNR as a function of K for fixed Kf_s=10^6.

Figure 4 shows results for the output SNR as a function of K where we have fixed the product Kf_s to a constant value of 10^6. Hence, for $K = 1$, the sampling frequency is chosen to be $f_s = 10^6$, whereas for $K = 3$, the sampling frequency is chosen to be $f_s = 10^3$. As can be seen from Figure 4, the performance is more or less constant for a constant product Kf_s from which we conclude that we can trade-off integration time, given by KT_0, and sampling frequency f_s.

The last experiment shows results, see Figure 5, of detection of the pulsar B0329+54 given an input SNR of -90 dB. The sampling frequency is set to 50 MHz and the number of periods $K = 1000$, resulting in a total integration time of less than two hours. The error in the location of the peak of the pulsar is less than 0.02% of the period T_0.

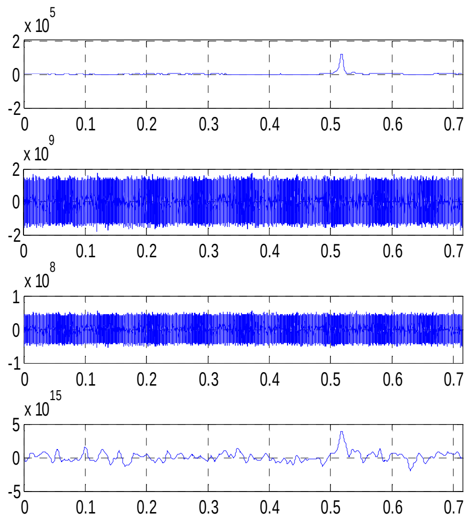

Figure 5: Result of detecting the pulsar B0329+54 at an input SNR of -90 dB

IV. CONCLUSIONS

Matched filtering as an approach to improve the detection speeds of pulsars was investigated in this contribution and compared to the commonly applied epoch folding approach. As a priori knowledge about the signal can be included in this detection scheme, the acquisition process of pulsar signals can be improved.

The matched filtering approach shows great promise for very weak signal powers; provided sufficient bandwidth is available in the sampled dataset. Both theoretically and in simulations, matched filtering outperforms the epoch folding approach in signal detection. Employing such techniques for weak signal detection can significantly reduce pulsar detection times making navigation with pulsars more feasible.

V. REFERENCES

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